

Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.1-Rational-functions

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September 20, 2021

Compiled on September 20, 2021 at 4:56pm

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [490]. This is test number [37].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (490)	0.00 (0)
Maple	99.80 (489)	0.20 (1)
Rubi	99.39 (487)	0.61 (3)
Mupad	98.37 (482)	1.63 (8)
Fricas	87.96 (431)	12.04 (59)
Sympy	87.76 (430)	% 12.24 (60)
Giac	85.92 (421)	14.08 (69)
Maxima	83.47 (409)	16.53 (81)
IntegrateAlgebraic	2.45 (12)	97.55 (478)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

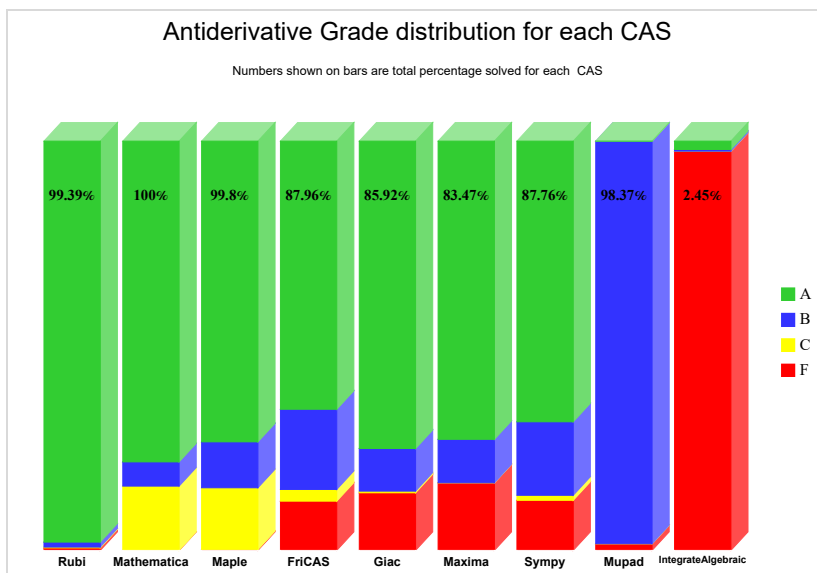
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

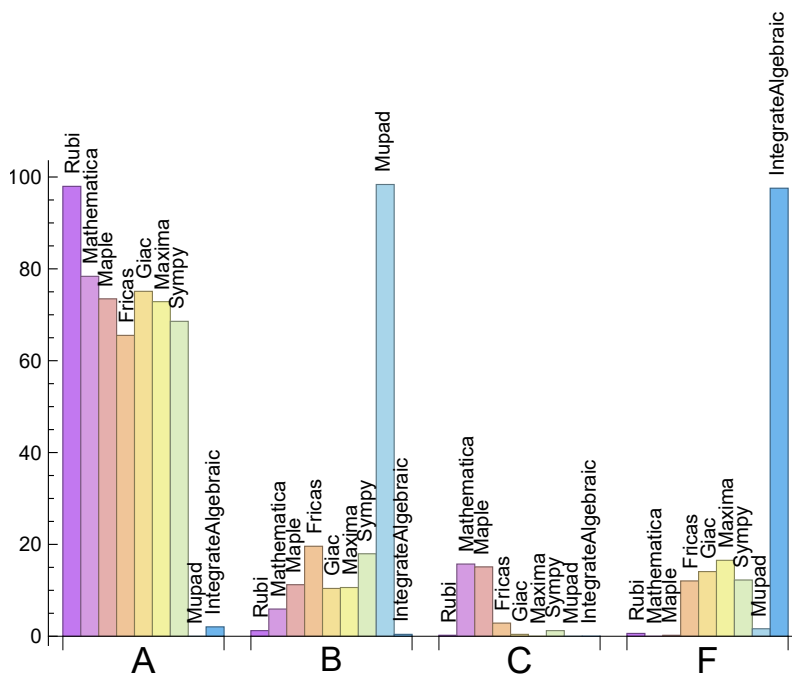
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.96	1.22	0.20	0.61
Mathematica	78.37	5.92	15.71	0.00
Giac	75.10	10.41	0.41	14.08
Maple	73.47	11.22	15.10	0.20
Maxima	72.86	10.61	0.00	16.53
Sympy	68.57	17.96	1.22	12.24
Fricas	65.51	19.59	2.86	12.04
IntegrateAlgebraic	2.04	0.41	0.00	97.55
Mupad	N/A	98.37	0.00	1.63

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	59	0.00 %	98.31 %	1.69 %
IntegrateAlgebraic	478	100.00 %	0.00 %	0.00 %
Giac	69	91.30 %	2.90 %	5.80 %
Maxima	81	93.83 %	3.70 %	2.47 %
Sympy	60	8.33 %	91.67 %	0.00 %
Mupad	8	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

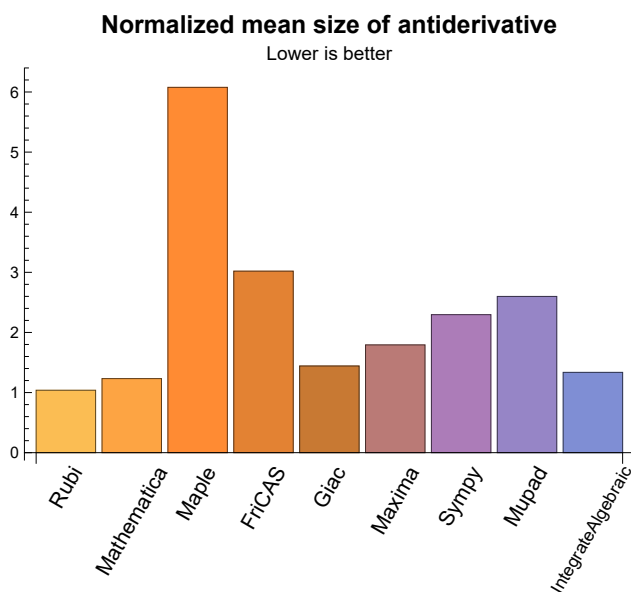
1.3 Performance

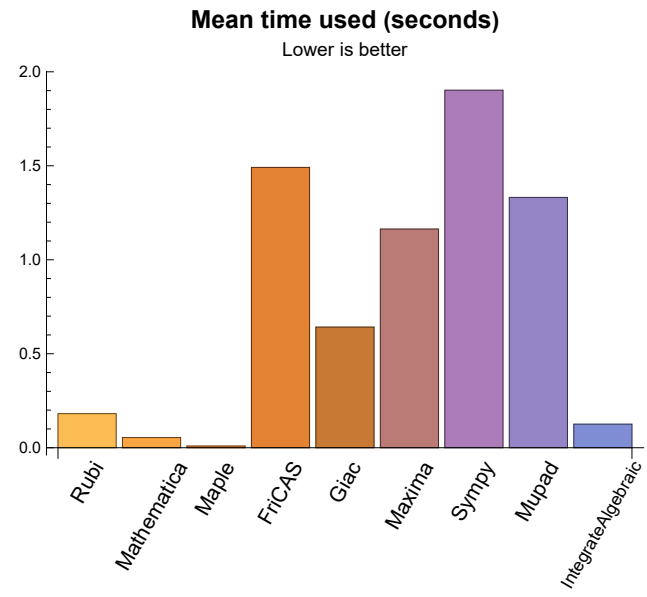
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	119.77	1.04	32.00	1.00
Mathematica	0.05	80.41	1.23	37.00	1.00
Maple	0.01	208.48	6.08	34.00	0.91
Maxima	1.16	115.91	1.79	28.00	0.88
Fricas	1.49	297.22	3.02	36.00	1.10
Sympy	1.90	142.16	2.30	39.00	0.91
Giac	0.64	110.16	1.44	28.00	0.92
Mupad	1.33	452.85	2.60	40.00	0.94
IntegrateAlgebraic	0.13	47.17	1.34	22.50	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {147, 148, 149, 150, 151, 152, 153}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each `integrate` call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the `integrate` command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by

failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

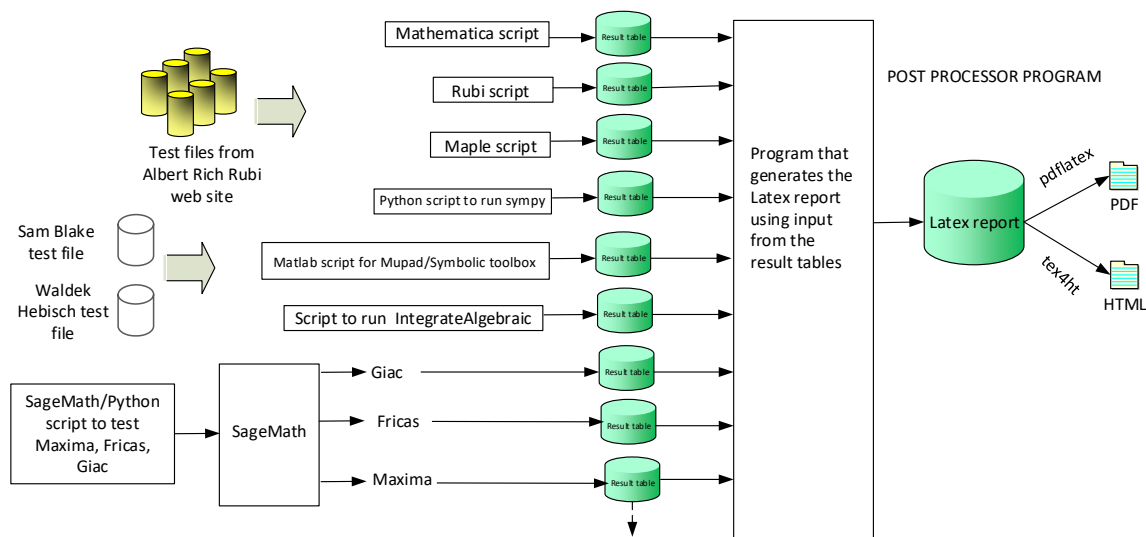
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. integer. 1 if result was verified or 0 if not verified.
The following field present only in Rubi and Mathematica Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 61, 73, 217, 218, 229, 420 }

C grade: { 170 }

F grade: { 389, 489, 490 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98, 102, 109, 112, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 189, 190, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419,

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B grade: { 61, 88, 91, 156, 157, 158, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 417 }

C grade: { 12, 13, 14, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 103, 104, 105, 106, 107, 108, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 169, 170, 171, 172, 180, 181, 223, 246, 247, 248, 249, 250, 251, 252, 253, 383, 384, 385, 386, 387, 388, 389, 487 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 160, 161, 162, 163, 168, 169, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488 }

B grade: { 3, 4, 9, 15, 20, 59, 60, 61, 88, 91, 97, 98, 112, 156, 157, 158, 159, 164, 165, 166, 167, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 223, 228, 333, 334, 396, 417, 420, 439, 487, 490 }

C grade: { 1, 12, 13, 14, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 170, 246, 247, 248, 249, 250, 251, 252, 253, 383, 384, 385, 386, 387, 388, 389, 489 }

F grade: { 172 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 89, 92, 93, 94, 95, 96, 112, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 156, 160, 161, 162, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 194, 195, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 488 }

B grade: { 3, 4, 7, 8, 9, 19, 20, 59, 60, 61, 62, 63, 64, 70, 83, 84, 87, 88, 90, 91, 97, 98, 157, 158, 159, 163, 165, 166, 167, 190, 193, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 417, 420, 473, 489, 490 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 223, 224, 246, 247, 248, 249, 250, 251, 252, 253, 331, 332, 333, 334, 383, 384, 385, 386, 387, 388, 389, 486, 487 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 92, 94, 95, 96, 97, 98, 102, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 204, 205, 212, 213, 214, 215, 216, 220, 221, 222, 224, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 335, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 393, 400, 407, 411, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448,

450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 481, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 3, 4, 7, 8, 9, 12, 13, 14, 15, 33, 34, 39, 40, 59, 60, 61, 62, 63, 64, 70, 71, 72, 73, 83, 87, 88, 89, 90, 91, 93, 112, 116, 117, 118, 142, 150, 156, 157, 158, 159, 164, 165, 166, 167, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 219, 225, 229, 246, 247, 248, 249, 250, 251, 252, 253, 264, 273, 339, 340, 383, 384, 385, 386, 417, 419, 420, 449, 451, 473, 480, 489, 490 }

C grade: { 45, 46, 51, 52, 99, 100, 101, 103, 104, 105, 330, 337, 364, 389 }

F grade: { 19, 20, 57, 58, 106, 107, 108, 110, 111, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 223, 333, 334, 338, 387, 388, 390, 391, 392, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 408, 409, 410, 479 }

2.1.6 Sympy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 65, 66, 67, 68, 69, 71, 72, 89, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 126, 127, 128, 129, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 168, 173, 174, 183, 184, 185, 187, 204, 215, 216, 220, 221, 222, 224, 225, 226, 227, 228, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 404, 405, 406, 407, 411, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 46, 51, 52, 58, 59, 60, 61, 62, 63, 64, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 88, 90, 91, 117, 118, 124, 125, 130, 131, 156, 157, 158, 164, 165, 166, 175, 180, 181, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 229, 247, 264, 335, 417, 419, 420, 455, 473, 485, 486, 489, 490 }

C grade: { 85, 86, 87, 253, 275, 454 }

F grade: { 2, 18, 19, 20, 28, 40, 96, 97, 98, 103, 104, 105, 110, 111, 132, 133, 134, 135, 136, 137, 138, 159, 167, 169, 170, 171, 172, 176, 177, 178, 179, 182, 186, 205, 213, 214, 223, 230, 231, 232, 233, 234, 235, 236, 237, 251, 333, 334, 336, 337, 338, 394, 395, 396, 401, 402, 403, 408, 409, 410 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98, 102, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 156, 160, 161, 162, 164, 165, 166, 168, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 185, 186, 187, 189, 190, 192, 193, 195, 196, 197, 202, 204, 205, 206, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 2, 3, 4, 9, 15, 19, 20, 59, 60, 61, 70, 88, 91, 112, 155, 157, 158, 159, 169, 170, 171, 178, 184, 188, 191, 194, 198, 199, 200, 201, 203, 207, 209, 211, 230, 231, 232, 233, 234, 253, 264, 278, 320, 333, 334, 355, 417, 420, 481, 489, 490 }

C grade: { 51, 52 }

F grade: { 39, 40, 45, 46, 57, 58, 99, 100, 101, 103, 104, 105, 106, 107, 108, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 163, 167, 172, 223, 235, 236, 237, 246, 247, 248, 249, 250, 251, 252, 383, 384, 385, 386, 387, 388, 389, 409 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283,

284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490 }

C grade: { }

F grade: { 96, 97, 98, 163, 167, 170, 172, 223 }

2.1.9 Integrate Algebraic

A grade: { 96, 154, 159, 163, 167, 173, 174, 359, 462, 483 }

B grade: { 97, 98 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 160, 161, 162, 164, 165, 166, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 488, 489, 490 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N. S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I. A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	143	43	0	76	60	55	51	0
N.S.	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66	0.00
time (sec)	N/A	0.134	0.074	0.054	0.000	0.719	0.334	0.221	0.196	0.001
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	23	46	25	43	0	73	52	0
N.S.	1	1.00	0.77	1.53	0.83	1.43	0.00	2.43	1.73	0.00
time (sec)	N/A	0.018	0.105	0.012	0.489	1.030	0.000	0.196	2.132	0.076
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	98	216	97	107	97	97	0
N.S.	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93	0.00
time (sec)	N/A	0.007	0.001	0.003	0.686	1.289	0.088	0.254	2.073	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	65	99	64	66	64	64	0
N.S.	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57	0.00
time (sec)	N/A	0.007	0.001	0.001	0.657	0.750	0.079	0.375	0.028	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	32	31	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89	0.00
time (sec)	N/A	0.007	0.000	0.001	0.735	0.513	0.066	0.283	0.037	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	24	24	26	12	26	0
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86	0.00
time (sec)	N/A	0.009	0.005	0.023	0.649	0.795	0.183	0.264	0.034	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	57	57	61	12	59	0
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21	0.00
time (sec)	N/A	0.009	0.004	0.003	0.714	0.926	0.349	0.295	2.048	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	90	90	97	12	92	0
N.S.	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57	0.00
time (sec)	N/A	0.008	0.003	0.005	0.538	1.110	0.543	0.392	2.069	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	159	295	204	166	175	166	149	0
N.S.	1	1.00	1.89	3.51	2.43	1.98	2.08	1.98	1.77	0.00
time (sec)	N/A	0.125	0.025	0.002	0.676	0.951	0.101	0.273	2.077	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	82	84	93	83	87	83	79	0
N.S.	1	1.00	1.46	1.50	1.66	1.48	1.55	1.48	1.41	0.00
time (sec)	N/A	0.068	0.010	0.002	0.636	0.626	0.083	0.344	0.039	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	28	31	28	28	0
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88	0.00
time (sec)	N/A	0.006	0.000	0.001	0.550	0.760	0.064	0.270	0.038	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	63	57	0	387	53	212	174	0
N.S.	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93	0.00
time (sec)	N/A	0.312	0.021	0.005	0.000	1.108	0.406	0.358	0.494	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	112	136	0	704	192	289	247	0
N.S.	1	1.00	0.46	0.56	0.00	2.87	0.78	1.18	1.01	0.00
time (sec)	N/A	0.249	0.066	0.020	0.000	1.104	1.225	0.357	2.645	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	305	149	276	0	1268	474	366	483	0
N.S.	1	1.00	0.49	0.90	0.00	4.16	1.55	1.20	1.58	0.00
time (sec)	N/A	0.302	0.090	0.018	0.000	1.173	2.542	0.397	2.983	0.001

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	653	861	461	987	1018	971	787	0
N.S.	1	1.00	1.81	2.39	1.28	2.73	2.82	2.69	2.18	0.00
time (sec)	N/A	0.658	0.224	0.002	0.680	0.865	0.246	0.268	2.226	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	241	188	180	346	345	346	270	0
N.S.	1	1.00	1.25	0.97	0.93	1.79	1.79	1.79	1.40	0.00
time (sec)	N/A	0.231	0.086	0.002	0.625	0.832	0.135	0.415	0.084	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	76	51	50	62	63	54	54	0
N.S.	1	1.00	1.36	0.91	0.89	1.11	1.12	0.96	0.96	0.00
time (sec)	N/A	0.016	0.000	0.000	0.827	0.951	0.073	0.232	0.045	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	80	87	112	112	0	137	106	0
N.S.	1	1.00	0.93	1.01	1.30	1.30	0.00	1.59	1.23	0.00
time (sec)	N/A	0.073	0.057	0.011	0.647	12.865	0.000	0.307	2.335	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	232	398	2096	0	0	1414	1940	0
N.S.	1	1.00	0.99	1.70	8.96	0.00	0.00	6.04	8.29	0.00
time (sec)	N/A	0.405	0.552	0.026	1.539	0.000	0.000	0.392	8.176	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	495	495	490	1076	11005	0	0	6908	82532	0
N.S.	1	1.00	0.99	2.17	22.23	0.00	0.00	13.96	166.73	0.00
time (sec)	N/A	1.462	1.192	0.034	5.723	0.000	0.000	1.103	20.456	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.015	0.009	0.013	1.453	1.116	0.131	0.305	2.201	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	25	25	24	26	25	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81	0.00
time (sec)	N/A	0.020	0.007	0.007	1.280	1.245	0.146	0.352	0.053	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.001	0.000	0.001	0.626	0.815	0.064	0.274	0.034	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	26	19	30	25	0
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89	0.00
time (sec)	N/A	0.015	0.006	0.009	0.587	0.569	0.180	0.362	0.060	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	18	15	24	18	0
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82	0.00
time (sec)	N/A	0.012	0.007	0.005	0.793	0.869	0.205	0.296	2.134	0.000
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	62	0	211	564	62	213	0
N.S.	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44	0.00
time (sec)	N/A	0.055	0.082	0.009	0.000	1.126	4.193	0.294	0.465	0.000
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	89	91	98	299	20	112	99	0
N.S.	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86	0.00
time (sec)	N/A	0.063	0.036	0.008	1.281	0.544	0.157	0.264	0.233	0.000
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	17	17	16	0	16	16	0
N.S.	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.004	0.002	0.002	0.574	1.168	0.000	0.309	2.500	0.052

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	285	392	372	277	299	277	261	0
N.S.	1	1.00	1.06	1.45	1.38	1.03	1.11	1.03	0.97	0.00
time (sec)	N/A	0.539	0.040	0.002	0.628	0.760	0.127	0.235	2.298	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	171	231	205	166	180	166	160	0
N.S.	1	1.00	1.00	1.35	1.20	0.97	1.05	0.97	0.94	0.00
time (sec)	N/A	0.092	0.028	0.002	0.802	0.744	0.102	0.243	2.160	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	84	94	83	95	83	82	0
N.S.	1	1.00	1.00	0.91	1.02	0.90	1.03	0.90	0.89	0.00
time (sec)	N/A	0.043	0.021	0.001	0.570	0.537	0.085	0.252	0.036	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	28	31	28	28	0
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88	0.00
time (sec)	N/A	0.006	0.000	0.002	0.651	0.463	0.067	0.268	0.037	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	529	529	71	64	0	905	88	603	1551	0
N.S.	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93	0.00
time (sec)	N/A	0.896	0.027	0.059	0.000	0.735	1.149	0.336	4.541	0.001
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	746	746	182	232	0	3222	427	1057	5844	0
N.S.	1	1.00	0.24	0.31	0.00	4.32	0.57	1.42	7.83	0.00
time (sec)	N/A	1.328	0.116	0.018	0.000	1.088	109.971	0.367	4.304	0.001
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	345	500	383	353	366	323	331	0
N.S.	1	1.00	1.17	1.69	1.30	1.20	1.24	1.09	1.12	0.00
time (sec)	N/A	0.532	0.055	0.002	0.745	0.659	0.137	0.308	0.268	0.000
Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	207	288	214	205	218	187	201	0
N.S.	1	1.00	1.02	1.42	1.05	1.01	1.07	0.92	0.99	0.00
time (sec)	N/A	0.123	0.029	0.001	0.598	0.703	0.109	0.238	2.243	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	109	100	101	99	112	90	98	0
N.S.	1	1.00	1.02	0.93	0.94	0.93	1.05	0.84	0.92	0.00
time (sec)	N/A	0.051	0.015	0.000	0.607	0.638	0.087	0.293	0.044	0.000
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	34	33	33	36	30	33	0
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.97	0.81	0.89	0.00
time (sec)	N/A	0.007	0.000	0.000	0.667	0.560	0.068	0.380	0.043	0.000
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	71	67	0	1115	122	0	1264	0
N.S.	1	1.00	0.46	0.44	0.00	7.29	0.80	0.00	8.26	0.00
time (sec)	N/A	0.254	0.033	0.066	0.000	0.888	1.835	0.000	3.731	0.001
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	342	342	234	288	0	4285	0	0	10351	0
N.S.	1	1.00	0.68	0.84	0.00	12.53	0.00	0.00	30.27	0.00
time (sec)	N/A	0.532	0.193	0.023	0.000	1.294	0.000	0.000	7.109	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	85	84	84	94	84	84	0
N.S.	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88	0.00
time (sec)	N/A	0.030	0.003	0.002	1.248	0.681	0.075	0.307	0.185	0.000
Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	65	64	64	71	64	64	0
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86	0.00
time (sec)	N/A	0.023	0.001	0.001	0.939	0.479	0.070	0.334	0.076	0.000
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	49	44	44	0
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.017	0.001	0.000	0.884	0.758	0.062	0.332	0.032	0.000
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83	0.00
time (sec)	N/A	0.003	0.000	0.000	0.911	0.731	0.057	0.315	0.031	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	45	41	0	1015	41	0	123	0
N.S.	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46	0.00
time (sec)	N/A	0.396	0.010	0.015	0.000	3.625	0.955	0.000	2.445	0.000
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	357	357	113	83	0	1201	3834	0	176	0
N.S.	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49	0.00
time (sec)	N/A	0.397	0.018	0.013	0.000	5.199	3.224	0.000	0.207	0.004
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	78	77	77	94	77	77	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79	0.00
time (sec)	N/A	0.028	0.002	0.003	0.570	0.721	0.071	0.364	0.150	0.000
Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.020	0.001	0.001	0.455	0.710	0.066	0.362	0.063	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	38	37	37	42	37	37	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82	0.00
time (sec)	N/A	0.015	0.001	0.001	0.568	0.736	0.062	0.255	0.026	0.000
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	19	17	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81	0.00
time (sec)	N/A	0.003	0.000	0.000	0.569	1.184	0.056	0.361	0.028	0.000
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	47	41	0	499	3432	265	87	0
N.S.	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37	0.00
time (sec)	N/A	0.315	0.016	0.007	0.000	4.088	2.572	0.520	2.359	0.000
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	108	79	0	704	3834	315	174	0
N.S.	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55	0.00
time (sec)	N/A	0.335	0.024	0.013	0.000	3.888	3.665	0.724	2.207	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	85	84	84	100	84	84	0
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81	0.00
time (sec)	N/A	0.034	0.002	0.002	0.661	0.614	0.081	0.364	2.230	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	65	64	64	73	64	64	0
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84	0.00
time (sec)	N/A	0.024	0.001	0.000	0.646	0.393	0.076	0.384	0.078	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	52	45	44	44	49	44	44	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85	0.00
time (sec)	N/A	0.016	0.001	0.002	0.610	0.857	0.066	0.357	0.033	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	27	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80	0.00
time (sec)	N/A	0.004	0.000	0.001	0.462	0.732	0.058	0.300	0.019	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	55	49	0	0	41	0	123	0
N.S.	1	1.00	0.21	0.19	0.00	0.00	0.16	0.00	0.47	0.00
time (sec)	N/A	0.493	0.015	0.008	0.000	0.000	2.372	0.000	0.411	0.000
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	366	366	128	96	0	0	3839	0	181	0
N.S.	1	1.00	0.35	0.26	0.00	0.00	10.49	0.00	0.49	0.00
time (sec)	N/A	0.508	0.029	0.011	0.000	0.000	3.946	0.000	0.205	0.000
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	164	592	163	185	163	163	0
N.S.	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64	0.00
time (sec)	N/A	0.017	0.002	0.003	0.683	0.923	0.114	0.283	0.167	0.000
Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	109	228	108	114	108	108	0
N.S.	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71	0.00
time (sec)	N/A	0.018	0.001	0.002	0.491	1.015	0.101	0.242	0.061	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	61	61	54	53	53	60	53	53	0
N.S.	1	4.36	4.36	3.86	3.79	3.79	4.29	3.79	3.79	0.00
time (sec)	N/A	0.014	0.000	0.001	0.458	0.845	0.074	0.292	0.024	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	46	46	49	12	48	0
N.S.	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43	0.00
time (sec)	N/A	0.018	0.004	0.006	0.690	0.773	0.288	0.354	0.047	0.001

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	101	101	109	12	103	0
N.S.	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36	0.00
time (sec)	N/A	0.017	0.004	0.005	0.602	1.169	0.593	0.276	2.105	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	156	156	168	12	158	0
N.S.	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29	0.00
time (sec)	N/A	0.018	0.005	0.005	0.544	1.128	0.936	0.355	3.033	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	31	30	30	29	31	36	0
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95	0.00
time (sec)	N/A	0.025	0.008	0.009	1.113	1.004	0.154	0.295	2.165	0.000
Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	65	64	64	80	64	64	0
N.S.	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76	0.00
time (sec)	N/A	0.086	0.002	0.000	0.488	1.068	0.077	0.380	2.166	0.000
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	50	49	49	60	49	49	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78	0.00
time (sec)	N/A	0.076	0.002	0.002	0.727	1.102	0.070	0.365	0.053	0.000
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77	0.00
time (sec)	N/A	0.069	0.001	0.003	0.749	0.995	0.064	0.362	0.024	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	22	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76	0.00
time (sec)	N/A	0.003	0.000	0.002	0.639	1.003	0.057	0.279	0.032	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	62	42	54	56	63	62	27	0
N.S.	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87	0.00
time (sec)	N/A	0.023	0.018	0.011	1.055	1.251	0.154	0.282	0.066	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	103	84	89	177	104	97	64	0
N.S.	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72	0.00
time (sec)	N/A	0.061	0.063	0.023	1.289	1.190	1.359	0.278	0.084	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	137	126	119	282	134	112	93	0
N.S.	1	1.00	0.85	0.78	0.74	1.75	0.83	0.70	0.58	0.00
time (sec)	N/A	0.119	0.092	0.023	1.320	1.301	1.453	0.308	0.089	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	205	132	116	114	223	296	134	126	0
N.S.	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38	0.00
time (sec)	N/A	0.130	0.103	0.022	1.271	1.316	1.426	0.435	2.184	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	73	127	81	198	209	77	87	0
N.S.	1	1.00	0.94	1.63	1.04	2.54	2.68	0.99	1.12	0.00
time (sec)	N/A	0.063	0.067	0.012	1.458	1.199	0.694	0.341	2.269	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	54	89	61	157	153	54	206	0
N.S.	1	1.00	1.08	1.78	1.22	3.14	3.06	1.08	4.12	0.00
time (sec)	N/A	0.039	0.033	0.007	1.648	1.226	0.462	0.308	0.090	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	38	54	50	136	124	43	46	0
N.S.	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12	0.00
time (sec)	N/A	0.021	0.016	0.003	1.362	1.155	0.250	0.356	2.076	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	28	24	83	54	17	17	0
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81	0.00
time (sec)	N/A	0.008	0.004	0.003	1.520	0.881	0.186	0.377	0.043	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	48	72	68	154	738	62	173	0
N.S.	1	1.00	0.81	1.22	1.15	2.61	12.51	1.05	2.93	0.00
time (sec)	N/A	0.035	0.041	0.007	1.518	1.244	3.403	0.377	2.588	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	81	123	123	229	1620	117	425	0
N.S.	1	1.00	1.03	1.56	1.56	2.90	20.51	1.48	5.38	0.00
time (sec)	N/A	0.086	0.053	0.011	1.591	1.138	11.116	0.367	2.583	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	106	198	197	371	3284	195	573	0
N.S.	1	1.00	0.88	1.64	1.63	3.07	27.14	1.61	4.74	0.00
time (sec)	N/A	0.125	0.166	0.010	1.612	1.438	38.258	0.415	2.771	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	34	30	109	61	24	27	0
N.S.	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87	0.00
time (sec)	N/A	0.024	0.012	0.006	1.561	1.273	0.207	0.453	0.057	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	60	86	75	253	117	65	76	0
N.S.	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21	0.00
time (sec)	N/A	0.033	0.035	0.004	1.560	1.118	0.580	0.359	0.098	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	75	147	184	595	257	103	181	0
N.S.	1	1.00	0.82	1.62	2.02	6.54	2.82	1.13	1.99	0.00
time (sec)	N/A	0.048	0.071	0.005	1.585	1.257	1.248	0.409	2.221	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	42	66	279	92	30	31	0
N.S.	1	1.00	1.00	1.20	1.89	7.97	2.63	0.86	0.89	0.00
time (sec)	N/A	0.035	0.019	0.010	1.404	1.464	0.218	0.446	0.103	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	18	10	24	10	10	0
N.S.	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00	0.00
time (sec)	N/A	0.003	0.005	0.003	1.569	1.447	0.168	0.318	0.040	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	31	59	51	55	56	41	42	0
N.S.	1	1.00	0.84	1.59	1.38	1.49	1.51	1.11	1.14	0.00
time (sec)	N/A	0.010	0.014	0.006	1.315	1.148	0.437	0.377	2.066	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	52	94	115	153	146	73	111	0
N.S.	1	1.00	0.87	1.57	1.92	2.55	2.43	1.22	1.85	0.00
time (sec)	N/A	0.016	0.020	0.006	1.505	0.900	0.931	0.304	0.121	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	32	26	25	22	22	27	10	0
N.S.	1	1.00	3.20	2.60	2.50	2.20	2.20	2.70	1.00	0.00
time (sec)	N/A	0.003	0.006	0.010	0.490	0.798	0.177	0.384	2.050	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	45	52	56	85	54	56	43	0
N.S.	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10	0.00
time (sec)	N/A	0.013	0.026	0.010	0.624	0.929	0.469	0.351	2.064	0.000
Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	65	78	122	220	141	88	114	0
N.S.	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78	0.00
time (sec)	N/A	0.022	0.032	0.010	0.641	1.212	1.030	0.357	2.118	0.001
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	15	12	11	11	10	13	4	0
N.S.	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00	0.00
time (sec)	N/A	0.002	0.002	0.005	0.571	1.282	0.099	0.355	0.149	0.000
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	24	25	39	24	27	23	0
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85	0.00
time (sec)	N/A	0.007	0.018	0.008	0.649	1.082	0.117	0.325	0.066	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	37	36	44	71	44	39	36	0
N.S.	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80	0.00
time (sec)	N/A	0.012	0.018	0.010	0.644	0.822	0.144	0.339	2.090	0.000
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	64	64	54	54	58	62	55	0
N.S.	1	1.00	1.08	1.08	0.92	0.92	0.98	1.05	0.93	0.00
time (sec)	N/A	0.055	0.028	0.004	0.542	1.196	0.171	0.310	0.050	0.000
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	11	12	11	11	10	11	11	0
N.S.	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10	0.00
time (sec)	N/A	0.011	0.007	0.001	0.678	1.221	0.086	0.353	0.032	0.000
Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	51	35	36	35	0	23	-1	45
N.S.	1	1.00	1.16	0.80	0.82	0.80	0.00	0.52	-0.02	1.02
time (sec)	N/A	0.026	0.032	0.006	1.289	1.259	0.000	0.386	0.000	0.132

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	55	152	139	92	0	55	-1	171
N.S.	1	1.00	0.82	2.27	2.07	1.37	0.00	0.82	-0.01	2.55
time (sec)	N/A	0.049	0.086	0.020	1.462	0.933	0.000	0.486	0.000	0.454
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	51	146	135	70	0	70	-1	176
N.S.	1	1.00	0.81	2.32	2.14	1.11	0.00	1.11	-0.02	2.79
time (sec)	N/A	0.040	0.088	0.011	0.587	1.883	0.000	0.417	0.000	0.547
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	132	108	0	6315	238	0	374	0
N.S.	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60	0.00
time (sec)	N/A	0.372	0.055	0.006	0.000	47.200	2.901	0.000	2.460	0.001
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	81	74	0	4759	158	0	437	0
N.S.	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08	0.00
time (sec)	N/A	0.228	0.035	0.004	0.000	4.037	0.992	0.000	2.300	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	79	72	0	1950	83	0	145	0
N.S.	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81	0.00
time (sec)	N/A	0.156	0.022	0.003	0.000	3.828	0.700	0.000	0.257	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	116	71	0	442	26	160	144	0
N.S.	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03	0.00
time (sec)	N/A	0.107	0.033	0.002	0.000	1.576	0.261	0.452	2.311	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	238	119	105	0	4370	0	0	553	0
N.S.	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47	0.00
time (sec)	N/A	0.482	0.055	0.010	0.000	3.965	0.000	0.000	0.123	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	314	312	173	144	0	8919	0	0	1588	0
N.S.	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06	0.00
time (sec)	N/A	0.545	0.105	0.010	0.000	3.925	0.000	0.000	2.329	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	393	393	244	217	0	14765	0	0	1328	0
N.S.	1	1.00	0.62	0.55	0.00	37.57	0.00	0.00	3.38	0.00
time (sec)	N/A	0.604	0.170	0.012	0.000	12.243	0.000	0.000	2.489	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	106	97	0	0	374	0	1003	0
N.S.	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82	0.00
time (sec)	N/A	0.426	0.055	0.016	0.000	0.000	3.736	0.000	2.692	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	106	97	0	0	274	0	625	0
N.S.	1	1.00	0.33	0.31	0.00	0.00	0.86	0.00	1.97	0.00
time (sec)	N/A	0.312	0.036	0.005	0.000	0.000	2.671	0.000	2.588	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	104	95	0	0	131	0	205	0
N.S.	1	1.00	0.40	0.36	0.00	0.00	0.50	0.00	0.79	0.00
time (sec)	N/A	0.264	0.034	0.007	0.000	0.000	0.879	0.000	2.357	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	161	94	0	189	26	103	60	0
N.S.	1	1.00	0.73	0.43	0.00	0.86	0.12	0.47	0.27	0.00
time (sec)	N/A	0.185	0.099	0.005	0.000	1.000	0.301	0.501	0.116	0.001

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	393	393	163	139	0	0	0	0	882	0
N.S.	1	1.00	0.41	0.35	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.466	0.073	0.008	0.000	0.000	0.000	0.000	2.183	0.001

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	496	496	238	188	0	0	0	0	2440	0
N.S.	1	1.00	0.48	0.38	0.00	0.00	0.00	0.00	4.92	0.00
time (sec)	N/A	0.894	0.132	0.013	0.000	0.000	0.000	0.000	2.479	0.001

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	195	264	192	219	199	219	175	0
N.S.	1	1.00	1.59	2.15	1.56	1.78	1.62	1.78	1.42	0.00
time (sec)	N/A	0.240	0.037	0.003	0.692	1.033	0.118	0.328	0.215	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	114	138	119	128	114	128	108	0
N.S.	1	1.00	0.95	1.15	0.99	1.07	0.95	1.07	0.90	0.00
time (sec)	N/A	0.063	0.020	0.000	0.667	1.075	0.090	0.296	0.095	0.000
Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	66	63	65	65	65	65	61	0
N.S.	1	1.00	0.92	0.88	0.90	0.90	0.90	0.90	0.85	0.00
time (sec)	N/A	0.030	0.008	0.000	0.707	1.097	0.076	0.313	0.039	0.000
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	22	22	22	22	22	0
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85	0.00
time (sec)	N/A	0.004	0.000	0.000	0.627	0.675	0.061	0.303	0.018	0.000
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	57	49	0	457	66	0	571	0
N.S.	1	1.00	0.64	0.55	0.00	5.13	0.74	0.00	6.42	0.00
time (sec)	N/A	0.087	0.023	0.016	0.000	0.883	0.933	0.000	2.576	0.001

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	150	158	0	1948	294	0	4591	0
N.S.	1	1.00	0.89	0.93	0.00	11.53	1.74	0.00	27.17	0.00
time (sec)	N/A	0.291	0.075	0.013	0.000	0.863	6.317	0.000	5.352	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	254	398	0	3971	697	0	8242	0
N.S.	1	1.00	1.01	1.58	0.00	15.76	2.77	0.00	32.71	0.00
time (sec)	N/A	0.532	0.150	0.023	0.000	1.667	15.616	0.000	6.405	0.001

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	204	267	182	222	212	222	178	0
N.S.	1	1.00	0.97	1.27	0.87	1.06	1.01	1.06	0.85	0.00
time (sec)	N/A	0.226	0.035	0.003	0.621	0.783	0.123	0.390	0.217	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	130	143	113	133	128	133	113	0
N.S.	1	1.00	0.97	1.07	0.84	0.99	0.96	0.99	0.84	0.00
time (sec)	N/A	0.140	0.020	0.002	0.624	0.982	0.098	0.351	2.120	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	75	66	59	68	70	68	64	0
N.S.	1	1.00	0.95	0.84	0.75	0.86	0.89	0.86	0.81	0.00
time (sec)	N/A	0.076	0.008	0.000	0.616	0.861	0.077	0.348	0.038	0.000
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	27	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.00
time (sec)	N/A	0.010	0.002	0.000	0.611	1.019	0.061	0.360	0.022	0.000
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	59	50	0	0	155	0	275	0
N.S.	1	1.00	0.51	0.43	0.00	0.00	1.34	0.00	2.37	0.00
time (sec)	N/A	0.083	0.017	0.004	0.000	0.000	4.430	0.000	2.582	0.001
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	166	162	0	0	539	0	1167	0
N.S.	1	1.00	0.72	0.70	0.00	0.00	2.33	0.00	5.05	0.00
time (sec)	N/A	0.241	0.072	0.012	0.000	0.000	31.420	0.000	2.817	0.001

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	284	405	0	0	1102	0	2200	0
N.S.	1	1.00	0.81	1.16	0.00	0.00	3.16	0.00	6.30	0.00
time (sec)	N/A	0.369	0.164	0.023	0.000	0.000	88.090	0.000	3.394	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	204	267	182	222	219	222	178	0
N.S.	1	1.00	0.97	1.27	0.87	1.06	1.04	1.06	0.85	0.00
time (sec)	N/A	0.164	0.036	0.001	0.639	1.246	0.145	0.243	2.294	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	132	143	113	133	134	133	113	0
N.S.	1	1.00	0.96	1.04	0.82	0.96	0.97	0.96	0.82	0.00
time (sec)	N/A	0.121	0.019	0.003	0.563	0.913	0.097	0.285	0.092	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	73	66	59	68	73	68	64	0
N.S.	1	1.00	0.92	0.84	0.75	0.86	0.92	0.86	0.81	0.00
time (sec)	N/A	0.077	0.010	0.001	0.546	0.935	0.077	0.287	0.042	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	27	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.00
time (sec)	N/A	0.009	0.004	0.000	0.609	1.060	0.062	0.313	0.021	0.000
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	61	52	0	0	172	0	878	0
N.S.	1	1.00	0.62	0.53	0.00	0.00	1.74	0.00	8.87	0.00
time (sec)	N/A	0.087	0.016	0.004	0.000	0.000	7.607	0.000	2.783	0.001
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	182	160	0	0	561	0	1218	0
N.S.	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41	0.00
time (sec)	N/A	0.213	0.076	0.014	0.000	0.000	43.733	0.000	2.846	0.001
Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	545	545	99	93	0	0	0	0	1563	0
N.S.	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87	0.00
time (sec)	N/A	1.476	0.069	0.015	0.000	0.000	0.000	0.000	3.175	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	487	487	99	93	0	0	0	0	1354	0
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.756	0.049	0.003	0.000	0.000	0.000	0.000	3.104	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	334	97	93	0	0	0	0	825	0
N.S.	1	1.00	0.29	0.28	0.00	0.00	0.00	0.00	2.47	0.00
time (sec)	N/A	0.470	0.059	0.005	0.000	0.000	0.000	0.000	3.342	0.001

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	469	469	95	91	0	0	0	0	1057	0
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.684	0.047	0.005	0.000	0.000	0.000	0.000	2.903	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	522	522	99	90	0	0	0	0	1394	0
N.S.	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67	0.00
time (sec)	N/A	0.861	0.066	0.005	0.000	0.000	0.000	0.000	0.711	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	563	563	157	134	0	0	0	0	4002	0
N.S.	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	7.11	0.00
time (sec)	N/A	1.157	0.115	0.009	0.000	0.000	0.000	0.000	2.546	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	645	640	163	133	0	0	0	0	2663	0
N.S.	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	4.13	0.00
time (sec)	N/A	1.380	0.139	0.008	0.000	0.000	0.000	0.000	2.721	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	395	395	61	56	0	0	70	0	427	0
N.S.	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08	0.00
time (sec)	N/A	1.438	0.023	0.008	0.000	0.000	0.257	0.000	0.651	0.001

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	61	56	0	0	65	0	390	0
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03	0.00
time (sec)	N/A	0.910	0.014	0.009	0.000	0.000	0.275	0.000	2.704	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	61	56	0	0	61	0	276	0
N.S.	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76	0.00
time (sec)	N/A	0.515	0.015	0.009	0.000	0.000	0.247	0.000	0.522	0.001

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	59	56	0	1277	48	0	247	0
N.S.	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00	0.00
time (sec)	N/A	0.323	0.013	0.008	0.000	4.126	0.205	0.000	2.679	0.001

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	57	54	0	0	61	0	176	0
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49	0.00
time (sec)	N/A	0.548	0.016	0.008	0.000	0.000	0.257	0.000	2.423	0.001

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	62	53	0	0	65	0	306	0
N.S.	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81	0.00
time (sec)	N/A	0.721	0.013	0.007	0.000	0.000	0.274	0.000	2.675	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	415	415	103	75	0	0	82	0	432	0
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04	0.00
time (sec)	N/A	0.902	0.019	0.013	0.000	0.000	0.417	0.000	2.328	0.001
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	448	448	109	74	0	0	70	0	340	0
N.S.	1	1.00	0.24	0.17	0.00	0.00	0.16	0.00	0.76	0.00
time (sec)	N/A	1.102	0.021	0.011	0.000	0.000	0.319	0.000	0.292	0.001
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	1064	1064	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36	0.00
time (sec)	N/A	2.504	0.040	0.015	0.000	0.000	0.397	0.000	2.341	0.001
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	1005	1005	167	122	0	0	112	0	387	0
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39	0.00
time (sec)	N/A	2.403	0.030	0.016	0.000	0.000	0.403	0.000	2.304	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	677	677	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57	0.00
time (sec)	N/A	1.553	0.045	0.015	0.000	0.000	0.378	0.000	2.328	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	682	682	167	122	0	1445	104	0	299	0
N.S.	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44	0.00
time (sec)	N/A	1.235	0.026	0.012	0.000	3.889	0.307	0.000	0.314	0.001

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	850	850	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46	0.00
time (sec)	N/A	1.470	0.036	0.013	0.000	0.000	0.388	0.000	2.420	0.001

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	873	873	167	122	0	0	112	0	387	0
N.S.	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44	0.00
time (sec)	N/A	1.916	0.028	0.013	0.000	0.000	0.397	0.000	2.416	0.001

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	986	986	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39	0.00
time (sec)	N/A	1.927	0.035	0.014	0.000	0.000	0.387	0.000	2.478	0.001
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	26
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	1.04
time (sec)	N/A	0.055	0.002	0.000	0.443	0.794	0.094	0.370	0.037	0.030
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	79	114	105	105	88	365	106	0
N.S.	1	1.00	0.84	1.21	1.12	1.12	0.94	3.88	1.13	0.00
time (sec)	N/A	0.127	0.037	0.005	0.439	0.935	0.308	0.263	0.059	0.001
Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	172	155	13	154	175	13	154	0
N.S.	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27	0.00
time (sec)	N/A	0.018	0.006	0.003	0.436	0.714	0.131	0.300	2.224	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	182	157	156	156	182	156	156	0
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75	0.00
time (sec)	N/A	0.055	0.006	0.002	0.608	1.241	0.136	0.313	0.143	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	186	157	156	156	185	156	156	0
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75	0.00
time (sec)	N/A	0.050	0.006	0.002	0.552	1.173	0.139	0.332	2.171	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	230	229	262	0	189	229	21
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90	1.00
time (sec)	N/A	0.031	0.118	0.052	0.645	0.979	0.000	1.857	4.019	0.051

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	9	9	10	10	8	11	8	0
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80	0.00
time (sec)	N/A	0.004	0.004	0.000	0.555	0.726	0.127	0.230	0.050	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	18	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87	0.00
time (sec)	N/A	0.026	0.006	0.004	0.644	0.767	0.179	0.334	2.081	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	15	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87	0.00
time (sec)	N/A	0.027	0.005	0.006	0.525	0.956	0.197	0.314	0.060	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	47	23	29	0	-1	24
N.S.	1	1.00	1.00	1.20	3.13	1.53	1.93	0.00	-0.07	1.60
time (sec)	N/A	0.025	0.011	0.018	0.577	0.889	1.483	0.000	0.000	0.042

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	14	177	13	81	87	13	12	0
N.S.	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80	0.00
time (sec)	N/A	0.004	0.021	0.018	0.528	0.943	0.933	0.270	4.377	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.023	0.029	0.019	0.670	0.911	1.428	0.406	2.225	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.023	0.036	0.016	0.800	1.125	2.061	0.304	7.220	0.001

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	203	612	143	0	0	-1	21
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.032	0.178	0.064	1.034	1.732	0.000	0.000	0.000	0.115

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	17	24	19	26	46	19	23	0
N.S.	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21	0.00
time (sec)	N/A	0.005	0.009	0.005	0.559	1.067	0.710	0.374	2.095	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	97	31	35	32	0	54	45	0
N.S.	1	1.00	3.59	1.15	1.30	1.19	0.00	2.00	1.67	0.00
time (sec)	N/A	0.023	0.075	0.004	0.990	1.231	0.000	0.334	2.213	0.073
Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	A	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	116	97	142	35	33	0	54	-1	0
N.S.	1	4.30	3.59	5.26	1.30	1.22	0.00	2.00	-0.04	0.00
time (sec)	N/A	0.102	0.027	0.259	1.073	0.827	0.000	0.494	0.000	0.265
Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	99	33	35	34	0	58	49	0
N.S.	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69	0.00
time (sec)	N/A	0.024	0.077	0.005	1.013	1.078	0.000	0.346	2.205	0.077
Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	108	0	39	42	0	0	-1	0
N.S.	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.086	0.166	0.135	1.072	0.850	0.000	0.000	0.000	0.178

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	26	29	26	26	29
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.91
time (sec)	N/A	0.034	0.002	0.001	0.781	0.934	0.095	0.313	2.142	0.041

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	14
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.17
time (sec)	N/A	0.049	0.001	0.001	0.472	0.928	0.095	0.396	0.019	0.041

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	32	31	98	124	31	32	0
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76	0.00
time (sec)	N/A	0.057	0.015	0.005	1.489	0.807	0.297	0.404	2.130	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	26	25	38	0	25	54	0
N.S.	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16	0.00
time (sec)	N/A	0.024	0.015	0.003	0.672	1.018	0.000	0.294	2.191	0.032

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	21	34	24	36	0	24	46	0
N.S.	1	1.00	0.88	1.42	1.00	1.50	0.00	1.00	1.92	0.00
time (sec)	N/A	0.018	0.047	0.005	0.629	0.859	0.000	0.307	2.124	0.088
Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	26	39	35	0	65	51	0
N.S.	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04	0.00
time (sec)	N/A	0.021	0.022	0.006	0.862	1.018	0.000	0.395	2.155	0.001
Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	20	28	0	20	39	0
N.S.	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95	0.00
time (sec)	N/A	0.010	0.008	0.004	0.651	0.954	0.000	0.282	2.143	0.027
Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	106	26	19	26	73	19	25	0
N.S.	1	1.00	5.58	1.37	1.00	1.37	3.84	1.00	1.32	0.00
time (sec)	N/A	0.009	0.066	0.005	0.534	1.379	11.344	0.341	2.127	0.069

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	108	23	31	27	76	39	26	0
N.S.	1	1.00	4.91	1.05	1.41	1.23	3.45	1.77	1.18	0.00
time (sec)	N/A	0.008	0.036	0.003	1.209	1.030	52.838	0.338	2.161	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	23	22	32	0	22	43	0
N.S.	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95	0.00
time (sec)	N/A	0.018	0.012	0.004	0.661	0.987	0.000	0.345	2.138	0.030

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	28	21	30	53	21	27	0
N.S.	1	1.00	0.90	1.33	1.00	1.43	2.52	1.00	1.29	0.00
time (sec)	N/A	0.012	0.011	0.004	0.594	0.912	1.160	0.310	2.162	0.084

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	28	32	31	56	51	28	0
N.S.	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17	0.00
time (sec)	N/A	0.023	0.014	0.003	0.826	0.792	6.170	0.460	2.199	0.098

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	23	32	29	53	41	26	0
N.S.	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18	0.00
time (sec)	N/A	0.009	0.010	0.005	0.895	0.970	6.154	0.314	2.188	0.064

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	23	32	32	0	22	43	0
N.S.	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95	0.00
time (sec)	N/A	0.013	0.006	0.004	0.751	1.072	0.000	0.323	2.142	0.036

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	28	32	30	53	21	27	0
N.S.	1	1.00	0.90	1.33	1.52	1.43	2.52	1.00	1.29	0.00
time (sec)	N/A	0.012	0.005	0.003	0.930	0.663	1.142	0.398	2.148	0.041

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	143	25686	19	1956	1771	160	1576	0
N.S.	1	1.00	6.81	1223.14	0.90	93.14	84.33	7.62	75.05	0.00
time (sec)	N/A	0.125	0.151	0.004	0.740	0.570	0.420	0.299	2.979	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	5596	18	496	469	18	418	0
N.S.	1	1.00	0.90	279.80	0.90	24.80	23.45	0.90	20.90	0.00
time (sec)	N/A	0.052	0.036	0.002	0.633	0.618	0.182	0.314	2.320	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	18	5596	441	496	469	18	418	0
N.S.	1	1.00	0.95	294.53	23.21	26.11	24.68	0.95	22.00	0.00
time (sec)	N/A	0.069	0.017	0.002	0.601	0.802	0.170	0.297	2.267	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	127	2185	14	486	483	120	438	0
N.S.	1	1.00	7.94	136.56	0.88	30.38	30.19	7.50	27.38	0.00
time (sec)	N/A	0.024	0.056	0.001	0.549	0.944	0.170	0.413	2.628	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	98	89	13	88	97	13	88	0
N.S.	1	1.00	6.53	5.93	0.87	5.87	6.47	0.87	5.87	0.00
time (sec)	N/A	0.013	0.003	0.000	0.511	0.713	0.096	0.301	0.050	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	98	89	88	88	97	13	88	0
N.S.	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50	0.00
time (sec)	N/A	0.028	0.003	0.000	0.645	0.801	0.088	0.268	0.040	0.000
Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	115	2205	16	488	484	136	440	0
N.S.	1	1.00	6.39	122.50	0.89	27.11	26.89	7.56	24.44	0.00
time (sec)	N/A	0.044	0.053	0.003	0.623	0.639	0.173	0.430	2.632	0.000
Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	98	89	15	88	97	15	88	0
N.S.	1	1.00	5.76	5.24	0.88	5.18	5.71	0.88	5.18	0.00
time (sec)	N/A	0.025	0.003	0.001	0.661	0.946	0.098	0.373	2.072	0.000
Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	15	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29	0.00
time (sec)	N/A	0.227	0.003	0.001	0.525	0.545	0.095	0.234	0.047	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	15	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29	0.00
time (sec)	N/A	0.011	0.003	0.002	0.659	0.731	0.088	0.256	0.041	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	115	2205	458	488	484	488	440	0
N.S.	1	1.00	6.39	122.50	25.44	27.11	26.89	27.11	24.44	0.00
time (sec)	N/A	0.060	0.011	0.002	0.595	0.628	0.169	0.317	0.568	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	88	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29	0.00
time (sec)	N/A	0.022	0.003	0.001	0.598	1.037	0.094	0.306	0.040	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	98	89	88	88	97	88	88	0
N.S.	1	1.00	5.44	4.94	4.89	4.89	5.39	4.89	4.89	0.00
time (sec)	N/A	0.019	0.003	0.002	0.570	1.011	0.096	0.294	0.040	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	88	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29	0.00
time (sec)	N/A	0.002	0.002	0.000	0.670	1.071	0.088	0.243	0.039	0.000
Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	80	67	66	66	70	24	66	0
N.S.	1	1.00	2.86	2.39	2.36	2.36	2.50	0.86	2.36	0.00
time (sec)	N/A	0.027	0.005	0.002	0.610	1.523	0.086	0.366	0.051	0.000
Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	108	325	187	208	194	88	180	0
N.S.	1	1.00	3.48	10.48	6.03	6.71	6.26	2.84	5.81	0.00
time (sec)	N/A	0.031	0.035	0.001	0.438	0.946	0.114	0.366	0.105	0.000
Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	31	52	48	230	30	31	0
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.76	0.88	0.91	0.00
time (sec)	N/A	0.008	0.059	0.004	1.133	0.633	50.746	0.403	2.121	0.283

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	33	54	52	0	32	58	0
N.S.	1	1.00	1.00	0.94	1.54	1.49	0.00	0.91	1.66	0.00
time (sec)	N/A	0.009	0.056	0.003	1.159	1.048	0.000	0.251	2.110	0.296

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	93	78	77	77	87	24	77	0
N.S.	1	1.00	3.10	2.60	2.57	2.57	2.90	0.80	2.57	0.00
time (sec)	N/A	0.020	0.006	0.002	0.438	0.967	0.095	0.220	0.055	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	140	618	280	291	314	105	266	0
N.S.	1	1.00	4.52	19.94	9.03	9.39	10.13	3.39	8.58	0.00
time (sec)	N/A	0.038	0.046	0.002	0.455	0.822	0.143	0.336	2.266	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	98	81	80	80	90	30	80	0
N.S.	1	1.00	2.88	2.38	2.35	2.35	2.65	0.88	2.35	0.00
time (sec)	N/A	0.033	0.008	0.002	0.439	0.644	0.098	0.294	0.065	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	146	646	289	298	321	126	273	0
N.S.	1	1.00	3.56	15.76	7.05	7.27	7.83	3.07	6.66	0.00
time (sec)	N/A	0.045	0.051	0.001	0.446	0.922	0.150	0.312	2.283	0.000
Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	244	1523	289	309	323	37	270	0
N.S.	1	1.00	5.30	33.11	6.28	6.72	7.02	0.80	5.87	0.00
time (sec)	N/A	0.047	0.063	0.002	0.453	0.934	0.157	0.307	2.237	0.000
Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	248	4284	773	928	930	153	753	0
N.S.	1	1.00	5.28	91.15	16.45	19.74	19.79	3.26	16.02	0.00
time (sec)	N/A	0.091	0.123	0.002	0.480	0.956	0.269	0.442	2.450	0.000
Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	36	31	54	48	201	30	33	0
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.91	0.88	0.97	0.00
time (sec)	N/A	0.009	0.068	0.004	1.143	1.030	112.002	0.417	2.111	0.299

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	42	37	71	57	0	36	37	0
N.S.	1	1.00	0.95	0.84	1.61	1.30	0.00	0.82	0.84	0.00
time (sec)	N/A	0.010	0.086	0.003	1.173	1.053	0.000	0.392	2.210	0.347

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	49	43	83	72	0	42	73	0
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46	0.00
time (sec)	N/A	0.009	0.193	0.003	1.218	1.161	0.000	0.391	2.170	0.375

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	33	30	17	29	29	30	29	0
N.S.	1	1.00	1.74	1.58	0.89	1.53	1.53	1.58	1.53	0.00
time (sec)	N/A	0.009	0.002	0.001	0.434	0.744	0.061	0.346	0.025	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	21	18	14	17	17	22	17	0
N.S.	1	1.00	1.31	1.12	0.88	1.06	1.06	1.38	1.06	0.00
time (sec)	N/A	0.007	0.002	0.000	0.439	0.936	0.057	0.233	0.034	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	96	96	87	86	86	94	28	86	0
N.S.	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61	0.00
time (sec)	N/A	0.198	0.006	0.001	0.437	0.728	0.082	0.288	0.218	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	96	96	87	86	86	94	28	86	0
N.S.	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61	0.00
time (sec)	N/A	0.154	0.005	0.001	0.449	0.662	0.091	0.360	0.191	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	57	56	12	12	0
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86	0.00
time (sec)	N/A	0.008	0.006	0.001	0.436	0.837	0.189	0.316	2.100	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87	0.00
time (sec)	N/A	0.009	0.004	0.002	0.442	1.020	0.089	0.371	0.049	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	20	14	15	15	14	18	13	0
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76	0.00
time (sec)	N/A	0.010	0.006	0.003	0.448	0.770	0.099	0.270	0.066	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	23	28	23	23	22	24	23	0
N.S.	1	1.00	0.58	0.70	0.58	0.58	0.55	0.60	0.58	0.00
time (sec)	N/A	0.097	0.059	0.013	0.503	0.997	32.564	0.536	0.108	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	605	605	98	2105	0	0	0	0	-1	0
N.S.	1	1.00	0.16	3.48	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	4.535	0.069	0.088	0.000	0.000	0.000	0.000	0.000	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	55	82	0	83	58	58	75	0
N.S.	1	1.00	0.87	1.30	0.00	1.32	0.92	0.92	1.19	0.00
time (sec)	N/A	0.066	0.029	0.049	0.000	1.870	0.126	0.274	0.185	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	22	38	20	13	12	0
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86	0.00
time (sec)	N/A	0.050	0.008	0.007	0.436	1.166	0.096	0.289	0.040	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	24	27	32	46	29	23	21	0
N.S.	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75	0.00
time (sec)	N/A	0.028	0.013	0.006	0.446	1.104	0.109	0.392	0.037	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	28	30	38	38	36	28	28	0
N.S.	1	1.00	0.47	0.51	0.64	0.64	0.61	0.47	0.47	0.00
time (sec)	N/A	0.091	0.012	0.006	0.463	1.867	0.181	1.436	0.054	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	41	11	11	8	11	11	0
N.S.	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00	0.00
time (sec)	N/A	0.007	0.007	0.009	0.438	1.221	0.117	0.331	2.301	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	205	132	116	114	223	272	134	124	0
N.S.	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36	0.00
time (sec)	N/A	0.149	0.090	0.019	0.987	2.227	1.362	0.372	2.369	0.001
Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	26	44	40	0	99	49	0
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96	0.00
time (sec)	N/A	0.025	0.375	0.010	0.699	2.024	0.000	2.090	2.656	1.127
Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	39	39	0	89	39	0
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70	0.00
time (sec)	N/A	0.082	0.233	0.010	0.656	1.806	0.000	1.887	2.549	0.959
Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	39	39	0	89	39	0
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70	0.00
time (sec)	N/A	0.059	0.177	0.008	0.629	0.803	0.000	0.728	2.319	0.738

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	22	37	37	0	87	37	0
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76	0.00
time (sec)	N/A	0.044	0.122	0.005	0.608	1.255	0.000	1.083	2.273	0.557
Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	17	20	33	33	0	52	19	0
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00	0.00
time (sec)	N/A	0.043	0.010	0.006	0.621	0.816	0.000	0.296	2.193	0.077
Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.034	0.176	0.009	0.619	1.277	0.000	0.000	3.202	0.464
Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.035	0.199	0.008	0.636	1.083	0.000	0.000	3.319	0.532

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.036	0.188	0.008	0.642	1.502	0.000	0.000	3.355	0.663
Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	83	74	73	79	97	73	97	0
N.S.	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00	0.00
time (sec)	N/A	0.138	0.036	0.007	1.520	1.341	0.245	0.309	0.194	0.001
Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	78	69	68	74	92	68	92	0
N.S.	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02	0.00
time (sec)	N/A	0.123	0.023	0.006	1.880	1.225	0.247	0.378	0.178	0.001
Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	72	62	61	67	78	61	85	0
N.S.	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10	0.00
time (sec)	N/A	0.121	0.031	0.005	1.441	0.828	0.232	0.391	2.290	0.001

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	69	57	56	62	75	56	80	0
N.S.	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11	0.00
time (sec)	N/A	0.093	0.023	0.005	1.586	1.290	0.232	0.307	2.292	0.001
Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	65	56	55	61	75	55	79	0
N.S.	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11	0.00
time (sec)	N/A	0.078	0.017	0.006	1.316	1.331	0.225	0.287	0.147	0.000
Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	69	60	59	65	78	60	83	0
N.S.	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11	0.00
time (sec)	N/A	0.135	0.019	0.007	1.571	1.164	0.288	0.307	0.149	0.001
Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	78	65	64	76	87	65	88	0
N.S.	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05	0.00
time (sec)	N/A	0.152	0.033	0.009	1.441	1.160	0.315	0.257	2.280	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	82	70	69	89	94	70	92	0
N.S.	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01	0.00
time (sec)	N/A	0.157	0.055	0.010	1.264	1.130	0.321	0.286	0.152	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	307	307	109	74	0	1202	61	0	128	0
N.S.	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42	0.00
time (sec)	N/A	0.578	0.021	0.008	0.000	3.804	0.985	0.000	2.169	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	101	67	0	1145	3662	0	188	0
N.S.	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70	0.00
time (sec)	N/A	0.392	0.016	0.007	0.000	4.347	2.732	0.000	0.134	0.001

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	94	62	0	1190	48	0	183	0
N.S.	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80	0.00
time (sec)	N/A	0.357	0.015	0.008	0.000	3.808	0.950	0.000	0.193	0.001

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	90	58	0	1189	46	0	181	0
N.S.	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91	0.00
time (sec)	N/A	0.194	0.013	0.007	0.000	5.343	0.914	0.000	2.342	0.001
Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	101	67	0	1143	60	0	237	0
N.S.	1	1.00	0.41	0.27	0.00	4.67	0.24	0.00	0.97	0.00
time (sec)	N/A	0.473	0.020	0.011	0.000	6.124	12.657	0.000	2.342	0.001
Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	109	72	0	1245	0	0	242	0
N.S.	1	1.00	0.39	0.26	0.00	4.43	0.00	0.00	0.86	0.00
time (sec)	N/A	0.467	0.018	0.013	0.000	6.702	0.000	0.000	2.302	0.001
Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	116	77	0	1274	70	0	246	0
N.S.	1	1.00	0.37	0.24	0.00	4.02	0.22	0.00	0.78	0.00
time (sec)	N/A	0.539	0.019	0.012	0.000	5.633	2.705	0.000	2.248	0.001

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	87	75	0	83	44	87	252	0
N.S.	1	1.00	4.58	3.95	0.00	4.37	2.32	4.58	13.26	0.00
time (sec)	N/A	0.105	0.047	0.095	0.000	2.113	1.045	4.080	2.270	0.001

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	57	34	33	47	37	34	38	0
N.S.	1	1.00	1.33	0.79	0.77	1.09	0.86	0.79	0.88	0.00
time (sec)	N/A	0.067	0.023	0.009	2.104	0.878	0.170	0.254	0.052	0.001

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	18	21	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24	0.00
time (sec)	N/A	0.037	0.005	0.008	1.116	1.152	0.131	0.311	2.200	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	20	24	30	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20	0.00
time (sec)	N/A	0.037	0.006	0.006	1.147	1.376	0.132	0.254	0.054	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	20	19	18	24	14	28	18	0
N.S.	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82	0.00
time (sec)	N/A	0.013	0.010	0.007	1.095	1.427	0.111	0.366	0.036	0.001
Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	38	22	21	21	26	22	25	0
N.S.	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93	0.00
time (sec)	N/A	0.039	0.011	0.007	2.309	1.516	0.144	0.286	2.260	0.001
Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	17	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81	0.00
time (sec)	N/A	0.015	0.004	0.001	2.281	1.489	0.082	0.270	0.031	0.001
Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	23	23	22	23	23	0
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85	0.00
time (sec)	N/A	0.026	0.007	0.005	2.077	1.445	0.109	0.278	2.140	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	26	25	25	26	29	25	0
N.S.	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64	0.00
time (sec)	N/A	0.020	0.018	0.009	1.273	1.128	0.241	0.317	2.141	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	23	19	18	18	17	19	18	0
N.S.	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82	0.00
time (sec)	N/A	0.015	0.004	0.002	1.133	1.015	0.075	0.281	0.027	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.029	0.005	0.003	2.301	0.998	0.091	0.341	2.131	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	24	21	20	26	17	22	10	0
N.S.	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83	0.00
time (sec)	N/A	0.021	0.011	0.008	1.078	1.078	0.098	0.297	0.072	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	22	19	23	19	0
N.S.	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76	0.00
time (sec)	N/A	0.036	0.006	0.007	1.129	1.193	0.102	0.269	2.139	0.001
Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.00
time (sec)	N/A	0.028	0.006	0.005	2.157	1.216	0.114	0.309	0.038	0.000
Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	31	35	55	36	30	35	0
N.S.	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00	0.00
time (sec)	N/A	0.034	0.017	0.007	2.223	1.613	0.147	0.329	2.125	0.001
Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	20	17	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74	0.00
time (sec)	N/A	0.037	0.005	0.009	1.405	2.088	0.138	0.239	2.188	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	19	20	19	0
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83	0.00
time (sec)	N/A	0.054	0.007	0.006	1.115	1.490	0.109	0.295	0.055	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	24	23	32	20	23	25	0
N.S.	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86	0.00
time (sec)	N/A	0.014	0.010	0.006	2.335	1.160	0.119	0.374	2.125	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	42	41	58	49	43	55	0
N.S.	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25	0.00
time (sec)	N/A	0.243	0.024	0.010	2.248	1.413	0.212	0.363	0.129	0.001

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	41	38	38	51	38	88	0
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91	0.00
time (sec)	N/A	0.133	0.020	0.005	2.287	1.702	0.213	0.304	0.160	0.001

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	34	44	72	41	44	56	0
N.S.	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70	0.00
time (sec)	N/A	0.166	0.022	0.012	1.156	2.043	0.186	0.301	2.162	0.001

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65	0.00
time (sec)	N/A	0.007	0.006	0.007	2.200	1.770	0.141	0.288	0.032	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	21	20	20	34	20	20	0
N.S.	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83	0.00
time (sec)	N/A	0.018	0.009	0.003	2.255	1.296	0.166	0.288	2.134	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	23	18	17	17	17	20	19	0
N.S.	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27	0.00
time (sec)	N/A	0.060	0.006	0.008	2.313	1.415	0.137	0.319	0.056	0.001

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	18	17	17	19	17	17	0
N.S.	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85	0.00
time (sec)	N/A	0.012	0.009	0.007	2.398	1.819	0.148	0.375	0.050	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	27	44	29	60	35	0
N.S.	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95	0.00
time (sec)	N/A	0.040	0.023	0.007	2.224	1.390	0.164	0.287	2.144	0.001

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	22	21	21	26	21	21	0
N.S.	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81	0.00
time (sec)	N/A	0.011	0.008	0.002	2.216	1.494	0.094	0.309	0.030	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	16	10	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83	0.00
time (sec)	N/A	0.032	0.004	0.006	1.105	1.643	0.095	0.287	2.125	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	20	21	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00	0.00
time (sec)	N/A	0.030	0.007	0.008	1.092	1.418	0.132	0.282	2.144	0.000
Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	34	17	20	20	0
N.S.	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91	0.00
time (sec)	N/A	0.015	0.008	0.005	2.231	1.671	0.112	0.282	2.124	0.000
Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	21	20	32	19	20	22	0
N.S.	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92	0.00
time (sec)	N/A	0.016	0.011	0.007	2.176	1.436	0.116	0.405	0.029	0.000
Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	34	33	33	39	33	56	0
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56	0.00
time (sec)	N/A	0.026	0.015	0.005	2.057	1.427	0.188	0.245	0.103	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	28	27	27	29	27	37	0
N.S.	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00	0.00
time (sec)	N/A	0.024	0.008	0.003	2.003	1.033	0.174	0.285	2.136	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	21	22	23	21	0
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72	0.00
time (sec)	N/A	0.017	0.006	0.006	1.083	1.190	0.109	0.367	2.214	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	14	16	15	0
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79	0.00
time (sec)	N/A	0.013	0.004	0.002	1.066	1.232	0.081	0.266	0.034	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	35	34	34	46	34	36	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88	0.00
time (sec)	N/A	0.028	0.014	0.005	2.178	1.527	0.118	0.288	0.043	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	39	32	31	31	34	31	31	0
N.S.	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76	0.00
time (sec)	N/A	0.028	0.007	0.004	2.074	1.759	0.118	0.278	2.116	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	24	21	20	20	24	23	20	0
N.S.	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67	0.00
time (sec)	N/A	0.058	0.011	0.007	1.038	0.690	0.152	0.250	2.125	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	31	30	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77	0.00
time (sec)	N/A	0.040	0.008	0.008	1.118	1.243	0.149	0.324	0.044	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	32	25	24	34	27	34	26	0
N.S.	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76	0.00
time (sec)	N/A	0.057	0.014	0.008	1.133	1.573	0.145	0.387	2.109	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	35	33	44	36	33	39	0
N.S.	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93	0.00
time (sec)	N/A	0.021	0.021	0.006	2.355	0.915	0.131	0.297	2.187	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	38	37	37	46	37	41	0
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84	0.00
time (sec)	N/A	0.157	0.013	0.006	2.803	1.424	0.227	0.315	0.072	0.001

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	20	19	19	24	22	19	0
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66	0.00
time (sec)	N/A	0.053	0.007	0.008	1.166	1.608	0.142	0.275	2.182	0.001

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	43	34	33	33	41	34	50	0
N.S.	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09	0.00
time (sec)	N/A	0.044	0.016	0.006	2.161	1.557	0.144	0.317	0.086	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	14	15	14	17	10	15	14	0
N.S.	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88	0.00
time (sec)	N/A	0.025	0.007	0.004	1.085	0.800	0.093	0.397	0.043	0.001
Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	14	16	17	17	14	12	12	0
N.S.	1	1.00	0.67	0.76	0.81	0.81	0.67	0.57	0.57	0.00
time (sec)	N/A	0.019	0.003	0.004	1.088	1.289	0.087	0.301	2.085	0.000
Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	23	23	17	30	15	0
N.S.	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00	0.00
time (sec)	N/A	0.027	0.011	0.005	1.268	1.298	0.114	0.262	2.089	0.001
Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	29	28	28	3	29	57	0
N.S.	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84	0.00
time (sec)	N/A	0.042	0.012	0.005	2.357	1.257	0.130	0.278	0.112	0.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	22	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76	0.00
time (sec)	N/A	0.042	0.006	0.007	1.088	1.071	0.141	0.287	0.060	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	25	24	36	20	26	22	0
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73	0.00
time (sec)	N/A	0.030	0.015	0.006	1.212	1.138	0.098	0.276	2.108	0.001

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	18	21	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91	0.00
time (sec)	N/A	0.038	0.005	0.005	2.075	1.037	0.137	0.331	2.121	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	93	73	77	136	88	74	96	0
N.S.	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93	0.00
time (sec)	N/A	0.476	0.043	0.013	2.248	0.979	0.515	0.344	2.200	0.001

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	30	28	25	36	24	25	32	0
N.S.	1	1.00	0.91	0.85	0.76	1.09	0.73	0.76	0.97	0.00
time (sec)	N/A	0.017	0.011	0.005	2.149	1.682	0.127	0.317	0.035	0.001
Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	29	44	27	30	33	0
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00	0.00
time (sec)	N/A	0.041	0.019	0.008	2.254	1.541	0.149	0.379	2.113	0.001
Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	23	19	17	26	19	0
N.S.	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76	0.00
time (sec)	N/A	0.041	0.005	0.007	1.046	1.278	0.096	0.291	0.043	0.000
Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	32	31	31	36	31	56	0
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56	0.00
time (sec)	N/A	0.115	0.015	0.004	2.096	1.585	0.202	0.336	0.106	0.001

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	33	22	21	23	0
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79	0.00
time (sec)	N/A	0.107	0.017	0.009	2.089	1.057	0.174	0.227	0.042	0.001

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	36	35	39	46	36	49	0
N.S.	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07	0.00
time (sec)	N/A	0.058	0.026	0.008	2.093	1.520	0.162	0.296	2.169	0.001

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	26	26	19	18	18	17	20	14	0
N.S.	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64	0.00
time (sec)	N/A	0.015	0.006	0.005	0.974	1.608	0.098	0.284	0.039	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	18	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.039	0.006	0.007	0.850	1.265	0.136	0.381	0.065	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	18	14	16	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00	0.00
time (sec)	N/A	0.025	0.004	0.007	0.999	0.958	0.115	0.373	2.120	0.000
Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	25	18	17	17	17	20	17	0
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89	0.00
time (sec)	N/A	0.027	0.007	0.007	1.029	1.342	0.131	0.292	2.116	0.001
Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	19	17	33	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43	0.00
time (sec)	N/A	0.029	0.009	0.006	2.122	1.086	0.183	0.364	0.054	0.000
Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	57	51	50	50	68	53	58	0
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92	0.00
time (sec)	N/A	0.088	0.027	0.010	2.112	0.903	0.346	0.306	2.213	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	86	67	54	59	103	65	59	71	0
N.S.	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03	0.00
time (sec)	N/A	0.099	0.047	0.014	2.070	1.242	0.213	0.236	2.177	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	19	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76	0.00
time (sec)	N/A	0.013	0.006	0.006	2.043	1.624	0.106	0.302	0.033	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	15	17	17	0
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89	0.00
time (sec)	N/A	0.027	0.005	0.004	2.030	0.809	0.101	0.269	2.098	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	7	7	28	19	0
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11	0.00
time (sec)	N/A	0.075	0.006	0.005	2.166	0.971	0.138	0.292	2.104	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	12	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.016	0.005	0.002	1.375	1.055	0.087	0.295	0.028	0.001
Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	41	51	66	99	55	48	0
N.S.	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74	0.00
time (sec)	N/A	0.062	0.036	0.008	1.771	1.513	0.448	0.295	0.099	0.001
Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	46	24	28	28	0
N.S.	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00	0.00
time (sec)	N/A	0.022	0.011	0.006	1.656	0.994	0.131	0.285	0.038	0.001
Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	26	29	26	26	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.040	0.023	0.004	1.515	0.613	0.103	0.374	0.025	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	41	39	24	23	23	26	27	23	0
N.S.	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74	0.00
time (sec)	N/A	0.049	0.007	0.008	0.797	1.403	0.194	0.308	0.050	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	26	19	23	30	0
N.S.	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25	0.00
time (sec)	N/A	0.167	0.009	0.006	1.223	1.266	0.155	0.292	2.129	0.001

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	24	35	20	19	23	0
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00	0.00
time (sec)	N/A	0.025	0.011	0.006	1.431	1.095	0.130	0.391	0.033	0.001

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	24	35	20	19	23	0
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00	0.00
time (sec)	N/A	0.039	0.006	0.005	1.416	1.877	0.127	0.274	0.030	0.001

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	15	10	13	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00	0.00
time (sec)	N/A	0.049	0.005	0.005	0.708	2.057	0.100	0.283	2.144	0.000
Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	193	236	199	4545	138	208	357	0
N.S.	1	1.00	0.94	1.15	0.97	22.06	0.67	1.01	1.73	0.00
time (sec)	N/A	0.274	0.107	0.008	1.652	5.144	1.238	0.442	0.231	0.001
Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	41	0	47	44	38	42	0
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.93	0.00
time (sec)	N/A	0.067	0.025	0.009	0.000	1.121	0.154	1.109	2.191	0.001
Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	44	41	0	75	46	42	47	0
N.S.	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80	0.00
time (sec)	N/A	0.064	0.021	0.010	0.000	0.892	0.184	1.803	0.049	0.001

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	234	616	0	0	0	1587	3942	0
N.S.	1	1.00	1.12	2.95	0.00	0.00	0.00	7.59	18.86	0.00
time (sec)	N/A	0.371	0.255	0.061	0.000	0.000	0.000	4.697	3.437	0.001

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	245	633	0	0	0	1625	3046	0
N.S.	1	1.00	1.09	2.83	0.00	0.00	0.00	7.25	13.60	0.00
time (sec)	N/A	0.389	0.248	0.060	0.000	0.000	0.000	4.563	3.220	0.001

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	57	60	65	190	62	61	0
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09	0.00
time (sec)	N/A	0.047	0.027	0.007	0.685	1.196	1.065	0.313	0.213	0.001

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	73	87	84	162	0	85	347	0
N.S.	1	1.00	0.76	0.91	0.88	1.69	0.00	0.89	3.61	0.00
time (sec)	N/A	0.108	0.035	0.008	1.222	1.231	0.000	0.257	1.131	0.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	228	336	279	5975	0	320	570	0
N.S.	1	1.00	0.86	1.27	1.06	22.63	0.00	1.21	2.16	0.00
time (sec)	N/A	0.472	0.090	0.007	1.706	4.868	0.000	0.302	2.496	0.001
Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	417	370	422	349	0	0	401	823	0
N.S.	1	1.00	0.89	1.01	0.84	0.00	0.00	0.96	1.97	0.00
time (sec)	N/A	0.547	0.230	0.014	1.481	0.000	0.000	0.567	2.449	0.001
Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	24	21	20	26	19	21	12	0
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75	0.00
time (sec)	N/A	0.008	0.009	0.007	0.753	1.298	0.105	0.239	0.039	0.000
Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	27	24	23	34	20	30	17	0
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89	0.00
time (sec)	N/A	0.013	0.011	0.009	0.651	0.652	0.118	0.303	2.138	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	85	90	75	106	92	77	103	0
N.S.	1	1.00	0.88	0.93	0.77	1.09	0.95	0.79	1.06	0.00
time (sec)	N/A	0.071	0.050	0.012	1.934	1.189	0.361	0.314	0.177	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87	0.00
time (sec)	N/A	0.106	0.008	0.005	1.400	1.063	0.120	0.289	2.120	0.001

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.00
time (sec)	N/A	0.094	0.007	0.004	1.327	1.053	0.117	0.298	0.035	0.001

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	25	24	24	29	24	51	0
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76	0.00
time (sec)	N/A	0.118	0.014	0.004	1.479	1.234	0.193	0.259	2.153	0.001

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	17	10	14	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00	0.00
time (sec)	N/A	0.011	0.008	0.004	2.074	1.148	0.138	0.253	0.042	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	12	7	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83	0.00
time (sec)	N/A	0.011	0.003	0.004	0.752	1.226	0.091	0.418	0.037	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	10	17	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55	0.00
time (sec)	N/A	0.034	0.005	0.006	1.608	1.130	0.131	0.292	2.166	0.001

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	13	8	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83	0.00
time (sec)	N/A	0.021	0.003	0.005	0.714	1.136	0.091	0.384	0.039	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	12	14	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00	0.00
time (sec)	N/A	0.018	0.003	0.005	0.596	0.932	0.105	0.229	0.047	0.000
Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	16	23	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35	0.00
time (sec)	N/A	0.037	0.005	0.005	1.532	1.257	0.133	0.243	2.284	0.000
Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	10	14	19	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46	0.00
time (sec)	N/A	0.035	0.008	0.006	1.410	1.177	0.132	0.343	0.060	0.000
Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	22	21	20	26	20	22	20	0
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71	0.00
time (sec)	N/A	0.011	0.015	0.007	0.662	1.102	0.112	0.297	0.069	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	37	26	43	22	0
N.S.	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69	0.00
time (sec)	N/A	0.026	0.017	0.009	0.746	1.157	0.134	0.298	2.226	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	28	20	19	19	19	20	25	0
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09	0.00
time (sec)	N/A	0.034	0.008	0.005	1.558	1.191	0.136	0.310	0.049	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	21	20	36	20	47	28	0
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17	0.00
time (sec)	N/A	0.035	0.014	0.007	1.600	1.352	0.141	0.303	2.111	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	40	39	37	44	39	41	0
N.S.	1	1.00	1.00	0.82	0.80	0.76	0.90	0.80	0.84	0.00
time (sec)	N/A	0.142	0.014	0.007	1.708	1.303	0.224	0.283	2.156	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	19	20	19	19	19	22	19	0
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76	0.00
time (sec)	N/A	0.055	0.008	0.006	1.067	1.135	0.145	0.392	2.135	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	54	48	47	60	63	60	61	0
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02	0.00
time (sec)	N/A	0.246	0.053	0.009	2.798	1.599	0.252	0.343	0.130	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	5	9	6	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.55	0.82
time (sec)	N/A	0.007	0.000	0.000	0.845	1.165	0.060	0.227	0.016	0.018

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	21	22	23	21	0
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72	0.00
time (sec)	N/A	0.016	0.005	0.005	1.096	1.136	0.108	0.355	2.124	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	37	37	52	37	37	49	0
N.S.	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09	0.00
time (sec)	N/A	0.025	0.013	0.007	1.839	0.745	0.137	0.299	0.041	0.000
Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	52	29	29	44	0
N.S.	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38	0.00
time (sec)	N/A	0.254	0.019	0.010	2.025	1.220	0.263	0.325	0.069	0.001
Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	159	110	132	178	146	122	124	0
N.S.	1	1.00	1.07	0.74	0.89	1.20	0.99	0.82	0.84	0.00
time (sec)	N/A	0.136	0.068	0.011	2.009	1.287	0.429	0.388	2.189	0.001
Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	101	79	99	515	61	92	170	0
N.S.	1	1.00	0.90	0.71	0.88	4.60	0.54	0.82	1.52	0.00
time (sec)	N/A	0.114	0.049	0.007	2.058	4.324	0.965	0.390	2.228	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	14	12	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86	0.00
time (sec)	N/A	0.016	0.003	0.005	0.907	1.348	0.103	0.262	0.044	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	12	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83	0.00
time (sec)	N/A	0.029	0.005	0.004	1.005	1.563	0.098	0.372	2.138	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	21	14	17	15	0
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88	0.00
time (sec)	N/A	0.030	0.004	0.007	0.892	1.183	0.108	0.319	0.027	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	17	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88	0.00
time (sec)	N/A	0.029	0.004	0.005	0.900	1.005	0.100	0.260	0.033	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	25	15	23	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89	0.00
time (sec)	N/A	0.030	0.004	0.006	1.921	1.017	0.105	0.267	0.050	0.000
Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	14	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00	0.00
time (sec)	N/A	0.019	0.005	0.005	0.892	1.029	0.086	0.361	0.034	0.000
Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	31	30	30	36	33	30	0
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71	0.00
time (sec)	N/A	0.040	0.007	0.008	0.878	0.928	0.151	0.287	0.053	0.000
Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	41	38	38	51	38	88	0
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91	0.00
time (sec)	N/A	0.125	0.017	0.002	1.972	1.154	0.212	0.355	0.002	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	14	17	8	0
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.89	0.42	0.00
time (sec)	N/A	0.062	0.003	0.007	1.026	1.083	0.109	0.281	0.084	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	31	30	30	34	32	26	0
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65	0.00
time (sec)	N/A	0.021	0.005	0.007	0.995	1.087	0.119	0.284	2.093	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	27	27	21	0
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64	0.00
time (sec)	N/A	0.020	0.004	0.006	1.020	0.929	0.117	0.343	0.033	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	20	20	20	22	16	0
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62	0.00
time (sec)	N/A	0.015	0.005	0.006	1.027	1.465	0.117	0.270	2.117	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	17	19	13	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62	0.00
time (sec)	N/A	0.007	0.003	0.003	1.173	0.935	0.109	0.286	2.110	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	19	8	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.38	0.00
time (sec)	N/A	0.010	0.003	0.005	0.926	1.104	0.110	0.275	0.078	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	21	24	24	17	0
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63	0.00
time (sec)	N/A	0.016	0.004	0.008	0.916	0.826	0.148	0.373	0.095	0.001

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	27	26	30	31	29	22	0
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65	0.00
time (sec)	N/A	0.030	0.004	0.008	1.062	0.967	0.162	0.266	0.035	0.001

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	32	31	39	36	34	26	0
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63	0.00
time (sec)	N/A	0.034	0.005	0.008	1.015	1.124	0.170	0.276	0.036	0.001

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	37	36	44	41	39	32	0
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67	0.00
time (sec)	N/A	0.041	0.005	0.009	1.071	1.018	0.182	0.238	0.037	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	61	54	0	1506	41	0	144	0
N.S.	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92	0.00
time (sec)	N/A	0.168	0.016	0.014	0.000	3.453	0.225	0.000	2.745	0.001

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	61	56	0	1546	41	0	142	0
N.S.	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90	0.00
time (sec)	N/A	0.112	0.017	0.015	0.000	4.054	0.227	0.000	2.805	0.001

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	61	54	0	2271	39	0	142	0
N.S.	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76	0.00
time (sec)	N/A	0.276	0.013	0.008	0.000	4.129	0.211	0.000	2.780	0.000
Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	61	56	0	2259	39	0	142	0
N.S.	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76	0.00
time (sec)	N/A	0.156	0.013	0.010	0.000	4.000	0.212	0.000	2.740	0.000
Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	663	663	63	69	0	0	133	0	328	0
N.S.	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49	0.00
time (sec)	N/A	1.108	0.037	0.074	0.000	0.000	3.924	0.000	2.699	0.001
Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	663	663	63	69	0	0	133	0	328	0
N.S.	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49	0.00
time (sec)	N/A	0.648	0.031	0.002	0.000	0.000	3.920	0.000	0.002	0.001

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	C	A	F	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	0	95	67	0	5653	42	0	504	0
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00	0.00
time (sec)	N/A	0.380	0.061	0.278	0.000	4.553	1.866	0.000	3.082	0.001

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	322	314	310	0	384	311	894	0
N.S.	1	1.00	1.01	0.98	0.97	0.00	1.20	0.97	2.79	0.00
time (sec)	N/A	0.256	0.249	0.011	1.956	0.000	5.289	0.361	2.843	0.001

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	243	292	275	0	277	285	556	0
N.S.	1	1.00	0.84	1.00	0.95	0.00	0.95	0.98	1.91	0.00
time (sec)	N/A	0.198	0.100	0.003	2.047	0.000	2.678	0.336	2.657	0.001

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	184	151	207	0	124	215	160	0
N.S.	1	1.00	0.84	0.69	0.95	0.00	0.57	0.98	0.73	0.00
time (sec)	N/A	0.174	0.059	0.002	2.666	0.000	0.823	0.316	2.323	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	134	128	169	121	20	179	33	0
N.S.	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18	0.00
time (sec)	N/A	0.098	0.018	0.003	2.367	1.073	0.161	0.325	0.083	0.000
Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	404	433	345	0	0	371	874	0
N.S.	1	1.00	0.97	1.04	0.83	0.00	0.00	0.89	2.10	0.00
time (sec)	N/A	0.428	0.149	0.007	2.727	0.000	0.000	0.422	0.418	0.001
Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	552	552	524	866	561	0	0	646	2436	0
N.S.	1	1.00	0.95	1.57	1.02	0.00	0.00	1.17	4.41	0.00
time (sec)	N/A	0.808	0.606	0.010	2.109	0.000	0.000	2.780	2.779	0.001
Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	680	680	738	1201	817	0	0	901	1955	0
N.S.	1	1.00	1.09	1.77	1.20	0.00	0.00	1.32	2.88	0.00
time (sec)	N/A	0.950	0.929	0.012	2.342	0.000	0.000	0.892	3.674	0.001

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	347	390	332	0	350	342	670	0
N.S.	1	1.00	0.99	1.12	0.95	0.00	1.00	0.98	1.92	0.00
time (sec)	N/A	0.298	0.374	0.004	2.200	0.000	8.334	0.342	0.433	0.001

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	321	362	318	0	318	323	391	0
N.S.	1	1.00	1.00	1.12	0.99	0.00	0.99	1.00	1.21	0.00
time (sec)	N/A	0.269	0.316	0.007	2.630	0.000	3.521	0.364	2.483	0.001

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	224	188	238	0	155	241	282	0
N.S.	1	1.00	0.93	0.78	0.99	0.00	0.64	1.00	1.17	0.00
time (sec)	N/A	0.189	0.189	0.005	2.433	0.000	1.092	0.339	0.274	0.001

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	183	143	189	173	39	194	58	0
N.S.	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29	0.00
time (sec)	N/A	0.116	0.107	0.005	2.108	1.162	0.297	0.246	0.089	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	855	855	558	1122	601	0	0	771	1591	0
N.S.	1	1.00	0.65	1.31	0.70	0.00	0.00	0.90	1.86	0.00
time (sec)	N/A	0.849	0.391	0.019	2.291	0.000	0.000	0.504	2.995	0.001

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1141	1141	807	1636	961	0	0	1104	2246	0
N.S.	1	1.00	0.71	1.43	0.84	0.00	0.00	0.97	1.97	0.00
time (sec)	N/A	1.659	0.865	0.023	2.541	0.000	0.000	103.636	4.104	0.001

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1384	1384	996	2121	1394	0	0	1488	3256	0
N.S.	1	1.00	0.72	1.53	1.01	0.00	0.00	1.08	2.35	0.00
time (sec)	N/A	1.963	1.357	0.026	2.602	0.000	0.000	1.148	5.038	0.001

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	388	470	392	0	413	389	721	0
N.S.	1	1.00	0.98	1.19	0.99	0.00	1.05	0.99	1.83	0.00
time (sec)	N/A	0.346	0.351	0.007	2.373	0.000	8.013	0.354	0.478	0.001

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	358	419	364	0	374	356	676	0
N.S.	1	1.00	0.99	1.16	1.01	0.00	1.04	0.99	1.88	0.00
time (sec)	N/A	0.326	0.294	0.007	2.181	0.000	4.878	0.433	0.471	0.001
Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	249	222	269	0	192	260	315	0
N.S.	1	1.00	0.94	0.83	1.01	0.00	0.72	0.98	1.18	0.00
time (sec)	N/A	0.249	0.206	0.007	2.204	0.000	1.483	0.320	0.303	0.000
Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	200	158	212	232	63	204	80	0
N.S.	1	1.00	0.91	0.72	0.97	1.06	0.29	0.93	0.37	0.00
time (sec)	N/A	0.142	0.083	0.005	2.704	1.459	0.461	0.358	0.096	0.000
Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1352	1352	835	2098	1015	0	0	1259	2720	0
N.S.	1	1.00	0.62	1.55	0.75	0.00	0.00	0.93	2.01	0.00
time (sec)	N/A	1.412	0.738	0.025	2.408	0.000	0.000	0.910	4.406	0.001

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1830	1830	1115	2769	1564	0	0	0	3572	0
N.S.	1	1.00	0.61	1.51	0.85	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	2.781	1.551	0.033	3.097	0.000	0.000	0.000	5.754	0.001
Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2204	2204	1338	3334	2198	0	0	2119	6280	0
N.S.	1	1.00	0.61	1.51	1.00	0.00	0.00	0.96	2.85	0.00
time (sec)	N/A	3.165	2.873	0.038	3.988	0.000	0.000	1.534	7.785	0.001
Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	33	29	28	28	34	28	30	0
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94	0.00
time (sec)	N/A	0.019	0.010	0.002	2.532	1.534	0.112	0.295	0.041	0.000
Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	33	29	28	28	34	28	30	0
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94	0.00
time (sec)	N/A	0.029	0.006	0.002	2.530	1.387	0.115	0.380	0.035	0.001

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	29	26	26	36	26	30	0
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94	0.00
time (sec)	N/A	0.017	0.009	0.003	2.156	1.652	0.116	0.388	0.043	0.001

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	29	26	26	36	26	30	0
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94	0.00
time (sec)	N/A	0.035	0.005	0.003	2.329	1.658	0.120	0.335	0.032	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	29	35	45	39	44	34	0
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76	0.00
time (sec)	N/A	0.012	0.022	0.003	2.621	1.613	0.111	0.415	2.319	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	29	35	45	39	44	34	0
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76	0.00
time (sec)	N/A	0.021	0.005	0.003	2.332	1.406	0.118	0.315	0.047	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	23	18	17	17	15	15	6	0
N.S.	1	1.00	3.83	3.00	2.83	2.83	2.50	2.50	1.00	0.00
time (sec)	N/A	0.002	0.003	0.005	1.061	1.393	0.094	0.310	2.270	0.000
Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	23	18	17	17	15	19	6	0
N.S.	1	1.00	1.10	0.86	0.81	0.81	0.71	0.90	0.29	0.00
time (sec)	N/A	0.004	0.003	0.002	0.942	1.162	0.098	0.416	0.146	0.000
Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	10	9	24	22	9	9	0
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69	0.00
time (sec)	N/A	0.002	0.003	0.001	0.882	1.346	0.125	0.335	2.319	0.000
Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	81	11	58	57	24	22	57	9	0
N.S.	1	6.23	0.85	4.46	4.38	1.85	1.69	4.38	0.69	0.00
time (sec)	N/A	0.012	0.002	0.002	0.937	1.104	0.314	0.312	0.025	0.001

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	78	67	66	66	85	68	94	0
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36	0.00
time (sec)	N/A	0.111	0.014	0.009	2.014	1.257	0.250	0.293	0.102	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	78	67	66	66	85	68	94	0
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36	0.00
time (sec)	N/A	0.124	0.006	0.003	1.990	1.359	0.254	0.307	0.039	0.001

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	31	21	20	20	20	21	20	0
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83	0.00
time (sec)	N/A	0.019	0.006	0.003	0.631	1.482	0.078	0.307	0.037	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	21	20	20	19	21	20	0
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77	0.00
time (sec)	N/A	0.016	0.004	0.002	0.686	1.351	0.073	0.295	0.033	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	22	16	15	18	15	15	17	0
N.S.	1	1.00	1.29	0.94	0.88	1.06	0.88	0.88	1.00	0.00
time (sec)	N/A	0.006	0.000	0.000	0.664	0.992	0.057	0.388	0.024	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	20	20	18	15	20	17	0
N.S.	1	1.00	0.92	0.83	0.83	0.75	0.62	0.83	0.71	0.00
time (sec)	N/A	0.005	0.001	0.001	0.568	1.000	0.058	0.368	0.021	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	24	23	22	25	20	23	28	0
N.S.	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27	0.00
time (sec)	N/A	0.019	0.006	0.007	1.378	0.958	0.132	0.393	0.045	0.001

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	27	16	15	15	15	18	15	0
N.S.	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.013	0.004	0.007	0.755	1.408	0.130	0.327	0.057	0.001

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	13	12	16	13	0
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68	0.00
time (sec)	N/A	0.021	0.004	0.006	0.659	1.279	0.096	0.305	2.336	0.001

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	17	11	10	10	8	13	10	0
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83	0.00
time (sec)	N/A	0.006	0.004	0.001	0.621	1.396	0.089	0.398	2.269	0.001

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	11	11	10	10	8	11	10	0
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00	0.00
time (sec)	N/A	0.009	0.006	0.002	0.610	1.160	0.132	0.332	0.056	0.001

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	18	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.018	0.006	0.005	0.614	0.994	0.125	0.356	0.056	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	18	21	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91	0.00
time (sec)	N/A	0.021	0.004	0.005	1.640	1.391	0.128	0.376	0.049	0.000
Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87	0.00
time (sec)	N/A	0.009	0.004	0.000	0.655	1.272	0.092	0.278	0.042	0.000
Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	20	20	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81	0.00
time (sec)	N/A	0.021	0.005	0.006	0.586	1.534	0.131	0.369	0.067	0.000
Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	9	14	9	9	7	9	10	0
N.S.	1	1.00	0.90	1.40	0.90	0.90	0.70	0.90	1.00	0.00
time (sec)	N/A	0.010	0.005	0.003	0.781	1.190	0.083	0.261	2.204	0.001

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	20	20	17	0
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68	0.00
time (sec)	N/A	0.021	0.005	0.006	0.646	1.421	0.132	0.362	0.107	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	14	12	14	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00	0.00
time (sec)	N/A	0.025	0.004	0.001	1.557	1.728	0.105	0.252	2.219	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	14	30	11	16	17	11	20	0
N.S.	1	1.00	1.08	2.31	0.85	1.23	1.31	0.85	1.54	0.00
time (sec)	N/A	0.007	0.007	0.010	0.582	1.448	0.130	0.371	2.245	0.001

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	32	26	25	20	27	25	0
N.S.	1	1.00	0.96	1.23	1.00	0.96	0.77	1.04	0.96	0.00
time (sec)	N/A	0.045	0.008	0.003	0.717	1.513	0.154	0.283	2.234	0.001

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	4	5	4	4	3	5	4	0
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67	0.00
time (sec)	N/A	0.008	0.001	0.001	0.570	1.491	0.064	0.280	0.018	0.000
Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	20	25	19	27	18	0
N.S.	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90	0.00
time (sec)	N/A	0.015	0.005	0.007	0.718	1.653	0.105	0.381	0.039	0.001
Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	40	28	27	27	31	28	44	0
N.S.	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16	0.00
time (sec)	N/A	0.025	0.010	0.005	1.973	1.270	0.141	0.389	0.158	0.000
Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	18	16	15	15	14	16	15	0
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88	0.00
time (sec)	N/A	0.008	0.004	0.002	0.440	1.158	0.073	0.367	0.031	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	36	22	21	21	22	22	25	0
N.S.	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81	0.00
time (sec)	N/A	0.012	0.005	0.005	0.995	1.455	0.141	0.274	0.053	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	17	15	17	17	13	0
N.S.	1	1.00	1.00	0.84	0.89	0.79	0.89	0.89	0.68	0.00
time (sec)	N/A	0.017	0.006	0.006	0.474	1.301	0.114	0.408	0.090	0.001

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	9	9	9	9	8	11	9	0
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82	0.00
time (sec)	N/A	0.005	0.004	0.000	0.447	1.330	0.088	0.287	2.217	0.001

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	13	12	12	14	12	12	0
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67	0.00
time (sec)	N/A	0.014	0.009	0.006	1.008	1.476	0.143	0.371	2.216	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	44	39	46	83	46	52	45	0
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98	0.00
time (sec)	N/A	0.023	0.019	0.010	0.448	1.321	0.195	0.363	0.042	0.000
Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	10	14	8	8	7	9	8	0
N.S.	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67	0.00
time (sec)	N/A	0.002	0.001	0.002	0.438	1.394	0.076	0.275	0.042	0.000
Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	27	28	23	34	20	25	17	0
N.S.	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81	0.00
time (sec)	N/A	0.003	0.008	0.008	0.451	1.292	0.108	0.271	2.216	0.000
Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	21	12	15	17	0
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89	0.00
time (sec)	N/A	0.004	0.008	0.006	0.996	1.045	0.101	0.290	0.027	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	7	9	6	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60	0.00
time (sec)	N/A	0.001	0.001	0.001	0.453	1.393	0.058	0.382	0.068	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	20	10	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00	0.00
time (sec)	N/A	0.003	0.002	0.003	0.995	1.350	0.112	0.261	2.245	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.007	0.004	0.002	0.982	1.205	0.134	0.379	2.243	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	17	16	16	26	16	16	0
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84	0.00
time (sec)	N/A	0.011	0.006	0.002	0.955	1.440	0.108	0.380	0.032	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	17	16	17	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77	0.00
time (sec)	N/A	0.007	0.001	0.000	0.460	1.130	0.057	0.310	0.024	0.000
Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	15	16	17	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77	0.00
time (sec)	N/A	0.005	0.001	0.000	0.456	1.119	0.056	0.287	0.029	0.000
Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	15	10	14	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94	0.00
time (sec)	N/A	0.005	0.001	0.003	0.456	1.303	0.069	0.306	0.028	0.000
Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	39	30	30	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81	0.00
time (sec)	N/A	0.021	0.010	0.005	0.976	1.259	0.118	0.266	2.219	0.001

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	13	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93	0.00
time (sec)	N/A	0.004	0.000	0.001	0.466	0.969	0.054	0.364	0.024	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	7	9	6	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.64	0.82	0.55	0.82
time (sec)	N/A	0.004	0.000	0.001	0.450	1.281	0.058	0.286	0.016	0.014

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	20	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00	0.00
time (sec)	N/A	0.010	0.006	0.005	0.994	1.444	0.136	0.365	0.049	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.010	0.005	0.003	1.359	2.123	0.129	0.292	2.228	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.017	0.006	0.005	1.432	1.268	0.123	0.249	2.209	0.000
Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	7	7	8	7	12	5	8	7	0
N.S.	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78	0.00
time (sec)	N/A	0.006	0.003	0.003	0.951	1.179	0.073	0.298	0.023	0.000
Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	20	16	16	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73	0.00
time (sec)	N/A	0.011	0.013	0.007	1.579	1.192	0.149	0.292	0.055	0.001
Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	44	30	30	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81	0.00
time (sec)	N/A	0.022	0.013	0.003	1.393	1.169	0.122	0.384	0.045	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	15	19	14	16	13	0
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87	0.00
time (sec)	N/A	0.004	0.002	0.004	0.969	1.342	0.086	0.269	0.026	0.000
Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	15	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87	0.00
time (sec)	N/A	0.004	0.001	0.005	0.948	0.903	0.076	0.280	0.023	0.000
Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	13	11	8	13	11	0
N.S.	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00	0.00
time (sec)	N/A	0.011	0.003	0.005	0.692	1.303	0.095	0.287	0.056	0.001
Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	19	18	17	17	17	17	17	0
N.S.	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77	0.00
time (sec)	N/A	0.004	0.001	0.000	0.876	0.524	0.056	0.356	0.030	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	19	19	19	9	9	0
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82	0.00
time (sec)	N/A	0.001	0.001	0.000	0.862	1.463	0.057	0.216	0.142	0.000
Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	16	14	13	13	12	14	11	0
N.S.	1	1.00	1.23	1.08	1.00	1.00	0.92	1.08	0.85	0.00
time (sec)	N/A	0.010	0.004	0.002	0.643	1.003	0.076	0.361	2.211	0.000
Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	18	14	16	12	0
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75	0.00
time (sec)	N/A	0.006	0.003	0.007	0.588	1.316	0.100	0.360	0.034	0.000
Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	24	25	39	24	27	23	0
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85	0.00
time (sec)	N/A	0.011	0.016	0.008	0.815	1.210	0.111	0.379	2.228	0.001

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	18	16	15	15	12	16	15	0
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88	0.00
time (sec)	N/A	0.008	0.003	0.002	0.740	1.578	0.073	0.357	0.026	0.000
Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	20	15	17	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89	0.00
time (sec)	N/A	0.006	0.001	0.005	0.728	1.031	0.078	0.371	0.029	0.000
Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	28	14	0	14	14	13	0
N.S.	1	1.00	1.00	1.56	0.78	0.00	0.78	0.78	0.72	0.00
time (sec)	N/A	0.010	0.001	0.001	1.708	0.000	0.057	0.351	0.024	0.000
Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	34	46	31	25	23	0
N.S.	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00	0.00
time (sec)	N/A	0.018	0.010	0.006	0.738	1.137	0.109	0.369	0.032	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	12	13	12	15	10	32	12	0
N.S.	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75	0.00
time (sec)	N/A	0.012	0.006	0.004	1.789	0.870	0.107	0.262	2.219	0.001
Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	30	26	25	25	24	26	25	0
N.S.	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86	0.00
time (sec)	N/A	0.020	0.008	0.001	0.765	1.409	0.080	0.283	0.026	0.001
Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	10	12	13	21
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	1.31
time (sec)	N/A	0.006	0.000	0.000	0.891	1.242	0.059	0.348	0.021	0.021
Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	17	25	14	12	17	0
N.S.	1	1.00	1.00	0.94	1.00	1.47	0.82	0.71	1.00	0.00
time (sec)	N/A	0.015	0.012	0.006	1.745	1.241	0.124	0.237	0.031	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	43	68	190	151	48	42	0
N.S.	1	1.00	1.00	0.91	1.45	4.04	3.21	1.02	0.89	0.00
time (sec)	N/A	0.066	0.025	0.006	1.021	1.325	0.296	0.281	0.097	0.001

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	61	61	0	240	294	60	82	0
N.S.	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44	0.00
time (sec)	N/A	0.082	0.027	0.004	0.000	0.773	0.350	0.358	2.227	0.001

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	39	308	0	247	24	147	101	0
N.S.	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54	0.00
time (sec)	N/A	0.189	0.031	0.102	0.000	1.509	0.517	1.488	2.300	0.001

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	27	28	65	65	60	30	27	0
N.S.	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45	0.00
time (sec)	N/A	0.136	0.015	0.007	1.249	1.287	0.229	0.397	2.337	0.001

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	B	B	B	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	0	27	250	65	65	60	197	27	0
N.S.	1	0.00	1.00	9.26	2.41	2.41	2.22	7.30	1.00	0.00
time (sec)	N/A	0.311	0.010	0.030	1.199	1.293	0.324	0.397	0.045	0.003

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	B	B	B	B	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	0	27	112	65	65	60	111	27	0
N.S.	1	0.00	1.00	4.15	2.41	2.41	2.22	4.11	1.00	0.00
time (sec)	N/A	0.433	0.010	0.019	0.975	1.357	0.288	0.447	0.043	0.002

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [408] had the largest ratio of [.8824]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154
25	A	5	5	1.00	11	0.454
26	A	7	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	2	1.00	29	0.069
30	A	2	1	1.00	29	0.034
31	A	2	1	1.00	29	0.034
32	A	1	0	1.00	27	0.000
33	A	10	6	1.00	29	0.207
34	A	11	7	1.00	29	0.241
35	A	3	2	1.00	32	0.062
36	A	2	1	1.00	32	0.031
37	A	2	1	1.00	32	0.031
38	A	1	0	1.00	30	0.000
39	A	4	3	1.00	32	0.094
40	A	5	4	1.00	32	0.125
41	A	2	1	1.00	17	0.059
42	A	2	1	1.00	17	0.059
43	A	2	1	1.00	17	0.059
44	A	1	0	1.00	15	0.000
45	A	16	9	1.00	17	0.529
46	A	18	11	1.00	17	0.647
47	A	2	1	1.00	17	0.059
48	A	2	1	1.00	17	0.059
49	A	2	1	1.00	17	0.059
50	A	1	0	1.00	15	0.000
51	A	15	9	1.00	17	0.529
52	A	17	11	1.00	17	0.647
53	A	2	1	1.00	22	0.045
54	A	2	1	1.00	22	0.045
55	A	2	1	1.00	22	0.045
56	A	1	0	1.00	20	0.000
57	A	16	9	1.00	22	0.409
58	A	18	11	1.00	22	0.500
59	A	2	2	1.00	51	0.039
60	A	2	2	1.00	51	0.039

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	B	1	0	4.36	49	0.000
62	A	2	2	1.00	51	0.039
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	A	6	5	1.00	13	0.385
66	A	5	3	1.00	19	0.158
67	A	5	3	1.00	19	0.158
68	A	5	3	1.00	19	0.158
69	A	1	0	1.00	17	0.000
70	A	5	2	1.00	19	0.105
71	A	7	3	1.00	19	0.158
72	A	10	3	1.00	19	0.158
73	B	15	7	2.25	17	0.412
74	A	6	5	1.00	15	0.333
75	A	6	5	1.00	15	0.333
76	A	4	4	1.00	13	0.308
77	A	2	2	1.00	11	0.182
78	A	6	6	1.00	15	0.400
79	A	7	6	1.00	15	0.400
80	A	7	6	1.00	15	0.400
81	A	2	2	1.00	13	0.154
82	A	3	3	1.00	13	0.231
83	A	4	3	1.00	13	0.231
84	A	2	2	1.00	19	0.105
85	A	2	2	1.00	11	0.182
86	A	3	3	1.00	11	0.273
87	A	4	3	1.00	11	0.273
88	A	2	2	1.00	13	0.154
89	A	3	3	1.00	13	0.231
90	A	4	3	1.00	13	0.231
91	A	2	2	1.00	11	0.182
92	A	3	3	1.00	11	0.273
93	A	4	3	1.00	11	0.273
94	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	4	3	1.00	13	0.231
96	A	4	4	1.00	17	0.235
97	A	4	4	1.00	19	0.210
98	A	4	4	1.00	17	0.235
99	A	11	10	1.00	17	0.588
100	A	9	9	1.00	17	0.529
101	A	7	7	1.00	15	0.467
102	A	7	7	1.00	13	0.538
103	A	11	10	1.06	17	0.588
104	A	11	10	0.99	17	0.588
105	A	11	10	1.00	17	0.588
106	A	16	12	1.00	17	0.706
107	A	14	10	1.00	17	0.588
108	A	14	10	1.00	15	0.667
109	A	10	7	1.00	13	0.538
110	A	18	13	1.00	17	0.765
111	A	18	13	1.00	17	0.765
112	A	3	2	1.00	22	0.091
113	A	2	1	1.00	22	0.045
114	A	2	1	1.00	22	0.045
115	A	1	0	1.00	20	0.000
116	A	4	3	1.00	22	0.136
117	A	5	4	1.00	22	0.182
118	A	6	5	1.00	22	0.227
119	A	2	1	1.00	24	0.042
120	A	2	1	1.00	24	0.042
121	A	2	1	1.00	24	0.042
122	A	2	1	1.00	22	0.045
123	A	8	7	1.00	24	0.292
124	A	10	9	1.00	24	0.375
125	A	12	10	1.00	24	0.417
126	A	2	1	1.00	26	0.038
127	A	2	1	1.00	26	0.038
128	A	2	1	1.00	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	1	1.00	24	0.042
130	A	9	8	1.00	26	0.308
131	A	11	10	1.00	26	0.385
132	A	14	5	1.00	46	0.109
133	A	14	5	1.00	46	0.109
134	A	8	3	1.00	46	0.065
135	A	14	5	1.00	44	0.114
136	A	14	5	1.00	42	0.119
137	A	14	5	1.00	46	0.109
138	A	14	5	0.99	46	0.109
139	A	14	6	1.00	26	0.231
140	A	14	6	1.00	26	0.231
141	A	14	6	1.00	26	0.231
142	A	8	4	1.00	26	0.154
143	A	14	6	1.00	24	0.250
144	A	14	6	1.00	22	0.273
145	A	14	6	1.00	26	0.231
146	A	14	6	1.00	26	0.231
147	A	23	7	1.00	26	0.269
148	A	23	7	1.00	26	0.269
149	A	14	6	1.00	26	0.231
150	A	17	5	1.00	26	0.192
151	A	23	7	1.00	26	0.269
152	A	23	7	1.00	26	0.269
153	A	23	7	1.00	26	0.269
154	A	2	1	1.00	52	0.019
155	A	4	3	1.00	52	0.058
156	A	1	1	1.00	18	0.056
157	A	3	3	1.00	23	0.130
158	A	3	3	1.00	23	0.130
159	A	3	3	1.00	29	0.103
160	A	1	1	1.00	18	0.056
161	A	4	3	1.00	20	0.150
162	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
163	A	4	3	1.00	22	0.136
164	A	1	1	1.00	18	0.056
165	A	3	3	1.00	23	0.130
166	A	3	3	1.00	23	0.130
167	A	3	3	1.00	29	0.103
168	A	1	1	1.00	18	0.056
169	A	1	1	1.00	25	0.040
170	C	7	3	4.30	38	0.079
171	A	1	1	1.00	27	0.037
172	A	1	1	1.00	31	0.032
173	A	2	1	1.00	54	0.019
174	A	3	1	1.00	54	0.019
175	A	5	4	1.00	54	0.074
176	A	1	1	1.00	30	0.033
177	A	1	1	1.00	29	0.034
178	A	1	1	1.00	28	0.036
179	A	1	1	1.00	21	0.048
180	A	1	1	1.00	20	0.050
181	A	1	1	1.00	21	0.048
182	A	1	1	1.00	26	0.038
183	A	1	1	1.00	25	0.040
184	A	2	2	1.00	26	0.077
185	A	1	1	1.00	24	0.042
186	A	1	1	1.00	24	0.042
187	A	1	1	1.00	23	0.043
188	A	1	1	1.00	30	0.033
189	A	1	1	1.00	29	0.034
190	A	1	1	1.00	28	0.036
191	A	1	1	1.00	21	0.048
192	A	1	1	1.00	20	0.050
193	A	2	2	1.00	21	0.095
194	A	1	1	1.00	26	0.038
195	A	1	1	1.00	25	0.040
196	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	1	1	1.00	22	0.045
198	A	1	1	1.00	24	0.042
199	A	2	2	1.00	23	0.087
200	A	1	1	1.00	23	0.043
201	A	1	1	1.00	19	0.053
202	A	2	1	1.00	22	0.045
203	A	2	1	1.00	23	0.043
204	A	2	1	1.00	22	0.045
205	A	2	1	1.00	23	0.043
206	A	2	1	1.00	24	0.042
207	A	2	1	1.00	25	0.040
208	A	2	1	1.00	31	0.032
209	A	2	1	1.00	32	0.031
210	A	2	1	1.00	35	0.029
211	A	2	1	1.00	36	0.028
212	A	2	1	1.00	24	0.042
213	A	2	1	1.00	31	0.032
214	A	2	1	1.00	35	0.029
215	A	1	1	1.00	22	0.045
216	A	1	1	1.00	18	0.056
217	B	3	2	2.91	26	0.077
218	B	2	1	2.91	28	0.036
219	A	1	1	1.00	18	0.056
220	A	1	1	1.00	20	0.050
221	A	1	1	1.00	21	0.048
222	A	3	3	1.00	52	0.058
223	A	9	5	1.00	38	0.132
224	A	3	2	1.00	32	0.062
225	A	4	3	1.00	33	0.091
226	A	3	2	1.00	34	0.059
227	A	6	6	1.00	43	0.140
228	A	1	1	1.00	16	0.062
229	B	15	7	2.25	25	0.280
230	A	1	1	1.00	56	0.018

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
231	A	1	1	1.00	51	0.020
232	A	1	1	1.00	49	0.020
233	A	1	1	1.00	46	0.022
234	A	2	2	1.00	48	0.042
235	A	1	1	1.00	49	0.020
236	A	1	1	1.00	48	0.021
237	A	1	1	1.00	48	0.021
238	A	10	5	1.00	35	0.143
239	A	10	5	1.00	35	0.143
240	A	10	5	1.00	35	0.143
241	A	10	5	1.00	33	0.152
242	A	10	5	1.00	32	0.156
243	A	13	6	1.00	35	0.171
244	A	13	6	1.00	35	0.171
245	A	13	6	1.00	35	0.171
246	A	13	6	1.00	35	0.171
247	A	13	6	1.00	35	0.171
248	A	11	6	1.00	33	0.182
249	A	9	5	1.00	32	0.156
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	13	6	1.00	35	0.171
253	A	2	2	1.00	40	0.050
254	A	6	5	1.00	20	0.250
255	A	3	2	1.00	20	0.100
256	A	3	2	1.00	20	0.100
257	A	2	1	1.00	16	0.062
258	A	5	4	1.00	22	0.182
259	A	3	2	1.00	21	0.095
260	A	6	5	1.00	26	0.192
261	A	2	1	1.00	20	0.050
262	A	2	1	1.00	11	0.091
263	A	4	3	1.00	22	0.136
264	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	3	2	1.00	25	0.080
266	A	6	6	1.00	22	0.273
267	A	5	5	1.00	31	0.161
268	A	3	2	1.00	21	0.095
269	A	4	3	1.00	33	0.091
270	A	4	4	1.00	14	0.286
271	A	7	6	1.00	33	0.182
272	A	7	6	1.00	29	0.207
273	A	6	5	1.00	44	0.114
274	A	3	2	1.00	15	0.133
275	A	5	4	1.00	15	0.267
276	A	4	3	1.00	18	0.167
277	A	3	2	1.00	20	0.100
278	A	5	4	1.00	26	0.154
279	A	3	2	1.00	13	0.154
280	A	3	2	1.00	18	0.111
281	A	2	1	1.00	26	0.038
282	A	5	5	1.00	19	0.263
283	A	5	5	1.00	24	0.208
284	A	8	6	1.00	20	0.300
285	A	8	6	1.00	18	0.333
286	A	5	3	1.00	19	0.158
287	A	4	3	1.00	13	0.231
288	A	6	5	1.00	22	0.227
289	A	6	5	1.00	24	0.208
290	A	2	1	1.00	29	0.034
291	A	2	1	1.00	30	0.033
292	A	2	1	1.00	19	0.053
293	A	4	4	1.00	16	0.250
294	A	10	5	1.00	36	0.139
295	A	2	1	1.00	21	0.048
296	A	5	4	1.00	16	0.250
297	A	2	1	1.00	24	0.042
298	A	2	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
299	A	2	1	1.00	24	0.042
300	A	6	5	1.00	26	0.192
301	A	3	2	1.00	25	0.080
302	A	2	1	1.00	29	0.034
303	A	6	5	1.00	20	0.250
304	A	14	10	1.00	32	0.312
305	A	4	4	1.00	23	0.174
306	A	6	5	1.00	26	0.192
307	A	4	3	1.00	26	0.115
308	A	8	4	1.00	25	0.160
309	A	6	3	1.00	23	0.130
310	A	7	6	1.00	23	0.261
311	A	5	3	1.18	20	0.150
312	A	3	2	1.00	25	0.080
313	A	3	2	1.00	22	0.091
314	A	2	1	1.00	24	0.042
315	A	7	6	1.00	24	0.250
316	A	6	5	1.00	43	0.116
317	A	7	5	1.25	50	0.100
318	A	3	2	1.00	16	0.125
319	A	6	5	1.00	15	0.333
320	A	5	4	1.00	20	0.200
321	A	3	2	1.00	24	0.083
322	A	5	3	1.00	27	0.111
323	A	5	5	1.00	26	0.192
324	A	3	2	1.00	16	0.125
325	A	11	8	1.32	22	0.364
326	A	5	4	1.00	24	0.167
327	A	4	3	1.00	26	0.115
328	A	5	3	1.00	36	0.083
329	A	4	3	1.00	26	0.115
330	A	10	9	1.00	20	0.450
331	A	6	6	1.00	27	0.222
332	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	8	7	1.00	25	0.280
334	A	8	7	1.00	22	0.318
335	A	2	1	1.00	18	0.056
336	A	5	4	1.00	20	0.200
337	A	10	9	1.00	20	0.450
338	A	16	12	1.00	20	0.600
339	A	3	2	1.00	14	0.143
340	A	2	2	1.00	20	0.100
341	A	14	8	1.00	20	0.400
342	A	4	3	1.00	26	0.115
343	A	4	3	1.00	24	0.125
344	A	6	4	1.00	30	0.133
345	A	4	4	1.00	21	0.190
346	A	2	1	1.00	15	0.067
347	A	4	3	1.00	18	0.167
348	A	3	2	1.00	22	0.091
349	A	3	2	1.00	16	0.125
350	A	6	5	1.00	16	0.312
351	A	3	2	1.00	25	0.080
352	A	3	2	1.00	19	0.105
353	A	2	1	1.00	23	0.043
354	A	5	4	1.00	23	0.174
355	A	5	4	1.00	21	0.190
356	A	10	6	1.00	28	0.214
357	A	2	1	1.00	24	0.042
358	A	6	5	1.00	26	0.192
359	A	2	1	1.00	14	0.071
360	A	5	3	1.00	16	0.188
361	A	5	5	1.00	16	0.312
362	A	7	5	1.00	43	0.116
363	A	17	13	1.00	26	0.500
364	A	18	13	1.00	16	0.812
365	A	3	2	1.00	15	0.133
366	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	3	2	1.00	17	0.118
368	A	3	2	1.00	15	0.133
369	A	4	3	1.00	15	0.200
370	A	4	3	1.00	18	0.167
371	A	3	2	1.00	20	0.100
372	A	7	6	1.00	29	0.207
373	A	4	3	1.00	22	0.136
374	A	6	4	1.00	16	0.250
375	A	6	4	1.00	16	0.250
376	A	5	4	1.00	14	0.286
377	A	4	3	1.00	12	0.250
378	A	4	3	1.00	16	0.188
379	A	6	5	1.00	16	0.312
380	A	4	3	1.00	16	0.188
381	A	4	3	1.00	16	0.188
382	A	4	3	1.00	16	0.188
383	A	8	5	1.00	17	0.294
384	A	8	5	1.00	19	0.263
385	A	8	5	1.00	15	0.333
386	A	8	5	1.00	17	0.294
387	A	16	10	1.00	23	0.435
388	A	17	10	1.00	21	0.476
389	F	0	0	N/A	0	N/A
390	A	15	11	1.00	17	0.647
391	A	13	9	1.00	17	0.529
392	A	13	9	1.00	15	0.600
393	A	9	6	1.00	9	0.667
394	A	17	12	1.00	17	0.706
395	A	17	12	1.00	17	0.706
396	A	17	12	1.00	17	0.706
397	A	16	12	1.00	17	0.706
398	A	14	10	1.00	17	0.588
399	A	14	10	1.00	15	0.667
400	A	10	7	1.00	9	0.778

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
401	A	31	14	1.00	17	0.824
402	A	31	14	1.00	17	0.824
403	A	31	14	1.00	17	0.824
404	A	15	11	1.00	17	0.647
405	A	15	10	1.00	17	0.588
406	A	15	10	1.00	15	0.667
407	A	11	7	1.00	9	0.778
408	A	46	15	1.00	17	0.882
409	A	46	15	1.00	17	0.882
410	A	46	15	1.00	17	0.882
411	A	4	4	1.00	14	0.286
412	A	5	5	1.00	13	0.385
413	A	4	4	1.00	16	0.250
414	A	5	5	1.00	18	0.278
415	A	3	2	1.00	14	0.143
416	A	4	3	1.00	16	0.188
417	A	2	2	1.00	11	0.182
418	A	1	0	1.00	17	0.000
419	A	1	1	1.00	11	0.091
420	B	1	0	6.23	73	0.000
421	A	11	7	1.00	13	0.538
422	A	13	9	1.00	19	0.474
423	A	3	2	1.00	15	0.133
424	A	3	2	1.00	11	0.182
425	A	1	0	1.00	11	0.000
426	A	1	0	1.00	11	0.000
427	A	5	4	1.00	16	0.250
428	A	2	1	1.00	16	0.062
429	A	4	3	1.00	15	0.200
430	A	1	1	1.00	15	0.067
431	A	1	1	1.00	20	0.050
432	A	3	2	1.00	15	0.133
433	A	6	5	1.00	13	0.385
434	A	1	1	1.00	22	0.045

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	3	2	1.00	18	0.111
436	A	3	3	1.00	20	0.150
437	A	3	2	1.00	16	0.125
438	A	6	6	1.00	17	0.353
439	A	1	1	1.00	17	0.059
440	A	4	3	1.00	25	0.120
441	A	2	2	1.00	20	0.100
442	A	3	2	1.00	18	0.111
443	A	6	5	1.00	15	0.333
444	A	2	1	1.00	11	0.091
445	A	5	5	1.00	13	0.385
446	A	3	2	1.00	20	0.100
447	A	2	1	1.00	16	0.062
448	A	4	3	1.00	16	0.188
449	A	2	1	1.00	16	0.062
450	A	1	1	1.00	9	0.111
451	A	2	2	1.00	7	0.286
452	A	2	2	1.00	11	0.182
453	A	1	1	1.00	7	0.143
454	A	1	1	1.00	9	0.111
455	A	1	1	1.00	9	0.111
456	A	2	2	1.00	10	0.200
457	A	2	1	1.00	13	0.077
458	A	2	1	1.00	11	0.091
459	A	2	1	1.00	14	0.071
460	A	4	4	1.00	16	0.250
461	A	2	1	1.00	7	0.143
462	A	2	1	1.00	11	0.091
463	A	5	5	1.00	13	0.385
464	A	5	5	1.00	13	0.385
465	A	5	4	1.00	14	0.286
466	A	2	1	0.78	13	0.077
467	A	3	2	1.00	20	0.100
468	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	2	1	1.00	12	0.083
470	A	2	1	1.00	10	0.100
471	A	3	2	1.00	16	0.125
472	A	2	1	1.00	11	0.091
473	A	1	1	1.00	7	0.143
474	A	2	1	1.00	15	0.067
475	A	2	1	1.00	12	0.083
476	A	3	3	1.00	16	0.188
477	A	2	1	1.00	11	0.091
478	A	2	1	1.00	17	0.059
479	A	2	1	1.00	29	0.034
480	A	2	1	1.00	18	0.056
481	A	3	2	1.00	14	0.143
482	A	2	1	1.00	24	0.042
483	A	2	1	1.00	11	0.091
484	A	3	2	1.00	18	0.111
485	A	3	3	1.00	15	0.200
486	A	3	3	1.00	16	0.188
487	A	10	7	1.00	15	0.467
488	A	5	2	1.00	50	0.040
489	F	0	0	N/A	0	N/A
490	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

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3.42	$\int (8 + 8x - x^3 + 8x^4)^3 dx$	408
3.43	$\int (8 + 8x - x^3 + 8x^4)^2 dx$	411
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3.45	$\int \frac{1}{8+8x-x^3+8x^4} dx$	417
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3.62	$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5} dx$	505
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3.65	$\int \frac{1}{1+x^2+x^3+x^5} dx$	516
3.66	$\int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx$	520
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3.69	$\int (3 - 19x^2 + 32x^4 - 16x^6) dx$	532
3.70	$\int \frac{1}{3-19x^2+32x^4-16x^6} dx$	535
3.71	$\int \frac{1}{(3-19x^2+32x^4-16x^6)^2} dx$	538
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3.75	$\int \frac{x^2}{c+(a+bx)^2} dx$	555

3.76	$\int \frac{x}{c+(a+bx)^2} dx$	559
3.77	$\int \frac{1}{c+(a+bx)^2} dx$	563
3.78	$\int \frac{1}{x(c+(a+bx)^2)} dx$	566
3.79	$\int \frac{1}{x^2(c+(a+bx)^2)} dx$	571
3.80	$\int \frac{1}{x^3(c+(a+bx)^2)} dx$	577
3.81	$\int \frac{1}{a+b(c+dx)^2} dx$	584
3.82	$\int \frac{1}{(a+b(c+dx)^2)^2} dx$	587
3.83	$\int \frac{1}{(a+b(c+dx)^2)^3} dx$	591
3.84	$\int \frac{1}{\sqrt{-a+b(c+dx)^2}} dx$	595
3.85	$\int \frac{1}{1+(c+dx)^2} dx$	599
3.86	$\int \frac{1}{(1+(c+dx)^2)^2} dx$	602
3.87	$\int \frac{1}{(1+(c+dx)^2)^3} dx$	606
3.88	$\int \frac{1}{1-(c+dx)^2} dx$	610
3.89	$\int \frac{1}{(1-(c+dx)^2)^2} dx$	613
3.90	$\int \frac{1}{(1-(c+dx)^2)^3} dx$	617
3.91	$\int \frac{1}{1-(1+x)^2} dx$	621
3.92	$\int \frac{1}{(1-(1+x)^2)^2} dx$	624
3.93	$\int \frac{1}{(1-(1+x)^2)^3} dx$	628
3.94	$\int \frac{(1+(a+bx)^2)^2}{x^2} dx$	632
3.95	$\int \frac{x^2}{1+(-1+x)^2} dx$	635
3.96	$\int \frac{x^2}{\sqrt{1-(1+x)^2}} dx$	638
3.97	$\int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$	642
3.98	$\int \frac{x^2}{\sqrt{1+(a+bx)^2}} dx$	646
3.99	$\int \frac{x^3}{a+b(c+dx)^3} dx$	650
3.100	$\int \frac{x^2}{a+b(c+dx)^3} dx$	658

3.101	$\int \frac{x}{a+b(c+dx)^3} dx$	665
3.102	$\int \frac{1}{a+b(c+dx)^3} dx$	671
3.103	$\int \frac{1}{x(a+b(c+dx)^3)} dx$	676
3.104	$\int \frac{1}{x^2(a+b(c+dx)^3)} dx$	683
3.105	$\int \frac{1}{x^3(a+b(c+dx)^3)} dx$	693
3.106	$\int \frac{x^3}{a+b(c+dx)^4} dx$	706
3.107	$\int \frac{x^2}{a+b(c+dx)^4} dx$	713
3.108	$\int \frac{x}{a+b(c+dx)^4} dx$	718
3.109	$\int \frac{1}{a+b(c+dx)^4} dx$	723
3.110	$\int \frac{1}{x(a+b(c+dx)^4)} dx$	728
3.111	$\int \frac{1}{x^2(a+b(c+dx)^4)} dx$	735
3.112	$\int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$	743
3.113	$\int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$	747
3.114	$\int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$	750
3.115	$\int (a + 8x - 8x^2 + 4x^3 - x^4) dx$	753
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3.117	$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} dx$	762
3.118	$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3} dx$	776
3.119	$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$	803
3.120	$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$	807
3.121	$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$	810
3.122	$\int x (a + 8x - 8x^2 + 4x^3 - x^4) dx$	813
3.123	$\int \frac{x}{a+8x-8x^2+4x^3-x^4} dx$	816
3.124	$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} dx$	821
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3.129	$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx$	846
3.130	$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4} dx$	849
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3.280	$\int \frac{2+2x+x^4}{x^4+x^5} dx$.1460
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3.284	$\int \frac{3+4x}{(1+x^2)(2+x^2)} dx$.1474
3.285	$\int \frac{2+x}{(1+x^2)(4+x^2)} dx$.1478

3.286	$\int \frac{2-x+x^3}{-7-6x+x^2} dx$.1482
3.287	$\int \frac{-1+x^5}{-1+x^2} dx$.1485
3.288	$\int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$.1488
3.289	$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} dx$.1492
3.290	$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx$.1496
3.291	$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$.1499
3.292	$\int \frac{2+x^2}{(-1+x)^2x(1+x)} dx$.1502
3.293	$\int \frac{3+x^2+x^3}{(2+x^2)^2} dx$.1505
3.294	$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)} dx$.1509
3.295	$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx$.1513
3.296	$\int \frac{x^4}{(-1+x)(2+x^2)} dx$.1516
3.297	$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx$.1520
3.298	$\int \frac{1+2x}{-1+3x-3x^2+x^3} dx$.1523
3.299	$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx$.1526
3.300	$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$.1529
3.301	$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$.1533
3.302	$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx$.1536
3.303	$\int \frac{4-x+2x^2}{4x+x^3} dx$.1539
3.304	$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx$.1543
3.305	$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx$.1548
3.306	$\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$.1552
3.307	$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$.1556
3.308	$\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx$.1560
3.309	$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$.1564
3.310	$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$.1568
3.311	$\int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx$.1572

- 3.312 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx \dots\dots\dots .1575$
- 3.313 $\int \frac{-2+3x+5x^2}{2x^2+x^3} dx \dots\dots\dots .1578$
- 3.314 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx \dots\dots\dots .1581$
- 3.315 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx \dots\dots\dots .1584$
- 3.316 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx \dots\dots\dots .1588$
- 3.317 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx \dots\dots\dots .1592$
- 3.318 $\int \frac{9+x^4}{x^2(9+x^2)} dx \dots\dots\dots .1596$
- 3.319 $\int \frac{2x+x^4}{1+x^2} dx \dots\dots\dots .1599$
- 3.320 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx \dots\dots\dots .1603$
- 3.321 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx \dots\dots\dots .1606$
- 3.322 $\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx \dots\dots\dots .1609$
- 3.323 $\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx \dots\dots\dots .1613$
- 3.324 $\int \frac{(-1+x)^4 x^4}{1+x^2} dx \dots\dots\dots .1617$
- 3.325 $\int \frac{-20x+4x^2}{9-10x^2+x^4} dx \dots\dots\dots .1620$
- 3.326 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx \dots\dots\dots .1624$
- 3.327 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx \dots\dots\dots .1628$
- 3.328 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} dx \dots\dots\dots .1632$
- 3.329 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx \dots\dots\dots .1636$
- 3.330 $\int \frac{x^2(c+dx)^2}{a+bx^3} dx \dots\dots\dots .1640$
- 3.331 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx \dots\dots\dots .1648$
- 3.332 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx \dots\dots\dots .1652$
- 3.333 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx \dots\dots\dots .1657$
- 3.334 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx \dots\dots\dots .1665$
- 3.335 $\int \frac{x^2}{(a+bx)(c+dx)} dx \dots\dots\dots .1672$
- 3.336 $\int \frac{x^2}{(c+dx)(a+bx^2)} dx \dots\dots\dots .1675$
- 3.337 $\int \frac{x^2}{(c+dx)(a+bx^3)} dx \dots\dots\dots .1679$

3.338	$\int \frac{x^2}{(c+dx)(a+bx^4)} dx$.1687
3.339	$\int \frac{x}{(1-x)(1+x)^2} dx$.1693
3.340	$\int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$.1696
3.341	$\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$.1699
3.342	$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx$.1704
3.343	$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$.1707
3.344	$\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} dx$.1710
3.345	$\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$.1714
3.346	$\int \frac{-3+x+x^2}{(-3+x)x^2} dx$.1718
3.347	$\int \frac{1+x+4x^2}{x+4x^3} dx$.1721
3.348	$\int \frac{1-x+3x^2}{-x^2+x^3} dx$.1724
3.349	$\int \frac{4+3x+x^2}{x+x^2} dx$.1727
3.350	$\int \frac{4+x+3x^2}{x+x^3} dx$.1730
3.351	$\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx$.1734
3.352	$\int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$.1737
3.353	$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx$.1740
3.354	$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$.1743
3.355	$\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} dx$.1746
3.356	$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} dx$.1750
3.357	$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} dx$.1754
3.358	$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx$.1757
3.359	$\int \frac{-1+x^3}{1+x+x^2} dx$.1761
3.360	$\int \frac{-3+x^3}{-7-6x+x^2} dx$.1764
3.361	$\int \frac{1+x^3}{(13+4x+x^2)^2} dx$.1767
3.362	$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$.1771

3.363	$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} dx$.1775
3.364	$\int \frac{1+x^3+x^6}{x+x^5} dx$.1781
3.365	$\int \frac{1+x^2}{-x+x^2} dx$.1788
3.366	$\int \frac{1+x^3}{-x+x^3} dx$.1791
3.367	$\int \frac{1+x^3}{-x^2+x^3} dx$.1794
3.368	$\int \frac{-1+x^5}{-x+x^3} dx$.1797
3.369	$\int \frac{1+x^4}{x^3+x^5} dx$.1800
3.370	$\int \frac{1+x^2}{x+2x^2+x^3} dx$.1804
3.371	$\int \frac{1+x^5}{-10x-3x^2+x^3} dx$.1808
3.372	$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$.1811
3.373	$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$.1815
3.374	$\int \frac{x^3}{13+\frac{2}{x}+15x} dx$.1818
3.375	$\int \frac{x^2}{13+\frac{2}{x}+15x} dx$.1822
3.376	$\int \frac{\frac{x}{x}}{13+\frac{2}{x}+15x} dx$.1826
3.377	$\int \frac{1}{13+\frac{2}{x}+15x} dx$.1830
3.378	$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)} dx$.1833
3.379	$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)} dx$.1836
3.380	$\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)} dx$.1840
3.381	$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)} dx$.1844
3.382	$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)} dx$.1848
3.383	$\int \frac{x^2}{2-(1+x^2)^4} dx$.1852
3.384	$\int \frac{x^2}{2-(1-x^2)^4} dx$.1857
3.385	$\int \frac{x^2}{2+(1+x^2)^4} dx$.1862
3.386	$\int \frac{x^2}{2+(1-x^2)^4} dx$.1868

3.387	$\int \frac{1-x^2}{a+b(1-x^2)^4} dx$.1874
3.388	$\int \frac{1-x^2}{a+b(-1+x^2)^4} dx$.1880
3.389	$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$.1886
3.390	$\int \frac{(d+ex)^3}{a+cx^4} dx$.1893
3.391	$\int \frac{(d+ex)^2}{a+cx^4} dx$.1899
3.392	$\int \frac{d+ex}{a+cx^4} dx$.1905
3.393	$\int \frac{1}{a+cx^4} dx$.1910
3.394	$\int \frac{1}{(d+ex)(a+cx^4)} dx$.1915
3.395	$\int \frac{1}{(d+ex)^2(a+cx^4)} dx$.1922
3.396	$\int \frac{1}{(d+ex)^3(a+cx^4)} dx$.1930
3.397	$\int \frac{(d+ex)^3}{(a+cx^4)^2} dx$.1939
3.398	$\int \frac{(d+ex)^2}{(a+cx^4)^2} dx$.1946
3.399	$\int \frac{d+ex}{(a+cx^4)^2} dx$.1952
3.400	$\int \frac{1}{(a+cx^4)^2} dx$.1958
3.401	$\int \frac{1}{(d+ex)(a+cx^4)^2} dx$.1963
3.402	$\int \frac{1}{(d+ex)^2(a+cx^4)^2} dx$.1971
3.403	$\int \frac{1}{(d+ex)^3(a+cx^4)^2} dx$.1980
3.404	$\int \frac{(d+ex)^3}{(a+cx^4)^3} dx$.1992
3.405	$\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$.1999
3.406	$\int \frac{d+ex}{(a+cx^4)^3} dx$.2006
3.407	$\int \frac{1}{(a+cx^4)^3} dx$.2012
3.408	$\int \frac{1}{(d+ex)(a+cx^4)^3} dx$.2018
3.409	$\int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$.2029

3.410	$\int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$.2041
3.411	$\int \frac{-1+x}{1-x+x^2} dx$.2056
3.412	$\int \frac{-1+x^2}{1+x^3} dx$.2060
3.413	$\int \frac{-4+3x}{4-2x+x^2} dx$.2064
3.414	$\int \frac{-8+2x+3x^2}{8+x^3} dx$.2068
3.415	$\int \frac{2+x}{-1+2x+x^2} dx$.2072
3.416	$\int \frac{-4+x^2}{2-5x+x^3} dx$.2075
3.417	$\int \frac{2}{-1+4x^2} dx$.2079
3.418	$\int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx$.2082
3.419	$\int \frac{x}{(1-x^2)^5} dx$.2085
3.420	$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2} \right) dx$.2088
3.421	$\int \frac{1+x^6}{-1+x^6} dx$.2091
3.422	$\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3} dx$.2095
3.423	$\int \frac{-x+x^3}{6+2x} dx$.2100
3.424	$\int \frac{x+x^3}{-1+x} dx$.2103
3.425	$\int (ac + (bc + d)x) dx$.2106
3.426	$\int (dx + c(a + bx)) dx$.2109
3.427	$\int \frac{4+4x}{x^2(1+x^2)} dx$.2112
3.428	$\int \frac{24+8x}{x(-4+x^2)} dx$.2116
3.429	$\int \frac{-1+x^2}{-2x+x^3} dx$.2119
3.430	$\int \frac{1+x^2}{3x+x^3} dx$.2123
3.431	$\int \frac{a+3bx^2}{ax+bx^3} dx$.2126
3.432	$\int \frac{-2+4x}{-x+x^3} dx$.2129
3.433	$\int \frac{4+x}{4x+x^3} dx$.2132
3.434	$\int \frac{-x+2x^3}{1-x^2+x^4} dx$.2136
3.435	$\int \frac{-3+x}{2x+3x^2+x^3} dx$.2139
3.436	$\int \frac{2+4x}{x^2+2x^3+x^4} dx$.2142
3.437	$\int \frac{1+x}{-6x+x^2+x^3} dx$.2145

3.438	$\int \frac{4x^2+x^3}{x+x^3} dx$2148
3.439	$\int \frac{x+2x^3}{(x^2+x^4)^3} dx$2152
3.440	$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$2155
3.441	$\int \frac{x+x^2}{-2x-x^2+x^3} dx$2158
3.442	$\int \frac{1-5x^2}{x^3(1+x^2)} dx$2161
3.443	$\int \frac{2x}{(-1+x)(5+x^2)} dx$2164
3.444	$\int \frac{2+x^2}{2+x} dx$2168
3.445	$\int \frac{1}{(-3+x)(4+x^2)} dx$2171
3.446	$\int \frac{-2+3x^6}{x(5+2x^6)} dx$2175
3.447	$\int \frac{3+2x}{(-2+x)(5+x)} dx$2178
3.448	$\int \frac{x^4}{4+5x^2+x^4} dx$2181
3.449	$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$2184
3.450	$\int \frac{x}{-1+x^2} dx$2187
3.451	$\int \frac{1}{(-1+x^2)^2} dx$2190
3.452	$\int \frac{x^2}{(1+x^2)^2} dx$2193
3.453	$\int \frac{1}{2+3x} dx$2196
3.454	$\int \frac{1}{a^2+x^2} dx$2199
3.455	$\int \frac{1}{a+bx^2} dx$2202
3.456	$\int \frac{1}{2-x+x^2} dx$2205
3.457	$\int x^2(4-x^2)^2 dx$2208
3.458	$\int x(1-x^3)^2 dx$2211
3.459	$\int \frac{-4+5x^2+x^3}{x^2} dx$2214
3.460	$\int \frac{-1+x}{3-4x+3x^2} dx$2217
3.461	$\int (2+x^3)^2 dx$2221
3.462	$\int \frac{-4+x^2}{2+x} dx$2224
3.463	$\int \frac{1}{(2+x)(1+x^2)} dx$2227
3.464	$\int \frac{1}{(1+x)(1+x^2)} dx$2231

- 3.465 $\int \frac{x}{(1+x)(1+x^2)} dx \dots\dots\dots .2235$
- 3.466 $\int \frac{2x+x^2}{(1+x)^2} dx \dots\dots\dots .2238$
- 3.467 $\int \frac{-10+x^2}{4+9x^2+2x^4} dx \dots\dots\dots .2241$
- 3.468 $\int \frac{31+5x}{11-4x+3x^2} dx \dots\dots\dots .2244$
- 3.469 $\int \frac{-2+x^2+x^3}{x^4} dx \dots\dots\dots .2248$
- 3.470 $\int \frac{1+x+x^3}{x^2} dx \dots\dots\dots .2251$
- 3.471 $\int \frac{-2+x^2}{x(2+x^2)} dx \dots\dots\dots .2254$
- 3.472 $\int (-3+x)(-7+4x^2) dx \dots\dots\dots .2257$
- 3.473 $\int (-2+7x)^3 dx \dots\dots\dots .2260$
- 3.474 $\int \frac{-7+4x^2}{3+2x} dx \dots\dots\dots .2263$
- 3.475 $\int \frac{1+x}{(-1+x)x^2} dx \dots\dots\dots .2266$
- 3.476 $\int \frac{1}{4x^2+4x^3+x^4} dx \dots\dots\dots .2269$
- 3.477 $\int \frac{1+x^2}{1+x} dx \dots\dots\dots .2273$
- 3.478 $\int \frac{-1+3x-3x^2+x^3}{x^2} dx \dots\dots\dots .2276$
- 3.479 $\int \left(\frac{1}{2}(3-\sqrt{37})+x \right) \left(\frac{1}{2}(3+\sqrt{37})+x \right) dx \dots\dots\dots .2279$
- 3.480 $\int \frac{4+3x^2+2x^3}{(1+x)^4} dx \dots\dots\dots .2282$
- 3.481 $\int \frac{x}{(1+x)^2(1+x^2)} dx \dots\dots\dots .2285$
- 3.482 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx \dots\dots\dots .2288$
- 3.483 $\int \frac{-1+x^3}{-1+x} dx \dots\dots\dots .2291$
- 3.484 $\int \frac{2+2x}{(-1+x)^3(1+x^2)} dx \dots\dots\dots .2294$
- 3.485 $\int \frac{1}{bx+c(d+ex)^2} dx \dots\dots\dots .2297$
- 3.486 $\int \frac{1}{a+bx+c(d+ex)^2} dx \dots\dots\dots .2301$
- 3.487 $\int \frac{x^2}{1+(-1+x^2)^2} dx \dots\dots\dots .2305$
- 3.488 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} dx \dots\dots\dots .2310$
- 3.489 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx \dots\dots\dots .2314$
- 3.490 $\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx \dots\dots\dots .2318$

$$3.1 \quad \int \frac{1}{2\sqrt{3}b^{3/2}-9bx+9x^3} dx$$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\sqrt{b}(\sqrt{3}\sqrt{b}-3x)} - \frac{\log(\sqrt{b}-\sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b}+\sqrt{3}x)}{27b}$$

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2063, 44}

$$\frac{1}{3\sqrt{3}\sqrt{b}(\sqrt{3}\sqrt{b}-3x)} - \frac{\log(\sqrt{b}-\sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b}+\sqrt{3}x)}{27b}$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]

[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(27*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 2063

Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[1/(3^(3*p)*a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} dx &= (324b^3) \int \frac{1}{(6\sqrt{3}b^{3/2} - 18bx)^2 (6\sqrt{3}b^{3/2} + 9bx)} dx \\
&= (324b^3) \int \left(\frac{1}{324\sqrt{3}b^{7/2} (\sqrt{3}\sqrt{b} - 3x)^2} + \frac{1}{2916b^4 (\sqrt{3}\sqrt{b} - 3x)} + \frac{1}{2916b^4 (2\sqrt{3}\sqrt{b} + 3x)} \right) dx \\
&= \frac{1}{3\sqrt{3}\sqrt{b} (\sqrt{3}\sqrt{b} - 3x)} - \frac{\log(\sqrt{b} - \sqrt{3}x)}{27b} + \frac{\log(2\sqrt{b} + \sqrt{3}x)}{27b}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 143, normalized size = 1.86

$$\frac{(3x - \sqrt{3}\sqrt{b})(2\sqrt{3}\sqrt{b} + 3x)((3x - \sqrt{3}\sqrt{b}) \log(3x - \sqrt{3}\sqrt{b}) + (\sqrt{3}\sqrt{b} - 3x) \log(2\sqrt{3}\sqrt{b} + 3x) + 3\sqrt{3}\sqrt{b})}{81b(2\sqrt{3}b^{3/2} - 9bx + 9x^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]

[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt[b] + (-Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*Sqrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2\sqrt{3}b^{3/2} - 9bx + 9x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]

fricas [A] time = 0.72, size = 76, normalized size = 0.99

$$\frac{3\sqrt{3}\sqrt{b}x - (3x^2 - b) \log(2\sqrt{3}\sqrt{b} + 3x) + (3x^2 - b) \log(-\sqrt{3}\sqrt{b} + 3x) + 3b}{27(3bx^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")

[Out] $-1/27*(3*\sqrt{3}*\sqrt{b}*x - (3*x^2 - b)*\log(2*\sqrt{3}*\sqrt{b} + 3*x) + (3*x^2 - b)*\log(-\sqrt{3}*\sqrt{b} + 3*x) + 3*b)/(3*b*x^2 - b^2)$

giac [A] time = 0.22, size = 55, normalized size = 0.71

$$\frac{\log\left(\left|9\sqrt{3}x + 18\sqrt{b}\right|\right)}{27b} - \frac{\log\left(\left|-\sqrt{3}x + \sqrt{b}\right|\right)}{27b} - \frac{1}{9(\sqrt{3}x - \sqrt{b})\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")`

[Out] $1/27*\log(\text{abs}(9*\sqrt{3}*x + 18*\sqrt{b}))/b - 1/27*\log(\text{abs}(-\sqrt{3}*x + \sqrt{b}))/b - 1/9/((\sqrt{3}*x - \sqrt{b})*\sqrt{b})$

maple [C] time = 0.05, size = 43, normalized size = 0.56

$$\frac{\ln\left(-\text{RootOf}\left(9_Z^3 - 9_Zb + 2\sqrt{3}b^{\frac{3}{2}}\right) + x\right)}{27\text{RootOf}\left(9_Z^3 - 9_Zb + 2\sqrt{3}b^{\frac{3}{2}}\right)^2 - 9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)`

[Out] $1/9*\text{sum}(1/(3*_R^2-b)*\ln(x-_R), _R=\text{RootOf}(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{9x^3 + 2\sqrt{3}b^{\frac{3}{2}} - 9bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")`

[Out] `integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)`

mupad [B] time = 0.20, size = 51, normalized size = 0.66

$$\frac{2\sqrt{3}\sqrt{27}\operatorname{atanh}\left(\frac{\sqrt{3}\sqrt{27}}{27} + \frac{2\sqrt{27}x}{9\sqrt{b}}\right)}{243b} - \frac{\sqrt{3}}{27\sqrt{b}\left(x - \frac{\sqrt{3}\sqrt{b}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3), x)`

[Out] $(2 \cdot 3^{1/2} \cdot 27^{1/2} \cdot \operatorname{atanh}((3^{1/2} \cdot 27^{1/2})/27 + (2 \cdot 27^{1/2} \cdot x)/(9 \cdot b^{1/2}))) / (243 \cdot b) - 3^{1/2} / (27 \cdot b^{1/2} \cdot (x - (3^{1/2} \cdot b^{1/2})/3))$

sympy [A] time = 0.33, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}x - 27\sqrt{3}b} + \frac{-\frac{\log\left(-\frac{\sqrt{3}\sqrt{b}}{3} + x\right)}{27} + \frac{\log\left(\frac{2\sqrt{3}\sqrt{b}}{3} + x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)), x)`

[Out] $-3 \cdot \sqrt{3} / (81 \cdot \sqrt{b} \cdot x - 27 \cdot \sqrt{3} \cdot b) + (-\log(-\sqrt{3} \cdot \sqrt{b})/3 + x)/27 + \log(2 \cdot \sqrt{3} \cdot \sqrt{b})/3 + x)/27 / b$

$$3.2 \quad \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p dx$$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b} + x\right) \left(b^3 \left(\frac{a}{b} + x\right)^3\right)^p}{3p + 1}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2067, 15, 30}

$$\frac{\left(\frac{a}{b} + x\right) \left(b^3 \left(\frac{a}{b} + x\right)^3\right)^p}{3p + 1}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]

[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2067

Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]

Rubi steps

$$\begin{aligned}
\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p dx &= \text{Subst} \left(\int (b^3x^3)^p dx, x, \frac{a}{b} + x \right) \\
&= \left(\left(\frac{a}{b} + x \right)^{-3p} \left(b^3 \left(\frac{a}{b} + x \right)^3 \right)^p \right) \text{Subst} \left(\int x^{3p} dx, x, \frac{a}{b} + x \right) \\
&= \frac{(a + bx) \left((a + bx)^3 \right)^p}{b(1 + 3p)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 23, normalized size = 0.77

$$\frac{(a + bx) \left((a + bx)^3 \right)^p}{3bp + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p, x]

[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p, x]

[Out] Defer[IntegrateAlgebraic][(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p, x]

fricas [A] time = 1.03, size = 43, normalized size = 1.43

$$\frac{(bx + a)(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p}{3bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")

[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)

giac [B] time = 0.20, size = 73, normalized size = 2.43

$$\frac{(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p bx + (b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p a}{3bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")

[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)

maple [A] time = 0.01, size = 46, normalized size = 1.53

$$\frac{(bx + a)(b^3x^3 + 3ab^2x^2 + 3a^2bx + a^3)^p}{(3p + 1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)

[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p

maxima [A] time = 0.49, size = 25, normalized size = 0.83

$$\frac{(bx + a)(bx + a)^{3p}}{b(3p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")

[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))

mupad [B] time = 2.13, size = 52, normalized size = 1.73

$$\left(\frac{x}{3p + 1} + \frac{a}{b(3p + 1)} \right) (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)

[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\left\{ \begin{array}{ll} \frac{x}{\sqrt[3]{a^3}} & \text{for } b = 0 \wedge p = -\frac{1}{3} \\ x (a^3)^p & \text{for } b = 0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}} dx & \text{for } p = -\frac{1}{3} \\ \frac{a(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} + \frac{bx(a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3bp+b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)

[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), Eq(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), True))

$$3.3 \quad \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3 dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^{10}}{10b}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2059, 32}

$$\frac{(a + bx)^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]

[Out] (a + b*x)^10/(10*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3 dx = \int (a + bx)^9 dx = \frac{(a + bx)^{10}}{10b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{10}}{10b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]

[Out] (a + b*x)^10/(10*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]

[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3, x]

fricas [B] time = 1.29, size = 97, normalized size = 6.93

$$\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2ba^8 + xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")

[Out] 1/10*x^10*b^9 + x^9*b^8*a + 9/2*x^8*b^7*a^2 + 12*x^7*b^6*a^3 + 21*x^6*b^5*a^4 + 126/5*x^5*b^4*a^5 + 21*x^4*b^3*a^6 + 12*x^3*b^2*a^7 + 9/2*x^2*b*a^8 + x*a^9

giac [B] time = 0.25, size = 97, normalized size = 6.93

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")

[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 + a^9*x

maple [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)

[Out] $1/10*b^9*x^{10}+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x$

maxima [B] time = 0.69, size = 216, normalized size = 15.43

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{27}{8}a^2b^7x^8 + \frac{27}{7}a^3b^6x^7 + \frac{27}{4}a^4b^5x^6 + a^5x^5 + \frac{3}{4}(b^3x^4 + 4ab^2x^3 + 6a^2bx^2)a^6 + \frac{9}{10}(5b^3x^6 + 18ab^2x^5)a^4b^2 + \frac{3}{70}(10b^6x^7 + 70ab^5x^6 + 126a^2b^4x^5 + 210a^3b^3x^4 + 21(4b^3x^5 + 15a^2b^2x^4)a^2b)a^3 + \frac{9}{56}(7b^6x^8 + 48ab^5x^7 + 84a^2b^4x^6)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")`

[Out] $1/10*b^9*x^{10} + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b$

mupad [B] time = 2.07, size = 97, normalized size = 6.93

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)`

[Out] $a^9*x + (b^9*x^{10})/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*a^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a^2*b^7*x^8)/2$

sympy [B] time = 0.09, size = 107, normalized size = 7.64

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)`

[Out] $a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 + a*b**8*x**9 + b**9*x**10/10$

$$3.4 \quad \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^7}{7b}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2059, 32}

$$\frac{(a + bx)^7}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 dx &= \int (a + bx)^6 dx \\ &= \frac{(a + bx)^7}{7b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^7}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] (a + b*x)^7/(7*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]

[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2, x]

fricas [B] time = 0.75, size = 64, normalized size = 4.57

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")

[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3*x^2*b*a^5 + x*a^6

giac [B] time = 0.37, size = 64, normalized size = 4.57

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")

[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3*a^5*b*x^2 + a^6*x

maple [B] time = 0.00, size = 65, normalized size = 4.64

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)

[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2+a^6*x

maxima [B] time = 0.66, size = 99, normalized size = 7.07

$$\frac{1}{7} b^6 x^7 + a b^5 x^6 + \frac{9}{5} a^2 b^4 x^5 + 3 a^4 b^2 x^3 + a^6 x + \frac{1}{2} (b^3 x^4 + 4 a b^2 x^3 + 6 a^2 b x^2) a^3 + \frac{3}{10} (4 b^3 x^5 + 15 a b^2 x^4) a^2 b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")

[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b

mupad [B] time = 0.03, size = 64, normalized size = 4.57

$$a^6 x + 3 a^5 b x^2 + 5 a^4 b^2 x^3 + 5 a^3 b^3 x^4 + 3 a^2 b^4 x^5 + a b^5 x^6 + \frac{b^6 x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)

[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x^4 + 3*a^2*b^4*x^5

sympy [B] time = 0.08, size = 66, normalized size = 4.71

$$a^6 x + 3 a^5 b x^2 + 5 a^4 b^2 x^3 + 5 a^3 b^3 x^4 + 3 a^2 b^4 x^5 + a b^5 x^6 + \frac{b^6 x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)

[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*x**5 + a*b**5*x**6 + b**6*x**7/7

$$3.5 \quad \int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3) dx$$

Optimal. Leaf size=35

$$a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{3}{2}a^2bx^2 + a^3x + ab^2x^3 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4

Rubi steps

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3) dx = a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x + \frac{3}{2}a^2bx^2 + ab^2x^3 + \frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

[Out] IntegrateAlgebraic[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]

fricas [A] time = 0.51, size = 31, normalized size = 0.89

$$\frac{1}{4}x^4b^3 + x^3b^2a + \frac{3}{2}x^2ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")

[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3

giac [A] time = 0.28, size = 31, normalized size = 0.89

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")

[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{4}b^3x^4 + a b^2x^3 + \frac{3}{2}a^2b x^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)

[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4

maxima [A] time = 0.74, size = 31, normalized size = 0.89

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")

[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x, x)`

[Out] `a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3`

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3, x)`

[Out] `a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4`

$$3.6 \quad \int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

[Out] -1/(2*b*(a + b*x)^2)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3} dx &= \int \frac{1}{(a+bx)^3} dx \\ &= -\frac{1}{2b(a+bx)^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

[Out] -1/2*1/(b*(a + b*x)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]

fricas [A] time = 0.79, size = 24, normalized size = 1.71

$$-\frac{1}{2(b^3x^2 + 2ab^2x + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3), x, algorithm="fricas")

[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)

giac [A] time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2(bx + a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3), x, algorithm="giac")

[Out] -1/2/((b*x + a)^2*b)

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$-\frac{1}{2(bx + a)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3), x)

[Out] -1/2/b/(b*x+a)^2

maxima [A] time = 0.65, size = 24, normalized size = 1.71

$$-\frac{1}{2(b^3x^2 + 2ab^2x + a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")

[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)

mupad [B] time = 0.03, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)

[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b + 4ab^2x + 2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)

[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)

$$3.7 \quad \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

[Out] -1/(5*b*(a + b*x)^5)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2} dx &= \int \frac{1}{(a + bx)^6} dx \\ &= -\frac{1}{5b(a + bx)^5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

[Out] -1/5*1/(b*(a + b*x)^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]

fricas [B] time = 0.93, size = 57, normalized size = 4.07

$$\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")

[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)

giac [A] time = 0.29, size = 12, normalized size = 0.86

$$-\frac{1}{5(bx + a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")

[Out] -1/5/((b*x + a)^5*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{5(bx + a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)`

[Out] `-1/5/b/(b*x+a)^5`

maxima [B] time = 0.71, size = 57, normalized size = 4.07

$$\frac{1}{5(b^6x^5 + 5ab^5x^4 + 10a^2b^4x^3 + 10a^3b^3x^2 + 5a^4b^2x + a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")`

[Out] `-1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)`

mupad [B] time = 2.05, size = 59, normalized size = 4.21

$$\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)`

[Out] `-1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50*a^2*b^4*x^3)`

sympy [B] time = 0.35, size = 61, normalized size = 4.36

$$\frac{1}{5a^5b + 25a^4b^2x + 50a^3b^3x^2 + 50a^2b^4x^3 + 25ab^5x^4 + 5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)`

[Out] `-1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*a*b**5*x**4 + 5*b**6*x**5)`

$$3.8 \quad \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2058, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

[Out] -1/(8*b*(a + b*x)^8)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx &= \int \frac{1}{(a + bx)^9} dx \\ &= -\frac{1}{8b(a + bx)^8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

[Out] -1/8*1/(b*(a + b*x)^8)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]

fricas [B] time = 1.11, size = 90, normalized size = 6.43

$$\frac{1}{8(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")

[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)

giac [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{1}{8(bx + a)^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")

[Out] -1/8/((b*x + a)^8*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{8(bx + a)^8 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)

[Out] -1/8/b/(b*x+a)^8

maxima [B] time = 0.54, size = 90, normalized size = 6.43

$$\frac{1}{8(b^9x^8 + 8ab^8x^7 + 28a^2b^7x^6 + 56a^3b^6x^5 + 70a^4b^5x^4 + 56a^5b^4x^3 + 28a^6b^3x^2 + 8a^7b^2x + a^8b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")

[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)

mupad [B] time = 2.07, size = 92, normalized size = 6.57

$$\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)

[Out] -1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 + 448*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)

sympy [B] time = 0.54, size = 97, normalized size = 6.93

$$\frac{1}{8a^8b + 64a^7b^2x + 224a^6b^3x^2 + 448a^5b^4x^3 + 560a^4b^5x^4 + 448a^3b^6x^5 + 224a^2b^7x^6 + 64ab^8x^7 + 8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)

[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 560*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**7 + 8*b**9*x**8)

$$3.9 \quad \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 dx$$

Optimal. Leaf size=84

$$-\frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{b^3x(b^2 - 3ac)^3}{c^3} + \frac{(b + cx)^{10}}{10c^4}$$

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2060, 194}

$$-\frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{b^3x(b^2 - 3ac)^3}{c^3} + \frac{(b + cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3, x]

[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2060

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]

Rubi steps

$$\begin{aligned} \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 dx &= \frac{1}{27} \text{Subst} \left(\int \left(3b \left(3a - \frac{b^2}{c} \right) + 3c^2x^3 \right)^3 dx, x, \frac{b}{c} + x \right) \\ &= \frac{1}{27} \text{Subst} \left(\int \left(\frac{27(-b^3 + 3abc)^3}{c^3} + 81(b^3 - 3abc)^2 x^3 - 81bc^3(b^2 - 3ac)x^6 \right. \right. \\ &\quad \left. \left. - \frac{b^3(b^2 - 3ac)^3 x}{c^3} + \frac{3b^2(b^2 - 3ac)^2(b + cx)^4}{4c^4} - \frac{3b(b^2 - 3ac)(b + cx)^7}{7c^4} + \dots \right) dx, x, \frac{b}{c} + x \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 159, normalized size = 1.89

$$27a^3b^3x + \frac{81}{2}a^2b^4x^2 + \frac{27}{4}b^2x^4(a^2c^2 + 6ab^2c + b^4) + \frac{9}{7}bc^3x^7(ac + 9b^2) + 9b^2c^2x^6(ac + 2b^2) + \frac{27}{5}b^3cx^5(5ac + 3b^2) + 27ab^3x^3(ac + b^2) + \frac{9}{2}b^2c^4x^8 + bc^5x^9 + \frac{c^6x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]

[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 + b*c^5*x^9 + (c^6*x^10)/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]

[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3, x]

fricas [B] time = 0.95, size = 166, normalized size = 1.98

$$\frac{1}{10}x^{10}c^6 + x^9c^5b + \frac{9}{2}x^8c^4b^2 + \frac{81}{7}x^7c^3b^3 + \frac{9}{7}x^7c^4ba + 18x^6c^2b^4 + 9x^6c^3b^2a + \frac{81}{5}x^5cb^5 + 27x^5c^2b^3a + \frac{27}{4}x^4b^6 + \frac{81}{2}x^4cb^4a + \frac{27}{4}x^4c^2b^2a^2 + 27x^3cb^3a^2 + \frac{81}{2}x^2b^4a^2 + 27xb^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")

[Out] 1/10*x^10*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 + 9/7*x^7*c^4*b*a + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b^5 + 27*x^5*c^2*b^3*a + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^4*c^2*b^2*a^2 + 27*x^3*b^5*a + 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^2 + 27*x*b^3*a^3

giac [B] time = 0.27, size = 166, normalized size = 1.98

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{9}{2}b^2c^4x^8 + \frac{81}{7}b^3c^3x^7 + \frac{9}{7}abc^4x^7 + 18b^4c^2x^6 + 9ab^2c^3x^6 + \frac{81}{5}b^5cx^5 + 27ab^3c^2x^5 + \frac{27}{4}b^6x^4 + \frac{81}{2}ab^4cx^4 + \frac{27}{4}a^2b^2c^2x^4 + 27ab^5x^3 + 27a^2b^3cx^3 + \frac{81}{2}a^2b^4x^2 + 27a^3b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")

[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^5

$$5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 + 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x$$

maple [B] time = 0.00, size = 295, normalized size = 3.51

$$\frac{c^6 x^{10}}{10} + b^6 c^5 x^9 + \frac{81 a^2 b^4 c^2}{2} + \frac{81 a^2 b^4 c^2}{2} + 27 a^3 b^3 x + \frac{(3 a b^4 c^3 + 63 b^3 c^3 + (6 a b^2 c^2 + 18 b^3 c^2) c^2)}{7} + \frac{(18 a b^2 c^3 + 45 b^4 c^2 + 3(6 a b^2 c^2 + 18 b^3 c^2) b c + (18 a b^2 c + 9 b^4 c^2) c^2)}{6} + \frac{(63 a b^3 c^2 + 3(6 a b^2 c^2 + 18 b^3 c^2) b^2 + 3(18 a b^2 c + 9 b^4 c^2) b c)}{5} + \frac{(9 a^2 b^2 c^2 + 54 a b^3 c + 3(6 a b^2 c^2 + 18 b^3 c^2) a b + 3(18 a b^2 c + 9 b^4 c^2) b^2)}{4} + \frac{(27 a^2 b^3 c + 54 a b^4 + 3(18 a b^2 c + 9 b^4 c^2) a b)}{3} + \frac{c^6 x^{10}}{10} + b^6 c^5 x^9 + \frac{81 a^2 b^4 c^2}{2} + \frac{81 a^2 b^4 c^2}{2} + 27 a^3 b^3 x + \frac{(3 a b^4 c^3 + 63 b^3 c^3 + (6 a b^2 c^2 + 18 b^3 c^2) c^2)}{7} + \frac{(18 a b^2 c^3 + 45 b^4 c^2 + 3(6 a b^2 c^2 + 18 b^3 c^2) b c + (18 a b^2 c + 9 b^4 c^2) c^2)}{6} + \frac{(63 a b^3 c^2 + 3(6 a b^2 c^2 + 18 b^3 c^2) b^2 + 3(18 a b^2 c + 9 b^4 c^2) b c)}{5} + \frac{(9 a^2 b^2 c^2 + 54 a b^3 c + 3(6 a b^2 c^2 + 18 b^3 c^2) a b + 3(18 a b^2 c + 9 b^4 c^2) b^2)}{4} + \frac{(27 a^2 b^3 c + 54 a b^4 + 3(18 a b^2 c + 9 b^4 c^2) a b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)

[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3+c^2*(6*a*b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c*(6*a*b*c^2+18*b^3*c)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c^2+3*b^2*(6*a*b*c^2+18*b^3*c)+3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b^2*c+9*b^4)+54*b^4*c*a+9*c^2*a^2*b^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*b^5*a+27*b^3*c*a^2)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x

maxima [B] time = 0.68, size = 204, normalized size = 2.43

$$\frac{1}{10} c^6 x^{10} + b^6 c^5 x^9 + \frac{27}{8} b^2 c^4 x^8 + \frac{27}{7} b^3 c^3 x^7 + \frac{27}{4} b^4 c^2 x^6 + 27 a^3 b^3 x + \frac{27}{4} (c^2 x^4 + 4 b c x^3 + 6 b^2 x^2) a^2 b^2 + \frac{9}{10} (5 c^2 x^6 + 18 b c x^5) b^4 + \frac{9}{70} (10 c^4 x^7 + 70 b c^3 x^6 + 126 b^2 c^2 x^5 + 210 b^4 x^4 + 21 (4 c^2 x^5 + 15 b c x^4) b^2) a b + \frac{9}{56} (7 c^4 x^8 + 48 b c^3 x^7 + 84 b^2 c^2 x^6) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")

[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^2*x^5 + 210*b^4*x^4 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2

mupad [B] time = 2.08, size = 149, normalized size = 1.77

$$x^4 \left(\frac{27 a^2 b^2 c^2}{4} + \frac{81 a b^4 c}{2} + \frac{27 b^6}{4} \right) + \frac{c^6 x^{10}}{10} + 27 a^3 b^3 x + b^6 c^5 x^9 + \frac{81 a^2 b^4 c^2}{2} + \frac{9 b^2 c^4 x^8}{2} + 9 b^2 c^2 x^6 (2 b^2 + a c) + 27 a b^3 x^3 (b^2 + a c) + \frac{27 b^3 c x^5 (3 b^2 + 5 a c)}{5} + \frac{9 b c^3 x^7 (9 b^2 + a c)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)

[Out] x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 27*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7

sympy [B] time = 0.10, size = 175, normalized size = 2.08

$$27 a^3 b^3 x + \frac{81 a^2 b^4 x^2}{2} + \frac{9 b^2 c^4 x^8}{2} + b^6 c^5 x^9 + \frac{c^6 x^{10}}{10} + x^7 \left(\frac{9 a b c^4}{7} + \frac{81 b^3 c^3}{7} \right) + x^6 (9 a b^2 c^3 + 18 b^4 c^2) + x^5 (27 a b^3 c^2 + \frac{81 b^5 c}{5}) + x^4 \left(\frac{27 a^2 b^2 c^2}{4} + \frac{81 a b^4 c}{2} + \frac{27 b^6}{4} \right) + x^3 (27 a^2 b^3 c + 27 a b^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c  
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +  
18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c  
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

$$3.10 \quad \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2 dx$$

Optimal. Leaf size=56

$$-\frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{b^2x(b^2 - 3ac)^2}{c^2} + \frac{(b + cx)^7}{7c^3}$$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2060, 194}

$$-\frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{b^2x(b^2 - 3ac)^2}{c^2} + \frac{(b + cx)^7}{7c^3}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]

[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b + c*x)^7/(7*c^3)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2060

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]

Rubi steps

$$\begin{aligned} \int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2 dx &= \frac{1}{9} \text{Subst} \left(\int \left(3b \left(3a - \frac{b^2}{c} \right) + 3c^2x^3 \right)^2 dx, x, \frac{b}{c} + x \right) \\ &= \frac{1}{9} \text{Subst} \left(\int \left(\frac{9(-b^3 + 3abc)^2}{c^2} - 18bc(b^2 - 3ac)x^3 + 9c^4x^6 \right) dx, x, \frac{b}{c} + x \right) \\ &= \frac{b^2(b^2 - 3ac)^2 x}{c^2} - \frac{b(b^2 - 3ac)(b + cx)^4}{2c^3} + \frac{(b + cx)^7}{7c^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.46

$$9a^2b^2x + 9ab^3x^2 + \frac{3}{2}bcx^4(ac + 3b^2) + 3b^2x^3(2ac + b^2) + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]

[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]

[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2, x]

fricas [A] time = 0.63, size = 83, normalized size = 1.48

$$\frac{1}{7}c^4x^7 + x^6c^3b + 3x^5c^2b^2 + \frac{9}{2}x^4cb^3 + \frac{3}{2}x^4c^2ba + 3x^3b^4 + 6x^3cb^2a + 9x^2b^3a + 9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")

[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4*c^2*b*a + 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2

giac [A] time = 0.34, size = 83, normalized size = 1.48

$$\frac{1}{7}c^4x^7 + bc^3x^6 + 3b^2c^2x^5 + \frac{9}{2}b^3cx^4 + \frac{3}{2}abc^2x^4 + 3b^4x^3 + 6ab^2cx^3 + 9ab^3x^2 + 9a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")

[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 + 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x

maple [A] time = 0.00, size = 84, normalized size = 1.50

$$\frac{c^4x^7}{7} + bc^3x^6 + 3b^2c^2x^5 + 9ab^3x^2 + 9a^2b^2x + \frac{(6abc^2 + 18b^3c)x^4}{4} + \frac{(18ab^2c + 9b^4)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)`

[Out] $1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x$

maxima [A] time = 0.64, size = 93, normalized size = 1.66

$$\frac{1}{7}c^4x^7 + bc^3x^6 + \frac{9}{5}b^2c^2x^5 + 3b^4x^3 + 9a^2b^2x + \frac{3}{2}(c^2x^4 + 4bcx^3 + 6b^2x^2)ab + \frac{3}{10}(4c^2x^5 + 15bcx^4)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")`

[Out] $1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2$

mupad [B] time = 0.04, size = 79, normalized size = 1.41

$$x^3(3b^4 + 6acb^2) + \frac{c^4x^7}{7} + 9a^2b^2x + 9ab^3x^2 + bc^3x^6 + 3b^2c^2x^5 + \frac{3bcx^4(3b^2 + ac)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)`

[Out] $x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2$

sympy [A] time = 0.08, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4\left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3(6ab^2c + 3b^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)`

[Out] $9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)$

$$3.11 \quad \int (3ab + 3b^2x + 3bcx^2 + c^2x^3) dx$$

Optimal. Leaf size=32

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3, x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

Rubi steps

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3, x]

[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3ab + 3b^2x + 3bcx^2 + c^2x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3, x]

[Out] IntegrateAlgebraic[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3, x]

fricas [A] time = 0.76, size = 28, normalized size = 0.88

$$\frac{1}{4}x^4c^2 + x^3cb + \frac{3}{2}x^2b^2 + 3xba$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")

[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")

[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)

[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4

maxima [A] time = 0.55, size = 28, normalized size = 0.88

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")

[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{3b^2x^2}{2} + bcx^3 + 3abx + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)`

[Out] $(3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3$

sympy [A] time = 0.06, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)`

[Out] $3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4$

$$3.12 \quad \int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3} dx$$

Optimal. Leaf size=188

$$\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}(b^2-3ac)^{2/3}} + \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}(b^2-3ac)^{2/3}} - \frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}(b^2-3ac)^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2067, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}(b^2-3ac)^{2/3}} + \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}(b^2-3ac)^{2/3}} - \frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}(b^2-3ac)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]

[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2 - 3*a*c)^(2/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3ab + 3b^2x + 3bcx^2 + c^2x^3} dx &= \text{Subst} \left(\int \frac{1}{b \left(3a - \frac{b^2}{c} \right) + c^2x^3} dx, x, \frac{b}{c} + x \right) \\
&= \frac{c^{2/3} \text{Subst} \left(\int \frac{1}{-\frac{\sqrt[3]{b} \sqrt[3]{b^2-3ac}}{\sqrt[3]{c}} + c^{2/3}x} dx, x, \frac{b}{c} + x \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} + \frac{c^{2/3} \text{Subst} \left(\int \frac{-\frac{2 \sqrt[3]{b} \sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}} + \sqrt[3]{b} \sqrt[3]{c}} dx, x, \frac{b}{c} + x \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} \\
&= \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\text{Subst} \left(\int \frac{\sqrt[3]{b} \sqrt[3]{c} \sqrt[3]{b^2-3ac} + 2c^{4/3}x}{\frac{b^{2/3}(b^2-3ac)^{2/3}}{c^{2/3}} + \sqrt[3]{b} \sqrt[3]{c} \sqrt[3]{b^2-3ac}x + c^{4/3}} dx, x, \frac{b}{c} + x \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}} \\
&= \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\log \left(b^{2/3} (b^2 - 3ac)^{2/3} + \sqrt[3]{b} \sqrt[3]{b^2 - 3ac} (cx + c^{4/3}) \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}} \\
&= -\frac{\tan^{-1} \left(\frac{1 + \frac{2(b+cx)}{\sqrt[3]{b} \sqrt[3]{b^2-3ac}}}{\sqrt{3}} \right)}{\sqrt{3} b^{2/3} (b^2 - 3ac)^{2/3}} + \frac{\log \left(\sqrt[3]{b} (b^{2/3} - \sqrt[3]{b^2 - 3ac}) + cx \right)}{3b^{2/3} (b^2 - 3ac)^{2/3}} - \frac{\log \left(b^{2/3} (b^2 - 3ac)^{2/3} + \sqrt[3]{b} \sqrt[3]{b^2 - 3ac} (cx + c^{4/3}) \right)}{6b^{2/3} (b^2 - 3ac)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.34

$$\frac{1}{3} \text{RootSum} \left[\#1^3 c^2 + 3\#1^2 bc + 3\#1 b^2 + 3ab \&, \frac{\log(x - \#1)}{\#1^2 c^2 + 2\#1 bc + b^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]

[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 &, Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3ab + 3b^2x + 3bcx^2 + c^2x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]

fricas [B] time = 1.11, size = 387, normalized size = 2.06

$$\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)\arctan\left(\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}\right)+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\log\left(-b^5+3a^2b^3c-\left(b^3c^2-3a^2b^2c\right)x-2\left(b^4c-3a^2b^2c\right)x-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(cx+b\right)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)-2\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\log\left(-b^4+3ab^2c-\left(b^3-3abc\right)x+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\right)}{6\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b), x, algorithm="fricas")

[Out]
$$-1/6*(2*\sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/6)}*(b^3 - 3*a*b*c)*\arctan(1/3*(2*\sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(5/6))} + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*\log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a^2*b^2*c)*x^2 - 2*(b^4*c - 3*a^2*b^2*c)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*\log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a^2*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)$$

giac [A] time = 0.36, size = 212, normalized size = 1.13

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)}{6\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{\log\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b), x, algorithm="giac")

[Out]
$$1/3*\sqrt{3}*\arctan((\sqrt{3}*c*x + \sqrt{3}*b - \sqrt{3}*(-b^3 + 3*a*b*c))^{(1/3)})/(c*x + b + (-b^3 + 3*a*b*c)^{(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)} - 1/6*\log(4*(\sqrt{3}*c*x + \sqrt{3}*b - \sqrt{3}*(-b^3 + 3*a*b*c))^{(1/3)})^2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^{(1/3)})^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)} + 1/3*\log(\text{abs}(c*x + b + (-b^3 + 3*a*b*c)^{(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)})$$

maple [C] time = 0.00, size = 57, normalized size = 0.30

$$\frac{\ln\left(-\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right) + x\right)}{3\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)^2 c^2 + 6\text{RootOf}\left(c^2_Z^3 + 3bc_Z^2 + 3b^2_Z + 3ab\right)bc + 3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b), x)

[Out] $\frac{1}{3} \sum \left(\frac{1}{(-R^2 c^2 + 2 R b c + b^2)} \ln(-R+x), R = \text{RootOf}(-Z^3 c^2 + 3 Z^2 b c + 3 Z b^2 + 3 a b) \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2 x^3 + 3 b c x^2 + 3 b^2 x + 3 a b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")`

[Out] `integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)`

mupad [B] time = 0.49, size = 174, normalized size = 0.93

$$\frac{\ln\left(b + b^{1/3}(3ac - b^2)^{1/3} + cx\right)}{3b^{2/3}(3ac - b^2)^{2/3}} + \frac{\ln\left(3bc^3 + 3c^4x + \frac{3b^{1/3}c^3(-1 + \sqrt{3}i)(3ac - b^2)^{1/3}}{2}\right)(-1 + \sqrt{3}i)}{6b^{2/3}(3ac - b^2)^{2/3}} - \frac{\ln\left(3bc^3 + 3c^4x - \frac{3b^{1/3}c^3(1 + \sqrt{3}i)(3ac - b^2)^{1/3}}{2}\right)(1 + \sqrt{3}i)}{6b^{2/3}(3ac - b^2)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)`

[Out] $\log(b + b^{1/3}(3ac - b^2)^{1/3} + cx) / (3b^{2/3}(3ac - b^2)^{2/3}) + (\log(3b^2c^3 + 3c^4x + (3b^{1/3}c^3(3^{1/2}i - 1)(3ac - b^2)^{1/3})/2) / (6b^{2/3}(3ac - b^2)^{2/3}) - (\log(3b^2c^3 + 3c^4x - (3b^{1/3}c^3(3^{1/2}i + 1)(3ac - b^2)^{1/3})/2) / (6b^{2/3}(3ac - b^2)^{2/3}))$

sympy [A] time = 0.41, size = 53, normalized size = 0.28

$$\text{RootSum}\left(t^3(243a^2b^2c^2 - 162ab^4c + 27b^6) - 1, \left(t \mapsto t \log\left(x + \frac{9tabc - 3tb^3 + b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)`

[Out] `RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t, _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))`

$$3.13 \quad \int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx$$

Optimal. Leaf size=245

$$\frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} + \frac{c \log\left(\sqrt[3]{b} c \sqrt[3]{b^2 - 3ac} \left(\frac{b}{c} + x\right) + b^{2/3} (b^2 - 3ac)^{2/3} + c^2 \left(\frac{b}{c} + x\right)^2\right)}{9b^{5/3} (b^2 - 3ac)^{5/3}}$$

Rubi [A] time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{c \left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} + \frac{c \log\left(\sqrt[3]{b} c \sqrt[3]{b^2 - 3ac} \left(\frac{b}{c} + x\right) + b^{2/3} (b^2 - 3ac)^{2/3} + c^2 \left(\frac{b}{c} + x\right)^2\right)}{9b^{5/3} (b^2 - 3ac)^{5/3}} - \frac{2c \log\left(-\sqrt[3]{b} \sqrt[3]{b^2 - 3ac} + b + cx\right)}{9b^{5/3} (b^2 - 3ac)^{5/3}} + \frac{2c \tan^{-1}\left(\frac{\sqrt[3]{2b^2 - 3ac} + \sqrt[3]{b}}{\sqrt{3} \sqrt[3]{b}}\right)}{3\sqrt{3} b^{5/3} (b^2 - 3ac)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)) + (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))]/(Sqrt[3]*b^(1/3)))]/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} dx &= \text{Subst} \left(\int \frac{1}{\left(b\left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^2} dx, x, \frac{b}{c} + x \right) \\
&= \frac{c\left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{(2c) \text{Subst} \left(\int \frac{1}{b\left(3a - \frac{b^2}{c}\right) + c^2x^3} \right)}{3b(b^2 - 3ac)} \\
&= \frac{c\left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{(2c^{5/3}) \text{Subst} \left(\int \frac{1}{-\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}} \right)}{9b^{5/3}(b^2 - 3ac)} \\
&= \frac{c\left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{2c \log\left(\sqrt[3]{b}(b^{2/3} - \sqrt[3]{b^2 - 3ac})\right)}{9b^{5/3}(b^2 - 3ac)^5} \\
&= \frac{c\left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{2c \log\left(\sqrt[3]{b}(b^{2/3} - \sqrt[3]{b^2 - 3ac})\right)}{9b^{5/3}(b^2 - 3ac)^5} \\
&= \frac{c\left(\frac{b}{c} + x\right)}{3b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)} + \frac{2c \tan^{-1}\left(\frac{1 + \frac{2(b+cx)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}(b^2 - 3ac)^{5/3}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 112, normalized size = 0.46

$$\frac{2c \text{RootSum}\left[\#1^3c^2 + 3\#1^2bc + 3\#1b^2 + 3ab\&, \frac{\log(x-\#1)}{\#1^2c^2 + 2\#1bc + b^2}\&\right] + \frac{3(b+cx)}{3ab+x(3b^2+3bcx+c^2x^2)}}{9(b^3 - 3abc)}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) &])/(b^3 - 3*a*b*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]

fricas [B] time = 1.10, size = 704, normalized size = 2.87

$$\frac{2\sqrt{3}\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\log\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)}{9(b^3-3abc)}-\frac{cx+b}{3(c^2x^3+3bcx^2+3b^2x+3ab)(b^3-3abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")

[Out]
$$-1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*\sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/6)}*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 + 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*\arctan(1/3*(2*\sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(5/6)} - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*\log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c)) + 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a*b*c)*\log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^{10} - 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^{10}*c - 9*a*b^8*c^2 + 27*a^2*b^6*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^{11} - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*b^5*c^3)*x)$$

giac [A] time = 0.36, size = 289, normalized size = 1.18

$$\frac{2\sqrt{3}\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}}{cx+b+(-b^3+3abc)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}(-b^3+3abc)^{\frac{1}{3}}\right)^2+4\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^3}{(b^2-6ab^2c+9a^2b^2c^2)}\right)^{\frac{1}{3}}\log\left(cx+b+(-b^3+3abc)^{\frac{1}{3}}\right)}{9(b^3-3abc)}-\frac{cx+b}{3(c^2x^3+3bcx^2+3b^2x+3ab)(b^3-3abc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")

[Out]
$$-1/9*(2*\sqrt{3}*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^{(1/3)}*\arctan((\sqrt{3}*(3*c*x + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(2/3)}*(c*x + b) + \sqrt{3}*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^{(1/3)}*(b^3 - 3*a*b*c))^{(1/3)})/(c*x + b + (-b^3 + 3*a*b*c)^{(1/3)}))$$

$(b^3 c) \sqrt[3]{c} - (c^3 / (b^6 - 6 a^2 b^4 c + 9 a^4 b^2 c^2)) \sqrt[3]{c} \log(4 (\sqrt{3} c x + \sqrt{3} b - \sqrt{3} (-b^3 + 3 a^2 b c) \sqrt[3]{c})^2 + 4 (c x + b + (-b^3 + 3 a^2 b c) \sqrt[3]{c}))^2) + 2 (c^3 / (b^6 - 6 a^2 b^4 c + 9 a^4 b^2 c^2)) \sqrt[3]{c} \log(\text{abs}(c x + b + (-b^3 + 3 a^2 b c) \sqrt[3]{c})) / (b^3 - 3 a^2 b c) - 1/3 (c x + b) / ((c^2 x^3 + 3 b c x^2 + 3 b^2 x + 3 a^2 b) (b^3 - 3 a^2 b c))$

maple [C] time = 0.02, size = 136, normalized size = 0.56

$$\frac{2c \ln(-\text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab) + x)}{9(3ac - b^2) b (\text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab)^2 c^2 + 2 \text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab) bc + b^2)} + \frac{\frac{cx}{3(3ac - b^2)b} + \frac{1}{9ac - 3b^2}}{c^2 x^3 + 3bc x^2 + 3b^2 x + 3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)

[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+2/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{1}{3} \left(\frac{2 \sqrt{3} \arctan\left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3abc)^{\frac{1}{3}}}\right)}{(b^6 - 6ab^4c + 9a^2b^2c^2)^{\frac{1}{3}}} - \frac{\log\left(4\left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}\right)^2 + 4\left(cx + b + (-b^3 + 3abc)^{\frac{1}{3}}\right)^2\right)}{(b^6 - 6ab^4c + 9a^2b^2c^2)^{\frac{1}{3}}} + \frac{2 \log\left(\left|cx + b + (-b^3 + 3abc)^{\frac{1}{3}}\right|\right)}{(b^6 - 6ab^4c + 9a^2b^2c^2)^{\frac{1}{3}}} \right) c}{3(b^3 - 3abc)} - \frac{cx + b}{3(3ab^4 - 9a^2b^2c + (b^3c^2 - 3abc^3)x^3 + 3(b^4c - 3ab^2c^2)x^2 + 3(b^5 - 3ab^3c)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")

[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)

mupad [B] time = 2.65, size = 247, normalized size = 1.01

$$\frac{\frac{1}{3(3ac - b^2)} + \frac{cx}{3b(3ac - b^2)}}{3b^2x + 3bcx^2 + 3ab + c^2x^3} + \frac{2c \ln(b + b^{1/3}(3ac - b^2)^{1/3} + cx)}{9b^{5/3}(3ac - b^2)^{5/3}} - \frac{\ln(2b - b^{1/3}(3ac - b^2)^{1/3} + 2cx - \sqrt{3}b^{1/3}(3ac - b^2)^{1/3}i)(c + \sqrt{3}ci)}{9b^{5/3}(3ac - b^2)^{5/3}} - \frac{\ln(2b - b^{1/3}(3ac - b^2)^{1/3} + 2cx + \sqrt{3}b^{1/3}(3ac - b^2)^{1/3}i)(c - \sqrt{3}ci)}{9b^{5/3}(3ac - b^2)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)

[Out] (1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5/3)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*

$b^{(1/3)}*(3*a*c - b^2)^{(1/3)*1i}*(c - 3^{(1/2)*c*1i})/(9*b^{(5/3)}*(3*a*c - b^2)^{(5/3)})$

sympy [A] time = 1.22, size = 192, normalized size = 0.78

$$\frac{b + cx}{27a^2b^2c - 9ab^4 + x^3(9abc^3 - 3b^3c^2) + x^2(27ab^2c^2 - 9b^4c) + x(27ab^3c - 9b^5)} + \text{RootSum}\left(t^3(177147a^5b^5c^5 - 295245a^4b^7c^4 + 196830a^3b^9c^3 - 65610a^2b^{11}c^2 + 10935ab^{13}c - 729b^{15}) - 8c^3, \left(t \mapsto t \log\left(x + \frac{81ta^2b^2c^2 - 54tab^4c + 9tb^6 + 2bc}{2c^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)

[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x**2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2))))

$$3.14 \quad \int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3} dx$$

Optimal. Leaf size=305

$$\frac{5c^2 \left(\frac{b}{c} + x\right)}{18b^2 (b^2 - 3ac)^2 (3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{c \left(\frac{b}{c} + x\right)}{6b (b^2 - 3ac) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c^2 \log\left(-\sqrt[3]{b} \sqrt[3]{b^2 - 3ac}\right)}{27b^{8/3} (b^2 - 3ac)^{8/3}}$$

Rubi [A] time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5c^2 \left(\frac{b}{c} + x\right)}{18b^2 (b^2 - 3ac)^2 (3ab + 3b^2x + 3bcx^2 + c^2x^3)} - \frac{c \left(\frac{b}{c} + x\right)}{6b (b^2 - 3ac) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c^2 \log\left(-\sqrt[3]{b} \sqrt[3]{b^2 - 3ac} + b + cx\right)}{27b^{8/3} (b^2 - 3ac)^{8/3}} - \frac{5c^2 \log\left(\sqrt[3]{b} c \sqrt[3]{b^2 - 3ac} \left(\frac{b}{c} + x\right) + b^{2/3} (b^2 - 3ac)^{2/3} + c^2 \left(\frac{b}{c} + x\right)^2\right)}{54b^{8/3} (b^2 - 3ac)^{8/3}} - \frac{5c^2 \tan^{-1}\left(\frac{\sqrt[3]{b^2 - 3ac} + \sqrt[3]{b}}{\sqrt{3} \sqrt[3]{b}}\right)}{9\sqrt{3} b^{8/3} (b^2 - 3ac)^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))]/(Sqrt[3]*b^(1/3)))/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} dx &= \text{Subst} \left(\int \frac{1}{\left(b\left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^3} dx, x, \frac{b}{c} + x \right) \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} - \frac{(5c) \text{Subst} \left(\int \frac{1}{\left(b\left(3a - \frac{b^2}{c}\right) + c^2x^3\right)^3} dx, x, \frac{b}{c} + x \right)}{6b(b^2 - 3ac)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b + \dots)}{18b^2(b^2 - 3ac)^2(3ab + \dots)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b + \dots)}{18b^2(b^2 - 3ac)^2(3ab + \dots)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b + \dots)}{18b^2(b^2 - 3ac)^2(3ab + \dots)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b + \dots)}{18b^2(b^2 - 3ac)^2(3ab + \dots)} \\
&= -\frac{c\left(\frac{b}{c} + x\right)}{6b(b^2 - 3ac)(3ab + 3b^2x + 3bcx^2 + c^2x^3)^2} + \frac{5c(b + \dots)}{18b^2(b^2 - 3ac)^2(3ab + \dots)}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 149, normalized size = 0.49

$$\frac{10c^2 \text{RootSum} \left[\#1^3 c^2 + 3\#1^2 bc + 3\#1 b^2 + 3ab \&, \frac{\log(x-\#1)}{\#1^2 c^2 + 2\#1 bc + b^2} \& \right] - \frac{3(b+cx)(-3bc(8a+5cx^2)+3b^3-15b^2cx-5c^3x^3)}{(3ab+3b^2+3bcx+c^2x^2)^2}}{54(b^3-3abc)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) &])/(54*(b^3 - 3*a*b*c)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]

fricas [B] time = 1.17, size = 1268, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")

[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c - 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1

$$2*a*b^{10}*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(b^{13}*c^3 - 12*a*b^{11}*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^7)*x^5 + 15*(b^{14}*c^2 - 12*a*b^{12}*c^3 + 54*a^2*b^{10}*c^4 - 108*a^3*b^8*c^5 + 81*a^4*b^6*c^6)*x^4 + 6*(3*b^{15}*c - 35*a*b^{13}*c^2 + 150*a^2*b^{11}*c^3 - 270*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^{16} - 10*a*b^{14}*c + 30*a^2*b^{12}*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^{15} - 12*a^2*b^{13}*c + 54*a^3*b^{11}*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)$$

giac [A] time = 0.40, size = 366, normalized size = 1.20

$$\frac{5 \left(2 \sqrt{3} \left(\frac{c^6}{(b^6 - 6ab^4c + 9a^2b^2c^2)^2} \right)^{\frac{1}{3}} \arctan \left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3abc)^{\frac{1}{3}}} \right) - \left(\frac{c^6}{(b^6 - 6ab^4c + 9a^2b^2c^2)^2} \right)^{\frac{1}{3}} \log \left(4 \left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}} \right)^2 + 4 \left(cx + b + (-b^3 + 3abc)^{\frac{1}{3}} \right)^2 \right) + 2 \left(\frac{c^6}{(b^6 - 6ab^4c + 9a^2b^2c^2)^2} \right)^{\frac{1}{3}} \log \left(\left| cx + b + (-b^3 + 3abc)^{\frac{1}{3}} \right| \right) \right)}{54 (b^6 - 6ab^4c + 9a^2b^2c^2)} + \frac{5c^4x^4 + 20bc^3x^3 + 30b^2c^2x^2 + 12b^3cx + 24abc^2x - 3b^4 + 24ab^2c}{18 (b^6 - 6ab^4c + 9a^2b^2c^2) (c^2x^3 + 3bcx^2 + 3b^2x + 3ab)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")

$$[Out] \frac{5}{54} * (2 * \sqrt{3}) * (c^6 / (b^6 - 6 * a * b^4 * c + 9 * a^2 * b^2 * c^2))^{1/3} * \arctan((\sqrt{3} * c * x + \sqrt{3} * b - \sqrt{3} * (-b^3 + 3 * a * b * c)^{1/3}) / (c * x + b + (-b^3 + 3 * a * b * c)^{1/3})) - (c^6 / (b^6 - 6 * a * b^4 * c + 9 * a^2 * b^2 * c^2))^{1/3} * \log(4 * (\sqrt{3} * c * x + \sqrt{3} * b - \sqrt{3} * (-b^3 + 3 * a * b * c)^{1/3})^2 + 4 * (c * x + b + (-b^3 + 3 * a * b * c)^{1/3})^2) + 2 * (c^6 / (b^6 - 6 * a * b^4 * c + 9 * a^2 * b^2 * c^2))^{1/3} * \log(\text{abs}(c * x + b + (-b^3 + 3 * a * b * c)^{1/3}))) / (b^6 - 6 * a * b^4 * c + 9 * a^2 * b^2 * c^2) + 1 / 18 * (5 * c^4 * x^4 + 20 * b * c^3 * x^3 + 30 * b^2 * c^2 * x^2 + 12 * b^3 * c * x + 24 * a * b * c^2 * x - 3 * b^4 + 24 * a * b^2 * c) / ((b^6 - 6 * a * b^4 * c + 9 * a^2 * b^2 * c^2) * (c^2 * x^3 + 3 * b * c * x^2 + 3 * b^2 * x + 3 * a * b)^2)$$

maple [C] time = 0.02, size = 276, normalized size = 0.90

$$\frac{5c^4 \ln(-\text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab) + x)}{27(9a^2c^2 - 6ab^2c + b^4)b^2 \left(\text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab)^2 c^2 + 2 \text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab)bc + b^2 \right)} + \frac{\frac{5c^4 x^4}{18(9a^2c^2 - 6ab^2c + b^4)b^2} + \frac{10c^3 x^3}{9(9a^2c^2 - 6ab^2c + b^4)b} + \frac{5c^2 x^2}{3(9a^2c^2 - 6ab^2c + b^4)b} + \frac{2(2ac + b^2)cx}{3(9a^2c^2 - 6ab^2c + b^4)b} + \frac{8ac - b^2}{54a^2c^2 - 36ab^2c + 6a^4}}{(c^2x^3 + 3bcx^2 + 3b^2x + 3ab)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)

$$[Out] \frac{5}{18} * c^4 / b^2 / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) * x^4 + 10 / 9 / b * c^3 / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) * x^3 + 5 / 3 * c^2 / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) * x^2 + 2 / 3 / b * (2 * a * c + b^2) * c / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) * x + 1 / 6 * (8 * a * c - b^2) / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) / (c^2 * x^3 + 3 * b * c * x^2 + 3 * b^2 * x + 3 * a * b)^2 + 5 / 27 * c^2 / b^2 / (9 * a^2 * c^2 - 6 * a * b^2 * c + b^4) * \text{sum}(1 / (R^2 * c^2 + 2 * R * b * c + b^2) * \ln(-R + x), R = \text{RootOf}(Z^3 * c^2 + 3 * Z^2 * b * c + 3 * Z * b^2 + 3 * a * b))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \sqrt{3} \arctan \left(\frac{\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}}}{cx + b + (-b^3 + 3abc)^{\frac{1}{3}}} \right) - \left(\frac{c^6}{(b^6 - 6ab^4c + 9a^2b^2c^2)^2} \right)^{\frac{1}{3}} \log \left(4 \left(\sqrt{3}cx + \sqrt{3}b - \sqrt{3}(-b^3 + 3abc)^{\frac{1}{3}} \right)^2 + 4 \left(cx + b + (-b^3 + 3abc)^{\frac{1}{3}} \right)^2 \right) + 2 \left(\frac{c^6}{(b^6 - 6ab^4c + 9a^2b^2c^2)^2} \right)^{\frac{1}{3}} \log \left(\left| cx + b + (-b^3 + 3abc)^{\frac{1}{3}} \right| \right)}{54 (b^6 - 6ab^4c + 9a^2b^2c^2)} + \frac{5c^4x^4 + 20bc^3x^3 + 30b^2c^2x^2 - 3b^4 + 24abc^2x + 12b^3cx + 24ab^2c}{18 (9a^2c^2 - 6ab^2c + b^4)b^2 \left((b^6 - 6ab^4c + 9a^2b^2c^2)^2 c^2 + 2 \text{RootOf}(c^2 Z^3 + 3bc Z^2 + 3b^2 Z + 3ab)bc + b^2 \right) + 15 (b^2c^3 - 6ab^2c^2 + 9a^2b^2c) + 6 (b^2c^3 - 6ab^2c^2 + 9a^2b^2c)^2 + 15 (b^2c^3 - 6ab^2c^2 + 9a^2b^2c)^3 + 6 (b^2c^3 - 17ab^2c^2 + 21a^2b^2c + 9a^2b^2c^3)^2 + 9 (b^6 - 6ab^4c - 3a^2b^2c^2 + 18a^2b^2c^3)^2 + 18 (ab^5 - 6a^2b^3c + 9a^2b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")

[Out] $\frac{5}{9}c^2 \int \frac{1}{(c^2x^3 + 3b^2cx^2 + 3b^2x + 3ab)^3} dx / (b^6 - 6a^2b^4c + 9a^2b^2c^2) + \frac{1}{18} \frac{(5c^4x^4 + 20b^2c^3x^3 + 30b^2c^2x^2 - 3b^4 + 24ab^2c + 12(b^3c + 2ab^2c^2)x)}{(9a^2b^8 - 54a^3b^6c + 81a^4b^4c^2 + (b^6c^4 - 6a^2b^4c^5 + 9a^2b^2c^6)x^6 + 6(b^7c^3 - 6a^2b^5c^4 + 9a^2b^3c^5)x^5 + 15(b^8c^2 - 6a^2b^6c^3 + 9a^2b^4c^4)x^4 + 6(3b^9c - 17a^2b^7c^2 + 21a^2b^5c^3 + 9a^3b^3c^4)x^3 + 9(b^{10} - 4a^2b^8c - 3a^2b^6c^2 + 18a^3b^4c^3)x^2 + 18(a^2b^9 - 6a^2b^7c + 9a^3b^5c^2)x}$

mupad [B] time = 2.98, size = 483, normalized size = 1.58

$$\frac{\frac{8ac^2}{9(b^2+18ac^2)} + \frac{5c^2}{3(9b^2+4a^2c^2)} + \frac{10c^2}{9(9b^2+4a^2c^2)} + \frac{5c^4}{18(9b^2+4a^2c^2)} + \frac{2c^2(b^2+2ac)}{21(9b^2+4a^2c^2)} \cdot \frac{5c^2 \ln(b(3ac-b)^{10} - b^{10} + cx(3ac-b)^{10} + 27b^2b^{10}c^2 - 27b^2b^{10}c^2 + 9ab^{10}c)}{27b^{10}(3ac-b)^{10}} \cdot \frac{5c^2 \ln(2b-b^{10}(3ac-b)^{10}) + 2cx - \sqrt{3}b^{10}(3ac-b)^{10}}{27b^{10}(3ac-b)^{10}} \left(\frac{1}{2} + \frac{\sqrt{3}a}{2}\right) + \frac{5c^2 \ln(2b-b^{10}(3ac-b)^{10}) + 2cx + \sqrt{3}b^{10}(3ac-b)^{10}}{27b^{10}(3ac-b)^{10}} \left(\frac{1}{2} + \frac{\sqrt{3}a}{2}\right)}{x^2(9b^4+18ac^2)+9b^2b^2+c^2x^2+z^2(18b^2c+6ab^2c)+6b^2c^2x^2+15b^2c^2x^2+18ab^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)

[Out] $\frac{(8ac - b^2)(6(b^4 + 9a^2c^2 - 6a^2b^2c)) + (5c^2x^2)(3(b^4 + 9a^2c^2 - 6a^2b^2c)) + (10c^3x^3)(9b(b^4 + 9a^2c^2 - 6a^2b^2c)) + (5c^4x^4)(18b^2(b^4 + 9a^2c^2 - 6a^2b^2c)) + (2c^2x(2ac + b^2))(3b(b^4 + 9a^2c^2 - 6a^2b^2c))}{(x^2(9b^4 + 18a^2b^2c) + 9a^2b^2 + c^4x^6 + x^3(18b^3c + 6a^2b^2c) + 6b^2c^3x^5 + 15b^2c^2x^4 + 18a^2b^3x) + (5c^2 \log(b(3ac - b^2)^{8/3} - b^{19/3} + cx(3ac - b^2)^{8/3} + 27a^3b^{1/3}c^3 - 27a^2b^{7/3}c^2 + 9a^2b^{13/3}c)) / (27b^{8/3}(3ac - b^2)^{8/3}) - (5c^2 \log(2b - b^{1/3})(3ac - b^2)^{1/3} + 2cx - 3^{1/2}b^{1/3}(3ac - b^2)^{1/3}i) * ((3^{1/2}i)/2 + 1/2)} / (27b^{8/3}(3ac - b^2)^{8/3}) + (5c^2 \log(2b - b^{1/3})(3ac - b^2)^{1/3} + 2cx + 3^{1/2}b^{1/3}(3ac - b^2)^{1/3}i) * ((3^{1/2}i)/2 - 1/2)} / (27b^{8/3}(3ac - b^2)^{8/3})$

sympy [A] time = 2.54, size = 474, normalized size = 1.55

$$\frac{2ab^2 - 3b^4 + 30b^2c^2x^2 + 20b^2c^3x^3 + 5c^4x^4 + x(2ab^2c^2 + 12b^3c)}{(1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4) + x^5(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3) + x^4(2430a^2b^4c^4 - 1620ab^6c^3 + 270b^8c^2) + x^3(972a^3b^3c^4 + 2268a^2b^5c^3 - 1836ab^7c^2 + 324b^9c) + x^2(2916a^3b^4c^3 - 486a^2b^6c^2 - 648ab^8c + 162b^{10}) + x(2916a^3b^5c^2 - 486a^2b^7c + 162b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)

[Out] $(24a^2b^2c - 3b^4 + 30b^2c^2x^2 + 20b^2c^3x^3 + 5c^4x^4 + x(24a^2b^2c^2 + 12b^3c)) / (1458a^4b^4c^2 - 972a^3b^6c + 162a^2b^8 + x^6(162a^2b^2c^6 - 108ab^4c^5 + 18b^6c^4) + x^5(972a^2b^3c^5 - 648ab^5c^4 + 108b^7c^3) + x^4(2430a^2b^4c^4 - 1620ab^6c^3 + 270b^8c^2) + x^3(972a^3b^3c^4 + 2268a^2b^5c^3 - 1836ab^7c^2 + 324b^9c) + x^2(2916a^3b^4c^3 - 486a^2b^6c^2 - 648ab^8c + 162b^{10}) + x(2916a^3b^5c^2 - 486a^2b^7c + 162b^9)$

$$c^{**2} - 1944*a^{**2}*b^{**7}*c + 324*a*b^{**9})) + \text{RootSum}(_t^{**3}*(129140163*a^{**8}*b^{**8} \\ *c^{**8} - 344373768*a^{**7}*b^{**10}*c^{**7} + 401769396*a^{**6}*b^{**12}*c^{**6} - 267846264*a \\ **5*b^{**14}*c^{**5} + 111602610*a^{**4}*b^{**16}*c^{**4} - 29760696*a^{**3}*b^{**18}*c^{**3} + 496 \\ 0116*a^{**2}*b^{**20}*c^{**2} - 472392*a*b^{**22}*c + 19683*b^{**24}) - 125*c^{**6}, \text{Lambda}(_ \\ t, _t*\log(x + (729*_t*a^{**3}*b^{**3}*c^{**3} - 729*_t*a^{**2}*b^{**5}*c^{**2} + 243*_t*a*b^{**} \\ 7*c - 27*_t*b^{**9} + 5*b*c^{**2})/(5*c^{**3}))))$$

3.15

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3 \right)^3 dx$$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8(5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7} + \frac{(a+bx)^7(-2adf + bcf + bde)(10a^2d^2f^2 - 10abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{b^7}$$

Rubi [A] time = 0.66, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{3df(a+bx)^8(5a^2d^2f^2 - 5abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{8b^7} + \frac{(a+bx)^7(-2adf + bcf + bde)(10a^2d^2f^2 - 10abdf(cf+de) + b^2(c^2f^2 + 3cdef + d^2e^2))}{b^7}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]

[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e - a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e - a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3 dx &= \int (a + bx)^3(c + dx)^3(e + fx)^3 dx \\
&= \int \left(\frac{(bc - ad)^3(be - af)^3(a + bx)^3}{b^6} + \frac{3(bc - ad)^3(a + bx)^2}{b^5} + \frac{3(bc - ad)^2(a + bx)}{b^4} + \frac{3(bc - ad)}{b^3} \right) dx \\
&= \frac{(bc - ad)^3(be - af)^3(a + bx)^4}{4b^7} + \frac{3(bc - ad)^3(a + bx)^3}{3b^5} + \frac{3(bc - ad)^2(a + bx)^2}{2b^4} + \frac{3(bc - ad)(a + bx)}{b^3} + \frac{3(bc - ad)x}{b^2} + \frac{3(bc - ad)}{b}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 653, normalized size = 1.81

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]

[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3 + ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3 + (b^3*d^3*f^3*x^10)/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]

[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3, x]

fricas [B] time = 0.87, size = 987, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3, x, algorithm="fricas")

[Out] $\frac{1}{10}x^{10}f^3d^3b^3 + \frac{1}{3}x^9f^2e^2d^3b^3 + \frac{1}{3}x^9f^3d^2c^2b^3 + \frac{1}{3}x^9f^3d^3b^2a + \frac{3}{8}x^8f^2e^2d^3b^3 + \frac{9}{8}x^8f^2e^2d^2c^2b^3 + \frac{3}{8}x^8f^3d^2c^2b^3 + \frac{9}{8}x^8f^2e^2d^3b^2a + \frac{9}{8}x^8f^3d^2c^2b^2a + \frac{3}{8}x^8f^3d^3b^2a^2 + \frac{1}{7}x^7f^2e^3d^3b^3 + \frac{9}{7}x^7f^2e^2d^2c^2b^3 + \frac{9}{7}x^7f^2e^2d^2c^2b^3 + \frac{1}{7}x^7f^3c^3b^3 + \frac{9}{7}x^7f^2e^2d^3b^2a + \frac{27}{7}x^7f^2e^2d^2c^2b^2a + \frac{9}{7}x^7f^3d^2c^2b^2a + \frac{9}{7}x^7f^2e^2d^3b^2a^2 + \frac{9}{7}x^7f^3d^2c^2b^2a^2 + \frac{1}{7}x^7f^3d^3a^3 + \frac{1}{2}x^6e^3d^2c^2b^3 + \frac{3}{2}x^6f^2e^2d^2c^2b^3 + \frac{1}{2}x^6f^2e^2c^3b^3 + \frac{1}{2}x^6e^3d^3b^2a + \frac{9}{2}x^6f^2e^2d^2c^2b^2a + \frac{9}{2}x^6f^2e^2d^2c^2b^2a + \frac{1}{2}x^6f^3c^3b^2a + \frac{3}{2}x^6f^2e^2d^3b^2a^2 + \frac{9}{2}x^6f^2e^2d^2c^2b^2a^2 + \frac{3}{2}x^6f^3d^2c^2b^2a^2 + \frac{1}{2}x^6f^2e^2d^3a^3 + \frac{1}{2}x^6f^3d^2c^2a^3 + \frac{3}{5}x^5e^3d^2c^2b^3 + \frac{3}{5}x^5f^2e^2c^3b^3 + \frac{9}{5}x^5e^3d^2c^2b^2a + \frac{27}{5}x^5f^2e^2d^2c^2b^2a + \frac{9}{5}x^5f^2e^2c^3b^2a + \frac{3}{5}x^5e^3d^3b^2a^2 + \frac{27}{5}x^5f^2e^2d^2c^2b^2a^2 + \frac{27}{5}x^5f^2e^2d^2c^2b^2a^2 + \frac{3}{5}x^5f^3c^3b^2a^2 + \frac{3}{5}x^5f^2e^2d^3a^3 + \frac{9}{5}x^5f^2e^2d^2c^2a^3 + \frac{3}{5}x^5f^3d^2c^2a^3 + \frac{1}{4}x^4e^3c^3b^3 + \frac{9}{4}x^4e^3d^2c^2b^2a + \frac{9}{4}x^4f^2e^2c^3b^2a + \frac{9}{4}x^4e^3d^2c^2b^2a^2 + \frac{27}{4}x^4f^2e^2d^2c^2b^2a^2 + \frac{9}{4}x^4f^2e^2c^3b^2a^2 + \frac{1}{4}x^4e^3d^3a^3 + \frac{9}{4}x^4f^2e^2d^2c^2a^3 + \frac{9}{4}x^4f^2e^2d^2c^2a^3 + \frac{1}{4}x^4f^3c^3a^3 + x^3e^3c^3b^2a + 3x^3e^3d^2c^2b^2a^2 + 3x^3f^2e^2c^3b^2a^2 + x^3e^3d^2c^2a^3 + 3x^3f^2e^2d^2c^2a^3 + x^3f^2e^2c^3a^3 + \frac{3}{2}x^2e^3c^3b^2a^2 + \frac{3}{2}x^2e^3d^2c^2a^3 + \frac{3}{2}x^2f^2e^2c^3a^3 + xe^3c^3a^3$

giac [B] time = 0.27, size = 971, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3, x, algorithm="giac")

[Out] $\frac{1}{10}b^3d^3f^3x^{10} + \frac{1}{3}b^3c^2d^2f^3x^9 + \frac{1}{3}a^2b^2d^3f^3x^9 + \frac{1}{3}b^3d^3f^2x^9e + \frac{3}{8}b^3c^2d^2f^3x^8 + \frac{9}{8}a^2b^2c^2d^2f^3x^8 + \frac{3}{8}a^2b^2d^3f^3x^8 + \frac{9}{8}b^3c^2d^2f^2x^8e + \frac{9}{8}a^2b^2d^3f^2x^8e + \frac{1}{7}b^3c^3f^3x^7 + \frac{9}{7}a^2b^2c^2d^2f^3x^7 + \frac{9}{7}a^2b^2c^2d^2f^3x^7 + \frac{1}{7}a^3d^3f^3x^7 + \frac{3}{8}b^3d^3f^2x^8e^2 + \frac{9}{7}b^3c^2d^2f^2x^7e + \frac{27}{7}a^2$

$$\begin{aligned}
& b^2*c*d^2*f^2*x^7*e + 9/7*a^2*b*d^3*f^2*x^7*e + 1/2*a*b^2*c^3*f^3*x^6 + 3/2 \\
& *a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 9/7*b^3*c*d^2*f*x^7*e^2 + 9/ \\
& 7*a*b^2*d^3*f*x^7*e^2 + 1/2*b^3*c^3*f^2*x^6*e + 9/2*a*b^2*c^2*d*f^2*x^6*e + \\
& 9/2*a^2*b*c*d^2*f^2*x^6*e + 1/2*a^3*d^3*f^2*x^6*e + 3/5*a^2*b*c^3*f^3*x^5 \\
& + 3/5*a^3*c^2*d*f^3*x^5 + 1/7*b^3*d^3*x^7*e^3 + 3/2*b^3*c^2*d*f*x^6*e^2 + 9 \\
& /2*a*b^2*c*d^2*f*x^6*e^2 + 3/2*a^2*b*d^3*f*x^6*e^2 + 9/5*a*b^2*c^3*f^2*x^5* \\
& e + 27/5*a^2*b*c^2*d*f^2*x^5*e + 9/5*a^3*c*d^2*f^2*x^5*e + 1/4*a^3*c^3*f^3*x \\
& x^4 + 1/2*b^3*c*d^2*x^6*e^3 + 1/2*a*b^2*d^3*x^6*e^3 + 3/5*b^3*c^3*f*x^5*e^2 \\
& + 27/5*a*b^2*c^2*d*f*x^5*e^2 + 27/5*a^2*b*c*d^2*f*x^5*e^2 + 3/5*a^3*d^3*f* \\
& x^5*e^2 + 9/4*a^2*b*c^3*f^2*x^4*e + 9/4*a^3*c^2*d*f^2*x^4*e + 3/5*b^3*c^2*d \\
& *x^5*e^3 + 9/5*a*b^2*c*d^2*x^5*e^3 + 3/5*a^2*b*d^3*x^5*e^3 + 9/4*a*b^2*c^3* \\
& f*x^4*e^2 + 27/4*a^2*b*c^2*d*f*x^4*e^2 + 9/4*a^3*c*d^2*f*x^4*e^2 + a^3*c^3*f \\
& f^2*x^3*e + 1/4*b^3*c^3*x^4*e^3 + 9/4*a*b^2*c^2*d*x^4*e^3 + 9/4*a^2*b*c*d^2 \\
& *x^4*e^3 + 1/4*a^3*d^3*x^4*e^3 + 3*a^2*b*c^3*f*x^3*e^2 + 3*a^3*c^2*d*f*x^3* \\
& e^2 + a*b^2*c^3*x^3*e^3 + 3*a^2*b*c^2*d*x^3*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a \\
& ^3*c^3*f*x^2*e^2 + 3/2*a^2*b*c^3*x^2*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^3* \\
& x*e^3
\end{aligned}$$

maple [B] time = 0.00, size = 861, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)$

[Out] $1/10*b^3*d^3*f^3*x^{10}+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8*((a*c*f+a$
 $*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e$
 $+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7*(a*c*e*b^2*d^2*f^2+2*(a*c*f+a$
 $*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b$
 $*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c$
 $e)*(a*d*f+b*c*f+b*d*e))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+$
 $a*d*e+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c$
 $*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*($
 $2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)*x^6+1/5*(a*c*e*(2*(a*c$
 $f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b$
 $d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(2*a*c*e$
 $*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c$
 $e))*x^5+1/4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)$
 $)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2$
 $*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+b*d*f*a^2*c^2*e^2)*x^4+1/3*($
 $a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*(a*c*f+a*d*e+b$
 $c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*a^2*c^2*e^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a$
 $d*e+b*c*e)*x^2+a^3*c^3*e^3*x$

maxima [A] time = 0.68, size = 461, normalized size = 1.28

$\frac{1}{10} b^3 d^3 f^3 x^{10} + \frac{1}{3} (b^2 d^2 f^2 x^9 + \frac{1}{3} (b^3 c^3 e^3 x^8 + \frac{1}{7} (b^2 d^2 f^2 x^7 + \frac{1}{4} (3 b^2 d^2 f^2 x^4 + 4 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^7 + 28 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^6) * (b^2 c e + a d e + a c f)) * a^2 c^2 e^2 + 1/4 * (b^2 c e + a d e + a c f)^3 x^4 + 1/70 * (30 b^2 d^2 f^2 x^7 + 70 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^6 + 42 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^5 + 70 (b^2 c e + a d e + a c f)^2 x^3 + 21 (4 b^2 d^2 f^2 x^5 + 5 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^4) * (b^2 c e + a d e + a c f)) * a^2 c^2 e^2 + 1/10 * (5 b^2 d^2 f^2 x^6 + 6 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^5) * (b^2 c e + a d e + a c f)^2 + 1/56 * (2 b^2 d^2 f^2 x^8 + 48 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^7 + 28 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^6) * (b^2 c e + a d e + a c f)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a^2*c^2*e^2 + 1/10*(5*b*d*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(2*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2)*x^6)*(b*c*e + a*d*e + a*c*f)
```

mupad [B] time = 2.23, size = 787, normalized size = 2.18

$\frac{1}{10} b^3 d^3 f^3 x^{10} + \frac{1}{3} (b^2 d^2 f^2 x^9 + \frac{1}{3} (b^3 c^3 e^3 x^8 + \frac{1}{7} (b^2 d^2 f^2 x^7 + \frac{1}{4} (3 b^2 d^2 f^2 x^4 + 4 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^7 + 28 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^6) * (b^2 c e + a d e + a c f)) * a^2 c^2 e^2 + 1/4 * (b^2 c e + a d e + a c f)^3 x^4 + 1/70 * (30 b^2 d^2 f^2 x^7 + 70 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^6 + 42 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^5 + 70 (b^2 c e + a d e + a c f)^2 x^3 + 21 (4 b^2 d^2 f^2 x^5 + 5 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^4) * (b^2 c e + a d e + a c f)) * a^2 c^2 e^2 + 1/10 * (5 b^2 d^2 f^2 x^6 + 6 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^5) * (b^2 c e + a d e + a c f)^2 + 1/56 * (2 b^2 d^2 f^2 x^8 + 48 (b^2 d^2 e f + (b^2 c d + a b d^2) f^2) x^7 + 28 (b^2 d^2 e^2 + 2 (b^2 c d + a b d^2) e f + (b^2 c^2 + 2 a b c d + a^2 d^2) f^2) x^6) * (b^2 c e + a d e + a c f)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^3,x)
```

```
[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5 + (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*d^2*e^2*f)/4 + (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2
```

$$\frac{b^2 c^2 e^2 + 3 a b c d e^2 + 3 a^2 b c^2 e f + 3 a^2 c d e f + (3 b d f x^8 (a^2 d^2 f^2 + b^2 c^2 f^2 + b^2 d^2 e^2 + 3 a b c d f^2 + 3 a b d^2 e f + 3 b^2 c d e f))}{8}$$

sympy [B] time = 0.25, size = 1018, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**3,x)

[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b**2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 + 3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 + 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 + 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 + 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)

3.16

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3 \right)^2 dx$$

Optimal. Leaf size=193

$$\frac{(a + bx)^5 (6a^2d^2f^2 - 6abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2))}{5b^5} + \frac{df(a + bx)^6(-2adf + bcf + bde)}{3b^5} + \frac{(a + bx)^4(bde + bcf + adf)}{3b^5}$$

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{(a + bx)^5 (6a^2d^2f^2 - 6abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2))}{5b^5} + \frac{df(a + bx)^6(-2adf + bcf + bde)}{3b^5} + \frac{(a + bx)^4(bc - ad)(be - af)(-2adf + bcf + bde)}{2b^5} + \frac{(a + bx)^3(bc - ad)^2(be - af)^2}{3b^5} + \frac{d^2f^2(a + bx)^7}{7b^5}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2,x]

[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5) + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)^7)/(7*b^5)

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2 dx = \int (a + bx)^2(c + dx)^2(e + fx)^2 dx$$

$$= \int \left(\frac{(bc - ad)^2(be - af)^2(a + bx)^2}{b^4} + \frac{2(bc - ad)(bde + bcf + adf)(a + bx)}{b^3} + \frac{b^2d^2f^2x^2}{b^2} \right) dx$$

$$= \frac{(bc - ad)^2(be - af)^2(a + bx)^3}{3b^5} + \frac{(bc - ad)(bde + bcf + adf)(a + bx)^2}{2b^3} + \frac{b^2d^2f^2x^3}{3b^2}$$

Mathematica [A] time = 0.09, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5 (a^2d^2f^2 + 4abdf(cf + de) + b^2(c^2f^2 + 4cdef + d^2e^2)) + \frac{1}{2}x^4 (a^2df(cf + de) + ab(c^2f^2 + 4cdef + d^2e^2) + b^2ce(cf + de)) + \frac{1}{3}x^3 (a^2(c^2f^2 + 4cdef + d^2e^2) + 4abce(cf + de) + b^2c^2e^2) + a^2c^2e^2x + \frac{1}{3}bdfx^6(adf + bcf + bde) + acex^2(acf + ade + bce) + \frac{1}{2}b^2d^2f^2x^7$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2, x]

[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2, x]

[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2, x]

fricas [A] time = 0.83, size = 346, normalized size = 1.79

$$\frac{1}{5}x^5 f^2 a^2 e^2 + \frac{1}{3}x^4 f a^2 e^2 + \frac{1}{3}x^4 f^2 a d c e^2 + \frac{1}{3}x^4 f^2 a^2 b e + \frac{1}{5}x^4 b^2 a^2 e^2 + \frac{4}{5}x^4 f^2 a d e^2 + \frac{1}{5}x^4 f^2 c^2 e^2 + \frac{4}{5}x^4 f^2 a d b e + \frac{1}{5}x^4 f^2 a^2 e^2 + \frac{1}{2}x^4 c^2 a d e^2 + \frac{1}{2}x^4 f a^2 e^2 + \frac{1}{2}x^4 f^2 a d e^2 + \frac{1}{3}x^4 f^2 c^2 e^2 + \frac{4}{5}x^4 f^2 a d b e + \frac{4}{5}x^4 f^2 a^2 e^2 + \frac{1}{3}x^4 b^2 a^2 e^2 + \frac{4}{5}x^4 f^2 a d e^2 + \frac{1}{5}x^4 f^2 c^2 e^2 + x^2 c^2 d e^2 + x^2 f c^2 e^2 + x^2 c^2 e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2, x, algorithm="fricas")


```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*x
+ 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e +
a*d*e + a*c*f)
```

mupad [B] time = 0.08, size = 270, normalized size = 1.40

$$x^4 \left(\frac{a^2 c d f^2}{2} + \frac{a^2 d^2 e f}{2} + \frac{a b c^2 f^2}{2} + 2 a b c d e f + \frac{a b d^2 e^2}{2} + \frac{b^2 c^2 e f}{2} + \frac{b^2 c d e^2}{2} \right) + x^3 \left(\frac{a^2 c^2 f^2}{3} + \frac{4 a^2 c d e f}{3} + \frac{a^2 d^2 e^2}{3} + \frac{4 a b c^2 e f}{3} + \frac{4 a b c d e^2}{3} + \frac{b^2 c^2 e^2}{3} \right) + x^2 \left(\frac{a^2 d^2 f^2}{5} + \frac{4 a b c d f^2}{5} + \frac{4 a b d^2 e f}{5} + \frac{b^2 c^2 f^2}{5} + \frac{4 b^2 c d e f}{5} + \frac{b^2 d^2 e^2}{5} \right) + a^2 c^2 e^2 x + \frac{b^2 d^2 f^2 x^2}{7} + a c e^2 (a c f + a d e + b c e) + \frac{b d f^2 (a d f + b c f + b d e)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^2,x)
```

```
[Out] x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
+ (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)/3
+ (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/
3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*
e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c
^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f
*x^6*(a*d*f + b*c*f + b*d*e))/3
```

sympy [A] time = 0.14, size = 345, normalized size = 1.79

$$a^2 c^2 e^2 x + \frac{b^2 d^2 f^2 x^2}{7} + x^2 \left(\frac{a b c^2 f^2}{3} + \frac{b^2 c d f^2}{3} + \frac{b^2 d^2 e f}{3} \right) + x^3 \left(\frac{a^2 d^2 f^2}{5} + \frac{4 a b c d f^2}{5} + \frac{4 a b d^2 e f}{5} + \frac{b^2 c^2 f^2}{5} + \frac{4 b^2 c d e f}{5} + \frac{b^2 d^2 e^2}{5} \right) + x \left(\frac{a^2 c d f^2}{2} + \frac{a^2 d^2 e f}{2} + \frac{a b c^2 f^2}{2} + 2 a b c d e f + \frac{a b d^2 e^2}{2} + \frac{b^2 c^2 e f}{2} + \frac{b^2 c d e^2}{2} \right) + x^2 \left(\frac{a^2 c^2 f^2}{3} + \frac{4 a^2 c d e f}{3} + \frac{a^2 d^2 e^2}{3} + \frac{4 a b c^2 e f}{3} + \frac{4 a b c d e^2}{3} + \frac{b^2 c^2 e^2}{3} \right) + x^2 (a^2 c^2 e f + a^2 c d e^2 + a b c^2 e^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

3.17

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx$$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3,x]

[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3 + (b*d*f*x^4)/4

Rubi steps

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{4}bdfx^4$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.36

$$acex + \frac{1}{2}acf x^2 + \frac{1}{2}adex^2 + \frac{1}{3}adf x^3 + \frac{1}{2}bcex^2 + \frac{1}{3}bcf x^3 + \frac{1}{3}bdex^3 + \frac{1}{4}bdf x^4$$

Antiderivative was successfully verified.

[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3,x]

[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3, x]

[Out] IntegrateAlgebraic[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3, x]

fricas [A] time = 0.95, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x, algorithm="fricas")

[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1/2*x^2*e*c*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a

giac [A] time = 0.23, size = 54, normalized size = 0.96

$$\frac{1}{4}bdfx^4 + \frac{1}{3}(bcf + adf + bde)x^3 + acxe + \frac{1}{2}(acf + bce + ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x, algorithm="giac")

[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*c*e + a*d*e)*x^2

maple [A] time = 0.00, size = 51, normalized size = 0.91

$$\frac{bdfx^4}{4} + acex + \frac{(adf + bcf + bde)x^3}{3} + \frac{(acf + ade + bce)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)

[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x^4

maxima [A] time = 0.83, size = 50, normalized size = 0.89

$$\frac{1}{4}bdfx^4 + acex + \frac{1}{3}(bde + bcf + adf)x^3 + \frac{1}{2}(bce + ade + acf)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")

[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*d*e + a*c*f)*x^2

mupad [B] time = 0.04, size = 54, normalized size = 0.96

$$\frac{bdfx^4}{4} + \left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right)x^3 + \left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)x^2 + acex$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3,x)

[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d*e)/3) + a*c*e*x + (b*d*f*x^4)/4

sympy [A] time = 0.07, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x)

[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2 + a*d*e/2 + b*c*e/2)

$$3.18 \quad \int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx$$

Optimal. Leaf size=86

$$\frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx = \int \left(\frac{b^2}{(bc - ad)(be - af)(a + bx)} + \frac{f}{(bc - ad)(de - cf)} \right) dx$$

$$= \frac{b \log(a + bx)}{(bc - ad)(be - af)} - \frac{d \log(c + dx)}{(bc - ad)(de - cf)} + \frac{f \log(e + fx)}{(be - af)(de - cf)}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.93

$$\frac{b \log(a + bx)(cf - de) + d(be - af) \log(c + dx) + f(ad - bc) \log(e + fx)}{(bc - ad)(be - af)(cf - de)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-1), x]

fricas [A] time = 12.86, size = 112, normalized size = 1.30

$$\frac{(bc - ad)f \log(fx + e) + (bde - bcf) \log(bx + a) - (bde - adf) \log(dx + c)}{(b^2cd - abd^2)e^2 - (b^2c^2 - a^2d^2)ef + (abc^2 - a^2cd)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3), x, algorithm="fricas")

[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c^2 - a^2*c*d)*f^2)

giac [A] time = 0.31, size = 137, normalized size = 1.59

$$-\frac{b^2 \log(|bx + a|)}{ab^2cf - a^2bdf - b^3ce + ab^2de} + \frac{d^2 \log(|dx + c|)}{bc^2df - acd^2f - bcd^2e + ad^3e} + \frac{f^2 \log(|fx + e|)}{acf^3 - bcf^2e - adf^2e + bdf^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3), x, algorithm="giac")

[Out] -b^2*log(abs(b*x + a))/(a*b^2*c*f - a^2*b*d*f - b^3*c*e + a*b^2*d*e) + d^2*log(abs(d*x + c))/(b*c^2*d*f - a*c*d^2*f - b*c*d^2*e + a*d^3*e) + f^2*log(abs(f*x + e))/(a*c*f^3 - b*c*f^2*e - a*d*f^2*e + b*d*f*e^2)

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{b \ln(bx + a)}{(af - be)(ad - bc)} - \frac{d \ln(dx + c)}{(cf - de)(ad - bc)} + \frac{f \ln(fx + e)}{(cf - de)(af - be)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)

[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)/(a*d-b*c)*ln(b*x+a)

maxima [A] time = 0.65, size = 112, normalized size = 1.30

$$\frac{b \log(bx + a)}{(b^2c - abd)e - (abc - a^2d)f} - \frac{d \log(dx + c)}{(bcd - ad^2)e - (bc^2 - acd)f} + \frac{f \log(fx + e)}{bde^2 + acf^2 - (bc + ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x, algorithm="maxima")

[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)

mupad [B] time = 2.33, size = 106, normalized size = 1.23

$$\frac{b \ln(a + bx)}{b^2ce + a^2df - abc f - abde} + \frac{d \ln(c + dx)}{ad^2e + bc^2f - acdf - bcde} + \frac{f \ln(e + fx)}{acf^2 + bde^2 - ade f - bcef}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3),x)

[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3),x)
```

```
[Out] Timed out
```

$$3.19 \quad \int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx$$

Optimal. Leaf size=234

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} + \frac{2d^3 \log(c+dx)(-2adf+bcf+bde)}{(bc-ad)^3(de-cf)^3}$$

Rubi [A] time = 0.41, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} + \frac{2d^3 \log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3} - \frac{f^3}{(e+fx)(be-af)^2(de-cf)^2} + \frac{2f^3 \log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2} - \frac{2b^4}{(bc-ad)^2} \right) dx$$

$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{2b^4}{(bc-ad)^2}$$

Mathematica [A] time = 0.55, size = 232, normalized size = 0.99

$$\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2} - \frac{2b^3 \log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3} - \frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2} - \frac{2d^3 \log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(cf-de)^3} - \frac{f^3}{(e+fx)(be-af)^2(de-cf)^2} - \frac{2f^3 \log(e+fx)(adf+bcf-2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

[Out] $-(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b*d*e + b*c*f - 2*a*d*f)*\text{Log}[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^3*(b*d*e - 2*b*c*f + a*d*f)*\text{Log}[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3) - (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*\text{Log}[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.39, size = 1414, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x, algorithm="giac")

[Out] $2*(b^5*c*f - 2*a*b^4*d*f + b^5*d*e)*\text{log}(\text{abs}(b*x + a))/(a^3*b^4*c^3*f^3 - 3*a^4*b^3*c^2*d*f^3 + 3*a^5*b^2*c*d^2*f^3 - a^6*b*d^3*f^3 - 3*a^2*b^5*c^3*f^2*e + 9*a^3*b^4*c^2*d*f^2*e - 9*a^4*b^3*c*d^2*f^2*e + 3*a^5*b^2*d^3*f^2*e + 3*a*b^6*c^3*f*e^2 - 9*a^2*b^5*c^2*d*f*e^2 + 9*a^3*b^4*c*d^2*f*e^2 - 3*a^4*b^3*d^3*f*e^2 - b^7*c^3*e^3 + 3*a*b^6*c^2*d*e^3 - 3*a^2*b^5*c*d^2*e^3 + a^3*$

$$\begin{aligned}
& b^4 d^3 e^3) + 2*(2*b*c*d^4*f - a*d^5*f - b*d^5*e)*\log(\text{abs}(d*x + c))/(b^3*c \\
& ^6*d*f^3 - 3*a*b^2*c^5*d^2*f^3 + 3*a^2*b*c^4*d^3*f^3 - a^3*c^3*d^4*f^3 - 3* \\
& b^3*c^5*d^2*f^2*e + 9*a*b^2*c^4*d^3*f^2*e - 9*a^2*b*c^3*d^4*f^2*e + 3*a^3*c \\
& ^2*d^5*f^2*e + 3*b^3*c^4*d^3*f*e^2 - 9*a*b^2*c^3*d^4*f*e^2 + 9*a^2*b*c^2*d^ \\
& 5*f*e^2 - 3*a^3*c*d^6*f*e^2 - b^3*c^3*d^4*e^3 + 3*a*b^2*c^2*d^5*e^3 - 3*a^2 \\
& *b*c*d^6*e^3 + a^3*d^7*e^3) - 2*(b*c*f^5 + a*d*f^5 - 2*b*d*f^4*e)*\log(\text{abs}(f \\
& *x + e))/(a^3*c^3*f^7 - 3*a^2*b*c^3*f^6*e - 3*a^3*c^2*d*f^6*e + 3*a*b^2*c^3 \\
& *f^5*e^2 + 9*a^2*b*c^2*d*f^5*e^2 + 3*a^3*c*d^2*f^5*e^2 - b^3*c^3*f^4*e^3 - \\
& 9*a*b^2*c^2*d*f^4*e^3 - 9*a^2*b*c*d^2*f^4*e^3 - a^3*d^3*f^4*e^3 + 3*b^3*c^2 \\
& *d*f^3*e^4 + 9*a*b^2*c*d^2*f^3*e^4 + 3*a^2*b*d^3*f^3*e^4 - 3*b^3*c*d^2*f^2* \\
& e^5 - 3*a*b^2*d^3*f^2*e^5 + b^3*d^3*f*e^6) - (2*b^3*c^2*d*f^3*x^2 - 2*a*b^2 \\
& *c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 - 2*b^3*c*d^2*f^2*x^2*e - 2*a*b^2*d^3* \\
& f^2*x^2*e + 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3 \\
& *d^3*f^3*x + 2*b^3*d^3*f*x^2*e^2 - b^3*c^2*d*f^2*x*e - a^2*b*d^3*f^2*x*e + \\
& a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3 - b^3*c*d^2*f*x*e^2 - a*b \\
& ^2*d^3*f*x*e^2 + b^3*c^3*f^2*e + a^3*d^3*f^2*e + 2*b^3*d^3*x*e^3 - 2*b^3*c^ \\
& 2*d*f*e^2 - 2*a^2*b*d^3*f*e^2 + b^3*c*d^2*e^3 + a*b^2*d^3*e^3)/((a^2*b^2*c^ \\
& 4*f^4 - 2*a^3*b*c^3*d*f^4 + a^4*c^2*d^2*f^4 - 2*a*b^3*c^4*f^3*e + 2*a^2*b^2 \\
& *c^3*d*f^3*e + 2*a^3*b*c^2*d^2*f^3*e - 2*a^4*c*d^3*f^3*e + b^4*c^4*f^2*e^2 \\
& + 2*a*b^3*c^3*d*f^2*e^2 - 6*a^2*b^2*c^2*d^2*f^2*e^2 + 2*a^3*b*c*d^3*f^2*e^2 \\
& + a^4*d^4*f^2*e^2 - 2*b^4*c^3*d*f*e^3 + 2*a*b^3*c^2*d^2*f*e^3 + 2*a^2*b^2* \\
& c*d^3*f*e^3 - 2*a^3*b*d^4*f*e^3 + b^4*c^2*d^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2 \\
& *b^2*d^4*e^4)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b* \\
& c*x*e + a*d*x*e + a*c*e)
\end{aligned}$$

maple [A] time = 0.03, size = 398, normalized size = 1.70

$$\frac{4a^3b^2d^3\ln(bx+a) + 2a^2d^4f\ln(dx+c) - 2ad^5f\ln(fx+e) - 2b^3c^2f\ln(bx+a) - 2b^3d^4e\ln(bx+a) - 4bc^2d^2f\ln(dx+c) - 2bc^2f\ln(fx+e) + 2bd^3e\ln(dx+c) + 4bdc^2f\ln(fx+e) - \frac{b^3}{(af-bc)^2(ad-bc)^2(bx+a)} - \frac{d^3}{(cf-de)^2(ad-bc)^2(dx+c)} - \frac{f^3}{(cf-de)^2(af-bc)^2(fx+e)}}{(af-bc)^3(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)

[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e) *a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*c+4*f^3/(c*f-d*e)^3/(a*f-b* e)^3*ln(f*x+e)*b*d*e-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/ (a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d ^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*e-b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a)+4*b^3/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c) ^3*ln(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*d*e

maxima [B] time = 1.54, size = 2096, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x, algorithm="maxima")

[Out]
$$-2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*\log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*\log(d*x + c)/((b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)*f^4)*\log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d - 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)$$

mupad [B] time = 8.18, size = 1940, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^2, x)$

[Out]
$$- \left((a^2*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*d*e^2*f) / (a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 + b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2)) / (a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3 - 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3*c*d^2*e^2*f + b^3*c^2*d*e*f^2)) / (a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) / (x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (\log(a + b*x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f)) / (b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (\log(c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f)) / (a^3*d^6*e^3 + b^3*c^6*f^3 - a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 - 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) - (\log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3)) / (a^3*c^3*f^6 + b^3*d^3*e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*f^4)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**2,x)
```

```
[Out] Timed out
```

$$3.20 \quad \int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3} dx$$

Optimal. Leaf size=495

$$\frac{3f^5 \log(e + fx) (2a^2d^2f^2 - abdf(7de - 3cf) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(be - af)^5(de - cf)^5} - \frac{3d^5 \log(c + dx) (2a^2d^2f^2 + abdf(7de - 3cf) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(bc - ad)^5(de - cf)^5}$$

Rubi [A] time = 1.46, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2058}

$$\frac{3f^5 \log(e + fx) (2a^2d^2f^2 - abdf(7de - 3cf) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(be - af)^5(de - cf)^5} - \frac{3d^5 \log(c + dx) (2a^2d^2f^2 + abdf(7de - 3cf) + b^2(2c^2f^2 - 7cdef + 7d^2e^2))}{(bc - ad)^5(de - cf)^5}$$

Antiderivative was successfully verified.

[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3), x]

[Out]
$$-b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 - a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5)$$

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3} dx = \int \left(\frac{b^6}{(bc - ad)^3(be - af)^3(a + bx)^3} - \frac{3b^5}{(bc - ad)^4} \right) dx$$

$$= -\frac{b^5}{2(bc - ad)^3(be - af)^3(a + bx)^2} + \frac{3b^5}{(bc - ad)^4}$$

Mathematica [A] time = 1.19, size = 490, normalized size = 0.99

$$\frac{1}{2} \left(\frac{b^5 \log(a+fx) (2d^2 f^2 + abd(3c-7d) + d^2(2f^2-7df+2d^2))}{(b^2c-ad^2)(b^2c-df^2)} + \frac{b^5 \log(a+dx) (2d^2 f^2 + abd(3d-7f) + d^2(2f^2-7df+2d^2))}{(b^2c-ad^2)(b^2c-df^2)} + \frac{b^5 \log(a+bx) (2d^2 f^2 - 7abd(f+d) + d^2(2f^2+3df+2d^2))}{(b^2c-ad^2)(b^2c-df^2)} + \frac{b^5 (-2df+bf+bd)}{(a+bx)(b^2c-ad^2)(b^2c-df^2)} + \frac{d^5}{(a+bx)(b^2c-ad^2)(b^2c-df^2)} + \frac{b^5(adf-2bf+bd)}{(c+dx)(b^2c-ad^2)(b^2c-df^2)} + \frac{d^5}{(c+dx)(b^2c-ad^2)(b^2c-df^2)} + \frac{b^5(adf+bf-2bd)}{(e+fx)(b^2c-ad^2)(b^2c-df^2)} + \frac{f^5}{(e+fx)(b^2c-ad^2)(b^2c-df^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3), x]

[Out]
$$\begin{aligned} & -(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f - 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) \\ & + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2))*\text{Log}[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f^2))*\text{Log}[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*\text{Log}[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2 \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3), x]

[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(-3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.10, size = 6908, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^8*c^2*f^2 - 7*a*b^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2 + 3*b^8*c*d*f*e - 7
*a*b^7*d^2*f*e + 2*b^8*d^2*e^2)*log(abs(b*x + a))/(a^5*b^6*c^5*f^5 - 5*a^6*
b^5*c^4*d*f^5 + 10*a^7*b^4*c^3*d^2*f^5 - 10*a^8*b^3*c^2*d^3*f^5 + 5*a^9*b^2
*c*d^4*f^5 - a^10*b*d^5*f^5 - 5*a^4*b^7*c^5*f^4*e + 25*a^5*b^6*c^4*d*f^4*e
- 50*a^6*b^5*c^3*d^2*f^4*e + 50*a^7*b^4*c^2*d^3*f^4*e - 25*a^8*b^3*c*d^4*f^
4*e + 5*a^9*b^2*d^5*f^4*e + 10*a^3*b^8*c^5*f^3*e^2 - 50*a^4*b^7*c^4*d*f^3*e
^2 + 100*a^5*b^6*c^3*d^2*f^3*e^2 - 100*a^6*b^5*c^2*d^3*f^3*e^2 + 50*a^7*b^4
*c*d^4*f^3*e^2 - 10*a^8*b^3*d^5*f^3*e^2 - 10*a^2*b^9*c^5*f^2*e^3 + 50*a^3*b
^8*c^4*d*f^2*e^3 - 100*a^4*b^7*c^3*d^2*f^2*e^3 + 100*a^5*b^6*c^2*d^3*f^2*e^
3 - 50*a^6*b^5*c*d^4*f^2*e^3 + 10*a^7*b^4*d^5*f^2*e^3 + 5*a*b^10*c^5*f*e^4
- 25*a^2*b^9*c^4*d*f*e^4 + 50*a^3*b^8*c^3*d^2*f*e^4 - 50*a^4*b^7*c^2*d^3*f*
e^4 + 25*a^5*b^6*c*d^4*f*e^4 - 5*a^6*b^5*d^5*f*e^4 - b^11*c^5*e^5 + 5*a*b^1
0*c^4*d*e^5 - 10*a^2*b^9*c^3*d^2*e^5 + 10*a^3*b^8*c^2*d^3*e^5 - 5*a^4*b^7*c
*d^4*e^5 + a^5*b^6*d^5*e^5) + 3*(7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^
2*d^8*f^2 - 7*b^2*c*d^7*f*e + 3*a*b*d^8*f*e + 2*b^2*d^8*e^2)*log(abs(d*x +
c))/(b^5*c^10*d*f^5 - 5*a*b^4*c^9*d^2*f^5 + 10*a^2*b^3*c^8*d^3*f^5 - 10*a^3
*b^2*c^7*d^4*f^5 + 5*a^4*b*c^6*d^5*f^5 - a^5*c^5*d^6*f^5 - 5*b^5*c^9*d^2*f^
4*e + 25*a*b^4*c^8*d^3*f^4*e - 50*a^2*b^3*c^7*d^4*f^4*e + 50*a^3*b^2*c^6*d^
5*f^4*e - 25*a^4*b*c^5*d^6*f^4*e + 5*a^5*c^4*d^7*f^4*e + 10*b^5*c^8*d^3*f^3
*e^2 - 50*a*b^4*c^7*d^4*f^3*e^2 + 100*a^2*b^3*c^6*d^5*f^3*e^2 - 100*a^3*b^2
*c^5*d^6*f^3*e^2 + 50*a^4*b*c^4*d^7*f^3*e^2 - 10*a^5*c^3*d^8*f^3*e^2 - 10*b
^5*c^7*d^4*f^2*e^3 + 50*a*b^4*c^6*d^5*f^2*e^3 - 100*a^2*b^3*c^5*d^6*f^2*e^3
+ 100*a^3*b^2*c^4*d^7*f^2*e^3 - 50*a^4*b*c^3*d^8*f^2*e^3 + 10*a^5*c^2*d^9*
f^2*e^3 + 5*b^5*c^6*d^5*f*e^4 - 25*a*b^4*c^5*d^6*f*e^4 + 50*a^2*b^3*c^4*d^7
*f*e^4 - 50*a^3*b^2*c^3*d^8*f*e^4 + 25*a^4*b*c^2*d^9*f*e^4 - 5*a^5*c*d^10*f
*e^4 - b^5*c^5*d^6*e^5 + 5*a*b^4*c^4*d^7*e^5 - 10*a^2*b^3*c^3*d^8*e^5 + 10*
a^3*b^2*c^2*d^9*e^5 - 5*a^4*b*c*d^10*e^5 + a^5*d^11*e^5) + 3*(2*b^2*c^2*f^8
+ 3*a*b*c*d*f^8 + 2*a^2*d^2*f^8 - 7*b^2*c*d*f^7*e - 7*a*b*d^2*f^7*e + 7*b^
2*d^2*f^6*e^2)*log(abs(f*x + e))/(a^5*c^5*f^11 - 5*a^4*b*c^5*f^10*e - 5*a^5
*c^4*d*f^10*e + 10*a^3*b^2*c^5*f^9*e^2 + 25*a^4*b*c^4*d*f^9*e^2 + 10*a^5*c^
3*d^2*f^9*e^2 - 10*a^2*b^3*c^5*f^8*e^3 - 50*a^3*b^2*c^4*d*f^8*e^3 - 50*a^4*
b*c^3*d^2*f^8*e^3 - 10*a^5*c^2*d^3*f^8*e^3 + 5*a*b^4*c^5*f^7*e^4 + 50*a^2*b
^3*c^4*d*f^7*e^4 + 100*a^3*b^2*c^3*d^2*f^7*e^4 + 50*a^4*b*c^2*d^3*f^7*e^4 +
5*a^5*c*d^4*f^7*e^4 - b^5*c^5*f^6*e^5 - 25*a*b^4*c^4*d*f^6*e^5 - 100*a^2*b
^3*c^3*d^2*f^6*e^5 - 100*a^3*b^2*c^2*d^3*f^6*e^5 - 25*a^4*b*c*d^4*f^6*e^5 -
a^5*d^5*f^6*e^5 + 5*b^5*c^4*d*f^5*e^6 + 50*a*b^4*c^3*d^2*f^5*e^6 + 100*a^2
*b^3*c^2*d^3*f^5*e^6 + 50*a^3*b^2*c*d^4*f^5*e^6 + 5*a^4*b*d^5*f^5*e^6 - 10*
b^5*c^3*d^2*f^4*e^7 - 50*a*b^4*c^2*d^3*f^4*e^7 - 50*a^2*b^3*c*d^4*f^4*e^7 -
10*a^3*b^2*d^5*f^4*e^7 + 10*b^5*c^2*d^3*f^3*e^8 + 25*a*b^4*c*d^4*f^3*e^8 +
10*a^2*b^3*d^5*f^3*e^8 - 5*b^5*c*d^4*f^2*e^9 - 5*a*b^4*d^5*f^2*e^9 + b^5*d
```

$$\begin{aligned}
& ^5f^e^{10}) + 1/2*(12*b^7*c^5*d^2*f^7*x^5 - 30*a*b^6*c^4*d^3*f^7*x^5 + 12*a^2*b^5*c^3*d^4*f^7*x^5 + 12*a^3*b^4*c^2*d^5*f^7*x^5 - 30*a^4*b^3*c*d^6*f^7*x^5 + 12*a^5*b^2*d^7*f^7*x^5 - 30*b^7*c^4*d^3*f^6*x^5*e + 96*a*b^6*c^3*d^4*f^6*x^5*e - 72*a^2*b^5*c^2*d^5*f^6*x^5*e + 96*a^3*b^4*c*d^6*f^6*x^5*e - 30*a^4*b^3*d^7*f^6*x^5*e + 24*b^7*c^6*d*f^7*x^4 - 42*a*b^6*c^5*d^2*f^7*x^4 - 21*a^2*b^5*c^4*d^3*f^7*x^4 + 42*a^3*b^4*c^3*d^4*f^7*x^4 - 21*a^4*b^3*c^2*d^5*f^7*x^4 - 42*a^5*b^2*c*d^6*f^7*x^4 + 24*a^6*b*d^7*f^7*x^4 + 12*b^7*c^3*d^4*f^5*x^5*e^2 - 72*a*b^6*c^2*d^5*f^5*x^5*e^2 - 72*a^2*b^5*c*d^6*f^5*x^5*e^2 + 12*a^3*b^4*d^7*f^5*x^5*e^2 - 42*b^7*c^5*d^2*f^6*x^4*e + 102*a*b^6*c^4*d^3*f^6*x^4*e + 18*a^2*b^5*c^3*d^4*f^6*x^4*e + 18*a^3*b^4*c^2*d^5*f^6*x^4*e + 102*a^4*b^3*c*d^6*f^6*x^4*e - 42*a^5*b^2*d^7*f^6*x^4*e + 12*b^7*c^7*f^7*x^3 + 6*a*b^6*c^6*d*f^7*x^3 - 74*a^2*b^5*c^5*d^2*f^7*x^3 + 38*a^3*b^4*c^4*d^3*f^7*x^3 + 38*a^4*b^3*c^3*d^4*f^7*x^3 - 74*a^5*b^2*c^2*d^5*f^7*x^3 + 6*a^6*b*c*d^6*f^7*x^3 + 12*a^7*d^7*f^7*x^3 + 12*b^7*c^2*d^5*f^4*x^5*e^3 + 96*a*b^6*c*d^6*f^4*x^5*e^3 + 12*a^2*b^5*d^7*f^4*x^5*e^3 - 21*b^7*c^4*d^3*f^5*x^4*e^2 + 18*a*b^6*c^3*d^4*f^5*x^4*e^2 - 234*a^2*b^5*c^2*d^5*f^5*x^4*e^2 + 18*a^3*b^4*c*d^6*f^5*x^4*e^2 - 21*a^4*b^3*d^7*f^5*x^4*e^2 + 6*b^7*c^6*d*f^6*x^3*e - 56*a*b^6*c^5*d^2*f^6*x^3*e + 172*a^2*b^5*c^4*d^3*f^6*x^3*e - 136*a^3*b^4*c^3*d^4*f^6*x^3*e + 172*a^4*b^3*c^2*d^5*f^6*x^3*e - 56*a^5*b^2*c*d^6*f^6*x^3*e + 6*a^6*b*d^7*f^6*x^3*e + 18*a*b^6*c^7*f^7*x^2 - 37*a^2*b^5*c^6*d*f^7*x^2 - 3*a^3*b^4*c^5*d^2*f^7*x^2 + 32*a^4*b^3*c^4*d^3*f^7*x^2 - 3*a^5*b^2*c^3*d^4*f^7*x^2 - 37*a^6*b*c^2*d^5*f^7*x^2 + 18*a^7*c*d^6*f^7*x^2 - 30*b^7*c*d^6*f^3*x^5*e^4 - 30*a*b^6*d^7*f^3*x^5*e^4 + 42*b^7*c^3*d^4*f^4*x^4*e^3 + 18*a*b^6*c^2*d^5*f^4*x^4*e^3 + 18*a^2*b^5*c*d^6*f^4*x^4*e^3 + 42*a^3*b^4*d^7*f^4*x^4*e^3 - 74*b^7*c^5*d^2*f^5*x^3*e^2 + 172*a*b^6*c^4*d^3*f^5*x^3*e^2 - 104*a^2*b^5*c^3*d^4*f^5*x^3*e^2 - 104*a^3*b^4*c^2*d^5*f^5*x^3*e^2 + 172*a^4*b^3*c*d^6*f^5*x^3*e^2 - 74*a^5*b^2*d^7*f^5*x^3*e^2 + 18*b^7*c^7*f^6*x^2*e - 34*a*b^6*c^6*d*f^6*x^2*e + 9*a^2*b^5*c^5*d^2*f^6*x^2*e + a^3*b^4*c^4*d^3*f^6*x^2*e + a^4*b^3*c^3*d^4*f^6*x^2*e + 9*a^5*b^2*c^2*d^5*f^6*x^2*e - 34*a^6*b*c*d^6*f^6*x^2*e + 18*a^7*d^7*f^6*x^2*e + 4*a^2*b^5*c^7*f^7*x - 12*a^3*b^4*c^6*d*f^7*x + 8*a^4*b^3*c^5*d^2*f^7*x + 8*a^5*b^2*c^4*d^3*f^7*x - 12*a^6*b*c^3*d^4*f^7*x + 4*a^7*c^2*d^5*f^7*x + 12*b^7*d^7*f^2*x^5*e^5 - 21*b^7*c^2*d^5*f^3*x^4*e^4 + 102*a*b^6*c*d^6*f^3*x^4*e^4 - 21*a^2*b^5*d^7*f^3*x^4*e^4 + 38*b^7*c^4*d^3*f^4*x^3*e^3 - 136*a*b^6*c^3*d^4*f^4*x^3*e^3 - 104*a^2*b^5*c^2*d^5*f^4*x^3*e^3 - 136*a^3*b^4*c*d^6*f^4*x^3*e^3 + 38*a^4*b^3*d^7*f^4*x^3*e^3 - 37*b^7*c^6*d*f^5*x^2*e^2 + 9*a*b^6*c^5*d^2*f^5*x^2*e^2 + 234*a^2*b^5*c^4*d^3*f^5*x^2*e^2 - 208*a^3*b^4*c^3*d^4*f^5*x^2*e^2 + 234*a^4*b^3*c^2*d^5*f^5*x^2*e^2 + 9*a^5*b^2*c*d^6*f^5*x^2*e^2 - 37*a^6*b*d^7*f^5*x^2*e^2 + 28*a*b^6*c^7*f^6*x*e - 66*a^2*b^5*c^6*d*f^6*x*e + 34*a^3*b^4*c^5*d^2*f^6*x*e - 16*a^4*b^3*c^4*d^3*f^6*x*e + 34*a^5*b^2*c^3*d^4*f^6*x*e - 66*a^6*b*c^2*d^5*f^6*x*e + 28*a^7*c*d^6*f^6*x*e - a^3*b^4*c^7*f^7 + 4*a^4*b^3*c^6*d*f^7 - 6*a^5*b^2*c^5*d^2*f^7 + 4*a^6*b*c^4*d^3*f^7 - a^7*c^3*d^4*f^7 - 42*b^7*c*d^6*f^2*x^4*e^5 - 42*a*b^6*d^7*f^2*x^4*e^5 + 38*b^7*c^3*d^4*f^3*x^3*e^4 + 172*a*b^6*c^2*d^5*f^3*x^3*e^4 + 172*a^2*b^5*c*d^6*f^3*x^3*e^4 + 38*a^3*b^4*d^7*f^3*x^3*e^4 - 3*b^7*c^5*d^2*f^4*x^2*e^3 + a*b^6*c^4*d^3*f^4*x^2*e^3 - 2
\end{aligned}$$

$$\begin{aligned}
& 08*a^2*b^5*c^3*d^4*f^4*x^2*e^3 - 208*a^3*b^4*c^2*d^5*f^4*x^2*e^3 + a^4*b^3*c*d^6*f^4*x^2*e^3 - 3*a^5*b^2*d^7*f^4*x^2*e^3 + 4*b^7*c^7*f^5*x*e^2 - 66*a*b^6*c^6*d*f^5*x*e^2 + 156*a^2*b^5*c^5*d^2*f^5*x*e^2 - 52*a^3*b^4*c^4*d^3*f^5*x*e^2 - 52*a^4*b^3*c^3*d^4*f^5*x*e^2 + 156*a^5*b^2*c^2*d^5*f^5*x*e^2 - 66*a^6*b*c*d^6*f^5*x*e^2 + 4*a^7*d^7*f^5*x*e^2 + 7*a^2*b^5*c^7*f^6*e - 21*a^3*b^4*c^6*d*f^6*e + 14*a^4*b^3*c^5*d^2*f^6*e + 14*a^5*b^2*c^4*d^3*f^6*e - 21*a^6*b*c^3*d^4*f^6*e + 7*a^7*c^2*d^5*f^6*e + 24*b^7*d^7*f*x^4*e^6 - 74*b^7*c^2*d^5*f^2*x^3*e^5 - 56*a*b^6*c*d^6*f^2*x^3*e^5 - 74*a^2*b^5*d^7*f^2*x^3*e^5 + 32*b^7*c^4*d^3*f^3*x^2*e^4 + a*b^6*c^3*d^4*f^3*x^2*e^4 + 234*a^2*b^5*c^2*d^5*f^3*x^2*e^4 + a^3*b^4*c*d^6*f^3*x^2*e^4 + 32*a^4*b^3*d^7*f^3*x^2*e^4 - 12*b^7*c^6*d*f^4*x*e^3 + 34*a*b^6*c^5*d^2*f^4*x*e^3 - 52*a^2*b^5*c^4*d^3*f^4*x*e^3 - 52*a^4*b^3*c^2*d^5*f^4*x*e^3 + 34*a^5*b^2*c*d^6*f^4*x*e^3 - 12*a^6*b*d^7*f^4*x*e^3 + 7*a*b^6*c^7*f^5*e^2 - 26*a^2*b^5*c^6*d*f^5*e^2 + 52*a^3*b^4*c^5*d^2*f^5*e^2 - 78*a^4*b^3*c^4*d^3*f^5*e^2 + 52*a^5*b^2*c^3*d^4*f^5*e^2 - 26*a^6*b*c^2*d^5*f^5*e^2 + 7*a^7*c*d^6*f^5*e^2 + 6*b^7*c*d^6*f*x^3*e^6 + 6*a*b^6*d^7*f*x^3*e^6 - 3*b^7*c^3*d^4*f^2*x^2*e^5 + 9*a*b^6*c^2*d^5*f^2*x^2*e^5 + 9*a^2*b^5*c*d^6*f^2*x^2*e^5 - 3*a^3*b^4*d^7*f^2*x^2*e^5 + 8*b^7*c^5*d^2*f^3*x*e^4 - 16*a*b^6*c^4*d^3*f^3*x*e^4 - 52*a^2*b^5*c^3*d^4*f^3*x*e^4 - 52*a^3*b^4*c^2*d^5*f^3*x*e^4 - 16*a^4*b^3*c*d^6*f^3*x*e^4 + 8*a^5*b^2*d^7*f^3*x*e^4 - b^7*c^7*f^4*e^3 - 21*a*b^6*c^6*d*f^4*e^3 + 52*a^2*b^5*c^5*d^2*f^4*e^3 + 52*a^5*b^2*c^2*d^5*f^4*e^3 - 21*a^6*b*c*d^6*f^4*e^3 - a^7*d^7*f^4*e^3 + 12*b^7*d^7*x^3*e^7 - 37*b^7*c^2*d^5*f*x^2*e^6 - 34*a*b^6*c*d^6*f*x^2*e^6 - 37*a^2*b^5*d^7*f*x^2*e^6 + 8*b^7*c^4*d^3*f^2*x*e^5 + 34*a*b^6*c^3*d^4*f^2*x*e^5 + 156*a^2*b^5*c^2*d^5*f^2*x*e^5 + 34*a^3*b^4*c*d^6*f^2*x*e^5 + 8*a^4*b^3*d^7*f^2*x*e^5 + 4*b^7*c^6*d*f^3*e^4 + 14*a*b^6*c^5*d^2*f^3*e^4 - 78*a^2*b^5*c^4*d^3*f^3*e^4 - 78*a^4*b^3*c^2*d^5*f^3*e^4 + 14*a^5*b^2*c*d^6*f^3*e^4 + 4*a^6*b*d^7*f^3*e^4 + 18*b^7*c*d^6*x^2*e^7 + 18*a*b^6*d^7*x^2*e^7 - 12*b^7*c^3*d^4*f*x*e^6 - 66*a*b^6*c^2*d^5*f*x*e^6 - 66*a^2*b^5*c*d^6*f*x*e^6 - 12*a^3*b^4*d^7*f*x*e^6 - 6*b^7*c^5*d^2*f^2*e^5 + 14*a*b^6*c^4*d^3*f^2*e^5 + 52*a^2*b^5*c^3*d^4*f^2*e^5 + 52*a^3*b^4*c^2*d^5*f^2*e^5 + 14*a^4*b^3*c*d^6*f^2*e^5 - 6*a^5*b^2*d^7*f^2*e^5 + 4*b^7*c^2*d^5*x*e^7 + 28*a*b^6*c*d^6*x*e^7 + 4*a^2*b^5*d^7*x*e^7 + 4*b^7*c^4*d^3*f*e^6 - 21*a*b^6*c^3*d^4*f*e^6 - 26*a^2*b^5*c^2*d^5*f*e^6 - 21*a^3*b^4*c*d^6*f*e^6 + 4*a^4*b^3*d^7*f*e^6 - b^7*c^3*d^4*e^7 + 7*a*b^6*c^2*d^5*e^7 + 7*a^2*b^5*c*d^6*e^7 - a^3*b^4*d^7*e^7)/((a^4*b^4*c^8*f^8 - 4*a^5*b^3*c^7*d*f^8 + 6*a^6*b^2*c^6*d^2*f^8 - 4*a^7*b*c^5*d^3*f^8 + a^8*c^4*d^4*f^8 - 4*a^3*b^5*c^8*f^7*e + 12*a^4*b^4*c^7*d*f^7*e - 8*a^5*b^3*c^6*d^2*f^7*e - 8*a^6*b^2*c^5*d^3*f^7*e + 12*a^7*b*c^4*d^4*f^7*e - 4*a^8*c^3*d^5*f^7*e + 6*a^2*b^6*c^8*f^6*e^2 - 8*a^3*b^5*c^7*d*f^6*e^2 - 22*a^4*b^4*c^6*d^2*f^6*e^2 + 48*a^5*b^3*c^5*d^3*f^6*e^2 - 22*a^6*b^2*c^4*d^4*f^6*e^2 - 8*a^7*b*c^3*d^5*f^6*e^2 + 6*a^8*c^2*d^6*f^6*e^2 - 4*a*b^7*c^8*f^5*e^3 - 8*a^2*b^6*c^7*d*f^5*e^3 + 48*a^3*b^5*c^6*d^2*f^5*e^3 - 36*a^4*b^4*c^5*d^3*f^5*e^3 - 36*a^5*b^3*c^4*d^4*f^5*e^3 + 48*a^6*b^2*c^3*d^5*f^5*e^3 - 8*a^7*b*c^2*d^6*f^5*e^3 - 4*a^8*c*d^7*f^5*e^3 + b^8*c^8*f^4*e^4 + 12*a*b^7*c^7*d*f^4*e^4 - 22*a^2*b^6*c^6*d^2*f^4*e^4 - 36*a^3*b^5*c^5*d^3*f^4*e^4 + 90*a^4*b^4*c^4*d^4*f^4*e^4 - 36*a^5*b^3*c^3*d^5*f^4*e^4 -
\end{aligned}$$

$$22*a^6*b^2*c^2*d^6*f^4*e^4 + 12*a^7*b*c*d^7*f^4*e^4 + a^8*d^8*f^4*e^4 - 4*b^8*c^7*d*f^3*e^5 - 8*a*b^7*c^6*d^2*f^3*e^5 + 48*a^2*b^6*c^5*d^3*f^3*e^5 - 36*a^3*b^5*c^4*d^4*f^3*e^5 - 36*a^4*b^4*c^3*d^5*f^3*e^5 + 48*a^5*b^3*c^2*d^6*f^3*e^5 - 8*a^6*b^2*c*d^7*f^3*e^5 - 4*a^7*b*d^8*f^3*e^5 + 6*b^8*c^6*d^2*f^2*e^6 - 8*a*b^7*c^5*d^3*f^2*e^6 - 22*a^2*b^6*c^4*d^4*f^2*e^6 + 48*a^3*b^5*c^3*d^5*f^2*e^6 - 22*a^4*b^4*c^2*d^6*f^2*e^6 - 8*a^5*b^3*c*d^7*f^2*e^6 + 6*a^6*b^2*d^8*f^2*e^6 - 4*b^8*c^5*d^3*f*e^7 + 12*a*b^7*c^4*d^4*f*e^7 - 8*a^2*b^6*c^3*d^5*f*e^7 - 8*a^3*b^5*c^2*d^6*f*e^7 + 12*a^4*b^4*c*d^7*f*e^7 - 4*a^5*b^3*d^8*f*e^7 + b^8*c^4*d^4*e^8 - 4*a*b^7*c^3*d^5*e^8 + 6*a^2*b^6*c^2*d^6*e^8 - 4*a^3*b^5*c*d^7*e^8 + a^4*b^4*d^8*e^8)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b*c*x*e + a*d*x*e + a*c*e)^2)$$

maple [B] time = 0.03, size = 1076, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
[Out] 21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c*e*f-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*c*d*f^2-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e*f+9*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*d*e*f+9*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*c*d-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*d^2*e-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c*d*e+21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*c*f^2-9*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*e*f-6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*e^2*b^2+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*c*f+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*d*e+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c^2*f^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*d^2*e^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*c+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c^2+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e-6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*a*d-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2+1/2*d^5/(c*f-d*e)^3/(a*d-b*c)^3/(d*x+c)^2-1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2-6*f^5/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*d*e+21*f^5/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*d^2*e^2*b^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c^2*f^2-6*b^5/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a*d*f+21*b^5/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a^2*d^2*f^2
```

maxima [B] time = 5.72, size = 11005, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x, algorithm="maxima")
```

[Out] $3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d + 7*a^2*b^5*d^2)*f^2)*\log(b*x + a)/((b^{10}*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2 - 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3 + 5*a^9*b*c*d^4 - a^{10}*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6 - 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*\log(d*x + c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*c*d^9 - a^5*d^{10})*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f + 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^{10} - 5*a*b^4*c^9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*f^7)*\log(f*x + e)/(b^5*d^5*e^{10} + a^5*c^5*f^{10} - 5*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b^3*d^5)*e^8*f^2 - 10*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 + 10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d + 100*a^2*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*e^5*f^5 + 5*(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^3*d^2 + 10*a^4*b*c^2*d^3 + a^5*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 + 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^3*f^7 + 5*(2*a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f^8 - 5*(a^4*b*c^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5 - 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*d^4 - 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*f + 2*(3*b^7*c^5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*a^3*b^4*c^2*d^5 - 7*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a^2*b^5*c^4*d^3 - 39*a^4*b^3*c^2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^3 + (b^7*c^7 + 21*a*b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 21*a^6*b*c*d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b*c^2*d^5 + 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*e*f^6 + (a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7 - 6*(2*b^7*d^7*e^5*f^2 - 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5$

$$\begin{aligned}
& - 6a^2b^5c^2d^6 + a^3b^4d^7)e^2f^5 - (5b^7c^4d^3 - 16a^2b^6c^3d^4 + 12a^2b^5c^2d^5 - 16a^3b^4c^2d^6 + 5a^4b^3d^7)e^2f^5 + (2b^7c^5d^2 - 5a^2b^6c^4d^3 + 2a^2b^5c^3d^4 + 2a^3b^4c^2d^5 - 5a^4b^3c^2d^6 + 2a^5b^2d^7) * f^7) * x^5 - 3(8b^7d^7e^6f - 14(b^7c^4d^6 + a^2b^6d^7)e^5f^2 - (7b^7c^2d^5 - 34a^2b^6c^2d^6 + 7a^2b^5d^7)e^4f^3 + 2(7b^7c^3d^4 + 3a^2b^6c^2d^5 + 3a^2b^5c^2d^6 + 7a^3b^4d^7)e^3f^4 - (7b^7c^4d^3 - 6a^2b^6c^3d^4 + 78a^2b^5c^2d^5 - 6a^3b^4c^2d^6 + 7a^4b^3d^7)e^2f^5 - 2(7b^7c^5d^2 - 17a^2b^6c^4d^3 - 3a^2b^5c^3d^4 - 3a^3b^4c^2d^5 - 17a^4b^3c^2d^6 + 7a^5b^2d^7)e^2f^6 + (8b^7c^6d - 14a^2b^6c^5d^2 - 7a^2b^5c^4d^3 + 14a^3b^4c^3d^4 - 7a^4b^3c^2d^5 - 14a^5b^2c^2d^6 + 8a^6b^2d^7) * f^7) * x^4 - 2(6b^7d^7e^7 + 3(b^7c^4d^6 + a^2b^6d^7)e^6f - (37b^7c^2d^5 + 28a^2b^6c^2d^6 + 37a^2b^5d^7)e^5f^2 + (19b^7c^3d^4 + 86a^2b^6c^2d^5 + 86a^2b^5c^2d^6 + 19a^3b^4d^7)e^4f^3 + (19b^7c^4d^3 - 68a^2b^6c^3d^4 - 52a^2b^5c^2d^5 - 68a^3b^4c^2d^6 + 19a^4b^3d^7)e^3f^4 - (37b^7c^5d^2 - 86a^2b^6c^4d^3 + 52a^2b^5c^3d^4 + 52a^3b^4c^2d^5 - 86a^4b^3c^2d^6 + 37a^5b^2d^7)e^2f^5 + (3b^7c^6d - 28a^2b^6c^5d^2 + 86a^2b^5c^4d^3 - 68a^3b^4c^3d^4 + 86a^4b^3c^2d^5 - 28a^5b^2c^2d^6 + 3a^6b^2d^7) * e^2f^6 + (6b^7c^7 + 3a^2b^6c^6d - 37a^2b^5c^5d^2 + 19a^3b^4c^4d^3 + 19a^4b^3c^3d^4 - 37a^5b^2c^2d^5 + 3a^6b^2c^2d^6 + 6a^7d^7) * f^7) * x^3 - (18(b^7c^4d^6 + a^2b^6d^7)e^7 - (37b^7c^2d^5 + 34a^2b^6c^2d^6 + 37a^2b^5d^7)e^6f - 3(b^7c^3d^4 - 3a^2b^6c^2d^5 - 3a^2b^5c^2d^6 + a^3b^4d^7)e^5f^2 + (32b^7c^4d^3 + a^2b^6c^3d^4 + 234a^2b^5c^2d^5 + a^3b^4c^2d^6 + 32a^4b^3d^7)e^4f^3 - (3b^7c^5d^2 - a^2b^6c^4d^3 + 208a^2b^5c^3d^4 + 208a^3b^4c^2d^5 - a^4b^3c^2d^6 + 3a^5b^2d^7)e^3f^4 - (37b^7c^6d - 9a^2b^6c^5d^2 - 234a^2b^5c^4d^3 + 208a^3b^4c^3d^4 - 234a^4b^3c^2d^5 - 9a^5b^2c^2d^6 + 37a^6b^2d^7) * e^2f^5 + (18b^7c^7 - 34a^2b^6c^6d + 9a^2b^5c^5d^2 + a^3b^4c^4d^3 + a^4b^3c^3d^4 + 9a^5b^2c^2d^5 - 34a^6b^2c^2d^6 + 18a^7d^7) * e^2f^6 + (18a^2b^6c^7 - 37a^2b^5c^6d - 3a^3b^4c^5d^2 + 32a^4b^3c^4d^3 - 3a^5b^2c^3d^4 - 37a^6b^2c^2d^5 + 18a^7c^2d^6) * f^7) * x^2 - 2(2(b^7c^2d^5 + 7a^2b^6c^2d^6 + a^2b^5d^7)e^7 - 3(2b^7c^3d^4 + 11a^2b^6c^2d^5 + 11a^2b^5c^2d^6 + 2a^3b^4d^7)e^6f + (4b^7c^4d^3 + 17a^2b^6c^3d^4 + 78a^2b^5c^2d^5 + 17a^3b^4c^2d^6 + 4a^4b^3d^7)e^5f^2 + 2(2b^7c^5d^2 - 4a^2b^6c^4d^3 - 13a^2b^5c^3d^4 - 13a^3b^4c^2d^5 - 4a^4b^3c^2d^6 + 2a^5b^2d^7)e^4f^3 - (6b^7c^6d - 17a^2b^6c^5d^2 + 26a^2b^5c^4d^3 + 26a^4b^3c^2d^5 - 17a^5b^2c^2d^6 + 6a^6b^2d^7) * e^3f^4 + (2b^7c^7 - 33a^2b^6c^6d + 78a^2b^5c^5d^2 - 26a^3b^4c^4d^3 - 26a^4b^3c^3d^4 + 78a^5b^2c^2d^5 - 33a^6b^2c^2d^6 + 2a^7d^7) * e^2f^5 + (14a^2b^6c^7 - 33a^2b^5c^6d + 17a^3b^4c^5d^2 - 8a^4b^3c^4d^3 + 17a^5b^2c^3d^4 - 33a^6b^2c^2d^5 + 14a^7c^2d^6) * e^2f^6 + 2(a^2b^5c^7 - 3a^3b^4c^6d + 2a^4b^3c^5d^2 + 2a^5b^2c^4d^3 - 3a^6b^2c^3d^4 + a^7c^2d^5) * f^7) * x) / ((a^2b^8c^6d^4 - 4a^3b^7c^5d^5 + 6a^4b^6c^4d^6 - 4a^5b^5c^3d^7 + a^6b^4c^2d^8) * e^10 - 4(a^2b^8c^7d^3 - 3a^3b^7c^6d^4 + 2a^4b^6c^5d^5 - 3a^5b^5c^5d^6 + a^6b^4c^4d^7) * e^9 - 4(a^2b^7c^6d^4 - 3a^3b^6c^5d^5 + 2a^4b^5c^4d^6 - 3a^5b^4c^3d^7 + a^6b^3c^2d^8) * e^8 - 4(a^2b^6c^5d^5 - 3a^3b^5c^4d^6 + 2a^4b^4c^3d^7 - 3a^5b^3c^2d^8) * e^7 - 4(a^2b^5c^4d^6 - 3a^3b^4c^3d^7 + 2a^4b^3c^2d^8) * e^6 - 4(a^2b^4c^3d^7 - 3a^3b^3c^2d^8) * e^5 - 4(a^2b^3c^2d^8) * e^4 - 4(a^2b^2c^2d^9) * e^3 - 4(a^2b^1c^2d^10) * e^2 - 4(a^2b^0c^2d^11) * e^1 - 4(a^2b^0c^1d^12) * e^0)
\end{aligned}$$

$$\begin{aligned}
&5*d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2* \\
&(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^ \\
&5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2 \\
&- 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^ \\
&6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b* \\
&c^2*d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 - \\
&36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c \\
&^4*d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4 \\
&*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 1 \\
&2*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6* \\
&c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b \\
&^2*c^6*d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 - \\
&3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + \\
&a^10*c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^ \\
&2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3* \\
&d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10 \\
&*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4* \\
&b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 1 \\
&1*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d \\
&^9 + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b \\
&^8*c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2 \\
&*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 - \\
&22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5* \\
&c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4 \\
&*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^ \\
&5 + 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c* \\
&d^9)*e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^ \\
&4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2 \\
&*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6* \\
&d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4 \\
&*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + \\
&a^8*b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8 \\
&*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^ \\
&9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b \\
&^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 \\
&- 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^ \\
&6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b \\
&^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6* \\
&a^2*b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7 \\
&- 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^ \\
&2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b \\
&^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^ \\
&9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d \\
&^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^ \\
&3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d -
\end{aligned}$$

$$\begin{aligned}
& a^3b^7c^8d^2 - 3a^4b^6c^7d^3 + 3a^5b^5c^6d^4 + 3a^6b^4c^5d^5 - 3a^7b^3c^4d^6 - a^8b^2c^3d^7 + a^9b^1c^2d^8) e^{2f^8} - (4a^3b^7c^9d - 9a^4b^6c^8d^2 + 10a^5b^4c^6d^4 - 9a^8b^2c^4d^6 + 4a^9b^1c^3d^7) e^{f^9} + (a^4b^6c^9d - 3a^5b^5c^8d^2 + 2a^6b^4c^7d^3 + 2a^7b^3c^6d^4 - 3a^8b^2c^5d^5 + a^9b^1c^4d^6) f^{10} x^5 + ((b^{10}c^4d^6 - 4a^1b^9c^3d^7 + 6a^2b^8c^2d^8 - 4a^3b^7c^1d^9 + a^4b^6d^{10}) e^{10} - 3(3b^{10}c^6d^4 - 8a^1b^9c^5d^5 + 5a^2b^8c^4d^6 + 5a^4b^6c^2d^8 - 8a^5b^5c^1d^9 + 3a^6b^4d^{10}) e^{8f^2} + 4(4b^{10}c^7d^3 - 5a^1b^9c^6d^4 - 9a^2b^8c^5d^5 + 10a^3b^7c^4d^6 + 10a^4b^6c^3d^7 - 9a^5b^5c^2d^8 - 5a^6b^4c^1d^9 + 4a^7b^3d^{10}) e^{7f^3} - (9b^{10}c^8d^2 + 20a^1b^9c^7d^3 - 90a^2b^8c^6d^4 + 36a^3b^7c^5d^5 + 50a^4b^6c^4d^6 + 36a^5b^5c^3d^7 - 90a^6b^4c^2d^8 + 20a^7b^3c^1d^9 + 9a^8b^2d^{10}) e^{6f^4} + 12(2a^1b^9c^8d^2 - 3a^2b^8c^7d^3 - 3a^3b^7c^6d^4 + 4a^4b^6c^5d^5 + 4a^5b^5c^4d^6 - 3a^6b^4c^3d^7 - 3a^7b^3c^2d^8 + 2a^8b^2c^1d^9) e^{5f^5} + (b^{10}c^{10} - 15a^1b^8c^8d^2 + 40a^3b^7c^7d^3 - 50a^4b^6c^6d^4 + 48a^5b^5c^5d^5 - 50a^6b^4c^4d^6 + 40a^7b^3c^3d^7 - 15a^8b^2c^2d^8 + a^{10}d^{10}) e^{4f^6} - 4(a^1b^9c^{10} - 10a^4b^6c^7d^3 + 9a^5b^5c^6d^4 + 9a^6b^4c^5d^5 - 10a^7b^3c^4d^6 + a^{10}c^1d^9) e^{3f^7} + 3(2a^2b^8c^{10} - 5a^4b^6c^8d^2 - 12a^5b^5c^7d^3 + 30a^6b^4c^6d^4 - 12a^7b^3c^5d^5 - 5a^8b^2c^4d^6 + 2a^{10}c^2d^8) e^{2f^8} - 4(a^3b^7c^{10} - 6a^5b^5c^8d^2 + 5a^6b^4c^7d^3 + 5a^7b^3c^6d^4 - 6a^8b^2c^5d^5 + a^{10}c^3d^7) e^{f^9} + (a^4b^6c^{10} - 9a^6b^4c^8d^2 + 16a^7b^3c^7d^3 - 9a^8b^2c^6d^4 + a^{10}c^4d^6) f^{10} x^4 + 2((b^{10}c^5d^5 - 3a^1b^9c^4d^6 + 2a^2b^8c^3d^7 + 2a^3b^7c^2d^8 - 3a^4b^6c^1d^9 + a^5b^5d^{10}) e^{10} - (3b^{10}c^6d^4 - 8a^1b^9c^5d^5 + 5a^2b^8c^4d^6 + 5a^4b^6c^2d^8 - 8a^5b^5c^1d^9 + 3a^6b^4d^{10}) e^{9f} + (2b^{10}c^7d^3 - 5a^1b^9c^6d^4 + 3a^2b^8c^5d^5 + 3a^5b^5c^2d^8 - 5a^6b^4c^1d^9 + 2a^7b^3d^{10}) e^{8f^2} + 2(b^{10}c^8d^2 - 16a^3b^7c^5d^5 + 30a^4b^6c^4d^6 - 16a^5b^5c^3d^7 + a^8b^2d^{10}) e^{7f^3} - (3b^{10}c^9d + 5a^1b^9c^8d^2 - 60a^3b^7c^6d^4 + 52a^4b^6c^5d^5 + 52a^5b^5c^4d^6 - 60a^6b^4c^3d^7 + 5a^8b^2c^1d^9 + 3a^9b^1d^{10}) e^{6f^4} + (b^{10}c^{10} + 8a^1b^9c^9d + 3a^2b^8c^8d^2 - 32a^3b^7c^7d^3 - 52a^4b^6c^6d^4 + 144a^5b^5c^5d^5 - 52a^6b^4c^4d^6 - 32a^7b^3c^3d^7 + 3a^8b^2c^2d^8 + 8a^9b^1c^1d^9 + a^{10}d^{10}) e^{5f^5} - (3a^1b^9c^{10} + 5a^2b^8c^9d - 60a^4b^6c^7d^3 + 52a^5b^5c^6d^4 + 52a^6b^4c^5d^5 - 60a^7b^3c^4d^6 + 5a^9b^1c^2d^8 + 3a^{10}c^1d^9) e^{4f^6} + 2(a^2b^8c^{10} - 16a^5b^5c^7d^3 + 30a^6b^4c^6d^4 - 16a^7b^3c^5d^5 + a^{10}c^2d^8) e^{3f^7} + (2a^3b^7c^{10} - 5a^4b^6c^9d + 3a^5b^5c^8d^2 + 3a^8b^2c^5d^5 - 5a^9b^1c^4d^6 + 2a^{10}c^3d^7) e^{2f^8} - (3a^4b^6c^{10} - 8a^5b^5c^9d + 5a^6b^4c^8d^2 + 5a^8b^2c^6d^4 - 8a^9b^1c^5d^5 + 3a^{10}c^4d^6) e^{f^9} + (a^5b^5c^{10} - 3a^6b^4c^9d + 2a^7b^3c^8d^2 + 2a^8b^2c^7d^3 - 3a^9b^1c^6d^4 + a^{10}c^5d^5) f^{10} x^3 + ((b^{10}c^6d^4 - 9a^2b^8c^4d^6 + 16a^3b^7c^3d^7 - 9a^4b^6c^2d^8 + a^6b^4d^{10}) e^{10} - 4(b^{10}c^7d^3 - 6a^2b^8c^5d^5 + 5a^3b^7c^4d^6 - 4a^4b^6c^3d^7 + 4a^5b^5c^2d^8 - 3a^6b^4c^1d^9) e^{9f} + 4(4b^{10}c^8d^2 - 5a^1b^9c^7d^3 - 5a^2b^8c^6d^4 + 4a^3b^7c^5d^5 + 4a^4b^6c^4d^6 - 4a^5b^5c^3d^7 + 4a^6b^4c^2d^8 - 4a^7b^3c^1d^9) e^{8f^2} + 4(4b^{10}c^9d - 5a^1b^9c^8d^2 - 5a^2b^8c^7d^3 + 4a^3b^7c^6d^4 + 4a^4b^6c^5d^5 - 4a^5b^5c^4d^6 + 4a^6b^4c^3d^7 - 4a^7b^3c^2d^8 + 4a^8b^2c^1d^9) e^{7f^3} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{6f^4} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{5f^5} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{4f^6} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{3f^7} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{2f^8} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{f^9} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{0f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-1f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-2f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-3f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-4f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-5f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-6f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-7f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-8f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-9f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-10f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-11f} + 4(4b^{10}c^{10} - 5a^1b^9c^9d - 5a^2b^8c^8d^2 + 4a^3b^7c^7d^3 + 4a^4b^6c^6d^4 - 4a^5b^5c^5d^5 + 4a^6b^4c^4d^6 - 4a^7b^3c^3d^7 + 4a^8b^2c^2d^8 - 4a^9b^1c^1d^9) e^{-12f} + \dots
\end{aligned}$$

$$\begin{aligned}
& b^7c^4d^6 + 5a^4b^6c^3d^7 - 6a^5b^5c^2d^8 + a^7b^3d^{10})e^9f + \\
& 3(2b^{10}c^8d^2 - 5a^2b^8c^6d^4 - 12a^3b^7c^5d^5 + 30a^4b^6c^4d^6 - 12a^5b^5c^3d^7 - 5a^6b^4c^2d^8 + 2a^8b^2d^{10})e^8f^2 - \\
& 4(b^{10}c^9d - 10a^3b^7c^6d^4 + 9a^4b^6c^5d^5 + 9a^5b^5c^4d^6 - 10a^6b^4c^3d^7 + a^9b^2d^{10})e^7f^3 + (b^{10}c^{10} - 15a^2b^8c^8d^2 + \\
& 40a^3b^7c^7d^3 - 50a^4b^6c^6d^4 + 48a^5b^5c^5d^5 - 50a^6b^4c^4d^6 + 40a^7b^3c^3d^7 - 15a^8b^2c^2d^8 + a^{10}d^{10})e^6f^4 + \\
& 12(2a^2b^8c^9d - 3a^3b^7c^8d^2 - 3a^4b^6c^7d^3 + 4a^5b^5c^6d^4 + 4a^6b^4c^5d^5 - 3a^7b^3c^4d^6 - 3a^8b^2c^3d^7 + 2a^9b^2c^2d^8) \\
& e^5f^5 - (9a^2b^8c^{10} + 20a^3b^7c^9d - 90a^4b^6c^8d^2 + 36a^5b^5c^7d^3 + 50a^6b^4c^6d^4 + 36a^7b^3c^5d^5 - 90a^8b^2c^4d^6 + \\
& 20a^9b^2c^3d^7 + 9a^{10}c^2d^8)e^4f^6 + 4(4a^3b^7c^{10} - 5a^4b^6c^9d - 9a^5b^5c^8d^2 + 10a^6b^4c^7d^3 + 10a^7b^3c^6d^4 - \\
& 9a^8b^2c^5d^5 - 5a^9b^2c^4d^6 + 4a^{10}c^3d^7)e^3f^7 - 3(3a^4b^6c^{10} - 8a^5b^5c^9d + 5a^6b^4c^8d^2 + 5a^8b^2c^6d^4 - 8a^9b^2c^5d^5 + \\
& 3a^{10}c^4d^6)e^2f^8 + (a^6b^4c^{10} - 4a^7b^3c^9d + 6a^8b^2c^8d^2 - 4a^9b^2c^7d^3 + a^{10}c^6d^4)f^{10})x^2 + 2((ab^9c^6d^4 - \\
& 3a^2b^8c^5d^5 + 2a^3b^7c^4d^6 + 2a^4b^6c^3d^7 - 3a^5b^5c^2d^8 + a^6b^4c^2d^9)e^{10} - (4ab^9c^7d^3 - 9a^2b^8c^6d^4 + 10a^4b^6c^4d^6 - \\
& 9a^6b^4c^2d^8 + 4a^7b^3c^2d^9)e^9f + 6(ab^9c^8d^2 - a^2b^8c^7d^3 - 3a^3b^7c^6d^4 + 3a^4b^6c^5d^5 + 3a^5b^5c^4d^6 - 3a^6b^4c^3d^7 - \\
& a^7b^3c^2d^8 + a^8b^2c^2d^9)e^8f^2 - 2(2ab^9c^9d + 3a^2b^8c^8d^2 - 16a^3b^7c^7d^3 + 5a^4b^6c^6d^4 + 12a^5b^5c^5d^5 + 5a^6b^4c^4d^6 - \\
& 16a^7b^3c^3d^7 + 3a^8b^2c^2d^8 + 2a^9b^2c^2d^9)e^7f^3 + (ab^9c^{10} + 9a^2b^8c^9d - 18a^3b^7c^8d^2 - 10a^4b^6c^7d^3 + 18a^5b^5c^6d^4 + \\
& 18a^6b^4c^5d^5 - 10a^7b^3c^4d^6 - 18a^8b^2c^3d^7 + 9a^9b^2c^2d^8 + a^{10}c^2d^9)e^6f^4 - 3(a^2b^8c^{10} - 6a^4b^6c^8d^2 + 8a^5b^5c^7d^3 - 6a^6b^4c^6d^4 + \\
& 8a^7b^3c^5d^5 - 6a^8b^2c^4d^6 + a^{10}c^2d^8)e^5f^5 + 2(a^3b^7c^{10} - 5a^4b^6c^9d + 9a^5b^5c^8d^2 - 5a^6b^4c^7d^3 - 5a^7b^3c^6d^4 + \\
& 9a^8b^2c^5d^5 - 5a^9b^2c^4d^6 + a^{10}c^3d^7)e^4f^6 + 2(a^4b^6c^{10} - 9a^6b^4c^8d^2 + 16a^7b^3c^7d^3 - 9a^8b^2c^6d^4 + a^{10}c^4d^6) \\
& e^3f^7 - 3(a^5b^5c^{10} - 3a^6b^4c^9d + 2a^7b^3c^8d^2 + 2a^8b^2c^7d^3 - 3a^9b^2c^6d^4 + a^{10}c^5d^5)e^2f^8 + (a^6b^4c^{10} - 4a^7b^3c^9d + \\
& 6a^8b^2c^8d^2 - 4a^9b^2c^7d^3 + a^{10}c^6d^4)e^1f^9)x)
\end{aligned}$$

mupad [B] time = 20.46, size = 82532, normalized size = 166.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3)^3,x)

```
[Out] symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*
c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11
*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300
*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 3
43980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*
z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^
11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*
d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^
7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*
a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 +
326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12
*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e
^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12
*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b
^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a
^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245
700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3
- 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11
*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^1
1*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^1
3*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^
8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^
12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a
^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123
900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3
- 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14
*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7
*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^
6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^
13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b
^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a
^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101
700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e^14*f^6*z^3
+ 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d^14*e^15*f^5
*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*c^9*d^11*e^1
5*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*b^15*c^11*d^
9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*a^14*b^6*c^6
*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 65820*a^10*b^
10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^3 - 65820*a^
6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^13*z^3 - 5670
0*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*f^12*z^3 + 5
6700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e^5*f^15*z^3
- 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^16*e^13*f^7*z
^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*d^15*e^13*f^
7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c^7*d^13*e^15
```


$$\begin{aligned}
& *f^5z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13z^3 - 56700*a^7*b^13*c^15*d^5*e \\
& ^8*f^12z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5z^3 + 56700*a^7*b^13*c^7*d^1 \\
& 3*e^16*f^4z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8z^3 - 56700*a^5*b^15*c^12 \\
& *d^8*e^13*f^7z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7z^3 - 48252*a^15*b^5*c \\
& ^10*d^10*e^5*f^15z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10z^3 - 48252*a^10* \\
& b^10*c^15*d^5*e^5*f^15z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5z^3 - 48252* \\
& a^5*b^15*c^15*d^5*e^10*f^10z^3 - 48252*a^5*b^15*c^10*d^10*e^15*f^5z^3 - 3 \\
& 2400*a^16*b^4*c^8*d^12*e^6*f^14z^3 - 32400*a^16*b^4*c^6*d^14*e^8*f^12z^3 \\
& - 32400*a^14*b^6*c^12*d^8*e^4*f^16z^3 - 32400*a^14*b^6*c^4*d^16*e^12*f^8z \\
& ^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16z^3 - 32400*a^12*b^8*c^4*d^16*e^14*f^ \\
& 6z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14z^3 - 32400*a^8*b^12*c^6*d^14*e^16 \\
& *f^4z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12z^3 - 32400*a^6*b^14*c^8*d^12*e \\
& ^16*f^4z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8z^3 - 32400*a^4*b^16*c^12*d^ \\
& 8*e^14*f^6z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16z^3 + 20565*a^16*b^4*c^4 \\
& *d^16*e^10*f^10z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16z^3 + 20565*a^10*b^ \\
& 10*c^4*d^16*e^16*f^4z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10z^3 + 20565*a^ \\
& 4*b^16*c^10*d^10*e^16*f^4z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^15z^3 + 1566 \\
& 0*a^17*b^3*c^5*d^15*e^8*f^12z^3 + 15660*a^15*b^5*c^12*d^8*e^3*f^17z^3 + 1 \\
& 5660*a^15*b^5*c^3*d^17*e^12*f^8z^3 + 15660*a^12*b^8*c^15*d^5*e^3*f^17z^3 \\
& + 15660*a^12*b^8*c^3*d^17*e^15*f^5z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15z \\
& ^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^1 \\
& 2z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3z^3 + 15660*a^3*b^17*c^15*d^5*e^12 \\
& *f^8z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5z^3 - 9750*a^17*b^3*c^9*d^11*e^ \\
& 4*f^16z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11z^3 - 9750*a^16*b^4*c^11*d^9*e \\
& ^3*f^17z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9z^3 - 9750*a^11*b^9*c^16*d^4* \\
& e^3*f^17z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4z^3 - 9750*a^9*b^11*c^17*d^3 \\
& *e^4*f^16z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3z^3 - 9750*a^4*b^16*c^17*d^ \\
& 3*e^9*f^11z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3z^3 - 9750*a^3*b^17*c^16*d \\
& ^4*e^11*f^9z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4z^3 - 8100*a^17*b^3*c^7*d \\
& ^13*e^6*f^14z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13z^3 - 8100*a^14*b^6*c^13 \\
& *d^7*e^3*f^17z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7z^3 - 8100*a^13*b^7*c^1 \\
& 4*d^6*e^3*f^17z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6z^3 - 8100*a^7*b^13*c^ \\
& 17*d^3*e^6*f^14z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3z^3 - 8100*a^6*b^14*c \\
& ^17*d^3*e^7*f^13z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3z^3 - 8100*a^3*b^17* \\
& c^14*d^6*e^13*f^7z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6z^3 - 7980*a^16*b^4 \\
& *c^9*d^11*e^5*f^15z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11z^3 - 7980*a^15*b^ \\
& 5*c^11*d^9*e^4*f^16z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9z^3 - 7980*a^11*b \\
& ^9*c^15*d^5*e^4*f^16z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5z^3 - 7980*a^9*b \\
& ^11*c^16*d^4*e^5*f^15z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4z^3 - 7980*a^5* \\
& b^15*c^16*d^4*e^9*f^11z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4z^3 - 7980*a^4 \\
& *b^16*c^15*d^5*e^11*f^9z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5z^3 + 6300*a^ \\
& 18*b^2*c^6*d^14*e^6*f^14z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18z^3 + 6300*a \\
& ^14*b^6*c^2*d^18*e^14*f^6z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14z^3 + 6300* \\
& a^6*b^14*c^6*d^14*e^18*f^2z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^6z^3 - 4260 \\
& *a^18*b^2*c^7*d^13*e^5*f^15z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^13z^3 - 426
\end{aligned}$$

$$\begin{aligned}
& 0*a^{15}*b^5*c^{13}*d^7*e^2*f^{18}*z^3 - 4260*a^{15}*b^5*c^2*d^{18}*e^{13}*f^7*z^3 - 42 \\
& 60*a^{13}*b^7*c^{15}*d^5*e^2*f^{18}*z^3 - 4260*a^{13}*b^7*c^2*d^{18}*e^{15}*f^5*z^3 - 4 \\
& 260*a^7*b^{13}*c^{18}*d^2*e^5*f^{15}*z^3 - 4260*a^7*b^{13}*c^5*d^{15}*e^{18}*f^2*z^3 - \\
& 4260*a^5*b^{15}*c^{18}*d^2*e^7*f^{13}*z^3 - 4260*a^5*b^{15}*c^7*d^{13}*e^{18}*f^2*z^3 - \\
& 4260*a^2*b^{18}*c^{15}*d^5*e^{13}*f^7*z^3 - 4260*a^2*b^{18}*c^{13}*d^7*e^{15}*f^5*z^3 \\
& + 1470*a^{17}*b^3*c^{10}*d^{10}*e^3*f^{17}*z^3 + 1470*a^{17}*b^3*c^3*d^{17}*e^{10}*f^{10}*z^3 \\
& + 1470*a^{10}*b^{10}*c^{17}*d^3*e^3*f^{17}*z^3 + 1470*a^{10}*b^{10}*c^3*d^{17}*e^{17}*f^3*z^3 \\
& + 1470*a^3*b^{17}*c^{17}*d^3*e^{10}*f^{10}*z^3 + 1470*a^3*b^{17}*c^{10}*d^{10}*e^{17} \\
& *f^3*z^3 + 1350*a^{18}*b^2*c^9*d^{11}*e^3*f^{17}*z^3 + 1350*a^{18}*b^2*c^3*d^{17}*e^9 \\
& *f^{11}*z^3 + 1350*a^{17}*b^3*c^{11}*d^9*e^2*f^{18}*z^3 + 1350*a^{17}*b^3*c^2*d^{18}*e^{11} \\
& *f^9*z^3 + 1350*a^{11}*b^9*c^{17}*d^3*e^2*f^{18}*z^3 + 1350*a^{11}*b^9*c^2*d^{18}*e^{17} \\
& *f^3*z^3 + 1350*a^9*b^{11}*c^{18}*d^2*e^3*f^{17}*z^3 + 1350*a^9*b^{11}*c^3*d^{17}*e^{18} \\
& *f^2*z^3 + 1350*a^3*b^{17}*c^{18}*d^2*e^9*f^{11}*z^3 + 1350*a^3*b^{17}*c^9*d^{11} \\
& *e^{18}*f^2*z^3 + 1350*a^2*b^{18}*c^{17}*d^3*e^{11}*f^9*z^3 + 1350*a^2*b^{18}*c^{11}*d^9 \\
& *e^{17}*f^3*z^3 - 1070*a^{18}*b^2*c^{10}*d^{10}*e^2*f^{18}*z^3 - 1070*a^{18}*b^2*c^2*d^{18} \\
& *e^{10}*f^{10}*z^3 - 1070*a^{10}*b^{10}*c^{18}*d^2*e^2*f^{18}*z^3 - 1070*a^{10}*b^{10}*c^2 \\
& *d^{18}*e^{18}*f^2*z^3 - 1070*a^2*b^{18}*c^{18}*d^2*e^{10}*f^{10}*z^3 - 1070*a^2*b^{18} \\
& *c^{10}*d^{10}*e^{18}*f^2*z^3 + 525*a^{18}*b^2*c^8*d^{12}*e^4*f^{16}*z^3 + 525*a^{18}*b^2 \\
& *c^4*d^{16}*e^8*f^{12}*z^3 + 525*a^{16}*b^4*c^{12}*d^8*e^2*f^{18}*z^3 + 525*a^{16}*b^4* \\
& c^2*d^{18}*e^{12}*f^8*z^3 + 525*a^{12}*b^8*c^{16}*d^4*e^2*f^{18}*z^3 + 525*a^{12}*b^8*c^2 \\
& *d^{18}*e^{16}*f^4*z^3 + 525*a^8*b^{12}*c^{18}*d^2*e^4*f^{16}*z^3 + 525*a^8*b^{12}*c^4 \\
& *d^{16}*e^{18}*f^2*z^3 + 525*a^4*b^{16}*c^{18}*d^2*e^8*f^{12}*z^3 + 525*a^4*b^{16}*c^8 \\
& *d^{12}*e^{18}*f^2*z^3 + 525*a^2*b^{18}*c^{16}*d^4*e^{12}*f^8*z^3 + 525*a^2*b^{18}*c^{12} \\
& *d^8*e^{16}*f^4*z^3 + 900*a^{19}*b*c^7*d^{13}*e^4*f^{16}*z^3 + 900*a^{19}*b*c^4*d^{16} \\
& *e^7*f^{13}*z^3 + 900*a^{16}*b^4*c^{13}*d^7*e*f^{19}*z^3 + 900*a^{16}*b^4*c*d^{19}*e^{13} \\
& *f^7*z^3 + 900*a^{13}*b^7*c^{16}*d^4*e*f^{19}*z^3 + 900*a^{13}*b^7*c*d^{19}*e^{16}*f^4*z^3 \\
& + 900*a^7*b^{13}*c^{19}*d*e^4*f^{16}*z^3 + 900*a^7*b^{13}*c^4*d^{16}*e^{19}*f*z^3 + \\
& 900*a^4*b^{16}*c^{19}*d*e^7*f^{13}*z^3 + 900*a^4*b^{16}*c^7*d^{13}*e^{19}*f*z^3 + 900*a \\
& *b^{19}*c^{16}*d^4*e^{13}*f^7*z^3 + 900*a*b^{19}*c^{13}*d^7*e^{16}*f^4*z^3 - 750*a^{19}*b \\
& *c^8*d^{12}*e^3*f^{17}*z^3 - 750*a^{19}*b*c^3*d^{17}*e^8*f^{12}*z^3 - 750*a^{17}*b^3*c^ \\
& 12*d^8*e*f^{19}*z^3 - 750*a^{17}*b^3*c*d^{19}*e^{12}*f^8*z^3 - 750*a^{12}*b^8*c^{17}*d^3 \\
& *e*f^{19}*z^3 - 750*a^{12}*b^8*c*d^{19}*e^{17}*f^3*z^3 - 750*a^8*b^{12}*c^{19}*d*e^3*f \\
& ^{17}*z^3 - 750*a^8*b^{12}*c^3*d^{17}*e^{19}*f*z^3 - 750*a^3*b^{17}*c^{19}*d*e^8*f^{12}*z^3 \\
& - 750*a^3*b^{17}*c^8*d^{12}*e^{19}*f*z^3 - 750*a*b^{19}*c^{17}*d^3*e^{12}*f^8*z^3 - \\
& 750*a*b^{19}*c^{12}*d^8*e^{17}*f^3*z^3 - 420*a^{19}*b*c^6*d^{14}*e^5*f^{15}*z^3 - 420*a \\
& ^{19}*b*c^5*d^{15}*e^6*f^{14}*z^3 - 420*a^{15}*b^5*c^{14}*d^6*e*f^{19}*z^3 - 420*a^{15}*b \\
& ^5*c*d^{19}*e^{14}*f^6*z^3 - 420*a^{14}*b^6*c^{15}*d^5*e*f^{19}*z^3 - 420*a^{14}*b^6*c* \\
& d^{19}*e^{15}*f^5*z^3 - 420*a^6*b^{14}*c^{19}*d*e^5*f^{15}*z^3 - 420*a^6*b^{14}*c^5*d^{15} \\
& *e^{19}*f*z^3 - 420*a^5*b^{15}*c^{19}*d*e^6*f^{14}*z^3 - 420*a^5*b^{15}*c^6*d^{14}*e^{19} \\
& *f*z^3 - 420*a*b^{19}*c^{15}*d^5*e^{14}*f^6*z^3 - 420*a*b^{19}*c^{14}*d^6*e^{15}*f^5*z^3 \\
& + 350*a^{19}*b*c^9*d^{11}*e^2*f^{18}*z^3 + 350*a^{19}*b*c^2*d^{18}*e^9*f^{11}*z^3 + \\
& 350*a^{18}*b^2*c^{11}*d^9*e*f^{19}*z^3 + 350*a^{18}*b^2*c*d^{19}*e^{11}*f^9*z^3 + 350*a \\
& ^{11}*b^9*c^{18}*d^2*e*f^{19}*z^3 + 350*a^{11}*b^9*c*d^{19}*e^{18}*f^2*z^3 + 350*a^9*b^{11} \\
& *c^{19}*d*e^2*f^{18}*z^3 + 350*a^9*b^{11}*c^2*d^{18}*e^{19}*f*z^3 + 350*a^2*b^{18}*c^{19} \\
& *d*e^9*f^{11}*z^3 + 350*a^2*b^{18}*c^9*d^{11}*e^{19}*f*z^3 + 350*a*b^{19}*c^{18}*d^2*
\end{aligned}$$

$$\begin{aligned}
& e^{11}f^9z^3 + 350a^9b^{19}c^{11}d^9e^{18}f^2z^3 - 90a^{19}b^9c^{10}d^{10}e^9f^{19}z^3 - 90a^{19}b^9c^9d^{19}e^{10}f^{10}z^3 - 90a^{10}b^{10}c^{19}d^9e^9f^{19}z^3 - 90a^{10}b^{10}c^9d^{19}e^{19}f^9z^3 - 90a^9b^{19}c^{19}d^9e^{10}f^{10}z^3 - 90a^9b^{19}c^{10}d^{10}e^{19}f^9z^3 + 10b^{20}c^{19}d^9e^{11}f^9z^3 + 10b^{20}c^{11}d^9e^{19}f^9z^3 + 10a^{20}c^9d^{11}e^9f^{19}z^3 + 10a^{20}c^9d^{19}e^9f^{11}z^3 + 10a^{19}b^9d^{20}e^{11}f^9z^3 + 10a^{11}b^9d^{20}e^{19}f^9z^3 + 10a^9b^{11}c^{20}e^9f^{19}z^3 + 10a^9b^{11}c^{20}e^9f^{11}z^3 + 10a^{19}b^9c^{11}d^9f^{20}z^3 + 10a^{11}b^9c^{19}d^9f^{20}z^3 + 10a^9b^{11}c^9d^{19}e^{20}z^3 + 10a^9b^{11}c^9d^{11}e^{20}z^3 + 252b^{20}c^{15}d^5e^{15}f^5z^3 - 210b^{20}c^{16}d^4e^{14}f^6z^3 - 210b^{20}c^{14}d^6e^{16}f^4z^3 + 120b^{20}c^{17}d^3e^{13}f^7z^3 + 120b^{20}c^{13}d^7e^{17}f^3z^3 - 45b^{20}c^{18}d^2e^{12}f^8z^3 - 45b^{20}c^{12}d^8e^{18}f^2z^3 + 252a^{20}c^5d^{15}e^5f^{15}z^3 - 210a^{20}c^6d^{14}e^4f^{16}z^3 - 210a^{20}c^4d^{16}e^6f^{14}z^3 + 120a^{20}c^7d^{13}e^3f^{17}z^3 + 120a^{20}c^3d^{17}e^7f^{13}z^3 - 45a^{20}c^8d^{12}e^2f^{18}z^3 - 45a^{20}c^2d^{18}e^8f^{12}z^3 + 252a^{15}b^5d^{20}e^{15}f^5z^3 - 210a^{16}b^4d^{20}e^{14}f^6z^3 - 210a^{14}b^6d^{20}e^{16}f^4z^3 + 120a^{17}b^3d^{20}e^{13}f^7z^3 + 120a^{13}b^7d^{20}e^{17}f^3z^3 - 45a^{18}b^2d^{20}e^{12}f^8z^3 - 45a^{12}b^8d^{20}e^{18}f^2z^3 + 252a^5b^{15}c^{20}e^5f^{15}z^3 - 210a^6b^{14}c^{20}e^4f^{16}z^3 - 210a^4b^{16}c^{20}e^6f^{14}z^3 + 120a^7b^{13}c^{20}e^3f^{17}z^3 + 120a^3b^{17}c^{20}e^7f^{13}z^3 - 45a^8b^{12}c^{20}e^2f^{18}z^3 - 45a^2b^{18}c^{20}e^8f^{12}z^3 + 252a^{15}b^5c^{15}d^5e^5f^{20}z^3 - 210a^{16}b^4c^{14}d^6e^6f^{20}z^3 - 210a^{14}b^6c^{16}d^4e^4f^{20}z^3 + 120a^{17}b^3c^{13}d^7e^7f^{20}z^3 + 120a^{13}b^7c^{17}d^3e^3f^{20}z^3 - 45a^{18}b^2c^{12}d^8e^8f^{20}z^3 - 45a^{12}b^8c^{18}d^2e^2f^{20}z^3 + 252a^5b^{15}c^5d^{15}e^{20}z^3 - 210a^6b^{14}c^4d^{16}e^{20}z^3 - 210a^4b^{16}c^6d^{14}e^{20}z^3 + 120a^7b^{13}c^3d^{17}e^{20}z^3 + 120a^3b^{17}c^7d^{13}e^{20}z^3 - 45a^8b^{12}c^2d^{18}e^{20}z^3 - 45a^2b^{18}c^8d^{12}e^{20}z^3 - b^{20}c^{20}e^{10}f^{10}z^3 - a^{20}d^{20}e^{10}f^{10}z^3 - b^{20}c^{10}d^{10}e^{20}z^3 - a^{20}c^{10}d^{10}e^{20}z^3 - a^{10}b^{10}d^{20}e^{20}z^3 - a^{10}b^{10}c^{20}f^{20}z^3 + 1890a^{12}b^2c^9d^{13}e^9f^{13}z + 1890a^9b^{13}c^{12}d^2e^9f^{13}z + 1890a^9b^{13}c^9d^{13}e^{12}f^2z + 92610a^6b^8c^4d^{10}e^4f^{10}z + 92610a^4b^{10}c^6d^8e^4f^{10}z + 92610a^4b^{10}c^4d^{10}e^6f^8z + 66150a^8b^6c^3d^{11}e^3f^{11}z - 66150a^7b^7c^4d^{10}e^3f^{11}z - 66150a^4b^{10}c^3d^{11}e^4f^{10}z - 66150a^4b^{10}c^7d^7e^3f^{11}z - 66150a^4b^{10}c^3d^{11}e^7f^7z + 66150a^3b^{11}c^8d^6e^3f^{11}z - 66150a^3b^{11}c^7d^7e^4f^{10}z - 66150a^3b^{11}c^4d^{10}e^7f^7z + 66150a^3b^{11}c^3d^{11}e^8f^6z - 55566a^5b^9c^5d^9e^4f^{10}z - 55566a^5b^9c^4d^{10}e^5f^9z - 55566a^4b^{10}c^5d^9e^5f^9z - 32130a^9b^5c^3d^{11}e^2f^{12}z - 32130a^9b^5c^2d^{12}e^3f^{11}z - 32130a^3b^{11}c^9d^5e^2f^{12}z - 32130a^3b^{11}c^2d^{12}e^9f^5z - 32130a^2b^{12}c^9d^5e^3f^{11}z - 32130a^2b^{12}c^3d^{11}e^9f^5z + 22680a^8b^6c^4d^{10}e^2f^{12}z + 22680a^8b^6c^2d^{12}e^4f^{10}z + 22680a^4b^{10}c^8d^6e^2f^{12}z + 22680a^4b^{10}c^2d^{12}e^8f^6z + 22680a^2b^{12}c^8d^6e^4f^{10}z + 22680a^2b^{12}c^4d^{10}e^8f^6z + 19278a^{10}b^4c^2d^{12}e^2f^{12}z + 19278a^2b^{12}c^{10}d^4e^2f^{12}z + 19278a^2b^{12}c^2d^{12}e^{10}f^4z + 18522a^6b^8c^5d^9e^3f^{11}z + 18522a^6b^8c^3d^{11}e^5
\end{aligned}$$

$$\begin{aligned}
& *f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8* \\
& z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 1 \\
& 3230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230* \\
& a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7 \\
& *c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^ \\
& 12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7* \\
& f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z + \\
& 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a \\
& *b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3 \\
& *c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d \\
& ^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^11*d^3*e^2*f^ \\
& 12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3 \\
& 780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b \\
& ^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^1 \\
& 0*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z \\
& - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b \\
& ^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^ \\
& 8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z \\
& + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^ \\
& 13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13*e^13*f*z \\
& - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^13*b*c*d^13*f \\
& ^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z - 882*b^14*c^ \\
& 11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^12*d^2*e^2*f^ \\
& 12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*z + 567*b^14 \\
& *c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^4*d^10*e^10* \\
& f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z - 918*a^7*b \\
& ^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11*d^14*e^11*f \\
& ^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^2*z + 567*a^ \\
& 8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^4*d^14*e^4*f \\
& ^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9*z - 126*a^5 \\
& *b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3*c^3*d^11*f^ \\
& 14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^14*z + 693*a^ \\
& 2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b^8*c^8*d^6*f \\
& ^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^14*z - 126*a \\
& ^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^14*e^14*z + \\
& 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d^9*e^2*f^9 + \\
& 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 9018*a^2*b^9*c \\
& ^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d^8*e^2*f^9 + \\
& 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 9018*a*b^10*c^4 \\
& *d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*f^10 + 2268* \\
& b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^11*e^5*f^6 + \\
& 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^11*c^3*d^8*e^ \\
& 3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 - 1458*a^3*b^ \\
& 8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*e^4*f^7 - 14 \\
& 58*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*b^9*c^4*d^7*
\end{aligned}$$

$$\begin{aligned}
& f^{11} - 756b^{11}d^{11}e^6f^5 - 756b^{11}c^6d^5f^{11} - 756a^6b^5d^{11}f^{11} \\
& 1, z, k) * ((39a^5b^{11}c^{14}d^2f^{16} - 102a^6b^{10}c^{13}d^3f^{16} + 132a^7 \\
& * b^9c^{12}d^4f^{16} - 84a^8b^8c^{11}d^5f^{16} + 21a^9b^7c^{10}d^6f^{16} + \\
& 21a^{10}b^6c^9d^7f^{16} - 84a^{11}b^5c^8d^8f^{16} + 132a^{12}b^4c^7d^9 \\
& f^{16} - 102a^{13}b^3c^6d^{10}f^{16} + 39a^{14}b^2c^5d^{11}f^{16} + 39a^5b^{11} \\
& * d^{16}e^{14}f^2 - 102a^6b^{10}d^{16}e^{13}f^3 + 132a^7b^9d^{16}e^{12}f^4 - 8 \\
& 4a^8b^8d^{16}e^{11}f^5 + 21a^9b^7d^{16}e^{10}f^6 + 21a^{10}b^6d^{16}e^9f^7 \\
& - 84a^{11}b^5d^{16}e^8f^8 + 132a^{12}b^4d^{16}e^7f^9 - 102a^{13}b^3d^{16} \\
& e^6f^{10} + 39a^{14}b^2d^{16}e^5f^{11} + 39b^{16}c^5d^{11}e^{14}f^2 - 102b \\
& ^{16}c^6d^{10}e^{13}f^3 + 132b^{16}c^7d^9e^{12}f^4 - 84b^{16}c^8d^8e^{11}f^5 \\
& + 21b^{16}c^9d^7e^{10}f^6 + 21b^{16}c^{10}d^6e^9f^7 - 84b^{16}c^{11}d^5 \\
& e^8f^8 + 132b^{16}c^{12}d^4e^7f^9 - 102b^{16}c^{13}d^3e^6f^{10} + 39b^{16} \\
& c^{14}d^2e^5f^{11} - 6a^4b^{12}c^{15}d^*f^{16} - 6a^{15}b^*c^4d^{12}f^{16} - 6a^4 \\
& * b^{12}d^{16}e^{15}f - 6a^{15}b^*d^{16}e^4f^{12} - 6b^{16}c^4d^{12}e^{15}f - 6b^{16} \\
& c^{15}d^*e^4f^{12} + 24a^*b^{15}c^3d^{13}e^{15}f + 24a^*b^{15}c^{15}d^*e^3f^{13} + \\
& 24a^3b^{13}c^*d^{15}e^{15}f + 24a^3b^{13}c^{15}d^*e^*f^{15} + 24a^{15}b^*c^*d^{15}e \\
& ^3f^{13} + 24a^{15}b^*c^3d^{13}e^*f^{15} - 117a^*b^{15}c^4d^{12}e^{14}f^2 + 150a^* \\
& b^{15}c^5d^{11}e^{13}f^3 + 159a^*b^{15}c^6d^{10}e^{12}f^4 - 546a^*b^{15}c^7d^9 \\
& e^{11}f^5 + 414a^*b^{15}c^8d^8e^{10}f^6 - 168a^*b^{15}c^9d^7e^9f^7 + 414a^* \\
& b^{15}c^{10}d^6e^8f^8 - 546a^*b^{15}c^{11}d^5e^7f^9 + 159a^*b^{15}c^{12}d^4 \\
& e^6f^{10} + 150a^*b^{15}c^{13}d^3e^5f^{11} - 117a^*b^{15}c^{14}d^2e^4f^{12} - 36 \\
& a^2b^{14}c^2d^{14}e^{15}f - 36a^2b^{14}c^{15}d^*e^2f^{14} - 117a^4b^{12}c^*d^{15} \\
& e^{14}f^2 - 117a^4b^{12}c^{14}d^2e^*f^{15} + 150a^5b^{11}c^*d^{15}e^{13}f^3 + \\
& 150a^5b^{11}c^{13}d^3e^*f^{15} + 159a^6b^{10}c^*d^{15}e^{12}f^4 + 159a^6b^{10} \\
& c^{12}d^4e^*f^{15} - 546a^7b^9c^*d^{15}e^{11}f^5 - 546a^7b^9c^{11}d^5e^*f^{15} \\
& + 414a^8b^8c^*d^{15}e^{10}f^6 + 414a^8b^8c^{10}d^6e^*f^{15} - 168a^9b^7 \\
& c^*d^{15}e^9f^7 - 168a^9b^7c^9d^7e^*f^{15} + 414a^{10}b^6c^*d^{15}e^8f^8 \\
& + 414a^{10}b^6c^8d^8e^*f^{15} - 546a^{11}b^5c^*d^{15}e^7f^9 - 546a^{11}b^5 \\
& c^7d^9e^*f^{15} + 159a^{12}b^4c^*d^{15}e^6f^{10} + 159a^{12}b^4c^6d^{10}e^*f^{15} \\
& + 150a^{13}b^3c^*d^{15}e^5f^{11} + 150a^{13}b^3c^5d^{11}e^*f^{15} - 117a^{14} \\
& b^2c^*d^{15}e^4f^{12} - 117a^{14}b^2c^4d^{12}e^*f^{15} - 36a^{15}b^*c^2d^{14}e^2 \\
& *f^{14} + 78a^2b^{14}c^3d^{13}e^{14}f^2 + 318a^2b^{14}c^4d^{12}e^{13}f^3 - 12 \\
& 69a^2b^{14}c^5d^{11}e^{12}f^4 + 1134a^2b^{14}c^6d^{10}e^{11}f^5 + 618a^2b^{14} \\
& c^7d^9e^{10}f^6 - 843a^2b^{14}c^8d^8e^9f^7 - 843a^2b^{14}c^9d^7 \\
& e^8f^8 + 618a^2b^{14}c^{10}d^6e^7f^9 + 1134a^2b^{14}c^{11}d^5e^6f^{10} - \\
& 1269a^2b^{14}c^{12}d^4e^5f^{11} + 318a^2b^{14}c^{13}d^3e^4f^{12} + 78a^2 \\
& b^{14}c^{14}d^2e^3f^{13} + 78a^3b^{13}c^2d^{14}e^{14}f^2 - 732a^3b^{13}c^3d^{13} \\
& e^{13}f^3 + 978a^3b^{13}c^4d^{12}e^{12}f^4 + 1722a^3b^{13}c^5d^{11}e^{11} \\
& *f^5 - 4548a^3b^{13}c^6d^{10}e^{10}f^6 + 1362a^3b^{13}c^7d^9e^9f^7 + 22 \\
& 32a^3b^{13}c^8d^8e^8f^8 + 1362a^3b^{13}c^9d^7e^7f^9 - 4548a^3b^{13} \\
& c^{10}d^6e^6f^{10} + 1722a^3b^{13}c^{11}d^5e^5f^{11} + 978a^3b^{13}c^{12}d^4 \\
& e^4f^{12} - 732a^3b^{13}c^{13}d^3e^3f^{13} + 78a^3b^{13}c^{14}d^2e^2f^{14} \\
& + 318a^4b^{12}c^2d^{14}e^{13}f^3 + 978a^4b^{12}c^3d^{13}e^{12}f^4 - 4452a^4 \\
& b^{12}c^4d^{12}e^{11}f^5 + 3495a^4b^{12}c^5d^{11}e^{10}f^6 + 4302a^4b^{12} \\
& c^6d^{10}e^9f^7 - 4518a^4b^{12}c^7d^9e^8f^8 - 4518a^4b^{12}c^8d^8e
\end{aligned}$$

$$\begin{aligned}
& ^7f^9 + 4302a^4b^{12}c^9d^7e^6f^{10} + 3495a^4b^{12}c^{10}d^6e^5f^{11} - \\
& 4452a^4b^{12}c^{11}d^5e^4f^{12} + 978a^4b^{12}c^{12}d^4e^3f^{13} + 318a^4 \\
& b^{12}c^{13}d^3e^2f^{14} - 1269a^5b^{11}c^2d^{14}e^{12}f^4 + 1722a^5b^{11}c \\
& ^3d^{13}e^{11}f^5 + 3495a^5b^{11}c^4d^{12}e^{10}f^6 - 9348a^5b^{11}c^5d^{11} \\
& e^9f^7 + 2799a^5b^{11}c^6d^{10}e^8f^8 + 4824a^5b^{11}c^7d^9e^7f^9 + \\
& 2799a^5b^{11}c^8d^8e^6f^{10} - 9348a^5b^{11}c^9d^7e^5f^{11} + 3495a^5 \\
& b^{11}c^{10}d^6e^4f^{12} + 1722a^5b^{11}c^{11}d^5e^3f^{13} - 1269a^5b^{11}c \\
& ^{12}d^4e^2f^{14} + 1134a^6b^{10}c^2d^{14}e^{11}f^5 - 4548a^6b^{10}c^3d^{13} \\
& e^{10}f^6 + 4302a^6b^{10}c^4d^{12}e^9f^7 + 2799a^6b^{10}c^5d^{11}e^8f^8 \\
& - 3744a^6b^{10}c^6d^{10}e^7f^9 - 3744a^6b^{10}c^7d^9e^6f^{10} + 2799a \\
& ^6b^{10}c^8d^8e^5f^{11} + 4302a^6b^{10}c^9d^7e^4f^{12} - 4548a^6b^{10}c \\
& ^{10}d^6e^3f^{13} + 1134a^6b^{10}c^{11}d^5e^2f^{14} + 618a^7b^9c^2d^{14}e \\
& ^{10}f^6 + 1362a^7b^9c^3d^{13}e^9f^7 - 4518a^7b^9c^4d^{12}e^8f^8 + 4 \\
& 824a^7b^9c^5d^{11}e^7f^9 - 3744a^7b^9c^6d^{10}e^6f^{10} + 4824a^7b^ \\
& 9c^7d^9e^5f^{11} - 4518a^7b^9c^8d^8e^4f^{12} + 1362a^7b^9c^9d^7e \\
& ^3f^{13} + 618a^7b^9c^{10}d^6e^2f^{14} - 843a^8b^8c^2d^{14}e^9f^7 + 22 \\
& 32a^8b^8c^3d^{13}e^8f^8 - 4518a^8b^8c^4d^{12}e^7f^9 + 2799a^8b^8c \\
& ^5d^{11}e^6f^{10} + 2799a^8b^8c^6d^{10}e^5f^{11} - 4518a^8b^8c^7d^9e \\
& ^4f^{12} + 2232a^8b^8c^8d^8e^3f^{13} - 843a^8b^8c^9d^7e^2f^{14} - 84 \\
& 3a^9b^7c^2d^{14}e^8f^8 + 1362a^9b^7c^3d^{13}e^7f^9 + 4302a^9b^7c \\
& ^4d^{12}e^6f^{10} - 9348a^9b^7c^5d^{11}e^5f^{11} + 4302a^9b^7c^6d^{10}e \\
& ^4f^{12} + 1362a^9b^7c^7d^9e^3f^{13} - 843a^9b^7c^8d^8e^2f^{14} + 61 \\
& 8a^{10}b^6c^2d^{14}e^7f^9 - 4548a^{10}b^6c^3d^{13}e^6f^{10} + 3495a^{10}b \\
& ^6c^4d^{12}e^5f^{11} + 3495a^{10}b^6c^5d^{11}e^4f^{12} - 4548a^{10}b^6c^6d \\
& ^{10}e^3f^{13} + 618a^{10}b^6c^7d^9e^2f^{14} + 1134a^{11}b^5c^2d^{14}e^6f \\
& ^{10} + 1722a^{11}b^5c^3d^{13}e^5f^{11} - 4452a^{11}b^5c^4d^{12}e^4f^{12} + \\
& 1722a^{11}b^5c^5d^{11}e^3f^{13} + 1134a^{11}b^5c^6d^{10}e^2f^{14} - 1269a^ \\
& ^{12}b^4c^2d^{14}e^5f^{11} + 978a^{12}b^4c^3d^{13}e^4f^{12} + 978a^{12}b^4c^ \\
& ^4d^{12}e^3f^{13} - 1269a^{12}b^4c^5d^{11}e^2f^{14} + 318a^{13}b^3c^2d^{14}e \\
& ^4f^{12} - 732a^{13}b^3c^3d^{13}e^3f^{13} + 318a^{13}b^3c^4d^{12}e^2f^{14} + \\
& 78a^{14}b^2c^2d^{14}e^3f^{13} + 78a^{14}b^2c^3d^{13}e^2f^{14}) / (56a^3b^1 \\
& 3c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8f^{16} - b^{16}c^8d^8e^{16} \\
& - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b \\
& ^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a \\
& ^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3f \\
& ^{16} - 70a^{12}b^4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2c \\
& ^{10}d^6f^{16} - 28a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a \\
& ^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} \\
& - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16} \\
& e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^ \\
& ^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} \\
& + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^ \\
& ^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16} \\
& c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^7b^9c^7d^9e^{16} + 8 \\
& a^7b^9c^7d^9e^{16} + 8a^9b^7c^{15}d^7f^{16} + 8a^{15}b^9c^9d^7f^{16} + 8a^
\end{aligned}$$

$$\begin{aligned}
& b^{15}c^{16}e^7f^9 + 8a^7b^9c^{16}e^7f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^*d^{16}e^9f^7 + 8a^{16}c^*d^{15}e^7f^9 + 8a^{16}c^7d^9e^7f^{15} + 8b^{16}c^9 \\
& *d^7e^{15}f + 8b^{16}c^{15}d^*e^9f^7 - 56a*b^{15}c^8d^8e^{15}f - 56a*b^{15}c^{15}d^*e^8f^8 - 56a^8b^8c^*d^{15}e^{15}f - 56a^8b^8c^{15}d^*e^7f^{15} - 56a^* \\
& ^{15}b^*c^*d^{15}e^8f^8 - 56a^{15}b^*c^8d^8e^7f^{15} + 160a*b^{15}c^9d^7e^{14}f^2 - 224a*b^{15}c^{10}d^6e^{13}f^3 + 112a*b^{15}c^{11}d^5e^{12}f^4 + 112a*b^{15}c^{12}d^4e^{11}f^5 - 224a*b^{15}c^{13}d^3e^{10}f^6 + 160a*b^{15}c^{14}d^2e^9f^7 + 160a^2b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d^*e^7f^9 - 224a^3b^{13}c^6d^{10}e^{15}f - 224a^3b^{13}c^{15}d^*e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112a^4b^{12}c^{15}d^*e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15}d^*e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10}c^{15}d^*e^3f^{13} + 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d^*e^2f^{14} + 160a^9b^7c^*d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e^7f^{15} - 224a^{10}b^6c^*d^{15}e^{13}f^3 - 224a^{10}b^6c^{13}d^3e^7f^{15} + 112a^{11}b^5c^*d^{15}e^{12}f^4 + 112a^{11}b^5c^{12}d^4e^7f^{15} + 112a^{12}b^4c^*d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e^7f^{15} - 224a^{13}b^3c^*d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6e^7f^{15} + 160a^{14}b^2c^*d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e^7f^{15} + 160a^{15}b^*c^2d^{14}e^7f^9 - 224a^{15}b^*c^3d^{13}e^6f^{10} + 112a^{15}b^*c^4d^{12}e^5f^{11} + 112a^{15}b^*c^5d^{11}e^4f^{12} - 224a^{15}b^*c^6d^{10}e^3f^{13} + 160a^{15}b^*c^7d^9e^2f^{14} - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 - 1344a^2b^{14}c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a^2b^{14}c^{14}d^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5e^{10}f^6 - 2800a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 + 840a^4b^{12}c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^4b^{12}c^8d^8e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e^8f^8 - 2800a^4b^{12}c^{13}d^3e^7f^9 + 840a^4b^{12}c^{14}d^2e^6f^{10} - 1344a^5b^{11}c^5d^{11}e^{14}f^2 + 1568a^5b^{11}c^6d^{10}e^{13}f^3 + 4480a^5b^{11}c^7d^9e^{12}f^4 - 12264a^5b^{11}c^8d^8e^{11}f^5 + 7392a^5b^{11}c^9d^7e^{10}f^6 + 7392a^5b^{11}c^{10}d^6e^9f^7 - 12264a^5b^{11}c^{11}d^5e^8f^8 + 4480a^5b^{11}c^{12}d^4e^7f^9 + 1568a^5b^{11}c^{13}d^3e^6f^{10} - 1344a^5b^{11}c^{14}d^2e^5f^{11} + 840a^6b^{10}c^4d^{12}e^{14}f^2 + 1568a^6b^{10}c^5d^{11}e^{13}f^3 - 8624a^6b^{10}c^6d^{10}e^{12}f^4 + 7392a^6b^{10}c^7d^9e^{11}f^5 + 11396a^6b^{10}c^8d^8e^{10}f^6 - 24640a^6b^{10}c^9d^7e^9f^7 + 11396a^6b^{10}c^{10}d^6e^8f^8 + 7392a^6b^{10}c^{11}d^5e^7f^9 - 8624a^6b^{10}c^{12}d^4e^6f^{10} + 1568a^6b^{10}c^{13}d^3e^5f^{11} + 840a^6b^{10}c^{14}d^2e^4f^{12} - 2800a^7b^9c^4d^{12}e^{13}f^3 + 4480a^7b^9c^5d^{11}e^{12}f^4 + 7392a^7b^9c^6d^{10}e^{11}f^5 - 24640a^7b^9c^7d^9e^{10}f^6 + 15400a^7b^9c^8d^8e^9f^7 + 15400a^7b^9c^9d^7e^8f^8 - 24640a^7b^9c^{10}d^6e^7f^9 + 7392a^7b^9c^{11}d^5e^6f^{10} + 4480a^7b^9c^{12}d^4e^5f^{11} - 2800a^7b^9c^{13}d^3e^4f^{12} - 300a^8b^8c^2d^{14}e^{14}f^2 + 1400a^8b^8c^3d^{13}e^{13}f^3 + 1750a^8b^8c^4d^{12}e^{12}f^4 - 12264a^8b^8c^5d^{11}e^{11}f^5 + 11396a^8b^8c^6d^{10}e^{10}f^6 + 15400a^8b^8c^7d^9e^9f^7 - 34650a^8b^8c^8d^8e^8f^8
\end{aligned}$$

$$\begin{aligned}
& 8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264 \\
& *a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8* \\
& c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}* \\
& ^{12}*f^4 + 4480*a^9*b^7*c^4*d^{12}*e^{11}*f^5 + 7392*a^9*b^7*c^5*d^{11}*e^{10}*f^6 - \\
& 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9 \\
& *b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}*d \\
& ^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f^ \\
& 13 + 840*a^{10}*b^6*c^2*d^{14}*e^{12}*f^4 + 1568*a^{10}*b^6*c^3*d^{13}*e^{11}*f^5 - 862 \\
& 4*a^{10}*b^6*c^4*d^{12}*e^{10}*f^6 + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 + 11396*a^{10}* \\
& b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8* \\
& d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4* \\
& f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - 1 \\
& 344*a^{11}*b^5*c^2*d^{14}*e^{11}*f^5 + 1568*a^{11}*b^5*c^3*d^{13}*e^{10}*f^6 + 4480*a^{11} \\
& *b^5*c^4*d^{12}*e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^ \\
& 6*d^{10}*e^7*f^9 + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^ \\
& 5*f^{11} + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - \\
& 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6 - 2800*a^ \\
& 12*b^4*c^3*d^{13}*e^9*f^7 + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480*a^{12}*b^4*c^ \\
& 5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^ \\
& 5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} + \\
& 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13} \\
& *b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568*a^{13}*b^3*c^6 \\
& *d^{10}*e^5*f^{11} - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3 \\
& *f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 13 \\
& 44*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b \\
& ^2*c^8*d^8*e^2*f^{14}) + \text{root}(756756*a^{10}*b^{10}*c^{10}*d^{10}*e^{10}*f^{10}*z^3 + 5733 \\
& 00*a^{12}*b^8*c^9*d^{11}*e^9*f^{11}*z^3 + 573300*a^{11}*b^9*c^{11}*d^9*e^8*f^{12}*z^3 + \\
& 573300*a^{11}*b^9*c^8*d^{12}*e^{11}*f^9*z^3 + 573300*a^9*b^{11}*c^{12}*d^8*e^9*f^{11}* \\
& z^3 + 573300*a^9*b^{11}*c^9*d^{11}*e^{12}*f^8*z^3 + 573300*a^8*b^{12}*c^{11}*d^9*e^{11} \\
& *f^9*z^3 - 343980*a^{11}*b^9*c^{10}*d^{10}*e^9*f^{11}*z^3 - 343980*a^{11}*b^9*c^9*d^{11} \\
& *e^{10}*f^{10}*z^3 - 343980*a^{10}*b^{10}*c^{11}*d^9*e^9*f^{11}*z^3 - 343980*a^{10}*b^{10} \\
& *c^9*d^{11}*e^{11}*f^9*z^3 - 343980*a^9*b^{11}*c^{11}*d^9*e^{10}*f^{10}*z^3 - 343980*a^ \\
& 9*b^{11}*c^{10}*d^{10}*e^{11}*f^9*z^3 + 326340*a^{13}*b^7*c^{10}*d^{10}*e^7*f^{13}*z^3 + 32 \\
& 6340*a^{13}*b^7*c^7*d^{13}*e^{10}*f^{10}*z^3 + 326340*a^{10}*b^{10}*c^{13}*d^7*e^7*f^{13}*z \\
& ^3 + 326340*a^{10}*b^{10}*c^7*d^{13}*e^{13}*f^7*z^3 + 326340*a^7*b^{13}*c^{13}*d^7*e^{10} \\
& *f^{10}*z^3 + 326340*a^7*b^{13}*c^{10}*d^{10}*e^{13}*f^7*z^3 - 267540*a^{12}*b^8*c^{10}*d \\
& ^{10}*e^8*f^{12}*z^3 - 267540*a^{12}*b^8*c^8*d^{12}*e^{10}*f^{10}*z^3 - 267540*a^{10}*b^{11} \\
& 0*c^{12}*d^8*e^8*f^{12}*z^3 - 267540*a^{10}*b^{10}*c^8*d^{12}*e^{12}*f^8*z^3 - 267540*a \\
& ^8*b^{12}*c^{12}*d^8*e^{10}*f^{10}*z^3 - 267540*a^8*b^{12}*c^{10}*d^{10}*e^{12}*f^8*z^3 + 2 \\
& 45700*a^{14}*b^6*c^8*d^{12}*e^8*f^{12}*z^3 + 245700*a^{12}*b^8*c^{12}*d^8*e^6*f^{14}*z^ \\
& 3 + 245700*a^{12}*b^8*c^6*d^{14}*e^{12}*f^8*z^3 + 245700*a^8*b^{12}*c^{14}*d^6*e^8*f^ \\
& 12*z^3 + 245700*a^8*b^{12}*c^8*d^{12}*e^{14}*f^6*z^3 + 245700*a^6*b^{14}*c^{12}*d^8*e \\
& ^{12}*f^8*z^3 - 191100*a^{13}*b^7*c^9*d^{11}*e^8*f^{12}*z^3 - 191100*a^{13}*b^7*c^8*d \\
& ^{12}*e^9*f^{11}*z^3 - 191100*a^{12}*b^8*c^{11}*d^9*e^7*f^{13}*z^3 - 191100*a^{12}*b^8* \\
& c^7*d^{13}*e^{11}*f^9*z^3 - 191100*a^{11}*b^9*c^{12}*d^8*e^7*f^{13}*z^3 - 191100*a^{11}
\end{aligned}$$

$$\begin{aligned}
& *b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100 \\
& *a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 1 \\
& 91100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^ \\
& 3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^ \\
& 13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e \\
& ^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13* \\
& d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11* \\
& c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7* \\
& b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900 \\
& *a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 1 \\
& 01700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^ \\
& 3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f \\
& ^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e \\
& ^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d \\
& ^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14* \\
& c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5* \\
& b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820* \\
& a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 6 \\
& 5820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^ \\
& 3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^1 \\
& 3*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8* \\
& f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e \\
& ^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^1 \\
& 6*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5* \\
& d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c \\
& ^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^1 \\
& 3*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7* \\
& b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a \\
& ^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 4825 \\
& 2*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 - \\
& 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5* \\
& z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15 \\
& *f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e \\
& ^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^1 \\
& 6*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4* \\
& d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c \\
& ^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^1 \\
& 4*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4* \\
& b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565* \\
& a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 2 \\
& 0565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^ \\
& 3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^1 \\
& 5*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3* \\
& f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e \\
& ^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^
\end{aligned}$$

$$\begin{aligned}
& 3e^5f^{15}z^3 + 15660a^8b^{12}c^5d^{15}e^{17}f^3z^3 + 15660a^5b^{15}c^{17} \\
& *d^3e^8f^{12}z^3 + 15660a^5b^{15}c^8d^{12}e^{17}f^3z^3 + 15660a^3b^{17}c \\
& ^{15}d^5e^{12}f^8z^3 + 15660a^3b^{17}c^{12}d^8e^{15}f^5z^3 - 9750a^{17}b^3 \\
& *c^9d^{11}e^4f^{16}z^3 - 9750a^{17}b^3c^4d^{16}e^9f^{11}z^3 - 9750a^{16}b^4 \\
& *c^{11}d^9e^3f^{17}z^3 - 9750a^{16}b^4c^3d^{17}e^{11}f^9z^3 - 9750a^{11}b \\
& ^9c^{16}d^4e^3f^{17}z^3 - 9750a^{11}b^9c^3d^{17}e^{16}f^4z^3 - 9750a^9b \\
& ^{11}c^{17}d^3e^4f^{16}z^3 - 9750a^9b^{11}c^4d^{16}e^{17}f^3z^3 - 9750a^4* \\
& b^{16}c^{17}d^3e^9f^{11}z^3 - 9750a^4b^{16}c^9d^{11}e^{17}f^3z^3 - 9750a^3 \\
& *b^{17}c^{16}d^4e^{11}f^9z^3 - 9750a^3b^{17}c^{11}d^9e^{16}f^4z^3 - 8100a^ \\
& ^{17}b^3c^7d^{13}e^6f^{14}z^3 - 8100a^{17}b^3c^6d^{14}e^7f^{13}z^3 - 8100a \\
& ^{14}b^6c^{13}d^7e^3f^{17}z^3 - 8100a^{14}b^6c^3d^{17}e^{13}f^7z^3 - 8100* \\
& a^{13}b^7c^{14}d^6e^3f^{17}z^3 - 8100a^{13}b^7c^3d^{17}e^{14}f^6z^3 - 8100 \\
& *a^7b^{13}c^{17}d^3e^6f^{14}z^3 - 8100a^7b^{13}c^6d^{14}e^{17}f^3z^3 - 810 \\
& 0a^6b^{14}c^{17}d^3e^7f^{13}z^3 - 8100a^6b^{14}c^7d^{13}e^{17}f^3z^3 - 81 \\
& 00a^3b^{17}c^{14}d^6e^{13}f^7z^3 - 8100a^3b^{17}c^{13}d^7e^{14}f^6z^3 - 7 \\
& 980a^{16}b^4c^9d^{11}e^5f^{15}z^3 - 7980a^{16}b^4c^5d^{15}e^9f^{11}z^3 - \\
& 7980a^{15}b^5c^{11}d^9e^4f^{16}z^3 - 7980a^{15}b^5c^4d^{16}e^{11}f^9z^3 - \\
& 7980a^{11}b^9c^{15}d^5e^4f^{16}z^3 - 7980a^{11}b^9c^4d^{16}e^{15}f^5z^3 \\
& - 7980a^9b^{11}c^{16}d^4e^5f^{15}z^3 - 7980a^9b^{11}c^5d^{15}e^{16}f^4z^3 \\
& - 7980a^5b^{15}c^{16}d^4e^9f^{11}z^3 - 7980a^5b^{15}c^9d^{11}e^{16}f^4z^ \\
& 3 - 7980a^4b^{16}c^{15}d^5e^{11}f^9z^3 - 7980a^4b^{16}c^{11}d^9e^{15}f^5z \\
& ^3 + 6300a^{18}b^2c^6d^{14}e^6f^{14}z^3 + 6300a^{14}b^6c^{14}d^6e^2f^{18} \\
& z^3 + 6300a^{14}b^6c^2d^{18}e^{14}f^6z^3 + 6300a^6b^{14}c^{18}d^2e^6f^{14} \\
& *z^3 + 6300a^6b^{14}c^6d^{14}e^{18}f^2z^3 + 6300a^2b^{18}c^{14}d^6e^{14}f^ \\
& 6z^3 - 4260a^{18}b^2c^7d^{13}e^5f^{15}z^3 - 4260a^{18}b^2c^5d^{15}e^7f^ \\
& ^{13}z^3 - 4260a^{15}b^5c^{13}d^7e^2f^{18}z^3 - 4260a^{15}b^5c^2d^{18}e^{13} \\
& *f^7z^3 - 4260a^{13}b^7c^{15}d^5e^2f^{18}z^3 - 4260a^{13}b^7c^2d^{18}e^{15} \\
& *f^5z^3 - 4260a^7b^{13}c^{18}d^2e^5f^{15}z^3 - 4260a^7b^{13}c^5d^{15}e^1 \\
& 8f^2z^3 - 4260a^5b^{15}c^{18}d^2e^7f^{13}z^3 - 4260a^5b^{15}c^7d^{13}e^ \\
& ^{18}f^2z^3 - 4260a^2b^{18}c^{15}d^5e^{13}f^7z^3 - 4260a^2b^{18}c^{13}d^7e \\
& ^{15}f^5z^3 + 1470a^{17}b^3c^{10}d^{10}e^3f^{17}z^3 + 1470a^{17}b^3c^3d^{17} \\
& *e^{10}f^{10}z^3 + 1470a^{10}b^{10}c^{17}d^3e^3f^{17}z^3 + 1470a^{10}b^{10}c^3* \\
& d^{17}e^{17}f^3z^3 + 1470a^3b^{17}c^{17}d^3e^{10}f^{10}z^3 + 1470a^3b^{17}c^ \\
& ^{10}d^{10}e^{17}f^3z^3 + 1350a^{18}b^2c^9d^{11}e^3f^{17}z^3 + 1350a^{18}b^2* \\
& c^3d^{17}e^9f^{11}z^3 + 1350a^{17}b^3c^{11}d^9e^2f^{18}z^3 + 1350a^{17}b^3 \\
& *c^2d^{18}e^{11}f^9z^3 + 1350a^{11}b^9c^{17}d^3e^2f^{18}z^3 + 1350a^{11}b^ \\
& 9c^2d^{18}e^{17}f^3z^3 + 1350a^9b^{11}c^{18}d^2e^3f^{17}z^3 + 1350a^9b^ \\
& ^{11}c^3d^{17}e^{18}f^2z^3 + 1350a^3b^{17}c^{18}d^2e^9f^{11}z^3 + 1350a^3b \\
& ^{17}c^9d^{11}e^{18}f^2z^3 + 1350a^2b^{18}c^{17}d^3e^{11}f^9z^3 + 1350a^2* \\
& b^{18}c^{11}d^9e^{17}f^3z^3 - 1070a^{18}b^2c^{10}d^{10}e^2f^{18}z^3 - 1070a^ \\
& ^{18}b^2c^2d^{18}e^{10}f^{10}z^3 - 1070a^{10}b^{10}c^{18}d^2e^2f^{18}z^3 - 1070 \\
& *a^{10}b^{10}c^2d^{18}e^{18}f^2z^3 - 1070a^2b^{18}c^{18}d^2e^{10}f^{10}z^3 - 1 \\
& 070a^2b^{18}c^{10}d^{10}e^{18}f^2z^3 + 525a^{18}b^2c^8d^{12}e^4f^{16}z^3 + \\
& 525a^{18}b^2c^4d^{16}e^8f^{12}z^3 + 525a^{16}b^4c^{12}d^8e^2f^{18}z^3 + 5 \\
& 25a^{16}b^4c^2d^{18}e^{12}f^8z^3 + 525a^{12}b^8c^{16}d^4e^2f^{18}z^3 + 52
\end{aligned}$$

$$\begin{aligned}
&5a^{12}b^8c^2d^{18}e^{16}f^4z^3 + 525a^8b^{12}c^{18}d^2e^4f^{16}z^3 + 525 \\
&a^8b^{12}c^4d^{16}e^{18}f^2z^3 + 525a^4b^{16}c^{18}d^2e^8f^{12}z^3 + 525a \\
&a^4b^{16}c^8d^{12}e^{18}f^2z^3 + 525a^2b^{18}c^{16}d^4e^{12}f^8z^3 + 525a \\
&{^2b^{18}c^{12}d^8e^{16}f^4z^3 + 900a^{19}b^3c^7d^{13}e^4f^{16}z^3 + 900a^{19} \\
&*b^3c^4d^{16}e^7f^{13}z^3 + 900a^{16}b^4c^{13}d^7e^7f^{19}z^3 + 900a^{16}b^4c \\
&*d^{19}e^{13}f^7z^3 + 900a^{13}b^7c^{16}d^4e^7f^{19}z^3 + 900a^{13}b^7c^4d^{1 \\
&9}e^{16}f^4z^3 + 900a^7b^{13}c^{19}d^4e^4f^{16}z^3 + 900a^7b^{13}c^4d^{16}e \\
&{^19}f^4z^3 + 900a^4b^{16}c^{19}d^4e^7f^{13}z^3 + 900a^4b^{16}c^7d^{13}e^{19}f \\
&{*z^3 + 900a^3b^{19}c^{16}d^4e^{13}f^7z^3 + 900a^3b^{19}c^{13}d^7e^{16}f^4z^3 \\
&- 750a^{19}b^3c^8d^{12}e^3f^{17}z^3 - 750a^{19}b^3c^3d^{17}e^8f^{12}z^3 - 750 \\
&*a^{17}b^3c^{12}d^8e^7f^{19}z^3 - 750a^{17}b^3c^4d^{19}e^{12}f^8z^3 - 750a^{12} \\
&{*b^8c^{17}d^3e^7f^{19}z^3 - 750a^{12}b^8c^4d^{19}e^{17}f^3z^3 - 750a^8b^{12}c \\
&{^19}d^4e^3f^{17}z^3 - 750a^8b^{12}c^3d^{17}e^{19}f^4z^3 - 750a^3b^{17}c^{19} \\
&d^4e^8f^{12}z^3 - 750a^3b^{17}c^8d^{12}e^{19}f^4z^3 - 750a^3b^{19}c^{17}d^3e^{1 \\
&2}f^8z^3 - 750a^3b^{19}c^{12}d^8e^{17}f^3z^3 - 420a^{19}b^3c^6d^{14}e^5f^{15} \\
&{*z^3 - 420a^{19}b^3c^5d^{15}e^6f^{14}z^3 - 420a^{15}b^5c^{14}d^6e^7f^{19}z^3 \\
&- 420a^{15}b^5c^4d^{19}e^{14}f^6z^3 - 420a^{14}b^6c^{15}d^5e^7f^{19}z^3 - 420 \\
&{*a^{14}b^6c^4d^{19}e^{15}f^5z^3 - 420a^{14}b^6c^{19}d^4e^5f^{15}z^3 - 420a^6 \\
&b^{14}c^5d^{15}e^{19}f^4z^3 - 420a^5b^{15}c^{19}d^4e^6f^{14}z^3 - 420a^5b^{15}c \\
&{^6}d^{14}e^{19}f^4z^3 - 420a^5b^{19}c^{15}d^5e^{14}f^6z^3 - 420a^5b^{19}c^{14}d^ \\
&6}e^{15}f^5z^3 + 350a^{19}b^3c^9d^{11}e^2f^{18}z^3 + 350a^{19}b^3c^2d^{18}e^9 \\
&{*f^{11}z^3 + 350a^{18}b^2c^{11}d^9e^7f^{19}z^3 + 350a^{18}b^2c^4d^{19}e^{11}f^9 \\
&{*z^3 + 350a^{11}b^9c^{18}d^2e^7f^{19}z^3 + 350a^{11}b^9c^4d^{19}e^{18}f^2z^3 \\
&+ 350a^9b^{11}c^{19}d^4e^2f^{18}z^3 + 350a^9b^{11}c^2d^{18}e^{19}f^4z^3 + 350 \\
&{*a^2b^{18}c^{19}d^4e^9f^{11}z^3 + 350a^2b^{18}c^9d^{11}e^{19}f^4z^3 + 350a^2b^{1 \\
&9}c^{18}d^2e^{11}f^9z^3 + 350a^2b^{19}c^{11}d^9e^{18}f^2z^3 - 90a^{19}b^3c^1 \\
&0}d^{10}e^7f^{19}z^3 - 90a^{19}b^3c^4d^{19}e^{10}f^{10}z^3 - 90a^{10}b^{10}c^{19}d^4e^ \\
&{*f^{19}z^3 - 90a^{10}b^{10}c^4d^{19}e^{19}f^4z^3 - 90a^3b^{19}c^{19}d^4e^{10}f^{10}z^3 \\
&- 90a^3b^{19}c^{10}d^{10}e^{19}f^4z^3 + 10b^{20}c^{19}d^4e^{11}f^9z^3 + 10b^{20}c^ \\
&11}d^9e^{19}f^4z^3 + 10a^{20}c^9d^{11}e^7f^{19}z^3 + 10a^{20}c^4d^{19}e^9f^{11}z \\
&{^3 + 10a^{19}b^3d^{20}e^{11}f^9z^3 + 10a^{11}b^9d^{20}e^{19}f^4z^3 + 10a^9b^{11} \\
&1}c^{20}e^7f^{19}z^3 + 10a^3b^{19}c^{20}e^9f^{11}z^3 + 10a^{19}b^3c^{11}d^9f^{20}z \\
&{^3 + 10a^{11}b^9c^{19}d^4f^{20}z^3 + 10a^9b^{11}c^4d^{19}e^{20}z^3 + 10a^3b^{19} \\
&c^9d^{11}e^{20}z^3 + 252b^{20}c^{15}d^5e^{15}f^5z^3 - 210b^{20}c^{16}d^4e^{14} \\
&{*f^6z^3 - 210b^{20}c^{14}d^6e^{16}f^4z^3 + 120b^{20}c^{17}d^3e^{13}f^7z^3 \\
&+ 120b^{20}c^{13}d^7e^{17}f^3z^3 - 45b^{20}c^{18}d^2e^{12}f^8z^3 - 45b^{20}c \\
&{^12}d^8e^{18}f^2z^3 + 252a^{20}c^5d^{15}e^5f^{15}z^3 - 210a^{20}c^6d^{14}e^ \\
&4}f^{16}z^3 - 210a^{20}c^4d^{16}e^6f^{14}z^3 + 120a^{20}c^7d^{13}e^3f^{17}z \\
&{^3 + 120a^{20}c^3d^{17}e^7f^{13}z^3 - 45a^{20}c^8d^{12}e^2f^{18}z^3 - 45a^ \\
&{^20}c^2d^{18}e^8f^{12}z^3 + 252a^{15}b^5d^{20}e^{15}f^5z^3 - 210a^{16}b^4d^ \\
&{^20}e^{14}f^6z^3 - 210a^{14}b^6d^{20}e^{16}f^4z^3 + 120a^{17}b^3d^{20}e^{13} \\
&f^7z^3 + 120a^{13}b^7d^{20}e^{17}f^3z^3 - 45a^{18}b^2d^{20}e^{12}f^8z^3 - \\
&45a^{12}b^8d^{20}e^{18}f^2z^3 + 252a^5b^{15}c^{20}e^5f^{15}z^3 - 210a^6b^{14} \\
&c^{20}e^4f^{16}z^3 - 210a^4b^{16}c^{20}e^6f^{14}z^3 + 120a^7b^{13}c^{20}e^ \\
&{^3}f^{17}z^3 + 120a^3b^{17}c^{20}e^7f^{13}z^3 - 45a^8b^{12}c^{20}e^2f^{18}z^
\end{aligned}$$

$$\begin{aligned} & 3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^20*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 210*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^13*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 92610*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^10*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z - 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 22680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 22680*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^11*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13$$

$$\begin{aligned}
& 13e^{13fz} - 252a^{13}bd^{14}e^{13fz} - 252a^3b^{13}d^{14}e^{13fz} - 252a^1 \\
& 3b^3cd^{13}f^{14}z - 252a^3b^{13}c^{13}d^{14}f^{14}z - 918b^{14}c^7d^7e^7f^7z - \\
& 882b^{14}c^{11}d^3e^3f^{11}z - 882b^{14}c^3d^{11}e^{11}f^3z + 693b^{14}c^1 \\
& 2d^2e^2f^{12}z + 693b^{14}c^2d^{12}e^{12}f^2z + 567b^{14}c^8d^6e^6f^8z \\
& z + 567b^{14}c^6d^8e^8f^6z + 441b^{14}c^{10}d^4e^4f^{10}z + 441b^{14}c^4 \\
& d^{10}e^{10}f^4z - 126b^{14}c^9d^5e^5f^9z - 126b^{14}c^5d^9e^9f^5z \\
& - 918a^7b^7d^{14}e^7f^7z - 882a^{11}b^3d^{14}e^3f^{11}z - 882a^3b^{11} \\
& d^{14}e^{11}f^3z + 693a^{12}b^2d^{14}e^2f^{12}z + 693a^2b^{12}d^{14}e^{12}f^2z \\
& + 567a^8b^6d^{14}e^6f^8z + 567a^6b^8d^{14}e^8f^6z + 441a^{10}b^4 \\
& d^{14}e^4f^{10}z + 441a^4b^{10}d^{14}e^{10}f^4z - 126a^9b^5d^{14}e^5f^9z \\
& z - 126a^5b^9d^{14}e^9f^5z - 918a^7b^7c^7d^7f^{14}z - 882a^{11}b^3 \\
& c^3d^{11}f^{14}z - 882a^3b^{11}c^{11}d^3f^{14}z + 693a^{12}b^2c^2d^{12}f^{14}z \\
& + 693a^2b^{12}c^{12}d^2f^{14}z + 567a^8b^6c^6d^8f^{14}z + 567a^6b^8 \\
& c^8d^6f^{14}z + 441a^{10}b^4c^4d^{10}f^{14}z + 441a^4b^{10}c^{10}d^4f^{14}z \\
& - 126a^9b^5c^5d^9f^{14}z - 126a^5b^9c^9d^5f^{14}z + 36b^{14}d^{14} \\
& e^{14}z + 36b^{14}c^{14}f^{14}z + 36a^{14}d^{14}f^{14}z - 27054a^2b^9c^2d^9 \\
& e^2f^9 + 9018a^3b^8c^2d^9e^2f^{10} + 9018a^3b^8c^2d^9e^2f^9 + 90 \\
& 18a^2b^9c^3d^8e^2f^{10} + 9018a^2b^9c^3d^8e^2f^8 + 9018a^2b^9c^3d^8 \\
& e^2f^9 + 9018a^2b^9c^2d^9e^3f^8 - 9018a^4b^7c^2d^10e^2f^{10} - 901 \\
& 8a^2b^10c^4d^7e^2f^{10} - 9018a^2b^10c^4d^7e^2f^7 + 2268b^{11}c^5d^6e^2 \\
& f^{10} + 2268b^{11}c^5d^6e^2f^6 + 2268a^5b^6d^{11}e^2f^{10} + 2268a^5b^6 \\
& d^{11}e^2f^6 + 2268a^5b^6c^2d^{10}f^{11} + 2268a^5b^6c^2d^{10}f^6 - 1458b^1 \\
& 1c^3d^8e^3f^8 - 1161b^{11}c^4d^7e^2f^9 - 1161b^{11}c^2d^9e^4f^7 - \\
& 1458a^3b^8d^{11}e^3f^8 - 1161a^4b^7d^{11}e^2f^9 - 1161a^2b^9d^{11}e^4 \\
& f^7 - 1458a^3b^8c^3d^8f^{11} - 1161a^4b^7c^2d^9f^{11} - 1161a^2b^9 \\
& c^4d^7f^{11} - 756b^{11}d^{11}e^6f^5 - 756b^{11}c^6d^5f^{11} - 756a^6b^5 \\
& d^{11}f^{11}, z, k) \cdot ((20a^{11}b^8c^{16}d^3f^{19} - 7a^{10}b^9c^{17}d^2f^{19} \\
& - 28a^{12}b^7c^{15}d^4f^{19} + 14a^{13}b^6c^{14}d^5f^{19} + 14a^{14}b^5c^{13} \\
& d^6f^{19} - 28a^{15}b^4c^{12}d^7f^{19} + 20a^{16}b^3c^{11}d^8f^{19} - 7a^{17}b^2 \\
& c^{10}d^9f^{19} - 7a^{10}b^9d^{19}e^{17}f^2 + 20a^{11}b^8d^{19}e^{16}f^3 - \\
& 28a^{12}b^7d^{19}e^{15}f^4 + 14a^{13}b^6d^{19}e^{14}f^5 + 14a^{14}b^5d^{19}e^{13} \\
& f^6 - 28a^{15}b^4d^{19}e^{12}f^7 + 20a^{16}b^3d^{19}e^{11}f^8 - 7a^{17}b^2 \\
& d^{19}e^{10}f^9 - 7b^{19}c^{10}d^9e^{17}f^2 + 20b^{19}c^{11}d^8e^{16}f^3 - 28b^{19} \\
& c^{12}d^7e^{15}f^4 + 14b^{19}c^{13}d^6e^{14}f^5 + 14b^{19}c^{14}d^5e^{13} \\
& f^6 - 28b^{19}c^{15}d^4e^{12}f^7 + 20b^{19}c^{16}d^3e^{11}f^8 - 7b^{19}c^{17}d^2 \\
& e^{10}f^9 + a^9b^{10}c^{18}d^5f^{19} + a^{18}b^9c^9d^{10}f^{19} + a^9b^{10}d^{19}e^{18} \\
& f + a^{18}b^9d^{19}e^9f^{10} + b^{19}c^9d^{10}e^{18}f + b^{19}c^{18}d^9e^9f^{10} \\
& - 7a^8b^{18}c^8d^{11}e^{18}f - 7a^8b^{18}c^8d^{11}e^{18}f^{11} - 7a^8b^{11}c^7d^{18}e^{18} \\
& f - 7a^8b^{11}c^7d^{18}e^{18}f^{11} - 7a^8b^8c^8d^{18}e^8f^{11} - 7a^8b^8c^8 \\
& d^{11}e^8f^{18} + 34a^8b^{18}c^9d^{10}e^{17}f^2 - 27a^8b^{18}c^{10}d^9e^{16}f^3 - 1 \\
& 68a^8b^{18}c^{11}d^8e^{15}f^4 + 546a^8b^{18}c^{12}d^7e^{14}f^5 - 756a^8b^{18}c^{13} \\
& d^6e^{13}f^6 + 546a^8b^{18}c^{14}d^5e^{12}f^7 - 168a^8b^{18}c^{15}d^4e^{11}f^8 - \\
& 27a^8b^{18}c^{16}d^3e^{10}f^9 + 34a^8b^{18}c^{17}d^2e^9f^{10} + 20a^2b^{17} \\
& c^7d^{12}e^{18}f + 20a^2b^{17}c^{18}d^6e^7f^{12} - 28a^3b^{16}c^6d^{13}e^{18} \\
& f - 28a^3b^{16}c^6d^{13}e^{18}f^{13} + 14a^4b^{15}c^5d^{14}e^{18}f + 14a^4b^{15}
\end{aligned}$$

$$\begin{aligned}
& *c^{18}*d^5*f^{14} + 14*a^5*b^{14}*c^4*d^{15}*e^{18}*f + 14*a^5*b^{14}*c^{18}*d^4*f^{15} - 28*a^6*b^{13}*c^3*d^{16}*e^{18}*f - 28*a^6*b^{13}*c^{18}*d^3*f^{16} + 20*a^7*b^{12} \\
& *c^2*d^{17}*e^{18}*f + 20*a^7*b^{12}*c^{18}*d^2*f^{17} + 34*a^9*b^{10}*c*d^{18}*e^{17}*f^2 + 34*a^9*b^{10}*c^{17}*d^2*e*f^{18} - 27*a^{10}*b^9*c*d^{18}*e^{16}*f^3 - 27*a^{10}*b^9 \\
& *c^{16}*d^3*e*f^{18} - 168*a^{11}*b^8*c*d^{18}*e^{15}*f^4 - 168*a^{11}*b^8*c^{15}*d^4*e*f^{18} + 546*a^{12}*b^7*c*d^{18}*e^{14}*f^5 + 546*a^{12}*b^7*c^{14}*d^5*e*f^{18} - 756*a^{13} \\
& *b^6*c*d^{18}*e^{13}*f^6 - 756*a^{13}*b^6*c^{13}*d^6*e*f^{18} + 546*a^{14}*b^5*c*d^{18}*e^{12}*f^7 + 546*a^{14}*b^5*c^{12}*d^7*e*f^{18} - 168*a^{15}*b^4*c*d^{18}*e^{11}*f^8 - 16 \\
& 8*a^{15}*b^4*c^{11}*d^8*e*f^{18} - 27*a^{16}*b^3*c*d^{18}*e^{10}*f^9 - 27*a^{16}*b^3*c^{10} \\
& *d^9*e*f^{18} + 34*a^{17}*b^2*c*d^{18}*e^9*f^{10} + 34*a^{17}*b^2*c^9*d^{10}*e*f^{18} + 2 \\
& 0*a^{18}*b*c^2*d^{17}*e^7*f^{12} - 28*a^{18}*b*c^3*d^{16}*e^6*f^{13} + 14*a^{18}*b*c^4*d^{15} \\
& *e^5*f^{14} + 14*a^{18}*b*c^5*d^{14}*e^4*f^{15} - 28*a^{18}*b*c^6*d^{13}*e^3*f^{16} + 2 \\
& 0*a^{18}*b*c^7*d^{12}*e^2*f^{17} - 27*a^2*b^{17}*c^8*d^{11}*e^{17}*f^2 - 371*a^2*b^{17}*c \\
& ^9*d^{10}*e^{16}*f^3 + 1560*a^2*b^{17}*c^{10}*d^9*e^{15}*f^4 - 2484*a^2*b^{17}*c^{11}*d^8 \\
& *e^{14}*f^5 + 1302*a^2*b^{17}*c^{12}*d^7*e^{13}*f^6 + 1302*a^2*b^{17}*c^{13}*d^6*e^{12}*f \\
& ^7 - 2484*a^2*b^{17}*c^{14}*d^5*e^{11}*f^8 + 1560*a^2*b^{17}*c^{15}*d^4*e^{10}*f^9 - 37 \\
& 1*a^2*b^{17}*c^{16}*d^3*e^9*f^{10} - 27*a^2*b^{17}*c^{17}*d^2*e^8*f^{11} - 168*a^3*b^{16} \\
& *c^7*d^{12}*e^{17}*f^2 + 1560*a^3*b^{16}*c^8*d^{11}*e^{16}*f^3 - 3464*a^3*b^{16}*c^9*d^{10} \\
& *e^{15}*f^4 + 924*a^3*b^{16}*c^{10}*d^9*e^{14}*f^5 + 7728*a^3*b^{16}*c^{11}*d^8*e^{13} \\
& *f^6 - 13104*a^3*b^{16}*c^{12}*d^7*e^{12}*f^7 + 7728*a^3*b^{16}*c^{13}*d^6*e^{11}*f^8 + \\
& 924*a^3*b^{16}*c^{14}*d^5*e^{10}*f^9 - 3464*a^3*b^{16}*c^{15}*d^4*e^9*f^{10} + 1560*a^3 \\
& *b^{16}*c^{16}*d^3*e^8*f^{11} - 168*a^3*b^{16}*c^{17}*d^2*e^7*f^{12} + 546*a^4*b^{15}*c^6 \\
& *d^{13}*e^{17}*f^2 - 2484*a^4*b^{15}*c^7*d^{12}*e^{16}*f^3 + 924*a^4*b^{15}*c^8*d^{11}*e^{15} \\
& *f^4 + 12550*a^4*b^{15}*c^9*d^{10}*e^{14}*f^5 - 26838*a^4*b^{15}*c^{10}*d^9*e^{13}*f^6 + \\
& 15288*a^4*b^{15}*c^{11}*d^8*e^{12}*f^7 + 15288*a^4*b^{15}*c^{12}*d^7*e^{11}*f^8 - 2 \\
& 6838*a^4*b^{15}*c^{13}*d^6*e^{10}*f^9 + 12550*a^4*b^{15}*c^{14}*d^5*e^9*f^{10} + 924*a^4 \\
& *b^{15}*c^{15}*d^4*e^8*f^{11} - 2484*a^4*b^{15}*c^{16}*d^3*e^7*f^{12} + 546*a^4*b^{15}*c \\
& ^{17}*d^2*e^6*f^{13} - 756*a^5*b^{14}*c^5*d^{14}*e^{17}*f^2 + 1302*a^5*b^{14}*c^6*d^{13} \\
& *e^{16}*f^3 + 7728*a^5*b^{14}*c^7*d^{12}*e^{15}*f^4 - 26838*a^5*b^{14}*c^8*d^{11}*e^{14}*f \\
& ^5 + 18004*a^5*b^{14}*c^9*d^{10}*e^{13}*f^6 + 39858*a^5*b^{14}*c^{10}*d^9*e^{12}*f^7 - \\
& 78624*a^5*b^{14}*c^{11}*d^8*e^{11}*f^8 + 39858*a^5*b^{14}*c^{12}*d^7*e^{10}*f^9 + 18004 \\
& *a^5*b^{14}*c^{13}*d^6*e^9*f^{10} - 26838*a^5*b^{14}*c^{14}*d^5*e^8*f^{11} + 7728*a^5*b \\
& ^{14}*c^{15}*d^4*e^7*f^{12} + 1302*a^5*b^{14}*c^{16}*d^3*e^6*f^{13} - 756*a^5*b^{14}*c^{17} \\
& *d^2*e^5*f^{14} + 546*a^6*b^{13}*c^4*d^{15}*e^{17}*f^2 + 1302*a^6*b^{13}*c^5*d^{14}*e^{16} \\
& *f^3 - 13104*a^6*b^{13}*c^6*d^{13}*e^{15}*f^4 + 15288*a^6*b^{13}*c^7*d^{12}*e^{14}*f^5 \\
& + 39858*a^6*b^{13}*c^8*d^{11}*e^{13}*f^6 - 110474*a^6*b^{13}*c^9*d^{10}*e^{12}*f^7 + 6 \\
& 6612*a^6*b^{13}*c^{10}*d^9*e^{11}*f^8 + 66612*a^6*b^{13}*c^{11}*d^8*e^{10}*f^9 - 110474 \\
& *a^6*b^{13}*c^{12}*d^7*e^9*f^{10} + 39858*a^6*b^{13}*c^{13}*d^6*e^8*f^{11} + 15288*a^6* \\
& b^{13}*c^{14}*d^5*e^7*f^{12} - 13104*a^6*b^{13}*c^{15}*d^4*e^6*f^{13} + 1302*a^6*b^{13}*c \\
& ^{16}*d^3*e^5*f^{14} + 546*a^6*b^{13}*c^{17}*d^2*e^4*f^{15} - 168*a^7*b^{12}*c^3*d^{16}*e \\
& ^{17}*f^2 - 2484*a^7*b^{12}*c^4*d^{15}*e^{16}*f^3 + 7728*a^7*b^{12}*c^5*d^{14}*e^{15}*f^4 \\
& + 15288*a^7*b^{12}*c^6*d^{13}*e^{14}*f^5 - 78624*a^7*b^{12}*c^7*d^{12}*e^{13}*f^6 + 66 \\
& 612*a^7*b^{12}*c^8*d^{11}*e^{12}*f^7 + 99736*a^7*b^{12}*c^9*d^{10}*e^{11}*f^8 - 216216* \\
& a^7*b^{12}*c^{10}*d^9*e^{10}*f^9 + 99736*a^7*b^{12}*c^{11}*d^8*e^9*f^{10} + 66612*a^7*b \\
& ^{12}*c^{12}*d^7*e^8*f^{11} - 78624*a^7*b^{12}*c^{13}*d^6*e^7*f^{12} + 15288*a^7*b^{12}*c
\end{aligned}$$

$$\begin{aligned}
& ^{14}d^5e^6f^{13} + 7728a^7b^{12}c^{15}d^4e^5f^{14} - 2484a^7b^{12}c^{16}d^3 \\
& *e^4f^{15} - 168a^7b^{12}c^{17}d^2e^3f^{16} - 27a^8b^{11}c^2d^{17}e^{17}f^2 \\
& + 1560a^8b^{11}c^3d^{16}e^{16}f^3 + 924a^8b^{11}c^4d^{15}e^{15}f^4 - 26838a \\
& a^8b^{11}c^5d^{14}e^{14}f^5 + 39858a^8b^{11}c^6d^{13}e^{13}f^6 + 66612a^8b \\
& ^{11}c^7d^{12}e^{12}f^7 - 216216a^8b^{11}c^8d^{11}e^{11}f^8 + 134134a^8b^{11} \\
& *c^9d^{10}e^{10}f^9 + 134134a^8b^{11}c^{10}d^9e^9f^{10} - 216216a^8b^{11}c^ \\
& 11d^8e^8f^{11} + 66612a^8b^{11}c^{12}d^7e^7f^{12} + 39858a^8b^{11}c^{13}d^ \\
& 6e^6f^{13} - 26838a^8b^{11}c^{14}d^5e^5f^{14} + 924a^8b^{11}c^{15}d^4e^4f \\
& ^{15} + 1560a^8b^{11}c^{16}d^3e^3f^{16} - 27a^8b^{11}c^{17}d^2e^2f^{17} - 371 \\
& *a^9b^{10}c^2d^{17}e^{16}f^3 - 3464a^9b^{10}c^3d^{16}e^{15}f^4 + 12550a^9b \\
& ^{10}c^4d^{15}e^{14}f^5 + 18004a^9b^{10}c^5d^{14}e^{13}f^6 - 110474a^9b^{10} \\
& c^6d^{13}e^{12}f^7 + 99736a^9b^{10}c^7d^{12}e^{11}f^8 + 134134a^9b^{10}c^8d \\
& ^{11}e^{10}f^9 - 300300a^9b^{10}c^9d^{10}e^9f^{10} + 134134a^9b^{10}c^{10}d^ \\
& 9e^8f^{11} + 99736a^9b^{10}c^{11}d^8e^7f^{12} - 110474a^9b^{10}c^{12}d^7e^ \\
& 6f^{13} + 18004a^9b^{10}c^{13}d^6e^5f^{14} + 12550a^9b^{10}c^{14}d^5e^4f^1 \\
& 5 - 3464a^9b^{10}c^{15}d^4e^3f^{16} - 371a^9b^{10}c^{16}d^3e^2f^{17} + 1560 \\
& *a^{10}b^9c^2d^{17}e^{15}f^4 + 924a^{10}b^9c^3d^{16}e^{14}f^5 - 26838a^{10}b \\
& ^9c^4d^{15}e^{13}f^6 + 39858a^{10}b^9c^5d^{14}e^{12}f^7 + 66612a^{10}b^9c^ \\
& 6d^{13}e^{11}f^8 - 216216a^{10}b^9c^7d^{12}e^{10}f^9 + 134134a^{10}b^9c^8d \\
& ^{11}e^9f^{10} + 134134a^{10}b^9c^9d^{10}e^8f^{11} - 216216a^{10}b^9c^{10}d^9 \\
& *e^7f^{12} + 66612a^{10}b^9c^{11}d^8e^6f^{13} + 39858a^{10}b^9c^{12}d^7e^5 \\
& f^{14} - 26838a^{10}b^9c^{13}d^6e^4f^{15} + 924a^{10}b^9c^{14}d^5e^3f^{16} + \\
& 1560a^{10}b^9c^{15}d^4e^2f^{17} - 2484a^{11}b^8c^2d^{17}e^{14}f^5 + 7728a^ \\
& 11b^8c^3d^{16}e^{13}f^6 + 15288a^{11}b^8c^4d^{15}e^{12}f^7 - 78624a^{11}b^ \\
& 8c^5d^{14}e^{11}f^8 + 66612a^{11}b^8c^6d^{13}e^{10}f^9 + 99736a^{11}b^8c^7 \\
& *d^{12}e^9f^{10} - 216216a^{11}b^8c^8d^{11}e^8f^{11} + 99736a^{11}b^8c^9d^1 \\
& 0e^7f^{12} + 66612a^{11}b^8c^{10}d^9e^6f^{13} - 78624a^{11}b^8c^{11}d^8e^5 \\
& *f^{14} + 15288a^{11}b^8c^{12}d^7e^4f^{15} + 7728a^{11}b^8c^{13}d^6e^3f^{16} \\
& - 2484a^{11}b^8c^{14}d^5e^2f^{17} + 1302a^{12}b^7c^2d^{17}e^{13}f^6 - 13104 \\
& *a^{12}b^7c^3d^{16}e^{12}f^7 + 15288a^{12}b^7c^4d^{15}e^{11}f^8 + 39858a^{12} \\
& *b^7c^5d^{14}e^{10}f^9 - 110474a^{12}b^7c^6d^{13}e^9f^{10} + 66612a^{12}b^7 \\
& *c^7d^{12}e^8f^{11} + 66612a^{12}b^7c^8d^{11}e^7f^{12} - 110474a^{12}b^7c^9 \\
& *d^{10}e^6f^{13} + 39858a^{12}b^7c^{10}d^9e^5f^{14} + 15288a^{12}b^7c^{11}d^8 \\
& *e^4f^{15} - 13104a^{12}b^7c^{12}d^7e^3f^{16} + 1302a^{12}b^7c^{13}d^6e^2f \\
& ^{17} + 1302a^{13}b^6c^2d^{17}e^{12}f^7 + 7728a^{13}b^6c^3d^{16}e^{11}f^8 - 2 \\
& 6838a^{13}b^6c^4d^{15}e^{10}f^9 + 18004a^{13}b^6c^5d^{14}e^9f^{10} + 39858a \\
& ^{13}b^6c^6d^{13}e^8f^{11} - 78624a^{13}b^6c^7d^{12}e^7f^{12} + 39858a^{13}b \\
& ^6c^8d^{11}e^6f^{13} + 18004a^{13}b^6c^9d^{10}e^5f^{14} - 26838a^{13}b^6c \\
& ^{10}d^9e^4f^{15} + 7728a^{13}b^6c^{11}d^8e^3f^{16} + 1302a^{13}b^6c^{12}d^7 \\
& *e^2f^{17} - 2484a^{14}b^5c^2d^{17}e^{11}f^8 + 924a^{14}b^5c^3d^{16}e^{10}f^ \\
& 9 + 12550a^{14}b^5c^4d^{15}e^9f^{10} - 26838a^{14}b^5c^5d^{14}e^8f^{11} + 1 \\
& 5288a^{14}b^5c^6d^{13}e^7f^{12} + 15288a^{14}b^5c^7d^{12}e^6f^{13} - 26838a \\
& ^{14}b^5c^8d^{11}e^5f^{14} + 12550a^{14}b^5c^9d^{10}e^4f^{15} + 924a^{14}b^ \\
& 5c^{10}d^9e^3f^{16} - 2484a^{14}b^5c^{11}d^8e^2f^{17} + 1560a^{15}b^4c^2d \\
& ^{17}e^{10}f^9 - 3464a^{15}b^4c^3d^{16}e^9f^{10} + 924a^{15}b^4c^4d^{15}e^8*
\end{aligned}$$

$$\begin{aligned}
& f^{11} + 7728a^{15}b^4c^5d^{14}e^7f^{12} - 13104a^{15}b^4c^6d^{13}e^6f^{13} + \\
& 7728a^{15}b^4c^7d^{12}e^5f^{14} + 924a^{15}b^4c^8d^{11}e^4f^{15} - 3464a^{15}b^4c^9d^{10}e^3f^{16} + 1560a^{15}b^4c^{10}d^9e^2f^{17} - 371a^{16}b^3c^8d^{17}e^9f^{10} + 1560a^{16}b^3c^3d^{16}e^8f^{11} - 2484a^{16}b^3c^4d^{15}e^7f^{12} + 1302a^{16}b^3c^5d^{14}e^6f^{13} + 1302a^{16}b^3c^6d^{13}e^5f^{14} - 2484a^{16}b^3c^7d^{12}e^4f^{15} + 1560a^{16}b^3c^8d^{11}e^3f^{16} - 371a^{16}b^3c^9d^{10}e^2f^{17} - 27a^{17}b^2c^2d^{17}e^8f^{11} - 168a^{17}b^2c^3d^{16}e^7f^{12} + 546a^{17}b^2c^4d^{15}e^6f^{13} - 756a^{17}b^2c^5d^{14}e^5f^{14} + 546a^{17}b^2c^6d^{13}e^4f^{15} - 168a^{17}b^2c^7d^{12}e^3f^{16} - 27a^{17}b^2c^8d^{11}e^2f^{17}) / (56a^3b^{13}c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8e^{16} - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2e^{16} + 56a^{11}b^5c^{13}d^3e^{16} - 70a^{12}b^4c^{12}d^4e^{16} + 56a^{13}b^3c^{11}d^5e^{16} - 28a^{14}b^2c^{10}d^6e^{16} - 28a^2b^{14}c^6e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16}e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16}c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^7b^{15}c^7d^9e^{16} + 8a^7b^9c^7d^{15}e^{16} + 8a^9b^7c^{15}d^7f^{16} + 8a^{15}b^9c^9d^7f^{16} + 8a^7b^9c^{16}e^7f^9 + 8a^7b^9c^{16}e^8f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^7d^{16}e^9f^7 + 8a^{16}c^7d^{15}e^7f^9 + 8a^{16}c^7d^9e^7f^{15} + 8b^{16}c^9d^7e^{15}f + 8b^{16}c^{15}d^9e^9f^7 - 56a^8b^{15}c^8d^8e^{15}f - 56a^8b^{15}c^{15}d^8e^8f^8 - 56a^8b^8c^8d^{15}e^{15}f - 56a^8b^8c^{15}d^8e^8f^8 - 56a^{15}b^8c^8d^8e^8f^{15} + 160a^8b^{15}c^9d^7e^{14}f^2 - 224a^8b^{15}c^{10}d^6e^{13}f^3 + 112a^8b^{15}c^{11}d^5e^{12}f^4 + 112a^8b^{15}c^{12}d^4e^{11}f^5 - 224a^8b^{15}c^{13}d^3e^{10}f^6 + 160a^8b^{15}c^{14}d^2e^9f^7 + 160a^8b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d^7e^7f^9 - 224a^3b^{13}c^6d^{10}e^{15}f - 224a^3b^{13}c^{15}d^6e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112a^4b^{12}c^{15}d^5e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15}d^4e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10}c^{15}d^3e^3f^{13} + 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d^2e^2f^{14} + 160a^9b^7c^7d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e^7f^{15} - 224a^{10}b^6c^6d^{15}e^{13}f^3 - 224a^{10}b^6c^{13}d^3e^7f^{15} + 112a^{11}b^5c^5d^{15}e^{12}f^4 + 112a^{11}b^5c^{12}d^4e^4f^{15} + 112a^{12}b^4c^3d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e^7f^{15} - 224a^{13}b^3c^3d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6e^7f^{15} + 160a^{14}b^2c^2d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e^7f^{15} + 160a^{15}b^2c^2d^{14}e^7f^9 - 224a^{15}b^2c^3d^{13}e^6f^{10} + 112a^{15}b^2c^4d^{12}e^5f^{11} + 112a^{15}b^2c^5d^{11}e^4f^{12} - 224a^{15}b^2c^6d^{10}e^3f^{13} + 160a^{15}b^2c^7d^9e^2f^{14} - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 - 1344a^2b^{14}c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a^2b^{14}c^{14}d
\end{aligned}$$

$$\begin{aligned}
&^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5e^{10}f^6 - 280 \\
&0a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 + 840a^4b^{12} \\
&c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^4b^{12}c^8d^8 \\
&e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e^8f^8 - 2800 \\
&a^4b^{12}c^{13}d^3e^7f^9 + 840a^4b^{12}c^{14}d^2e^6f^{10} - 1344a^5b^{11} \\
&c^5d^{11}e^{14}f^2 + 1568a^5b^{11}c^6d^{10}e^{13}f^3 + 4480a^5b^{11}c^7d^9 \\
&e^{12}f^4 - 12264a^5b^{11}c^8d^8e^{11}f^5 + 7392a^5b^{11}c^9d^7e^{10}f^6 + 7392a^5b^{11}c^{10}d^6e^9f^7 - 12264a^5b^{11}c^{11}d^5e^8f^8 + 4480 \\
&a^5b^{11}c^{12}d^4e^7f^9 + 1568a^5b^{11}c^{13}d^3e^6f^{10} - 1344a^5b^{11} \\
&c^{14}d^2e^5f^{11} + 840a^6b^{10}c^4d^{12}e^{14}f^2 + 1568a^6b^{10}c^5d^{11} \\
&e^{13}f^3 - 8624a^6b^{10}c^6d^{10}e^{12}f^4 + 7392a^6b^{10}c^7d^9e^{11} \\
&f^5 + 11396a^6b^{10}c^8d^8e^{10}f^6 - 24640a^6b^{10}c^9d^7e^9f^7 + 11 \\
&396a^6b^{10}c^{10}d^6e^8f^8 + 7392a^6b^{10}c^{11}d^5e^7f^9 - 8624a^6b^{10} \\
&c^{12}d^4e^6f^{10} + 1568a^6b^{10}c^{13}d^3e^5f^{11} + 840a^6b^{10}c^{14} \\
&d^2e^4f^{12} - 2800a^7b^9c^4d^{12}e^{13}f^3 + 4480a^7b^9c^5d^{11}e^{12} \\
&f^4 + 7392a^7b^9c^6d^{10}e^{11}f^5 - 24640a^7b^9c^7d^9e^{10}f^6 + 15 \\
&400a^7b^9c^8d^8e^9f^7 + 15400a^7b^9c^9d^7e^8f^8 - 24640a^7b^9 \\
&c^{10}d^6e^7f^9 + 7392a^7b^9c^{11}d^5e^6f^{10} + 4480a^7b^9c^{12}d^4 \\
&e^5f^{11} - 2800a^7b^9c^{13}d^3e^4f^{12} - 300a^8b^8c^2d^{14}e^{14}f^2 + \\
&1400a^8b^8c^3d^{13}e^{13}f^3 + 1750a^8b^8c^4d^{12}e^{12}f^4 - 12264a^8 \\
&b^8c^5d^{11}e^{11}f^5 + 11396a^8b^8c^6d^{10}e^{10}f^6 + 15400a^8b^8c^7 \\
&d^9e^9f^7 - 34650a^8b^8c^8d^8e^8f^8 + 15400a^8b^8c^9d^7e^7 \\
&f^9 + 11396a^8b^8c^{10}d^6e^6f^{10} - 12264a^8b^8c^{11}d^5e^5f^{11} + 1 \\
&750a^8b^8c^{12}d^4e^4f^{12} + 1400a^8b^8c^{13}d^3e^3f^{13} - 300a^8b^8 \\
&c^{14}d^2e^2f^{14} - 2800a^9b^7c^3d^{13}e^{12}f^4 + 4480a^9b^7c^4d^{12} \\
&e^{11}f^5 + 7392a^9b^7c^5d^{11}e^{10}f^6 - 24640a^9b^7c^6d^{10}e^9f^7 + \\
&15400a^9b^7c^7d^9e^8f^8 + 15400a^9b^7c^8d^8e^7f^9 - 24640a^9 \\
&b^7c^9d^7e^6f^{10} + 7392a^9b^7c^{10}d^6e^5f^{11} + 4480a^9b^7c^{11} \\
&d^5e^4f^{12} - 2800a^9b^7c^{12}d^4e^3f^{13} + 840a^{10}b^6c^2d^{14}e^{11} \\
&f^2 + 1568a^{10}b^6c^3d^{13}e^{11}f^5 - 8624a^{10}b^6c^4d^{12}e^{10}f^6 + \\
&7392a^{10}b^6c^5d^{11}e^9f^7 + 11396a^{10}b^6c^6d^{10}e^8f^8 - 24640a^{10} \\
&b^6c^7d^9e^7f^9 + 11396a^{10}b^6c^8d^8e^6f^{10} + 7392a^{10}b^6c^9 \\
&d^7e^5f^{11} - 8624a^{10}b^6c^{10}d^6e^4f^{12} + 1568a^{10}b^6c^{11}d^5 \\
&e^3f^{13} + 840a^{10}b^6c^{12}d^4e^2f^{14} - 1344a^{11}b^5c^2d^{14}e^{11}f^5 \\
&+ 1568a^{11}b^5c^3d^{13}e^{10}f^6 + 4480a^{11}b^5c^4d^{12}e^9f^7 - 12264 \\
&a^{11}b^5c^5d^{11}e^8f^8 + 7392a^{11}b^5c^6d^{10}e^7f^9 + 7392a^{11}b^5 \\
&c^7d^9e^6f^{10} - 12264a^{11}b^5c^8d^8e^5f^{11} + 4480a^{11}b^5c^9d^7 \\
&e^4f^{12} + 1568a^{11}b^5c^{10}d^6e^3f^{13} - 1344a^{11}b^5c^{11}d^5e^2f^{14} \\
&+ 840a^{12}b^4c^2d^{14}e^{10}f^6 - 2800a^{12}b^4c^3d^{13}e^9f^7 + 1750 \\
&a^{12}b^4c^4d^{12}e^8f^8 + 4480a^{12}b^4c^5d^{11}e^7f^9 - 8624a^{12}b^4 \\
&c^6d^{10}e^6f^{10} + 4480a^{12}b^4c^7d^9e^5f^{11} + 1750a^{12}b^4c^8d^8 \\
&e^4f^{12} - 2800a^{12}b^4c^9d^7e^3f^{13} + 840a^{12}b^4c^{10}d^6e^2f^{14} \\
&+ 1400a^{13}b^3c^3d^{13}e^8f^8 - 2800a^{13}b^3c^4d^{12}e^7f^9 + 1568a
\end{aligned}$$

$$\begin{aligned}
& ^{13}b^3c^5d^{11}e^6f^{10} + 1568a^{13}b^3c^6d^{10}e^5f^{11} - 2800a^{13}b^3 \\
& *c^7d^9e^4f^{12} + 1400a^{13}b^3c^8d^8e^3f^{13} - 300a^{14}b^2c^2d^{14} \\
& e^8f^8 + 840a^{14}b^2c^4d^{12}e^6f^{10} - 1344a^{14}b^2c^5d^{11}e^5f^{11} \\
& + 840a^{14}b^2c^6d^{10}e^4f^{12} - 300a^{14}b^2c^8d^8e^2f^{14}) - (x(18* \\
& a^9b^{10}c^{17}d^2f^{19} - 74a^{10}b^9c^{16}d^3f^{19} + 184a^{11}b^8c^{15}d^4* \\
& f^{19} - 308a^{12}b^7c^{14}d^5f^{19} + 364a^{13}b^6c^{13}d^6f^{19} - 308a^{14}b \\
& ^5c^{12}d^7f^{19} + 184a^{15}b^4c^{11}d^8f^{19} - 74a^{16}b^3c^{10}d^9f^{19} + \\
& 18a^{17}b^2c^9d^{10}f^{19} + 18a^9b^{10}d^{19}e^{17}f^2 - 74a^{10}b^9d^{19}e \\
& ^{16}f^3 + 184a^{11}b^8d^{19}e^{15}f^4 - 308a^{12}b^7d^{19}e^{14}f^5 + 364a^1 \\
& 3b^6d^{19}e^{13}f^6 - 308a^{14}b^5d^{19}e^{12}f^7 + 184a^{15}b^4d^{19}e^{11}f \\
& ^8 - 74a^{16}b^3d^{19}e^{10}f^9 + 18a^{17}b^2d^{19}e^9f^{10} + 18b^{19}c^9d^ \\
& ^{10}e^{17}f^2 - 74b^{19}c^{10}d^9e^{16}f^3 + 184b^{19}c^{11}d^8e^{15}f^4 - 308* \\
& b^{19}c^{12}d^7e^{14}f^5 + 364b^{19}c^{13}d^6e^{13}f^6 - 308b^{19}c^{14}d^5e^{1 \\
& 2}f^7 + 184b^{19}c^{15}d^4e^{11}f^8 - 74b^{19}c^{16}d^3e^{10}f^9 + 18b^{19}c^ \\
& ^{17}d^2e^9f^{10} - 2a^8b^{11}c^{18}d^f^{19} - 2a^{18}b^c^8d^{11}f^{19} - 2a^8b \\
& ^{11}d^{19}e^{18}f - 2a^{18}b^d^{19}e^8f^{11} - 2b^{19}c^8d^{11}e^{18}f - 2b^{19} \\
& c^{18}d^e^8f^{11} + 16a*b^{18}c^7d^{12}e^{18}f + 16a*b^{18}c^{18}d^e^7f^{12} + 1 \\
& 6a^7b^{12}c^d^{18}e^{18}f + 16a^7b^{12}c^{18}d^e^f^{18} + 16a^{18}b^c^d^{18}e^7 \\
& *f^{12} + 16a^{18}b^c^7d^{12}e^f^{18} - 126a*b^{18}c^8d^{11}e^{17}f^2 + 434a*b^ \\
& ^{18}c^9d^{10}e^{16}f^3 - 840a*b^{18}c^{10}d^9e^{15}f^4 + 936a*b^{18}c^{11}d^8e \\
& ^{14}f^5 - 420a*b^{18}c^{12}d^7e^{13}f^6 - 420a*b^{18}c^{13}d^6e^{12}f^7 + 936 \\
& *a*b^{18}c^{14}d^5e^{11}f^8 - 840a*b^{18}c^{15}d^4e^{10}f^9 + 434a*b^{18}c^{16} \\
& d^3e^9f^{10} - 126a*b^{18}c^{17}d^2e^8f^{11} - 56a^2b^{17}c^6d^{13}e^{18}f - \\
& 56a^2b^{17}c^{18}d^e^6f^{13} + 112a^3b^{16}c^5d^{14}e^{18}f + 112a^3b^{16} \\
& c^{18}d^e^5f^{14} - 140a^4b^{15}c^4d^{15}e^{18}f - 140a^4b^{15}c^{18}d^e^4f^ \\
& ^{15} + 112a^5b^{14}c^3d^{16}e^{18}f + 112a^5b^{14}c^{18}d^e^3f^{16} - 56a^6b \\
& ^{13}c^2d^{17}e^{18}f - 56a^6b^{13}c^{18}d^e^2f^{17} - 126a^8b^{11}c^d^{18}e^1 \\
& 7f^2 - 126a^8b^{11}c^{17}d^2e^f^{18} + 434a^9b^{10}c^d^{18}e^{16}f^3 + 434a \\
& ^9b^{10}c^{16}d^3e^f^{18} - 840a^{10}b^9c^d^{18}e^{15}f^4 - 840a^{10}b^9c^{15} \\
& d^4e^f^{18} + 936a^{11}b^8c^d^{18}e^{14}f^5 + 936a^{11}b^8c^{14}d^5e^f^{18} - \\
& 420a^{12}b^7c^d^{18}e^{13}f^6 - 420a^{12}b^7c^{13}d^6e^f^{18} - 420a^{13}b^6* \\
& c^d^{18}e^{12}f^7 - 420a^{13}b^6c^{12}d^7e^f^{18} + 936a^{14}b^5c^d^{18}e^{11}f \\
& ^8 + 936a^{14}b^5c^{11}d^8e^f^{18} - 840a^{15}b^4c^d^{18}e^{10}f^9 - 840a^{15} \\
& *b^4c^{10}d^9e^f^{18} + 434a^{16}b^3c^d^{18}e^9f^{10} + 434a^{16}b^3c^9d^{10} \\
& *e^f^{18} - 126a^{17}b^2c^d^{18}e^8f^{11} - 126a^{17}b^2c^8d^{11}e^f^{18} - 56* \\
& a^{18}b^c^2d^{17}e^6f^{13} + 112a^{18}b^c^3d^{16}e^5f^{14} - 140a^{18}b^c^4d^ \\
& ^{15}e^4f^{15} + 112a^{18}b^c^5d^{14}e^3f^{16} - 56a^{18}b^c^6d^{13}e^2f^{17} + \\
& 360a^2b^{17}c^7d^{12}e^{17}f^2 - 882a^2b^{17}c^8d^{11}e^{16}f^3 + 728a^2b \\
& ^{17}c^9d^{10}e^{15}f^4 + 1152a^2b^{17}c^{10}d^9e^{14}f^5 - 4032a^2b^{17}c^1 \\
& 1d^8e^{13}f^6 + 5460a^2b^{17}c^{12}d^7e^{12}f^7 - 4032a^2b^{17}c^{13}d^6e \\
& ^{11}f^8 + 1152a^2b^{17}c^{14}d^5e^{10}f^9 + 728a^2b^{17}c^{15}d^4e^9f^{10} \\
& - 882a^2b^{17}c^{16}d^3e^8f^{11} + 360a^2b^{17}c^{17}d^2e^7f^{12} - 504a^3 \\
& *b^{16}c^6d^{13}e^{17}f^2 + 312a^3b^{16}c^7d^{12}e^{16}f^3 + 2520a^3b^{16}c^ \\
& 8d^{11}e^{15}f^4 - 7480a^3b^{16}c^9d^{10}e^{14}f^5 + 9408a^3b^{16}c^{10}d^9* \\
& e^{13}f^6 - 4368a^3b^{16}c^{11}d^8e^{12}f^7 - 4368a^3b^{16}c^{12}d^7e^{11}f^
\end{aligned}$$

$$\begin{aligned}
& 8 + 9408a^3b^{16}c^{13}d^6e^{10}f^9 - 7480a^3b^{16}c^{14}d^5e^9f^{10} + 2520a^3b^{16}c^{15}d^4e^8f^{11} + 312a^3b^{16}c^{16}d^3e^7f^{12} - 504a^3b^{16}c^{17}d^2e^6f^{13} + 252a^4b^{15}c^5d^{14}e^{17}f^2 + 1596a^4b^{15}c^6d^{13}e^{16}f^3 - 6288a^4b^{15}c^7d^{12}e^{15}f^4 + 7380a^4b^{15}c^8d^{11}e^{14}f^5 + 2660a^4b^{15}c^9d^{10}e^{13}f^6 - 18564a^4b^{15}c^{10}d^9e^{12}f^7 + 26208a^4b^{15}c^{11}d^8e^{11}f^8 - 18564a^4b^{15}c^{12}d^7e^{10}f^9 + 2660a^4b^{15}c^{13}d^6e^9f^{10} + 7380a^4b^{15}c^{14}d^5e^8f^{11} - 6288a^4b^{15}c^{15}d^4e^7f^{12} + 1596a^4b^{15}c^{16}d^3e^6f^{13} + 252a^4b^{15}c^{17}d^2e^5f^{14} + 252a^5b^{14}c^4d^{15}e^{17}f^2 - 2772a^5b^{14}c^5d^{14}e^{16}f^3 + 3696a^5b^{14}c^6d^{13}e^{15}f^4 + 7056a^5b^{14}c^7d^{12}e^{14}f^5 - 25452a^5b^{14}c^8d^{11}e^{13}f^6 + 30212a^5b^{14}c^9d^{10}e^{12}f^7 - 13104a^5b^{14}c^{10}d^9e^{11}f^8 - 13104a^5b^{14}c^{11}d^8e^{10}f^9 + 30212a^5b^{14}c^{12}d^7e^9f^{10} - 25452a^5b^{14}c^{13}d^6e^8f^{11} + 7056a^5b^{14}c^{14}d^5e^7f^{12} + 3696a^5b^{14}c^{15}d^4e^6f^{13} - 2772a^5b^{14}c^{16}d^3e^5f^{14} + 252a^5b^{14}c^{17}d^2e^4f^{15} - 504a^6b^{13}c^3d^{16}e^{17}f^2 + 1596a^6b^{13}c^4d^{15}e^{16}f^3 + 3696a^6b^{13}c^5d^{14}e^{15}f^4 - 17472a^6b^{13}c^6d^{13}e^{14}f^5 + 17472a^6b^{13}c^7d^{12}e^{13}f^6 + 9828a^6b^{13}c^8d^{11}e^{12}f^7 - 38584a^6b^{13}c^9d^{10}e^{11}f^8 + 48048a^6b^{13}c^{10}d^9e^{10}f^9 - 38584a^6b^{13}c^{11}d^8e^9f^{10} + 9828a^6b^{13}c^{12}d^7e^8f^{11} + 17472a^6b^{13}c^{13}d^6e^7f^{12} - 17472a^6b^{13}c^{14}d^5e^6f^{13} + 3696a^6b^{13}c^{15}d^4e^5f^{14} + 1596a^6b^{13}c^{16}d^3e^4f^{15} - 504a^6b^{13}c^{17}d^2e^3f^{16} + 360a^7b^{12}c^2d^{17}e^{17}f^2 + 312a^7b^{12}c^3d^{16}e^{16}f^3 - 6288a^7b^{12}c^4d^{15}e^{15}f^4 + 7056a^7b^{12}c^5d^{14}e^{14}f^5 + 17472a^7b^{12}c^6d^{13}e^{13}f^6 - 43680a^7b^{12}c^7d^{12}e^{12}f^7 + 32760a^7b^{12}c^8d^{11}e^{11}f^8 - 8008a^7b^{12}c^9d^{10}e^{10}f^9 - 8008a^7b^{12}c^{10}d^9e^9f^{10} + 32760a^7b^{12}c^{11}d^8e^8f^{11} - 43680a^7b^{12}c^{12}d^7e^7f^{12} + 17472a^7b^{12}c^{13}d^6e^6f^{13} + 7056a^7b^{12}c^{14}d^5e^5f^{14} - 6288a^7b^{12}c^{15}d^4e^4f^{15} + 312a^7b^{12}c^{16}d^3e^3f^{16} + 360a^7b^{12}c^{17}d^2e^2f^{17} - 882a^8b^{11}c^2d^{17}e^{16}f^3 + 2520a^8b^{11}c^3d^{16}e^{15}f^4 + 7380a^8b^{11}c^4d^{15}e^{14}f^5 - 25452a^8b^{11}c^5d^{14}e^{13}f^6 + 9828a^8b^{11}c^6d^{13}e^{12}f^7 + 32760a^8b^{11}c^7d^{12}e^{11}f^8 - 36036a^8b^{11}c^8d^{11}e^{10}f^9 + 20020a^8b^{11}c^9d^{10}e^9f^{10} - 36036a^8b^{11}c^{10}d^9e^8f^{11} + 32760a^8b^{11}c^{11}d^8e^7f^{12} + 9828a^8b^{11}c^{12}d^7e^6f^{13} - 25452a^8b^{11}c^{13}d^6e^5f^{14} + 7380a^8b^{11}c^{14}d^5e^4f^{15} + 2520a^8b^{11}c^{15}d^4e^3f^{16} - 882a^8b^{11}c^{16}d^3e^2f^{17} + 728a^9b^{10}c^2d^{17}e^{15}f^4 - 7480a^9b^{10}c^3d^{16}e^{14}f^5 + 2660a^9b^{10}c^4d^{15}e^{13}f^6 + 30212a^9b^{10}c^5d^{14}e^{12}f^7 - 38584a^9b^{10}c^6d^{13}e^{11}f^8 - 8008a^9b^{10}c^7d^{12}e^{10}f^9 + 20020a^9b^{10}c^8d^{11}e^9f^{10} + 20020a^9b^{10}c^9d^{10}e^8f^{11} - 8008a^9b^{10}c^{10}d^9e^7f^{12} - 38584a^9b^{10}c^{11}d^8e^6f^{13} + 30212a^9b^{10}c^{12}d^7e^5f^{14} + 2660a^9b^{10}c^{13}d^6e^4f^{15} - 7480a^9b^{10}c^{14}d^5e^3f^{16} + 728a^9b^{10}c^{15}d^4e^2f^{17} + 1152a^{10}b^9c^2d^{17}e^{14}f^5 + 9408a^{10}b^9c^3d^{16}e^{13}f^6 - 18564a^{10}b^9c^4d^{15}e^{12}f^7 - 13104a^{10}b^9c^5d^{14}e^{11}f^8 + 48048a^{10}b^9c^6d^{13}e^{10}f^9 - 8008a^{10}b^9c^7d^{12}e^9f^{10} - 36036a^{10}b^9c^8d^{11}e^8f^{11} + 20020a^{10}b^9c^9d^{10}e^7f^{12} - 36036a^{10}b^9c^{10}d^9e^6f^{13} + 20020a^{10}b^9c^{11}d^8e^5f^{14} - 8008a^{10}b^9c^{12}d^7e^4f^{15} + 728a^{10}b^9c^{13}d^6e^3f^{16} - 7480a^{10}b^9c^{14}d^5e^2f^{17} + 1152a^{10}b^9c^{15}d^4e^1f^{18}
\end{aligned}$$

$$\begin{aligned}
& 8*d^{11}*e^8*f^{11} - 8008*a^{10}*b^9*c^9*d^{10}*e^7*f^{12} + 48048*a^{10}*b^9*c^{10}*d^9 \\
& *e^6*f^{13} - 13104*a^{10}*b^9*c^{11}*d^8*e^5*f^{14} - 18564*a^{10}*b^9*c^{12}*d^7*e^4* \\
& f^{15} + 9408*a^{10}*b^9*c^{13}*d^6*e^3*f^{16} + 1152*a^{10}*b^9*c^{14}*d^5*e^2*f^{17} - \\
& 4032*a^{11}*b^8*c^2*d^{17}*e^{13}*f^6 - 4368*a^{11}*b^8*c^3*d^{16}*e^{12}*f^7 + 26208*a \\
& ^{11}*b^8*c^4*d^{15}*e^{11}*f^8 - 13104*a^{11}*b^8*c^5*d^{14}*e^{10}*f^9 - 38584*a^{11}*b \\
& ^8*c^6*d^{13}*e^9*f^{10} + 32760*a^{11}*b^8*c^7*d^{12}*e^8*f^{11} + 32760*a^{11}*b^8*c^ \\
& 8*d^{11}*e^7*f^{12} - 38584*a^{11}*b^8*c^9*d^{10}*e^6*f^{13} - 13104*a^{11}*b^8*c^{10}*d^ \\
& 9*e^5*f^{14} + 26208*a^{11}*b^8*c^{11}*d^8*e^4*f^{15} - 4368*a^{11}*b^8*c^{12}*d^7*e^3* \\
& f^{16} - 4032*a^{11}*b^8*c^{13}*d^6*e^2*f^{17} + 5460*a^{12}*b^7*c^2*d^{17}*e^{12}*f^7 - \\
& 4368*a^{12}*b^7*c^3*d^{16}*e^{11}*f^8 - 18564*a^{12}*b^7*c^4*d^{15}*e^{10}*f^9 + 30212* \\
& a^{12}*b^7*c^5*d^{14}*e^9*f^{10} + 9828*a^{12}*b^7*c^6*d^{13}*e^8*f^{11} - 43680*a^{12}*b \\
& ^7*c^7*d^{12}*e^7*f^{12} + 9828*a^{12}*b^7*c^8*d^{11}*e^6*f^{13} + 30212*a^{12}*b^7*c^9 \\
& *d^{10}*e^5*f^{14} - 18564*a^{12}*b^7*c^{10}*d^9*e^4*f^{15} - 4368*a^{12}*b^7*c^{11}*d^8* \\
& e^3*f^{16} + 5460*a^{12}*b^7*c^{12}*d^7*e^2*f^{17} - 4032*a^{13}*b^6*c^2*d^{17}*e^{11}*f^ \\
& 8 + 9408*a^{13}*b^6*c^3*d^{16}*e^{10}*f^9 + 2660*a^{13}*b^6*c^4*d^{15}*e^9*f^{10} - 254 \\
& 52*a^{13}*b^6*c^5*d^{14}*e^8*f^{11} + 17472*a^{13}*b^6*c^6*d^{13}*e^7*f^{12} + 17472*a^ \\
& ^{13}*b^6*c^7*d^{12}*e^6*f^{13} - 25452*a^{13}*b^6*c^8*d^{11}*e^5*f^{14} + 2660*a^{13}*b^6 \\
& *c^9*d^{10}*e^4*f^{15} + 9408*a^{13}*b^6*c^{10}*d^9*e^3*f^{16} - 4032*a^{13}*b^6*c^{11}*d \\
& ^8*e^2*f^{17} + 1152*a^{14}*b^5*c^2*d^{17}*e^{10}*f^9 - 7480*a^{14}*b^5*c^3*d^{16}*e^9* \\
& f^{10} + 7380*a^{14}*b^5*c^4*d^{15}*e^8*f^{11} + 7056*a^{14}*b^5*c^5*d^{14}*e^7*f^{12} - \\
& 17472*a^{14}*b^5*c^6*d^{13}*e^6*f^{13} + 7056*a^{14}*b^5*c^7*d^{12}*e^5*f^{14} + 7380*a \\
& ^{14}*b^5*c^8*d^{11}*e^4*f^{15} - 7480*a^{14}*b^5*c^9*d^{10}*e^3*f^{16} + 1152*a^{14}*b^5 \\
& *c^{10}*d^9*e^2*f^{17} + 728*a^{15}*b^4*c^2*d^{17}*e^9*f^{10} + 2520*a^{15}*b^4*c^3*d^{1 \\
& 6}*e^8*f^{11} - 6288*a^{15}*b^4*c^4*d^{15}*e^7*f^{12} + 3696*a^{15}*b^4*c^5*d^{14}*e^6*f \\
& ^{13} + 3696*a^{15}*b^4*c^6*d^{13}*e^5*f^{14} - 6288*a^{15}*b^4*c^7*d^{12}*e^4*f^{15} + 2 \\
& 520*a^{15}*b^4*c^8*d^{11}*e^3*f^{16} + 728*a^{15}*b^4*c^9*d^{10}*e^2*f^{17} - 882*a^{16}* \\
& b^3*c^2*d^{17}*e^8*f^{11} + 312*a^{16}*b^3*c^3*d^{16}*e^7*f^{12} + 1596*a^{16}*b^3*c^4* \\
& d^{15}*e^6*f^{13} - 2772*a^{16}*b^3*c^5*d^{14}*e^5*f^{14} + 1596*a^{16}*b^3*c^6*d^{13}*e^ \\
& 4*f^{15} + 312*a^{16}*b^3*c^7*d^{12}*e^3*f^{16} - 882*a^{16}*b^3*c^8*d^{11}*e^2*f^{17} + \\
& 360*a^{17}*b^2*c^2*d^{17}*e^7*f^{12} - 504*a^{17}*b^2*c^3*d^{16}*e^6*f^{13} + 252*a^{17}* \\
& b^2*c^4*d^{15}*e^5*f^{14} + 252*a^{17}*b^2*c^5*d^{14}*e^4*f^{15} - 504*a^{17}*b^2*c^6*d \\
& ^{13}*e^3*f^{16} + 360*a^{17}*b^2*c^7*d^{12}*e^2*f^{17}))/ (56*a^3*b^{13}*c^5*d^{11}*e^{16} \\
& - a^8*b^8*d^{16}*e^{16} - a^{16}*c^8*d^8*f^{16} - b^{16}*c^8*d^8*e^{16} - a^{16}*d^{16}*e^8 \\
& *f^8 - b^{16}*c^{16}*e^8*f^8 - 28*a^2*b^{14}*c^6*d^{10}*e^{16} - a^8*b^8*c^{16}*f^{16} - \\
& 70*a^4*b^{12}*c^4*d^{12}*e^{16} + 56*a^5*b^{11}*c^3*d^{13}*e^{16} - 28*a^6*b^{10}*c^2*d^{1 \\
& 4}*e^{16} - 28*a^{10}*b^6*c^{14}*d^2*f^{16} + 56*a^{11}*b^5*c^{13}*d^3*f^{16} - 70*a^{12}*b^ \\
& 4*c^{12}*d^4*f^{16} + 56*a^{13}*b^3*c^{11}*d^5*f^{16} - 28*a^{14}*b^2*c^{10}*d^6*f^{16} - 2 \\
& 8*a^2*b^{14}*c^{16}*e^6*f^{10} + 56*a^3*b^{13}*c^{16}*e^5*f^{11} - 70*a^4*b^{12}*c^{16}*e^4 \\
& *f^{12} + 56*a^5*b^{11}*c^{16}*e^3*f^{13} - 28*a^6*b^{10}*c^{16}*e^2*f^{14} - 28*a^{10}*b^6 \\
& *d^{16}*e^{14}*f^2 + 56*a^{11}*b^5*d^{16}*e^{13}*f^3 - 70*a^{12}*b^4*d^{16}*e^{12}*f^4 + 56 \\
& *a^{13}*b^3*d^{16}*e^{11}*f^5 - 28*a^{14}*b^2*d^{16}*e^{10}*f^6 - 28*a^{16}*c^2*d^{14}*e^6* \\
& f^{10} + 56*a^{16}*c^3*d^{13}*e^5*f^{11} - 70*a^{16}*c^4*d^{12}*e^4*f^{12} + 56*a^{16}*c^5* \\
& d^{11}*e^3*f^{13} - 28*a^{16}*c^6*d^{10}*e^2*f^{14} - 28*b^{16}*c^{10}*d^6*e^{14}*f^2 + 56* \\
& b^{16}*c^{11}*d^5*e^{13}*f^3 - 70*b^{16}*c^{12}*d^4*e^{12}*f^4 + 56*b^{16}*c^{13}*d^3*e^{11}* \\
& f^5 - 28*b^{16}*c^{14}*d^2*e^{10}*f^6 + 8*a*b^{15}*c^7*d^9*e^{16} + 8*a^7*b^9*c*d^{15}*
\end{aligned}$$

$$\begin{aligned}
& e^{16} + 8a^9b^7c^{15}d^7f^{16} + 8a^{15}b^7c^9d^7f^{16} + 8a^7b^{15}c^{16}e^7f^9 \\
& + 8a^7b^9c^{16}e^7f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^7d^{16}e^9f^7 + \\
& 8a^{16}c^7d^{15}e^7f^9 + 8a^{16}c^7d^9e^7f^{15} + 8b^{16}c^9d^7e^{15}f + 8b^{16}c^{15}d^7e^9f^7 \\
& - 56a^8b^{15}c^8d^8e^{15}f - 56a^8b^{15}c^{15}d^8e^8f^8 - 56a^8b^8c^8d^{15}e^{15}f \\
& - 56a^8b^8c^{15}d^8e^8f^{15} - 56a^{15}b^8c^8d^{15}e^8f^8 - 56a^{15}b^8c^8d^8e^8f^{15} \\
& + 160a^8b^{15}c^9d^7e^{14}f^2 - 224a^8b^{15}c^{10}d^6e^{13}f^3 + 112a^8b^{15}c^{11}d^5e^{12}f^4 \\
& + 112a^8b^{15}c^{12}d^4e^{11}f^5 - 224a^8b^{15}c^{13}d^3e^{10}f^6 + 160a^8b^{15}c^{14}d^2e^9f^7 + 160a^2 \\
& *b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d^2e^9f^7 - 224a^3b^{13}c^6d^{10}e^{15}f \\
& - 224a^3b^{13}c^{15}d^6e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112a^4b^{12}c^{15}d^5e^5f^{11} \\
& + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15}d^4e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f \\
& - 224a^6b^{10}c^{15}d^3e^3f^{13} + 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d^2e^2f^{14} \\
& + 160a^9b^7c^2d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e^2f^{15} - 224a^{10}b^6c^3d^{15}e^{13}f^3 \\
& - 224a^{10}b^6c^{13}d^3e^3f^{15} + 112a^{11}b^5c^4d^{15}e^{12}f^4 + 112a^{11}b^5c^{12}d^4e^2f^{15} \\
& + 112a^{12}b^4c^3d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e^2f^{15} - 224a^{13}b^3c^3d^{15}e^{10}f^6 \\
& - 224a^{13}b^3c^{10}d^6e^2f^{15} + 160a^{14}b^2c^3d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e^2f^{15} \\
& + 160a^{15}b^2c^2d^{14}e^7f^9 - 224a^{15}b^2c^3d^{13}e^6f^{10} + 112a^{15}b^2c^4d^{12}e^5f^{11} \\
& + 112a^{15}b^2c^5d^{11}e^4f^{12} - 224a^{15}b^2c^6d^{10}e^3f^{13} + 160a^{15}b^2c^7d^9e^2f^{14} \\
& - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 - 1344a^2b^{14}c^{11}d^5e^{11}f^5 \\
& + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a^2b^{14}c^{14}d^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 \\
& - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5e^{10}f^6 \\
& - 2800a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 + 840a^4b^{12}c^6d^{10}e^{14}f^2 \\
& - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^4b^{12}c^8d^8e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 \\
& - 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e^8f^8 \\
& - 2800a^4b^{12}c^{13}d^3e^7f^9 + 840a^4b^{12}c^{14}d^2e^6f^{10} - 1344a^5b^{11}c^5d^{11}e^{14}f^2 \\
& + 1568a^5b^{11}c^6d^{10}e^{13}f^3 + 4480a^5b^{11}c^7d^9e^{12}f^4 - 12264a^5b^{11}c^8d^8e^{11}f^5 \\
& + 7392a^5b^{11}c^9d^7e^{10}f^6 + 7392a^5b^{11}c^{10}d^6e^9f^7 - 12264a^5b^{11}c^{11}d^5e^8f^8 \\
& + 4480a^5b^{11}c^{12}d^4e^7f^9 + 1568a^5b^{11}c^{13}d^3e^6f^{10} - 1344a^5b^{11}c^{14}d^2e^5f^{11} \\
& + 840a^6b^{10}c^4d^{12}e^{14}f^2 + 1568a^6b^{10}c^5d^{11}e^{13}f^3 - 8624a^6b^{10}c^6d^{10}e^{12}f^4 \\
& + 7392a^6b^{10}c^7d^9e^{11}f^5 + 11396a^6b^{10}c^8d^8e^{10}f^6 - 24640a^6b^{10}c^9d^7e^9f^7 \\
& + 11396a^6b^{10}c^{10}d^6e^8f^8 + 7392a^6b^{10}c^{11}d^5e^7f^9 - 8624a^6b^{10}c^{12}d^4e^6f^{10} \\
& + 1568a^6b^{10}c^{13}d^3e^5f^{11} + 840a^6b^{10}c^{14}d^2e^4f^{12} - 2800a^7b^9c^4d^{12}e^{13}f^3 \\
& + 4480a^7b^9c^5d^{11}e^{12}f^4 + 7392a^7b^9c^6d^{10}e^{11}f^5 - 24640a^7b^9c^7d^9e^{10}f^6 \\
& + 15400a^7b^9c^8d^8e^9f^7 + 15400a^7b^9c^9d^7e^8f^8 - 24640a^7b^9c^{10}d^6e^7f^9 \\
& + 7392a^7b^9c^{11}d^5e^6f^{10} + 4480a^7b^9c^{12}d^4e^5f^{11} - 2800a^7b^9c^{13}d^3e^4f^{12} \\
& - 300a^8b^8c^2d^{14}e^{14}f^2 + 1400a^8b^8c^3d^{13}e^{13}f^3 + 1750a^8b^8c^4d^{12}e^{12}f^4 \\
& - 12264a^8b^8c^5d^{11}e^{11}f^5 + 11396a^8b^8c^6d^{10}e^{10}f^6 + 1
\end{aligned}$$

$$\begin{aligned}
& 5400a^8b^8c^7d^9e^9f^7 - 34650a^8b^8c^8d^8e^8f^8 + 15400a^8b^8 \\
& 8c^9d^7e^7f^9 + 11396a^8b^8c^10d^6e^6f^10 - 12264a^8b^8c^11d^5 \\
& e^5f^11 + 1750a^8b^8c^12d^4e^4f^12 + 1400a^8b^8c^13d^3e^3f^13 \\
& - 300a^8b^8c^14d^2e^2f^14 - 2800a^9b^7c^3d^13e^12f^4 + 4480a^9 \\
& b^7c^4d^12e^11f^5 + 7392a^9b^7c^5d^11e^10f^6 - 24640a^9b^7c^6 \\
& d^10e^9f^7 + 15400a^9b^7c^7d^9e^8f^8 + 15400a^9b^7c^8d^8e^7 \\
& f^9 - 24640a^9b^7c^9d^7e^6f^10 + 7392a^9b^7c^10d^6e^5f^11 + 44 \\
& 80a^9b^7c^11d^5e^4f^12 - 2800a^9b^7c^12d^4e^3f^13 + 840a^10b^6 \\
& c^2d^14e^12f^4 + 1568a^10b^6c^3d^13e^11f^5 - 8624a^10b^6c^4d^12 \\
& e^10f^6 + 7392a^10b^6c^5d^11e^9f^7 + 11396a^10b^6c^6d^10e^8 \\
& f^8 - 24640a^10b^6c^7d^9e^7f^9 + 11396a^10b^6c^8d^8e^6f^10 + 7 \\
& 392a^10b^6c^9d^7e^5f^11 - 8624a^10b^6c^10d^6e^4f^12 + 1568a^10 \\
& b^6c^11d^5e^3f^13 + 840a^10b^6c^12d^4e^2f^14 - 1344a^11b^5c^2 \\
& d^14e^11f^5 + 1568a^11b^5c^3d^13e^10f^6 + 4480a^11b^5c^4d^12e^9 \\
& f^7 - 12264a^11b^5c^5d^11e^8f^8 + 7392a^11b^5c^6d^10e^7f^9 + \\
& 7392a^11b^5c^7d^9e^6f^10 - 12264a^11b^5c^8d^8e^5f^11 + 4480a^11 \\
& b^5c^9d^7e^4f^12 + 1568a^11b^5c^10d^6e^3f^13 - 1344a^11b^5c^11 \\
& d^5e^2f^14 + 840a^12b^4c^2d^14e^10f^6 - 2800a^12b^4c^3d^13e^9 \\
& f^7 + 1750a^12b^4c^4d^12e^8f^8 + 4480a^12b^4c^5d^11e^7f^9 - \\
& 8624a^12b^4c^6d^10e^6f^10 + 4480a^12b^4c^7d^9e^5f^11 + 1750a^12 \\
& b^4c^8d^8e^4f^12 - 2800a^12b^4c^9d^7e^3f^13 + 840a^12b^4c^10 \\
& d^6e^2f^14 + 1400a^13b^3c^3d^13e^8f^8 - 2800a^13b^3c^4d^12e^7 \\
& f^9 + 1568a^13b^3c^5d^11e^6f^10 + 1568a^13b^3c^6d^10e^5f^11 - \\
& 2800a^13b^3c^7d^9e^4f^12 + 1400a^13b^3c^8d^8e^3f^13 - 300a^14 \\
& b^2c^2d^14e^8f^8 + 840a^14b^2c^4d^12e^6f^10 - 1344a^14b^2c^5d^11 \\
& e^5f^11 + 840a^14b^2c^6d^10e^4f^12 - 300a^14b^2c^8d^8e^2f^14) \\
& + (x*(84a^5b^11c^13d^3f^16 - 12a^4b^12c^14d^2f^16 - 243a^6 \\
& b^10c^12d^4f^16 + 366a^7b^9c^11d^5f^16 - 321a^8b^8c^10d^6f^16 \\
& + 252a^9b^7c^9d^7f^16 - 321a^10b^6c^8d^8f^16 + 366a^11b^5c^7 \\
& d^9f^16 - 243a^12b^4c^6d^10f^16 + 84a^13b^3c^5d^11f^16 - 12a^14 \\
& b^2c^4d^12f^16 - 12a^4b^12d^16e^14f^2 + 84a^5b^11d^16e^13f^3 \\
& - 243a^6b^10d^16e^12f^4 + 366a^7b^9d^16e^11f^5 - 321a^8b^8d^16 \\
& e^10f^6 + 252a^9b^7d^16e^9f^7 - 321a^10b^6d^16e^8f^8 + 366a^11 \\
& b^5d^16e^7f^9 - 243a^12b^4d^16e^6f^10 + 84a^13b^3d^16e^5f^11 \\
& - 12a^14b^2d^16e^4f^12 - 12b^16c^4d^12e^14f^2 + 84b^16c^5d^11 \\
& e^13f^3 - 243b^16c^6d^10e^12f^4 + 366b^16c^7d^9e^11f^5 - 321b^16 \\
& c^8d^8e^10f^6 + 252b^16c^9d^7e^9f^7 - 321b^16c^10d^6e^8f^8 + \\
& 366b^16c^11d^5e^7f^9 - 243b^16c^12d^4e^6f^10 + 84b^16c^13d^3 \\
& e^5f^11 - 12b^16c^14d^2e^4f^12 + 48a*b^15c^3d^13e^14f^2 - 252a* \\
& b^15c^4d^12e^13f^3 + 366a*b^15c^5d^11e^12f^4 + 354a*b^15c^6d^10 \\
& e^11f^5 - 1458a*b^15c^7d^9e^10f^6 + 942a*b^15c^8d^8e^9f^7 + 942 \\
& a*b^15c^9d^7e^8f^8 - 1458a*b^15c^10d^6e^7f^9 + 354a*b^15c^11d^5 \\
& e^6f^10 + 366a*b^15c^12d^4e^5f^11 - 252a*b^15c^13d^3e^4f^12 + \\
& 48a*b^15c^14d^2e^3f^13 + 48a^3b^13c*d^15e^14f^2 + 48a^3b^13c^1 \\
& 4d^2e*f^15 - 252a^4b^12c*d^15e^13f^3 - 252a^4b^12c^13d^3e*f^15
\end{aligned}$$

$$\begin{aligned}
& + 366*a^5*b^{11}*c*d^{15}*e^{12}*f^4 + 366*a^5*b^{11}*c^{12}*d^4*e*f^{15} + 354*a^6*b^{10}*c*d^{15}*e^{11}*f^5 + 354*a^6*b^{10}*c^{11}*d^5*e*f^{15} - 1458*a^7*b^9*c*d^{15}*e^{10}*f^6 - 1458*a^7*b^9*c^{10}*d^6*e*f^{15} + 942*a^8*b^8*c*d^{15}*e^9*f^7 + 942*a^8*b^8*c^9*d^7*e*f^{15} + 942*a^9*b^7*c*d^{15}*e^8*f^8 + 942*a^9*b^7*c^8*d^8*e*f^{15} - 1458*a^{10}*b^6*c*d^{15}*e^7*f^9 - 1458*a^{10}*b^6*c^7*d^9*e*f^{15} + 354*a^{11}*b^5*c*d^{15}*e^6*f^{10} + 354*a^{11}*b^5*c^6*d^{10}*e*f^{15} + 366*a^{12}*b^4*c*d^{15}*e^5*f^{11} + 366*a^{12}*b^4*c^5*d^{11}*e*f^{15} - 252*a^{13}*b^3*c*d^{15}*e^4*f^{12} - 252*a^{13}*b^3*c^4*d^{12}*e*f^{15} + 48*a^{14}*b^2*c*d^{15}*e^3*f^{13} + 48*a^{14}*b^2*c^3*d^{13}*e*f^{15} - 72*a^2*b^{14}*c^2*d^{14}*e^{14}*f^2 + 168*a^2*b^{14}*c^3*d^{13}*e^{13}*f^3 + 723*a^2*b^{14}*c^4*d^{12}*e^{12}*f^4 - 3258*a^2*b^{14}*c^5*d^{11}*e^{11}*f^5 + 3156*a^2*b^{14}*c^6*d^{10}*e^{10}*f^6 + 3522*a^2*b^{14}*c^7*d^9*e^9*f^7 - 8478*a^2*b^{14}*c^8*d^8*e^8*f^8 + 3522*a^2*b^{14}*c^9*d^7*e^7*f^9 + 3156*a^2*b^{14}*c^{10}*d^6*e^6*f^{10} - 3258*a^2*b^{14}*c^{11}*d^5*e^5*f^{11} + 723*a^2*b^{14}*c^{12}*d^4*e^4*f^{12} + 168*a^2*b^{14}*c^{13}*d^3*e^3*f^{13} - 72*a^2*b^{14}*c^{14}*d^2*e^2*f^{14} + 168*a^3*b^{13}*c^2*d^{14}*e^{13}*f^3 - 1692*a^3*b^{13}*c^3*d^{13}*e^{12}*f^4 + 2538*a^3*b^{13}*c^4*d^{12}*e^{11}*f^5 + 5634*a^3*b^{13}*c^5*d^{11}*e^{10}*f^6 - 18738*a^3*b^{13}*c^6*d^{10}*e^9*f^7 + 12042*a^3*b^{13}*c^7*d^9*e^8*f^8 + 12042*a^3*b^{13}*c^8*d^8*e^7*f^9 - 18738*a^3*b^{13}*c^9*d^7*e^6*f^{10} + 5634*a^3*b^{13}*c^{10}*d^6*e^5*f^{11} + 2538*a^3*b^{13}*c^{11}*d^5*e^4*f^{12} - 1692*a^3*b^{13}*c^{12}*d^4*e^3*f^{13} + 168*a^3*b^{13}*c^{13}*d^3*e^2*f^{14} + 723*a^4*b^{12}*c^2*d^{14}*e^{12}*f^4 + 2538*a^4*b^{12}*c^3*d^{13}*e^{11}*f^5 - 14022*a^4*b^{12}*c^4*d^{12}*e^{10}*f^6 + 14022*a^4*b^{12}*c^5*d^{11}*e^9*f^7 + 21087*a^4*b^{12}*c^6*d^{10}*e^8*f^8 - 48168*a^4*b^{12}*c^7*d^9*e^7*f^9 + 21087*a^4*b^{12}*c^8*d^8*e^6*f^{10} + 14022*a^4*b^{12}*c^9*d^7*e^5*f^{11} - 14022*a^4*b^{12}*c^{10}*d^6*e^4*f^{12} + 2538*a^4*b^{12}*c^{11}*d^5*e^3*f^{13} + 723*a^4*b^{12}*c^{12}*d^4*e^2*f^{14} - 3258*a^5*b^{11}*c^2*d^{14}*e^{11}*f^5 + 5634*a^5*b^{11}*c^3*d^{13}*e^{10}*f^6 + 14022*a^5*b^{11}*c^4*d^{12}*e^9*f^7 - 50544*a^5*b^{11}*c^5*d^{11}*e^8*f^8 + 33696*a^5*b^{11}*c^6*d^{10}*e^7*f^9 + 33696*a^5*b^{11}*c^7*d^9*e^6*f^{10} - 50544*a^5*b^{11}*c^8*d^8*e^5*f^{11} + 14022*a^5*b^{11}*c^9*d^7*e^4*f^{12} + 5634*a^5*b^{11}*c^{10}*d^6*e^3*f^{13} - 3258*a^5*b^{11}*c^{11}*d^5*e^2*f^{14} + 3156*a^6*b^{10}*c^2*d^{14}*e^{10}*f^6 - 18738*a^6*b^{10}*c^3*d^{13}*e^9*f^7 + 21087*a^6*b^{10}*c^4*d^{12}*e^8*f^8 + 33696*a^6*b^{10}*c^5*d^{11}*e^7*f^9 - 78624*a^6*b^{10}*c^6*d^{10}*e^6*f^{10} + 33696*a^6*b^{10}*c^7*d^9*e^5*f^{11} + 21087*a^6*b^{10}*c^8*d^8*e^4*f^{12} - 18738*a^6*b^{10}*c^9*d^7*e^3*f^{13} + 3156*a^6*b^{10}*c^{10}*d^6*e^2*f^{14} + 3522*a^7*b^9*c^2*d^{14}*e^9*f^7 + 12042*a^7*b^9*c^3*d^{13}*e^8*f^8 - 48168*a^7*b^9*c^4*d^{12}*e^7*f^9 + 33696*a^7*b^9*c^5*d^{11}*e^6*f^{10} + 33696*a^7*b^9*c^6*d^{10}*e^5*f^{11} - 48168*a^7*b^9*c^7*d^9*e^4*f^{12} + 12042*a^7*b^9*c^8*d^8*e^3*f^{13} + 3522*a^7*b^9*c^9*d^7*e^2*f^{14} - 8478*a^8*b^8*c^2*d^{14}*e^8*f^8 + 12042*a^8*b^8*c^3*d^{13}*e^7*f^9 + 21087*a^8*b^8*c^4*d^{12}*e^6*f^{10} - 50544*a^8*b^8*c^5*d^{11}*e^5*f^{11} + 21087*a^8*b^8*c^6*d^{10}*e^4*f^{12} + 12042*a^8*b^8*c^7*d^9*e^3*f^{13} - 8478*a^8*b^8*c^8*d^8*e^2*f^{14} + 3522*a^9*b^7*c^2*d^{14}*e^7*f^9 - 18738*a^9*b^7*c^3*d^{13}*e^6*f^{10} + 14022*a^9*b^7*c^4*d^{12}*e^5*f^{11} + 14022*a^9*b^7*c^5*d^{11}*e^4*f^{12} - 18738*a^9*b^7*c^6*d^{10}*e^3*f^{13} + 3522*a^9*b^7*c^7*d^9*e^2*f^{14} + 3156*a^{10}*b^6*c^2*d^{14}*e^6*f^{10} + 5634*a^{10}*b^6*c^3*d^{13}*e^5*f^{11} - 14022*a^{10}*b^6*c^4*d^{12}*e^4*f^{12} + 5634*a^{10}*b^6*c^5*d^{11}*e^3*f^{13} + 3156*a^{10}*b^6*c^6*d^{10}*e^2*f^{14} - 3258*a^{11}*b^5*c^2*d^{14}*e^5*f^{11} + 2538*a^{11}*
\end{aligned}$$

$$\begin{aligned}
& b^5c^3d^{13}e^4f^{12} + 2538a^{11}b^5c^4d^{12}e^3f^{13} - 3258a^{11}b^5c^5 \\
& *d^{11}e^2f^{14} + 723a^{12}b^4c^2d^{14}e^4f^{12} - 1692a^{12}b^4c^3d^{13}e^3 \\
& *f^{13} + 723a^{12}b^4c^4d^{12}e^2f^{14} + 168a^{13}b^3c^2d^{14}e^3f^{13} + \\
& 168a^{13}b^3c^3d^{13}e^2f^{14} - 72a^{14}b^2c^2d^{14}e^2f^{14})) / (56a^3b^6 \\
& *c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8f^{16} - b^{16}c^8d^8e^{16} \\
& - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8 \\
& *b^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28 \\
& *a^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3 \\
& *f^{16} - 70a^{12}b^4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2 \\
& *c^{10}d^6f^{16} - 28a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70 \\
& *a^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f \\
& ^{14} - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d \\
& ^{16}e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a \\
& ^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} \\
& + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d \\
& ^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16} \\
& *c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^7b^9c^{15}d^9e^{16} + \\
& 8a^7b^9c^{15}d^9e^{16} + 8a^9b^7c^{15}d^7f^{16} + 8a^15 \\
& *b^{15}c^{16}e^7f^9 + 8a^7b^9c^{16}e^7f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15} \\
& *b^{15}c^{16}e^9f^7 + 8a^{16}c^7d^9e^7f^9 + 8a^{16}c^7d^9e^7f^9 + 8b^{16}c^9 \\
& *d^7e^{15}f + 8b^{16}c^{15}d^9e^9f^7 - 56a^7b^9c^8d^8e^{15}f - 56a^7b^9 \\
& *c^{15}d^8e^8f^8 - 56a^8b^8c^8d^{15}e^{15}f - 56a^8b^8c^{15}d^8e^7f^{15} - 56 \\
& *a^{15}b^7c^8d^8e^8f^8 - 56a^{15}b^7c^8d^8e^8f^{15} + 160a^7b^9c^9d^7e^{14} \\
& *f^2 - 224a^7b^9c^{10}d^6e^{13}f^3 + 112a^7b^9c^{11}d^5e^{12}f^4 + 112a^7b^9 \\
& *c^{12}d^4e^{11}f^5 - 224a^7b^9c^{13}d^3e^{10}f^6 + 160a^7b^9c^{14}d^2 \\
& *e^9f^7 + 160a^2b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d^7e^7f^9 - 224a^3 \\
& *b^{13}c^6d^{10}e^{15}f - 224a^3b^{13}c^{15}d^8e^6f^{10} + 112a^4b^{12}c^5d \\
& ^{11}e^{15}f + 112a^4b^{12}c^{15}d^8e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + \\
& 112a^5b^{11}c^{15}d^7e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10} \\
& *c^{15}d^6e^3f^{13} + 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d^6e^2f^{14} \\
& + 160a^9b^7c^2d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e^7f^{15} - 224a^{10}b^6 \\
& *c^2d^{15}e^{13}f^3 - 224a^{10}b^6c^{13}d^3e^7f^{15} + 112a^{11}b^5c^2d^{15}e^{12} \\
& *f^4 + 112a^{11}b^5c^{12}d^4e^7f^{15} + 112a^{12}b^4c^2d^{15}e^{11}f^5 + 112a^{12} \\
& *b^4c^{11}d^5e^7f^{15} - 224a^{13}b^3c^2d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6 \\
& *e^7f^{15} + 160a^{14}b^2c^2d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e^7f^{15} + 160 \\
& *a^{15}b^2c^2d^{14}e^7f^9 - 224a^{15}b^2c^3d^{13}e^6f^{10} + 112a^{15}b^2c^4d^{12} \\
& *e^5f^{11} + 112a^{15}b^2c^5d^{11}e^4f^{12} - 224a^{15}b^2c^6d^{10}e^3f^{13} + \\
& 160a^{15}b^2c^7d^9e^2f^{14} - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14} \\
& *c^{10}d^6e^{12}f^4 - 1344a^2b^{14}c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4 \\
& *e^{10}f^6 - 300a^2b^{14}c^{14}d^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 \\
& - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3 \\
& *b^{13}c^{11}d^5e^{10}f^6 - 2800a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13} \\
& *c^{13}d^3e^8f^8 + 840a^4b^{12}c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^8 \\
& *f^3 + 1750a^4b^{12}c^8d^8e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - \\
& 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4
\end{aligned}$$

$$\begin{aligned}
& 4*b^{12}*c^{12}*d^4*e^8*f^8 - 2800*a^4*b^{12}*c^{13}*d^3*e^7*f^9 + 840*a^4*b^{12}*c^{14}*d^2*e^6*f^{10} - 1344*a^5*b^{11}*c^5*d^{11}*e^{14}*f^2 + 1568*a^5*b^{11}*c^6*d^{10}*e^{13}*f^3 + 4480*a^5*b^{11}*c^7*d^9*e^{12}*f^4 - 12264*a^5*b^{11}*c^8*d^8*e^{11}*f^5 \\
& + 7392*a^5*b^{11}*c^9*d^7*e^{10}*f^6 + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c^{13}*d^3*e^6*f^{10} - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12}*e^{14}*f^2 + 1568*a^6*b^{10}*c^5*d^{11}*e^{13}*f^3 - 8624*a^6*b^{10}*c^6*d^{10}*e^{12}*f^4 \\
& + 7392*a^6*b^{10}*c^7*d^9*e^{11}*f^5 + 11396*a^6*b^{10}*c^8*d^8*e^{10}*f^6 - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10}*c^{11}*d^5*e^7*f^9 - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13}*f^3 \\
& + 4480*a^7*b^9*c^5*d^{11}*e^{12}*f^4 + 7392*a^7*b^9*c^6*d^{10}*e^{11}*f^5 - 24640*a^7*b^9*c^7*d^9*e^{10}*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5*e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - 300*a^8*b^8*c^2*d^{14}*e^{14}*f^2 + 1400*a^8*b^8*c^3*d^{13}*e^{13}*f^3 + 1750*a^8*b^8*c^4*d^{12}*e^{12}*f^4 - 12264*a^8*b^8*c^5*d^{11}*e^{11}*f^5 + 11396*a^8*b^8*c^6*d^{10}*e^{10}*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8*c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}*e^{12}*f^4 + 4480*a^9*b^7*c^4*d^{12}*e^{11}*f^5 + 7392*a^9*b^7*c^5*d^{11}*e^{10}*f^6 - 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}*d^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f^{13} + 840*a^{10}*b^6*c^2*d^{14}*e^{12}*f^4 + 1568*a^{10}*b^6*c^3*d^{13}*e^{11}*f^5 - 8624*a^{10}*b^6*c^4*d^{12}*e^{10}*f^6 + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 + 11396*a^{10}*b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8*d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4*f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - 1344*a^{11}*b^5*c^2*d^{14}*e^{11}*f^5 + 1568*a^{11}*b^5*c^3*d^{13}*e^{10}*f^6 + 4480*a^{11}*b^5*c^4*d^{12}*e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^6*d^{10}*e^7*f^9 + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6 - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568*a^{13}*b^3*c^6*d^{10}*e^5*f^{11} - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 1344*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b^2*c^8*d^8*e^2*f^{14})) - (36*a^{11}*b^2*d^{13}*f^{13} + 36*b^{13}*c^{11}*d^2*f^{13} + 36*b^{13}*d^{13}*e^{11}*f^2 + 297*a^2*b^{11}*c^9*d^4*f^{13} - 108*a^3*b^{10}*c^8*d^5*f^1
\end{aligned}$$

$$\begin{aligned}
& 3 - 198a^4b^9c^7d^6f^{13} + 153a^5b^8c^6d^7f^{13} + 153a^6b^7c^5d^8f^{13} - 198a^7b^6c^4d^9f^{13} - 108a^8b^5c^3d^{10}f^{13} + 297a^9b^4c^2d^{11}f^{13} + 297a^2b^{11}d^{13}e^9f^4 - 108a^3b^{10}d^{13}e^8f^5 - 198a^4b^9d^{13}e^7f^6 + 153a^5b^8d^{13}e^6f^7 + 153a^6b^7d^{13}e^5f^8 - 198a^7b^6d^{13}e^4f^9 - 108a^8b^5d^{13}e^3f^{10} + 297a^9b^4d^{13}e^2f^{11} + 297b^{13}c^2d^{11}e^9f^4 - 108b^{13}c^3d^{10}e^8f^5 - 198b^{13}c^4d^9e^7f^6 + 153b^{13}c^5d^8e^6f^7 + 153b^{13}c^6d^7e^5f^8 - 198b^{13}c^7d^6e^4f^9 - 108b^{13}c^8d^5e^3f^{10} + 297b^{13}c^9d^4e^2f^{11} - 180a^ab^{12}c^{10}d^3f^{13} - 180a^{10}b^3c^d^{12}f^{13} - 180a^ab^{12}d^{13}e^{10}f^3 - 180a^{10}b^3d^{13}e^f^{12} - 180b^{13}c^d^{12}e^{10}f^3 - 180b^13c^{10}d^3e^f^{12} + 1026a^ab^{12}c^d^{12}e^9f^4 + 1026a^ab^{12}c^9d^4e^f^{12} + 1026a^9b^4c^d^{12}e^f^{12} - 2052a^ab^{12}c^2d^{11}e^8f^5 + 1548a^ab^{12}c^3d^{10}e^7f^6 + 297a^ab^{12}c^4d^9e^6f^7 - 1242a^ab^{12}c^5d^8e^5f^8 + 297a^ab^{12}c^6d^7e^4f^9 + 1548a^ab^{12}c^7d^6e^3f^{10} - 2052a^ab^{12}c^8d^5e^2f^{11} - 2052a^2b^{11}c^d^{12}e^8f^5 - 2052a^2b^{11}c^8d^5e^f^{12} + 1548a^3b^{10}c^d^{12}e^7f^6 + 1548a^3b^{10}c^7d^6e^f^{12} + 297a^4b^9c^d^{12}e^6f^7 + 297a^4b^9c^6d^7e^f^{12} - 1242a^5b^8c^d^{12}e^5f^8 - 1242a^5b^8c^5d^8e^f^{12} + 297a^6b^7c^d^{12}e^4f^9 + 297a^6b^7c^4d^9e^f^{12} + 1548a^7b^6c^d^{12}e^3f^{10} + 1548a^7b^6c^3d^{10}e^f^{12} - 2052a^8b^5c^d^{12}e^2f^{11} - 2052a^8b^5c^2d^{11}e^f^{12} + 4860a^2b^{11}c^2d^{11}e^7f^6 - 4986a^2b^{11}c^3d^{10}e^6f^7 + 1701a^2b^{11}c^4d^9e^5f^8 + 1701a^2b^{11}c^5d^8e^4f^9 - 4986a^2b^{11}c^6d^7e^3f^{10} + 4860a^2b^{11}c^7d^6e^2f^{11} - 4986a^3b^{10}c^2d^{11}e^6f^7 + 6336a^3b^{10}c^3d^{10}e^5f^8 - 3960a^3b^{10}c^4d^9e^4f^9 + 6336a^3b^{10}c^5d^8e^3f^{10} - 4986a^3b^{10}c^6d^7e^2f^{11} + 1701a^4b^9c^2d^{11}e^5f^8 - 3960a^4b^9c^3d^{10}e^4f^9 - 3960a^4b^9c^4d^9e^3f^{10} + 1701a^4b^9c^5d^8e^2f^{11} + 1701a^5b^8c^2d^{11}e^4f^9 + 6336a^5b^8c^3d^{10}e^3f^{10} + 1701a^5b^8c^4d^9e^2f^{11} - 4986a^6b^7c^2d^{11}e^3f^{10} - 4986a^6b^7c^3d^{10}e^2f^{11} + 4860a^7b^6c^2d^{11}e^2f^{11}) / (56a^3b^{13}c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8f^{16} - b^{16}c^8d^8e^{16} - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3f^{16} - 70a^{12}b^4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2c^{10}d^6f^{16} - 28a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16}e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16}c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^ab^{15}c^7d^9e^{16} + 8a^7b^9c^d^{15}e^{16} + 8a^9b^7c^{15}d^f^{16} + 8a^{15}b^c^9d^7f^{16} + 8a^ab^{15}c^{16}e^7f^9 + 8a^7b^9c^{16}e^f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^d^{16}e^9f^7 + 8a^{16}c^d^{15}e^7f^9 + 8a^{16}c^7d^9e^f^{15}
\end{aligned}$$

$$\begin{aligned}
& + 8*b^{16}*c^9*d^7*e^{15}*f + 8*b^{16}*c^{15}*d^8*e^9*f^7 - 56*a*b^{15}*c^8*d^8*e^{15}*f \\
& - 56*a*b^{15}*c^{15}*d^8*e^8*f^8 - 56*a^8*b^8*c^8*d^{15}*e^{15}*f - 56*a^8*b^8*c^{15}*d^8* \\
& *f^{15} - 56*a^{15}*b^8*c^8*d^8*e^8*f^8 - 56*a^{15}*b^8*c^8*d^8*e^8*f^{15} + 160*a*b^{15}*c^9* \\
& d^7*e^{14}*f^2 - 224*a*b^{15}*c^{10}*d^6*e^{13}*f^3 + 112*a*b^{15}*c^{11}*d^5*e^{12}*f^4 + 112*a*b^{15}*c^{12}* \\
& d^4*e^{11}*f^5 - 224*a*b^{15}*c^{13}*d^3*e^{10}*f^6 + 160*a*b^{15}*c^{14}*d^2*e^9*f^7 + 160*a^2*b^{14}*c^7*d^9* \\
& e^{15}*f + 160*a^2*b^{14}*c^{15}*d^7*f^9 - 224*a^3*b^{13}*c^6*d^{10}*e^{15}*f - 224*a^3*b^{13}*c^{15}*d^6*f^{10} + 112*a^4* \\
& b^{12}*c^5*d^{11}*e^{15}*f + 112*a^4*b^{12}*c^{15}*d^5*f^{11} + 112*a^5*b^{11}*c^4*d^{12}*e^{15}*f + 112*a^5*b^{11}*c^{15}*d^4*f^{12} - 224*a^6*b^{10}*c^3*d^{13}* \\
& e^{15}*f - 224*a^6*b^{10}*c^{15}*d^3*e^3*f^{13} + 160*a^7*b^9*c^2*d^{14}*e^{15}*f + 160*a^7*b^9*c^{15}*d^2*f^{14} + 160*a^9*b^7*c^{14}*d^2* \\
& e^2*f^{15} - 224*a^{10}*b^6*c^8*d^{15}*e^{13}*f^3 - 224*a^{10}*b^6*c^{13}*d^3*e^3*f^{15} + 112*a^{11}*b^5*c^8*d^{15}*e^{12}*f^4 + 112*a^{11}*b^5*c^{12}*d^4* \\
& e^3*f^{15} + 112*a^{12}*b^4*c^8*d^{15}*e^{11}*f^5 + 112*a^{12}*b^4*c^{11}*d^5*e^3*f^{15} - 224*a^{13}*b^3*c^8*d^{15}*e^{10}*f^6 - 224*a^{13}*b^3*c^{10}*d^6* \\
& e^3*f^{15} + 160*a^{14}*b^2*c^8*d^{15}*e^9*f^7 + 160*a^{14}*b^2*c^9*d^7*e^3*f^{15} + 160*a^{15}*b*c^2*d^{14}*e^7*f^9 - 224*a^{15}*b*c^3*d^{13}*e^6*f^{10} + 112*a^{15}* \\
& b*c^4*d^{12}*e^5*f^{11} + 112*a^{15}*b*c^5*d^{11}*e^4*f^{12} - 224*a^{15}*b*c^6*d^{10}*e^3*f^{13} + 160*a^{15}*b*c^7*d^9*e^2*f^{14} - 300*a^2*b^{14}*c^8*d^8* \\
& e^{14}*f^2 + 840*a^2*b^{14}*c^{10}*d^6*e^{12}*f^4 - 1344*a^2*b^{14}*c^{11}*d^5*e^{11}*f^5 + 840*a^2*b^{14}*c^{12}*d^4*e^{10}*f^6 - 300*a^2*b^{14}*c^{14}*d^2* \\
& e^8*f^8 + 1400*a^3*b^{13}*c^8*d^8*e^{13}*f^3 - 2800*a^3*b^{13}*c^9*d^7*e^{12}*f^4 + 1568*a^3*b^{13}*c^{10}*d^6*e^{11}*f^5 + 1568*a^3*b^{13}*c^{11}*d^5* \\
& e^{10}*f^6 - 2800*a^3*b^{13}*c^{12}*d^4*e^9*f^7 + 1400*a^3*b^{13}*c^{13}*d^3*e^8*f^8 + 840*a^4*b^{12}*c^6*d^{10}*e^{14}*f^2 - 2800*a^4*b^{12}*c^7*d^9* \\
& e^{13}*f^3 + 1750*a^4*b^{12}*c^8*d^8*e^{12}*f^4 + 4480*a^4*b^{12}*c^9*d^7*e^{11}*f^5 - 8624*a^4*b^{12}*c^{10}*d^6*e^{10}*f^6 + 4480*a^4*b^{12}*c^{11}*d^5* \\
& e^9*f^7 + 1750*a^4*b^{12}*c^{12}*d^4*e^8*f^8 - 2800*a^4*b^{12}*c^{13}*d^3*e^7*f^9 + 840*a^4*b^{12}*c^{14}*d^2*e^6*f^{10} - 1344*a^5*b^{11}*c^5*d^{11}* \\
& e^{14}*f^2 + 1568*a^5*b^{11}*c^6*d^{10}*e^{13}*f^3 + 4480*a^5*b^{11}*c^7*d^9*e^{12}*f^4 - 12264*a^5*b^{11}*c^8*d^8*e^{11}*f^5 + 7392*a^5*b^{11}*c^9*d^7* \\
& e^{10}*f^6 + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c^{13}*d^3* \\
& e^6*f^{10} - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12}*e^{14}*f^2 + 1568*a^6*b^{10}*c^5*d^{11}*e^{13}*f^3 - 8624*a^6*b^{10}*c^6*d^{10}* \\
& e^{12}*f^4 + 7392*a^6*b^{10}*c^7*d^9*e^{11}*f^5 + 11396*a^6*b^{10}*c^8*d^8*e^{10}*f^6 - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6* \\
& e^8*f^8 + 7392*a^6*b^{10}*c^{11}*d^5*e^7*f^9 - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2* \\
& e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13}*f^3 + 4480*a^7*b^9*c^5*d^{11}*e^{12}*f^4 + 7392*a^7*b^9*c^6*d^{10}*e^{11}*f^5 - 24640*a^7*b^9*c^7*d^9* \\
& e^{10}*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5* \\
& e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - 300*a^8*b^8*c^2*d^{14}*e^{14}*f^2 + 1400*a^8*b^8*c^3*d^{13}* \\
& e^{13}*f^3 + 1750*a^8*b^8*c^4*d^{12}*e^{12}*f^4 - 12264*a^8*b^8*c^5*d^{11}*e^{11}*f^5 + 11396*a^8*b^8*c^6*d^{10}*e^{10}*f^6 + 15400*a^8*b^8*c^7*d^9* \\
& e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5* \\
& e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1
\end{aligned}$$

$$\begin{aligned}
& 400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) + (x*(108*a^3*b^10*c^7*d^6*f^13 - 36*b^13*c^10*d^3*f^13 - 36*b^13*d^13*e^10*f^3 - 297*a^2*b^11*c^8*d^5*f^13 - 36*a^10*b^3*d^13*f^13 + 324*a^4*b^9*c^6*d^7*f^13 - 594*a^5*b^8*c^5*d^8*f^13 + 324*a^6*b^7*c^4*d^9*f^13 + 108*a^7*b^6*c^3*d^10*f^13 - 297*a^8*b^5*c^2*d^11*f^13 - 297*a^2*b^11*d^13*e^8*f^5 + 108*a^3*b^10*d^13*e^7*f^6 + 324*a^4*b^9*d^13*e^6*f^7 - 594*a^5*b^8*d^13*e^5*f^8 + 324*a^6*b^7*d^13*e^4*f^9 + 108*a^7*b^6*d^13*e^3*f^10 - 297*a^8*b^5*d^13*e^2*f^11 - 297*b^13*c^2*d^11*e^8*f^5 + 108*b^13*c^3*d^10*e^7*f^6 + 324*b^13*c^4*d^9*e^6*f^7 - 594*b^13*c^5*d^8*e^5*f^8 + 324*b^13*c^6*d^7*e^4*f^9 + 108*b^13*c^7*d^6*e^3*f^10 - 297*b^13*c^8*d^5*e^2*f^11 + 180*a*b^12*c^9*d^4*f^13 + 180*a^9*b^4*c*d^12*f^13 + 180*a*b^12*d^13*e^9*f^4 + 180*a^9*b^4*d^13*e*f^12 + 180*b^13*c*d^12*e^9*f^4 + 180*b^13*c^9*d^4*e*f^12 - 1026*a*b^12*c*d^12*e^8*f^5 - 1026*a*b^12*c^8*d^5*e*f^12 - 1026*a^8*b^5*c*d^12*e*f^12 + 2052*a*b^12*c^2*d^11*e^7*f^6 - 2052*a*b^12*c^3*d^10*e^6*f^7 + 1026*a*b^12*c^4*d^9*e^5*f^8 + 1026*a*b^12*c^5*d^8*e^4*f^9 - 2052*a*b^12*c^6*d^7*e^3*f^10 + 2052*a*b^12*c^7*d^6*e^2*f^11 + 2052*a^2*b^11*c*d^12*e^7*f^6 + 2052*a^2*b^11*c^7*d^6*e*f^12 - 2052*a^3*b^10*c*d^12*e^6*f^7 - 2052*a^3*b^10*c^6*d^7*e*f^12 + 1026*a^4*b^9*c*d^12*e^5*f^8 + 1026*a^4*b^9*c^5*d^8*e*f^12 + 1026*a^5*b^8*c*d^12*e^4*f^9 + 1026*a^5*b^8*c^4*d^9*e*f^12 - 2052*a^6*b^7*c*d^12*e^3*f^10 - 2052*a^6*b^7*c^3*d^10*e*f^12 + 2052*a^7*b^6*c*d^12*e^2*f^11 + 2052*a^7*b^6*c^2*d^11*e*f^12 - 4104*a^2*b^11*c^2*d^11*e^6*f^7 + 4104*a^2*b^11*c^3*d^10*e^5*f^8 - 5130*a^2*b^11*c^4*d^9*e^4*f^9 + 4104*a^2*b^11*c^5*d^8*e^3*f^10 - 4104*a^2*b^11*c^6*d^7*e^2*f^11 + 4
\end{aligned}$$

$$\begin{aligned}
& 104a^3b^{10}c^2d^{11}e^5f^8 + 4104a^3b^{10}c^5d^8e^2f^{11} - 5130a^4b^9c^2d^{11}e^4f^9 - 5130a^4b^9c^4d^9e^2f^{11} + 4104a^5b^8c^2d^{11}e^3f^{10} + 4104a^5b^8c^3d^{10}e^2f^{11} - 4104a^6b^7c^2d^{11}e^2f^{11} \\
&) / (56a^3b^{13}c^5d^{11}e^{16} - a^8b^8d^{16}e^{16} - a^{16}c^8d^8f^{16} - b^{16}c^8d^8e^{16} - a^{16}d^{16}e^8f^8 - b^{16}c^{16}e^8f^8 - 28a^2b^{14}c^6d^{10}e^{16} - a^8b^8c^{16}f^{16} - 70a^4b^{12}c^4d^{12}e^{16} + 56a^5b^{11}c^3d^{13}e^{16} - 28a^6b^{10}c^2d^{14}e^{16} - 28a^{10}b^6c^{14}d^2f^{16} + 56a^{11}b^5c^{13}d^3f^{16} - 70a^{12}b^4c^{12}d^4f^{16} + 56a^{13}b^3c^{11}d^5f^{16} - 28a^{14}b^2c^{10}d^6f^{16} - 28a^2b^{14}c^{16}e^6f^{10} + 56a^3b^{13}c^{16}e^5f^{11} - 70a^4b^{12}c^{16}e^4f^{12} + 56a^5b^{11}c^{16}e^3f^{13} - 28a^6b^{10}c^{16}e^2f^{14} - 28a^{10}b^6d^{16}e^{14}f^2 + 56a^{11}b^5d^{16}e^{13}f^3 - 70a^{12}b^4d^{16}e^{12}f^4 + 56a^{13}b^3d^{16}e^{11}f^5 - 28a^{14}b^2d^{16}e^{10}f^6 - 28a^{16}c^2d^{14}e^6f^{10} + 56a^{16}c^3d^{13}e^5f^{11} - 70a^{16}c^4d^{12}e^4f^{12} + 56a^{16}c^5d^{11}e^3f^{13} - 28a^{16}c^6d^{10}e^2f^{14} - 28b^{16}c^{10}d^6e^{14}f^2 + 56b^{16}c^{11}d^5e^{13}f^3 - 70b^{16}c^{12}d^4e^{12}f^4 + 56b^{16}c^{13}d^3e^{11}f^5 - 28b^{16}c^{14}d^2e^{10}f^6 + 8a^*b^{15}c^7d^9e^{16} + 8a^7b^9c^*d^{15}e^{16} + 8a^9b^7c^{15}d^*f^{16} + 8a^{15}b^*c^9d^7f^{16} + 8a^*b^{15}c^{16}e^7f^9 + 8a^7b^9c^{16}e^*f^{15} + 8a^9b^7d^{16}e^{15}f + 8a^{15}b^*d^{16}e^9f^7 + 8a^{16}c^*d^{15}e^7f^9 + 8a^{16}c^7d^9e^*f^{15} + 8b^{16}c^9d^7e^{15}f + 8b^{16}c^{15}d^*e^9f^7 - 56a^*b^{15}c^8d^8e^{15}f - 56a^*b^{15}c^{15}d^*e^8f^8 - 56a^8b^8c^*d^{15}e^{15}f - 56a^8b^8c^{15}d^*e^*f^{15} - 56a^{15}b^*c^*d^{15}e^8f^8 - 56a^{15}b^*c^8d^8e^*f^{15} + 160a^*b^{15}c^9d^7e^{14}f^2 - 224a^*b^{15}c^{10}d^6e^{13}f^3 + 112a^*b^{15}c^{11}d^5e^{12}f^4 + 112a^*b^{15}c^{12}d^4e^{11}f^5 - 224a^*b^{15}c^{13}d^3e^{10}f^6 + 160a^*b^{15}c^{14}d^2e^9f^7 + 160a^2b^{14}c^7d^9e^{15}f + 160a^2b^{14}c^{15}d^*e^7f^9 - 224a^3b^{13}c^6d^{10}e^{15}f - 224a^3b^{13}c^{15}d^*e^6f^{10} + 112a^4b^{12}c^5d^{11}e^{15}f + 112a^4b^{12}c^{15}d^*e^5f^{11} + 112a^5b^{11}c^4d^{12}e^{15}f + 112a^5b^{11}c^{15}d^*e^4f^{12} - 224a^6b^{10}c^3d^{13}e^{15}f - 224a^6b^{10}c^{15}d^*e^3f^{13} + 160a^7b^9c^2d^{14}e^{15}f + 160a^7b^9c^{15}d^*e^2f^{14} + 160a^9b^7c^*d^{15}e^{14}f^2 + 160a^9b^7c^{14}d^2e^*f^{15} - 224a^{10}b^6c^*d^{15}e^{13}f^3 - 224a^{10}b^6c^{13}d^3e^*f^{15} + 112a^{11}b^5c^*d^{15}e^{12}f^4 + 112a^{11}b^5c^{12}d^4e^*f^{15} + 112a^{12}b^4c^*d^{15}e^{11}f^5 + 112a^{12}b^4c^{11}d^5e^*f^{15} - 224a^{13}b^3c^*d^{15}e^{10}f^6 - 224a^{13}b^3c^{10}d^6e^*f^{15} + 160a^{14}b^2c^*d^{15}e^9f^7 + 160a^{14}b^2c^9d^7e^*f^{15} + 160a^{15}b^*c^2d^{14}e^7f^9 - 224a^{15}b^*c^3d^{13}e^6f^{10} + 112a^{15}b^*c^4d^{12}e^5f^{11} + 112a^{15}b^*c^5d^{11}e^4f^{12} - 224a^{15}b^*c^6d^{10}e^3f^{13} + 160a^{15}b^*c^7d^9e^2f^{14} - 300a^2b^{14}c^8d^8e^{14}f^2 + 840a^2b^{14}c^{10}d^6e^{12}f^4 - 1344a^2b^{14}c^{11}d^5e^{11}f^5 + 840a^2b^{14}c^{12}d^4e^{10}f^6 - 300a^2b^{14}c^{14}d^2e^8f^8 + 1400a^3b^{13}c^8d^8e^{13}f^3 - 2800a^3b^{13}c^9d^7e^{12}f^4 + 1568a^3b^{13}c^{10}d^6e^{11}f^5 + 1568a^3b^{13}c^{11}d^5e^{10}f^6 - 2800a^3b^{13}c^{12}d^4e^9f^7 + 1400a^3b^{13}c^{13}d^3e^8f^8 + 840a^4b^{12}c^6d^{10}e^{14}f^2 - 2800a^4b^{12}c^7d^9e^{13}f^3 + 1750a^4b^{12}c^8d^8e^{12}f^4 + 4480a^4b^{12}c^9d^7e^{11}f^5 - 8624a^4b^{12}c^{10}d^6e^{10}f^6 + 4480a^4b^{12}c^{11}d^5e^9f^7 + 1750a^4b^{12}c^{12}d^4e^8f^8 - 2800a^4b^{12}c^{13}d^3e^7f^9 + 840
\end{aligned}$$

$$\begin{aligned}
& *a^4*b^{12}*c^{14}*d^2*e^6*f^{10} - 1344*a^5*b^{11}*c^5*d^{11}*e^{14}*f^2 + 1568*a^5*b^{11}*c^6*d^{10}*e^{13}*f^3 + 4480*a^5*b^{11}*c^7*d^9*e^{12}*f^4 - 12264*a^5*b^{11}*c^8*d^8*e^{11}*f^5 + 7392*a^5*b^{11}*c^9*d^7*e^{10}*f^6 + 7392*a^5*b^{11}*c^{10}*d^6*e^9*f^7 - 12264*a^5*b^{11}*c^{11}*d^5*e^8*f^8 + 4480*a^5*b^{11}*c^{12}*d^4*e^7*f^9 + 1568*a^5*b^{11}*c^{13}*d^3*e^6*f^{10} - 1344*a^5*b^{11}*c^{14}*d^2*e^5*f^{11} + 840*a^6*b^{10}*c^4*d^{12}*e^{14}*f^2 + 1568*a^6*b^{10}*c^5*d^{11}*e^{13}*f^3 - 8624*a^6*b^{10}*c^6*d^{10}*e^{12}*f^4 + 7392*a^6*b^{10}*c^7*d^9*e^{11}*f^5 + 11396*a^6*b^{10}*c^8*d^8*e^{10}*f^6 - 24640*a^6*b^{10}*c^9*d^7*e^9*f^7 + 11396*a^6*b^{10}*c^{10}*d^6*e^8*f^8 + 7392*a^6*b^{10}*c^{11}*d^5*e^7*f^9 - 8624*a^6*b^{10}*c^{12}*d^4*e^6*f^{10} + 1568*a^6*b^{10}*c^{13}*d^3*e^5*f^{11} + 840*a^6*b^{10}*c^{14}*d^2*e^4*f^{12} - 2800*a^7*b^9*c^4*d^{12}*e^{13}*f^3 + 4480*a^7*b^9*c^5*d^{11}*e^{12}*f^4 + 7392*a^7*b^9*c^6*d^{10}*e^{11}*f^5 - 24640*a^7*b^9*c^7*d^9*e^{10}*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^{10}*d^6*e^7*f^9 + 7392*a^7*b^9*c^{11}*d^5*e^6*f^{10} + 4480*a^7*b^9*c^{12}*d^4*e^5*f^{11} - 2800*a^7*b^9*c^{13}*d^3*e^4*f^{12} - 300*a^8*b^8*c^2*d^{14}*e^{14}*f^2 + 1400*a^8*b^8*c^3*d^{13}*e^{13}*f^3 + 1750*a^8*b^8*c^4*d^{12}*e^{12}*f^4 - 12264*a^8*b^8*c^5*d^{11}*e^{11}*f^5 + 11396*a^8*b^8*c^6*d^{10}*e^{10}*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^{10}*d^6*e^6*f^{10} - 12264*a^8*b^8*c^{11}*d^5*e^5*f^{11} + 1750*a^8*b^8*c^{12}*d^4*e^4*f^{12} + 1400*a^8*b^8*c^{13}*d^3*e^3*f^{13} - 300*a^8*b^8*c^{14}*d^2*e^2*f^{14} - 2800*a^9*b^7*c^3*d^{13}*e^{12}*f^4 + 4480*a^9*b^7*c^4*d^{12}*e^{11}*f^5 + 7392*a^9*b^7*c^5*d^{11}*e^{10}*f^6 - 24640*a^9*b^7*c^6*d^{10}*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^{10} + 7392*a^9*b^7*c^{10}*d^6*e^5*f^{11} + 4480*a^9*b^7*c^{11}*d^5*e^4*f^{12} - 2800*a^9*b^7*c^{12}*d^4*e^3*f^{13} + 840*a^{10}*b^6*c^2*d^{14}*e^{12}*f^4 + 1568*a^{10}*b^6*c^3*d^{13}*e^{11}*f^5 - 8624*a^{10}*b^6*c^4*d^{12}*e^{10}*f^6 + 7392*a^{10}*b^6*c^5*d^{11}*e^9*f^7 + 11396*a^{10}*b^6*c^6*d^{10}*e^8*f^8 - 24640*a^{10}*b^6*c^7*d^9*e^7*f^9 + 11396*a^{10}*b^6*c^8*d^8*e^6*f^{10} + 7392*a^{10}*b^6*c^9*d^7*e^5*f^{11} - 8624*a^{10}*b^6*c^{10}*d^6*e^4*f^{12} + 1568*a^{10}*b^6*c^{11}*d^5*e^3*f^{13} + 840*a^{10}*b^6*c^{12}*d^4*e^2*f^{14} - 1344*a^{11}*b^5*c^2*d^{14}*e^{11}*f^5 + 1568*a^{11}*b^5*c^3*d^{13}*e^{10}*f^6 + 4480*a^{11}*b^5*c^4*d^{12}*e^9*f^7 - 12264*a^{11}*b^5*c^5*d^{11}*e^8*f^8 + 7392*a^{11}*b^5*c^6*d^{10}*e^7*f^9 + 7392*a^{11}*b^5*c^7*d^9*e^6*f^{10} - 12264*a^{11}*b^5*c^8*d^8*e^5*f^{11} + 4480*a^{11}*b^5*c^9*d^7*e^4*f^{12} + 1568*a^{11}*b^5*c^{10}*d^6*e^3*f^{13} - 1344*a^{11}*b^5*c^{11}*d^5*e^2*f^{14} + 840*a^{12}*b^4*c^2*d^{14}*e^{10}*f^6 - 2800*a^{12}*b^4*c^3*d^{13}*e^9*f^7 + 1750*a^{12}*b^4*c^4*d^{12}*e^8*f^8 + 4480*a^{12}*b^4*c^5*d^{11}*e^7*f^9 - 8624*a^{12}*b^4*c^6*d^{10}*e^6*f^{10} + 4480*a^{12}*b^4*c^7*d^9*e^5*f^{11} + 1750*a^{12}*b^4*c^8*d^8*e^4*f^{12} - 2800*a^{12}*b^4*c^9*d^7*e^3*f^{13} + 840*a^{12}*b^4*c^{10}*d^6*e^2*f^{14} + 1400*a^{13}*b^3*c^3*d^{13}*e^8*f^8 - 2800*a^{13}*b^3*c^4*d^{12}*e^7*f^9 + 1568*a^{13}*b^3*c^5*d^{11}*e^6*f^{10} + 1568*a^{13}*b^3*c^6*d^{10}*e^5*f^{11} - 2800*a^{13}*b^3*c^7*d^9*e^4*f^{12} + 1400*a^{13}*b^3*c^8*d^8*e^3*f^{13} - 300*a^{14}*b^2*c^2*d^{14}*e^8*f^8 + 840*a^{14}*b^2*c^4*d^{12}*e^6*f^{10} - 1344*a^{14}*b^2*c^5*d^{11}*e^5*f^{11} + 840*a^{14}*b^2*c^6*d^{10}*e^4*f^{12} - 300*a^{14}*b^2*c^8*d^8*e^2*f^{14}))*root(756756*a^{10}*b^{10}*c^{10}*d^{10}*e^{10}*f^{10}*z^3 + 573300*a^{12}*b^8*c^9*d^{11}*e^9*f^{11}*z^3 + 573300*a^{11}*b^9*c^{11}*d^9*e^8*f^{12}*z^3 + 573300*a^9*b^{11}*c^{12}
\end{aligned}$$

$$\begin{aligned}
& d^8 e^9 f^{11} z^3 + 573300 a^9 b^{11} c^9 d^{11} e^{12} f^8 z^3 + 573300 a^8 b^{12} c^{11} d^9 e^{11} f^9 z^3 - 343980 a^{11} b^9 c^{10} d^{10} e^9 f^{11} z^3 - 343980 a^{11} b^9 c^9 d^{11} e^{10} f^{10} z^3 - 343980 a^{10} b^{10} c^{11} d^9 e^9 f^{11} z^3 - 343980 a^{10} b^{10} c^9 d^{11} e^{11} f^9 z^3 - 343980 a^9 b^{11} c^{11} d^9 e^{10} f^{10} z^3 - 343980 a^9 b^{11} c^{10} d^{10} e^{11} f^9 z^3 + 326340 a^{13} b^7 c^{10} d^{10} e^7 f^{13} z^3 + 326340 a^{13} b^7 c^7 d^{13} e^{10} f^{10} z^3 + 326340 a^{10} b^{10} c^{13} d^7 e^7 f^{13} z^3 + 326340 a^{10} b^{10} c^7 d^{13} e^{13} f^7 z^3 + 326340 a^7 b^{13} c^{13} d^7 e^{10} f^{10} z^3 + 326340 a^7 b^{13} c^{10} d^{10} e^{13} f^7 z^3 - 267540 a^{12} b^8 c^{10} d^{10} e^8 f^{12} z^3 - 267540 a^{12} b^8 c^8 d^{12} e^{10} f^{10} z^3 - 267540 a^{10} b^{10} c^{12} d^8 e^8 f^{12} z^3 - 267540 a^{10} b^{10} c^8 d^{12} e^{12} f^8 z^3 - 267540 a^8 b^{12} c^{12} d^8 e^{10} f^{10} z^3 - 267540 a^8 b^{12} c^{10} d^{10} e^{12} f^8 z^3 + 245700 a^{14} b^6 c^8 d^{12} e^8 f^{12} z^3 + 245700 a^{12} b^8 c^{12} d^8 e^6 f^{14} z^3 + 245700 a^{12} b^8 c^6 d^{14} e^{12} f^8 z^3 + 245700 a^8 b^{12} c^{14} d^6 e^8 f^{12} z^3 + 245700 a^8 b^{12} c^8 d^{12} e^{14} f^6 z^3 + 245700 a^6 b^{14} c^{12} d^8 e^{12} f^8 z^3 - 191100 a^{13} b^7 c^9 d^{11} e^8 f^{12} z^3 - 191100 a^{13} b^7 c^8 d^{12} e^9 f^{11} z^3 - 191100 a^{12} b^8 c^{11} d^9 e^7 f^{13} z^3 - 191100 a^{12} b^8 c^7 d^{13} e^{11} f^9 z^3 - 191100 a^{11} b^9 c^{12} d^8 e^7 f^{13} z^3 - 191100 a^{11} b^9 c^7 d^{13} e^{12} f^8 z^3 - 191100 a^9 b^{11} c^{13} d^7 e^8 f^{12} z^3 - 191100 a^9 b^{11} c^8 d^{12} e^{13} f^7 z^3 - 191100 a^8 b^{12} c^{13} d^7 e^9 f^{11} z^3 - 191100 a^8 b^{12} c^9 d^{11} e^{13} f^7 z^3 - 191100 a^7 b^{13} c^{12} d^8 e^{11} f^9 z^3 - 191100 a^7 b^{13} c^{11} d^9 e^{12} f^8 z^3 - 123900 a^{14} b^6 c^9 d^{11} e^7 f^{13} z^3 - 123900 a^{14} b^6 c^7 d^{13} e^9 f^{11} z^3 - 123900 a^{13} b^7 c^{11} d^9 e^6 f^{14} z^3 - 123900 a^{13} b^7 c^6 d^{14} e^{11} f^9 z^3 - 123900 a^{11} b^9 c^{13} d^7 e^6 f^{14} z^3 - 123900 a^{11} b^9 c^6 d^{14} e^{13} f^7 z^3 - 123900 a^9 b^{11} c^{14} d^6 e^7 f^{13} z^3 - 123900 a^9 b^{11} c^7 d^{13} e^{14} f^6 z^3 - 123900 a^7 b^{13} c^{14} d^6 e^9 f^{11} z^3 - 123900 a^7 b^{13} c^9 d^{11} e^{14} f^6 z^3 - 123900 a^6 b^{14} c^{13} d^7 e^{11} f^9 z^3 - 123900 a^6 b^{14} c^{11} d^9 e^{13} f^7 z^3 + 101700 a^{15} b^5 c^9 d^{11} e^6 f^{14} z^3 + 101700 a^{15} b^5 c^6 d^{14} e^9 f^{11} z^3 + 101700 a^{14} b^6 c^{11} d^9 e^5 f^{15} z^3 + 101700 a^{14} b^6 c^5 d^{15} e^{11} f^9 z^3 + 101700 a^{11} b^9 c^{14} d^6 e^5 f^{15} z^3 + 101700 a^{11} b^9 c^5 d^{15} e^{14} f^6 z^3 + 101700 a^9 b^{11} c^{15} d^5 e^6 f^{14} z^3 + 101700 a^9 b^{11} c^6 d^{14} e^{15} f^5 z^3 + 101700 a^6 b^{14} c^{15} d^5 e^9 f^{11} z^3 + 101700 a^6 b^{14} c^9 d^{11} e^{15} f^5 z^3 + 101700 a^5 b^{15} c^{14} d^6 e^{11} f^9 z^3 + 101700 a^5 b^{15} c^{11} d^9 e^{14} f^6 z^3 - 65820 a^{14} b^6 c^{10} d^{10} e^6 f^{14} z^3 - 65820 a^{14} b^6 c^6 d^{14} e^{10} f^{10} z^3 - 65820 a^{10} b^{10} c^{14} d^6 e^6 f^{14} z^3 - 65820 a^{10} b^{10} c^6 d^{14} e^{14} f^6 z^3 - 65820 a^6 b^{14} c^{14} d^6 e^{10} f^{10} z^3 - 65820 a^6 b^{14} c^{10} d^{10} e^{14} f^6 z^3 + 56700 a^{16} b^4 c^7 d^{13} e^7 f^{13} z^3 - 56700 a^{15} b^5 c^8 d^{12} e^7 f^{13} z^3 - 56700 a^{15} b^5 c^7 d^{13} e^8 f^{12} z^3 + 56700 a^{13} b^7 c^{13} d^7 e^4 f^{16} z^3 - 56700 a^{13} b^7 c^{12} d^8 e^5 f^{15} z^3 - 56700 a^{13} b^7 c^5 d^{15} e^{12} f^8 z^3 + 56700 a^{13} b^7 c^4 d^{16} e^{13} f^7 z^3 - 56700 a^{12} b^8 c^{13} d^7 e^5 f^{15} z^3 - 56700 a^{12} b^8 c^5 d^{15} e^{13} f^7 z^3 - 56700 a^8 b^{12} c^{15} d^5 e^7 f^{13} z^3 - 56700 a^8 b^{12} c^7 d^{13} e^{15} f^5 z^3 + 56700 a^7 b^{13} c^{16} d^4 e^7 f^{13} z^3 - 56700 a^7 b^{13} c^{15} d^5 e^8 f^{12} z^3 - 56700 a^7 b^{13} c^8 d^{12} e^{15} f^5 z^3 + 56700 a^7 b^{13} c^7 d^{13} e^{16} f^4 z^3 - 56700 a^5 b^{15} c^{13} d^7 e^{12} f^8 z^3
\end{aligned}$$

$$\begin{aligned}
& z^3 - 56700a^5b^{15}c^{12}d^8e^{13}f^7z^3 + 56700a^4b^{16}c^{13}d^7e^{13}f^7z^3 - 48252a^{15}b^5c^{10}d^{10}e^5f^{15}z^3 - 48252a^{15}b^5c^5d^{15}e^{10}f^{10}z^3 - 48252a^{10}b^{10}c^{15}d^5e^5f^{15}z^3 - 48252a^{10}b^{10}c^5d^{15}e^{15}f^5z^3 - 48252a^5b^{15}c^{15}d^5e^{10}f^{10}z^3 - 48252a^5b^{15}c^{10}d^{10}e^{15}f^5z^3 - 32400a^{16}b^4c^8d^{12}e^6f^{14}z^3 - 32400a^{16}b^4c^6d^{14}e^8f^{12}z^3 - 32400a^{14}b^6c^{12}d^8e^4f^{16}z^3 - 32400a^{14}b^6c^4d^{16}e^{12}f^8z^3 - 32400a^{12}b^8c^{14}d^6e^4f^{16}z^3 - 32400a^{12}b^8c^4d^{16}e^{14}f^6z^3 - 32400a^8b^{12}c^{16}d^4e^6f^{14}z^3 - 32400a^8b^{12}c^6d^{14}e^{16}f^4z^3 - 32400a^6b^{14}c^{16}d^4e^8f^{12}z^3 - 32400a^6b^{14}c^8d^{12}e^{16}f^4z^3 - 32400a^4b^{16}c^{14}d^6e^{12}f^8z^3 - 32400a^4b^{16}c^{12}d^8e^{14}f^6z^3 + 20565a^{16}b^4c^{10}d^{10}e^4f^{16}z^3 + 20565a^{16}b^4c^4d^{16}e^{10}f^{10}z^3 + 20565a^{10}b^{10}c^{16}d^4e^4f^{16}z^3 + 20565a^{10}b^{10}c^4d^{16}e^{16}f^4z^3 + 20565a^4b^{16}c^{16}d^4e^{10}f^{10}z^3 + 20565a^4b^{16}c^{10}d^{10}e^{16}f^4z^3 + 15660a^{17}b^3c^8d^{12}e^5f^{15}z^3 + 15660a^{17}b^3c^5d^{15}e^8f^{12}z^3 + 15660a^{15}b^5c^{12}d^8e^3f^{17}z^3 + 15660a^{15}b^5c^3d^{17}e^{12}f^8z^3 + 15660a^{12}b^8c^{15}d^5e^3f^{17}z^3 + 15660a^{12}b^8c^3d^{17}e^{15}f^5z^3 + 15660a^8b^{12}c^{17}d^3e^5f^{15}z^3 + 15660a^8b^{12}c^5d^{15}e^{17}f^3z^3 + 15660a^5b^{15}c^{17}d^3e^8f^{12}z^3 + 15660a^5b^{15}c^8d^{12}e^{17}f^3z^3 + 15660a^3b^{17}c^{15}d^5e^{12}f^8z^3 + 15660a^3b^{17}c^{12}d^8e^{15}f^5z^3 - 9750a^{17}b^3c^9d^{11}e^4f^{16}z^3 - 9750a^{17}b^3c^4d^{16}e^9f^{11}z^3 - 9750a^{16}b^4c^{11}d^9e^3f^{17}z^3 - 9750a^{16}b^4c^3d^{17}e^{11}f^9z^3 - 9750a^{11}b^9c^{16}d^4e^3f^{17}z^3 - 9750a^{11}b^9c^3d^{17}e^{16}f^4z^3 - 9750a^9b^{11}c^{17}d^3e^4f^{16}z^3 - 9750a^9b^{11}c^4d^{16}e^{17}f^3z^3 - 9750a^4b^{16}c^{17}d^3e^9f^{11}z^3 - 9750a^4b^{16}c^9d^{11}e^{17}f^3z^3 - 9750a^3b^{17}c^{16}d^4e^{11}f^9z^3 - 9750a^3b^{17}c^{11}d^9e^{16}f^4z^3 - 8100a^{17}b^3c^7d^{13}e^6f^{14}z^3 - 8100a^{17}b^3c^6d^{14}e^7f^{13}z^3 - 8100a^{14}b^6c^{13}d^7e^3f^{17}z^3 - 8100a^{14}b^6c^3d^{17}e^{13}f^7z^3 - 8100a^{13}b^7c^{14}d^6e^3f^{17}z^3 - 8100a^{13}b^7c^3d^{17}e^{14}f^6z^3 - 8100a^7b^{13}c^{17}d^3e^6f^{14}z^3 - 8100a^7b^{13}c^6d^{14}e^{17}f^3z^3 - 8100a^6b^{14}c^{17}d^3e^7f^{13}z^3 - 8100a^6b^{14}c^7d^{13}e^{17}f^3z^3 - 8100a^3b^{17}c^{14}d^6e^{13}f^7z^3 - 8100a^3b^{17}c^{13}d^7e^{14}f^6z^3 - 7980a^{16}b^4c^9d^{11}e^5f^{15}z^3 - 7980a^{16}b^4c^5d^{15}e^9f^{11}z^3 - 7980a^{15}b^5c^{11}d^9e^4f^{16}z^3 - 7980a^{15}b^5c^4d^{16}e^{11}f^9z^3 - 7980a^{11}b^9c^{15}d^5e^4f^{16}z^3 - 7980a^{11}b^9c^4d^{16}e^{15}f^5z^3 - 7980a^9b^{11}c^{16}d^4e^5f^{15}z^3 - 7980a^9b^{11}c^5d^{15}e^{16}f^4z^3 - 7980a^5b^{15}c^{16}d^4e^9f^{11}z^3 - 7980a^5b^{15}c^9d^{11}e^{16}f^4z^3 - 7980a^4b^{16}c^{15}d^5e^{11}f^9z^3 - 7980a^4b^{16}c^{11}d^9e^{15}f^5z^3 + 6300a^{18}b^2c^6d^{14}e^6f^{14}z^3 + 6300a^{14}b^6c^{14}d^6e^2f^{18}z^3 + 6300a^{14}b^6c^2d^{18}e^{14}f^6z^3 + 6300a^6b^{14}c^{18}d^2e^6f^{14}z^3 + 6300a^6b^{14}c^6d^{14}e^{18}f^2z^3 + 6300a^2b^{18}c^{14}d^6e^{14}f^6z^3 - 4260a^{18}b^2c^7d^{13}e^5f^{15}z^3 - 4260a^{18}b^2c^5d^{15}e^7f^{13}z^3 - 4260a^{15}b^5c^{13}d^7e^2f^{18}z^3 - 4260a^{15}b^5c^2d^{18}e^{13}f^7z^3 - 4260a^{13}b^7c^{15}d^5e^2f^{18}z^3 - 4260a^{13}b^7c^2d^{18}e^{15}f^5z^3 - 4260a^7b^{13}c^{18}d^2e^5f^{15}z^3 - 4260a^7b^{13}
\end{aligned}$$

$$\begin{aligned}
& *c^5*d^{15}*e^{18}*f^2*z^3 - 4260*a^5*b^{15}*c^{18}*d^2*e^7*f^{13}*z^3 - 4260*a^5*b^{15} \\
& *c^7*d^{13}*e^{18}*f^2*z^3 - 4260*a^2*b^{18}*c^{15}*d^5*e^{13}*f^7*z^3 - 4260*a^2*b^{18} \\
& *c^{13}*d^7*e^{15}*f^5*z^3 + 1470*a^{17}*b^3*c^{10}*d^{10}*e^3*f^{17}*z^3 + 1470*a^{17} \\
& *b^3*c^3*d^{17}*e^{10}*f^{10}*z^3 + 1470*a^{10}*b^{10}*c^{17}*d^3*e^3*f^{17}*z^3 + 1470*a \\
& ^{10}*b^{10}*c^3*d^{17}*e^{17}*f^3*z^3 + 1470*a^3*b^{17}*c^{17}*d^3*e^{10}*f^{10}*z^3 + 147 \\
& 0*a^3*b^{17}*c^{10}*d^{10}*e^{17}*f^3*z^3 + 1350*a^{18}*b^2*c^9*d^{11}*e^3*f^{17}*z^3 + 1 \\
& 350*a^{18}*b^2*c^3*d^{17}*e^9*f^{11}*z^3 + 1350*a^{17}*b^3*c^{11}*d^9*e^2*f^{18}*z^3 + \\
& 1350*a^{17}*b^3*c^2*d^{18}*e^{11}*f^9*z^3 + 1350*a^{11}*b^9*c^{17}*d^3*e^2*f^{18}*z^3 + \\
& 1350*a^{11}*b^9*c^2*d^{18}*e^{17}*f^3*z^3 + 1350*a^9*b^{11}*c^{18}*d^2*e^3*f^{17}*z^3 \\
& + 1350*a^9*b^{11}*c^3*d^{17}*e^{18}*f^2*z^3 + 1350*a^3*b^{17}*c^{18}*d^2*e^9*f^{11}*z^3 \\
& + 1350*a^3*b^{17}*c^9*d^{11}*e^{18}*f^2*z^3 + 1350*a^2*b^{18}*c^{17}*d^3*e^{11}*f^9*z^ \\
& 3 + 1350*a^2*b^{18}*c^{11}*d^9*e^{17}*f^3*z^3 - 1070*a^{18}*b^2*c^{10}*d^{10}*e^2*f^{18} \\
& z^3 - 1070*a^{18}*b^2*c^2*d^{18}*e^{10}*f^{10}*z^3 - 1070*a^{10}*b^{10}*c^{18}*d^2*e^2*f^{18} \\
& z^3 - 1070*a^{10}*b^{10}*c^2*d^{18}*e^{18}*f^2*z^3 - 1070*a^2*b^{18}*c^{18}*d^2*e^{10} \\
& *f^{10}*z^3 - 1070*a^2*b^{18}*c^{10}*d^{10}*e^{18}*f^2*z^3 + 525*a^{18}*b^2*c^8*d^{12}*e^ \\
& 4*f^{16}*z^3 + 525*a^{18}*b^2*c^4*d^{16}*e^8*f^{12}*z^3 + 525*a^{16}*b^4*c^{12}*d^8*e^2 \\
& *f^{18}*z^3 + 525*a^{16}*b^4*c^2*d^{18}*e^{12}*f^8*z^3 + 525*a^{12}*b^8*c^{16}*d^4*e^2* \\
& f^{18}*z^3 + 525*a^{12}*b^8*c^2*d^{18}*e^{16}*f^4*z^3 + 525*a^8*b^{12}*c^{18}*d^2*e^4*f \\
& ^{16}*z^3 + 525*a^8*b^{12}*c^4*d^{16}*e^{18}*f^2*z^3 + 525*a^4*b^{16}*c^{18}*d^2*e^8*f^ \\
& ^{12}*z^3 + 525*a^4*b^{16}*c^8*d^{12}*e^{18}*f^2*z^3 + 525*a^2*b^{18}*c^{16}*d^4*e^{12}*f^ \\
& 8*z^3 + 525*a^2*b^{18}*c^{12}*d^8*e^{16}*f^4*z^3 + 900*a^{19}*b*c^7*d^{13}*e^4*f^{16}*z \\
& ^3 + 900*a^{19}*b*c^4*d^{16}*e^7*f^{13}*z^3 + 900*a^{16}*b^4*c^{13}*d^7*e*f^{19}*z^3 + \\
& 900*a^{16}*b^4*c*d^{19}*e^{13}*f^7*z^3 + 900*a^{13}*b^7*c^{16}*d^4*e*f^{19}*z^3 + 900*a \\
& ^{13}*b^7*c*d^{19}*e^{16}*f^4*z^3 + 900*a^7*b^{13}*c^{19}*d*e^4*f^{16}*z^3 + 900*a^7*b^ \\
& ^{13}*c^4*d^{16}*e^{19}*f*z^3 + 900*a^4*b^{16}*c^{19}*d*e^7*f^{13}*z^3 + 900*a^4*b^{16}*c^ \\
& ^7*d^{13}*e^{19}*f*z^3 + 900*a*b^{19}*c^{16}*d^4*e^{13}*f^7*z^3 + 900*a*b^{19}*c^{13}*d^7* \\
& e^{16}*f^4*z^3 - 750*a^{19}*b*c^8*d^{12}*e^3*f^{17}*z^3 - 750*a^{19}*b*c^3*d^{17}*e^8*f \\
& ^{12}*z^3 - 750*a^{17}*b^3*c^{12}*d^8*e*f^{19}*z^3 - 750*a^{17}*b^3*c*d^{19}*e^{12}*f^8*z \\
& ^3 - 750*a^{12}*b^8*c^{17}*d^3*e*f^{19}*z^3 - 750*a^{12}*b^8*c*d^{19}*e^{17}*f^3*z^3 - \\
& 750*a^8*b^{12}*c^{19}*d*e^3*f^{17}*z^3 - 750*a^8*b^{12}*c^3*d^{17}*e^{19}*f*z^3 - 750*a \\
& ^3*b^{17}*c^{19}*d*e^8*f^{12}*z^3 - 750*a^3*b^{17}*c^8*d^{12}*e^{19}*f*z^3 - 750*a*b^{19} \\
& *c^{17}*d^3*e^{12}*f^8*z^3 - 750*a*b^{19}*c^{12}*d^8*e^{17}*f^3*z^3 - 420*a^{19}*b*c^6* \\
& d^{14}*e^5*f^{15}*z^3 - 420*a^{19}*b*c^5*d^{15}*e^6*f^{14}*z^3 - 420*a^{15}*b^5*c^{14}*d^ \\
& 6*e*f^{19}*z^3 - 420*a^{15}*b^5*c*d^{19}*e^{14}*f^6*z^3 - 420*a^{14}*b^6*c^{15}*d^5*e*f \\
& ^{19}*z^3 - 420*a^{14}*b^6*c*d^{19}*e^{15}*f^5*z^3 - 420*a^6*b^{14}*c^{19}*d*e^5*f^{15}*z \\
& ^3 - 420*a^6*b^{14}*c^5*d^{15}*e^{19}*f*z^3 - 420*a^5*b^{15}*c^{19}*d*e^6*f^{14}*z^3 - \\
& 420*a^5*b^{15}*c^6*d^{14}*e^{19}*f*z^3 - 420*a*b^{19}*c^{15}*d^5*e^{14}*f^6*z^3 - 420*a \\
& *b^{19}*c^{14}*d^6*e^{15}*f^5*z^3 + 350*a^{19}*b*c^9*d^{11}*e^2*f^{18}*z^3 + 350*a^{19}*b \\
& *c^2*d^{18}*e^9*f^{11}*z^3 + 350*a^{18}*b^2*c^{11}*d^9*e*f^{19}*z^3 + 350*a^{18}*b^2*c* \\
& d^{19}*e^{11}*f^9*z^3 + 350*a^{11}*b^9*c^{18}*d^2*e*f^{19}*z^3 + 350*a^{11}*b^9*c*d^{19} \\
& e^{18}*f^2*z^3 + 350*a^9*b^{11}*c^{19}*d*e^2*f^{18}*z^3 + 350*a^9*b^{11}*c^2*d^{18}*e^1 \\
& 9*f*z^3 + 350*a^2*b^{18}*c^{19}*d*e^9*f^{11}*z^3 + 350*a^2*b^{18}*c^9*d^{11}*e^{19}*f*z \\
& ^3 + 350*a*b^{19}*c^{18}*d^2*e^{11}*f^9*z^3 + 350*a*b^{19}*c^{11}*d^9*e^{18}*f^2*z^3 - \\
& 90*a^{19}*b*c^{10}*d^{10}*e*f^{19}*z^3 - 90*a^{19}*b*c*d^{19}*e^{10}*f^{10}*z^3 - 90*a^{10}*b \\
& ^{10}*c^{19}*d*e*f^{19}*z^3 - 90*a^{10}*b^{10}*c*d^{19}*e^{19}*f*z^3 - 90*a*b^{19}*c^{19}*d*e
\end{aligned}$$

$$\begin{aligned}
& ^{10}f^{10}z^3 - 90ab^{19}c^{10}d^{10}e^{19}f^*z^3 + 10b^{20}c^{19}d^*e^{11}f^9z^3 \\
& + 10b^{20}c^{11}d^9e^{19}f^*z^3 + 10a^{20}c^9d^{11}e^*f^{19}z^3 + 10a^{20}c^*d^{19}e^9f^{11}z^3 + 10a^{19}b^*d^{20}e^{11}f^9z^3 + 10a^{11}b^9d^{20}e^{19}f^*z^3 \\
& + 10a^9b^{11}c^{20}e^*f^{19}z^3 + 10a^*b^{19}c^{20}e^9f^{11}z^3 + 10a^{19}b^*c^{11}d^9f^{20}z^3 + 10a^{11}b^9c^{19}d^*f^{20}z^3 + 10a^9b^{11}c^*d^{19}e^{20}z^3 \\
& + 10a^*b^{19}c^9d^{11}e^{20}z^3 + 252b^{20}c^{15}d^5e^{15}f^5z^3 - 210b^{20}c^{16}d^4e^{14}f^6z^3 - 210b^{20}c^{14}d^6e^{16}f^4z^3 + 120b^{20}c^{17}d^3e^{13}f^7z^3 + 120b^{20}c^{13}d^7e^{17}f^3z^3 - 45b^{20}c^{18}d^2e^{12}f^8z^3 \\
& - 45b^{20}c^{12}d^8e^{18}f^2z^3 + 252a^{20}c^5d^{15}e^5f^{15}z^3 - 210a^{20}c^6d^{14}e^4f^{16}z^3 - 210a^{20}c^4d^{16}e^6f^{14}z^3 + 120a^{20}c^7d^{13}e^3f^{17}z^3 + 120a^{20}c^3d^{17}e^7f^{13}z^3 - 45a^{20}c^8d^{12}e^2f^{18}z^3 - 45a^{20}c^2d^{18}e^8f^{12}z^3 + 252a^{15}b^5d^{20}e^{15}f^5z^3 - 210a^{16}b^4d^{20}e^{14}f^6z^3 - 210a^{14}b^6d^{20}e^{16}f^4z^3 + 120a^{17}b^3d^{20}e^{13}f^7z^3 + 120a^{13}b^7d^{20}e^{17}f^3z^3 - 45a^{18}b^2d^{20}e^{12}f^8z^3 - 45a^{12}b^8d^{20}e^{18}f^2z^3 + 252a^5b^{15}c^{20}e^5f^{15}z^3 - 210a^6b^{14}c^{20}e^4f^{16}z^3 - 210a^4b^{16}c^{20}e^6f^{14}z^3 + 120a^7b^{13}c^{20}e^3f^{17}z^3 + 120a^3b^{17}c^{20}e^7f^{13}z^3 - 45a^8b^{12}c^20e^2f^{18}z^3 - 45a^2b^{18}c^{20}e^8f^{12}z^3 + 252a^{15}b^5c^{15}d^5f^{15}z^3 - 210a^{16}b^4c^{14}d^6f^{20}z^3 - 210a^{14}b^6c^{16}d^4f^{20}z^3 + 120a^{17}b^3c^{13}d^7f^{20}z^3 + 120a^{13}b^7c^{17}d^3f^{20}z^3 - 45a^{18}b^2c^{12}d^8f^{20}z^3 - 45a^{12}b^8c^{18}d^2f^{20}z^3 + 252a^5b^{15}c^5d^{15}e^{20}z^3 - 210a^6b^{14}c^4d^{16}e^{20}z^3 - 210a^4b^{16}c^6d^{14}e^{20}z^3 + 120a^7b^{13}c^3d^{17}e^{20}z^3 + 120a^3b^{17}c^7d^{13}e^{20}z^3 - 45a^8b^{12}c^2d^{18}e^{20}z^3 - 45a^2b^{18}c^8d^{12}e^{20}z^3 - b^{20}c^{20}e^{10}f^{10}z^3 - a^{20}d^{20}e^{10}f^{10}z^3 - b^{20}c^{10}d^{10}e^{20}z^3 - a^{20}c^{10}d^{10}f^{20}z^3 - a^{10}b^{10}d^{20}e^{20}z^3 - a^{10}b^{10}c^{20}f^{20}z^3 + 1890a^{12}b^2c^*d^{13}e^*f^{13}z + 1890a^*b^{13}c^{12}d^2e^*f^{13}z + 1890a^*b^{13}c^*d^{13}e^{12}f^2z + 92610a^6b^8c^4d^{10}e^4f^{10}z + 92610a^4b^{10}c^6d^8e^4f^{10}z + 92610a^4b^{10}c^4d^{10}e^6f^8z + 66150a^8b^6c^3d^{11}e^3f^{11}z - 66150a^7b^7c^4d^{10}e^3f^{11}z - 66150a^7b^7c^3d^{11}e^4f^{10}z - 66150a^4b^{10}c^7d^7e^3f^{11}z - 66150a^4b^{10}c^3d^{11}e^7f^7z + 66150a^3b^{11}c^8d^6e^3f^{11}z - 66150a^3b^{11}c^7d^7e^4f^{10}z - 66150a^3b^{11}c^4d^{10}e^7f^7z + 66150a^3b^{11}c^3d^{11}e^8f^6z - 55566a^5b^9c^5d^9e^4f^{10}z - 55566a^5b^9c^4d^{10}e^5f^9z - 55566a^4b^{10}c^5d^9e^5f^9z - 32130a^9b^5c^3d^{11}e^2f^{12}z - 32130a^9b^5c^2d^{12}e^3f^{11}z - 32130a^3b^{11}c^9d^5e^2f^{12}z - 32130a^3b^{11}c^2d^{12}e^9f^5z - 32130a^2b^{12}c^9d^5e^3f^{11}z - 32130a^2b^{12}c^3d^{11}e^9f^5z + 22680a^8b^6c^4d^{10}e^2f^{12}z + 22680a^8b^6c^2d^{12}e^4f^{10}z + 22680a^4b^{10}c^8d^6e^2f^{12}z + 22680a^4b^{10}c^2d^{12}e^8f^6z + 22680a^2b^{12}c^8d^6e^4f^{10}z + 22680a^2b^{12}c^4d^{10}e^8f^6z + 19278a^{10}b^4c^2d^{12}e^2f^{12}z + 19278a^2b^{12}c^{10}d^4e^2f^{12}z + 19278a^2b^{12}c^2d^{12}e^{10}f^4z + 18522a^6b^8c^5d^9e^3f^{11}z + 18522a^6b^8c^3d^{11}e^5f^9z + 18522a^5b^9c^6d^8e^3f^{11}z + 18522a^5b^9c^3d^{11}e^6f^8z + 18522a^3b^{11}c^6d^8e^5f^9z + 18522a^3b^{11}c^5d^9e^6f^8z - 13230a^6b^8c^6d^8e^2f^{12}z - 13230a^6b^8c^
\end{aligned}$$

$$\begin{aligned}
& 2*d^{12}*e^6*f^8*z - 13230*a^2*b^{12}*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9* \\
& e^2*f^{12}*z + 3402*a^7*b^7*c^2*d^{12}*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^1 \\
& 2*z + 3402*a^5*b^9*c^2*d^{12}*e^7*f^7*z + 3402*a^2*b^{12}*c^7*d^7*e^5*f^9*z + 3 \\
& 402*a^2*b^{12}*c^5*d^9*e^7*f^7*z + 7938*a^{10}*b^4*c^3*d^{11}*e*f^{13}*z + 7938*a^1 \\
& 0*b^4*c*d^{13}*e^3*f^{11}*z + 7938*a^3*b^{11}*c^{10}*d^4*e*f^{13}*z + 7938*a^3*b^{11}*c \\
& *d^{13}*e^{10}*f^4*z + 7938*a*b^{13}*c^{10}*d^4*e^3*f^{11}*z + 7938*a*b^{13}*c^3*d^{11}*e \\
& ^{10}*f^4*z - 5670*a^{11}*b^3*c^2*d^{12}*e*f^{13}*z - 5670*a^{11}*b^3*c*d^{13}*e^2*f^{12} \\
& *z - 5670*a^2*b^{12}*c^{11}*d^3*e*f^{13}*z - 5670*a^2*b^{12}*c*d^{13}*e^{11}*f^3*z - 56 \\
& 70*a*b^{13}*c^{11}*d^3*e^2*f^{12}*z - 5670*a*b^{13}*c^2*d^{12}*e^{11}*f^3*z - 3780*a^9* \\
& b^5*c^4*d^{10}*e*f^{13}*z - 3780*a^9*b^5*c*d^{13}*e^4*f^{10}*z - 3780*a^4*b^{10}*c^9* \\
& d^5*e*f^{13}*z - 3780*a^4*b^{10}*c*d^{13}*e^9*f^5*z - 3780*a*b^{13}*c^9*d^5*e^4*f^1 \\
& 0*z - 3780*a*b^{13}*c^4*d^{10}*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^{13}*z - 2268 \\
& *a^8*b^6*c*d^{13}*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^{13}*z - 2268*a^5*b^9*c* \\
& d^{13}*e^8*f^6*z - 2268*a*b^{13}*c^8*d^6*e^5*f^9*z - 2268*a*b^{13}*c^5*d^9*e^8*f^ \\
& 6*z + 1890*a^7*b^7*c^6*d^8*e*f^{13}*z + 1890*a^7*b^7*c*d^{13}*e^6*f^8*z + 1890* \\
& a^6*b^8*c^7*d^7*e*f^{13}*z + 1890*a^6*b^8*c*d^{13}*e^7*f^7*z + 1890*a*b^{13}*c^7* \\
& d^7*e^6*f^8*z + 1890*a*b^{13}*c^6*d^8*e^7*f^7*z - 252*b^{14}*c^{13}*d*e*f^{13}*z - \\
& 252*b^{14}*c*d^{13}*e^{13}*f*z - 252*a^{13}*b*d^{14}*e*f^{13}*z - 252*a*b^{13}*d^{14}*e^{13} \\
& f*z - 252*a^{13}*b*c*d^{13}*f^{14}*z - 252*a*b^{13}*c^{13}*d*f^{14}*z - 918*b^{14}*c^7*d^ \\
& 7*e^7*f^7*z - 882*b^{14}*c^{11}*d^3*e^3*f^{11}*z - 882*b^{14}*c^3*d^{11}*e^{11}*f^3*z + \\
& 693*b^{14}*c^{12}*d^2*e^2*f^{12}*z + 693*b^{14}*c^2*d^{12}*e^{12}*f^2*z + 567*b^{14}*c^8 \\
& *d^6*e^6*f^8*z + 567*b^{14}*c^6*d^8*e^8*f^6*z + 441*b^{14}*c^{10}*d^4*e^4*f^{10}*z \\
& + 441*b^{14}*c^4*d^{10}*e^{10}*f^4*z - 126*b^{14}*c^9*d^5*e^5*f^9*z - 126*b^{14}*c^5* \\
& d^9*e^9*f^5*z - 918*a^7*b^7*d^{14}*e^7*f^7*z - 882*a^{11}*b^3*d^{14}*e^3*f^{11}*z - \\
& 882*a^3*b^{11}*d^{14}*e^{11}*f^3*z + 693*a^{12}*b^2*d^{14}*e^2*f^{12}*z + 693*a^2*b^{12} \\
& *d^{14}*e^{12}*f^2*z + 567*a^8*b^6*d^{14}*e^6*f^8*z + 567*a^6*b^8*d^{14}*e^8*f^6*z \\
& + 441*a^{10}*b^4*d^{14}*e^4*f^{10}*z + 441*a^4*b^{10}*d^{14}*e^{10}*f^4*z - 126*a^9*b^5 \\
& *d^{14}*e^5*f^9*z - 126*a^5*b^9*d^{14}*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^{14}*z - \\
& 882*a^{11}*b^3*c^3*d^{11}*f^{14}*z - 882*a^3*b^{11}*c^{11}*d^3*f^{14}*z + 693*a^{12}*b^2 \\
& *c^2*d^{12}*f^{14}*z + 693*a^2*b^{12}*c^{12}*d^2*f^{14}*z + 567*a^8*b^6*c^6*d^8*f^{14}* \\
& z + 567*a^6*b^8*c^8*d^6*f^{14}*z + 441*a^{10}*b^4*c^4*d^{10}*f^{14}*z + 441*a^4*b^1 \\
& 0*c^{10}*d^4*f^{14}*z - 126*a^9*b^5*c^5*d^9*f^{14}*z - 126*a^5*b^9*c^9*d^5*f^{14}*z \\
& + 36*b^{14}*d^{14}*e^{14}*z + 36*b^{14}*c^{14}*f^{14}*z + 36*a^{14}*d^{14}*f^{14}*z - 27054* \\
& a^2*b^9*c^2*d^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^{10} + 9018*a^3*b^8*c*d^{10} \\
& *e^2*f^9 + 9018*a^2*b^9*c^3*d^8*e*f^{10} + 9018*a^2*b^9*c*d^{10}*e^3*f^8 + 9018 \\
& *a*b^{10}*c^3*d^8*e^2*f^9 + 9018*a*b^{10}*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^{10} \\
& *e*f^{10} - 9018*a*b^{10}*c^4*d^7*e*f^{10} - 9018*a*b^{10}*c*d^{10}*e^4*f^7 + 2268*b^ \\
& 11*c^5*d^6*e*f^{10} + 2268*b^{11}*c*d^{10}*e^5*f^6 + 2268*a^5*b^6*d^{11}*e*f^{10} + 2 \\
& 268*a*b^{10}*d^{11}*e^5*f^6 + 2268*a^5*b^6*c*d^{10}*f^{11} + 2268*a*b^{10}*c^5*d^6*f^ \\
& 11 - 1458*b^{11}*c^3*d^8*e^3*f^8 - 1161*b^{11}*c^4*d^7*e^2*f^9 - 1161*b^{11}*c^2* \\
& d^9*e^4*f^7 - 1458*a^3*b^8*d^{11}*e^3*f^8 - 1161*a^4*b^7*d^{11}*e^2*f^9 - 1161* \\
& a^2*b^9*d^{11}*e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^{11} - 1161*a^4*b^7*c^2*d^9*f^1 \\
& 1 - 1161*a^2*b^9*c^4*d^7*f^{11} - 756*b^{11}*d^{11}*e^6*f^5 - 756*b^{11}*c^6*d^5*f^ \\
& 11 - 756*a^6*b^5*d^{11}*f^{11}, z, k), k, 1, 3) - ((7*a*b^6*c^2*d^5*e^7 - a^3*b \\
& ^4*d^7*e^7 - a^7*c^3*d^4*f^7 - b^7*c^3*d^4*e^7 - a^7*d^7*e^3*f^4 - b^7*c^7*
\end{aligned}$$

$$\begin{aligned}
& e^3 f^4 - 6a^5 b^2 c^5 d^2 f^7 - 6a^5 b^2 d^7 e^5 f^2 - 6b^7 c^5 d^2 e^5 \\
& f^2 - a^3 b^4 c^7 f^7 + 7a^2 b^5 c^6 d^6 e^7 + 4a^4 b^3 c^6 d^6 f^7 + 4a^6 b^3 \\
& c^4 d^3 f^7 + 7a^2 b^6 c^7 e^2 f^5 + 7a^2 b^5 c^7 e^2 f^6 + 4a^4 b^3 d^7 e^6 \\
& f^6 + 4a^6 b^3 d^7 e^4 f^3 + 7a^7 c^6 d^6 e^2 f^5 + 7a^7 c^2 d^5 e^2 f^6 + 4b^7 \\
& c^4 d^3 e^6 f^6 + 4b^7 c^6 d^6 e^4 f^3 - 21a^2 b^6 c^3 d^4 e^6 f^6 - 21a^2 b^6 \\
& c^6 d^6 e^3 f^4 - 21a^3 b^4 c^6 d^6 e^6 f^6 - 21a^3 b^4 c^6 d^6 e^2 f^6 - 21a^6 b^3 \\
& c^4 d^6 e^3 f^4 - 21a^6 b^3 c^3 d^4 e^2 f^6 + 14a^2 b^6 c^4 d^3 e^5 f^2 + 14a^2 b^6 \\
& c^5 d^2 e^4 f^3 - 26a^2 b^5 c^2 d^5 e^6 f^6 - 26a^2 b^5 c^6 d^6 e^2 f^5 + 14a^4 \\
& b^3 c^6 d^6 e^5 f^2 + 14a^4 b^3 c^5 d^2 e^2 f^6 + 14a^5 b^2 c^6 d^6 e^4 f^3 + 14a^5 \\
& b^2 c^4 d^3 e^2 f^6 - 26a^6 b^3 c^2 d^5 e^2 f^5 + 52a^2 b^5 c^3 d^4 e^5 f^2 - 78a^2 \\
& b^5 c^4 d^3 e^4 f^3 + 52a^2 b^5 c^5 d^2 e^3 f^4 + 52a^3 b^4 c^2 d^5 e^5 f^2 + 52a^3 \\
& b^4 c^5 d^2 e^2 f^5 - 78a^4 b^3 c^2 d^5 e^4 f^3 - 78a^4 b^3 c^4 d^3 e^2 f^5 + 52a^5 \\
& b^2 c^2 d^5 e^3 f^4 + 52a^5 b^2 c^3 d^4 e^2 f^5) / (2(4a^2 b^7 c^3 d^5 e^8 - a^4 b^4 d^8 e^8 - a^8 \\
& c^4 d^4 f^8 - b^8 c^4 d^4 e^8 - a^8 d^8 e^4 f^4 - b^8 c^8 e^4 f^4 - 6a^2 b^6 c^2 d^6 \\
& e^8 - 6a^6 b^2 c^6 d^2 f^8 - 6a^2 b^6 c^8 e^2 f^6 - 6a^6 b^2 d^8 e^6 f^2 - 6a^8 c^2 d^6 \\
& e^2 f^6 - 6b^8 c^6 d^2 e^6 f^2 - a^4 b^4 c^8 f^8 + 4a^3 b^5 c^6 d^7 e^8 + 4a^5 b^3 c^7 \\
& d^6 f^8 + 4a^7 b^3 c^5 d^3 f^8 + 4a^2 b^7 c^8 e^3 f^5 + 4a^3 b^5 c^8 e^2 f^7 + 4a^5 b^3 \\
& d^8 e^7 f^6 + 4a^7 b^3 d^8 e^5 f^3 + 4a^8 c^6 d^7 e^3 f^5 + 4a^8 c^3 d^5 e^2 f^7 + 4b^8 \\
& c^5 d^3 e^7 f^6 + 4b^8 c^7 d^6 e^5 f^3 - 12a^2 b^7 c^4 d^4 e^7 f^6 - 12a^2 b^7 c^7 d^6 e^4 \\
& f^4 - 12a^4 b^4 c^7 d^6 e^2 f^7 - 12a^7 b^3 c^6 d^7 e^4 f^4 - 12a^7 b^3 c^4 d^4 e^2 f^7 \\
& + 8a^2 b^7 c^5 d^3 e^6 f^2 + 8a^2 b^7 c^6 d^2 e^5 f^3 + 8a^3 b^5 c^3 d^5 e^7 f^6 + 8a^3 \\
& b^5 c^7 d^6 e^2 f^6 + 8a^5 b^3 c^6 d^2 e^2 f^7 + 8a^7 b^3 c^2 d^6 e^3 f^5 + 8a^7 b^3 c^3 \\
& d^5 e^2 f^6 + 22a^2 b^6 c^4 d^4 e^6 f^2 - 48a^2 b^6 c^5 d^3 e^5 f^3 + 22a^2 b^6 c^6 d^2 \\
& e^4 f^4 - 48a^3 b^5 c^3 d^5 e^6 f^2 + 36a^3 b^5 c^4 d^4 e^5 f^3 + 36a^3 b^5 c^5 d^3 e^4 f^4 - \\
& 48a^3 b^5 c^6 d^2 e^3 f^5 + 22a^4 b^4 c^2 d^6 e^6 f^2 + 36a^4 b^4 c^3 d^5 e^5 f^3 - 90a^4 \\
& b^4 c^4 d^4 e^4 f^4 + 36a^4 b^4 c^5 d^3 e^3 f^5 + 22a^4 b^4 c^6 d^2 e^2 f^6 - 48a^5 b^3 \\
& c^2 d^6 e^5 f^3 + 36a^5 b^3 c^3 d^5 e^4 f^4 + 36a^5 b^3 c^4 d^4 e^3 f^5 - 48a^5 b^3 c^5 d^3 \\
& e^2 f^6 + 22a^6 b^2 c^2 d^6 e^4 f^4 - 48a^6 b^2 c^3 d^5 e^3 f^5 + 22a^6 b^2 c^4 d^4 e^2 f^6) \\
& + (3x^5(2a^5 b^2 d^7 f^7 + 2b^7 c^5 d^2 f^7 + 2b^7 d^7 e^5 f^2 + 2a^2 b^5 c^3 d^4 f^7 + 2a^3 \\
& b^4 c^2 d^5 f^7 + 2a^2 b^5 d^7 e^3 f^4 + 2a^3 b^4 d^7 e^2 f^5 + 2b^7 c^2 d^5 e^3 f^4 + 2b^7 \\
& c^3 d^4 e^2 f^5 - 5a^2 b^6 c^4 d^3 f^7 - 5a^4 b^3 c^6 d^6 f^7 - 5a^2 b^6 d^7 e^4 f^3 - 5a^4 \\
& b^3 d^7 e^4 f^3 - 5b^7 c^4 d^3 e^6 f^6 + 16a^2 b^6 c^6 d^6 e^3 f^4 + 16a^2 b^6 c^3 d^4 e^6 f^6 + 1 \\
& 6a^3 b^4 c^6 d^6 e^2 f^6 - 12a^2 b^6 c^2 d^5 e^2 f^5 - 12a^2 b^5 c^6 d^6 e^2 f^5 - 12a^2 b^5 \\
& c^2 d^5 e^2 f^6)) / (4a^2 b^7 c^3 d^5 e^8 - a^4 b^4 d^8 e^8 - a^8 c^4 d^4 f^8 - b^8 c^4 d^4 e^8 - \\
& a^8 d^8 e^4 f^4 - b^8 c^8 e^4 f^4 - 6a^2 b^6 c^2 d^6 e^8 - 6a^6 b^2 c^6 d^2 f^8 - 6a^2 b^6 c^8 e^2 \\
& f^6 - 6a^6 b^2 d^8 e^6 f^2 - 6a^8 c^2 d^6 e^2 f^6 - 6b^8 c^6 d^2 e^6 f^2 - a^4 b^4 c^8 f^8 + 4a^3 \\
& b^5 c^6 d^7 e^8 + 4a^5 b^3 c^7 d^6 f^8 + 4a^7 b^3 c^5 d^3 f^8 + 4a^2 b^7 c^8 e^3 f^5 + 4a^3 \\
& b^5 c^8 e^2 f^7 + 4a^5 b^3 d^8 e^7 f^6 + 4a^7 b^3 d^8 e^5 f^3 + 4a^8 c^6 d^7 e^3 f^5 + 4a^8 \\
& c^3 d^5 e^2 f^7 + 4b^8 c^5 d^3 e^7 f^6 + 4b^8 c^7 d^6 e^5 f^3 - 12a^2 b^7 c^4 d^4 e^7 f^6 - 12a^2 \\
& b^7 c^7 d^6 e^4 f^4 - 12a^4 b^4 c^7 d^6 e^2 f^7 - 12a^7 b^3 c^6 d^7 e^4 f^4 - 12a^7 b^3 c^4 d^4 \\
& e^2 f^7 + 8a^2 b^7 c^5 d^3 e^6 f^2 + 8a^2 b^7 c^6 d^2 e^5 f^3 + 8a^3 b^5 c^3 d^5 e^7 f^6 + 8a^3 \\
& b^5 c^7 d^6 e^2 f^6 + 8a^5 b^3 c^6 d^2 e^2 f^7 + 8a^7 b^3 c^2 d^6 e^3 f^5 + 8a^7 b^3 c^3 d^5 e^2 \\
& f^6 + 22a^2 b^6 c^4 d^4 e^6 f^2 - 48a^2 b^6 c^5 d^3 e^5 f^3 + 22a^2 b^6 c^6 d^2 e^4 f^4 - 48a^3 \\
& b^5 c^3 d^5 e^6 f^2 + 36a^3 b^5 c^4 d^4 e^5 f^3 + 36a^3 b^5 c^5 d^3 e^4 f^4 - 48a^3 b^5 c^6 d^2 e^3 f^5 + \\
& 22a^4 b^4 c^2 d^6 e^6 f^2 + 36a^4 b^4 c^3 d^5 e^5 f^3 - 90a^4 b^4 c^4 d^4 e^4 f^4 + 36a^4 b^4 c^5 d^3 \\
& e^3 f^5 + 22a^4 b^4 c^6 d^2 e^2 f^6 - 48a^5 b^3 c^2 d^6 e^5 f^3 + 36a^5 b^3 c^3 d^5 e^4 f^4 + 36a^5 \\
& b^3 c^4 d^4 e^3 f^5 - 48a^5 b^3 c^5 d^3 e^2 f^6 + 22a^6 b^2 c^2 d^6 e^4 f^4 - 48a^6 b^2 c^3 d^5 e^3 f^5 + \\
& 22a^6 b^2 c^4 d^4 e^2 f^6)
\end{aligned}$$

$$\begin{aligned}
& b^7c^8e^3f^5 + 4a^3b^5c^8ef^7 + 4a^5b^3d^8e^7f + 4a^7b^1d^8e^5f^3 + 4a^8c^1d^7e^3f^5 + 4a^8c^3d^5ef^7 + 4b^8c^5d^3e^7f + \\
& 4b^8c^7d^1e^5f^3 - 12a^1b^7c^4d^4e^7f - 12a^1b^7c^7d^1e^4f^4 - 12a^4b^4c^1d^7e^7f - 12a^4b^4c^7d^1ef^7 - 12a^7b^1c^1d^7e^4f^4 - 12a^7b^1c^4d^4ef^7 + 8a^1b^7c^5d^3e^6f^2 + 8a^1b^7c^6d^2e^5f^3 + 8a^2b^6c^3d^5e^7f + 8a^2b^6c^7d^1e^3f^5 + 8a^3b^5c^2d^6e^7f + 8a^3b^5c^7d^1e^2f^6 + 8a^5b^3c^1d^7e^6f^2 + 8a^5b^3c^6d^2ef^7 + 8a^6b^2c^1d^7e^5f^3 + 8a^6b^2c^5d^3ef^7 + 8a^7b^1c^2d^6e^3f^5 + 8a^7b^1c^3d^5e^2f^6 + 22a^2b^6c^4d^4e^6f^2 - 48a^2b^6c^5d^3e^5f^3 + 22a^2b^6c^6d^2e^4f^4 - 48a^3b^5c^3d^5e^6f^2 + 36a^3b^5c^4d^4e^5f^3 + 36a^3b^5c^5d^3e^4f^4 - 48a^3b^5c^6d^2e^3f^5 + 22a^4b^4c^2d^6e^6f^2 + 36a^4b^4c^3d^5e^5f^3 - 90a^4b^4c^4d^4e^4f^4 + 36a^4b^4c^5d^3e^3f^5 + 22a^4b^4c^6d^2e^2f^6 - 48a^5b^3c^2d^6e^5f^3 + 36a^5b^3c^3d^5e^4f^4 + 36a^5b^3c^4d^4e^3f^5 - 48a^5b^3c^5d^3e^2f^6 + 22a^6b^2c^2d^6e^4f^4 - 48a^6b^2c^3d^5e^3f^5 + 22a^6b^2c^4d^4e^2f^6) + (3x^4*(8a^6b^1d^7f^7 + 8b^7c^6d^1f^7 + 8b^7d^7e^6f - 7a^2b^5c^4d^3f^7 + 14a^3b^4c^3d^4f^7 - 7a^4b^3c^2d^5f^7 - 7a^2b^5d^7e^4f^3 + 14a^3b^4d^7e^3f^4 - 7a^4b^3d^7e^2f^5 - 7b^7c^2d^5e^4f^3 + 14b^7c^3d^4e^3f^4 - 7b^7c^4d^3e^2f^5 - 14a^1b^6c^5d^2f^7 - 14a^5b^2c^1d^6f^7 - 14a^1b^6d^7e^5f^2 - 14a^5b^2d^7ef^6 - 14b^7c^1d^6e^5f^2 - 14b^7c^5d^2ef^6 + 34a^1b^6c^1d^6e^4f^3 + 34a^1b^6c^4d^3ef^6 + 34a^4b^3c^1d^6ef^6 + 6a^1b^6c^2d^5e^3f^4 + 6a^1b^6c^3d^4e^2f^5 + 6a^2b^5c^1d^6e^3f^4 + 6a^2b^5c^3d^4ef^6 + 6a^3b^4c^1d^6e^2f^5 + 6a^3b^4c^2d^5ef^6 - 78a^2b^5c^2d^5e^2f^5))/(2*(4a^1b^7c^3d^5e^8 - a^4b^4d^8e^8 - a^8c^4d^4f^8 - b^8c^4d^4e^8 - a^8d^8e^4f^4 - b^8c^8e^4f^4 - 6a^2b^6c^2d^6e^8 - 6a^6b^2c^6d^2f^8 - 6a^2b^6c^8e^2f^6 - 6a^6b^2d^8e^6f^2 - 6a^8c^2d^6e^2f^6 - 6b^8c^6d^2e^6f^2 - a^4b^4c^8f^8 + 4a^3b^5c^1d^7e^8 + 4a^5b^3c^7d^1ef^8 + 4a^7b^1c^5d^3f^8 + 4a^1b^7c^8e^3f^5 + 4a^3b^5c^8ef^7 + 4a^5b^3d^8e^7f + 4a^7b^1d^8e^5f^3 + 4a^8c^1d^7e^3f^5 + 4a^8c^3d^5ef^7 + 4b^8c^5d^3e^7f + 4b^8c^7d^1e^5f^3 - 12a^1b^7c^4d^4e^7f - 12a^1b^7c^7d^1e^4f^4 - 12a^4b^4c^1d^7e^7f - 12a^4b^4c^7d^1ef^7 - 12a^7b^1c^1d^7e^4f^4 - 12a^7b^1c^4d^4ef^7 + 8a^1b^7c^5d^3e^6f^2 + 8a^1b^7c^6d^2e^5f^3 + 8a^2b^6c^3d^5e^7f + 8a^2b^6c^7d^1e^3f^5 + 8a^3b^5c^2d^6e^7f + 8a^3b^5c^7d^1e^2f^6 + 8a^5b^3c^1d^7e^6f^2 + 8a^5b^3c^6d^2ef^7 + 8a^6b^2c^1d^7e^5f^3 + 8a^6b^2c^5d^3ef^7 + 8a^7b^1c^2d^6e^3f^5 + 8a^7b^1c^3d^5e^2f^6 + 22a^2b^6c^4d^4e^6f^2 - 48a^2b^6c^5d^3e^5f^3 + 22a^2b^6c^6d^2e^4f^4 - 48a^3b^5c^3d^5e^6f^2 + 36a^3b^5c^4d^4e^5f^3 + 36a^3b^5c^5d^3e^4f^4 - 48a^3b^5c^6d^2e^3f^5 + 22a^4b^4c^2d^6e^6f^2 + 36a^4b^4c^3d^5e^5f^3 - 90a^4b^4c^4d^4e^4f^4 + 36a^4b^4c^5d^3e^3f^5 + 22a^4b^4c^6d^2e^2f^6 - 48a^5b^3c^2d^6e^5f^3 + 36a^5b^3c^3d^5e^4f^4 + 36a^5b^3c^4d^4e^3f^5 - 48a^5b^3c^5d^3e^2f^6 + 22a^6b^2c^2d^6e^4f^4 - 48a^6b^2c^3d^5e^3f^5 + 22a^6b^2c^4d^4e^2f^6 - 48a^6b^2c^5d^3e^2f^6 + 22a^6b^2c^6d^2e^2f^6 - 48a^6b^2c^7d^1e^2f^6 + 22a^6b^2c^8e^2f^6 + 22a^6b^2c^9e^2f^6 - 48a^6b^2c^10e^2f^6 + 22a^6b^2c^11e^2f^6 - 48a^6b^2c^12e^2f^6 + 22a^6b^2c^13e^2f^6 - 48a^6b^2c^14e^2f^6 + 22a^6b^2c^15e^2f^6 - 48a^6b^2c^16e^2f^6 + 22a^6b^2c^17e^2f^6 - 48a^6b^2c^18e^2f^6 + 22a^6b^2c^19e^2f^6 - 48a^6b^2c^20e^2f^6 + 22a^6b^2c^21e^2f^6 - 48a^6b^2c^22e^2f^6 + 22a^6b^2c^23e^2f^6 - 48a^6b^2c^24e^2f^6 + 22a^6b^2c^25e^2f^6 - 48a^6b^2c^26e^2f^6 + 22a^6b^2c^27e^2f^6 - 48a^6b^2c^28e^2f^6 + 22a^6b^2c^29e^2f^6 - 48a^6b^2c^30e^2f^6 + 22a^6b^2c^31e^2f^6 - 48a^6b^2c^32e^2f^6 + 22a^6b^2c^33e^2f^6 - 48a^6b^2c^34e^2f^6 + 22a^6b^2c^35e^2f^6 - 48a^6b^2c^36e^2f^6 + 22a^6b^2c^37e^2f^6 - 48a^6b^2c^38e^2f^6 + 22a^6b^2c^39e^2f^6 - 48a^6b^2c^40e^2f^6 + 22a^6b^2c^41e^2f^6 - 48a^6b^2c^42e^2f^6 + 22a^6b^2c^43e^2f^6 - 48a^6b^2c^44e^2f^6 + 22a^6b^2c^45e^2f^6 - 48a^6b^2c^46e^2f^6 + 22a^6b^2c^47e^2f^6 - 48a^6b^2c^48e^2f^6 + 22a^6b^2c^49e^2f^6 - 48a^6b^2c^50e^2f^6 + 22a^6b^2c^51e^2f^6 - 48a^6b^2c^52e^2f^6 + 22a^6b^2c^53e^2f^6 - 48a^6b^2c^54e^2f^6 + 22a^6b^2c^55e^2f^6 - 48a^6b^2c^56e^2f^6 + 22a^6b^2c^57e^2f^6 - 48a^6b^2c^58e^2f^6 + 22a^6b^2c^59e^2f^6 - 48a^6b^2c^60e^2f^6 + 22a^6b^2c^61e^2f^6 - 48a^6b^2c^62e^2f^6 + 22a^6b^2c^63e^2f^6 - 48a^6b^2c^64e^2f^6 + 22a^6b^2c^65e^2f^6 - 48a^6b^2c^66e^2f^6 + 22a^6b^2c^67e^2f^6 - 48a^6b^2c^68e^2f^6 + 22a^6b^2c^69e^2f^6 - 48a^6b^2c^70e^2f^6 + 22a^6b^2c^71e^2f^6 - 48a^6b^2c^72e^2f^6 + 22a^6b^2c^73e^2f^6 - 48a^6b^2c^74e^2f^6 + 22a^6b^2c^75e^2f^6 - 48a^6b^2c^76e^2f^6 + 22a^6b^2c^77e^2f^6 - 48a^6b^2c^78e^2f^6 + 22a^6b^2c^79e^2f^6 - 48a^6b^2c^80e^2f^6 + 22a^6b^2c^81e^2f^6 - 48a^6b^2c^82e^2f^6 + 22a^6b^2c^83e^2f^6 - 48a^6b^2c^84e^2f^6 + 22a^6b^2c^85e^2f^6 - 48a^6b^2c^86e^2f^6 + 22a^6b^2c^87e^2f^6 - 48a^6b^2c^88e^2f^6 + 22a^6b^2c^89e^2f^6 - 48a^6b^2c^90e^2f^6 + 22a^6b^2c^91e^2f^6 - 48a^6b^2c^92e^2f^6 + 22a^6b^2c^93e^2f^6 - 48a^6b^2c^94e^2f^6 + 22a^6b^2c^95e^2f^6 - 48a^6b^2c^96e^2f^6 + 22a^6b^2c^97e^2f^6 - 48a^6b^2c^98e^2f^6 + 22a^6b^2c^99e^2f^6 - 48a^6b^2c^100e^2f^6)
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^4*d^4*e^2*f^6)) + (x^2*(18*a*b^6*c^7*f^7 + 18*a*b^6*d^7*e^7 + 18*a^7*c*d^6*f^7 + 18*b^7*c*d^6*e^7 + 18*a^7*d^7*e*f^6 + 18*b^7*c^7*e*f^6 - 3*a^3*b^4*c^5*d^2*f^7 + 32*a^4*b^3*c^4*d^3*f^7 - 3*a^5*b^2*c^3*d^4*f^7 - 3*a^3*b^4*d^7*e^5*f^2 + 32*a^4*b^3*d^7*e^4*f^3 - 3*a^5*b^2*d^7*e^3*f^4 - 3*b^7*c^3*d^4*e^5*f^2 + 32*b^7*c^4*d^3*e^4*f^3 - 3*b^7*c^5*d^2*e^3*f^4 - 37*a^2*b^5*c^6*d*f^7 - 37*a^6*b*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^6*f - 37*a^6*b*d^7*e^2*f^5 - 37*b^7*c^2*d^5*e^6*f - 37*b^7*c^6*d*e^2*f^5 + 9*a*b^6*c^2*d^5*e^5*f^2 + a*b^6*c^3*d^4*e^4*f^3 + a*b^6*c^4*d^3*e^3*f^4 + 9*a*b^6*c^5*d^2*e^2*f^5 + 9*a^2*b^5*c*d^6*e^5*f^2 + 9*a^2*b^5*c^5*d^2*e*f^6 + a^3*b^4*c*d^6*e^4*f^3 + a^3*b^4*c^4*d^3*e*f^6 + a^4*b^3*c^3*d^4*e*f^6 + 9*a^5*b^2*c*d^6*e^2*f^5 + 9*a^5*b^2*c^2*d^5*e*f^6 - 34*a*b^6*c*d^6*e^6*f - 34*a*b^6*c^6*d*e*f^6 - 34*a^6*b*c*d^6*e*f^6 + 234*a^2*b^5*c^2*d^5*e^4*f^3 - 208*a^2*b^5*c^3*d^4*e^3*f^4 + 234*a^2*b^5*c^4*d^3*e^2*f^5 - 208*a^3*b^4*c^2*d^5*e^3*f^4 - 208*a^3*b^4*c^3*d^4*e^2*f^5 + 234*a^4*b^3*c^2*d^5*e^2*f^5)) / ((2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x*(2*a^2*b^5*c^7*f^7 + 2*a^2*b^5*d^7*e^7 + 2*a^7*c^2*d^5*f^7 + 2*b^7*c^2*d^5*e^7 + 2*a^7*d^7*e^2*f^5 + 2*b^7*c^7*e^2*f^5 + 4*a^4*b^3*c^5*d^2*f^7 + 4*a^5*b^2*c^4*d^3*f^7 + 4*a^4*b^3*d^7*e^5*f^2 + 4*a^5*b^2*d^7*e^4*f^3 + 4*b^7*c^4*d^3*e^5*f^2 + 4*b^7*c^5*d^2*e^4*f^3 + 14*a*b^6*c*d^6*e^7 + 14*a*b^6*c^7*e*f^6 + 14*a^7*c*d^6*e*f^6 - 6*a^3*b^4*c^6*d*f^7 - 6*a^6*b*c^3*d^4*f^7 - 6*a^3*b^4*d^7*e^6*f - 6*a^6*b*d^7*e^3*f^4 - 6*b^7*c^3*d^4*e^6*f - 6*b^7*c^6*d*e^3*f^4 - 33*a*b^6*c^2*d^5*e^6*f - 33*a*b^6*c^6*d*e^2*f^5 - 33*a^2*b^5*c*d^6*e^6*f - 33*a^2*b^5*c^6*d*e*f^6 - 33*a^6*b*c*d^6*e^2*f^5 - 33*a^6*b*c^2*d^5*e*f^6 + 17*a*b^6*c^3*d^4*e^5*f^2 - 8*a*b^6*c^4*d^3*e^4*f^3 + 17*a*b^6*c^5*d^2*e^3*f^4 + 17*a^3*b^4*c*d^6*e^5*f^2 + 17*a^3*b^4*c^5*d^2*e*f^6 - 8*a^4*b^3*c^4*d^
\end{aligned}$$

$$\begin{aligned}
 &3*ef^6 + 17*a^5*b^2*c*d^6*e^3*f^4 + 17*a^5*b^2*c^3*d^4*ef^6 + 78*a^2*b^5* \\
 &c^2*d^5*e^5*f^2 - 26*a^2*b^5*c^3*d^4*e^4*f^3 - 26*a^2*b^5*c^4*d^3*e^3*f^4 + \\
 &78*a^2*b^5*c^5*d^2*e^2*f^5 - 26*a^3*b^4*c^2*d^5*e^4*f^3 - 26*a^3*b^4*c^4*d \\
 &^3*e^2*f^5 - 26*a^4*b^3*c^2*d^5*e^3*f^4 - 26*a^4*b^3*c^3*d^4*e^2*f^5 + 78*a \\
 &^5*b^2*c^2*d^5*e^2*f^5)/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d \\
 &^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^ \\
 &2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e \\
 &^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + \\
 &4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c \\
 &^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^ \\
 &3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8 \\
 &*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b \\
 &^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b \\
 &*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2* \\
 &b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a \\
 &^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + \\
 &8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 \\
 &+ 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^ \\
 &3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^ \\
 &3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3 \\
 &*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4 \\
 &*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 \\
 &- 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4* \\
 &d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48* \\
 &a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (x^3*(6*a^7*d^7*f^7 \\
 &+ 6*b^7*c^7*f^7 + 6*b^7*d^7*e^7 - 37*a^2*b^5*c^5*d^2*f^7 + 19*a^3*b^4*c^4* \\
 &d^3*f^7 + 19*a^4*b^3*c^3*d^4*f^7 - 37*a^5*b^2*c^2*d^5*f^7 - 37*a^2*b^5*d^7* \\
 &e^5*f^2 + 19*a^3*b^4*d^7*e^4*f^3 + 19*a^4*b^3*d^7*e^3*f^4 - 37*a^5*b^2*d^7* \\
 &e^2*f^5 - 37*b^7*c^2*d^5*e^5*f^2 + 19*b^7*c^3*d^4*e^4*f^3 + 19*b^7*c^4*d^3* \\
 &e^3*f^4 - 37*b^7*c^5*d^2*e^2*f^5 + 3*a*b^6*c^6*d*f^7 + 3*a^6*b*c*d^6*f^7 + \\
 &3*a*b^6*d^7*e^6*f + 3*a^6*b*d^7*e*f^6 + 3*b^7*c*d^6*e^6*f + 3*b^7*c^6*d*e*f \\
 &^6 - 28*a*b^6*c*d^6*e^5*f^2 - 28*a*b^6*c^5*d^2*e*f^6 - 28*a^5*b^2*c*d^6*e*f \\
 &^6 + 86*a*b^6*c^2*d^5*e^4*f^3 - 68*a*b^6*c^3*d^4*e^3*f^4 + 86*a*b^6*c^4*d^3 \\
 &*e^2*f^5 + 86*a^2*b^5*c*d^6*e^4*f^3 + 86*a^2*b^5*c^4*d^3*e*f^6 - 68*a^3*b^4 \\
 &*c*d^6*e^3*f^4 - 68*a^3*b^4*c^3*d^4*e*f^6 + 86*a^4*b^3*c*d^6*e^2*f^5 + 86*a \\
 &^4*b^3*c^2*d^5*e*f^6 - 52*a^2*b^5*c^2*d^5*e^3*f^4 - 52*a^2*b^5*c^3*d^4*e^2* \\
 &f^5 - 52*a^3*b^4*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - \\
 &a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6* \\
 &a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6 \\
 &b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4* \\
 &c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + \\
 &4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b* \\
 &d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7 \\
 &*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 \\
 &- 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4
 \end{aligned}$$

$$\begin{aligned}
& - 12a^7bc^4d^4e^7f^7 + 8a^7b^7c^5d^3e^6f^2 + 8a^7b^7c^6d^2e^5f^3 + 8a^7b^6c^3d^5e^7f + 8a^7b^6c^7d^2e^3f^5 + 8a^7b^5c^2d^6e^7f + 8a^7b^5c^7d^2e^6f^6 + 8a^7b^3c^4d^7e^6f^2 + 8a^7b^3c^6d^2e^7f^7 + 8a^7b^2c^5d^7e^5f^3 + 8a^7b^2c^5d^3e^7f^7 + 8a^7b^2c^6e^3f^5 + 8a^7b^2c^3d^5e^2f^6 + 22a^7b^6c^4d^4e^6f^2 - 48a^7b^6c^5d^3e^5f^3 + 22a^7b^6c^6d^2e^4f^4 - 48a^7b^5c^3d^5e^6f^2 + 36a^7b^5c^4d^4e^5f^3 + 36a^7b^5c^5d^3e^4f^4 - 48a^7b^5c^6d^2e^3f^5 + 22a^7b^4c^2d^6e^6f^2 + 36a^7b^4c^3d^5e^5f^3 - 90a^7b^4c^4d^4e^4f^4 + 36a^7b^4c^5d^3e^3f^5 + 22a^7b^4c^6d^2e^2f^6 - 48a^7b^3c^2d^6e^5f^3 + 36a^7b^3c^3d^5e^4f^4 + 36a^7b^3c^4d^4e^3f^5 - 48a^7b^3c^5d^3e^2f^6 + 22a^7b^2c^2d^6e^4f^4 - 48a^7b^2c^3d^5e^3f^5 + 22a^7b^2c^4d^4e^2f^6) / (x(2abc^2e^2 + 2a^2cde^2 + 2a^2c^2ef) + x^3(2abc^2f^2 + 2abd^2e^2 + 2a^2cdf^2 + 2b^2cde^2 + 2a^2d^2ef + 2b^2c^2ef + 8abcde) + x^2(a^2c^2f^2 + a^2d^2e^2 + b^2c^2e^2 + 4abcde^2 + 4abc^2ef + 4a^2cde^2) + x^5(2abd^2f^2 + 2b^2cdf^2 + 2b^2d^2ef) + x^4(a^2d^2f^2 + b^2c^2f^2 + b^2d^2e^2 + 4abcdf^2 + 4abd^2ef + 4b^2cde^2) + a^2c^2e^2 + b^2d^2f^2x^6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)**3,x)

[Out] Timed out

$$3.21 \quad \int \frac{1}{1+x+x^2+x^3} dx$$

Optimal. Leaf size=25

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^2 + x^3)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x+x^2+x^3} dx &= \int \left(\frac{1}{2(1+x)} + \frac{1-x}{2(1+x^2)} \right) dx \\
&= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1-x}{1+x^2} dx \\
&= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^2 + x^3)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x+x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)^(-1), x]

[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)^(-1), x]

fricas [A] time = 1.12, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1), x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

giac [A] time = 0.31, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln(x+1)}{2} - \frac{\ln(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3+x^2+x+1),x)

[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)

maxima [A] time = 1.45, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

mupad [B] time = 2.20, size = 25, normalized size = 1.00

$$\frac{\ln(x+1)}{2} + \ln(x-i) \left(-\frac{1}{4} - \frac{1}{4}i \right) + \ln(x+1i) \left(-\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + x^2 + x^3 + 1),x)

[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**3+x**2+x+1),x)

[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2

$$3.22 \quad \int \frac{1}{-1+4x-4x^2+16x^3} dx$$

Optimal. Leaf size=31

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1 + 4x - 4x^2 + 16x^3} dx &= \int \left(\frac{4}{5(-1 + 4x)} + \frac{-1 - 4x}{5(1 + 4x^2)} \right) dx \\
&= \frac{1}{5} \log(1 - 4x) + \frac{1}{5} \int \frac{-1 - 4x}{1 + 4x^2} dx \\
&= \frac{1}{5} \log(1 - 4x) - \frac{1}{5} \int \frac{1}{1 + 4x^2} dx - \frac{4}{5} \int \frac{x}{1 + 4x^2} dx \\
&= -\frac{1}{10} \tan^{-1}(2x) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \log(1 + 4x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(1 - 4x) - \frac{1}{10} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-1 + 4x - 4x^2 + 16x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]

fricas [A] time = 1.25, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1), x, algorithm="fricas")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)

giac [A] time = 0.35, size = 26, normalized size = 0.84

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2 + 1) + \frac{1}{5} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$-\frac{\arctan(2x)}{10} + \frac{\ln(4x-1)}{5} - \frac{\ln(4x^2+1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(16*x^3-4*x^2+4*x-1),x)

[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)

maxima [A] time = 1.28, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan(2x) - \frac{1}{10} \log(4x^2+1) + \frac{1}{5} \log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")

[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x - \frac{1}{4}\right)}{5} + \ln\left(x - \frac{1}{2}i\right) \left(-\frac{1}{10} + \frac{1}{20}i\right) + \ln\left(x + \frac{1}{2}i\right) \left(-\frac{1}{10} - \frac{1}{20}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)

[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20)

sympy [A] time = 0.15, size = 24, normalized size = 0.77

$$\frac{\log\left(x - \frac{1}{4}\right)}{5} - \frac{\log\left(x^2 + \frac{1}{4}\right)}{10} - \frac{\operatorname{atan}(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)

[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10

$$3.23 \quad \int \frac{1}{dx^3} dx$$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(d*x^3),x]

[Out] -1/(2*d*x^2)

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{dx^3} dx &= \int \frac{1}{x^3} dx \\ &= -\frac{1}{2dx^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(d*x^3),x]

[Out] -1/2*1/(d*x^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(d*x^3),x]

[Out] IntegrateAlgebraic[1/(d*x^3), x]

fricas [A] time = 0.81, size = 8, normalized size = 0.80

$$-\frac{1}{2 dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x^3,x, algorithm="fricas")

[Out] -1/2/(d*x^2)

giac [A] time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{2 dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/d/x^3,x, algorithm="giac")

[Out] -1/2/(d*x^2)

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{2d x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/d/x^3,x)

[Out] -1/2/d/x^2

maxima [A] time = 0.63, size = 8, normalized size = 0.80

$$-\frac{1}{2 dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/d/x^3,x, algorithm="maxima")`

[Out] `-1/2/(d*x^2)`

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*x^3),x)`

[Out] `-1/(2*d*x^2)`

sympy [A] time = 0.06, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/d/x**3,x)`

[Out] `-1/(2*d*x**2)`

$$3.24 \quad \int \frac{1}{cx^2+dx^3} dx$$

Optimal. Leaf size=28

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 44}

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2 + d*x^3)^(-1),x]

[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2

Rule 44

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & & NeQ[b*c - a*d, 0] & & ILtQ[m, 0] & & IntegerQ[n] & & !(IGtQ[n, 0] & & LtQ[m + n + 2, 0])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] & & IntegerQ[n] & & PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{cx^2 + dx^3} dx &= \int \frac{1}{x^2(c + dx)} dx \\ &= \int \left(\frac{1}{cx^2} - \frac{d}{c^2x} + \frac{d^2}{c^2(c + dx)} \right) dx \\ &= -\frac{1}{cx} - \frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{d \log(x)}{c^2} + \frac{d \log(c + dx)}{c^2} - \frac{1}{cx}$$

Antiderivative was successfully verified.

[In] Integrate[(c*x^2 + d*x^3)^(-1),x]

[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^2 + dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c*x^2 + d*x^3)^(-1),x]

[Out] IntegrateAlgebraic[(c*x^2 + d*x^3)^(-1), x]

fricas [A] time = 0.57, size = 26, normalized size = 0.93

$$\frac{dx \log(dx + c) - dx \log(x) - c}{c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")

[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)

giac [A] time = 0.36, size = 30, normalized size = 1.07

$$\frac{d \log(|dx + c|)}{c^2} - \frac{d \log(|x|)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")

[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{d \ln(x)}{c^2} + \frac{d \ln(dx + c)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*x^3+c*x^2),x)`

[Out] $-1/c/x - d*\ln(x)/c^2 + d*\ln(d*x+c)/c^2$

maxima [A] time = 0.59, size = 28, normalized size = 1.00

$$\frac{d \log(dx + c)}{c^2} - \frac{d \log(x)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")`

[Out] $d*\log(d*x + c)/c^2 - d*\log(x)/c^2 - 1/(c*x)$

mupad [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{2d \operatorname{atanh}\left(\frac{2dx}{c} + 1\right)}{c^2} - \frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2 + d*x^3),x)`

[Out] $(2*d*\operatorname{atanh}((2*d*x)/c + 1))/c^2 - 1/(c*x)$

sympy [A] time = 0.18, size = 19, normalized size = 0.68

$$-\frac{1}{cx} + \frac{d(-\log(x) + \log(\frac{c}{d} + x))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x**3+c*x**2),x)`

[Out] $-1/(c*x) + d*(-\log(x) + \log(c/d + x))/c**2$

$$3.25 \quad \int \frac{1}{bx+dx^3} dx$$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{b} - \frac{\log(b+dx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(b*x + d*x^3)^(-1), x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&

PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{bx + dx^3} dx &= \int \frac{1}{x(b + dx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(b + dx)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2b} - \frac{d \text{Subst} \left(\int \frac{1}{b+dx} dx, x, x^2 \right)}{2b} \\
&= \frac{\log(x)}{b} - \frac{\log(b + dx^2)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b} - \frac{\log(b + dx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + d*x^3)^(-1), x]

[Out] Log[x]/b - Log[b + d*x^2]/(2*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx + dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + d*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(b*x + d*x^3)^(-1), x]

fricas [A] time = 0.87, size = 18, normalized size = 0.82

$$-\frac{\log(dx^2 + b) - 2 \log(x)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")

[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b

giac [A] time = 0.30, size = 24, normalized size = 1.09

$$\frac{\log(x^2)}{2b} - \frac{\log(|dx^2 + b|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")

[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b} - \frac{\ln(dx^2 + b)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+b*x),x)

[Out] ln(x)/b-1/2*ln(d*x^2+b)/b

maxima [A] time = 0.79, size = 20, normalized size = 0.91

$$-\frac{\log(dx^2 + b)}{2b} + \frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")

[Out] -1/2*log(d*x^2 + b)/b + log(x)/b

mupad [B] time = 2.13, size = 18, normalized size = 0.82

$$-\frac{\ln(dx^2 + b) - 2 \ln(x)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x + d*x^3),x)

[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{b}{d} + x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x**3+b*x),x)

[Out] log(x)/b - log(b/d + x**2)/(2*b)

$$3.26 \quad \int \frac{1}{bx+cx^2+dx^3} dx$$

Optimal. Leaf size=62

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log(b+cx+dx^2)}{2b} + \frac{\log(x)}{b}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1594, 705, 29, 634, 618, 206, 628}

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log(b+cx+dx^2)}{2b} + \frac{\log(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2 + d*x^3)^(-1), x]

[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b - Log[b + c*x + d*x^2]/(2*b)

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{bx + cx^2 + dx^3} dx &= \int \frac{1}{x(b + cx + dx^2)} dx \\
 &= \frac{\int \frac{1}{x} dx}{b} + \frac{\int \frac{-c-dx}{b+cx+dx^2} dx}{b} \\
 &= \frac{\log(x)}{b} - \frac{\int \frac{c+2dx}{b+cx+dx^2} dx}{2b} - \frac{c \int \frac{1}{b+cx+dx^2} dx}{2b} \\
 &= \frac{\log(x)}{b} - \frac{\log(b + cx + dx^2)}{2b} + \frac{c \operatorname{Subst}\left(\int \frac{1}{c^2-4bd-x^2} dx, x, c + 2dx\right)}{b} \\
 &= \frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} + \frac{\log(x)}{b} - \frac{\log(b + cx + dx^2)}{2b}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.98

$$\frac{\frac{2c \tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}} + \log(b + x(c + dx)) - 2 \log(x)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1), x]

[Out] $-1/2 * ((2 * c * \text{ArcTan}[(c + 2 * d * x) / \text{Sqrt}[-c^2 + 4 * b * d]]) / \text{Sqrt}[-c^2 + 4 * b * d] - 2 * \text{Log}[x] + \text{Log}[b + x * (c + d * x)]) / b$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx + cx^2 + dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)^(-1), x]

fricas [A] time = 1.13, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{c^2 - 4bd} c \log\left(\frac{2d^2x^2 + 2cdx + c^2 - 2bd + \sqrt{c^2 - 4bd}(2dx + c)}{dx^2 + cx + b}\right) - (c^2 - 4bd) \log(dx^2 + cx + b) + 2(c^2 - 4bd) \log(x)}{2(bc^2 - 4b^2d)}, \frac{2\sqrt{-c^2 + 4bd} c \arctan\left(\frac{-\sqrt{-c^2 + 4bd}(2dx + c)}{c^2 - 4bd}\right) - (c^2 - 4bd) \log(dx^2 + cx + b) + 2(c^2 - 4bd) \log(x)}{2(bc^2 - 4b^2d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2+b*x), x, algorithm="fricas")

[Out] $[1/2 * (\text{sqrt}(c^2 - 4 * b * d) * c * \log((2 * d^2 * x^2 + 2 * c * d * x + c^2 - 2 * b * d + \text{sqrt}(c^2 - 4 * b * d)) * (2 * d * x + c)) / (d * x^2 + c * x + b)) - (c^2 - 4 * b * d) * \log(d * x^2 + c * x + b) + 2 * (c^2 - 4 * b * d) * \log(x)) / (b * c^2 - 4 * b^2 * d), 1/2 * (2 * \text{sqrt}(-c^2 + 4 * b * d) * c * \arctan(-\text{sqrt}(-c^2 + 4 * b * d) * (2 * d * x + c)) / (c^2 - 4 * b * d)) - (c^2 - 4 * b * d) * \log(d * x^2 + c * x + b) + 2 * (c^2 - 4 * b * d) * \log(x)) / (b * c^2 - 4 * b^2 * d)]$

giac [A] time = 0.29, size = 62, normalized size = 1.00

$$-\frac{c \arctan\left(\frac{2dx+c}{\sqrt{-c^2+4bd}}\right)}{\sqrt{-c^2+4bd} b} - \frac{\log(dx^2 + cx + b)}{2b} + \frac{\log(|x|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+c*x^2+b*x), x, algorithm="giac")

[Out] $-c * \arctan((2 * d * x + c) / \text{sqrt}(-c^2 + 4 * b * d)) / (\text{sqrt}(-c^2 + 4 * b * d) * b) - 1/2 * \log(d * x^2 + c * x + b) / b + \log(\text{abs}(x)) / b$

maple [A] time = 0.01, size = 62, normalized size = 1.00

$$-\frac{c \arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2} b} + \frac{\ln(x)}{b} - \frac{\ln(dx^2 + cx + b)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*x^3+c*x^2+b*x),x)`

[Out] $-1/2*\ln(d*x^2+c*x+b)/b-1/b*c/(4*b*d-c^2)^{(1/2)}*\arctan((2*d*x+c)/(4*b*d-c^2)^{(1/2)})+1/b*\ln(x)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for more details)Is 4*b*d-c^2 positive or negative?

mupad [B] time = 0.47, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{b} - \ln\left(x(6bd^2 - 2c^2d) - bcd\right) \left(\frac{1}{2b} - \frac{c\sqrt{c^2 - 4bd}}{2(bc^2 - 4b^2d)}\right) - cd - 3d^2x \left(\frac{1}{2b} - \frac{c\sqrt{c^2 - 4bd}}{2(bc^2 - 4b^2d)}\right) - \ln\left(x(6bd^2 - 2c^2d) - bcd\right) \left(\frac{1}{2b} + \frac{c\sqrt{c^2 - 4bd}}{2(bc^2 - 4b^2d)}\right) - cd - 3d^2x \left(\frac{1}{2b} + \frac{c\sqrt{c^2 - 4bd}}{2(bc^2 - 4b^2d)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x + c*x^2 + d*x^3),x)`

[Out] $\log(x)/b - \log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)^{(1/2)})/(2*(b*c^2 - 4*b^2*d)))) - c*d - 3*d^2*x*(1/(2*b) - (c*(c^2 - 4*b*d)^{(1/2)})/(2*(b*c^2 - 4*b^2*d)))) - \log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) + (c*(c^2 - 4*b*d)^{(1/2)})/(2*(b*c^2 - 4*b^2*d)))) - c*d - 3*d^2*x*(1/(2*b) + (c*(c^2 - 4*b*d)^{(1/2)})/(2*(b*c^2 - 4*b^2*d))))$

sympy [B] time = 4.19, size = 564, normalized size = 9.10

$$\left(\frac{c\sqrt{4bd-c^2}}{2(4bd-c^2)} - \frac{1}{2b}\right) \log\left(x + \frac{24bd^2\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 14b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 12b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 2b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 3b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 12b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 11b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 2b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}}}{9bd^2-2b^2d}\right) + \left(\frac{c\sqrt{4bd-c^2}}{2(4bd-c^2)} + \frac{1}{2b}\right) \log\left(x + \frac{24bd^2\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 14b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 12b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 2b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 3b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 12b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} + 11b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}} - 2b^2d\sqrt{\frac{c^2-4bd}{4bd-c^2}}}{9bd^2-2b^2d}\right) + \frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x**3+c*x**2+b*x),x)`

[Out] $(-c*\sqrt{-4*b*d + c**2})/(2*b*(4*b*d - c**2)) - 1/(2*b))*\log(x + (24*b**4*d**2*(-c*\sqrt{-4*b*d + c**2})/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(-c*\sqrt{-4*b*d + c**2})/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(-c*\sqrt{-4*b*d + c**2})/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-c*\sqrt{-4*b*d + c**2})/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-$

$$\begin{aligned}
& c\sqrt{-4bd + c^2}/(2b(4bd - c^2)) - 1/(2b) - 12b^2d^2 + 11b \\
& *c^2d - 2c^4/(9b^2cd^2 - 2c^3d) + (c\sqrt{-4bd + c^2}/(2b(4 \\
& *bd - c^2)) - 1/(2b))\log(x + (24b^4d^2(c\sqrt{-4bd + c^2})/(2b \\
& (4bd - c^2)) - 1/(2b))^2 - 14b^3c^2d(c\sqrt{-4bd + c^2})/(2b \\
& (4bd - c^2)) - 1/(2b))^2 - 12b^3d^2(c\sqrt{-4bd + c^2})/(2b(4 \\
& *bd - c^2)) - 1/(2b) + 2b^2c^4(c\sqrt{-4bd + c^2})/(2b(4bd - \\
& c^2)) - 1/(2b))^2 + 3b^2c^2d(c\sqrt{-4bd + c^2})/(2b(4bd - \\
& c^2)) - 1/(2b) - 12b^2d^2 + 11b^2c^2d - 2c^4/(9b^2cd^2 - 2c \\
& *3d) + \log(x)/b
\end{aligned}$$

$$3.27 \quad \int \frac{1}{a+dx^3} dx$$

Optimal. Leaf size=115

$$-\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{d} x + d^{2/3} x^2)}{6a^{2/3} \sqrt[3]{d}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{d} x)}{3a^{2/3} \sqrt[3]{d}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} \sqrt[3]{d}}$$

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{d} x + d^{2/3} x^2)}{6a^{2/3} \sqrt[3]{d}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{d} x)}{3a^{2/3} \sqrt[3]{d}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} \sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + d*x^3)^(-1), x]

[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{a + dx^3} dx &= \frac{\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{d}x} dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a} - \sqrt[3]{d}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3a^{2/3}} \\
 &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} + \frac{\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\sqrt[3]{d} + 2d^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2} dx}{6a^{2/3}\sqrt[3]{d}} \\
 &= \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2)}{6a^{2/3}\sqrt[3]{d}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}} \\
 &= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{d}} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{d}x)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2)}{6a^{2/3}\sqrt[3]{d}}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.77

$$\frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{d}x + d^{2/3}x^2) - 2\log(\sqrt[3]{a} + \sqrt[3]{d}x) + 2\sqrt{3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + d*x^3)^(-1), x]

[Out] $-\frac{1}{6} \cdot (2 \sqrt[3]{3} \operatorname{ArcTan}[(1 - (2d^{1/3}x)/a^{1/3})/\sqrt[3]{3}] - 2 \operatorname{Log}[a^{1/3} + d^{1/3}x] + \operatorname{Log}[a^{2/3} - a^{1/3}d^{1/3}x + d^{2/3}x^2]) / (a^{2/3}d^{1/3})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + d*x^3)^(-1), x]

[Out] IntegrateAlgebraic[(a + d*x^3)^(-1), x]

fricas [A] time = 0.54, size = 299, normalized size = 2.60

$$\frac{3 \sqrt[3]{\frac{a}{d}} \sqrt{\frac{(a^2d)^2}{d}} \operatorname{Log} \left(\frac{2 a^{2/3} d^{1/3} a x^2 + 3 \sqrt[3]{\frac{a^2 d^2}{d}} \sqrt{\frac{2 a^{2/3} d^{1/3} x - (a^2 d)^{1/3}}{d}} \sqrt{\frac{(a^2 d)^2}{d}}}{6 a^2 d} \right) - (a^2 d)^{2/3} \operatorname{Log} \left(a d x^2 - (a^2 d)^{1/3} x + (a^2 d)^{1/3} a \right) + 2 (a^2 d)^{2/3} \operatorname{Log} \left(a d x + (a^2 d)^{1/3} \right)}{6 a^2 d} + \frac{6 \sqrt[3]{\frac{a}{d}} \sqrt{\frac{(a^2d)^2}{d}} \operatorname{arctan} \left(\frac{\sqrt[3]{\frac{a^2 d^2}{d}} \left(2 (a^2 d)^{1/3} x - (a^2 d)^{1/3} \right) \sqrt{\frac{(a^2 d)^2}{d}}}{2} \right) - (a^2 d)^{2/3} \operatorname{Log} \left(a d x^2 - (a^2 d)^{1/3} x + (a^2 d)^{1/3} a \right) + 2 (a^2 d)^{2/3} \operatorname{Log} \left(a d x + (a^2 d)^{1/3} \right)}{6 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+a), x, algorithm="fricas")

[Out] $\left[\frac{1}{6} \cdot (3 \sqrt[3]{1/3} \cdot a \cdot d \cdot \sqrt[3]{-(a^2 \cdot d)^{1/3}/d}) \cdot \operatorname{Log} \left(\frac{2 \cdot a \cdot d \cdot x^3 - 3 \cdot (a^2 \cdot d)^{1/3} \cdot a \cdot x - a^2 + 3 \cdot \sqrt[3]{1/3} \cdot (2 \cdot a \cdot d \cdot x^2 + (a^2 \cdot d)^{2/3} \cdot x - (a^2 \cdot d)^{1/3} \cdot a) \cdot \sqrt[3]{-(a^2 \cdot d)^{1/3}/d}}{d} \right) / (d \cdot x^3 + a) - (a^2 \cdot d)^{2/3} \cdot \operatorname{Log} \left(\frac{a \cdot d \cdot x^2 - (a^2 \cdot d)^{2/3} \cdot x + (a^2 \cdot d)^{1/3} \cdot a}{d} \right) + 2 \cdot (a^2 \cdot d)^{2/3} \cdot \operatorname{Log} \left(\frac{a \cdot d \cdot x + (a^2 \cdot d)^{2/3}}{d} \right) \right] / (a^2 \cdot d) + \frac{1}{6} \cdot (6 \cdot \sqrt[3]{1/3} \cdot a \cdot d \cdot \sqrt[3]{(a^2 \cdot d)^{1/3}/d}) \cdot \operatorname{arctan} \left(\frac{\sqrt[3]{1/3} \cdot (2 \cdot (a^2 \cdot d)^{2/3} \cdot x - (a^2 \cdot d)^{1/3} \cdot a) \cdot \sqrt[3]{(a^2 \cdot d)^{1/3}/d}}{a^2} \right) - (a^2 \cdot d)^{2/3} \cdot \operatorname{Log} \left(\frac{a \cdot d \cdot x^2 - (a^2 \cdot d)^{2/3} \cdot x + (a^2 \cdot d)^{1/3} \cdot a}{d} \right) + 2 \cdot (a^2 \cdot d)^{2/3} \cdot \operatorname{Log} \left(\frac{a \cdot d \cdot x + (a^2 \cdot d)^{2/3}}{d} \right) \right] / (a^2 \cdot d)$

giac [A] time = 0.26, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}} \operatorname{Log} \left(\left| x - \left(-\frac{a}{d}\right)^{\frac{1}{3}} \right| \right)}{3a} + \frac{\sqrt{3} \left(-ad^2\right)^{\frac{1}{3}} \operatorname{arctan} \left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{d}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{d}\right)^{\frac{1}{3}}} \right)}{3ad} + \frac{\left(-ad^2\right)^{\frac{1}{3}} \operatorname{Log} \left(x^2 + x \left(-\frac{a}{d}\right)^{\frac{1}{3}} + \left(-\frac{a}{d}\right)^{\frac{2}{3}} \right)}{6ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+a),x, algorithm="giac")

[Out] $-1/3*(-a/d)^{(1/3)}*\log(\text{abs}(x - (-a/d)^{(1/3)}))/a + 1/3*\sqrt{3}*(-a*d^2)^{(1/3)}*$
 $\arctan(1/3*\sqrt{3}*(2*x + (-a/d)^{(1/3)})/(-a/d)^{(1/3)})/(a*d) + 1/6*(-a*d^2)^{(1/3)}*$
 $\log(x^2 + x*(-a/d)^{(1/3)} + (-a/d)^{(2/3)})/(a*d)$

maple [A] time = 0.01, size = 91, normalized size = 0.79

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(\frac{2x - \frac{1}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3}\right)}{3 \left(\frac{a}{d}\right)^{\frac{2}{3}} d}\right)}{3 \left(\frac{a}{d}\right)^{\frac{2}{3}} d} + \frac{\ln\left(x + \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{d}\right)^{\frac{2}{3}} d} - \frac{\ln\left(x^2 - \left(\frac{a}{d}\right)^{\frac{1}{3}} x + \left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{d}\right)^{\frac{2}{3}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x^3+a),x)

[Out] $1/3*d/(a/d)^{(2/3)}*\ln(x+(a/d)^{(1/3)})-1/6*d/(a/d)^{(2/3)}*\ln(x^2-(a/d)^{(1/3)}*x+$
 $(a/d)^{(2/3}))+1/3*d/(a/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/d)^{(1/3)}*x-$
 $1))$

maxima [A] time = 1.28, size = 98, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3} \left(2x - \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3 d \left(\frac{a}{d}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x \left(\frac{a}{d}\right)^{\frac{1}{3}} + \left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6 d \left(\frac{a}{d}\right)^{\frac{2}{3}}} + \frac{\log\left(x + \left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3 d \left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x^3+a),x, algorithm="maxima")

[Out] $1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - (a/d)^{(1/3)})/(a/d)^{(1/3)})/(d*(a/d)^{(2/3)}) -$
 $1/6*\log(x^2 - x*(a/d)^{(1/3)} + (a/d)^{(2/3)})/(d*(a/d)^{(2/3)}) + 1/3*\log$
 $(x + (a/d)^{(1/3)})/(d*(a/d)^{(2/3)})$

mupad [B] time = 0.23, size = 99, normalized size = 0.86

$$\frac{\ln\left(d^{1/3} x + a^{1/3}\right)}{3 a^{2/3} d^{1/3}} + \frac{\ln\left(3 d^2 x + \frac{3 a^{1/3} d^{5/3} (-1 + \sqrt{3} i i)}{2}\right) (-1 + \sqrt{3} i i)}{6 a^{2/3} d^{1/3}} - \frac{\ln\left(3 d^2 x - \frac{3 a^{1/3} d^{5/3} (1 + \sqrt{3} i i)}{2}\right) (1 + \sqrt{3} i i)}{6 a^{2/3} d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + d*x^3),x)`

[Out] $\log(d^{1/3}x + a^{1/3})/(3a^{2/3}d^{1/3}) + (\log(3d^2x + (3a^{1/3}d^{5/3})(3^{1/2}i - 1))/2)(3^{1/2}i - 1)/(6a^{2/3}d^{1/3}) - (\log(3d^2x - (3a^{1/3}d^{5/3})(3^{1/2}i + 1))/2)(3^{1/2}i + 1)/(6a^{2/3}d^{1/3})$

sympy [A] time = 0.16, size = 20, normalized size = 0.17

$$\text{RootSum}\left(27t^3a^2d - 1, (t \mapsto t \log(3ta + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x**3+a),x)`

[Out] `RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x))`

$$3.28 \quad \int (dx^3)^n dx$$

Optimal. Leaf size=16

$$\frac{x(dx^3)^n}{3n+1}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\frac{x(dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Int[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (dx^3)^n dx &= \left(x^{-3n} (dx^3)^n \right) \int x^{3n} dx \\ &= \frac{x(dx^3)^n}{1+3n} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(dx^3)^n}{3n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x^3)^n,x]

[Out] (x*(d*x^3)^n)/(1 + 3*n)

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(d*x^3)^n, x]

fricas [A] time = 1.17, size = 16, normalized size = 1.00

$$\frac{(dx^3)^n x}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3)^n,x, algorithm="fricas")

[Out] (d*x^3)^n*x/(3*n + 1)

giac [A] time = 0.31, size = 16, normalized size = 1.00

$$\frac{(dx^3)^n x}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3)^n,x, algorithm="giac")

[Out] (d*x^3)^n*x/(3*n + 1)

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x(dx^3)^n}{3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3)^n,x)

[Out] $x*(d*x^3)^n/(1+3*n)$

maxima [A] time = 0.57, size = 17, normalized size = 1.06

$$\frac{d^n x x^{3n}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3)^n,x, algorithm="maxima")`

[Out] $d^n*x*x^{(3*n)/(3*n+1)}$

mupad [B] time = 2.50, size = 16, normalized size = 1.00

$$\frac{x(d x^3)^n}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3)^n,x)`

[Out] $(x*(d*x^3)^n)/(3*n+1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{d^n x (x^3)^n}{3n+1} & \text{for } n \neq -\frac{1}{3} \\ \int \frac{1}{\sqrt[3]{dx^3}} dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3)**n,x)`

[Out] `Piecewise((d**n*x*(x**3)**n/(3*n+1), Ne(n, -1/3)), (Integral((d*x**3)**(-1/3), x), True))`

$$3.29 \quad \int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^4 dx$$

Optimal. Leaf size=270

$$\frac{2}{9}c^2(48a^2d^4 + 120ac^3d^2 + 35c^6)\left(\frac{c}{d} + x\right)^9 - \frac{8}{11}c^3d^2(12ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{11} + \frac{4}{13}cd^4(4ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{13} + \frac{4c^3}{15}\left(\frac{c}{d} + x\right)^{15} + \frac{1}{17}d^6\left(\frac{c}{d} + x\right)^{17}$$

Rubi [A] time = 0.54, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1106, 1090}

$$\frac{2}{9}c^2(48a^2d^4 + 120ac^3d^2 + 35c^6)\left(\frac{c}{d} + x\right)^9 + \frac{4}{13}cd^4(4ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{13} - \frac{8}{11}c^3d^2(12ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^{11} - \frac{8c^4(4ad^2 + c^3)(12ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^7}{7d^2} + \frac{4c^3(4ad^2 + c^3)^2(4ad^2 + 7c^3)\left(\frac{c}{d} + x\right)^5}{5d^4} - \frac{8c^5(4ad^2 + c^3)^3\left(\frac{c}{d} + x\right)^3}{3d^6} + \frac{c^4x(4ad^2 + c^3)^4}{d^8} - \frac{8}{15}c^2d^6\left(\frac{c}{d} + x\right)^{15} + \frac{1}{17}d^6\left(\frac{c}{d} + x\right)^{17}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]

[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned}
\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^4 dx &= \text{Subst} \left(\int \left(c \left(4a + \frac{c^3}{d^2} \right) - 2c^2x^2 + d^2x^4 \right)^4 dx, x, \frac{c}{d} + x \right) \\
&= \text{Subst} \left(\int \left(\frac{(c^4 + 4acd^2)^4}{d^8} - \frac{8c^5(c^3 + 4ad^2)^3 x^2}{d^6} + \frac{24c^6(c^3 + 4ad^2)^2 \left(\frac{7}{6}\right)}{d^4} \right. \right. \\
&= \frac{c^4(c^3 + 4ad^2)^4 x}{d^8} - \frac{8c^5(c^3 + 4ad^2)^3 \left(\frac{c}{d} + x\right)^3}{3d^6} + \frac{4c^3(c^3 + 4ad^2)^2 (7c^3 + 5d^4)}{5d^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 285, normalized size = 1.06

$$\frac{256a^4c^4x + 1024a^3c^3x^3 + 256a^3c^3dx^4 + 512a^2c^2d^2x^5 + \frac{256}{5}a^2c^2d^2(a^2 + 6c^2) + \frac{32}{5}c^2d^2(3a^2d^4 + 120ac^3d^2 + 8c^6) + \frac{64}{11}c^3d^2d^2(15ad^2 + 28c^2) + 96ac^3d^2(a^2 + 4c^2) + \frac{16}{13}cd^4d^2(16a^2 + 70c^2) + \frac{256}{5}c^4d^2(5ad^2 + 2c^2) + \frac{256}{7}a^4d^2(9ad^2 + 4c^2) + \frac{16}{3}c^2d^2d^2(3ad^2 + 28c^2) + 32c^2d^2d^2 + \frac{112}{15}c^2d^2d^2 + cd^3d^6 + \frac{d^8x^{17}}{17}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]

[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3 + 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*x^16 + (d^8*x^17)/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4, x]

fricas [A] time = 0.76, size = 277, normalized size = 1.03

$$\frac{1}{17}a^{17}d^8 + a^{16}d^7c + \frac{112}{15}a^{15}d^6c^2 + 32a^{14}d^5c^3 + \frac{1120}{15}a^{13}d^4c^4 + \frac{16}{15}a^{13}d^3c^5 + \frac{448}{3}a^{12}d^2c^6 + 16a^{12}d^2c^6 + \frac{1792}{11}a^{11}d^2c^7 + \frac{960}{11}a^{11}d^2c^7 + \frac{512}{5}a^{10}d^2c^8 + 256a^{10}d^2c^8 + \frac{256}{9}a^9d^2c^9 + \frac{1280}{3}a^9d^2c^9 + \frac{32}{3}a^8d^2c^{10} + 384a^8d^2c^{10} + 96a^8d^2c^{10} + \frac{1024}{7}a^7d^2c^{11} + \frac{2804}{7}a^7d^2c^{11} + 512a^7d^2c^{11} + \frac{1536}{5}a^6d^2c^{12} + \frac{256}{5}a^6d^2c^{12} + 256a^6d^2c^{12} + \frac{1024}{3}a^5d^2c^{13} + 256a^5d^2c^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")

[Out] $1/17*x^{17}*d^8 + x^{16}*d^7*c + 112/15*x^{15}*d^6*c^2 + 32*x^{14}*d^5*c^3 + 1120/13*x^{13}*d^4*c^4 + 16/13*x^{13}*d^6*c*a + 448/3*x^{12}*d^3*c^5 + 16*x^{12}*d^5*c^2*a + 1792/11*x^{11}*d^2*c^6 + 960/11*x^{11}*d^4*c^3*a + 512/5*x^{10}*d*c^7 + 256*x^{10}*d^3*c^4*a + 256/9*x^9*c^8 + 1280/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2 + 384*x^8*d*c^6*a + 96*x^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2*c^4*a^2 + 512*x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 + 256*x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4$

giac [A] time = 0.24, size = 277, normalized size = 1.03

$\frac{1}{17}d^{17}x^{17} + cd^{16}x^{16} + \frac{112}{15}d^2x^{15} + 32d^5x^{14} + \frac{1120}{13}d^4x^{13} + \frac{16}{13}ad^6x^{13} + \frac{448}{3}d^3x^{12} + 16ad^5x^{12} + \frac{1792}{11}d^2x^{11} + \frac{960}{11}ad^4x^{11} + \frac{512}{5}d^7x^{10} + 256ad^3x^{10} + \frac{256}{9}d^8x^9 + \frac{1280}{3}ad^2x^9 + \frac{32}{3}d^4x^9 + 384ad^6x^8 + 96d^3x^8 + \frac{1024}{7}d^7x^7 + \frac{2304}{7}d^2x^7 + 512d^5x^6 + \frac{1536}{5}d^2x^6 + \frac{256}{5}d^3x^5 + 256ad^4x^4 + \frac{1024}{3}d^5x^3 + 256d^4x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")

[Out] $1/17*d^8*x^{17} + c*d^7*x^{16} + 112/15*c^2*d^6*x^{15} + 32*c^3*d^5*x^{14} + 1120/13*c^4*d^4*x^{13} + 16/13*a*c*d^6*x^{13} + 448/3*c^5*d^3*x^{12} + 16*a*c^2*d^5*x^{12} + 1792/11*c^6*d^2*x^{11} + 960/11*a*c^3*d^4*x^{11} + 512/5*c^7*d*x^{10} + 256*a*c^4*d^3*x^{10} + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 + 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x$

maple [A] time = 0.00, size = 392, normalized size = 1.45

$\frac{d^{17}x^{17}}{17} + cd^{16}x^{16} + \frac{112d^2x^{15}}{15} + 32d^5x^{14} + \frac{1120d^4x^{13}}{13} + \frac{16ad^6x^{13}}{13} + \frac{448d^3x^{12}}{3} + 16ad^5x^{12} + \frac{1792d^2x^{11}}{11} + \frac{960ad^4x^{11}}{11} + \frac{512d^7x^{10}}{5} + \frac{256ad^3x^{10}}{9} + \frac{1280d^2x^9}{3} + \frac{32d^4x^9}{3} + 384ad^6x^8 + 96d^3x^8 + \frac{1024d^7x^7}{7} + \frac{2304d^2x^7}{7} + 512d^5x^6 + \frac{1536d^2x^6}{5} + \frac{256d^3x^5}{5} + 256ad^4x^4 + \frac{1024d^5x^3}{3} + 256d^4x$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)

[Out] $1/17*d^8*x^{17}+c*d^7*x^{16}+112/15*c^2*d^6*x^{15}+32*c^3*d^5*x^{14}+1/13*(2*(8*a*c*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^{13}+1/12*(64*a*c^2*d^5+16*(8*a*c*d^2+16*c^4)*c*d^3+1536*c^5*d^3)*x^{12}+1/11*(576*a*c^3*d^4+48*(8*a*c*d^2+16*c^4)*c^2*d^2+1024*c^6*d^2)*x^{11}+1/10*(2048*a*c^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^{10}+1/9*(32*a^2*c^2*d^4+3584*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*c^3*d^3+2048*a*c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d^2+64*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*(8*a*c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3*c^5*x^3+256*a^4*c^4*x$

maxima [A] time = 0.63, size = 372, normalized size = 1.38

$\frac{1}{17}d^{17}x^{17} + cd^{16}x^{16} + \frac{112}{15}d^2x^{15} + 32d^5x^{14} + \frac{1120}{13}d^4x^{13} + \frac{16}{13}ad^6x^{13} + \frac{448}{3}d^3x^{12} + 16ad^5x^{12} + \frac{1792}{11}d^2x^{11} + \frac{960}{11}ad^4x^{11} + \frac{512}{5}d^7x^{10} + \frac{256}{9}ad^3x^{10} + \frac{1280}{3}d^2x^9 + \frac{32}{3}d^4x^9 + 384ad^6x^8 + 96d^3x^8 + \frac{1024}{7}d^7x^7 + \frac{2304}{7}d^2x^7 + 512d^5x^6 + \frac{1536}{5}d^2x^6 + \frac{256}{5}d^3x^5 + 256ad^4x^4 + \frac{1024}{3}d^5x^3 + 256d^4x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")

[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/13*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c + 16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x^12)*c^2

mupad [B] time = 2.30, size = 261, normalized size = 0.97

$$x^{10} \left(\frac{512d^8}{5} + 256acd^4 \right) + x^{13} \left(\frac{1120c^4d^4}{13} + \frac{16acd^6}{13} \right) + x^9 \left(\frac{32a^2d^2}{3} + \frac{1280a^2d^2}{3} - \frac{256c^4}{9} \right) + x^{11} \left(\frac{448c^5d^3}{3} + \frac{16a^2c^2d^5}{3} \right) + x^{12} \left(\frac{448c^5d^3}{3} + \frac{16a^2c^2d^5}{3} \right) + x^{11} \left(\frac{1792c^6d^2}{11} + \frac{960a^3c^3d^4}{11} \right) + \frac{d^8x^{17}}{17} + 256a^4c^4x + cd^7x^{16} + \frac{1024a^3c^5x^3}{3} + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + 256a^3c^4d^4x^4 + 512a^2c^5d^4x^6 + \frac{256a^3c^4d^4}{7} + \frac{256a^2c^5d^4}{5} + \frac{256a^3c^4d^4}{5} + 96acd^4(4c^2 + ad^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)

[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d^4*x^4 + 512*a^2*c^5*d^4*x^6 + (256*a*c^4*d^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)

sympy [A] time = 0.13, size = 299, normalized size = 1.11

$$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4d^4x^4 + 512a^2c^5d^4x^6 + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + cd^7x^{16} + \frac{d^8x^{17}}{17} + x^{13} \left(\frac{16acd^6}{13} + \frac{1120c^4d^4}{13} \right) + x^{12} \left(\frac{448c^5d^3}{3} + \frac{16a^2c^2d^5}{3} \right) + x^{11} \left(\frac{1792c^6d^2}{11} + \frac{960a^3c^3d^4}{11} \right) + x^{10} \left(\frac{256c^8}{9} + \frac{1280a^2c^5d^2}{3} + \frac{32a^2c^2d^4}{3} \right) + x^9 \left(\frac{32a^2d^2}{3} + \frac{1280a^2d^2}{3} - \frac{256c^4}{9} \right) + x^8 \left(\frac{96a^2c^3d}{7} + \frac{384acd}{7} \right) + x^7 \left(\frac{2304a^2c^4d^2}{7} + \frac{1024a^2c^4d^2}{7} \right) + x^5 \left(\frac{256a^3c^3d^2}{5} + \frac{1536a^2c^4d^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)

[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11) + x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x**7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1536*a**2*c**6/5)

$$3.30 \quad \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^3 dx$$

Optimal. Leaf size=171

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{48}{5}a$$

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2061}

$$48a^2c^3dx^4 + 64a^2c^4x^3 + 64a^3c^3x + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 4c^3) + 64ac^4dx^6 + \frac{60}{11}c^2d^4x^{11} + 16c^3d^3x^{10} + cd^5x^{12} + \frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3, x]

[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^3 dx &= \int (64a^3c^3 + 192a^2c^4x^2 + 192a^2c^3dx^3 + 48ac^2(4c^3 + ad^2)x^4 + 384ac^4dx^5 \\ &\quad + 64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + \frac{48}{5}ac^2(4c^3 + ad^2)x^5 + 64ac^4dx^6 + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) \\ &\quad + 12c^2dx^8(ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 4c^3) + 64ac^4dx^6 + \frac{60}{11}c^2d^4x^{11} + 16c^3d^3x^{10} + cd^5x^{12} + \frac{d^6x^{13}}{13}) dx \end{aligned}$$

Mathematica [A] time = 0.03, size = 171, normalized size = 1.00

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + \frac{4}{3}cd^2x^9(ad^2 + 20c^3) + \frac{32}{7}c^3x^7(9ad^2 + 2c^3) + 12c^2dx^8(ad^2 + 2c^3) + \frac{48}{5}ac^2x^5(ad^2 + 4c^3) + 16c^3d^3x^{10} + \frac{60}{11}c^2d^4x^{11} + cd^5x^{12} + \frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3, x]

[Out] $64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3, x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3, x]

fricas [A] time = 0.74, size = 166, normalized size = 0.97

$$\frac{1}{13}x^{13}d^6 + x^{12}d^5c + \frac{60}{11}x^{11}d^4c^2 + 16x^{10}d^3c^3 + \frac{80}{3}x^9d^2c^4 + \frac{4}{3}x^9d^4ca + 24x^8dc^5 + 12x^8d^3c^2a + \frac{64}{7}x^7c^6 + \frac{288}{7}x^7d^2c^3a + 64x^6dc^4a + \frac{192}{5}x^5c^5a + \frac{48}{5}x^5d^2c^2a^2 + 48x^4dc^3a^2 + 64x^3c^4a^2 + 64xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3, x, algorithm="fricas")

[Out] $1/13*x^{13}*d^6 + x^{12}*d^5*c + 60/11*x^{11}*d^4*c^2 + 16*x^{10}*d^3*c^3 + 80/3*x^9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3*c^2*a + 64/7*x^7*c^6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 192/5*x^5*c^5*a + 48/5*x^5*d^2*c^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*c^4*a^2 + 64*x*c^3*a^3$

giac [A] time = 0.24, size = 166, normalized size = 0.97

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{60}{11}c^2d^4x^{11} + 16c^3d^3x^{10} + \frac{80}{3}c^4d^2x^9 + \frac{4}{3}acd^4x^9 + 24c^5dx^8 + 12ac^2d^3x^8 + \frac{64}{7}c^6x^7 + \frac{288}{7}ac^3d^2x^7 + 64ac^4dx^6 + \frac{192}{5}ac^5x^5 + \frac{48}{5}a^2c^2d^2x^5 + 48a^2c^3dx^4 + 64a^2c^4x^3 + 64a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3, x, algorithm="giac")

[Out] $1/13*d^6*x^{13} + c*d^5*x^{12} + 60/11*c^2*d^4*x^{11} + 16*c^3*d^3*x^{10} + 80/3*c^4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x$

maple [A] time = 0.00, size = 231, normalized size = 1.35

$$\frac{d^6x^{13}}{13} + cd^5x^{12} + \frac{60c^2d^4x^{11}}{11} + 16c^3d^3x^{10} + 64acd^4x^9 + 48a^2c^3dx^8 + 64a^2c^4x^3 + \frac{(4acd^4 + 224c^4d^2 + (8ac^2d^2 + 16c^4)d^2)x^9}{9} + \frac{(64ac^2d^3 + 128c^5d + 4(8ac^2d^2 + 16c^4)cd)x^8}{8} + 64a^2c^3x + \frac{(256a^2c^2d^2 + 4(8ac^2d^2 + 16c^4)c^2)x^7}{7} + \frac{(16a^2c^2d^2 + 128ac^5 + 4(8ac^2d^2 + 16c^4)ac)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)

[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*c^5*a+16*d^2*a^2*c^2)*x^5+48*a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x

maxima [A] time = 0.80, size = 205, normalized size = 1.20

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{48}{11}c^2d^4x^{11} + \frac{32}{5}c^3d^3x^{10} + \frac{64}{7}c^4d^2x^9 + 64a^3c^3x^8 + \frac{16}{5}(3d^2x^5 + 15cdx^4 + 20c^2x^3)d^2c^2 + \frac{8}{3}(2d^2x^9 + 9cdx^8)c^4 + \frac{4}{105}(35d^4x^9 + 315cd^3x^8 + 720c^2d^2x^7 + 1008c^4x^5 + 120(3d^2x^7 + 14cdx^6)c^2)ac + \frac{4}{165}(45d^4x^{11} + 396cd^3x^{10} + 880c^2d^2x^9)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")

[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*c^4*d^2*x^9 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/165*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2

mupad [B] time = 2.16, size = 160, normalized size = 0.94

$$x^8(24c^5d + 12ac^2d^3) + x^9\left(\frac{80c^4d^2}{3} + \frac{4ac^4d^4}{3}\right) + \frac{d^6x^{13}}{13} + x^7\left(\frac{64c^6}{7} + \frac{288ac^3d^2}{7}\right) + 64a^3c^3x + cd^5x^{12} + 64a^2c^4x^3 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + 48a^2c^3dx^4 + \frac{48a^2c^2x^5(4c^3 + ad^2)}{5} + 64ac^4dx^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)

[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^12 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3*d*x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6

sympy [A] time = 0.10, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9\left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8(12ac^2d^3 + 24c^5d) + x^7\left(\frac{288ac^3d^2}{7} + \frac{64c^6}{7}\right) + x^5\left(\frac{48a^2c^2d^2}{5} + \frac{192ac^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)

[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/13 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*a*c**5/5)

$$3.31 \quad \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2 dx$$

Optimal. Leaf size=92

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2061}

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + 8ac^2dx^4 + \frac{32}{3}ac^3x^3 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3dx^6 + cd^3x^8 + \frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2, x]

[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2 dx &= \int (16a^2c^2 + 32ac^3x^2 + 32ac^2dx^3 + 8c(2c^3 + ad^2)x^4 + 32c^3dx^5 + 24c^2d^2x^6 + 16cd^3x^7 + d^4x^8) dx \\ &= 16a^2c^2x + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{8}{5}c(2c^3 + ad^2)x^5 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.02, size = 92, normalized size = 1.00

$$16a^2c^2x + \frac{8}{5}cx^5(ad^2 + 2c^3) + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2, x]

[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2, x]

fricas [A] time = 0.54, size = 83, normalized size = 0.90

$$\frac{1}{9}x^9d^4 + x^8d^3c + \frac{24}{7}x^7d^2c^2 + \frac{16}{3}x^6dc^3 + \frac{16}{5}x^5c^4 + \frac{8}{5}x^5d^2ca + 8x^4dc^2a + \frac{32}{3}x^3c^3a + 16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")

[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/5*x^5*c^4 + 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16*x*c^2*a^2

giac [A] time = 0.25, size = 83, normalized size = 0.90

$$\frac{1}{9}d^4x^9 + cd^3x^8 + \frac{24}{7}c^2d^2x^7 + \frac{16}{3}c^3dx^6 + \frac{16}{5}c^4x^5 + \frac{8}{5}acd^2x^5 + 8ac^2dx^4 + \frac{32}{3}ac^3x^3 + 16a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5 + 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x

maple [A] time = 0.00, size = 84, normalized size = 0.91

$$\frac{d^4x^9}{9} + cd^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3dx^6}{3} + 8ac^2dx^4 + \frac{32ac^3x^3}{3} + 16a^2c^2x + \frac{(8acd^2 + 16c^4)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)

[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x

maxima [A] time = 0.57, size = 94, normalized size = 1.02

$$\frac{1}{9}d^4x^9 + cd^3x^8 + \frac{16}{7}c^2d^2x^7 + \frac{16}{5}c^4x^5 + 16a^2c^2x + \frac{8}{15}(3d^2x^5 + 15cdx^4 + 20c^2x^3)ac + \frac{8}{21}(3d^2x^7 + 14cdx^6)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")

[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x + 8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x^6)*c^2

mupad [B] time = 0.04, size = 82, normalized size = 0.89

$$x^5 \left(\frac{16c^4}{5} + \frac{8acd^2}{5} \right) + \frac{d^4x^9}{9} + 16a^2c^2x + \frac{32ac^3x^3}{3} + \frac{16c^3dx^6}{3} + cd^3x^8 + \frac{24c^2d^2x^7}{7} + 8a^2cdx^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)

[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4

sympy [A] time = 0.08, size = 95, normalized size = 1.03

$$16a^2c^2x + \frac{32ac^3x^3}{3} + 8ac^2dx^4 + \frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7} + cd^3x^8 + \frac{d^4x^9}{9} + x^5 \left(\frac{8acd^2}{5} + \frac{16c^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)

[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**4/5)

$$3.32 \quad \int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) dx$$

Optimal. Leaf size=32

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5

Rubi steps

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) dx = 4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]

[Out] IntegrateAlgebraic[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4, x]

fricas [A] time = 0.46, size = 28, normalized size = 0.88

$$\frac{1}{5}x^5d^2 + x^4dc + \frac{4}{3}x^3c^2 + 4xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")

[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")

[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)

[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5

maxima [A] time = 0.65, size = 28, normalized size = 0.88

$$\frac{1}{5}d^2x^5 + cdx^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")

[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{4c^2x^3}{3} + cdx^4 + 4acx + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)`

[Out] `(4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4`

sympy [A] time = 0.07, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)`

[Out] `4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5`

$$3.33 \quad \int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4} dx$$

Optimal. Leaf size=529

$$\frac{d \log \left(\sqrt{c} \sqrt{4ad^2 + c^3} - \sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}} + \frac{d \log \left(\sqrt{c} \sqrt{4ad^2 + c^3} + \sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}}$$

Rubi [A] time = 0.90, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, number of rules / integrand size = 0.207, Rules used = {1106, 1094, 634, 618, 206, 628}

$$\frac{d \log \left(-\sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + \sqrt{c} \sqrt{4ad^2 + c^3} + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}} + \frac{d \log \left(\sqrt{2} \sqrt[4]{c} d \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} \left(\frac{c}{d} + x \right) + \sqrt{c} \sqrt{4ad^2 + c^3} + d^2 \left(\frac{c}{d} + x \right)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}}} - \frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} + \sqrt{2} c + \sqrt{2} dx}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{4ad^2 + c^3}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{c^{3/2} - \sqrt{4ad^2 + c^3}}} + \frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \sqrt{\sqrt{4ad^2 + c^3} + c^{3/2}} - \sqrt{2} (c + dx)}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{4ad^2 + c^3}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{4ad^2 + c^3} \sqrt{c^{3/2} - \sqrt{4ad^2 + c^3}}}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1), x]

[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2]/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2]/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
+ x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} dx &= \text{Subst} \left(\int \frac{1}{c \left(4a + \frac{c^3}{d^2} \right) - 2c^2x^2 + d^2x^4} dx, x, \frac{c}{d} + x \right) \\
&= \frac{d \text{Subst} \left(\int \frac{\frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d}^{-x}}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}} + \frac{d \text{Subst} \left(\int \frac{1}{\sqrt{c} \sqrt{c^3 + 4ad^2}} dx, x, \frac{c}{d} + x \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2}} \\
&= \frac{\text{Subst} \left(\int \frac{1}{\frac{\sqrt{c} \sqrt{c^3 + 4ad^2}}{d^2} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}}{d} x + x^2} dx, x, \frac{c}{d} + x \right)}{4\sqrt{c} \sqrt{c^3 + 4ad^2}} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{c} \sqrt{c^3 + 4ad^2}} dx, x, \frac{c}{d} + x \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2}} \\
&= -\frac{d \log \left(\sqrt{c} \sqrt{c^3 + 4ad^2} - \sqrt{2} \sqrt[4]{c} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} (c + dx) + (c + dx)^2 \right)}{4\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}}} \\
&= -\frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \left(\sqrt{2} c^{3/4} - \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} \right) + \sqrt{2} dx}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} - \frac{d \tanh^{-1} \left(\frac{\sqrt[4]{c} \left(\sqrt{2} c^{3/4} + \sqrt{c^{3/2} + \sqrt{c^3 + 4ad^2}} \right)}{\sqrt[4]{c} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}} \right)}{2\sqrt{2} c^{3/4} \sqrt{c^3 + 4ad^2} \sqrt{c^{3/2} - \sqrt{c^3 + 4ad^2}}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.13

$$\frac{1}{4} \text{RootSum} \left[\#1^4 d^2 + 4\#1^3 cd + 4\#1^2 c^2 + 4ac \&, \frac{\log(x - \#1)}{\#1^3 d^2 + 3\#1^2 cd + 2\#1 c^2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1), x]

[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1), x]

fricas [B] time = 0.74, size = 905, normalized size = 1.71

$$\frac{1}{8} \sqrt{-\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd + (2a^3c^3d^2 + (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + 1}\right) - \frac{1}{8} \sqrt{-\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd - (2a^3c^3d^2 + (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + 1}\right) + \frac{1}{8} \sqrt{\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd + (2a^3c^3d^2 - (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - 1}\right) - \frac{1}{8} \sqrt{\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd - (2a^3c^3d^2 - (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")

[Out]
$$\frac{1}{8} \sqrt{-\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd + (2a^3c^3d^2 + (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + 1}\right) - \frac{1}{8} \sqrt{-\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd - (2a^3c^3d^2 + (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} + 1}\right) + \frac{1}{8} \sqrt{\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd + (2a^3c^3d^2 - (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - 1}\right) - \frac{1}{8} \sqrt{\frac{2(a^3c^3 + 4a^2d^2)}{(a^3c^3 + 4a^2d^2)}} \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - \frac{1}{(a^3c^3 + 4a^2d^2)} \log\left(\frac{d^2x + cd - (2a^3c^3d^2 - (a^7c^7 + 4a^2c^4d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}}}{(2(a^3c^3 + 4a^2d^2)) \sqrt{-\frac{d^2}{a^9c^9 + 8a^2c^6d^2 + 16a^3c^3d^4}} - 1}\right)$$

giac [A] time = 0.34, size = 603, normalized size = 1.14

$$\frac{\log\left(x + \sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}\right)}{4\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} + 3\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} + 2\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}}} + \frac{\log\left(x - \sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}\right)}{4\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} - 3\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}} + 2\sqrt{\frac{2d^2 + 2\sqrt{-ac}d^3}{d^4}}} + \frac{\log\left(x + \sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}\right)}{4\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} + 3\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} + 2\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}}} + \frac{\log\left(x - \sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}\right)}{4\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} - 3\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}} + 2\sqrt{\frac{2d^2 - 2\sqrt{-ac}d^3}{d^4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")

[Out]
$$-\frac{1}{4} \log\left(\frac{x + \sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}{\sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}\right) - \frac{3cd}{\sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}} + \frac{1}{4} \log\left(\frac{x - \sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}{\sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} - \frac{c}{d}}\right) - \frac{3cd}{\sqrt{\frac{c^2d^2 + 2\sqrt{-ac}d^3}{d^4}} - \frac{c}{d}} + \frac{1}{4} \log\left(\frac{x + \sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}{\sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}\right) - \frac{3cd}{\sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}} + \frac{1}{4} \log\left(\frac{x - \sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} + \frac{c}{d}}{\sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} - \frac{c}{d}}\right) - \frac{3cd}{\sqrt{\frac{c^2d^2 - 2\sqrt{-ac}d^3}{d^4}} - \frac{c}{d}}$$

)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d))

maple [C] time = 0.06, size = 64, normalized size = 0.12

$$\frac{\ln(-\text{RootOf}(d^2_Z^4 + 4dc_Z^3 + 4c^2_Z^2 + 4ac) + x)}{4\text{RootOf}(d^2_Z^4 + 4dc_Z^3 + 4c^2_Z^2 + 4ac)^3 d^2 + 12\text{RootOf}(d^2_Z^4 + 4dc_Z^3 + 4c^2_Z^2 + 4ac)^2 cd + 8\text{RootOf}(d^2_Z^4 + 4dc_Z^3 + 4c^2_Z^2 + 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c), x)

[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x), _R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^2+4*a*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d^2 x^4 + 4 c d x^3 + 4 c^2 x^2 + 4 a c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c), x, algorithm="maxima")

[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)

mupad [B] time = 4.54, size = 1551, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3), x)

[Out] atan((((-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) * (((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + ((-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) * (((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) * (((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) * (((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)

$$\begin{aligned} &^2))^{1/2} - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^{1/2} + a*c^3)/(64*(a^2*c^6 + 4 \\ &*a^3*c^3*d^2)))^{1/2} + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^{1/2} + a*c^3 \\ &)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2}*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x \\ &)*(-(2*d*(-a^3*c^3)^{1/2} + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} + \\ &64*a*c*d^6)*(-(2*d*(-a^3*c^3)^{1/2} + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))) \\ &)^{1/2} + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^{1/2} + a*c^3)/(64*(a^2*c^ \\ &6 + 4*a^3*c^3*d^2)))^{1/2}*2i + \operatorname{atan}(\frac{(2*d*(-a^3*c^3)^{1/2} - a*c^3)}{64*(\\ &a^2*c^6 + 4*a^3*c^3*d^2)})^{1/2}*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d* \\ &(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} - 64*a*c*d^ \\ &6)*((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} + \\ &4*c*d^5 + 4*d^6*x)*1i + ((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^ \\ &3*c^3*d^2)))^{1/2}*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^{1/ \\ &2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} + 64*a*c*d^6)*((2*d*(-a^3 \\ &*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} + 4*c*d^5 + 4*d^ \\ &6*x)*1i)/(((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{ \\ &1/2}*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(6 \\ &4*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} - 64*a*c*d^6)*((2*d*(-a^3*c^3)^{1/2} - \\ &a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2} + 4*c*d^5 + 4*d^6*x) - ((2*d*(\\ &-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2}*(((256*a*c^4 \\ &*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c^6 + 4*a^ \\ &3*c^3*d^2)))^{1/2} + 64*a*c*d^6)*((2*d*(-a^3*c^3)^{1/2} - a*c^3)/(64*(a^2*c \\ &^6 + 4*a^3*c^3*d^2)))^{1/2} + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^{1/2} - \\ &a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^{1/2}*2i \end{aligned}$$

sympy [A] time = 1.15, size = 88, normalized size = 0.17

$$\operatorname{RootSum}\left(t^4(16384a^3c^3d^2 + 4096a^2c^6) + 128t^2ac^3 + 1, \left(t \mapsto t \log\left(x + \frac{-1024t^3a^2c^4d^2 - 256t^3ac^7 + 16tacd^2 - 4tc^4 + cd}{d^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c), x)

[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 + 1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 16*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))

$$3.34 \quad \int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2} dx$$

Optimal. Leaf size=746

$$\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

Rubi [A] time = 1.33, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, number of rules / integrand size = 0.241, Rules used = {1106, 1092, 1169, 634, 618, 206, 628}

$$\frac{\left(\frac{c}{d}+x\right)\left(4ad^2+c^3\right)\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}} - \frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4*a*d^2)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])]/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2]/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2]/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} dx &= \text{Subst} \left(\int \frac{1}{\left(c \left(4a + \frac{c^3}{d^2}\right) - 2c^2x^2 + d^2x^4\right)^2} dx, x, \frac{c}{d} + x \right) \\
&= \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} + \text{Subst} \left(\int \frac{4c^4 - 2c \left(4a + \frac{c^3}{d^2}\right)}{\dots} \right) \\
&= \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} + d \text{Subst} \left(\int \frac{\sqrt{2} \sqrt[4]{c} \sqrt{c^3}}{\dots} \right) \\
&= \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 - c^{3/2}\right)}{\dots} \\
&= \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 - c^{3/2}\right)}{\dots} \\
&= \frac{\left(\frac{c}{d} + x\right) \left(c^3 - 4ad^2 - cd^2 \left(\frac{c}{d} + x\right)^2\right)}{16ac \left(c^3 + 4ad^2\right) \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)} - \frac{d \left(c^3 + 12ad^2 + c^{3/2}\right)}{32\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 182, normalized size = 0.24

$$\frac{\text{RootSum} \left[\#1^4 d^2 + 4 \#1^3 cd + 4 \#1^2 c^2 + 4ac \&, \frac{\#1^2 cd^2 \log(x-\#1) + 12ad^2 \log(x-\#1) + 2c^3 \log(x-\#1) + 2\#1 c^2 d \log(x-\#1)}{\#1^3 d^2 + 3\#1^2 cd + 2\#1 c^2} \& \right] + \frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2}}{64ac(4ad^2+c^3)}$$

Antiderivative was successfully verified.

[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

[Out] $((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + \text{RootSum}[4*a*c + 4*c^2*\#1^2 + 4*c*d*\#1^3 + d^2*\#1^4 \& , (2*c^3*\text{Log}[x - \#1] + 12*a*d^2*\text{Log}[x - \#1] + 2*c^2*d*\text{Log}[x - \#1]*\#1 + c*d^2*\text{Log}[x - \#1]*\#1^2)/(2*c^2*\#1 + 3*c*d*\#1^2 + d^2*\#1^3) \&])/(64*a*c*(c^3 + 4*a*d^2))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]

fricas [B] time = 1.09, size = 3222, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")

[Out] $\frac{1}{64} \cdot (4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) * \text{sqrt}(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6) * \text{sqrt}(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \text{log}(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8) * \text{sqrt}(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) * \text{sqrt}(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6) * \text{sqrt}(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) * \text{sqrt}(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6) * \text{sqrt}(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))$

$$\begin{aligned}
&^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*\sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) * \sqrt{-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) * \sqrt{-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*\sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) * \sqrt{-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) * \sqrt{-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*\sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) * \sqrt{-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) * \sqrt{-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)}) / (a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12))) / (a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)) + 8*(c^3 + 2*a*d^2)*x) / (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)
\end{aligned}$$

5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)

giac [A] time = 0.37, size = 1057, normalized size = 1.42

$$\frac{\left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \sqrt{c^2d^2+2c^2d+4ac} \sqrt{\frac{2d^2+4ac}{d^2+4ac}}}{d\sqrt{\frac{2d^2+4ac}{d^2+4ac}}} - \frac{\left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \sqrt{c^2d^2+2c^2d+4ac} \sqrt{\frac{2d^2+4ac}{d^2+4ac}}}{d\sqrt{\frac{2d^2+4ac}{d^2+4ac}}} + \frac{\left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \sqrt{c^2d^2+2c^2d+4ac} \sqrt{\frac{2d^2+4ac}{d^2+4ac}}}{d\sqrt{\frac{2d^2+4ac}{d^2+4ac}}} - \frac{\left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \left(\sqrt{\frac{2d^2+4ac}{d^2+4ac}}\right)^{\frac{1}{2}} \sqrt{c^2d^2+2c^2d+4ac} \sqrt{\frac{2d^2+4ac}{d^2+4ac}}}{d\sqrt{\frac{2d^2+4ac}{d^2+4ac}}} + \frac{c^2d^2+2c^2d+4ac}{16(d^2+4ac)^2} \sqrt{\frac{2d^2+4ac}{d^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")

[Out] -1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 2*c^3*x + 4*a*d^2*x + 4*a*c*d)/((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)*(a*c^4 + 4*a^2*c*d^2))

maple [C] time = 0.02, size = 232, normalized size = 0.31

$$\frac{\left(\text{RootOf}(d^2 Z^4 + 4dc Z^3 + 4c^2 Z^2 + 4ac)\right)^2 c d^2 + 2 \text{RootOf}(d^2 Z^4 + 4dc Z^3 + 4c^2 Z^2 + 4ac) c^2 d + 12 a d^2 + 2c^3}{64(4a d^2 + c^3)ac} \ln\left(-\text{RootOf}(d^2 Z^4 + 4dc Z^3 + 4c^2 Z^2 + 4ac) + x\right) + \frac{d^2 x^3}{16(4a d^2 + c^3)d} + \frac{3cd^2}{16(4a d^2 + c^3)d} + \frac{d}{16a d^2 + 4c^3} + \frac{(2a d^2 + c^3)x}{8(4a d^2 + c^3)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)

[Out] (1/16*d^2/a/(4*a*d^2+c^3)*x^3+3/16/a*c*d/(4*a*d^2+c^3)*x^2+1/8/c*(2*a*d^2+c^3)/(4*a*d^2+c^3)/a*x+1/4*d/(4*a*d^2+c^3))/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)+1/64/(4*a*d^2+c^3)/a/c*sum((R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x), _R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^2+4*a*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")

[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c^4 + 4*a^2*c*d^2)

mupad [B] time = 4.30, size = 5844, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)

[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))*i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2)))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4))

$$\begin{aligned}
& \frac{1}{(1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4))} * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} + (64a^6c^7d^5 + 2304a^3c^4d^9 + 704a^2c^4d^7) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(36a^2d^{10} + c^6d^6 + 11a^3c^3d^8)) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4)) * 1i) / ((9a^6d^8 + c^3d^6) / (512(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) - (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} * (((262144a^4c^{12}d^5 + 2097152a^5c^9d^7 + 4194304a^6c^6d^9) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(4096a^3c^{11}d^6 + 32768a^4c^8d^8 + 65536a^5c^5d^{10})) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4))) * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} - (4096a^3c^8d^6 + 65536a^4c^5d^8 + 196608a^5c^2d^{10}) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4))) * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} + (64a^6c^7d^5 + 2304a^3c^4d^9 + 704a^2c^4d^7) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(36a^2d^{10} + c^6d^6 + 11a^3c^3d^8)) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4)) + (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} * (((262144a^4c^{12}d^5 + 2097152a^5c^9d^7 + 4194304a^6c^6d^9) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(4096a^3c^{11}d^6 + 32768a^4c^8d^8 + 65536a^5c^5d^{10})) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4))) * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} + (4096a^3c^8d^6 + 65536a^4c^5d^8 + 196608a^5c^2d^{10}) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4))) * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} + (64a^6c^7d^5 + 2304a^3c^4d^9 + 704a^2c^4d^7) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(36a^2d^{10} + c^6d^6 + 11a^3c^3d^8)) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4)) * (- (a^3c^{11} + 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 + 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} * 2i - \operatorname{atan}(((- (a^3c^{11} - 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2} * (((262144a^4c^{12}d^5 + 2097152a^5c^9d^7 + 4194304a^6c^6d^9) / (1024(a^3c^8 + 8a^4c^5d^2 + 16a^5c^2d^4)) + (x(4096a^3c^{11}d^6 + 32768a^4c^8d^8 + 65536a^5c^5d^{10})) / (16(a^2c^8 + 8a^3c^5d^2 + 16a^4c^2d^4))) * (- (a^3c^{11} - 10c^3d^3(-a^9c^7)^{1/2} + 15a^4c^8d^2 + 60a^5c^5d^4 - 72a^6d^5(-a^9c^7)^{1/2})) / (4096(a^6c^{16} + 12a^7c^{13}d^2 + 48a^8c^{10}d^4 + 64a^9c^7d^6))^{1/2}
\end{aligned}$$

$$5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)}/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6))^{(1/2)} + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) * (- (a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6))^{(1/2)} + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)) * (- (a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^{(1/2)} + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^{(1/2)})/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6))^{(1/2)} * 2i$$

sympy [A] time = 109.97, size = 427, normalized size = 0.57

$\frac{4ad^4 - 72a^2d^5 + 144d^6}{256c^8d^4 - 144c^7d^5 + 196608c^2d^10} + \frac{1}{1024} \operatorname{RootSum}\left(\frac{1}{t^3} (1073741824a^9c^7d^6 + 805306368a^8c^10d^4 + 201326592a^7c^13d^2 + 16777216a^6c^16) + t^2 (491520a^5c^5d^4 + 122880a^4c^8d^2 + 8192a^3c^11) + 81a^2d^4 + 18a^3d^2 + c^6, \operatorname{Lambda}(t, t \log(x + (-67108864t^3a^7c^7d^8 - 58720256t^3a^6c^10d^6 - 18874368t^3a^5c^13d^4 - 2621440t^3a^4c^16d^2 - 131072t^3a^3c^19 + 27648t^2a^4c^2d^8 - 9216t^2a^3c^5d^6 - 5440t^2a^2c^8d^4 - 736t^2a^2c^11d^2 - 32t^2c^14 + 324a^2c^7d^5 + 81a^3c^4d^5 + 5c^7d^3)) / (324a^2d^8 + 81a^3c^3d^6 + 5c^6d^4))\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)

[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*c**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(256*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6) + RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 + 201326592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720256*_t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t**3*a**4*c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t**2*a**4*c**2*d**8 - 9216*_t**2*a**3*c**5*d**6 - 5440*_t**2*a**2*c**8*d**4 - 736*_t**2*a*c**11*d**2 - 32*_t**2*c**14 + 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3))/(324*a**2*d**8 + 81*a*c**3*d**6 + 5*c**6*d**4))

$$3.35 \quad \int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4 \right)^4 dx$$

Optimal. Leaf size=295

$$\frac{1}{24}e^4 \left(65536a^2e^6 + 20992ad^4e^3 + 601d^8 \right) \left(\frac{d}{4e} + x \right)^9 + \frac{(256ae^3 + 5d^4)^2 (256ae^3 + 59d^4) \left(\frac{d}{4e} + x \right)^5}{5120} + \frac{64}{13}e^8 (256ae^3$$

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1106, 1090}

$$\frac{1}{24}e^4 \left(65536a^2e^6 + 20992ad^4e^3 + 601d^8 \right) \left(\frac{d}{4e} + x \right)^9 + \frac{64}{13}e^8 (256ae^3 + 59d^4) \left(\frac{d}{4e} + x \right)^{13} - \frac{72}{11}d^2e^6 (256ae^3 + 17d^4) \left(\frac{d}{4e} + x \right)^{11} - \frac{9}{224}d^2e^2 (256ae^3 + 5d^4) (256ae^3 + 17d^4) \left(\frac{d}{4e} + x \right)^7 + \frac{(256ae^3 + 5d^4)^2 (256ae^3 + 59d^4) \left(\frac{d}{4e} + x \right)^5}{5120} - \frac{d^2 (256ae^3 + 5d^4) \left(\frac{d}{4e} + x \right)^3}{8192e^2} + \frac{x (256ae^3 + 5d^4)^4}{1048576e^4} - \frac{2048}{5}d^2e^{10} \left(\frac{d}{4e} + x \right)^{15} + \frac{4096}{17}e^{12} \left(\frac{d}{4e} + x \right)^{17}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]

[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e) + x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned}
\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 dx &= \text{Subst} \left(\int \left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2 \right) - 3d^2ex^2 + 8e^3x^4 \right)^4 dx, x, \frac{d}{4e} + x \right) \\
&= \text{Subst} \left(\int \left(\frac{(5d^4 + 256ae^3)^4}{1048576e^4} - \frac{3d^2(5d^4 + 256ae^3)^3 x^2}{8192e^2} + \frac{27}{512}d^4(5d^4 + 256ae^3)^2 x^4 \right. \right. \\
&= \frac{(5d^4 + 256ae^3)^4 x}{1048576e^4} - \frac{d^2(5d^4 + 256ae^3)^3 \left(\frac{d}{4e} + x \right)^3}{8192e^2} + \frac{(5d^4 + 256ae^3)^2 (5d^4 + 256ae^3)^2 x^5}{1048576e^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 345, normalized size = 1.17

4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4, x]

[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4, x]

[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4, x]

fricas [A] time = 0.66, size = 353, normalized size = 1.20

4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")

[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 1024*x^14*e^9*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 512*x^12*e^7*d^5 + 4096*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 49152/11*x^11*e^9*d^2*a + 384/5*x^10*e^5*d^7 + 1024*x^10*e^8*d^3*a + 128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a + 8192/3*x^9*e^10*a^2 - 4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d*a^2 - 32/7*x^7*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 + 128*x^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e^6*d^4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8*d*a^3 + 128*x^3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*a^4

giac [A] time = 0.31, size = 323, normalized size = 1.09

$\frac{4096}{17}e^{12}x^{17} + 1024e^{11}dx^{16} + \frac{8192}{5}e^{10}d^2x^{15} + 1024e^9d^3x^{14} - \frac{2048}{13}e^8d^4x^{13} + \frac{16384}{13}e^{11}ax^{13} - 512e^7d^5x^{12} + 4096e^{10}dax^{12} - \frac{1664}{11}e^6d^6x^{11} + \frac{49152}{11}e^9d^2ax^{11} + \frac{384}{5}e^5d^7x^{10} + 1024e^8d^3ax^{10} + \frac{128}{3}e^4d^8x^9 - \frac{4096}{3}e^7d^4ax^9 + \frac{8192}{3}e^{10}a^2x^9 - 4e^3d^9x^8 - 768e^6d^5ax^8 + 6144e^9da^2x^8 - \frac{32}{7}e^2d^{10}x^7 + \frac{768}{7}e^5d^6ax^7 + \frac{24576}{7}e^8d^2a^2x^7 + 128e^4d^7ax^6 - 1024e^7d^3a^2x^6 + \frac{1}{5}d^{12}x^5 - \frac{6144}{5}e^6d^4a^2x^5 + \frac{16384}{5}e^9a^3x^5 - 8e^2d^9ax^4 + 4096e^8da^3x^4 + 128e^4d^6a^2x^3 - 1024e^6d^3a^3x^2 + 4096e^8a^4x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")

[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1/5*d^12*x^5 + 16384/13*a*x^13*e^11 + 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^11*e^9 + 1024*a*d^3*x^10*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 768/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e^10 + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 - 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8

maple [A] time = 0.00, size = 500, normalized size = 1.69

$\frac{4096}{17}e^{12}x^{17} + 1024de^{11}x^{16} + \frac{8192}{5}d^2e^{10}x^{15} + 1024d^3e^9x^{14} - \frac{2048}{13}d^4e^8x^{13} + \frac{16384}{13}ae^{11}x^{13} - 512d^5e^7x^{12} + 4096dae^{10}x^{12} - \frac{1664}{11}d^6e^6x^{11} + \frac{49152}{11}a^2d^2e^9x^{11} + \frac{384}{5}d^7e^5x^{10} + 1024ad^3e^8x^{10} + \frac{128}{3}d^8e^4x^9 - \frac{4096}{3}d^4ae^7x^9 + \frac{8192}{3}a^2d^2e^{10}x^9 - 4d^9e^3x^8 - 768ad^5e^6x^8 + 6144a^2dx^8e^9 + \frac{24576}{7}a^2d^2e^8x^7 - 1024a^2d^3e^7x^6 - \frac{6144}{5}a^2d^4e^6x^5 + 128a^2d^6e^4x^3 + \frac{16384}{5}a^3e^9x^5 + 4096a^3dx^4e^8 - 1024a^3d^3e^6x^2 + 4096a^4xe^8$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)

[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+128/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^10*d+256*(128*a*e^5-16*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*a*e^9*d^2+128*(128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*e^8*d^3+256*d^7*e^5-32*(128*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*e^7*d^4+128*d^8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*e^6*d^5-32*d^9*e^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2+512*a*e^5*d^6+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*e^2*d^3*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-

$$16*d^4*e^2)-4096*a^2*e^6*d^4+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x$$

maxima [A] time = 0.75, size = 383, normalized size = 1.30

$$\frac{4096}{17}d^{12}e^2 + 1024a^2d^6e^4 + \frac{8192}{5}d^2e^{10} + \frac{8192}{7}d^3e^9x + 4096a^4e^8x - \frac{4}{7}(7e^3x^8 + 8d^2e^2x^7)d^9 + 1024/5(16e^3x^5 + 20d^2e^2x^4 - 5d^3x^2)a^3e^6 + 128/165(45e^6x^{11} + 99d^5e^5x^{10} + 55d^2e^4x^9)d^6 + 128/105(2240e^6x^9 + 5040d^5e^5x^8 + 2880d^2e^4x^7 + 105d^6x^3 - 168(5e^3x^6 + 6d^2e^2x^5)d^3)a^2e^4 - 512/1001(286e^9x^{14} + 924d^8e^8x^{13} + 1001d^2e^7x^{12} + 364d^3e^6x^{11})d^3 + 8/15015(2365440e^9x^{13} + 7687680d^8e^8x^{12} + 8386560d^2e^7x^{11} + 3075072d^3e^6x^{10} - 15015d^9x^4 + 34320(6e^3x^7 + 7d^2e^2x^6)d^6 - 32032(36e^6x^{10} + 80d^5e^5x^9 + 45d^2e^4x^8)d^3)a^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")

[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^13 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 + 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3)*a^2e^2

mupad [B] time = 0.27, size = 331, normalized size = 1.12

$$x^5 \left(\frac{16384a^3e^9}{5} + \frac{1664d^6e^6}{11} - \frac{49152ad^2e^9}{11} + \frac{4096e^{12}x^{17}}{17} + \frac{2048e^8x^{13}(8a^3e^3 - d^4)}{13} + \frac{128e^4x^9(d^8 + 64a^2e^6 - 32ad^4e^3)}{3} + \frac{4096a^4e^8x}{1} + \frac{1024d^5e^5x^{10}}{1} + \frac{8192d^2e^4x^7}{1} + \frac{1024d^6x^3}{1} - \frac{168(5e^3x^6 + 6d^2e^2x^5)d^3}{1} \right) a^2e^4 - \frac{512}{1001} (286e^9x^{14} + 924d^8e^8x^{13} + 1001d^2e^7x^{12} + 364d^3e^6x^{11})d^3 + \frac{8}{15015} (2365440e^9x^{13} + 7687680d^8e^8x^{12} + 8386560d^2e^7x^{11} + 3075072d^3e^6x^{10} - 15015d^9x^4 + 34320(6e^3x^7 + 7d^2e^2x^6)d^6 - 32032(36e^6x^{10} + 80d^5e^5x^9 + 45d^2e^4x^8)d^3)a^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)

[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) + (4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) + (32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 - 4*d^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3) - 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d^2*x^4*(d^8 - 512*a^2*e^6)

sympy [A] time = 0.14, size = 366, normalized size = 1.24

$$4096x^5d^{12}e^2 + 1024a^2d^6e^4 + \frac{8192}{5}d^2e^{10} + \frac{8192}{7}d^3e^9x + 4096a^4e^8x - \frac{4}{7}(7e^3x^8 + 8d^2e^2x^7)d^9 + 1024/5(16e^3x^5 + 20d^2e^2x^4 - 5d^3x^2)a^3e^6 + 128/165(45e^6x^{11} + 99d^5e^5x^{10} + 55d^2e^4x^9)d^6 + 128/105(2240e^6x^9 + 5040d^5e^5x^8 + 2880d^2e^4x^7 + 105d^6x^3 - 168(5e^3x^6 + 6d^2e^2x^5)d^3)a^2e^4 - 512/1001(286e^9x^{14} + 924d^8e^8x^{13} + 1001d^2e^7x^{12} + 364d^3e^6x^{11})d^3 + 8/15015(2365440e^9x^{13} + 7687680d^8e^8x^{12} + 8386560d^2e^7x^{11} + 3075072d^3e^6x^{10} - 15015d^9x^4 + 34320(6e^3x^7 + 7d^2e^2x^6)d^6 - 32032(36e^6x^{10} + 80d^5e^5x^9 + 45d^2e^4x^8)d^3)a^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
+ x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
8*a*d**9*e**2)
```

$$3.36 \quad \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 dx$$

Optimal. Leaf size=203

$$512a^3e^6x - \frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{128}{3}e^5x^9(d^4 - 4ae^3) - 24de^4x^8(d^4 - 16ae^3) - \frac{384}{5}ae^4x^5(d^4 - 4ae^3)$$

Rubi [A] time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2061}

$$-\frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 512a^3e^6x - \frac{128}{3}e^5x^9(d^4 - 4ae^3) - 24de^4x^8(d^4 - 16ae^3) + \frac{24}{7}d^2e^3x^7(64ae^3 + d^4) + 4d^3e^2x^6(d^4 - 16ae^3) - \frac{384}{5}ae^4x^5(d^4 - 4ae^3) + 8ad^6e^2x^3 + \frac{1536}{11}d^2e^7x^{11} + 32d^3e^6x^{10} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8 - (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 dx &= \int (512a^3e^6 - 192a^2d^3e^4x + 24ad^6e^2x^2 - d(d^8 - 1536a^2e^6)x^3 - 384ae^4(d^4 - 4ae^3)x^4 \\ &\quad + 4d^3e^2(d^4 - 16ae^3)x^5 + 24d^2e^3(d^4 + 64ae^3)x^6 - 24de^4(d^4 - 16ae^3)x^7 - 128e^5(d^4 - 4ae^3)x^8 \\ &\quad + 32d^3e^6x^9 + 128de^8x^{10} + 512e^9x^{11}) dx \\ &= 512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 - \frac{1}{4}d(d^8 - 1536a^2e^6)x^4 - \frac{384}{5}ae^4(d^4 - 4ae^3)x^5 \\ &\quad + 4d^3e^2(d^4 - 16ae^3)x^6 + \frac{24}{7}d^2e^3(d^4 + 64ae^3)x^7 - 24de^4(d^4 - 16ae^3)x^8 - \frac{128}{3}e^5(d^4 - 4ae^3)x^9 \\ &\quad + 32d^3e^6x^{10} + 128de^8x^{11} + \frac{512e^9x^{12}}{12} \end{aligned}$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.02

$$512a^3e^6x - \frac{1}{4}dx^4(d^8 - 1536a^2e^6) - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + \frac{128}{3}e^5x^9(4ae^3 - d^4) - 24de^4x^8(d^4 - 16ae^3) + \frac{384}{5}ae^4x^5(4ae^3 - d^4) + 4d^3e^2x^6(d^4 - 16ae^3) + \frac{24}{7}d^2e^3x^7(64ae^3 + d^4) + 32d^3e^6x^{10} + \frac{1536}{11}d^2e^7x^{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

[Out] $512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^{10} + (1536*d^2*e^7*x^{11})/11 + 128*d*e^8*x^{12} + (512*e^9*x^{13})/13$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]

fricas [A] time = 0.70, size = 205, normalized size = 1.01

$$\frac{512}{13}x^{13}e^9 + 128x^{12}e^8d + \frac{1536}{11}x^{11}e^7d^2 + 32x^{10}e^6d^3 - \frac{128}{3}x^9e^5d^4 + \frac{512}{3}x^9e^8a - 24x^8e^4d^5 + 384x^8e^7da + \frac{24}{7}x^7e^3d^6 + \frac{1536}{7}x^7e^6d^2a + 4x^6e^2d^7 - 64x^6e^5d^3a - \frac{384}{5}x^5e^4d^4a + \frac{1536}{5}x^5e^7a^2 - \frac{1}{4}x^4d^9 + 384x^4e^6da^2 + 8x^3e^2d^6a - 96x^2e^4d^3a^2 + 512xe^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")

[Out] $512/13*x^{13}*e^9 + 128*x^{12}*e^8*d + 1536/11*x^{11}*e^7*d^2 + 32*x^{10}*e^6*d^3 - 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 + 384*x^8*e^7*d*a + 24/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^6*e^2*d^7 - 64*x^6*e^5*d^3*a - 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2 + 8*x^3*e^2*d^6*a - 96*x^2*e^4*d^3*a^2 + 512*x*e^6*a^3$

giac [A] time = 0.24, size = 187, normalized size = 0.92

$$\frac{512}{13}x^{13}e^9 + 128dx^{12}e^8 + \frac{1536}{11}d^2x^{11}e^7 + 32d^3x^{10}e^6 - \frac{128}{3}d^4x^9e^5 + \frac{512}{3}ax^9e^8 + 384adx^8e^7 + \frac{1536}{7}ad^2x^7e^6 - 64ad^3x^6e^5 - \frac{384}{5}ad^4x^5e^4 + 8ad^5x^4e^3 + \frac{1536}{5}a^2x^5e^7 + 384a^2dx^4e^6 - 96a^2d^3x^2e^4 + 512a^3xe^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")

[Out] $512/13*x^{13}*e^9 + 128*d*x^{12}*e^8 + 1536/11*d^2*x^{11}*e^7 + 32*d^3*x^{10}*e^6 - 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 + 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6$

maple [A] time = 0.00, size = 288, normalized size = 1.42

$$\frac{512e^{9x^{13}} + 128d^8e^{8x^{12}} + \frac{1536d^2e^{7x^{11}}}{11} + 32d^3e^{6x^{10}} + 8a^3d^6e^{2x^3} - 96a^2d^3e^{4x^2} + 512a^3e^6x}{(512a^9d^9 - 256a^8d^8 + 8(128a^8d^8 - 16a^8d^8)x^2)^2} + \frac{(2048ad^7 - 64d^8 + 8(128a^8d^8 - 16a^8d^8)d^2)x^2}{8} + \frac{(1536ad^6d^2 + 24d^6d^2)x^2}{7} + \frac{(-256ad^5 + 8d^7d^2 - (128a^8d^8 - 16a^8d^8)d^2)x^2}{6} + \frac{(512d^7d^2 - 256a^8d^8 + 8(128a^8d^8 - 16a^8d^8)ad^2)x^2}{5} + \frac{(1536a^2d^4d^4 - d^4)x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((8e^3x^4+8d^2e^2x^3-d^3x+8a^2e^2)^3,x)$

[Out] $512/13e^9x^{13}+128d^2e^8x^{12}+1536/11d^2e^7x^{11}+32d^3e^6x^{10}+1/9(512a^2e^8-256d^4e^5+8e^3(128a^2e^5-16d^4e^2))x^9+1/8(2048a^2e^7d-64d^5e^4+8d^2e^2(128a^2e^5-16d^4e^2))x^8+1/7(1536ad^2e^6+24d^6e^3)x^7+1/6(-256a^2e^5d^3-d^3(128a^2e^5-16d^4e^2)+8d^7e^2)x^6+1/5(8a^2e^2(128a^2e^5-16d^4e^2)-256d^4a^2e^4+512e^7a^2)x^5+1/4(1536a^2d^2e^6-d^9)x^4+8ad^6e^2x^3-96a^2d^3e^4x^2+512a^3e^6x$

maxima [A] time = 0.60, size = 214, normalized size = 1.05

$$\frac{512}{13}e^9x^{13} + 128d^2e^8x^{12} + \frac{1536}{11}d^2e^7x^{11} + \frac{256}{5}d^6e^3x^{10} - \frac{1}{4}d^9x^9 + 512a^2e^8x^8 + \frac{4}{7}(6e^3x^7 + 7d^2e^4x^6) + \frac{96}{5}(16e^3x^5 + 20d^2e^4x^4 - 5d^3e^3x^2) + \frac{8}{15}(36e^6x^{10} + 80d^2e^5x^9 + 45d^2e^4x^8) + \frac{8}{105}(2240e^6x^9 + 5040d^2e^5x^8 + 2880d^2e^4x^7 + 105d^6e^3x^3 - 168(5e^3x^6 + 6d^2e^2x^5)d^3)a^2e^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((8e^3x^4+8d^2e^2x^3-d^3x+8a^2e^2)^3,x, \text{algorithm}=\text{"maxima"})$

[Out] $512/13e^9x^{13} + 128d^2e^8x^{12} + 1536/11d^2e^7x^{11} + 256/5d^3e^6x^{10} - 1/4d^9x^9 + 512a^3e^6x^8 + 4/7(6e^3x^7 + 7d^2e^2x^6)d^6 + 96/5(16e^3x^5 + 20d^2e^2x^4 - 5d^3x^2)a^2e^4 - 8/15(36e^6x^{10} + 80d^2e^5x^9 + 45d^2e^4x^8)d^3 + 8/105(2240e^6x^9 + 5040d^2e^5x^8 + 2880d^2e^4x^7 + 105d^6e^3x^3 - 168(5e^3x^6 + 6d^2e^2x^5)d^3)a^2e^2$

mupad [B] time = 2.24, size = 201, normalized size = 0.99

$$\frac{512}{13}e^9x^{13} - x^4\left(\frac{d^9}{4} - 384a^2d^6e^6\right) + \frac{128e^5x^9(4a^2e^3 - d^4)}{3} + 512a^3e^6x^8 + 128d^2e^8x^{12} + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 8ad^6e^2x^3 + \frac{384a^2e^4x^5(4a^2e^3 - d^4)}{5} + 24d^2e^4x^8(16a^2e^3 - d^4) + \frac{24d^2e^3x^7(d^4 + 64a^2e^3)}{7} - 96d^2d^3e^4x^2 - 4d^3e^2x^6(16a^2e^3 - d^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((8a^2e^2 - d^3x + 8e^3x^4 + 8d^2e^2x^3)^3,x)$

[Out] $(512e^9x^{13})/13 - x^4(d^9/4 - 384a^2d^6e^6) + (128e^5x^9(4a^2e^3 - d^4))/3 + 512a^3e^6x^8 + 128d^2e^8x^{12} + 32d^3e^6x^{10} + (1536d^2e^7x^{11})/11 + 8a^2d^6e^2x^3 + (384a^2e^4x^5(4a^2e^3 - d^4))/5 + 24d^2e^4x^8(16a^2e^3 - d^4) + (24d^2e^3x^7(64a^2e^3 + d^4))/7 - 96a^2d^3e^4x^2 - 4d^3e^2x^6(16a^2e^3 - d^4)$

sympy [A] time = 0.11, size = 218, normalized size = 1.07

$$512a^3e^6x^8 - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 128d^2e^8x^{12} + \frac{512e^9x^{13}}{13} + x^8\left(\frac{512a^2e^8}{3} - \frac{128d^4e^5}{3}\right) + x^8(384ad^6e^2 - 24d^6e^4) + x^7\left(\frac{1536ad^2e^6}{7} + \frac{24d^6e^3}{7}\right) + x^6(-64ad^3e^5 + 4d^7e^2) + x^5\left(\frac{1536ad^2e^7}{5} - \frac{384ad^4e^4}{5}\right) + x^4(384a^2d^6e^6 - \frac{d^9}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((8e^{**3}x^{**4}+8d^2e^{**2}x^{**3}-d^{**3}x+8a^2e^{**2})^{**3},x)$

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

$$3.37 \quad \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 dx$$

Optimal. Leaf size=107

$$64a^2e^4x - \frac{16}{5}e^2x^5(d^4 - 8ae^3) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2061}

$$64a^2e^4x - \frac{16}{5}e^2x^5(d^4 - 8ae^3) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{d^6x^3}{3} + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]

[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8 + (64*e^6*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 dx &= \int (64a^2e^4 - 16ad^3e^2x + d^6x^2 + 128ade^4x^3 - 16e^2(d^4 - 8ae^3)x^4 - 16d^3e^3x^5 \\ &\quad + 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 - \frac{16}{5}e^2(d^4 - 8ae^3)x^5 - \frac{8}{3}d^3e^3x^6 \end{aligned}$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.02

$$64a^2e^4x + \frac{16}{5}e^2x^5(8ae^3 - d^4) - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]

[Out] $64a^2e^4x - 8ad^3e^2x^2 + (d^6x^3)/3 + 32ad^2e^4x^4 + (16e^2(-d^4 + 8ae^3)x^5)/5 - (8d^3e^3x^6)/3 + (64d^2e^4x^7)/7 + 16d^2e^5x^8 + (64e^6x^9)/9$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]

[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]

fricas [A] time = 0.64, size = 99, normalized size = 0.93

$$\frac{64}{9}x^9e^6 + 16x^8e^5d + \frac{64}{7}x^7e^4d^2 - \frac{8}{3}x^6e^3d^3 - \frac{16}{5}x^5e^2d^4 + \frac{128}{5}x^5e^5a + 32x^4e^4da + \frac{1}{3}x^3d^6 - 8x^2e^2d^3a + 64xe^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")

[Out] $64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3 - 16/5*x^5*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d^6 - 8*x^2*e^2*d^3*a + 64*x*e^4*a^2$

giac [A] time = 0.29, size = 90, normalized size = 0.84

$$\frac{64}{9}x^9e^6 + 16dx^8e^5 + \frac{64}{7}d^2x^7e^4 - \frac{8}{3}d^3x^6e^3 - \frac{16}{5}d^4x^5e^2 + \frac{1}{3}d^6x^3 + \frac{128}{5}ax^5e^5 + 32adx^4e^4 - 8ad^3x^2e^2 + 64a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")

[Out] $64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2 + 64*a^2*x*e^4$

maple [A] time = 0.00, size = 100, normalized size = 0.93

$$\frac{64e^6x^9}{9} + 16de^5x^8 + \frac{64d^2e^4x^7}{7} - \frac{8d^3e^3x^6}{3} + 32ade^4x^4 + \frac{d^6x^3}{3} - 8ad^3e^2x^2 + 64a^2e^4x + \frac{(128ae^5 - 16d^4e^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)

[Out] $64/9e^{6x^9} + 16de^5x^8 + 64/7d^2e^4x^7 - 8/3d^3e^3x^6 + 1/5(128ae^5 - 16d^4e^2)x^5 + 32ad^3e^2x^4 + 1/3d^6x^3 - 8ad^3e^2x^2 + 64a^2e^4x$

maxima [A] time = 0.61, size = 101, normalized size = 0.94

$$\frac{64}{9}e^6x^9 + 16de^5x^8 + \frac{64}{7}d^2e^4x^7 + \frac{1}{3}d^6x^3 + 64a^2e^4x - \frac{8}{15}(5e^3x^6 + 6de^2x^5)d^3 + \frac{8}{5}(16e^3x^5 + 20de^2x^4 - 5d^3x^2)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")

[Out] $64/9e^{6x^9} + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a*e^2$

mupad [B] time = 0.04, size = 98, normalized size = 0.92

$$x^5 \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5} \right) + \frac{d^6x^3}{3} + \frac{64e^6x^9}{9} + 64a^2e^4x + 16de^5x^8 - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} - 8ad^3e^2x^2 + 32ade^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)

[Out] $x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e^2*x^2 + 32*a*d*e^4*x^4$

sympy [A] time = 0.09, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5 \left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8***3*x**4+8*d***2*x**3-d**3*x+8*a*e**2)**2,x)

[Out] $64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d**3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x**5*(128*a*e**5/5 - 16*d**4*e**2/5)$

$$3.38 \quad \int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) dx$$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5

Rubi steps

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

[Out] IntegrateAlgebraic[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]

fricas [A] time = 0.56, size = 33, normalized size = 0.89

$$\frac{8}{5}x^5e^3 + 2x^4e^2d - \frac{1}{2}x^2d^3 + 8xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")

[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{8}{5}x^5e^3 + 2dx^4e^2 - \frac{1}{2}d^3x^2 + 8axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")

[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{8}{5}e^3x^5 + 2de^2x^4 - \frac{1}{2}d^3x^2 + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)

[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5

maxima [A] time = 0.67, size = 33, normalized size = 0.89

$$\frac{8}{5}e^3x^5 + 2de^2x^4 - \frac{1}{2}d^3x^2 + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")

[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x

mupad [B] time = 0.04, size = 33, normalized size = 0.89

$$-\frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5} + 8ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)`

[Out] $(8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x$

sympy [A] time = 0.07, size = 36, normalized size = 0.97

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)`

[Out] $8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5$

$$3.39 \quad \int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx$$

Optimal. Leaf size=153

$$\frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1106, 1093, 208}

$$\frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2 \tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]

[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &

& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx &= \text{Subst} \left(\int \frac{1}{\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2 \right) - 3d^2ex^2 + 8e^3x^4} dx, x, \frac{d}{4e} + x \right) \\ &= \frac{(4e^2) \text{Subst} \left(\int \frac{1}{-\frac{3d^2e}{2} - e\sqrt{d^4 - 64ae^3} + 8e^3x^2} dx, x, \frac{d}{4e} + x \right)}{\sqrt{d^4 - 64ae^3}} - \frac{(4e^2) \text{Subst} \left(\int \frac{1}{-\frac{3d^2e}{2}} dx, x, \frac{d}{4e} + x \right)}{\sqrt{d^4 - 64ae^3}} \\ &= \frac{2 \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} \right)}{\sqrt{d^4 - 64ae^3} \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} - \frac{2 \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}} \right)}{\sqrt{d^4 - 64ae^3} \sqrt{3d^2 + 2\sqrt{d^4 - 64ae^3}}} \end{aligned}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.46

$$-\text{RootSum} \left[8\#1^4e^3 + 8\#1^3de^2 - \#1d^3 + 8ae^2 \&, \frac{\log(x - \#1)}{-32\#1^3e^3 - 24\#1^2de^2 + d^3} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]

[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3 - 24*d*e^2*#1^2 - 32*e^3*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]

fricas [B] time = 0.89, size = 1115, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")

[Out]
$$\frac{-\sqrt{(3d^2 + 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} \log(8ex + 2(2d^4 - 128ae^3 - 3(5d^{10} - 64ad^6e^3 - 16384a^2d^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9})) \sqrt{(3d^2 + 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} + 2d) + \sqrt{(3d^2 + 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} \log(8ex - 2(2d^4 - 128ae^3 - 3(5d^{10} - 64ad^6e^3 - 16384a^2d^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9})) \sqrt{(3d^2 + 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} + 2d) - \sqrt{(3d^2 - 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} \log(8ex + 2(2d^4 - 128ae^3 + 3(5d^{10} - 64ad^6e^3 - 16384a^2d^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9})) \sqrt{(3d^2 - 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} + 2d) + \sqrt{(3d^2 - 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} \log(8ex - 2(2d^4 - 128ae^3 + 3(5d^{10} - 64ad^6e^3 - 16384a^2d^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9})) \sqrt{(3d^2 - 2(5d^8 - 64ad^4e^3 - 16384a^2e^6))/\sqrt{25d^{12} + 960ad^8e^3 - 98304a^2d^4e^6 - 4194304a^3e^9}}/\sqrt{(5d^8 - 64ad^4e^3 - 16384a^2e^6))} + 2d)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")

[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)

maple [C] time = 0.07, size = 67, normalized size = 0.44

$$\frac{\ln(-\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2) + x)}{32\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^3 e^3 + 24\text{RootOf}(8e^3_Z^4 + 8e^2d_Z^3 - d^3_Z + 8ae^2)^2 d e^2 - d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)`

[Out] `sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8*a*e^2))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8e^3x^4 + 8de^2x^3 - d^3x + 8ae^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")`

[Out] `integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)`

mupad [B] time = 3.73, size = 1264, normalized size = 8.26

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)`

[Out] `atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256i + a^2*d*e^6*8192i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*1024i)/(5*d^12*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6`

$$\begin{aligned} & \left. \right)^{(1/2)} + 1048576*a^3*e^9*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 122 \\ & 88*a^2*d^4*e^6)^{(1/2)} + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - \\ & 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^{(1/2)} - 384*a*d^8*e^3*((2*(d^12 - 26214 \\ & 4*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^{(1/2)} + 3*d^6 - 192*a*d^2*e^ \\ & 3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^{(1/2)} - \\ & 12288*a^2*d^4*e^6*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^ \\ & 4*e^6)^{(1/2)} + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8 \\ & *e^3 - 12288*a^2*d^4*e^6))^{(1/2)})) * ((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e \\ & ^3 + 12288*a^2*d^4*e^6)^{(1/2)} + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^ \\ & 3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^{(1/2)} * 2i \end{aligned}$$

sympy [A] time = 1.83, size = 122, normalized size = 0.80

$$\text{RootSum}\left(t^4(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}) + t^2(384ad^2e^3 - 6d^6) + 1, \left(t \mapsto t \log\left(x + \frac{-49152t^3a^2d^2e^6 - 192t^3ad^6e^3 + 15t^3d^{10} + 256tae^3 - 13td^4 + 2d}{8e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)

[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 + 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-49152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))

$$3.40 \quad \int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx$$

Optimal. Leaf size=342

$$\frac{2e \left(\frac{d}{4e} + x \right) \left(-256ae^3 + 13d^4 - 48d^2e^2 \left(\frac{d}{4e} + x \right)^2 \right)}{(-16384a^2e^6 - 64ad^4e^3 + 5d^8)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e \left(-d^2\sqrt{d^4 - 64ae^3} + 128ae^3 + d^4 \right) \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} \right)}{(d^4 - 64ae^3)^{3/2} (256ae^3 + 5d^4) \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}}$$

Rubi [A] time = 0.53, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1106, 1092, 1166, 208}

$$\frac{2e \left(\frac{d}{4e} + x \right) \left(-256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x \right)^2 + 13d^4 \right)}{(-16384a^2e^6 - 64ad^4e^3 + 5d^8)(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e \left(-d^2\sqrt{d^4 - 64ae^3} + 128ae^3 + d^4 \right) \tanh^{-1} \left(\frac{d+4ex}{\sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} \right)}{(d^4 - 64ae^3)^{3/2} (256ae^3 + 5d^4) \sqrt{3d^2 - 2\sqrt{d^4 - 64ae^3}}} + \frac{24e \left(d^2\sqrt{d^4 - 64ae^3} + 128ae^3 + d^4 \right) \tanh^{-1} \left(\frac{d+4ex}{\sqrt{2\sqrt{d^4 - 64ae^3} + 3d^2}} \right)}{(d^4 - 64ae^3)^{3/2} (256ae^3 + 5d^4) \sqrt{2\sqrt{d^4 - 64ae^3} + 3d^2}}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2), x]

[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/sqrt[3*d^2 - 2*sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*sqrt[3*d^2 - 2*sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 + d^2*sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/sqrt[3*d^2 + 2*sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*sqrt[3*d^2 + 2*sqrt[d^4 - 64*a*e^3]])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1092

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx = \text{Subst} \left(\int \frac{1}{\left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^2} dx, x, \frac{d}{4e} + x \right)$$

$$= \frac{2e \left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{4 \text{Subst} \left(\int \frac{1}{\left(\frac{1}{32} \left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^2} dx, x, \frac{d}{4e} + x \right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} + \frac{(48e^3 (d^4 - 4d^2e^2x^2 + 4e^3x^4))}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

$$= \frac{2e \left(\frac{d}{4e} + x\right) \left(13d^4 - 256ae^3 - 48d^2e^2 \left(\frac{d}{4e} + x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)} - \frac{24e (d^4 - 4d^2e^2x^2 + 4e^3x^4)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

Mathematica [C] time = 0.19, size = 234, normalized size = 0.68

$$\frac{48e^2 \text{RootSum} \left[8\#1^4 e^3 + 8\#1^3 d e^2 - \#1 d^3 + 8ae^2 \&, \frac{2\#1^2 d^2 e \log(x-\#1) + 32ne^2 \log(x-\#1) + \#1 d^3 \log(x-\#1)}{32\#1^3 e^3 + 24\#1^2 d e^2 - d^3} \& \right]}{16384a^2e^6 + 64ad^4e^3 - 5d^8} + \frac{(d + 4ex) (-128ae^3 + 5d^4 - 12d^3ex - 24d^2e^2x^2)}{(d^4 - 64ae^3) (256ae^3 + 5d^4) (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)}$$

Antiderivative was successfully verified.

[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]

[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*Log[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^2*#1^2 + 32*e^3*#1^3) &])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]

[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2), x]

fricas [B] time = 1.29, size = 4285, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")

[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2))*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18))

$$\begin{aligned}
& 8 - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27})) * \sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))) - 12*\sqrt{2}*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^{11} - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*\sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))*\log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^{10})*x - 13824*\sqrt{2}*(d^{16}*e^2 - 128*a*d^{12}*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^{11} - 268435456*a^4*e^{14} - (125*d^{30} + 59200*a*d^{26}*e^3 - 3624960*a^2*d^{22}*e^6 - 566493184*a^3*d^{18}*e^9 + 19797114880*a^4*d^{14}*e^{12} + 1906965479424*a^5*d^{10}*e^{15} - 30786325577728*a^6*d^6*e^{18} - 2251799813685248*a^7*d^2*e^{21})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))*\sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))) + 12*\sqrt{2}*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^{11} - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*\sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})))
\end{aligned}$$

$$\begin{aligned}
& e^8 - (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))*\log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^{10})*x + 13824*\sqrt{2}*(d^{16}*e^2 - 128*a*d^{12}*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^{11} - 268435456*a^4*e^{14} + (125*d^{30} + 59200*a*d^{26}*e^3 - 3624960*a^2*d^{22}*e^6 - 566493184*a^3*d^{18}*e^9 + 19797114880*a^4*d^{14}*e^{12} + 1906965479424*a^5*d^{10}*e^{15} - 30786325577728*a^6*d^6*e^{18} - 2251799813685248*a^7*d^2*e^{21})*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))*\sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))) - 12*\sqrt{2}*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^{11} - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*\sqrt{(d^{10}*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))*\sqrt{(d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^{10})/(15625*d^{36} + 1800000*a*d^{32}*e^3 - 115200000*a^2*d^{28}*e^6 - 21135360000*a^3*d^{24}*e^9 - 150994944000*a^4*d^{20}*e^{12} + 78082505441280*a^5*d^{16}*e^{15} + 2744381022928896*a^6*d^{12}*e^{18} - 70931694131085312*a^7*d^8*e^{21} - 5188146770730811392*a^8*d^4*e^{24} - 73786976294838206464*a^9*e^{27}))/((125*d^{24} - 4800*a*d^{20}*e^3 - 1167360*a^2*d^{16}*e^6 + 31195136*a^3*d^{12}*e^9 + 3825205248*a^4*d^8*e^{12} - 51539607552*a^5*d^4*e^{15} - 4398046511104*a^6*e^{18}))*\log(884736*a*d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^{10})*x - 13824*\sqrt{2}*(d^{16}*e^2 - 128*a*d^{12}*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^{11} - 268435456*a^4*e^{14} + (125*d^{30} + 59200*a*d^{26}*e^3 - 3624960*a^2*d^{22}*e^6 - 566493184*a^3*d^{18}*e^9 + 19797114880*a^4*d^{14}*e^{12} + 1906965479424*a^5*d^{10}*e^{15} - 30786325577728*a^6*d^6*e^{18} - 2251799813685248*a^7*d^2*e^{21})*\sqrt{(d^8*e^4 + 512*a*d^4*
\end{aligned}$$

$$\frac{e^7 + 65536a^2e^{10}}{(15625d^{36} + 1800000a*d^{32}e^3 - 115200000a^2*d^{28}e^6 - 21135360000a^3*d^{24}e^9 - 150994944000a^4*d^{20}e^{12} + 78082505441280a^5*d^{16}e^{15} + 2744381022928896a^6*d^{12}e^{18} - 70931694131085312a^7*d^8e^{21} - 5188146770730811392a^8*d^4e^{24} - 73786976294838206464a^9e^{27})} \sqrt{(d^{10}e^2 + 160a*d^6e^5 + 40960a^2*d^2e^8 - (125d^{24} - 4800a*d^{20}e^3 - 1167360a^2*d^{16}e^6 + 31195136a^3*d^{12}e^9 + 3825205248a^4*d^8e^{12} - 51539607552a^5*d^4e^{15} - 4398046511104a^6e^{18}))} \sqrt{(d^8e^4 + 512a*d^4e^7 + 65536a^2e^{10})} / (15625d^{36} + 1800000a*d^{32}e^3 - 115200000a^2*d^{28}e^6 - 21135360000a^3*d^{24}e^9 - 150994944000a^4*d^{20}e^{12} + 78082505441280a^5*d^{16}e^{15} + 2744381022928896a^6*d^{12}e^{18} - 70931694131085312a^7*d^8e^{21} - 5188146770730811392a^8*d^4e^{24} - 73786976294838206464a^9e^{27})) / (125d^{24} - 4800a*d^{20}e^3 - 1167360a^2*d^{16}e^6 + 31195136a^3*d^{12}e^9 + 3825205248a^4*d^8e^{12} - 51539607552a^5*d^4e^{15} - 4398046511104a^6e^{18})) - 8*(d^4e - 64a*e^4)*x / (40a*d^8e^2 - 512a^2*d^4e^5 - 131072a^3e^8 + 8*(5*d^8e^3 - 64a*d^4e^6 - 16384a^2e^9)*x^4 + 8*(5*d^9e^2 - 64a*d^5e^5 - 16384a^2*d^1e^8)*x^3 - (5*d^{11} - 64a*d^7e^3 - 16384a^2*d^3e^6)*x)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.02, size = 288, normalized size = 0.84

$$\frac{384e^2 \left(2 \operatorname{RootOf}(8e^3Z^4 + 8e^2dZ^3 - d^3Z + 8ae^2)^2 d^2e + \operatorname{RootOf}(8e^3Z^4 + 8e^2dZ^3 - d^3Z + 8ae^2)^3 e^3 + 32ae^2 \right) \ln(-\operatorname{RootOf}(8e^3Z^4 + 8e^2dZ^3 - d^3Z + 8ae^2) + x)}{(2048ae^3 + 40d^4)(64ae^3 - d^4) \left(32 \operatorname{RootOf}(8e^3Z^4 + 8e^2dZ^3 - d^3Z + 8ae^2)^3 e^3 + 24 \operatorname{RootOf}(8e^3Z^4 + 8e^2dZ^3 - d^3Z + 8ae^2)^2 d^2e - d^3 \right)} + \frac{12d^2e^3}{(256ae^3 + 5d^4)(64ae^3 - d^4)} + \frac{9d^2e^2}{(256ae^3 + 5d^4)(64ae^3 - d^4)} + \frac{e^2}{256ae^3 + 5d^4} + \frac{(128ae^3 - 5d^4)d}{131072d^2e^2 + 512ae^3 - 40d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)

[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(2048*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+8*a*e^2))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 7.11, size = 10351, normalized size = 30.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)

[Out]
$$\begin{aligned} & ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256 \\ & *a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)) \\ & + (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x \\ & + 8*e^3*x^4 + 8*d*e^2*x^3) + \operatorname{atan}\left(\frac{((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - \\ & d^4)^9)^{1/2} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} \\ & + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - \\ & d^4)^9)^{1/2}))}{(125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32} \\ & *e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20} \\ & *e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 79164837 \\ & 1998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{1/2} * ((1536*(6871 \\ & 9476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - \\ & 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/ (25*d^{20} - 17179869184*a^5 \\ & *e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 1342 \\ & 17728*a^4*d^4*e^{12}) - ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^{19} \\ & *e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^5 \\ & *d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/ (25*d^{20} - 17179869184*a^5*e^{15} - \\ & 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4 \\ & *d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^{15} \\ & + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2* \\ & e^{24}))/ (25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 \\ & + 2097152*a^3*d^4*e^9) * ((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{1/2} \\ & - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440 \\ & *a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{1/2} \\ &))/ (125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 12902 \\ & 40*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 9663 \\ & 6764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7* \\ & d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{1/2} * ((288*(d^{22}*e^2 + d^4*e^2 \\ & *(-(64*a*e^3 - d^4)^9)^{1/2} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 616038 \\ & 4*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a \\ & *e^5*(-(64*a*e^3 - d^4)^9)^{1/2}))/ (125*d^{36} + 1152921504606846976*a^9*e^{27} \\ & - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250 \\ & 489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12} \\ & *e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{1/2} \end{aligned}$$

$$\begin{aligned}
&) + (1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(1i + ((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)}*((1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) - ((1536*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2*e^{24}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)}*((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(1i)/((((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(1i)/((((288*(d^{22}*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - 32*a*d^{18}*e^5 + 22528*a^2*d^{14}*e^8 - 6160384*a^3*d^{10}*e^{11} + 461373440*a^4*d^6*e^{14} - 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(1i)
\end{aligned}$$

$$\begin{aligned}
& \left(8e^{21} - 40532396646334464a^8d^4e^{24} \right)^{1/2} * \left(\left((1536*(68719476736a^5e^{24} + 20d^{20}e^9 - 7936a*d^{16}e^{12} + 770048a^2d^{12}e^{15} - 5242880a^3d^8e^{18} - 2147483648a^4d^4e^{21})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) - ((1536*(25d^{27}e^8 - 3840a*d^{23}e^{11} + 24576a^2d^{19}e^{14} + 19922944a^3d^{15}e^{17} - 654311424a^4d^{11}e^{20} - 25769803776a^5d^7e^{23} + 1099511627776a^6d^3e^{26})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) \right. \right. \\
& + (6144*x*(25d^{22}e^9 - 2240a*d^{18}e^{12} - 118784a^2d^{14}e^{15} + 12320768a^3d^{10}e^{18} + 134217728a^4d^6e^{21} - 17179869184a^5d^2e^{24})) / (25d^{16} + 268435456a^4e^{12} - 640a*d^{12}e^3 - 159744a^2d^8e^6 + 2097152a^3d^4e^9) \left. \right) * \left((288*(d^{22}e^2 + d^4e^2*(-(64a*e^3 - d^4)^9)^{1/2} - 32a*d^18e^5 + 22528a^2d^{14}e^8 - 6160384a^3d^{10}e^{11} + 461373440a^4d^6e^{14} - 10737418240a^5d^2e^{17} + 256a*e^5*(-(64a*e^3 - d^4)^9)^{1/2}) \right) / (125d^{36} + 1152921504606846976a^9e^{27} - 28800a*d^{32}e^3 + 1290240a^2d^{28}e^6 + 163577856a^3d^{24}e^9 - 15250489344a^4d^{20}e^{12} - 96636764160a^5d^{16}e^{15} + 44324062494720a^6d^{12}e^{18} - 791648371998720a^7d^8e^{21} - 40532396646334464a^8d^4e^{24}) \left. \right)^{1/2} * \left((288*(d^{22}e^2 + d^4e^2*(-(64a*e^3 - d^4)^9)^{1/2} - 32a*d^{18}e^5 + 22528a^2d^{14}e^8 - 6160384a^3d^{10}e^{11} + 461373440a^4d^6e^{14} - 10737418240a^5d^2e^{17} + 256a*e^5*(-(64a*e^3 - d^4)^9)^{1/2}) \right) / (125d^{36} + 1152921504606846976a^9e^{27} - 28800a*d^{32}e^3 + 1290240a^2d^{28}e^6 + 163577856a^3d^{24}e^9 - 15250489344a^4d^{20}e^{12} - 96636764160a^5d^{16}e^{15} + 44324062494720a^6d^{12}e^{18} - 791648371998720a^7d^8e^{21} - 40532396646334464a^8d^4e^{24}) \left. \right)^{1/2} + (1536*(96d^{13}e^{10} + 3072a*d^9e^{13} - 50331648a^3d^5e^{16} + 196608a^2d^5e^{16})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) + (6144*x*(786432a^2e^{17} + 96d^8e^{11} + 9216a*d^4e^{14})) / (25d^{16} + 268435456a^4e^{12} - 640a*d^{12}e^3 - 159744a^2d^8e^6 + 2097152a^3d^4e^9) - ((288*(d^{22}e^2 + d^4e^2*(-(64a*e^3 - d^4)^9)^{1/2} - 32a*d^{18}e^5 + 22528a^2d^{14}e^8 - 6160384a^3d^{10}e^{11} + 461373440a^4d^6e^{14} - 10737418240a^5d^2e^{17} + 256a*e^5*(-(64a*e^3 - d^4)^9)^{1/2}) \left. \right) / (125d^{36} + 1152921504606846976a^9e^{27} - 28800a*d^{32}e^3 + 1290240a^2d^{28}e^6 + 163577856a^3d^{24}e^9 - 15250489344a^4d^{20}e^{12} - 96636764160a^5d^{16}e^{15} + 44324062494720a^6d^{12}e^{18} - 791648371998720a^7d^8e^{21} - 40532396646334464a^8d^4e^{24}) \left. \right)^{1/2} * \left((1536*(96d^{13}e^{10} + 3072a*d^9e^{13} - 50331648a^3d^5e^{16} + 196608a^2d^5e^{16})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) - ((1536*(68719476736a^5e^{24} + 20d^{20}e^9 - 7936a*d^{16}e^{12} + 770048a^2d^{12}e^{15} - 5242880a^3d^8e^{18} - 2147483648a^4d^4e^{21})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) + ((1536*(25d^{27}e^8 - 3840a*d^{23}e^{11} + 24576a^2d^{19}e^{14} + 19922944a^3d^{15}e^{17} - 654311424a^4d^{11}e^{20} - 25769803776a^5d^7e^{23} + 1099511627776a^6d^3e^{26})) / (25d^{20} - 17179869184a^5e^{15} - 2240a*d^{16}e^3 - 118784a^2d^{12}e^6 + 12320768a^3d^8e^9 + 134217728a^4d^4e^{12}) \right. \left. \right)
\end{aligned}$$

$$\begin{aligned}
& d^4 e^{12}) + (6144 x (25 d^{22} e^9 - 2240 a d^{18} e^{12} - 118784 a^2 d^{14} e^{15} \\
& + 12320768 a^3 d^{10} e^{18} + 134217728 a^4 d^6 e^{21} - 17179869184 a^5 d^2 e^{24}) / (25 d^{16} + 268435456 a^4 e^{12} - 640 a d^{12} e^3 - 159744 a^2 d^8 e^6 + 2 \\
& 097152 a^3 d^4 e^9)) * ((288 (d^{22} e^2 + d^4 e^2 * (- (64 a e^3 - d^4)^9)^{1/2}) \\
& - 32 a d^{18} e^5 + 22528 a^2 d^{14} e^8 - 6160384 a^3 d^{10} e^{11} + 461373440 a^4 d^6 e^{14} - 10737418240 a^5 d^2 e^{17} + 256 a e^5 * (- (64 a e^3 - d^4)^9)^{1/2})) / (125 d^{36} + 1152921504606846976 a^9 e^{27} - 28800 a d^{32} e^3 + 1290240 a^2 d^{28} e^6 + 163577856 a^3 d^{24} e^9 - 15250489344 a^4 d^{20} e^{12} - 96636764160 a^5 d^{16} e^{15} + 44324062494720 a^6 d^{12} e^{18} - 791648371998720 a^7 d^8 e^{21} - 40532396646334464 a^8 d^4 e^{24}))^{1/2}) * ((288 (d^{22} e^2 + d^4 e^2 * (- (64 a e^3 - d^4)^9)^{1/2}) - 32 a d^{18} e^5 + 22528 a^2 d^{14} e^8 - 6160384 a^3 d^{10} e^{11} + 461373440 a^4 d^6 e^{14} - 10737418240 a^5 d^2 e^{17} + 256 a e^5 * (- (64 a e^3 - d^4)^9)^{1/2})) / (125 d^{36} + 1152921504606846976 a^9 e^{27} - 28800 a d^{32} e^3 + 1290240 a^2 d^{28} e^6 + 163577856 a^3 d^{24} e^9 - 15250489344 a^4 d^{20} e^{12} - 96636764160 a^5 d^{16} e^{15} + 44324062494720 a^6 d^{12} e^{18} - 791648371998720 a^7 d^8 e^{21} - 40532396646334464 a^8 d^4 e^{24}))^{1/2} + (6144 x (786432 a^2 e^{17} + 96 d^8 e^{11} + 9216 a d^4 e^{14})) / (25 d^{16} + 268435456 a^4 e^{12} - 640 a d^{12} e^3 - 159744 a^2 d^8 e^6 + 2097152 a^3 d^4 e^9) + (113246208 a d^2 e^{14}) / (25 d^{20} - 17179869184 a^5 e^{15} - 2240 a d^{16} e^3 - 118784 a^2 d^{12} e^6 + 12320768 a^3 d^8 e^9 + 134217728 a^4 d^4 e^{12})) * ((288 (d^{22} e^2 + d^4 e^2 * (- (64 a e^3 - d^4)^9)^{1/2}) - 32 a d^{18} e^5 + 22528 a^2 d^{14} e^8 - 6160384 a^3 d^{10} e^{11} + 461373440 a^4 d^6 e^{14} - 10737418240 a^5 d^2 e^{17} + 256 a e^5 * (- (64 a e^3 - d^4)^9)^{1/2})) / (125 d^{36} + 1152921504606846976 a^9 e^{27} - 28800 a d^{32} e^3 + 1290240 a^2 d^{28} e^6 + 163577856 a^3 d^{24} e^9 - 15250489344 a^4 d^{20} e^{12} - 96636764160 a^5 d^{16} e^{15} + 44324062494720 a^6 d^{12} e^{18} - 791648371998720 a^7 d^8 e^{21} - 40532396646334464 a^8 d^4 e^{24}))^{1/2} * i + \operatorname{atan}(((- (288 (d^4 e^2 * (- (64 a e^3 - d^4)^9)^{1/2}) - d^{22} e^2 + 32 a d^{18} e^5 - 22528 a^2 d^{14} e^8 + 6160384 a^3 d^{10} e^{11} - 461373440 a^4 d^6 e^{14} + 10737418240 a^5 d^2 e^{17} + 256 a e^5 * (- (64 a e^3 - d^4)^9)^{1/2})) / (125 d^{36} + 1152921504606846976 a^9 e^{27} - 28800 a d^{32} e^3 + 1290240 a^2 d^{28} e^6 + 163577856 a^3 d^{24} e^9 - 15250489344 a^4 d^{20} e^{12} - 96636764160 a^5 d^{16} e^{15} + 44324062494720 a^6 d^{12} e^{18} - 791648371998720 a^7 d^8 e^{21} - 40532396646334464 a^8 d^4 e^{24}))^{1/2} * ((1536 * (68719476736 a^5 e^{24} + 20 d^{20} e^9 - 7936 a d^{16} e^{12} + 770048 a^2 d^{12} e^{15} - 5242880 a^3 d^8 e^{18} - 2147483648 a^4 d^4 e^{21})) / (25 d^{20} - 17179869184 a^5 e^{15} - 2240 a d^{16} e^3 - 118784 a^2 d^{12} e^6 + 12320768 a^3 d^8 e^9 + 134217728 a^4 d^4 e^{12}) - ((1536 * (25 d^{27} e^8 - 3840 a d^{23} e^{11} + 24576 a^2 d^{19} e^{14} + 19922944 a^3 d^{15} e^{17} - 654311424 a^4 d^{11} e^{20} - 25769803776 a^5 d^7 e^{23} + 1099511627776 a^6 d^3 e^{26})) / (25 d^{20} - 17179869184 a^5 e^{15} - 2240 a d^{16} e^3 - 118784 a^2 d^{12} e^6 + 12320768 a^3 d^8 e^9 + 134217728 a^4 d^4 e^{12}) + (6144 x (25 d^{22} e^9 - 2240 a d^{18} e^{12} - 118784 a^2 d^{14} e^{15} + 12320768 a^3 d^{10} e^{18} + 134217728 a^4 d^6 e^{21} - 17179869184 a^5 d^2 e^{24})) / (25 d^{16} + 268435456 a^4 e^{12} - 640 a d^{12} e^3 - 159744 a^2 d^8 e^6 + 2097152 a^3 d^4 e^9)) * (- (288 (d^4 e^2 * (- (64 a e^3 - d^4)^9)^{1/2}) - d^2 e^2 + 32 a d^{18} e^5 - 22528 a^2 d^{14} e^8 + 6160384 a^3 d^{10} e^{11} - 461373
\end{aligned}$$

$$\begin{aligned}
& 440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})/(125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 12 \\
& 90240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 9 \\
& 6636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a \\
& ^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2))*(-(288*(d^4*e^2*(-(64 \\
& *a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 61 \\
& 60384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 2 \\
& 56*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})))/(125*d^{36} + 1152921504606846976*a^9* \\
& e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 1 \\
& 5250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d \\
& ^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2)} + (1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 19660 \\
& 8*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784 \\
& *a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(7 \\
& 86432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e \\
& ^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + (- (2 \\
& 88*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528* \\
& a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240 \\
& *a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})))/(125*d^{36} + 1152921 \\
& 504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856 \\
& *a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 443 \\
& 24062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 4053239664633446 \\
& 4*a^8*d^4*e^{24}))^{(1/2))*((1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^ \\
& 3*d*e^{19} + 196608*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d \\
& ^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^ \\
& ^{12}) - ((1536*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 77004 \\
& 8*a^2*d^{12}*e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} \\
& - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768* \\
& a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e \\
& ^{11} + 24576*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^2 \\
& 0 - 25769803776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 1717 \\
& 9869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8 \\
& *e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - \\
& 118784*a^2*d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17 \\
& 179869184*a^5*d^2*e^{24}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 1 \\
& 59744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4 \\
&)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^ \\
& ^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(\\
& 64*a*e^3 - d^4)^9)^{(1/2)})))/(125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800 \\
& *a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a \\
& ^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 7 \\
& 91648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24}))^{(1/2))*(-(28 \\
& 8*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a \\
& ^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240* \\
& a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)})))/(125*d^{36} + 11529215
\end{aligned}$$

$$\begin{aligned}
& 04606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856* \\
& a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 4432 \\
& 4062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464 \\
& *a^8*d^4*e^{24})^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4 \\
& *e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 \\
& + 2097152*a^3*d^4*e^9))*i)/((-288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - \\
& d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - 46 \\
& 1373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 \\
& + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} \\
& - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 7916483719987 \\
& 20*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*((1536*(687194767 \\
& 36*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12}*e^{15} - 52428 \\
& 80*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} - 17179869184*a^5*e^{15} \\
& - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728 \\
& *a^4*d^4*e^{12}) - ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 24576*a^2*d^{19}*e^{14} \\
& + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 25769803776*a^5*d^7 \\
& *e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240 \\
& *a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4 \\
& *e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2*d^{14}*e^{15} + \\
& 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184*a^5*d^2*e^{24} \\
&))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 209 \\
& 7152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + \\
& 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4 \\
& *d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)} \\
&))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 \\
& + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764 \\
& 160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8* \\
& e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*((288*(d^4*e^2*(-(64*a*e^3 \\
& - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} \\
& - 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 \\
& - d^4)^9)^{(1/2)}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 \\
& + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489 \\
& 344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} \\
& - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)} + \\
& (1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + 196608*a^2*d^5 \\
& *e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 \\
& + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) + (6144*x*(786432*a^2*e^{17} \\
& + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((25*d^{16} + 268435456*a^4*e^{12} - 6 \\
& 40*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - ((288*(d^4*e^2 \\
& *(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 \\
& + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} \\
& + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2)}))/((125*d^{36} + 11529215046068469 \\
& 76*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}* \\
& e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 4432406249472
\end{aligned}$$

$$\begin{aligned}
& 0*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4* \\
& e^{24})^{(1/2)}*((1536*(96*d^{13}*e^{10} + 3072*a*d^9*e^{13} - 50331648*a^3*d*e^{19} + \\
& 196608*a^2*d^5*e^{16}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2240*a*d^{16}*e^3 - \\
& 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^{12}) - ((15 \\
& 36*(68719476736*a^5*e^{24} + 20*d^{20}*e^9 - 7936*a*d^{16}*e^{12} + 770048*a^2*d^{12} \\
& *e^{15} - 5242880*a^3*d^8*e^{18} - 2147483648*a^4*d^4*e^{21}))/((25*d^{20} - 1717986 \\
& 9184*a^5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^ \\
& 9 + 134217728*a^4*d^4*e^{12}) + ((1536*(25*d^{27}*e^8 - 3840*a*d^{23}*e^{11} + 2457 \\
& 6*a^2*d^{19}*e^{14} + 19922944*a^3*d^{15}*e^{17} - 654311424*a^4*d^{11}*e^{20} - 257698 \\
& 03776*a^5*d^7*e^{23} + 1099511627776*a^6*d^3*e^{26}))/((25*d^{20} - 17179869184*a^ \\
& 5*e^{15} - 2240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134 \\
& 217728*a^4*d^4*e^{12}) + (6144*x*(25*d^{22}*e^9 - 2240*a*d^{18}*e^{12} - 118784*a^2 \\
& *d^{14}*e^{15} + 12320768*a^3*d^{10}*e^{18} + 134217728*a^4*d^6*e^{21} - 17179869184* \\
& a^5*d^2*e^{24}))/((25*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2* \\
& d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} \\
& - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - \\
& 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - \\
& d^4)^9)^{(1/2}))/((125*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^ \\
& 3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^ \\
& 12 - 96636764160*a^5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 79164837199 \\
& 8720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e^{24})^{(1/2)})*(-(288*(d^4*e^2 \\
& *(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a*d^{18}*e^5 - 22528*a^2*d^{14}*e^ \\
& 8 + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e^{14} + 10737418240*a^5*d^2*e^ \\
& 17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2}))/((125*d^{36} + 115292150460684697 \\
& 6*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^{28}*e^6 + 163577856*a^3*d^{24}*e \\
& ^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^5*d^{16}*e^{15} + 44324062494720 \\
& *a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - 40532396646334464*a^8*d^4*e \\
& ^{24})^{(1/2)} + (6144*x*(786432*a^2*e^{17} + 96*d^8*e^{11} + 9216*a*d^4*e^{14}))/((2 \\
& 5*d^{16} + 268435456*a^4*e^{12} - 640*a*d^{12}*e^3 - 159744*a^2*d^8*e^6 + 2097152 \\
& *a^3*d^4*e^9)) + (113246208*a*d^2*e^{14}))/((25*d^{20} - 17179869184*a^5*e^{15} - 2 \\
& 240*a*d^{16}*e^3 - 118784*a^2*d^{12}*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4 \\
& *d^4*e^{12}))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^{(1/2)} - d^{22}*e^2 + 32*a* \\
& d^{18}*e^5 - 22528*a^2*d^{14}*e^8 + 6160384*a^3*d^{10}*e^{11} - 461373440*a^4*d^6*e \\
& ^{14} + 10737418240*a^5*d^2*e^{17} + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^{(1/2}))/((1 \\
& 25*d^{36} + 1152921504606846976*a^9*e^{27} - 28800*a*d^{32}*e^3 + 1290240*a^2*d^2 \\
& 8*e^6 + 163577856*a^3*d^{24}*e^9 - 15250489344*a^4*d^{20}*e^{12} - 96636764160*a^ \\
& 5*d^{16}*e^{15} + 44324062494720*a^6*d^{12}*e^{18} - 791648371998720*a^7*d^8*e^{21} - \\
& 40532396646334464*a^8*d^4*e^{24})^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)

[Out] Timed out

$$3.41 \quad \int (8 + 8x - x^3 + 8x^4)^4 dx$$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + 6784x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^4, x]

[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^4 dx &= \int (4096 + 16384x + 24576x^2 + 14336x^3 + 14336x^4 + 43008x^5 + 47488x^6 + 11008x^7 + 1376x^8 + 1408x^9 + 21488x^{10} + 25312x^{11} + 1168x^{12} + 128x^{13} + 128x^{14} + 4096x^{15} + 128x^{16} + 4096x^{17}) dx \\ &= 4096x + 8192x^2 + 8192x^3 + 3584x^4 + \frac{14336x^5}{5} + 7168x^6 + 6784x^7 + 1376x^8 + 1408x^9 + \frac{21488x^{10}}{5} + \frac{25312x^{11}}{11} + 1168x^{12} + \frac{10241x^{13}}{13} + 1168x^{14} + \frac{128x^{15}}{5} - 128x^{16} + \frac{4096x^{17}}{17} \end{aligned}$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4, x]

[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 8x - x^3 + 8x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^4, x]

[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^4, x]

fricas [A] time = 0.68, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")

[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x

giac [A] time = 0.31, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")

[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x

maple [A] time = 0.00, size = 85, normalized size = 0.89

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^4, x)

[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+1408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^15-128*x^16+4096/17*x^17

maxima [A] time = 1.25, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336}{5}x^5 + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")

[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x

mupad [B] time = 0.19, size = 84, normalized size = 0.88

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)

[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17

sympy [A] time = 0.07, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**4,x)

[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x

$$3.42 \quad \int (8 + 8x - x^3 + 8x^4)^3 dx$$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^3, x]

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^3 dx &= \int (512 + 1536x + 1536x^2 + 320x^3 + 1152x^4 + 2880x^5 + 1560x^6 - 360x^7 + 1152x^8 \\ &\quad + 512x + 768x^2 + 512x^3 + 80x^4 + \frac{1152x^5}{5} + 480x^6 + \frac{1560x^7}{7} - 45x^8 + 128x^9 + \frac{307x^{10}}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 74, normalized size = 1.00

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3, x]

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 8x - x^3 + 8x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^3,x]

[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^3, x]

fricas [A] time = 0.48, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^9 - 45x^8 + \frac{1560}{7}x^7 + 480x^6 + \frac{1152}{5}x^5 + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

giac [A] time = 0.33, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^9 - 45x^8 + \frac{1560}{7}x^7 + 480x^6 + \frac{1152}{5}x^5 + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

maple [A] time = 0.00, size = 65, normalized size = 0.88

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^9 - 45x^8 + \frac{1560}{7}x^7 + 480x^6 + \frac{1152}{5}x^5 + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^3,x)

[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+307/2*x^10+24/11*x^11-16*x^12+512/13*x^13

maxima [A] time = 0.94, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^9 - 45x^8 + \frac{1560}{7}x^7 + 480x^6 + \frac{1152}{5}x^5 + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")

[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x

mupad [B] time = 0.08, size = 64, normalized size = 0.86

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)

[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13

sympy [A] time = 0.07, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^9 - 45x^8 + \frac{1560x^7}{7} + 480x^6 + \frac{1152x^5}{5} + 80x^4 + 512x^3 + 768x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**3,x)

[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 + 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*x

$$3.43 \quad \int (8 + 8x - x^3 + 8x^4)^2 dx$$

Optimal. Leaf size=54

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^2, x]

[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 8x - x^3 + 8x^4)^2 dx &= \int (64 + 128x + 64x^2 - 16x^3 + 112x^4 + 128x^5 + x^6 - 16x^7 + 64x^8) dx \\ &= 64x + 64x^2 + \frac{64x^3}{3} - 4x^4 + \frac{112x^5}{5} + \frac{64x^6}{3} + \frac{x^7}{7} - 2x^8 + \frac{64x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2, x]

[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 8x - x^3 + 8x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^2,x]

[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^2, x]

fricas [A] time = 0.76, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")

[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x

giac [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")

[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-x^3+8*x+8)^2,x)

[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9

maxima [A] time = 0.88, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")

[Out] $64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x^2 + 64*x$

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)

[Out] $64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x^8 + (64*x^9)/9$

sympy [A] time = 0.06, size = 49, normalized size = 0.91

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-x**3+8*x+8)**2,x)

[Out] $64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 + 64*x**2 + 64*x$

$$3.44 \quad \int (8 + 8x - x^3 + 8x^4) dx$$

Optimal. Leaf size=23

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Antiderivative was successfully verified.

[In] Int[8 + 8*x - x^3 + 8*x^4, x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

Rubi steps

$$\int (8 + 8x - x^3 + 8x^4) dx = 8x + 4x^2 - \frac{x^4}{4} + \frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Antiderivative was successfully verified.

[In] Integrate[8 + 8*x - x^3 + 8*x^4, x]

[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 8x - x^3 + 8x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[8 + 8*x - x^3 + 8*x^4, x]

[Out] IntegrateAlgebraic[8 + 8*x - x^3 + 8*x^4, x]

fricas [A] time = 0.73, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")

[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x

giac [A] time = 0.32, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")

[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(8*x^4-x^3+8*x+8,x)

[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5

maxima [A] time = 0.91, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{1}{4}x^4 + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")

[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x

mupad [B] time = 0.03, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x - x^3 + 8*x^4 + 8, x)
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

sympy [A] time = 0.06, size = 19, normalized size = 0.83

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8, x)
```

```
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```


$$3.45 \quad \int \frac{1}{8+8x-x^3+8x^4} dx$$

Optimal. Leaf size=268

$$-\frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 - \sqrt{6(1+\sqrt{29})} \left(\frac{4}{x}+1\right) + 3\sqrt{29}\right) + \frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 + \sqrt{6(1+\sqrt{29})} \left(\frac{4}{x}+1\right) + 3\sqrt{29}\right)$$

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 - \sqrt{6(1+\sqrt{29})} \left(\frac{4}{x}+1\right) + 3\sqrt{29}\right) + \frac{1}{24} \sqrt{\frac{67\sqrt{29}-109}{1218}} \log\left(\left(\frac{4}{x}+1\right)^2 + \sqrt{6(1+\sqrt{29})} \left(\frac{4}{x}+1\right) + 3\sqrt{29}\right) - \frac{\tan^{-1}\left(\frac{3\left(\frac{4}{x}+1\right)}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12} \sqrt{\frac{109+67\sqrt{29}}{1218}} \tan^{-1}\left(\frac{\frac{2}{3} - \sqrt{6(1+\sqrt{29})} + 2}{\sqrt{6(\sqrt{29}-1)}}\right) - \frac{1}{12} \sqrt{\frac{109+67\sqrt{29}}{1218}} \tan^{-1}\left(\frac{\frac{2}{3} + \sqrt{6(1+\sqrt{29})} + 2}{\sqrt{6(\sqrt{29}-1)}}\right)$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1), x]

[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])]] + 8/x)/Sqrt[6*(-1 + Sqrt[29])])]/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]] + 8/x)/Sqrt[6*(-1 + Sqrt[29])])]/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]]*(1 + 4/x) + (1 + 4/x)^2)/24 + (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]]*(1 + 4/x) + (1 + 4/x)^2)/24

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^(p)/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{8 + 8x - x^3 + 8x^4} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(8 - 32x)^2}{8(1069056 - 393216x^2 + 1048576x^4)} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int \frac{(8 - 32x)^2}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int -\frac{512x}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) - 128 \operatorname{Subst} \left(\int \frac{x}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) \\
&= 65536 \operatorname{Subst} \left(\int \frac{x}{1069056 - 393216x^2 + 1048576x^4} dx, x, \frac{1}{4} + \frac{1}{x} \right) - \frac{\operatorname{Subst} \left(\int \frac{16\sqrt{6}(1 + \sqrt{29})}{3\sqrt{29}} dx, x, \frac{1}{4} + \frac{1}{x} \right)}{76} \\
&= 32768 \operatorname{Subst} \left(\int \frac{1}{1069056 - 393216x + 1048576x^2} dx, x, \left(\frac{1}{4} + \frac{1}{x} \right)^2 \right) - \frac{(87 + \sqrt{29}) \operatorname{Subst} \left(\int \frac{1}{1069056 - 393216x + 1048576x^2} dx, x, \frac{1}{4} + \frac{1}{x} \right)}{76} \\
&= -\frac{1}{24} \sqrt{\frac{-109 + 67\sqrt{29}}{1218}} \log \left(3\sqrt{29} - \sqrt{6(1 + \sqrt{29})} \left(1 + \frac{4}{x} \right) + \left(1 + \frac{4}{x} \right)^2 \right) + \frac{1}{24} \sqrt{\frac{-109 - 67\sqrt{29}}{1218}} \log \left(3\sqrt{29} + \sqrt{6(1 + \sqrt{29})} \left(1 + \frac{4}{x} \right) + \left(1 + \frac{4}{x} \right)^2 \right) \\
&= -\frac{\tan^{-1} \left(\frac{3 - \left(1 + \frac{4}{x} \right)^2}{6\sqrt{7}} \right)}{12\sqrt{7}} - \frac{1}{12} \sqrt{\frac{109 + 67\sqrt{29}}{1218}} \tan^{-1} \left(\frac{2 + \sqrt{6(1 + \sqrt{29})} + \frac{8}{x}}{\sqrt{6(-1 + \sqrt{29})}} \right) - \frac{1}{12} \sqrt{\frac{109 - 67\sqrt{29}}{1218}} \tan^{-1} \left(\frac{2 + \sqrt{6(1 + \sqrt{29})} + \frac{8}{x}}{\sqrt{6(-1 + \sqrt{29})}} \right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.17

$$\operatorname{RootSum} \left[8\#1^4 - \#1^3 + 8\#1 + 8\&, \frac{\log(x - \#1)}{32\#1^3 - 3\#1^2 + 8} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1), x]

[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8 + 8x - x^3 + 8x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-1),x]

[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-1), x]

fricas [C] time = 3.62, size = 1015, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/168*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} - 109/87696})*\log(287314195 \\ & 392*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} - 109/87696})^3 - 120389 \\ & 06880*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} - 109/87696})^2 + 1687 \\ & 8104*x + 4897683*I*\sqrt{7} - 411405372*\sqrt{65/43848*I*\sqrt{7} - 109/87696} \\ & + 6055613) - 1/168*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} - 109/87696})* \\ & \log(-35914274424*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} - 109/87696} \\ &))^3 + 16443*(-1/168*I*\sqrt{7} - 1/2*\sqrt{-65/43848*I*\sqrt{7} - 109/87696}) \\ & ^2*(-13001*I*\sqrt{7} + 1092084*\sqrt{65/43848*I*\sqrt{7} - 109/87696} - 91520 \\ &) + 609*(351027*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} - 109/87696} \\ &)^2 - 613)*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} - 109/87696}) + 2109763 \\ & *x - 1911147/8*I*\sqrt{7} + 40134087/2*\sqrt{65/43848*I*\sqrt{7} - 109/87696} \\ & - 1461344) + 1/1044*(\sqrt{174}*\sqrt{-4698*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43 \\ & 848*I*\sqrt{7} - 109/87696})^2 - 4698*(-1/168*I*\sqrt{7} - 1/2*\sqrt{-65/43848 \\ & *I*\sqrt{7} - 109/87696})^2 - 87/784*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} \\ &) - 109/87696})*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} - 109/87696}) - 7) \\ & + 261*\sqrt{65/43848*I*\sqrt{7} - 109/87696} + 261*\sqrt{-65/43848*I*\sqrt{7} \\ & - 109/87696})*\log(-16443/2*(-1/168*I*\sqrt{7} - 1/2*\sqrt{-65/43848*I*\sqrt{7} \\ & - 109/87696})^2*(-13001*I*\sqrt{7} + 1092084*\sqrt{65/43848*I*\sqrt{7} - 109/ \\ & 87696} - 91520) - 609/2*(351027*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} \\ &) - 109/87696})^2 - 613)*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} - 109/8 \\ & 7696}) + 752431680*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/43848*I*\sqrt{7} - 109/876 \\ & 96})^2 + 1/32*(3*(13001*\sqrt{174}*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} \\ & - 109/87696}) - 91520*\sqrt{174})*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} - \\ & 109/87696}) - 274560*\sqrt{174}*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} - \\ & 109/87696}) + 1922368*\sqrt{174}*\sqrt{-4698*(1/168*I*\sqrt{7} - 1/2*\sqrt{65/ \\ & 43848*I*\sqrt{7} - 109/87696})^2 - 4698*(-1/168*I*\sqrt{7} - 1/2*\sqrt{-65/438 \\ & 48*I*\sqrt{7} - 109/87696})^2 - 87/784*(I*\sqrt{7} + 84*\sqrt{-65/43848*I*\sqrt{7} \\ &) - 109/87696})*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} - 109/87696}) - \\ & 7) + 2109763*x - 373317/2*I*\sqrt{7} + 15679314*\sqrt{65/43848*I*\sqrt{7} - 10 \\ & 9/87696} + 220336) - 1/1044*(\sqrt{174}*\sqrt{-4698*(1/168*I*\sqrt{7} - 1/2*\sqrt{ \\ & 65/43848*I*\sqrt{7} - 109/87696})^2 - 4698*(-1/168*I*\sqrt{7} - 1/2*\sqrt{- \\ & 65/43848*I*\sqrt{7} - 109/87696})^2 - 87/784*(I*\sqrt{7} + 84*\sqrt{-65/43848* \\ & I*\sqrt{7} - 109/87696})*(-I*\sqrt{7} + 84*\sqrt{65/43848*I*\sqrt{7} - 109/8769} \end{aligned}$$

6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 1/32*(3*(13001*sqrt(174))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) - 274560*sqrt(174))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 109/87696) + 220336)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - x^3 + 8x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

maple [C] time = 0.02, size = 41, normalized size = 0.15

$$\frac{\ln\left(-\text{RootOf}\left(8_Z^4 - _Z^3 + 8_Z + 8\right) + x\right)}{32 \text{RootOf}\left(8_Z^4 - _Z^3 + 8_Z + 8\right)^3 - 3 \text{RootOf}\left(8_Z^4 - _Z^3 + 8_Z + 8\right)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8),x)

[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - x^3 + 8x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)

mupad [B] time = 2.45, size = 123, normalized size = 0.46

$$\sum_{k=1}^4 \ln \left(\frac{\text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right) \left(8064 \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right) + 256x + \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right) x 12285 + \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right)^2 x 148176 + 198072 \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right)^2 - 8\right)}{4096} \right) \text{root}\left(z^4 + \frac{7z^2}{6264} + \frac{65z}{1052352} + \frac{1}{1035909}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x - x^3 + 8*x^4 + 8), x)

[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 12285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)

sympy [A] time = 0.96, size = 41, normalized size = 0.15

$$\text{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t \log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x + \frac{6055613}{16878104}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-x**3+8*x+8), x)

[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(35914274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x + 6055613/16878104)))

$$3.46 \quad \int \frac{1}{(8+8x-x^3+8x^4)^2} dx$$

Optimal. Leaf size=357

$$\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} + \frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \log\left(\left(\frac{4}{x}+1\right)\right)}{175392}$$

Rubi [A] time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} + \frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \log\left(\left(\frac{4}{x}+1\right)\right)}{175392} + \frac{17 \operatorname{Im}\left(\frac{1}{x}\right)}{1008\sqrt{7}} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \operatorname{Im}\left(\frac{1}{x}\right)}{87696} - \frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}} \operatorname{Im}\left(\frac{1}{x}\right)}{87696}$$

Antiderivative was successfully verified.

[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2), x]

[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(5157 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*sqrt(7))])/(1008*sqrt(7)) - (sqrt((180983329 + 45923327*sqrt(29))/1218)*ArcTan[(2 - sqrt(6*(1 + sqrt(29)))) + 8/x]/sqrt(6*(-1 + sqrt(29)))))/87696 - (sqrt((180983329 + 45923327*sqrt(29))/1218)*ArcTan[(2 + sqrt(6*(1 + sqrt(29)))) + 8/x]/sqrt(6*(-1 + sqrt(29)))))/87696 - (sqrt((-180983329 + 45923327*sqrt(29))/1218)*Log[3*sqrt(29) - sqrt(6*(1 + sqrt(29)))]*(1 + 4/x) + (1 + 4/x)^2)/175392 + (sqrt((-180983329 + 45923327*sqrt(29))/1218)*Log[3*sqrt(29) + sqrt(6*(1 + sqrt(29)))]*(1 + 4/x) + (1 + 4/x)^2)/175392

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1673


```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(8 + 8x - x^3 + 8x^4)^2} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(8 - 32x)^6}{64 (1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(16 \operatorname{Subst} \left(\int \frac{(8 - 32x)^6}{(1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) \\
&= - \left(16 \operatorname{Subst} \left(\int \frac{x(-6291456 - 335544320x^2 - 1610612736x^4)}{(1069056 - 393216x^2 + 1048576x^4)^2} dx, x, \frac{1}{4} + \frac{1}{x} \right) \right) - 16 \\
&= \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{\operatorname{Subst} \left(\int \frac{2789277407614152474624 + 7758008804499}{1069056 - 393216x^2 + 1048576x^4} dx \right)}{57853663025666457} \\
&= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{\operatorname{Subst} \left(\int \frac{2789277407614152474624 + 7758008804499}{1069056 - 393216x^2 + 1048576x^4} dx \right)}{57853663025666457} \\
&= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{1392}{21924} \\
&= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{\sqrt{-1}}{21924} \\
&= - \frac{207 + 29 \left(1 + \frac{4}{x} \right)^2}{336 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} + \frac{5 \left(5157 + 199 \left(1 + \frac{4}{x} \right)^2 \right) \left(1 + \frac{4}{x} \right)}{87696 \left(261 - 6 \left(1 + \frac{4}{x} \right)^2 + \left(1 + \frac{4}{x} \right)^4 \right)} - \frac{17 \operatorname{ta}}{21924}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 113, normalized size = 0.32

$$\frac{\operatorname{RootSum} \left[8\#1^4 - \#1^3 + 8\#1 + 8\&, \frac{392\#1^2 \log(x-\#1) - 1097\#1 \log(x-\#1) + 2243 \log(x-\#1)}{32\#1^3 - 3\#1^2 + 8} \& \right]}{21924} + \frac{784x^3 - 1146x^2 + 1539x + 544}{43848(8x^4 - x^3 + 8x + 8)}$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2), x]

[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 392*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) &]/21924

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8 + 8x - x^3 + 8x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-2), x]

fricas [C] time = 5.20, size = 1201, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")

[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(6217850567873065654359973859328*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^3 - 10028767243179717478632775680*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 + 67481665655469287031416*x + 320944207138750561964778*I*sqrt(7) - 133210725033589645013145504*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(-777231320984133206794996732416*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) + 7569*(7276511507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 8435208206933660878927*x - 148449195141328682772633/4*I*sqrt(7) + 15403787072311988024172036*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) - 47393606606696595067616) - 5583312*x^2 + (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)

$$\begin{aligned}
& - 1/2*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344))^2 - \\
& 125452723536*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{(-4550065/334540596096*I*\sqrt{7} \\
&) - 180983329/4683568345344))^2 - 658503/1568*(17*I*\sqrt{7} + 7056*\sqrt{(-45 \\
& 50065/334540596096*I*\sqrt{7} - 180983329/4683568345344)})*(-17*I*\sqrt{7} + 7 \\
& 056*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) - 66301 \\
& 91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*\sqrt{7} + 7056*\sqrt{(-4550065/33454 \\
& 0596096*I*\sqrt{7} - 180983329/4683568345344)}) + 7569*(8*x^4 - x^3 + 8*x + 8 \\
&)*(-17*I*\sqrt{7} + 7056*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/468 \\
& 3568345344)}))*\log(-439084532376*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{(-4550065/33 \\
& 4540596096*I*\sqrt{7} - 180983329/4683568345344)})^2*(-1066184864424603*I*\sqrt{ \\
& 7} + 442529435492941104*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/4 \\
& 683568345344)} - 1427510892508480) - 7569/2*(7276511507810430573072*(17/1411 \\
& 2*I*\sqrt{7} - 1/2*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/468356834 \\
& 5344)})^2 - 23359423554371543)*(17*I*\sqrt{7} + 7056*\sqrt{(-4550065/3345405960 \\
& 96*I*\sqrt{7} - 180983329/4683568345344)}) + 626797952698732342414548480*(17/ \\
& 14112*I*\sqrt{7} - 1/2*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/46835 \\
& 68345344)})^2 + 1/16*(261*(62716756730859*\sqrt{87})*(-17*I*\sqrt{7} + 7056*\sqrt{ \\
& (4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) - 142751089250 \\
& 8480*\sqrt{87})*(17*I*\sqrt{7} + 7056*\sqrt{(-4550065/334540596096*I*\sqrt{7} - \\
& 180983329/4683568345344)}) - 372580342944713280*\sqrt{87})*(-17*I*\sqrt{7} + 70 \\
& 56*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) + 104650 \\
& 21752358451264*\sqrt{87})*\sqrt{(-125452723536*(17/14112*I*\sqrt{7} - 1/2*\sqrt{ \\
& (4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)})^2 - 125452723536 \\
& *(-17/14112*I*\sqrt{7} - 1/2*\sqrt{(-4550065/334540596096*I*\sqrt{7} - 18098332 \\
& 9/4683568345344)})^2 - 658503/1568*(17*I*\sqrt{7} + 7056*\sqrt{(-4550065/334540 \\
& 596096*I*\sqrt{7} - 180983329/4683568345344)})*(-17*I*\sqrt{7} + 7056*\sqrt{(455 \\
& 0065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) - 6630191) + 843520 \\
& 8206933660878927*x - 3005727107011649552439/2*I*\sqrt{7} + 62377677844335880 \\
& 1235576*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)} + 22 \\
& 95910220839785410704) - (56*\sqrt{87})*(8*x^4 - x^3 + 8*x + 8)*\sqrt{(-12545272 \\
& 3536*(17/14112*I*\sqrt{7} - 1/2*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983 \\
& 329/4683568345344)})^2 - 125452723536*(-17/14112*I*\sqrt{7} - 1/2*\sqrt{(-45500 \\
& 65/334540596096*I*\sqrt{7} - 180983329/4683568345344)})^2 - 658503/1568*(17*I \\
& *\sqrt{7} + 7056*\sqrt{(-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345 \\
& 344)})*(-17*I*\sqrt{7} + 7056*\sqrt{(4550065/334540596096*I*\sqrt{7} - 180983329 \\
& /4683568345344)}) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*\sqrt{7} + \\
& 7056*\sqrt{(-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) - 756 \\
& 9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*\sqrt{7} + 7056*\sqrt{(4550065/334540596096*I \\
& *\sqrt{7} - 180983329/4683568345344)}))*\log(-439084532376*(-17/14112*I*\sqrt{7} \\
&) - 1/2*\sqrt{(-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)})^2* \\
& (-1066184864424603*I*\sqrt{7} + 442529435492941104*\sqrt{(4550065/334540596096 \\
& *I*\sqrt{7} - 180983329/4683568345344)} - 1427510892508480) - 7569/2*(7276511 \\
& 507810430573072*(17/14112*I*\sqrt{7} - 1/2*\sqrt{(4550065/334540596096*I*\sqrt{7} \\
&) - 180983329/4683568345344)})^2 - 23359423554371543)*(17*I*\sqrt{7} + 7056* \\
& \sqrt{(-4550065/334540596096*I*\sqrt{7} - 180983329/4683568345344)}) + 62679795
\end{aligned}$$

2698732342414548480*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 1/16*(261*(62716756730859*sqrt(87))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 1427510892508480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 10465021752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*sqrt(7) + 623776778443358801235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 2295910220839785410704) + 7498008*x + 2650368)/(8*x^4 - x^3 + 8*x + 8)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - x^3 + 8x + 8)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")

[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)

maple [C] time = 0.01, size = 83, normalized size = 0.23

$$\frac{(392 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8)^2 - 1097 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8) + 2243) \ln(-\operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8) + x) + \frac{7}{3132}x^3 - \frac{191}{58464}x^2 + \frac{57}{12992}x + \frac{17}{10962}}{701568 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8)^3 - 65772 \operatorname{RootOf}(8_Z^4 - _Z^3 + 8_Z + 8)^2 + 175392} + \frac{1}{x^4 - \frac{1}{8}x^3 + x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-x^3+8*x+8)^2,x)

[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*sum((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{784x^3 - 1146x^2 + 1539x + 544}{43848(8x^4 - x^3 + 8x + 8)} + \frac{1}{21924} \int \frac{392x^2 - 1097x + 2243}{8x^4 - x^3 + 8x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")

[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/21924*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)

mupad [B] time = 0.21, size = 176, normalized size = 0.49



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)

[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962)/(x - x^3/8 + x^4 + 1)

sympy [B] time = 3.22, size = 3834, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)

[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x + 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))) + 40699873480352667)/227008323264998681573683424 - 267658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))) + 40699873480352667)/8435208206933660878927 - 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/113504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 435853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sq

$$\begin{aligned}
& \text{rt}(-180983329 + 45923327*\text{sqrt}(29))/85227636229779664) - 2942814074101429415 \\
& 084030510182204250067556953*\text{sqrt}(214095423017213*\text{sqrt}(29) + 47106822945*\text{sqr} \\
& \text{t}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 40699873480352667)/888496186 \\
& 751485201253966401139075287452416534006272 - 142576256328563148358311429727 \\
& 65102609010539559351093/27765505835983912539186450035596102732888016687696 \\
& - 75184631502818837388875900060881355871*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923 \\
& 327*\text{sqrt}(29))*\text{sqrt}(214095423017213*\text{sqrt}(29) + 47106822945*\text{sqrt}(1218)*\text{sqrt}(- \\
& 180983329 + 45923327*\text{sqrt}(29)) + 40699873480352667)/30637799543154662112205 \\
& 737970312940946635052896768 - 963314181796141259748858766106570487809406229 \\
& 9*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))/3063779954315466211220573 \\
& 7970312940946635052896768 - 1398888334001652366855237255*\text{sqrt}(42)*\text{sqrt}(-180 \\
& 983329 + 45923327*\text{sqrt}(29))*\text{sqrt}(214095423017213*\text{sqrt}(29) + 47106822945*\text{sqr} \\
& \text{t}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 40699873480352667)/359456428 \\
& 291497016547944746810895370264 + 91245981690030498967778233214015591679*\text{sqr} \\
& \text{t}(42)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))/23005211410655809059068463795897 \\
& 303696896 + 10304175351841941260676745569701505519*\text{sqrt}(29)*\text{sqrt}(2140954230 \\
& 17213*\text{sqrt}(29) + 47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29) \\
&) + 40699873480352667)/19347382796361535418676578052349632409089536 + 63911 \\
& 1088489748962499984017403917984374085485*\text{sqrt}(29)/4836845699090383854669144 \\
& 513087408102272384) + \text{sqrt}(-180983329/37468546762752 + 1583563*\text{sqrt}(29)/129 \\
& 2018853888)*\log(x**2 + x*(-62716756730859*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 4592 \\
& 3327*\text{sqrt}(29))*\text{sqrt}(-47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt} \\
& (29)) + 214095423017213*\text{sqrt}(29) + 40699873480352667)/227008323264998681573 \\
& 683424 - 20905585576953*\text{sqrt}(42)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))/85227 \\
& 636229779664 + 3881045239007430*\text{sqrt}(29)/5326727264361229 + 267658292345340 \\
& *\text{sqrt}(-47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 214095 \\
& 423017213*\text{sqrt}(29) + 40699873480352667)/8435208206933660878927 + 2157374520 \\
& 970352866823*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))/11350416163249 \\
& 9340786841712 + 435853770857118353330297/33740832827734643515708) - 1425762 \\
& 5632856314835831142972765102609010539559351093/2776550583598391253918645003 \\
& 5596102732888016687696 - 10304175351841941260676745569701505519*\text{sqrt}(29)*\text{sq} \\
& \text{rt}(-47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 214095423 \\
& 017213*\text{sqrt}(29) + 40699873480352667)/19347382796361535418676578052349632409 \\
& 089536 - 91245981690030498967778233214015591679*\text{sqrt}(42)*\text{sqrt}(-180983329 + \\
& 45923327*\text{sqrt}(29))/23005211410655809059068463795897303696896 - 751846315028 \\
& 18837388875900060881355871*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))* \\
& \text{sqrt}(-47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 2140954 \\
& 23017213*\text{sqrt}(29) + 40699873480352667)/306377995431546621122057379703129409 \\
& 46635052896768 - 1398888334001652366855237255*\text{sqrt}(42)*\text{sqrt}(-180983329 + 45 \\
& 923327*\text{sqrt}(29))*\text{sqrt}(-47106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sq} \\
& \text{rt}(29)) + 214095423017213*\text{sqrt}(29) + 40699873480352667)/3594564282914970165 \\
& 47944746810895370264 + 9633141817961412597488587661065704878094062299*\text{sqrt}(\\
& 1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29))/30637799543154662112205737970312 \\
& 940946635052896768 + 2942814074101429415084030510182204250067556953*\text{sqrt}(-4 \\
& 7106822945*\text{sqrt}(1218)*\text{sqrt}(-180983329 + 45923327*\text{sqrt}(29)) + 21409542301721
\end{aligned}$$

$$\begin{aligned}
& 3\sqrt{29} + 40699873480352667)/8884961867514852012539664011390752874524165 \\
& 34006272 + 639111088489748962499984017403917984374085485\sqrt{29}/483684569 \\
& 9090383854669144513087408102272384) - 2\sqrt{(199631405/37468546762752 + \sqrt{ \\
& t(-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29}))} + 2140954230 \\
& 17213\sqrt{29} + 40699873480352667)/9367136690688 + 1583563\sqrt{29}/430672 \\
& 951296)*\operatorname{atan}(454016646529997363147366848*x/(-4509673516272467429860\sqrt{12 \\
& 18})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923 \\
& 327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667) + 137769981\sqrt{ \\
& 29))} + 3601609981798895040\sqrt{(-180983329 + 45923327\sqrt{29})}\sqrt{(19 \\
& 9631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29} \\
&))} + 214095423017213\sqrt{29} + 40699873480352667) + 137769981\sqrt{29))} + \\
& 20905585576953\sqrt{1218})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{ \\
& (-180983329 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 4069987348 \\
& 0352667) + 137769981\sqrt{29})}\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 \\
& + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667)) + 29 \\
& 32424170326692281206238216/(-4509673516272467429860\sqrt{1218})\sqrt{(1996314 \\
& 05 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29}))} + \\
& 214095423017213\sqrt{29} + 40699873480352667) + 137769981\sqrt{29))} + 36016 \\
& 09981798895040\sqrt{(-180983329 + 45923327\sqrt{29})}\sqrt{(199631405 + 4\sqrt{ \\
& (-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29}))} + 21409542301 \\
& 7213\sqrt{29} + 40699873480352667) + 137769981\sqrt{29))} + 20905585576953\sqrt{ \\
& 1218})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + \\
& 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667) + 13776 \\
& 9981\sqrt{29})}\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{ \\
& 29}))} + 214095423017213\sqrt{29} + 40699873480352667)) + 431474904194070573 \\
& 3646\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29})}/(-45096735162724674298 \\
& 60\sqrt{1218})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-1809833 \\
& 29 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667) + 1 \\
& 37769981\sqrt{29))} + 3601609981798895040\sqrt{(-180983329 + 45923327\sqrt{29} \\
&)}\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + 459233 \\
& 27\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667) + 137769981\sqrt{ \\
& 29))} + 20905585576953\sqrt{1218})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{ \\
& 1218})\sqrt{(-180983329 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + \\
& 40699873480352667) + 137769981\sqrt{29})}\sqrt{(-47106822945\sqrt{1218})\sqrt{ \\
& (-180983329 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352 \\
& 667)) + 7203219963597790080\sqrt{(-47106822945\sqrt{1218})\sqrt{(-180983329 + \\
& 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667)/(-450967 \\
& 3516272467429860\sqrt{1218})\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218}) \\
& }\sqrt{(-180983329 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873 \\
& 480352667) + 137769981\sqrt{29))} + 3601609981798895040\sqrt{(-180983329 + 45 \\
& 923327\sqrt{29})}\sqrt{(199631405 + 4\sqrt{(-47106822945\sqrt{1218})\sqrt{(-1809 \\
& 83329 + 45923327\sqrt{29}))} + 214095423017213\sqrt{29} + 40699873480352667) \\
& + 137769981\sqrt{29))} + 20905585576953\sqrt{1218})\sqrt{(199631405 + 4\sqrt{ \\
& (-47106822945\sqrt{1218})\sqrt{(-180983329 + 45923327\sqrt{29}))} + 2140954230172 \\
& 13\sqrt{29} + 40699873480352667) + 137769981\sqrt{29})}\sqrt{(-47106822945\sqrt{ \\
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(1218) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}} + 214095423017213 \cdot \sqrt{29} + \\
& 40699873480352667) + 165397912920614705160598080 \cdot \sqrt{29} / (-45096735162724 \\
& 67429860 \cdot \sqrt{1218} \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-1} \\
& 80983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 4069987348035266 \\
& 7) + 137769981 \cdot \sqrt{29}) + 3601609981798895040 \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}} \\
& \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}})} \\
& + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 137769 \\
& 981 \cdot \sqrt{29}) + 20905585576953 \cdot \sqrt{1218} \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822} \\
& 945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} \\
& + 40699873480352667) + 137769981 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \\
& \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 40699873 \\
& 480352667)) - 55683134469459984392598 \cdot \sqrt{42} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}} \\
& \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106} \\
& 822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} \\
& + 40699873480352667) + 137769981 \cdot \sqrt{29}) + 3601609981798895040 \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}} \\
& \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}})} \\
& (1218) \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 40 \\
& 699873480352667) + 137769981 \cdot \sqrt{29}) + 20905585576953 \cdot \sqrt{1218} \cdot \sqrt{199 \\
& 631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}})} \\
&) + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + 137769981 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 2140954230 \\
& 17213 \cdot \sqrt{29} + 40699873480352667)) - 62716756730859 \cdot \sqrt{1218} \cdot \sqrt{-1809 \\
& 83329 + 45923327 \cdot \sqrt{29}}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 4 \\
& 5923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 40699873480352667) / (-4509673 \\
& 516272467429860 \cdot \sqrt{1218} \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}})} \\
& + 214095423017213 \cdot \sqrt{29} + 406998734 \\
& 80352667) + 137769981 \cdot \sqrt{29}) + 3601609981798895040 \cdot \sqrt{-180983329 + 459 \\
& 23327 \cdot \sqrt{29}}) \cdot \sqrt{199631405 + 4 \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-18098 \\
& 3329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 40699873480352667) + \\
& 137769981 \cdot \sqrt{29}) + 20905585576953 \cdot \sqrt{1218} \cdot \sqrt{199631405 + 4 \cdot \sqrt{-4 \\
& 7106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 21409542301721 \\
& 3 \cdot \sqrt{29} + 40699873480352667) + 137769981 \cdot \sqrt{29}) \cdot \sqrt{-47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 214095423017213 \cdot \sqrt{29} + 4 \\
& 0699873480352667)) + 2 \cdot \sqrt{-\sqrt{214095423017213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 40699873480352667} / 9367136 \\
& 690688 + 199631405 / 37468546762752 + 1583563 \cdot \sqrt{29} / 430672951296) \cdot \text{atan}(454 \\
& 016646529997363147366848 \cdot x / (3601609981798895040 \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) \cdot \sqrt{-4 \cdot \sqrt{214095423017213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 40699873480352667} + 199631405 + 13776 \\
& 9981 \cdot \sqrt{29}) + 4509673516272467429860 \cdot \sqrt{1218} \cdot \sqrt{-4 \cdot \sqrt{21409542301 \\
& 7213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}})} \\
& + 40699873480352667) + 199631405 + 137769981 \cdot \sqrt{29}) + 20905585576953 \cdot \sqrt{1218} \cdot \sqrt{214095423017213 \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218} \cdot \sqrt{-18098 \\
& 3329 + 45923327 \cdot \sqrt{29}}) + 40699873480352667) \cdot \sqrt{-4 \cdot \sqrt{214095423017213 \\
& \cdot \sqrt{29} + 47106822945 \cdot \sqrt{1218} \cdot \sqrt{-180983329 + 45923327 \cdot \sqrt{29}}) + 4
\end{aligned}$$

$$\begin{aligned}
& (-4\sqrt{214095423017213\sqrt{29}} + 47106822945\sqrt{1218})\sqrt{-180983329} \\
& + 45923327\sqrt{29}) + 40699873480352667) + 199631405 + 137769981\sqrt{29}) \\
& + 4509673516272467429860\sqrt{1218})\sqrt{-4\sqrt{214095423017213\sqrt{29}} \\
& + 47106822945\sqrt{1218})\sqrt{-180983329 + 45923327\sqrt{29})} + 40699873480 \\
& 352667) + 199631405 + 137769981\sqrt{29}) + 20905585576953\sqrt{1218})\sqrt{(\\
& 214095423017213\sqrt{29} + 47106822945\sqrt{1218})\sqrt{-180983329 + 4592332 \\
& 7\sqrt{29})} + 40699873480352667)\sqrt{-4\sqrt{214095423017213\sqrt{29}} + 47 \\
& 106822945\sqrt{1218})\sqrt{-180983329 + 45923327\sqrt{29})} + 406998734803526 \\
& 67) + 199631405 + 137769981\sqrt{29})) + 55683134469459984392598\sqrt{42})\sqrt{ \\
& -180983329 + 45923327\sqrt{29})/(3601609981798895040\sqrt{-180983329 + \\
& 45923327\sqrt{29})}\sqrt{-4\sqrt{214095423017213\sqrt{29}} + 47106822945\sqrt{ \\
& 1218})\sqrt{-180983329 + 45923327\sqrt{29})} + 40699873480352667) + 19963140 \\
& 5 + 137769981\sqrt{29}) + 4509673516272467429860\sqrt{1218})\sqrt{-4\sqrt{21 \\
& 4095423017213\sqrt{29} + 47106822945\sqrt{1218})\sqrt{-180983329 + 45923327\sqrt{ \\
& 29})} + 40699873480352667) + 199631405 + 137769981\sqrt{29}) + 20905585 \\
& 576953\sqrt{1218})\sqrt{214095423017213\sqrt{29} + 47106822945\sqrt{1218})\sqrt{ \\
& -180983329 + 45923327\sqrt{29})} + 40699873480352667)\sqrt{-4\sqrt{214095 \\
& 423017213\sqrt{29} + 47106822945\sqrt{1218})\sqrt{-180983329 + 45923327\sqrt{ \\
& 29})} + 40699873480352667) + 199631405 + 137769981\sqrt{29}))
\end{aligned}$$

$$3.47 \quad \int (1 + 4x + 4x^2 + 4x^4)^4 dx$$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + 4x + 4x^2 + 4x^4)^4 dx &= \int (1 + 16x + 112x^2 + 448x^3 + 1136x^4 + 1984x^5 + 2752x^6 + 3584x^7 + 4192x^8 + 384x^9 + 256x^{10} + 112x^{11} + 56x^{12} + 16x^{13} + 4x^{14} + x^{15}) dx \\ &= x + 8x^2 + \frac{112x^3}{3} + 112x^4 + \frac{1136x^5}{5} + \frac{992x^6}{3} + \frac{2752x^7}{7} + 448x^8 + \frac{4192x^9}{9} + 384x^{10} + \frac{3328x^{11}}{11} + 256x^{12} + \frac{1792x^{13}}{13} + \frac{512x^{14}}{7} + \frac{1024x^{15}}{15} + \frac{256x^{17}}{17} \end{aligned}$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 4x + 4x^2 + 4x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]

fricas [A] time = 0.72, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + 112x^4 + \frac{112}{3}x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")

[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x

giac [A] time = 0.36, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + 112x^4 + \frac{112}{3}x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")

[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x

maple [A] time = 0.00, size = 78, normalized size = 0.80

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + 112x^4 + \frac{112}{3}x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^4, x)

[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/17*x^17

maxima [A] time = 0.57, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^9 + 448x^8 + \frac{2752}{7}x^7 + \frac{992}{3}x^6 + \frac{1136}{5}x^5 + 112x^4 + \frac{112}{3}x^3 + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")

[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x

mupad [B] time = 0.15, size = 77, normalized size = 0.79

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)

[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17

sympy [A] time = 0.07, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^9}{9} + 448x^8 + \frac{2752x^7}{7} + \frac{992x^6}{3} + \frac{1136x^5}{5} + 112x^4 + \frac{112x^3}{3} + 8x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)

[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 3328*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x

$$3.48 \quad \int (1 + 4x + 4x^2 + 4x^4)^3 dx$$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + 4x + 4x^2 + 4x^4)^3 dx &= \int (1 + 12x + 60x^2 + 160x^3 + 252x^4 + 288x^5 + 352x^6 + 384x^7 + 240x^8 + 192x^9 \\ &\quad + 48x^{10} + 48x^{11} + 64x^{13}) dx \\ &= x + 6x^2 + 20x^3 + 40x^4 + \frac{252x^5}{5} + 48x^6 + \frac{352x^7}{7} + 48x^8 + \frac{80x^9}{3} + \frac{96x^{10}}{5} + \frac{192x^{11}}{11} + \frac{64x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 4x + 4x^2 + 4x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]

fricas [A] time = 0.71, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")

[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x

giac [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")

[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^3,x)

[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+192/11*x^11+64/13*x^13

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13} + \frac{192}{11}x^{11} + \frac{96}{5}x^{10} + \frac{80}{3}x^9 + 48x^8 + \frac{352}{7}x^7 + 48x^6 + \frac{252}{5}x^5 + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")

[Out] $64/13*x^{13} + 192/11*x^{11} + 96/5*x^{10} + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x$

mupad [B] time = 0.06, size = 57, normalized size = 0.83

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)

[Out] $x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 + (80*x^9)/3 + (96*x^{10})/5 + (192*x^{11})/11 + (64*x^{13})/13$

sympy [A] time = 0.07, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^9}{3} + 48x^8 + \frac{352x^7}{7} + 48x^6 + \frac{252x^5}{5} + 40x^4 + 20x^3 + 6x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)

[Out] $64*x^{13}/13 + 192*x^{11}/11 + 96*x^{10}/5 + 80*x^9/3 + 48*x^8 + 352*x^7/7 + 48*x^6 + 252*x^5/5 + 40*x^4 + 20*x^3 + 6*x^2 + x$

$$3.49 \quad \int (1 + 4x + 4x^2 + 4x^4)^2 dx$$

Optimal. Leaf size=45

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]

[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (1 + 4x + 4x^2 + 4x^4)^2 dx &= \int (1 + 8x + 24x^2 + 32x^3 + 24x^4 + 32x^5 + 32x^6 + 16x^8) dx \\ &= x + 4x^2 + 8x^3 + 8x^4 + \frac{24x^5}{5} + \frac{16x^6}{3} + \frac{32x^7}{7} + \frac{16x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]

[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 4x + 4x^2 + 4x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]

[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]

fricas [A] time = 0.74, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

giac [A] time = 0.25, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

maple [A] time = 0.00, size = 38, normalized size = 0.84

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^4+4*x^2+4*x+1)^2,x)

[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9

maxima [A] time = 0.57, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9 + \frac{32}{7}x^7 + \frac{16}{3}x^6 + \frac{24}{5}x^5 + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")

[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x

mupad [B] time = 0.03, size = 37, normalized size = 0.82

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)

[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)/9

sympy [A] time = 0.06, size = 42, normalized size = 0.93

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)

[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 + x

$$3.50 \quad \int (1 + 4x + 4x^2 + 4x^4) dx$$

Optimal. Leaf size=21

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

[In] Int[1 + 4*x + 4*x^2 + 4*x^4, x]

[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5

Rubi steps

$$\int (1 + 4x + 4x^2 + 4x^4) dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4, x]

[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 4x + 4x^2 + 4x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1 + 4*x + 4*x^2 + 4*x^4, x]

[Out] IntegrateAlgebraic[1 + 4*x + 4*x^2 + 4*x^4, x]

fricas [A] time = 1.18, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")

[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x

giac [A] time = 0.36, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")

[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(4*x^4+4*x^2+4*x+1,x)

[Out] x+2*x^2+4/3*x^3+4/5*x^5

maxima [A] time = 0.57, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")

[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 4*x^2 + 4*x^4 + 1,x)
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

```
sympy [A] time = 0.06, size = 19, normalized size = 0.90
```

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
```

```
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

$$3.51 \quad \int \frac{1}{1+4x+4x^2+4x^4} dx$$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2(1+\sqrt{5})}\right)$$

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2069, 1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}}(\sqrt{5}-2)\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2(1+\sqrt{5})}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(\frac{1}{x}+1\right)^2-1\right)-\frac{1}{2}\sqrt{\frac{1}{5}}(2+\sqrt{5})\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2(1+\sqrt{5})}+2}{\sqrt{2(\sqrt{5}-1)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}}(2+\sqrt{5})\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2(1+\sqrt{5})}+2}{\sqrt{2(\sqrt{5}-1)}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])]) + 2/x]/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])]) + 2/x]/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2)/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]]*(1 + x^(-1)) + (1 + x^(-1))^2)/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628


```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]* (a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]* (a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c))*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p]/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+4x+4x^2+4x^4} dx &= -\left(16 \operatorname{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right)\right) \\
&= -\left(16 \operatorname{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right)\right) - 16 \operatorname{Subst}\left(\int \frac{16+16x}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right) \\
&= 512 \operatorname{Subst}\left(\int \frac{x}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right) - \frac{\operatorname{Subst}\left(\int \frac{16\sqrt{2(1+\sqrt{5})}-(16-16\sqrt{5})x}{\sqrt{5}-\sqrt{2(1+\sqrt{5})}x+x^2} dx, x, 1+\frac{1}{x}\right)}{32\sqrt{10}(1+\sqrt{5})} \\
&= 256 \operatorname{Subst}\left(\int \frac{1}{1280-512x+256x^2} dx, x, \left(1+\frac{1}{x}\right)^2\right) + \frac{(1-\sqrt{5}) \operatorname{Subst}\left(\int \frac{-\sqrt{2(1+\sqrt{5})}}{\sqrt{5}-\sqrt{2(1+\sqrt{5})}x+x^2} dx, x, 1+\frac{1}{x}\right)}{4\sqrt{10}(1+\sqrt{5})} \\
&= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}} \log\left(\sqrt{5}-\sqrt{2(1+\sqrt{5})}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right) + \frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}} \log\left(\sqrt{5}+\sqrt{2(1+\sqrt{5})}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right) \\
&= \frac{1}{2} \tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{(1+\sqrt{5})^{3/2} \tan^{-1}\left(\frac{2-\sqrt{2(1+\sqrt{5})}+\frac{2}{x}}{\sqrt{2(-1+\sqrt{5})}}\right)}{4\sqrt{10}} - \frac{(1+\sqrt{5})^{3/2} \tan^{-1}\left(\frac{2+\sqrt{2(1+\sqrt{5})}+\frac{2}{x}}{\sqrt{2(-1+\sqrt{5})}}\right)}{4\sqrt{10}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.20

$$\frac{1}{4} \operatorname{RootSum}\left[4\#1^4 + 4\#1^2 + 4\#1 + 1 \&, \frac{\log(x - \#1)}{4\#1^3 + 2\#1 + 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+4x+4x^2+4x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]

fricas [C] time = 4.09, size = 499, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/20*(\sqrt{10}*\sqrt{-15/8*(2*\sqrt{1/10*I} - 1/5) - I})^2 - 5/4*(2*\sqrt{1/10*I} \\ & I - 1/5) - I)*(2*\sqrt{-1/10*I} - 1/5) + I) - 15/8*(2*\sqrt{-1/10*I} - 1/5) + I \\ &)^2 - 9) - 5*\sqrt{1/10*I} - 1/5) - 5*\sqrt{-1/10*I} - 1/5))*\log(5/2*(2*\sqrt{1/ \\ & 10*I} - 1/5) - I)^2*(12*\sqrt{-1/10*I} - 1/5) + 6*I - 1) + 15*(2*\sqrt{1/10*I} - \\ & 1/5) - I)*(2*\sqrt{-1/10*I} - 1/5) + I)^2 - 5/2*(2*\sqrt{-1/10*I} - 1/5) + I)^ \\ & 2 + ((6*\sqrt{10})*(2*\sqrt{-1/10*I} - 1/5) + I) - \sqrt{10})*(2*\sqrt{1/10*I} - 1 \\ & /5) - I) - \sqrt{10}*(2*\sqrt{-1/10*I} - 1/5) + I))*\sqrt{-15/8*(2*\sqrt{1/10*I} \\ & - 1/5) - I})^2 - 5/4*(2*\sqrt{1/10*I} - 1/5) - I)*(2*\sqrt{-1/10*I} - 1/5) + I) \\ & - 15/8*(2*\sqrt{-1/10*I} - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(\sqrt{10}*\sqrt{ \\ & -15/8*(2*\sqrt{1/10*I} - 1/5) - I})^2 - 5/4*(2*\sqrt{1/10*I} - 1/5) - I)*(2*\sqrt{ \\ & (-1/10*I} - 1/5) + I) - 15/8*(2*\sqrt{-1/10*I} - 1/5) + I)^2 - 9) + 5*\sqrt{1/1 \\ & 0*I} - 1/5) + 5*\sqrt{-1/10*I} - 1/5))*\log(5/2*(2*\sqrt{1/10*I} - 1/5) - I)^2*(1 \\ & 2*\sqrt{-1/10*I} - 1/5) + 6*I - 1) + 15*(2*\sqrt{1/10*I} - 1/5) - I)*(2*\sqrt{-1 \\ & /10*I} - 1/5) + I)^2 - 5/2*(2*\sqrt{-1/10*I} - 1/5) + I)^2 - ((6*\sqrt{10})*(2*s \\ & \sqrt{-1/10*I} - 1/5) + I) - \sqrt{10})*(2*\sqrt{1/10*I} - 1/5) - I) - \sqrt{10}*(\\ & 2*\sqrt{-1/10*I} - 1/5) + I))*\sqrt{-15/8*(2*\sqrt{1/10*I} - 1/5) - I})^2 - 5/4*(\\ & 2*\sqrt{1/10*I} - 1/5) - I)*(2*\sqrt{-1/10*I} - 1/5) + I) - 15/8*(2*\sqrt{-1/10* \\ & I} - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*\sqrt{1/10*I} - 1/5) - I)*\log(-5*(2* \\ & \sqrt{1/10*I} - 1/5) - I)^2*(12*\sqrt{-1/10*I} - 1/5) + 6*I - 1) - 30*(2*\sqrt{1 \\ & /10*I} - 1/5) - I)*(2*\sqrt{-1/10*I} - 1/5) + I)^2 - 30*(2*\sqrt{-1/10*I} - 1/5) \\ & + I)^3 + 8*x - 216*\sqrt{-1/10*I} - 1/5) - 108*I + 21) - 1/4*(2*\sqrt{-1/10*I} \\ & - 1/5) + I)*\log(30*(2*\sqrt{-1/10*I} - 1/5) + I)^3 + 5*(2*\sqrt{-1/10*I} - 1/5 \\ &) + I)^2 + 8*x + 216*\sqrt{-1/10*I} - 1/5) + 108*I - 27) \end{aligned}$$

giac [C] time = 0.52, size = 265, normalized size = 1.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/20*((I + 2)*\sqrt{\sqrt{5} - 2}*(I/(\sqrt{5} - 2) + 1) + 5*I)*\log((406*I + \\ & 174)*\sqrt{5}*x + (868*I + 372)*x + 29*\sqrt{5})*\sqrt{29*\sqrt{5} + 62} + (87*I \\ & - 203)*\sqrt{5} + (19*I + 62)*\sqrt{29*\sqrt{5} + 62} + 186*I - 434) - 1/20*(\\ & (I + 2)*\sqrt{\sqrt{5} - 2}*(-I/(\sqrt{5} - 2) - 1) + 5*I)*\log((406*I + 174)*s \\ & \sqrt{5}*x + (868*I + 372)*x - 29*\sqrt{5})*\sqrt{29*\sqrt{5} + 62} + (87*I - 203 \\ &)*\sqrt{5} - (19*I + 62)*\sqrt{29*\sqrt{5} + 62} + 186*I - 434) - 1/20*((2*I + \end{aligned}$$

1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) - (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)

maple [C] time = 0.01, size = 41, normalized size = 0.18

$$\frac{\ln\left(-\text{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right) + x\right)}{16 \text{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right)^3 + 8 \text{RootOf}\left(4_Z^4 + 4_Z^2 + 4_Z + 1\right) + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^4+4*x^2+4*x+1),x)

[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4x^4 + 4x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")

[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)

mupad [B] time = 2.36, size = 87, normalized size = 0.37

$$\sum_{k=1}^4 \ln\left(-\text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right)\left(\frac{x}{4} + \text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right)\left(6x + \text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right)(36x + 16)\right)\right)\right) \text{root}\left(z^4 + \frac{9z^2}{40} + \frac{z}{40} + \frac{1}{1280}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)

[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k, 1, 4)

sympy [B] time = 2.57, size = 3432, normalized size = 14.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)

[Out] $\sqrt{-1/40 + \sqrt{5}/80} \cdot \log(x^2 + x(-8 - 21\sqrt{5})\sqrt{-2 + \sqrt{5}}) / 10 - \sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 2 - \sqrt{5} / 2 + 12\sqrt{-2 + \sqrt{5}} + 9\sqrt{5}\sqrt{-2 + \sqrt{5}}\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 5 - 841\sqrt{5}\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 20 - 14351/40 - 441\sqrt{-2 + \sqrt{5}} / 4 - 75\sqrt{5}\sqrt{-2 + \sqrt{5}}\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 8 - 3\sqrt{-2 + \sqrt{5}}\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) + 301\sqrt{5}\sqrt{-2 + \sqrt{5}} / 10 + 7407\sqrt{5} / 40 + 3913\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 40 - \sqrt{-1/40 + \sqrt{5}/80} \cdot \log(x^2 + x(-8 - 12\sqrt{-2 + \sqrt{5}}) - \sqrt{5} / 2 + 21\sqrt{5})\sqrt{-2 + \sqrt{5}}) / 10 + \sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 2 + 9\sqrt{5}\sqrt{-2 + \sqrt{5}}\sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 5 - 3913\sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 40 - 14351/40 - 75\sqrt{5}\sqrt{-2 + \sqrt{5}}\sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 8 - 301\sqrt{5}\sqrt{-2 + \sqrt{5}} / 10 - 3\sqrt{-2 + \sqrt{5}}\sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) + 441\sqrt{-2 + \sqrt{5}} / 4 + 7407\sqrt{5} / 40 + 841\sqrt{5}\sqrt{2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 20 - 2\sqrt{3/80 + 3\sqrt{5}/80 + \sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / 40} \cdot \operatorname{atan}(-20x / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 6\sqrt{5}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19))\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) - 18\sqrt{5}\sqrt{-2 + \sqrt{5}}\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19) / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 6\sqrt{5}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19))\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) - 120\sqrt{-2 + \sqrt{5}} / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 6\sqrt{5}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19))\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{5} / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 6\sqrt{5}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19))\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 5\sqrt{-2 + \sqrt{5}}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 6\sqrt{5}\sqrt{3 + 3\sqrt{5}} + 2\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19))\sqrt{-2\sqrt{5}}\sqrt{-2 + \sqrt{5}} + \sqrt{5} + 19)) + 21\sqrt{5}\sqrt{-2 + \sqrt{5}} / (-27\sqrt{5})\sqrt{3 + 3\sqrt{5}}$

$$3.52 \quad \int \frac{1}{(1+4x+4x^2+4x^4)^2} dx$$

Optimal. Leaf size=317

$$-\frac{17 - \left(\frac{1}{x} + 1\right)^2}{2\left(\left(\frac{1}{x} + 1\right)^4 - 2\left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{\left(59 - 17\left(\frac{1}{x} + 1\right)^2\right)\left(\frac{1}{x} + 1\right)}{10\left(\left(\frac{1}{x} + 1\right)^4 - 2\left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{1}{40} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \log\left(\left(\frac{1}{x} + 1\right)^2 - \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)}\right)$$

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2069, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$\frac{17 - \left(\frac{1}{x} + 1\right)^2}{2\left(\left(\frac{1}{x} + 1\right)^4 - 2\left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{\left(59 - 17\left(\frac{1}{x} + 1\right)^2\right)\left(\frac{1}{x} + 1\right)}{10\left(\left(\frac{1}{x} + 1\right)^4 - 2\left(\frac{1}{x} + 1\right)^2 + 5\right)} + \frac{1}{40} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \log\left(\left(\frac{1}{x} + 1\right)^2 - \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)}\right) - \frac{1}{20} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \tan^{-1}\left(\frac{\frac{1}{x} + 1}{\sqrt{2(\sqrt{5} - 1)}}\right) - \frac{1}{20} \sqrt{\frac{1}{10} (2665\sqrt{5} - 5959)} \tan^{-1}\left(\frac{\frac{1}{x} + 1}{\sqrt{2(\sqrt{5} - 1)}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

[Out] $-(17 - (1 + x^{-1})^2)/(2*(5 - 2*(1 + x^{-1})^2 + (1 + x^{-1})^4)) + ((59 - 17*(1 + x^{-1})^2)*(1 + x^{-1}))/((10*(5 - 2*(1 + x^{-1})^2 + (1 + x^{-1})^4)) + (7*ArcTan[(-1 + (1 + x^{-1})^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])]] + 2/x]/Sqrt[2*(-1 + Sqrt[5])]))/20 - (Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])]] + 2/x]/Sqrt[2*(-1 + Sqrt[5])]))/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^{-1}) + (1 + x^{-1})^2])/40 - (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^{-1}) + (1 + x^{-1})^2])/40$

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] :> \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1169

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4}, x_Symbol] :> \text{With}[\{q = \text{Rt}[a/c, 2]\}, \text{With}[\{r = \text{Rt}[2*q - b/c, 2]\}, \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2*c*q*r), \text{Int}[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NegQ}[b^2 - 4*a*c]$

Rule 1660

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{With}[\{Q = \text{PolynomialQuotient}[Pq, a + b*x + c*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x + c*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x + c*x^2, x], x, 1]\}, \text{Simp}[(b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^{(p + 1)}/((p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/((p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x + c*x^2)^{(p + 1)*\text{ExpandToSum}[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g)], x], x]]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

Rule 1663

$\text{Int}[(Pq_)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] :> \text{Dist}[1/2, \text{Subst}[\text{Int}[x^{(m - 1)/2}*\text{SubstFor}[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{PolyQ}[Pq, x^2] \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 1673

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] :> \text{Module}[\{q = \text{Expon}[Pq, x], k\}, \text{Int}[\text{Sum}[\text{Coeff}[Pq, x, 2*k]*x^{(2*k)}, \{k, 0, q/2\}](a + b$

$*x^2 + c*x^4)^p, x] + \text{Int}[x*\text{Sum}[\text{Coeff}[\text{Pq}, x, 2*k + 1]*x^{(2*k)}, \{k, 0, (q - 1)/2\}](a + b*x^2 + c*x^4)^p, x]] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{!PolyQ}[\text{Pq}, x^2]$

Rule 1678

$\text{Int}[(\text{Pq}_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \text{ :> With}[\{d = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2 + c*x^4, x], x, 0], e = \text{Coeff}[\text{PolynomialRemainder}[\text{Pq}, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[(x*(a + b*x^2 + c*x^4)^{(p + 1)}*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[(a + b*x^2 + c*x^4)^{(p + 1)}*\text{ExpandToSum}[2*a*(p + 1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[\text{Pq}, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{PolyQ}[\text{Pq}, x^2] \&\& \text{Expon}[\text{Pq}, x^2] > 1 \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1]$

Rule 2069

$\text{Int}[(\text{P4}_.)^{(p_.)}, x_Symbol] \text{ :> With}[\{a = \text{Coeff}[\text{P4}, x, 0], b = \text{Coeff}[\text{P4}, x, 1], c = \text{Coeff}[\text{P4}, x, 2], d = \text{Coeff}[\text{P4}, x, 3], e = \text{Coeff}[\text{P4}, x, 4]\}, \text{Dist}[-16*a^2, \text{Subst}[\text{Int}[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^p]/(b - 4*a*x)^2, x], x, b/(4*a) + 1/x], x] /; \text{NeQ}[a, 0] \&\& \text{NeQ}[b, 0] \&\& \text{EqQ}[b^3 - 4*a*b*c + 8*a^2*d, 0]] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[\text{P4}, x, 4] \&\& \text{IntegerQ}[2*p] \&\& \text{!IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+4x+4x^2+4x^4)^2} dx &= -\left(16 \operatorname{Subst}\left(\int \frac{(4-4x)^6}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x}\right)\right) \\
&= -\left(16 \operatorname{Subst}\left(\int \frac{x(-24576-81920x^2-24576x^4)}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x}\right)\right) - 16 \operatorname{Subst}\left(\int \frac{4}{(1280-512x^2+256x^4)^2} dx, x, 1+\frac{1}{x}\right) \\
&= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right)}{167772160} \\
&= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\operatorname{Subst}\left(\int -\frac{4}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right)}{167772160} \\
&= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896 \operatorname{Subst}\left(\int \frac{4}{1280-512x^2+256x^4} dx, x, 1+\frac{1}{x}\right) \\
&= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40} \sqrt{-\frac{5959}{10}} + \frac{7}{4} \tan^{-1}\left(\frac{1}{2}\left(-\frac{17-\left(1+\frac{1}{x}\right)^2}{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}\right)\right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 108, normalized size = 0.34

$$\frac{1}{40} \left(\operatorname{RootSum}\left[4\#1^4 + 4\#1^2 + 4\#1 + 1 \&, \frac{18\#1^2 \log(x - \#1) - 16\#1 \log(x - \#1) + 27 \log(x - \#1)}{4\#1^3 + 2\#1 + 1} \&\right] + \frac{72x^3 - 32x^2 + 84x + 38}{4x^4 + 4x^2 + 4x + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 &, (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2)/(1 + 2*#1 + 4*#1^3) &])/40

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 + 4x + 4x^2 + 4x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]

fricas [C] time = 3.89, size = 704, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")

[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 45160856496*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I))

$$\begin{aligned} & /1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*\sqrt{-19/1000} \\ & *I - 5959/2000) - 7*I))*\log(125/8*(4271136*\sqrt{19/1000*I - 5959/2000} + 74 \\ & 74488*I + 94043)*(4*\sqrt{-19/1000*I - 5959/2000} - 7*I)^2 + 11755375/8*(4*s \\ & \sqrt{19/1000*I - 5959/2000} + 7*I)^2 + 25/2*(1334730*(4*\sqrt{19/1000*I - 595} \\ & 9/2000) + 7*I)^2 + 219601)*(4*\sqrt{-19/1000*I - 5959/2000} - 7*I) - 1/2*\sqrt{ \\ & t(-375/32*(4*\sqrt{19/1000*I - 5959/2000} + 7*I)^2 - 125/16*(4*\sqrt{19/1000* \\ & I - 5959/2000} + 7*I)*(4*\sqrt{-19/1000*I - 5959/2000} - 7*I) - 375/32*(4*\sqrt{ \\ & rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*\sqrt{10})*(4*\sqrt{19 \\ & /1000*I - 5959/2000} + 7*I) + 94043*\sqrt{10}))*\sqrt{10})*\sqrt{-19/1000*I - 5959/200} \\ & 0) - 7*I) + 470215*\sqrt{10}*(4*\sqrt{19/1000*I - 5959/2000} + 7*I) - 878404* \\ & \sqrt{10}) + 541735337*x + 10980050*\sqrt{19/1000*I - 5959/2000} + 38430175/2 \\ & *I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1) \end{aligned}$$

giac [C] time = 0.72, size = 315, normalized size = 0.99

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & -1/400*(-(I + 3)*\sqrt{2665*\sqrt{5} - 4790}*(709*I/(533*\sqrt{5} - 958) + 1) \\ & - 350*I)*\log((2534636224790*I + 16853816172010)*\sqrt{5}*x - (3913528401620* \\ & I + 26022625108780)*x + 5049076145*\sqrt{5}*\sqrt{1424281*\sqrt{5} - 2199118} \\ & - (8426908086005*I - 1267318112395)*\sqrt{5} + (8166407345*I - 7795873310)*\sqrt{ \\ & rt(1424281*\sqrt{5} - 2199118) + 13011312554390*I - 1956764200810) - 1/400* \\ & ((I + 3)*\sqrt{2665*\sqrt{5} - 4790}*(709*I/(533*\sqrt{5} - 958) + 1) - 350*I) \\ & *\log((2534636224790*I + 16853816172010)*\sqrt{5}*x - (3913528401620*I + 2602 \\ & 2625108780)*x - 5049076145*\sqrt{5}*\sqrt{1424281*\sqrt{5} - 2199118} - (84269 \\ & 08086005*I - 1267318112395)*\sqrt{5} - (8166407345*I - 7795873310)*\sqrt{1424 \\ & 281*\sqrt{5} - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I + \\ & 1)*\sqrt{2665*\sqrt{5} + 4790}*(709*I/(533*\sqrt{5} + 958) + 1) + 350*I)*\log((\\ & 16722951192450*I + 2480822188910)*\sqrt{5}*x + (25712356272300*I + 381438558 \\ & 5140)*x + 5021907265*\sqrt{5}*\sqrt{1416617*\sqrt{5} + 2178118} + (12404110944 \\ & 55*I - 8361475596225)*\sqrt{5} + (8153361745*I + 7721428310)*\sqrt{1416617*\sqrt{ \\ & rt(5) + 2178118} + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*\sqrt{ \\ & rt(2665*\sqrt{5} + 4790}*(709*I/(533*\sqrt{5} + 958) + 1) + 350*I)*\log((16722 \\ & 951192450*I + 2480822188910)*\sqrt{5}*x + (25712356272300*I + 3814385585140) \\ & *x - 5021907265*\sqrt{5}*\sqrt{1416617*\sqrt{5} + 2178118} + (1240411094455*I \\ & - 8361475596225)*\sqrt{5} - (8153361745*I + 7721428310)*\sqrt{1416617*\sqrt{5} \\ & + 2178118} + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 4 \\ & 2*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) \end{aligned}$$

maple [C] time = 0.01, size = 79, normalized size = 0.25

$$\frac{(18\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1)^2 - 16\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1) + 27)\ln(-\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1) + x)}{160\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1)^3 + 80\text{RootOf}(4_Z^4 + 4_Z^2 + 4_Z + 1) + 40} + \frac{\frac{9}{20}x^3 - \frac{1}{5}x^2 + \frac{21}{40}x + \frac{19}{80}}{x^4 + x^2 + x + \frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x^4+4*x^2+4*x+1)^2,x)`

[Out] $(9/20*x^3 - 1/5*x^2 + 21/40*x + 19/80)/(x^4 + x^2 + x + 1/4) + 1/40*\text{sum}((18*_R^2 - 16*_R + 27)/(4*_R^3 + 2*_R + 1)*\ln(-_R + x), _R = \text{RootOf}(4*_Z^4 + 4*_Z^2 + 4*_Z + 1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{36x^3 - 16x^2 + 42x + 19}{20(4x^4 + 4x^2 + 4x + 1)} + \frac{1}{10} \int \frac{18x^2 - 16x + 27}{4x^4 + 4x^2 + 4x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")`

[Out] $1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*\text{integrate}((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)$

mupad [B] time = 2.21, size = 174, normalized size = 0.55

$$\left(\sum \ln \left(\frac{169 \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}}}{100} + \frac{11x}{1600} \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} + 131 \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} + 72 \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} + 72 \right) - \text{root} \left(z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}, z, k \right)^3 x^{36} + \frac{59 \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}}}{20} - 16 \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} \right) \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} + \frac{11x}{1600} \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} + \frac{19}{1600} \sqrt{z^4 + \frac{3021z^2}{1000} - \frac{133z}{8000} + \frac{29}{64000}} \right) / (z^4 + z^2 + z + \frac{1}{4})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)`

[Out] $\text{symsum}(\log((11*x)/1600 - (169*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k))/100 + (131*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)*x)/100 - (72*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^2*x)/5 - 36*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3*x + (59*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^2)/20 - 16*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3 + 27/1600)*\text{root}(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k, 1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)$

sympy [B] time = 3.66, size = 3834, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)`

[Out] $(36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - \text{sqrt}(-5959/16000 + 533*\text{sqrt}(5)/3200)*\log(x**2 + x*(-1601676*\text{sqrt}(10)*\text{sqrt}(-5959 + 2665*\text{sqrt}(5)))*\text{sqrt}(-665*\text{sqrt}(10)*\text{sqrt}(-5959 + 2665*\text{sqrt}(5)) + 221195*\text{sqrt}(5) + 36004639)/13543383425 - 1067784*\text{sqrt}(2)*\text{sqrt}(-5959 + 2665*\text{sqrt}(5))/101638$

$$\begin{aligned}
& 9 + 3131659367*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}/13543383425 + 291689395/ \\
& 1083470674 + 470215*\sqrt{5}/2032778 + 94043*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + \\
& 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/541735337) - 40634464149111451* \\
& \sqrt{5}*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36 \\
& 004639)/27530691871904650 - 2885835544225227917282997/146738587677251784500 \\
& - 83803227754187*\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}/100111606806926 - 5020 \\
& 8805356*\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + \\
& 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/550613837438093 - 53848575489193 \\
& 3*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665* \\
& \sqrt{5}}) + 221195*\sqrt{5} + 36004639)/14673858767725178450 - 92532195509690 \\
& 1411*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}/29347717535450356900 + 48430461193 \\
& 8766076267*\sqrt{5}/55061383743809300 + 22013036087014785403*\sqrt{-665*\sqrt{ \\
& 10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/666993580351144 \\
& 4750) + \sqrt{-5959/16000 + 533*\sqrt{5}/3200}*\log(x**2 + x*(-94043*\sqrt{665* \\
& \sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/541735337 \\
& - 1601676*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{665*\sqrt{10}*\sqrt{-5959 \\
& + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/13543383425 - 3131659367*\sqrt{ \\
& 10}*\sqrt{-5959 + 2665*\sqrt{5}}/13543383425 + 291689395/1083470674 + 1067784 \\
& *\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}/1016389 + 470215*\sqrt{5}/2032778) - 220 \\
& 13036087014785403*\sqrt{665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{ \\
& 5} + 36004639)/6669935803511444750 - 2885835544225227917282997/1467385876 \\
& 77251784500 - 50208805356*\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{665*\sqrt{ \\
& 10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/550613837438093 \\
& - 538485754891933*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{665*\sqrt{10}*\sqrt{ \\
& -5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/14673858767725178450 \\
& + 925321955096901411*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}/293477175354503569 \\
& 00 + 83803227754187*\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}/100111606806926 + 48 \\
& 4304611938766076267*\sqrt{5}/55061383743809300 + 40634464149111451*\sqrt{5}*\sqrt{ \\
& 665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/27 \\
& 530691871904650) + 2*\sqrt{6291/16000 + 1599*\sqrt{5}/3200} + \sqrt{-665*\sqrt{1 \\
& 0}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/4000)*\operatorname{atan}(54173 \\
& 533700*x/(-6440570878*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5}} + 4*\sqrt{-665*\sqrt{ \\
& 10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{ \\
& (-5959 + 2665*\sqrt{5})*\sqrt{6291 + 7995*\sqrt{5}} + 4*\sqrt{-665*\sqrt{10}*\sqrt{ \\
& (-5959 + 2665*\sqrt{5}) + 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{ \\
& t(6291 + 7995*\sqrt{5}) + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 2 \\
& 21195*\sqrt{5} + 36004639))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + \\
& 221195*\sqrt{5} + 36004639)) - 3203352*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{ \\
& -665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)/ \\
& (-6440570878*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5}} + 4*\sqrt{-665*\sqrt{10}*\sqrt{ \\
& -5959 + 2665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{-5959 + 2 \\
& 665*\sqrt{5}}*\sqrt{6291 + 7995*\sqrt{5}} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2 \\
& 665*\sqrt{5}}) + 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{6291 + 7 \\
& 995*\sqrt{5}} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{ \\
& 5} + 36004639))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}) + 221195*\sqrt{
\end{aligned}$$

$$\begin{aligned}
& t(5) + 36004639)) - 28456443600*\sqrt{2}*\sqrt{-5959 + 2665*\sqrt{5}}/(-644057 \\
& 0878*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + \\
& 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5))*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} \\
& t(5) + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 3 \\
& 6004639))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + \\
& 36004639)) + 6263318734*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}}/(-6440570878*\sqrt{10} \\
& t(5))*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{6291 \\
& t(5) + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + \\
& 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} + \\
& 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639 \\
&))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639 \\
& 9)) + 7292234875/(-6440570878*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-6 \\
& 65*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) + 2351 \\
& 075*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10} \\
& t(5))*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10} \\
& t(5))*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) + 6265614875*\sqrt{5}/(-6440570878*\sqrt{10} \\
& t(5))*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{6291 \\
& t(5) + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 22 \\
& 1195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665 \\
& t(5))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639))* \\
& \sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) \\
& + 4702150*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + \\
& 36004639}/(-6440570878*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10} \\
& t(5))*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) + 2351075*\sqrt{-5959 \\
& t(5))*\sqrt{6291 + 7995*\sqrt{5} + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) + 1067784*\sqrt{10}*\sqrt{6291 + 7995*\sqrt{5} \\
& t(5) + 4*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639)) \\
& + 221195*\sqrt{5} + 36004639))*\sqrt{-665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} \\
& t(5) + 221195*\sqrt{5} + 36004639)) - 2*\sqrt{-\sqrt{665*\sqrt{10}*\sqrt{-5959 + 266 \\
& 5*\sqrt{5}} + 221195*\sqrt{5} + 36004639}/4000 + 6291/16000 + 1599*\sqrt{5}/32 \\
& 00)*\operatorname{atan}(54173533700*x/(2351075*\sqrt{-5959 + 2665*\sqrt{5}}*\sqrt{-4*\sqrt{665 \\
& t(5))*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639} + 6291 + \\
& 7995*\sqrt{5}) + 6440570878*\sqrt{10}*\sqrt{-4*\sqrt{665*\sqrt{10}*\sqrt{-5959 + \\
& 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639} + 6291 + 7995*\sqrt{5}) + 1067784 \\
& t(5))*\sqrt{665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 3 \\
& 6004639))*\sqrt{-4*\sqrt{665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5} \\
& t(5) + 36004639} + 6291 + 7995*\sqrt{5})) - 4702150*\sqrt{665*\sqrt{10}*\sqrt{-5 \\
& 959 + 2665*\sqrt{5}} + 221195*\sqrt{5} + 36004639}/(2351075*\sqrt{-5959 + 2665 \\
& t(5))*\sqrt{-4*\sqrt{665*\sqrt{10}*\sqrt{-5959 + 2665*\sqrt{5}} + 221195*\sqrt{5}}
\end{aligned}$$

$$3.53 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^4 dx$$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8$$

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - \frac{566912x^7}{7} - 30720x^6 + \frac{538624x^5}{5} + 139776x^4 + \frac{237568x^3}{3} + 24576x^2 + 4096x$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^4 dx = \int (4096 + 49152x + 237568x^2 + 559104x^3 + 538624x^4 - 184320x^5 - 566912x^6 + 36384x^7 + 641152x^8 - 169584x^9 - 331040x^{10} + 31128x^{11} - 75504x^{12} + 102784x^{13} - 1920x^{14} + 4096x^{15}) dx$$

$$= 4096x + 24576x^2 + \frac{237568x^3}{3} + 139776x^4 + \frac{538624x^5}{5} - 30720x^6 - \frac{566912x^7}{7} + 36384x^8 + \frac{641152x^9}{9} - \frac{169584x^{10}}{5} - \frac{331040x^{11}}{11} + 31128x^{12} - \frac{75504x^{13}}{7} + \frac{102784x^{14}}{15} - 1920x^{15} + \frac{4096x^{16}}{17}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.00

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - \frac{566912x^7}{7} - 30720x^6 + \frac{538624x^5}{5} + 139776x^4 + \frac{237568x^3}{3} + 24576x^2 + 4096x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

[Out] $4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^{10})/5 - (331040*x^{11})/11 + 31128*x^{12} - (12095*x^{13})/13 - (75504*x^{14})/7 + (102784*x^{15})/15 - 1920*x^{16} + (4096*x^{17})/17$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]

fricas [A] time = 0.61, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")

[Out] $4096/17*x^{17} - 1920*x^{16} + 102784/15*x^{15} - 75504/7*x^{14} - 12095/13*x^{13} + 31128*x^{12} - 331040/11*x^{11} - 169584/5*x^{10} + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x$

giac [A] time = 0.36, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")

[Out] $4096/17*x^{17} - 1920*x^{16} + 102784/15*x^{15} - 75504/7*x^{14} - 12095/13*x^{13} + 31128*x^{12} - 331040/11*x^{11} - 169584/5*x^{10} + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x$

maple [A] time = 0.00, size = 85, normalized size = 0.82

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)

[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17

maxima [A] time = 0.66, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")

[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 + 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x^2 + 4096*x

mupad [B] time = 2.23, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)

[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/15 - 1920*x^16 + (4096*x^17)/17

sympy [A] time = 0.08, size = 100, normalized size = 0.96

$$\frac{4096}{17}x^{17} - 1920x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384x^8 - \frac{566912}{7}x^7 - 30720x^6 + \frac{538624}{5}x^5 + 139776x^4 + \frac{237568}{3}x^3 + 24576x^2 + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)

[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x**3/3 + 24576*x**2 + 4096*x

$$3.54 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^3 dx$$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12 + (512*x^13)/13

Rule 2061

Int[(P_)^(p_), x_Symbol] := Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && IntegerQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^3 dx &= \int (512 + 4608x + 15360x^2 + 20160x^3 - 384x^4 - 17856x^5 + 5528x^6 + 16128x^7 - 2976x^8 + 5120x^9 + 2304x^{10} + 512x^{11}) dx \\ &= 512x + 2304x^2 + 5120x^3 + 5040x^4 - \frac{384x^5}{5} - 2976x^6 + \frac{5528x^7}{7} + 2097x^8 - \frac{2936x^9}{3} - \frac{4527x^{10}}{10} + \frac{6936x^{11}}{11} - 240x^{12} + \frac{512x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

[Out] $512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^{10})/10 + (6936*x^{11})/11 - 240*x^{12} + (512*x^{13})/13$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]

fricas [A] time = 0.39, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")

[Out] $512/13*x^{13} - 240*x^{12} + 6936/11*x^{11} - 4527/10*x^{10} - 2936/3*x^9 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 512*x$

giac [A] time = 0.38, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")

[Out] $512/13*x^{13} - 240*x^{12} + 6936/11*x^{11} - 4527/10*x^{10} - 2936/3*x^9 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 512*x$

maple [A] time = 0.00, size = 65, normalized size = 0.86

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3, x)

[Out] $512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-2936/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13$

maxima [A] time = 0.65, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13} - 240x^{12} + \frac{6936}{11}x^{11} - \frac{4527}{10}x^{10} - \frac{2936}{3}x^9 + 2097x^8 + \frac{5528}{7}x^7 - 2976x^6 - \frac{384}{5}x^5 + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")`

[Out] $512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 512*x$

mupad [B] time = 0.08, size = 64, normalized size = 0.84

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)`

[Out] $512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12 + (512*x^13)/13$

sympy [A] time = 0.08, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^9}{3} + 2097x^8 + \frac{5528x^7}{7} - 2976x^6 - \frac{384x^5}{5} + 5040x^4 + 5120x^3 + 2304x^2 + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)`

[Out] $512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 2097*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 2304*x**2 + 512*x$

$$3.55 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2 dx$$

Optimal. Leaf size=52

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]

[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2 dx &= \int (64 + 384x + 704x^2 + 144x^3 - 528x^4 + 144x^5 + 353x^6 - 240x^7 + 64x^8) dx \\ &= 64x + 192x^2 + \frac{704x^3}{3} + 36x^4 - \frac{528x^5}{5} + 24x^6 + \frac{353x^7}{7} - 30x^8 + \frac{64x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]

[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]

[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]

fricas [A] time = 0.86, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")

[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x

giac [A] time = 0.36, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")

[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)

[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9

maxima [A] time = 0.61, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9 - 30x^8 + \frac{353}{7}x^7 + 24x^6 - \frac{528}{5}x^5 + 36x^4 + \frac{704}{3}x^3 + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")

[Out] $64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 192*x^2 + 64*x$

mupad [B] time = 0.03, size = 44, normalized size = 0.85

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)

[Out] $64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7 - 30*x^8 + (64*x^9)/9$

sympy [A] time = 0.07, size = 49, normalized size = 0.94

$$\frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)

[Out] $64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**3/3 + 192*x**2 + 64*x$

$$3.56 \quad \int (8 + 24x + 8x^2 - 15x^3 + 8x^4) dx$$

Optimal. Leaf size=30

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4, x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5

Rubi steps

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4) dx = 8x + 12x^2 + \frac{8x^3}{3} - \frac{15x^4}{4} + \frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Antiderivative was successfully verified.

[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4, x]

[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (8 + 24x + 8x^2 - 15x^3 + 8x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4, x]

[Out] IntegrateAlgebraic[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4, x]

fricas [A] time = 0.73, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")

[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x

giac [A] time = 0.30, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")

[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x)

[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5

maxima [A] time = 0.46, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")

[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)`

[Out] `8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5`

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)`

[Out] `8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x`

$$3.57 \quad \int \frac{1}{8+24x+8x^2-15x^3+8x^4} dx$$

Optimal. Leaf size=263

$$\frac{1}{4} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39} x^2} \right) - \frac{1}{8} \sqrt{\frac{235\sqrt{517} - 5167}{40326}} \log \left(\left(\frac{4}{x} + 3 \right)^2 - \sqrt{2(19 + \sqrt{517})} \left(\frac{4}{x} + 3 \right) + \sqrt{517} \right) + \frac{1}{8} \sqrt{\frac{235\sqrt{517} - 5167}{40326}} \log \left(\left(\frac{4}{x} + 3 \right)^2 + \sqrt{2(19 + \sqrt{517})} \left(\frac{4}{x} + 3 \right) + \sqrt{517} \right) + \frac{1}{4} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39} x^2} \right)$$

Rubi [A] time = 0.49, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{4} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39} x^2} \right) - \frac{1}{8} \sqrt{\frac{235\sqrt{517} - 5167}{40326}} \log \left(\left(\frac{4}{x} + 3 \right)^2 - \sqrt{2(19 + \sqrt{517})} \left(\frac{4}{x} + 3 \right) + \sqrt{517} \right) + \frac{1}{8} \sqrt{\frac{235\sqrt{517} - 5167}{40326}} \log \left(\left(\frac{4}{x} + 3 \right)^2 + \sqrt{2(19 + \sqrt{517})} \left(\frac{4}{x} + 3 \right) + \sqrt{517} \right) - \frac{1}{4} \sqrt{\frac{5167 + 235\sqrt{517}}{40326}} \tan^{-1} \left(\frac{\frac{4}{x} - \sqrt{2(19 + \sqrt{517})} + 6}{\sqrt{2(\sqrt{517} - 19)}} \right) + \frac{1}{4} \sqrt{\frac{5167 + 235\sqrt{517}}{40326}} \tan^{-1} \left(\frac{\frac{4}{x} + \sqrt{2(19 + \sqrt{517})} + 6}{\sqrt{2(\sqrt{517} - 19)}} \right)$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])])]/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])])]/4 + (Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)]/4 - (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2)/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]]*(3 + 4/x) + (3 + 4/x)^2)/8

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^(p_)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{8 + 24x + 8x^2 - 15x^3 + 8x^4} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(24 - 32x)^2}{8(2117632 - 2490368x^2 + 1048576x^4)} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int \frac{(24 - 32x)^2}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\
&= - \left(128 \operatorname{Subst} \left(\int -\frac{1536x}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) - 12 \\
&= 196608 \operatorname{Subst} \left(\int \frac{x}{2117632 - 2490368x^2 + 1048576x^4} dx, x, \frac{3}{4} + \frac{1}{x} \right) - \dots \\
&= 98304 \operatorname{Subst} \left(\int \frac{1}{2117632 - 2490368x + 1048576x^2} dx, x, \left(\frac{3}{4} + \frac{1}{x} \right)^2 \right) - \dots \\
&= -\frac{1}{8} \sqrt{\frac{-5167 + 235\sqrt{517}}{40326}} \log \left(\sqrt{517} - \sqrt{2(19 + \sqrt{517})} \left(3 + \frac{4}{x} \right) + \left(3 + \frac{4}{x} \right)^2 \right) \\
&= -\frac{1}{4} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{19 - \left(3 + \frac{4}{x} \right)^2}{2\sqrt{39}} \right) - \frac{1}{4} \sqrt{\frac{5167 + 235\sqrt{517}}{40326}} \tan^{-1} \left(\frac{6 + \sqrt{2(19 + \sqrt{517})}}{\sqrt{2(-19 + \sqrt{517})}} \right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.21

$$\operatorname{RootSum} \left[8\#1^4 - 15\#1^3 + 8\#1^2 + 24\#1 + 8\&, \frac{\log(x - \#1)}{32\#1^3 - 45\#1^2 + 16\#1 + 24} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 - 45*#1^2 + 32*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8 + 24x + 8x^2 - 15x^3 + 8x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="giac")

[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)

maple [C] time = 0.01, size = 49, normalized size = 0.19

$$\frac{\ln(-\text{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8) + x)}{32\text{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^3 - 45\text{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^2 + 16\text{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8) + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8), x)

[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(-_R+x), _R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8), x, algorithm="maxima")

[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)

mupad [B] time = 0.41, size = 123, normalized size = 0.47

$$\sum_{k=1}^4 \ln \left(\frac{\text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right) \left(2184 \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right) + 256x + \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right) x 38259 + \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right)^2 x 1531920 + 805896 \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right)^2 - 120\right)}{4096} \right) \text{root}\left(z^4 + \frac{2455z^2}{161304} + \frac{109z}{430144} + \frac{1}{786357}, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8), x)

[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) * (2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 256*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) * x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k), k, 1, 4)

sympy [A] time = 2.37, size = 41, normalized size = 0.16

$$\text{RootSum}\left(50326848t^4 + 765960t^2 + 12753t + 64, \left(t \mapsto t \log\left(\frac{100785893208t^3}{4758335} - \frac{1430512512t^2}{4758335} + \frac{72982352521t}{223641745} + x + \frac{2270349121}{1789133960}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8), x)

[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(100785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641745 + x + 2270349121/1789133960)))

$$3.58 \quad \int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2} dx$$

Optimal. Leaf size=366

$$\frac{73}{208} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39} x^2} \right) - \frac{3 \left(3359 - 107 \left(\frac{4}{x} + 3 \right)^2 \right)}{208 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)} + \frac{\left(3327931 - 129631 \left(\frac{4}{x} + 3 \right)^2 \right) \left(\frac{4}{x} + 3 \right)}{322608 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)}$$

Rubi [A] time = 0.51, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{73}{208} \sqrt{\frac{3}{13}} \tan^{-1} \left(\frac{-5x^2 + 12x + 8}{\sqrt{39} x^2} \right) - \frac{3 \left(3359 - 107 \left(\frac{4}{x} + 3 \right)^2 \right)}{208 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)} + \frac{\left(3327931 - 129631 \left(\frac{4}{x} + 3 \right)^2 \right) \left(\frac{4}{x} + 3 \right)}{322608 \left(\left(\frac{4}{x} + 3 \right)^4 - 38 \left(\frac{4}{x} + 3 \right)^2 + 517 \right)}$$

Antiderivative was successfully verified.

[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) + ((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])]])/645216 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 + Sqrt[2*(19 + Sqrt[517])]) + 8/x]/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)]/208 - (Sqrt[(-59644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]])*(3 + 4/x) + (3 + 4/x)^2)/645216 + (Sqrt[(-59644114671451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]])*(3 + 4/x) + (3 + 4/x)^2)/645216

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx &= - \left(1024 \operatorname{Subst} \left(\int \frac{(24 - 32x)^6}{64 (2117632 - 2490368x^2 + 1048576x^4)^2} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right. \\
&= - \left(16 \operatorname{Subst} \left(\int \frac{(24 - 32x)^6}{(2117632 - 2490368x^2 + 1048576x^4)^2} dx, x, \frac{3}{4} + \frac{1}{x} \right) \right) \\
&= - \left(16 \operatorname{Subst} \left(\int \frac{x(-1528823808 - 9059696640x^2 - 4831838208x^4)}{(2117632 - 2490368x^2 + 1048576x^4)^2} dx \right. \right. \\
&= \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right) \left(3 + \frac{4}{x} \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} - \frac{\operatorname{Subst} \left(\int \frac{120925685220163941564}{2117632 - 2490368x^2 + 1048576x^4} dx \right)}{70944} \\
&= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\
&= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\
&= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} \\
&= - \frac{3 \left(3359 - 107 \left(3 + \frac{4}{x} \right)^2 \right)}{208 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)} + \frac{\left(3327931 - 129631 \left(3 + \frac{4}{x} \right)^2 \right)}{322608 \left(517 - 38 \left(3 + \frac{4}{x} \right)^2 + \left(3 + \frac{4}{x} \right)^4 \right)}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 128, normalized size = 0.35

$$\frac{\operatorname{RootSum} \left[8\#1^4 - 15\#1^3 + 8\#1^2 + 24\#1 + 8 \&, \frac{19640\#1^2 \log(x-\#1) - 57489\#1 \log(x-\#1) + 74897 \log(x-\#1)}{32\#1^3 - 45\#1^2 + 16\#1 + 24} \& \right]}{80652} + \frac{39280x^3 - 94314x^2 + 89033x + 72888}{161304(8x^4 - 15x^3 + 8x^2 + 24x + 8)}$$

Antiderivative was successfully verified.

[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#1^2 + 32*#1^3) &]/80652

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8 + 24x + 8x^2 - 15x^3 + 8x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")

[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)

maple [C] time = 0.01, size = 96, normalized size = 0.26

$$\frac{(19640 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^2 - 57489 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8) + 74897) \ln(-\operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8) + x)}{2580864 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^3 - 3629340 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8)^2 + 1290432 \operatorname{RootOf}(8_Z^4 - 15_Z^3 + 8_Z^2 + 24_Z + 8) + 1935648} + \frac{2455}{80652} x^3 - \frac{1429}{19552} x^2 + \frac{89033}{1290432} x + \frac{3037}{53768} x^4 - \frac{15}{8} x^3 + x^2 + 3x + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)

[Out] $(2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^2+3*x+1)+1/80652*\text{sum}((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)*\ln(-_R+x),_R=\text{RootOf}(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{39280x^3 - 94314x^2 + 89033x + 72888}{161304(8x^4 - 15x^3 + 8x^2 + 24x + 8)} + \frac{1}{80652} \int \frac{19640x^2 - 57489x + 74897}{8x^4 - 15x^3 + 8x^2 + 24x + 8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")`

[Out] $1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8) + 1/80652*\text{integrate}((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)$

mupad [B] time = 0.21, size = 181, normalized size = 0.49

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)`

[Out] $((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x + x^2 - (15*x^3)/8 + x^4 + 1) + \text{symsum}(\log((2146659825*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (924124364159*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72451101*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k)^2)/264704 - (100737*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k)^3)/512 + 271033/624455529984)*\text{root}(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/44204510553294663, z, k), k, 1, 4)$

sympy [B] time = 3.95, size = 3839, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)

[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 + 1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-112396995020468503306932567484755463/603722125611976319526135612861060 - 29643869829812833230907750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)))/1936419398792394461637855141912238396080 - 181533261043120360732*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/150930531402994079881533903215265 - 46926347979646613249222*sqrt(517)/29746860362632912338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517)))/1427849297406379792240272 + 994065243322493861977*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1290946265861596307758570094608158930720) - 4597149706773066968921854791223560238809189313591735176029*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1843276718699869862645060404837476389014874805380627572841955840 - 1022132763720267175882780425063613131088601935958303878081158710949715459967411486447/302201812380681690634631534385892067350866441656553353409624708723614680800 - 106380947173342801261761115266827766437283828356533836993*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/6896193703069972436744723519626607862706189949382980866560 - 890360389298500646731845595593034670326044595870824169313*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/9352473884121677079601749613489889898672849258229891014611209554309158400 - 45113976327488809325094501633826014671791*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/107753026610468319324136304994165747854784217959109076040 + 42698009636515468718900942734274005212255282299552802371283821308121207*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/9352473884121677079601749613489889898672849258229891014611209554309158400 + 68548776709669674081892851407413209373218007060934353137152573209405073*sqrt(517)/460819179674967465661265101209369097253718701345156893210488960 + 274196431933554153007434570764680602132735098624644758583157528631167033*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/4835228998090907050154104550174273077613863066504853654553995339577834892800) - sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-112396995020468503306932567484755463/603722125611976319526135612861060 - 994065243322493861977*sqrt(78)*sqrt(-59644114671451

$$\begin{aligned}
& + 2623170438295*\sqrt{517})/1427849297406379792240272 - 4692634797964661324 \\
& 9222*\sqrt{517}/29746860362632912338339 + 181533261043120360732*\sqrt{7120427} \\
& 417275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 626 \\
& 3621568587150042935*\sqrt{517} + 3557579971691991294769382675)/1509305314029 \\
& 94079881533903215265 + 29643869829812833230907750777733957*\sqrt{40326}*\sqrt{ \\
& (-59644114671451 + 2623170438295*\sqrt{517})/1936419398792394461637855141912 \\
& 238396080 + 994065243322493861977*\sqrt{40326}*\sqrt{-59644114671451 + 262317} \\
& 0438295*\sqrt{517}}*\sqrt{7120427417275887*\sqrt{40326}*\sqrt{-59644114671451 + \\
& 2623170438295*\sqrt{517}} + 6263621568587150042935*\sqrt{517} + 355757997169 \\
& 1991294769382675)/1290946265861596307758570094608158930720) - 2741964319335 \\
& 54153007434570764680602132735098624644758583157528631167033*\sqrt{7120427417} \\
& 275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 626362 \\
& 1568587150042935*\sqrt{517} + 3557579971691991294769382675)/4835228998090907 \\
& 050154104550174273077613863066504853654553995339577834892800 - 102213276372 \\
& 0267175882780425063613131088601935958303878081158710949715459967411486447/3 \\
& 02201812380681690634631534385892067350866441656553353409624708723614680800 \\
& - 890360389298500646731845595593034670326044595870824169313*\sqrt{40326}*\sqrt{ \\
& (-59644114671451 + 2623170438295*\sqrt{517})*\sqrt{7120427417275887*\sqrt{403} \\
& 26}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 626362156858715004293 \\
& 5*\sqrt{517} + 3557579971691991294769382675)/9352473884121677079601749613489 \\
& 889898672849258229891014611209554309158400 - 426980096365154687189009427342 \\
& 74005212255282299552802371283821308121207*\sqrt{40326}*\sqrt{-59644114671451} \\
& + 2623170438295*\sqrt{517})/935247388412167707960174961348988989867284925822 \\
& 9891014611209554309158400 - 45113976327488809325094501633826014671791*\sqrt{(} \\
& 78)*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}*\sqrt{7120427417275887*} \\
& \sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 62636215685871 \\
& 50042935*\sqrt{517} + 3557579971691991294769382675)/107753026610468319324136 \\
& 304994165747854784217959109076040 + 106380947173342801261761115266827766437 \\
& 2838283556533836993*\sqrt{78}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} \\
&)/6896193703069972436744723519626607862706189949382980866560 + 685487767096 \\
& 69674081892851407413209373218007060934353137152573209405073*\sqrt{517}/46081 \\
& 9179674967465661265101209369097253718701345156893210488960 + 45971497067730 \\
& 66968921854791223560238809189313591735176029*\sqrt{517}*\sqrt{712042741727588} \\
& 7*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 62636215685 \\
& 87150042935*\sqrt{517} + 3557579971691991294769382675)/184327671869986986264 \\
& 5060404837476389014874805380627572841955840) - 2*\sqrt{59653665894623/167878} \\
& 62468089856 + 5073830635*\sqrt{517}/10823895853056 + \sqrt{-7120427417275887*} \\
& \sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 6263621568587 \\
& 150042935*\sqrt{517} + 3557579971691991294769382675)/4196965617022464)*\operatorname{atan}(\\
& -7745677595169577846551420567648953584320*x/(-59292486929118917272637172801 \\
& 533436*\sqrt{40326}*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-} \\
& 7120427417275887*\sqrt{40326}*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} \\
&) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 232 \\
& 9048502925820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}* \\
& \sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-7120427417275887*} \\
\end{aligned}$$

$$\begin{aligned}
& t(40326)*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 6263621568587150 \\
& 042935*\sqrt{517} + 3557579971691991294769382675) + 994065243322493861977*s \\
& \text{qrt}(40326)*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-71204274} \\
& 17275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 6263 \\
& 621568587150042935*\sqrt{517} + 3557579971691991294769382675))*\sqrt{-7120427} \\
& 417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}} + 626 \\
& 3621568587150042935*\sqrt{517} + 3557579971691991294769382675)) - 2982195729 \\
& 967481585931*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{ \\
& \text{rt}(-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{ \\
& 517}} + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)/(- \\
& 59292486929118917272637172801533436*\sqrt{40326})*\sqrt{59653665894623 + 78695} \\
& 11314885*\sqrt{517} + 4*\sqrt{-7120427417275887*\sqrt{40326})*\sqrt{-59644114671} \\
& 451 + 2623170438295*\sqrt{517}} + 6263621568587150042935*\sqrt{517} + 3557579 \\
& 971691991294769382675)) + 2329048502925820708785386304*\sqrt{-59644114671451} \\
& + 2623170438295*\sqrt{517}})*\sqrt{59653665894623 + 7869511314885*\sqrt{517} +} \\
& 4*\sqrt{-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*} \\
& \sqrt{517}} + 6263621568587150042935*\sqrt{517} + 355757997169199129476938267 \\
& 5)) + 994065243322493861977*\sqrt{40326})*\sqrt{59653665894623 + 7869511314885} \\
& *\sqrt{517} + 4*\sqrt{-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 26} \\
& 23170438295*\sqrt{517}} + 6263621568587150042935*\sqrt{517} + 355757997169199 \\
& 1294769382675))*\sqrt{-7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2} \\
& 623170438295*\sqrt{517}} + 6263621568587150042935*\sqrt{517} + 35575799716919 \\
& 91294769382675)) - 2696261047060775175517112572266328310*\sqrt{78})*\sqrt{-596} \\
& 44114671451 + 2623170438295*\sqrt{517}})/(-5929248692911891727263717280153343 \\
& 6*\sqrt{40326})*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-71204} \\
& 27417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6 \\
& 263621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 23290485 \\
& 02925820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{(5} \\
& 9653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-7120427417275887*\sqrt{403} \\
& 26})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362156858715004293 \\
& 5*\sqrt{517} + 3557579971691991294769382675)) + 994065243322493861977*\sqrt{(4} \\
& 0326)*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-7120427417275} \\
& 887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362156 \\
& 8587150042935*\sqrt{517} + 3557579971691991294769382675))*\sqrt{-712042741727} \\
& 5887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215 \\
& 68587150042935*\sqrt{517} + 3557579971691991294769382675)) + 610949118223023 \\
& 8698537149348154570111680*\sqrt{517}})/(-59292486929118917272637172801533436*s \\
& \text{qrt}(40326)*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-71204274} \\
& 17275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263 \\
& 621568587150042935*\sqrt{517} + 3557579971691991294769382675)) + 23290485029 \\
& 25820708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{5965} \\
& 3665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-7120427417275887*\sqrt{40326} \\
& }*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*s \\
& \text{qrt}(517) + 3557579971691991294769382675)) + 994065243322493861977*\sqrt{4032} \\
& 6)*\sqrt{59653665894623 + 7869511314885*\sqrt{517} + 4*\sqrt{-7120427417275887}
\end{aligned}$$

$$\begin{aligned}
& * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 626362156858 \\
& 7150042935 * \sqrt{517} + 3557579971691991294769382675) * \sqrt{-712042741727588 \\
& 7 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 62636215685 \\
& 87150042935 * \sqrt{517} + 3557579971691991294769382675) + 465809700585164141 \\
& 7570772608 * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 262317 \\
& 0438295 * \sqrt{517}} + 6263621568587150042935 * \sqrt{517} + 3557579971691991294 \\
& 769382675) / (-59292486929118917272637172801533436 * \sqrt{40326} * \sqrt{596536658 \\
& 94623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{ \\
& (-59644114671451 + 2623170438295 * \sqrt{517})} + 6263621568587150042935 * \sqrt{5 \\
& 17} + 3557579971691991294769382675) + 2329048502925820708785386304 * \sqrt{-5 \\
& 9644114671451 + 2623170438295 * \sqrt{517}}) * \sqrt{59653665894623 + 786951131488 \\
& 5 * \sqrt{517} + 4 * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2 \\
& 623170438295 * \sqrt{517}} + 6263621568587150042935 * \sqrt{517} + 35575799716919 \\
& 91294769382675) + 994065243322493861977 * \sqrt{40326} * \sqrt{59653665894623 + \\
& 7869511314885 * \sqrt{517} + 4 * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{-596441 \\
& 14671451 + 2623170438295 * \sqrt{517}} + 6263621568587150042935 * \sqrt{517} + 35 \\
& 57579971691991294769382675) * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{-59644 \\
& 114671451 + 2623170438295 * \sqrt{517}} + 6263621568587150042935 * \sqrt{517} + 3 \\
& 557579971691991294769382675) + 59287739659625666461815501555467914 * \sqrt{40 \\
& 326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} / (-59292486929118917272 \\
& 637172801533436 * \sqrt{40326} * \sqrt{59653665894623 + 7869511314885 * \sqrt{517} + \\
& 4 * \sqrt{-7120427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \\
& \sqrt{517}} + 6263621568587150042935 * \sqrt{517} + 355757997169199129476938267 \\
& 5) + 2329048502925820708785386304 * \sqrt{-59644114671451 + 2623170438295 * \sqrt{ \\
& 517}}) * \sqrt{59653665894623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{-71204274172 \\
& 75887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 6263621 \\
& 568587150042935 * \sqrt{517} + 3557579971691991294769382675) + 99406524332249 \\
& 3861977 * \sqrt{40326} * \sqrt{59653665894623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{ \\
& -7120427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517 \\
& }} + 6263621568587150042935 * \sqrt{517} + 3557579971691991294769382675) * \sqrt{ \\
& (-7120427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{51 \\
& 7}} + 6263621568587150042935 * \sqrt{517} + 3557579971691991294769382675) + 7 \\
& 21019529648624138729760776730387270795368 / (-5929248692911891727263717280153 \\
& 3436 * \sqrt{40326} * \sqrt{59653665894623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{-71 \\
& 20427417275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}}) \\
& + 6263621568587150042935 * \sqrt{517} + 3557579971691991294769382675) + 23290 \\
& 48502925820708785386304 * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}}) * \sqrt{ \\
& 59653665894623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{-7120427417275887 * \sqrt{ \\
& 40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 626362156858715004 \\
& 2935 * \sqrt{517} + 3557579971691991294769382675) + 994065243322493861977 * \sqrt{ \\
& 40326} * \sqrt{59653665894623 + 7869511314885 * \sqrt{517} + 4 * \sqrt{-7120427417 \\
& 275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 626362 \\
& 1568587150042935 * \sqrt{517} + 3557579971691991294769382675) * \sqrt{-712042741 \\
& 7275887 * \sqrt{40326} * \sqrt{-59644114671451 + 2623170438295 * \sqrt{517}} + 62636 \\
& 21568587150042935 * \sqrt{517} + 3557579971691991294769382675) - 2 * \sqrt{-\sqrt{
\end{aligned}$$

$$\begin{aligned}
& t(7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675)/4196 \\
& 965617022464 + 59653665894623/16787862468089856 + 5073830635*\sqrt{517})/1082 \\
& 3895853056)*\operatorname{atan}(7745677595169577846551420567648953584320*x/(23290485029258 \\
& 20708785386304*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{-4*\sqrt{ \\
& (7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517} \\
&)) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769382675) + 596 \\
& 53665894623 + 7869511314885*\sqrt{517})) + 5929248692911891727263717280153343 \\
& 6*\sqrt{40326})*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-5964411467145 \\
& 1 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997 \\
& 1691991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) + 99406524 \\
& 3322493861977*\sqrt{40326})*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-596441146 \\
& 71451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 35575 \\
& 79971691991294769382675)*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59 \\
& 644114671451 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} \\
& + 3557579971691991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) \\
&) - 721019529648624138729760776730387270795368/(232904850292582070878538630 \\
& 4*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}})*\sqrt{-4*\sqrt{712042741727 \\
& 5887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215 \\
& 68587150042935*\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + \\
& 7869511314885*\sqrt{517})) + 59292486929118917272637172801533436*\sqrt{40326} \\
& *\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 262317043 \\
& 8295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 3557579971691991294769 \\
& 382675) + 59653665894623 + 7869511314885*\sqrt{517})) + 994065243322493861977 \\
& *\sqrt{40326})*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 26231 \\
& 70438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997169199129 \\
& 4769382675)*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 \\
& + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 35575799716 \\
& 91991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) - 269626104 \\
& 7060775175517112572266328310*\sqrt{78})*\sqrt{-59644114671451 + 2623170438295* \\
& \sqrt{517}})/(2329048502925820708785386304*\sqrt{-59644114671451 + 26231704382 \\
& 95*\sqrt{517}})*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-5964411467145 \\
& 1 + 2623170438295*\sqrt{517}}) + 6263621568587150042935*\sqrt{517} + 355757997 \\
& 1691991294769382675) + 59653665894623 + 7869511314885*\sqrt{517})) + 59292486 \\
& 929118917272637172801533436*\sqrt{40326})*\sqrt{-4*\sqrt{7120427417275887*\sqrt{ \\
& 40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 626362156858715004 \\
& 2935*\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 786951131 \\
& 4885*\sqrt{517})) + 994065243322493861977*\sqrt{40326})*\sqrt{7120427417275887*s \\
& \sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 62636215685871 \\
& 50042935*\sqrt{517} + 3557579971691991294769382675)*\sqrt{-4*\sqrt{71204274172 \\
& 75887*\sqrt{40326})*\sqrt{-59644114671451 + 2623170438295*\sqrt{517}}) + 6263621 \\
& 568587150042935*\sqrt{517} + 3557579971691991294769382675) + 59653665894623 \\
& + 7869511314885*\sqrt{517})) - 6109491182230238698537149348154570111680*\sqrt{ \\
& (517)/(2329048502925820708785386304*\sqrt{-59644114671451 + 2623170438295*s \\
& \sqrt{517}})*\sqrt{-4*\sqrt{7120427417275887*\sqrt{40326})*\sqrt{-59644114671451 + 2
\end{aligned}$$

$623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 35575799716919$
 $91294769382675) + 59653665894623 + 7869511314885\sqrt{517}) + 5929248692911$
 $8917272637172801533436\sqrt{40326})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326}}$
 $)\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} +$
 $3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}) +$
 $994065243322493861977\sqrt{40326})\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)$
 $\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) +$
 $6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 786$
 $9511314885\sqrt{517})) + 4658097005851641417570772608\sqrt{7120427417275887}$
 $\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} +$
 $3557579971691991294769382675)/(2329048502925820708785386304\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) +$
 $59653665894623 + 7869511314885\sqrt{517}) + 59292486929118917272637172801533436\sqrt{40326})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) +$
 $59653665894623 + 7869511314885\sqrt{517}) + 994065243322493861977\sqrt{40326})\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 +$
 $2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) +$
 $59653665894623 + 7869511314885\sqrt{517})) + 59287739659625666461815501555467914\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517})/(2329048502925820708785386304\sqrt{-59644114671451 + 2623170438295\sqrt{517})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}) + 59292486929118917272637172801533436\sqrt{40326})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}) + 994065243322493861977\sqrt{40326})\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517})) + 2982195729967481585931\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517})\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)/(2329048502925820708785386304\sqrt{-59644114671451 + 2623170438295\sqrt{517})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517}) + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 78$

$$\begin{aligned} & 69511314885\sqrt{517}) + 59292486929118917272637172801533436\sqrt{40326})\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517})} \\ & + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517}) + 994065243322493861977\sqrt{40326})\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517})} \\ & + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675)\sqrt{-4\sqrt{7120427417275887}\sqrt{40326})\sqrt{-59644114671451 + 2623170438295\sqrt{517})} \\ & + 6263621568587150042935\sqrt{517} + 3557579971691991294769382675) + 59653665894623 + 7869511314885\sqrt{517})) \end{aligned}$$

3.59

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^3 dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^{16}}{16b}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2059, 32}

$$\frac{(a + bx)^{16}}{16b}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

[Out] (a + b*x)^16/(16*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^3 dx = \int (a + bx)^{15} dx = \frac{(a + bx)^{16}}{16b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{16}}{16b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

[Out] (a + b*x)^16/(16*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^3, x]

fricas [B] time = 0.92, size = 163, normalized size = 11.64

$$\frac{1}{16}x^{16}b^{15} + x^{15}b^{14}a + \frac{15}{2}x^{14}b^{13}a^2 + 35x^{13}b^{12}a^3 + \frac{455}{4}x^{12}b^{11}a^4 + 273x^{11}b^{10}a^5 + \frac{1001}{2}x^{10}b^9a^6 + 715x^9b^8a^7 + \frac{6435}{8}x^8b^7a^8 + 715x^7b^6a^9 + \frac{1001}{2}x^6b^5a^{10} + 273x^5b^4a^{11} + \frac{455}{4}x^4b^3a^{12} + 35x^3b^2a^{13} + \frac{15}{2}x^2ba^{14} + xa^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3, x, algorithm="fricas")

[Out] 1/16*x^16*b^15 + x^15*b^14*a + 15/2*x^14*b^13*a^2 + 35*x^13*b^12*a^3 + 455/4*x^12*b^11*a^4 + 273*x^11*b^10*a^5 + 1001/2*x^10*b^9*a^6 + 715*x^9*b^8*a^7 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 1001/2*x^6*b^5*a^10 + 273*x^5*b^4*a^11 + 455/4*x^4*b^3*a^12 + 35*x^3*b^2*a^13 + 15/2*x^2*b*a^14 + x*a^15

giac [B] time = 0.28, size = 163, normalized size = 11.64

$$\frac{1}{16}b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}a^2b^{13}x^{14} + 35a^3b^{12}x^{13} + \frac{455}{4}a^4b^{11}x^{12} + 273a^5b^{10}x^{11} + \frac{1001}{2}a^6b^9x^{10} + 715a^7b^8x^9 + \frac{6435}{8}a^8b^7x^8 + 715a^9b^6x^7 + \frac{1001}{2}a^{10}b^5x^6 + 273a^{11}b^4x^5 + \frac{455}{4}a^{12}b^3x^4 + 35a^{13}b^2x^3 + \frac{15}{2}a^{14}bx^2 + a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3, x, algorithm="giac")

[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x

maple [B] time = 0.00, size = 164, normalized size = 11.71

$$\frac{1}{16}b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}a^2b^{13}x^{14} + 35a^3b^{12}x^{13} + \frac{455}{4}a^4b^{11}x^{12} + 273a^5b^{10}x^{11} + \frac{1001}{2}a^6b^9x^{10} + 715a^7b^8x^9 + \frac{6435}{8}a^8b^7x^8 + 715a^9b^6x^7 + \frac{1001}{2}a^{10}b^5x^6 + 273a^{11}b^4x^5 + \frac{455}{4}a^{12}b^3x^4 + 35a^{13}b^2x^3 + \frac{15}{2}a^{14}bx^2 + a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)

[Out] 1/16*b^15*x^16+a*b^14*x^15+15/2*a^2*b^13*x^14+35*a^3*b^12*x^13+455/4*a^4*b^11*x^12+273*a^5*b^10*x^11+1001/2*a^6*b^9*x^10+715*a^7*b^8*x^9+6435/8*a^8*b^7*x^8+715*a^9*b^6*x^7+1001/2*a^10*b^5*x^6+273*a^11*b^4*x^5+455/4*a^12*b^3*x^4+35*a^13*b^2*x^3+15/2*a^14*b*x^2+a^15*x

maxima [B] time = 0.68, size = 592, normalized size = 42.29

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="maxima")

[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 + 100*a^4*b^11*x^12 + 1000/7*a^5*b^10*x^11 + 125/4*a^6*b^9*x^10 + a^15*x + 1/2*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 385*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3

mupad [B] time = 0.17, size = 163, normalized size = 11.64

$$a^{15}x + \frac{15a^{14}bx^2}{2} + 35a^{13}b^2x^3 + \frac{455a^{12}b^3x^4}{4} + 273a^{11}b^4x^5 + \frac{1001a^{10}b^5x^6}{2} + 715a^9b^6x^7 + \frac{6435a^8b^7x^8}{8} + 715a^7b^8x^9 + \frac{1001a^6b^9x^{10}}{2} + 273a^5b^{10}x^{11} + \frac{455a^4b^{11}x^{12}}{4} + 35a^3b^{12}x^{13} + \frac{15a^2b^{13}x^{14}}{2} + ab^{14}x^{15} + \frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^3,x)

[Out] $a^{15}x + (b^{15}x^{16})/16 + (15a^{14}b^2x^2)/2 + a^2b^{14}x^{15} + 35a^{13}b^2x^3 + (455a^{12}b^3x^4)/4 + 273a^{11}b^4x^5 + (1001a^{10}b^5x^6)/2 + 715a^9b^6x^7 + (6435a^8b^7x^8)/8 + 715a^7b^8x^9 + (1001a^6b^9x^{10})/2 + 273a^5b^{10}x^{11} + (455a^4b^{11}x^{12})/4 + 35a^3b^{12}x^{13} + (15a^2b^{13}x^{14})/2$

sympy [B] time = 0.11, size = 185, normalized size = 13.21

$$a^{15}x + \frac{15a^{14}bx^2}{2} + 35a^{13}b^2x^3 + \frac{455a^{12}b^3x^4}{4} + 273a^{11}b^4x^5 + \frac{1001a^{10}b^5x^6}{2} + 715a^9b^6x^7 + \frac{6435a^8b^7x^8}{8} + 715a^7b^8x^9 + \frac{1001a^6b^9x^{10}}{2} + 273a^5b^{10}x^{11} + \frac{455a^4b^{11}x^{12}}{4} + 35a^3b^{12}x^{13} + \frac{15a^2b^{13}x^{14}}{2} + ab^{14}x^{15} + \frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5)**3,x)

[Out] $a^{15}x + 15a^{14}b^2x^3 + 35a^{13}b^2x^3 + 455a^{12}b^3x^4/4 + 273a^{11}b^4x^5 + 1001a^{10}b^5x^6/2 + 715a^9b^6x^7 + 6435a^8b^7x^8/8 + 715a^7b^8x^9 + 1001a^6b^9x^{10}/2 + 273a^5b^{10}x^{11} + 455a^4b^{11}x^{12}/4 + 35a^3b^{12}x^{13} + 15a^2b^{13}x^{14}/2 + a^2b^{14}x^{15} + b^{15}x^{16}/16$

3.60

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^2 dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^{11}}{11b}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2059, 32}

$$\frac{(a + bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2,x]

[Out] (a + b*x)^11/(11*b)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2059

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonFreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\int \left(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \right)^2 dx = \int (a + bx)^{10} dx = \frac{(a + bx)^{11}}{11b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2,x]

[Out] (a + b*x)^11/(11*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2,x]

[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^2, x]

fricas [B] time = 1.02, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^9a + 5x^9b^8a^2 + 15x^8b^7a^3 + 30x^7b^6a^4 + 42x^6b^5a^5 + 42x^5b^4a^6 + 30x^4b^3a^7 + 15x^3b^2a^8 + 5x^2ba^9 + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="fricas")

[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*x^2*b*a^9 + x*a^10

giac [B] time = 0.24, size = 108, normalized size = 7.71

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15a^8b^2x^3 + 5a^9bx^2 + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="giac")

[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*a^9*b*x^2 + a^10*x

maple [B] time = 0.00, size = 109, normalized size = 7.79

$$\frac{1}{11}b^{10}x^{11} + ab^9x^{10} + 5a^2b^8x^9 + 15a^3b^7x^8 + 30a^4b^6x^7 + 42a^5b^5x^6 + 42a^6b^4x^5 + 30a^7b^3x^4 + 15a^8b^2x^3 + 5a^9bx^2 + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)$

[Out] $1/11*b^{10}*x^{11}+a*b^9*x^{10}+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^{10}*x$

maxima [B] time = 0.49, size = 228, normalized size = 16.29

$$\frac{1}{11} b^{10} x^{11} + a b^9 x^{10} + \frac{25}{9} a^2 b^8 x^9 + \frac{100}{7} a^3 b^7 x^8 + 20 a^4 b^6 x^7 + \frac{25}{3} a^5 b^5 x^6 + a^{10} x + \frac{1}{3} (b^5 x^6 + 6 a b^4 x^5 + 15 a^2 b^3 x^4 + 20 a^3 b^2 x^3 + 15 a^4 b x^2) a^5 + \frac{5}{21} (6 b^5 x^7 + 35 a b^4 x^6 + 84 a^2 b^3 x^5 + 105 a^3 b^2 x^4) a^4 b + \frac{5}{42} (21 b^5 x^8 + 120 a b^4 x^7 + 280 a^2 b^3 x^6) a^3 b^2 + \frac{5}{18} (8 b^5 x^9 + 45 a b^4 x^8) a^2 b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, \text{algorithm}="maxima")$

[Out] $1/11*b^{10}*x^{11} + a*b^9*x^{10} + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^{10}*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3$

mupad [B] time = 0.06, size = 108, normalized size = 7.71

$$a^{10} x + 5 a^9 b x^2 + 15 a^8 b^2 x^3 + 30 a^7 b^3 x^4 + 42 a^6 b^4 x^5 + 42 a^5 b^5 x^6 + 30 a^4 b^6 x^7 + 15 a^3 b^7 x^8 + 5 a^2 b^8 x^9 + a b^9 x^{10} + \frac{b^{10} x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x)^2,x)$

[Out] $a^{10}*x + (b^{10}*x^{11})/11 + 5*a^9*b*x^2 + a*b^9*x^{10} + 15*a^8*b^2*x^3 + 30*a^7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x^8 + 5*a^2*b^8*x^9$

sympy [B] time = 0.10, size = 114, normalized size = 8.14

$$a^{10} x + 5 a^9 b x^2 + 15 a^8 b^2 x^3 + 30 a^7 b^3 x^4 + 42 a^6 b^4 x^5 + 42 a^5 b^5 x^6 + 30 a^4 b^6 x^7 + 15 a^3 b^7 x^8 + 5 a^2 b^8 x^9 + a b^9 x^{10} + \frac{b^{10} x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b^{**5}*x^{**5}+5*a*b^{**4}*x^{**4}+10*a^{**2}*b^{**3}*x^{**3}+10*a^{**3}*b^{**2}*x^{**2}+5*a^{**4}*b*x+a^{**5})^{**2},x)$

[Out] $a^{**10}*x + 5*a^{**9}*b*x^{**2} + 15*a^{**8}*b^{**2}*x^{**3} + 30*a^{**7}*b^{**3}*x^{**4} + 42*a^{**6}*b^{**4}*x^{**5} + 42*a^{**5}*b^{**5}*x^{**6} + 30*a^{**4}*b^{**6}*x^{**7} + 15*a^{**3}*b^{**7}*x^{**8} + 5*a^{**2}*b^{**8}*x^{**9} + a*b^{**9}*x^{**10} + b^{**10}*x^{**11}/11$

3.61

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx$$

Optimal. Leaf size=14

$$\frac{(a + bx)^6}{6b}$$

Rubi [B] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x + ab^4x^5 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^5 + (b^5*x^6)/6

Rubi steps

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx = a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{b^5x^6}{6}$$

Mathematica [B] time = 0.00, size = 61, normalized size = 4.36

$$a^5x + \frac{5}{2}a^4bx^2 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^2b^3x^4 + ab^4x^5 + \frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^5 + (b^5*x^6)/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

[Out] IntegrateAlgebraic[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5, x]

fricas [B] time = 0.85, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5, x, algorithm="fricas")

[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^4 + x*a^5

giac [B] time = 0.29, size = 53, normalized size = 3.79

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5, x, algorithm="giac")

[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^2 + a^5*x

maple [B] time = 0.00, size = 54, normalized size = 3.86

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5, x)

[Out] a^5*x+5/2*a^4*b*x^2+10/3*a^3*b^2*x^3+5/2*a^2*b^3*x^4+a*b^4*x^5+1/6*b^5*x^6

maxima [B] time = 0.46, size = 53, normalized size = 3.79

$$\frac{1}{6}b^5x^6 + ab^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^2 + a^5*x
```

mupad [B] time = 0.02, size = 53, normalized size = 3.79

$$a^5 x + \frac{5 a^4 b x^2}{2} + \frac{10 a^3 b^2 x^3}{3} + \frac{5 a^2 b^3 x^4}{2} + a b^4 x^5 + \frac{b^5 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2
```

sympy [B] time = 0.07, size = 60, normalized size = 4.29

$$a^5 x + \frac{5 a^4 b x^2}{2} + \frac{10 a^3 b^2 x^3}{3} + \frac{5 a^2 b^3 x^4}{2} + a b^4 x^5 + \frac{b^5 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**4*x**5 + b**5*x**6/6
```


$$3.62 \quad \int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5} dx$$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

[Out] -1/(4*b*(a + b*x)^4)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5} dx &= \int \frac{1}{(a+bx)^5} dx \\ &= -\frac{1}{4b(a+bx)^4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

[Out] -1/4*1/(b*(a + b*x)^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-1), x]

fricas [B] time = 0.77, size = 46, normalized size = 3.29

$$-\frac{1}{4(b^5x^4 + 4ab^4x^3 + 6a^2b^3x^2 + 4a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5), x, algorithm="fricas")

[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{4(bx + a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5), x, algorithm="giac")

[Out] -1/4/((b*x + a)^4*b)

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{1}{4(bx + a)^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)`

[Out] `-1/4/b/(b*x+a)^4`

maxima [B] time = 0.69, size = 46, normalized size = 3.29

$$-\frac{1}{4(b^5x^4 + 4ab^4x^3 + 6a^2b^3x^2 + 4a^3b^2x + a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x, algorithm="maxima")`

[Out] `-1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)`

mupad [B] time = 0.05, size = 48, normalized size = 3.43

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x),x)`

[Out] `-1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)`

sympy [B] time = 0.29, size = 49, normalized size = 3.50

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5),x)`

[Out] `-1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5*x**4)`

$$3.63 \quad \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2} dx$$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

[Out] -1/(9*b*(a + b*x)^9)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2} dx = \int \frac{1}{(a+bx)^{10}} dx$$

$$= -\frac{1}{9b(a+bx)^9}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

[Out] -1/9*1/(b*(a + b*x)^9)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-2), x]

fricas [B] time = 1.17, size = 101, normalized size = 7.21

$$\frac{1}{9(b^{10}x^9 + 9ab^9x^8 + 36a^2b^8x^7 + 84a^3b^7x^6 + 126a^4b^6x^5 + 126a^5b^5x^4 + 84a^6b^4x^3 + 36a^7b^3x^2 + 9a^8b^2x + a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="fricas")

[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a^9*b)

giac [A] time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{9(bx + a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="giac")

[Out] -1/9/((b*x + a)^9*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{9(bx+a)^9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)

[Out] -1/9/b/(b*x+a)^9

maxima [B] time = 0.60, size = 101, normalized size = 7.21

$$\frac{1}{9(b^{10}x^9 + 9ab^9x^8 + 36a^2b^8x^7 + 84a^3b^7x^6 + 126a^4b^6x^5 + 126a^5b^5x^4 + 84a^6b^4x^3 + 36a^7b^3x^2 + 9a^8b^2x + a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x, algorithm="maxima")

[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a^9*b)

mupad [B] time = 2.10, size = 103, normalized size = 7.36

$$\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + 9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^2,x)

[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 + 756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 324*a^2*b^8*x^7)

sympy [B] time = 0.59, size = 109, normalized size = 7.79

$$\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + 9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5)**2,x)

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1  
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*  
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

$$3.64 \quad \int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3} dx$$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

[Out] -1/(14*b*(a + b*x)^14)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2058

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3} dx = \int \frac{1}{(a + bx)^{15}} dx$$

$$= -\frac{1}{14b(a + bx)^{14}}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

[Out] -1/14*1/(b*(a + b*x)^14)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x^5)^(-3), x]

fricas [B] time = 1.13, size = 156, normalized size = 11.14

$$\frac{1}{14(b^{15}x^{14} + 14ab^{14}x^{13} + 91a^2b^{13}x^{12} + 364a^3b^{12}x^{11} + 1001a^4b^{11}x^{10} + 2002a^5b^{10}x^9 + 3003a^6b^9x^8 + 3432a^7b^8x^7 + 3003a^8b^7x^6 + 2002a^9b^6x^5 + 1001a^{10}b^5x^4 + 364a^{11}b^4x^3 + 91a^{12}b^3x^2 + 14a^{13}b^2x + a^{14}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="fricas")

[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{14(bx + a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="giac")

[Out] -1/14/((b*x + a)^14*b)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{14 (bx + a)^{14} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)

[Out] -1/14/b/(b*x+a)^14

maxima [B] time = 0.54, size = 156, normalized size = 11.14

$$-\frac{1}{14 (b^{15}x^{14} + 14ab^{14}x^{13} + 91a^2b^{13}x^{12} + 364a^3b^{12}x^{11} + 1001a^4b^{11}x^{10} + 2002a^5b^{10}x^9 + 3003a^6b^9x^8 + 3432a^7b^8x^7 + 3003a^8b^7x^6 + 2002a^9b^6x^5 + 1001a^{10}b^5x^4 + 364a^{11}b^4x^3 + 91a^{12}b^3x^2 + 14a^{13}b^2x + a^{14}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x, algorithm="maxima")

[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 + 1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)

mupad [B] time = 3.03, size = 158, normalized size = 11.29

$$-\frac{1}{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048a^7b^8x^7 + 42042a^6b^9x^8 + 28028a^5b^{10}x^9 + 14014a^4b^{11}x^{10} + 5096a^3b^{12}x^{11} + 1274a^2b^{13}x^{12} + 196ab^{14}x^{13} + 14b^{15}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x)^3,x)

[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 42042*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)

sympy [B] time = 0.94, size = 168, normalized size = 12.00

$$-\frac{1}{14a^{14}b + 196a^{13}b^2x + 1274a^{12}b^3x^2 + 5096a^{11}b^4x^3 + 14014a^{10}b^5x^4 + 28028a^9b^6x^5 + 42042a^8b^7x^6 + 48048a^7b^8x^7 + 42042a^6b^9x^8 + 28028a^5b^{10}x^9 + 14014a^4b^{11}x^{10} + 5096a^3b^{12}x^{11} + 1274a^2b^{13}x^{12} + 196ab^{14}x^{13} + 14b^{15}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**4*b*x+a**5)**3,x)

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*  
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6  
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140  
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a  
*b**14*x**13 + 14*b**15*x**14)
```

$$3.65 \quad \int \frac{1}{1+x^2+x^3+x^5} dx$$

Optimal. Leaf size=38

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2058, 635, 203, 260, 628}

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{6} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3 + x^5)^(-1), x]

[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 2058

```
Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x^2+x^3+x^5} dx &= \int \left(\frac{1}{6(1+x)} + \frac{1+x}{2(1+x^2)} + \frac{1-2x}{3(1-x+x^2)} \right) dx \\ &= \frac{1}{6} \log(1+x) + \frac{1}{3} \int \frac{1-2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1+x}{1+x^2} dx \\ &= \frac{1}{6} \log(1+x) - \frac{1}{3} \log(1-x+x^2) + \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{6} \log(1+x) + \frac{1}{4} \log(1+x^2) - \frac{1}{3} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{4} \log(x^2+1) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{6} \log(x+1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1), x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x^2+x^3+x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^3 + x^5)^(-1), x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2 + x^3 + x^5)^(-1), x]
```

fricas [A] time = 1.00, size = 30, normalized size = 0.79

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2-x+1) + \frac{1}{4} \log(x^2+1) + \frac{1}{6} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
```

[Out] $1/2*\arctan(x) - 1/3*\log(x^2 - x + 1) + 1/4*\log(x^2 + 1) + 1/6*\log(x + 1)$

giac [A] time = 0.29, size = 31, normalized size = 0.82

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{4} \log(x^2 + 1) + \frac{1}{6} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")`

[Out] $1/2*\arctan(x) - 1/3*\log(x^2 - x + 1) + 1/4*\log(x^2 + 1) + 1/6*\log(\text{abs}(x + 1))$

maple [A] time = 0.01, size = 31, normalized size = 0.82

$$\frac{\arctan(x)}{2} + \frac{\ln(x + 1)}{6} + \frac{\ln(x^2 + 1)}{4} - \frac{\ln(x^2 - x + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5+x^3+x^2+1),x)`

[Out] $1/2*\arctan(x)+1/6*\ln(x+1)+1/4*\ln(x^2+1)-1/3*\ln(x^2-x+1)$

maxima [A] time = 1.11, size = 30, normalized size = 0.79

$$\frac{1}{2} \arctan(x) - \frac{1}{3} \log(x^2 - x + 1) + \frac{1}{4} \log(x^2 + 1) + \frac{1}{6} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")`

[Out] $1/2*\arctan(x) - 1/3*\log(x^2 - x + 1) + 1/4*\log(x^2 + 1) + 1/6*\log(x + 1)$

mupad [B] time = 2.16, size = 36, normalized size = 0.95

$$\frac{\ln(x + 1)}{6} - \frac{\ln(x^2 - x + 1)}{3} + \ln(x - i) \left(\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + 1i) \left(\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 + x^3 + x^5 + 1),x)`

[Out] $\log(x + 1)/6 + \log(x - 1i)*(1/4 - 1i/4) + \log(x + 1i)*(1/4 + 1i/4) - \log(x^2 - x + 1)/3$

sympy [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{\log(x+1)}{6} + \frac{\log(x^2+1)}{4} - \frac{\log(x^2-x+1)}{3} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**5+x**3+x**2+1),x)

[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2

$$3.66 \quad \int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx$$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9} - \frac{149700x^7}{7} + 4590x^5 - 684x^3 + 81x$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

Rule 517

```
Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] := Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (
f_)*(x_)^(n_))^(r_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx &= \int (-1 + x)^4(1 + x)^4(-1 + 2x)^4(1 + 2x)^4(-3 + 4x^2)^4 dx \\
&= \int (-1 + 2x)^4(1 + 2x)^4(-1 + x^2)^4(-3 + 4x^2)^4 dx \\
&= \int (-1 + x^2)^4(-3 + 4x^2)^4(-1 + 4x^2)^4 dx \\
&= \int (81 - 2052x^2 + 22950x^4 - 149700x^6 + 634321x^8 - 1841600x^{10} + 3764416x^{12} - 1497000x^{14} + 459000x^{16} - 68400x^{18} + 81x^{20}) dx \\
&= 81x - 684x^3 + 4590x^5 - \frac{149700x^7}{7} + \frac{634321x^9}{9} - \frac{1841600x^{11}}{11} + \frac{3764416x^{13}}{13} - \frac{1497000x^{15}}{15} + \frac{459000x^{17}}{17} - \frac{68400x^{19}}{19} + \frac{81x^{21}}{21}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9} - \frac{149700x^7}{7} + 4590x^5 - 684x^3 + 81x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4, x]

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - 19x^2 + 32x^4 - 16x^6)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4, x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4, x]

fricas [A] time = 1.07, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

giac [A] time = 0.38, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

maple [A] time = 0.00, size = 65, normalized size = 0.77

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x)

[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/23*x^23+65536/25*x^25

maxima [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25} - \frac{524288}{23}x^{23} + \frac{1884160}{21}x^{21} - \frac{4014080}{19}x^{19} + \frac{5633536}{17}x^{17} - \frac{1094656}{3}x^{15} + \frac{3764416}{13}x^{13} - \frac{1841600}{11}x^{11} + \frac{634321}{9}x^9 - \frac{149700}{7}x^7 + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")

[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 5633536/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x

mupad [B] time = 2.17, size = 64, normalized size = 0.76

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9} - \frac{149700x^7}{7} + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)

[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25

sympy [A] time = 0.08, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25} - \frac{524288x^{23}}{23} + \frac{1884160x^{21}}{21} - \frac{4014080x^{19}}{19} + \frac{5633536x^{17}}{17} - \frac{1094656x^{15}}{3} + \frac{3764416x^{13}}{13} - \frac{1841600x^{11}}{11} + \frac{634321x^9}{9} - \frac{149700x^7}{7} + 4590x^5 - 684x^3 + 81x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)

[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 5633536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 634321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x

$$3.67 \quad \int (3 - 19x^2 + 32x^4 - 16x^6)^3 dx$$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3, x]

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19

Rule 517

Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] :> Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 521

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 2059

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (3 - 19x^2 + 32x^4 - 16x^6)^3 dx &= - \int (-1 + x)^3 (1 + x)^3 (-1 + 2x)^3 (1 + 2x)^3 (-3 + 4x^2)^3 dx \\
&= - \int (-1 + 2x)^3 (1 + 2x)^3 (-1 + x^2)^3 (-3 + 4x^2)^3 dx \\
&= - \int (-1 + x^2)^3 (-3 + 4x^2)^3 (-1 + 4x^2)^3 dx \\
&= - \int (-27 + 513x^2 - 4113x^4 + 18235x^6 - 49344x^8 + 84912x^{10} - 93440x^{12} + 21248x^{14} - 24576x^{16} + 4096x^{18}) dx \\
&= 27x - 171x^3 + \frac{4113x^5}{5} - 2605x^7 + \frac{16448x^9}{3} - \frac{84912x^{11}}{11} + \frac{93440x^{13}}{13} - \frac{21248x^{15}}{5} + \frac{24576x^{17}}{17} - \frac{4096x^{19}}{19}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3, x]

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - 19x^2 + 32x^4 - 16x^6)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3, x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3, x]

fricas [A] time = 1.10, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

giac [A] time = 0.36, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

maple [A] time = 0.00, size = 50, normalized size = 0.79

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x)

[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21248/5*x^15+24576/17*x^17-4096/19*x^19

maxima [A] time = 0.73, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19} + \frac{24576}{17}x^{17} - \frac{21248}{5}x^{15} + \frac{93440}{13}x^{13} - \frac{84912}{11}x^{11} + \frac{16448}{3}x^9 - 2605x^7 + \frac{4113}{5}x^5 - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")

[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^11 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x

mupad [B] time = 0.05, size = 49, normalized size = 0.78

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)

[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11 + (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19

sympy [A] time = 0.07, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19} + \frac{24576x^{17}}{17} - \frac{21248x^{15}}{5} + \frac{93440x^{13}}{13} - \frac{84912x^{11}}{11} + \frac{16448x^9}{3} - 2605x^7 + \frac{4113x^5}{5} - 171x^3 + 27x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)

[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x**11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x

$$3.68 \quad \int (3 - 19x^2 + 32x^4 - 16x^6)^2 dx$$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2059, 517, 521}

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

Rule 517

```
Int[(u_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.)*((a1_.) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] := Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))
```

Rule 521

```
Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.)*((e_.) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P]}, Int[u^p, x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (3 - 19x^2 + 32x^4 - 16x^6)^2 dx &= \int (-1 + x)^2(1 + x)^2(-1 + 2x)^2(1 + 2x)^2(-3 + 4x^2)^2 dx \\
&= \int (-1 + 2x)^2(1 + 2x)^2(-1 + x^2)^2(-3 + 4x^2)^2 dx \\
&= \int (-1 + x^2)^2(-3 + 4x^2)^2(-1 + 4x^2)^2 dx \\
&= \int (9 - 114x^2 + 553x^4 - 1312x^6 + 1632x^8 - 1024x^{10} + 256x^{12}) dx \\
&= 9x - 38x^3 + \frac{553x^5}{5} - \frac{1312x^7}{7} + \frac{544x^9}{3} - \frac{1024x^{11}}{11} + \frac{256x^{13}}{13}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - 19x^2 + 32x^4 - 16x^6)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2, x]

fricas [A] time = 1.00, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

giac [A] time = 0.36, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x)

[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13

maxima [A] time = 0.75, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13} - \frac{1024}{11}x^{11} + \frac{544}{3}x^9 - \frac{1312}{7}x^7 + \frac{553}{5}x^5 - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")

[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 + 9*x

mupad [B] time = 0.02, size = 34, normalized size = 0.77

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)

[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 + (256*x^13)/13

sympy [A] time = 0.06, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13} - \frac{1024x^{11}}{11} + \frac{544x^9}{3} - \frac{1312x^7}{7} + \frac{553x^5}{5} - 38x^3 + 9x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)

[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x**3 + 9*x

$$3.69 \quad \int (3 - 19x^2 + 32x^4 - 16x^6) dx$$

Optimal. Leaf size=25

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Antiderivative was successfully verified.

[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6, x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7

Rubi steps

$$\int (3 - 19x^2 + 32x^4 - 16x^6) dx = 3x - \frac{19x^3}{3} + \frac{32x^5}{5} - \frac{16x^7}{7}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Antiderivative was successfully verified.

[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6, x]

[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3 - 19x^2 + 32x^4 - 16x^6) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[3 - 19*x^2 + 32*x^4 - 16*x^6, x]

[Out] IntegrateAlgebraic[3 - 19*x^2 + 32*x^4 - 16*x^6, x]

fricas [A] time = 1.00, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")

[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")

[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-16*x^6+32*x^4-19*x^2+3,x)

[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7

maxima [A] time = 0.64, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7 + \frac{32}{5}x^5 - \frac{19}{3}x^3 + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")

[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)`

[Out] `3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7`

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$-\frac{16x^7}{7} + \frac{32x^5}{5} - \frac{19x^3}{3} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-16*x**6+32*x**4-19*x**2+3,x)`

[Out] `-16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x`

$$3.70 \quad \int \frac{1}{3-19x^2+32x^4-16x^6} dx$$

Optimal. Leaf size=31

$$\frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2057, 207}

$$\frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{3-19x^2+32x^4-16x^6} dx &= \int \left(-\frac{1}{3(-1+x^2)} + \frac{2}{-3+4x^2} - \frac{2}{3(-1+4x^2)} \right) dx \\ &= -\left(\frac{1}{3} \int \frac{1}{-1+x^2} dx \right) - \frac{2}{3} \int \frac{1}{-1+4x^2} dx + 2 \int \frac{1}{-3+4x^2} dx \\ &= \frac{1}{3} \tanh^{-1}(x) + \frac{1}{3} \tanh^{-1}(2x) - \frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 2.00

$$\frac{1}{6} \left(-\log(2x^2 - 3x + 1) + \log(2x^2 + 3x + 1) + \sqrt{3} \log(\sqrt{3} - 2x) - \sqrt{3} \log(2x + \sqrt{3}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3 - 19x^2 + 32x^4 - 16x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]

fricas [B] time = 1.25, size = 56, normalized size = 1.81

$$\frac{1}{6} \sqrt{3} \log\left(\frac{4x^2 - 4\sqrt{3}x + 3}{4x^2 - 3}\right) + \frac{1}{6} \log(2x^2 + 3x + 1) - \frac{1}{6} \log(2x^2 - 3x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)

giac [B] time = 0.28, size = 62, normalized size = 2.00

$$\frac{1}{6} \sqrt{3} \log\left(\frac{|8x - 4\sqrt{3}|}{|8x + 4\sqrt{3}|}\right) + \frac{1}{6} \log(|2x + 1|) - \frac{1}{6} \log(|2x - 1|) + \frac{1}{6} \log(|x + 1|) - \frac{1}{6} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")

[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))

maple [A] time = 0.01, size = 42, normalized size = 1.35

$$-\frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3} - \frac{\ln(x-1)}{6} - \frac{\ln(2x-1)}{6} + \frac{\ln(x+1)}{6} + \frac{\ln(2x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x)

[Out] -1/6*ln(-1+x)-1/6*ln(-1+2*x)+1/6*ln(1+2*x)+1/6*ln(x+1)-1/3*arctanh(2/3*x*3^(1/2))*3^(1/2)

maxima [B] time = 1.05, size = 54, normalized size = 1.74

$$\frac{1}{6} \sqrt{3} \log\left(\frac{2x - \sqrt{3}}{2x + \sqrt{3}}\right) + \frac{1}{6} \log(2x + 1) - \frac{1}{6} \log(2x - 1) + \frac{1}{6} \log(x + 1) - \frac{1}{6} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")

[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)

mupad [B] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912} + \frac{1}{13824}\right)}\right)}{3} - \frac{\sqrt{3} \operatorname{atanh}\left(\frac{2\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)

[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3

sympy [B] time = 0.15, size = 63, normalized size = 2.03

$$\frac{\sqrt{3} \log\left(x - \frac{\sqrt{3}}{2}\right)}{6} - \frac{\sqrt{3} \log\left(x + \frac{\sqrt{3}}{2}\right)}{6} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{1}{2}\right)}{6} + \frac{\log\left(x^2 + \frac{3x}{2} + \frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)

[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6

$$3.71 \quad \int \frac{1}{(3-19x^2+32x^4-16x^6)^2} dx$$

Optimal. Leaf size=89

$$\frac{2x}{3(3-4x^2)} + \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(x+1)} - \frac{1}{18(2x+1)} + \frac{67}{54} \tanh^{-1}(x) - \frac{7}{27} \tanh^{-1}(2x) - \frac{5 \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 207, 199}

$$\frac{2x}{3(3-4x^2)} + \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(x+1)} - \frac{1}{18(2x+1)} + \frac{67}{54} \tanh^{-1}(x) - \frac{7}{27} \tanh^{-1}(2x) - \frac{5 \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[(2*x)/Sqrt[3]])/(3*Sqrt[3])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^2} dx &= \int \left(\frac{1}{36(-1+x)^2} + \frac{1}{36(1+x)^2} + \frac{1}{9(-1+2x)^2} + \frac{1}{9(1+2x)^2} - \frac{67}{54(-1+x^2)} \right) dx \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{14}{27} \int \frac{1}{-1+4x^2} dx - \frac{67}{54} \int \frac{1}{-1+x^2} dx \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{2x}{3(3-4x^2)} + \frac{67}{54} \tanh^{-1} \left(\frac{x}{\sqrt{3}} \right) \\
&= \frac{1}{18(1-2x)} + \frac{1}{36(1-x)} - \frac{1}{36(1+x)} - \frac{1}{18(1+2x)} + \frac{2x}{3(3-4x^2)} + \frac{67}{54} \tanh^{-1} \left(\frac{x}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{108} \left(-\frac{6x(80x^4 - 104x^2 + 27)}{16x^6 - 32x^4 + 19x^2 - 3} + 14 \log(1-2x) + 30\sqrt{3} \log(\sqrt{3}-2x) - 67 \log(1-x) + 67 \log(x+1) - 14 \log(2x+1) - 30\sqrt{3} \log(2x+\sqrt{3}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1 - 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]

fricas [B] time = 1.19, size = 177, normalized size = 1.99

$$\frac{480x^5 - 624x^3 - 30\sqrt{3}(16x^6 - 32x^4 + 19x^2 - 3) \log\left(\frac{x^2 - 4\sqrt{3}x + 3}{4x^2 - 3}\right) + 14(16x^6 - 32x^4 + 19x^2 - 3) \log(2x + 1) - 14(16x^6 - 32x^4 + 19x^2 - 3) \log(2x - 1) - 67(16x^6 - 32x^4 + 19x^2 - 3) \log(x + 1) + 67(16x^6 - 32x^4 + 19x^2 - 3) \log(x - 1) + 162x}{108(16x^6 - 32x^4 + 19x^2 - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")

[Out] $-1/108*(480*x^5 - 624*x^3 - 30*\sqrt{3}*(16*x^6 - 32*x^4 + 19*x^2 - 3)*\log((4*x^2 - 4*\sqrt{3}*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*\log(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*\log(2*x - 1) - 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*\log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*\log(x - 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)$

giac [A] time = 0.28, size = 97, normalized size = 1.09

$$\frac{5}{18}\sqrt{3}\log\left(\left|\frac{8x-4\sqrt{3}}{8x+4\sqrt{3}}\right|\right) - \frac{80x^5-104x^3+27x}{18(16x^6-32x^4+19x^2-3)} - \frac{7}{54}\log(|2x+1|) + \frac{7}{54}\log(|2x-1|) + \frac{67}{108}\log(|x+1|) - \frac{67}{108}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")

[Out] $5/18*\sqrt{3}*\log(\text{abs}(8*x - 4*\sqrt{3})/\text{abs}(8*x + 4*\sqrt{3})) - 1/18*(80*x^5 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*\log(\text{abs}(2*x + 1)) + 7/54*\log(\text{abs}(2*x - 1)) + 67/108*\log(\text{abs}(x + 1)) - 67/108*\log(\text{abs}(x - 1))$

maple [A] time = 0.02, size = 84, normalized size = 0.94

$$-\frac{x}{6\left(x^2-\frac{3}{4}\right)} - \frac{5\sqrt{3}\operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{9} - \frac{67\ln(x-1)}{108} + \frac{7\ln(2x-1)}{54} + \frac{67\ln(x+1)}{108} - \frac{7\ln(2x+1)}{54} - \frac{1}{36(x-1)} - \frac{1}{18(2x-1)} - \frac{1}{18(2x+1)} - \frac{1}{36(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)

[Out] $-1/36/(x-1) - 67/108*\ln(x-1) - 1/18/(2*x-1) + 7/54*\ln(2*x-1) - 1/18/(2*x+1) - 7/54*\ln(2*x+1) - 1/36/(x+1) + 67/108*\ln(x+1) - 1/6*x/(x^2-3/4) - 5/9*\operatorname{arctanh}(2/3*3^{(1/2)}*x)*3^{(1/2)}$

maxima [A] time = 1.29, size = 89, normalized size = 1.00

$$\frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) - \frac{80x^5-104x^3+27x}{18(16x^6-32x^4+19x^2-3)} - \frac{7}{54}\log(2x+1) + \frac{7}{54}\log(2x-1) + \frac{67}{108}\log(x+1) - \frac{67}{108}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")

[Out] $5/18*\sqrt{3}*\log((2*x - \sqrt{3})/(2*x + \sqrt{3})) - 1/18*(80*x^5 - 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*\log(2*x + 1) + 7/54*\log(2*x - 1) + 67/108*\log(x + 1) - 67/108*\log(x - 1)$

mupad [B] time = 0.08, size = 64, normalized size = 0.72

$$-\frac{\operatorname{atan}(x1i) 67i}{54} + \frac{\operatorname{atan}(x2i) 7i}{27} - \frac{\frac{5x^5}{18} - \frac{13x^3}{36} + \frac{3x}{32}}{x^6 - 2x^4 + \frac{19x^2}{16} - \frac{3}{16}} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x2i}{3}\right) 5i}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)`

[Out] `(atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*5i)/9`

sympy [A] time = 1.36, size = 104, normalized size = 1.17

$$\frac{-80x^5 + 104x^3 - 27x}{288x^6 - 576x^4 + 342x^2 - 54} - \frac{67 \log(x-1)}{108} + \frac{7 \log\left(x - \frac{1}{2}\right)}{54} - \frac{7 \log\left(x + \frac{1}{2}\right)}{54} + \frac{67 \log(x+1)}{108} + \frac{5\sqrt{3} \log\left(x - \frac{\sqrt{3}}{2}\right)}{18} - \frac{5\sqrt{3} \log\left(x + \frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)`

[Out] `(-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18`

$$3.72 \quad \int \frac{1}{(3-19x^2+32x^4-16x^6)^3} dx$$

Optimal. Leaf size=161

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{1}{432(1-x)^2}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2057, 207, 199}

$$\frac{5x}{3(3-4x^2)} - \frac{2x}{3(3-4x^2)^2} - \frac{7}{108(1-2x)} + \frac{67}{432(1-x)} - \frac{67}{432(x+1)} + \frac{7}{108(2x+1)} + \frac{1}{108(1-2x)^2} + \frac{1}{432(1-x)^2} - \frac{1}{432(x+1)^2} - \frac{1}{108(2x+1)^2} + \frac{3913}{648} \tanh^{-1}(x) + \frac{67}{162} \tanh^{-1}(2x) - 4\sqrt{3} \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right) + \frac{5 \tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 - x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) - 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^3} dx &= \int \left(-\frac{1}{216(-1+x)^3} + \frac{67}{432(-1+x)^2} + \frac{1}{216(1+x)^3} + \frac{67}{432(1+x)^2} - \frac{1}{27(-1+x)} \right. \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)} \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)} \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)} \\
&= \frac{1}{108(1-2x)^2} - \frac{7}{108(1-2x)} + \frac{1}{432(1-x)^2} + \frac{67}{432(1-x)} - \frac{1}{432(1+x)^2} - \frac{1}{432(1+x)}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.85

$$\frac{\frac{36x(80x^4-104x^2+27)}{(-16x^6+32x^4-19x^2+3)^2} - \frac{6x(2288x^4-2384x^2+345)}{16x^6-32x^4+19x^2-3} - 268 \log(1-2x) + 2412\sqrt{3} \log(\sqrt{3}-2x) - 3913 \log(1-x) + 3913 \log(x+1) + 268 \log(2x+1) - 2412\sqrt{3} \log(2x+\sqrt{3})}{1296}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(345 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x] + 2412*sqrt[3]*Log[sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 268*Log[1 + 2*x] - 2412*sqrt[3]*Log[sqrt[3] + 2*x])/1296

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3 - 19x^2 + 32x^4 - 16x^6)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]

fricas [B] time = 1.30, size = 282, normalized size = 1.75

$$\frac{20441x^{11} - 668160x^{10} + 751680x^9 - 382080x^8 + 85986x^7 - 2412\sqrt{3}(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)\log\left(\frac{4x^2 - 3}{4x^2 - 3}\right) - 268(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)\log(2x + 1) + 268(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)\log(2x - 1) - 3913(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)\log(x + 1) + 3913(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)\log(x - 1) - 7182x}{324(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")

[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1) - 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)

giac [A] time = 0.31, size = 112, normalized size = 0.70

$$\frac{67}{36}\sqrt{3}\log\left(\frac{8x-4\sqrt{3}}{8x+4\sqrt{3}}\right) - \frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{216(16x^6 - 32x^4 + 19x^2 - 3)^2} + \frac{67}{324}\log(2x+1) - \frac{67}{324}\log(2x-1) + \frac{3913}{1296}\log(x+1) - \frac{3913}{1296}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")

[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))

maple [A] time = 0.02, size = 126, normalized size = 0.78

$$\frac{67\sqrt{3}\operatorname{arctanh}\left(\frac{2\sqrt{3}x}{3}\right)}{18} - \frac{3913\ln(x-1)}{1296} - \frac{67\ln(2x-1)}{324} + \frac{3913\ln(x+1)}{1296} + \frac{67\ln(2x+1)}{324} + \frac{1}{432(x-1)^2} - \frac{67}{432(x-1)} + \frac{1}{108(2x-1)^2} + \frac{7}{108(2x-1)} - \frac{1}{108(2x+1)^2} + \frac{7}{108(2x+1)} - \frac{1}{432(x+1)^2} - \frac{67}{432(x+1)} + \frac{-\frac{20}{3}x^3 + \frac{13}{3}x}{(4x^2-3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)

[Out] 1/432/(x-1)^2-67/432/(x-1)-3913/1296*ln(x-1)+1/108/(2*x-1)^2+7/108/(2*x-1)-67/324*ln(2*x-1)-1/108/(2*x+1)^2+7/108/(2*x+1)+67/324*ln(2*x+1)-1/432/(x+1)^2-67/432/(x+1)+3913/1296*ln(x+1)+64*(-5/48*x^3+13/192*x)/(4*x^2-3)^2-67/18*arctanh(2/3*3^(1/2)*x)*3^(1/2)

maxima [A] time = 1.32, size = 119, normalized size = 0.74

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) - \frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{216(256x^{12} - 1024x^{10} + 1632x^8 - 1312x^6 + 553x^4 - 114x^2 + 9)} + \frac{67}{324}\log(2x+1) - \frac{67}{324}\log(2x-1) + \frac{3913}{1296}\log(x+1) - \frac{3913}{1296}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")

[Out] $67/36*\sqrt{3}*\log((2*x - \sqrt{3})/(2*x + \sqrt{3})) - 1/216*(36608*x^{11} - 11360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^{12} - 1024*x^{10} + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*\log(2*x + 1) - 67/324*\log(2*x - 1) + 3913/1296*\log(x + 1) - 3913/1296*\log(x - 1)$

mupad [B] time = 0.09, size = 93, normalized size = 0.58

$$-\frac{143x^{11}}{216} + \frac{145x^9}{72} - \frac{145x^7}{64} + \frac{995x^5}{864} - \frac{4777x^3}{18432} + \frac{133x}{6144} - \frac{\operatorname{atan}(x2i)67i}{162} - \frac{\operatorname{atan}(x1i)3913i}{648} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x2i}{3}\right)67i}{18}$$

$$x^{12} - 4x^{10} + \frac{51x^8}{8} - \frac{41x^6}{8} + \frac{553x^4}{256} - \frac{57x^2}{128} + \frac{9}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)

[Out] $((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9)/72 - (143*x^{11})/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^8)/8 - 4*x^{10} + x^{12} + 9/256) - (\operatorname{atan}(x*2i)*67i)/162 - (\operatorname{atan}(x*1i)*3913i)/648 + (3^{(1/2)}*\operatorname{atan}((3^{(1/2)}*x*2i)/3)*67i)/18$

sympy [A] time = 1.45, size = 134, normalized size = 0.83

$$-\frac{36608x^{11} - 111360x^9 + 125280x^7 - 63680x^5 + 14331x^3 - 1197x}{55296x^{12} - 221184x^{10} + 352512x^8 - 283392x^6 + 119448x^4 - 24624x^2 + 1944} - \frac{3913\log(x-1)}{1296} - \frac{67\log\left(x-\frac{1}{2}\right)}{324} + \frac{67\log\left(x+\frac{1}{2}\right)}{324} + \frac{3913\log(x+1)}{1296} + \frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36} - \frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)

[Out] $-(36608*x^{11} - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(55296*x^{12} - 221184*x^{10} + 352512*x^8 - 283392*x^6 + 119448*x^4 - 24624*x^2 + 1944) - 3913*\log(x - 1)/1296 - 67*\log(x - 1/2)/324 + 67*\log(x + 1/2)/324 + 3913*\log(x + 1)/1296 + 67*\sqrt{3}*\log(x - \sqrt{3}/2)/36 - 67*\sqrt{3}*\log(x + \sqrt{3}/2)/36$

$$3.73 \quad \int \frac{1}{(-1+7x^2-7x^4+x^6)^2} dx$$

Optimal. Leaf size=91

$$\frac{x}{32(1-x^2)} + \frac{(99-17x^2)x}{128(x^4-6x^2+1)} + \frac{5}{32} \tanh^{-1}(x) + \frac{1}{512} (3\sqrt{2}-4) \tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right) + \frac{1}{512} (4+3\sqrt{2}) \tanh^{-1}\left(\left(\sqrt{2}+1\right)x\right)$$

Rubi [B] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2057, 207, 638, 618, 206, 632, 31}

$$\frac{41-17x}{256(-x^2+2x+1)} + \frac{17x+41}{256(-x^2-2x+1)} + \frac{1}{64(1-x)} - \frac{1}{64(x+1)} + \frac{1}{512}(2-7\sqrt{2})\log(-x-\sqrt{2}+1) + \frac{1}{512}(2+7\sqrt{2})\log(-x+\sqrt{2}+1) - \frac{1}{512}(2-7\sqrt{2})\log(x-\sqrt{2}+1) - \frac{1}{512}(2+7\sqrt{2})\log(x+\sqrt{2}+1) - \frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}} + \frac{5}{32}\tanh^{-1}(x) + \frac{17\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 - 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2]) + (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x])/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] + x])/512

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTanh[Rt[b, 2]*x]/Rt[-a, 2]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 2057

Int[(P_)^(p_), x_Symbol] := With[{u = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(u /. x -> x^2)^p, x], x] /; !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x^2] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(-1 + 7x^2 - 7x^4 + x^6)^2} dx &= \int \left(\frac{1}{64(-1 + x)^2} + \frac{1}{64(1 + x)^2} - \frac{5}{32(-1 + x^2)} + \frac{29 - 12x}{64(-1 - 2x + x^2)^2} + \frac{6}{128(-1 + x^2)} \right) dx \\
 &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{1}{128} \int \frac{6 + x}{-1 - 2x + x^2} dx + \frac{1}{128} \int \frac{6 - x}{-1 + 2x + x^2} dx + \frac{6}{128} \int \frac{1}{-1 + x^2} dx \\
 &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} + \frac{5}{32} \tanh^{-1}(x) \\
 &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} + \frac{5}{32} \tanh^{-1}(x) \\
 &= \frac{1}{64(1 - x)} - \frac{1}{64(1 + x)} + \frac{41 + 17x}{256(1 - 2x - x^2)} - \frac{41 - 17x}{256(1 + 2x - x^2)} - \frac{17 \tanh^{-1}\left(\frac{1 - x}{\sqrt{-1 + x^2}}\right)}{256\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 1.45

$$\frac{-\frac{8x(21x^4-140x^2+103)}{x^6-7x^4+7x^2-1} - 80 \log(1-x) - (4+3\sqrt{2}) \log(-x+\sqrt{2}-1) + (4-3\sqrt{2}) \log(-x+\sqrt{2}+1) + 80 \log(x+1) + (4+3\sqrt{2}) \log(x+\sqrt{2}-1) + (3\sqrt{2}-4) \log(x+\sqrt{2}+1)}{1024}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x] - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/1024

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-1 + 7x^2 - 7x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

[Out] IntegrateAlgebraic[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]

fricas [B] time = 1.32, size = 223, normalized size = 2.45

$$\frac{168x^5 - 1120x^3 - 3\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 + \sqrt{2}(x+1) + 2x^3}{x^2 + 2x - 1}\right) - 3\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 + \sqrt{2}(x-1) + 2x^3}{x^2 - 2x - 1}\right) + 4(x^6 - 7x^4 + 7x^2 - 1) \log(x^2 + 2x - 1) - 4(x^6 - 7x^4 + 7x^2 - 1) \log(x^2 - 2x - 1) - 80(x^6 - 7x^4 + 7x^2 - 1) \log(x+1) + 80(x^6 - 7x^4 + 7x^2 - 1) \log(x-1) + 824x}{1024(x^6 - 7x^4 + 7x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")

[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)

giac [A] time = 0.44, size = 134, normalized size = 1.47

$$-\frac{3}{1024} \sqrt{2} \log\left(\frac{2x-2\sqrt{2}+2}{2x+2\sqrt{2}+2}\right) - \frac{3}{1024} \sqrt{2} \log\left(\frac{2x-2\sqrt{2}-2}{2x+2\sqrt{2}-2}\right) - \frac{21x^5-140x^3+103x}{128(x^6-7x^4+7x^2-1)} - \frac{1}{256} \log(|x^2+2x-1|) + \frac{1}{256} \log(|x^2-2x-1|) + \frac{5}{64} \log(|x+1|) - \frac{5}{64} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")

[Out] $-3/1024*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} + 2)/\text{abs}(2*x + 2*\sqrt{2} + 2)) - 3/1024*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} - 2)/\text{abs}(2*x + 2*\sqrt{2} - 2)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*\log(\text{abs}(x^2 + 2*x - 1)) + 1/256*\log(\text{abs}(x^2 - 2*x - 1)) + 5/64*\log(\text{abs}(x + 1)) - 5/64*\log(\text{abs}(x - 1))$

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512} + \frac{3\sqrt{2} \operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{512} - \frac{5\ln(x-1)}{64} + \frac{5\ln(x+1)}{64} + \frac{\ln(x^2-2x-1)}{256} - \frac{\ln(x^2+2x-1)}{256} - \frac{1}{64(x-1)} - \frac{\frac{17x}{2} + \frac{41}{2}}{128(x^2+2x-1)} - \frac{1}{64(x+1)} + \frac{-\frac{17x}{2} + \frac{41}{2}}{128x^2-256x-128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^6-7*x^4+7*x^2-1)^2, x)$

[Out] $-1/64/(x-1) - 5/64*\ln(x-1) - 1/128*(17/2*x+41/2)/(x^2+2*x-1) - 1/256*\ln(x^2+2*x-1) + 3/512*2^{(1/2)}*\operatorname{arctanh}(1/4*(2+2*x)*2^{(1/2)}) - 1/64/(x+1) + 5/64*\ln(x+1) + 1/128*(-17/2*x+41/2)/(x^2-2*x-1) + 1/256*\ln(x^2-2*x-1) + 3/512*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*x-2)*2^{(1/2)})$

maxima [A] time = 1.27, size = 114, normalized size = 1.25

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) - \frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{21x^5-140x^3+103x}{128(x^6-7x^4+7x^2-1)} - \frac{1}{256}\log(x^2+2x-1) + \frac{1}{256}\log(x^2-2x-1) + \frac{5}{64}\log(x+1) - \frac{5}{64}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(x^6-7*x^4+7*x^2-1)^2, x, \text{algorithm}=\text{"maxima"})$

[Out] $-3/1024*\sqrt{2}*\log((x - \sqrt{2} + 1)/(x + \sqrt{2} + 1)) - 3/1024*\sqrt{2}*\log((x - \sqrt{2} - 1)/(x + \sqrt{2} - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*\log(x^2 + 2*x - 1) + 1/256*\log(x^2 - 2*x - 1) + 5/64*\log(x + 1) - 5/64*\log(x - 1)$

mupad [B] time = 2.18, size = 126, normalized size = 1.38

$$-\frac{\operatorname{atan}(x1i)5i}{32} - \frac{21x^5 - 35x^3 + 103x}{128x^6 - 7x^4 + 7x^2 - 1} - \operatorname{atan}\left(\frac{x940311i}{134217728\left(\frac{275445\sqrt{2}}{134217728} - \frac{389421}{134217728}\right)} - \frac{\sqrt{2}x332433i}{67108864\left(\frac{275445\sqrt{2}}{134217728} - \frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}3i}{512} - \frac{1}{128}i\right) - \operatorname{atan}\left(\frac{x940311i}{134217728\left(\frac{275445\sqrt{2}}{134217728} + \frac{389421}{134217728}\right)} + \frac{\sqrt{2}x332433i}{67108864\left(\frac{275445\sqrt{2}}{134217728} + \frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}3i}{512} + \frac{1}{128}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(7*x^2 - 7*x^4 + x^6 - 1)^2, x)$

[Out] $-(\operatorname{atan}(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 - 7*x^4 + x^6 - 1) - \operatorname{atan}((x*940311i)/(134217728*((275445*2^{(1/2)})/134217728 - 389421/134217728))) - (2^{(1/2)}*x*332433i)/(67108864*((275445*2^{(1/2)})/134217728 - 389421/134217728)))*((2^{(1/2)}*3i)/512 - 1i/128) - \operatorname{atan}((x*940311i)/(134217728*((275445*2^{(1/2)})/134217728 + 389421/134217728))) + (2^{(1/2)}*x*33$

$2433i)/(67108864*((275445*2^{(1/2)})/134217728 + 389421/134217728)))*((2^{(1/2)})*3i)/512 + 1i/128)$

sympy [B] time = 1.43, size = 296, normalized size = 3.25

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)

[Out] $(-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*\log(x - 1)/64 + 5*\log(x + 1)/64 + (-1/256 + 3*\sqrt{2}/1024)*\log(x - 8071264001/202624020 - 471550901878784*(-1/256 + 3*\sqrt{2}/1024)**3/2979765 + 1299552375287054336*(-1/256 + 3*\sqrt{2}/1024)**5/50656005 + 8071264001*\sqrt{2}/270165360) + (-3*\sqrt{2}/1024 - 1/256)*\log(x - 8071264001*\sqrt{2}/270165360 - 8071264001/202624020 + 1299552375287054336*(-3*\sqrt{2}/1024 - 1/256)**5/50656005 - 471550901878784*(-3*\sqrt{2}/1024 - 1/256)**3/2979765) + (1/256 - 3*\sqrt{2}/1024)*\log(x - 8071264001*\sqrt{2}/270165360 + 1299552375287054336*(1/256 - 3*\sqrt{2}/1024)**5/50656005 - 471550901878784*(1/256 - 3*\sqrt{2}/1024)**3/2979765 + 8071264001/202624020) + (1/256 + 3*\sqrt{2}/1024)*\log(x - 471550901878784*(1/256 + 3*\sqrt{2}/1024)**3/2979765 + 1299552375287054336*(1/256 + 3*\sqrt{2}/1024)**5/50656005 + 8071264001/202624020 + 8071264001*\sqrt{2}/270165360)$

$$3.74 \quad \int \frac{x^3}{c+(a+bx)^2} dx$$

Optimal. Leaf size=78

$$\frac{(3a^2 - c) \log((a + bx)^2 + c)}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(a + bx)^2}{2b^4} - \frac{3ax}{b^3}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 702, 635, 203, 260}

$$\frac{(3a^2 - c) \log((a + bx)^2 + c)}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(a + bx)^2}{2b^4} - \frac{3ax}{b^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(c + (a + b*x)^2), x]

[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned} \int \frac{x^3}{c + (a + bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2} dx, x, a + bx\right)}{b^4} \\ &= \frac{\text{Subst}\left(\int \left(-3a + x - \frac{a^3-3ac-(3a^2-c)x}{c+x^2}\right) dx, x, a + bx\right)}{b^4} \\ &= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{\text{Subst}\left(\int \frac{a^3-3ac-(3a^2-c)x}{c+x^2} dx, x, a + bx\right)}{b^4} \\ &= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{(a(a^2 - 3c)) \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a + bx\right)}{b^4} + \frac{(3a^2 - c) \text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a + bx\right)}{b^4} \\ &= -\frac{3ax}{b^3} + \frac{(a + bx)^2}{2b^4} - \frac{a(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{(3a^2 - c) \log(c + (a + bx)^2)}{2b^4} \end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.94

$$\frac{-\frac{2(a^3-3ac) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + (3a^2 - c) \log(a^2 + 2abx + b^2x^2 + c) + bx(bx - 4a)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(c + (a + b*x)^2), x]

[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2]/(2*b^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{c + (a + bx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(c + (a + b*x)^2), x]

[Out] IntegrateAlgebraic[x^3/(c + (a + b*x)^2), x]

fricas [A] time = 1.20, size = 198, normalized size = 2.54

$$\left[\frac{b^2cx^2 - 4abx + (a^3 - 3ac)\sqrt{-c} \log\left(\frac{b^2x^2 + 2abx + a^2 - 2(bx+a)\sqrt{-c}}{b^2x^2 + 2abx + a^2 + c}\right) + (3a^2c - c^2) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4c}, \frac{b^2cx^2 - 4abx - 2(a^3 - 3ac)\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right) + (3a^2c - c^2) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]

giac [A] time = 0.34, size = 77, normalized size = 0.99

$$\frac{(3a^2 - c) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{(a^3 - 3ac) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{b^2x^2 - 4abx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2), x, algorithm="giac")

[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4

maple [A] time = 0.01, size = 127, normalized size = 1.63

$$\frac{x^2}{2b^2} - \frac{a^3 \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4\sqrt{c}} + \frac{3a^2 \ln(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{2ax}{b^3} + \frac{3a\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4} - \frac{c \ln(b^2x^2 + 2abx + a^2 + c)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c+(b*x+a)^2), x)

[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a

maxima [A] time = 1.46, size = 81, normalized size = 1.04

$$\frac{bx^2 - 4ax}{2b^3} + \frac{(3a^2 - c) \log(b^2x^2 + 2abx + a^2 + c)}{2b^4} - \frac{(a^3 - 3ac) \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))

mupad [B] time = 2.27, size = 87, normalized size = 1.12

$$\frac{x^2}{2b^2} - \frac{2ax}{b^3} - \frac{\ln(a^2 + 2abx + b^2x^2 + c) (4b^4c^2 - 12a^2b^4c)}{8b^8c} + \frac{a \operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right) (3c - a^2)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c + (a + b*x)^2),x)

[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 - 12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(1/2))

sympy [B] time = 0.69, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3} + \left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) \log\left(x + \frac{a^4-2b^4c\left(-\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b-3abc}\right) + \left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) \log\left(x + \frac{a^4-2b^4c\left(\frac{a\sqrt{-c}(a^2-3c)}{2b^4c} + \frac{3a^2-c}{2b^4}\right) - c^2}{a^3b-3abc}\right) + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c+(b*x+a)**2),x)

[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) *log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/(2*b**2)

$$3.75 \quad \int \frac{x^2}{c+(a+bx)^2} dx$$

Optimal. Leaf size=50

$$\frac{(a^2 - c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3 \sqrt{c}} - \frac{a \log((a+bx)^2 + c)}{b^3} + \frac{x}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 702, 635, 203, 260}

$$\frac{(a^2 - c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3 \sqrt{c}} - \frac{a \log((a+bx)^2 + c)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(c + (a + b*x)^2), x]

[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a + b*x)^2])/b^3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e

}, x] && !NiceSqrtQ[-(a*c)]

Rule 702

Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{c + (a + bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2} dx, x, a + bx\right)}{b^3} \\
 &= \frac{\text{Subst}\left(\int \left(1 + \frac{a^2-c-2ax}{c+x^2}\right) dx, x, a + bx\right)}{b^3} \\
 &= \frac{x}{b^2} + \frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2} dx, x, a + bx\right)}{b^3} \\
 &= \frac{x}{b^2} - \frac{(2a) \text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a + bx\right)}{b^3} + \frac{(a^2 - c) \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a + bx\right)}{b^3} \\
 &= \frac{x}{b^2} + \frac{(a^2 - c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3 \sqrt{c}} - \frac{a \log(c + (a + bx)^2)}{b^3}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.08

$$\frac{-a \log(a^2 + 2abx + b^2x^2 + c) + \frac{(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + bx}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(c + (a + b*x)^2), x]

[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*b*x + b^2*x^2])/b^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{c + (a + bx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(c + (a + b*x)^2), x]

[Out] IntegrateAlgebraic[x^2/(c + (a + b*x)^2), x]

fricas [A] time = 1.23, size = 157, normalized size = 3.14

$$\left[\frac{2bcx - 2ac \log(b^2x^2 + 2abx + a^2 + c) + (a^2 - c)\sqrt{-c} \log\left(\frac{b^2x^2 + 2abx + a^2 + 2(bx+a)\sqrt{-c}}{b^2x^2 + 2abx + a^2 + c}\right)}{2b^3c}, \frac{bcx - ac \log(b^2x^2 + 2abx + a^2 + c) + (a^2 - c)\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c) *log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]

giac [A] time = 0.31, size = 54, normalized size = 1.08

$$\frac{x}{b^2} - \frac{a \log(b^2x^2 + 2abx + a^2 + c)}{b^3} + \frac{(a^2 - c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2), x, algorithm="giac")

[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)/sqrt(c))/(b^3*sqrt(c))

maple [A] time = 0.01, size = 89, normalized size = 1.78

$$\frac{a^2 \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3\sqrt{c}} - \frac{a \ln(b^2x^2 + 2abx + a^2 + c)}{b^3} + \frac{x}{b^2} - \frac{\sqrt{c} \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c+(b*x+a)^2), x)

[Out] x/b^2 - 1/b^3*a*ln(b^2*x^2 + 2*a*b*x + a^2 + c) + 1/b^3/c^(1/2)*arctan(1/2*(2*b^2*x + 2*a*b)/b/c^(1/2)) * a^2 - 1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x + 2*a*b)/b/c^(1/2))

maxima [A] time = 1.65, size = 61, normalized size = 1.22

$$\frac{x}{b^2} - \frac{a \log(b^2 x^2 + 2 abx + a^2 + c)}{b^3} + \frac{(a^2 - c) \arctan\left(\frac{b^2 x + ab}{b \sqrt{c}}\right)}{b^3 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^3*sqrt(c))

mupad [B] time = 0.09, size = 206, normalized size = 4.12

$$\frac{x}{b^2} - \frac{a \ln(a^2 + 2 abx + b^2 x^2 + c)}{b^3} + \frac{\sqrt{c} \operatorname{atan}\left(\frac{a^3}{\sqrt{c}(c-a^2)} - \frac{\sqrt{c}x}{\frac{c}{b} - \frac{a^2}{b}} - \frac{a\sqrt{c}}{c-a^2} + \frac{a^2x}{\sqrt{c}\left(\frac{c}{b} - \frac{a^2}{b}\right)}\right)}{b^3} - \frac{a^2 \operatorname{atan}\left(\frac{a^3}{\sqrt{c}(c-a^2)} - \frac{\sqrt{c}x}{\frac{c}{b} - \frac{a^2}{b}} - \frac{a\sqrt{c}}{c-a^2} + \frac{a^2x}{\sqrt{c}\left(\frac{c}{b} - \frac{a^2}{b}\right)}\right)}{b^3 \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c + (a + b*x)^2),x)

[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/2)*(c - a^2))) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b)))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2))) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b)))/b^3*c^(1/2)

sympy [B] time = 0.46, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3} - \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right) \log\left(x + \frac{a^3 + ac + 2b^3c\left(-\frac{a}{b^3} - \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right)}{a^2b - bc}\right) + \left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right) \log\left(x + \frac{a^3 + ac + 2b^3c\left(-\frac{a}{b^3} + \frac{\sqrt{-c}(a^2 - c)}{2b^3c}\right)}{a^2b - bc}\right) + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c+(b*x+a)**2),x)

[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2

$$3.76 \quad \int \frac{x}{c+(a+bx)^2} dx$$

Optimal. Leaf size=41

$$\frac{\log((a+bx)^2+c)}{2b^2} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {371, 635, 203, 260}

$$\frac{\log((a+bx)^2+c)}{2b^2} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[x/(c + (a + b*x)^2), x]

[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^2)

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rubi steps

$$\begin{aligned} \int \frac{x}{c + (a + bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2} dx, x, a + bx\right)}{b^2} \\ &= \frac{\text{Subst}\left(\int \frac{x}{c+x^2} dx, x, a + bx\right)}{b^2} - \frac{a \text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a + bx\right)}{b^2} \\ &= -\frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2 \sqrt{c}} + \frac{\log(c + (a + bx)^2)}{2b^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.93

$$\frac{\log((a + bx)^2 + c) - \frac{2a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(c + (a + b*x)^2), x]

[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{c + (a + bx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(c + (a + b*x)^2), x]

[Out] IntegrateAlgebraic[x/(c + (a + b*x)^2), x]

fricas [A] time = 1.15, size = 136, normalized size = 3.32

$$\left[\frac{a\sqrt{-c} \log\left(\frac{b^2x^2+2abx+a^2+2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right) - c \log(b^2x^2 + 2abx + a^2 + c)}{2b^2c}, -\frac{2a\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right) - c \log(b^2x^2 + 2abx + a^2 + c)}{2b^2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] $[-1/2*(a*\sqrt{-c})*\log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*\sqrt{-c} - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*\log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c), -1/2*(2*a*\sqrt{c})*\arctan((b*x + a)/\sqrt{c}) - c*\log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c)]$

giac [A] time = 0.36, size = 43, normalized size = 1.05

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c+(b*x+a)^2),x, algorithm="giac")`

[Out] $-a*\arctan((b*x + a)/\sqrt{c})/(b^2*\sqrt{c}) + 1/2*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^2$

maple [A] time = 0.00, size = 54, normalized size = 1.32

$$-\frac{a \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\ln(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c+(b*x+a)^2),x)`

[Out] $1/2/b^2*\ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^{(1/2)}*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^{(1/2)})$

maxima [A] time = 1.36, size = 50, normalized size = 1.22

$$-\frac{a \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}} + \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")`

[Out] $-a*\arctan((b^2*x + a*b)/(b*\sqrt{c}))/b^2*\sqrt{c} + 1/2*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^2$

mupad [B] time = 2.08, size = 46, normalized size = 1.12

$$\frac{\ln(a^2 + 2abx + b^2x^2 + c)}{2b^2} - \frac{a \operatorname{atan}\left(\frac{a}{\sqrt{c}} + \frac{bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c + (a + b*x)^2), x)`

[Out] $\log(c + a^2 + b^2x^2 + 2abx)/(2b^2) - (a \operatorname{atan}(a/c^{1/2} + (bx)/c^{1/2}))/b^2c^{1/2}$

sympy [B] time = 0.25, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c} + \frac{1}{2b^2}\right) \log\left(x + \frac{a^2 - 2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c} + \frac{1}{2b^2}\right) + c}{ab}\right) + \left(\frac{a\sqrt{-c}}{2b^2c} + \frac{1}{2b^2}\right) \log\left(x + \frac{a^2 - 2b^2c\left(\frac{a\sqrt{-c}}{2b^2c} + \frac{1}{2b^2}\right) + c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c+(b*x+a)**2), x)`

[Out] $(-a\sqrt{-c}/(2b^2c) + 1/(2b^2)) \log(x + (a^2 - 2b^2c(-a\sqrt{-c})/(2b^2c) + 1/(2b^2)) + c)/(ab) + (a\sqrt{-c}/(2b^2c) + 1/(2b^2)) \log(x + (a^2 - 2b^2c(a\sqrt{-c})/(2b^2c) + 1/(2b^2)) + c)/(ab)$

$$3.77 \quad \int \frac{1}{c+(a+bx)^2} dx$$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 203}

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(c + (a + b*x)^2)^(-1), x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{c+(a+bx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{c+x^2} dx, x, a+bx\right)}{b} \\ &= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + (a + b*x)^2)^(-1), x]

[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c + (a + bx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + (a + b*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(c + (a + b*x)^2)^(-1), x]

fricas [A] time = 0.88, size = 83, normalized size = 3.95

$$\left[-\frac{\sqrt{-c} \log\left(\frac{b^2x^2+2abx+a^2-2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right)}{2bc}, \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]

giac [A] time = 0.38, size = 17, normalized size = 0.81

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)^2), x, algorithm="giac")

[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c+(b*x+a)^2),x)`

[Out] `1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))`

maxima [A] time = 1.52, size = 24, normalized size = 1.14

$$\frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")`

[Out] `arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))`

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$\frac{\operatorname{atan}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c + (a + b*x)^2),x)`

[Out] `atan((a + b*x)/c^(1/2))/(b*c^(1/2))`

sympy [B] time = 0.19, size = 54, normalized size = 2.57

$$\frac{\frac{\sqrt{-\frac{1}{c}} \log\left(x + \frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2} + \frac{\sqrt{-\frac{1}{c}} \log\left(x + \frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+(b*x+a)**2),x)`

[Out] `(-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sqrt(-1/c))/b)/2)/b`

$$3.78 \quad \int \frac{1}{x(c+(a+bx)^2)} dx$$

Optimal. Leaf size=59

$$-\frac{\log((a+bx)^2+c)}{2(a^2+c)} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)} + \frac{\log(x)}{a^2+c}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 706, 31, 635, 203, 260}

$$-\frac{\log((a+bx)^2+c)}{2(a^2+c)} - \frac{a \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)} + \frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(c + (a + b*x)^2)),x]

[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]]/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - Log[c + (a + b*x)^2]/(2*(a^2 + c)))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;

FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(c + (a + bx)^2)} dx &= \text{Subst} \left(\int \frac{1}{(-a + x)(c + x^2)} dx, x, a + bx \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{-a+x} dx, x, a + bx \right)}{a^2 + c} + \frac{\text{Subst} \left(\int \frac{-a-x}{c+x^2} dx, x, a + bx \right)}{a^2 + c} \\ &= \frac{\log(x)}{a^2 + c} - \frac{\text{Subst} \left(\int \frac{x}{c+x^2} dx, x, a + bx \right)}{a^2 + c} - \frac{a \text{Subst} \left(\int \frac{1}{c+x^2} dx, x, a + bx \right)}{a^2 + c} \\ &= -\frac{a \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c} (a^2 + c)} + \frac{\log(x)}{a^2 + c} - \frac{\log(c + (a + bx)^2)}{2(a^2 + c)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.81

$$-\frac{\log((a + bx)^2 + c) + \frac{2a \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c}} - 2 \log(bx)}{2(a^2 + c)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(c + (a + b*x)^2)),x]

[Out] -1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b*x)^2))/(a^2 + c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(c + (a + bx)^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(c + (a + b*x)^2)), x]

[Out] IntegrateAlgebraic[1/(x*(c + (a + b*x)^2)), x]

fricas [A] time = 1.24, size = 154, normalized size = 2.61

$$\left[\frac{a\sqrt{-c} \log\left(\frac{b^2x^2+2abx+a^2+2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right) + c \log(b^2x^2 + 2abx + a^2 + c) - 2c \log(x)}{2(a^2c + c^2)}, \frac{2a\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right) + c \log(b^2x^2 + 2abx + a^2 + c) - 2c \log(x)}{2(a^2c + c^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]

giac [A] time = 0.38, size = 62, normalized size = 1.05

$$-\frac{a \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^2 + c)\sqrt{c}} - \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2(a^2 + c)} + \frac{\log(|x|)}{a^2 + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c+(b*x+a)^2), x, algorithm="giac")

[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$-\frac{a \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{(a^2 + c)\sqrt{c}} + \frac{\ln(x)}{a^2 + c} - \frac{\ln(b^2x^2 + 2abx + a^2 + c)}{2(a^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c+(b*x+a)^2), x)

[Out] $\ln(x)/(a^2+c)-1/2/(a^2+c)*\ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^{(1/2)}*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^{(1/2)})$

maxima [A] time = 1.52, size = 68, normalized size = 1.15

$$-\frac{a \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^2+c)\sqrt{c}} - \frac{\log(b^2x^2 + 2abx + a^2 + c)}{2(a^2+c)} + \frac{\log(x)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")`

[Out] $-a*\arctan((b^2*x + a*b)/(b*\sqrt{c}))/((a^2 + c)*\sqrt{c}) - 1/2*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^2 + c) + \log(x)/(a^2 + c)$

mupad [B] time = 2.59, size = 173, normalized size = 2.93

$$\frac{\ln(x)}{a^2+c} - \frac{\ln\left(2ab^3 + 3b^4x + \frac{b^3(c+a\sqrt{-c})(a^3+bx^2+c-3bcx)}{c(a^2+c)}\right)(c+a\sqrt{-c})}{2(a^2c+c^2)} - \frac{\ln\left(2ab^3 + 3b^4x + \frac{b^3(c-a\sqrt{-c})(a^3+bx^2+c-3bcx)}{c(a^2+c)}\right)(c-a\sqrt{-c})}{2(a^2c+c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(c + (a + b*x)^2)),x)`

[Out] $\log(x)/(c + a^2) - (\log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^{(1/2)})*(a*c + a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^{(1/2)}))/(2*(a^2*c + c^2)) - (\log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^{(1/2)})*(a*c + a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^{(1/2)}))/(2*(a^2*c + c^2))$

sympy [B] time = 3.40, size = 738, normalized size = 12.51

$$\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) \log\left(x + \frac{-4a^3c\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) + 4a^4c^2\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 6a^4c\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) + 20a^2c^3\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 12a^2c^2\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) + 10a^2c + 12c^4\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right)^2 - 6c^3\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) - 6c^2\left(\frac{-a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}\right) + \frac{a\sqrt{-c}}{2c(a^2+c)} - \frac{1}{2(a^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c+(b*x+a)**2),x)`

[Out] $(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*\log(x + (-4*a**6*c*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*2 + 4*a**4*c**2*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*2 - 6*a**4*c*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*2 - 12*a**2*c**2*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*2 - 6*c**3*(-a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2)/(a**3*b + 9*a*b*c) + (a*\sqrt{-c}/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*\log$

$$\begin{aligned}
&g(x + (-4*a**6*c*(a*\sqrt{-c})/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a* \\
&*4*c**2*(a*\sqrt{-c})/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s \\
&qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*\sqrt{-c})/(2* \\
&c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*\sqrt{-c})/(2*c*(a**2 \\
&+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*\sqrt{-c})/(2*c*(a**2 + c) \\
&) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*\sqrt{-c})/(2*c*(a**2 + c)) - 1/(2*(a**2 \\
&+ c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + \log(x + (-4*a**6*c/(a**2 + c)**2 + \\
&4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)** \\
&2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a \\
&**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
\end{aligned}$$

$$3.79 \quad \int \frac{1}{x^2(c+(a+bx)^2)} dx$$

Optimal. Leaf size=79

$$-\frac{2ab \log(x)}{(a^2+c)^2} + \frac{ab \log((a+bx)^2+c)}{(a^2+c)^2} + \frac{b(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2} - \frac{1}{x(a^2+c)}$$

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 710, 801, 635, 203, 260}

$$-\frac{2ab \log(x)}{(a^2+c)^2} + \frac{ab \log((a+bx)^2+c)}{(a^2+c)^2} + \frac{b(a^2-c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^2} - \frac{1}{x(a^2+c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(c + (a + b*x)^2)),x]

[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)^2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2(c + (a + bx)^2)} dx &= b \operatorname{Subst} \left(\int \frac{1}{(-a + x)^2(c + x^2)} dx, x, a + bx \right) \\
 &= -\frac{1}{(a^2 + c)x} + \frac{b \operatorname{Subst} \left(\int \frac{-a - x}{(-a + x)(c + x^2)} dx, x, a + bx \right)}{a^2 + c} \\
 &= -\frac{1}{(a^2 + c)x} + \frac{b \operatorname{Subst} \left(\int \left(\frac{2a}{(a^2 + c)(a - x)} + \frac{a^2 - c + 2ax}{(a^2 + c)(c + x^2)} \right) dx, x, a + bx \right)}{a^2 + c} \\
 &= -\frac{1}{(a^2 + c)x} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{b \operatorname{Subst} \left(\int \frac{a^2 - c + 2ax}{c + x^2} dx, x, a + bx \right)}{(a^2 + c)^2} \\
 &= -\frac{1}{(a^2 + c)x} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{(2ab) \operatorname{Subst} \left(\int \frac{x}{c + x^2} dx, x, a + bx \right)}{(a^2 + c)^2} + \frac{(b(a^2 - c)) \operatorname{Subst} \left(\int \frac{1}{c + x^2} dx, x, a + bx \right)}{(a^2 + c)^2} \\
 &= -\frac{1}{(a^2 + c)x} + \frac{b(a^2 - c) \tan^{-1} \left(\frac{a + bx}{\sqrt{c}} \right)}{\sqrt{c}(a^2 + c)^2} - \frac{2ab \log(x)}{(a^2 + c)^2} + \frac{ab \log(c + (a + bx)^2)}{(a^2 + c)^2}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.03

$$\frac{bx(a^2 - c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right) - \sqrt{c}(-abx \log(a^2 + 2abx + b^2x^2 + c) + a^2 + 2abx \log(x) + c)}{\sqrt{c}x(a^2 + c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(c + (a + b*x)^2)), x]

[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(c + (a + bx)^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(c + (a + b*x)^2)), x]

[Out] IntegrateAlgebraic[1/(x^2*(c + (a + b*x)^2)), x]

fricas [A] time = 1.14, size = 229, normalized size = 2.90

$$\left[\frac{2abx \log(b^2x^2 + 2abx + a^2 + c) - 4abcx \log(x) + (a^2b - bc)\sqrt{-c}x \log\left(\frac{b^2x^2 + 2abx + a^2 + 2(bx+a)\sqrt{-c} - c}{b^2x^2 + 2abx + a^2 + c}\right) - 2a^2c - 2c^2}{2(a^4c + 2a^2c^2 + c^3)x}, \frac{abx \log(b^2x^2 + 2abx + a^2 + c) - 2abcx \log(x) + (a^2b - bc)\sqrt{c}x \arctan\left(\frac{bx+a}{\sqrt{c}}\right) - a^2c - c^2}{(a^4c + 2a^2c^2 + c^3)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^2*c^2 + c^3)*x)]

giac [A] time = 0.37, size = 117, normalized size = 1.48

$$\frac{ab \log(b^2x^2 + 2abx + a^2 + c)}{a^4 + 2a^2c + c^2} - \frac{2ab \log(|x|)}{a^4 + 2a^2c + c^2} + \frac{(a^2b^2 - b^2c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^4 + 2a^2c + c^2)b\sqrt{c}} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")

[Out] $a*b*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*\log(\text{abs}(x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*\arctan((b*x + a)/\text{sqrt}(c))/((a^4 + 2*a^2*c + c^2)*b*\text{sqrt}(c)) - 1/((a^2 + c)*x)$

maple [A] time = 0.01, size = 123, normalized size = 1.56

$$\frac{a^2 b \arctan\left(\frac{2b^2 x + 2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^2 \sqrt{c}} - \frac{2ab \ln(x)}{(a^2 + c)^2} + \frac{ab \ln(b^2 x^2 + 2abx + a^2 + c)}{(a^2 + c)^2} - \frac{b\sqrt{c} \arctan\left(\frac{2b^2 x + 2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^2} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c+(b*x+a)^2),x)

[Out] $-1/(a^2+c)/x - 2*a*b*\ln(x)/(a^2+c)^2 + b/(a^2+c)^2*a*\ln(b^2*x^2+2*a*b*x+a^2+c) + b/(a^2+c)^2/c^(1/2)*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2 - b/(a^2+c)^2*c^(1/2)*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))$

maxima [A] time = 1.59, size = 123, normalized size = 1.56

$$\frac{ab \log(b^2 x^2 + 2 abx + a^2 + c)}{a^4 + 2 a^2 c + c^2} - \frac{2 ab \log(x)}{a^4 + 2 a^2 c + c^2} + \frac{(a^2 b^2 - b^2 c) \arctan\left(\frac{b^2 x + ab}{b\sqrt{c}}\right)}{(a^4 + 2 a^2 c + c^2) b \sqrt{c}} - \frac{1}{(a^2 + c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] $a*b*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*\log(x)/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*\arctan((b^2*x + a*b)/(b*\text{sqrt}(c)))/((a^4 + 2*a^2*c + c^2)*b*\text{sqrt}(c)) - 1/((a^2 + c)*x)$

mupad [B] time = 2.58, size = 425, normalized size = 5.38

$$\frac{b \log\left(\frac{b^2 x^2 + 2 abx + a^2 + c}{b^2 x^2 + 2 abx + a^2 + c}\right) - 2 ab \log(x) + (a^2 b^2 - b^2 c) \arctan\left(\frac{b^2 x + ab}{b \sqrt{c}}\right) - 1}{(a^4 + 2 a^2 c + c^2) b \sqrt{c} (a^2 + c) x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c + (a + b*x)^2)),x)

[Out] $(\log((-c)^{(13/2)} - 35*a^2*(-c)^{(11/2)} + 34*a^4*(-c)^{(9/2)} + 34*a^6*(-c)^{(7/2)} - 35*a^8*(-c)^{(5/2)} + a^{10}*(-c)^{(3/2)} + a*c^6 - a^{11}*c + 35*a^3*c^5 + 34*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^{10}*b*c*x + 35*a^2*b*c^5*x + 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^{(3/2)} + 2*a*b*c + a^2*b*(-c)^{(1/2)}))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (10$

$$g((-c)^{(13/2)} - 35a^2(-c)^{(11/2)} + 34a^4(-c)^{(9/2)} + 34a^6(-c)^{(7/2)} - 35a^8(-c)^{(5/2)} + a^{10}(-c)^{(3/2)} - a^2c^6 + a^{11}c - 35a^3c^5 - 34a^5c^4 + 34a^7c^3 + 35a^9c^2 - b^2c^6x + a^{10}b^2c^2x - 35a^2b^2c^5x - 34a^4b^2c^4x + 34a^6b^2c^3x + 35a^8b^2c^2x)(b(-c)^{(3/2)} - 2ab^2c + a^2b^2(-c)^{(1/2)})) / (2(a^4c + c^3 + 2a^2c^2)) - (2ab^2 \log(x)) / (c + a^2)^2$$

sympy [B] time = 11.12, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c+(b*x+a)**2),x)

[Out]
$$\begin{aligned} & -2ab^2 \log(x + (-16a^{11}b^2c^2/(a^2 + c)^4 + 48a^{11}b^2c^2/(a^2 + c)^4 + 352a^9b^2c^3/(a^2 + c)^4 - 20a^9b^2c/(a^2 + c)^2 + 608a^7b^2c^4/(a^2 + c)^4 - 64a^7b^2c^2/(a^2 + c)^2 + 432a^5b^2c^5/(a^2 + c)^4 - 72a^5b^2c^3/(a^2 + c)^2 + 36a^5b^2c + 112a^3b^2c^6/(a^2 + c)^4 - 32a^3b^2c^4/(a^2 + c)^2 - 88a^3b^2c^2 - 4ab^2c^5/(a^2 + c)^2 + 4ab^2c^3)/(a^6b^3 + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3)) / (a^2 + c)^2 + (ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) \log(x + (-4a^{11}c(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 12a^9c^2(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 10a^8b^2c(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 88a^7c^3(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 32a^6b^2c^2(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) + 36a^5b^2c + 152a^5c^4(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 36a^4b^2c^3(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) - 88a^3b^2c^2 + 108a^3c^5(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 16a^2b^2c^4(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 4ab^2c^3 + 28a^2c^6(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 2b^2c^5(ab/(a^2 + c)^2 - b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) / (a^6b^3 + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3)) + (ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) \log(x + (-4a^{11}c(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 12a^9c^2(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 10a^8b^2c(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 88a^7c^3(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 32a^6b^2c^2(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) + 36a^5b^2c + 152a^5c^4(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) + 36a^4b^2c^3(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) + 88a^3b^2c^2 + 108a^3c^5(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 16a^2b^2c^4(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))) + 4ab^2c^3 + 28a^2c^6(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2))))^2 + 2b^2c^5(ab/(a^2 + c)^2 + b\sqrt{-c}(a^2 - c)/(2c(a^4 + 2a^2c + c^2)))) / (a^6b^3 + 33a^4b^3c - 33a^2b^3c^2 - b^3c^3)) \end{aligned}$$

$$\begin{aligned}
& c)(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2}))^{**2} + 36*a^{**4}*b*c^{**3}*(a*b/(a^{**2} + c)^{**2} + b*sqrt(-c)*(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2}))) - 88*a^{**3} \\
& *b^{**2}*c^{**2} + 108*a^{**3}*c^{**5}*(a*b/(a^{**2} + c)^{**2} + b*sqrt(-c)*(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2})))^{**2} + 16*a^{**2}*b*c^{**4}*(a*b/(a^{**2} + c)^{**2} + b*sqrt(-c)*(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2}))) \\
& + 4*a*b^{**2}*c^{**3} + 28*a*c^{**6}*(a*b/(a^{**2} + c)^{**2} + b*sqrt(-c)*(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2})))^{**2} + 2*b*c^{**5}*(a*b/(a^{**2} + c)^{**2} + b*sqrt(-c)*(a^{**2} - c)/(2*c*(a^{**4} + 2*a^{**2}*c + c^{**2}))))/(a^{**6}*b^{**3} + 33*a^{**4}*b^{**3}*c - 33*a^{**2}*b^{**3}*c^{**2} - b^{**3}*c^{**3}) \\
&) - 1/(x*(a^{**2} + c))
\end{aligned}$$

$$3.80 \quad \int \frac{1}{x^3(c+(a+bx)^2)} dx$$

Optimal. Leaf size=121

$$\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^3} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3} + \frac{2ab}{x(a^2+c)^2} - \frac{1}{2x^2(a^2+c)}$$

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {371, 710, 801, 635, 203, 260}

$$\frac{b^2(3a^2-c)\log(x)}{(a^2+c)^3} - \frac{b^2(3a^2-c)\log((a+bx)^2+c)}{2(a^2+c)^3} - \frac{ab^2(a^2-3c)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}(a^2+c)^3} + \frac{2ab}{x(a^2+c)^2} - \frac{1}{2x^2(a^2+c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(c + (a + b*x)^2)),x]

[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2 + c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(d - e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (c + (a + bx)^2)} dx &= b^2 \operatorname{Subst} \left(\int \frac{1}{(-a + x)^3 (c + x^2)} dx, x, a + bx \right) \\
 &= -\frac{1}{2(a^2 + c)x^2} + \frac{b^2 \operatorname{Subst} \left(\int \frac{-a-x}{(-a+x)^2(c+x^2)} dx, x, a + bx \right)}{a^2 + c} \\
 &= -\frac{1}{2(a^2 + c)x^2} + \frac{b^2 \operatorname{Subst} \left(\int \left(-\frac{2a}{(a^2+c)(a-x)^2} + \frac{-3a^2+c}{(a^2+c)^2(a-x)} + \frac{-a(a^2-3c)-(3a^2-c)x}{(a^2+c)^2(c+x^2)} \right) dx, x, a + bx \right)}{a^2 + c} \\
 &= -\frac{1}{2(a^2 + c)x^2} + \frac{2ab}{(a^2 + c)^2 x} + \frac{b^2 (3a^2 - c) \log(x)}{(a^2 + c)^3} + \frac{b^2 \operatorname{Subst} \left(\int \frac{-a(a^2-3c)-(3a^2-c)x}{c+x^2} dx, x, a + bx \right)}{(a^2 + c)^3} \\
 &= -\frac{1}{2(a^2 + c)x^2} + \frac{2ab}{(a^2 + c)^2 x} + \frac{b^2 (3a^2 - c) \log(x)}{(a^2 + c)^3} - \frac{(ab^2 (a^2 - 3c)) \operatorname{Subst} \left(\int \frac{1}{c+x^2} dx, x, a + bx \right)}{(a^2 + c)^3} \\
 &= -\frac{1}{2(a^2 + c)x^2} + \frac{2ab}{(a^2 + c)^2 x} - \frac{ab^2 (a^2 - 3c) \tan^{-1} \left(\frac{a+bx}{\sqrt{c}} \right)}{\sqrt{c} (a^2 + c)^3} + \frac{b^2 (3a^2 - c) \log(x)}{(a^2 + c)^3} - \frac{b^2 (3a^2 - c)}{(a^2 + c)^3}
 \end{aligned}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.88

$$\frac{b^2(3a^2 - c) \log(a^2 + 2abx + b^2x^2 + c) + 2b^2(c - 3a^2) \log(x) + \frac{2ab^2(a^2 - 3c) \tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{(a^2+c)(a^2-4abx+c)}{x^2}}{2(a^2 + c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(c + (a + b*x)^2)), x]

[Out] -1/2*((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c]]/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(c + (a + bx)^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(c + (a + b*x)^2)), x]

[Out] IntegrateAlgebraic[1/(x^3*(c + (a + b*x)^2)), x]

fricas [A] time = 1.44, size = 371, normalized size = 3.07

$$\frac{\frac{a^4c - (a^3b^2 - 3ab^2c)\sqrt{c} \log\left(\frac{b^2x^2 + 2abx + a^2 + c}{2a^2b^2 + 3a^2c}\right) + 2a^2c^2 + (3a^2b^2c - b^2c^2)x^2 \log(b^2x^2 + 2abx + a^2 + c) - 2(3a^2b^2c - b^2c^2)x^2 \log(x) + c^3 - 4(a^3bc + ab^2c)x}{2(a^6c + 3a^4c^2 + 3a^2c^3 + c^4)^2} - \frac{a^4c + 2(a^3b^2 - 3ab^2c)\sqrt{c} \arctan\left(\frac{bx+a}{\sqrt{c}}\right) + 2a^2c^2 + (3a^2b^2c - b^2c^2)x^2 \log(b^2x^2 + 2abx + a^2 + c) - 2(3a^2b^2c - b^2c^2)x^2 \log(x) + c^3 - 4(a^3bc + ab^2c)x}{2(a^6c + 3a^4c^2 + 3a^2c^3 + c^4)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2), x, algorithm="fricas")

[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]

giac [A] time = 0.42, size = 195, normalized size = 1.61

$$\frac{(3a^2b^2 - b^2c) \log(b^2x^2 + 2abx + a^2 + c)}{2(a^6 + 3a^4c + 3a^2c^2 + c^3)} + \frac{(3a^2b^2 - b^2c) \log(|x|)}{a^6 + 3a^4c + 3a^2c^2 + c^3} - \frac{(a^3b^3 - 3ab^3c) \arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{(a^6 + 3a^4c + 3a^2c^2 + c^3)b\sqrt{c}} - \frac{a^4 + 2a^2c + c^2 - 4(a^3b + abc)x}{2(a^2 + c)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")

[Out] $-1/2*(3*a^2*b^2 - b^2*c)*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*\log(\text{abs}(x))/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*\arctan((b*x + a)/\sqrt{c})/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*b*\sqrt{c}) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*c)*x)/((a^2 + c)^3*x^2)$

maple [A] time = 0.01, size = 198, normalized size = 1.64

$$-\frac{a^3 b^2 \arctan\left(\frac{2b^2 x + 2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^3 \sqrt{c}} + \frac{3a^2 b^2 \ln(x)}{(a^2 + c)^3} - \frac{3a^2 b^2 \ln(b^2 x^2 + 2abx + a^2 + c)}{2(a^2 + c)^3} + \frac{3a b^2 \sqrt{c} \arctan\left(\frac{2b^2 x + 2ba}{2b\sqrt{c}}\right)}{(a^2 + c)^3} - \frac{b^2 c \ln(x)}{(a^2 + c)^3} + \frac{b^2 c \ln(b^2 x^2 + 2abx + a^2 + c)}{2(a^2 + c)^3} + \frac{2ab}{(a^2 + c)^2 x} - \frac{1}{2(a^2 + c)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c+(b*x+a)^2),x)

[Out] $-1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*\ln(x)*a^2-b^2/(a^2+c)^3*\ln(x)*c+2*a*b/(a^2+c)^2/x-3/2*b^2/(a^2+c)^3*\ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^3*\ln(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*\arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a$

maxima [A] time = 1.61, size = 197, normalized size = 1.63

$$-\frac{(3a^2b^2 - b^2c)\log(b^2x^2 + 2abx + a^2 + c)}{2(a^6 + 3a^4c + 3a^2c^2 + c^3)} + \frac{(3a^2b^2 - b^2c)\log(x)}{a^6 + 3a^4c + 3a^2c^2 + c^3} - \frac{(a^3b^3 - 3ab^3c)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{(a^6 + 3a^4c + 3a^2c^2 + c^3)b\sqrt{c}} + \frac{4abx - a^2 - c}{2(a^4 + 2a^2c + c^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")

[Out] $-1/2*(3*a^2*b^2 - b^2*c)*\log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*\log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*\arctan((b^2*x + a*b)/(b*\sqrt{c}))/((a^6 + 3*a^4*c + 3*a^2*c^2 + c^3)*b*\sqrt{c}) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c + c^2)*x^2)$

mupad [B] time = 2.77, size = 573, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(c + (a + b*x)^2)),x)

```
[Out] log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2))) - (
2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*a^
4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(5/2
) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*a^7*
c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c^6*x
- 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x)*(a^3
*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*c +
c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) +
9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(
5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*c^5 + 324*
a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x - 90*a^2*b*c^
6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^10*b*c^2*x)*
(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*
c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

sympy [B] time = 38.26, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
+ c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
+ 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
```

$$\begin{aligned}
& 3a^{**2}c^{**2} + c^{**3}) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}) + 568a^{**10}c^{**} \\
& *4*(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**} \\
& 3)) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 78a^{**8}b^{**4}c - 42a^{**8}b^{**} \\
& *2c^{**3}*(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} \\
& + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}) + 720a^{**8}c^{**5}*(-a*b^{**2}\sqrt{ \\
& rt(-c)*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3* \\
& a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 464a^{**6}b^{**4}c^{**2} - 8a^{**6}b^{**2}c^{**4}*(-a \\
& *b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - \\
& b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}) + 456a^{**6}c^{**6}*(-a*b^{**2}\sqrt{-c}*(a^{**} \\
& 2 - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(\\
& 2*(a^{**2} + c)^{**3}))^{**2} + 380a^{**4}b^{**4}c^{**3} - 2a^{**4}b^{**2}c^{**5}*(-a*b^{**2}\sqrt{ \\
& (-c)*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**} \\
& 2 - c)/(2*(a^{**2} + c)^{**3}) + 104a^{**4}c^{**7}*(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2 \\
& *c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c \\
&)^{**3}))^{**2} - 96a^{**2}b^{**4}c^{**4} - 12a^{**2}b^{**2}c^{**6}*(-a*b^{**2}\sqrt{-c}*(a^{**2} - \\
& 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(\\
& a^{**2} + c)^{**3})) - 24a^{**2}c^{**8}*(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3 \\
& a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + \\
& 6b^{**4}c^{**5} - 6b^{**2}c^{**7}*(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**} \\
& 4c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3})) - 12c^{**9} \\
& *(-a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3}) \\
&) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2})/(a^{**9}b^{**5} + 72a^{**7}b^{**5}c - \\
& 270a^{**5}b^{**5}c^{**2} + 144a^{**3}b^{**5}c^{**3} - 27a*b^{**5}c^{**4})) + (a*b^{**2}\sqrt{ \\
& (-c)*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} \\
& - c)/(2*(a^{**2} + c)^{**3}))*\log(x + (-4a^{**16}c*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/ \\
& (2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + \\
& c)^{**3}))^{**2} + 24a^{**14}c^{**2}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**} \\
& 4c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 14* \\
& a^{**12}b^{**2}c*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**} \\
& **2 + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 216a^{**12}c^{**3}*(a*b^{**} \\
& 2*\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2} \\
& *(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 44a^{**10}b^{**2}c^{**2}*(a*b^{**2}\sqrt{-c}*(\\
& a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c \\
&)/(2*(a^{**2} + c)^{**3})) + 568a^{**10}c^{**4}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a \\
& **6 + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3} \\
&))^{**2} + 78a^{**8}b^{**4}c - 42a^{**8}b^{**2}c^{**3}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2* \\
& c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c) \\
& **3)) + 720a^{**8}c^{**5}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + \\
& 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} - 464a^{**6} \\
& b^{**4}c^{**2} - 8a^{**6}b^{**2}c^{**4}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a \\
& **4c + 3a^{**2}c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 456a \\
& **6c^{**6}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**2}c^{**2} \\
& + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3}))^{**2} + 380a^{**4}b^{**4}c^{**3} - 2 \\
& *a^{**4}b^{**2}c^{**5}*(a*b^{**2}\sqrt{-c}*(a^{**2} - 3c)/(2*c*(a^{**6} + 3a^{**4}c + 3a^{**} \\
& 2*c^{**2} + c^{**3})) - b^{**2}(3a^{**2} - c)/(2*(a^{**2} + c)^{**3})) + 104a^{**4}c^{**7}*(a*b
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 \sqrt{-c} (a^2 - 3c)}{2c(a^6 + 3a^4c + 3a^2c^2 + c^3)} - b^2 \frac{(3a^2 - c)}{2(a^2 + c)^3} - 96a^2b^4c^4 - 12a^2b^2c^6 \frac{(a^2 - 3c)}{2c(a^6 + 3a^4c + 3a^2c^2 + c^3)} \\
& - b^2 \frac{(3a^2 - c)}{2(a^2 + c)^3} - 24a^2c^8 \frac{(a^2 - 3c)}{2c(a^6 + 3a^4c + 3a^2c^2 + c^3)} - b^2 \frac{(3a^2 - c)}{2(a^2 + c)^3} + 6b^4c^5 - 6b^2c^7 \frac{(a^2 - 3c)}{2c(a^6 + 3a^4c + 3a^2c^2 + c^3)} \\
& - b^2 \frac{(3a^2 - c)}{2(a^2 + c)^3} - 12c^9 \frac{(a^2 - 3c)}{2c(a^6 + 3a^4c + 3a^2c^2 + c^3)} - b^2 \frac{(3a^2 - c)}{2(a^2 + c)^3} \Big/ (a^9b^5 + 72a^7b^5c - 270a^5b^5c^2 + 144a^3b^5c^3 - 27ab^5c^4) \\
& + (-a^2 + 4abx - c) / (x^2(2a^4 + 4a^2c + 2c^2))
\end{aligned}$$

$$3.81 \quad \int \frac{1}{a+b(c+dx)^2} dx$$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-1), x]

fricas [A] time = 1.27, size = 109, normalized size = 3.52

$$\left[\frac{\sqrt{-ab} \log\left(\frac{bd^2x^2 + 2bcdx + bc^2 - 2\sqrt{-ab}(dx+c) - a}{bd^2x^2 + 2bcdx + bc^2 + a}\right)}{2abd}, \frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{abd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2), x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)/(a*b*d)]

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2), x, algorithm="giac")

[Out] $\arctan((b*d*x + b*c)/\sqrt{a*b})/(\sqrt{a*b}*d)$

maple [A] time = 0.01, size = 34, normalized size = 1.10

$$\frac{\arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a+b*(d*x+c)^2), x)$

[Out] $1/d/(b*a)^{(1/2)}*\arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*a)^{(1/2)})$

maxima [A] time = 1.56, size = 30, normalized size = 0.97

$$\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(a+b*(d*x+c)^2), x, \text{algorithm}=\text{"maxima"})$

[Out] $\arctan((b*d^2*x + b*c*d)/(\sqrt{a*b}*d))/(\sqrt{a*b}*d)$

mupad [B] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\text{atan}\left(\frac{\sqrt{b}c+\sqrt{b}dx}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a + b*(c + d*x)^2), x)$

[Out] $\text{atan}((b^{(1/2)}*c + b^{(1/2)}*d*x)/a^{(1/2)})/(a^{(1/2)}*b^{(1/2)}*d)$

sympy [B] time = 0.21, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}} \log\left(x + \frac{-a\sqrt{-\frac{1}{ab}} + c}{d}\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(x + \frac{a\sqrt{-\frac{1}{ab}} + c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(a+b*(d*x+c)**2), x)$

[Out] $(-\sqrt{-1/(a*b)}*\log(x + (-a*\sqrt{-1/(a*b)} + c)/d)/2 + \sqrt{-1/(a*b)}*\log(x + (a*\sqrt{-1/(a*b)} + c)/d)/2)/d$

$$3.82 \quad \int \frac{1}{(a+b(c+dx)^2)^2} dx$$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{c+dx}{2ad(a+b(c+dx)^2)}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{c+dx}{2ad(a+b(c+dx)^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b(c + dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, c + dx\right)}{d} \\
&= \frac{c + dx}{2ad(a + b(c + dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c + dx\right)}{2ad} \\
&= \frac{c + dx}{2ad(a + b(c + dx)^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^2)^(-2), x]

[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/Sqrt[b])/(2*a^(3/2)*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b(c + dx)^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-2), x]

[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-2), x]

fricas [A] time = 1.12, size = 253, normalized size = 4.02

$$\left[\frac{2 abdx + 2 abc - (bd^2x^2 + 2 bcdx + bc^2 + a)\sqrt{-ab} \log\left(\frac{bd^2x^2 + 2 bcdx + bc^2 - 2\sqrt{-ab}(dx+c) - a}{bd^2x^2 + 2 bcdx + bc^2 + a}\right)}{4(a^2b^2d^3x^2 + 2a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)}, \frac{abdx + abc + (bd^2x^2 + 2 bcdx + bc^2 + a)\sqrt{ab} \arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{2(a^2b^2d^3x^2 + 2a^2b^2cd^2x + (a^2b^2c^2 + a^3b)d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2*c^2 + a^3*b)*d)]

giac [A] time = 0.36, size = 65, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\sqrt{ab}ad} + \frac{dx+c}{2(bd^2x^2+2bcdx+bc^2+a)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)

maple [A] time = 0.00, size = 86, normalized size = 1.37

$$\frac{\arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{ab}d}\right)}{2\sqrt{ab}ad} + \frac{2bd^2x+2bdc}{4(bd^2x^2+2bcdx+bc^2+a)abd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^2)^2,x)

[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)

maxima [A] time = 1.56, size = 75, normalized size = 1.19

$$\frac{dx+c}{2(abd^3x^2+2abcd^2x+(abc^2+a^2)d)} + \frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}d}\right)}{2\sqrt{ab}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)

mupad [B] time = 0.10, size = 76, normalized size = 1.21

$$\frac{\frac{x}{2a} + \frac{c}{2ad}}{bc^2 + 2bcdx + bd^2x^2 + a} + \frac{\operatorname{atan}\left(2a\left(\frac{\sqrt{b}c}{2a^{3/2}} + \frac{\sqrt{b}dx}{2a^{3/2}}\right)\right)}{2a^{3/2}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*(c + d*x)^2)^2,x)`

[Out] $(x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + \operatorname{atan}(2*a*((b^{(1/2)}*c)/(2*a^{(3/2)}) + (b^{(1/2)}*d*x)/(2*a^{(3/2)})))/(2*a^{(3/2)}*b^{(1/2)}*d)$

sympy [B] time = 0.58, size = 117, normalized size = 1.86

$$\frac{c + dx}{2a^2d + 2abc^2d + 4abcd^2x + 2abd^3x^2} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(x + \frac{-a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}} \log\left(x + \frac{a^2\sqrt{-\frac{1}{a^3b}} + c}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*(d*x+c)**2)**2,x)`

[Out] $(c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-\operatorname{sqrt}(-1/(a**3*b))*\log(x + (-a**2*\operatorname{sqrt}(-1/(a**3*b)) + c)/d)/4 + \operatorname{sqrt}(-1/(a**3*b))*\log(x + (a**2*\operatorname{sqrt}(-1/(a**3*b)) + c)/d)/4)/d$

$$3.83 \quad \int \frac{1}{(a+b(c+dx)^2)^3} dx$$

Optimal. Leaf size=91

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{c+dx}{4ad(a+b(c+dx)^2)^2}$$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 205}

$$\frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{c+dx}{4ad(a+b(c+dx)^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2 + (3*(c + d*x))/(8*a^2*d*(a + b*(c + d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)

Rule 199

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+b(c+dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3} dx, x, c+dx\right)}{d} \\ &= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3\text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, c+dx\right)}{4ad} \\ &= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{8a^2d} \\ &= \frac{c+dx}{4ad(a+b(c+dx)^2)^2} + \frac{3(c+dx)}{8a^2d(a+b(c+dx)^2)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} \end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(c+dx)(5a+3b(c+dx)^2)}{(a+b(c+dx)^2)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3), x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/Sqrt[b])/(8*a^(5/2)*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b(c+dx)^2)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-3), x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-3), x]
```

fricas [B] time = 1.26, size = 595, normalized size = 6.54

$$\frac{1}{16(a^2b^2d^2 + 4ab^3cd^2 + 2(3a^2b^2 + a^2b^2)d^2 + 4(a^2b^2 + a^2b^2)dx + (a^2b^2 + 2a^2b^2 + a^2b^2))} \log\left(\frac{\sqrt{a}(c+dx) + \sqrt{b}(c+dx)^2}{\sqrt{a}(c+dx) - \sqrt{b}(c+dx)^2}\right) + \frac{3ab^2d^2 + 9ab^2cd^2 + 3ab^2c^2 + 5b^3c + (9ab^2c^2 + 5b^3d)dx + 3(b^2d^2 + 4b^2cd^2 + b^2c^2 + 2(3b^2c^2 + ab)d^2 + 2ab^2c + 4(b^2c^2 + ab^2)dx + 2ab^2c^2 + 4(b^2c^2 + ab^2)dx + 2)}{8(a^2b^2d^2 + 4ab^3cd^2 + 2(3a^2b^2 + a^2b^2)d^2 + 4(a^2b^2 + a^2b^2)dx + (a^2b^2 + 2a^2b^2 + a^2b^2))} \sqrt{ab} \arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d)]

giac [A] time = 0.41, size = 103, normalized size = 1.13

$$\frac{3 \arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2d} + \frac{3bd^3x^3 + 9bcd^2x^2 + 9bc^2dx + 3bc^3 + 5adx + 5ac}{8(bd^2x^2 + 2bcdx + bc^2 + a)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")

[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)^2*a^2*d)

maple [A] time = 0.00, size = 147, normalized size = 1.62

$$\frac{3x}{8(bd^2x^2 + 2bcdx + bc^2 + a)a^2} + \frac{3c}{8(bd^2x^2 + 2bcdx + bc^2 + a)a^2d} + \frac{3 \arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{ab}d}\right)}{8\sqrt{ab}a^2d} + \frac{2bd^2x + 2bdc}{8(bd^2x^2 + 2bcdx + bc^2 + a)^2abd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^2)^3,x)

[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/8/a^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*x+3/8/a^2/d/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*c+3/8/a^2/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)

maxima [B] time = 1.59, size = 184, normalized size = 2.02

$$\frac{3bd^3x^3 + 9bcd^2x^2 + 3bc^3 + (9bc^2 + 5a)dx + 5ac}{8(a^2b^2d^5x^4 + 4a^2b^2cd^4x^3 + 2(3a^2b^2c^2 + a^3b)d^3x^2 + 4(a^2b^2c^3 + a^3bc)d^2x + (a^2b^2c^4 + 2a^3bc^2 + a^4)d)} + \frac{3 \arctan\left(\frac{bd^2x + bcd}{\sqrt{abd}}\right)}{8\sqrt{ab}a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} \frac{(3b^3d^3x^3 + 9b^2cd^2x^2 + 3b^2c^3 + (9b^2c^2 + 5a)d^2x + 5a^2c)}{a^2b^2d^5x^4 + 4a^2b^2cd^4x^3 + 2(3a^2b^2c^2 + a^3b)d^3x^2 + 4(a^2b^2c^3 + a^3bc)d^2x + (a^2b^2c^4 + 2a^3bc^2 + a^4)d} + \frac{3}{8} \frac{\arctan\left(\frac{bd^2x + bcd}{\sqrt{abd}}\right)}{\sqrt{ab}a^2d}$

mupad [B] time = 2.22, size = 181, normalized size = 1.99

$$\frac{\frac{x(9bc^2+5a)}{8a^2} + \frac{3bc^3+5ac}{8a^2d} + \frac{3bd^2x^3}{8a^2} + \frac{9bcdx^2}{8a^2}}{x^2(6b^2c^2d^2 + 2abd^2) + x(4db^2c^3 + 4adbc) + a^2 + b^2c^4 + b^2d^4x^4 + 2abc^2 + 4b^2cd^3x^3} + \frac{3 \operatorname{atan}\left(\frac{8a^2\left(\frac{3\sqrt{b}c}{8a^{5/2}} + \frac{3\sqrt{b}dx}{8a^{5/2}}\right)}{3}\right)}{8a^{5/2}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^2)^3,x)

[Out] $\frac{(x(5a + 9b^2c^2))/(8a^2) + (5a^2c + 3b^2c^3)/(8a^2d) + (3b^2d^2x^3)/(8a^2) + (9b^2cd^2x^2)/(8a^2d) + x^2(6b^2c^2d^2 + 2a^2bd^2) + x(4b^2c^3d + 4a^2b^2cd) + a^2 + b^2c^4 + b^2d^4x^4 + 2a^2b^2c^2 + 4b^2cd^3x^3 + (3 \operatorname{atan}\left(\frac{8a^2\left(\frac{3b^{1/2}c}{8a^{5/2}}\right) + (3b^{1/2}d^2x)/(8a^{5/2})}{3}\right))/(8a^{5/2}b^{1/2}d)}$

sympy [B] time = 1.25, size = 257, normalized size = 2.82

$$\frac{5ac + 3bc^3 + 9bcd^2x^2 + 3bd^3x^3 + x(5ad + 9bc^2d)}{8a^4d + 16a^3bc^2d + 8a^2b^2c^4d + 32a^2b^2cd^4x^3 + 8a^2b^2d^5x^4 + x^2(16a^3bd^3 + 48a^2b^2c^2d^3) + x(32a^3bcd^2 + 32a^2b^2c^3d^2)} + \frac{3\sqrt{\frac{1}{a^5b}} \log\left(x + \frac{-3a^3\sqrt{\frac{1}{a^5b}} + 3c}{3d}\right)}{16} + \frac{3\sqrt{\frac{1}{a^5b}} \log\left(x + \frac{3a^3\sqrt{\frac{1}{a^5b}} + 3c}{3d}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**2)**3,x)

[Out] $\frac{(5a^2c + 3b^2c^3 + 9b^2cd^2x^2 + 3b^2d^3x^3 + x(5a^2d + 9b^2c^2d))}{8a^4d + 16a^3bc^2d + 8a^2b^2c^4d + 32a^2b^2cd^4x^3 + 8a^2b^2d^5x^4 + x^2(16a^3bd^3 + 48a^2b^2c^2d^3) + x(32a^3bcd^2 + 32a^2b^2c^3d^2)} + (-3\sqrt{-1/(a^5b)}) \log(x + (-3a^3\sqrt{-1/(a^5b)} + 3c)/(3d))/16 + 3\sqrt{-1/(a^5b)} \log(x + (3a^3\sqrt{-1/(a^5b)} + 3c)/(3d))/16/d$

$$3.84 \quad \int \frac{1}{\sqrt{-a} + b(c+dx)^2} dx$$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-a} + b(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a} + bx^2} dx, x, c + dx\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a} \sqrt{b} d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a} + b(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]

fricas [A] time = 1.46, size = 279, normalized size = 7.97

$$\left[\frac{\sqrt{\frac{-a}{ab}} \log\left(\frac{b^2d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2c^3dx + b^2c^4 - 2(bd^2x^2 + 2bcdx + bc^2)\sqrt{-a} + 2(abdx + abc + (b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3)\sqrt{-a})\sqrt{\frac{-a}{ab}} - a}{b^2d^4x^4 + 4b^2cd^3x^3 + 6b^2c^2d^2x^2 + 4b^2c^3dx + b^2c^4 + a}\right)}{2d}, \frac{\sqrt{-\frac{-a}{ab}} \arctan\left(\frac{bdx + bc}{d}\sqrt{-\frac{-a}{ab}}\right)}{d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")

[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b))*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]

giac [A] time = 0.45, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")

[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)

maple [A] time = 0.01, size = 42, normalized size = 1.20

$$\frac{\arctan\left(\frac{2bd^2x+2bdc}{2\sqrt{\sqrt{-a}bd}}\right)}{\sqrt{\sqrt{-a}bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)

[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1/2))

maxima [B] time = 1.40, size = 66, normalized size = 1.89

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}bd}}{bd^2x+bcd+\sqrt{-\sqrt{-a}bd}}\right)}{2\sqrt{-\sqrt{-a}bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")

[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sqrt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)

mupad [B] time = 0.10, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}c+\sqrt{b}dx}{(-a)^{1/4}}\right)}{(-a)^{1/4}\sqrt{b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)

[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)

sympy [B] time = 0.22, size = 92, normalized size = 2.63

$$\frac{\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}}{2} \log\left(x + \frac{c - \sqrt{-a} \sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right) + \frac{\sqrt{-\frac{1}{b\sqrt{-a}}}}{2} \log\left(x + \frac{c + \sqrt{-a} \sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)

[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a)))))/d)/2 + sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a)))))/d)/2)/d

$$3.85 \quad \int \frac{1}{1+(c+dx)^2} dx$$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 203}

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-1), x]

[Out] ArcTan[c + d*x]/d

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{d} \\ &= \frac{\tan^{-1}(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-1), x]

[Out] ArcTan[c + d*x]/d

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 + (c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-1), x]

fricas [A] time = 1.45, size = 10, normalized size = 1.00

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2), x, algorithm="fricas")

[Out] arctan(d*x + c)/d

giac [A] time = 0.32, size = 10, normalized size = 1.00

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2), x, algorithm="giac")

[Out] arctan(d*x + c)/d

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(d*x+c)^2), x)

[Out] arctan(d*x+c)/d

maxima [A] time = 1.57, size = 18, normalized size = 1.80

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")

[Out] arctan((d^2*x + c*d)/d)/d

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c + d*x)^2 + 1),x)

[Out] atan(c + d*x)/d

sympy [C] time = 0.17, size = 24, normalized size = 2.40

$$\frac{-\frac{i \log\left(x + \frac{c-i}{d}\right)}{2} + \frac{i \log\left(x + \frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)**2),x)

[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d

$$3.86 \quad \int \frac{1}{(1+(c+dx)^2)^2} dx$$

Optimal. Leaf size=37

$$\frac{c+dx}{2d((c+dx)^2+1)} + \frac{\tan^{-1}(c+dx)}{2d}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 203}

$$\frac{c+dx}{2d((c+dx)^2+1)} + \frac{\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/
(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/
(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1],
Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+(c+dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, c+dx\right)}{d} \\ &= \frac{c+dx}{2d(1+(c+dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{2d} \\ &= \frac{c+dx}{2d(1+(c+dx)^2)} + \frac{\tan^{-1}(c+dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1} + \tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-2), x]

[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+(c+dx)^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-2), x]

[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-2), x]

fricas [A] time = 1.15, size = 55, normalized size = 1.49

$$\frac{dx + (d^2x^2 + 2cdx + c^2 + 1) \arctan(dx + c) + c}{2(d^3x^2 + 2cd^2x + (c^2 + 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*\arctan(d*x + c) + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d)$

giac [A] time = 0.38, size = 41, normalized size = 1.11

$$\frac{\arctan(dx + c)}{2d} + \frac{dx + c}{2(d^2x^2 + 2cdx + c^2 + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $\frac{1}{2}*\arctan(d*x + c)/d + \frac{1}{2}*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)$

maple [A] time = 0.01, size = 59, normalized size = 1.59

$$\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d} + \frac{2d^2x + 2cd}{4(d^2x^2 + 2cdx + c^2 + 1)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+(d*x+c)^2)^2,x)`

[Out] $\frac{1}{4}*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+\frac{1}{2}/d*\arctan(1/2*(2*d^2*x+2*c*d)/d)$

maxima [A] time = 1.32, size = 51, normalized size = 1.38

$$\frac{dx + c}{2(d^3x^2 + 2cd^2x + (c^2 + 1)d)} + \frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{2}*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + \frac{1}{2}*\arctan((d^2*x + c*d)/d)/d$

mupad [B] time = 2.07, size = 42, normalized size = 1.14

$$\frac{\frac{x}{2} + \frac{c}{2d}}{c^2 + 2cdx + d^2x^2 + 1} + \frac{\operatorname{atan}(c + dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((c + d*x)^2 + 1)^2,x)`

[Out] $(x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + \operatorname{atan}(c + d*x)/(2*d)$

sympy [C] time = 0.44, size = 56, normalized size = 1.51

$$\frac{c + dx}{2c^2d + 4cd^2x + 2d^3x^2 + 2d} + \frac{-\frac{i \log\left(x + \frac{c-i}{d}\right)}{4} + \frac{i \log\left(x + \frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+(d*x+c)**2)**2,x)`

[Out] $(c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*\log(x + (c - I)/d)/4 + I*\log(x + (c + I)/d)/4)/d$

$$3.87 \quad \int \frac{1}{(1+(c+dx)^2)^3} dx$$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d((c+dx)^2+1)} + \frac{c+dx}{4d((c+dx)^2+1)^2} + \frac{3 \tan^{-1}(c+dx)}{8d}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 203}

$$\frac{3(c+dx)}{8d((c+dx)^2+1)} + \frac{c+dx}{4d((c+dx)^2+1)^2} + \frac{3 \tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(1 + (c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2)) + (3*ArcTan[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+(c+dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3} dx, x, c+dx\right)}{d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, c+dx\right)}{4d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1+(c+dx)^2)} + \frac{3 \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, c+dx\right)}{8d} \\
&= \frac{c+dx}{4d(1+(c+dx)^2)^2} + \frac{3(c+dx)}{8d(1+(c+dx)^2)} + \frac{3 \tan^{-1}(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1} + \frac{2(c+dx)}{((c+dx)^2+1)^2} + 3 \tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (c + d*x)^2)^(-3), x]

[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Arctan[c + d*x])/(8*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+(c+dx)^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-3), x]

[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-3), x]

fricas [B] time = 0.90, size = 153, normalized size = 2.55

$$\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 + 5)dx + 3(d^4x^4 + 4cd^3x^3 + 2(3c^2 + 1)d^2x^2 + c^4 + 4(c^3 + c)dx + 2c^2 + 1) \arctan(dx + c) + 5c}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 + 1)d^3x^2 + 4(c^3 + c)d^2x + (c^4 + 2c^2 + 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $\frac{1}{8}*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*\arctan(d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d)$

giac [A] time = 0.30, size = 73, normalized size = 1.22

$$\frac{3 \arctan(dx + c)}{8d} + \frac{3d^3x^3 + 9cd^2x^2 + 9c^2dx + 3c^3 + 5dx + 5c}{8(d^2x^2 + 2cdx + c^2 + 1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{3}{8}*\arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 + 5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)$

maple [A] time = 0.01, size = 94, normalized size = 1.57

$$\frac{3 \arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d} + \frac{2d^2x + 2cd}{8(d^2x^2 + 2cdx + c^2 + 1)^2d^2} + \frac{\frac{3}{8}d^2x + \frac{3}{8}cd}{(d^2x^2 + 2cdx + c^2 + 1)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+(d*x+c)^2)^3,x)

[Out] $\frac{1}{8}*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2*c*d)/(d^2*x^2+2*c*d*x+c^2+1)/d^2+3/8/d*\arctan(1/2*(2*d^2*x+2*c*d)/d)$

maxima [B] time = 1.51, size = 115, normalized size = 1.92

$$\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 + 5)dx + 5c}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 + 1)d^3x^2 + 4(c^3 + c)d^2x + (c^4 + 2c^2 + 1)d)} + \frac{3 \arctan\left(\frac{d^2x+cd}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{8}*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d) + 3/8*\arctan((d^2*x + c*d)/d)/d$

mupad [B] time = 0.12, size = 111, normalized size = 1.85

$$\frac{3 \operatorname{atan}(c + dx)}{8d} + \frac{x \left(\frac{9c^2}{8} + \frac{5}{8} \right) + \frac{3c^3 + 5c}{8d} + \frac{3d^2 x^3}{8} + \frac{9cdx^2}{8}}{x^2 (6c^2 d^2 + 2d^2) + 2c^2 + c^4 + x(4dc^3 + 4dc) + d^4 x^4 + 4cd^3 x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((c + d*x)^2 + 1)^3, x)

[Out] (3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*d^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*c*d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)

sympy [C] time = 0.93, size = 146, normalized size = 2.43

$$\frac{3c^3 + 9cd^2x^2 + 5c + 3d^3x^3 + x(9c^2d + 5d)}{8c^4d + 16c^2d + 32cd^4x^3 + 8d^5x^4 + 8d + x^2(48c^2d^3 + 16d^3) + x(32c^3d^2 + 32cd^2)} + \frac{-\frac{3i \log\left(x + \frac{3c-3i}{3d}\right)}{16} + \frac{3i \log\left(x + \frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+(d*x+c)**2)**3, x)

[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/16 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d

$$3.88 \quad \int \frac{1}{1-(c+dx)^2} dx$$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {247, 206}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-1), x]

[Out] ArcTanh[c + d*x]/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(c+dx)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c+dx\right)}{d} \\ &= \frac{\tanh^{-1}(c+dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 3.20

$$\frac{\log(c+dx+1)}{2d} - \frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-1), x]

[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 - (c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-1), x]

fricas [B] time = 0.80, size = 22, normalized size = 2.20

$$\frac{\log(dx + c + 1) - \log(dx + c - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2), x, algorithm="fricas")

[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d

giac [B] time = 0.38, size = 27, normalized size = 2.70

$$\frac{\log(|dx + c + 1|)}{2d} - \frac{\log(|dx + c - 1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2), x, algorithm="giac")

[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d

maple [B] time = 0.01, size = 26, normalized size = 2.60

$$-\frac{\ln(dx + c - 1)}{2d} + \frac{\ln(dx + c + 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(d*x+c)^2), x)

[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)

maxima [B] time = 0.49, size = 25, normalized size = 2.50

$$\frac{\log(dx + c + 1)}{2d} - \frac{\log(dx + c - 1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d

mupad [B] time = 2.05, size = 10, normalized size = 1.00

$$\frac{\operatorname{atanh}(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((c + d*x)^2 - 1),x)

[Out] atanh(c + d*x)/d

sympy [B] time = 0.18, size = 22, normalized size = 2.20

$$-\frac{\frac{\log\left(x + \frac{c-1}{d}\right)}{2} - \frac{\log\left(x + \frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)**2),x)

[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d

$$3.89 \quad \int \frac{1}{(1-(c+dx)^2)^2} dx$$

Optimal. Leaf size=39

$$\frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 206}

$$\frac{c+dx}{2d(1-(c+dx)^2)} + \frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-2), x]

[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1 - (c + dx)^2)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x, c + dx\right)}{d} \\ &= \frac{c + dx}{2d(1 - (c + dx)^2)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{2d} \\ &= \frac{c + dx}{2d(1 - (c + dx)^2)} + \frac{\tanh^{-1}(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1} - \log(-c - dx + 1) + \log(c + dx + 1)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-2), x]

[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(4*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - (c + dx)^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-2), x]

[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-2), x]

fricas [B] time = 0.93, size = 85, normalized size = 2.18

$$\frac{2 dx - (d^2 x^2 + 2 c dx + c^2 - 1) \log(dx + c + 1) + (d^2 x^2 + 2 c dx + c^2 - 1) \log(dx + c - 1) + 2 c}{4 (d^3 x^2 + 2 c d^2 x + (c^2 - 1) d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $-1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*\log(d*x + c + 1) + (d^2*x^2 + 2*c*d*x + c^2 - 1)*\log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d)$

giac [A] time = 0.35, size = 56, normalized size = 1.44

$$\frac{\log(|dx + c + 1|)}{4d} - \frac{\log(|dx + c - 1|)}{4d} - \frac{dx + c}{2(d^2x^2 + 2cdx + c^2 - 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $1/4*\log(\text{abs}(d*x + c + 1))/d - 1/4*\log(\text{abs}(d*x + c - 1))/d - 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 - 1)*d)$

maple [A] time = 0.01, size = 52, normalized size = 1.33

$$-\frac{\ln(dx + c - 1)}{4d} + \frac{\ln(dx + c + 1)}{4d} - \frac{1}{4(dx + c - 1)d} - \frac{1}{4(dx + c + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-(d*x+c)^2)^2,x)`

[Out] $-1/4/d/(d*x+c-1)-1/4/d*\ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*\ln(d*x+c+1)$

maxima [A] time = 0.62, size = 56, normalized size = 1.44

$$-\frac{dx + c}{2(d^3x^2 + 2cd^2x + (c^2 - 1)d)} + \frac{\log(dx + c + 1)}{4d} - \frac{\log(dx + c - 1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*\log(d*x + c + 1)/d - 1/4*\log(d*x + c - 1)/d$

mupad [B] time = 2.06, size = 43, normalized size = 1.10

$$\frac{\operatorname{atanh}(c + dx)}{2d} - \frac{\frac{x}{2} + \frac{c}{2d}}{c^2 + 2cdx + d^2x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((c + d*x)^2 - 1)^2,x)`

[Out] $\operatorname{atanh}(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)$

sympy [A] time = 0.47, size = 54, normalized size = 1.38

$$\frac{-c - dx}{2c^2d + 4cd^2x + 2d^3x^2 - 2d} + \frac{-\frac{\log\left(x + \frac{c-1}{d}\right)}{4} + \frac{\log\left(x + \frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(d*x+c)**2)**2,x)`

[Out] $(-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-\log(x + (c - 1)/d)/4 + \log(x + (c + 1)/d)/4)/d$

$$3.90 \quad \int \frac{1}{(1-(c+dx)^2)^3} dx$$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3 \tanh^{-1}(c+dx)}{8d}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 206}

$$\frac{3(c+dx)}{8d(1-(c+dx)^2)} + \frac{c+dx}{4d(1-(c+dx)^2)^2} + \frac{3 \tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(1 - (c + d*x)^2)^(-3), x]

[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2)) + (3*ArcTanh[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 - (c + dx)^2)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3} dx, x, c + dx\right)}{d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x, c + dx\right)}{4d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3(c + dx)}{8d(1 - (c + dx)^2)} + \frac{3 \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, c + dx\right)}{8d} \\
&= \frac{c + dx}{4d(1 - (c + dx)^2)^2} + \frac{3(c + dx)}{8d(1 - (c + dx)^2)} + \frac{3 \tanh^{-1}(c + dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1} + \frac{4(c+dx)}{((c+dx)^2-1)^2} - 3 \log(-c - dx + 1) + 3 \log(c + dx + 1)}{16d}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (c + d*x)^2)^(-3), x]

[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - (c + dx)^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-3), x]

[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-3), x]

fricas [B] time = 1.21, size = 220, normalized size = 3.44

$$\frac{6d^3x^3 + 18cd^2x^2 + 6c^3 + 2(9c^2 - 5)dx - 3(d^4x^4 + 4cd^3x^3 + 2(3c^2 - 1)d^2x^2 + c^4 + 4(c^3 - c)dx - 2c^2 + 1) \log(dx + c + 1) + 3(d^4x^4 + 4cd^3x^3 + 2(3c^2 - 1)d^2x^2 + c^4 + 4(c^3 - c)dx - 2c^2 + 1) \log(dx + c - 1) - 10c}{16(d^5x^4 + 4cd^4x^3 + 2(3c^2 - 1)d^3x^2 + 4(c^3 - c)d^2x + (c^4 - 2c^2 + 1)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$-1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*\log(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*\log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)$$

giac [A] time = 0.36, size = 88, normalized size = 1.38

$$\frac{3 \log(|dx + c + 1|)}{16d} - \frac{3 \log(|dx + c - 1|)}{16d} - \frac{3d^3x^3 + 9cd^2x^2 + 9c^2dx + 3c^3 - 5dx - 5c}{8(d^2x^2 + 2cdx + c^2 - 1)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$3/16*\log(\text{abs}(d*x + c + 1))/d - 3/16*\log(\text{abs}(d*x + c - 1))/d - 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^2 - 1)^2*d)$$

maple [A] time = 0.01, size = 78, normalized size = 1.22

$$-\frac{3 \ln(dx + c - 1)}{16d} + \frac{3 \ln(dx + c + 1)}{16d} + \frac{1}{16(dx + c - 1)^2d} - \frac{3}{16(dx + c - 1)d} - \frac{1}{16(dx + c + 1)^2d} - \frac{3}{16(dx + c + 1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(d*x+c)^2)^3,x)

[Out]
$$1/16/d/(d*x+c-1)^2-3/16/(d*x+c-1)/d-3/16/d*\ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3/16/(d*x+c+1)/d+3/16/d*\ln(d*x+c+1)$$

maxima [B] time = 0.64, size = 122, normalized size = 1.91

$$\frac{3d^3x^3 + 9cd^2x^2 + 3c^3 + (9c^2 - 5)dx - 5c}{8(d^5x^4 + 4cd^4x^3 + 2(3c^2 - 1)d^3x^2 + 4(c^3 - c)d^2x + (c^4 - 2c^2 + 1)d)} + \frac{3 \log(dx + c + 1)}{16d} - \frac{3 \log(dx + c - 1)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d) + 3/16*\log(d*x + c + 1)/d - 3/16*\log(d*x + c - 1)/d$$

mupad [B] time = 2.12, size = 114, normalized size = 1.78

$$\frac{3 \operatorname{atanh}(c + dx)}{8d} - \frac{x \left(\frac{9c^2}{8} - \frac{5}{8} \right) - \frac{5c - 3c^3}{8d} + \frac{3d^2 x^3}{8} + \frac{9cdx^2}{8}}{c^4 - 2c^2 - x^2 (2d^2 - 6c^2 d^2) - x (4cd - 4c^3 d) + d^4 x^4 + 4cd^3 x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/((c + d*x)^2 - 1)^3, x)`

[Out] $(3 \operatorname{atanh}(c + dx))/(8d) - (x((9c^2)/8 - 5/8) - (5c - 3c^3)/(8d) + (3d^2 x^3)/8 + (9cdx^2)/8)/(c^4 - 2c^2 - x^2(2d^2 - 6c^2 d^2) - x(4cd - 4c^3 d) + d^4 x^4 + 4cd^3 x^3 + 1)$

sympy [B] time = 1.03, size = 141, normalized size = 2.20

$$\frac{3c^3 + 9cd^2 x^2 - 5c + 3d^3 x^3 + x(9c^2 d - 5d)}{8c^4 d - 16c^2 d + 32cd^4 x^3 + 8d^5 x^4 + 8d + x^2(48c^2 d^3 - 16d^3) + x(32c^3 d^2 - 32cd^2)} - \frac{\frac{3 \log\left(x + \frac{3c-3}{3d}\right)}{16} - \frac{3 \log\left(x + \frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(d*x+c)**2)**3, x)`

[Out] $-(3c^3 + 9cd^2 x^2 - 5c + 3d^3 x^3 + x(9c^2 d - 5d))/(8c^4 d - 16c^2 d + 32cd^4 x^3 + 8d^5 x^4 + 8d + x^2(48c^2 d^3 - 16d^3) + x(32c^3 d^2 - 32cd^2)) - (3 \log(x + (3c - 3)/(3d))/16 - 3 \log(x + (3c + 3)/(3d))/16)/d$

$$3.91 \quad \int \frac{1}{1-(1+x)^2} dx$$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 206}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-1), x]

[Out] ArcTanh[1 + x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\int \frac{1}{1-(1+x)^2} dx = \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, 1+x\right) \\ = \tanh^{-1}(1+x)$$

Mathematica [B] time = 0.00, size = 15, normalized size = 3.75

$$\frac{1}{2} \log(x+2) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-1), x]

[Out] -1/2*Log[x] + Log[2 + x]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 - (1 + x)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-1), x]

fricas [B] time = 1.28, size = 11, normalized size = 2.75

$$\frac{1}{2} \log(x + 2) - \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2), x, algorithm="fricas")

[Out] 1/2*log(x + 2) - 1/2*log(x)

giac [B] time = 0.35, size = 13, normalized size = 3.25

$$\frac{1}{2} \log(|x + 2|) - \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2), x, algorithm="giac")

[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))

maple [B] time = 0.00, size = 12, normalized size = 3.00

$$-\frac{\ln(x)}{2} + \frac{\ln(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2), x)

[Out] 1/2*ln(2+x)-1/2*ln(x)

maxima [B] time = 0.57, size = 11, normalized size = 2.75

$$\frac{1}{2} \log(x + 2) - \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")

[Out] 1/2*log(x + 2) - 1/2*log(x)

mupad [B] time = 0.15, size = 4, normalized size = 1.00

$$\operatorname{atanh}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((x + 1)^2 - 1),x)

[Out] atanh(x + 1)

sympy [B] time = 0.10, size = 10, normalized size = 2.50

$$-\frac{\log(x)}{2} + \frac{\log(x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)**2),x)

[Out] -log(x)/2 + log(x + 2)/2

$$3.92 \quad \int \frac{1}{(1-(1+x)^2)^2} dx$$

Optimal. Leaf size=27

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 206}

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-2), x]

[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/
(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 - (1 + x)^2)^2} dx &= \text{Subst} \left(\int \frac{1}{(1 - x^2)^2} dx, x, 1 + x \right) \\
&= \frac{1 + x}{2(1 - (1 + x)^2)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, 1 + x \right) \\
&= \frac{1 + x}{2(1 - (1 + x)^2)} + \frac{1}{2} \tanh^{-1}(1 + x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4} \left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-2), x]

[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - (1 + x)^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-2), x]

[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-2), x]

fricas [A] time = 1.08, size = 39, normalized size = 1.44

$$\frac{(x^2 + 2x) \log(x + 2) - (x^2 + 2x) \log(x) - 2x - 2}{4(x^2 + 2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)

giac [A] time = 0.33, size = 27, normalized size = 1.00

$$-\frac{x+1}{2(x^2+2x)} + \frac{1}{4} \log(|x+2|) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4} + \frac{\ln(x+2)}{4} - \frac{1}{4x} - \frac{1}{4(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2)^2,x)

[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)

maxima [A] time = 0.65, size = 25, normalized size = 0.93

$$-\frac{x+1}{2(x^2+2x)} + \frac{1}{4} \log(x+2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)

mupad [B] time = 0.07, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x+1)}{2} - \frac{x+1}{2((x+1)^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)^2 - 1)^2,x)

[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))

sympy [A] time = 0.12, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

$$3.93 \quad \int \frac{1}{(1-(1+x)^2)^3} dx$$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8} \tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 199, 206}

$$\frac{3(x+1)}{8(1-(x+1)^2)} + \frac{x+1}{4(1-(x+1)^2)^2} + \frac{3}{8} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 - (1 + x)^2)^(-3), x]

[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTanh[1 + x])/8

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 - (1 + x)^2)^3} dx &= \text{Subst} \left(\int \frac{1}{(1 - x^2)^3} dx, x, 1 + x \right) \\
&= \frac{1 + x}{4(1 - (1 + x)^2)^2} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{(1 - x^2)^2} dx, x, 1 + x \right) \\
&= \frac{1 + x}{4(1 - (1 + x)^2)^2} + \frac{3(1 + x)}{8(1 - (1 + x)^2)} + \frac{3}{8} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, 1 + x \right) \\
&= \frac{1 + x}{4(1 - (1 + x)^2)^2} + \frac{3(1 + x)}{8(1 - (1 + x)^2)} + \frac{3}{8} \tanh^{-1}(1 + x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.82

$$\frac{1}{16} \left(\frac{1}{x^2} - \frac{3}{x} - \frac{3}{x+2} - \frac{1}{(x+2)^2} - 3 \log(x) + 3 \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - (1 + x)^2)^(-3), x]

[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - (1 + x)^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-3), x]

[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-3), x]

fricas [B] time = 0.82, size = 71, normalized size = 1.58

$$\frac{6x^3 + 18x^2 - 3(x^4 + 4x^3 + 4x^2) \log(x+2) + 3(x^4 + 4x^3 + 4x^2) \log(x) + 8x - 4}{16(x^4 + 4x^3 + 4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")

[Out] $-1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*\log(x + 2) + 3*(x^4 + 4*x^3 + 4*x^2)*\log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)$

giac [A] time = 0.34, size = 39, normalized size = 0.87

$$-\frac{3x^3 + 9x^2 + 4x - 2}{8(x^2 + 2x)^2} + \frac{3}{16} \log(|x + 2|) - \frac{3}{16} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")

[Out] $-1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*\log(\text{abs}(x + 2)) - 3/16*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{3 \ln(x)}{16} + \frac{3 \ln(x + 2)}{16} - \frac{3}{16x} + \frac{1}{16x^2} - \frac{1}{16(x + 2)^2} - \frac{3}{16(x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(x+1)^2)^3,x)

[Out] $-1/16/(x+2)^2 - 3/16/(x+2) + 3/16*\ln(x+2) + 1/16/x^2 - 3/16/x - 3/16*\ln(x)$

maxima [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{3x^3 + 9x^2 + 4x - 2}{8(x^4 + 4x^3 + 4x^2)} + \frac{3}{16} \log(x + 2) - \frac{3}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")

[Out] $-1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*\log(x + 2) - 3/16*\log(x)$

mupad [B] time = 2.09, size = 36, normalized size = 0.80

$$\frac{3 \operatorname{atanh}(x + 1)}{8} + \frac{\frac{5x}{8} - \frac{3(x+1)^3}{8} + \frac{5}{8}}{(x+1)^4 - 2(x+1)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/((x + 1)^2 - 1)^3,x)`

[Out] $(3*\operatorname{atanh}(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x + 1)^2 + 1)$

sympy [A] time = 0.14, size = 44, normalized size = 0.98

$$-\frac{3 \log(x)}{16} + \frac{3 \log(x + 2)}{16} - \frac{3x^3 + 9x^2 + 4x - 2}{8x^4 + 32x^3 + 32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-(1+x)**2)**3,x)`

[Out] $-3*\log(x)/16 + 3*\log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x**3 + 32*x**2)$

$$3.94 \quad \int \frac{(1+(a+bx)^2)^2}{x} dx$$

Optimal. Leaf size=59

$$\frac{1}{2}(a^2+2)(a+bx)^2 + a(a^2+2)bx + (a^2+1)^2 \log(x) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {371, 697}

$$\frac{1}{2}(a^2+2)(a+bx)^2 + a(a^2+2)bx + (a^2+1)^2 \log(x) + \frac{1}{4}(a+bx)^4 + \frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

[In] Int[(1 + (a + b*x)^2)^2/x, x]

[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)^4/4 + (1 + a^2)^2*Log[x]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{(1+(a+bx)^2)^2}{x} dx &= \text{Subst} \left(\int \frac{(1+x^2)^2}{-a+x} dx, x, a+bx \right) \\ &= \text{Subst} \left(\int \left(a(2+a^2) - \frac{(1+a^2)^2}{a-x} + (2+a^2)x + ax^2 + x^3 \right) dx, x, a+bx \right) \\ &= a(2+a^2)bx + \frac{1}{2}(2+a^2)(a+bx)^2 + \frac{1}{3}a(a+bx)^3 + \frac{1}{4}(a+bx)^4 + (1+a^2)^2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.08

$$\frac{1}{2}(a^2 + 2)(a + bx)^2 + a(a^2 + 2)(a + bx) + (a^2 + 1)^2 \log(bx) + \frac{1}{4}(a + bx)^4 + \frac{1}{3}a(a + bx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (a + b*x)^2)^2/x, x]

[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)^4/4 + (1 + a^2)^2*Log[b*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1 + (a + bx)^2)^2}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + (a + b*x)^2)^2/x, x]

[Out] IntegrateAlgebraic[(1 + (a + b*x)^2)^2/x, x]

fricas [A] time = 1.20, size = 54, normalized size = 0.92

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + (3a^2 + 1)b^2x^2 + 4(a^3 + a)bx + (a^4 + 2a^2 + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")

[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4 + 2*a^2 + 1)*log(x)

giac [A] time = 0.31, size = 62, normalized size = 1.05

$$\frac{1}{4}b^4x^4 + \frac{4}{3}ab^3x^3 + 3a^2b^2x^2 + 4a^3bx + b^2x^2 + 4abx + (a^4 + 2a^2 + 1)\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")

[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x + (a^4 + 2*a^2 + 1)*log(abs(x))

maple [A] time = 0.00, size = 64, normalized size = 1.08

$$\frac{b^4 x^4}{4} + \frac{4a b^3 x^3}{3} + 3a^2 b^2 x^2 + a^4 \ln(x) + 4a^3 b x + b^2 x^2 + 2a^2 \ln(x) + 4abx + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+(b*x+a)^2)^2/x,x)

[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*x^2*a^2*b^2+4*a^3*b*x+b^2*x^2+4*a*b*x+ln(x)*a^4+2*ln(x)*a^2+ln(x)

maxima [A] time = 0.54, size = 54, normalized size = 0.92

$$\frac{1}{4} b^4 x^4 + \frac{4}{3} a b^3 x^3 + (3a^2 + 1)b^2 x^2 + 4(a^3 + a)bx + (a^4 + 2a^2 + 1)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")

[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4 + 2*a^2 + 1)*log(x)

mupad [B] time = 0.05, size = 55, normalized size = 0.93

$$\ln(x) (a^4 + 2a^2 + 1) + \frac{b^4 x^4}{4} + \frac{4a b^3 x^3}{3} + b^2 x^2 (3a^2 + 1) + 4a b x (a^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)^2 + 1)^2/x,x)

[Out] log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 + 1) + 4*a*b*x*(a^2 + 1)

sympy [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2(3a^2b^2 + b^2) + x(4a^3b + 4ab) + (a^2 + 1)^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b*x+a)**2)**2/x,x)

[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4*a*b) + (a**2 + 1)**2*log(x)

$$3.95 \quad \int \frac{x^2}{1+(-1+x)^2} dx$$

Optimal. Leaf size=10

$$x + \log((x-1)^2 + 1)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {371, 702, 260}

$$x + \log((x-1)^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + (-1 + x)^2), x]

[Out] x + Log[1 + (-1 + x)^2]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 702

Int[((d_) + (e_)*(x_)^(m_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+(-1+x)^2} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{1+x^2} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \left(1 + \frac{2x}{1+x^2} \right) dx, x, -1+x \right) \\
&= x + 2 \text{Subst} \left(\int \frac{x}{1+x^2} dx, x, -1+x \right) \\
&= x + \log(1+(-1+x)^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log(x^2 - 2x + 2) + x$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + (-1 + x)^2), x]

[Out] x + Log[2 - 2*x + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+(-1+x)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 + (-1 + x)^2), x]

[Out] IntegrateAlgebraic[x^2/(1 + (-1 + x)^2), x]

fricas [A] time = 1.22, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(-1+x)^2), x, algorithm="fricas")

[Out] x + log(x^2 - 2*x + 2)

giac [A] time = 0.35, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")`

[Out] `x + log(x^2 - 2*x + 2)`

maple [A] time = 0.00, size = 12, normalized size = 1.20

$$x + \ln(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(1+(x-1)^2),x)`

[Out] `x+ln(x^2-2*x+2)`

maxima [A] time = 0.68, size = 11, normalized size = 1.10

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")`

[Out] `x + log(x^2 - 2*x + 2)`

mupad [B] time = 0.03, size = 11, normalized size = 1.10

$$x + \ln(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((x - 1)^2 + 1),x)`

[Out] `x + log(x^2 - 2*x + 2)`

sympy [A] time = 0.09, size = 10, normalized size = 1.00

$$x + \log(x^2 - 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1+(-1+x)**2),x)`

[Out] `x + log(x**2 - 2*x + 2)`

$$3.96 \quad \int \frac{x^2}{\sqrt{1-(1+x)^2}} dx$$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x + \frac{3}{2}\sqrt{1-(x+1)^2} + \frac{3}{2}\sin^{-1}(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {371, 671, 641, 216}

$$-\frac{1}{2}\sqrt{1-(x+1)^2}x + \frac{3}{2}\sqrt{1-(x+1)^2} + \frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 - (1 + x)^2], x]

[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 671

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(1+x)^2}} dx &= \text{Subst} \left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}} dx, x, 1+x \right) \\
&= -\frac{1}{2}x\sqrt{1-(1+x)^2} - \frac{3}{2} \text{Subst} \left(\int \frac{-1+x}{\sqrt{1-x^2}} dx, x, 1+x \right) \\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1+x \right) \\
&= \frac{3}{2}\sqrt{1-(1+x)^2} - \frac{1}{2}x\sqrt{1-(1+x)^2} + \frac{3}{2} \sin^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.16

$$\frac{x(x^2 - x - 6) + 6\sqrt{x}\sqrt{x+2} \sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 - (1 + x)^2], x]

[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]])/(2*Sqrt[-(x*(2 + x))])

IntegrateAlgebraic [A] time = 0.13, size = 45, normalized size = 1.02

$$\frac{1}{2}(3-x)\sqrt{-x^2-2x} - 3 \tan^{-1}\left(\frac{\sqrt{-x^2-2x}}{x+2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[1 - (1 + x)^2], x]

[Out] ((3 - x)*Sqrt[-2*x - x^2])/2 - 3*ArcTan[Sqrt[-2*x - x^2]/(2 + x)]

fricas [A] time = 1.26, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2-2x}(x-3) - 3 \arctan\left(\frac{\sqrt{-x^2-2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)

giac [A] time = 0.39, size = 23, normalized size = 0.52

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}(x-3) + \frac{3}{2}\arcsin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)

maple [A] time = 0.01, size = 35, normalized size = 0.80

$$-\frac{\sqrt{-x^2-2x}x}{2} + \frac{3\arcsin(x+1)}{2} + \frac{3\sqrt{-x^2-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1-(x+1)^2)^(1/2),x)

[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(x+1)

maxima [A] time = 1.29, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2-2x}x + \frac{3}{2}\sqrt{-x^2-2x} - \frac{3}{2}\arcsin(-x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1 - (x + 1)^2)^(1/2),x)

[Out] int(x^2/(1 - (x + 1)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x(x+2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1-(1+x)**2)**(1/2), x)

[Out] Integral(x**2/sqrt(-x*(x + 2)), x)

$$3.97 \quad \int \frac{x^2}{\sqrt{1-(a+bx)^2}} dx$$

Optimal. Leaf size=67

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{2b^3} + \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {371, 743, 641, 216}

$$\frac{(2a^2 + 1) \sin^{-1}(a + bx)}{2b^3} + \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 - (a + b*x)^2], x]

[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (1 + 2*a^2)*ArcSin[a + b*x]/(2*b^3)

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 743

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m

- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{1 - (a + bx)^2}} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}} dx, x, a + bx\right)}{b^3} \\ &= -\frac{x\sqrt{1 - (a + bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}} dx, x, a + bx\right)}{2b^3} \\ &= \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2} + \frac{(1 + 2a^2)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, a + bx\right)}{2b^3} \\ &= \frac{3a\sqrt{1 - (a + bx)^2}}{2b^3} - \frac{x\sqrt{1 - (a + bx)^2}}{2b^2} + \frac{(1 + 2a^2)\sin^{-1}(a + bx)}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 0.82

$$\frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}(3a - bx) + (2a^2 + 1)\sin^{-1}(a + bx)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2], x]

[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcSin[a + b*x])/ (2*b^3)

IntegrateAlgebraic [B] time = 0.45, size = 171, normalized size = 2.55

$$\frac{(2a^2 + 1)\sqrt{-b^2} \log\left(2\sqrt{-b^2}x\sqrt{-a^2 - 2abx - b^2x^2 + 1} + 2abx + 2b^2x^2 - 1\right)}{4b^4} + \frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1}(3a - bx)}{2b^3} + \frac{(-2a^2 - 1)\tan^{-1}\left(\frac{\sqrt{-a^2 - 2abx - b^2x^2 + 1} - \sqrt{-b^2}x}{a}\right)}{2b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[1 - (a + b*x)^2], x]

[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/ (2*b^3) + ((-1 - 2*a^2)*ArcTan[(-Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a])/ (2*b^3) + ((1 + 2*a^2)*Sqrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]])/(4*b^4)

fricas [A] time = 0.93, size = 92, normalized size = 1.37

$$\frac{(2a^2 + 1) \arctan\left(\frac{\sqrt{-b^2x^2 - 2abx - a^2 + 1}(bx+a)}{b^2x^2 + 2abx + a^2 - 1}\right) + \sqrt{-b^2x^2 - 2abx - a^2 + 1}(bx - 3a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))/b^3

giac [A] time = 0.49, size = 55, normalized size = 0.82

$$-\frac{1}{2} \sqrt{-(bx+a)^2 + 1} \left(\frac{x}{b^2} - \frac{3a}{b^3} \right) - \frac{(2a^2 + 1) \arcsin(-bx - a) \operatorname{sgn}(b)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*arcsin(-b*x - a)*sgn(b)/(b^2*abs(b))

maple [B] time = 0.02, size = 152, normalized size = 2.27

$$\frac{a^2 \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2} b^2} - \frac{\sqrt{-b^2x^2-2abx-a^2+1} x}{2b^2} + \frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2\sqrt{b^2} b^2} + \frac{3\sqrt{-b^2x^2-2abx-a^2+1} a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1-(b*x+a)^2)^(1/2),x)

[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))

maxima [B] time = 1.46, size = 139, normalized size = 2.07

$$-\frac{3a^2 \arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2b^3} - \frac{\sqrt{-b^2x^2-2abx-a^2+1} x}{2b^2} + \frac{(a^2-1) \arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2b^3} + \frac{3\sqrt{-b^2x^2-2abx-a^2+1} a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")

[Out]
$$-3/2*a^2*\arcsin(-(b^2*x + a*b)/\sqrt{a^2*b^2 - (a^2 - 1)*b^2})/b^3 - 1/2*\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1}*x/b^2 + 1/2*(a^2 - 1)*\arcsin(-(b^2*x + a*b)/\sqrt{a^2*b^2 - (a^2 - 1)*b^2})/b^3 + 3/2*\sqrt{-b^2*x^2 - 2*a*b*x - a^2 + 1}*a/b^3$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{1 - (a + bx)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1 - (a + b*x)^2)^(1/2),x)

[Out] int(x^2/(1 - (a + b*x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(a + bx - 1)(a + bx + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)

[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)

$$3.98 \quad \int \frac{x^2}{\sqrt{1+(a+bx)^2}} dx$$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3} - \frac{3a\sqrt{(a+bx)^2+1}}{2b^3} + \frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {371, 743, 641, 215}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3} - \frac{3a\sqrt{(a+bx)^2+1}}{2b^3} + \frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[1 + (a + b*x)^2], x]

[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) - ((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 743

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m

- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{1 + (a + bx)^2}} dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}} dx, x, a + bx\right)}{b^3} \\ &= \frac{x\sqrt{1 + (a + bx)^2}}{2b^2} + \frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}} dx, x, a + bx\right)}{2b^3} \\ &= -\frac{3a\sqrt{1 + (a + bx)^2}}{2b^3} + \frac{x\sqrt{1 + (a + bx)^2}}{2b^2} - \frac{(1 - 2a^2) \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, a + bx\right)}{2b^3} \\ &= -\frac{3a\sqrt{1 + (a + bx)^2}}{2b^3} + \frac{x\sqrt{1 + (a + bx)^2}}{2b^2} - \frac{(1 - 2a^2) \sinh^{-1}(a + bx)}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2 + 2abx + b^2x^2 + 1} (bx - 3a) + (2a^2 - 1) \sinh^{-1}(a + bx)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2], x]

[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*ArcSinh[a + b*x])/(2*b^3)

IntegrateAlgebraic [B] time = 0.55, size = 176, normalized size = 2.79

$$\frac{(-2a^2\sqrt{b^2} - 2a^2b + \sqrt{b^2} + b) \log(\sqrt{a^2 + 2abx + b^2x^2 + 1} - a - \sqrt{b^2}x)}{4b^4} + \frac{(-2a^2\sqrt{b^2} + 2a^2b + \sqrt{b^2} - b) \log(\sqrt{a^2 + 2abx + b^2x^2 + 1} + a - \sqrt{b^2}x)}{4b^4} + \frac{\sqrt{a^2 + 2abx + b^2x^2 + 1} (bx - 3a)}{2b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[1 + (a + b*x)^2], x]

[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^3) + ((b - 2*a^2*b + Sqrt[b^2] - 2*a^2*Sqrt[b^2])*Log[-a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(4*b^4) + ((-b + 2*a^2*b + Sqrt[b^2] - 2*a^2*Sqrt[b^2])*Log[a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(4*b^4)

fricas [A] time = 1.88, size = 70, normalized size = 1.11

$$\frac{(2a^2 - 1) \log(-bx - a + \sqrt{b^2x^2 + 2abx + a^2 + 1}) - \sqrt{b^2x^2 + 2abx + a^2 + 1}(bx - 3a)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3

giac [A] time = 0.42, size = 70, normalized size = 1.11

$$\frac{1}{2} \sqrt{(bx + a)^2 + 1} \left(\frac{x}{b^2} - \frac{3a}{b^3} \right) - \frac{(2a^2 - 1) \log\left(-ab - \left(x|b| - \sqrt{(bx + a)^2 + 1}\right)|b|\right)}{2b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*log(-a*b - (x*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))

maple [B] time = 0.01, size = 146, normalized size = 2.32

$$\frac{a^2 \ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{\sqrt{b^2} b^2} + \frac{\sqrt{b^2x^2 + 2abx + a^2 + 1} x}{2b^2} - \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{2\sqrt{b^2} b^2} - \frac{3\sqrt{b^2x^2 + 2abx + a^2 + 1} a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+(b*x+a)^2)^(1/2),x)

[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2)^(1/2)

maxima [B] time = 0.59, size = 135, normalized size = 2.14

$$\frac{3a^2 \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{2b^3} + \frac{\sqrt{b^2x^2 + 2abx + a^2 + 1} x}{2b^2} - \frac{(a^2 + 1) \operatorname{arsinh}\left(\frac{2(b^2x+ab)}{\sqrt{-4a^2b^2+4(a^2+1)b^2}}\right)}{2b^3} - \frac{3\sqrt{b^2x^2 + 2abx + a^2 + 1} a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{3}{2}a^2\operatorname{arcsinh}\left(\frac{2(b^2x + a*b)}{\sqrt{-4a^2b^2 + 4(a^2 + 1)b^2}}\right)/b^3 + \frac{1}{2}\sqrt{b^2x^2 + 2a*b*x + a^2 + 1}*x/b^2 - \frac{1}{2}(a^2 + 1)\operatorname{arcsinh}\left(\frac{2(b^2x + a*b)}{\sqrt{-4a^2b^2 + 4(a^2 + 1)b^2}}\right)/b^3 - \frac{3}{2}\sqrt{b^2x^2 + 2a*b*x + a^2 + 1}*a/b^3$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{(a + bx)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^2 + 1)^(1/2),x)

[Out] int(x^2/((a + b*x)^2 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2 + 2abx + b^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)

[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)

$$3.99 \quad \int \frac{x^3}{a+b(c+dx)^3} dx$$

Optimal. Leaf size=234

$$\frac{(3\sqrt[3]{a} b^{2/3} c^2 + a + bc^3) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} b^{4/3} d^4} + \frac{(3\sqrt[3]{a} b^{2/3} c^2 + a + bc^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}(c + dx) + b^{2/3}(c + dx))}{6a^{2/3} b^{4/3} d^4}$$

Rubi [A] time = 0.37, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{(3\sqrt[3]{a} b^{2/3} c^2 + a + bc^3) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} b^{4/3} d^4} + \frac{(3\sqrt[3]{a} b^{2/3} c^2 + a + bc^3) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2)}{6a^{2/3} b^{4/3} d^4} + \frac{(-3\sqrt[3]{a} b^{2/3} c^2 + a + bc^3) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c + dx)}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{4/3} d^4} - \frac{c \log(a + b(c + dx)^3)}{bd^4} + \frac{x}{bd^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*(c + d*x)^3), x]

[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a + b*(c + d*x)^3])/(b*d^4)

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(-p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim

plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1860

Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]

Rule 1871

Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rule 1887

Int[(Pq_)/((a_) + (b_)*(x_)^n), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a+b(c+dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3} dx, x, c+dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b(a+bx^3)}\right) dx, x, c+dx\right)}{d^4} \\
&= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3} dx, x, c+dx\right)}{bd^4} \\
&= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3} dx, x, c+dx\right)}{bd^4} - \frac{(3c) \text{Subst}\left(\int \frac{x^2}{a+bx^3} dx, x, c+dx\right)}{d^4} \\
&= \frac{x}{bd^3} - \frac{c \log(a+b(c+dx)^3)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}bc^2+2\sqrt[3]{b}(a+bc^3)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}bc^2-\sqrt[3]{b}(a+bc^3)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c+dx\right)}{3a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4} - \frac{c \log(a+b(c+dx)^3)}{bd^4} - \frac{(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4} + \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6a^{2/3}b^{4/3}d^4} \\
&= \frac{x}{bd^3} + \frac{(a-3\sqrt[3]{a}b^{2/3}c^2+bc^3) \tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}d^4} - \frac{(a+3\sqrt[3]{a}b^{2/3}c^2+bc^3) \log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{4/3}d^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 132, normalized size = 0.56

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{3\#1^2bcd^2 \log(x-\#1) + a \log(x-\#1) + bc^3 \log(x-\#1) + 3\#1bc^2d \log(x-\#1)}{\#1^2d^2 + 2\#1cd + c^2} \&\right] - 3bdx}{3b^2d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*(c + d*x)^3), x]

[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 &, (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) &])/(b^2*d^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b(c + dx)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^3), x]

[Out] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^3), x]

fricas [C] time = 47.20, size = 6315, normalized size = 26.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^3), x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/12*(2*((-I*\sqrt{3}) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)) \\ &)/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + \\ & 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2 \\ & *c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{1/3} + 3*(I*\sqrt{3}) + 1)*(-c^3/(\\ & b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2* \\ & c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3* \\ & a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{1/3} + 6*c/(b*d^4))*b*d^4*\log(-3/4*((-I*s \\ & \text{qrt}(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^ \\ & 12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + \\ & 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b* \\ & c^3 + a^3)/(a^2*b^4*d^12))^{1/3} + 3*(I*\sqrt{3}) + 1)*(-c^3/(b^3*d^12) - 1/2 \\ & *(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\ & c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3 \\ &)/(a^2*b^4*d^12))^{1/3} + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + b^3*c^10 - 9*a*b \\ & ^2*c^7 - 12*a^2*b*c^4 + 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*\sqrt{3} \\ & (3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) \\ & - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^ \\ & 2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 \\ & + a^3)/(a^2*b^4*d^12))^{1/3} + 3*(I*\sqrt{3}) + 1)*(-c^3/(b^3*d^12) - 1/2*(b* \\ & c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + \\ & a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a \\ & ^2*b^4*d^12))^{1/3} + 6*c/(b*d^4))*d^4 - 2*a^3*c + (b^3*c^9 - 24*a*b^2*c^6 \\ & + 3*a^2*b*c^3 + a^3)*d*x) - 12*d*x - (((-I*\sqrt{3}) + 1)*(3*c^2/(b^2*d^8) + \\ & (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(\\ & a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^1 \\ & 2) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{1/3} \\ &) + 3*(I*\sqrt{3}) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^1 \\ & 2) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54 \end{aligned}$$

$$\begin{aligned}
&*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 6c/(\\
&bd^4)*bd^4 - 3\sqrt{1/3}*bd^4*\sqrt{(-((-I*\sqrt{3}) + 1)*(3c^2/(b^2d^8) \\
&+ (b^5c - 2a^2c^2)/(ab^2d^8)))/(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)* \\
&c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})} \\
&+ 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} \\
&+ 3*(I*\sqrt{3} + 1)*(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d^{12}) \\
&- 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1 \\
&/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 6* \\
&c/(bd^4))^{2*ab^2*d^8 - 12*((-I*\sqrt{3}) + 1)*(3c^2/(b^2d^8) + (b^5c - 2 \\
&a^2c^2)/(ab^2d^8)))/(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d^{12})} \\
&- 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1/54* \\
&(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 3*(I*s \\
&qrt(3) + 1)*(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d^{12}) - 1/54* \\
&(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1/54*(b^3c^9 \\
&- 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 6*c/(bd^4))*a \\
&b^3*c^4 - 48*b^5c - 12*a^2*c^2)/(ab^2d^8) - 18*c)*\log(3/4*((-I*\sqrt{3}) + 1) \\
&*(3c^2/(b^2d^8) + (b^5c - 2a^2c^2)/(ab^2d^8)))/(-c^3/(b^3d^{12}) - 1/2 \\
&*(b^5c - 2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^ \\
&^3 + a^3)/(a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3) \\
&)/(a^2b^4d^{12})^{1/3} + 3*(I*\sqrt{3} + 1)*(-c^3/(b^3d^{12}) - 1/2*(b^5c - \\
&2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3) \\
&/a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^ \\
&4*d^{12})^{1/3} + 6*c/(bd^4))^{2*a^2*b^3*c^2*d^8 + 2*b^3*c^{10} - 63*a*b^2*c^7 \\
&+ 21*a^2*b^3*c^4 - 1/2*(ab^3c^6 + 20a^2b^2c^3 + a^3b)*((-I*\sqrt{3}) + 1) \\
&)*(3c^2/(b^2d^8) + (b^5c - 2a^2c^2)/(ab^2d^8)))/(-c^3/(b^3d^{12}) - 1/2* \\
&(b^5c - 2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^ \\
&^9 + a^3)/(a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3) \\
&/a^2b^4d^{12})^{1/3} + 3*(I*\sqrt{3} + 1)*(-c^3/(b^3d^{12}) - 1/2*(b^5c - \\
&2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/ \\
&a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4 \\
&*d^{12})^{1/3} + 6*c/(bd^4))*d^4 + 5a^3*c + 2*(b^3c^9 - 24ab^2c^6 + 3 \\
&a^2b^2c^3 + a^3)*d*x + 3/4*\sqrt{1/3}*(3*((-I*\sqrt{3}) + 1)*(3c^2/(b^2d^8) \\
&+ (b^5c - 2a^2c^2)/(ab^2d^8)))/(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c \\
&/ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12} \\
&^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1 \\
&/3} + 3*(I*\sqrt{3} + 1)*(-c^3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d \\
&^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1/ \\
&54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 6*c \\
&/bd^4))*a^2*b^3*c^2*d^8 + 2*(ab^3c^6 - 7a^2b^2c^3 + a^3b)*d^4)*\sqrt{ \\
&-((-I*\sqrt{3}) + 1)*(3c^2/(b^2d^8) + (b^5c - 2a^2c^2)/(ab^2d^8)))/(-c^ \\
&3/(b^3d^{12}) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^ \\
&^2c^6 + 3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + \\
&3a^2b^2c^3 + a^3)/(a^2b^4d^{12})^{1/3} + 3*(I*\sqrt{3} + 1)*(-c^3/(b^3d^ \\
&12) - 1/2*(b^5c - 2a^2c^2)*c/(ab^3d^{12}) - 1/54*(b^3c^9 + 3a^2b^2c^6 + \\
&3a^2b^2c^3 + a^3)/(a^2b^4d^{12}) + 1/54*(b^3c^9 - 24ab^2c^6 + 3a^2b^2c^
\end{aligned}$$

$$\begin{aligned}
& c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8))) - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))*b*d^4 + 3*sqrt(1/3)*b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7 + 21*a^2*b*c^4 - 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b))*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^{(1/3)} + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*d*x - 3/4*s
\end{aligned}$$

```

sqrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a^2*b^3*c^2*d^8 + 2*(a*b^3*c^6 - 7*a^2*b^2*c^3 + a^3*b)*d^4)*sqrt(-(((I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8))))/(b*d^4)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(x^3/((d*x + c)^3*b + a), x)

maple [C] time = 0.01, size = 108, normalized size = 0.46

$$\frac{x}{b d^3} + \frac{\left(-3 \operatorname{RootOf}\left(b d^3 Z^3+3 b d^2 c Z^2+3 b d c^2 Z+b c^3+a\right)^2 b c d^2-3 \operatorname{RootOf}\left(b d^3 Z^3+3 b d^2 c Z^2+3 b d c^2 Z+b c^3+a\right) b c^2 d-b c^3-a\right) \ln\left(-\operatorname{RootOf}\left(b d^3 Z^3+3 b d^2 c Z^2+3 b d c^2 Z+b c^3+a\right)+x\right)}{3 b^2 d^4\left(d^2 \operatorname{RootOf}\left(b d^3 Z^3+3 b d^2 c Z^2+3 b d c^2 Z+b c^3+a\right)^2+2 c d \operatorname{RootOf}\left(b d^3 Z^3+3 b d^2 c Z^2+3 b d c^2 Z+b c^3+a\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b*(d*x+c)^3),x)

[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{bd^3} - \frac{\int \frac{3bcd^2x^2 + 3bc^2dx + bc^3 + a}{bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + a} dx}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)

mupad [B] time = 2.46, size = 374, normalized size = 1.60

$$\left(\sum_{k=0}^2 \frac{3(b^k c^{3-k} + a^k)}{d^k} - \text{root}(27a^2b^4d^{12}z^3 + 81a^2b^3c^5d^4z^2 + 54a^2b^2c^2d^4z - 27a^2b^3c^5d^4z + 3a^2b^2c^6 + 3a^2b^3c^9 + a^3, z, k) \left(\frac{3(b^2c^4d^4 - 5ab^3c^5d^4)}{d^2} - \text{root}(27a^2b^4d^{12}z^3 + 81a^2b^3c^5d^4z^2 + 54a^2b^2c^2d^4z - 27a^2b^3c^5d^4z + 3a^2b^2c^6 + 3a^2b^3c^9 + a^3, z, k) \right) \right) \frac{3a(b^2c^4d^4 - 5ab^3c^5d^4)}{d^2} - \text{root}(27a^2b^4d^{12}z^3 + 81a^2b^3c^5d^4z^2 + 54a^2b^2c^2d^4z - 27a^2b^3c^5d^4z + 3a^2b^2c^6 + 3a^2b^3c^9 + a^3, z, k) \frac{3a(b^2c^4d^4 - 5ab^3c^5d^4)}{d^2} + \frac{x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*(c + d*x)^3),x)

[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c^5*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b^3*c^9 + b^3*c^9 + a^3, z, k))*((3*(b^2*c^4*d^4 - 5*a*b^3*c^5*d^4))/d^2 + (3*x*(b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c^5*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b^3*c^9 + b^3*c^9 + a^3, z, k))*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c^5*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b^3*c^9 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)

sympy [A] time = 2.90, size = 238, normalized size = 1.02

$$\text{RootSum}\left(27t^2a^2b^4d^{12} + 81t^2a^2b^3c^5d^8 + t(54a^2b^2c^2d^4 - 27ab^3c^5d^4) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t \log\left(x + \frac{-27t^2a^2b^3c^2d^8 - 3a^3bd^4 - 60ta^2b^2c^3d^4 - 3tab^3c^6d^4 - 2a^3c - 12a^2bc^4 - 9ab^2c^7 + b^3c^{10}}{a^3d + 3a^2bc^3d - 24ab^2c^6d + b^3c^9d}\right)\right)\right) + \frac{x}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+b*(d*x+c)**3),x)

[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c - 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d - 24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)

$$3.100 \quad \int \frac{x^2}{a+b(c+dx)^3} dx$$

Optimal. Leaf size=210

$$\frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{\log(a+b(c+dx)^3)}{3bd^3}$$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {371, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{\log(a+b(c+dx)^3)}{3bd^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*(c + d*x)^3), x]

[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + b(c + dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3} dx, x, c + dx\right)}{d^3} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3} dx, x, c + dx\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3} dx, x, c + dx\right)}{d^3} \\
&= \frac{\log(a + b(c + dx)^3)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}(-2\sqrt[3]{a}c+2\sqrt[3]{b}c^2)+\sqrt[3]{b}(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c + dx\right)}{3a^{2/3}\sqrt[3]{b}d^3} + \dots \\
&= \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} + \frac{\log(a + b(c + dx)^3)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}} - \frac{c}{\sqrt[3]{a}}\right)\right) \text{Subst}\left(\dots\right)}{3bd^3} \\
&= \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3})}{6a^{2/3}b^{2/3}d^3} \\
&= \frac{c(2\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^3} + \frac{c(2\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}b^{2/3}d^3} - \frac{c(2\sqrt[3]{a} + \dots)}{3bd^3}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 81, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{\#1^2 \log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \&\right]}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*(c + d*x)^3),x]

[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) &]/(3*b*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b(c + dx)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^3),x]

[Out] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^3), x]

fricas [C] time = 4.04, size = 4759, normalized size = 22.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out]
$$-1/12*(2*(2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)*b*d^3*\log(-1/2*(2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*a^2*b)*2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2 - ((2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)*b*d^3 - 3*\sqrt{1/3}*b*d^3*\sqrt{-((2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)^2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3)*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*\log(1/2*(2*(1/2)^{(2/3)}*(-I*\sqrt{3}) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)}$$

$$\begin{aligned}
 & 9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))^{2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 + 3/2*\sqrt{1/3)*((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*d^3)*\sqrt{-((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))^{2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) - ((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*b*d^3 + 3*\sqrt{1/3)*b*d^3*\sqrt{-((2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))^{2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)} + 6)*\log(1/2*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} + (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b*d^3))^{2*a*b^2*d^6 + 4*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6)))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^{(1/3)} - 2
 \end{aligned}$$

$$\frac{1}{(b^3 d^3)^2} a^2 b^2 d^6 + 2 b^2 c^6 - 23 a b c^3 + \frac{1}{2} (a b^2 c^3 + 4 a^2 b) \cdot \left(\frac{2^{1/2}}{3} (-\sqrt{3} + 1) \left(\frac{2 b^2 c^3 - a}{a b^2 d^6} + \frac{1}{b^2 d^6} \right) \right) / \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} + \frac{1}{2} \left(\frac{1}{3} \right)^{1/3} (I \sqrt{3} + 1) \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} - \frac{2}{(b^3 d^9)^{1/3}} \cdot d^3 + 2 (b^2 c^5 - 8 a b c^2) d x + 2 a^2 - \frac{3}{2} \sqrt{1/3} \left(\frac{2^{1/2}}{3} (-\sqrt{3} + 1) \left(\frac{2 b^2 c^3 - a}{a b^2 d^6} + \frac{1}{b^2 d^6} \right) \right) / \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} + \frac{1}{2} \left(\frac{1}{3} \right)^{1/3} (I \sqrt{3} + 1) \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} - \frac{2}{(b^3 d^9)^{1/3}} \cdot a^2 b^2 d^6 - (a b^2 c^3 - 2 a^2 b) d^3 \sqrt{-\left(\frac{2^{1/2}}{3} (-\sqrt{3} + 1) \left(\frac{2 b^2 c^3 - a}{a b^2 d^6} + \frac{1}{b^2 d^6} \right) \right) / \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} + \frac{1}{2} \left(\frac{1}{3} \right)^{1/3} (I \sqrt{3} + 1) \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} - \frac{2}{(b^3 d^9)^{1/3}} \cdot a^2 b^2 d^6 + 4 \left(\frac{2^{1/2}}{3} (-\sqrt{3} + 1) \left(\frac{2 b^2 c^3 - a}{a b^2 d^6} + \frac{1}{b^2 d^6} \right) \right) / \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} + \frac{1}{2} \left(\frac{1}{3} \right)^{1/3} (I \sqrt{3} + 1) \left(\frac{b^2 c^3 - 8 a}{a^2 b^2 d^9} + \frac{3 (2 b^2 c^3 - a)}{a b^3 d^9} + \frac{2}{b^3 d^9} \right) + \frac{b^2 c^6 + 2 a b c^3 + a^2}{(a^2 b^3 d^9)^{1/3}} - \frac{2}{(b^3 d^9)^{1/3}} \cdot a^2 b^2 d^6 + 4 a) / (a b^2 d^6)) / (b^3 d^3)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(x^2/((d*x + c)^3*b + a), x)

maple [C] time = 0.00, size = 74, normalized size = 0.35

$$\frac{\text{RootOf}(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a)^2 \ln(-\text{RootOf}(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a) + x)}{3 b d \left(d^2 \text{RootOf}(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a) \right)^2 + 2 c d \text{RootOf}(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a) + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*(d*x+c)^3),x)

[Out] $1/3/b/d*\text{sum}(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*\ln(-_R+x), _R=\text{RootOf}(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^3 b+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")`

[Out] `integrate(x^2/((d*x + c)^3*b + a), x)`

mupad [B] time = 2.30, size = 437, normalized size = 2.08

$\sum_{k=1}^{n-1} \dots$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a + b*(c + d*x)^3),x)`

[Out] `symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^3 + 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k), k, 1, 3)`

sympy [A] time = 0.99, size = 158, normalized size = 0.75

`RootSum(27t^3a^2b^3d^9 - 27t^2a^2b^2d^6 + t(9a^2bd^3 - 18ab^2c^3d^3) - a^2 - 2abc^3 - b^2c^6, (t -> t*log(x + (18t^2a^2b^2d^6 - 12ta^2bd^3 - 3tab^2c^3d^3 + 2a^2 + abc^3 - b^2c^6))))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a+b*(d*x+c)**3),x)`

[Out] `RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d**3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*log(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3 + 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))`

$$3.101 \quad \int \frac{x}{a+b(c+dx)^3} dx$$

Optimal. Leaf size=180

$$\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a} - \sqrt[3]{b}c)}{\sqrt{3}a^{2/3}b^{2/3}d^2}$$

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {371, 1860, 31, 634, 617, 204, 628}

$$\frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a} + \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a} - \sqrt[3]{b}c) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*(c + d*x)^3), x]

[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(2/3)*d^2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a+b(c+dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3} dx, x, c+dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}(\sqrt[3]{a}-2\sqrt[3]{b}c)+\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{b}c)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}d^2} - \frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx, x, c+dx\right)}{3a^{2/3}d^2} \\
&= -\frac{(\sqrt[3]{a}+\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c+dx\right)}{2d^2} \\
&= -\frac{(\sqrt[3]{a}+\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a}+\sqrt[3]{b}c)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3})}{6a^{2/3}b^{2/3}d^2} \\
&= -\frac{(\sqrt[3]{a}-\sqrt[3]{b}c)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2} - \frac{(\sqrt[3]{a}+\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}b^{2/3}d^2} + \frac{(\sqrt[3]{a}+\sqrt[3]{b}c)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3})}{6a^{2/3}b^{2/3}d^2}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{\#1 \log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \& \right]}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*(c + d*x)^3), x]

[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x - #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) &]/(3*b*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b(c+dx)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + b*(c + d*x)^3), x]

[Out] IntegrateAlgebraic[x/(a + b*(c + d*x)^3), x]

fricas [C] time = 3.83, size = 1950, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
+ 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))^2*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + 1/36*(9*(I*sqrt(3) + 1)*(-
1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
+ c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4)) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
```

)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c^3 - a)*d*x + a*c)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(x/((d*x + c)^3*b + a), x)

maple [C] time = 0.00, size = 72, normalized size = 0.40

$$\frac{\text{RootOf}(b d^3 _Z^3 + 3b d^2 c _Z^2 + 3bd c^2 _Z + b c^3 + a) \ln(-\text{RootOf}(b d^3 _Z^3 + 3b d^2 c _Z^2 + 3bd c^2 _Z + b c^3 + a) + x)}{3bd \left(d^2 \text{RootOf}(b d^3 _Z^3 + 3b d^2 c _Z^2 + 3bd c^2 _Z + b c^3 + a) \right)^2 + 2cd \text{RootOf}(b d^3 _Z^3 + 3b d^2 c _Z^2 + 3bd c^2 _Z + b c^3 + a) + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*(d*x+c)^3),x)

[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x), _R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(x/((d*x + c)^3*b + a), x)

mupad [B] time = 0.26, size = 145, normalized size = 0.81

$$\sum_{k=1}^3 \ln(-\text{root}(27 a^2 b^2 d^6 z^3 - 9 a b c d^2 z + b c^3 + a, z, k) (3 b^2 c^2 d^4 - \text{root}(27 a^2 b^2 d^6 z^3 - 9 a b c d^2 z + b c^3 + a, z, k) a b^2 d^6 9 + 3 b^2 c d^5 x) + b d^3 x) \text{root}(27 a^2 b^2 d^6 z^3 - 9 a b c d^2 z + b c^3 + a, z, k)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*(c + d*x)^3),x)`

[Out] `symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z, k), k, 1, 3)`

sympy [A] time = 0.70, size = 83, normalized size = 0.46

$$\text{RootSum}\left(27t^3a^2b^2d^6 - 9tabcd^2 + a + bc^3, \left(t \mapsto t \log\left(x + \frac{9t^2a^2bd^4 + 3tabc^2d^2 - ac - bc^4}{ad - bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*(d*x+c)**3),x)`

[Out] `RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t, _t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d - b*c**3*d))))`

$$3.102 \quad \int \frac{1}{a+b(c+dx)^3} dx$$

Optimal. Leaf size=140

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}d} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^3)^(-1), x]

[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(1/3)*d) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(1/3)*d)

Rule 31

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1],
Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b(c + dx)^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, c + dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx, x, c + dx\right)}{3a^{2/3}d} \\
&= \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx, x, c + dx\right)}{2\sqrt[3]{a}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx, x, c + dx\right)}{6a^{2/3}\sqrt[3]{b}d} \\
&= \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2)}{6a^{2/3}\sqrt[3]{b}d} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, \frac{2\sqrt[3]{b}(c+dx) - \sqrt[3]{a}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}d} \\
&= -\frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}d} + \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3}\sqrt[3]{b}d} - \frac{\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2)}{6a^{2/3}\sqrt[3]{b}d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.83

$$\frac{-\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c + dx) + b^{2/3}(c + dx)^2) + 2\log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx)) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}(c+dx) - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^3)^(-1), x]

[Out] (2*sqrt(3)*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(sqrt(3)*a^(1/3))] + 2*Log[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b(c + dx)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*(c + d*x)^3)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*(c + d*x)^3)^(-1), x]

fricas [A] time = 1.58, size = 442, normalized size = 3.16

$$\frac{\sqrt[3]{a} \sqrt{\frac{\sqrt{3} \sqrt{a^2 b^3}}{a^2 b^3}} \left(\frac{2 a b^2 d^2 x^2 + 2 a b^2 d^2 x + 2 a b^2 d^2 c - \sqrt[3]{a^2 b^3} \sqrt{\frac{a^2 b^3}{a^2 b^3}}}{2 a b^2 d^2 x^2 + 2 a b^2 d^2 x + 2 a b^2 d^2 c - \sqrt[3]{a^2 b^3} \sqrt{\frac{a^2 b^3}{a^2 b^3}}} \right) - (a^2 b)^{\frac{1}{3}} \log \left(\frac{a b^2 d^2 + 2 a b c d + a b c^2 - (a^2 b)^{\frac{1}{3}} (d x + c) + (a^2 b)^{\frac{1}{3}}}{(a^2 b)^{\frac{1}{3}} \log(a b d x + a b c + (a^2 b)^{\frac{1}{3}})} \right) + \sqrt[3]{a} \sqrt{\frac{\sqrt{3} \sqrt{a^2 b^3}}{a^2 b^3}} \arctan \left(\frac{\sqrt[3]{a^2 b^3} \sqrt{\frac{a^2 b^3}{a^2 b^3}}}{\sqrt[3]{a^2 b^3}} \right) - (a^2 b)^{\frac{1}{3}} \log \left(\frac{a b^2 d^2 + 2 a b c d + a b c^2 - (a^2 b)^{\frac{1}{3}} (d x + c) + (a^2 b)^{\frac{1}{3}}}{(a^2 b)^{\frac{1}{3}} \log(a b d x + a b c + (a^2 b)^{\frac{1}{3}})} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]

giac [A] time = 0.45, size = 160, normalized size = 1.14

$$\frac{1}{3} \sqrt[3]{a} \left(\frac{1}{a^2 b d^3} \right)^{\frac{1}{3}} \arctan \left(- \frac{b d x + b c + (a b^2)^{\frac{1}{3}}}{\sqrt[3]{a^2 b d^3} \sqrt{3 b d x + \sqrt[3]{a^2 b d^3}}} \right) - \frac{1}{6} \left(\frac{1}{a^2 b d^3} \right)^{\frac{1}{3}} \log \left(4 \left(\sqrt[3]{a^2 b d^3} b d x + \sqrt[3]{a^2 b d^3} b c - \sqrt[3]{a^2 b d^3} (a b^2)^{\frac{1}{3}} \right)^2 + 4 \left(b d x + b c + (a b^2)^{\frac{1}{3}} \right)^2 \right) + \frac{1}{3} \left(\frac{1}{a^2 b d^3} \right)^{\frac{1}{3}} \log \left(\left(b d x + b c + (a b^2)^{\frac{1}{3}} \right)^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*c + (a*b^2)^(1/3)))

maple [C] time = 0.00, size = 71, normalized size = 0.51

$$\frac{\ln \left(- \text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right) + x \right)}{3 b d \left(d^2 \text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right)^2 + 2 c d \text{RootOf} \left(b d^3 _Z^3 + 3 b d^2 c _Z^2 + 3 b d c^2 _Z + b c^3 + a \right) + c^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^3),x)

[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^3 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] integrate(1/((d*x + c)^3*b + a), x)

mupad [B] time = 2.31, size = 144, normalized size = 1.03

$$\frac{\ln(b^{1/3}c + a^{1/3} + b^{1/3}dx)}{3a^{2/3}b^{1/3}d} + \frac{\ln\left(3b^2cd^5 + 3b^2d^6x + \frac{3a^{1/3}b^{5/3}d^5(-1+\sqrt{3}i)}{2}\right)(-1+\sqrt{3}i)}{6a^{2/3}b^{1/3}d} - \frac{\ln\left(3b^2cd^5 + 3b^2d^6x - \frac{3a^{1/3}b^{5/3}d^5(1+\sqrt{3}i)}{2}\right)(1+\sqrt{3}i)}{6a^{2/3}b^{1/3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^3),x)

[Out] $\log(b^{1/3}c + a^{1/3} + b^{1/3}d*x)/(3*a^{2/3}*b^{1/3}*d) + (\log(3*b^2*c*d^5 + 3*b^2*d^6*x + (3*a^{1/3}*b^{5/3}*d^5*(3^{1/2}*1i - 1))/2)*(3^{1/2}*1i - 1))/(6*a^{2/3}*b^{1/3}*d) - (\log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^{1/3}*b^{5/3}*d^5*(3^{1/2}*1i + 1))/2)*(3^{1/2}*1i + 1))/(6*a^{2/3}*b^{1/3}*d)$

sympy [A] time = 0.26, size = 26, normalized size = 0.19

$$\frac{\text{RootSum}\left(27t^3a^2b - 1, \left(t \mapsto t \log\left(x + \frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**3),x)

[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d

$$3.103 \quad \int \frac{1}{x(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=224

$$\frac{(2\sqrt[3]{a} - \sqrt[3]{b}c) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}(c+dx) + b^{2/3}(c+dx)^2)}{6a^{2/3}(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}c + b^{2/3}c^2)} + \frac{\sqrt[3]{b}c \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}c + b^{2/3}c^2)} - \frac{\log(\sqrt[3]{a} + \sqrt[3]{b}(c+dx))}{3a^{2/3}(\sqrt[3]{a} + \sqrt[3]{b}c)}$$

Rubi [A] time = 0.48, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2)}{6a^{2/3}(a+bc^3)} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}+\sqrt[3]{b}c)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)} - \frac{\log(a+b(c+dx)^3)}{3(a+bc^3)} + \frac{\log(x)}{a+bc^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c + d*x)^3)),x]

[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*(a + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c + d*x)^3]/(3*(a + b*c^3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

```
Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+b(c+dx)^3)} dx &= \text{Subst} \left(\int \frac{1}{(-c+x)(a+bx^3)} dx, x, c+dx \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{(a+bc^3)(c-x)} - \frac{b(c^2+cx+x^2)}{(a+bc^3)(a+bx^3)} \right) dx, x, c+dx \right) \\
&= \frac{\log(x)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{c^2+cx+x^2}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} \\
&= \frac{\log(x)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{x^2}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} - \frac{b \text{Subst} \left(\int \frac{c^2+cx}{a+bx^3} dx, x, c+dx \right)}{a+bc^3} \\
&= \frac{\log(x)}{a+bc^3} - \frac{\log(a+b(c+dx)^3)}{3(a+bc^3)} - \frac{b^{2/3} \text{Subst} \left(\int \frac{\sqrt[3]{a}(\sqrt[3]{a}c+2\sqrt[3]{b}c^2)+\sqrt[3]{b}(\sqrt[3]{a}c-\sqrt[3]{b}c^2)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx, x, c+dx \right)}{3a^{2/3}(a+bc^3)} \\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)} - \frac{\log(a+b(c+dx)^3)}{3(a+bc^3)} - \frac{(\sqrt[3]{b}c)^2}{3a^{2/3}(a+bc^3)} \\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)} - \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{6a^{2/3}(a+bc^3)} \\
&= \frac{\sqrt[3]{b}c(\sqrt[3]{a}+\sqrt[3]{b}c)\tan^{-1}\left(\frac{1-2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(a+bc^3)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}c(\sqrt[3]{a}-\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}(c+dx))}{3a^{2/3}(a+bc^3)}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 119, normalized size = 0.53

$$\frac{\text{RootSum} \left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3 \&, \frac{\#1^2d^2 \log(x-\#1)+3c^2 \log(x-\#1)+3\#1cd \log(x-\#1)}{\#1^2d^2+2\#1cd+c^2} \& \right] - 3 \log(x)}{3(a+bc^3)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]

[Out]
$$-1/3*(-3*\text{Log}[x] + \text{RootSum}[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 \& , (3*c^2*\text{Log}[x - #1] + 3*c*d*\text{Log}[x - #1]*#1 + d^2*\text{Log}[x - #1]*#1^2) / (c^2 + 2*c*d*#1 + d^2*#1^2) \&])/(a + b*c^3)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b(c + dx)^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^3)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^3)), x]

fricas [C] time = 3.97, size = 4370, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/12*(2*(b*c^3 + a)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(1/(a*b*c^3 + a^2) - 1/ \\ & (b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c \\ & ^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\text{sqrt}(3) + \\ & 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b \\ & *c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a))*\text{log}(b*c^2*d*x + b*c^3 \\ & + 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(1/(a*b*c^3 + a^2) \\ & - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a \\ & *b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\text{sqrt}(3) \\ &) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2) \\ &)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 - \\ & 2*a^2)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^ \\ & 2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b \\ & *c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(b*c^3/((\\ & b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - \\ & 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^{(2/3)} \\ & *(-I*\text{sqrt}(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a) \\ & ^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 \\ & + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\text{sqrt}(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/ \\ & (a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} \\ &) - 2/(b*c^3 + a)) + 3*\text{sqrt}(1/3)*(b*c^3 + a)*\text{sqrt}(-16*b*c^3 + (a*b^2*c^6 + \\ & 2*a^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(-I*\text{sqrt}(3) + 1)*(1/(a*b*c^3 + a^2) - 1/ \\ & (b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c \\ & ^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\text{sqrt}(3) + \\ & 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b \end{aligned}$$

$$\begin{aligned}
& *c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)* \\
& (2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^ \\
& 3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a \\
&)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + \\
& a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^ \\
& 3 + a)^3)^{(1/3)} - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + \\
& 6)*\log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} \\
& t(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) \\
& - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} \\
& - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^ \\
& ^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b \\
& *c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(1/(a* \\
& b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + \\
& a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1 \\
& /3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((\\
& a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a)) + 3/4 \\
& *sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(\\
& 1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b* \\
& c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/ \\
& 2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + \\
& 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a)) \\
& + 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(\\
& -I*\sqrt{3} + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2 \\
& *a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + \\
& a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a \\
& ^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} \\
& - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(1/(\\
& a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 \\
& + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(\\
& 1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/ \\
& ((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a)) + 4 \\
& *a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a - ((b*c^3 + a)*(2*(1/2)^{(2/3)}*(-I \\
& *sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a \\
& ^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a) \\
& ^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2 \\
& *b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - \\
& 2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a \\
& ^2*b*c^3 + a^3)*(2*(1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c \\
& ^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + \\
& a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(\\
& b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 \\
& + a)) - 2/(b*c^3 + a)^3)^{(1/3)} - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(\\
& 1/2)^{(2/3)}*(-I*\sqrt{3} + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((\\
& b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - \\
& 2/(b*c^3 + a)^3)^{(1/3)} - (1/2)^{(1/3)}*(I*\sqrt{3} + 1)*(b*c^3/((b*c^3 + a)^2
\end{aligned}$$

$a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*\log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^{2/3})*(-I*\sqrt{3}) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - (1/2)^{1/3}*(I*\sqrt{3}) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a)^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^{2/3})*(-I*\sqrt{3}) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - (1/2)^{1/3}*(I*\sqrt{3}) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a) - 3/4*\sqrt{1/3}*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^{2/3})*(-I*\sqrt{3}) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - (1/2)^{1/3}*(I*\sqrt{3}) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a) + 2*a^2*\sqrt{-16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^{2/3})*(-I*\sqrt{3}) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - (1/2)^{1/3}*(I*\sqrt{3}) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a)^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^{2/3})*(-I*\sqrt{3}) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - (1/2)^{1/3}*(I*\sqrt{3}) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^{1/3} - 2/(b*c^3 + a) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) + 12*\log(x))/(b*c^3 + a)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx+c)^3 b+a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)

maple [C] time = 0.01, size = 105, normalized size = 0.47

$$\frac{\ln(x)}{b^3+a} - \frac{(d^2 \operatorname{RootOf}(b d^3 Z^3 + 3b d^2 c Z^2 + 3bd c^2 Z + b c^3 + a)^2 + 3cd \operatorname{RootOf}(b d^3 Z^3 + 3b d^2 c Z^2 + 3bd c^2 Z + b c^3 + a) + 3c^2) \ln(-\operatorname{RootOf}(b d^3 Z^3 + 3b d^2 c Z^2 + 3bd c^2 Z + b c^3 + a) + x)}{3(b^3+a)(d^2 \operatorname{RootOf}(b d^3 Z^3 + 3b d^2 c Z^2 + 3bd c^2 Z + b c^3 + a)^2 + 2cd \operatorname{RootOf}(b d^3 Z^3 + 3b d^2 c Z^2 + 3bd c^2 Z + b c^3 + a) + c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(d*x+c)^3),x)

[Out] $-1/3/(b*c^3+a)*\text{sum}((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*\ln(-_R+x), _R=\text{RootOf}(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))+\ln(x)/(b*c^3+a)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd \int \frac{d^2x^2+3cdx+3c^2}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a} dx}{bc^3+a} + \frac{\log(x)}{bc^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] $-b*d*\text{integrate}((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + \log(x)/(b*c^3 + a)$

mupad [B] time = 0.12, size = 553, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^3)),x)

[Out] $\log(x)/(a + b*c^3) + \text{symsum}(\log(3*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x + 9*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*c*d^8 + 9*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^4*d^8 - 36*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*\text{root}(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)**3),x)

[Out] Timed out

$$3.104 \quad \int \frac{1}{x^2(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=314

$$\frac{\sqrt[3]{b} d \left(\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3) \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} (c + dx) + b^{2/3} (c + dx)^2 \right) + \sqrt[3]{b} d \left(\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3) \right)}{6a^{2/3} (a + bc^3)^2} + \dots$$

Rubi [A] time = 0.54, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, number of rules / integrand size = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3} d \left(\frac{\sqrt[3]{a} (a - 2bc^3)}{\sqrt[3]{b}} + 2ac - bc^4 \right) \log \left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} (c + dx) + b^{2/3} (c + dx)^2 \right) + \frac{\sqrt[3]{b} d \left(\sqrt[3]{a} (a - 2bc^3) - \sqrt[3]{b} c (2a - bc^3) \right) \log \left(\sqrt[3]{a} + \sqrt[3]{b} (c + dx) \right)}{3a^{2/3} (a + bc^3)^2} + \frac{\sqrt[3]{b} d \left(\sqrt[3]{a} - \sqrt[3]{b} c \right) \left(\sqrt[3]{a} + \sqrt[3]{b} c \right)^3 \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} (c + dx)}{\sqrt[3]{3} \sqrt[3]{a}} \right)}{\sqrt[3]{3} a^{2/3} (a + bc^3)^2} - \frac{3bc^2 d \log(x) + bc^2 d \log(a + b(c + dx)^3)}{(a + bc^3)^2} - \frac{1}{x(a + bc^3)}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]

[Out] $-(1/((a + b*c^3)*x)) + (b^{(1/3)}*(a^{(1/3)} - b^{(1/3)*c})*(a^{(1/3)} + b^{(1/3)*c})^{3*d*ArcTan[(a^{(1/3)} - 2*b^{(1/3)}*(c + d*x))/(Sqrt[3]*a^{(1/3)})]}/(Sqrt[3]*a^{(2/3)}*(a + b*c^3)^2 - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^{(1/3)}*(a^{(1/3)}*(a - 2*b*c^3) - b^{(1/3)*c}*(2*a - b*c^3))*d*Log[a^{(1/3)} + b^{(1/3)}*(c + d*x)])/(3*a^{(2/3)}*(a + b*c^3)^2 + (b^{(2/3)}*(2*a*c - b*c^4 - (a^{(1/3)}*(a - 2*b*c^3))/b^{(1/3)})*d*Log[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*(c + d*x) + b^{(2/3)}*(c + d*x)^2])/(6*a^{(2/3)}*(a + b*c^3)^2 + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c^3)^2$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
```

x₁ = expand[u/(a + b*x^n), x], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^2 (a + b(c + dx)^3)} dx &= d \operatorname{Subst} \left(\int \frac{1}{(-c + x)^2 (a + bx^3)} dx, x, c + dx \right) \\
 &= d \operatorname{Subst} \left(\int \left(\frac{1}{(a + bc^3)(c - x)^2} + \frac{3bc^2}{(a + bc^3)^2 (c - x)} + \frac{b(-c(2a - bc^3) - (a - 2bc^3))}{(a + bc^3)^2 (a + bx^3)} \right) dx, x, c + dx \right) \\
 &= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c(2a - bc^3) - (a - 2bc^3)x + 3bc^2 x^2}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} \\
 &= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c(2a - bc^3) + (-a + 2bc^3)x}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} + \frac{(b^2/3 d) \operatorname{Subst} \left(\int \frac{\sqrt[3]{a}(-2x^2 + 3cx - c^2)}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} \\
 &= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} + \frac{bc^2 d \log(a + b(c + dx)^3)}{(a + bc^3)^2} + \frac{(b^2/3 d) \operatorname{Subst} \left(\int \frac{\sqrt[3]{a}(-2x^2 + 3cx - c^2)}{a + bx^3} dx, x, c + dx \right)}{(a + bc^3)^2} \\
 &= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} - \frac{b^{2/3} \left(2ac - bc^4 - \frac{\sqrt[3]{a}(a - 2bc^3)}{\sqrt[3]{b}} \right) d \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} (a + bc^3)^2} \\
 &= -\frac{1}{(a + bc^3)x} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2} - \frac{b^{2/3} \left(2ac - bc^4 - \frac{\sqrt[3]{a}(a - 2bc^3)}{\sqrt[3]{b}} \right) d \log(\sqrt[3]{a} + \sqrt[3]{b}(c + dx))}{3a^{2/3} (a + bc^3)^2} \\
 &= -\frac{1}{(a + bc^3)x} + \frac{\sqrt[3]{b} (\sqrt[3]{a} - \sqrt[3]{b}c) (\sqrt[3]{a} + \sqrt[3]{b}c)^3 d \tan^{-1} \left(\frac{1 - 2\sqrt[3]{b}(c + dx)}{\sqrt[3]{a}} \right)}{\sqrt{3} a^{2/3} (a + bc^3)^2} - \frac{3bc^2 d \log(x)}{(a + bc^3)^2}
 \end{aligned}$$

Mathematica [C] time = 0.10, size = 173, normalized size = 0.55

$$\frac{dx \operatorname{RootSum} \left[\#1^3 b d^3 + 3 \#1^2 b c d^2 + 3 \#1 b c^2 d + a + b c^3 \&, \frac{3 \#1^2 b c^2 d^2 \log(x - \#1) - 3 a c \log(x - \#1) - \#1 a d \log(x - \#1) + 6 b c^4 \log(x - \#1) + 8 \#1 b c^3 d \log(x - \#1)}{\#1^2 d^2 + 2 \#1 c d + c^2} \& \right] - 3(a + bc^3 + 3bc^2 dx \log(x))}{3x(a + bc^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
+ 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b(c + dx)^3)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^3)), x]
```

fricas [C] time = 3.93, size = 8919, normalized size = 28.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*
(I*sqrt(3) + 1))*x*log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3*
b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^
2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^
3 + a^3)))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)
) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a
)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
- 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a
^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
+ a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1
0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*
```

$$\begin{aligned}
& c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 - 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 \\
& * b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) \\
& + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - \\
& 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) \\
&) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} * (I \sqrt{3} + 1) * d + (b^3 c^7 + 5 a b^2 c^4 - 5 a^2 b c) * d \\
& ^2 - (18 b c^2 d * x - (b^2 c^6 + 2 a b c^3 + a^2) * (6 b c^2 d / (b^2 c^6 + 2 a \\
& * b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 \\
& - 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 \\
& + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a \\
& ^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 \\
& + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 \\
& + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} * (I \sqrt{3} + 1) * x - \\
& 3 * \sqrt{1/3} * (b^2 c^6 + 2 a b c^3 + a^2) * x * \sqrt{-((a b^4 c^{12} + 4 a^2 b^3 c^9 \\
& + 6 a^3 b^2 c^6 + 4 a^4 b c^3 + a^5) * (6 b c^2 d / (b^2 c^6 + 2 a b c^3 + a^2) \\
& - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 - 2 b c d^2 / (a b^2 \\
& * c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 \\
& + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 \\
& c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 \\
& - a) b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 \\
& + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 \\
& + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) \\
& * b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} * (I \sqrt{3} + 1)^2 - 12 * (a b^3 c^8 + 2 a^2 b^2 c^5 \\
& + a^3 b c^2) * (6 b c^2 d / (b^2 c^6 + 2 a b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 - 2 b c d^2 / (a b^2 \\
& * c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b \\
& c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + \\
& 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) * \\
& b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 \\
& * a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 \\
& + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - \\
& a) * b d^3 / ((b c^3 + a)^3 a^2)^{(1/3)} * (I \sqrt{3} + 1) * d + 4 * (8 b^3 c^7 - 11 * \\
& a b^2 c^4 + 8 a^2 b c) * d^2 / (a b^4 c^{12} + 4 a^2 b^3 c^9 + 6 a^3 b^2 c^6 + 4 \\
& * a^4 b c^3 + a^5)) * \log(2 * (b^3 c^6 - a^2 b) * d^3 * x - 1/4 * (2 a^2 b^3 c^9 + 3 * \\
& a^3 b^2 c^6 - a^5) * (6 b c^2 d / (b^2 c^6 + 2 a b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (\\
& 9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 - 2 b c d^2 / (a b^2 c^6 + 2 a^2 * \\
& b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 \\
& - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + \\
& a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 \\
& + a)^3 a^2)^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 \\
& - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3
\end{aligned}$$

$$\begin{aligned} & + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1))^2 - 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^{(2/3)}*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2))^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)} - (1/2)^{(1/3)}*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1))*d + (2*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*b*c)*d^2 + 3/4*\sqrt{1/3)*((2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^{(2/3)}*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2))^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)} - (1/2)^{(1/3)}*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*d)*\sqrt{-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^{(2/3)}*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2))^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)} - (1/2)^{(1/3)}*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^{(2/3)}*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2))^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^{(2/3)}*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2))^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*\sqrt{3} + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^{(1/3)}*(I*\sqrt{3} + 1))$$

$$\begin{aligned}
& / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2) \\
& ^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 \\
& * c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b \\
& d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2) \\
&)^{(1/3)} * (I \sqrt{3} + 1) * x + 3 \sqrt{3} * (b^2 c^6 + 2 a b c^3 + a^2) * x \sqrt{3} \\
& \text{rt}(-((a b^4 c^{12} + 4 a^2 b^3 c^9 + 6 a^3 b^2 c^6 + 4 a^4 b c^3 + a^5) * (6 b \\
& c^2 d / (b^2 c^6 + 2 a b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + \\
& 2 a b c^3 + a^2)^2 - 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} \\
&) + 1) / (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b \\
& ^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 \\
& + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} - (1 \\
& / 2)^{(1/3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((\\
& a b^2 c^6 + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 \\
& + 2 a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} * (\\
& I \sqrt{3} + 1)^2 - 12 * (a b^3 c^8 + 2 a^2 b^2 c^5 + a^3 b c^2) * (6 b c^2 d / (\\
& b^2 c^6 + 2 a b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b \\
& c^3 + a^2)^2 - 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / \\
& (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 \\
& + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 \\
& a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} - (1/2)^{(1/ \\
& 3)} * (54 b^3 c^6 d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 \\
& + 2 a^2 b c^3 + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 \\
& a^3 b c^3 + a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} * (I \sqrt{3} \\
& (3) + 1) * d + 4 * (8 b^3 c^7 - 11 a b^2 c^4 + 8 a^2 b c) * d^2 / (a b^4 c^{12} + 4 \\
& a^2 b^3 c^9 + 6 a^3 b^2 c^6 + 4 a^4 b c^3 + a^5)) * \log(2 * (b^3 c^6 - a^2 b) * \\
& d^3 * x - 1/4 * (2 a^2 b^3 c^9 + 3 a^3 b^2 c^6 - a^5) * (6 b c^2 d / (b^2 c^6 + 2 a \\
& * b c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 \\
& - 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d \\
& ^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 \\
& + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a \\
& ^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 \\
& d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 \\
& + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + \\
& a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} * (I \sqrt{3} + 1)^2 - \\
& 1/2 * (a b^3 c^8 - 16 a^2 b^2 c^5 + 10 a^3 b c^2) * (6 b c^2 d / (b^2 c^6 + 2 a b \\
& * c^3 + a^2) - 2 * (1/2)^{(2/3)} * (9 b^2 c^4 d^2 / (b^2 c^6 + 2 a b c^3 + a^2)^2 - \\
& 2 b c d^2 / (a b^2 c^6 + 2 a^2 b c^3 + a^3)) * (-I \sqrt{3} + 1) / (54 b^3 c^6 d^3 \\
& / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 + \\
& a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + a^4 \\
&) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} - (1/2)^{(1/3)} * (54 b^3 c^6 * \\
& d^3 / (b^2 c^6 + 2 a b c^3 + a^2)^3 - 18 b^2 c^3 d^3 / ((a b^2 c^6 + 2 a^2 b c^3 \\
& + a^3) * (b^2 c^6 + 2 a b c^3 + a^2)) + b d^3 / (a^2 b^2 c^6 + 2 a^3 b c^3 + \\
& a^4) + (b c^3 - a) b d^3 / ((b c^3 + a)^3 a^2))^{(1/3)} * (I \sqrt{3} + 1) * d + (2 \\
& * b^3 c^7 - 5 a b^2 c^4 + 2 a^2 b c) * d^2 - 3/4 * \sqrt{3} * ((2 a^2 b^3 c^9 + 3 \\
& * a^3 b^2 c^6 - a^5) * (6 b c^2 d / (b^2 c^6 + 2 a b c^3 + a^2) - 2 * (1/2)^{(2/3)} *
\end{aligned}$$

$$\begin{aligned}
& (9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2* \\
& *b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2) \\
& ^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + \\
& a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 \\
& + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2) \\
& ^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + \\
& a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b \\
& *c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a \\
& ^3*b*c^2)*d)*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 \\
& + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b \\
& ^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(- \\
& I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d \\
& ^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a \\
& ^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1 \\
& /3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^ \\
& 3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3 \\
& /((a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2)) \\
& ^3)*sqrt(3) + 1) - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 \\
& + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2) \\
& ^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3 \\
& /((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a \\
& ^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1 \\
& /3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^ \\
& 3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3 \\
& /((a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2)) \\
& ^3)*(I*sqrt(3) + 1)*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2/((\\
& a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) + 12*a)/(\\
& (b^2*c^6 + 2*a*b*c^3 + a^2)*x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^3 b + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)

maple [C] time = 0.01, size = 144, normalized size = 0.46

$$\frac{3b^2c^2d \ln(x)}{(b^2c^3 + a)^2} + \frac{d \left(3 \operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) b^2c^2d^2 + 8 \operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) b^2c^2d + 6b^2c^4 - \operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) ad - 3ac \right) \ln(-\operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) x)}{3(b^2c^3 + a)^2 \left(d^3 \operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) + 2cd \operatorname{RootOf}(b^3d^3Z^3 + 3bd^2cZ^2 + 3bd^2cZ + b^3c^3 + a^3) \right)} - \frac{1}{(b^2c^3 + a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d/(b*c^3+a)^2*sum((3*_R^2*b*c^2*d^2+8*_R*b*c^3*d+6*b*c^4-_R*a*d-3*a*c)/
(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*
b*c^2*d+b*c^3+a))-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3bc^2d \log(x)}{b^2c^6 + 2abc^3 + a^2} + \frac{bd^2 \int \frac{3bc^2d^2x^2+6bc^4+(8bc^3-a)dx-3ac}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a} dx}{b^2c^6 + 2abc^3 + a^2} - \frac{1}{(bc^3 + a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d^
2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)*x)
```

mupad [B] time = 2.33, size = 1588, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*
a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^9
- 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*
d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2*c^6
*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z
- b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3
+ 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^
4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81
*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8 + 27*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
```

```

*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
  3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
*z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
  51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
  18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*log(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)

[Out] Timed out

$$3.105 \quad \int \frac{1}{x^3(a+b(c+dx)^3)} dx$$

Optimal. Leaf size=393

$$\frac{b^{2/3}d^2 \left(-3a^{2/3} \sqrt[3]{b}c + a + bc^3 \right) \left(\sqrt[3]{a} + \sqrt[3]{b}c \right)^3 \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c+dx)}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} a^{2/3} (a + bc^3)^3} - \frac{b^{2/3}d^2 \left(6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3 \right)}{3a^{2/3} (a + bc^3)}$$

Rubi [A] time = 0.60, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}d^2 (6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3)}{3a^{2/3} (a + bc^3)^3} + \frac{b^{2/3}d^2 (6a^{4/3}b^{2/3}c^2 + a^2 - 3\sqrt[3]{a}b^{5/3}c^5 - 7abc^3) \log(\sqrt[3]{a} + \sqrt[3]{b}c + dx)}{6a^{2/3} (a + bc^3)^3} + \frac{b^{2/3}d^2 (-3a^{2/3}\sqrt[3]{b}c + a + bc^3) \left(\sqrt[3]{a} + \sqrt[3]{b}c \right)^3 \tan^{-1} \left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}(c+dx)}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt{3} a^{2/3} (a + bc^3)^3} - \frac{3bcd^2 \log(a - 2bc^3)}{(a + bc^3)^3} + \frac{bcd^2 (a - 2bc^3) \log(a + b(c + dx)^3)}{(a + bc^3)^3} - \frac{3bd^2d}{x(a + bc^3)^2} - \frac{1}{2d^2(a + bc^3)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*(c + d*x)^3)), x]

[Out] $-1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^{(2/3)}*(a^{(1/3)} + b^{(1/3)*c}^3*(a - 3*a^{(2/3)*b^{(1/3)*c} + b*c^3)*d^2*ArcTan[(a^{(1/3)} - 2*b^{(1/3)*c}*(c + d*x))/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(2/3)}*(a + b*c^3)^3) - (3*b*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^{(2/3)}*(a^2 + 6*a^{(4/3)*b^{(2/3)*c}^2 - 7*a*b*c^3 - 3*a^{(1/3)*b^{(5/3)*c}^5 + b^2*c^6)*d^2*Log[a^{(1/3)} + b^{(1/3)*c}*(c + d*x)])/(3*a^{(2/3)}*(a + b*c^3)^3) + (b^{(2/3)}*(a^2 + 6*a^{(4/3)*b^{(2/3)*c}^2 - 7*a*b*c^3 - 3*a^{(1/3)*b^{(5/3)*c}^5 + b^2*c^6)*d^2*Log[a^{(2/3)} - a^{(1/3)*b^{(1/3)*c}*(c + d*x) + b^{(2/3)*c}*(c + d*x)^2])/(6*a^{(2/3)}*(a + b*c^3)^3) + (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]}

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
```

xpand[u/(a + b*x^n), x]], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + b(c + dx)^3)} dx &= d^2 \operatorname{Subst} \left(\int \frac{1}{(-c + x)^3 (a + bx^3)} dx, x, c + dx \right) \\
 &= d^2 \operatorname{Subst} \left(\int \left(-\frac{1}{(a + bc^3)(c - x)^3} - \frac{3bc^2}{(a + bc^3)^2 (c - x)^2} - \frac{3bc(-a + 2bc^3)}{(a + bc^3)^3 (c - x)} + \frac{b(-a + 2bc^3)}{(a + bc^3)^3} \right) dx, x, c + dx \right) \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} - \frac{3bc(a - 2bc^3)d^2 \log(x)}{(a + bc^3)^3} + \frac{(bd^2) \operatorname{Subst} \left(\int \frac{-a^2 + 7abc}{(a + bc^3)^3} dx, x, c + dx \right)}{(a + bc^3)^3} \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} - \frac{3bc(a - 2bc^3)d^2 \log(x)}{(a + bc^3)^3} + \frac{(bd^2) \operatorname{Subst} \left(\int \frac{-a^2 + 7abc}{(a + bc^3)^3} dx, x, c + dx \right)}{(a + bc^3)^3} \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} - \frac{3bc(a - 2bc^3)d^2 \log(x)}{(a + bc^3)^3} + \frac{bc(a - 2bc^3)d^2 \log(a - bc^3)}{(a + bc^3)^3} \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} - \frac{3bc(a - 2bc^3)d^2 \log(x)}{(a + bc^3)^3} - \frac{b^{2/3}(a^2 + 6a^{4/3}b^{2/3}c^2 - 6a^{2/3}b^2c^2)}{(a + bc^3)^3} \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} - \frac{3bc(a - 2bc^3)d^2 \log(x)}{(a + bc^3)^3} - \frac{b^{2/3}(a^2 + 6a^{4/3}b^{2/3}c^2 - 6a^{2/3}b^2c^2)}{(a + bc^3)^3} \\
 &= -\frac{1}{2(a + bc^3)x^2} + \frac{3bc^2d}{(a + bc^3)^2 x} + \frac{b^{2/3}(\sqrt[3]{a} + \sqrt[3]{bc})^3 (a - 3a^{2/3}\sqrt[3]{bc} + bc^3)d^2 \tan^{-1} \left(\frac{c + dx}{\sqrt[3]{a + bc^3}} \right)}{\sqrt{3} a^{2/3} (a + bc^3)^3}
 \end{aligned}$$

Mathematica [C] time = 0.17, size = 244, normalized size = 0.62

$$\frac{2d^2x^2\operatorname{RootSum}\left[\#1^3bd^3 + 3\#1^2bcd^2 + 3\#1bc^2d + a + bc^3\&, \frac{-3\#1^2akc^2\log(x-\#1)+6\#1^2b^2c^2d^2\log(x-\#1)+a^2\log(x-\#1)-16ab^2\log(x-\#1)-12\#1ak^2d\log(x-\#1)+10d^2c^6\log(x-\#1)+15\#1b^2c^5d\log(x-\#1)\&}{\#1^2d^2+2\#1d+c^2}\right] + 18bcd^2x^2\log(x)(a-2bc^3) + 3(a+bc^3)(a+bc^2(c-6dx))}{6x^2(a+bc^3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]

[Out]
$$-1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*\text{Log}[x] + 2*d^2*x^2*\text{RootSum}[a + b*c^3 + 3*b*c^2*d*\#1 + 3*b*c*d^2*\#1^2 + b*d^3*\#1^3 \& , (a^2*\text{Log}[x - \#1] - 16*a*b*c^3*\text{Log}[x - \#1] + 10*b^2*c^6*\text{Log}[x - \#1] - 12*a*b*c^2*d*\text{Log}[x - \#1]*\#1 + 15*b^2*c^5*d*\text{Log}[x - \#1]*\#1 - 3*a*b*c*d^2*\text{Log}[x - \#1]*\#1^2 + 6*b^2*c^4*d^2*\text{Log}[x - \#1]*\#1^2)/(c^2 + 2*c*d*\#1 + d^2*\#1^2) \&])/(a + b*c^3)^3*x^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a + b(c + dx)^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(a + b*(c + d*x)^3)),x]

[Out] IntegrateAlgebraic[1/(x^3*(a + b*(c + d*x)^3)), x]

fricas [C] time = 12.24, size = 14765, normalized size = 37.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")

[Out]
$$-1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*\log(x) + 12*a*b*c^3 - 2*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3}) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*\sqrt{3}) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2*\log((b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (b^4*c^10 + 15*a*b^3*c^7 - 63*a^2*b^2*c^4 + 4*a^3*b*c)*d^4 - 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a$$

$$\begin{aligned}
& *b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3} - (1/2)^{1/3}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + 3/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^{2/3}*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3} - (1/2)^{1/3}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2) - 36*(b^2*c^5 + a*b*c^2)*d*x + 6*a^2 + (18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^{2/3}*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3} - (1/2)^{1/3}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{1/3}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2 + 3*\sqrt{1/3}*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2*\sqrt{-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 -
\end{aligned}$$

$$\begin{aligned} & a^4 b^2 c^4 - a^5 b^* c) * (6 * (1/2)^{(2/3)} * (b^2 c^2 d^4 / (a^3 b^3 c^9 + 3 a^2 b^2 c^6 \\ & + 3 a^3 b^* c^3 + a^4) - 3 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^2 / (b^3 c^9 + 3 a b^2 c^6 \\ & + 3 a^2 b^* c^3 + a^3)^2) * (-I \sqrt{3} + 1) / (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 \\ & + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 \\ & / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)} * (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 \\ & + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 \\ & / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} * (I \sqrt{3} + 1) - 6 * (2 b^2 c^4 d^2 - a b^* c^* d^2) / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3) * d^2 + (a b^6 c^18 + 6 a^2 b^5 c^15 + 15 a^3 b^4 c^12 + 20 a^4 b^3 c^9 + 15 a^5 b^2 c^6 + 6 a^6 b^* c^3 + a^7) * (6 * (1/2)^{(2/3)} * (b^2 c^2 d^4 / (a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) \\ & - 3 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^2 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2) * (-I \sqrt{3} + 1) / (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) \\ & + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)} * (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) \\ & + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} * (I \sqrt{3} + 1) - 6 * (2 b^2 c^4 d^2 - a b^* c^* d^2) / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2 / (a b^6 c^18 + 6 a^2 b^5 c^15 + 15 a^3 b^4 c^12 + 20 a^4 b^3 c^9 + 15 a^5 b^2 c^6 + 6 a^6 b^* c^3 + a^7) * \log(2 \\ & * (b^4 c^9 + 3 a b^3 c^6 - 24 a^2 b^2 c^3 + a^3 b) * d^5 * x + (2 b^4 c^10 - 6 a b^3 c^7 - 9 a^2 b^2 c^4 - a^3 b^* c) * d^4 + 1/2 * (a b^4 c^12 - 50 a^2 b^3 c^9 + 141 a^3 b^2 c^6 - 50 a^4 b^* c^3 + a^5) * (6 * (1/2)^{(2/3)} * (b^2 c^2 d^4 / (a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) \\ & - 3 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^2 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2) * (-I \sqrt{3} + 1) / (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) \\ & + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)} * (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) \\ & + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} * (I \sqrt{3} + 1) - 6 * (2 b^2 \\ & c^4 d^2 - a b^* c^* d^2) / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2 * \log(2 * (b^4 c^9 + 3 a b^3 c^6 - 24 a^2 b^2 c^3 + a^3 b) * d^5 * x + (2 b^4 c^10 - 6 a b^3 c^7 - 9 a^2 b^2 c^4 - a^3 b^* c) * d^4 + 1/2 * (a b^4 c^12 - 50 a^2 b^3 c^9 + 141 a^3 b^2 c^6 - 50 a^4 b^* c^3 + a^5) * (6 * (1/2)^{(2/3)} * (b^2 c^2 d^4 / (a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) \\ & - 3 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^2 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2) * (-I \sqrt{3} + 1) / (27 * (2 b^2 c^4 d^2 - a b^* c^* d^2) * b^2 c^2 d^4 / ((a^3 b^3 c^9 + 3 a^2 b^2 c^6 + 3 a^3 b^* c^3 + a^4) * (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)) - b^2 d^6 / (a^2 b^3 c^9 + 3 a^3 b^2 c^6 + 3 a^4 b^* c^3 + a^5) \\ & + (b^3 c^9 + 3 a b^2 c^6 - 24 a^2 b^* c^3 + a^3) * b^2 d^6 / ((b^* c^3 + a)^6 a^2) - 54 * (2 b^2 c^4 d^2 - a b^* c^* d^2)^3 / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^3)^{(1/3)} * (I \sqrt{3} + 1) - 6 * (2 b^2 \\ & c^4 d^2 - a b^* c^* d^2) / (b^3 c^9 + 3 a b^2 c^6 + 3 a^2 b^* c^3 + a^3)^2 \end{aligned}$$

$$\begin{aligned}
& 2c^4d^2 - a*b*c*d^2)/(b^3c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3 \\
& /4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)* \\
& (6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) \\
& - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a \\
& ^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^ \\
& 2*c^6 + 3*a^2*b*c^3 + a^3))^2 + 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^ \\
& 9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 \\
& - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a* \\
& b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2) \\
&)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2* \\
& b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b* \\
& c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3* \\
& c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2* \\
& b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3 \\
& /((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2* \\
& b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b* \\
& c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3* \\
& c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2* \\
& b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3 \\
& /((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(\\
& 2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*sq \\
& rt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + \\
& 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)* \\
& (6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) \\
& - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a \\
& ^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) \\
& ^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3 \\
& *c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b* \\
& c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (\\
& b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5 \\
& 4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
\end{aligned}$$

$$\begin{aligned}
& *c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(\\
& 2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3) \\
& ^{(1/3)}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c \\
& ^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^ \\
& 12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^{(2/3)}*(b \\
& ^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d \\
& ^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} \\
&) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2* \\
& c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2 \\
& *d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2 \\
& *c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 \\
& - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2) \\
& ^{(1/3)}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2* \\
& c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2 \\
& *d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2 \\
& *c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 \\
& - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{ \\
& 3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c \\
& ^3 + a^3))^2/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c \\
& ^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))*\log(2*(b^4*c^9 + 3*a*b^3*c^6 - 2 \\
& 4*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a*b^3*c^7 - 9*a^2*b^2*c^4 - \\
& a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9 + 141*a^3*b^2*c^6 - 50*a^4* \\
& b*c^3 + a^5)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3 \\
& *b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3* \\
& a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^ \\
& 2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c \\
& ^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c \\
& ^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + \\
& a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2 \\
& *b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^ \\
& 2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c \\
& ^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c \\
& ^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + \\
& a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2 \\
& *b*c^3 + a^3)^3)^{(1/3)}*(I*\sqrt{3} + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3 \\
& *c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3/4*(a^2*b^4*c^14 + a^3*b^3* \\
& c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^{(2/3)}*(b^2*c^2*d^4 \\
& /((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c \\
& *d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*\sqrt{3} + 1)/(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^ \\
& 3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^ \\
& 2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^{(1/3)}*(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
\end{aligned}$$

$$\begin{aligned}
& 3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^ \\
& 2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - \\
& 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) \\
& ^2 - 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b \\
& *c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b* \\
& c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3 \\
& *a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 \\
& + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^ \\
& 2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b \\
& ^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4 \\
& *b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^ \\
& 3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3 \\
& *a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^ \\
& 2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b \\
& ^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4 \\
& *b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^ \\
& 3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3 \\
& *a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/ \\
& (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sqrt(-(12*(4*b^5*c^11 - 24*a* \\
& b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^ \\
& 4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4 \\
& /((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c \\
& *d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^ \\
& 3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^ \\
& 2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^(1/3)*(27 \\
& *(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^ \\
& 3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2* \\
& b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24* \\
& a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^ \\
& 2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - \\
& 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) \\
& *d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15 \\
& *a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + \\
& 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c \\
& ^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^ \\
& 2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) \\
& *(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^ \\
& 3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^ \\
& 3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 \\
& + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)} - (1/2)^(1/3)*(27*(2*b^2*c^4*d^
\end{aligned}$$

$$2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) * (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^{(1/3)}*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))))/((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^3 b + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)

maple [C] time = 0.01, size = 217, normalized size = 0.55

$$\frac{6b^2d^2 \ln(a)}{(b^3+c^3)} - \frac{3abc^2 \ln(a)}{(b^3+c^3)} + \frac{3b^2d^2}{(b^3+c^3)^2} + \frac{d^2 \left(-6 \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a)^2 \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a) - 10b^2d^2 + 3 \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a) \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a) \right)}{3(b^3+c^3)^2} + \frac{12 \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a) \operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a) \ln(-\operatorname{RootOf}(b^3d^2x^2 + 3bd^2x + b^3 + a))}{2(b^3+c^3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b*(d*x+c)^3),x)

[Out] $\frac{1}{3}d^2/(b*c^3+a)^3*\sum((-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_R^2*a*b*c*d^2+12*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*\ln(-_R+x),_R=\operatorname{RootOf}(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))-1/2/(b*c^3+a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*d^2*c^4/(b*c^3+a)^3*\ln(x)-3*b*d^2*c/(b*c^3+a)^3*\ln(x)*a$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^3 \int \frac{10b^2c^6-16abc^3+3(2b^2c^4-abc)d^2x^2+3(5b^2c^5-4abc^2)dx+a^2}{b^3c^9+3ab^2c^6+3a^2bc^3+a^3} dx}{b^3c^9+3ab^2c^6+3a^2bc^3+a^3} + \frac{3(2b^2c^4-abc)d^2 \log(x)}{b^3c^9+3ab^2c^6+3a^2bc^3+a^3} + \frac{6bc^2dx-bc^3-a}{2(b^2c^6+2abc^3+a^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")

[Out] $-b*d^3*\operatorname{integrate}((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 + 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2*c^4 - a*b*c)*d^2*\log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + (6*b*c^2*d*x - b*c^3 - a)/(2*(b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)$

$*c^4 - a*b*c)*d^2*\log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)$

mupad [B] time = 2.49, size = 1328, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + b*(c + d*x)^3)),x)`

[Out] `symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k))*((a^3*b^4*d^12 + 19*b^7*c^9*d^12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k))*(root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k))*((9*a^6*b^3*c*d^8 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 + 90*a^3*b^6*c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 126*a^5*b^4*c^3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7*c^12*d^9))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (3*b^8*c^14*d^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5*c^5*d^10 - 63*a^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c^10*d^11 + 39*a^3*b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) + (x*(18*b^7*c^8*d^13 + 90*a*b^6*c^5*d^13 - 9*a^2*b^5*c^2*d^13))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - (x*(a*b^5*d^15 + b^6*c^3*d^15))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6))*root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k), k, 1, 3) - 1/(2*(a*x^2 + b*c^3*x^2)) + (3*b*c^2*d)/(a^2*x + b^2*c^6*x + 2*a*b*c^3*x) + (6*b^2*c^4*d^2*log(x))/(a^3 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6) - (3*a*b*c*d^2*log(x))/(a^3 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

$$3.106 \quad \int \frac{x^3}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=356

$$\frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4}$$

Rubi [A] time = 0.43, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {371, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt{a}}\right)}{2\sqrt{2} a^{3/4} b^{3/4} d^4} - \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt{a}} + 1\right)}{2\sqrt{2} a^{3/4} b^{3/4} d^4} + \frac{3c^2 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} d^4} + \frac{\log(a + b(c+dx)^4)}{4bd^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*(c + d*x)^4), x]

[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

```
Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

$c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[-(a*c)]$

Rule 1248

$\text{Int}[(x_*)*((d_*) + (e_*)*(x_*)^2)^{(q_*)}*((a_*) + (c_*)*(x_*)^4)^{(p_*)}, x_Symbol]$
 $:\> \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /;$ $\text{FreeQ}[\{a, c, d, e, p, q\}, x]$

Rule 1876

$\text{Int}[(Pq_)/((a_*) + (b_*)*(x_*)^n), x_Symbol] :\> \text{With}[\{v = \text{Sum}[(x^{ii}*(\text{Coeff}[Pq, x, ii] + \text{Coeff}[Pq, x, n/2 + ii]*x^{(n/2)}))]/(a + b*x^n), \{ii, 0, n/2 - 1\}]\}, \text{Int}[v, x] /;$ $\text{SumQ}[v] /;$ $\text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[n/2, 0] \&\& \text{Expon}[Pq, x] < n$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4} dx, x, c + dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right) dx, x, c + dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4} dx, x, c + dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4} dx, x, c + dx\right)}{d^4} \\
&= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2} dx, x, (c + dx)^2\right)}{2d^4} + \frac{\left(c\left(3 - \frac{\sqrt{b}c^2}{\sqrt{a}}\right)\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4} dx, x, c + dx\right)}{2bd^4} - \frac{c(3\sqrt{a} - \sqrt{b}c^2)}{2d^4} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2} dx, x, (c + dx)^2\right)}{2d^4} + \frac{(3c^2) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c + dx)^2\right)}{2d^4} - \frac{c(3\sqrt{a} - \sqrt{b}c^2)}{2d^4} \\
&= \frac{3c^2 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} - \frac{c(3\sqrt{a} - \sqrt{b}c^2) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^4} \\
&= \frac{3c^2 \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c(3\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 106, normalized size = 0.30

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1^3 \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b(c + dx)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^4), x]

[Out] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="giac")

[Out] integrate(x^3/((d*x + c)^4*b + a), x)

maple [C] time = 0.02, size = 97, normalized size = 0.27

$$\frac{\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^3 \ln(-\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a) + x)}{4bd^4 \text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^3 + 3 \text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^2 c d^2 + 3 \text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a) c^2 d + c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b*(d*x+c)^4), x)

[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x), _R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^4), x, algorithm="maxima")

[Out] integrate(x^3/((d*x + c)^4*b + a), x)

mupad [B] time = 2.69, size = 1003, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(a + b*(c + d*x)^4), x)$

[Out] $\text{symsum}(\log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))^2*a*b^2*c*d^8 + 24*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k))^2*a*b^2*d^9*x - 2*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^4*d^5*x + 38*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*c*d^4 + 6*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*d^5*x)))*\text{root}(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k), k, 1, 4)$

sympy [A] time = 3.74, size = 374, normalized size = 1.05

$\text{RootSum}\left(256a^3b^4d^{16}z^4 - 256a^3b^3d^{12}z^3 + 480a^2b^3c^4d^8z^2 + 96a^3b^2d^8z^2 + 192a^2b^2c^4d^4z - 48ab^3c^8d^4z - 16a^3bd^4z + 3ab^2c^8 + 3a^2bc^4 + b^3c^{12} + a^3, z\right) \log\left(x + \frac{-1728a^3b^4c^4d^{12}z^3 - 960a^3b^3c^4d^8z^2 + 1296a^2b^3c^4d^4z + 2016a^2b^2c^4d^4z - 48a^2b^3c^8d^4z - 324a^2b^4c^8d^4z - 4716a^2b^3c^8d^4z - 1452a^2b^2c^8d^4z - 4ab^4c^{12}z + 27a^4 - 390b^4c^4 - 444b^2c^8 - 26ab^2c^{12} + b^4c^{12}}{729a^3b^4d^{16}z^4 - 1053a^2b^3c^4d^8z^2 - 117a^2b^3c^4d^4z + b^3c^{12}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3/(a+b*(d*x+c)**4), x)$

[Out] $\text{RootSum}(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8 + b**3*c**12, \text{Lambda}(_t, _t*\log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t**3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2$

$$\frac{7a^4 - 390a^3bc^4 - 444a^2b^2c^8 - 26ab^3c^{12} + b^4c^{16}}{(729a^3bc^3d - 1053a^2b^2c^7d - 117ab^3c^{11}d + b^4c^{15}d)}$$

$$3.107 \quad \int \frac{x^2}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=318

$$\frac{(\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{(\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}$$

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {371, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{a} - \sqrt{b}c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{(\sqrt{a} - \sqrt{b}c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{(\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{(\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt{a}} + 1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*(c + d*x)^4), x]

[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coefficient
```

[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n), {ii, 0, n/2 - 1 }], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4} dx, x, c + dx\right)}{d^3} \\
 &= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4} + \frac{c^2+x^2}{a+bx^4}\right) dx, x, c + dx\right)}{d^3} \\
 &= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4} dx, x, c + dx\right)}{d^3} - \frac{(2c) \text{Subst}\left(\int \frac{x}{a+bx^4} dx, x, c + dx\right)}{d^3} \\
 &= -\frac{c \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c + dx)^2\right)}{d^3} - \frac{\left(1 - \frac{\sqrt{b}c^2}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx, x, c + dx\right)}{2bd^3} + \frac{\left(1 - \frac{\sqrt{b}c^2}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx, x, c + dx\right)}{2bd^3} \\
 &= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}+2x}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx, x, c + dx\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx, x, c + dx\right)}{2bd^3} \\
 &= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} + \frac{(\sqrt{a} - \sqrt{b}c^2) \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{(\sqrt{a} - \sqrt{b}c^2) \text{Subst}\left(\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx, x, c + dx\right)}{2bd^3} \\
 &= -\frac{c \tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{(\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{(\sqrt{a} + \sqrt{b}c^2) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1^2 \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b(c + dx)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^4), x]

[Out] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4), x, algorithm="giac")

[Out] integrate(x^2/((d*x + c)^4*b + a), x)

maple [C] time = 0.00, size = 97, normalized size = 0.31

$$\frac{\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^2 \ln(-\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a) + x)}{4bd^4(\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^3 + 3\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a)^2 c d^2 + 3\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + bc^4 + a) c^2 d + c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b*(d*x+c)^4), x)

[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x), _R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(x^2/((d*x + c)^4*b + a), x)

mupad [B] time = 2.59, size = 625, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*(c + d*x)^4),x)

[Out] symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k), k, 1, 4)

sympy [A] time = 2.67, size = 274, normalized size = 0.86

RootSum(256*t^4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lambda(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9 + 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d - 33*a*b**2*c**8*d + b**3*c**12*d))))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*(d*x+c)**4),x)

[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lambda(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9 + 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d - 33*a*b**2*c**8*d + b**3*c**12*d))))

$$3.108 \quad \int \frac{x}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=261

$$\frac{c \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c+dx) + \sqrt{a} + \sqrt{b} (c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} - \frac{c \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c+dx) + \sqrt{a} + \sqrt{b} (c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} (c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^3}$$

Rubi [A] time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {371, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{c \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c+dx) + \sqrt{a} + \sqrt{b} (c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} - \frac{c \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c+dx) + \sqrt{a} + \sqrt{b} (c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} (c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} - \frac{c \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} (c+dx)}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{b} d^2} + \frac{\tan^{-1}\left(\frac{\sqrt{b} (c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{b} d^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*(c + d*x)^4), x]

[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&

AtomQ[SplitProduct[SumBaseQ, b]])

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 371

Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4} dx, x, c + dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4} + \frac{x}{a+bx^4}\right) dx, x, c + dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4} dx, x, c + dx\right)}{d^2} - \frac{c \text{Subst}\left(\int \frac{1}{a+bx^4} dx, x, c + dx\right)}{d^2} \\
&= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, (c + dx)^2\right)}{2d^2} - \frac{c \text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d^2} - \frac{c \text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} - \frac{c \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d^2} - \frac{c \text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{c \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} - \frac{c \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2} + \frac{c \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 104, normalized size = 0.40

$$\frac{\text{RootSum}\left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1 \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \&\right]}{4bd}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*(c + d*x)^4), x]

[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &]/(4*b*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b(c + dx)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + b*(c + d*x)^4), x]

[Out] IntegrateAlgebraic[x/(a + b*(c + d*x)^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] integrate(x/((d*x + c)^4*b + a), x)

maple [C] time = 0.01, size = 95, normalized size = 0.36

$$\frac{\text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a) \ln(-\text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a) + x)}{4 b d \left(d^3 \text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a)^3 + 3 \text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a)^2 c d^2 + 3 \text{RootOf}(b d^4 Z^4 + 4 b d^3 c Z^3 + 6 b d^2 c^2 Z^2 + 4 b c^3 d Z + b c^4 + a) c^2 d + c^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*(d*x+c)^4), x)

[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x), _R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(x/((d*x + c)^4*b + a), x)

mupad [B] time = 2.36, size = 205, normalized size = 0.79

$$\sum_{k=1}^4 \ln\left(-\sqrt{(256a^3b^2d^6z^4 + 32a^2bd^4z^2 - 16abc^2d^2z + bc^4 + a, z, k)} \left(-\sqrt{(256a^3b^2d^6z^4 + 32a^2bd^4z^2 - 16abc^2d^2z + bc^4 + a, z, k)} (16axb^3d^2 + 32ac^3d^4) + 4b^3c^3d^0 + 4b^3c^2d^0x\right) \sqrt{(256a^3b^2d^6z^4 + 32a^2bd^4z^2 - 16abc^2d^2z + bc^4 + a, z, k)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*(c + d*x)^4),x)

[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k))*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k))*(32*a*b^3*c*d^11 + 16*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)

sympy [A] time = 0.88, size = 131, normalized size = 0.50

$$\text{RootSum}\left(256t^4a^3b^2d^8 + 32t^2a^2bd^4 - 16tabc^2d^2 + a + bc^4, \left(t \mapsto t \log\left(x + \frac{128t^3a^3bd^6 + 16t^2a^2bc^2d^4 + 8ta^2d^2 + 4tabc^4d^2 - ac^2 - bc^6}{4acd - bc^5d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)**4),x)

[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d**2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a**2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*a*c*d - b*c**5*d))))

$$3.109 \quad \int \frac{1}{a+b(c+dx)^4} dx$$

Optimal. Leaf size=221

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{b} d}$$

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {247, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}(c+dx) + \sqrt{a} + \sqrt{b}(c+dx)^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{b} d} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{b} d} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b}(c+dx)}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{b} d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c + d*x)^4)^(-1), x]

[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b(c + dx)^4} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^4} dx, x, c + dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx, x, c + dx\right)}{2\sqrt{a}d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx, x, c + dx\right)}{4\sqrt{a}\sqrt{b}d} \\
&= -\frac{\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d} \\
&= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} - \frac{\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{b}(c + dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{a} + \sqrt{b}(c + dx)^2\right) + \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c + dx) + \sqrt{a} + \sqrt{b}(c + dx)^2\right) - 2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right) + 2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c + d*x)^4)^(-1), x]

[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b(c + dx)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*(c + d*x)^4)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*(c + d*x)^4)^(-1), x]

fricas [A] time = 1.00, size = 189, normalized size = 0.86

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(a^2bd^4\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}} + d^2x^2 + 2cdx + c^2}{d^2}}\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}} - (a^2bd^4x + a^2bcd^3)\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} + dx + c\right) - \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} + dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] $(-1/(a^3*b*d^4))^{1/4}*\arctan(a^2*b*d^4*\sqrt{((a^2*d^2*\sqrt{-1/(a^3*b*d^4)}) + d^2*x^2 + 2*c*d*x + c^2)/d^2}*(-1/(a^3*b*d^4))^{3/4} - (a^2*b*d^4*x + a^2*b*c*d^3)*(-1/(a^3*b*d^4))^{3/4}) + 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(a*d*(-1/(a^3*b*d^4))^{1/4} + d*x + c) - 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(-a*d*(-1/(a^3*b*d^4))^{1/4} + d*x + c)$

giac [A] time = 0.50, size = 103, normalized size = 0.47

$$-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \arctan\left(\frac{bdx + bc}{(-ab^3)^{\frac{1}{4}}}\right) + \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(\left|bdx + bc + (-ab^3)^{\frac{1}{4}}\right|\right) - \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}} \log\left(\left|-bdx - bc + (-ab^3)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] $-1/2*(-1/(a^3*b*d^4))^{1/4}*\arctan(-(b*d*x + b*c)/(-a*b^3)^{1/4}) + 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(\text{abs}(b*d*x + b*c + (-a*b^3)^{1/4})) - 1/4*(-1/(a^3*b*d^4))^{1/4}*\log(\text{abs}(-b*d*x - b*c + (-a*b^3)^{1/4}))$

maple [C] time = 0.00, size = 94, normalized size = 0.43

$$\frac{\ln(-\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + b^4c^4 + a) + x)}{4bd(d^3\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + b^4c^4 + a)^3 + 3\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + b^4c^4 + a)^2c^2 + 3\text{RootOf}(bd^4Z^4 + 4bd^3cZ^3 + 6bd^2c^2Z^2 + 4bc^3dZ + b^4c^4 + a)c^2d + c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*(d*x+c)^4),x)

[Out] $1/4/b/d*\text{sum}(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*\ln(-_R*x), _R=\text{RootOf}(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] integrate(1/((d*x + c)^4*b + a), x)

mupad [B] time = 0.12, size = 60, normalized size = 0.27

$$\frac{\operatorname{atan}\left(\frac{b^{1/4}c}{(-a)^{1/4}} + \frac{b^{1/4}dx}{(-a)^{1/4}}\right) + \operatorname{atanh}\left(\frac{b^{1/4}c}{(-a)^{1/4}} + \frac{b^{1/4}dx}{(-a)^{1/4}}\right)}{2(-a)^{3/4}b^{1/4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^4), x)

[Out] $-(\operatorname{atan}((b^{1/4}c)/(-a)^{1/4} + (b^{1/4}d*x)/(-a)^{1/4}) + \operatorname{atanh}((b^{1/4}c)/(-a)^{1/4} + (b^{1/4}d*x)/(-a)^{1/4}))/ (2*(-a)^{3/4}*b^{1/4}*d)$

sympy [A] time = 0.30, size = 26, normalized size = 0.12

$$\frac{\operatorname{RootSum}\left(256t^4a^3b + 1, \left(t \mapsto t \log\left(x + \frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**4), x)

[Out] $\operatorname{RootSum}(256*_t**4*a**3*b + 1, \operatorname{Lambda}(_t, _t*\log(x + (4*_t*a + c)/d)))/d$

$$3.110 \quad \int \frac{1}{x(a+b(c+dx)^4)} dx$$

Optimal. Leaf size=393

$$\frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)}$$

Rubi [A] time = 0.47, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2)}{4\sqrt{2}a^{3/4}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}+\sqrt{b}c^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(a+bc^4)} - \frac{\sqrt[4]{b}c(\sqrt{a}+\sqrt{b}c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}}+1\right)}{2\sqrt{2}a^{3/4}(a+bc^4)} - \frac{\log(a+b(c+dx)^4)}{4(a+bc^4)} - \frac{\sqrt{b}c^2\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}(a+bc^4)} + \frac{\log(x)}{a+bc^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c + d*x)^4)),x]

[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

```
Int[((a_) + (b_)*(v_)^(n_))^(p_)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
```

c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+b(c+dx)^4)} dx &= \text{Subst} \left(\int \frac{1}{(-c+x)(a+bx^4)} dx, x, c+dx \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{(a+bc^4)(c-x)} - \frac{b(c^3+c^2x+cx^2+x^3)}{(a+bc^4)(a+bx^4)} \right) dx, x, c+dx \right) \\
&= \frac{\log(x)}{a+bc^4} - \frac{b \text{Subst} \left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4} dx, x, c+dx \right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4} - \frac{b \text{Subst} \left(\int \left(\frac{x(c^2+x^2)}{a+bx^4} + \frac{c^3+cx^2}{a+bx^4} \right) dx, x, c+dx \right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4} - \frac{b \text{Subst} \left(\int \frac{x(c^2+x^2)}{a+bx^4} dx, x, c+dx \right)}{a+bc^4} - \frac{b \text{Subst} \left(\int \frac{c^3+cx^2}{a+bx^4} dx, x, c+dx \right)}{a+bc^4} \\
&= \frac{\log(x)}{a+bc^4} - \frac{b \text{Subst} \left(\int \frac{c^2+x}{a+bx^2} dx, x, (c+dx)^2 \right)}{2(a+bc^4)} + \frac{\left(c \left(1 - \frac{\sqrt{b}c^2}{\sqrt{a}} \right) \right) \text{Subst} \left(\int \frac{\sqrt{a}\sqrt{b-bx^2}}{a+bx^4} dx, x, (c+dx)^2 \right)}{2(a+bc^4)} \\
&= \frac{\log(x)}{a+bc^4} - \frac{b \text{Subst} \left(\int \frac{x}{a+bx^2} dx, x, (c+dx)^2 \right)}{2(a+bc^4)} - \frac{(bc^2) \text{Subst} \left(\int \frac{1}{a+bx^2} dx, x, (c+dx)^2 \right)}{2(a+bc^4)} \\
&= -\frac{\sqrt{b}c^2 \tan^{-1} \left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}} \right)}{2\sqrt{a}(a+bc^4)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}c(\sqrt{a}-\sqrt{b}c^2) \log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)} \\
&= -\frac{\sqrt{b}c^2 \tan^{-1} \left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}} \right)}{2\sqrt{a}(a+bc^4)} + \frac{\sqrt[4]{b}c(\sqrt{a}+\sqrt{b}c^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}(a+bc^4)} - \frac{\sqrt[4]{b}c(\sqrt{a}+\sqrt{b}c^2) \log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx))}{4\sqrt{2}a^{3/4}(a+bc^4)}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 163, normalized size = 0.41

$$\frac{\text{RootSum} \left[\#1^4bd^4 + 4\#1^3bcd^3 + 6\#1^2bc^2d^2 + 4\#1bc^3d + a + bc^4 \&, \frac{\#1^3d^3 \log(x-\#1) + 4\#1^2cd^2 \log(x-\#1) + 4c^3 \log(x-\#1) + 6\#1c^2d \log(x-\#1)}{\#1^3d^3 + 3\#1^2cd^2 + 3\#1c^2d + c^3} \& \right] - 4 \log(x)}{4(a+bc^4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c + d*x)^4)), x]

[Out] $-1/4*(-4*\text{Log}[x] + \text{RootSum}[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 \& , (4*c^3*\text{Log}[x - #1] + 6*c^2*d*\text{Log}[x - #1]*#1 + 4*c*d^2*\text{Log}[x - #1]*#1^2 + d^3*\text{Log}[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) \&])/(a + b*c^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b(c + dx)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^4)),x]

[Out] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^4 b + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)

maple [C] time = 0.01, size = 139, normalized size = 0.35

$$\frac{\ln(x) \left(b^4 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a)^3 + 4 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a)^2 c d^2 + 6 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a) c^2 d + 4c^3 \right) \ln(-\text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a) + x)}{b^4 c^4 + a^4 \left(b^4 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a)^3 + 3 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a) c^2 d + 3 \text{RootOf}(b^4 d^4 Z^4 + 4b^3 d^3 c Z^3 + 6b^2 d^2 c^2 Z^2 + 4b c^2 d Z + b^4 + a) c^2 d + c^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(d*x+c)^4),x)

[Out] $-1/4/(b*c^4+a)*\text{sum}((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*\ln(-_R+x), _R=\text{RootOf}(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))+\ln(x)/(b*c^4+a)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bd \int \frac{d^3x^3 + 4cd^2x^2 + 6c^2dx + 4c^3}{bd^4x^4 + 4bcd^3x^3 + 6bc^2d^2x^2 + 4bc^3dx + bc^4 + a} dx}{bc^4 + a} + \frac{\log(x)}{bc^4 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) + 1*og(x)/(b*c^4 + a)

mupad [B] time = 2.18, size = 882, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^4)),x)

[Out] log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k), k, 1, 4)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

$$3.111 \quad \int \frac{1}{x^2(a+b(c+dx)^4)} dx$$

Optimal. Leaf size=496

$$\frac{\sqrt[4]{b} d \left(\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4) \right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2 \right) + \sqrt[4]{b} d \left(\sqrt{a} (a - 3bc^4) \right)}{4\sqrt{2} a^{3/4} (a + bc^4)^2}$$

Rubi [A] time = 0.89, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, number of rules / integrand size = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b} d \left(\sqrt{a} (a - 3bc^4) - \sqrt{b} c^2 (3a - bc^4) \right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (c + dx) + \sqrt{a} + \sqrt{b} (c + dx)^2 \right) + \sqrt[4]{b} d \left(\sqrt{a} (a - 3bc^4) \right)}{4\sqrt{2} a^{3/4} (a + bc^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]

[Out] $-(1/((a + b*c^4)*x)) - (\text{Sqrt}[b]*c*(a - b*c^4)*d*\text{ArcTan}[(\text{Sqrt}[b]*(c + d*x)^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(a + b*c^4)^2) + (b^{(1/4)}*(\text{Sqrt}[a]*(a - 3*b*c^4) + \text{Sqrt}[b]*c^2*(3*a - b*c^4))*d*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*(c + d*x))/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(a + b*c^4)^2) - (b^{(1/4)}*(\text{Sqrt}[a]*(a - 3*b*c^4) + \text{Sqrt}[b]*c^2*(3*a - b*c^4))*d*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*(c + d*x))/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(a + b*c^4)^2) - (4*b*c^3*d*\text{Log}[x])/(a + b*c^4)^2 - (b^{(1/4)}*(\text{Sqrt}[a]*(a - 3*b*c^4) - \text{Sqrt}[b]*c^2*(3*a - b*c^4))*d*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*(c + d*x) + \text{Sqrt}[b]*(c + d*x)^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(a + b*c^4)^2) + (b^{(1/4)}*(\text{Sqrt}[a]*(a - 3*b*c^4) - \text{Sqrt}[b]*c^2*(3*a - b*c^4))*d*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*(c + d*x) + \text{Sqrt}[b]*(c + d*x)^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(a + b*c^4)^2) + (b*c^3*d*\text{Log}[a + b*(c + d*x)^4])/(a + b*c^4)^2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 371

$\text{Int}(((a_) + (b_.)*(v_)^{(n_.)})^{(p_.)}*(x_)^{(m_.)}, x_Symbol] \rightarrow \text{With}\{c = \text{Coefficient}[v, x, 0], d = \text{Coefficient}[v, x, 1]\}, \text{Dist}[1/d^{(m+1)}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(x - c)^m*(a + b*x^n)^p, x], x, v], x] /; \text{NeQ}[c, 0] /; \text{FreeQ}\{a, b, n, p\}, x\} \ \&\& \ \text{LinearQ}[v, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}), x_Symbol] \rightarrow \text{With}\{q = 1 - 4*Simplify[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}(((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 635

$\text{Int}(((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 1162

$\text{Int}(((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}(((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1168


```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b(c + dx)^4)} dx &= d \operatorname{Subst} \left(\int \frac{1}{(-c + x)^2 (a + bx^4)} dx, x, c + dx \right) \\
&= d \operatorname{Subst} \left(\int \left(\frac{1}{(a + bc^4)(c - x)^2} + \frac{4bc^3}{(a + bc^4)^2 (c - x)} + \frac{b(-c^2(3a - bc^4) - 2c(a - bc^4))}{(a + bc^4)^2} \right) dx, \right. \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-c^2(3a - bc^4) - 2c(a - bc^4)x - (a - 3bc^4)x^2 + 4bc^3x^3}{a + bx^4} dx, \right. \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \left(\frac{x(-2c(a - bc^4) + 4bc^3x^2)}{a + bx^4} + \frac{-c^2(3a - bc^4) + (-a + 3bc^4)x}{a + bx^4} \right) dx, \right. \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{x(-2c(a - bc^4) + 4bc^3x^2)}{a + bx^4} dx, x, c + dx \right)}{(a + bc^4)^2} + \frac{(bd) \operatorname{Subst} \left(\int \frac{-2c(a - bc^4) + 4bc^3x}{a + bx^2} dx, x, (c + dx)^2 \right)}{2(a + bc^4)^2} + \frac{\left((a - bc^4) \operatorname{Subst} \left(\int \frac{x}{a + bx^2} dx, x, (c + dx)^2 \right) \right)}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} + \frac{(2b^2c^3d) \operatorname{Subst} \left(\int \frac{x}{a + bx^2} dx, x, (c + dx)^2 \right)}{(a + bc^4)^2} - \frac{(bc(a - bc^4) \operatorname{Subst} \left(\int \frac{x}{a + bx^2} dx, x, (c + dx)^2 \right))}{(a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{\sqrt{b}c(a - bc^4)d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{\sqrt{a}(a + bc^4)^2} - \frac{4bc^3 d \log(x)}{(a + bc^4)^2} - \frac{\sqrt[4]{b} \left(a - 3bc^4 - \frac{bc^2(3a - bc^4)}{\sqrt{a}} \right) d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{2\sqrt{2} \sqrt[4]{a} (a + bc^4)^2} \\
&= -\frac{1}{(a + bc^4)x} - \frac{\sqrt{b}c(a - bc^4)d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{\sqrt{a}(a + bc^4)^2} + \frac{\sqrt[4]{b} \left(a - 3bc^4 + \frac{\sqrt{b}c^2(3a - bc^4)}{\sqrt{a}} \right) d \tan^{-1} \left(\frac{\sqrt{b}(c + dx)^2}{\sqrt{a}} \right)}{2\sqrt{2} \sqrt[4]{a} (a + bc^4)^2}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 238, normalized size = 0.48

$$\frac{dx \operatorname{RootSum} \left[\#1^4 b d^4 + 4 \#1^3 b c d^3 + 6 \#1^2 b c^2 d^2 + 4 \#1 b c^3 d + a + b c^4 \&, \frac{4 \#1^3 b c^3 d^3 \log(x - \#1) - \#1^2 a d^2 \log(x - \#1) + 15 \#1^2 b c^4 d^2 \log(x - \#1) - 6 a c^2 \log(x - \#1) - 4 \#1 a c d \log(x - \#1) + 10 b c^6 \log(x - \#1) + 20 \#1 b c^5 d \log(x - \#1) \&}{\#1^3 d^3 + 3 \#1^2 c d^2 + 3 \#1 c^2 d + c^3} \right] - 4(a + bc^4 + 4bc^3 dx \log(x))}{4x(a + bc^4)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]

[Out] $(-4*(a + b*c^4 + 4*b*c^3*d*x*\text{Log}[x]) + d*x*\text{RootSum}[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^4*x^4 \& , (-6*a*c^2*\text{Log}[x - \#1] + 10*b*c^6*\text{Log}[x - \#1] - 4*a*c*d*\text{Log}[x - \#1]*\#1 + 20*b*c^5*d*\text{Log}[x - \#1]*\#1 - a*d^2*\text{Log}[x - \#1]*\#1^2 + 15*b*c^4*d^2*\text{Log}[x - \#1]*\#1^2 + 4*b*c^3*d^3*\text{Log}[x - \#1]*\#1^3)/(c^3 + 3*c^2*d*\#1 + 3*c*d^2*\#1^2 + d^3*\#1^3) \&])/(4*(a + b*c^4)^2*x)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^4)),x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((dx + c)^4 b + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")

[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)

maple [C] time = 0.01, size = 188, normalized size = 0.38

$$\frac{4b^2c^2 \ln(x)}{(b^4 + a)^2} + \frac{d \left(4 \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) b^2 d^2 + 10b^2 c + (5b^4 - a) \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) d^2 + 4(5b^4 - a) \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) d^2 - 4b^2 c^2 \right) \ln(-\text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) + \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a))}{4(b^4 + a)^2 \left(d^2 \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) + 3 \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) \right)^2 c d^2 + 3 \text{RootOf}(b^2 d^2 Z^2 + 4b^2 d Z + b^4 + a) c d + c^2} \frac{1}{(b^4 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b*(d*x+c)^4),x)

[Out] $\frac{1}{4} \frac{d}{(bc^4+a)^2} \sum((4*b*d^3*c^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^4-a)*_R+10*b*c^6-6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*\ln(-_R+x),$
 $_R=\text{RootOf}(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a)$
 $-1/(b*c^4+a)/x-4*b*c^3*d*\ln(x)/(b*c^4+a)^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4bc^3d \log(x)}{b^2c^8 + 2abc^4 + a^2} + \frac{bd^2 \int \frac{4bc^3d^3x^3 + 10bc^6 + (15bc^4 - a)d^2x^2 - 6ac^2 + 4(5bc^5 - ac)dx}{bd^4x^4 + 4bcd^3x^3 + 6bc^2d^2x^2 + 4bc^3dx + bc^4 + a} dx}{b^2c^8 + 2abc^4 + a^2} - \frac{1}{(bc^4 + a)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")

[Out] $-4*b*c^3*d*\log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*\text{integrate}((4*b*c^3*d^3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)$

mupad [B] time = 2.48, size = 2440, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*(c + d*x)^4)),x)

[Out] $\text{symsum}(\log(-(4*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^{11}*d^{17} - 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^4*b^4*d^{16} - b^5*d^{20}*x + 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^{18} - 60*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^{17} + 176*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^{16} + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^5*d^{15} + 144*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^{16} + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^{15} + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32$

$$\begin{aligned}
& *a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^19*x + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^15 - 184*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^17 - 48*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^12*d^16 - 320*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*d^16*x + 4*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*b^7*c^10*d^18*x - 248*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^18*x - 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^11*d^17*x + 32*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^19*x - 316*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2*d^18*x + 704*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^3*d^17*x - 448*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*c^4*d^16*x + 640*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^17*x + 64*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^16*x + 192*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^12*d^16*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*\text{root}(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*log(x))/(a^2 + b^2*c^8 + 2*a*b*c^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)

[Out] Timed out

$$3.112 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=123

$$-\frac{2}{9}(-3a^2 + 6a + 37)(x-1)^9 + \frac{4}{13}(3-a)(x-1)^{13} - \frac{8}{11}(3a+5)(x-1)^{11} + \frac{8}{7}(a+3)(3a+5)(x-1)^7 + \frac{4}{5}(3-a)(a+3)^2(x-1)^5 - \dots$$

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1106, 1090}

$$-\frac{2}{9}(-3a^2 + 6a + 37)(x-1)^9 + \frac{4}{13}(3-a)(x-1)^{13} - \frac{8}{11}(3a+5)(x-1)^{11} + \frac{8}{7}(a+3)(3a+5)(x-1)^7 + \frac{4}{5}(3-a)(a+3)^2(x-1)^5 - \frac{8}{3}(a+3)^3(x-1)^3 + (a+3)^4x + \frac{1}{17}(x-1)^{17} + \frac{8}{15}(x-1)^{15}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] $(-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 + a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 + 3*a)*(-1 + x)^{11})/11 + (4*(3 - a)*(-1 + x)^{13})/13 + (8*(-1 + x)^{15})/15 + (-1 + x)^{17}/17 + (3 + a)^4*x$

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx &= \text{Subst} \left(\int (3 + a - 2x^2 - x^4)^4 dx, x, -1 + x \right) \\ &= \text{Subst} \left(\int \left(81 \left(1 + \frac{1}{81}a(108 + 54a + 12a^2 + a^3) \right) \right) - 216 \left(1 + a \left(1 + \frac{1}{27}a(9 + \dots) \right) \right) \right) \\ &= -\frac{8}{3}(3 + a)^3(-1 + x)^3 + \frac{4}{5}(3 - a)(3 + a)^2(-1 + x)^5 + \frac{8}{7}(3 + a)(5 + 3a)(-1 + x)^7 - \dots \end{aligned}$$

Mathematica [A] time = 0.04, size = 195, normalized size = 1.59

$$a^4x + 16a^3x^2 + \frac{2}{3}(3a^2 - 1536a + 20480)x^3 - 6(a^2 - 128a + 896)x^4 + \frac{64}{7}(3a^2 - 140a + 512)x^5 - \frac{16}{3}(15a^2 - 288a + 512)x^6 + 4a(a^2 - 48a + 128)x^7 - \frac{32}{3}(a - 12)a^2x^8 - \frac{4}{5}(a^3 - 192a^2 + 1536a - 1024)x^9 - \frac{4}{13}(a - 640)x^{10} + \frac{4}{3}(3a - 464)x^{11} - \frac{32}{11}(9a - 524)x^{11} + \frac{16}{5}(35a - 928)x^{10} + \frac{x^{17}}{17} - \frac{128x^{15}}{15} - 48x^{14}$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

fricas [B] time = 1.03, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} - \frac{4}{13}x^{13}a + \frac{2560}{13}x^{13} + 4x^{12}a^2 - \frac{1856}{3}x^{12} - \frac{288}{11}x^{11}a + \frac{16768}{11}x^{11} + 112x^{10}a^2 + \frac{2}{3}x^9a^3 - \frac{14848}{5}x^{10} - \frac{1024}{3}x^9a^2 - \frac{14848}{5}x^{10} - \frac{1024}{3}x^9a^2 + \frac{40960}{9}x^9 + 768x^8a + \frac{192}{7}x^7a^2 - 5376x^8 - 1280x^7a - 80x^6a^2 - \frac{4}{5}x^5a^3 + \frac{32768}{7}x^7 + 1536x^6a + \frac{768}{5}x^5a^2 + 4x^4a^3 - \frac{8192}{3}x^6 - \frac{6144}{5}x^5a - 192x^4a^2 - \frac{32}{3}x^3a^3 + \frac{4096}{5}x^5 + 512x^4a + 128x^3a^2 + 16x^2a^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 - 4/13*x^13*a + 2560/13*x^13 + 4*x^12*a - 1856/3*x^12 - 288/11*x^11*a + 16768/11*x^11 + 112*x^10*a + 2/3*x^9*a^2 - 14848/5*x^10 - 1024/3*x^9*a - 6*x^8*a^2 + 40960/9*x^9 + 768*x^8*a + 192/7*x^7*a^2 - 5376*x^8 - 1280*x^7*a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x^7 + 1536*x^6*a + 768/5*x^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 192*x^4*a^2 - 32/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a^3 + xa^4

giac [B] time = 0.33, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + 4x^{12}a^2 + \frac{2560}{13}x^{13} - \frac{288}{11}x^{11}a - \frac{1856}{3}x^{12} + \frac{2}{3}x^9a^3 + 112x^{10}a + \frac{16768}{11}x^{11} - 6x^8a^2 - \frac{1024}{3}x^9a^2 - \frac{14848}{5}x^{10} + \frac{192}{7}x^7a^2 + 768x^8a + \frac{40960}{9}x^9 - \frac{4}{5}x^5a^3 - 80x^6a^2 - 1280x^7a - 5376x^8 + 4x^4a^3 + \frac{768}{5}x^5a^2 + 1536x^6a + \frac{32768}{7}x^7 - \frac{32}{3}x^3a^3 - 192x^4a^2 - \frac{6144}{5}x^5 - \frac{8192}{3}x^6 + a^3x + 16a^2x^2 + 128a^2x^3 + 512ax^4 + \frac{4096}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

[Out] $\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} - 6a^2x^8 - \frac{1024}{3}ax^9 - \frac{14848}{5}x^{10} + \frac{192}{7}a^2x^7 + 768ax^8 + \frac{40960}{9}x^9 - \frac{4}{5}a^3x^5 - 80a^2x^6 - 1280ax^7 - 5376x^8 + 4a^3x^4 + \frac{768}{5}a^2x^5 + 1536ax^6 + \frac{32768}{7}x^7 - \frac{32}{3}a^3x^3 - 192a^2x^4 - \frac{6144}{5}ax^5 - \frac{8192}{3}x^6 + a^4x + 16a^3x^2 + 128a^2x^3 + 512ax^4 + \frac{4096}{5}x^5$

maple [B] time = 0.00, size = 264, normalized size = 2.15

$\frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 4ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560x^{13}}{13} - \frac{288ax^{11}}{11} - \frac{1856x^{12}}{3} + \frac{2a^2x^9}{3} + 112ax^{10} + \frac{16768x^{11}}{11} - 6a^2x^8 - \frac{1024ax^9}{3} - \frac{14848x^{10}}{5} + \frac{192a^2x^7}{7} + 768ax^8 + \frac{40960x^9}{9} - \frac{4a^3x^5}{5} - 80a^2x^6 - 1280ax^7 - 5376x^8 + 4a^3x^4 + \frac{768a^2x^5}{5} + 1536ax^6 + \frac{32768x^7}{7} - \frac{32a^3x^3}{3} - 192a^2x^4 - \frac{6144ax^5}{5} - \frac{8192x^6}{3} + a^4x + 16a^3x^2 + 128a^2x^3 + 512ax^4 + \frac{4096x^5}{5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)

[Out] $\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{1}{13}(-4a+2560)x^{13} + \frac{1}{12}(48a-7424)x^{12} + \frac{1}{11}(-288a+16768)x^{11} + \frac{1}{10}(1120a-29696)x^{10} + \frac{1}{9}(2a^2-2560a+24576+(-2a+128)^2)x^9 + \frac{1}{8}(-16a^2+3584a-10240+2(8a-128)(-2a+128))x^8 + \frac{1}{7}(64a^2-2560a+2(-16a+64)(-2a+128)+(8a-128)^2)x^7 + \frac{1}{6}(-160a^2+32a(-2a+128)+2(-16a+64)(8a-128))x^6 + \frac{1}{5}(2a^2(-2a+128)+32a(8a-128)+(-16a+64)^2)x^5 + \frac{1}{4}(2a^2(8a-128)+32a(-16a+64))x^4 + \frac{1}{3}(2a^2(-16a+64)+256a^2)x^3 + 16a^3x^2 + a^4x$

maxima [A] time = 0.69, size = 192, normalized size = 1.56

$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{1}{13}(-4a+2560)x^{13} + \frac{1}{12}(48a-7424)x^{12} + \frac{1}{11}(-288a+16768)x^{11} + \frac{1}{10}(1120a-29696)x^{10} + \frac{1}{9}(2a^2-2560a+24576+(-2a+128)^2)x^9 + \frac{1}{8}(-16a^2+3584a-10240+2(8a-128)(-2a+128))x^8 + \frac{1}{7}(64a^2-2560a+2(-16a+64)(-2a+128)+(8a-128)^2)x^7 + \frac{1}{6}(-160a^2+32a(-2a+128)+2(-16a+64)(8a-128))x^6 + \frac{1}{5}(2a^2(-2a+128)+32a(8a-128)+(-16a+64)^2)x^5 + \frac{1}{4}(2a^2(8a-128)+32a(-16a+64))x^4 + \frac{1}{3}(2a^2(-16a+64)+256a^2)x^3 + 16a^3x^2 + a^4x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

[Out] $\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + 16768/11x^{11} - 14848/5x^{10} + \frac{40960}{9}x^9 - 5376x^8 + \frac{32768}{7}x^7 - 8192/3x^6 + a^4x + \frac{4096}{5}x^5 - \frac{4}{15}(3x^5 - 15x^4 + 40x^3 - 60x^2)a^3 + \frac{2}{1}05(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3)a^2 - \frac{4}{2145}(165x^{13} - 2145x^{12} + 14040x^{11} - 60060x^{10} + 183040x^9 - 411840x^8 + 686400x^7 - 823680x^6 + 658944x^5 - 274560x^4)a$

mupad [B] time = 0.21, size = 175, normalized size = 1.42

$x^{12} \left(4a - \frac{1856}{3} \right) - x^{11} \left(\frac{4a}{13} - \frac{2560}{13} \right) + x^{10} \left(\frac{1120a}{10} - \frac{14848}{5} \right) - x^9 \left(\frac{288a}{11} - \frac{16768}{11} \right) - x^8 (6a^2 - 768a + 5376) - x^7 (80a^2 - 1536a + \frac{8192}{3}) + x^6 \left(\frac{192a^2}{7} - 1280a + \frac{32768}{7} \right) + x^5 \left(\frac{2a^2}{3} - \frac{1024a}{3} + \frac{40960}{9} \right) - x^4 \left(\frac{4a^3}{5} - \frac{768a^2}{5} + \frac{6144a}{5} - \frac{4096}{5} \right) + a^4x - 48x^{14} + \frac{128x^{15}}{15} - \frac{x^{17}}{17} + 16a^3x^2 + 4ax^4 + x^2(a^2 - 48a + 128) - \frac{32a^2x^3(a-12)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] $x^{12}(4a - 1856/3) - x^{13}((4a)/13 - 2560/13) + x^{10}(112a - 14848/5) - x^{11}((288a)/11 - 16768/11) - x^8(6a^2 - 768a + 5376) - x^6(80a^2 - 1536a + 8192/3) + x^7((192a^2)/7 - 1280a + 32768/7) + x^9((2a^2)/3 - (1024a)/3 + 40960/9) - x^5((6144a)/5 - (768a^2)/5 + (4a^3)/5 - 4096/5) + a^4x - 48x^{14} + (128x^{15})/15 - x^{16} + x^{17}/17 + 16a^3x^2 + 4ax^4(a^2 - 48a + 128) - (32a^2x^3(a - 12))/3$

sympy [A] time = 0.12, size = 199, normalized size = 1.62

$$a^4x + 16a^3x^2 + \frac{x^{17}}{17} - x^{16} + \frac{128a^{15}}{15} - 48a^{14} + x^{15}\left(\frac{2560}{13} - \frac{4a}{13}\right) + x^{14}\left(4a - \frac{1856}{3}\right) + x^{13}\left(\frac{16768}{11} - \frac{288a}{11}\right) + x^{12}\left(112a - \frac{14848}{5}\right) + x^{11}\left(\frac{2a^2}{3} - \frac{1024a}{3} + \frac{40960}{9}\right) + x^9(-6a^2 + 768a - 5376) + x^7\left(\frac{192a^2}{7} - 1280a + \frac{32768}{7}\right) + x^6(-80a^2 + 1536a - \frac{8192}{3}) + x^5\left(\frac{4a^3}{5} + \frac{768a^2}{5} - \frac{6144a}{5} + \frac{4096}{5}\right) + x^4(4a^3 - 192a^2 + 512a) + x^3\left(-\frac{32a^3}{3} + 128a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] $a^{**4}x + 16*a^{**3}*x^{**2} + x^{**17}/17 - x^{**16} + 128*x^{**15}/15 - 48*x^{**14} + x^{**13}*(2560/13 - 4*a/13) + x^{**12}*(4*a - 1856/3) + x^{**11}*(16768/11 - 288*a/11) + x^{**10}*(112*a - 14848/5) + x^{**9}*(2*a^{**2}/3 - 1024*a/3 + 40960/9) + x^{**8}*(-6*a^{**2} + 768*a - 5376) + x^{**7}*(192*a^{**2}/7 - 1280*a + 32768/7) + x^{**6}*(-80*a^{**2} + 1536*a - 8192/3) + x^{**5}*(-4*a^{**3}/5 + 768*a^{**2}/5 - 6144*a/5 + 4096/5) + x^{**4}*(4*a^{**3} - 192*a^{**2} + 512*a) + x^{**3}*(-32*a^{**3}/3 + 128*a^{**2})$

$$3.113 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Optimal. Leaf size=120

$$a^3x - \frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 - \frac{1}{3}(256 - a)x^9 + 3(64 - a)x^8 - \frac{32}{7}(70 - 3a)x^7 + 8(48 - 5a)x^6$$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$-\frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 + a^3x - \frac{1}{3}(256 - a)x^9 + 3(64 - a)x^8 - \frac{32}{7}(70 - 3a)x^7 + 8(48 - 5a)x^6 + 8(8 - a)ax^3 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 - 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx &= \int (a^3 + 24a^2x + 24(8 - a)ax^2 + 4(128 - 96a + 3a^2)x^3 - 3(512 - 128a + a^2)x^4 \\ &\quad + 8(48 - 5a)x^5 - (256 - a)x^6 + 3(64 - a)x^7 - (72 - 3a)x^8 + 28x^9 - 72x^{10} + 11x^{11} - 11x^{12} + 11x^{13}) dx \\ &= a^3x + 12a^2x^2 + 8(8 - a)ax^3 + (128 - 96a + 3a^2)x^4 - \frac{3}{5}(512 - 128a + a^2)x^5 \\ &\quad + 8(48 - 5a)x^6 - \frac{32}{7}(70 - 3a)x^7 + 3(64 - a)x^8 - \frac{(256 - a)x^9}{3} + 28x^{10} - \frac{72x^{11}}{11} + x^{12} - \frac{x^{13}}{13} \end{aligned}$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.95

$$a^3x - \frac{3}{5}(a^2 - 128a + 512)x^5 + (3a^2 - 96a + 128)x^4 + 12a^2x^2 + \frac{1}{3}(a - 256)x^9 - 3(a - 64)x^8 + \frac{32}{7}(3a - 70)x^7 - 8(5a - 48)x^6 - 8(a - 8)ax^3 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 - 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

fricas [A] time = 1.08, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} + \frac{1}{3}x^9a - \frac{256}{3}x^9 - 3x^8a + 192x^8 + \frac{96}{7}x^7a - 320x^7 - 40x^6a - \frac{3}{5}x^5a^2 + 384x^6 + \frac{384}{5}x^5a + 3x^4a^2 - \frac{1536}{5}x^5 - 96x^4a - 8x^3a^2 + 128x^4 + 64x^3a + 12x^2a^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 + 1/3*x^9*a - 256/3*x^9 - 3*x^8*a + 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a^2 + 384*x^6 + 384/5*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a - 8*x^3*a^2 + 128*x^4 + 64*x^3*a + 12*x^2*a^2 + x*a^3

giac [A] time = 0.30, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}ax^5 + 384x^6 - 8a^2x^3 - 96ax^4 - \frac{1536}{5}x^5 + a^3x + 12a^2x^2 + 64ax^3 + 128x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9 + 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2 + 64*a*x^3 + 128*x^4

maple [A] time = 0.00, size = 138, normalized size = 1.15

$$\frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + \frac{(3a-768)x^9}{9} + \frac{(-24a+1536)x^8}{8} + \frac{(96a-2240)x^7}{7} + \frac{(-240a+2304)x^6}{6} + \frac{(-a^2+(-2a+128)a+256a-1536)x^5}{5} + a^3x + 12a^2x^2 + \frac{(4a^2+(8a-128)a-256a+512)x^4}{4} + \frac{(-8a^2+(-16a+64)a+128a)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3, x)

[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*((-2*a+128)*a+256*a-1536-a^2)*

$$x^5 + 1/4 * ((8*a - 128) * a - 256*a + 512 + 4*a^2) * x^4 + 1/3 * ((-16*a + 64) * a + 128*a - 8*a^2) * x^3 + 12*a^2 * x^2 + a^3 * x$$

maxima [A] time = 0.67, size = 119, normalized size = 0.99

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} - \frac{256}{3}x^9 + 192x^8 - 320x^7 + 384x^6 - \frac{1536}{5}x^5 + a^3x + 128x^4 - \frac{1}{5}(3x^5 - 15x^4 + 40x^3 - 60x^2)a^2 + \frac{1}{105}(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

$$[Out] -1/13*x^{13} + x^{12} - 72/11*x^{11} + 28*x^{10} - 256/3*x^9 + 192*x^8 - 320*x^7 + 384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^3)*a$$

mupad [B] time = 0.10, size = 108, normalized size = 0.90

$$x^9 \left(\frac{a}{3} - \frac{256}{3} \right) - x^8 (3a - 192) - x^6 (40a - 384) + x^7 \left(\frac{96a}{7} - 320 \right) + x^4 (3a^2 - 96a + 128) - x^5 \left(\frac{3a^2}{5} - \frac{384a}{5} + \frac{1536}{5} \right) + a^3x + 28x^{10} - \frac{72x^{11}}{11} + x^{12} - \frac{x^{13}}{13} + 12a^2x^2 - 8ax^3(a - 8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)

$$[Out] x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 320) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*x + 28*x^{10} - (72*x^{11})/11 + x^{12} - x^{13}/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)$$

sympy [A] time = 0.09, size = 114, normalized size = 0.95

$$a^3x + 12a^2x^2 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + x^9 \left(\frac{a}{3} - \frac{256}{3} \right) + x^8 (192 - 3a) + x^7 \left(\frac{96a}{7} - 320 \right) + x^6 (384 - 40a) + x^5 \left(-\frac{3a^2}{5} + \frac{384a}{5} - \frac{1536}{5} \right) + x^4 (3a^2 - 96a + 128) + x^3 (-8a^2 + 64a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)

$$[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) + x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8*a**2 + 64*a)$$

$$3.114 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=72

$$a^2x + \frac{2}{5}(64 - a)x^5 - 2(16 - a)x^4 + \frac{16}{3}(4 - a)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2061}

$$a^2x + \frac{2}{5}(64 - a)x^5 - 2(16 - a)x^4 + \frac{16}{3}(4 - a)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

Rule 2061

Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I GtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2 + 16ax + 16(4 - a)x^2 - 8(16 - a)x^3 + 2(64 - a)x^4 - 80x^5 + 32x^6 - 8x^7 \\ &+ a^2x + 8ax^2 + \frac{16}{3}(4 - a)x^3 - 2(16 - a)x^4 + \frac{2}{5}(64 - a)x^5 - \frac{40x^6}{3} + \frac{32x^7}{7} - x^8 + \end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.92

$$a^2x - \frac{2}{5}(a - 64)x^5 + 2(a - 16)x^4 - \frac{16}{3}(a - 4)x^3 + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

fricas [A] time = 1.10, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 - \frac{2}{5}x^5a + \frac{128}{5}x^5 + 2x^4a - 32x^4 - \frac{16}{3}x^3a + \frac{64}{3}x^3 + 8x^2a + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x^4*a - 32*x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2

giac [A] time = 0.31, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{2}{5}ax^5 - \frac{40}{3}x^6 + 2ax^4 + \frac{128}{5}x^5 - \frac{16}{3}ax^3 - 32x^4 + a^2x + 8ax^2 + \frac{64}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3

maple [A] time = 0.00, size = 63, normalized size = 0.88

$$\frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + \frac{(-2a + 128)x^5}{5} + \frac{(8a - 128)x^4}{4} + a^2x + 8ax^2 + \frac{(-16a + 64)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16*a+64)*x^3+8*a*x^2+a^2*x

maxima [A] time = 0.71, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9 - x^8 + \frac{32}{7}x^7 - \frac{40}{3}x^6 + \frac{128}{5}x^5 - 32x^4 + a^2x + \frac{64}{3}x^3 - \frac{2}{15}(3x^5 - 15x^4 + 40x^3 - 60x^2)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a

mupad [B] time = 0.04, size = 61, normalized size = 0.85

$$x^4 (2a - 32) - x^3 \left(\frac{16a}{3} - \frac{64}{3} \right) - x^5 \left(\frac{2a}{5} - \frac{128}{5} \right) + 8ax^2 + a^2x - \frac{40x^6}{3} + \frac{32x^7}{7} - x^8 + \frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 + a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9

sympy [A] time = 0.08, size = 65, normalized size = 0.90

$$a^2x + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + x^5 \left(\frac{128}{5} - \frac{2a}{5} \right) + x^4 (2a - 32) + x^3 \left(\frac{64}{3} - \frac{16a}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)

$$3.115 \quad \int (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Optimal. Leaf size=26

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5

Rubi steps

$$\int (a + 8x - 8x^2 + 4x^3 - x^4) dx = ax + 4x^2 - \frac{8x^3}{3} + x^4 - \frac{x^5}{5}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Antiderivative was successfully verified.

[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

[Out] IntegrateAlgebraic[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]

fricas [A] time = 0.68, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + 4x^2 + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")

[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a

giac [A] time = 0.30, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")

[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^4+4*x^3-8*x^2+a+8*x,x)

[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5

maxima [A] time = 0.63, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + ax + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")

[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2

mupad [B] time = 0.02, size = 22, normalized size = 0.85

$$-\frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + 8*x - 8*x^2 + 4*x^3 - x^4, x)`

[Out] `a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5`

sympy [A] time = 0.06, size = 22, normalized size = 0.85

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x**4+4*x**3-8*x**2+a+8*x, x)`

[Out] `a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2`

$$3.116 \quad \int \frac{1}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1106, 1093, 204}

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4 + a]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 1106

Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &

& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + 8x - 8x^2 + 4x^3 - x^4} dx &= \text{Subst} \left(\int \frac{1}{3 + a - 2x^2 - x^4} dx, x, -1 + x \right) \\ &= -\frac{\text{Subst} \left(\int \frac{1}{-1 - \sqrt{4+a} - x^2} dx, x, -1 + x \right)}{2\sqrt{4+a}} + \frac{\text{Subst} \left(\int \frac{1}{-1 + \sqrt{4+a} - x^2} dx, x, -1 + x \right)}{2\sqrt{4+a}} \\ &= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1+\sqrt{4+a}}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.64

$$-\frac{1}{4} \text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, Log[x - #1]/(-2 + 4*#1 - 3*#1^2 + #1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]

fricas [B] time = 0.88, size = 457, normalized size = 5.13

$$\frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} + 1\right) - \frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} - 1\right) + \frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} + 1\right) - \frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} - 1\right) + \frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} + 1\right) - \frac{1}{4} \sqrt{\frac{a+12}{a^2+7a+12}} \log\left(\left(-\frac{a+12}{\sqrt{a^2+7a+12}} + 1\right) \sqrt{\frac{a+12}{a^2+7a+12}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-(a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-(a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-(a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-(a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1)
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34]-sqrt(1/16)*sqrt(1/256*(256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3
-11*a^2-40*a-48))*ln(abs(-a^5*sqrt(a+4)+a^5+a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*x-a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-17*a^4*sqrt(a+
4)+17*a^4-a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x+a^3*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+14*a^3*sqrt(sqrt(a+4)*(-a^2-
7*a-12)+a^2+7*a+12)*x-14*a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-111*a
^3*sqrt(a+4)+111*a^3-10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a
+4)*x+10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+69*a^2*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-69*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+
a^2+7*a+12)-351*a^2*sqrt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2
+7*a+12)*sqrt(a+4)*x+29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4
)+144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-144*a*sqrt(sqrt(a+4)*(-a
^2-7*a-12)+a^2+7*a+12)-544*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*sqrt(a+4)*x+28*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(
```

$$\begin{aligned}
& a+4)+112*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}*x-112*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-336*\sqrt{a+4}+336))+\sqrt{1/16}*\sqrt{1/256*(256*\sqrt{a+4}*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*\ln(\text{abs}(-a^5*\sqrt{a+4}+a^5-a^4*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*x+a^4*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-17*a^4*\sqrt{a+4}+17*a^4+a^3*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}*x-a^3*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}-14*a^3*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*x+14*a^3*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-111*a^3*\sqrt{a+4}+111*a^3+10*a^2*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}*x-10*a^2*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}-69*a^2*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*x+69*a^2*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-351*a^2*\sqrt{a+4}+351*a^2+29*a*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}*x-29*a*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}-144*a*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*x+144*a*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-544*a*\sqrt{a+4}+544*a+28*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}*x-28*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*\sqrt{a+4}-112*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12})*x+112*\sqrt{\sqrt{a+4}*(-a^2-7*a-12)+a^2+7*a+12}-336*\sqrt{a+4}+336))-\sqrt{1/16}*\sqrt{1/256*(-256*\sqrt{a+4}*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*\ln(\text{abs}(a^5*\sqrt{a+4}+a^5+a^4*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x-a^4*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+17*a^4*\sqrt{a+4}+17*a^4+a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x-a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+14*a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x-14*a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+111*a^3*\sqrt{a+4}+111*a^3+10*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x-10*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+69*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x-69*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+351*a^2*\sqrt{a+4}+351*a^2+29*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x-29*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+144*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x-144*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+544*a*\sqrt{a+4}+544*a+28*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x-28*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+112*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x-112*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+336*\sqrt{a+4}+336)))+\sqrt{1/16}*\sqrt{1/256*(-256*\sqrt{a+4}*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*\ln(\text{abs}(a^5*\sqrt{a+4}+a^5-a^4*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x+a^4*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+17*a^4*\sqrt{a+4}+17*a^4-a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x+a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}-14*a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x+14*a^3*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+111*a^3*\sqrt{a+4}+111*a^3-10*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x+10*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}-69*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x+69*a^2*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+351*a^2*\sqrt{a+4}+351*a^2-29*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x+29*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}-144*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*x+144*a*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}+544*a*\sqrt{a+4}+544*a-28*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12})*\sqrt{a+4}*x+2
\end{aligned}$$

$8*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12}*\sqrt{a+4}-112*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12}*x+112*\sqrt{\sqrt{a+4}*(a^2+7*a+12)+a^2+7*a+12}+336*\sqrt{a+4}+336)$

maple [C] time = 0.02, size = 49, normalized size = 0.55

$$\frac{\ln(-\text{RootOf}(-Z^4-4Z^3+8Z^2-8Z-a)+x)}{4(\text{RootOf}(-Z^4-4Z^3+8Z^2-8Z-a)^3-3\text{RootOf}(-Z^4-4Z^3+8Z^2-8Z-a)^2+4\text{RootOf}(-Z^4-4Z^3+8Z^2-8Z-a)-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-x^4+4*x^3-8*x^2+a+8*x), x)`

[Out] `-1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x), _R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="maxima")`

[Out] `-integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)`

mupad [B] time = 2.58, size = 571, normalized size = 6.42

$$-\text{atan}\left(\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{44a^2\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}}}\right)\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}} - \text{atan}\left(\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{160a\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}}}\right)\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}} - \text{atan}\left(\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{192\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}}}\right)\sqrt{\frac{a8i - x16i + x(48a + 12a^2 + a^3 + 64)^{1/2}1i - a*x*8i - (48a + 12a^2 + a^3 + 64)^{1/2}1i - a^2*x*1i + a^2*1i + 16i}{16a^2 + 176a^2 + 640a + 768}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x)`

[Out] `-atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2))`

$$\begin{aligned} &)^{(1/2)} + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64))^{(1/2)} + 4)/(640*a + 176*a \\ &^2 + 16*a^3 + 768))^{(1/2)} + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64))^{(1/2)} + \\ &4)/(640*a + 176*a^2 + 16*a^3 + 768))^{(1/2)}))*((a + (48*a + 12*a^2 + a^3 + 6 \\ &4))^{(1/2)} + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^{(1/2)}*2i \end{aligned}$$

sympy [A] time = 0.93, size = 66, normalized size = 0.74

$$-\text{RootSum}\left(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-32a - 128) - 1, (t \mapsto t \log(64t^3a^2 + 448t^3a + 768t^3 - 4ta - 20t + x - 1))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x), x)

[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))

$$3.117 \quad \int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=169

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{(3a-\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1106, 1092, 1166, 204}

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{(3a-\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]

[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] & & NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx = \text{Subst} \left(\int \frac{1}{(3 + a - 2x^2 - x^4)^2} dx, x, -1 + x \right)$$

$$= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} - \frac{\text{Subst} \left(\int \frac{4+2(3+a)-2(4+a)}{3+a-2x^2} \right)}{8(12 + 7a + a^2)}$$

$$= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} - \frac{(10 + 3a - \sqrt{4 + a}) \text{Subst} \left(\int \frac{1}{3+a-2x^2} \right)}{8(12 + 7a + a^2)}$$

$$= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{4(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)} + \frac{(10 + 3a + \sqrt{4 + a}) \text{Subst} \left(\int \frac{1}{3+a-2x^2} \right)}{8(3 + a)(4 + a)^{3/2} \sqrt{4 + a}}$$

Mathematica [C] time = 0.07, size = 150, normalized size = 0.89

$$\frac{(x-1)(a+x^2-2x+6)}{4(a+3)(a+4)(a-x(x^3-4x^2+8x-8))} - \frac{\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x-\#1) + 3a \log(x-\#1) - 2\#1 \log(x-\#1) + 12 \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]}{16(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (12*Log[x - #1] + 3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]

fricas [B] time = 0.86, size = 1948, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 5800*a + 5456)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 5800*a + 5456)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{(15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))} + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 \end{aligned}$$

$$\begin{aligned}
& 3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{((15* \\
& a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(8 \\
& 1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598 \\
& *a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2 \\
& 1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^ \\
& 2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513 \\
& *a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 5 \\
& 58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11 \\
& 1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*\sqrt{((15*a^2 - \\
& (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 \\
& + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + \\
& 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 \\
& + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7 \\
& *a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^ \\
& 2 - 8*(a^2 + 7*a + 12)*x - 12*a)*\sqrt{((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8 \\
& 47*a^3 + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^ \\
& 8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + \\
& 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196* \\
& a^2 + 3024*a + 1728))*\log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40 \\
& 8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214 \\
& 20*a^2 + 24624*a + 12096)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a \\
& ^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a \\
& + 46656)) + 5800*a + 5456)*\sqrt{((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 \\
& + 2196*a^2 + 3024*a + 1728)*\sqrt{(81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39 \\
& 9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304 \\
& *a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + \\
& 3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7 \\
& *a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^ \\
& 2 - 8*(a^2 + 7*a + 12)*x - 12*a)
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[48]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done ass

uming [a]=[34] $(x^3 - 3x^2 + xa + 8x - a - 6) / (-4a^2 - 28a - 48) / (x^4 - 4x^3 + 8x^2 - 8x - a) + (\sqrt{1/16}) * \sqrt{1/256 * (256 * \sqrt{a+4}) * (9a^3 + 103a^2 + 392a + 496) + 3840a^3 + 42240a^2 + 154624a + 188416)} / (a^3 + 11a^2 + 40a + 48) * \ln(\text{abs}(243a^{10} * \sqrt{a+4}) + 324a^{10} + 8640a^9 * \sqrt{a+4} + 11466a^9 + 81a^8 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 81a^8 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 138027a^8 * \sqrt{a+4} + 182314a^8 + 81a^7 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 81a^7 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 2340a^7 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 2340a^7 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 1304648a^7 * \sqrt{a+4} + 1715172a^7 + 2016a^6 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 2016a^6 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 29518a^6 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 29518a^6 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 8079749a^6 * \sqrt{a+4} + 10572392a^6 + 21454a^5 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 21454a^5 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 212356a^5 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 212356a^5 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 34255200a^5 * \sqrt{a+4} + 44613658a^5 + 126540a^4 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 126540a^4 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 952845a^4 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 952845a^4 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 100679657a^4 * \sqrt{a+4} + 130513730a^4 + 446685a^3 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 446685a^3 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 2730184a^3 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 2730184a^3 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 202540404a^3 * \sqrt{a+4} + 261341928a^3 + 943444a^2 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 943444a^2 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} + 4877364a^2 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * x - 4877364a^2 * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) + 266882676a^2 * \sqrt{a+4} + 342778384a^2 + 1103588a * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)} + 15a^4 + 210a^3 + 1099a^2 + 2548a + 2208) * \sqrt{a+4} * x - 1103588a * \sqrt{\sqrt{a+4} * (9a^4 + 130a^3 + 701a^2 + 1672a + 1488)}$

$$\begin{aligned}
& a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+4965684*a*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4965684*a*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+207974132*a*\sqrt{a+4}+265897256*a+551332*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x-551332*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+2205328*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2205328*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+72775824*\sqrt{a+4}+92623776))-\sqrt{1/16}*\sqrt{1/256*(256*\sqrt{a+4}} \\
& (9*a^3+103*a^2+392*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*\ln(\text{abs}(243*a^{10}*\sqrt{a+4}+324*a^{10}+8640*a^9*\sqrt{a+4}+11466*a^9-81*a^8*\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+81*a^8*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+138027*a^8*\sqrt{a+4}+182314*a^8-81*a^7*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+81*a^7*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-2340*a^7*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2340*a^7*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+1304648*a^7*\sqrt{a+4}+1715172*a^7-2016*a^6*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+2016*a^6*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-29518*a^6*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+29518*a^6*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+8079749*a^6*\sqrt{a+4}+10572392*a^6-21454*a^5*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+21454*a^5*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-212356*a^5*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+212356*a^5*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34255200*a^5*\sqrt{a+4}+44613658*a^5-126540*a^4*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+126540*a^4*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-952845*a^4*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+952845*a^4*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+100679657*a^4*\sqrt{a+4}+130513730*a^4-446685*a^3*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+446685*a^3*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-2730184*a^3*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2730184*a^3*\sqrt{\sqrt{a+4}} \\
& (9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+202540404*a^3*\sqrt{a+4}
\end{aligned}$$

$$\begin{aligned}
& (a+4)+261341928*a^3-943444*a^2*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a \\
& +1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+943444*a^2*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-4877364*a^2*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+26688267 \\
& 6*a^2*\sqrt{a+4}+342778384*a^2-1103588*a*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+1103588*a* \\
& \sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}-4965684*a*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672 \\
& *a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*\sqrt{\sqrt{a+4}}*(9 \\
& *a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+2079 \\
& 74132*a*\sqrt{a+4}+265897256*a-551332*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+ \\
& 1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}*x+551332*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a \\
& +2208)*\sqrt{a+4}-2205328*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488) \\
& +15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*\sqrt{\sqrt{a+4}}*(9*a^4+130*a \\
& ^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+72775824*\sqrt{\sqrt{a+4}}+92623776))+\sqrt{1/16}*\sqrt{1/256*(-256*\sqrt{a+4}}*(9*a^3+103*a^2+392*a+ \\
& 496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*\ln(\text{abs}(-243* \\
& a^{10}*\sqrt{a+4}+324*a^{10}-8640*a^9*\sqrt{a+4}+11466*a^9+81*a^8*\sqrt{\sqrt{a+4}}* \\
& (-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x \\
& -81*a^8*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+ \\
& 1099*a^2+2548*a+2208)-138027*a^8*\sqrt{a+4}+182314*a^8-81*a^7*\sqrt{\sqrt{a+4}} \\
& *(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)* \\
& \sqrt{a+4}*x+81*a^7*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a \\
& ^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+2340*a^7*\sqrt{\sqrt{a+4}}*(-9*a^4- \\
& 130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^ \\
& 7*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a \\
& ^2+2548*a+2208)-1304648*a^7*\sqrt{a+4}+1715172*a^7-2016*a^6*\sqrt{\sqrt{a+4}}*(\\
& -9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4} \\
& *x+2016*a^6*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a \\
& ^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+29518*a^6*\sqrt{\sqrt{a+4}}*(-9*a^4 \\
& -130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-29518* \\
& a^6*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099 \\
& *a^2+2548*a+2208)-8079749*a^6*\sqrt{a+4}+10572392*a^6-21454*a^5*\sqrt{\sqrt{a+4}} \\
& *(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208) \\
&)*\sqrt{a+4}*x+21454*a^5*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488) \\
& +15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+212356*a^5*\sqrt{\sqrt{a+4}}*(\\
& -9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x- \\
& 212356*a^5*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a \\
& ^3+1099*a^2+2548*a+2208)-34255200*a^5*\sqrt{a+4}+44613658*a^5-126540*a^4*\sqrt{\sqrt{a+4}} \\
& *(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+25 \\
& 48*a+2208)*\sqrt{a+4}*x+126540*a^4*\sqrt{\sqrt{a+4}}*(-9*a^4-130*a^3-701*a^2-16 \\
& 72*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}+952845*a^4*\sqrt{\sqrt{a+4}}*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\sqrt{a+4}
\end{aligned}$$

$$\begin{aligned} & \text{qrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548* \\ & a+2208)*x-952845*a^4*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15 \\ & *a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*\text{sqrt}(a+4)+130513730*a^4-44 \\ & 6685*a^3*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3 \\ & +1099*a^2+2548*a+2208)*\text{sqrt}(a+4)*x+446685*a^3*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^ \\ & 3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)+27301 \\ & 84*a^3*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1 \\ & 099*a^2+2548*a+2208)*x-2730184*a^3*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1 \\ & 672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*\text{sqrt}(a+4)+26 \\ & 1341928*a^3-943444*a^2*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+ \\ & 15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)*x+943444*a^2*\text{sqrt}(\text{sqrt}(a+4)* \\ & (-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*s \\ & \text{qrt}(a+4)+4877364*a^2*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15 \\ & *a^4+210*a^3+1099*a^2+2548*a+2208)*x-4877364*a^2*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130 \\ & *a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-266882676*a^ \\ & 2*\text{sqrt}(a+4)+342778384*a^2-1103588*a*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2- \\ & 1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)*x+1103588*a*sqr \\ & \text{t}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+25 \\ & 48*a+2208)*\text{sqrt}(a+4)+4965684*a*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672* \\ & a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4965684*a*\text{sqrt}(\text{sqrt}(a+4)*(-9 \\ & *a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-2079 \\ & 74132*a*\text{sqrt}(a+4)+265897256*a-551332*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2 \\ & -1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)*x+551332*\text{sqrt} \\ & (\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548 \\ & *a+2208)*\text{sqrt}(a+4)+2205328*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14 \\ & 88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2205328*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-1 \\ & 30*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-72775824*s \\ & \text{qrt}(a+4)+92623776)-\text{sqrt}(1/16)*\text{sqrt}(1/256*(-256*\text{sqrt}(a+4)*(9*a^3+103*a^2+39 \\ & 2*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*\ln(\text{abs}(- \\ & 243*a^10*\text{sqrt}(a+4)+324*a^10-8640*a^9*\text{sqrt}(a+4)+11466*a^9-81*a^8*\text{sqrt}(\text{sqrt}(a \\ & +4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220 \\ & 8)*x+81*a^8*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210* \\ & a^3+1099*a^2+2548*a+2208)-138027*a^8*\text{sqrt}(a+4)+182314*a^8+81*a^7*\text{sqrt}(\text{sqrt} \\ & (a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+22 \\ & 08)*\text{sqrt}(a+4)*x-81*a^7*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+ \\ & 15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)-2340*a^7*\text{sqrt}(\text{sqrt}(a+4)*(-9* \\ & a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+234 \\ & 0*a^7*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+10 \\ & 99*a^2+2548*a+2208)-1304648*a^7*\text{sqrt}(a+4)+1715172*a^7+2016*a^6*\text{sqrt}(\text{sqrt}(a \\ & +4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208 \\ &)*\text{sqrt}(a+4)*x-2016*a^6*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+ \\ & 15*a^4+210*a^3+1099*a^2+2548*a+2208)*\text{sqrt}(a+4)-29518*a^6*\text{sqrt}(\text{sqrt}(a+4)*(-9 \\ & *a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+29 \\ & 518*a^6*\text{sqrt}(\text{sqrt}(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+ \\ & 1099*a^2+2548*a+2208)-8079749*a^6*\text{sqrt}(a+4)+10572392*a^6+21454*a^5*\text{sqrt}(\text{sqrt}(a+4) \end{aligned}$$

```

t(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+
2208)*sqrt(a+4)*x-21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1
488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^5*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*x+212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+2
10*a^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)*x-126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952845*a^4*sq
rt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2
548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
))+15*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^
4+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-13
0*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2
730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a
^3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a
^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4
)+261341928*a^3+943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
))+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4
-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-26688267
6*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*
a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-1103588*a
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)
)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-
207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701
*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-551332*s
qrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(-9*a
^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-727758
24*sqrt(a+4)+92623776)))/(4*a^2+28*a+48)

```

maple [C] time = 0.01, size = 158, normalized size = 0.93

$$\frac{\left(-\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)^2+2\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)-3a-12\right)\ln\left(-\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)+x\right)}{16(a+3)(a+4)\left(\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)^3-3\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)^2+4\text{RootOf}\left(\mathcal{Z}^4-4\mathcal{Z}^3+8\mathcal{Z}^2-8\mathcal{Z}-a\right)-2\right)}+\frac{-\frac{x^3}{4(a^2+7a+12)}+\frac{3x^2}{4(a^2+7a+12)}-\frac{(a+8)x}{4(a^2+7a+12)}+\frac{a+6}{4a^2+28a+48}}{x^4-4x^3+8x^2-a-8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] $(-1/4/(a^2+7a+12)*x^3+3/4/(a^2+7a+12)*x^2-1/4*(8+a)/(a^2+7a+12)*x+1/4*(6+a)/(a^2+7a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*\text{sum}((-R^2+2*_R-3*a-12)/(_R^3-3*_R^2+4*_R-2)*\ln(-R+x),_R=\text{RootOf}(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x^3 + (a+8)x - 3x^2 - a - 6}{4((a^2+7a+12)x^4 - 4(a^2+7a+12)x^3 - a^3 + 8(a^2+7a+12)x^2 - 7a^2 - 8(a^2+7a+12)x - 12a)} - \frac{\int \frac{x^2+3a-2x+12}{x^4-4x^3+8x^2-a-8x} dx}{4(a^2+7a+12)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")`

[Out] $-1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a) - 1/4*\text{integrate}((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)$

mupad [B] time = 5.35, size = 4591, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)`

[Out] $\text{atan}\left(\frac{-(((15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} * (((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))) * ((15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) * ((15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))} * i + \frac{((15552*a - 9*a*((a + 4)^9)^{(1/2)} - 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} * (((15728640*a + 10878976*a^2 + 3997$

$$\begin{aligned}
& 696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184) / (64(816a + 460a^2 \\
& + 129a^3 + 18a^4 + a^5 + 576)) - (x(208896a + 117760a^2 + 33024a^3 + \\
& 4608a^4 + 256a^5 + 147456)) / (4(168a + 73a^2 + 14a^3 + a^4 + 144))) * (\\
& (15552a - 9a((a + 4)^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a \\
& ^3 + 285a^4 + 15a^5 + 11776) / (256(276480a + 306432a^2 + 197632a^3 + 8 \\
& 1744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} \\
& + (733184a + 396288a^2 + 106752a^3 + 14336a^4 + 768a^5 + 540672) / (64(\\
& 816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) * ((15552a - 9a((a + 4) \\
& ^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + \\
& 11776) / (256(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4 \\
& 115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 1 \\
& 44a^3 + 6656) / (64(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x(\\
& 61a + 9a^2 + 104)) / (4(168a + 73a^2 + 14a^3 + a^4 + 144))) * i) / ((9a + \\
& 32) / (32(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) + ((15552a - 9a \\
& * ((a + 4)^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 \\
& + 15a^5 + 11776) / (256(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22 \\
& 488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} * (((15728640 * \\
& a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 943718 \\
& 4) / (64(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) - (x(208896a + 1 \\
& 17760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 147456)) / (4(168a + 73a^2 + \\
& 14a^3 + a^4 + 144))) * ((15552a - 9a((a + 4)^9)^{(1/2)} - 31((a + 4)^9)^{(1 \\
& /2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776) / (256(276480a + 3064 \\
& 32a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + \\
& a^9 + 110592)))^{(1/2)} - (733184a + 396288a^2 + 106752a^3 + 14336a^4 + \\
& 768a^5 + 540672) / (64(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) * ((\\
& 15552a - 9a((a + 4)^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^ \\
& 3 + 285a^4 + 15a^5 + 11776) / (256(276480a + 306432a^2 + 197632a^3 + 81 \\
& 744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + \\
& (5568a + 1552a^2 + 144a^3 + 6656) / (64(816a + 460a^2 + 129a^3 + 18a \\
& ^4 + a^5 + 576)) - (x(61a + 9a^2 + 104)) / (4(168a + 73a^2 + 14a^3 + a \\
& ^4 + 144))) - ((15552a - 9a((a + 4)^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 82 \\
& 08a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776) / (256(276480a + 306432a^2 + \\
& 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 1 \\
& 10592)))^{(1/2)} * (((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 9 \\
& 0112a^5 + 4096a^6 + 9437184) / (64(816a + 460a^2 + 129a^3 + 18a^4 + a^ \\
& 5 + 576))) - (x(208896a + 117760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 14 \\
& 7456)) / (4(168a + 73a^2 + 14a^3 + a^4 + 144))) * ((15552a - 9a((a + 4) \\
& ^9)^{(1/2)} - 31((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + \\
& 11776) / (256(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4 \\
& 115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (733184a + 396288a^2 \\
& + 106752a^3 + 14336a^4 + 768a^5 + 540672) / (64(816a + 460a^2 + 129a^ \\
& 3 + 18a^4 + a^5 + 576))) * ((15552a - 9a((a + 4)^9)^{(1/2)} - 31((a + 4)^9 \\
&)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776) / (256(276480a + \\
& 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a \\
& ^8 + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 144a^3 + 6656) / (64(816 *
\end{aligned}$$

$$\begin{aligned}
& a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576) - (x(61a + 9a^2 + 104))/(4* \\
& (168a + 73a^2 + 14a^3 + a^4 + 144))) * ((15552a - 9a*((a + 4)^9)^{(1/2)} \\
& - 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(\\
& 256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 \\
& + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} * i + \operatorname{atan}(-(((15552a + 9a*((a \\
& + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a \\
& ^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^ \\
& 5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} * (((15728640a + 10 \\
& 878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64 \\
& *(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(208896a + 117760* \\
& a^2 + 33024a^3 + 4608a^4 + 256a^5 + 147456))/(4*(168a + 73a^2 + 14a^3 \\
& + a^4 + 144))) * ((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + \\
& 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 \\
& + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + \\
& 110592)))^{(1/2)} - (733184a + 396288a^2 + 106752a^3 + 14336a^4 + 768a^ \\
& 5 + 540672)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) * ((15552* \\
& a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 28 \\
& 5a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^ \\
& 4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (5568 \\
& *a + 1552a^2 + 144a^3 + 6656)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a \\
& ^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(168a + 73a^2 + 14a^3 + a^4 + 1 \\
& 44))) * i + ((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208* \\
& a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306432a^2 + 19 \\
& 7632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 + a^9 + 1105 \\
& 92)))^{(1/2)} * (((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 9011 \\
& 2a^5 + 4096a^6 + 9437184)/(64*(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + \\
& 576)) - (x*(208896a + 117760a^2 + 33024a^3 + 4608a^4 + 256a^5 + 14745 \\
& 6))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144))) * ((15552a + 9a*((a + 4)^9)^{ \\
& (1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 117 \\
& 76)/(256*(276480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115 \\
& *a^6 + 483a^7 + 33a^8 + a^9 + 110592)))^{(1/2)} + (733184a + 396288a^2 + \\
& 106752a^3 + 14336a^4 + 768a^5 + 540672)/(64*(816a + 460a^2 + 129a^3 + \\
& 18a^4 + a^5 + 576))) * ((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(\\
& 1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276480a + 306 \\
& 432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^7 + 33a^8 \\
& + a^9 + 110592)))^{(1/2)} + (5568a + 1552a^2 + 144a^3 + 6656)/(64*(816a + \\
& 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x*(61a + 9a^2 + 104))/(4*(16 \\
& 8a + 73a^2 + 14a^3 + a^4 + 144))) * i + ((9a + 32)/(32*(816a + 460a^2 + \\
& 129a^3 + 18a^4 + a^5 + 576)) + ((15552a + 9a*((a + 4)^9)^{(1/2)} + 31*((\\
& a + 4)^9)^{(1/2)} + 8208a^2 + 2164a^3 + 285a^4 + 15a^5 + 11776)/(256*(276 \\
& 480a + 306432a^2 + 197632a^3 + 81744a^4 + 22488a^5 + 4115a^6 + 483a^ \\
& 7 + 33a^8 + a^9 + 110592)))^{(1/2)} * (((15728640a + 10878976a^2 + 3997696* \\
& a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64*(816a + 460a^2 + 1 \\
& 29a^3 + 18a^4 + a^5 + 576)) - (x*(208896a + 117760a^2 + 33024a^3 + 460 \\
& 8a^4 + 256a^5 + 147456))/(4*(168a + 73a^2 + 14a^3 + a^4 + 144))) * ((155
\end{aligned}$$

$$\begin{aligned}
& 52*a + 9*a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + \\
& 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744 \\
& *a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} - (7 \\
& 33184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816* \\
& a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^{(1/2)} + \\
& 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 1177 \\
& 6)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115* \\
& a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} + (5568*a + 1552*a^2 + 144*a \\
& ^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a \\
& + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)) - ((15552*a + 9 \\
& *a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a^3 + 285*a^4 \\
& + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 2 \\
& 2488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)}*(((15728640 \\
& *a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 94371 \\
& 84)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + \\
& 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + \\
& 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + \\
& 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306 \\
& 432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 \\
& + a^9 + 110592)))^{(1/2)} + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + \\
& 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))* \\
& ((15552*a + 9*a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8208*a^2 + 2164*a \\
& ^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8 \\
& 1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^{(1/2)} \\
& + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18* \\
& a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + \\
& a^4 + 144)))))*((15552*a + 9*a*((a + 4)^9)^{(1/2)} + 31*((a + 4)^9)^{(1/2)} + 8 \\
& 208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 \\
& + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + \\
& 110592)))^{(1/2)}*2i + (x^3/(4*(7*a + a^2 + 12))) - (a + 6)/(4*(a + 3)*(a + 4) \\
&) - (3*x^2)/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(4*(a + 3)*(a + 4)))/(a + 8*x \\
& - 8*x^2 + 4*x^3 - x^4)
\end{aligned}$$

sympy [B] time = 6.32, size = 294, normalized size = 1.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7

$$\begin{aligned} & 680*a^{**5} - 145920*a^{**4} - 1107968*a^{**3} - 4202496*a^{**2} - 7962624*a - 6029312) \\ & - 81*a^{**2} - 576*a - 1024, \text{Lambda}(_t, _t*\log(x + (-16384*_t^{**3}*a^{**7} - 40140 \\ & 8*_t^{**3}*a^{**6} - 4202496*_t^{**3}*a^{**5} - 24371200*_t^{**3}*a^{**4} - 84549632*_t^{**3}*a^{**3} \\ & - 175472640*_t^{**3}*a^{**2} - 201719808*_t^{**3}*a - 99090432*_t^{**3} + 432*_t*a^{**4} \\ & + 7488*_t*a^{**3} + 47024*_t*a^{**2} + 128096*_t*a + 128512*_t - 81*a^{**2} - 567*a - 992)/(81*a^{**2} + 567*a + 992)))) \end{aligned}$$

$$3.118 \quad \int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal. Leaf size=252

$$\frac{(x-1)(a+(x-1)^2+5) \left(3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80) \tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right) \right)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2 \sqrt{1-\sqrt{a+4}}} - \frac{64(a+3)^2(a+4)^{5/2} \sqrt{1-\sqrt{a+4}}}{64(a+3)^2(a+4)^{5/2} \sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.53, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1106, 1092, 1178, 1166, 204}

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} - \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80) \tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2} \sqrt{1-\sqrt{a+4}}} - \frac{3\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right) \tan^{-1}\left(\frac{x-1}{\sqrt{a+4}+1}\right)}{64(a+3)^2(a+4)^2 \sqrt{a+4}+1} + \frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{32(a+3)^2(a+4)^2(a-(x-1)^4-2(x-1)^2+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2*(-1 + x))/(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14*sqrt[4 + a] + a*(47 + 4*sqrt[4 + a]))*ArcTan[(-1 + x)/sqrt[1 - sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*sqrt[1 - sqrt[4 + a]]) - (3*(14 + 4*a - (80 + 47*a + 7*a^2)/sqrt[4 + a])*ArcTan[(-1 + x)/sqrt[1 + sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^2*sqrt[1 + sqrt[4 + a]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :- Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :- Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1106


```
Int[(P4_)^(p_), x_Symbol] := With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1],
c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx &= \text{Subst} \left(\int \frac{1}{(3 + a - 2x^2 - x^4)^3} dx, x, -1 + x \right) \\
&= \frac{(5 + a + (-1 + x)^2)(-1 + x)}{8(12 + 7a + a^2)(3 + a - 2(-1 + x)^2 - (-1 + x)^4)^2} - \frac{\text{Subst} \left(\int \frac{4+2(3+a)-4(4+4)}{(3+a-2x^2-} \right.}{16(12 +} \\
&= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1}}{8(12 + 7a + a^2)(3 + a} \\
&= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1}}{8(12 + 7a + a^2)(3 + a} \\
&= -\frac{((6 + a)(25 + 7a) + 6(7 + 2a)(1 - x)^2)(1 - x)}{32(12 + 7a + a^2)^2(3 + a - 2(1 - x)^2 - (1 - x)^4)} + \frac{(5 + a + (-1}}{8(12 + 7a + a^2)(3 + a}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 254, normalized size = 1.01

$$\frac{1}{128} \left(\frac{3\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{4\#1^2 a \log(x-\#1) + 14\#1^2 \log(x-\#1) + 7a^2 \log(x-\#1) + 55a \log(x-\#1) - 8\#1 a \log(x-\#1) + 108 \log(x-\#1) - 28\#1 \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \right] + \frac{4(x-1)(7a^2 + a(12x^2 - 24x + 79) + 6(7x^2 - 14x + 32))}{(a+3)^2(a+4)^2(a-x(x^3 - 4x^2 + 8x - 8))} + \frac{16(x-1)(a+x^2 - 2x + 6)}{(a+3)(a+4)(a-x(x^3 - 4x^2 + 8x - 8))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x + 12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (108*Log[x - #1] + 55*a*Log[x - #1] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &])/(12 + 7*a + a^2)^2)/128

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]

fricas [B] time = 1.67, size = 3971, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5 \\ & - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 - \\ & 8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 \\ & - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 \\ & + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + \\ & 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 \\ & + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16 \\ & *(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + \\ & 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*\sqrt{((105*a^4 + 1470*a^3 + 7749*a^2 + \\ & (a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 \\ & + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*\sqrt{((2401*a^4 + 33124*a^3 + \\ & 171966*a^2 + 398164*a + 346921))/(a^{15} + 50*a^{14} + 1165*a^{13} + 16780*a^{12} + \\ & 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2 \\ & 41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 \\ & + 277136640*a + 60466176)) + 18228*a + 16144)/(a^{10} + 35*a^9 + 550*a^8 + \\ & 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + \\ & 725760*a + 248832))*\log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 \\ & + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 \\ & + 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^{12} + 462*a^{11} \\ & + 8881*a^{10} + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410 \\ & 692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338 \\ & 41152))*\sqrt{((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921))/(a^{15} + \\ & 50*a^{14} + 1165*a^{13} + 16780*a^{12} + 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 \\ & + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940 \\ & *a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a \\ & + 3543424)*\sqrt{((105*a^4 + 1470*a^3 + 7749*a^2 + (a^{10} + 35*a^9 + 550*a^8 \\ & + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 \\ & + 725760*a + 248832))*\sqrt{((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3 \\ & 46921))/(a^{15} + 50*a^{14} + 1165*a^{13} + 16780*a^{12} + 167090*a^{11} + 1218460*a^{10} \\ & + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 \\ & + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176 \\ &)) + 18228*a + 16144)/(a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129 \\ & 367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122 \end{aligned}$$

$$\begin{aligned}
& 8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
\end{aligned}$$

$$\begin{aligned} & -64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a \\ & \quad \wedge 2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^ \\ & \quad 4 + 2354874*a^3 + 5293208*a^2 + (11*a^{12} + 462*a^{11} + 8881*a^{10} + 103320*a^ \\ & \quad 9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 \\ & \quad + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*\text{sqrt}((2401*a^4 + \\ & \quad 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^{15} + 50*a^{14} + 1165*a^{13} + 1 \\ & \quad 6780*a^{12} + 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 + 28570320*a^8 + 94320 \\ & \quad 045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5 \\ & \quad 92064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*\text{sqrt}((105*a^ \\ & \quad 4 + 1470*a^3 + 7749*a^2 - (a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + \\ & \quad 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*\text{sqr} \\ & \quad \text{t}((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^{15} + 50*a^{14} + \\ & \quad 1165*a^{13} + 16780*a^{12} + 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 + 285703 \\ & \quad 20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782 \\ & \quad 071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a \\ & \quad ^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 7 \\ & \quad 35840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((\\ & \quad a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a \\ & \quad + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 \\ & \quad + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - \\ & \quad 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000* \\ & \quad a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - \\ & \quad 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x \\ & \quad)*\text{sqrt}((105*a^4 + 1470*a^3 + 7749*a^2 - (a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 \\ & \quad + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a \\ & \quad + 248832)*\text{sqrt}((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^ \\ & \quad 15 + 50*a^{14} + 1165*a^{13} + 16780*a^{12} + 167090*a^{11} + 1218460*a^{10} + 672213 \\ & \quad 0*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 71431 \\ & \quad 7940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228 \\ & \quad *a + 16144)/(a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + \\ & \quad 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*\log(-64827*a^4 - \\ & \quad 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884* \\ & \quad a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874* \\ & \quad a^3 + 5293208*a^2 + (11*a^{12} + 462*a^{11} + 8881*a^{10} + 103320*a^9 + 810205*a \\ & \quad ^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400* \\ & \quad a^3 + 187875072*a^2 + 117863424*a + 33841152)*\text{sqrt}((2401*a^4 + 33124*a^3 + \\ & \quad 171966*a^2 + 398164*a + 346921)/(a^{15} + 50*a^{14} + 1165*a^{13} + 16780*a^{12} + \\ & \quad 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 24 \\ & \quad 1870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 \\ & \quad + 277136640*a + 60466176)) + 6613472*a + 3543424)*\text{sqrt}((105*a^4 + 1470*a^3 \\ & \quad + 7749*a^2 - (a^{10} + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 \\ & \quad + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*\text{sqrt}((2401*a^4 \\ & \quad + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^{15} + 50*a^{14} + 1165*a^{13} + \\ & \quad 16780*a^{12} + 167090*a^{11} + 1218460*a^{10} + 6722130*a^9 + 28570320*a^8 + 943 \\ & \quad 20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + \end{aligned}$$

$$\begin{aligned} & (592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^{10} + 35*a^9 \\ & + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + \\ & 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 524*a^2 - 4*(11* \\ & a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a + 1152)/((a^4 + 14*a^3 + 73*a^2 + 1 \\ & 68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 1 \\ & 4*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a \\ & + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921 \\ & 6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 \\ & + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 \\ & + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x) \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[86]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[12]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[48]Warning, need to choose a branch for the root of a polynomial with parameters. This might be wrong.The choice was done assuming [a]=[34]-(-12*x^7*a-42*x^7+84*x^6*a+294*x^6-7*x^5*a^2-343*x^5*a-1116*x^5+35*x^4*a^2+875*x^4*a+2640*x^4-68*x^3*a^2-1358*x^3*a-3936*x^3+64*x^2*a^2+1246*x^2*a+3600*x^2+11*x*a^3+107*x*a^2-84*x*a-1152*x-11*a^3-131*a^2-408*a-288)/(-32*a^4-448*a^3-2336*a^2-5376*a-4608)/(x^4-4*x^3+8*x^2-8*x-a)^2-(sqrt(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+26411*a^15-908950*a^14*sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-357887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728

$$\begin{aligned}
&) * x - 105154 * a^{11} * \sqrt{\sqrt{a+4} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088)} + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728 - 3865394166 * a^{11} * \sqrt{\sqrt{a+4}} + 5945365998 * a^{11} - 95550 * a^{10} * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} * x + 95550 * a^{10} * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} + 2109279 * a^{10} * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * x - 2109279 * a^{10} * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) - 30600511272 * a^{10} * \sqrt{\sqrt{a+4}} + 46810709868 * a^{10} - 1727079 * a^9 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} * x + 1727079 * a^9 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} + 25624212 * a^9 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * x - 25624212 * a^9 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) - 183431725500 * a^9 * \sqrt{\sqrt{a+4}} + 279067335420 * a^9 - 18715896 * a^8 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} * x + 18715896 * a^8 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} + 209972223 * a^8 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * x - 209972223 * a^8 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) - 847810669320 * a^8 * \sqrt{\sqrt{a+4}} + 1282741275000 * a^8 - 135108639 * a^7 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} * x + 135108639 * a^7 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} + 1222644882 * a^7 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * x - 1222644882 * a^7 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) - 3046208716923 * a^7 * \sqrt{\sqrt{a+4}} + 4583471076759 * a^7 - 682210326 * a^6 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} * x + 682210326 * a^6 * \sqrt{\sqrt{a+4}} * (-49 * a^6 - 1073 * a^5 - 9775 * a^4 - 47419 * a^3 - 129188 * a^2 - 187408 * a - 113088) + 105 * a^6 + 2205 * a^5 + 19299 * a^4 + 90111 * a^3 + 236728 * a^2 + 331744 * a + 193728) * \sqrt{\sqrt{a+4}} + 5187446733 * a^6 * \sqrt{\sqrt{a+4}}
\end{aligned}$$

$$\begin{aligned}
& 4) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 \\
& + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * x - 5187446733*a^6 * \\
& \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113 \\
& 088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) - 85085 \\
& 27261290*a^6 * \text{sqrt}(a+4) + 12731345334296*a^6 - 2458605429*a^5 * \text{sqrt}(\text{sqrt}(a+4)) * (-4 \\
& 9*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205* \\
& a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * \text{sqrt}(a+4) * x + 2458605429* \\
& a^5 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a \\
& - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * s \\
& \text{qrt}(a+4) + 16158435972*a^5 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^ \\
& 3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a \\
& ^2 + 331744*a + 193728) * x - 16158435972*a^5 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775 \\
& *a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111 \\
& *a^3 + 236728*a^2 + 331744*a + 193728) - 18324012543204*a^5 * \text{sqrt}(a+4) + 2726574704738 \\
& 0*a^5 - 6324014256*a^4 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 12 \\
& 9188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 3 \\
& 31744*a + 193728) * \text{sqrt}(a+4) * x + 6324014256*a^4 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 \\
& - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + \\
& 90111*a^3 + 236728*a^2 + 331744*a + 193728) * \text{sqrt}(a+4) + 36673732452*a^4 * \text{sqrt}(\text{sqrt}(a \\
& + 4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^ \\
& 6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * x - 36673732452*a^ \\
& 4 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 1 \\
& 13088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) - 298 \\
& 80339194272*a^4 * \text{sqrt}(a+4) + 44213263379848*a^4 - 11377675428*a^3 * \text{sqrt}(\text{sqrt}(a+4)) \\
& * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2 \\
& 205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * \text{sqrt}(a+4) * x + 1137767 \\
& 5428*a^3 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187 \\
& 408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 1937 \\
& 28) * \text{sqrt}(a+4) + 59146408708*a^3 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 474 \\
& 19*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236 \\
& 728*a^2 + 331744*a + 193728) * x - 59146408708*a^3 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 \\
& - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + \\
& 90111*a^3 + 236728*a^2 + 331744*a + 193728) - 35712575419864*a^3 * \text{sqrt}(a+4) + 52547642 \\
& 661032*a^3 - 13635706996*a^2 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419* \\
& a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728 \\
& *a^2 + 331744*a + 193728) * \text{sqrt}(a+4) * x + 13635706996*a^2 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1 \\
& 073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^6 + 2205*a^5 + 192 \\
& 99*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * \text{sqrt}(a+4) + 64340036640*a^2 * \text{sqrt} \\
& (\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) \\
& + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * x - 6434003 \\
& 6640*a^2 * \text{sqrt}(\text{sqrt}(a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187 \\
& 408*a - 113088) + 105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 1937 \\
& 28) - 29533943648028*a^2 * \text{sqrt}(a+4) + 43213040637212*a^2 - 9797208656*a * \text{sqrt}(\text{sqrt}(\\
& a+4)) * (-49*a^6 - 1073*a^5 - 9775*a^4 - 47419*a^3 - 129188*a^2 - 187408*a - 113088) + 105*a^ \\
& ^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + 331744*a + 193728) * \text{sqrt}(a+4) * x + 979
\end{aligned}$$

$$\begin{aligned}
& 7208656*a*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+42385864836*a*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-42385864836*a*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-15111479733208*a*\sqrt{a+4}+21986673204304*a-3197030212*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+3197030212*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+12788120848*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*\sqrt{\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3606250079136*\sqrt{a+4}+5217553305984))-\sqrt{9/16}*\sqrt{1/256*(256*\sqrt{a+4}*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*\ln(\text{abs}(-16807*a^15*\sqrt{a+4}+26411*a^15-908950*a^14*\sqrt{a+4}+1420804*a^14-22929088*a^13*\sqrt{a+4}+35650176*a^13-2401*a^12*\sqrt{a+4}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2401*a^12*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-357887692*a^12*\sqrt{a+4}+553458148*a^12+2401*a^11*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}}*x-2401*a^11*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}}-105154*a^11*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+105154*a^11*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3865394166*a^11*\sqrt{a+4}+5945365998*a^11+95550*a^10*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}}*x-95550*a^10*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}}-2109279*a^10*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2109279*a^10*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-30600511272*a^10*\sqrt{a+4}+46810709868*a^10+1727079*a^9*\sqrt{a+4}}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3
\end{aligned}$$

$$\begin{aligned}
& +236728a^2+331744a+193728) \sqrt{a+4} x - 1727079a^9 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + \\
& 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 25624212a^9 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) \\
& + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) x + 2562421 \\
& 2a^9 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \\
& - 183431725500a^9 \sqrt{a+4} + 279067335420a^9 + 18715896a^8 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205 \\
& a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} x - 18715896a^8 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - \\
& 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 209972223a^8 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 1 \\
& 29188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + \\
& 331744a + 193728) x + 209972223a^8 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + \\
& 236728a^2 + 331744a + 193728) - 847810669320a^8 \sqrt{a+4} + 1282741275000a^8 + 13 \\
& 5108639a^7 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - \\
& 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 1 \\
& 93728) \sqrt{a+4} x - 135108639a^7 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + \\
& 236728a^2 + 331744a + 193728) \sqrt{a+4} - 1222644882a^7 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + \\
& 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) x + 1222644882a^7 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) - 3046208716923a^7 \sqrt{a+4} + 4583471076759a^7 + 682210326a^6 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} x - 682210326a^6 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 5187446733a^6 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) x + 5187446733a^6 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) - 8508527261290a^6 \sqrt{a+4} + 12731345334296a^6 + 2458605429a^5 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} x - 2458605429a^5 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 16158435972a^5 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) x + 16158435972a^5 \sqrt{\sqrt{a+4}} (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) - 18324012543204a^5 *
\end{aligned}$$

$$\begin{aligned}
& \sqrt{a+4} + 27265747047380a^5 + 6324014256a^4 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 \\
& + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} * x - 6324014256a^4 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 \\
& + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} - 36673732452a^4 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 1 \\
& 93728 * x + 36673732452a^4 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \\
& - 29880339194272a^4 \sqrt{\sqrt{a+4}} + 44213263379848a^4 + 11377675428a^3 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 1937 \\
& 28 \sqrt{\sqrt{a+4}} * x - 11377675428a^3 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 2 \\
& 36728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} - 59146408708a^3 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \\
& * x + 59146408708a^3 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 - 35712575419864 \\
& a^3 \sqrt{\sqrt{a+4}} + 52547642661032a^3 + 13635706996a^2 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \\
& \sqrt{\sqrt{a+4}} * x - 13635706996a^2 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} \\
& - 64340036640a^2 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 * x + 64340036640a^2 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 2 \\
& 36728a^2 + 331744a + 193728 - 29533943648028a^2 \sqrt{\sqrt{a+4}} + 43213040637212a^2 + 9797208656a \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \\
& \sqrt{\sqrt{a+4}} * x - 9797208656a \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} - 42385864836a \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 1 \\
& 9299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 * x + 42385864836a \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 - 15111479733208a * \\
& \sqrt{\sqrt{a+4}} + 21986673204304a + 3197030212 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} * x - 3197030212 \sqrt{\sqrt{a+4} * (-49 \\
& a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 \sqrt{\sqrt{a+4}} - 12788120848 \sqrt{\sqrt{a+4} * (-49a^6 - 1073a^5 - 9775a^4 - 47419a^3 - 129188a^2 - 187408a - 113088)} + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728 * x + 12788
\end{aligned}$$

$$\begin{aligned}
& 120848*\sqrt{\sqrt{a+4}*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)} \\
& -3606250079136*\sqrt{a+4}+5217553305984))+\sqrt{9/16}*\sqrt{1/256*(-256*\sqrt{a+4} \\
& *(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3 \\
& -12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*\ln(\text{abs}(16807*a^{15}*\sqrt{a+4} \\
& +26411*a^{15}+908950*a^{14}*\sqrt{a+4}+1420804*a^{14}+22929088*a^{13}*\sqrt{a+4}+35650176*a^{13} \\
& +2401*a^{12}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)} \\
& +105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728))*x-2401*a^{12}*\sqrt{\sqrt{a+4} \\
& *(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4 \\
& +90111*a^3+236728*a^2+331744*a+193728)+357887692*a^{12}*\sqrt{a+4}+553458148*a^{12}+2401*a^{11} \\
& *\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5 \\
& +19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x-2401*a^{11}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5 \\
& +9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+2 \\
& 36728*a^2+331744*a+193728)*\sqrt{a+4}+105154*a^{11}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3 \\
& +129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) \\
& *x-105154*a^{11}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)} \\
& +105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3865394166*a^{11}*\sqrt{a+4} \\
&)+5945365998*a^{11}+95550*a^{10}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a \\
& +113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x-95550*a^{10} \\
& *\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5 \\
& +19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+2109279*a^{10}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5 \\
& +9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2 \\
& +331744*a+193728)*x-2109279*a^{10}*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a \\
& +113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+30600511272*a^{10} \\
& *\sqrt{a+4}+46810709868*a^{10}+1727079*a^9*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2 \\
& +187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4} \\
& *x-1727079*a^9*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6 \\
& +2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+25624212*a^9*\sqrt{\sqrt{a+4} \\
& *(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4 \\
& +90111*a^3+236728*a^2+331744*a+193728)*x-25624212*a^9*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3 \\
& +129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) \\
& +183431725500*a^9*\sqrt{a+4}+279067335420*a^9+18715896*a^8*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3 \\
& +129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728) \\
& *x-18715896*a^8*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)} \\
& +105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+209972223*a^8*\sqrt{\sqrt{a+4} \\
& *(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3 \\
& +236728*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+187408*a+113088) \\
& +105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+187408*a+113088)
\end{aligned}$$

$$\begin{aligned} &8a^2+331744a+193728)x-209972223a^8\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775 \\ &a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111 \\ &a^3+236728a^2+331744a+193728)+847810669320a^8\sqrt{\sqrt{a+4}}+1282741275000a \\ &^8+135108639a^7\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188 \\ &a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744 \\ &a+193728)}\sqrt{\sqrt{a+4}}x-135108639a^7\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a \\ &^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a \\ &^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}}+1222644882a^7\sqrt{\sqrt{a+4}(49 \\ &a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^ \\ &5+19299a^4+90111a^3+236728a^2+331744a+193728)}x-1222644882a^7\sqrt{\sqrt{a+4}} \\ &\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105 \\ &a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}+3046208716923 \\ &a^7\sqrt{\sqrt{a+4}}+4583471076759a^7+682210326a^6\sqrt{\sqrt{a+4}(49a^6+1073a \\ &^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^ \\ &4+90111a^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}}x-682210326a^6\sqrt{\sqrt{a+4}} \\ &\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105 \\ &a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}}+51874 \\ &46733a^6\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187 \\ &408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+1937 \\ &28)}x-5187446733a^6\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129 \\ &188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+33 \\ &1744a+193728)}+8508527261290a^6\sqrt{\sqrt{a+4}}+12731345334296a^6+2458605429a^ \\ &5\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+11 \\ &3088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}} \\ &x-2458605429a^5\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+1 \\ &29188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+ \\ &331744a+193728)}\sqrt{\sqrt{a+4}}+16158435972a^5\sqrt{\sqrt{a+4}(49a^6+1073a^5+ \\ &9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+9 \\ &0111a^3+236728a^2+331744a+193728)}x-16158435972a^5\sqrt{\sqrt{a+4}(49a \\ &^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5 \\ &+19299a^4+90111a^3+236728a^2+331744a+193728)}+18324012543204a^5\sqrt{\sqrt{a+ \\ &4}}+27265747047380a^5+6324014256a^4\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a \\ &^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a \\ &^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}}x-6324014256a^4\sqrt{\sqrt{a+4}(4 \\ &9a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088)+105a^6+2205 \\ &a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}}+36673732452 \\ &a^4\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+1 \\ &13088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}x-3 \\ &6673732452a^4\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^ \\ &2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744 \\ &a+193728)}+29880339194272a^4\sqrt{\sqrt{a+4}}+44213263379848a^4+11377675428a^3 \\ &\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+129188a^2+187408a+113088 \\ &)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331744a+193728)}\sqrt{\sqrt{a+4}} \\ &)}x-11377675428a^3\sqrt{\sqrt{a+4}(49a^6+1073a^5+9775a^4+47419a^3+1291 \\ &88a^2+187408a+113088)+105a^6+2205a^5+19299a^4+90111a^3+236728a^2+331 \end{aligned}$$

$$\begin{aligned}
& 744*a+193728)*\sqrt{a+4}+59146408708*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-59146408708*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+35712575419864*a^3*\sqrt{a+4}+52547642661032*a^3+13635706996*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x-13635706996*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+64340036640*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-64340036640*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*\sqrt{a+4}+43213040637212*a^2+9797208656*a*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x-9797208656*a*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+42385864836*a*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-42385864836*a*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+15111479733208*a*\sqrt{a+4}+21986673204304*a+3197030212*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x-3197030212*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}+12788120848*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3606250079136*\sqrt{a+4}+5217553305984))-\sqrt{9/16}*\sqrt{1/256*(-256*\sqrt{a+4}*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696))-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48)}*\ln(\text{abs}(16807*a^15*\sqrt{a+4}+26411*a^15+908950*a^14*\sqrt{a+4}+1420804*a^14+22929088*a^13*\sqrt{a+4}+35650176*a^13-2401*a^12*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2401*a^12*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+357887692*a^12*\sqrt{a+4}+553458148*a^12-2401*a^11*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)}+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+2401*a^11*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*
\end{aligned}$$

$$\begin{aligned}
& a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)* \\
& \text{sqrt}(a+4)-105154*a^{11}*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12 \\
& 9188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3 \\
& 31744*a+193728)*x+105154*a^{11}*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+4741 \\
& 9*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367 \\
& 28*a^2+331744*a+193728)+3865394166*a^{11}*\text{sqrt}(a+4)+5945365998*a^{11}-95550*a^1 \\
& 0*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11 \\
& 3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\text{sqrt} \\
& (a+4)*x+95550*a^{10}*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918 \\
& 8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317 \\
& 44*a+193728)*\text{sqrt}(a+4)-2109279*a^{10}*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^ \\
& 4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^ \\
& 3+236728*a^2+331744*a+193728)*x+2109279*a^{10}*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^ \\
& 5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4 \\
& +90111*a^3+236728*a^2+331744*a+193728)+30600511272*a^{10}*\text{sqrt}(a+4)+468107098 \\
& 68*a^{10}-1727079*a^9*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+1291 \\
& 88*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331 \\
& 744*a+193728)*\text{sqrt}(a+4)*x+1727079*a^9*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775* \\
& a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111* \\
& a^3+236728*a^2+331744*a+193728)*\text{sqrt}(a+4)-25624212*a^9*\text{sqrt}(\text{sqrt}(a+4))*(49*a \\
& ^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5 \\
& +19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+25624212*a^9*\text{sqrt}(\text{sqrt}(a \\
& +4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6 \\
& +2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+183431725500*a^9* \\
& \text{sqrt}(a+4)+279067335420*a^9-18715896*a^8*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+977 \\
& 5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011 \\
& 1*a^3+236728*a^2+331744*a+193728)*\text{sqrt}(a+4)*x+18715896*a^8*\text{sqrt}(\text{sqrt}(a+4))*(\\
& 49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205 \\
& *a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\text{sqrt}(a+4)-209972223*a^ \\
& 8*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11 \\
& 3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+20 \\
& 9972223*a^8*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+1 \\
& 87408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19 \\
& 3728)+847810669320*a^8*\text{sqrt}(a+4)+1282741275000*a^8-135108639*a^7*\text{sqrt}(\text{sqrt}(\\
& a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^ \\
& 6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\text{sqrt}(a+4)*x+1351 \\
& 08639*a^7*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187 \\
& 408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937 \\
& 28)*\text{sqrt}(a+4)-1222644882*a^7*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419 \\
& *a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672 \\
& 8*a^2+331744*a+193728)*x+1222644882*a^7*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+977 \\
& 5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011 \\
& 1*a^3+236728*a^2+331744*a+193728)+3046208716923*a^7*\text{sqrt}(a+4)+4583471076759 \\
& *a^7-682210326*a^6*\text{sqrt}(\text{sqrt}(a+4))*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918 \\
& 8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317
\end{aligned}$$

$44*a+193728)*\sqrt{a+4}*x+682210326*a^6*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}-5187446733*a^6*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+5187446733*a^6*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+8508527261290*a^6*\sqrt{a+4}+12731345334296*a^6-2458605429*a^5*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+2458605429*a^5*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}-16158435972*a^5*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+16158435972*a^5*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+18324012543204*a^5*\sqrt{a+4}+27265747047380*a^5-6324014256*a^4*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+6324014256*a^4*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}-36673732452*a^4*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+36673732452*a^4*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29880339194272*a^4*\sqrt{a+4}+44213263379848*a^4-11377675428*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+11377675428*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}-59146408708*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+59146408708*a^3*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+35712575419864*a^3*\sqrt{a+4}+52547642661032*a^3-13635706996*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}*x+13635706996*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*\sqrt{a+4}-64340036640*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+64340036640*a^2*\sqrt{\sqrt{a+4}*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*$

$\sqrt{a+4} + 43213040637212a^2 - 9797208656a\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} \cdot x + 9797208656a\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 42385864836a\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \cdot x + 42385864836a\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) + 15111479733208a\sqrt{\sqrt{a+4}} + 21986673204304a - 3197030212\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} \cdot x + 3197030212\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \sqrt{a+4} - 12788120848\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) \cdot x + 12788120848\sqrt{\sqrt{a+4}} + (49a^6 + 1073a^5 + 9775a^4 + 47419a^3 + 129188a^2 + 187408a + 113088) + 105a^6 + 2205a^5 + 19299a^4 + 90111a^3 + 236728a^2 + 331744a + 193728) + 3606250079136\sqrt{\sqrt{a+4}} + 5217553305984) / (32a^4 + 448a^3 + 2336a^2 + 5376a + 4608)$

maple [C] time = 0.02, size = 398, normalized size = 1.58

$$\frac{3(2(2a+7)\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)^2+7Z^2+4(-2a-7)\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)+55a+108)\ln(-\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)+Z)}{128(a^3+10a^2+35a+36)(a+4)\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)^2-3\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)^2+4\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)^2} \cdot \frac{32a+7Z^2}{10(a^3+14a^2+75a+144)} + \frac{212a+7Z^2}{10(a^3+14a^2+75a+144)} + \frac{(7Z^2+94a+116)Z^2}{20(a^3+14a^2+75a+144)} + \frac{5Z^2+175+25Z^2}{20(a^3+14a^2+75a+144)} + \frac{(24a^2+279a+198)Z^2}{20(a^3+14a^2+75a+144)} + \frac{(32a^2+423a+180)Z^2}{20(a^3+14a^2+75a+144)} + \frac{(11a^3+107Z^2-8a-1152)}{20(a^3+14a^2+75a+144)} + \frac{11a^3+112Z^2+49a+288}{20(a^3+14a^2+75a+144)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144))/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(a+4)*\text{sum}((108+2*(7+2*a)*_R^2+4*(-2*a-7)*_R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)*\ln(-_R+x),_R=\text{RootOf}(Z^4-4*_Z^3+8*_Z^2-8*_Z-a))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{6(2a+7Z^2-4Z^2+8Z^2-8Z-a)^2+7Z^2+4(-2a-7)\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)+55a+108)\ln(-\text{RootOf}(Z^2-4Z^2+8Z^2-8Z-a)+Z)}{32((a^3+14a^2+75a+144)^2-3(a^3+14a^2+75a+144)^2+3(2a^2+679a+1968)a^2-11a^2-2(12a^2+623a+1800)a-11Z^2-107Z^2-8a-1152)+488a+588} \cdot \frac{32a+7Z^2}{10(a^3+14a^2+75a+144)} + \frac{212a+7Z^2}{10(a^3+14a^2+75a+144)} + \frac{(7Z^2+94a+116)Z^2}{20(a^3+14a^2+75a+144)} + \frac{5Z^2+175+25Z^2}{20(a^3+14a^2+75a+144)} + \frac{(24a^2+279a+198)Z^2}{20(a^3+14a^2+75a+144)} + \frac{(32a^2+423a+180)Z^2}{20(a^3+14a^2+75a+144)} + \frac{(11a^3+107Z^2-8a-1152)}{20(a^3+14a^2+75a+144)} + \frac{11a^3+112Z^2+49a+288}{20(a^3+14a^2+75a+144)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 408*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x + 55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)
```

mupad [B] time = 6.41, size = 8242, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] atan((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
```

$$\begin{aligned}
& 4016a^5 + 28224a^6 + 66207744)/(16384*(940032a + 1195776a^2 + 899328a^3 \\
& + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} \\
& + 331776)) - (x*(73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656))/(256* \\
& (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736)))*((9*(39329792a - 338a*((a + 4)^{15})^{(1/2)} - 589*((a + 4)^{15})^{(1/2)} \\
& - 49a^2*((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187 \\
& 840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531 \\
& 456))/(16384*(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 \\
& + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119442 \\
& 00a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} \\
& + 254803968)))^{(1/2)}*i - (((52357496832a + 57139003392a^2 + 363221483 \\
& 52a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + \\
& 5726208a^8 + 172032a^9 + 21290287104)/(16384*(940032a + 1195776a^2 + 89 \\
& 9328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 \\
& + a^{10} + 331776)) - ((4290672328704a + 6001143054336a^2 + 5025917042688 \\
& a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 598621552 \\
& 64a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 13 \\
& 91569403904)/(16384*(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149 \\
& 208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x*(3 \\
& 510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 \\
& + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496))/(2 \\
& 56*(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736)))*((9*(39329792a - 338a*((a + 4)^{15})^{(1/2)} - 589*((a + \\
& 4)^{15})^{(1/2)} - 49a^2*((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10 \\
& 187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16 \\
& 531456))/(16384*(1061683200a + 2061434880a^2 + 2474311680a^3 + 205320192 \\
& 0a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119 \\
& 44200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + \\
& a^{15} + 254803968)))^{(1/2)})*((9*(39329792a - 338a*((a + 4)^{15})^{(1/2)} - 58 \\
& 9*((a + 4)^{15})^{(1/2)} - 49a^2*((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960* \\
& a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105* \\
& a^9 + 16531456))/(16384*(1061683200a + 2061434880a^2 + 2474311680a^3 + 2 \\
& 053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 \\
& + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 5 \\
& 5a^{14} + a^{15} + 254803968)))^{(1/2)} - (108343296a + 74059776a^2 + 27065088 \\
& a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744)/(16384*(940032a + \\
& 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + \\
& 582a^8 + 36a^9 + a^{10} + 331776)) + (x*(73476a + 31545a^2 + 6066a^3 + \\
& 441a^4 + 64656))/(256*(48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380* \\
& a^5 + 342a^6 + 28a^7 + a^8 + 20736)))*((9*(39329792a - 338a*((a + 4)^{15})^{(1/2)} \\
&)^{(1/2)} - 589*((a + 4)^{15})^{(1/2)} - 49a^2*((a + 4)^{15})^{(1/2)} + 41598976a^2 \\
& + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 357 \\
& 0a^8 + 105a^9 + 16531456))/(16384*(1061683200a + 2061434880a^2 + 247431 \\
& 1680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 \\
& + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1
\end{aligned}$$

$$\begin{aligned}
& (410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} * i) / (((((52357496832a + 5713 \\
& 9003392a^2 + 36322148352a^3 + 14822473728a^4 + 4027170816a^5 + 72850636 \\
& 8a^6 + 84615168a^7 + 5726208a^8 + 172032a^9 + 21290287104) / (16384 * (9400 \\
& 32a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 556 \\
& 4a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) + ((4290672328704a + 6001143054 \\
& 336a^2 + 5025917042688a^3 + 2800520003584a^4 + 1090200272896a^5 + 30255 \\
& 6119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266176a^9 + 41943040a \\
& ^{10} + 1048576a^{11} + 1391569403904) / (16384 * (940032a + 1195776a^2 + 899328 \\
& * a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + \\
& a^{10} + 331776)) - (x * (3510632448a + 4020240384a^2 + 2678587392a^3 + 1144 \\
& 324096a^4 + 325074944a^5 + 61407232a^6 + 7438336a^7 + 524288a^8 + 1638 \\
& 4a^9 + 1358954496)) / (256 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 23 \\
& 80a^5 + 342a^6 + 28a^7 + a^8 + 20736))) * ((9 * (39329792a - 338a * ((a + 4) \\
& ^{15})^{(1/2)} - 589 * ((a + 4)^{15})^{(1/2)} - 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976 * \\
& a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + \\
& 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 247 \\
& 4311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a \\
& ^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} \\
& + 1410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} * ((9 * (39329792a - 338a * \\
& ((a + 4)^{15})^{(1/2)} - 589 * ((a + 4)^{15})^{(1/2)} - 49a^2 * ((a + 4)^{15})^{(1/2)} + 4 \\
& 1598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 5394 \\
& 9a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a \\
& ^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203 \\
& 166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 223 \\
& 50a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} + (108343296a + \\
& 74059776a^2 + 27065088a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 662077 \\
& 44) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + \\
& 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (73476a + \\
& 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256 * (48384a + 49248a^2 + 28560 * \\
& a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736))) * ((9 * (3932979 \\
& 2a - 338a * ((a + 4)^{15})^{(1/2)} - 589 * ((a + 4)^{15})^{(1/2)} - 49a^2 * ((a + 4)^{1 \\
& 5})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 47560 \\
& 8a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + \\
& 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 5736217 \\
& 60a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 24496 \\
& 5a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968))^{(1/2)} + (((\\
& 52357496832a + 57139003392a^2 + 36322148352a^3 + 14822473728a^4 + 40271 \\
& 70816a^5 + 728506368a^6 + 84615168a^7 + 5726208a^8 + 172032a^9 + 21290 \\
& 287104) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a \\
& ^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - ((42906723 \\
& 28704a + 6001143054336a^2 + 5025917042688a^3 + 2800520003584a^4 + 10902 \\
& 00272896a^5 + 302556119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266 \\
& 176a^9 + 41943040a^{10} + 1048576a^{11} + 1391569403904) / (16384 * (940032a + \\
& 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + \\
& 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (3510632448a + 4020240384a^2 + 2
\end{aligned}$$

$$\begin{aligned}
& 678587392a^3 + 1144324096a^4 + 325074944a^5 + 61407232a^6 + 7438336a^7 \\
& + 524288a^8 + 16384a^9 + 1358954496) / (256(48384a + 49248a^2 + 28560a^3 \\
& + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736)) * ((9(39329792a - 338a * ((a + 4)^{15})^{1/2} - 589 * ((a + 4)^{15})^{1/2} - 49a^2 * ((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^10 + 244965a^11 + 22350a^12 + 1410a^13 + 55a^14 + a^15 + 254803968)))^{1/2}) * ((9(39329792a - 338a * ((a + 4)^{15})^{1/2} - 589 * ((a + 4)^{15})^{1/2} - 49a^2 * ((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^10 + 244965a^11 + 22350a^12 + 1410a^13 + 55a^14 + a^15 + 254803968)))^{1/2} - (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^10 + 331776)) + (x * (73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 + a^8 + 20736)) * ((9(39329792a - 338a * ((a + 4)^{15})^{1/2} - 589 * ((a + 4)^{15})^{1/2} - 49a^2 * ((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^10 + 244965a^11 + 22350a^12 + 1410a^13 + 55a^14 + a^15 + 254803968)))^{1/2} - (99468a + 28053a^2 + 2646a^3 + 117936) / (8192 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^10 + 331776))) * ((9(39329792a - 338a * ((a + 4)^{15})^{1/2} - 589 * ((a + 4)^{15})^{1/2} - 49a^2 * ((a + 4)^{15})^{1/2} + 41598976a^2 + 25672960a^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^10 + 244965a^11 + 22350a^12 + 1410a^13 + 55a^14 + a^15 + 254803968)))^{1/2} * 2i + \operatorname{atan}((((52357496832a + 57139003392a^2 + 36322148352a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + 5726208a^8 + 172032a^9 + 21290287104) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^10 + 331776)) + ((4290672328704a + 6001143054336a^2 + 5025917042688a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^10 + 1048576a^11 + 1391569403904) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^10 + 331776))) - (x * (3510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a^5 + 61407232a^6 + 7438336a^7 + 524288a^8
\end{aligned}$$

$$\begin{aligned}
& + 16384*a^9 + 1358954496)) / (256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 \\
& + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736))) * ((9*(39329792*a + 338*a*((a + 4)^{15})^{(1/2)} \\
& + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 415 \\
& 98976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949* \\
& a^7 + 3570*a^8 + 105*a^9 + 16531456)) / (16384*(1061683200*a + 2061434880*a^2 \\
& + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 20316 \\
& 6720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350 \\
& *a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} * ((9*(39329792*a + \\
& 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} \\
& + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 \\
& + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456)) / (16384*(1061683200*a + 206143 \\
& 4880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 \\
& + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 \\
& + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} + (10834329 \\
& 6*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + \\
& 66207744) / (16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208 \\
& *a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(7347 \\
& 6*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656)) / (256*(48384*a + 49248*a^2 + \\
& 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736))) * ((9*(3 \\
& 9329792*a + 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a \\
& + 4)^{15})^{(1/2)} + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + \\
& 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456)) / (16384*(106168320 \\
& 0*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5 \\
& 73621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + \\
& 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} \\
& * i - (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 \\
& + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 \\
& + 21290287104) / (16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + \\
& 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (\\
& (4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 \\
& + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 \\
& + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904) / (16384*(94 \\
& 0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5 \\
& 564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038 \\
& 4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74 \\
& 38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496)) / (256*(48384*a + 49248*a^2 \\
& + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736))) * ((9 \\
& *(39329792*a + 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*(\\
& (a + 4)^{15})^{(1/2)} + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 \\
& + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456)) / (16384*(106168 \\
& 3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 \\
& + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 \\
& + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} \\
& * ((9*(39329792*a + 338*a*((a + 4)^{15})^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + \\
& 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
\end{aligned}$$

$$\begin{aligned}
& (95744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456)) / (16384 \\
& * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703 \\
& 040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 196 \\
& 6491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 2548039 \\
& 68)))^{(1/2)} - (108343296a + 74059776a^2 + 27065088a^3 + 5576256a^4 + 61 \\
& 4016a^5 + 28224a^6 + 66207744) / (16384 * (940032a + 1195776a^2 + 899328a^ \\
& 3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} \\
& + 331776)) + (x * (73476a + 31545a^2 + 6066a^3 + 441a^4 + 64656)) / (256 * \\
& (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^7 \\
& + a^8 + 20736))) * ((9 * (39329792a + 338a * ((a + 4)^{15})^{(1/2)} + 589 * ((a + 4)^ \\
& 15)^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10187 \\
& 840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531 \\
& 456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a \\
& ^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119442 \\
& 00a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} \\
& + 254803968)))^{(1/2)} * i) / (((52357496832a + 57139003392a^2 + 363221483 \\
& 52a^3 + 14822473728a^4 + 4027170816a^5 + 728506368a^6 + 84615168a^7 + \\
& 5726208a^8 + 172032a^9 + 21290287104) / (16384 * (940032a + 1195776a^2 + 89 \\
& 9328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^ \\
& 9 + a^{10} + 331776)) + ((4290672328704a + 6001143054336a^2 + 5025917042688 \\
& * a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 598621552 \\
& 64a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 13 \\
& 91569403904) / (16384 * (940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149 \\
& 208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (3 \\
& 510632448a + 4020240384a^2 + 2678587392a^3 + 1144324096a^4 + 325074944a \\
& ^5 + 61407232a^6 + 7438336a^7 + 524288a^8 + 16384a^9 + 1358954496)) / (2 \\
& 56 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a^5 + 342a^6 + 28a^ \\
& ^7 + a^8 + 20736))) * ((9 * (39329792a + 338a * ((a + 4)^{15})^{(1/2)} + 589 * ((a + \\
& 4)^{15})^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a^3 + 10 \\
& 187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16 \\
& 531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 205320192 \\
& 0a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 119 \\
& 44200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + \\
& a^{15} + 254803968)))^{(1/2)} * ((9 * (39329792a + 338a * ((a + 4)^{15})^{(1/2)} + 58 \\
& 9 * ((a + 4)^{15})^{(1/2)} + 49a^2 * ((a + 4)^{15})^{(1/2)} + 41598976a^2 + 25672960a \\
& ^3 + 10187840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a \\
& ^9 + 16531456)) / (16384 * (1061683200a + 2061434880a^2 + 2474311680a^3 + 2 \\
& 053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a \\
& ^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 5 \\
& 5a^{14} + a^{15} + 254803968)))^{(1/2)} + (108343296a + 74059776a^2 + 27065088 \\
& * a^3 + 5576256a^4 + 614016a^5 + 28224a^6 + 66207744) / (16384 * (940032a + \\
& 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + \\
& 582a^8 + 36a^9 + a^{10} + 331776)) - (x * (73476a + 31545a^2 + 6066a^3 + \\
& 441a^4 + 64656)) / (256 * (48384a + 49248a^2 + 28560a^3 + 10321a^4 + 2380a \\
& ^5 + 342a^6 + 28a^7 + a^8 + 20736))) * ((9 * (39329792a + 338a * ((a + 4)^{15}
\end{aligned}$$

$$\begin{aligned}
&)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 \\
& + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357 \\
& 0*a^8 + 105*a^9 + 16531456))/((16384*(1061683200*a + 2061434880*a^2 + 247431 \\
& 1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 \\
& + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1 \\
& 410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} + (((52357496832*a + 5713900 \\
& 3392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a \\
& ^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032* \\
& a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a \\
& ^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336 \\
& *a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255611 \\
& 9040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 \\
& + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^ \\
& 3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1 \\
& 0 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324 \\
& 096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a \\
& ^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380* \\
& a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15 \\
&)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 \\
& + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357 \\
& 0*a^8 + 105*a^9 + 16531456))/((16384*(1061683200*a + 2061434880*a^2 + 247431 \\
& 1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 \\
& + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1 \\
& 410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)}*((9*(39329792*a + 338*a*((a \\
& + 4)^15)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 4159 \\
& 8976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a \\
& ^7 + 3570*a^8 + 105*a^9 + 16531456))/((16384*(1061683200*a + 2061434880*a^2 \\
& + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166 \\
& 720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350* \\
& a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} - (108343296*a + 740 \\
& 59776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744) \\
& /((16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34 \\
& 833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 315 \\
& 45*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 \\
& + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a \\
& + 338*a*((a + 4)^15)^{(1/2)} + 589*((a + 4)^{15})^{(1/2)} + 49*a^2*((a + 4)^{15})^{ \\
& (1/2)} + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a \\
& ^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/((16384*(1061683200*a + 206 \\
& 1434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760* \\
& a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a \\
& ^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^{(1/2)} - (99468 \\
& *a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a + 1195776*a^2 + 899328* \\
& a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a \\
& ^10 + 331776))))*((9*(39329792*a + 338*a*((a + 4)^15)^{(1/2)} + 589*((a + 4)^ \\
& 15)^{(1/2)} + 49*a^2*((a + 4)^{15})^{(1/2)} + 41598976*a^2 + 25672960*a^3 + 10187
\end{aligned}$$

$$\frac{840a^4 + 2695744a^5 + 475608a^6 + 53949a^7 + 3570a^8 + 105a^9 + 16531456}{(16384(1061683200a + 2061434880a^2 + 2474311680a^3 + 2053201920a^4 + 1247703040a^5 + 573621760a^6 + 203166720a^7 + 55893360a^8 + 11944200a^9 + 1966491a^{10} + 244965a^{11} + 22350a^{12} + 1410a^{13} + 55a^{14} + a^{15} + 254803968))^{1/2}} \cdot 2i - \frac{(408a + 131a^2 + 11a^3 + 288)}{(32(a + 4)(33a + 10a^2 + a^3 + 36))} - \frac{(21x^6(2a + 7))}{(16(168a + 73a^2 + 14a^3 + a^4 + 144))} + \frac{(3x^7(2a + 7))}{(16(168a + 73a^2 + 14a^3 + a^4 + 144))} + \frac{(x(84a - 107a^2 - 11a^3 + 1152))}{(32(a + 4)(33a + 10a^2 + a^3 + 36))} - \frac{(5x^4(175a + 7a^2 + 528))}{(32(a + 4)(33a + 10a^2 + a^3 + 36))} + \frac{(x^5(343a + 7a^2 + 1116))}{(32(a + 4)(33a + 10a^2 + a^3 + 36))} - \frac{(x^2(623a + 32a^2 + 1800))}{(16(a + 4)(33a + 10a^2 + a^3 + 36))} + \frac{(x^3(679a + 34a^2 + 1968))}{(16(a + 4)(33a + 10a^2 + a^3 + 36))} \cdot \frac{(16ax - x^2(16a - 64) - x^4(2a - 128) + x^3(8a - 128) + a^2 - 80x^5 + 32x^6 - 8x^7 + x^8)}{(16(a + 4)(33a + 10a^2 + a^3 + 36))}$$

sympy [B] time = 15.62, size = 697, normalized size = 2.77

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out] $-\frac{(11a^3 + 131a^2 + 408a + x^7(12a + 42) + x^6(-84a - 294) + x^5(7a^2 + 343a + 1116) + x^4(-35a^2 - 875a - 2640) + x^3(68a^2 + 1358a + 3936) + x^2(-64a^2 - 1246a - 3600) + x(-11a^3 - 107a^2 + 84a + 1152) + 288)}{(32a^6 + 448a^5 + 2336a^4 + 5376a^3 + 4608a^2 + x^8(32a^4 + 448a^3 + 2336a^2 + 5376a + 4608) + x^7(-256a^4 - 3584a^3 - 18688a^2 - 43008a - 36864) + x^6(1024a^4 + 14336a^3 + 74752a^2 + 172032a + 147456) + x^5(-2560a^4 - 35840a^3 - 186880a^2 - 430080a - 368640) + x^4(-64a^5 + 3200a^4 + 52672a^3 + 288256a^2 + 678912a + 589824) + x^3(256a^5 - 512a^4 - 38656a^3 - 256000a^2 - 651264a - 589824) + x^2(-512a^5 - 5120a^4 - 8704a^3 + 63488a^2 + 270336a + 294912) + x(512a^5 + 7168a^4 + 37376a^3 + 86016a^2 + 73728a)) - \text{RootSum}(_t^4(268435456a^{15} + 14763950080a^{14} + 378493992960a^{13} + 5999532441600a^{12} + 65757291479040a^{11} + 527875908304896a^{10} + 3206246773555200a^9 + 15003759578972160a^8 + 54537151127224320a^7 + 153980418717122560a^6 + 334927734494986240a^5 + 551152193655275520a^4 + 664192984106926080a^3 + 553362212027105280a^2 + 284993413919539200a + 68398419340689408) + _t^2(-30965760a^9 - 1052835840a^8 - 15910207488a^7 - 140262506496a^6 - 795007254528a^5 - 3004516270080a^4 - 7571263979520a^3 - 12268037210112a^2 - 11598827618304a - 4875324751872) - 194481a^4 - 2762424a^3 - 14762736a^2 - 35178624a - 31539456, \text{Lambda}(_t, _t \log(x + \frac{23068672_t^3 a^{12} + 968884224_t^3 a^{11} + 18624806912_t^3 a^{10} + 216677744640_t^3 a^9 + 1699123036160_t^3 a^8 + 9461389328384_t^3 a^7 + 38361186172928_t^3 a^6 + 1141074915491$

$$\begin{aligned} & 84*_t^{**3}*a^{**5} + 247138458009600*_t^{**3}*a^{**4} + 380084473036800*_t^{**3}*a^{**3} + 3 \\ & 94002582994944*_t^{**3}*a^{**2} + 247177515368448*_t^{**3}*a + 70970039599104*_t^{**3} \\ & - 395136*_t*a^{**7} - 11676672*_t*a^{**6} - 144076032*_t*a^{**5} - 969518592*_t*a^{**4} \\ & - 3861475200*_t*a^{**3} - 9133300224*_t*a^{**2} - 11906574336*_t*a - 6611337216*_ \\ & _t - 64827*a^{**4} - 907578*a^{**3} - 4780647*a^{**2} - 11228868*a - 9923472)/(64827 \\ & *a^{**4} + 907578*a^{**3} + 4780647*a^{**2} + 11228868*a + 9923472)))) \end{aligned}$$

$$3.119 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=210

$$\frac{a^4 x^2}{2} + \frac{32 a^3 x^3}{3} + \frac{1}{5} (3 a^2 - 1536 a + 20480) x^{10} - \frac{16}{3} (a^2 - 128 a + 896) x^9 - \frac{32}{7} (15 a^2 - 288 a + 512) x^7 + \frac{16}{5} a (a^2 - 4$$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6742}

$$\frac{1}{5}(3a^2 - 1536a + 20480)x^{10} - \frac{16}{3}(a^2 - 128a + 896)x^9 - \frac{32}{7}(15a^2 - 288a + 512)x^7 + \frac{2}{3}(a^3 + 192a^2 - 1536a + 1024)x^6 + \frac{16}{5}a(a^2 - 48a + 128)x^5 + 8(12 - a)a^2x^4 + \frac{32a^3x^3}{3} + \frac{a^4x^2}{2} + \frac{2}{5}(640 - a)x^{14} - \frac{16}{13}(464 - 3a)x^{13} + \frac{8}{3}(524 - 9a)x^{12} - \frac{32}{11}(928 - 35a)x^{11} + 8(128 - 3a)(4 - a)x^8 + \frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 15*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 - 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx = \int (a^4 x + 32a^3 x^2 - 32(-12 + a)a^2 x^3 + 16a(128 - 48a + a^2)x^4 - 4(-1024 + 16a(128 - 48a + a^2))x^5 + 2(1024 - 1536a + 192a^2 - a^3)x^6 - 32(512 - 288a + 15a^2)x^7 + 8(128 - 3a)(4 - a)x^8 - 16(896 - 128a + a^2)x^9 + (20480 - 1536a + 3a^2)x^{10} - 32(928 - 35a)x^{11} + 8(524 - 9a)x^{12} - 16(464 - 3a)x^{13} + 2(640 - a)x^{14} - 224x^{15} + 8x^{16} - 16x^{17} + x^{18}) dx$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4 x^2}{2} + \frac{32 a^3 x^3}{3} + \frac{1}{5} (3 a^2 - 1536 a + 20480) x^{10} - \frac{16}{3} (a^2 - 128 a + 896) x^9 + 8 (3 a^2 - 140 a + 512) x^8 - \frac{32}{7} (15 a^2 - 288 a + 512) x^7 + \frac{16}{5} a (a^2 - 48 a + 128) x^5 - 8 (a - 12) a^2 x^4 - \frac{2}{3} (a^3 - 192 a^2 + 1536 a - 1024) x^6 - \frac{2}{7} (a - 640) x^{14} + \frac{16}{13} (3 a - 464) x^{13} - \frac{8}{3} (9 a - 524) x^{12} + \frac{32}{11} (35 a - 928) x^{11} + \frac{x^{18}}{18} - \frac{16 x^{17}}{17} + 8 x^{16} - \frac{224 x^{15}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

[Out] $(a^4x^2)/2 + (32a^3x^3)/3 - 8(-12 + a)a^2x^4 + (16a(128 - 48a + a^2)x^5)/5 - (2(-1024 + 1536a - 192a^2 + a^3)x^6)/3 - (32(512 - 288a + 15a^2)x^7)/7 + 8(512 - 140a + 3a^2)x^8 - (16(896 - 128a + a^2)x^9)/3 + ((20480 - 1536a + 3a^2)x^{10})/5 + (32(-928 + 35a)x^{11})/11 - (8(-524 + 9a)x^{12})/3 + (16(-464 + 3a)x^{13})/13 - (2(-640 + a)x^{14})/7 - (224x^{15})/5 + 8x^{16} - (16x^{17})/17 + x^{18}/18$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

fricas [A] time = 0.78, size = 222, normalized size = 1.06

$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{224}{5}x^{15} - \frac{2}{7}x^{14}a + \frac{1280}{7}x^{14} + \frac{48}{13}x^{13}a - \frac{7424}{13}x^{13} - 24x^{12}a + \frac{4192}{3}x^{12} + \frac{1120}{11}x^{11}a + \frac{3}{5}x^{10}a^2 - \frac{29696}{11}x^{11} - 1536/5x^{10}a - \frac{16}{3}x^9a^2 + 4096x^{10} + \frac{2048}{3}x^9a + 24x^8a^2 - \frac{14336}{3}x^9 - 1120x^8a - \frac{480}{7}x^7a^2 - \frac{2}{3}x^6a^3 + 4096x^8 + \frac{9216}{7}x^7a + 128x^6a^2 + \frac{16}{5}x^5a^3 - \frac{16384}{7}x^7 - 1024x^6a - \frac{768}{5}x^5a^2 - 8x^4a^3 + \frac{2048}{3}x^6 + \frac{2048}{5}x^5a + 96x^4a^2 + \frac{32}{3}x^3a^3 + \frac{1}{2}x^2a^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

[Out] $1/18x^{18} - 16/17x^{17} + 8x^{16} - 224/5x^{15} - 2/7x^{14}a + 1280/7x^{14} + 48/13x^{13}a - 7424/13x^{13} - 24x^{12}a + 4192/3x^{12} + 1120/11x^{11}a + 3/5x^{10}a^2 - 29696/11x^{11} - 1536/5x^{10}a - 16/3x^9a^2 + 4096x^{10} + 2048/3x^9a + 24x^8a^2 - 14336/3x^9 - 1120x^8a - 480/7x^7a^2 - 2/3x^6a^3 + 4096x^8 + 9216/7x^7a + 128x^6a^2 + 16/5x^5a^3 - 16384/7x^7 - 1024x^6a - 768/5x^5a^2 - 8x^4a^3 + 2048/3x^6 + 2048/5x^5a + 96x^4a^2 + 32/3x^3a^3 + 1/2x^2a^4$

giac [A] time = 0.39, size = 222, normalized size = 1.06

$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{224}{5}x^{15} - \frac{2}{7}x^{14}a + \frac{1280}{7}x^{14} + \frac{48}{13}x^{13}a - \frac{7424}{13}x^{13} - 24x^{12}a + \frac{4192}{3}x^{12} + \frac{1120}{11}x^{11}a + \frac{3}{5}x^{10}a^2 - \frac{29696}{11}x^{11} - 1536/5x^{10}a - \frac{16}{3}x^9a^2 + 4096x^{10} + \frac{2048}{3}x^9a + 24x^8a^2 - \frac{14336}{3}x^9 - 1120x^8a - \frac{480}{7}x^7a^2 - \frac{2}{3}x^6a^3 + 4096x^8 + \frac{9216}{7}x^7a + 128x^6a^2 + \frac{16}{5}x^5a^3 - \frac{16384}{7}x^7 - 1024x^6a - \frac{768}{5}x^5a^2 - 8x^4a^3 + \frac{2048}{3}x^6 + \frac{2048}{5}x^5a + 96x^4a^2 + \frac{32}{3}x^3a^3 + \frac{1}{2}x^2a^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

[Out] $1/18x^{18} - 16/17x^{17} + 8x^{16} - 2/7ax^{14} - 224/5x^{15} + 48/13ax^{13} + 1280/7x^{14} - 24ax^{12} - 7424/13x^{13} + 3/5a^2x^{10} + 1120/11ax^{11} + 4192/3x^{12} - 16/3a^2x^9 - 1536/5ax^{10} - 29696/11x^{11} + 24a^2x^8 + 2048/3ax^9 + 4096x^{10} - 2/3a^3x^6 - 480/7a^2x^7 - 1120ax^8 - 14336/3x^9 + 16/5a^3x^5 + 128a^2x^6 + 9216/7ax^7 + 4096x^8 - 8a^3x^4 - 76$

$$8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^2*x^4 + 2048/5*a*x^5 + 2048/3*x^6$$

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{a^{18}}{18} - \frac{16a^{17}}{17} + 8a^{16} - \frac{224}{5}a^{15} + \frac{16}{13}(3a - 464)a^{14} - \frac{8}{3}(9a - 524)a^{13} + \frac{32}{11}(35a - 928)a^{12} + \frac{1}{5}(3a^2 - 1536a + 20480)a^{11} - \frac{16}{3}(a^2 - 128a + 896)a^9 + 8(3a^2 - 140a + 512)a^8 - \frac{32}{7}(15a^2 - 288a + 512)a^7 - \frac{2}{3}(a^3 - 192a^2 + 1536a - 1024)a^6 + \frac{1}{2}a^4 + \frac{32}{3}a^3 + \frac{16}{5}(a^3 - 48a^2 + 128a)a^5 - 8(a^3 - 12a^2)a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)

[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13*(48*a-7424)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^11+1/10*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^9+1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^7+1/6*(2*(-2*a+128)*a^2+32*(8*a-128)*a+(-16*a+64)^2)*x^6+1/5*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^5+1/4*(2*(-16*a+64)*a^2+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2

maxima [A] time = 0.62, size = 182, normalized size = 0.87

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{224}{5}x^{15} + \frac{16}{13}(3a - 464)x^{14} - \frac{8}{3}(9a - 524)x^{13} + \frac{32}{11}(35a - 928)x^{12} + \frac{1}{5}(3a^2 - 1536a + 20480)x^{11} - \frac{16}{3}(a^2 - 128a + 896)a^9 + 8(3a^2 - 140a + 512)a^8 - \frac{32}{7}(15a^2 - 288a + 512)a^7 - \frac{2}{3}(a^3 - 192a^2 + 1536a - 1024)a^6 + \frac{1}{2}a^4 + \frac{32}{3}a^3 + \frac{16}{5}(a^3 - 48a^2 + 128a)a^5 - 8(a^3 - 12a^2)a^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5 - 8*(a^3 - 12*a^2)*x^4

mupad [B] time = 0.22, size = 178, normalized size = 0.85

$$x^{18} \left(\frac{48a}{13} - \frac{7424}{13} \right) - x^{17} \left(24a - \frac{4192}{3} \right) - x^{16} \left(\frac{2a}{7} - \frac{1280}{7} \right) + x^{15} \left(\frac{1120a}{11} - \frac{29696}{11} \right) + x^{14} (24a^2 - 1120a + 4096) + x^{13} \left(\frac{3a^2}{5} - \frac{1536a}{5} + 4096 \right) - x^{12} \left(\frac{16a^2}{3} - \frac{2048a}{3} + \frac{14336}{3} \right) - x^{11} \left(\frac{480a^2}{7} - \frac{9216a}{7} + \frac{16384}{7} \right) - x^{10} \left(\frac{2a^3}{3} - 128a^2 + 1024a - \frac{2048}{3} \right) - \frac{224a^{15}}{5} + 8a^{14} - \frac{16a^{17}}{17} + \frac{32a^3}{3} + \frac{a^4}{2} + \frac{16a^2(a^2 - 48a + 128)}{5} - 8a^2x^4(a - 12)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7) + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 - 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3 + (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)

sympy [A] time = 0.12, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2} + \frac{32a^3x^3}{3} + \frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5} + x^{14}\left(\frac{1280}{7} - \frac{2a}{7}\right) + x^{13}\left(\frac{48a}{13} - \frac{7424}{13}\right) + x^{12}\left(\frac{4192}{3} - 24a\right) + x^{11}\left(\frac{1120a}{11} - \frac{29696}{11}\right) + x^{10}\left(\frac{3a^2}{5} - \frac{1536a}{5} + 4096\right) + x^9\left(\frac{16a^2}{3} - \frac{2048a}{3} - \frac{14336}{3}\right) + x^8(24a^2 - 1120a + 4096) + x^7\left(\frac{480a^2}{7} - \frac{9216a}{7} - \frac{16384}{7}\right) + x^6\left(\frac{2a^3}{3} + 128a^2 - 1024a + \frac{2048}{3}\right) + x^5\left(\frac{16a^3}{5} - \frac{768a^2}{5} + \frac{2048a}{5}\right) + x^4(-8a^3 + 96a^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 24*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) + x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96*a**2)

$$3.120 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Optimal. Leaf size=134

$$\frac{a^3 x^2}{2} - \frac{1}{2} (a^2 - 128a + 512) x^6 + \frac{4}{5} (3a^2 - 96a + 128) x^5 + 8a^2 x^3 - \frac{3}{10} (256 - a) x^{10} + \frac{8}{3} (64 - a) x^9 - 4(70 - 3a) x^8 + \frac{48}{7} (48 - 5a) x^7 + 6(8 - a) a x^4 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11}$$

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6742}

$$-\frac{1}{2} (a^2 - 128a + 512) x^6 + \frac{4}{5} (3a^2 - 96a + 128) x^5 + 8a^2 x^3 + \frac{a^3 x^2}{2} - \frac{3}{10} (256 - a) x^{10} + \frac{8}{3} (64 - a) x^9 - 4(70 - 3a) x^8 + \frac{48}{7} (48 - 5a) x^7 + 6(8 - a) a x^4 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5 - ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 + (8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx = \int (a^3 x + 24a^2 x^2 - 24(-8 + a) a x^3 + 4(128 - 96a + 3a^2) x^4 - 3(512 - 128a + a^2) x^5 + 8a^2 x^3 + 6(8 - a) a x^4 + \frac{4}{5} (128 - 96a + 3a^2) x^5 - \frac{1}{2} (512 - 128a + a^2) x^6) dx$$

Mathematica [A] time = 0.02, size = 130, normalized size = 0.97

$$\frac{a^3 x^2}{2} + \frac{1}{2} (-a^2 + 128a - 512) x^6 + \frac{4}{5} (3a^2 - 96a + 128) x^5 + 8a^2 x^3 + \frac{3}{10} (a - 256) x^{10} - \frac{8}{3} (a - 64) x^9 + 4(3a - 70) x^8 - \frac{48}{7} (5a - 48) x^7 - 6(a - 8) a x^4 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] $(a^3x^2)/2 + 8a^2x^3 - 6(-8 + a)ax^4 + (4(128 - 96a + 3a^2)x^5)/5 + ((-512 + 128a - a^2)x^6)/2 - (48(-48 + 5a)x^7)/7 + 4(-70 + 3a)x^8 - (8(-64 + a)x^9)/3 + (3(-256 + a)x^{10})/10 + (280x^{11})/11 - 6x^{12} + (12x^{13})/13 - x^{14}/14$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

fricas [A] time = 0.98, size = 133, normalized size = 0.99

$$\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{280}{11}x^{11} + \frac{3}{10}x^{10}a - \frac{384}{5}x^{10} - \frac{8}{3}x^9a + \frac{512}{3}x^9 + 12x^8a - 280x^8 - \frac{240}{7}x^7a - \frac{1}{2}x^6a^2 + \frac{2304}{7}x^7 + 64x^6a + \frac{12}{5}x^5a^2 - 256x^6 - \frac{384}{5}x^5a - 6x^4a^2 + \frac{512}{5}x^5 + 48x^4a + 8x^3a^2 + \frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 280/11*x^{11} + 3/10*x^{10}*a - 384/5*x^{10} - 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7*a - 1/2*x^6*a^2 + 2304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6 - 384/5*x^5*a - 6*x^4*a^2 + 512/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1/2*x^2*a^3$

giac [A] time = 0.35, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}ax^{10} + \frac{280}{11}x^{11} - \frac{8}{3}ax^9 - \frac{384}{5}x^{10} + 12ax^8 + \frac{512}{3}x^9 - \frac{1}{2}a^2x^6 - \frac{240}{7}ax^7 - 280x^8 + \frac{12}{5}a^2x^5 + 64ax^6 + \frac{2304}{7}x^7 - 6a^2x^4 - \frac{384}{5}ax^5 - 256x^6 + \frac{1}{2}a^3x^2 + 8a^2x^3 + 48ax^4 + \frac{512}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 3/10*a*x^{10} + 280/11*x^{11} - 8/3*a*x^9 - 384/5*x^{10} + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 12/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5$

maple [A] time = 0.00, size = 143, normalized size = 1.07

$$\frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + \frac{(3a-768)x^{10}}{10} + \frac{(-24a+1536)x^9}{9} + \frac{(96a-2240)x^8}{8} + \frac{(-240a+2304)x^7}{7} + \frac{(-a^2+(-2a+128)a+256a-1536)x^6}{6} + \frac{a^3x^2}{2} + 8a^2x^3 + \frac{(4a^2+(8a-128)a-256a+512)x^5}{5} + \frac{(-8a^2+(-16a+64)a+128a)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-1/14*x^{14}+12/13*x^{13}-6*x^{12}+280/11*x^{11}+1/10*(3*a-768)*x^{10}+1/9*(-24*a+1536)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(-a^2+(-2*a+128)*a+256*a-1536)*x^6+1/5*(4*a^2+(8*a-128)*a-256*a+512)*x^5+1/4*(-8*a^2+(-16*a+64)*a+128*a)*x^4+8*a^2*x^3+1/2*a^3*x^2$

maxima [A] time = 0.62, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14} + \frac{12}{13}x^{13} - 6x^{12} + \frac{3}{10}(a-256)x^{10} + \frac{280}{11}x^{11} - \frac{8}{3}(a-64)x^9 + 4(3a-70)x^8 - \frac{48}{7}(5a-48)x^7 - \frac{1}{2}(a^2-128a+512)x^6 + \frac{4}{5}(3a^2-96a+128)x^5 + \frac{1}{2}a^3x^2 + 8a^2x^3 - 6(a^2-8a)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out] $-1/14*x^{14} + 12/13*x^{13} - 6*x^{12} + 3/10*(a - 256)*x^{10} + 280/11*x^{11} - 8/3*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a + 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2 - 8*a)*x^4$

mupad [B] time = 2.12, size = 113, normalized size = 0.84

$$x^8(12a-280) + x^{10}\left(\frac{3a}{10} - \frac{384}{5}\right) - x^9\left(\frac{8a}{3} - \frac{512}{3}\right) - x^7\left(\frac{240a}{7} - \frac{2304}{7}\right) - x^6\left(\frac{a^2}{2} - 64a + 256\right) + x^5\left(\frac{12a^2}{5} - \frac{384a}{5} + \frac{512}{5}\right) + \frac{280x^{11}}{11} - 6x^{12} + \frac{12x^{13}}{13} - \frac{x^{14}}{14} + 8a^2x^3 + \frac{a^3x^2}{2} - 6ax^4(a-8)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)

[Out] $x^8*(12*a - 280) + x^{10}*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5 + 512/5) + (280*x^{11})/11 - 6*x^{12} + (12*x^{13})/13 - x^{14}/14 + 8*a^2*x^3 + (a^3*x^2)/2 - 6*a*x^4*(a - 8)$

sympy [A] time = 0.10, size = 128, normalized size = 0.96

$$\frac{a^3x^2}{2} + 8a^2x^3 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + x^{10}\left(\frac{3a}{10} - \frac{384}{5}\right) + x^9\left(\frac{512}{3} - \frac{8a}{3}\right) + x^8(12a-280) + x^7\left(\frac{2304}{7} - \frac{240a}{7}\right) + x^6\left(-\frac{a^2}{2} + 64a - 256\right) + x^5\left(\frac{12a^2}{5} - \frac{384a}{5} + \frac{512}{5}\right) + x^4(-6a^2 + 48a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out] $a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a/5 + 512/5) + x**4*(-6*a**2 + 48*a)$

$$3.121 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2} + \frac{1}{3}(64 - a)x^6 - \frac{8}{5}(16 - a)x^5 + 4(4 - a)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {6742}

$$\frac{a^2x^2}{2} + \frac{1}{3}(64 - a)x^6 - \frac{8}{5}(16 - a)x^5 + 4(4 - a)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10

Rule 6742

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2x + 16ax^2 - 16(-4 + a)x^3 + 8(-16 + a)x^4 - 2(-64 + a)x^5 - 80x^6 + 32x^7 - 80x^8 + 4x^9 - x^{10}) dx \\ &= \frac{a^2x^2}{2} + \frac{16ax^3}{3} + 4(4 - a)x^4 - \frac{8}{5}(16 - a)x^5 + \frac{1}{3}(64 - a)x^6 - \frac{80x^7}{7} + 4x^8 - \frac{80x^9}{9} + \frac{x^{10}}{10} \end{aligned}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2} + \frac{1}{3}(64 - a)x^6 + \frac{8}{5}(a - 16)x^5 - 4(a - 4)x^4 + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 - a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

fricas [A] time = 0.86, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{80}{7}x^7 - \frac{1}{3}x^6a + \frac{64}{3}x^6 + \frac{8}{5}x^5a - \frac{128}{5}x^5 - 4x^4a + 16x^4 + \frac{16}{3}x^3a + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8/5*x^5*a - 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^2

giac [A] time = 0.35, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}ax^6 - \frac{80}{7}x^7 + \frac{8}{5}ax^5 + \frac{64}{3}x^6 - 4ax^4 - \frac{128}{5}x^5 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3 + 16x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 - 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + \frac{(-2a + 128)x^6}{6} + \frac{(8a - 128)x^5}{5} + \frac{a^2x^2}{2} + \frac{16ax^3}{3} + \frac{(-16a + 64)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2

maxima [A] time = 0.62, size = 59, normalized size = 0.75

$$\frac{1}{10}x^{10} - \frac{8}{9}x^9 + 4x^8 - \frac{1}{3}(a - 64)x^6 - \frac{80}{7}x^7 + \frac{8}{5}(a - 16)x^5 - 4(a - 4)x^4 + \frac{1}{2}a^2x^2 + \frac{16}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^5 \left(\frac{8a}{5} - \frac{128}{5} \right) - x^6 \left(\frac{a}{3} - \frac{64}{3} \right) - x^4 (4a - 16) + \frac{16ax^3}{3} - \frac{80x^7}{7} + 4x^8 - \frac{8x^9}{9} + \frac{x^{10}}{10} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2

sympy [A] time = 0.08, size = 70, normalized size = 0.89

$$\frac{a^2x^2}{2} + \frac{16ax^3}{3} + \frac{x^{10}}{10} - \frac{8x^9}{9} + 4x^8 - \frac{80x^7}{7} + x^6 \left(\frac{64}{3} - \frac{a}{3} \right) + x^5 \left(\frac{8a}{5} - \frac{128}{5} \right) + x^4 (16 - 4a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)

$$3.122 \quad \int x (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Optimal. Leaf size=35

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {14}

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

Rule 14

Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x (a + 8x - 8x^2 + 4x^3 - x^4) dx &= \int (ax + 8x^2 - 8x^3 + 4x^4 - x^5) dx \\ &= \frac{ax^2}{2} + \frac{8x^3}{3} - 2x^4 + \frac{4x^5}{5} - \frac{x^6}{6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

fricas [A] time = 1.02, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{8}{3}x^3 + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a

giac [A] time = 0.36, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3} + \frac{ax^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3

$$3.123 \quad \int \frac{x}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}} + \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1680, 1673, 1093, 204, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}} + \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{a + 8x - 8x^2 + 4x^3 - x^4} dx &= \text{Subst} \left(\int \frac{1+x}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{3+a-2x^2-x^4} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{3+a-2x-x^2} dx, x, (-1+x)^2 \right) - \frac{\text{Subst} \left(\int \frac{1}{-1-\sqrt{4+a}-x^2} dx, x, -1+x \right)}{2\sqrt{4+a}} \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1+\sqrt{4+a}}} - \text{Subst} \left(\int \frac{1}{4(4+a)-x^2} dx, x, -1+x \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{4+a} \sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1} \left(\frac{1+(-1+x)^2}{\sqrt{4+a}} \right)}{2\sqrt{4+a}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.51

$$-\frac{1}{4} \text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1 \log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

maple [C] time = 0.00, size = 50, normalized size = 0.43

$$\frac{\text{RootOf}(-Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) \ln(-\text{RootOf}(-Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) + x)}{4\left(\text{RootOf}(-Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^3 - 3\text{RootOf}(-Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^2 + 4\text{RootOf}(-Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(-Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

mupad [B] time = 2.58, size = 275, normalized size = 2.37

$\sum_{i=1}^n (-1)^{i+1} \text{root}(2816a^2z^4 + 256a^3z^4 + 10240a^2z^4 + 12288z^4 - 32a^2z^2 - 256a^2z^2 - 512z^2 + 16az + 64z + a, z, i) \text{root}(2816a^2z^4 + 256a^3z^4 + 10240a^2z^4 + 12288z^4 - 32a^2z^2 - 256a^2z^2 - 512z^2 + 16az + 64z + a, z, i) (2i - \text{root}(2816a^2z^4 + 256a^3z^4 + 10240a^2z^4 + 12288z^4 - 32a^2z^2 - 256a^2z^2 - 512z^2 + 16az + 64z + a, z, i)) (4i - 1) (4i + 256) + (8i + 64 + 128) - 8) \text{root}(2816a^2z^4 + 256a^3z^4 + 10240a^2z^4 + 12288z^4 - 32a^2z^2 - 256a^2z^2 - 512z^2 + 16az + 64z + a, z, i)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*a^2

```

2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 51
2*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 +
10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z
+ a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))*root(
2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z
^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)

```

sympy [A] time = 4.43, size = 155, normalized size = 1.34

$$-\text{RootSum}\left(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-32a^2 - 256a - 512) + t(-16a - 64) + a, \left(t \mapsto t \log\left(x + \frac{-128t^3a^4 - 1728t^3a^3 - 8640t^3a^2 - 18944t^3a - 15360t^3 + 48t^2a^3 + 464t^2a^2 + 1472t^2a + 1536t^2 + 8ta^3 + 88ta^2 + 312a + 352t - a^2 - 2a}{4a^2 + 21a + 28}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
- 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

$$3.124 \quad \int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=231

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{4(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{a+4}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.24, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1680, 1673, 1092, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)(a+(x-1)^2+5)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{4(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(3a+\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}} + \frac{(3a-\sqrt{a+4}+10)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free

$Q\{a, b, c\}, x \} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[p, -3/2] \&\& \text{IntegerQ}[4*p]$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1092

$\text{Int}[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]^{(p_)}, x_Symbol] \rightarrow -\text{Simp}[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^{(p+1)})/(2*a*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p+1)*(b^2 - 4*a*c)), \text{Int}[(b^2 - 2*a*c + 2*(p+1)*(b^2 - 4*a*c) + b*c*(4*p+7)*x^2)*(a + b*x^2 + c*x^4)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2*p]$

Rule 1107

$\text{Int}[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x] /; \text{FreeQ}\{a, b, c, p, x\}$

Rule 1166

$\text{Int}[(d_) + (e_.)*(x_)^2]/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^2), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 1673

$\text{Int}[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Module}\{q = \text{Expon}[Pq, x], k\}, \text{Int}[\text{Sum}[\text{Coeff}[Pq, x, 2*k]*x^{(2*k)}, \{k, 0, q/2\}]*(a + b*x^2 + c*x^4)^p, x] + \text{Int}[x*\text{Sum}[\text{Coeff}[Pq, x, 2*k + 1]*x^{(2*k)}, \{k, 0, (q-1)/2\}]*(a + b*x^2 + c*x^4)^p, x]] /; \text{FreeQ}\{a, b, c, p, x\} \&\& \text{PolyQ}[Pq, x] \&\& \text{!PolyQ}[Pq, x^2]$

Rule 1680

$\text{Int}[(Pq_)*(Q4_)^{(p_)}, x_Symbol] \rightarrow \text{With}\{a = \text{Coeff}[Q4, x, 0], b = \text{Coeff}[Q4, x, 1], c = \text{Coeff}[Q4, x, 2], d = \text{Coeff}[Q4, x, 3], e = \text{Coeff}[Q4, x, 4]\}, \text{Subst}[\text{Int}[\text{SimplifyIntegrand}[(Pq /. x \rightarrow -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; \text{EqQ}[d^3 - 4*c*d*e + 8*b*e^2, 0] \&\& \text{NeQ}[d, 0]] /; \text{FreeQ}[p, x] \&\& \text{PolyQ}[Pq,$

x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx &= \text{Subst} \left(\int \frac{1+x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
 &= \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
 &= \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
 &= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
 &= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
 &= \frac{1+(-1+x)^2}{4(4+a)(3+a-2(1-x)^2-(1-x)^4)} + \frac{(5+a+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)}
 \end{aligned}$$

Mathematica [C] time = 0.07, size = 166, normalized size = 0.72

$$\frac{ax^2 - ax + a + x^3 + 2x}{4(a+3)(a+4)(a-x(x^3-4x^2+8x-8))} - \frac{\text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x-\#1) + 2\#1a \log(x-\#1) + a \log(x-\#1) + 4\#1 \log(x-\#1) + 6 \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]}{16(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (6*Log[x - #1] + a*Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]/(16*(12 + 7*a + a^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x^4 - 4x^3 + 8x^2 - a - 8x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)

maple [C] time = 0.01, size = 162, normalized size = 0.70

$$\frac{(-\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 + 2(-a - 2)\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) - a - 6)\ln(-\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) + x)}{16(a^2 + 7a + 12)\left(\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^3 - 3\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a)^2 + 4\text{RootOf}(-Z^4 - 4Z^3 + 8Z^2 - 8Z - a) - 2\right)} + \frac{-\frac{ax^2}{4(a^2+7a+12)} - \frac{x^3}{4(a^2+7a+12)} - \frac{a}{4(a^2+7a+12)} + \frac{(a-2)x}{4a^2+28a+48}}{x^4 - 4x^3 + 8x^2 - a - 8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+12)*x-1/4*a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a^2+7*a+12)*sum((-6-_R^2+2*(-a-2)*_R-a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(-Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ax^2 + x^3 - (a-2)x + a}{4((a^2 + 7a + 12)x^4 - 4(a^2 + 7a + 12)x^3 - a^3 + 8(a^2 + 7a + 12)x^2 - 7a^2 - 8(a^2 + 7a + 12)x - 12a)} - \frac{\int \frac{2(a+2)x+x^2+a+6}{x^4-4x^3+8x^2-a-8x} dx}{4(a^2 + 7a + 12)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)

mupad [B] time = 2.82, size = 1167, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(104448*a +

$$\frac{58880a^2 + 16512a^3 + 2304a^4 + 128a^5 + 73728)}{(16(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))} - \frac{(x(864a + 228a^2 + 20a^3 + 1088))}{(16(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))} + \frac{(x(9a + 2a^2 + 8))}{(16(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))} \cdot \text{root}(12952010752a^3z^4 + 31653888a^7z^4 + 2162688a^8z^4 + 65536a^9z^4 + 18119393280a^2z^4 + 20082327552a^2z^4 + 1473773568a^5z^4 + 5357174784a^4z^4 + 269680640a^6z^4 + 7247757312z^4 - 8699904a^2z^2 - 2842624a^3z^2 - 520704a^4z^2 - 50688a^5z^2 - 2048a^6z^2 - 14155776a^2z^2 - 9568256z^2 + 102912a^2z + 17792a^3z + 1152a^4z + 264192a^2z + 253952z - 984a - 57a^2 + 16a^3 - 2064, z, k), k, 1, 4) + \frac{x^3}{(4(7a + a^2 + 12))} + \frac{a}{(4(a + 3)(a + 4))} - \frac{(x(a - 2))}{(4(a + 3)(a + 4))} + \frac{(ax^2)}{(4(a + 3)(a + 4))} / (a + 8x - 8x^2 + 4x^3 - x^4)$$

sympy [B] time = 31.42, size = 539, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out]
$$\frac{(-ax^2 - a - x^3 + x(a - 2))}{(-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48))} + x^3 \frac{(-16a^2 - 112a - 192)}{(-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48))} + x^2 \frac{(32a^2 + 224a + 384)}{(-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48))} + x \frac{(-32a^2 - 224a - 384)}{(-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48))} + \text{RootSum}(_t^4(65536a^9 + 2162688a^8 + 31653888a^7 + 269680640a^6 + 1473773568a^5 + 5357174784a^4 + 12952010752a^3 + 20082327552a^2 + 18119393280a + 7247757312) + _t^2(-2048a^6 - 50688a^5 - 520704a^4 - 2842624a^3 - 8699904a^2 - 14155776a - 9568256) + _t(1152a^4 + 17792a^3 + 102912a^2 + 264192a + 253952) + 16a^3 - 57a^2 - 984a - 2064, \text{Lambda}(_t, _t \log(x + (98304_t^3a^{11} + 3948544_t^3a^{11} + 72196096_t^3a^{10} + 793837568_t^3a^9 + 5839372288_t^3a^8 + 30226464768_t^3a^7 + 112668450816_t^3a^6 + 303864643584_t^3a^5 + 586157391872_t^3a^4 + 784017129472_t^3a^3 + 683648483328_t^3a^2 + 343136010240_t^3a + 72477573120_t^3 + 30208_t^2a^{10} + 986624_t^2a^9 + 14420992_t^2a^8 + 124156928_t^2a^7 + 696815104_t^2a^6 + 2661758464_t^2a^5 + 7001485312_t^2a^4 + 12506562560_t^2a^3 + 14494924800_t^2a^2 + 9820569600_t^2a + 2944401408_t^2 - 1536_t a^9 - 52048_t a^8 - 757040_t a^7 - 6200656_t a^6 - 31380496_t a^5 - 100736416_t a^4 - 200813696_t a^3 - 228144640_t a^2 - 114632704_t a - 2490368_t + 248a^7 + 6797a^6 + 71132a^5 + 369745a^4 + 987758a^3 + 1128896a^2 - 129568a - 956416)/(576a^7 + 10985a^6 + 88746a^5 + 396609a^4 + 1076268a^3 + 1826304a^2 + 186776a + 917504))))$$

$$3.125 \quad \int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3} dx$$

Optimal. Leaf size=349

$$\frac{(x-1)(a+(x-1)^2+5) \left(3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80) \tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right) \right)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2 \sqrt{1-\sqrt{a+4}}} \quad 64(a+3)^2(a+4)^{5/2} \sqrt{1-\sqrt{a+4}}$$

Rubi [A] time = 0.37, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, number of rules / integrand size = 0.417, Rules used = {1680, 1673, 1092, 1178, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2} + \frac{3(7a^2+(4\sqrt{a+4}+47)a+14\sqrt{a+4}+80) \tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2} \sqrt{1-\sqrt{a+4}}} + \frac{3\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right) \tan^{-1}\left(\frac{x-1}{\sqrt{a+4}}\right)}{64(a+3)^2(a+4)^{5/2} \sqrt{a+4}} + \frac{3(x-1)^2+1}{16(a+4)^2(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{8(a+4)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{32(a+3)^2(a+4)^2(a-(x-1)^4-2(x-1)^2+3)} + \frac{3 \tanh^{-1}\left(\frac{x-1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1 + (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2*(-1 + x))/(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14*sqrt[4 + a] + a*(47 + 4*sqrt[4 + a]))*ArcTan[(-1 + x)/sqrt[1 - sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(14 + 4*a - (80 + 47*a + 7*a^2)/sqrt[4 + a])*ArcTan[(-1 + x)/sqrt[1 + sqrt[4 + a]]])/(64*(3 + a)^2*(4 + a)^2*sqrt[1 + sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/sqrt[4 + a]])/(16*(4 + a)^(5/2))

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1092

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1178

Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx &= \text{Subst} \left(\int \frac{1+x}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \frac{(5+a+(-1+x)^2)(-1+x)}{8(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^3} dx, x, -1+x \right) \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} - \frac{((6+a)(25+7a)+6(7+2a)(1-x))}{32(12+7a+a^2)^2(3+a-2(1-x)^2-(1-x)^4)^2} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)^2} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)^2} \\
&= \frac{1+(-1+x)^2}{8(4+a)(3+a-2(1-x)^2-(1-x)^4)^2} + \frac{3(1+(-1+x)^2)}{16(4+a)^2(3+a-2(1-x)^2-(1-x)^4)^2}
\end{aligned}$$

Mathematica [C] time = 0.16, size = 284, normalized size = 0.81

$$\frac{1}{128} \left[\frac{3\text{RootSum}[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a\&, \frac{4\#1^2 a \log(x-\#1) + 14\#1^2 \log(x-\#1) + 3a^2 \log(x-\#1) + 4\#1 a^2 \log(x-\#1) + 31a \log(x-\#1) + 16\#1 a \log(x-\#1) + 72 \log(x-\#1) + 8\#1 \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2}}{\&}] + \frac{4(a^2(6a^2 - 5x + 5) + a(12x^3 + 31x - 7) + 6(7x^3 - 12x^2 + 28x - 14))}{(a+3)^2(a+4)^2(a-x(x^3 - 4x^2 + 8x - 8))} + \frac{16(ax^2 - ax + a + x^3 + 2x)}{(a+3)(a+4)(a-x(x^3 - 4x^2 + 8x - 8))^2} \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3) + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (72*Log[x - #1] + 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1

] * #1 + 4 * a ^ 2 * Log [x - #1] * #1 + 14 * Log [x - #1] * #1 ^ 2 + 4 * a * Log [x - #1] * #1 ^ 2) / (- 2 + 4 * #1 - 3 * #1 ^ 2 + #1 ^ 3) &]) / (12 + 7 * a + a ^ 2) ^ 2) / 128

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + 8x - 8x^2 + 4x^3 - x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{(x^4 - 4x^3 + 8x^2 - a - 8x)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)

maple [C] time = 0.02, size = 405, normalized size = 1.16

$$\frac{3(2(2a+7)\text{RootOf}[Z^4-4Z^2+8Z^2-8Z-a]^2+3a^2+(a+2)\text{RootOf}[Z^4-4Z^2+8Z^2-8Z-a]+31a+72)\ln(-\text{RootOf}[Z^4-4Z^2+8Z^2-8Z-a]+a)}{128(a^4+14a^3+73a^2+168a+144)\sqrt{\text{RootOf}[Z^4-4Z^2+8Z^2-8Z-a]}} + \frac{3(a^2-4a-8)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-127a-792)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-227a-1668)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-301a-1104)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-367a-1088)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-571a-112)a^2}{32(a^4+14a^3+73a^2+168a+144)} + \frac{3(a^2-117a-102)a^2}{32(a^4+14a^3+73a^2+168a+144)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)

[Out] $-(3/16*(2*a+7)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6*2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*$

$$\frac{a+1008}{(a^4+14a^3+73a^2+168a+144)}x^2+\frac{3}{32}\frac{(3a^3-17a^2-40a+192)}{(a^4+14a^3+73a^2+168a+144)}x-\frac{3}{32}\frac{a(3a^2+7a-12)}{(a^4+14a^3+73a^2+168a+144)}\frac{\sum((72+2*(2a+7)*_R^2+4*(a^2+4a+2)*_R+3a^2+31a)/(_R^3-3*_R^2+4*_R-2)*\ln(-_R+x), _R=\text{RootOf}(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))}{(a^4+14a^3+73a^2+168a+144)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$\frac{8(2a^2+7a^2+6(a^2-8a-40)a^2-(29a^2-127a-792)a^2+73a^2-227a-1668)a^2-2(62a^2-103a-1104)a^2-9a^2-2(5a^2-26a+140a+1008)a^2-21a^2+3(3a^3-17a^2-40a+192)a^2+36a}{32(a^4+14a^3+73a^2+168a+144)a^2-8(a^4+14a^3+73a^2+168a+144)a^2+32(a^4+14a^3+73a^2+168a+144)a^2-2(62a^2-103a-1104)a^2-9a^2-2(5a^2-26a+140a+1008)a^2-21a^2+3(3a^3-17a^2-40a+192)a^2+36a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")

[Out]
$$\frac{-1/32*(6*(2a+7)*x^7+6*(a^2-8a-40)*x^6-(29a^2-127a-792)*x^5+(73a^2-227a-1668)*x^4-2*(62a^2-103a-1104)*x^3-9a^3-2*(5a^3-26a^2+140a+1008)*x^2-21a^2+3*(3a^3-17a^2-40a+192)*x+36a)}{((a^4+14a^3+73a^2+168a+144)*x^8-8*(a^4+14a^3+73a^2+168a+144)*x^7+32*(a^4+14a^3+73a^2+168a+144)*x^6+a^6-80*(a^4+14a^3+73a^2+168a+144)*x^5+14a^5-2*(a^5-50a^4-823a^3-4504a^2-10608a-9216)*x^4+73a^4+8*(a^5-2a^4-151a^3-1000a^2-2544a-2304)*x^3+168a^3-16*(a^5+10a^4+17a^3-124a^2-528a-576)*x^2+144a^2+16*(a^5+14a^4+73a^3+168a^2+144a)*x)-3/32*\text{integrate}((2*(2a+7)*x^2+3a^2+4*(a^2+4a+2)*x+31a+72)/(x^4-4*x^3+8*x^2-a-8*x),x)/(a^4+14a^3+73a^2+168a+144)}$$

mupad [B] time = 3.39, size = 2200, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+8*x-8*x^2+4*x^3-x^4)^3,x)

[Out]
$$\text{symsum}(\log(\text{root}(15003759578972160*a^8*z^4+54537151127224320*a^7*z^4+153980418717122560*a^6*z^4+334927734494986240*a^5*z^4+551152193655275520*a^4*z^4+664192984106926080*a^3*z^4+553362212027105280*a^2*z^4+5999532441600*a^{12}*z^4+527875908304896*a^{10}*z^4+284993413919539200*a*z^4+3206246773555200*a^9*z^4+14763950080*a^{14}*z^4+65757291479040*a^{11}*z^4+378493992960*a^{13}*z^4+268435456*a^{15}*z^4+68398419340689408*z^4-4718592*a^{10}*z^2-3648061440*a^8*z^2-286939938816*a^6*z^2-15023392948224*a*z^2-16752587046912*a^2*z^2-4764645457920*a^4*z^2-40022212608*a^7*z^2-11043392716800*a^3*z^2-1405437345792*a^5*z^2-196116480*a^9*z^2-6049461436416*z^2+5375877120*a^4*z+839890944*a^5*z+47542173696*a^2*z+72880128*a^6*z+2709504*a^7*z+20640890880*a^3*z+60827369472*a*z+33351008256*z-74027520*a-29249424*a^2-4706424*a^3-155601*a^4+20736*a^5-68345856,z,k)*((242823168*a+170044416*a^2+63509760*a^3+13340736*a^4+$$

$$\begin{aligned}
& (1494144a^5 + 69696a^6 + 144506880)/(16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) + \text{root}(15003759578972160a^8z^4 + 54537151127224320a^7z^4 + 153980418717122560a^6z^4 + 334927734494986240a^5z^4 + 551152193655275520a^4z^4 + 664192984106926080a^3z^4 + 553362212027105280a^2z^4 + 5999532441600a^{12}z^4 + 527875908304896a^{10}z^4 + 284993413919539200a^*z^4 + 3206246773555200a^9z^4 + 14763950080a^{14}z^4 + 65757291479040a^{11}z^4 + 378493992960a^{13}z^4 + 268435456a^{15}z^4 + 68398419340689408z^4 - 4718592a^{10}z^2 - 3648061440a^8z^2 - 286939938816a^6z^2 - 15023392948224a^*z^2 - 16752587046912a^2z^2 - 4764645457920a^4z^2 - 40022212608a^7z^2 - 11043392716800a^3z^2 - 1405437345792a^5z^2 - 196116480a^9z^2 - 6049461436416z^2 + 5375877120a^4z + 839890944a^5z + 47542173696a^2z + 72880128a^6z + 2709504a^7z + 20640890880a^3z + 60827369472a^*z + 3351008256z - 74027520a - 29249424a^2 - 4706424a^3 - 155601a^4 + 20736a^5 - 68345856, z, k) * (\text{root}(15003759578972160a^8z^4 + 54537151127224320a^7z^4 + 153980418717122560a^6z^4 + 334927734494986240a^5z^4 + 551152193655275520a^4z^4 + 664192984106926080a^3z^4 + 553362212027105280a^2z^4 + 5999532441600a^{12}z^4 + 527875908304896a^{10}z^4 + 284993413919539200a^*z^4 + 3206246773555200a^9z^4 + 14763950080a^{14}z^4 + 65757291479040a^{11}z^4 + 378493992960a^{13}z^4 + 268435456a^{15}z^4 + 68398419340689408z^4 - 4718592a^{10}z^2 - 3648061440a^8z^2 - 286939938816a^6z^2 - 15023392948224a^*z^2 - 16752587046912a^2z^2 - 4764645457920a^4z^2 - 40022212608a^7z^2 - 11043392716800a^3z^2 - 1405437345792a^5z^2 - 196116480a^9z^2 - 6049461436416z^2 + 5375877120a^4z + 839890944a^5z + 47542173696a^2z + 72880128a^6z + 2709504a^7z + 20640890880a^3z + 60827369472a^*z + 33351008256z - 74027520a - 29249424a^2 - 4706424a^3 - 155601a^4 + 20736a^5 - 68345856, z, k) * ((4290672328704a + 6001143054336a^2 + 5025917042688a^3 + 2800520003584a^4 + 1090200272896a^5 + 302556119040a^6 + 59862155264a^7 + 8275361792a^8 + 761266176a^9 + 41943040a^{10} + 1048576a^{11} + 1391569403904)/(16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x*(536334041088a + 750142881792a^2 + 628239630336a^3 + 350065000448a^4 + 136275034112a^5 + 37819514880a^6 + 7482769408a^7 + 1034420224a^8 + 95158272a^9 + 5242880a^{10} + 131072a^{11} + 173946175488))/(2048(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776))) - (73421291520a + 81260445696a^2 + 52393672704a^3 + 21688418304a^4 + 5977620480a^5 + 1096949760a^6 + 129245184a^7 + 8871936a^8 + 270336a^9 + 29444014080)/(16384(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) + (x*(2632974336a + 3015180288a^2 + 2008940544a^3 + 858243072a^4 + 243806208a^5 + 46055424a^6 + 5578752a^7 + 393216a^8 + 12288a^9 + 1019215872))/(2048(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776))) - (x*(10805760a + 7173504a^2 + 2539872a^3 + 505800a^4 + 53712a^5 + 2376a^6 + 6782976))/(2048(940032a + 1195776a^2 + 899328a^3
\end{aligned}$$

$$\begin{aligned}
& + 442864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} \\
& + 331776)) - (133812a + 56187a^2 + 10098a^3 + 648a^4 + 115776)/(16384 \\
& *(940032a + 1195776a^2 + 899328a^3 + 442864a^4 + 149208a^5 + 34833a^6 \\
& + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331776)) - (x*(1971a^2 - 1539a + \\
& 918a^3 + 108a^4 - 6372))/(2048*(940032a + 1195776a^2 + 899328a^3 + 442 \\
& 864a^4 + 149208a^5 + 34833a^6 + 5564a^7 + 582a^8 + 36a^9 + a^{10} + 331 \\
& 776)))*\text{root}(15003759578972160a^8z^4 + 54537151127224320a^7z^4 + 1539804 \\
& 18717122560a^6z^4 + 334927734494986240a^5z^4 + 551152193655275520a^4z \\
& ^4 + 664192984106926080a^3z^4 + 553362212027105280a^2z^4 + 599953244160 \\
& 0a^{12}z^4 + 527875908304896a^{10}z^4 + 284993413919539200a^9z^4 + 32062467 \\
& 73555200a^9z^4 + 14763950080a^{14}z^4 + 65757291479040a^{11}z^4 + 3784939 \\
& 92960a^{13}z^4 + 268435456a^{15}z^4 + 68398419340689408z^4 - 4718592a^{10} \\
& z^2 - 3648061440a^8z^2 - 286939938816a^6z^2 - 15023392948224a^5z^2 - 16 \\
& 752587046912a^2z^2 - 4764645457920a^4z^2 - 40022212608a^7z^2 - 110433 \\
& 92716800a^3z^2 - 1405437345792a^5z^2 - 196116480a^9z^2 - 604946143641 \\
& 6z^2 + 5375877120a^4z + 839890944a^5z + 47542173696a^2z + 72880128a^ \\
& ^6z + 2709504a^7z + 20640890880a^3z + 60827369472a^2z + 33351008256z \\
& - 74027520a - 29249424a^2 - 4706424a^3 - 155601a^4 + 20736a^5 - 683458 \\
& 56, z, k), k, 1, 4) + ((3*(7a^2 - 12a + 3a^3))/(32*(6a + a^2 + 9)*(8a \\
& + a^2 + 16)) - (3*x^7*(2a + 7))/(16*(168a + 73a^2 + 14a^3 + a^4 + 144)) \\
& + (x^2*(140a - 26a^2 + 5a^3 + 1008))/(16*(6a + a^2 + 9)*(8a + a^2 + 1 \\
& 6)) + (3*x*(40a + 17a^2 - 3a^3 - 192))/(32*(6a + a^2 + 9)*(8a + a^2 + \\
& 16)) + (3*x^6*(8a - a^2 + 40))/(16*(6a + a^2 + 9)*(8a + a^2 + 16)) - (x^ \\
& 5*(127a - 29a^2 + 792))/(32*(6a + a^2 + 9)*(8a + a^2 + 16)) - (x^3*(103 \\
& *a - 62a^2 + 1104))/(16*(6a + a^2 + 9)*(8a + a^2 + 16)) + (x^4*(227a - \\
& 73a^2 + 1668))/(32*(6a + a^2 + 9)*(8a + a^2 + 16)))/(16*a*x - x^2*(16*a \\
& - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 + \\
& x^8)
\end{aligned}$$

sympy [B] time = 88.09, size = 1102, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)

[Out]
$$\begin{aligned}
& -(-9a^3 - 21a^2 + 36a + x^7*(12a + 42) + x^6*(6a^2 - 48a - 240) \\
& + x^5*(-29a^2 + 127a + 792) + x^4*(73a^2 - 227a - 1668) + x^3*(-12 \\
& 4a^2 + 206a + 2208) + x^2*(-10a^3 + 52a^2 - 280a - 2016) + x*(9a^ \\
& *3 - 51a^2 - 120a + 576))/(32a^6 + 448a^5 + 2336a^4 + 5376a^3 + \\
& 4608a^2 + x^8*(32a^4 + 448a^3 + 2336a^2 + 5376a + 4608) + x^7*(- \\
& 256a^4 - 3584a^3 - 18688a^2 - 43008a - 36864) + x^6*(1024a^4 + 14 \\
& 336a^3 + 74752a^2 + 172032a + 147456) + x^5*(-2560a^4 - 35840a^3 \\
& - 186880a^2 - 430080a - 368640) + x^4*(-64a^5 + 3200a^4 + 52672a^ \\
& 3 + 288256a^2 + 678912a + 589824) + x^3*(256a^5 - 512a^4 - 38656a^
\end{aligned}$$

$$\begin{aligned}
& *3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704* \\
& a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a* \\
& *3 + 86016*a**2 + 73728*a)) - \text{RootSum}(_t**4*(268435456*a**15 + 14763950080* \\
& a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5 \\
& 27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545 \\
& 37151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 5 \\
& 5115219365275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 \\
& + 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611 \\
& 6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543 \\
& 7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a* \\
& *2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6 \\
& - 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 - \\
& 60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29 \\
& 249424*a**2 - 74027520*a - 68345856, \text{Lambda}(_t, _t*\log(x + (-469762048*_t** \\
& 3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292 \\
& 48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15 \\
& - 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931 \\
& 54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730 \\
& 0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_ \\
& t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a* \\
& *6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1 \\
& 97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098 \\
& 68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17 \\
& + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2* \\
& a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396 \\
& 56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t* \\
& *2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5 \\
& 61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498 \\
& 750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t** \\
& 2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t* \\
& a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9 \\
& 50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12 \\
& 5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 - \\
& 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a \\
& **3 - 10371340262965248*_t*a**2 - 7285247072796672*_t*a - 2270140431335424*_ \\
& _t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5 \\
& 0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a** \\
& 5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928 \\
& 92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a** \\
& 10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432* \\
& a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167 \\
& 60449024*a**2 + 7467334520832*a + 2773884911616))))))
\end{aligned}$$

$$3.126 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Optimal. Leaf size=210

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{2}{11} (3a^2 - 1536a + 20480) x^{11} - \frac{24}{5} (a^2 - 128a + 896) x^{10} - 4(15a^2 - 288a + 512) x^8 + \frac{8}{3} a (a^2 - 48a +$$

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$\frac{2}{11}(3a^2 - 1536a + 20480)x^{11} - \frac{24}{5}(a^2 - 128a + 896)x^{10} - 4(15a^2 - 288a + 512)x^8 + \frac{4}{7}(a^2 - 192a^2 - 1536a + 1024)x^7 + \frac{8}{3}a(a^2 - 48a + 128)x^6 + \frac{32}{5}(12 - a)a^2x^5 + 8a^3x^4 + \frac{a^4x^3}{3} + \frac{4}{15}(640 - a)x^{15} - \frac{8}{7}(464 - 3a)x^{14} + \frac{32}{13}(524 - 9a)x^{13} - \frac{8}{3}(928 - 35a)x^{12} + \frac{64}{9}(128 - 3a)(4 - a)x^9 + \frac{x^{19}}{9} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx &= \int (a^4 x^2 + 32a^3 x^3 - 32(-12 + a)a^2 x^4 + 16a(128 - 48a + a^2)x^5 - 4(-1024 + 1536a - 192a^2 + a^3)x^6 + 4(1024 - 1536a + 192a^2 - a^3)x^7 - 4(512 - 288a + 15a^2)x^8 + 64(128 - 3a)(4 - a)x^9 - 24(896 - 128a + a^2)x^{10} + 2(20480 - 1536a + 3a^2)x^{11} - 8(928 - 35a)x^{12} + 32(524 - 9a)x^{13} - 8(464 - 3a)x^{14} + 4(640 - a)x^{15} - 42x^{16} + 128x^{17} - 8x^{18} + x^{19}) dx \\ &= \frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{32}{5}(12 - a)a^2 x^5 + \frac{8}{3}a(128 - 48a + a^2)x^6 + \frac{4}{7}(1024 - 1536a + 192a^2 - a^3)x^7 - 4(512 - 288a + 15a^2)x^8 + \frac{64}{9}(128 - 3a)(4 - a)x^9 - \frac{24}{5}(896 - 128a + a^2)x^{10} + \frac{2}{11}(20480 - 1536a + 3a^2)x^{11} - \frac{8}{3}(928 - 35a)x^{12} + \frac{32}{13}(524 - 9a)x^{13} - \frac{8}{7}(464 - 3a)x^{14} + \frac{4}{15}(640 - a)x^{15} - 42x^{16} + \frac{128x^{17}}{17} - \frac{8x^{18}}{9} + \frac{x^{19}}{19} \end{aligned}$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{2}{11}(3a^2 - 1536a + 20480)x^{11} - \frac{24}{5}(a^2 - 128a + 896)x^{10} + \frac{64}{9}(3a^2 - 140a + 512)x^9 - 4(15a^2 - 288a + 512)x^8 + \frac{8}{3}a(a^2 - 48a + 128)x^6 - \frac{32}{5}(a - 12)a^2 x^5 - \frac{4}{7}(a^2 - 192a^2 + 1536a - 1024)x^7 - \frac{4}{15}(a - 640)x^{15} + \frac{8}{3}(3a - 464)x^{14} - \frac{32}{13}(9a - 524)x^{13} + \frac{8}{3}(35a - 928)x^{12} + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] $(a^4x^3)/3 + 8a^3x^4 - (32(-12 + a)a^2x^5)/5 + (8a(128 - 48a + a^2)x^6)/3 - (4(-1024 + 1536a - 192a^2 + a^3)x^7)/7 - 4(512 - 288a + 15a^2)x^8 + (64(512 - 140a + 3a^2)x^9)/9 - (24(896 - 128a + a^2)x^{10})/5 + (2(20480 - 1536a + 3a^2)x^{11})/11 + (8(-928 + 35a)x^{12})/3 - (32(-524 + 9a)x^{13})/13 + (8(-464 + 3a)x^{14})/7 - (4(-640 + a)x^{15})/15 - 42x^{16} + (128x^{17})/17 - (8x^{18})/9 + x^{19}/19$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^4 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]

[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]

fricas [A] time = 1.25, size = 222, normalized size = 1.06

$\frac{1}{19}x^{19} - \frac{8}{9}a^{18}x^{18} + \frac{128}{17}a^{17}x^{17} - 42a^{16}x^{16} + \frac{4}{15}a^{15}x^{15} + \frac{512}{3}a^{14}x^{14} - \frac{3712}{7}a^{13}x^{13} - \frac{288}{13}a^{12}x^{12} + \frac{16768}{13}a^{11}x^{11} + \frac{280}{3}a^{10}x^{10} + \frac{6}{11}a^9x^9 - \frac{7424}{3}a^8x^8 - \frac{3072}{11}a^7x^7 + \frac{24}{5}a^6x^6 + \frac{40960}{11}a^5x^5 + \frac{3072}{5}a^4x^4 + \frac{64}{3}a^3x^3 - \frac{21504}{5}a^2x^2 - \frac{8960}{9}ax - 60x^2 - \frac{4}{7}x^3 + \frac{32768}{9}x^4 + 1152a^2x^5 + \frac{768}{7}a^2x^6 + \frac{8}{3}a^2x^7 - 2048a^3x^8 - \frac{6144}{7}a^3x^9 - 128a^3x^{10} - \frac{32}{5}a^3x^{11} + \frac{4096}{7}a^3x^{12} + \frac{1024}{3}a^3x^{13} + \frac{384}{5}a^3x^{14} + \frac{1}{3}a^3x^{15}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")

[Out] $1/19x^{19} - 8/9x^{18} + 128/17x^{17} - 42x^{16} - 4/15x^{15}a + 512/3x^{15} + 24/7x^{14}a - 3712/7x^{14} - 288/13x^{13}a + 16768/13x^{13} + 280/3x^{12}a + 6/11x^{11}a^2 - 7424/3x^{12} - 3072/11x^{11}a - 24/5x^{10}a^2 + 40960/11x^{11} + 3072/5x^{10}a + 64/3x^9a^2 - 21504/5x^{10} - 8960/9x^9a - 60x^8a^2 - 4/7x^7a^3 + 32768/9x^9 + 1152x^8a + 768/7x^7a^2 + 8/3x^6a^3 - 2048x^8 - 6144/7x^7a - 128x^6a^2 - 32/5x^5a^3 + 4096/7x^7 + 1024/3x^6a + 384/5x^5a^2 + 8x^4a^3 + 1/3x^3a^4$

giac [A] time = 0.24, size = 222, normalized size = 1.06

$\frac{1}{19}x^{19} - \frac{8}{9}a^{18}x^{18} + \frac{128}{17}a^{17}x^{17} - \frac{4}{15}a^{15}x^{15} - 42a^{16}x^{16} + \frac{24}{7}a^{14}x^{14} + \frac{512}{3}a^{14}x^{14} - \frac{288}{13}a^{13}x^{13} - \frac{3712}{7}a^{13}x^{13} + \frac{6}{11}a^9x^9 + \frac{16768}{13}a^{11}x^{11} + \frac{280}{3}a^{10}x^{10} + \frac{6}{11}a^9x^9 - \frac{7424}{3}a^8x^8 - \frac{3072}{11}a^7x^7 + \frac{24}{5}a^6x^6 + \frac{40960}{11}a^5x^5 + \frac{3072}{5}a^4x^4 + \frac{64}{3}a^3x^3 - \frac{21504}{5}a^2x^2 - \frac{8960}{9}ax - 60x^2 - \frac{4}{7}x^3 + \frac{32768}{9}x^4 + 1152a^2x^5 + \frac{768}{7}a^2x^6 + \frac{8}{3}a^2x^7 - 2048a^3x^8 - \frac{6144}{7}a^3x^9 - 128a^3x^{10} - \frac{32}{5}a^3x^{11} + \frac{4096}{7}a^3x^{12} + \frac{1024}{3}a^3x^{13} + \frac{384}{5}a^3x^{14} + \frac{1}{3}a^3x^{15}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")

[Out] $1/19x^{19} - 8/9x^{18} + 128/17x^{17} - 4/15a*x^{15} - 42x^{16} + 24/7a*x^{14} + 512/3x^{15} - 288/13a*x^{13} - 3712/7x^{14} + 6/11a^2*x^{11} + 280/3a*x^{12} + 16768/13x^{13} - 24/5a^2*x^{10} - 3072/11a*x^{11} - 7424/3x^{12} + 64/3a^2*x^9 + 3072/5a*x^{10} + 40960/11x^{11} - 4/7a^3*x^7 - 60a^2*x^8 - 8960/9a*x^9 - 21504/5x^{10} + 8/3a^3*x^6 + 768/7a^2*x^7 + 1152a*x^8 + 32768/9x^9 - 32$

$$/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7$$

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}(a-640)x^{15} - 42x^{16} + \frac{8}{7}(3a-464)x^{14} - \frac{32}{13}(9a-524)x^{13} + \frac{8}{3}(35a-928)x^{12} + \frac{2}{11}(3a^2-1536a+20480)x^{11} - \frac{24}{5}(a^2-128a+896)x^{10} + \frac{64}{9}(3a^2-140a+512)x^9 - 4(15a^2-288a+512)x^8 - \frac{4}{7}(a^3-192a^2+1536a-1024)x^7 + \frac{1}{3}a^4x^3 + 8a^3x^4 + \frac{8}{3}(a^3-48a^2+128a)x^6 - \frac{32}{5}(a^3-12a^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)

[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14*(48*a-7424)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^12+1/11*(2*a^2-2560*a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^10+1/9*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-160*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^8+1/7*(2*(-2*a+128)*a^2+32*(8*a-128)*a+(-16*a+64)^2)*x^7+1/6*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^6+1/5*(2*(-16*a+64)*a^2+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3

maxima [A] time = 0.64, size = 182, normalized size = 0.87

$$\frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}(a-640)x^{15} - 42x^{16} + \frac{8}{7}(3a-464)x^{14} - \frac{32}{13}(9a-524)x^{13} + \frac{8}{3}(35a-928)x^{12} + \frac{2}{11}(3a^2-1536a+20480)x^{11} - \frac{24}{5}(a^2-128a+896)x^{10} + \frac{64}{9}(3a^2-140a+512)x^9 - 4(15a^2-288a+512)x^8 - \frac{4}{7}(a^3-192a^2+1536a-1024)x^7 + \frac{1}{3}a^4x^3 + 8a^3x^4 + \frac{8}{3}(a^3-48a^2+128a)x^6 - \frac{32}{5}(a^3-12a^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")

[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 140*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 - 32/5*(a^3 - 12*a^2)*x^5

mupad [B] time = 2.29, size = 178, normalized size = 0.85

$$x^{14} \left(\frac{24a}{7} - \frac{3712}{7} \right) - x^{15} \left(\frac{4a}{15} - \frac{512}{3} \right) + x^{12} \left(\frac{280a}{3} - \frac{7424}{3} \right) - x^{13} \left(\frac{288a}{13} - \frac{16768}{13} \right) - x^8 (60a^2 - 1152a + 2048) - x^{10} \left(\frac{24a^2}{5} - \frac{3072a}{5} + \frac{21504}{5} \right) + x^9 \left(\frac{64a^2}{3} - \frac{8960a}{9} + \frac{32768}{9} \right) + x^{11} \left(\frac{6a^2}{11} - \frac{3072a}{11} + \frac{40960}{11} \right) - x^7 \left(\frac{4a^3}{7} - \frac{768a^2}{7} + \frac{6144a}{7} - \frac{4096}{7} \right) - 42x^{16} + \frac{128x^{17}}{17} - \frac{8x^{18}}{9} + \frac{x^{19}}{19} + 8a^3x^4 + \frac{a^4x^3}{3} + \frac{8ax^6(a^2-48a+128)}{3} - \frac{32a^2x^5(a-12)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)

[Out] x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 7424/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*((24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768/9) + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*a^2)/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19 +

$$8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a - 12))/5$$

sympy [A] time = 0.15, size = 219, normalized size = 1.04

$$\frac{a^4 x^3}{3} + 8a^3 x^4 + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16} + x^{15} \left(\frac{512}{3} - \frac{4a}{15} \right) + x^{14} \left(\frac{24a}{7} - \frac{3712}{7} \right) + x^{13} \left(\frac{16768}{13} - \frac{288a}{13} \right) + x^{12} \left(\frac{280a}{3} - \frac{7424}{3} \right) + x^{11} \left(\frac{6a^2}{11} - \frac{3072a}{11} + \frac{40960}{11} \right) + x^{10} \left(\frac{24a^2}{5} - \frac{3072a}{5} - \frac{21504}{5} \right) + x^9 \left(\frac{64a^2}{3} - \frac{8960a}{9} + \frac{32768}{9} \right) + x^8 (-60a^2 + 1152a - 2048) + x^7 \left(\frac{4a^3}{7} + \frac{768a^2}{7} - \frac{6144a}{7} + \frac{4096}{7} \right) + x^6 \left(\frac{8a^3}{3} - 128a^2 + \frac{1024a}{3} \right) + x^5 \left(\frac{32a^3}{5} + \frac{384a^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)

[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16 + x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11) + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 32768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 - 6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3/5 + 384*a**2/5)

$$3.127 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Optimal. Leaf size=138

$$\frac{a^3 x^3}{3} - \frac{3}{7} (a^2 - 128a + 512) x^7 + \frac{2}{3} (3a^2 - 96a + 128) x^6 + 6a^2 x^4 - \frac{3}{11} (256 - a) x^{11} + \frac{12}{5} (64 - a) x^{10} - \frac{32}{9} (70 - 3a) x^9 + 6(48 - 5a) x^8 + \frac{24}{5} (8 - a) a x^5 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3}$$

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$-\frac{3}{7}(a^2-128a+512)x^7 + \frac{2}{3}(3a^2-96a+128)x^6 + 6a^2x^4 + \frac{a^3x^3}{3} - \frac{3}{11}(256-a)x^{11} + \frac{12}{5}(64-a)x^{10} - \frac{32}{9}(70-3a)x^9 + 6(48-5a)x^8 + \frac{24}{5}(8-a)ax^5 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx &= \int (a^3 x^2 + 24a^2 x^3 - 24(-8 + a)ax^4 + 4(128 - 96a + 3a^2)x^5 - 3(512 - 128a + a^2)x^6 - 3(256 - a)x^7 + 6(48 - 5a)x^8 - 32(70 - 3a)x^9 + 12(64 - a)x^{10} - 3(256 - a)x^{11} + 70x^{12} - 72x^{13} + 70x^{12}) dx \\ &= \frac{a^3 x^3}{3} + 6a^2 x^4 + \frac{24}{5}(8 - a)ax^5 + \frac{2}{3}(128 - 96a + 3a^2)x^6 - \frac{3}{7}(512 - 128a + a^2)x^7 + 6(48 - 5a)x^8 - \frac{32}{9}(70 - 3a)x^9 + \frac{12}{5}(64 - a)x^{10} - \frac{3}{11}(256 - a)x^{11} + \frac{70}{3}x^{12} - \frac{72}{13}x^{13} + \frac{70}{3}x^{12} \end{aligned}$$

Mathematica [A] time = 0.02, size = 132, normalized size = 0.96

$$\frac{a^3 x^3}{3} - \frac{3}{7} (a^2 - 128a + 512) x^7 + \frac{2}{3} (3a^2 - 96a + 128) x^6 + 6a^2 x^4 + \frac{3}{11} (a - 256) x^{11} - \frac{12}{5} (a - 64) x^{10} + \frac{32}{9} (3a - 70) x^9 - 6(5a - 48) x^8 - \frac{24}{5} (a - 8) a x^5 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] $(a^3x^3)/3 + 6a^2x^4 - (24*(-8 + a)*ax^5)/5 + (2*(128 - 96a + 3a^2)*x^6)/3 - (3*(512 - 128a + a^2)*x^7)/7 - 6*(-48 + 5a)*x^8 + (32*(-70 + 3a)*x^9)/9 - (12*(-64 + a)*x^{10})/5 + (3*(-256 + a)*x^{11})/11 + (70*x^{12})/3 - (7*2*x^{13})/13 + (6*x^{14})/7 - x^{15}/15$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]

[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]

fricas [A] time = 0.91, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{70}{3}x^{12} + \frac{3}{11}x^{11}a - \frac{768}{11}x^{11} - \frac{12}{5}x^{10}a + \frac{768}{5}x^{10} + \frac{32}{3}x^9a - \frac{2240}{9}x^9 - 30x^8a - \frac{3}{7}x^7a^2 + 288x^8 + \frac{384}{7}x^7a + 2x^6a^2 - \frac{1536}{7}x^7 - 64x^6a - \frac{24}{5}x^5a^2 + \frac{256}{3}x^6 + \frac{192}{5}x^5a + 6x^4a^2 + \frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 70/3*x^{12} + 3/11*x^{11}*a - 768/11*x^{11} - 12/5*x^{10}*a + 768/5*x^{10} + 32/3*x^9*a - 2240/9*x^9 - 30*x^8*a - 3/7*x^7*a^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536/7*x^7 - 64*x^6*a - 24/5*x^5*a^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^4*a^2 + 1/3*x^3*a^3$

giac [A] time = 0.29, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}ax^{11} + \frac{70}{3}x^{12} - \frac{12}{5}ax^{10} - \frac{768}{11}x^{11} + \frac{32}{3}ax^9 + \frac{768}{5}x^{10} - \frac{3}{7}a^2x^7 - 30ax^8 - \frac{2240}{9}x^9 + 2a^2x^6 + \frac{384}{7}ax^7 + 288x^8 - \frac{24}{5}a^2x^5 - 64ax^6 - \frac{1536}{7}x^7 + \frac{1}{3}a^3x^3 + 6a^2x^4 + \frac{192}{5}ax^5 + \frac{256}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 3/11*a*x^{11} + 70/3*x^{12} - 12/5*a*x^{10} - 768/11*x^{11} + 32/3*a*x^9 + 768/5*x^{10} - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6$

maple [A] time = 0.00, size = 143, normalized size = 1.04

$$\frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + \frac{(3a-768)x^{11}}{11} + \frac{(-24a+1536)x^{10}}{10} + \frac{(96a-2240)x^9}{9} + \frac{(-240a+2304)x^8}{8} + \frac{(-a^2+(-2a+128)a+256a-1536)x^7}{7} + \frac{a^3x^3}{3} + 6a^2x^4 + \frac{(4a^2+(8a-128)a-256a+512)x^6}{6} + \frac{(-8a^2+(-16a+64)a+128a)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)`

[Out] $-1/15*x^{15}+6/7*x^{14}-72/13*x^{13}+70/3*x^{12}+1/11*(3*a-768)*x^{11}+1/10*(-24*a+1536)*x^{10}+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*(-a^2+(-2*a+128)*a+256*a-1536)*x^7+1/6*(4*a^2+(8*a-128)*a-256*a+512)*x^6+1/5*(-8*a^2+(-16*a+64)*a+128*a)*x^5+6*a^2*x^4+1/3*a^3*x^3$

maxima [A] time = 0.56, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}(a-256)x^{11} + \frac{70}{3}x^{12} - \frac{12}{5}(a-64)x^{10} + \frac{32}{9}(3a-70)x^9 - 6(5a-48)x^8 - \frac{3}{7}(a^2-128a+512)x^7 + \frac{2}{3}(3a^2-96a+128)x^6 + \frac{1}{3}a^3x^3 + 6a^2x^4 - \frac{24}{5}(a^2-8a)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")`

[Out] $-1/15*x^{15} + 6/7*x^{14} - 72/13*x^{13} + 3/11*(a - 256)*x^{11} + 70/3*x^{12} - 12/5*(a - 64)*x^{10} + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*(a^2 - 8*a)*x^5$

mupad [B] time = 0.09, size = 113, normalized size = 0.82

$$x^{11} \left(\frac{3a}{11} - \frac{768}{11} \right) - x^{10} \left(\frac{12a}{5} - \frac{768}{5} \right) - x^8 (30a - 288) + x^9 \left(\frac{32a}{3} - \frac{2240}{9} \right) + x^6 \left(2a^2 - 64a + \frac{256}{3} \right) - x^7 \left(\frac{3a^2}{7} - \frac{384a}{7} + \frac{1536}{7} \right) + \frac{70x^{12}}{3} - \frac{72x^{13}}{13} + \frac{6x^{14}}{7} - \frac{x^{15}}{15} + 6a^2x^4 + \frac{a^3x^3}{3} - \frac{24ax^5(a-8)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)`

[Out] $x^{11}*((3*a)/11 - 768/11) - x^{10}*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a)/7 + 1536/7) + (70*x^{12})/3 - (72*x^{13})/13 + (6*x^{14})/7 - x^{15}/15 + 6*a^2*x^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5$

sympy [A] time = 0.10, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3} + 6a^2x^4 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + x^{11} \left(\frac{3a}{11} - \frac{768}{11} \right) + x^{10} \left(\frac{12a}{5} - \frac{768}{5} \right) + x^9 \left(\frac{32a}{3} - \frac{2240}{9} \right) + x^8 (288 - 30a) + x^7 \left(-\frac{3a^2}{7} + \frac{384a}{7} - \frac{1536}{7} \right) + x^6 \left(2a^2 - 64a + \frac{256}{3} \right) + x^5 \left(-\frac{24a^2}{5} + \frac{192a}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)`

[Out] $a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2 - 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)$

$$3.128 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3} + \frac{2}{7}(64-a)x^7 - \frac{4}{3}(16-a)x^6 + \frac{16}{5}(4-a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$\frac{a^2x^3}{3} + \frac{2}{7}(64-a)x^7 - \frac{4}{3}(16-a)x^6 + \frac{16}{5}(4-a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 - a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx &= \int (a^2x^2 + 16ax^3 - 16(-4 + a)x^4 + 8(-16 + a)x^5 - 2(-64 + a)x^6 - 80x^7 \\ &= \frac{a^2x^3}{3} + 4ax^4 + \frac{16}{5}(4-a)x^5 - \frac{4}{3}(16-a)x^6 + \frac{2}{7}(64-a)x^7 - 10x^8 + \frac{32x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} - \frac{2}{7}(a-64)x^7 + \frac{4}{3}(a-16)x^6 - \frac{16}{5}(a-4)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

fricas [A] time = 0.93, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - 10x^8 - \frac{2}{7}x^7a + \frac{128}{7}x^7 + \frac{4}{3}x^6a - \frac{64}{3}x^6 - \frac{16}{5}x^5a + \frac{64}{5}x^5 + 4x^4a + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7 + 4/3*x^6*a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^3*a^2

giac [A] time = 0.29, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}ax^7 - 10x^8 + \frac{4}{3}ax^6 + \frac{128}{7}x^7 - \frac{16}{5}ax^5 - \frac{64}{3}x^6 + \frac{1}{3}a^2x^3 + 4ax^4 + \frac{64}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \frac{(-2a + 128)x^7}{7} + \frac{(8a - 128)x^6}{6} + \frac{a^2x^3}{3} + 4ax^4 + \frac{(-16a + 64)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3

maxima [A] time = 0.55, size = 59, normalized size = 0.75

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}(a - 64)x^7 - 10x^8 + \frac{4}{3}(a - 16)x^6 - \frac{16}{5}(a - 4)x^5 + \frac{1}{3}a^2x^3 + 4ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^6 \left(\frac{4a}{3} - \frac{64}{3} \right) - x^5 \left(\frac{16a}{5} - \frac{64}{5} \right) - x^7 \left(\frac{2a}{7} - \frac{128}{7} \right) + 4ax^4 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)

[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3

sympy [A] time = 0.08, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + x^7 \left(\frac{128}{7} - \frac{2a}{7} \right) + x^6 \left(\frac{4a}{3} - \frac{64}{3} \right) + x^5 \left(\frac{64}{5} - \frac{16a}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)

$$3.129 \quad \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Optimal. Leaf size=35

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {14}

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx &= \int (ax^2 + 8x^3 - 8x^4 + 4x^5 - x^6) dx \\ &= \frac{ax^3}{3} + 2x^4 - \frac{8x^5}{5} + \frac{2x^6}{3} - \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + 8x - 8x^2 + 4x^3 - x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

fricas [A] time = 1.06, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + 2x^4 + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="fricas")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a

giac [A] time = 0.31, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x), x, algorithm="giac")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x), x)

[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4 + \frac{ax^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)

[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)

[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4

$$3.130 \quad \int \frac{x^2}{a+8x-8x^2+4x^3-x^4} dx$$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1680, 1673, 1166, 204, 12, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}} - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a + 8x - 8x^2 + 4x^3 - x^4} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{2x}{3+a-2x^2-x^4} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{1+x^2}{3+a-2x^2-x^4} dx, x, -1+x \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1-\sqrt{4+a}-x^2} dx, x, -1+x \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+\sqrt{4+a}-x^2} dx, x, -1+x \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} + \text{Subst} \left(\int \frac{1}{3+a-2x-x^2} dx, x, (-1+x) \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} - 2 \text{Subst} \left(\int \frac{1}{4(4+a)-x^2} dx, x, -2(1+x) \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}} \right)}{2\sqrt{1-\sqrt{4+a}}} + \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}} \right)}{2\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1} \left(\frac{1+(-1+x)^2}{\sqrt{4+a}} \right)}{\sqrt{4+a}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.62

$$-\frac{1}{4} \text{RootSum} \left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a \&, \frac{\#1^2 \log(x - \#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + 8x - 8x^2 + 4x^3 - x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

[Out] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")

[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

maple [C] time = 0.00, size = 52, normalized size = 0.53

$$\frac{\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^2 \ln(-\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) + x)}{4(\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^3 - 3\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a)^2 + 4\text{RootOf}(_Z^4 - 4_Z^3 + 8_Z^2 - 8_Z - a) - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)

[Out] -1/4*sum(_R^2/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4 - 4x^3 + 8x^2 - a - 8x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")

[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)

mupad [B] time = 2.78, size = 878, normalized size = 8.87

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x)$

[Out] $\text{symsum}(\log(64*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k) - a - 8*x + 20*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a - 48*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a + 64*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a + 128*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x - 256*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 192*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*\text{root}(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)$

sympy [B] time = 7.61, size = 172, normalized size = 1.74

$-\text{RootSum}\left(t^4(256a^3 + 2816a^2 + 10240a + 12288) + t^2(-160a^2 - 1152a - 2048) + t(-32a^2 - 256a - 512) - a^2, \left(t \mapsto t \log\left(x + \frac{-64t^3a^4 - 448t^3a^3 - 256t^3a^2 + 3584t^3a + 6144t^3 - 224t^2a^3 - 2208t^2a^2 - 7168t^2a - 7680t^2 + 56t^2 + 400ta^2 + 864ta + 512t + 5a^3 + 34a^2 + 56a}{a^3 + 60a^2 + 320a + 448}\right)\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2/(-x**4+4*x**3-8*x**2+a+8*x), x)$

[Out] $-\text{RootSum}(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2 - 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, \text{Lambda}(_t, _t*\log(x + (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t**2*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 + 60*a**2 + 320*a + 448))))$

$$3.131 \quad \int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2} dx$$

Optimal. Leaf size=225

$$\frac{(a+4)((x-1)^2+2)(x-1)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(a+\sqrt{a+4}+4)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1680, 1673, 1178, 1166, 204, 12, 1107, 614, 618, 206}

$$\frac{(a+4)((x-1)^2+2)(x-1)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)} + \frac{(x-1)^2+1}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)} - \frac{(a+\sqrt{a+4}+4)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}} - \frac{(a-\sqrt{a+4}+4)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}} + \frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]

[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
  x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /;
EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx &= \text{Subst} \left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \text{Subst} \left(\int \frac{2x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{1+x^2}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{(4+a)(2+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + 2 \text{Subst} \left(\int \frac{x}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right) \\
&= \frac{(4+a)(2+(-1+x)^2)(-1+x)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a-\sqrt{4+a}) \text{Subst} \left(\int \frac{1}{(3+a-2x^2-x^4)^2} dx, x, -1+x \right)}{8(12+7a+a^2)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)} \\
&= \frac{1+(-1+x)^2}{2(4+a)(3+a-2(-1+x)^2-(-1+x)^4)} + \frac{(4+a)(2+(-1+x)^2)}{4(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 182, normalized size = 0.81

$$\frac{a(x^3 - x^2 + x + 1) + 2x(2x^2 - 3x + 4)}{4(a+3)(a+4)(a-x(x^3 - 4x^2 + 8x - 8))} - \frac{\text{RootSum}\left[-\#1^4 + 4\#1^3 - 8\#1^2 + 8\#1 + a\&, \frac{\#1^2 a \log(x-\#1) + 4\#1^2 \log(x-\#1) + 2\#1 a \log(x-\#1) - a \log(x-\#1) + 4\#1 \log(x-\#1)}{\#1^3 - 3\#1^2 + 4\#1 - 2}\right]}{16(a^2 + 7a + 12)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 &, (-a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) &]/(16*(12 + 7*a + a^2))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

[Out] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^4 - 4x^3 + 8x^2 - a - 8x)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")

[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)

maple [C] time = 0.01, size = 160, normalized size = 0.71

$$\frac{(-a-4)\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)^2+2(-a-2)\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)+a)\ln(-\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)+x)}{16(a+3)(a+4)(\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)^3-3\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)^2+4\text{RootOf}(_Z^4-4_Z^3+8_Z^2-8_Z-a)-2)} + \frac{-\frac{x^3}{4(a+3)} + \frac{(a+6)x^2}{4(a+3)(a+4)} - \frac{a}{4(a+3)(a+4)} - \frac{(a+8)x}{4(a+3)(a+4)}}{x^4-4x^3+8x^2-a-8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)

[Out] (-1/4/(a+3)*x^3+1/4*(a+6)/(a+3)/(a+4)*x^2-1/4*(a+8)/(a+3)/(a+4)*x-1/4*a/(a+3)/(a+4))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a+3)/(a+4)*sum(((a-4)*_R^2+2*(a-2)*_R+a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a+4)x^3 - (a+6)x^2 + (a+8)x + a}{4((a^2+7a+12)x^4 - 4(a^2+7a+12)x^3 - a^3 + 8(a^2+7a+12)x^2 - 7a^2 - 8(a^2+7a+12)x - 12a)} - \frac{\int \frac{(a+4)x^2+2(a+2)x-a}{x^4-4x^3+8x^2-a-8x} dx}{4(a^2+7a+12)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")

[Out] -1/4*((a+4)*x^3 - (a+6)*x^2 + (a+8)*x + a)/((a^2+7*a+12)*x^4 - 4*(a^2+7*a+12)*x^3 - a^3 + 8*(a^2+7*a+12)*x^2 - 7*a^2 - 8*(a^2+7*a+12)*x - 12*a) - 1/4*integrate(((a+4)*x^2 + 2*(a+2)*x - a)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)

mupad [B] time = 2.85, size = 1218, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+8*x-8*x^2+4*x^3-x^4)^2,x)

[Out] symsum(log((x*(40*a+7*a^2+56))/(8*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) - (48*a+12*a^2-a^3)/(64*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) - root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-24215552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k)*((28160*a+11328*a^2+2064*a^3+144*a^4+26624)/(64*(816*a+460*a^2+129*a^3+18*a^4+a^5+576)) + root(12952010752*a^3*z^4+31653888*a^7*z^4+2162688*a^8*z^4+65536*a^9*z^4+18119393280*a*z^4+20082327552*a^2*z^4+1473773568*a^5*z^4+5357174784*a^4*z^4+269680640*a^6*z^4+7247757312*z^4-24215552*a^2*z^2-8986624*a^3*z^2-1878016*a^4*z^2-209408*a^5*z^2-9728*a^6*z^2-34865152*a*z^2-20971520*z^2+237568*a^2*z+53248*a^3*z+5888*a^4*z+256*a^5*z+524288*a*z+458752*z+1792*a+1024*a^2+144*a^3-a^4, z, k))

$$\begin{aligned}
& 247757312z^4 - 24215552a^2z^2 - 8986624a^3z^2 - 1878016a^4z^2 - 209408a^5z^2 - 9728a^6z^2 - 34865152az^2 - 20971520z^2 + 237568a^2z + 53248a^3z + 5888a^4z + 256a^5z + 524288az + 458752z + 1792a + 1024a^2 + 144a^3 - a^4, z, k) \cdot (\text{root}(12952010752a^3z^4 + 31653888a^7z^4 + 2162688a^8z^4 + 65536a^9z^4 + 18119393280az^4 + 20082327552a^2z^4 + 1473773568a^5z^4 + 5357174784a^4z^4 + 269680640a^6z^4 + 7247757312z^4 - 24215552a^2z^2 - 8986624a^3z^2 - 1878016a^4z^2 - 209408a^5z^2 - 9728a^6z^2 - 34865152az^2 - 20971520z^2 + 237568a^2z + 53248a^3z + 5888a^4z + 256a^5z + 524288az + 458752z + 1792a + 1024a^2 + 144a^3 - a^4, z, k) \cdot ((15728640a + 10878976a^2 + 3997696a^3 + 823296a^4 + 90112a^5 + 4096a^6 + 9437184)/(64(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) - (x(1966080a + 1359872a^2 + 499712a^3 + 102912a^4 + 11264a^5 + 512a^6 + 1179648))/(8(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) - (1359872a + 749568a^2 + 205824a^3 + 28160a^4 + 1536a^5 + 983040)/(64(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576)) + (x(104448a + 58880a^2 + 16512a^3 + 2304a^4 + 128a^5 + 73728))/(8(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) + (x(448a + 104a^2 - 2a^3 - 2a^4 + 512))/(8(816a + 460a^2 + 129a^3 + 18a^4 + a^5 + 576))) \cdot \text{root}(12952010752a^3z^4 + 31653888a^7z^4 + 2162688a^8z^4 + 65536a^9z^4 + 18119393280az^4 + 20082327552a^2z^4 + 1473773568a^5z^4 + 5357174784a^4z^4 + 269680640a^6z^4 + 7247757312z^4 - 24215552a^2z^2 - 8986624a^3z^2 - 1878016a^4z^2 - 209408a^5z^2 - 9728a^6z^2 - 34865152az^2 - 20971520z^2 + 237568a^2z + 53248a^3z + 5888a^4z + 256a^5z + 524288az + 458752z + 1792a + 1024a^2 + 144a^3 - a^4, z, k), k, 1, 4) + (x^3/(4(a + 3)) + a/(4(a + 3)(a + 4)) - (x^2(a + 6))/(4(a + 3)(a + 4)) + (x(a + 8))/(4(a + 3)(a + 4)))/(a + 8x - 8x^2 + 4x^3 - x^4)
\end{aligned}$$

sympy [B] time = 43.73, size = 561, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)

[Out] $(-a + x^{**3}(-a - 4) + x^{**2}(a + 6) + x(-a - 8))/(-4a^{**3} - 28a^{**2} - 48a + x^{**4}(4a^{**2} + 28a + 48) + x^{**3}(-16a^{**2} - 112a - 192) + x^{**2}(32a^{**2} + 224a + 384) + x(-32a^{**2} - 224a - 384)) + \text{RootSum}(_t^{**4}(65536a^{**9} + 2162688a^{**8} + 31653888a^{**7} + 269680640a^{**6} + 1473773568a^{**5} + 5357174784a^{**4} + 12952010752a^{**3} + 20082327552a^{**2} + 18119393280a + 7247757312) + _t^{**2}(-9728a^{**6} - 209408a^{**5} - 1878016a^{**4} - 8986624a^{**3} - 24215552a^{**2} - 34865152a - 20971520) + _t(256a^{**5} + 5888a^{**4} + 53248a^{**3} + 237568a^{**2} + 524288a + 458752) - a^{**4} + 144a^{**3} + 1024a^{**2} + 1792a, \text{Lambd}a(_t, _t \cdot \log(x + (4096 \cdot _t^{**3}a^{**12} - 61440 \cdot _t^{**3}a^{**11} - 5480448 \cdot _t^{**3}a^{**10} - 111403008 \cdot _t^{**3}a^{**9} - 1227173888 \cdot _t^{**3}a^{**8} - 8682876928 \cdot _t^{**3}a^{**7} - 42187440128 \cdot _t^{**3}a^{**6} - 144630284288 \cdot _t^{**3}a^{**5} - 350972280832 \cdot _t^{**3}a^{**4}$

$$\begin{aligned}
& - 591750234112*_t^{**3}*a^{**3} - 660716126208*_t^{**3}*a^{**2} - 439848271872*_t^{**3}*a \\
& - 132271570944*_t^{**3} - 28672*_t^{**2}*a^{**10} - 993280*_t^{**2}*a^{**9} - 15400960*_t \\
& **2*a^{**8} - 140742656*_t^{**2}*a^{**7} - 839462912*_t^{**2}*a^{**6} - 3414427648*_t^{**2}*a \\
& **5 - 9590087680*_t^{**2}*a^{**4} - 18363547648*_t^{**2}*a^{**3} - 22938255360*_t^{**2}*a \\
& *2 - 16873684992*_t^{**2}*a - 5549064192*_t^{**2} - 848*_t*a^{**9} - 6096*_t*a^{**8} + \\
& 174608*_t*a^{**7} + 3323792*_t*a^{**6} + 26276224*_t*a^{**5} + 119009280*_t*a^{**4} + 3 \\
& 32017664*_t*a^{**3} + 566497280*_t*a^{**2} + 544112640*_t*a + 225837056*_t + 11*a \\
& **8 + 958*a^{**7} + 17419*a^{**6} + 142964*a^{**5} + 632632*a^{**4} + 1567552*a^{**3} + 20 \\
& 49792*a^{**2} + 1100800*a)/(a^{**8} + 870*a^{**7} + 18289*a^{**6} + 165176*a^{**5} + 82456 \\
& 0*a^{**4} + 2452288*a^{**3} + 4340224*a^{**2} + 4229120*a + 1748992)))))
\end{aligned}$$

$$3.132 \quad \int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=545

$$\frac{\sqrt[3]{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b \right) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) \left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right) (-1)^{2/3}}{3\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} - 9\sqrt{3} a^{5/6} b^2 c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} \quad 3\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

Rubi [A] time = 1.48, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, number of rules / integrand size = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\sqrt[3]{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b \right) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) \left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right) (-1)^{2/3}}{3\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} - 9\sqrt{3} a^{5/6} b^2 c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} \quad 3\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{5/6} b^2 c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c^(1/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{-(-1)^{2/3} \sqrt[3]{a} - \sqrt[3]{c} x}{59049 (1 + \sqrt[3]{-1})^2 a^{20/3} bc^{2/3} (-3a + 3\sqrt[3]{-1} a^{2/3} x)} \right. \\
&= \frac{\int \frac{-\sqrt[3]{a} - \sqrt[3]{c} x}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{9a^{2/3} bc^{2/3}} - \frac{(-1)^{2/3} \int \frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{c} x}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{9a^{2/3} bc^{2/3}} + \dots \\
&= \frac{\left(3 - \frac{2b}{\sqrt[3]{a} c^{2/3}}\right) \int \frac{1}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{18b^2} + \frac{\left(3 - \frac{2(-1)^{2/3} b}{\sqrt[3]{a} c^{2/3}}\right) \int \frac{1}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{18b^2} \\
&= -\frac{\log\left(3a + 3a^{2/3} \sqrt[3]{c} x + bx^2\right)}{18a^{2/3} b^2 \sqrt[3]{c}} + \frac{\log\left(3a - 3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} x + bx^2\right)}{6(1 + \sqrt[3]{-1})^2 a^{2/3} b^2 \sqrt[3]{c}} \\
&= -\frac{\left(3ib + \sqrt{3} (b + 3\sqrt[3]{a} c^{2/3})\right) \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{27a^{5/6} b^2 \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.18

$$\frac{1}{3} \text{RootSum}\left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\#1^3 \log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] IntegrateAlgebraic[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="giac")

[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.02, size = 93, normalized size = 0.17

$$\frac{\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) \ln(-\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) + x)}{6\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^5 b^3 + 36\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^4 a^2 b^2 + 81\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^3 a^2 c + 54\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^2 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x)

[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x), _R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)
```

mupad [B] time = 3.18, size = 1563, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^9*c^3
```

```

4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b
^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1,
z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489*
a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 53144
1*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^
4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*
a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*
b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

[Out] Timed out

$$3.133 \quad \int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=487

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{18\left(1 + \sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54a^{4/3}bc^{2/3}}$$

Rubi [A] time = 0.76, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{18\left(1 + \sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x - 2bx}{\sqrt{3}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{c}x^2}}\right)}{3\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{2/3}b\sqrt[3]{c}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{c}x^2}} - \frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}x + 2bx}{\sqrt{3}\sqrt{4b - 3\sqrt[3]{c}x^2}}\right)}{9\sqrt{3}a^{2/3}b\sqrt[3]{c}\sqrt{4b - 3\sqrt[3]{c}x^2}} + \frac{\sqrt{-1} \tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x - 2bx}{\sqrt{3}\sqrt{4b - 3\sqrt[3]{-1}\sqrt[3]{c}x^2}}\right)}{3\sqrt{3}\left(1 - \sqrt[3]{-1}\right)\left(1 + \sqrt[3]{-1}\right)^2 a^{2/3}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{c}x^2 + 4b}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/((18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/((54*a^(4/3)*b*c^(2/3)))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{(-1)^{2/3}x}{177147(1 + \sqrt[3]{-1})^2 a^{22/3}c^{2/3}(-3a + 3\sqrt[3]{-1}a^2)} \right. \\
&= \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \dots \\
&= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{54a^{4/3}bc^{2/3}} - \dots \\
&= \frac{\log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3} \log(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2)}{18(1 + \sqrt[3]{-1})^2 a^{4/3}bc^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 99, normalized size = 0.20

$$\frac{1}{3} \text{RootSum}\left[\#1^6b^3 + 9\#1^4ab^2 + 27\#1^3a^2c + 27\#1^2a^2b + 27a^3\&, \frac{\#1^2 \log(x - \#1)}{2\#1^4b^3 + 12\#1^2ab^2 + 27\#1a^2c + 18a^2b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] IntegrateAlgebraic[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 93, normalized size = 0.19

$$\frac{\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3) \ln(-\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3) + x)}{6\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3)^5 b^3 + 36\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3)^4 a^2 b^2 + 81\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3)^3 a^2 c + 54\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2aZ^2 + 27a^3)^2 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x), _R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 3.10, size = 1354, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3), x)

[Out] symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 129140163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 - 94143178827*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^7*c + 4374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^9*c - 59049*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^6*c^2*x - 531441*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^8*c^2*x - 688747536*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^10*b^12*c^2*x + 1162261467*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^11*b^9*c^4*x - 20920706406*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^11*c^4*x)*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1

$4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k), k, 1, 6)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.134 \quad \int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{9\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{11/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}}\right)}{27\sqrt{3} a^{11/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} + \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3(-1)^{2/3} \sqrt[3]{-1} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{9\sqrt{3} (1 - \sqrt[3]{-1}) (1 + \sqrt[3]{-1})^2}$$

Rubi [A] time = 0.47, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2097, 618, 204}

$$\frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{9\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{11/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}}\right)}{27\sqrt{3} a^{11/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} + \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{3(-1)^{2/3} \sqrt[3]{-1} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{9\sqrt{3} (1 - \sqrt[3]{-1}) (1 + \sqrt[3]{-1})^2 a^{11/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p))*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx = (19683a^6) \int \left(\frac{(-1)^{2/3}}{177147 (1 + \sqrt[3]{-1})^2 a^{22/3} c^{2/3} (-3a + 3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} x + bx^2)} \right) dx$$

$$= \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2} dx}{27a^{4/3}c^{2/3}} + \dots$$

$$= \frac{2 \operatorname{Subst} \left(\int \frac{1}{-3a(4b-3\sqrt[3]{a}c^{2/3})-x^2} dx, x, 3a^{2/3}\sqrt[3]{c} + 2bx \right)}{27a^{4/3}c^{2/3}} - \dots$$

$$= \frac{2(-1)^{2/3} \tan^{-1} \left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} \right)}{9\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{11/6} \sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} c^{2/3}} + \frac{2 \tan^{-1} \dots}{27\sqrt{3} a^{11/6} \dots}$$

Mathematica [C] time = 0.06, size = 97, normalized size = 0.29

$$\frac{1}{3} \operatorname{RootSum} \left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\#1 \log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] IntegrateAlgebraic[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x, algorithm="giac")

[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 93, normalized size = 0.28

$$\frac{\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) \ln(-\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) + x)}{6\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^5 b^3 + 36\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^4 a^2 b^2 + 81\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^3 a^2 c + 54\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^2 a^2 c + 27\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3), x)

[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x), _R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 3.34, size = 825, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)

[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 - 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3*c^3*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3 + 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k), k, 1, 6)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.135 \quad \int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=469

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54\left(1 + \sqrt[3]{-1}\right)^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{162a^{7/3}c^{2/3}}$$

Rubi [A] time = 0.68, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{54\left(1 + \sqrt[3]{-1}\right)^2 a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{162a^{7/3}c^{2/3}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{c}a^{2/3}}}\right)}{9\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{1/3}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{c}a^{2/3}}} - \frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{c}a^{2/3}}}\right)}{27\sqrt{3}a^{1/3}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{c}a^{2/3}}} + \frac{\sqrt{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{c}a^{2/3}}}\right)}{9\sqrt{3}\left(1 - \sqrt[3]{-1}\right)\left(1 + \sqrt[3]{-1}\right)^2 a^{1/3}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{c}a^{2/3} + 4b}}$$

Antiderivative was successfully verified.

[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(7/3)*c^(2/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{-3a^{2/3}\sqrt[3]{c} - (-1)^{2/3}bx}{531441(1 + \sqrt[3]{-1})^2 a^{25/3}c^{2/3}(-3a + 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2)} \right. \\
&= \frac{\int \frac{-3a^{2/3}\sqrt[3]{c} - bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{81a^{7/3}c^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + bx}{3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2} dx}{81a^{7/3}c^{2/3}} + \frac{\int \frac{3a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{162a^{7/3}c^{2/3}} \\
&= -\frac{\int \frac{3a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{162a^{7/3}c^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x + bx^2} dx}{162a^{7/3}c^{2/3}} + \frac{\log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{162a^{7/3}c^{2/3}} \\
&= -\frac{\log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3} \log(3a - 3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + bx^2)}{54(1 + \sqrt[3]{-1})^2 a^{7/3}c^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{13/6}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} + 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 95, normalized size = 0.20

$$\frac{1}{3} \text{RootSum} \left[\#1^6 b^3 + 9\#1^4 ab^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\log(x - \#1)}{2\#1^4 b^3 + 12\#1^2 ab^2 + 27\#1 a^2 c + 18a^2 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

[Out] IntegrateAlgebraic[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 91, normalized size = 0.19

$$\frac{\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)\ln(-\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3) + x)}{6\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^3 b^3 + 36\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^2 a b^2 + 81\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^2 a^2 c + 54\text{RootOf}(b^3Z^6 + 9b^2aZ^4 + 27a^2cZ^3 + 27b^2a^2Z^2 + 27a^3)^2 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x), _R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(x/(b³*x⁶ + 9*a*b²*x⁴ + 27*a²*c*x³ + 27*a²*b*x² + 27*a³),
x)

mupad [B] time = 2.90, size = 1057, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(27*a³ + b³*x⁶ + 27*a²*b*x² + 9*a*b²*x⁴ + 27*a²*c*x³),x)

[Out] symsum(log(b¹²*x + 1033121304*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁴*a¹⁰*b¹¹*c³ + 167365651248*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁵*a¹²*b¹²*c³ - 94143178827*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁵*a¹³*b⁹*c⁵ + 54*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)*a²*b¹³*x + 177147*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)²*a⁵*b¹¹*c²*x + 17006112*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)³*a⁷*b¹²*c²*x - 14348907*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)³*a⁸*b⁹*c⁴*x + 229582512*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁴*a⁹*b¹³*c²*x + 387420489*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁴*a¹⁰*b¹⁰*c⁴*x - 20920706406*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k)⁵*a¹²*b¹¹*c⁴*x)*root(18075490334784*a¹⁴*b³*c⁴*z⁶ - 7625597484987*a¹⁵*c⁶*z⁶ + 1162261467*a¹⁰*b*c⁴*z⁴ + 8503056*a⁷*b³*c²*z³ - 14348907*a⁸*c⁴*z³ + 177147*a⁵*b²*c²*z² + b³, z, k), k, 1, 6)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)
```

```
[Out] Timed out
```

$$3.136 \quad \int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Optimal. Leaf size=522

$$\frac{\sqrt[3]{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt[3]{-1} b \right) \tan^{-1} \left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) \left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right)}{27\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} - 81\sqrt{3} a^{17/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} - 27\sqrt{3} \left(1 - \sqrt[3]{-1} \right)^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}} + 81\sqrt{3} a^{17/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}} - 27\sqrt{3} \left(1 + \sqrt[3]{-1} \right)^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}$$

Rubi [A] time = 0.86, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 42, number of rules / integrand size = 0.119, Rules used = {2070, 634, 618, 204, 628}

$$\frac{\sqrt{-1} \left(3\sqrt[3]{a} c^{2/3} + 2\sqrt{-1} b \right) \tan^{-1} \left(\frac{3\sqrt{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right) \left(2b - 3\sqrt[3]{a} c^{2/3} \right) \tan^{-1} \left(\frac{3a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} \right) + \frac{(2(-1)^{2/3} b - 3\sqrt[3]{a} c^{2/3}) \tan^{-1} \left(\frac{3(-1)^{2/3} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right)}{27\sqrt{3} (1 + \sqrt{-1})^2 a^{17/6} c^{2/3} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} - \frac{(2(-1)^{2/3} b - 3\sqrt[3]{a} c^{2/3}) \tan^{-1} \left(\frac{3(-1)^{2/3} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right)}{81\sqrt{3} a^{17/6} c^{2/3} \sqrt{4b-3\sqrt[3]{a} c^{2/3}}} - \frac{(2(-1)^{2/3} b - 3\sqrt[3]{a} c^{2/3}) \tan^{-1} \left(\frac{3(-1)^{2/3} a^{2/3} \sqrt[3]{c} + 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b-3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}} \right)}{27\sqrt{3} (1 - \sqrt{-1}) (1 + \sqrt{-1})^2 a^{17/6} c^{2/3} \sqrt{3\sqrt[3]{a} \sqrt[3]{c} + 4b}} + \frac{\log(3a^{2/3} \sqrt[3]{c} x + 3a + bx^2)}{162a^{8/3} \sqrt[3]{c}} - \frac{\log(-3\sqrt{-1} a^{2/3} \sqrt[3]{c} x + 3a + bx^2)}{54(1 + \sqrt{-1})^2 a^{8/3} \sqrt[3]{c}} - \frac{\sqrt{-1} \log(3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + 3a + bx^2)}{162a^{8/3} \sqrt[3]{c}}$$

Antiderivative was successfully verified.

[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1), x]

[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(81*Sqrt[3]*a^(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p))*a^(2*p)], Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx &= (19683a^6) \int \left(\frac{-(-1)^{2/3} \sqrt[3]{a} b - 3\sqrt[3]{-1} a^{2/3} c^{2/3} + b\sqrt[3]{c}}{531441 (1 + \sqrt[3]{-1})^2 a^{26/3} c^{2/3} (-3a + 3\sqrt[3]{-1} a^{2/3} c^{2/3})} \right. \\
&= \frac{\int \frac{-\sqrt[3]{a} b + 3a^{2/3} c^{2/3} + b\sqrt[3]{c} x}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{81a^{8/3} c^{2/3}} - \frac{\int \frac{(-1)^{2/3} \sqrt[3]{a} b - 3a^{2/3} c^{2/3} + \sqrt[3]{-1} b\sqrt[3]{c} x}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{81a^{8/3} c^{2/3}} \\
&= -\frac{(2b - 3\sqrt[3]{a} c^{2/3}) \int \frac{1}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{162a^{7/3} c^{2/3}} - \frac{(2(-1)^{2/3} b - 3\sqrt[3]{a} c^{2/3}) \int \frac{1}{3a + 3(-1)^{2/3} a^{2/3} \sqrt[3]{c} x + bx^2} dx}{162a^{7/3} c^{2/3}} \\
&= \frac{\log(3a + 3a^{2/3} \sqrt[3]{c} x + bx^2)}{162a^{8/3} \sqrt[3]{c}} - \frac{\log(3a - 3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} x + bx^2)}{54(1 + \sqrt[3]{-1})^2 a^{8/3} \sqrt[3]{c}} \\
&= -\frac{(2(-1)^{2/3} b + 3\sqrt[3]{-1} \sqrt[3]{a} c^{2/3}) \tan^{-1}\left(\frac{3\sqrt[3]{-1} a^{2/3} \sqrt[3]{c} - 2bx}{\sqrt{3} \sqrt{a} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}\right)}{27\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{17/6} \sqrt{4b - 3(-1)^{2/3} \sqrt[3]{a} c^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.19

$$\frac{1}{3} \text{RootSum}\left[\#1^6 b^3 + 9\#1^4 a b^2 + 27\#1^3 a^2 c + 27\#1^2 a^2 b + 27a^3 \&, \frac{\log(x - \#1)}{2\#1^5 b^3 + 12\#1^3 a b^2 + 27\#1^2 a^2 c + 18\#1 a^2 b} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1), x]

[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1), x]

[Out] IntegrateAlgebraic[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3 x^6 + 9 a b^2 x^4 + 27 a^2 c x^3 + 27 a^2 b x^2 + 27 a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

maple [C] time = 0.00, size = 90, normalized size = 0.17

$$\frac{\ln(-\text{RootOf}(b^3 Z^6 + 9 b^2 a Z^4 + 27 a^2 c Z^3 + 27 b a^2 Z^2 + 27 a^3) + x)}{6 \text{RootOf}(b^3 Z^6 + 9 b^2 a Z^4 + 27 a^2 c Z^3 + 27 b a^2 Z^2 + 27 a^3)^5 b^3 + 36 \text{RootOf}(b^3 Z^6 + 9 b^2 a Z^4 + 27 a^2 c Z^3 + 27 b a^2 Z^2 + 27 a^3)^3 a b^2 + 81 \text{RootOf}(b^3 Z^6 + 9 b^2 a Z^4 + 27 a^2 c Z^3 + 27 b a^2 Z^2 + 27 a^3)^2 a^2 c + 54 \text{RootOf}(b^3 Z^6 + 9 b^2 a Z^4 + 27 a^2 c Z^3 + 27 b a^2 Z^2 + 27 a^3)^2 a^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3 x^6 + 9 a b^2 x^4 + 27 a^2 c x^3 + 27 a^2 b x^2 + 27 a^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="maxima")

[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)

mupad [B] time = 0.71, size = 1394, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3), x)

[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 94143178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 209207064


```
06*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 1
0460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^
5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11
*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1,
6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

$$3.137 \quad \int \frac{1}{x(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx$$

Optimal. Leaf size=563

$$\frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}} - \frac{\left(6\sqrt[3]{a}c^{2/3} + i\sqrt{3}b + b\right) \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{2/3}}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}}$$

Rubi [A] time = 1.16, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, number of rules / integrand size = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}} - \frac{\left(6\sqrt[3]{a}c^{2/3} + i\sqrt{3}b + b\right) \log\left(-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{972a^{10/3}c^{2/3}} - \frac{\left(3\sqrt[3]{a} - \frac{(-1)^{2/3}}{c^{2/3}}\right) \log\left(3a^{2/3}\sqrt[3]{c}x + 3a + bx^2\right)}{486a^{10/3}} + \frac{\left(b - (-1)^{2/3}\sqrt[3]{a}c^{2/3}\right) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x - 2bx}{\sqrt{3}\sqrt[3]{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 + \sqrt[3]{-1})a^{10/3}\sqrt[3]{c}\sqrt[3]{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}} + \frac{\left(b - \sqrt[3]{a}c^{2/3}\right) \tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}x - 2bx}{\sqrt{3}\sqrt[3]{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{10/3}\sqrt[3]{c}\sqrt[3]{4b - 3\sqrt[3]{a}c^{2/3}}} + \frac{(-1)^{2/3}\left((-1)^{2/3}b - \sqrt[3]{a}c^{2/3}\right) \tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x - 2bx}{\sqrt{3}\sqrt[3]{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 - \sqrt[3]{-1})(1 + \sqrt[3]{-1})a^{10/3}\sqrt[3]{c}\sqrt[3]{4b - 3\sqrt[3]{a}c^{2/3}}} + \frac{\log(x)}{27a^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(10/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{ :> S}$
 $\text{imp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; FreeQ}\{a, b, c, d,$
 $e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \text{ :> D}$
 $\text{ist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$
 $\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}$
 $[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2097

$\text{Int}[(Q6_)^(p_)*(u_), x_Symbol] \text{ :> With}\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6,$
 $x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}$
 $[1/(3^(3*p)*a^(2*p)), \text{Int}[\text{ExpandIntegrand}[u*(3*a + 3*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x$
 $+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p*(3*a + 3*(-$
 $1)^(2/3)*\text{Rt}[a, 3]^2*\text{Rt}[c, 3]*x + b*x^2)^p, x], x] \text{ /; EqQ}[b^2 - 3*a*d, 0$
 $] \&\& \text{EqQ}[b^3 - 27*a^2*e, 0]] \text{ /; ILtQ}[p, 0] \&\& \text{PolyQ}[Q6, x, 6] \&\& \text{EqQ}[\text{Coeff}[$
 $Q6, x, 1], 0] \&\& \text{EqQ}[\text{Coeff}[Q6, x, 5], 0] \&\& \text{RationalFunctionQ}[u, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx &= (19683a^6) \int \left(\frac{1}{531441a^9x} + \frac{3a^{2/3}(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c} + b}{4782969a^{28/3}c^{2/3}(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)} \right) dx \\
&= \frac{\log(x)}{27a^3} + \frac{\int \frac{3a^{2/3}(2b - 3\sqrt[3]{a}c^{2/3})\sqrt[3]{c} + b(b - 3\sqrt[3]{a}c^{2/3})x}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{243a^{10/3}c^{2/3}} + \frac{(-1)^{2/3}}{27a^3} \\
&= \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \int \frac{3a^{2/3}\sqrt[3]{c} + 2bx}{3a + 3a^{2/3}\sqrt[3]{c}x + bx^2} dx}{486a^{10/3}} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right)}{27a^3} \\
&= \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a} - \frac{b}{c^{2/3}}\right) \log(3a + 3a^{2/3}\sqrt[3]{c}x + bx^2)}{486a^{10/3}} - \frac{(b - (-1)^{2/3}\sqrt[3]{a}c^{2/3}) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c} - 2bx}{\sqrt{3}\sqrt{a}\sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}(1 + \sqrt[3]{-1})^2 a^{19/6} \sqrt{4b - 3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} \sqrt[3]{c}} + \frac{(-1)^{2/3}}{27a^3}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 157, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^6b^3 + 9\#1^4ab^2 + 27\#1^3a^2c + 27\#1^2a^2b + 27a^3\&, \frac{\#1^4b^3 \log(x-\#1) + 9\#1^2ab^2 \log(x-\#1) + 27a^2b \log(x-\#1) + 27\#1a^2c \log(x-\#1)}{2\#1^4b^3 + 12\#1^2ab^2 + 27\#1a^2c + 18a^2b}\right] - 3 \log(x)}{81a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &])/a^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] IntegrateAlgebraic[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x), x)

maple [C] time = 0.01, size = 134, normalized size = 0.24

$$\frac{\ln(x) \left(\text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) b^3 + 9 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a b^2 + 27 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a^2 c + 27 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a^2 b \right) \ln(-\text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) + x)}{81a^3 \left(2 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) b^3 + 12 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a b^2 + 27 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a^2 c + 18 \text{RootOf}(b^3z^6 + 9a^2bz^4 + 27a^2cz^3 + 27a^3) a^2 b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] -1/81/a^3*sum((R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 05891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 254186582832 \\
& 9*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 \\
& - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2 \\
& *z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^{15}*b^{10}*c^5 - 67783088755440*ro \\
& ot(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488 \\
& 038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 611930662375 \\
& 5*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4* \\
& z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8* \\
& b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6* \\
& a^{17}*b^{13}*c^3 + 22876792454961*root(13177032454057536*a^{20}*b^3*c^4*z^6 - 55 \\
& 59060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 2058911320 \\
& 94649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^ \\
& 6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 1046035 \\
& 3203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6 \\
& 561*a^4*b^6*c^2*z + b^9, z, k)^6*a^{18}*b^{10}*c^5 + 17496*root(131770324540575 \\
& 36*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}* \\
& b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 \\
& - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^ \\
& 10*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009 \\
& 7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^{16}*c - 47239 \\
& 2*root(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + \\
& 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 61193066 \\
& 23755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3* \\
& c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907* \\
& a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k) \\
& ^3*a^7*b^{16}*c - 39366*root(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566 \\
& 555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^ \\
& 18*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + \\
& 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^1 \\
& 2*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^ \\
& 6*c^2*z + b^9, z, k)^2*a^4*b^{15}*c^2*x + 51372630*root(13177032454057536*a \\
& ^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3* \\
& c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2 \\
& 541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b \\
& ^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379 \\
& *a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^7*b^{15}*c^2*x + 71744 \\
& 535*root(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 \\
& + 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 611930 \\
& 6623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^ \\
& 3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 1434890 \\
& 7*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, \\
& k)^3*a^8*b^{12}*c^4*x - 2008846980*root(13177032454057536*a^{20}*b^3*c^4*z^6 - \\
& 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 2058911 \\
& 32094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15} \\
& *c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 1046
\end{aligned}$$

$$\begin{aligned}
& 0353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 \\
& - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^{10}*b^{15}*c^2*x + 108477736920*\text{root}(131 \\
& 77032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239 \\
& 039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14} \\
& *b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - \\
& 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4* \\
& z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^{11}*b \\
& ^{12}*c^4*x - 41841412812*\text{root}(13177032454057536*a^{20}*b^3*c^4*z^6 - 555906056 \\
& 6555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a \\
& ^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + \\
& 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^ \\
& ^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4* \\
& *b^6*c^2*z + b^9, z, k)^4*a^{12}*b^9*c^6*x + 18596183472*\text{root}(131770324540575 \\
& 36*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}* \\
& b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 \\
& - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^ \\
& ^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009 \\
& 7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^{13}*b^{15}*c^2*x + \\
& 16129864639026*\text{root}(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a \\
& ^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z \\
& ^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854 \\
& 719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^ \\
& ^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2* \\
& z + b^9, z, k)^5*a^{14}*b^{12}*c^4*x - 6778308875544*\text{root}(13177032454057536*a^2 \\
& 0*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^ \\
& ^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 254 \\
& 1865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6 \\
& *c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a \\
& ^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^{15}*b^9*c^6*x + 6456339 \\
& 20395566*\text{root}(13177032454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^ \\
& ^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6 \\
& 119306623755*a^{14}*b^3*c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^ \\
& ^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14 \\
& 348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^ \\
& ^9, z, k)^6*a^{17}*b^{12}*c^4*x - 274521509459532*\text{root}(13177032454057536*a^{20}*b^ \\
& ^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 488038239039168*a^{17}*b^3*c^4*z^ \\
& ^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3*c^4*z^4 - 2541865 \\
& 828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 229582512*a^{10}*b^6*c^2 \\
& *z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b \\
& ^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^{18}*b^9*c^6*x)*\text{root}(1317703 \\
& 2454057536*a^{20}*b^3*c^4*z^6 - 5559060566555523*a^{21}*c^6*z^6 + 4880382390391 \\
& 68*a^{17}*b^3*c^4*z^5 - 205891132094649*a^{18}*c^6*z^5 + 6119306623755*a^{14}*b^3 \\
& *c^4*z^4 - 2541865828329*a^{15}*c^6*z^4 + 27506854719*a^{11}*b^3*c^4*z^3 - 2295 \\
& 82512*a^{10}*b^6*c^2*z^3 - 10460353203*a^{12}*c^6*z^3 + 14348907*a^8*b^3*c^4*z^ \\
& ^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k), k, 1, 6)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3),x)

[Out] Timed out

$$3.138 \quad \int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)} dx$$

Optimal. Leaf size=645

$$\frac{(9a^{2/3}c^{4/3} + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 2(-1)^{2/3}b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right) + (9a^{2/3}c^{4/3} - 12\sqrt[3]{a}bc^{2/3} + 2b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} + 243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

Rubi [A] time = 1.38, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, number of rules / integrand size = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{(9a^{2/3}c^{4/3} + 12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3} + 2(-1)^{2/3}b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right) + (9a^{2/3}c^{4/3} - 12\sqrt[3]{a}bc^{2/3} + 2b^2) \tan^{-1}\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1 + \sqrt[3]{-1}\right)^2 a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}} + 243\sqrt{3}a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)), x]

[Out]
$$-1/(27*a^3*x) + ((2*(-1)^{(2/3)}*b^2 + 12*(-1)^{(1/3)}*a^{(1/3)}*b*c^{(2/3)} + 9*a^{(2/3)}*c^{(4/3)})*ArcTan[(3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)} - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}])]/(81*Sqrt[3]*(1 + (-1)^{(1/3)})^2*a^{(23/6)}*Sqrt[4*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) + ((2*b^2 - 12*a^{(1/3)}*b*c^{(2/3)} + 9*a^{(2/3)}*c^{(4/3)})*ArcTan[(3*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^{(1/3)}*c^{(2/3)}])]/(243*Sqrt[3]*a^{(23/6)}*Sqrt[4*b - 3*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) + ((2*(-1)^{(2/3)}*b^2 - 12*a^{(1/3)}*b*c^{(2/3)} - 9*(-1)^{(1/3)}*a^{(2/3)}*c^{(4/3)})*ArcTan[(3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)} + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}])]/(81*Sqrt[3]*(1 - (-1)^{(1/3)})*(1 + (-1)^{(1/3)})^2*a^{(23/6)}*Sqrt[4*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)}]*c^{(2/3)}) - ((2*b - 3*a^{(1/3)}*c^{(2/3)})*Log[3*a + 3*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(486*a^{(11/3)}*c^{(1/3)}) + ((2*b - 3*(-1)^{(2/3)}*a^{(1/3)}*c^{(2/3)})*Log[3*a - 3*(-1)^{(1/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(162*(1 + (-1)^{(1/3)})^2*a^{(11/3)}*c^{(1/3)}) + ((-1)^{(1/3)}*(2*b + 3*(-1)^{(1/3)}*a^{(1/3)}*c^{(2/3)})*Log[3*a + 3*(-1)^{(2/3)}*a^{(2/3)}*c^{(1/3)}*x + b*x^2])/(486*a^{(11/3)}*c^{(1/3)})$$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx &= (19683a^6) \int \left(\frac{1}{531441a^9x^2} + \frac{\sqrt[3]{a} (b^2 - 9\sqrt[3]{a} bc^{2/3} + 9a^{2/3}c^4/3)}{1594323 (1 - \sqrt[3]{-1}) (1 + \sqrt[3]{-1})} \right) dx \\
&= -\frac{1}{27a^3x} + \frac{\int \frac{\sqrt[3]{a} (b^2 - 9\sqrt[3]{a} bc^{2/3} + 9a^{2/3}c^4/3) - b(2b - 3\sqrt[3]{a} c^{2/3}) \sqrt[3]{c} x}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{243a^{11/3} c^{2/3}} \\
&= -\frac{1}{27a^3x} - \frac{(2b - 3\sqrt[3]{a} c^{2/3}) \int \frac{3a^{2/3} \sqrt[3]{c} + 2bx}{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2} dx}{486a^{11/3} \sqrt[3]{c}} + \frac{(\sqrt[3]{-1})}{\dots} \\
&= -\frac{1}{27a^3x} - \frac{(2b - 3\sqrt[3]{a} c^{2/3}) \log(3a + 3a^{2/3} \sqrt[3]{c} x + bx^2)}{486a^{11/3} \sqrt[3]{c}} + \dots \\
&= -\frac{1}{27a^3x} + \frac{(2(-1)^{2/3}b^2 + 12\sqrt[3]{-1} \sqrt[3]{a} bc^{2/3} + 9a^{2/3}c^4/3) \tan^{-1} \left(\frac{3a + 3a^{2/3} \sqrt[3]{c} x + bx^2}{\dots} \right)}{81\sqrt{3} (1 + \sqrt[3]{-1})^2 a^{23/6} \sqrt{4b - 3(\dots)}}
\end{aligned}$$

Mathematica [C] time = 0.14, size = 163, normalized size = 0.25

$$\frac{x \text{RootSum} \left[\#1^6 b^3 + 9 \#1^4 a b^2 + 27 \#1^3 a^2 c + 27 \#1^2 a^2 b + 27 a^3 \&, \frac{\#1^4 b^3 \log(x - \#1) + 9 \#1^2 a b^2 \log(x - \#1) + 27 a^2 b \log(x - \#1) + 27 \#1 a^2 c \log(x - \#1)}{2 \#1^5 b^3 + 12 \#1^3 a b^2 + 27 \#1^2 a^2 c + 18 \#1 a^2 b} \& \right] + 3}{81 a^3 x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &, (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &])/(a^3*x)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]

[Out] IntegrateAlgebraic[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b^3x^6 + 9ab^2x^4 + 27a^2cx^3 + 27a^2bx^2 + 27a^3)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algorithm="giac")

[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)*x^2), x)

maple [C] time = 0.01, size = 133, normalized size = 0.21

$$\frac{(-\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3) - 9\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)ab^2 - 27\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)a^2c - 27b\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)) \ln(-\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3) + x)}{81a^3(2\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3) + 12\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)ab^2 + 27\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)a^2c + 18\text{RootOf}(b^3z^6 + 9b^2az^4 + 27a^2cz^3 + 27a^2bz^2 + 27a^3)ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)

[Out] -1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")

[Out] Timed out

mupad [B] time = 2.72, size = 2663, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),
x)

[Out] symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946
35296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*
a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^
3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a
^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x + 2541
865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^2
4*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^
4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 2824295
36481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 387420489*root
(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45
753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 7531454306
16*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7
*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^
10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876259553472*a^23*
b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z
^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207
657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*
c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^
9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635
296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^
16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*
z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8
*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^5*c^2 + 6973568802
*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6
- 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 75314
5430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^1
4*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a
^4*b^10*c*z + b^12, z, k)^3*a^13*b^2*c^4 - 4518872583696*root(3557798762595
53472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a
^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^
5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 2582803
26*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12,

$$\begin{aligned}
& z, k)^4 a^{16} b^3 c^3 - 328050 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 1 \\
& 50094635296999121 a^{24} c^6 z^6 - 45753584909922 a^{17} b^3 c^6 z^4 + 1093002306 \\
& 18147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 + 207657382104 a^{12} b^6 c^3 z^3 \\
& + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 + 10044 \\
& 2349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^5 b^6 c^2 - 17714 \\
& 7 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 \\
& - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 7531 \\
& 45430616 a^{13} b^3 c^5 z^3 + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 \\
& + 258280326 a^9 b^5 c^4 z^2 + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^6 b^3 c^4 \\
& + 387420489 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 \\
& - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 \\
& + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 \\
& + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^{10} b^3 c^4 + 387420489 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 \\
& - 150094635296999121 a^{24} c^6 z^6 - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 \\
& - 753145430616 a^{13} b^3 c^5 z^3 + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 \\
& + 258280326 a^9 b^5 c^4 z^2 + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^2 a^{10} b^3 c^5 \\
& + 23328 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 \\
& - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 \\
& + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 \\
& + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^4 b^8 c^3 x + 196830 \operatorname{root}(35 \\
& 5779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 - 45753 \\
& 584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 \\
& + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 \\
& + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^5 b^5 c^3 x - 20920706406 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 \\
& - 150094635296999121 a^{24} c^6 z^6 - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 \\
& - 753145430616 a^{13} b^3 c^5 z^3 + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 \\
& + 258280326 a^9 b^5 c^4 z^2 + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^3 a^{13} b^3 c^5 x \\
& + 74401740 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 \\
& - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 \\
& + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 \\
& + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^2 a^8 b^6 c^2 x - 746143164 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 \\
& - 150094635296999121 a^{24} c^6 z^6 - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 \\
& - 753145430616 a^{13} b^3 c^5 z^3 + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 \\
& + 258280326 a^9 b^5 c^4 z^2 + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^2 a^9 b^3 c^4 x \\
& + 55788550416 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 - 150094635296999121 a^{24} c^6 z^6 \\
& - 45753584909922 a^{17} b^3 c^6 z^4 + 109300230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 \\
& + 207657382104 a^{12} b^6 c^3 z^3 + 282429536481 a^{14} c^7 z^3 + 258280326 a^9 b^5 c^4 z^2 \\
& + 100442349 a^8 b^8 c^2 z^2 + 17496 a^4 b^{10} c z + b^{12}, z, k) a^3 a^{12} b^4 c^3 x + 564859072962 \operatorname{root}(355779876259553472 a^{23} b^3 c^4 z^6 \\
& - 150094635296999121 a^{24} c^6 z^6 - 45753584909922 a^{17} b^3 c^6 z^4 + 1093 \\
& 00230618147 a^{16} b^4 c^4 z^4 - 753145430616 a^{13} b^3 c^5 z^3 + 207657382104
\end{aligned}$$

```
*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 +
  100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^2*c^
  4*x))*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^
  6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 -
  753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 28242953648
  1*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17
  496*a^4*b^10*c*z + b^12, z, k), k, 1, 6) - 1/(27*a^3*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```


$$3.139 \quad \int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=395

$$\frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6) + \frac{1}{108} (18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6) + \frac{1}{108} (18 - 2^{2/3} \sqrt[3]{3}) \log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + 6) - \frac{\sqrt{-2} (1 + \sqrt{-2} 3^{2/3}) \tan^{-1} \left(\frac{2x + 3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{4 + 3(-2)^{2/3} \sqrt[3]{3}}} \right) + \frac{\sqrt{3} (1 - (-3)^{2/3} \sqrt[3]{3}) \tan^{-1} \left(\frac{\sqrt{3} (x + \sqrt{-3} - \sqrt[3]{3})}{\sqrt{4 + 3(-3)^{2/3} \sqrt[3]{3}}} \right) - (1 - \sqrt[3]{2} 3^{2/3}) \tanh^{-1} \left(\frac{\sqrt{3} (\sqrt[3]{2} x + \sqrt[3]{3})}{\sqrt{4 + 3 \sqrt[3]{2} 3^{2/3} - 4}} \right)}{3^{5/6} \sqrt{8 + 9i\sqrt{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}}} + \frac{(1 + \sqrt{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{3}}}{\sqrt[3]{2} 3^{5/6} \sqrt{3 \sqrt[3]{2} 3^{2/3} - 4}}$$

Rubi [A] time = 1.44, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6) + \frac{1}{108} (18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6) + \frac{1}{108} (18 - 2^{2/3} \sqrt[3]{3}) \log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + 6) - \frac{\sqrt{-2} (1 + \sqrt{-2} 3^{2/3}) \tan^{-1} \left(\frac{2x + 3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{4 + 3(-2)^{2/3} \sqrt[3]{3}}} \right) + \frac{\sqrt{3} (1 - (-3)^{2/3} \sqrt[3]{3}) \tan^{-1} \left(\frac{\sqrt{3} (x + \sqrt{-3} - \sqrt[3]{3})}{\sqrt{4 + 3(-3)^{2/3} \sqrt[3]{3}}} \right) - (1 - \sqrt[3]{2} 3^{2/3}) \tanh^{-1} \left(\frac{\sqrt{3} (\sqrt[3]{2} x + \sqrt[3]{3})}{\sqrt{4 + 3 \sqrt[3]{2} 3^{2/3} - 4}} \right)}{3^{5/6} \sqrt{8 + 9i\sqrt{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}}} + \frac{(1 + \sqrt{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{3}}}{\sqrt[3]{2} 3^{5/6} \sqrt{3 \sqrt[3]{2} 3^{2/3} - 4}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] -(((−2)^(1/3)*(1 + (−2)^(1/3)*3^(2/3))*ArcTan[(3*(−2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(−2)^(1/3)*3^(2/3))])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ((3/2)^(1/6)*(1 − (−3)^(2/3)*2^(1/3))*ArcTan[(2^(1/6)*(3*(−3)^(1/3) − 2^(1/3)*x))/Sqrt[3*(4 − 3*(−3)^(2/3)*2^(1/3))])/(1 + (−1)^(1/3))^2*Sqrt[4 − 3*(−3)^(2/3)*2^(1/3)] − ((1 − 2^(1/3)*3^(2/3))*ArcTanH[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(−4 + 3*2^(1/3)*3^(2/3))])/(2^(1/6)*3^(5/6)*Sqrt[−4 + 3*2^(1/3)*3^(2/3)] + ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 − 3*(−3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 − (−2)^(2/3)*3^(1/3))*Log[6 + 3*(−2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 − 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanH[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (3\sqrt[3]{-3} 2^{2/3} + (1 - 3(-3)^{2/3} \sqrt[3]{2}) x)}{3779136 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} + \dots \right) \\
&= \frac{1}{54} \int \frac{6\sqrt[3]{2} 3^{2/3} + (18 - 2^{2/3} \sqrt[3]{3}) x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx + \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3} - (1 + 3\sqrt[3]{-2} 3^{2/3}) x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2}}{9\sqrt[3]{2} 3^{2/3}} \\
&= \frac{((-1)^{2/3} (1 - 3(-3)^{2/3} \sqrt[3]{2})) \int \frac{-3\sqrt[3]{-3} 2^{2/3} + 2x}{6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2} dx}{6\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{((-1)^{2/3} \sqrt[3]{\frac{3}{2}} (6 + 3\sqrt[3]{-2} 3^{2/3})) \int \frac{3(-2)^{2/3} \sqrt[3]{3} - (1 + 3\sqrt[3]{-2} 3^{2/3}) x}{6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{9\sqrt[3]{2} 3^{2/3}} \\
&= \frac{1}{216} (36 + 2^{2/3} \sqrt[3]{3} + i 2^{2/3} 3^{5/6}) \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2) + \frac{1}{108} (18 - 3\sqrt[3]{-3} 2^{2/3}) \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2) \\
&\quad - \frac{(-1)^{2/3} ((-2)^{2/3} - 2 \cdot 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{\sqrt{6(4 + 3\sqrt[3]{-2} 3^{2/3})}} \right)}{6^{5/6} \sqrt{4 + 3\sqrt[3]{-2} 3^{2/3}}} - \frac{(-1)^{2/3} (\sqrt[3]{-3} + 3\sqrt[3]{-2} 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3} \sqrt[3]{3} - (1 + 3\sqrt[3]{-2} 3^{2/3}) x}{\sqrt{6(4 + 3\sqrt[3]{-2} 3^{2/3})}} \right)}{\sqrt[6]{6} (1 + \sqrt[3]{-1})^2}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + 36, \frac{\#1^4 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.14

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 \ln(-\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) + x)}{6\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 972\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 216\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.65, size = 427, normalized size = 1.08

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)`

[Out] `symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 17047422*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/72662865048, z, k), k, 1, 6)`

sympy [A] time = 0.26, size = 70, normalized size = 0.18

`RootSum(72662865048*t^6 - 72662865048*t^5 + 24163559388*t^4 - 2646786132*t^3 - 6626610*t^2 - 4374*t - 1, t -> t*log(-89236417131047376*t^5/833243797 + 89301949532998128*t^4/833243797 - 29740560281805852*t^3/833243797 + 192466080408420*t^2/49014341 + 5867255361684*t/833243797 + x + 5365044886/2499731391))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)`

[Out] `RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-89236417131047376*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/833243797 + x + 5365044886/2499731391)))`

$$3.140 \quad \int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=377

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{6 \cdot 2^{2/3}\sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}\sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3} - 2^2)}{9\sqrt[6]{3} (1 + \sqrt[3]{-1})^2}$$

Rubi [A] time = 0.91, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{6 \cdot 2^{2/3}\sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3}\sqrt[3]{3}x + 6)}{18 \cdot 2^{2/3}\sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3} - 2^2) \tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{(4-3(-3)^{2/3}\sqrt[3]{2}})}\right)}{9\sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}} + \frac{(9 - (-2)^{2/3}\sqrt[3]{3}) \tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{(4+3\sqrt[3]{-2}3^{2/3})}}\right)}{27\sqrt[6]{3} (8 + 9i\sqrt[3]{2}\sqrt[3]{3} + 3\sqrt[3]{2}3^{2/3})} - \frac{(9 - 2^{2/3}\sqrt[3]{3}) \tanh^{-1}\left(\frac{\sqrt[3]{2}(\sqrt[3]{2}x + 3\sqrt[3]{3})}{\sqrt{(3\sqrt[3]{2}3^{2/3}-4)}}\right)}{27\sqrt[6]{3} (3\sqrt[3]{2}3^{2/3} - 4)}$$

Antiderivative was successfully verified.

[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] $((-1)^{(2/3)} * (3 * (-3)^{(2/3)} - 2^{(2/3)}) * \text{ArcTan}[(3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / \text{Sqrt}[6 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})]]) / (9 * 3^{(1/6)} * (1 + (-1)^{(1/3)})^2 * \text{Sqrt}[2 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})]) + ((9 - (-2)^{(2/3)} * 3^{(1/3)}) * \text{ArcTan}[(3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / \text{Sqrt}[6 * (4 + 3 * (-2)^{(1/3)} * 3^{(2/3)})]]) / (27 * \text{Sqrt}[3 * (8 + (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)})]) - ((9 - 2^{(2/3)} * 3^{(1/3)}) * \text{ArcTanh}[(2^{(1/6)} * (3 * 3^{(1/3)} + 2^{(1/3)} * x)) / \text{Sqrt}[3 * (-4 + 3 * 2^{(1/3)} * 3^{(2/3)})]]) / (27 * \text{Sqrt}[6 * (-4 + 3 * 2^{(1/3)} * 3^{(2/3)})]) + \text{Log}[6 - 3 * (-3)^{(1/3)} * 2^{(2/3)} * x + x^2] / (6 * 2^{(2/3)} * 3^{(1/3)} * (1 + (-1)^{(1/3)})^2) + ((-1/3)^{(1/3)} * \text{Log}[6 + 3 * (-2)^{(2/3)} * 3^{(1/3)} * x + x^2]) / (18 * 2^{(2/3)}) - \text{Log}[6 + 3 * 2^{(2/3)} * 3^{(1/3)} * x + x^2] / (18 * 2^{(2/3)} * 3^{(1/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (-2 + \sqrt[3]{-3} 2^{2/3} x)}{7558272 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3 \sqrt[3]{-3} 2^{2/3} x - x^2)} + \dots \right) \\
&= -\frac{(-1)^{2/3} \int \frac{2+(-2)^{2/3} \sqrt[3]{3} x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{18 \sqrt[3]{2} 3^{2/3}} - \frac{\int \frac{\sqrt[3]{2} + \sqrt[3]{3} x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{9 6^{2/3}} + \frac{(-1)^{2/3} \int \frac{-2+}{-6+3 \sqrt[3]{3}}}{6 \sqrt[3]{2} 3^{2/3} (1 - \dots)} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{18 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3} + 2x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{18 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{-6+3 \sqrt[3]{-3} 2^{2/3} x-x^2}}{6 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})} \\
&= \frac{\log(6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)}{6 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} + \frac{\sqrt[3]{-\frac{1}{3}} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{18 2^{2/3}} - \frac{\log(\dots)}{\dots} \\
&= -\frac{\sqrt[3]{-1} (9 + \sqrt[3]{-3} 2^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{9 (1 + \sqrt[3]{-1})^2 \sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} - \frac{((-2)^{2/3} - 3 3^{2/3}) \tan^{-1}(\dots)}{27 \sqrt[3]{3} \sqrt{2(4 - \dots)}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.16

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + 36, \frac{\#1^3 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.15

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^4 \ln(-\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) + x)}{6 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 972 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 216 \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216), x)

[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x), _R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.70, size = 390, normalized size = 1.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 7558272*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^4/7596 + (217*z^3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k), k, 1, 6)
```

sympy [A] time = 0.28, size = 65, normalized size = 0.17

$$\text{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t \log\left(\frac{614714526178551746208t^5}{57121295165} - \frac{1270857362386176t^4}{57121295165} - \frac{80483053187684376t^3}{57121295165} + \frac{72431318325103884t^2}{57121295165} - \frac{45358602689088t}{57121295165} + x - \frac{44532180783}{57121295165}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 1180980*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/57121295165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/57121295165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165 + x - 44532180783/57121295165)))
```

$$3.141 \quad \int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{36\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} + \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} - \frac{\text{ta}}{6\sqrt[3]{2} 3^{5/6}}$$

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{36\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} + \frac{\log(x^2 + 3 2^{2/3} \sqrt[3]{3} x + 6)}{108\sqrt[3]{2} 3^{2/3}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{6\sqrt[3]{2} 3^{5/6} (1 + \sqrt[3]{-1})^2 \sqrt{4-3(-3)^{2/3} \sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{2x+3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{6(4+3\sqrt[3]{-2} 3^{2/3})}}\right)}{9 2^{2/3} 3^{5/6} \sqrt{8+9\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{5}(\sqrt[3]{2}x+3\sqrt[3]{5})}{\sqrt{3(3\sqrt[3]{2} 3^{2/3}-4)}}\right)}{18\sqrt[3]{2} 3^{5/6} \sqrt{3\sqrt[3]{2} 3^{2/3}-4}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} x}{22674816 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} \right) dx \\
&= \frac{\int \frac{x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{x}{-6+3 \sqrt[3]{3} x+x^2} dx}{18 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{18 \cdot 2^{2/3}} + \frac{\int \frac{3 \cdot 2^{2/3} \sqrt[3]{3} + 2x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{108 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{108 \sqrt[3]{2} 3^{2/3}} \\
&= -\frac{(-1)^{2/3} \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{36 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(-1)^{2/3} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x - x^2)}{108 \sqrt[3]{2} 3^{2/3}} \\
&= -\frac{\tan^{-1}\left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}}\right)}{6 \sqrt[6]{2} 3^{5/6} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{\sqrt{6(4 + 3 \sqrt[3]{-2} 3^{2/3} x - x^2)}}\right)}{18 \sqrt[6]{2} 3^{5/6} \sqrt{4 + 3 \sqrt[3]{-2} 3^{2/3} x - x^2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.17

$$\frac{1}{6} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + 36, \frac{\#1^2 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.16

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 \ln(-\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) + x)}{6\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 972\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 216\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.52, size = 276, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z
^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z,
k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3
390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 7438
4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
+ 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)
```

sympy [A] time = 0.25, size = 61, normalized size = 0.17

$$\text{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t \log\left(-\frac{8482372214243328t^5}{415817} + \frac{2216055910930560t^4}{415817} - \frac{2062546612992t^3}{415817} - \frac{57027208896t^2}{415817} - \frac{416583756t}{415817} + x - \frac{89938}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

$$3.142 \quad \int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3} \tan^{-1} \left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3} \tan^{-1} \left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6(4+3\sqrt[3]{-2} 3^{2/3})}} \right)}{81 \sqrt[3]{2} \sqrt[6]{3} \sqrt{8 + 9i\sqrt[3]{2} \sqrt[6]{3} + 3\sqrt[3]{2} 3^{2/3}}} - \frac{\tanh^{-1} \left(\frac{\sqrt[6]{2} (\sqrt[3]{2} x + 3\sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2} 3^{2/3} - 4)}} \right)}{81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{3\sqrt[3]{2} 3^{2/3} - 4}}$$

Rubi [A] time = 0.32, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2097, 618, 204, 206}

$$\frac{(-1)^{2/3} \tan^{-1} \left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2})}} \right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3} \tan^{-1} \left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6(4+3\sqrt[3]{-2} 3^{2/3})}} \right)}{81 \sqrt[3]{2} \sqrt[6]{3} \sqrt{8 + 9i\sqrt[3]{2} \sqrt[6]{3} + 3\sqrt[3]{2} 3^{2/3}}} - \frac{\tanh^{-1} \left(\frac{\sqrt[6]{2} (\sqrt[3]{2} x + 3\sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2} 3^{2/3} - 4)}} \right)}{81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{3\sqrt[3]{2} 3^{2/3} - 4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] $((-1)^{(2/3)} * \text{ArcTan}[(3 * (-3)^{(1/3)} * 2^{(2/3)} - 2 * x) / \text{Sqrt}[6 * (4 - 3 * (-3)^{(2/3)} * 2^{(1/3)})])]) / (27 * 2^{(5/6)} * 3^{(1/6)} * (1 + (-1)^{(1/3)})^2 * \text{Sqrt}[4 - 3 * (-3)^{(2/3)} * 2^{(1/3)}]) + ((-1)^{(2/3)} * \text{ArcTan}[(3 * (-2)^{(2/3)} * 3^{(1/3)} + 2 * x) / \text{Sqrt}[6 * (4 + 3 * (-2)^{(1/3)} * 3^{(2/3)})])]) / (81 * 2^{(1/3)} * 3^{(1/6)} * \text{Sqrt}[8 + (9 * I) * 2^{(1/3)} * 3^{(1/6)} + 3 * 2^{(1/3)} * 3^{(2/3)}]) - \text{ArcTanh}[(2^{(1/6)} * (3 * 3^{(1/3)} + 2^{(1/3)} * x)) / \text{Sqrt}[3 * (-4 + 3 * 2^{(1/3)} * 3^{(2/3)})])]) / (81 * 2^{(5/6)} * 3^{(1/6)} * \text{Sqrt}[-4 + 3 * 2^{(1/3)} * 3^{(2/3)}])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618


```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx = 1259712 \int \left(\frac{(-1)^{2/3}}{22674816 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} \right) dx$$

$$= \frac{\int \frac{1}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{54 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{1}{-6+3 \sqrt[3]{3} x-x^2} dx}{18 \sqrt[3]{2} 3^{2/3}}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-6(4-3 \sqrt[3]{2} 3^{2/3})-x^2} dx, x, 3 \cdot 2^{2/3} \sqrt[3]{3} + 2x\right)}{27 \sqrt[3]{2} 3^{2/3}} - \frac{(-1)^{2/3} \text{Subst}\left(\int \frac{1}{-6(4-3 \sqrt[3]{2} 3^{2/3})-x^2} dx, x, 3 \cdot 2^{2/3} \sqrt[3]{3} + 2x\right)}{27 \sqrt[3]{2} 3^{2/3}}$$

$$= \frac{(-1)^{2/3} \tan^{-1}\left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{27 \cdot 2^{5/6} \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{4-3(-3)^{2/3} \sqrt[3]{2}}} + \frac{(-1)^{2/3} \tan^{-1}\left(\frac{3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{6(4+3 \sqrt[3]{-1})}}\right)}{81 \cdot 2^{5/6} \sqrt[6]{3} \sqrt{4+3 \sqrt[3]{-1}}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.24

$$\frac{1}{6} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1, \frac{\#1 \log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

fricas [B] time = 4.13, size = 1277, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)

$2/3) + 48*18^{(1/3)} + 371) - 8/9*18^{(1/3)} + 18) + 3376*x - 2916*18^{(2/3)} - 3$
 $888*18^{(1/3)} - 16578) - 1/136728*\sqrt{1266}*\sqrt{-2/3*18^{(2/3)} - \sqrt{-1/27$
 $*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371) - 8/9*$
 $18^{(1/3)} + 18)*\log(2*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 - 18*\sqrt{-1/27*(6*18$
 $^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371)*(6*18^{(2/3)}$
 $+ 8*18^{(1/3)} + 81) + 1/211*(6*\sqrt{1266}*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 -$
 $9*\sqrt{-1/27*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)}$
 $+ 371)*(6*\sqrt{1266}*(6*18^{(2/3)} + 8*18^{(1/3)} + 81) - 211*\sqrt{1266})) - 124$
 $7*\sqrt{1266}*(6*18^{(2/3)} + 8*18^{(1/3)} + 81) + 51273*\sqrt{1266})*\sqrt{-2/3*1$
 $8^{(2/3)} - \sqrt{-1/27*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18$
 $^{(1/3)} + 371) - 8/9*18^{(1/3)} + 18) + 3376*x - 2916*18^{(2/3)} - 3888*18^{(1/3)}$
 $- 16578) + 1/136728*\sqrt{1266}*\sqrt{-2/3*18^{(2/3)} - \sqrt{-1/27*(6*18^{(2/3)}$
 $+ 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371) - 8/9*18^{(1/3)} + 1$
 $8)*\log(2*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 - 18*\sqrt{-1/27*(6*18^{(2/3)} + 8*1$
 $8^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371)*(6*18^{(2/3)} + 8*18^{(1/3)}$
 $+ 81) - 1/211*(6*\sqrt{1266}*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 - 9*\sqrt{-1/2$
 $7*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371)*(6*\sqrt{$
 $\sqrt{1266}*(6*18^{(2/3)} + 8*18^{(1/3)} + 81) - 211*\sqrt{1266})) - 1247*\sqrt{1266}$
 $*(6*18^{(2/3)} + 8*18^{(1/3)} + 81) + 51273*\sqrt{1266})*\sqrt{-2/3*18^{(2/3)} - \sqrt{$
 $\sqrt{-1/27*(6*18^{(2/3)} + 8*18^{(1/3)} + 81)^2 + 36*18^{(2/3)} + 48*18^{(1/3)} + 371$
 $) - 8/9*18^{(1/3)} + 18) + 3376*x - 2916*18^{(2/3)} - 3888*18^{(1/3)} - 16578)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.23

$$\frac{\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 \ln(-\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) + x)}{6\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 72\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 972\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 216\text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(-Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.68, size = 247, normalized size = 1.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 204073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 - z^4/273456 + z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

sympy [A] time = 0.21, size = 48, normalized size = 0.19

$$\text{RootSum}\left(732274264442769408t^6 - 2677850419968t^4 + 2834352t^2 - 1, \left(t \mapsto t \log\left(10170475895038464t^5 - 5231726283456t^4 - 31809932496t^3 + 19131876t^2 + 19683t + x - \frac{27}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1, Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

$$3.143 \quad \int \frac{x}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{216\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - 36\sqrt[6]{2} 3^{5/6} \left(\tan \right.$$

Rubi [A] time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{216\sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + 6)}{648\sqrt[3]{2} 3^{2/3}} - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{36\sqrt[6]{2} 3^{5/6} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{2x+3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{6(4+3(-2)^{2/3} \sqrt[3]{3})}}\right)}{54 \cdot 2^{2/3} 3^{5/6} \sqrt{8+9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}(\sqrt[3]{2}x+3\sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2} 3^{2/3}-4)}}\right)}{108\sqrt[6]{2} 3^{5/6} \sqrt{3\sqrt[3]{2} 3^{2/3}-4}}$$

Antiderivative was successfully verified.

[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3*6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (3\sqrt[3]{-3} 2^{2/3} - x)}{136048896 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} \right) \\
&= -\frac{(-1)^{2/3} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + x}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{324 \sqrt[3]{2} 3^{2/3}} - \frac{\int \frac{6 \sqrt[3]{3} + \sqrt[3]{2} x}{6+3 2^{2/3} \sqrt[3]{3} x + x^2} dx}{324 6^{2/3}} + \frac{(-1)^{2/3} \int \frac{3 \sqrt[3]{-3}}{-6+3 \sqrt[3]{-3} x - x^2} dx}{108 \sqrt[3]{2} 3^{2/3}} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{108 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3} + 2x}{6+3 2^{2/3} \sqrt[3]{3} x + x^2} dx}{648 \sqrt[3]{2} 3^{2/3}} - \frac{(-1)^{2/3} \int \frac{3(-2)^{2/3}}{6+3(-2)^{2/3} \sqrt[3]{3} x + x^2} dx}{648 \sqrt[3]{2} 3^{2/3}} \\
&= \frac{(-1)^{2/3} \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{216 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{(-1)^{2/3} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{648 \sqrt[3]{2} 3^{2/3}} \\
&\quad - \frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}}\right)}{36 \sqrt[6]{2} 3^{5/6} (1 + \sqrt[3]{-1})^2 \sqrt{4 - 3(-3)^{2/3} \sqrt[3]{2}}} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{6(4 + 3\sqrt[3]{-2})}}\right)}{108 \sqrt[6]{2} 3^{5/6} \sqrt{4 + 3\sqrt[3]{-2}}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.16

$$\frac{1}{6} \text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\log(x - \#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16 2*#1 + 12*#1^2 + #1^4) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)`

[Out] `symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x + 168897381688221696) + 28563737812992)))))*root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k, 1, 6)`

sympy [A] time = 0.26, size = 61, normalized size = 0.17

`RootSum(158171241119638192128*t^6 - 96402615118848*t^4 + 287743415040*t^3 - 51018336*t^2 - 1, (t -> t*log(65418399445721140961280*t^5/415817 + 2480926457425102848*t^4/415817 - 39451802929737984*t^3/415817 + 118071997444800*t^2/415817 - 16745884920*t/415817 + x - 268790/415817)))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)`

[Out] `RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(65418399445721140961280*_t**5/415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817 + 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)))`

$$3.144 \quad \int \frac{1}{216+108x^2+324x^3+18x^4+x^6} dx$$

Optimal. Leaf size=377

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3}} + \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3} \sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3} - 2^{2/3}) \log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + 6)}{324 \sqrt[6]{3} (1 + \sqrt[3]{-1})^2}$$

Rubi [A] time = 0.72, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 22, number of rules / integrand size = 0.273, Rules used = {2070, 634, 618, 204, 628, 206}

$$\frac{\log(x^2 - 3\sqrt[3]{-3}2^{2/3}x + 6)}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3}} + \frac{\log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + 6)}{648 \cdot 2^{2/3} \sqrt[3]{3}} + \frac{(-1)^{2/3} (3(-3)^{2/3} - 2^{2/3}) \tan^{-1}\left(\frac{3\sqrt[3]{-3}2^{2/3} - 2x}{\sqrt{4 - 3(-3)^{2/3}\sqrt[3]{2}}}\right)}{324 \sqrt[6]{3} (1 + \sqrt[3]{-1})^2 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}} + \frac{(9 - (-2)^{2/3} \sqrt[3]{3}) \tan^{-1}\left(\frac{2x + 3(-2)^{2/3} \sqrt[3]{3}}{\sqrt{4 + 3 \cdot 2^{2/3} \sqrt[3]{3}}}\right)}{972 \sqrt{3(8 + 9i\sqrt{2}\sqrt[3]{3} + 3\sqrt[3]{2}3^{2/3})}} - \frac{(9 - 2^{2/3} \sqrt[3]{3}) \tanh^{-1}\left(\frac{\sqrt[3]{2}(\sqrt[3]{2}x + \sqrt[3]{3})}{\sqrt{3(3\sqrt[3]{2}3^{2/3} - 4)}}\right)}{972 \sqrt{6(3\sqrt[3]{2}3^{2/3} - 4)}}$$

Antiderivative was successfully verified.

[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]

[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))])/(972*Sqrt[3*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x)/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))])/(972*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(216*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2]/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(2/3)*3^(1/3))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx &= 1259712 \int \left(\frac{(-1)^{2/3} (-2 + 6(-3)^{2/3} \sqrt[3]{2} - \sqrt[3]{-3} 2^{2/3} x)}{272097792 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (-6 + 3\sqrt[3]{-3} 2^{2/3} x - x^2)} + \dots \right) \\
&= \frac{\int \frac{18-2^{2/3} \sqrt[3]{3} + \sqrt[3]{2} 3^{2/3} x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{1944} - \frac{\int \frac{2(-1)^{2/3} - 6 \sqrt[3]{2} 3^{2/3} + \sqrt[3]{-3} 2^{2/3} x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{648 \sqrt[3]{2} 3^{2/3}} + \frac{(-1)^{2/3} \int \frac{-2+6(-3)^{2/3} \sqrt[3]{2} - \sqrt[3]{-3} 2^{2/3} x}{-6+3 \sqrt[3]{-3} 2^{2/3} x-x^2} dx}{216 \sqrt[3]{2} 3^{2/3}} \\
&= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{6+3(-2)^{2/3} \sqrt[3]{3} x+x^2} dx}{648 \cdot 2^{2/3}} + \frac{\int \frac{3 \cdot 2^{2/3} \sqrt[3]{3} + 2x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{648 \cdot 2^{2/3} \sqrt[3]{3}} - \frac{\int \frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{-6+3 \sqrt[3]{-3} 2^{2/3} x-x^2} dx}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} \\
&= -\frac{\log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{216 \cdot 2^{2/3} \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} - \frac{\sqrt[3]{-\frac{1}{3}} \log(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)}{648 \cdot 2^{2/3}} + \frac{\log(x - 1)}{36 \cdot \sqrt[3]{3} (1 + \sqrt[3]{-1})^2} \\
&= -\frac{\sqrt[3]{-1} (9 + \sqrt[3]{-3} 2^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{324 (1 + \sqrt[3]{-1})^2 \sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} - \frac{((-2)^{2/3} - 3 \cdot 3^{2/3}) \tan^{-1} \left(\frac{3 \sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} \sqrt[3]{2})}} \right)}{972 \sqrt[3]{3} \sqrt{2(4 - 3(-3)^{2/3} \sqrt[3]{2})}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.16

$$\frac{1}{6} \text{RootSum} \left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\#1 + \frac{\log(x - \#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]

[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]

[Out] IntegrateAlgebraic[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 53, normalized size = 0.14

$$\frac{\ln(-\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216) + x)}{6\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 72\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^3 + 972\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^2 + 216\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.67, size = 306, normalized size = 0.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216), x)
```

```
[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 *x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 6122200320*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 258263796059136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 168897381688221696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k), k, 1, 6)
```

sympy [A] time = 0.27, size = 65, normalized size = 0.17

```
RootSum(34164988081841849499648*t**6 - 3470494144278528*t**4 - 86087932019712*t**3 - 1530550080*t**2 + 69984*t - 1, Lambda(t, _t*log(18590444669109611410573787136*t**5/57121295165 + 6377301253267917382766592*t**4/57121295165 - 18904636002388564311552*t**3/57121295165 - 46908055291518172396*t**2/57121295165 - 24358640509989936*t/57121295165 + x + 152427895956/57121295165)))
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216), x)
```

```
[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 86087932019712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/57121295165 - 18904636002388564311552*_t**3/57121295165 - 46908055291518172396*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/57121295165)))
```

$$3.145 \quad \int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)} dx$$

Optimal. Leaf size=415

$$\frac{(36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{46656} - \frac{(18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{23328} - \frac{(18 - 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{46656} - \frac{(18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{23328}$$

Rubi [A] time = 0.90, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(36 + 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{46656} - \frac{(18 - (-2)^{2/3} \sqrt[3]{3}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{23328} - \frac{(18 - 2^{2/3} \sqrt[3]{3} (1 + i\sqrt{3})) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{46656} + \frac{\log(x)}{216} + \frac{(-1)^{2/3} ((-2)^{2/3} - 2^{2/3}) \tan^{-1}\left(\frac{2x+3-2^{2/3}\sqrt[3]{3}}{\sqrt{(1+i\sqrt{3})3^{2/3}}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9\sqrt{2}\sqrt{3}+3\sqrt{2}3^{2/3}}} - \frac{(-1)^{2/3} (\sqrt[3]{-3} + 3\sqrt{2}) \tan^{-1}\left(\frac{\sqrt[3]{3}(\sqrt[3]{-3}-\sqrt{2})}{\sqrt{(4-3i)3^{2/3}\sqrt{2}}}\right)}{216\sqrt[3]{6}(1+\sqrt{-1})^2\sqrt{4-3(-3)^{2/3}\sqrt{2}}} - \frac{(1-\sqrt[3]{2}3^{2/3}) \tanh^{-1}\left(\frac{\sqrt[3]{3}(\sqrt[3]{2}+i\sqrt{3})}{\sqrt{(1+i\sqrt{3})3^{2/3}}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{3\sqrt{2}3^{2/3}-4}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] $((-1)^{2/3} * ((-2)^{2/3} - 2 * 3^{2/3}) * \text{ArcTan}[(3 * (-2)^{2/3} * 3^{1/3} + 2 * x) / \text{Sqrt}[6 * (4 + 3 * (-2)^{1/3} * 3^{2/3})]]) / (216 * 2^{1/3} * 3^{5/6} * \text{Sqrt}[8 + (9 * I) * 2^{1/3} * 3^{1/6} + 3 * 2^{1/3} * 3^{2/3}]) - ((-1)^{2/3} * ((-3)^{1/3} + 3 * 2^{1/3}) * \text{ArcTan}[(2^{1/6} * (3 * (-3)^{1/3} - 2^{1/3} * x)) / \text{Sqrt}[3 * (4 - 3 * (-3)^{2/3} * 2^{1/3})]]) / (216 * 6^{1/6} * (1 + (-1)^{1/3})^2 * \text{Sqrt}[4 - 3 * (-3)^{2/3} * 2^{1/3}]) - ((1 - 2^{1/3} * 3^{2/3}) * \text{ArcTanh}[(2^{1/6} * (3 * 3^{1/3} + 2^{1/3} * x)) / \text{Sqrt}[3 * (-4 + 3 * 2^{1/3} * 3^{2/3})]]) / (216 * 2^{1/6} * 3^{5/6} * \text{Sqrt}[-4 + 3 * 2^{1/3} * 3^{2/3}]) + \text{Log}[x] / 216 - ((36 + 2^{2/3} * 3^{1/3} * (1 + I * \text{Sqrt}[3])) * \text{Log}[6 - 3 * (-3)^{1/3} * 2^{2/3} * x + x^2]) / 46656 - ((18 - (-2)^{2/3} * 3^{1/3}) * \text{Log}[6 + 3 * (-2)^{2/3} * 3^{1/3} * x + x^2]) / 23328 - ((18 - 2^{2/3} * 3^{1/3}) * \text{Log}[6 + 3 * 2^{2/3} * 3^{1/3} * x + x^2]) / 23328$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1 * ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx &= 1259712 \int \left(\frac{1}{272097792x} + \frac{(-1)^{2/3} (6(9 + \sqrt[3]{-3} 2^{2/3}) - (1 - 3\sqrt[3]{-3}))}{816293376 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2 (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} \right) dx \\
&= \frac{\log(x)}{216} + \frac{\int \frac{-6 \sqrt[3]{6} (9 \sqrt[3]{2} - 2 \sqrt[3]{3}) - (18 - 2^{2/3} \sqrt[3]{3}) x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{11664} + \frac{(-1)^{2/3} \int \frac{-6(9 - (-2)^{2/3})}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{1944} \\
&= \frac{\log(x)}{216} + \frac{\left(\left(-\frac{1}{6} \right)^{2/3} (\sqrt[3]{-3} + 3\sqrt[3]{2}) \right) \int \frac{1}{6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2} dx}{72 (1 + \sqrt[3]{-1})^2} + \frac{((-1)^{2/3})}{1944} \\
&= \frac{\log(x)}{216} - \frac{(-1)^{2/3} (1 - 3(-3)^{2/3} \sqrt[3]{2}) \log(6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{((-1)^{2/3})}{1944} \\
&= \frac{(-1)^{2/3} ((-2)^{2/3} - 2 \cdot 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^{2/3} \sqrt[3]{3} + 2x}{\sqrt{6(4 + 3 \sqrt[3]{-2} 3^{2/3})}} \right)}{216 \cdot 6^{5/6} \sqrt{4 + 3 \sqrt[3]{-2} 3^{2/3}}} - \frac{(-1)^{2/3} (\sqrt[3]{-3})}{216 \sqrt[3]{6}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1^4 \log(x-\#1) + 18\#1^2 \log(x-\#1) + 324\#1 \log(x-\#1) + 108 \log(x-\#1)}{\#1^4 + 12\#1^2 + 162\#1 + 36}\right] \&}{1296}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &, (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(36 + 162*#1 + 12*#1^2 + #1^4) &]/1296

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] IntegrateAlgebraic[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)

maple [C] time = 0.01, size = 75, normalized size = 0.18

$$\frac{\ln(x)}{216} \frac{\left(\text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 18 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 324 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 108 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) \right) \ln(-\text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) + x)}{1296 \left(\text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^5 + 12 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^3 + 162 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)^2 + 36 \text{RootOf}(Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/216*ln(x)-1/1296*sum((R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x), _R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216} \int \frac{x^5 + 18x^3 + 324x^2 + 108x}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx + \frac{1}{216} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x) + 1/216*log(x)

mupad [B] time = 2.33, size = 432, normalized size = 1.04

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)$

[Out] $\log(x)/216 + \text{symsum}(\log(7*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)*x - 5670000*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 1546875947520*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^3*x - 106961147905609728*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4*x - 140511995854134018048*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^5*x - 45607290567387619000320*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2 + 594896472576*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^3 - 8483430130458624*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 3831425535283494912*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^5 + 1217393817906599165952*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*\text{root}(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k), k, 1, 6)$

sympy [A] time = 0.42, size = 82, normalized size = 0.20

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)$

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(81455700996688179367833621151
19297360560128*_t**6/143425799309052440063 + 977068766770806381087358257564
745728*_t**5/143425799309052440063 - 11652952660885126428840097153906153881
6*_t**4/143425799309052440063 - 239359794985242202542501440710766592*_t**3/
143425799309052440063 - 136678312638137094439887341418240*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/143425799309052440063 + x - 31
64446315075236190044/143425799309052440063)))
```

$$3.146 \quad \int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)} dx$$

Optimal. Leaf size=448

$$\frac{(-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3}) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(3(-6)^{2/3} + 2\sqrt[3]{-2}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{7776 \sqrt[3]{3}} (2^{2/3} - 3)$$

Rubi [A] time = 1.10, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3}) \log(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} + \frac{(3(-6)^{2/3} + 2\sqrt[3]{-2}) \log(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)}{7776 \sqrt[3]{3}} + \frac{(2^{2/3} - 3)^{2/3} \log(x^2 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + 6)}{3888 \sqrt[3]{6}} - \frac{1}{216x} - \frac{(27\sqrt[3]{-6} - (-2)^{2/3} + 12 \cdot 3^{2/3}) \tan^{-1}\left(\frac{2+3x-2^{2/3}\sqrt[3]{3}}{\sqrt{4+3\sqrt[3]{3}x^2}}\right)}{5832\sqrt[3]{3}\sqrt{6+9\sqrt[3]{2}\sqrt[3]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3} (6(-6)^{2/3} + 27\sqrt[3]{-3} - \sqrt[3]{2}) \tan^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{-3}}{\sqrt{4-3(-3)^{2/3}\sqrt[3]{3}}}\right)}{1944\sqrt[3]{6}(1+\sqrt[3]{-3})\sqrt{4-3(-3)^{2/3}\sqrt[3]{3}}} - \frac{(\sqrt[3]{2} + 27\sqrt[3]{3} - 6 \cdot 6^{2/3}) \tanh^{-1}\left(\frac{\sqrt[3]{3}\sqrt[3]{3}\sqrt[3]{3}}{\sqrt{3}\sqrt[3]{3}\sqrt[3]{3}}\right)}{5832\sqrt[3]{6}\sqrt[3]{3}\sqrt[3]{2}3^{2/3}-4}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] $-1/(216*x) - ((27*(-6)^{(1/3)} - (-2)^{(2/3)} + 12*3^{(2/3)})*ArcTan[(3*(-2)^{(2/3)})*3^{(1/3)} + 2*x]/Sqrt[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})])/(5832*3^{(1/6)}*Sqrt[8 + (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)}]) - ((-1)^{(2/3)}*(6*(-6)^{(2/3)} + 27*(-3)^{(1/3)} - 2^{(1/3)})*ArcTan[(2^{(1/6)}*(3*(-3)^{(1/3)} - 2^{(1/3)}*x))/Sqrt[3*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})])]/(1944*6^{(1/6)}*(1 + (-1)^{(1/3)})^2*Sqrt[4 - 3*(-3)^{(2/3)}*2^{(1/3)}]) - ((2^{(1/3)} + 27*3^{(1/3)} - 6*6^{(2/3)})*ArcTanh[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/Sqrt[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})])]/(5832*6^{(1/6)}*Sqrt[-4 + 3*2^{(1/3)}*3^{(2/3)}]) - ((-1)^{(2/3)}*(9 + (-3)^{(1/3)}*2^{(2/3)})*Log[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/(1296*2^{(1/3)}*3^{(2/3)}*(1 + (-1)^{(1/3)})^2) + ((3*(-6)^{(2/3)} + 2*(-2)^{(1/3)})*Log[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(7776*3^{(1/3)}) - ((2^{(2/3)} - 3*3^{(2/3)})*Log[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2])/(3888*6^{(1/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx &= 1259712 \int \left(\frac{1}{272097792x^2} + \frac{(-1)^{2/3} (-1 + 9(-3)^{2/3} \sqrt[3]{2} + 27\sqrt[3]{-1})}{816293376 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} \right. \\
&= -\frac{1}{216x} + \frac{\int \frac{-54+2^{2/3} \sqrt[3]{3} + 54 \sqrt[3]{2} 3^{2/3} - 6^{2/3} (2^{2/3} - 3 \cdot 3^{2/3}) x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{11664} + \frac{(-1)^{2/3} \int \frac{1+}{-}}{11664} \\
&= -\frac{1}{216x} - \frac{((-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3})) \int \frac{-3 \sqrt[3]{-3} 2^{2/3} + 2x}{6-3 \sqrt[3]{-3} 2^{2/3} x+x^2} dx}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{((-1)^{2/3} \int \frac{1+}{-}}{11664} \\
&= -\frac{1}{216x} - \frac{(-1)^{2/3} (9 + \sqrt[3]{-3} 2^{2/3}) \log(6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)}{1296 \sqrt[3]{2} 3^{2/3} (1 + \sqrt[3]{-1})^2} - \frac{((-1)^{2/3} \int \frac{1+}{-}}{11664} \\
&= -\frac{1}{216x} + \frac{(-1)^{2/3} (2 + 27(-2)^{2/3} \sqrt[3]{3} + 12 \sqrt[3]{-2} 3^{2/3}) \tan^{-1} \left(\frac{3(-2)^2}{\sqrt{6(4+3x)}} \right)}{5832 \cdot 2^{5/6} \sqrt[3]{3} \sqrt{4 + 3 \sqrt[3]{-2} 3^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{\#1^4 \log(x-\#1) + 18\#1^2 \log(x-\#1) + 324\#1 \log(x-\#1) + 108 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\&\right]}{1296} - \frac{1}{216x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]

[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &, (108*Log[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/1296

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
[Out] IntegrateAlgebraic[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)), x]
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
[Out] Timed out
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
maple [C] time = 0.01, size = 74, normalized size = 0.17
```

$$\frac{(-\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^4 - 18\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^2 - 324\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216) - 108)\ln(-\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216) + x)}{1296\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^5 + 15552\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^3 + 209952\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^2 + 46656\text{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)} - \frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
[Out] -1/216/x+1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R)
)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$-\frac{1}{216x} - \frac{1}{216} \int \frac{x^4 + 18x^2 + 324x + 108}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```


mupad [B] time = 0.29, size = 340, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)`

[Out] `symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^2*x - 598229670528*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^5*x + 2344464*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^4 - 168897381688221696*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413330255577088, z, k), k, 1, 6) - 1/(216*x)`

sympy [A] time = 0.32, size = 70, normalized size = 0.16

`RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 - 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_t, _t*log(-49875532761902496003293561236914468028416*_t**5/12350449784703991795`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)`

[Out] `RootSum(1594001683946413330255577088*_t**6 + 3791612026460331638784*_t**4 - 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_t, _t*log(-49875532761902496003293561236914468028416*_t**5/12350449784703991795`

$$\begin{aligned} & 1795 + 12625489872431620388005975200497664*_t^{**4}/12350449784703991795 - 118 \\ & 637692607573771238550798852644864*_t^{**3}/12350449784703991795 + 270486324927 \\ & 832147818193778754816*_t^{**2}/12350449784703991795 + 273914194897479402961199 \\ & 352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795 \\ &))) - 1/(216*x) \end{aligned}$$

$$3.147 \quad \int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=1064

$$\frac{\sqrt[3]{-\frac{1}{3}} \left((2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) x + 9(6 - (-2)^{2/3} \sqrt[3]{3}) \right) \sqrt[3]{-1} (2 + 27(-2)^{2/3} \sqrt[3]{3} + 12\sqrt[3]{-2} 3^{2/3}) \tan^{-1}}{729 2^{2/3} (8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6)} \frac{162 \sqrt[6]{2} 3^{5/6} (1 - \sqrt[3]{-1})^2 (1 + \sqrt[3]{-1})^4 (4 +$$

Rubi [A] time = 2.50, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

Antiderivative was successfully verified.

[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $-\left(\left(-\frac{1}{3}\right)^{\frac{1}{3}} \left(9 \left(6 + (-3)^{\frac{1}{3}} 2^{\frac{2}{3}}\right) + (2 - 3 \cdot 2^{\frac{2}{3}}) (2 \cdot (-6)^{\frac{2}{3}} + 9 \cdot (-3)^{\frac{1}{3}})\right) x\right) / (162 \cdot 2^{\frac{2}{3}} (1 + (-1)^{\frac{1}{3}})^4 (4 - 3 \cdot (-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}})) \cdot (6 - 3 \cdot (-3)^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} \cdot x + x^2) - \left(\left(-\frac{1}{3}\right)^{\frac{1}{3}} \left(9 \left(6 - (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}\right) + (2 + 27 \cdot (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} + 12 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}\right) x\right) / (729 \cdot 2^{\frac{2}{3}} (8 + (9 \cdot I) \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{1}{6}} + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})) \cdot (6 + 3 \cdot (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot x + x^2) + (9 \cdot (6 - 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}})) + (2 + 2^{\frac{2}{3}} \cdot (27 \cdot 3^{\frac{1}{3}} - 6 \cdot 6^{\frac{2}{3}})) \cdot x\right) / (1458 \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot (4 - 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})) \cdot (6 + 3 \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot x + x^2) - \left(\frac{I}{162} \cdot ((-2)^{\frac{2}{3}} + 6 \cdot 3^{\frac{2}{3}}) \cdot \text{ArcTan}\left[\frac{3 \cdot (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} + 2 \cdot x}{\sqrt{6 \cdot (4 + 3 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right]\right) / (2^{\frac{5}{6}} \cdot 3^{\frac{1}{3}} \cdot (1 + (-1)^{\frac{1}{3}})^5 \cdot \sqrt{4 + 3 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}}) - \left(\left(-\frac{1}{3}\right)^{\frac{1}{3}} \left(2 + 27 \cdot (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} + 12 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}\right) \cdot \text{ArcTan}\left[\frac{3 \cdot (-2)^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} + 2 \cdot x}{\sqrt{6 \cdot (4 + 3 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right]\right) / (162 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{5}{6}} \cdot (1 - (-1)^{\frac{1}{3}})^2 \cdot (1 + (-1)^{\frac{1}{3}})^4 \cdot (4 + 3 \cdot (-2)^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})^{\frac{3}{2}}) - \left(\left(-\frac{1}{3}\right)^{\frac{1}{3}} \left(6 \cdot (-6)^{\frac{2}{3}} + 27 \cdot (-3)^{\frac{1}{3}} - 2^{\frac{1}{3}}\right) \cdot \text{ArcTan}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot (-3)^{\frac{1}{3}} - 2^{\frac{1}{3}} \cdot x)}{\sqrt{3 \cdot (4 - 3 \cdot (-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}})}}\right]\right) / (81 \cdot \sqrt{2} \cdot 3^{\frac{5}{6}} \cdot (1 + (-1)^{\frac{1}{3}})^4 \cdot (4 - 3 \cdot (-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}})^{\frac{3}{2}}) + \left(\left(I \cdot 2^{\frac{2}{3}} - 9 \cdot 3^{\frac{1}{6}} - (3 \cdot I) \cdot 3^{\frac{2}{3}}\right) \cdot \text{ArcTan}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot (-3)^{\frac{1}{3}} - 2^{\frac{1}{3}} \cdot x)}{\sqrt{3 \cdot (4 - 3 \cdot (-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}})}}\right]\right) / (162 \cdot 2^{\frac{5}{6}} \cdot 3^{\frac{1}{3}} \cdot (1 + (-1)^{\frac{1}{3}})^5 \cdot \sqrt{4 - 3 \cdot (-3)^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}}) - \left(\left(1 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}\right) \cdot \text{ArcTanh}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \cdot x)}{\sqrt{3 \cdot (-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right]\right) / (1458 \cdot 2^{\frac{1}{6}} \cdot 3^{\frac{5}{6}} \cdot \sqrt{-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}}}) + \left(\left(2^{\frac{1}{3}} + 27 \cdot 3^{\frac{1}{3}} - 6 \cdot 6^{\frac{2}{3}}\right) \cdot \text{ArcTanh}\left[\frac{2^{\frac{1}{6}} \cdot (3 \cdot 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \cdot x)}{\sqrt{3 \cdot (-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})}}\right]\right) / (81 \cdot \sqrt{2} \cdot 3^{\frac{5}{6}} \cdot (1 - (-1)^{\frac{1}{3}})^2 \cdot (1 + (-1)^{\frac{1}{3}})^4 \cdot (-4 + 3 \cdot 2^{\frac{1}{3}} \cdot 3^{\frac{2}{3}})^{\frac{3}{2}}) - \text{Log}[6 - 3$

$$\frac{(-3)^{1/3} \cdot 2^{2/3} \cdot x + x^2}{(972 \cdot 2^{1/3} \cdot 3^{2/3} \cdot (1 + (-1)^{1/3})^4)} + \left(\frac{(I/972) \cdot \text{Log}[6 + 3 \cdot (-2)^{2/3} \cdot 3^{1/3} \cdot x + x^2]}{(2^{1/3} \cdot 3^{1/6} \cdot (1 + (-1)^{1/3}))^5} - \text{Log}[6 + 3 \cdot 2^{2/3} \cdot 3^{1/3} \cdot x + x^2] \right) / (8748 \cdot 2^{1/3} \cdot 3^{2/3})$$

Rule 204

$$\text{Int}[(a_) + (b_.) \cdot (x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 206

$$\text{Int}[(a_) + (b_.) \cdot (x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2])] / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$

Rule 618

$$\text{Int}[(a_.) + (b_.) \cdot (x_) + (c_.) \cdot (x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$$

Rule 628

$$\text{Int}[(d_) + (e_.) \cdot (x_)] / ((a_.) + (b_.) \cdot (x_) + (c_.) \cdot (x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$

Rule 634

$$\text{Int}[(d_.) + (e_.) \cdot (x_)] / ((a_) + (b_.) \cdot (x_) + (c_.) \cdot (x_)^2), x_Symbol] \rightarrow \text{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \text{Int}[1 / (a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$$

Rule 638

$$\text{Int}[(d_.) + (e_.) \cdot (x_)] \cdot ((a_.) + (b_.) \cdot (x_) + (c_.) \cdot (x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(b \cdot d - 2 \cdot a \cdot e + (2 \cdot c \cdot d - b \cdot e) \cdot x) \cdot (a + b \cdot x + c \cdot x^2)^{p+1}] / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), x] - \text{Dist}[(2 \cdot p + 3) \cdot (2 \cdot c \cdot d - b \cdot e) / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[(a + b \cdot x + c \cdot x^2)^{p+1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$$

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^8}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}} \left(-1 + 3(-3)^{2/3} \sqrt[3]{2} + (9 + \sqrt[3]{-3}) \right)}{42845606719488 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (6 - 3\sqrt[3]{-3})} \right) dx$$

$$= \frac{\sqrt[3]{-\frac{1}{3}} \int \frac{-1 - 3\sqrt[3]{-2} 3^{2/3} + (9 - (-2)^{2/3} \sqrt[3]{3})x}{(6 + 3(-2)^{2/3} \sqrt[3]{3}x + x^2)^2} dx}{243 \cdot 2^{2/3}} - \frac{\int \frac{-27 + x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3}x + x^2} dx}{4374 \sqrt[3]{2} \cdot 3^{2/3}} + \dots$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}} \left(9(6 + \sqrt[3]{-3} 2^{2/3}) + (2 - 3 \cdot 2^{2/3} (2(-6)^{2/3} + 9\sqrt[3]{-3}))x \right)}{162 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)}$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.16

$$\frac{-9x^5 - 203x^4 - 11610x^3 - 3990x^2 + 324x - 7884}{34182(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{9\#1^4 \log(x-\#1) + 406\#1^3 \log(x-\#1) + 324\#1^2 \log(x-\#1) - 96\#1 \log(x-\#1) + 324 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{205092}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $(-7884 + 324x - 3990x^2 - 11610x^3 - 203x^4 - 9x^5)/(34182(216 + 108x^2 + 324x^3 + 18x^4 + x^6)) - \text{RootSum}[216 + 108\#1^2 + 324\#1^3 + 18\#1^4 + \#1^6 \& , (324\text{Log}[x - \#1] - 96\text{Log}[x - \#1]\#1 + 324\text{Log}[x - \#1]\#1^2 + 406\text{Log}[x - \#1]\#1^3 + 9\text{Log}[x - \#1]\#1^4)/(36\#1 + 162\#1^2 + 12\#1^3 + \#1^5) \&]/205092$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.02, size = 122, normalized size = 0.11

$\frac{(-9\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^4 - 406\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^3 - 324\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^2 + 96\text{RootOf}(z^2+18z^2+324z^3+108z^2+216) - 324)\ln(-\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)+x) - \frac{1}{108}x^8 - \frac{203}{18}x^4 - \frac{215}{27}x^3 - \frac{665}{54}x^2 + \frac{2}{27}x - \frac{116}{81}}{205092\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^5 + 2465104\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^3 + 33224904\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)^2 + 7383312\text{RootOf}(z^2+18z^2+324z^3+108z^2+216)} + \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] $(-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/205092*\text{sum}((-9*_R^4-406*_R^3-324*_R^2+96*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*\ln(-_R+x), _R=\text{RootOf}(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^5 + 203x^4 + 11610x^3 + 3990x^2 - 324x + 7884}{34182(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{34182} \int \frac{9x^4 + 406x^3 + 324x^2 - 96x + 324}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")`

[Out] $-1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*\text{integrate}((9*x^4 + 406*x^3 + 324*x^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)$

mupad [B] time = 2.34, size = 388, normalized size = 0.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)`

[Out] $\text{symsum}(\log((239491904*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)*x)/876306843 - (275536*x)/638827688547 - (3848128*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k))/3606201 - (152363520*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^2*x)/44521 - (698075283456*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^3*x)/44521 + (130789789876224*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^4*x)/211 - 6940988288557056*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^5*x - (4264220928*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, k)^2)/44521 - (5086414725120*\text{root}(z^6 + (326*z^4)/5$

$$\begin{aligned}
& 54702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883 \\
& 83387712 + (505*z)/13368686435083133627113728 + 4513/8525601705296418741512 \\
& 3360664576, z, k)^3)/44521 + (243585208571904*\text{root}(z^6 + (326*z^4)/55470223 \\
& 1619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/50939972698838338771 \\
& 2 + (505*z)/13368686435083133627113728 + 4513/85256017052964187415123360664 \\
& 576, z, k)^4)/211 - 168897381688221696*\text{root}(z^6 + (326*z^4)/554702231619 + \\
& (8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (50 \\
& 5*z)/13368686435083133627113728 + 4513/85256017052964187415123360664576, z, \\
& k)^5 - 48160/23660284761)*\text{root}(z^6 + (326*z^4)/554702231619 + (8113597*z^3 \\
&)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868 \\
& 6435083133627113728 + 4513/85256017052964187415123360664576, z, k), k, 1, 6 \\
&) - ((665*x^2)/5697 - (2*x)/211 + (215*x^3)/633 + (203*x^4)/34182 + x^5/379 \\
& 8 + 146/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
\end{aligned}$$

sympy [A] time = 0.40, size = 112, normalized size = 0.11

RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_t**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t + 4513, Lambda(_t, _t*log(35492036204084174404119193135483487466590764032*_t**5/356900697070792948475845 - 19474160067218837086826809631017022308224*_t**4/71380139414158589695169 + 20779963076545132233894582764903396544*_t**3/356900697070792948475845 + 20265219154367004972162198012037344*_t**2/356900697070792948475845 + 275192468949210532049075145372*_t/356900697070792948475845 + x + 1290285191292177289622012/1070702091212378845427535))) + (-9*x**5 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383312)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_t**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t + 4513, Lambda(_t, _t*log(35492036204084174404119193135483487466590764032*_t**5/356900697070792948475845 - 19474160067218837086826809631017022308224*_t**4/71380139414158589695169 + 20779963076545132233894582764903396544*_t**3/356900697070792948475845 + 20265219154367004972162198012037344*_t**2/356900697070792948475845 + 275192468949210532049075145372*_t/356900697070792948475845 + x + 1290285191292177289622012/1070702091212378845427535))) + (-9*x**5 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383312)

$$3.148 \quad \int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=1005

$$\frac{2(2-3\sqrt[3]{2}3^{2/3})-3(6-2^{2/3}\sqrt[3]{3})x}{2916 \cdot 2^{2/3} \sqrt[3]{3} (4-3\sqrt[3]{2}3^{2/3})(x^2+3 \cdot 2^{2/3} \sqrt[3]{3}x+6)} + \frac{(9i + \sqrt[3]{3} (2i2^{2/3} - 9\sqrt[3]{3} + 2 \cdot 2^{2/3} \sqrt[3]{3})) \tan^{-1} \left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6(4-3(-3)^{2/3}\sqrt[3]{2}})} \right)}{5832(1+\sqrt[3]{-1})^5 \sqrt{2(4-3(-3)^{2/3}\sqrt[3]{2})}}$$

Rubi [A] time = 2.40, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 634, 618, 204, 628, 638, 206}

Antiderivative was successfully verified.

[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $-(2*(2*(-1)^{1/3}*3^{2/3} + 9*6^{1/3}) - 9*((-2)^{2/3} + 2*(-1)^{1/3}*3^{2/3}))*x/(972*2^{2/3}*(1 + (-1)^{1/3}))^4*(4 - 3*(-3)^{2/3}*2^{1/3})*(6 - 3*(-3)^{1/3}*2^{2/3}*x + x^2)) - ((-6)^{1/3}*(9*(-2)^{1/3} + 2*3^{1/3}) - 9*(1 + (-2)^{1/3}*3^{2/3})*x)/(4374*(8 + (9*I)*2^{1/3}*3^{1/6} + 3*2^{1/3}*3^{2/3}))*x/(2916*2^{2/3}*3^{1/3}*(4 - 3*2^{1/3}*3^{2/3}))*x + ((9*I + 3^{1/3}*((2*I)*2^{2/3} - 9*3^{1/6} + 2*2^{2/3}*Sqrt[3]))*ArcTan[(3*(-3)^{1/3}*2^{2/3} - 2*x)/Sqrt[6*(4 - 3*(-3)^{2/3}*2^{1/3})]])/(5832*(1 + (-1)^{1/3}))^5*Sqrt[2*(4 - 3*(-3)^{2/3}*2^{1/3})]) + (((1 + (-2)^{1/3}*3^{2/3})*ArcTan[(3*(-2)^{2/3}*3^{1/3} + 2*x)/Sqrt[6*(4 + 3*(-2)^{1/3}*3^{2/3})]])/(54*Sqrt[6]*(1 - (-1)^{1/3}))^2*(1 + (-1)^{1/3}))^4*(4 + 3*(-2)^{1/3}*3^{2/3})^{3/2} + ((9*3^{1/6} + I*(4*2^{2/3} - 3*3^{2/3}))*ArcTan[(3*(-2)^{2/3}*3^{1/3} + 2*x)/Sqrt[6*(4 + 3*(-2)^{1/3}*3^{2/3})]])/(1944*3^{2/3}*(1 + (-1)^{1/3}))^5*Sqrt[2*(4 + 3*(-2)^{1/3}*3^{2/3})]) - ((-1)^{1/3}*((-3)^{1/3} + 3*2^{1/3})*ArcTan[(2^{1/6}*(3*(-3)^{1/3} - 2^{1/3})*x)/Sqrt[3*(4 - 3*(-3)^{2/3}*2^{1/3})]])/(54*Sqrt[2]*3^{5/6}*(1 + (-1)^{1/3}))^4*(4 - 3*(-3)^{2/3}*2^{1/3})^{3/2} + (((1 - 2^{1/3}*3^{2/3})*ArcTanh[(2^{1/6}*(3*3^{1/3} + 2^{1/3})*x)/Sqrt[3*(-4 + 3*2^{1/3}*3^{2/3})]])/(54*Sqrt[6]*(1 - (-1)^{1/3}))^2*(1 + (-1)^{1/3}))^4*(-4 + 3*2^{1/3}*3^{2/3})^{3/2} + ((2*2^{2/3} + 3*3^{2/3})*ArcTanh[(2^{1/6}*(3*3^{1/3} + 2^{1/3})*x)/Sqrt[3*(-4 + 3*2^{1/3}*3^{2/3})]])/(26244*3^{1/6}*Sqrt[2*(-4 + 3*2^{1/3}*3^{2/3})]) + ((I/648)*Log[6 - 3*(-3)^{1/3}*2^{2/3}*x + x^2])/(2^{2/3}*3^{5/6}*(1 + (-1)^{1/3}))^5 - ((I + Sqrt[3])*Log[6 + 3*(-2)^{2/3}*3^{1/3}*x + x^2])/(129$

$$6 \cdot 2^{2/3} \cdot 3^{5/6} \cdot (1 + (-1)^{1/3})^5 - \text{Log}[6 + 3 \cdot 2^{2/3} \cdot 3^{1/3} \cdot x + x^2] / (17496 \cdot 2^{2/3} \cdot 3^{1/3})$$

Rule 204

$$\text{Int}[(a_.) + (b_.) \cdot (x_.)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

Rule 206

$$\text{Int}[(a_.) + (b_.) \cdot (x_.)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]] / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$

Rule 618

$$\text{Int}[(a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$$

Rule 628

$$\text{Int}[(d_.) + (e_.) \cdot (x_.)] / ((a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$

Rule 634

$$\text{Int}[(d_.) + (e_.) \cdot (x_.)] / ((a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2), x_Symbol] \rightarrow \text{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \text{Int}[1 / (a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$$

Rule 638

$$\text{Int}[(d_.) + (e_.) \cdot (x_.)] \cdot ((a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(b \cdot d - 2 \cdot a \cdot e + (2 \cdot c \cdot d - b \cdot e) \cdot x) \cdot (a + b \cdot x + c \cdot x^2)^{(p+1)} / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), x] - \text{Dist}[(2 \cdot p + 3) \cdot (2 \cdot c \cdot d - b \cdot e) / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[(a + b \cdot x + c \cdot x^2)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \&\& \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[p, -3/2]$$

Rule 2097

$$\text{Int}[(Q6_.)^{p_} \cdot (u_), x_Symbol] \rightarrow \text{With}[\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}$$

[1/(3^(3*p))*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^7}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{-27 + 3 \cdot 2^{2/3} \sqrt[3]{3} + 9i\sqrt{3} + i2^{2/3}3^{5/6} - 3i\sqrt[3]{-3}}{9254651051409408 (1 + \sqrt[3]{-1})^5 (-6 + 3\sqrt[3]{-3})} \right. \\ = \frac{\int \frac{-18 - 2 \cdot 2^{2/3} \sqrt[3]{3} - \sqrt[3]{2} 3^{2/3} x}{6 + 3 \cdot 2^{2/3} \sqrt[3]{3} x + x^2} dx}{52488} + \frac{\int \frac{9 \cdot 2^{2/3} + \sqrt[3]{-1} 3^{2/3} (1 + 3 \sqrt[3]{-2} 3^{2/3}) x}{(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{4374 \cdot 2^{2/3}} + \dots \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \dots \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \dots \\ = -\frac{2(2\sqrt[3]{-1} 3^{2/3} + 9\sqrt[3]{6}) - 9((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) x}{972 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \dots$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{73\#1^4 \log(x-\#1) - 36\#1^3 \log(x-\#1) + 96\#1^2 \log(x-\#1) - 216\#1 \log(x-\#1) + 96 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right] \&}{410184} + \frac{73x^5 - 18x^4 + 908x^3 + 432x^2 - 96x + 648}{68364(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1

$\wedge 6 \& , (96*\text{Log}[x - \#1] - 216*\text{Log}[x - \#1]*\#1 + 96*\text{Log}[x - \#1]*\#1^2 - 36*\text{Log}[x - \#1]*\#1^3 + 73*\text{Log}[x - \#1]*\#1^4)/(36*\#1 + 162*\#1^2 + 12*\#1^3 + \#1^5) \&] / 410184$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.02, size = 122, normalized size = 0.12

$$\frac{(73\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^5 - 36\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^4 + 96\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 - 216\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 + 96\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) - 36)\ln(-\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + x) + \frac{73}{108}x^5 - \frac{1}{36}x^4 + \frac{227}{17091}x^3 + \frac{4}{633}x^2 - \frac{8}{5697}x + \frac{2}{211}}{410184\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^5 + 4922208\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^4 + 66449808\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 34766624\text{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R

$\sqrt[5]{12R^3 + 162R^2 + 36R} \ln(-R+x)$, $R = \text{RootOf}(-Z^6 + 18Z^4 + 324Z^3 + 108Z^2 + 216)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{73x^5 - 18x^4 + 908x^3 + 432x^2 - 96x + 648}{68364(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{68364} \int \frac{73x^4 - 36x^3 + 96x^2 - 216x + 96}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 216*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.30, size = 387, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/((108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)*x)/7886761587 - (207565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5*x - (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289589870088463413332668913549312

```
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
(11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899
008 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884
63413332668913549312, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 71
97829/589289589870088463413332668913549312, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5 +
2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (198
9787*z)/311864717157619341253309046784 - 7197829/58928958987008846341333266
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 5396402902660752484057
37472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 37
59837842016*_t - 7197829, Lambda(_t, _t*log(4299602763972744771400374330516
0746111018438501025999323136*_t**5/154206009791052044490694380303237521 - 4
2584766259508194684689715474422251405157209835847680*_t**4/1542060097910520
44490694380303237521 - 37512446128849588150108369449323754078317341082112*_
t**3/154206009791052044490694380303237521 + 7152037594021675267638890715531
672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835
297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 1
74573349036676047734132569583024855/154206009791052044490694380303237521)))
+ (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 123
0552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

$$3.149 \quad \int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=677

$$\frac{\sqrt[3]{6} \left(9 + \sqrt[3]{-3} 2^{2/3}\right) x + 9(-2)^{2/3}}{2916 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} 3^{2/3} \left(2 + 3\sqrt[3]{-2} 3^{2/3}\right) x + 9 2^2}{13122 2^{2/3} \left(8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right) \left(x^2 + 3(-\right)}$$

Rubi [A] time = 1.55, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 638, 618, 204, 628, 206}

$$\frac{\sqrt[3]{6} \left(9 + \sqrt[3]{-3} 2^{2/3}\right) x + 9(-2)^{2/3}}{2916 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} 3^{2/3} \left(2 + 3\sqrt[3]{-2} 3^{2/3}\right) x + 9 2^2}{13122 2^{2/3} \left(8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right) \left(x^2 + 3(-\right)}$$

Antiderivative was successfully verified.

[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $(9*(-2)^{(2/3)} + 6^{(1/3)}*(9 + (-3)^{(1/3)}*2^{(2/3)})*x)/(2916*2^{(2/3)}*(1 + (-1)^{(1/3)})^4*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})*(6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2)) + (9*2^{(2/3)} + (-1)^{(1/3)}*3^{(2/3)}*(2 + 3*(-2)^{(1/3)}*3^{(2/3)})*x)/(13122*2^{(2/3)}*(8 + (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)}*(6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2)) + (3*2^{(2/3)}*3^{(1/3)} - (2 - 3*2^{(1/3)}*3^{(2/3)})*x)/(8748*2^{(2/3)}*3^{(1/3)}*(4 - 3*2^{(1/3)}*3^{(2/3)})*(6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2)) + ((-1)^{(1/3)}*(3*(-3)^{(2/3)} - 2^{(2/3)})*ArcTan[(3*(-3)^{(1/3)}*2^{(2/3)} - 2*x)/Sqrt[6*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(486*6^{(5/6)}*(1 + (-1)^{(1/3)})^4*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})^{(3/2)}) + ((3*(-3)^{(2/3)} + (-1)^{(1/3)}*2^{(2/3)})*ArcTan[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/Sqrt[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(486*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})^{(3/2)}) - ((2^{(2/3)} - 3*3^{(2/3)})*ArcTanh[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/Sqrt[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(486*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(-4 + 3*2^{(1/3)}*3^{(2/3)})^{(3/2)}) + ((-1/3)^{(1/6)}*Log[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/(5832*2^{(1/3)}*(1 + (-1)^{(1/3)})^5) - ((I/5832)*Log[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(2^{(1/3)}*3^{(1/6)}*(1 + (-1)^{(1/3)})^5) + Log[6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2]/(52488*2^{(1/3)}*3^{(2/3)})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(\frac{-2\sqrt[3]{-1} 3^{2/3} + 3(-2)^{2/3}x}{1542441841901568 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + 3x)} \right. \\
&= -\frac{\int \frac{-2\sqrt[3]{-1} 3^{2/3} + 3 2^{2/3}x}{(6+3(-2)^{2/3} \sqrt[3]{3}x+x^2)^2} dx}{8748 2^{2/3}} - \frac{\int \frac{2+2^{2/3} \sqrt[3]{3}x}{(6+3 2^{2/3} \sqrt[3]{3}x+x^2)^2} dx}{2916 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{3 \sqrt[3]{3} + \sqrt[3]{2}x}{6+3 2^{2/3} \sqrt[3]{3}x}}{26244 6^2} \\
&= \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)} + \\
&= \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)} + \\
&= \frac{9(-2)^{2/3} + \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{2916 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3}x + x^2)} +
\end{aligned}$$

Mathematica [C] time = 0.05, size = 167, normalized size = 0.25

$$\frac{-3x^5 + 73x^4 - 72x^3 - 64x^2 + 108x - 96}{68364 (x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{3\#1^4 \log(x-\#1) - 146\#1^3 \log(x-\#1) + 108\#1^2 \log(x-\#1) - 32\#1 \log(x-\#1) + 108 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{410184}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/410184

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.02, size = 122, normalized size = 0.18

$$\frac{(-3\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)^3+146\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)^2-108\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)-32\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)-108)\ln(-\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)+x)-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{410184\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)^3+4922208\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)^2+66449808\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)+14766624\text{RootOf}(Z^6+18Z^4+324Z^3+108Z^2+216)}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] $(-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/410184*\text{sum}((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)/(*_R^5+12*_R^3+162*_R^2+36*_R)*\ln(-_R+x),_R=\text{RootOf}(Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3x^5 - 73x^4 + 72x^3 + 64x^2 - 108x + 96}{68364(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{68364} \int \frac{3x^4 - 146x^3 + 108x^2 - 32x + 108}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] $-1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/68364*\int((3*x^4 - 146*x^3 + 108*x^2 - 32*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)$

mupad [B] time = 2.33, size = 388, normalized size = 0.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] $\text{symsum}(\log((7028852*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)*x)/70980854283 - (6628544*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^2*x)/44521 - (141776759808*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^3*x)/44521 + (183701926508544*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^4*x)/211 - 6940988288557056*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^5*x + (100886752*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^2)/133563 + (1715052538368*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^3)/44521 + (115004308571136*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^4)/211 - 168897381688221696*\text{root}(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*\text{root}(z$

$$\begin{aligned} &^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - \\ &(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090 \\ &46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1 \\ &6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/ \\ &(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216) \end{aligned}$$

sympy [A] time = 0.38, size = 112, normalized size = 0.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 101031431941529596105
0951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 -
50648453064*_t - 880007, Lambda(_t, _t*log(-2736555670900189915706499414143
95560986199688040644608*_t**5/49797855396139900267573395695 + 1183700847019
6046085308646230764354292805044570112*_t**4/49797855396139900267573395695 -
10570581900446717266374077482873315047787008*_t**3/49797855396139900267573
395695 - 1552547411569469872387563218792789323392*_t**2/4979785539613990026
7573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573
395695 + x - 23066533870320322410834348296/49797855396139900267573395695))
+ (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 12305
52*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)

$$3.150 \quad \int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}} \left(4 - \sqrt[3]{-3} 2^{2/3} x\right)}{1944 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-\frac{1}{3}} \left((-2)^{2/3} \sqrt[3]{3} x + 4\right)}{8748 2^{2/3} \left(8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right) \left(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6\right)}$$

Rubi [A] time = 1.24, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2097, 638, 618, 204, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}} \left(4 - \sqrt[3]{-3} 2^{2/3} x\right)}{1944 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-\frac{1}{3}} \left((-2)^{2/3} \sqrt[3]{3} x + 4\right)}{8748 2^{2/3} \left(8 + 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}\right) \left(x^2 + 3(-2)^{2/3} \sqrt[3]{3} x + 6\right)}$$

Antiderivative was successfully verified.

[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $\left(\frac{(-1/3)^{1/3} (4 - (-3)^{1/3} 2^{2/3} x)}{(1944 2^{2/3} (1 + (-1)^{1/3}))^4 (4 - 3(-3)^{2/3} 2^{1/3}) (6 - 3(-3)^{1/3} 2^{2/3} x + x^2)} + \frac{(-1/3)^{1/3} (4 + (-2)^{2/3} 3^{1/3} x)}{(8748 2^{2/3} (8 + (9I) 2^{1/3} 3^{1/6} + 3 2^{1/3} 3^{2/3}) (6 + 3(-2)^{2/3} 3^{1/3} x + x^2)} - \frac{(4 + 2^{2/3} 3^{1/3} x)}{(17496 2^{2/3} 3^{1/3} (4 - 3 2^{1/3} 3^{2/3}) (6 + 3 2^{2/3} 3^{1/3} x + x^2)} - \text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} 2^{1/3})}}\right]\right) / (4374 2^{5/6} 3^{1/6} (1 + (-1)^{1/3})^4 \sqrt{4 - 3(-3)^{2/3} 2^{1/3}}) + \text{ArcTan}\left[\frac{3(-3)^{1/3} 2^{2/3} - 2x}{\sqrt{6(4 - 3(-3)^{2/3} 2^{1/3})}}\right] / (4374 \sqrt{3} (8 - (9I) 2^{1/3} 3^{1/6} + 3 2^{1/3} 3^{2/3})^{3/2}) - ((I/1458) \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4 + 3(-2)^{1/3} 3^{2/3})}}\right]) / (2^{5/6} 3^{2/3} (1 + (-1)^{1/3})^5 \sqrt{4 + 3(-2)^{1/3} 3^{2/3}}) - \text{ArcTan}\left[\frac{3(-2)^{2/3} 3^{1/3} + 2x}{\sqrt{6(4 + 3(-2)^{1/3} 3^{2/3})}}\right] / (4374 \sqrt{3} (8 + (9I) 2^{1/3} 3^{1/6} + 3 2^{1/3} 3^{2/3})^{3/2}) - \text{ArcTanh}\left[\frac{2^{1/6} (3 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4 + 3 2^{1/3} 3^{2/3})}}\right] / (8748 \sqrt{6} (-4 + 3 2^{1/3} 3^{2/3})^{3/2}) - \text{ArcTanh}\left[\frac{2^{1/6} (3 3^{1/3} + 2^{1/3} x)}{\sqrt{3(-4 + 3 2^{1/3} 3^{2/3})}}\right] / (39366 2^{5/6} 3^{1/6} \sqrt{4 + 3(-2)^{1/3} 3^{2/3}})$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}} x}{1542441841901568 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + \dots)} \right. \\
&= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{x}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748 \cdot 2^{2/3}} + \frac{\int \frac{1}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{26244 \sqrt[3]{2} \cdot 3^{2/3}} + \frac{\int \frac{1}{(6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748} \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} \cdot 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3} x + x^2)} + \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} \cdot 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3} x + x^2)} + \\
&= \frac{\sqrt[3]{-\frac{1}{3}} (4 - \sqrt[3]{-3} \cdot 2^{2/3} x)}{1944 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} \cdot 2^{2/3} x + x^2)} +
\end{aligned}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{4\#1^4 \log(x-\#1) - 54\#1^3 \log(x-\#1) + 2043\#1^2 \log(x-\#1) - 324\#1 \log(x-\#1) + 144 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right] \&}{3691656} + \frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/3691656

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [B] time = 3.89, size = 1445, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] $\frac{1}{28041818976} \cdot (182304x^5 - 1230552x^4 + 33224904x^3 + 422\sqrt{1/633})(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)\sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} \cdot \log\left(\frac{2}{1982119441}\sqrt{1/633} \cdot (7238020557 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 - 4479023748400406176979673 \cdot 18^{2/3} - 8334306522507661258645112 \cdot 18^{1/3} - 26862559811422885347120477)\sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} - 7383041510/9393931 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 247458158879850620x + 5132255454960803463351330/9393931 \cdot 18^{2/3} + 9549802036377046040753520/9393931 \cdot 18^{1/3} + 27278928233033940032425830/9393931) - 422\sqrt{1/633} \cdot (x^6 + 18x^4 + 324x^3 + 108x^2 + 216)\sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} \cdot \log\left(-\frac{2}{1982119441}\sqrt{1/633} \cdot (7238020557 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 - 4479023748400406176979673 \cdot 18^{2/3} - 8334306522507661258645112 \cdot 18^{1/3} - 26862559811422885347120477)\sqrt{5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457} - 7383041510/9393931 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 247458158879850620x + 5132255454960803463351330/9393931 \cdot 18^{2/3} + 9549802036377046040753520/9393931 \cdot 18^{1/3} + 27278928233033940032425830/9393931) - 9\sqrt{422} \cdot (x^6 + 18x^4 + 324x^3 + 108x^2 + 216)\sqrt{-20718 \cdot 18^{2/3} + \sqrt{-1/19683} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699} - 9367856/243 \cdot 18^{1/3} + 367798) \cdot \log\left(\frac{14766083020}{211} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 3064230/211 \cdot \sqrt{-1/19683} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) \cdot (5895278433468 \cdot 18^{2/3} + 10969590754592 \cdot 18^{1/3} + 57028339027521) + 9/9393931 \cdot (14476041114 \cdot \sqrt{422}) \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 243 \cdot \sqrt{-1/19683} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) \cdot (14476041114 \cdot \sqrt{422}) \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) - 161351097450615865 \cdot \sqrt{422}) - 1779341296985705429 \cdot \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) + 26505855880569051992480475 \cdot \sqrt{422}) \cdot \sqrt{-20718 \cdot 18^{2/3} + \sqrt{-1/19683} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699} - 9367856/243 \cdot 18^{1/3} + 367798) + 44068338765959317812080x - 10264510909921606926702660/211 \cdot 18^{2/3} - 19099604072754$

$$\begin{aligned}
& 092081507040/211*18^{(1/3)} - 54557856466067880064851660/211) + 9*\text{sqrt}(422)*(\\
& x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*\text{sqrt}(-20718*18^{(2/3)} + \text{sqrt}(-1/1968 \\
& 3*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + 22860116892*18^{(2/3)} \\
& + 3445478701088/81*18^{(1/3)} + 273974962699) - 9367856/243*18^{(1/3)} + 36779 \\
& 8)*\log(14766083020/211*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + \\
& 3064230/211*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457) \\
& ^2 + 22860116892*18^{(2/3)} + 3445478701088/81*18^{(1/3)} + 273974962699)*(5895 \\
& 278433468*18^{(2/3)} + 10969590754592*18^{(1/3)} + 57028339027521) - 9/9393931* \\
& (14476041114*\text{sqrt}(422)*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + \\
& 243*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + 228 \\
& 60116892*18^{(2/3)} + 3445478701088/81*18^{(1/3)} + 273974962699)*(14476041114* \\
& \text{sqrt}(422)*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457) - 16135109745061 \\
& 5865*\text{sqrt}(422)) - 1779341296985705429*\text{sqrt}(422)*(5034474*18^{(2/3)} + 9367856 \\
& *18^{(1/3)} + 44687457) + 26505855880569051992480475*\text{sqrt}(422))*\text{sqrt}(-20718*1 \\
& 8^{(2/3)} + \text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 \\
& + 22860116892*18^{(2/3)} + 3445478701088/81*18^{(1/3)} + 273974962699) - 936785 \\
& 6/243*18^{(1/3)} + 367798) + 44068338765959317812080*x - 10264510909921606926 \\
& 702660/211*18^{(2/3)} - 19099604072754092081507040/211*18^{(1/3)} - 54557856466 \\
& 067880064851660/211) - 9*\text{sqrt}(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) \\
& *\text{sqrt}(-20718*18^{(2/3)} - \text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} \\
& + 44687457)^2 + 22860116892*18^{(2/3)} + 3445478701088/81*18^{(1/3)} + 27397496 \\
& 2699) - 9367856/243*18^{(1/3)} + 367798)*\log(14766083020/211*(5034474*18^{(2/3)} \\
&) + 9367856*18^{(1/3)} + 44687457)^2 - 3064230/211*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} \\
& (2/3) + 9367856*18^{(1/3)} + 44687457)^2 + 22860116892*18^{(2/3)} + 34454787010 \\
& 88/81*18^{(1/3)} + 273974962699)*(5895278433468*18^{(2/3)} + 10969590754592*18^{(1/3)} \\
& + 57028339027521) + 9/9393931*(14476041114*\text{sqrt}(422)*(5034474*18^{(2/3)} \\
&) + 9367856*18^{(1/3)} + 44687457)^2 - 243*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} + \\
& 9367856*18^{(1/3)} + 44687457)^2 + 22860116892*18^{(2/3)} + 3445478701088/81*18 \\
& ^{(1/3)} + 273974962699)*(14476041114*\text{sqrt}(422)*(5034474*18^{(2/3)} + 9367856*1 \\
& 8^{(1/3)} + 44687457) - 161351097450615865*\text{sqrt}(422)) - 1779341296985705429*s \\
& \text{qrt}(422)*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457) + 265058558805690 \\
& 51992480475*\text{sqrt}(422))*\text{sqrt}(-20718*18^{(2/3)} - \text{sqrt}(-1/19683*(5034474*18^{(2/ \\
& 3) + 9367856*18^{(1/3)} + 44687457)^2 + 22860116892*18^{(2/3)} + 3445478701088/ \\
& 81*18^{(1/3)} + 273974962699) - 9367856/243*18^{(1/3)} + 367798) + 440683387659 \\
& 59317812080*x - 10264510909921606926702660/211*18^{(2/3)} - 19099604072754092 \\
& 081507040/211*18^{(1/3)} - 54557856466067880064851660/211) + 9*\text{sqrt}(422)*(x^6 \\
& + 18*x^4 + 324*x^3 + 108*x^2 + 216)*\text{sqrt}(-20718*18^{(2/3)} - \text{sqrt}(-1/19683*(\\
& 5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + 22860116892*18^{(2/3)} + \\
& 3445478701088/81*18^{(1/3)} + 273974962699) - 9367856/243*18^{(1/3)} + 367798)* \\
& \log(14766083020/211*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 - 30 \\
& 64230/211*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 \\
& + 22860116892*18^{(2/3)} + 3445478701088/81*18^{(1/3)} + 273974962699)*(5895278 \\
& 433468*18^{(2/3)} + 10969590754592*18^{(1/3)} + 57028339027521) - 9/9393931*(14 \\
& 476041114*\text{sqrt}(422)*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 - 24 \\
& 3*\text{sqrt}(-1/19683*(5034474*18^{(2/3)} + 9367856*18^{(1/3)} + 44687457)^2 + 228601
\end{aligned}$$

$16892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) \cdot (14476041114 \cdot \sqrt{422}) \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) - 161351097450615865 \cdot \sqrt{422}) - 1779341296985705429 \cdot \sqrt{422} \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457) + 26505855880569051992480475 \cdot \sqrt{422}) \cdot \sqrt{-20718 \cdot 18^{2/3} - \sqrt{-1/19683 \cdot (5034474 \cdot 18^{2/3} + 9367856 \cdot 18^{1/3} + 44687457)^2 + 22860116892 \cdot 18^{2/3} + 3445478701088/81 \cdot 18^{1/3} + 273974962699) - 9367856/243 \cdot 18^{1/3} + 367798} + 44068338765959317812080 \cdot x - 10264510909921606926702660/211 \cdot 18^{2/3} - 19099604072754092081507040/211 \cdot 18^{1/3} - 54557856466067880064851660/211) + 29533248 \cdot x^2 - 6562944 \cdot x + 44299872) / (x^6 + 18 \cdot x^4 + 324 \cdot x^3 + 108 \cdot x^2 + 216)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.18

$$\frac{(4 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) - 54 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)) + 2043 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) - 324 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + 144) \ln(-\operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + \frac{1}{10381z^5} - \frac{1}{22788z^4} + \frac{1}{844z^3} + \frac{2}{1899z^2} - \frac{4}{17091z} + \frac{1}{633})}{3691656 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 + 44299872 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 598048272 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^4 + 132899616 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{615276} \int \frac{4x^4 - 54x^3 + 2043x^2 - 324x + 144}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.31, size = 299, normalized size = 0.44

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5/(108x^2 + 324x^3 + 18x^4 + x^6 + 216)^2, x)$

[Out] $\text{symsum}(\log((6305x)/4967524106141472 - (4477969\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k))/189282278088 - (16340881\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)x)/5110621508376 - (43348696\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^2x)/10818603 - (65333687616\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^3x)/44521 - (40024496812032\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^4x)/211 - 6940988288557056\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^5x + (5943884\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^2)/400689 + (224442467136\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^3)/44521 - (137087493272064\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^4)/211 - 168897381688221696\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k)^5 - 13082875/178830867821092992\sqrt[3]{z^6 - (183899z^4)/3834101824950528 + (6209z^2)/14083883651774823903461376 - 39753025/27493895104978847349012449000830556700672}, z, k), k, 1, 6) + ((2x^2)/1899 - (4x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/633)/(108x^2 + 324x^3 + 18x^4 + x^6 + 216)$

sympy [A] time = 0.31, size = 104, normalized size = 0.15

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2, x)$

[Out] $\text{RootSum}(27493895104978847349012449000830556700672*_t**6 - 1318718189226950088862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, \text{Lambda}(_t, _t$

$$\begin{aligned} & * \log(947842259001288723909832054550209950242045952*_t^{**5}/61864539719962655 \\ & - 243458646817775607639654889480814592*_t^{**4}/9811980923071 - 41682556475067 \\ & 500431787310779667456*_t^{**3}/61864539719962655 + 12026877442664328616462272* \\ & _t^{**2}/9811980923071 + 216142618488859793668428*_t/61864539719962655 + x - 3 \\ & 08574300024117/39247923692284))) + (4*x^{**5} - 27*x^{**4} + 729*x^{**3} + 648*x^{**2} \\ & - 144*x + 972)/(615276*x^{**6} + 11074968*x^{**4} + 199349424*x^{**3} + 66449808*x^{**} \\ & 2 + 132899616) \end{aligned}$$

$$3.151 \quad \int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=850

$$\frac{\sqrt[3]{-\frac{1}{3}} \left(3\sqrt[3]{-3} 2^{2/3} - 2x\right)}{5832 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{729 2^{2/3} 3^{5/6} \left(1 + \sqrt[3]{-1}\right)^4 \left(8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2}\right)}$$

Rubi [A] time = 1.47, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 614, 618, 204, 634, 628, 206}

$\frac{\sqrt[3]{-\frac{1}{3}} \left(3\sqrt[3]{-3} 2^{2/3} - 2x\right)}{5832 2^{2/3} \left(1 + \sqrt[3]{-1}\right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2}\right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6\right)} + \frac{\sqrt[3]{-1} \tan^{-1}\left(\frac{3\sqrt[3]{-3} 2^{2/3} - 2x}{\sqrt{6(4-3(-3)^{2/3} \sqrt[3]{2})}}\right)}{729 2^{2/3} 3^{5/6} \left(1 + \sqrt[3]{-1}\right)^4 \left(8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2}\right)}$

Antiderivative was successfully verified.

[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $\left(\left(-\frac{1}{3}\right)^{1/3} \left(3 \left(-3\right)^{1/3} 2^{2/3} - 2x\right) / \left(5832 2^{2/3} \left(1 + \left(-1\right)^{1/3}\right)^4 \left(4 - 3 \left(-3\right)^{2/3} 2^{1/3}\right) \left(6 - 3 \left(-3\right)^{1/3} 2^{2/3} x + x^2\right)\right) - \left(\left(-1\right)^{1/3} \left(3 \left(-2\right)^{2/3} 3^{1/3} + 2x\right) / \left(26244 2^{2/3} \left(8 + \left(9I\right) 2^{1/3} 3^{1/3} \left(6 + 3 2^{1/3} 3^{2/3}\right) \left(6 + 3 \left(-2\right)^{2/3} 3^{1/3} x + x^2\right)\right) - \left(3 3^{1/3} + 2^{1/3} x\right) / \left(52488 \left(9 2^{1/3} - 4 3^{1/3}\right) \left(6 + 3 2^{2/3} 3^{1/3} x + x^2\right)\right) + \left(\left(-1\right)^{1/3} \text{ArcTan}\left[\frac{3 \left(-3\right)^{1/3} 2^{2/3} - 2x}{\sqrt{6 \left(4 - 3 \left(-3\right)^{2/3} 2^{1/3}\right)}}\right]\right) / \left(729 2^{2/3} 3^{5/6} \left(1 + \left(-1\right)^{1/3}\right)^4 \left(8 - \left(9I\right) 2^{1/3} 3^{1/3} \left(6 + 3 2^{1/3} 3^{2/3}\right)\right)^{3/2}\right) - \left(\left(-1\right)^{1/3} \text{ArcTan}\left[\frac{3 \left(-2\right)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 \left(-2\right)^{1/3} 3^{2/3}\right)}}\right]\right) / \left(2916 2^{1/6} 3^{5/6} \left(1 - \left(-1\right)^{1/3}\right)^2 \left(1 + \left(-1\right)^{1/3}\right)^4 \left(4 + 3 \left(-2\right)^{1/3} 3^{2/3}\right)^{3/2}\right) - \left(\left(I + \sqrt{3}\right) \text{ArcTan}\left[\frac{3 \left(-2\right)^{2/3} 3^{1/3} + 2x}{\sqrt{6 \left(4 + 3 \left(-2\right)^{1/3} 3^{2/3}\right)}}\right]\right) / \left(11664 2^{1/6} 3^{1/3} \left(1 + \left(-1\right)^{1/3}\right)^5 \sqrt{4 + 3 \left(-2\right)^{1/3} 3^{2/3}}\right) - \left(\left(I / 5832\right) \text{ArcTan}\left[\frac{2^{1/6} \left(3 \left(-3\right)^{1/3} - 2^{1/3} x\right)}{\sqrt{3 \left(4 - 3 \left(-3\right)^{2/3} 2^{1/3}\right)}}\right]\right) / \left(2^{1/6} 3^{1/3} \left(1 + \left(-1\right)^{1/3}\right)^5 \sqrt{4 - 3 \left(-3\right)^{2/3} 2^{1/3}}\right) + \text{ArcTanh}\left[\frac{2^{1/6} \left(3 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 \left(-4 + 3 2^{1/3} 3^{2/3}\right)}}\right] / \left(26244 2^{1/6} 3^{5/6} \left(-4 + 3 2^{1/3} 3^{2/3}\right)^{3/2}\right) + \text{ArcTanh}\left[\frac{2^{1/6} \left(3 3^{1/3} + 2^{1/3} x\right)}{\sqrt{3 \left(-4 + 3 2^{1/3} 3^{2/3}\right)}}\right] / \left(52488 2^{1/6} 3^{5/6} \sqrt{-4 + 3 2^{1/3} 3^{2/3}}\right) - \text{Log}\left[6 - 3 \left(-3\right)^{1/3} 2^{2/3} x + x^2\right] / \left(34992 2^{1/3} 3^{2/3} \left(1 + \left(-1\right)^{1/3}\right)^4\right) + \left(\left(I / 34992\right) \text{Log}\left[6 + 3 \left(-2\right)^{2/3} 3^{1/3} x + x^2\right]\right) / \left(2^{1/3} 3^{1/6} \left(1 + \left(-1\right)^{1/3}\right)^5\right) - \text{Log}\left[6 + 3 2^{2/3} 3^{1/3} x + x^2\right] / \left(314928 2^{1/3} 3^{2/3}\right)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[

Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx &= 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + 3x)} \right. \\
 &= -\frac{\sqrt[3]{-\frac{1}{3}} \int \frac{1}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748 \cdot 2^{2/3}} - \frac{\int \frac{3 \cdot 2^{2/3} \sqrt[3]{3}+x}{6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2} dx}{157464 \sqrt[3]{2} \cdot 3^{2/3}} + \frac{\int \frac{1}{(6+3 \cdot 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{8748} \\
 &= \frac{\sqrt[3]{-\frac{1}{3}} (3 \sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)} - \\
 &= \frac{\sqrt[3]{-\frac{1}{3}} (3 \sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)} - \\
 &= \frac{\sqrt[3]{-\frac{1}{3}} (3 \sqrt[3]{-3} 2^{2/3} - 2x)}{5832 \cdot 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3 \sqrt[3]{-3} 2^{2/3} x + x^2)} -
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.20

$$\frac{-9x^5 + 8x^4 - 216x^3 - 1458x^2 + 324x - 288}{1230552(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{9\#1^4 \log(x-\#1) - 16\#1^3 \log(x-\#1) + 324\#1^2 \log(x-\#1) - 2628\#1 \log(x-\#1) + 324 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{7383312} \&$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/7383312

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{(-9 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^5 + 16 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^4 - 324 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 2628 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 - 324) \ln(-\operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + x) - \frac{1}{136728} x^5 + \frac{1}{153819} x^4 - \frac{1}{5697} x^3 - \frac{1}{844} x^2 + \frac{1}{3798} x - \frac{4}{17091}}{7383312 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^5 + 88599744 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^4 + 1196096544 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 26579232 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 + 108 x^6 + 18 x^4 + 324 x^3 + 108 x^2 + 216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^5 - 8x^4 + 216x^3 + 1458x^2 - 324x + 288}{1230552(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{1230552} \int \frac{9x^4 - 16x^3 + 324x^2 - 2628x + 324}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.42, size = 388, normalized size = 0.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (1090723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (64554687936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77003

```

63386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880
007/185583791958607219605834030755606257729536, z, k)^3)/44521 + (253679492
45952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
040617647997585731147792384 - 880007/18558379195860721960583403075560625772
9536, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^4)/862672910613
8688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607
884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/1855
83791958607219605834030755606257729536, z, k)^5 - 971/22353858477636624)*ro
ot(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018204617694487047
4752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/242506004061764
7997585731147792384 - 880007/185583791958607219605834030755606257729536, z,
k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/
17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)

```

sympy [A] time = 0.39, size = 112, normalized size = 0.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

```

[Out] RootSum(185583791958607219605834030755606257729536*_t**6 - 1309367357962223
565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 40529828454803873
28*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593476
841044707333565976412952759280634691584*_t**5/49797855396139900267573395695
+ 8836979346223785538912817601414711102396804462575616*_t**4/4979785539613
9900267573395695 - 264930581348308532588844249597134695706805067776*_t**3/4
9797855396139900267573395695 + 886135333547363185201515109826158376250624*_
t**2/49797855396139900267573395695 - 682321479574909906511394635855601936*_
t/49797855396139900267573395695 + x - 21375560770846486224291519568/4979785
5396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*
x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 +
265799232)

```

$$3.152 \quad \int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=873

$$\frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[6]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 - 3\sqrt[3]{-3} 2^{2/3}x + 6)} \frac{(9i - \sqrt[3]{3} (2i2^{2/3} + 9\sqrt[6]{3} + 2 2^{2/3}\sqrt{3})) \tan^{-1} \left(\frac{3\sqrt[3]{-3}}{\sqrt{6(4-3)}} \right)}{209952 (1 + \sqrt[3]{-1})^5 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}}$$

Rubi [A] time = 1.92, antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$\frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[6]{3} + 3\sqrt[3]{2} 3^{2/3}) (x^2 - 3\sqrt[3]{-3} 2^{2/3}x + 6)} \frac{(9i - \sqrt[3]{3} (2i2^{2/3} + 9\sqrt[6]{3} + 2 2^{2/3}\sqrt{3})) \tan^{-1} \left(\frac{3\sqrt[3]{-3}}{\sqrt{6(4-3)}} \right)}{209952 (1 + \sqrt[3]{-1})^5 \sqrt{2(4 - 3(-3)^{2/3}\sqrt[3]{2})}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $((-6)^{1/3}*(2*(-3)^{1/3} + 9*2^{1/3}) - 3*x)/(157464*(8 - (9*I)*2^{1/3}*3^{1/6} + 3*2^{1/3}*3^{2/3})*(6 - 3*(-3)^{1/3}*2^{2/3}*x + x^2)) - ((-6)^{1/3})*(9*(-2)^{1/3} + 2*3^{1/3}) + 3*x)/(157464*(8 + (9*I)*2^{1/3}*3^{1/6} + 3*2^{1/3}*3^{2/3})*(6 + 3*(-2)^{2/3}*3^{1/3}*x + x^2)) - (2*2^{1/3} - 3*6^{2/3} - 3^{1/3}*x)/(104976*(9*2^{1/3} - 4*3^{1/3})*(6 + 3*2^{2/3}*3^{1/3}*x + x^2)) + \text{ArcTan}[(3*(-3)^{1/3}*2^{2/3} - 2*x)/\text{Sqrt}[6*(4 - 3*(-3)^{2/3}*2^{1/3})]]/(26244*\text{Sqrt}[3]*(8 - (9*I)*2^{1/3}*3^{1/6} + 3*2^{1/3}*3^{2/3})^{3/2}) - ((9*I - 3^{1/3})*((2*I)*2^{2/3} + 9*3^{1/6} + 2*2^{2/3}*\text{Sqrt}[3]))*\text{ArcTan}[(3*(-3)^{1/3}*2^{2/3} - 2*x)/\text{Sqrt}[6*(4 - 3*(-3)^{2/3}*2^{1/3})]]/(209952*(1 + (-1)^{1/3})^5*\text{Sqrt}[2*(4 - 3*(-3)^{2/3}*2^{1/3})]) - \text{ArcTan}[(3*(-2)^{2/3}*3^{1/3} + 2*x)/\text{Sqrt}[6*(4 + 3*(-2)^{1/3}*3^{2/3})]]/(26244*\text{Sqrt}[3]*(8 + (9*I)*2^{1/3}*3^{1/6} + 3*2^{1/3}*3^{2/3})^{3/2}) + ((9*I + 3^{1/3})*((4*I)*2^{2/3} - 9*3^{1/6}))*\text{ArcTan}[(3*(-2)^{2/3}*3^{1/3} + 2*x)/\text{Sqrt}[6*(4 + 3*(-2)^{1/3}*3^{2/3})]]/(209952*(1 + (-1)^{1/3})^5*\text{Sqrt}[2*(4 + 3*(-2)^{1/3}*3^{2/3})]) - \text{ArcTanh}[(2^{1/6}*(3*3^{1/3} + 2^{1/3}*x))/\text{Sqrt}[3*(-4 + 3*2^{1/3}*3^{2/3})]]/(52488*\text{Sqrt}[6]*(-4 + 3*2^{1/3}*3^{2/3})^{3/2}) + ((2*2^{2/3} - 3*3^{2/3})*\text{ArcTanh}[(2^{1/6}*(3*3^{1/3} + 2^{1/3}*x))/\text{Sqrt}[3*(-4 + 3*2^{1/3}*3^{2/3})]]/(944784*3^{1/6}*\text{Sqrt}[2*(-4 + 3*2^{1/3}*3^{2/3})]) - ((I/23328)*\text{Log}[6 - 3*(-3)^{1/3}*2^{2/3}*x + x^2])/(2^{2/3}*3^{5/6}*(1 + (-1)^{1/3})^5) + ((I + \text{Sqrt}[3])*\text{Log}[6 + 3*(-2)^{2/3}*3^{1/3}*x + x^2])/(46656*2^{2/3}*3^{5/6})*(1 + (-1)^{1/3})^5 + \text{Log}[6 + 3*2^{2/3}*3^{1/3}*x + x^2]/(629856*2^{2/3}*3^{1/3}))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 638

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 2097

Int[(Q6_)^(p_)*(u_), x_Symbol] := With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[

Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(-\frac{9(-2)^{2/3} - \sqrt[3]{-1} 3^{2/3} x}{27763953154228224 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + \dots)} \right. \\ = \frac{\int \frac{18-2 2^{2/3} \sqrt[3]{3} + \sqrt[3]{2} 3^{2/3} x}{6+3 2^{2/3} \sqrt[3]{3} x+x^2} dx}{1889568} - \frac{\int \frac{9 2^{2/3} - \sqrt[3]{-1} 3^{2/3} x}{(6+3(-2)^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{157464 2^{2/3}} - \frac{\int \frac{3 2^{2/3} \sqrt[3]{3}}{(6+3 2^{2/3} \sqrt[3]{3} x+x^2)^2} dx}{52488 2^{2/3}} \\ = \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{\dots}{31492} \\ = \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{\dots}{31492} \\ = \frac{\sqrt[3]{-6} (2\sqrt[3]{-3} + 9\sqrt[3]{2}) - 3x}{157464 (8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3\sqrt[3]{2} 3^{2/3}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} - \frac{\dots}{31492}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{2\#1^4 \log(x-\#1) - 27\#1^3 \log(x-\#1) + 72\#1^2 \log(x-\#1) - 162\#1 \log(x-\#1) + 1971 \log(x-\#1)}{\#1^5 + 12\#1^3 + 162\#1^2 + 36\#1}\right] \& + \frac{4x^5 - 27x^4 + 96x^3 + 648x^2 - 3942x + 972}{3691656(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)}}{11074968}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/11074968

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{(2 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) - 27 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 72 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 - 162 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + 1971) \ln(-\operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + x) + \frac{1}{522914} x^5 - \frac{1}{136728} x^4 + \frac{4}{153819} x^3 + \frac{1}{5697} x^2 - \frac{73}{68364} x + \frac{1}{3798}}{11074968 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216) + 13289936 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^3 + 1794144816 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)^2 + 398698848 \operatorname{RootOf}(z^6 + 18z^4 + 324z^3 + 108z^2 + 216)}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4x^5 - 27x^4 + 96x^3 + 648x^2 - 3942x + 972}{3691656(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} + \frac{1}{1845828} \int \frac{2x^4 - 27x^3 + 72x^2 - 162x + 1971}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 - 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.42, size = 387, normalized size = 0.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k))/30663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)*x)/6623365474855296 + (6603523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^5*x + (166697*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k)^6*x)/1845828

$$\begin{aligned}
& 832, z, k)^2)/43274412 + (639193032*\text{root}(z^6 - (292589*z^4)/414082997094657 \\
& 024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068 \\
& 6141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 - \\
& 7197829/1282755170017893101915524820582750453426552832, z, k)^3)/44521 - (9 \\
& 815247601920*\text{root}(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35 \\
& 20970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (\\
& 1989787*z)/18857266875842830829226645405233577984 - 7197829/128275517001789 \\
& 3101915524820582750453426552832, z, k)^4)/211 - 168897381688221696*\text{root}(z^6 \\
& - (292589*z^4)/414082997094657024 - (11805253*z^3)/35209709129437059758653 \\
& 44 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/1885726687 \\
& 5842830829226645405233577984 - 7197829/128275517001789310191552482058275045 \\
& 3426552832, z, k)^5 + 661/28970600587017064704)*\text{root}(z^6 - (292589*z^4)/414 \\
& 082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/ \\
& 4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405 \\
& 233577984 - 7197829/1282755170017893101915524820582750453426552832, z, k), \\
& k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922 \\
& 914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
\end{aligned}$$

sympy [A] time = 0.40, size = 112, normalized size = 0.13

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775
544244426251149770752*_t**4 - 4300873166389987741684137984*_t**3 - 71700090
8921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257
935592810449901409556597891882995604001083339368041361480613888*_t**5/15420
6009791052044490694380303237521 + 23896074006209855243763588535726522071819
56324560587684052992*_t**4/154206009791052044490694380303237521 - 122860721
60883283930711715948878260078996992193488388096*_t**3/154206009791052044490
694380303237521 - 59490553573959173161125496013527909754156558410752*_t**2/
154206009791052044490694380303237521 - 175201496798366911123670641977137530
04827200*_t/154206009791052044490694380303237521 + x + 76642298870722961505
5855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**
4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196
096544*x**3 + 398698848*x**2 + 797397696)

$$3.153 \quad \int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$$

Optimal. Leaf size=986

$$\frac{27 \left((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3} \right) - \sqrt[3]{6} \left(9 + \sqrt[3]{-3} 2^{2/3} \right) x}{104976 2^{2/3} \left(1 + \sqrt[3]{-1} \right)^4 \left(4 - 3(-3)^{2/3} \sqrt[3]{2} \right) \left(x^2 - 3\sqrt[3]{-3} 2^{2/3} x + 6 \right)} \frac{(1 + i\sqrt{3} + 3\sqrt[3]{2} 3^{2/3}) \tan^{-1} \left(\frac{3\sqrt[3]{-3} 2^{2/3}}{\sqrt{6(4-3(-3))}} \right)}{8748 2^{2/3} 3^{5/6} \left(1 + \sqrt[3]{-1} \right)^4 \left(8 - 9i\sqrt[3]{2} \sqrt[3]{3} + 3 \right)}$$

Rubi [A] time = 1.93, antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2097, 638, 618, 204, 634, 628, 206}

Antiderivative was successfully verified.

[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] $-(27*((-2)^{(2/3)} + 2*(-1)^{(1/3)}*3^{(2/3)}) - 6^{(1/3)}*(9 + (-3)^{(1/3)}*2^{(2/3)}) * x)/(104976*2^{(2/3)}*(1 + (-1)^{(1/3)})^4*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})*(6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2)) - (27*2^{(2/3)}*(1 + (-2)^{(1/3)}*3^{(2/3)}) - (-1)^{(1/3)}*3^{(2/3)}*(2 + 3*(-2)^{(1/3)}*3^{(2/3)})*x)/(472392*2^{(2/3)}*(8 + (9*I)*2^{(1/3)})*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)}*(6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2)) + (9*(6 - 2^{(2/3)}*3^{(1/3)}) - (2 - 3*2^{(1/3)}*3^{(2/3)})*x)/(314928*2^{(2/3)}*3^{(1/3)}*(4 - 3*2^{(1/3)}*3^{(2/3)}*(6 + 3*2^{(2/3)}*3^{(1/3)}*x + x^2)) - ((1 + I*Sqrt[3] + 3*2^{(1/3)}*3^{(2/3)})*ArcTan[(3*(-3)^{(1/3)}*2^{(2/3)} - 2*x)/Sqrt[6*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(8748*2^{(2/3)}*3^{(5/6)}*(1 + (-1)^{(1/3)})^4*(8 - (9*I)*2^{(1/3)}*3^{(1/6)} + 3*2^{(1/3)}*3^{(2/3)})^{(3/2)}) + ((3*(-3)^{(2/3)} + (-1)^{(1/3)}*2^{(2/3)}) * ArcTan[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/Sqrt[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(17496*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})^{(3/2)}) + ((I + Sqrt[3])*ArcTan[(3*(-2)^{(2/3)}*3^{(1/3)} + 2*x)/Sqrt[6*(4 + 3*(-2)^{(1/3)}*3^{(2/3)})]])/(34992*2^{(1/6)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^5*Sqrt[4 + 3*(-2)^{(1/3)}*3^{(2/3)}]) + ((I/17496)*ArcTan[(2^{(1/6)}*(3*(-3)^{(1/3)} - 2^{(1/3)}*x))/Sqrt[3*(4 - 3*(-3)^{(2/3)}*2^{(1/3)})]])/(2^{(1/6)}*3^{(1/3)}*(1 + (-1)^{(1/3)})^5*Sqrt[4 - 3*(-3)^{(2/3)}*2^{(1/3)}]) - ((2^{(2/3)} - 3*3^{(2/3)})*ArcTanh[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/Sqrt[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]])/(17496*6^{(5/6)}*(1 - (-1)^{(1/3)})^2*(1 + (-1)^{(1/3)})^4*(-4 + 3*2^{(1/3)}*3^{(2/3)})^{(3/2)}) - ArcTanh[(2^{(1/6)}*(3*3^{(1/3)} + 2^{(1/3)}*x))/Sqrt[3*(-4 + 3*2^{(1/3)}*3^{(2/3)})]]/(157464*2^{(1/6)}*3^{(5/6)}*Sqrt[-4 + 3*2^{(1/3)}*3^{(2/3)}]) + ((I + Sqrt[3])*Log[6 - 3*(-3)^{(1/3)}*2^{(2/3)}*x + x^2])/(419904*2^{(1/3)}*3^{(1/6)}*(1 + (-1)^{(1/3)})^5) - ((I/209952)*Log[6 + 3*(-2)^{(2/3)}*3^{(1/3)}*x + x^2])/(2^{(1/3)}*3^{(1/6)})$

$(1 + (-1)^{1/3})^5 + \text{Log}[6 + 3 \cdot 2^{2/3} \cdot 3^{1/3} \cdot x + x^2] / (1889568 \cdot 2^{1/3} \cdot 3^{2/3})$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 206

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]] / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

$\text{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \text{Int}[1 / (a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4 \cdot a \cdot c]$

Rule 638

$\text{Int}[(d + (e \cdot x)) \cdot (a + (b \cdot x) + (c \cdot x)^2)^{p}, x_Symbol] \rightarrow \text{Simp}[(b \cdot d - 2 \cdot a \cdot e + (2 \cdot c \cdot d - b \cdot e) \cdot x) \cdot (a + b \cdot x + c \cdot x^2)^{(p+1)} / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), x] - \text{Dist}[(2 \cdot p + 3) \cdot (2 \cdot c \cdot d - b \cdot e) / ((p+1) \cdot (b^2 - 4 \cdot a \cdot c)), \text{Int}[(a + b \cdot x + c \cdot x^2)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[2 \cdot c \cdot d - b \cdot e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

Rule 2097

$\text{Int}[(Q6)^{p} \cdot (u), x_Symbol] \rightarrow \text{With}\{a = \text{Coeff}[Q6, x, 0], b = \text{Coeff}[Q6, x, 2], c = \text{Coeff}[Q6, x, 3], d = \text{Coeff}[Q6, x, 4], e = \text{Coeff}[Q6, x, 6]\}, \text{Dist}$

[1/(3^(3*p))*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x] /; EqQ[b^2 - 3*a*d, 0] && EqQ[b^3 - 27*a^2*e, 0] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]

Rubi steps

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = 1586874322944 \int \left(\frac{2\sqrt[3]{-1} 3^{2/3} + 18\sqrt[3]{6} + 3(-2)^{2/3}}{55527906308456448 2^{2/3} (1 + \sqrt[3]{-1})^4 (-6 + \dots)} \right. \\ = \frac{\int \frac{2\sqrt[3]{-1} 3^{2/3} + 18(-1)^{2/3} \sqrt[3]{6} + 3 2^{2/3} x}{(6 + 3(-2)^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{314928 2^{2/3}} + \frac{\int \frac{-2 + 6\sqrt[3]{2} 3^{2/3} + 2^{2/3} \sqrt[3]{3} x}{(6 + 3 2^{2/3} \sqrt[3]{3} x + x^2)^2} dx}{104976 2^{2/3} \sqrt[3]{3}} + \frac{\int \frac{\dots}{6 + \dots}}{9} \\ = -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} \\ = -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)} \\ = -\frac{27((-2)^{2/3} + 2\sqrt[3]{-1} 3^{2/3}) - \sqrt[3]{6} (9 + \sqrt[3]{-3} 2^{2/3}) x}{104976 2^{2/3} (1 + \sqrt[3]{-1})^4 (4 - 3(-3)^{2/3} \sqrt[3]{2}) (6 - 3\sqrt[3]{-3} 2^{2/3} x + x^2)}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{-9x^5 + 8x^4 - 216x^3 - 2724x^2 + 324x - 7884}{7383312(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{\text{RootSum}\left[\#1^6 + 18\#1^4 + 324\#1^3 + 108\#1^2 + 216\&, \frac{9\#1^4 \log(x-\#1) - 16\#1^3 \log(x-\#1) + 324\#1^2 \log(x-\#1) + 2436\#1 \log(x-\#1) + 324 \log(x-\#1)}{\#1^3 + 12\#1^3 + 162\#1^2 + 36\#1}\right]}{44299872}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 &, (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 -

16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) &]/44299872

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]

[Out] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")

[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)

maple [C] time = 0.01, size = 122, normalized size = 0.12

$$\frac{(-9 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^4 + 16 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^3 - 324 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^2 - 2436 \operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216) - 324) \ln(-\operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216) + x) + \frac{1}{44299872} \frac{1}{\operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)} + \frac{1}{531598464} \frac{1}{\operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^2} + \frac{1}{7176579264} \frac{1}{\operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^3} + \frac{1}{1594795392} \frac{1}{\operatorname{RootOf}(_Z^6 + 18_Z^4 + 324_Z^3 + 108_Z^2 + 216)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)

[Out] $(-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*\operatorname{sum}((-9*_R^4+16*_R^3-324*_R^2-436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*\ln(-_R+x),_R=\operatorname{RootOf}(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{9x^5 - 8x^4 + 216x^3 + 2724x^2 - 324x + 7884}{7383312(x^6 + 18x^4 + 324x^3 + 108x^2 + 216)} - \frac{1}{7383312} \int \frac{9x^4 - 16x^3 + 324x^2 + 2436x + 324}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")

[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.48, size = 388, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)

[Out] symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k))/1103894245809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)*x)/29805144636848832 + (452809*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2*x)/194734854 - (1241776944*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^3*x)/44521 + (452407928832*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^5*x + (114155*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^2)/292102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/

$$\begin{aligned}
& 142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - \\
& (505*z)/6285755625280943609742215135077859328 + 4513/8658597397620778437929 \\
& 792538933565560629231616, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4 \\
&)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2 \\
&)/1108852327671535435567321055232 - (505*z)/6285755625280943609742215135077 \\
& 859328 + 4513/8658597397620778437929792538933565560629231616, z, k)^4)/211 \\
& - 168897381688221696*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3) \\
& /142599321974220092022546432 + (5171*z^2)/1108852327671535435567321055232 - \\
& (505*z)/6285755625280943609742215135077859328 + 4513/865859739762077843792 \\
& 9792538933565560629231616, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16 \\
& 3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517 \\
& 1*z^2)/1108852327671535435567321055232 - (505*z)/62857556252809436097422151 \\
& 35077859328 + 4513/8658597397620778437929792538933565560629231616, z, k), k \\
& , 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368 \\
& + 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
\end{aligned}$$

sympy [A] time = 0.39, size = 112, normalized size = 0.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)

[Out] RootSum(8658597397620778437929792538933565560629231616*_t**6 + 109068095871770168248838645612544*_t**4 - 492655707593366915713499136*_t**3 + 40378331745144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(101442531561804181113161287039859349851881619653631712165888*_t**5/356900697070792948475845 - 149796550082359335112709434971975088967050210050048*_t**4/356900697070792948475845 + 1222409754458272818505898777768670783617236992*_t**3/356900697070792948475845 - 5775055524251595723022901938558261453824*_t**2/356900697070792948475845 + 96165242200260265765603930470432*_t/71380139414158589695169 + x - 17059152341129698120545584/1070702091212378845427535))) + (-9*x**5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 132899616*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)

$$3.154 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx$$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x), x]

[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx &= \int (a^2 + 2abx^2 + b^2x^4) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x), x]

[Out] $a^2x + (2abx^3)/3 + (b^2x^5)/5$

IntegrateAlgebraic [A] time = 0.03, size = 26, normalized size = 1.04

$$\frac{1}{15} (15a^2x + 10abx^3 + 3b^2x^5)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x),x]

[Out] $(15a^2x + 10abx^3 + 3b^2x^5)/15$

fricas [A] time = 0.79, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x, algorithm="fricas")

[Out] $1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x$

giac [A] time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x, algorithm="giac")

[Out] $1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x$

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)

[Out] $a^2*x+2/3*a*b*x^3+1/5*b^2*x^5$

maxima [A] time = 0.44, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x, algorithm="maxima")

[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(c + d*x),x)

[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3

sympy [A] time = 0.09, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(d*x+c),x)

[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5

$$3.155 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2} dx$$

Optimal. Leaf size=94

$$\frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5} - \frac{bcx(2ad^2 + bc^2)}{d^4} + \frac{bx^2(2ad^2 + bc^2)}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}$$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {1586, 28, 697}

$$\frac{bx^2(2ad^2 + bc^2)}{2d^3} - \frac{bcx(2ad^2 + bc^2)}{d^4} + \frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2,x]

[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c + dx)^2} dx &= \int \frac{a^2 + 2abx^2 + b^2x^4}{c + dx} dx \\
&= \frac{\int \frac{(ab+b^2x^2)^2}{c+dx} dx}{b^2} \\
&= \frac{\int \left(-\frac{b^3c(bc^2+2ad^2)}{d^4} + \frac{b^3(bc^2+2ad^2)x}{d^3} - \frac{b^4cx^2}{d^2} + \frac{b^4x^3}{d} + \frac{b^2(bc^2+ad^2)}{d^4(c+dx)} \right) dx}{b^2} \\
&= -\frac{bc(bc^2 + 2ad^2)x}{d^4} + \frac{b(bc^2 + 2ad^2)x^2}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.84

$$\frac{12(ad^2 + bc^2)^2 \log(c + dx) + bdx(12ad^2(dx - 2c) + b(-12c^3 + 6c^2dx - 4cd^2x^2 + 3d^3x^3))}{12d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2,x]

[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c + dx)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2,x]

[Out] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(c + d*x)^2, x]

fricas [A] time = 0.94, size = 105, normalized size = 1.12

$$\frac{3b^2d^4x^4 - 4b^2cd^3x^3 + 6(b^2c^2d^2 + 2abd^4)x^2 - 12(b^2c^3d + 2abcd^3)x + 12(b^2c^4 + 2abc^2d^2 + a^2d^4) \log(dx + c)}{12d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="fricas")

[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 12*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*x + c))/d^5

giac [B] time = 0.26, size = 365, normalized size = 3.88

$$\frac{1}{12} \frac{1}{d^5} \left(\frac{(dx+c)^2 \left(\frac{20c}{dx+c} + \frac{60c^2}{(dx+c)^2} + \frac{120c^3}{(dx+c)^3} - 3 \right)}{d^6} + \frac{60c^4 \log\left(\frac{dx+c}{(dx+c)^2 d}\right)}{d^6} - \frac{12c^5}{(dx+c)d^6} \right) - \frac{1}{3} \frac{1}{d^5} \left(\frac{(dx+c)^2 \left(\frac{6c}{dx+c} - \frac{18c^2}{(dx+c)^2} - 1 \right)}{d^6} - \frac{12c^3 \log\left(\frac{dx+c}{(dx+c)^2 d}\right)}{d^6} + \frac{3c^4}{(dx+c)d^6} \right) - \frac{abd}{d^5} \left(\frac{(dx+c)^2 \left(\frac{6c}{dx+c} - 1 \right)}{d^4} + \frac{6c^2 \log\left(\frac{dx+c}{(dx+c)^2 d}\right)}{d^4} - \frac{2c^3}{(dx+c)d^4} \right) + 2 \frac{abd}{d^5} \left(\frac{2c \log\left(\frac{dx+c}{(dx+c)^2 d}\right)}{d^4} + \frac{dx+c}{d^4} - \frac{c^2}{(dx+c)d^4} \right) + \frac{1}{d^5} \left(\frac{\log\left(\frac{dx+c}{(dx+c)^2 d}\right)}{d} - \frac{c}{(dx+c)d} \right) - \frac{a^2 c}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="giac")

[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x + c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x + c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d)) - a^2*c/((d*x + c)*d)

maple [A] time = 0.00, size = 114, normalized size = 1.21

$$\frac{b^2 x^4}{4d} - \frac{b^2 c x^3}{3d^2} + \frac{ab x^2}{d} + \frac{b^2 c^2 x^2}{2d^3} + \frac{a^2 \ln(dx+c)}{d} + \frac{2abc^2 \ln(dx+c)}{d^3} - \frac{2abcx}{d^2} + \frac{b^2 c^4 \ln(dx+c)}{d^5} - \frac{b^2 c^3 x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x)

[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b/d^2*a*c*x-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^2+1/d^5*ln(d*x+c)*b^2*c^4

maxima [A] time = 0.44, size = 105, normalized size = 1.12

$$\frac{3b^2d^3x^4 - 4b^2cd^2x^3 + 6(b^2c^2d + 2abd^3)x^2 - 12(b^2c^3 + 2abcd^2)x}{12d^4} + \frac{(b^2c^4 + 2abc^2d^2 + a^2d^4) \log(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x, algorithm="maxima")

[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*x + c)/d^5

mupad [B] time = 0.06, size = 106, normalized size = 1.13

$$x^2 \left(\frac{b^2 c^2}{2d^3} + \frac{ab}{d} \right) + \frac{\ln(c + dx) (a^2 d^4 + 2ab c^2 d^2 + b^2 c^4)}{d^5} + \frac{b^2 x^4}{4d} - \frac{b^2 c x^3}{3d^2} - \frac{cx \left(\frac{b^2 c^2}{d^3} + \frac{2ab}{d} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(c + d*x)^2,x)

[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d^3 + (2*a*b)/d))/d

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{b^2 c x^3}{3d^2} + \frac{b^2 x^4}{4d} + x^2 \left(\frac{ab}{d} + \frac{b^2 c^2}{2d^3} \right) + x \left(-\frac{2abc}{d^2} - \frac{b^2 c^3}{d^4} \right) + \frac{(ad^2 + bc^2)^2 \log(c + dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(d*x+c)**2,x)

[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3)) + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d**5

$$3.156 \quad \int (b + 2cx) (bx + cx^2)^{13} dx$$

Optimal. Leaf size=15

$$\frac{1}{14} (bx + cx^2)^{14}$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{1}{14} (bx + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^{13} dx = \frac{1}{14} (bx + cx^2)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^6c^8x^{22} + 143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 2cx)(bx + cx^2)^{13} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13, x]

[Out] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13, x]

fricas [B] time = 0.71, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14} + x^{27}c^{13}b + \frac{13}{2}x^{26}c^{12}b^2 + 26x^{25}c^{11}b^3 + \frac{143}{2}x^{24}c^{10}b^4 + 143x^{23}c^9b^5 + \frac{429}{2}x^{22}c^8b^6 + \frac{1716}{7}x^{21}c^7b^7 + \frac{429}{2}x^{20}c^6b^8 + 143x^{19}c^5b^9 + \frac{143}{2}x^{18}c^4b^{10} + 26x^{17}c^3b^{11} + \frac{13}{2}x^{16}c^2b^{12} + x^{15}cb^{13} + \frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")

[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{14} (cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28} + b c^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}cx^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)

[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+b^13*c*x^15+1/14*b^14*x^14

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14} (cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")

[Out] 1/14*(c*x^2 + b*x)^14

mupad [B] time = 2.22, size = 154, normalized size = 10.27

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)

[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)

[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14

$$3.157 \quad \int x^{14} (b + 2cx^2) (bx + cx^3)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{14} (b + 2cx^2) (bx + cx^3)^{13} dx &= \int x^{27} (b + cx^2)^{13} (b + 2cx^2) dx \\
&= \frac{1}{2} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^2 \right) \\
&= \frac{1}{28} x^{28} (b + cx^2)^{14}
\end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{1}{2}b^{13}cx^{30} + \frac{13}{4}b^{12}c^2x^{32} + 13b^{11}c^3x^{34} + \frac{143}{4}b^{10}c^4x^{36} + \frac{143}{2}b^9c^5x^{38} + \frac{429}{4}b^8c^6x^{40} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^6c^8x^{44} + \frac{143}{2}b^5c^9x^{46} + \frac{143}{4}b^4c^{10}x^{48} + 13b^3c^{11}x^{50} + \frac{13}{4}b^2c^{12}x^{52} + \frac{1}{2}bc^{13}x^{54} + \frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]

[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 + (c^14*x^56)/28

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14} (b + 2cx^2) (bx + cx^3)^{13} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]

[Out] IntegrateAlgebraic[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13, x]

fricas [B] time = 1.24, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8 + \frac{143}{2}x^{38}c^5b^9 + \frac{143}{4}x^{36}c^4b^{10} + 13x^{34}c^3b^{11} + \frac{13}{4}x^{32}c^2b^{12} + \frac{1}{2}x^{30}cb^{13} + \frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")

[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 + 143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 13*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14

giac [B] time = 0.31, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/28*c^{14}*x^{56} + 1/2*b*c^{13}*x^{54} + 13/4*b^2*c^{12}*x^{52} + 13*b^3*c^{11}*x^{50} + \\ & 143/4*b^4*c^{10}*x^{48} + 143/2*b^5*c^9*x^{46} + 429/4*b^6*c^8*x^{44} + 858/7*b^7*c \\ & ^7*x^{42} + 429/4*b^8*c^6*x^{40} + 143/2*b^9*c^5*x^{38} + 143/4*b^{10}*c^4*x^{36} + 1 \\ & 3*b^{11}*c^3*x^{34} + 13/4*b^{12}*c^2*x^{32} + 1/2*b^{13}*c*x^{30} + 1/28*b^{14}*x^{28} \end{aligned}$$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)

$$\begin{aligned} \text{[Out]} & 1/28*c^{14}*x^{56}+1/2*b*c^{13}*x^{54}+13/4*b^2*c^{12}*x^{52}+13*b^3*c^{11}*x^{50}+143/4*b^4 \\ & *c^{10}*x^{48}+143/2*b^5*c^9*x^{46}+429/4*b^6*c^8*x^{44}+858/7*b^7*c^7*x^{42}+429/4* \\ & b^8*c^6*x^{40}+143/2*b^9*c^5*x^{38}+143/4*b^{10}*c^4*x^{36}+13*b^{11}*c^3*x^{34}+13/4*b \\ & ^{12}*c^2*x^{32}+1/2*b^{13}*c*x^{30}+1/28*b^{14}*x^{28} \end{aligned}$$

maxima [B] time = 0.61, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/28*c^{14}*x^{56} + 1/2*b*c^{13}*x^{54} + 13/4*b^2*c^{12}*x^{52} + 13*b^3*c^{11}*x^{50} + \\ & 143/4*b^4*c^{10}*x^{48} + 143/2*b^5*c^9*x^{46} + 429/4*b^6*c^8*x^{44} + 858/7*b^7*c \\ & ^7*x^{42} + 429/4*b^8*c^6*x^{40} + 143/2*b^9*c^5*x^{38} + 143/4*b^{10}*c^4*x^{36} + 1 \\ & 3*b^{11}*c^3*x^{34} + 13/4*b^{12}*c^2*x^{32} + 1/2*b^{13}*c*x^{30} + 1/28*b^{14}*x^{28} \end{aligned}$$

mupad [B] time = 0.14, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)

[Out] $(b^{14}x^{28})/28 + (c^{14}x^{56})/28 + (b^{13}c*x^{30})/2 + (b*c^{13}x^{54})/2 + (13*b^{12}c^2*x^{32})/4 + 13*b^{11}c^3*x^{34} + (143*b^{10}c^4*x^{36})/4 + (143*b^9*c^5*x^{38})/2 + (429*b^8*c^6*x^{40})/4 + (858*b^7*c^7*x^{42})/7 + (429*b^6*c^8*x^{44})/4 + (143*b^5*c^9*x^{46})/2 + (143*b^4*c^{10}x^{48})/4 + 13*b^3*c^{11}x^{50} + (13*b^2*c^{12}x^{52})/4$

sympy [B] time = 0.14, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)`

[Out] $b^{14}x^{28}/28 + b^{13}c*x^{30}/2 + 13*b^{12}c^2*x^{32}/4 + 13*b^{11}c^3*x^{34} + 143*b^{10}c^4*x^{36}/4 + 143*b^9*c^5*x^{38}/2 + 429*b^8*c^6*x^{40}/4 + 858*b^7*c^7*x^{42}/7 + 429*b^6*c^8*x^{44}/4 + 143*b^5*c^9*x^{46}/2 + 143*b^4*c^{10}x^{48}/4 + 13*b^3*c^{11}x^{50} + 13*b^2*c^{12}x^{52}/4 + b*c^{13}x^{54}/2 + c^{14}x^{56}/28$

$$3.158 \quad \int x^{28} (b + 2cx^3) (bx + cx^4)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$\frac{1}{42} x^{42} (b + cx^3)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]

[Out] (x^42*(b + c*x^3)^14)/42

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int x^{28} (b + 2cx^3) (bx + cx^4)^{13} dx &= \int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx \\
 &= \frac{1}{3} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^3 \right) \\
 &= \frac{1}{42} x^{42} (b + cx^3)^{14}
 \end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42} + \frac{1}{3}b^{13}cx^{45} + \frac{13}{6}b^{12}c^2x^{48} + \frac{26}{3}b^{11}c^3x^{51} + \frac{143}{6}b^{10}c^4x^{54} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{2}b^8c^6x^{60} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^6c^8x^{66} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{6}b^4c^{10}x^{72} + \frac{26}{3}b^3c^{11}x^{75} + \frac{13}{6}b^2c^{12}x^{78} + \frac{1}{3}bc^{13}x^{81} + \frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]

[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^81)/3 + (c^14*x^84)/42

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{28} (b + 2cx^3) (bx + cx^4)^{13} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]

[Out] IntegrateAlgebraic[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13, x]

fricas [B] time = 1.17, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6b^8 + \frac{143}{3}x^{57}c^5b^9 + \frac{143}{6}x^{54}c^4b^{10} + \frac{26}{3}x^{51}c^3b^{11} + \frac{13}{6}x^{48}c^2b^{12} + \frac{1}{3}x^{45}cb^{13} + \frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")

[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3 + 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^63*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 + 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14

giac [B] time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")

$$\begin{aligned} \text{[Out]} & 1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} \\ & + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7* \\ & *c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + \\ & 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42} \end{aligned}$$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)

$$\begin{aligned} \text{[Out]} & 1/42*c^{14}*x^{84}+1/3*b*c^{13}*x^{81}+13/6*b^2*c^{12}*x^{78}+26/3*b^3*c^{11}*x^{75}+143/6* \\ & b^4*c^{10}*x^{72}+143/3*b^5*c^9*x^{69}+143/2*b^6*c^8*x^{66}+572/7*b^7*c^7*x^{63}+143/ \\ & 2*b^8*c^6*x^{60}+143/3*b^9*c^5*x^{57}+143/6*b^{10}*c^4*x^{54}+26/3*b^{11}*c^3*x^{51}+13 \\ & /6*b^{12}*c^2*x^{48}+1/3*b^{13}*c*x^{45}+1/42*b^{14}*x^{42} \end{aligned}$$

maxima [B] time = 0.55, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b^{10}c^4x^{54} + \frac{26}{3}b^{11}c^3x^{51} + \frac{13}{6}b^{12}c^2x^{48} + \frac{1}{3}b^{13}cx^{45} + \frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")

$$\begin{aligned} \text{[Out]} & 1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} \\ & + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7* \\ & *c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + \\ & 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42} \end{aligned}$$

mupad [B] time = 2.17, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)

[Out] $(b^{14}x^{42})/42 + (c^{14}x^{84})/42 + (b^{13}c*x^{45})/3 + (b*c^{13}x^{81})/3 + (13*b^{12}c^2*x^{48})/6 + (26*b^{11}c^3*x^{51})/3 + (143*b^{10}c^4*x^{54})/6 + (143*b^9*c^5*x^{57})/3 + (143*b^8*c^6*x^{60})/2 + (572*b^7*c^7*x^{63})/7 + (143*b^6*c^8*x^{66})/2 + (143*b^5*c^9*x^{69})/3 + (143*b^4*c^{10}x^{72})/6 + (26*b^3*c^{11}x^{75})/3 + (13*b^2*c^{12}x^{78})/6$

sympy [B] time = 0.14, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3} + \frac{143b^4c^{10}x^{72}}{6} + \frac{26b^3c^{11}x^{75}}{3} + \frac{13b^2c^{12}x^{78}}{6} + \frac{bc^{13}x^{81}}{3} + \frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)`

[Out] $b^{14}x^{42}/42 + b^{13}c*x^{45}/3 + 13*b^{12}c^2*x^{48}/6 + 26*b^{11}c^3*x^{51}/3 + 143*b^{10}c^4*x^{54}/6 + 143*b^9*c^5*x^{57}/3 + 143*b^8*c^6*x^{60}/2 + 572*b^7*c^7*x^{63}/7 + 143*b^6*c^8*x^{66}/2 + 143*b^5*c^9*x^{69}/3 + 143*b^4*c^{10}x^{72}/6 + 26*b^3*c^{11}x^{75}/3 + 13*b^2*c^{12}x^{78}/6 + b*c^{13}x^{81}/3 + c^{14}x^{84}/42$

$$3.159 \quad \int x^{14(-1+n)} (b + 2cx^n) (bx + cx^{1+n})^{13} dx$$

Optimal. Leaf size=21

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{14(-1+n)} (b + 2cx^n) (bx + cx^{1+n})^{13} dx &= \int x^{13+14(-1+n)} (b + cx^n)^{13} (b + 2cx^n) dx \\ &= \frac{\text{Subst}\left(\int x^{13}(b + cx)^{13}(b + 2cx) dx, x, x^n\right)}{n} \\ &= \frac{x^{14n} (b + cx^n)^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

IntegrateAlgebraic [A] time = 0.05, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

fricas [B] time = 0.98, size = 262, normalized size = 12.48

$$\frac{b^{14}x^{14n+14} + 14b^{13}cx^{13n+15} + 91b^{12}c^2x^{12n+16} + 364b^{11}c^3x^{11n+17} + 1001b^{10}c^4x^{10n+18} + 2002b^9c^5x^{9n+19} + 3003b^8c^6x^{8n+20} + 3432b^7c^7x^{7n+21} + 3003b^6c^8x^{6n+22} + 2002b^5c^9x^{5n+23} + 1001b^4c^{10}x^{4n+24} + 364b^3c^{11}x^{3n+25} + 91b^2c^{12}x^{2n+26} + 14bc^{13}x^{n+27} + c^{14}x^{28}}{14n^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas")

[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n + 20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 2002*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^11*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n + 27) + c^14*x^28)

$$1*x^3*x^{(25*n + 25)} + 91*b^2*c^{12}*x^{(26*n + 26)} + 14*b*c^{13}*x*x^{(27*n + 27)} + c^{14}*x^{(28*n + 28)}/(n*x^{28})$$

giac [B] time = 1.86, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")

$$[Out] \frac{1}{14} * (c^{14} * x^{(28*n)} + 14 * b * c^{13} * x^{(27*n)} + 91 * b^2 * c^{12} * x^{(26*n)} + 364 * b^3 * c^{11} * x^{(25*n)} + 1001 * b^4 * c^{10} * x^{(24*n)} + 2002 * b^5 * c^9 * x^{(23*n)} + 3003 * b^6 * c^8 * x^{(22*n)} + 3432 * b^7 * c^7 * x^{(21*n)} + 3003 * b^8 * c^6 * x^{(20*n)} + 2002 * b^9 * c^5 * x^{(19*n)} + 1001 * b^{10} * c^4 * x^{(18*n)} + 364 * b^{11} * c^3 * x^{(17*n)} + 91 * b^{12} * c^2 * x^{(16*n)} + 14 * b^{13} * c * x^{(15*n)} + b^{14} * x^{(14*n)}) / n$$

maple [B] time = 0.05, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n} + \frac{b^{13}cx^{15n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{14b^1cx^{27n}}{n} + \frac{b^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)

$$[Out] \frac{1}{14} * c^{14} / n * (x^n)^{28} + b * c^{13} / n * (x^n)^{27} + 13/2 * b^2 * c^{12} / n * (x^n)^{26} + 26 * b^3 * c^{11} / n * (x^n)^{25} + 143/2 * b^4 * c^{10} / n * (x^n)^{24} + 143 * b^5 * c^9 / n * (x^n)^{23} + 429/2 * b^6 * c^8 / n * (x^n)^{22} + 1716/7 * b^7 * c^7 / n * (x^n)^{21} + 429/2 * b^8 * c^6 / n * (x^n)^{20} + 143 * b^9 * c^5 / n * (x^n)^{19} + 143/2 * b^{10} * c^4 / n * (x^n)^{18} + 26 * b^{11} * c^3 / n * (x^n)^{17} + 13/2 * b^{12} * c^2 / n * (x^n)^{16} + b^{13} * c / n * (x^n)^{15} + 1/14 * b^{14} / n * (x^n)^{14}$$

maxima [B] time = 0.65, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14n} + \frac{bc^{13}x^{27n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{b^{13}cx^{15n}}{n} + \frac{b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima")

$$[Out] \frac{1}{14} * c^{14} * x^{(28*n)} / n + b * c^{13} * x^{(27*n)} / n + 13/2 * b^2 * c^{12} * x^{(26*n)} / n + 26 * b^3 * c^{11} * x^{(25*n)} / n + 143/2 * b^4 * c^{10} * x^{(24*n)} / n + 143 * b^5 * c^9 * x^{(23*n)} / n + 429/2 * b^6 * c^8 * x^{(22*n)} / n + 1716/7 * b^7 * c^7 * x^{(21*n)} / n + 429/2 * b^8 * c^6 * x^{(20*n)} / n + 143 * b^9 * c^5 * x^{(19*n)} / n + 143/2 * b^{10} * c^4 * x^{(18*n)} / n + 26 * b^{11} * c^3 * x^{(17*n)} / n + 13/2 * b^{12} * c^2 * x^{(16*n)} / n + b^{13} * c * x^{(15*n)} / n + 1/14 * b^{14} * x^{(14*n)} / n$$

mupad [B] time = 4.02, size = 229, normalized size = 10.90

$$\frac{b^{14}x^{14n}}{14n} + \frac{c^{14}x^{28n}}{14n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{b^{13}cx^{15n}}{n} + \frac{b^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

$$3.160 \quad \int \frac{b+2cx}{bx+cx^2} dx$$

Optimal. Leaf size=10

$$\log(bx + cx^2)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {628}

$$\log(bx + cx^2)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(b*x + c*x^2), x]

[Out] Log[b*x + c*x^2]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{b + 2cx}{bx + cx^2} dx = \log(bx + cx^2)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b + cx) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(b*x + c*x^2), x]

[Out] Log[x] + Log[b + c*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx}{bx + cx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2), x]

[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2), x]

fricas [A] time = 0.73, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x), x, algorithm="fricas")

[Out] log(c*x^2 + b*x)

giac [A] time = 0.23, size = 11, normalized size = 1.10

$$\log(|cx^2 + bx|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x), x, algorithm="giac")

[Out] log(abs(c*x^2 + b*x))

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln((cx + b)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x), x)

[Out] ln(x*(c*x+b))

maxima [A] time = 0.55, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x), x, algorithm="maxima")

[Out] log(c*x^2 + b*x)

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln(x(b + cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
```

```
[Out] log(x*(b + c*x))
```

```
sympy [A] time = 0.13, size = 8, normalized size = 0.80
```

$$\log(bx + cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
```

```
[Out] log(b*x + c*x**2)
```

$$3.161 \quad \int \frac{b+2cx^2}{bx+cx^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1593, 446, 72}

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_) * ((c_.) + (d_.)*(x_))),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.),
x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{b + 2cx^2}{bx + cx^3} dx &= \int \frac{b + 2cx^2}{x(b + cx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{b + 2cx}{x(b + cx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b + cx} \right) dx, x, x^2 \right) \\
&= \log(x) + \frac{1}{2} \log(b + cx^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3), x]

[Out] Log[x] + Log[b + c*x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2}{bx + cx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^2)/(b*x + c*x^3), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(b*x + c*x^3), x]

fricas [A] time = 0.77, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x), x, algorithm="fricas")

[Out] 1/2*log(c*x^2 + b) + log(x)

giac [A] time = 0.33, size = 18, normalized size = 1.20

$$\frac{1}{2} \log(x^2) + \frac{1}{2} \log(|cx^2 + b|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")

[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^2 + b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2+b)/(c*x^3+b*x),x)

[Out] ln(x)+1/2*ln(c*x^2+b)

maxima [A] time = 0.64, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")

[Out] 1/2*log(c*x^2 + b) + log(x)

mupad [B] time = 2.08, size = 13, normalized size = 0.87

$$\frac{\ln(cx^2 + b)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)

[Out] log(b + c*x^2)/2 + log(x)

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

$$3.162 \quad \int \frac{b+2cx^3}{bx+cx^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1593, 446, 72}

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(b*x + c*x^4), x]

[Out] Log[x] + Log[b + c*x^3]/3

Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_) * ((c_.) + (d_.)*(x_))),
  x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.),
  x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
  *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
  b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
  ^((n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
  PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{b + 2cx^3}{bx + cx^4} dx &= \int \frac{b + 2cx^3}{x(b + cx^3)} dx \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{b + 2cx}{x(b + cx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b + cx} \right) dx, x, x^3 \right) \\
&= \log(x) + \frac{1}{3} \log(b + cx^3)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4), x]

[Out] Log[x] + Log[b + c*x^3]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^3}{bx + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^3)/(b*x + c*x^4), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(b*x + c*x^4), x]

fricas [A] time = 0.96, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x), x, algorithm="fricas")

[Out] 1/3*log(c*x^3 + b) + log(x)

giac [A] time = 0.31, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(|cx^3 + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")

[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln(cx^3 + b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^3+b)/(c*x^4+b*x),x)

[Out] ln(x)+1/3*ln(c*x^3+b)

maxima [A] time = 0.52, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")

[Out] 1/3*log(c*x^3 + b) + log(x)

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln(cx^3 + b)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)

[Out] log(b + c*x^3)/3 + log(x)

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

$$3.163 \quad \int \frac{b+2cx^n}{bx+cx^{1+n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1593, 446, 72}

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
  x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.),
  x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
  *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
  b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
  ^((n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
  PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{b + 2cx^n}{bx + cx^{1+n}} dx &= \int \frac{b + 2cx^n}{x(b + cx^n)} dx \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x} + \frac{c}{b+cx}\right) dx, x, x^n\right)}{n} \\
&= \log(x) + \frac{\log(b + cx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b + cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)), x]

[Out] Log[x] + Log[b + c*x^n]/n

IntegrateAlgebraic [A] time = 0.04, size = 24, normalized size = 1.60

$$\frac{\log(bn + cnx^n)}{n} + \frac{\log(x^n)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b + 2*c*x^n)/(b*x + c*x^(1 + n)), x]

[Out] Log[x^n]/n + Log[b*n + c*n*x^n]/n

fricas [A] time = 0.89, size = 23, normalized size = 1.53

$$\frac{(n - 1)\log(x) + \log(bx + cx^{n+1})}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)), x, algorithm="fricas")

[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2cx^n + b}{bx + cx^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")

[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x) + \frac{\ln(c e^{n \ln(x)} + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)

[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)

maxima [B] time = 0.58, size = 47, normalized size = 3.13

$$b \left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n + b}{c}\right)}{bn} \right) + \frac{2 \log\left(\frac{cx^n + b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")

[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{b + 2cx^n}{bx + cx^{n+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)

[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)

sympy [A] time = 1.48, size = 29, normalized size = 1.93

$$\left\{ \begin{array}{ll} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c} + x^n\right)}{n} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)

[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))

$$3.164 \quad \int \frac{b+2cx}{(bx+cx^2)^8} dx$$

Optimal. Leaf size=15

$$-\frac{1}{7(bx+cx^2)^7}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$-\frac{1}{7(bx+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*(b*x + c*x^2)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx}{(bx + cx^2)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8, x]

fricas [B] time = 0.94, size = 81, normalized size = 5.40

$$-\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 21b^2c^5x^{12} + 35b^3c^4x^{11} + 35b^4c^3x^{10} + 21b^5c^2x^9 + 7b^6cx^8 + b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)

giac [A] time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x)^7

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7(cx+b)^7b^7} + \frac{c^7}{(cx+b)^6b^8} + \frac{4c^7}{(cx+b)^5b^9} + \frac{12c^7}{(cx+b)^4b^{10}} + \frac{30c^7}{(cx+b)^3b^{11}} + \frac{66c^7}{(cx+b)^2b^{12}} + \frac{132c^7}{(cx+b)b^{13}} - \frac{132c^6}{b^{13}x} + \frac{66c^5}{b^{12}x^2} - \frac{30c^4}{b^{11}x^3} + \frac{12c^3}{b^{10}x^4} - \frac{4c^2}{b^9x^5} + \frac{c}{b^8x^6} - \frac{1}{7b^7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)

[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+1/b^8*c^7/(c*x+b)^6+1/7/b^7*c^7/(c*x+b)^7-1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6

maxima [A] time = 0.53, size = 13, normalized size = 0.87

$$-\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")

[Out] -1/7/(c*x^2 + b*x)^7

mupad [B] time = 4.38, size = 12, normalized size = 0.80

$$-\frac{1}{7x^7(b + cx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)

[Out] -1/(7*x^7*(b + c*x)^7)

sympy [B] time = 0.93, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)

[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**14)

$$3.165 \quad \int \frac{b+2cx^2}{x^7(bx+cx^3)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]

[Out] -1/(14*x^14*(b + c*x^2)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b + 2cx^2}{x^7 (bx + cx^3)^8} dx &= \int \frac{b + 2cx^2}{x^{15} (b + cx^2)^8} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{b + 2cx}{x^8 (b + cx)^8} dx, x, x^2 \right) \\ &= -\frac{1}{14x^{14} (b + cx^2)^7} \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14} (b + cx^2)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]

[Out] -1/14*1/(x^14*(b + c*x^2)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2}{x^7 (bx + cx^3)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]

fricas [B] time = 0.91, size = 81, normalized size = 5.06

$$\frac{1}{14(c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

giac [A] time = 0.41, size = 15, normalized size = 0.94

$$-\frac{1}{14 (cx^4 + bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2)^7

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(\frac{b^6}{7(cx^2+b)^7c} - \frac{b^5}{(cx^2+b)^6c} - \frac{4b^4}{(cx^2+b)^5c} - \frac{12b^3}{(cx^2+b)^4c} - \frac{30b^2}{(cx^2+b)^3c} - \frac{66b}{(cx^2+b)^2c} - \frac{132}{(cx^2+b)c}\right)c^8}{2b^{13}} - \frac{66c^6}{b^{13}x^2} + \frac{33c^5}{b^{12}x^4} - \frac{15c^4}{b^{11}x^6} + \frac{6c^3}{b^{10}x^8} - \frac{2c^2}{b^9x^{10}} + \frac{c}{2b^8x^{12}} - \frac{1}{14b^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)

[Out] -1/2/b^13*c^8*(-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-132/c/(c*x^2+b)-66*b/c/(c*x^2+b)^2-4*b^4/c/(c*x^2+b)^5-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)^4)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12

maxima [B] time = 0.67, size = 81, normalized size = 5.06

$$-\frac{1}{14 (c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

mupad [B] time = 2.22, size = 14, normalized size = 0.88

$$-\frac{1}{14x^{14}(cx^2 + b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)

[Out] $-1/(14*x^{14}*(b + c*x^2)^7)$

sympy [B] time = 1.43, size = 87, normalized size = 5.44

$$\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)`

[Out] $-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**7*x**28)$

$$3.166 \quad \int \frac{b+2cx^3}{x^{14}(bx+cx^4)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{b + 2cx^3}{x^{14}(bx + cx^4)^8} dx &= \int \frac{b + 2cx^3}{x^{22}(b + cx^3)^8} dx \\ &= \frac{1}{3} \text{Subst} \left(\int \frac{b + 2cx}{x^8(b + cx)^8} dx, x, x^3 \right) \\ &= -\frac{1}{21x^{21}(b + cx^3)^7} \end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}(b + cx^3)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]

[Out] -1/21*1/(x^21*(b + c*x^3)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^3}{x^{14}(bx + cx^4)^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]

[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]

fricas [B] time = 1.12, size = 81, normalized size = 5.06

$$\frac{1}{21(c^7x^{42} + 7bc^6x^{39} + 21b^2c^5x^{36} + 35b^3c^4x^{33} + 35b^4c^3x^{30} + 21b^5c^2x^{27} + 7b^6cx^{24} + b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

giac [A] time = 0.30, size = 15, normalized size = 0.94

$$-\frac{1}{21 (cx^6 + bx^3)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")

[Out] -1/21/(c*x^6 + b*x^3)^7

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$\frac{\left(\frac{b^6}{7(cx^3+b)^7c} - \frac{b^5}{(cx^3+b)^6c} - \frac{4b^4}{(cx^3+b)^5c} - \frac{12b^3}{(cx^3+b)^4c} - \frac{30b^2}{(cx^3+b)^3c} - \frac{66b}{(cx^3+b)^2c} - \frac{132}{(cx^3+b)c}\right)c^8 - \frac{44c^6}{b^{13}x^3} + \frac{22c^5}{b^{12}x^6} - \frac{10c^4}{b^{11}x^9} + \frac{4c^3}{b^{10}x^{12}} - \frac{4c^2}{3b^9x^{15}} + \frac{c}{3b^8x^{18}} - \frac{1}{21b^7x^{21}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)

[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-132/c/(c*x^3+b)-66*b/c/(c*x^3+b)^2-4*b^4/c/(c*x^3+b)^5-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)^4)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18

maxima [B] time = 0.80, size = 81, normalized size = 5.06

$$-\frac{1}{21 (c^7x^{42} + 7bc^6x^{39} + 21b^2c^5x^{36} + 35b^3c^4x^{33} + 35b^4c^3x^{30} + 21b^5c^2x^{27} + 7b^6cx^{24} + b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

mupad [B] time = 7.22, size = 14, normalized size = 0.88

$$-\frac{1}{21 x^{21} (cx^3 + b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)

[Out] $-1/(21*x^{21}*(b + c*x^3)^7)$

sympy [B] time = 2.06, size = 87, normalized size = 5.44

$$\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)`

[Out] $-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c**7*x**42)$

$$3.167 \quad \int \frac{x^{-7(-1+n)}(b+2cx^n)}{(bx+cx^{1+n})^8} dx$$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8), x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)

Rule 74

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\int \frac{x^{-7(-1+n)} (b + 2cx^n)}{(bx + cx^{1+n})^8} dx = \int \frac{x^{-8-7(-1+n)} (b + 2cx^n)}{(b + cx^n)^8} dx$$

$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-7n}}{7n(b + cx^n)^7}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b + cx^n)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8), x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

IntegrateAlgebraic [A] time = 0.11, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b + cx^n)^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8), x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

fricas [B] time = 1.73, size = 143, normalized size = 6.81

$$\frac{x^{14}}{7(b^7 n x^7 x^{7n+7} + 7 b^6 c n x^6 x^{8n+8} + 21 b^5 c^2 n x^5 x^{9n+9} + 35 b^4 c^3 n x^4 x^{10n+10} + 35 b^3 c^4 n x^3 x^{11n+11} + 21 b^2 c^5 n x^2 x^{12n+12} + 7 b c^6 n x x^{13n+13} + c^7 n x^{14n+14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")

[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*n*x^(14*n + 14))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2cx^n + b}{(bx + cx^{n+1})^8 x^{7n-7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)

maple [B] time = 0.06, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^{7n}} + \frac{cx^{-6n}}{b^8n} - \frac{4c^2x^{-5n}}{b^9n} + \frac{12c^3x^{-4n}}{b^{10n}} - \frac{30c^4x^{-3n}}{b^{11n}} + \frac{66c^5x^{-2n}}{b^{12n}} + \frac{(9009b^5cx^n + 20020b^4c^2x^{2n} + 24024b^3c^3x^{3n} + 16380b^2c^4x^{4n} + 6006b^5c^5x^{5n} + 924c^6x^{6n} + 1716b^6)c^7}{7(cx^n + b)^7 b^{13n}} - \frac{132c^6x^{-n}}{b^{13n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)

[Out]
$$-132/b^{13}c^6/n/(x^n)+66/b^{12}c^5/n/(x^n)^2-30/b^{11}c^4/n/(x^n)^3+12/b^{10}c^3/n/(x^n)^4-4/b^9c^2/n/(x^n)^5+1/b^8c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^{13}n/(b+c*x^n)^7$$

maxima [B] time = 1.03, size = 612, normalized size = 29.14

$$\frac{1}{105} \left(\frac{360360c^{13}x^{13n} + 2342340b^2c^{12}x^{12n} + 6426420b^3c^{11}x^{11n} + 9579570b^4c^{10}x^{10n} + 8270262b^5c^9x^{9n} + 4018014b^6c^8x^{8n} + 934362b^7c^7x^{7n} + 45045b^8c^6x^{6n} - 5005b^9c^5x^{5n} + 1001b^{10}c^4x^{4n} - 273b^{11}c^3x^{3n} + 91b^{12}c^2x^{2n} - 35b^{13}c}{b^{14}c^7n x^{14n} + 7b^{15}c^6n x^{13n} + 21b^{16}c^5n x^{12n} + 35b^{17}c^4n x^{11n} + 35b^{18}c^3n x^{10n} + 21b^{19}c^2n x^{9n} + 7b^{20}c^1n x^{8n} + b^{21}n x^{7n}} + \frac{360360c^7 \log(x)}{b^{15}} - \frac{360360c^7 \log((cx^n + b)/c)}{b^{15}n} + \frac{1}{105} c \left(\frac{360360c^{12}x^{12n} + 2342340b^2c^{11}x^{11n} + 6426420b^3c^{10}x^{10n} + 9579570b^4c^9x^{9n} + 8270262b^5c^8x^{8n} + 4018014b^6c^7x^{7n} + 934362b^7c^6x^{6n} + 45045b^8c^5x^{5n} - 5005b^9c^4x^{4n} + 1001b^{10}c^3x^{3n} - 273b^{11}c^2x^{2n} + 91b^{12}c}{b^{13}c^7n x^{13n} + 7b^{14}c^6n x^{12n} + 21b^{15}c^5n x^{11n} + 35b^{16}c^4n x^{10n} + 35b^{17}c^3n x^{9n} + 21b^{18}c^2n x^{8n} + 7b^{19}c^1n x^{7n}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")

[Out]
$$-1/105*b*((360360*c^{13}*x^{(13*n)} + 2342340*b*c^{12}*x^{(12*n)} + 6426420*b^2*c^{11}*x^{(11*n)} + 9579570*b^3*c^{10}*x^{(10*n)} + 8270262*b^4*c^9*x^{(9*n)} + 4018014*b^5*c^8*x^{(8*n)} + 934362*b^6*c^7*x^{(7*n)} + 45045*b^7*c^6*x^{(6*n)} - 5005*b^8*c^5*x^{(5*n)} + 1001*b^9*c^4*x^{(4*n)} - 273*b^{10}*c^3*x^{(3*n)} + 91*b^{11}*c^2*x^{(2*n)} - 35*b^{12}*c)/b^{14}*c^7*n*x^{(14*n)} + 7*b^{15}*c^6*n*x^{(13*n)} + 21*b^{16}*c^5*n*x^{(12*n)} + 35*b^{17}*c^4*n*x^{(11*n)} + 35*b^{18}*c^3*n*x^{(10*n)} + 21*b^{19}*c^2*n*x^{(9*n)} + 7*b^{20}*c^1*n*x^{(8*n)} + b^{21}*n*x^{(7*n)}) + 360360*c^7*log(x)/b^{15} - 360360*c^7*log((c*x^n + b)/c)/b^{15*n} + 1/105*c*((360360*c^{12}*x^{(12*n)} + 2342340*b*c^{11}*x^{(11*n)} + 6426420*b^2*c^{10}*x^{(10*n)} + 9579570*b^3*c^9*x^{(9*n)} + 8270262*b^4*c^8*x^{(8*n)} + 4018014*b^5*c^7*x^{(7*n)} + 934362*b^6*c^6*x^{(6*n)} + 45045*b^7*c^5*x^{(5*n)} - 5005*b^8*c^4*x^{(4*n)} + 1001*b^9*c^3*x^{(3*n)} - 273*b^{10}*c^2*x^{(2*n)} + 91*b^{11}*c*x^n - 35*b^{12})/b^{13}*c^7*n*x^{(13*n)} + 7*b^{14}*c^6*n*x^{(12*n)} + 21*b^{15}*c^5*n*x^{(11*n)} + 35*b^{16}*c^4*n*x^{(10*n)} + 35*b^{17}*c^3*n*x^{(9*n)} + 21*b^{18}*c^2*n*x^{(8*n)} + 7*b^{19}*c^1*n*x^{(7*n)})$$

$x^{(7*n)} + b^{20*n}*x^{(6*n)} + 360360*c^6*\log(x)/b^{14} - 360360*c^6*\log((c*x^n + b)/c)/(b^{14*n})$

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{7-7n} (b + 2c x^n)}{(bx + c x^{n+1})^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)

[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)

[Out] Timed out

$$3.168 \quad \int (b + 2cx) (bx + cx^2)^p dx$$

Optimal. Leaf size=19

$$\frac{(bx + cx^2)^{p+1}}{p + 1}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{(bx + cx^2)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (b*x + c*x^2)^(1 + p)/(1 + p)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^p dx = \frac{(bx + cx^2)^{1+p}}{1 + p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b + cx))^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (x*(b + c*x))^(1 + p)/(1 + p)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 2cx)(bx + cx^2)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] Defer[IntegrateAlgebraic] [(b + 2*c*x)*(b*x + c*x^2)^p, x]

fricas [A] time = 1.07, size = 26, normalized size = 1.37

$$\frac{(cx^2 + bx)(cx^2 + bx)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)

giac [A] time = 0.37, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")

[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx + b)x(c x^2 + bx)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)

[Out] (c*x+b)*x/(1+p)*(c*x^2+b*x)^p

maxima [A] time = 0.56, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")`

[Out] $(c*x^2 + b*x)^{(p + 1)}/(p + 1)$

mupad [B] time = 2.10, size = 23, normalized size = 1.21

$$\frac{x(c x^2 + b x)^p (b + c x)}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x + c*x^2)^p*(b + 2*c*x),x)`

[Out] $(x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)$

sympy [A] time = 0.71, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)*(c*x**2+b*x)**p,x)`

[Out] `Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))`

$$3.169 \quad \int x^{1+p} (b + 2cx^2) (bx + cx^3)^p dx$$

Optimal. Leaf size=27

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1590}

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]

[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int x^{1+p} (b + 2cx^2) (bx + cx^3)^p dx = \frac{x^{1+p} (bx + cx^3)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 3.59

$$\frac{x^{p+2} (x(b + cx^2))^p \left(\frac{cx^2}{b} + 1\right)^{-p} \left(2c(p+1)x^2 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]

[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{1+p} (b + 2cx^2) (bx + cx^3)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]

[Out] Defer[IntegrateAlgebraic][x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p, x]

fricas [A] time = 1.23, size = 32, normalized size = 1.19

$$\frac{(cx^3 + bx)(cx^3 + bx)^p x^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)

giac [B] time = 0.33, size = 54, normalized size = 2.00

$$\frac{cx^3 e^{(p \log(cx^2+b)+2p \log(x)+\log(x))} + bxe^{(p \log(cx^2+b)+2p \log(x)+\log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")

[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)))/(p + 1)

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{(cx^2 + b)x^{p+2}(cx^3 + bx)^p}{2 + 2p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)`

[Out] `1/2*x^(2+p)*(c*x^2+b)/(p+1)*(c*x^3+b*x)^p`

maxima [A] time = 0.99, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b)+2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")`

[Out] `1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)`

mupad [B] time = 2.21, size = 45, normalized size = 1.67

$$(cx^3 + bx)^p \left(\frac{bx^{p+1}}{2p+2} + \frac{cx^{p+1}x^3}{2p+2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)`

[Out] `(b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)`

[Out] Timed out

$$3.170 \quad \int \left(bx^{1+p} (bx + cx^3)^p + 2cx^{3+p} (bx + cx^3)^p \right) dx$$

Optimal. Leaf size=27

$$\frac{x^{p+1} (bx + cx^3)^{p+1}}{2(p+1)}$$

Rubi [C] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2032, 365, 364}

$$\frac{bx^{p+2} (bx + cx^3)^p \left(\frac{cx^2}{b} + 1\right)^{-p} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)}{2(p+1)} + \frac{cx^{p+4} (bx + cx^3)^p \left(\frac{cx^2}{b} + 1\right)^{-p} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]

[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 2032

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rubi steps

$$\begin{aligned}
\int \left(bx^{1+p} (bx + cx^3)^p + 2cx^{3+p} (bx + cx^3)^p \right) dx &= b \int x^{1+p} (bx + cx^3)^p dx + (2c) \int x^{3+p} (bx + cx^3)^p dx \\
&= \left(bx^{-p} (b + cx^2)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} (b + cx^2)^p dx + \left(2cx^{-p} (b + cx^2)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} (b + cx^2)^p dx \\
&= \left(bx^{-p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} \left(1 + \frac{cx^2}{b} \right)^p dx + \left(2cx^{-p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p \right) \int x^{1+2p} \left(1 + \frac{cx^2}{b} \right)^p dx \\
&= \frac{bx^{2+p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p {}_2F_1 \left(-p, 1 + p; 2 + p; -\frac{cx^2}{b} \right)}{2(1 + p)} + \frac{2cx^{2+p} \left(1 + \frac{cx^2}{b} \right)^{-p} (bx + cx^3)^p {}_2F_1 \left(-p, 1 + p; 2 + p; -\frac{cx^2}{b} \right)}{2(1 + p)}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 97, normalized size = 3.59

$$\frac{x^{p+2} (x(b + cx^2))^p \left(\frac{cx^2}{b} + 1 \right)^{-p} \left(2c(p+1)x^2 {}_2F_1 \left(-p, p+2; p+3; -\frac{cx^2}{b} \right) + b(p+2) {}_2F_1 \left(-p, p+1; p+2; -\frac{cx^2}{b} \right) \right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]

[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -(c*x^2)/b]) + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -(c*x^2)/b])/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(bx^{1+p} (bx + cx^3)^p + 2cx^{3+p} (bx + cx^3)^p \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]

[Out] Defer[IntegrateAlgebraic][b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p, x]

fricas [A] time = 0.83, size = 33, normalized size = 1.22

$$\frac{(cx^2 + b)(cx^3 + bx)^p x^{p+3}}{2(p+1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)

giac [B] time = 0.49, size = 54, normalized size = 2.00

$$\frac{cx^3 e^{(p \log(cx^2+b)+2p \log(x)+\log(x))} + bxe^{(p \log(cx^2+b)+2p \log(x)+\log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="giac")

[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)))/(p + 1)

maple [C] time = 0.26, size = 142, normalized size = 5.26

$$\frac{(cx^2 + b)x x^{p+1} e^{\frac{(-i\pi \operatorname{csgn}(ix) \operatorname{csgn}(i(cx^2+b))) \operatorname{csgn}(i(cx^2+b)x) + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(i(cx^2+b)x)^2 + i\pi \operatorname{csgn}(i(cx^2+b)) \operatorname{csgn}(i(cx^2+b)x)^2 - i\pi \operatorname{csgn}(i(cx^2+b)x)^3 + 2 \ln(x) + 2 \ln(cx^2+b))}{2}}}{2 + 2p}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)

[Out] 1/2*(c*x^2+b)*x*x^(p+1)/(p+1)*exp(1/2*p*(-I*csgn(I*x*(c*x^2+b)))^3*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*x)*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))*Pi-I*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))*Pi+2*ln(x)+2*ln(c*x^2+b))

maxima [A] time = 1.07, size = 35, normalized size = 1.30

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b)+2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int b x^{p+1} (c x^3 + b x)^p + 2 c x^{p+3} (c x^3 + b x)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)

[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p, x)

[Out] Timed out

$$3.171 \quad \int x^{2(1+p)} (b + 2cx^3) (bx + cx^4)^p dx$$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)} (bx + cx^4)^{p+1}}{3(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1590}

$$\frac{x^{2(p+1)} (bx + cx^4)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]

[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int x^{2(1+p)} (b + 2cx^3) (bx + cx^4)^p dx = \frac{x^{2(1+p)} (bx + cx^4)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.08, size = 99, normalized size = 3.41

$$\frac{x^{2p+3} (x(b + cx^3))^p \left(\frac{cx^3}{b} + 1\right)^{-p} \left(2c(p+1)x^3 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^3}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]

[Out] (x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{2(1+p)} (b + 2cx^3) (bx + cx^4)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]

[Out] Defer[IntegrateAlgebraic][x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p, x]

fricas [A] time = 1.08, size = 34, normalized size = 1.17

$$\frac{(cx^4 + bx)(cx^4 + bx)^p x^{2p+2}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")

[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)

giac [B] time = 0.35, size = 58, normalized size = 2.00

$$\frac{cx^4 e^{(p \log(cx^3+b)+3p \log(x)+2 \log(x))} + bxe^{(p \log(cx^3+b)+3p \log(x)+2 \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")

[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)

maple [A] time = 0.00, size = 33, normalized size = 1.14

$$\frac{(cx^3 + b)x^{2p+3}(cx^4 + bx)^p}{3 + 3p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)`

[Out] $\frac{1}{3}x^{(3+2p)}(cx^3+b)/(p+1)(cx^4+bx)^p$

maxima [A] time = 1.01, size = 35, normalized size = 1.21

$$\frac{(cx^6 + bx^3)e^{(p \log(cx^3+b)+3p \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")`

[Out] $\frac{1}{3}(cx^6 + bx^3)e^{(p \log(cx^3 + b) + 3p \log(x))}/(p + 1)$

mupad [B] time = 2.21, size = 49, normalized size = 1.69

$$(cx^4 + bx)^p \left(\frac{cx^{2p+2}x^4}{3p+3} + \frac{bx^{2p+2}}{3p+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)`

[Out] $(b*x + c*x^4)^p((c*x^{(2*p + 2)}*x^4)/(3*p + 3) + (b*x*x^{(2*p + 2)})/(3*p + 3))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)`

[Out] Timed out

$$3.172 \quad \int x^{(-1+n)(1+p)} (b + 2cx^n) (bx + cx^{1+n})^p dx$$

Optimal. Leaf size=36

$$\frac{x^{-((1-n)(p+1))} (bx + cx^{n+1})^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2036}

$$\frac{x^{-(1-n)(p+1)} (bx + cx^{n+1})^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))

Rule 2036

Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j + b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m, n, p}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || IntegerQ[j]) && NeQ[m + j*p + 1, 0]

Rubi steps

$$\int x^{(-1+n)(1+p)} (b + 2cx^n) (bx + cx^{1+n})^p dx = \frac{x^{-((1-n)(1+p))} (bx + cx^{1+n})^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.17, size = 108, normalized size = 3.00

$$\frac{x^{-p} (x(b + cx^n))^p \left(\frac{cx^n}{b} + 1\right)^{-p} \left(b(p+2)x^{n(p+1)} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^n}{b}\right) + 2c(p+1)x^{n(p+2)} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]

[Out] $((x*(b + c*x^n))^p*(b*(2 + p)*x^{n*(1 + p)}*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^{n*(2 + p)}*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p$

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{(-1+n)(1+p)} (b + 2cx^n) (bx + cx^{1+n})^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p, x]

[Out] Defer[IntegrateAlgebraic][x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p, x]

fricas [A] time = 0.85, size = 42, normalized size = 1.17

$$\frac{(bx + cx^{n+1})(bx + cx^{n+1})^p x^{(n-1)p+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fricas")

[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx^n + b)(bx + cx^{n+1})^p x^{(n-1)(p+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac")

[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (2cx^n + b)x^{(n-1)(p+1)} (bx + cx^{n+1})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)`

[Out] `int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)`

maxima [A] time = 1.07, size = 39, normalized size = 1.08

$$\frac{(cx^{2n} + bx^n)e^{(np \log(x) + p \log(cx^n + b))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxima")`

[Out] `(c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{(n-1)(p+1)} (bx + cx^{n+1})^p (b + 2cx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^((n-1)*(p+1))*(b*x+c*x^(n+1))^p*(b+2*c*x^n),x)`

[Out] `int(x^((n-1)*(p+1))*(b*x+c*x^(n+1))^p*(b+2*c*x^n),x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)`

[Out] Timed out

$$3.173 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} dx$$

Optimal. Leaf size=32

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2), x]

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} dx &= \int (ac + adx + bcx^2 + bdx^3) dx \\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4 \end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2), x]

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

IntegrateAlgebraic [A] time = 0.04, size = 29, normalized size = 0.91

$$\frac{1}{12}x(12ac + 6adx + 4bcx^2 + 3bdx^3)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2),x]

[Out] (x*(12*a*c + 6*a*d*x + 4*b*c*x^2 + 3*b*d*x^3))/12

fricas [A] time = 0.93, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="fricas")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

giac [A] time = 0.31, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="giac")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x)

[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4

maxima [A] time = 0.78, size = 26, normalized size = 0.81

$$\frac{1}{4} bdx^4 + \frac{1}{3} bcx^3 + \frac{1}{2} adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x, algorithm="maxima")

[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x

mupad [B] time = 2.14, size = 26, normalized size = 0.81

$$\frac{bdx^4}{4} + \frac{bcx^3}{3} + \frac{adx^2}{2} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2),x)

[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a),x)

[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4

$$3.174 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a+bx^2)^2} dx$$

Optimal. Leaf size=12

$$cx + \frac{dx^2}{2}$$

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {1586}

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^2,x]

[Out] c*x + (d*x^2)/2

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx &= \int \frac{ac + adx + bcx^2 + bdx^3}{a + bx^2} dx \\ &= \int (c + dx) dx \\ &= cx + \frac{dx^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^2,x]

[Out] c*x + (d*x^2)/2

IntegrateAlgebraic [A] time = 0.04, size = 14, normalized size = 1.17

$$\frac{(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^2,x]

[Out] (c + d*x)^2/(2*d)

fricas [A] time = 0.93, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/2*d*x^2 + c*x

giac [A] time = 0.40, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] 1/2*d*x^2 + c*x

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x)

[Out] $c*x+1/2*d*x^2$

maxima [A] time = 0.47, size = 10, normalized size = 0.83

$$\frac{1}{2} dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2,x, algorithm="maxima")`

[Out] $1/2*d*x^2 + c*x$

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{dx^2}{2} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2)^2,x)`

[Out] $c*x + (d*x^2)/2$

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a)**2,x)`

[Out] $c*x + d*x**2/2$

$$3.175 \quad \int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a+bx^2)^3} dx$$

Optimal. Leaf size=42

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1586, 635, 205, 260}

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned}
\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} dx &= \int \frac{ac + adx + bcx^2 + bdx^3}{(a + bx^2)^2} dx \\
&= \int \frac{c + dx}{a + bx^2} dx \\
&= c \int \frac{1}{a + bx^2} dx + d \int \frac{x}{a + bx^2} dx \\
&= \frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d \log(a + bx^2)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3,x]

[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3,x]

[Out] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(a + b*x^2)^3, x]

fricas [A] time = 0.81, size = 98, normalized size = 2.33

$$\left[\frac{ad \log(bx^2 + a) - \sqrt{-ab} c \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2ab}, \frac{ad \log(bx^2 + a) + 2\sqrt{ab} c \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)*x/a))/(a*b)]
```

giac [A] time = 0.40, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.76

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \ln(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

maxima [A] time = 1.49, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log(bx^2 + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

mupad [B] time = 2.13, size = 32, normalized size = 0.76

$$\frac{d \ln(bx^2 + a)}{2b} + \frac{c \operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(a + b*x^2)^3,x)`

[Out] `(d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))`

sympy [B] time = 0.30, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) \log\left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right) + \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) \log\left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*c)/(b*x**2+a)**3,x)`

[Out] `(d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))`

$$3.176 \quad \int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=25

$$\frac{(a + bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1588}

$$\frac{(a + bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^n dx = \frac{(a + bx + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{(a + x(b + x(c + dx)))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (b + 2cx + 3dx^2)(a + bx + cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n, x]

fricas [A] time = 1.02, size = 38, normalized size = 1.52

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)

giac [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)

[Out] (d*x^3+c*x^2+b*x+a)^(n+1)/(n+1)

maxima [A] time = 0.67, size = 25, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)

mupad [B] time = 2.19, size = 54, normalized size = 2.16

$$(dx^3 + cx^2 + bx + a)^n \left(\frac{a}{n+1} + \frac{bx}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)

[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)

[Out] Timed out

$$3.177 \quad \int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=24

$$\frac{(bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1588}

$$\frac{(bx + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^n dx = \frac{(bx + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.88

$$\frac{(x(b + x(c + dx)))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (b + 2cx + 3dx^2)(bx + cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n, x]

fricas [A] time = 0.86, size = 36, normalized size = 1.50

$$\frac{(dx^3 + cx^2 + bx)(dx^3 + cx^2 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)

giac [A] time = 0.31, size = 24, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 34, normalized size = 1.42

$$\frac{(dx^2 + cx + b)x(dx^3 + cx^2 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)

[Out] x*(d*x^2+c*x+b)/(n+1)*(d*x^3+c*x^2+b*x)^n

maxima [A] time = 0.63, size = 24, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)

mupad [B] time = 2.12, size = 46, normalized size = 1.92

$$\left(\frac{bx}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1} \right) (dx^3 + cx^2 + bx)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)

[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)

[Out] Timed out

$$3.178 \quad \int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) dx$$

Optimal. Leaf size=25

$$\frac{x^{n+1} (b + cx + dx^2)^{n+1}}{n + 1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1590}

$$\frac{x^{n+1} (b + cx + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) dx = \frac{x^{1+n} (b + cx + dx^2)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.96

$$\frac{x^{n+1} (b + x(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]

[Out] $(x^{(1+n)}(b+x(c+dx))^{(1+n)})/(1+n)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n (b + cx + dx^2)^n (b + 2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2), x]

[Out] Defer[IntegrateAlgebraic][x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2), x]

fricas [A] time = 1.02, size = 35, normalized size = 1.40

$$\frac{(dx^3 + cx^2 + bx)(dx^2 + cx + b)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b), x, algorithm="fricas")

[Out] $(d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)$

giac [B] time = 0.40, size = 65, normalized size = 2.60

$$\frac{(dx^2 + cx + b)^n dx^3 x^n + (dx^2 + cx + b)^n cx^2 x^n + (dx^2 + cx + b)^n bxx^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b), x, algorithm="giac")

[Out] $((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c*x + b)^n*b*x*x^n)/(n + 1)$

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$\frac{x^{n+1} (dx^2 + cx + b)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b), x)

[Out] $x^{(n+1)}*(d*x^2+c*x+b)^{(n+1)}/(n+1)$

maxima [A] time = 0.86, size = 39, normalized size = 1.56

$$\frac{(dx^3 + cx^2 + bx)e^{(n \log(dx^2 + cx + b) + n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)

mupad [B] time = 2.15, size = 51, normalized size = 2.04

$$\left(\frac{c x^n x^2}{n+1} + \frac{d x^n x^3}{n+1} + \frac{b x x^n}{n+1}\right) (d x^2 + c x + b)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)

[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x + d*x^2)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)

[Out] Timed out

$$3.179 \quad \int (b + 3dx^2) (a + bx + dx^3)^n dx$$

Optimal. Leaf size=20

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2) (a + bx + dx^3)^n dx = \frac{(a + bx + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(a + bx + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] $(a + b*x + d*x^3)^{(1 + n)/(1 + n)}$

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (b + 3dx^2)(a + bx + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(b + 3*d*x^2)*(a + b*x + d*x^3)^n, x]

fricas [A] time = 0.95, size = 28, normalized size = 1.40

$$\frac{(dx^3 + bx + a)(dx^3 + bx + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")

[Out] $(d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)$

giac [A] time = 0.28, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")

[Out] $(d*x^3 + b*x + a)^{(n + 1)/(n + 1)}$

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)

[Out] $(d*x^3+b*x+a)^{(n+1)/(n+1)}$

maxima [A] time = 0.65, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)

mupad [B] time = 2.14, size = 39, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{bx}{n+1} + \frac{dx^3}{n+1} \right) (dx^3 + bx + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)

[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)

[Out] Timed out

$$3.180 \quad \int (b + 3dx^2) (bx + dx^3)^n dx$$

Optimal. Leaf size=19

$$\frac{(bx + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1588}

$$\frac{(bx + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] (b*x + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b + 3dx^2) (bx + dx^3)^n dx = \frac{(bx + dx^3)^{1+n}}{1 + n}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 5.58

$$\frac{x \left(x (b + dx^2) \right)^n \left(\frac{dx^2}{b} + 1 \right)^{-n} \left(3d(n+1)x^2 {}_2F_1 \left(-n, \frac{n+3}{2}; \frac{n+5}{2}; -\frac{dx^2}{b} \right) + b(n+3) {}_2F_1 \left(-n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{dx^2}{b} \right) \right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] $(x*(x*(b + d*x^2))^n*(b*(3 + n)*\text{Hypergeometric2F1}[-n, (1 + n)/2, (3 + n)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*\text{Hypergeometric2F1}[-n, (3 + n)/2, (5 + n)/2, -((d*x^2)/b)])/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)$

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (b + 3dx^2)(bx + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(b + 3*d*x^2)*(b*x + d*x^3)^n, x]

fricas [A] time = 1.38, size = 26, normalized size = 1.37

$$\frac{(dx^3 + bx)(dx^3 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")

[Out] $(d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)$

giac [A] time = 0.34, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")

[Out] $(d*x^3 + b*x)^{(n + 1)}/(n + 1)$

maple [A] time = 0.00, size = 26, normalized size = 1.37

$$\frac{(dx^2 + b)x(dx^3 + bx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x)

[Out] $x*(d*x^2+b)/(n+1)*(d*x^3+b*x)^n$

maxima [A] time = 0.53, size = 19, normalized size = 1.00

$$\frac{(dx^3 + bx)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")

[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)

mupad [B] time = 2.13, size = 25, normalized size = 1.32

$$\frac{x(dx^3 + bx)^n(dx^2 + b)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + d*x^3)^n*(b + 3*d*x^2), x)

[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)

sympy [B] time = 11.34, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx(bx+dx^3)^n}{n+1} + \frac{dx^3(bx+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)

[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(1/d) + x), True))

$$3.181 \quad \int x^n (b + dx^2)^n (b + 3dx^2) dx$$

Optimal. Leaf size=22

$$\frac{x^{n+1} (b + dx^2)^{n+1}}{n + 1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {449}

$$\frac{x^{n+1} (b + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\int x^n (b + dx^2)^n (b + 3dx^2) dx = \frac{x^{1+n} (b + dx^2)^{1+n}}{1 + n}$$

Mathematica [C] time = 0.04, size = 108, normalized size = 4.91

$$\frac{x^{n+1} (b + dx^2)^n \left(\frac{dx^2}{b} + 1\right)^{-n} \left(3d(n+1)x^2 {}_2F_1\left(-n, \frac{n+3}{2}; \frac{n+5}{2}; -\frac{dx^2}{b}\right) + b(n+3) {}_2F_1\left(-n, \frac{n+1}{2}; \frac{n+3}{2}; -\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]

[Out] $(x^{(1+n)}(b+dx^2)^n(b*(3+n)*\text{Hypergeometric2F1}[-n, (1+n)/2, (3+n)/2, -((d*x^2)/b)] + 3*d*(1+n)*x^2*\text{Hypergeometric2F1}[-n, (3+n)/2, (5+n)/2, -((d*x^2)/b)]) / ((1+n)*(3+n)*(1+(d*x^2)/b)^n)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n (b + dx^2)^n (b + 3dx^2) dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[x^n*(b + d*x^2)^n*(b + 3*d*x^2), x]`

[Out] `Defer[IntegrateAlgebraic][x^n*(b + d*x^2)^n*(b + 3*d*x^2), x]`

fricas [A] time = 1.03, size = 27, normalized size = 1.23

$$\frac{(dx^3 + bx)(dx^2 + b)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b), x, algorithm="fricas")`

[Out] $(d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)$

giac [A] time = 0.34, size = 39, normalized size = 1.77

$$\frac{(dx^2 + b)^n dx^3 x^n + (dx^2 + b)^n bxx^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b), x, algorithm="giac")`

[Out] $((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)$

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{n+1} (dx^2 + b)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^n*(d*x^2+b)^n*(3*d*x^2+b), x)`

[Out] $x^{(n+1)}*(d*x^2+b)^{(n+1)}/(n+1)$

maxima [A] time = 1.21, size = 31, normalized size = 1.41

$$\frac{(dx^3 + bx)e^{(n \log(dx^2+b) + n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")

[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)

mupad [B] time = 2.16, size = 26, normalized size = 1.18

$$\frac{xx^n(dx^2 + b)^n(dx^2 + b)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)

[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)

sympy [B] time = 52.84, size = 76, normalized size = 3.45

$$\begin{cases} \frac{bx^n(b+dx^2)^n}{n+1} + \frac{dx^3x^n(b+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ \log(x) + \log\left(-i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) + \log\left(i\sqrt{b}\sqrt{\frac{1}{d}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)

[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(1/d) + x), True))

$$3.182 \quad \int (2cx + 3dx^2) (a + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=22

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1588}

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :=> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^n dx = \frac{(a + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(a + x^2(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]

[Out] $(a + x^2(c + dx))^{(1 + n)/(1 + n)}$

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (2cx + 3dx^2)(a + cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n, x]

fricas [A] time = 0.99, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")

[Out] $(d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)$

giac [A] time = 0.34, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")

[Out] $(d*x^3 + c*x^2 + a)^{(n + 1)}/(n + 1)$

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)

[Out] $(d*x^3+c*x^2+a)^{(n+1)}/(n+1)$

maxima [A] time = 0.66, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1}\right)(dx^3 + cx^2 + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)

[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)

[Out] Timed out

$$3.183 \quad \int (2cx + 3dx^2)(cx^2 + dx^3)^n dx$$

Optimal. Leaf size=21

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1588}

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2)(cx^2 + dx^3)^n dx = \frac{(cx^2 + dx^3)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{(x^2(c + dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] $(x^2(c + dx))^{(1 + n)/(1 + n)}$

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (2cx + 3dx^2)(cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic] [(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n, x]

fricas [A] time = 0.91, size = 30, normalized size = 1.43

$$\frac{(dx^3 + cx^2)(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] $(d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)$

giac [A] time = 0.31, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] $(d*x^3 + c*x^2)^{(n + 1)}/(n + 1)$

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)x^2(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)

[Out] $(d*x^3+c*x^2)^n*x^2*(d*x+c)/(n+1)$

maxima [A] time = 0.59, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

mupad [B] time = 2.16, size = 27, normalized size = 1.29

$$\frac{x^2 (dx^3 + cx^2)^n (c + dx)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)

[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)

sympy [A] time = 1.16, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)

[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.184 \quad \int x^n (cx + dx^2)^n (2cx + 3dx^2) dx$$

Optimal. Leaf size=24

$$\frac{x^{n+1} (cx + dx^2)^{n+1}}{n + 1}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 763}

$$\frac{x^{n+1} (cx + dx^2)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)

Rule 763

Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p + 2), 0] && NeQ[m + 2*p + 2, 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^n (cx + dx^2)^n (2cx + 3dx^2) dx &= \int x^{1+n} (2c + 3dx) (cx + dx^2)^n dx \\ &= \frac{x^{1+n} (cx + dx^2)^{1+n}}{1 + n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^{n+1} (x(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^n (cx + dx^2)^n (2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

[Out] Defer[IntegrateAlgebraic][x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]

fricas [A] time = 0.79, size = 31, normalized size = 1.29

$$\frac{(dx^3 + cx^2)(dx^2 + cx)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x), x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)

giac [B] time = 0.46, size = 51, normalized size = 2.12

$$\frac{dx^3 x^n e^{(n \log(dx+c)+n \log(x))} + cx^2 x^n e^{(n \log(dx+c)+n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x), x, algorithm="giac")

[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*log(x)))/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.17

$$\frac{(dx + c) x^{n+2} (dx^2 + cx)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x), x)

[Out] $(d*x^2+c*x)^n*x^{(2+n)}*(d*x+c)/(n+1)$

maxima [A] time = 0.83, size = 32, normalized size = 1.33

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c) + 2n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2)*e^{(n*\log(d*x + c) + 2*n*\log(x))}/(n + 1)$

mupad [B] time = 2.20, size = 28, normalized size = 1.17

$$\frac{x^n x^2 (d x^2 + c x)^n (c + d x)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)`

[Out] $(x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)$

sympy [A] time = 6.17, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^2x^n(cx+dx^2)^n}{n+1} + \frac{dx^3x^n(cx+dx^2)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)`

[Out] `Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))`

$$3.185 \quad \int x^{2n}(c + dx)^n (2cx + 3dx^2) dx$$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c + dx)^{n+1}}{n + 1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {845}

$$\frac{x^{2(n+1)}(c + dx)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]

[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)

Rule 845

Int[(x_)^(m_.)*((f_) + (g_.)*(x_)^(n_.))*((b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n + 3, 0]

Rubi steps

$$\int x^{2n}(c + dx)^n (2cx + 3dx^2) dx = \frac{x^{2(1+n)}(c + dx)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{2n+2}(c + dx)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]

[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^{2n}(c + dx)^n(2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2), x]

[Out] Defer[IntegrateAlgebraic][x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2), x]

fricas [A] time = 0.97, size = 29, normalized size = 1.32

$$\frac{(dx^3 + cx^2)(dx + c)^n x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x), x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)

giac [A] time = 0.31, size = 41, normalized size = 1.86

$$\frac{(dx + c)^n dx^3 x^{2n} + (dx + c)^n cx^2 x^{2n}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x), x, algorithm="giac")

[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{2n+2} (dx + c)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x), x)

[Out] x^(2+2*n)*(d*x+c)^(n+1)/(n+1)

maxima [A] time = 0.90, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c) + 2n \log(x))}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2)*e^{(n*\log(d*x + c) + 2*n*\log(x))}/(n + 1)$

mupad [B] time = 2.19, size = 26, normalized size = 1.18

$$\frac{x^{2n} x^2 (c + d x)^n (c + d x)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)`

[Out] $(x^{(2*n)}*x^2*(c + d*x)^n*(c + d*x))/(n + 1)$

sympy [A] time = 6.15, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)`

[Out] `Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))`

$$3.186 \quad \int x(2c + 3dx) (a + cx^2 + dx^3)^n dx$$

Optimal. Leaf size=22

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1588}

$$\frac{(a + cx^2 + dx^3)^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]

[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c + 3dx) (a + cx^2 + dx^3)^n dx = \frac{(a + cx^2 + dx^3)^{1+n}}{1 + n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(a + x^2(c + dx))^{n+1}}{n + 1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]

[Out] $(a + x^2(c + dx))^{(1 + n)/(1 + n)}$

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x(2c + 3dx)(a + cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic][x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n, x]

fricas [A] time = 1.07, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")

[Out] $(d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)$

giac [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")

[Out] $(d*x^3 + c*x^2 + a)^{(n + 1)/(n + 1)}$

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(dx^3 + cx^2 + a)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)

[Out] $1/(n+1)*(d*x^3+c*x^2+a)^{(n+1)}$

maxima [A] time = 0.75, size = 32, normalized size = 1.45

$$\frac{(dx^3 + cx^2 + a)(dx^3 + cx^2 + a)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{cx^2}{n+1} + \frac{dx^3}{n+1}\right)(dx^3 + cx^2 + a)^n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)

[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)

[Out] Timed out

$$3.187 \quad \int x(2c + 3dx) (cx^2 + dx^3)^n dx$$

Optimal. Leaf size=21

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1588}

$$\frac{(cx^2 + dx^3)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x(2c + 3dx) (cx^2 + dx^3)^n dx = \frac{(cx^2 + dx^3)^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{(x^2(c + dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] $(x^2(c + dx))^{(1 + n)/(1 + n)}$

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x(2c + 3dx)(cx^2 + dx^3)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]

[Out] Defer[IntegrateAlgebraic][x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n, x]

fricas [A] time = 0.66, size = 30, normalized size = 1.43

$$\frac{(dx^3 + cx^2)(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)

giac [A] time = 0.40, size = 21, normalized size = 1.00

$$\frac{(dx^3 + cx^2)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")

[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)x^2(dx^3 + cx^2)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)

[Out] (d*x+c)/(n+1)*x^2*(d*x^3+c*x^2)^n

maxima [A] time = 0.93, size = 32, normalized size = 1.52

$$\frac{(dx^3 + cx^2)e^{(n \log(dx+c)+2n \log(x))}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")

[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)

mupad [B] time = 2.15, size = 27, normalized size = 1.29

$$\frac{x^2 (dx^3 + cx^2)^n (c + dx)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)

[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)

sympy [A] time = 1.14, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2(cx^2+dx^3)^n}{n+1} + \frac{dx^3(cx^2+dx^3)^n}{n+1} & \text{for } n \neq -1 \\ 2 \log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)

[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))

$$3.188 \quad \int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=21

$$\frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

Rubi [A] time = 0.13, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1588}

$$\frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (a + b*x + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :=> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (a + bx + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + bx + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.15, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b + x(c + dx))(8a^7 + 28a^6x(b + x(c + dx)) + 56a^5x^2(b + x(c + dx))^2 + 70a^4x^3(b + x(c + dx))^3 + 56a^3x^4(b + x(c + dx))^4 + 28a^2x^5(b + x(c + dx))^5 + 8ax^6(b + x(c + dx))^6 + x^7(b + x(c + dx))^7)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b + x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*(b + x*(c + d*x))^7)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 2cx + 3dx^2)(a + bx + cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7, x]

fricas [B] time = 0.57, size = 1956, normalized size = 93.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")

[Out] $1/8*x^{24}*d^8 + x^{23}*d^7*c + 7/2*x^{22}*d^6*c^2 + x^{22}*d^7*b + 7*x^{21}*d^5*c^3 + 7*x^{21}*d^6*c*b + x^{21}*d^7*a + 35/4*x^{20}*d^4*c^4 + 21*x^{20}*d^5*c^2*b + 7/2*x^{20}*d^6*b^2 + 7*x^{20}*d^6*c*a + 7*x^{19}*d^3*c^5 + 35*x^{19}*d^4*c^3*b + 21*x^{19}*d^5*c*b^2 + 21*x^{19}*d^5*c^2*a + 7*x^{19}*d^6*b*a + 7/2*x^{18}*d^2*c^6 + 35*x^{18}*d^3*c^4*b + 105/2*x^{18}*d^4*c^2*b^2 + 7*x^{18}*d^5*b^3 + 35*x^{18}*d^4*c^3*a + 42*x^{18}*d^5*c*b*a + 7/2*x^{18}*d^6*a^2 + x^{17}*d*c^7 + 21*x^{17}*d^2*c^5*b + 70*x^{17}*d^3*c^3*b^2 + 35*x^{17}*d^4*c*b^3 + 35*x^{17}*d^3*c^4*a + 105*x^{17}*d^4*c^2*b*a + 21*x^{17}*d^5*b^2*a + 21*x^{17}*d^5*c*a^2 + 1/8*x^{16}*c^8 + 7*x^{16}*d*c^6*b + 105/2*x^{16}*d^2*c^4*b^2 + 70*x^{16}*d^3*c^2*b^3 + 35/4*x^{16}*d^4*b^4 + 21*x^{16}*d^2*c^5*a + 140*x^{16}*d^3*c^3*b*a + 105*x^{16}*d^4*c*b^2*a + 105/2*x^{16}*d^4*c^2*a^2 + 21*x^{16}*d^5*b*a^2 + x^{15}*c^7*b + 21*x^{15}*d*c^5*b^2 + 70*x^{15}*d^2*c^3*b^3 + 35*x^{15}*d^3*c*b^4 + 7*x^{15}*d*c^6*a + 105*x^{15}*d^2*c^4*b*a + 210*x^{15}*d^3*c^2*b^2*a + 35*x^{15}*d^4*b^3*a + 70*x^{15}*d^3*c^3*a^2 + 105*x^{15}*d^4*c*b*a^2 + 7*x^{15}*d^5*a^3 + 7/2*x^{14}*c^6*b^2 + 35*x^{14}*d*c^4*b^3 + 105/2*x^{14}*d^2*c^2*b^4 + 7*x^{14}*d^3*b^5 + x^{14}*c^7*a + 42*x^{14}*d*c^5*b*a + 210*x^{14}*d^2*c^3*b^2*a + 140*x^{14}*d^3*c*b^3*a + 105/2*x^{14}*d^2*c^4*a^2 + 210*x^{14}*d^3*c^2*b*a^2 + 105/2*x^{14}*d^4*b^2*a^2 + 35*x^{14}*d^4*c*a^3 + 7*x^{13}*c^5*b^3 + 35*x^{13}*d*c^3*b^4 + 21*x^{13}*d^2*c*b^5 + 7*x^{13}*c^6*b*a + 105*x^{13}*d*c^4*b^2*a + 210*x^{13}*d^2*c^2*b^3*a + 35*x^{13}*d^3*b^4*a + 21*x^{13}*d*c^5*a^2 + 210*x^{13}*d^2*c^3*b*a^2 + 210*x^{13}*d^3*c*b^2*a^2 + 70*x^{13}*d^3*c^2*a^3 + 35*x^{13}*d^4*b*a^3 + 35/4*x^{12}*c^4*b^4 + 21*x^{12}*d*c^2*b^5 + 7/2*x^{12}*d^2*b^6 + 21*x^{12}*c^5*b^2*a + 140*x^{12}*d*c^3*b^3*a + 105*x^{12}*d^2*c*b^4*a + 7/2*x^{12}*c^6*a^2 + 105*x^{12}*d*c^4*b*a^2 + 315*x^{12}*d^2*c^2*b^2*a^2 + 70*x^{12}*d^3*b^3*a^2 + 70*x^{12}*d^2*c^3*a^3 + 140*x^{12}*d^3*c*b*a^3 + 35/4*x^{12}*d^4*a^4 + 7*x^{11}*c^3*b^5 + 7*x^{11}*d*c*b^6 + 35*x^{11}*c^4*b^3*a + 105*x^{11}*d*c^2*b^4*a + 21*x^{11}*d^2*b^5*a + 21*x^{11}*c^5*b*a^2 + 210*x^{11}*d*c^3*b^2*a^2 + 210*x^{11}*d^2*c*b^3*a^2 + 35*x^{11}*d*c^4*a^3 + 210*x^{11}*d^2*c^2*b*a^3 + 70*x^{11}*d^3*b^2*a^3 + 35*x^{11}*d^3*c*a^4 + 7/2*x^{10}*c^2*b^6 + x^{10}*d*b^7 + 35*x^{10}*c^3*b^4$

$a + 42x^{10}d^2c^2b^5a + 105/2x^{10}d^2c^4b^2a^2 + 210x^{10}d^2c^2b^3a^2 + 105/2x^{10}d^2b^4a^2 + 7x^{10}c^5a^3 + 140x^{10}d^2c^3b^2a^3 + 210x^{10}d^2c^2b^2a^3 + 105/2x^{10}d^2c^2a^4 + 35x^{10}d^3b^2a^4 + x^9c^2b^7 + 21x^9c^2b^5a + 7x^9d^2b^6a + 70x^9c^3b^3a^2 + 105x^9d^2c^2b^4a^2 + 35x^9c^4b^2a^3 + 210x^9d^2c^2b^2a^3 + 70x^9d^2b^3a^3 + 35x^9d^2c^3a^4 + 105x^9d^2c^2b^2a^4 + 7x^9d^3a^5 + 1/8x^8b^8 + 7x^8c^2b^6a + 105/2x^8c^2b^4a^2 + 21x^8d^2b^5a^2 + 70x^8c^3b^2a^3 + 140x^8d^2c^2b^3a^3 + 35/4x^8c^4a^4 + 105x^8d^2c^2b^2a^4 + 105/2x^8d^2b^2a^4 + 21x^8d^2c^2a^5 + x^7b^7a + 21x^7c^2b^5a^2 + 70x^7c^2b^3a^3 + 35x^7d^2b^4a^3 + 35x^7c^3b^2a^4 + 105x^7d^2c^2b^2a^4 + 21x^7d^2c^2a^5 + 21x^7d^2b^2a^5 + 7/2x^6b^6a^2 + 35x^6c^2b^4a^3 + 105/2x^6c^2b^2a^4 + 35x^6d^2b^3a^4 + 7x^6c^3a^5 + 42x^6d^2c^2b^2a^5 + 7/2x^6d^2a^6 + 7x^5b^5a^3 + 35x^5c^2b^3a^4 + 21x^5c^2b^2a^5 + 21x^5d^2b^2a^5 + 7x^5d^2c^2a^6 + 35/4x^4b^4a^4 + 21x^4c^2b^2a^5 + 7/2x^4c^2a^6 + 7x^4d^2b^2a^6 + 7x^3b^3a^5 + 7x^3c^2b^2a^6 + x^3d^2a^7 + 7/2x^2b^2a^6 + x^2c^2a^7 + x^2b^2a^7$

giac [B] time = 0.30, size = 160, normalized size = 7.62

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8 + (dx^3 + cx^2 + bx)^7a + \frac{7}{2}(dx^3 + cx^2 + bx)^6a^2 + 7(dx^3 + cx^2 + bx)^5a^3 + \frac{35}{4}(dx^3 + cx^2 + bx)^4a^4 + 7(dx^3 + cx^2 + bx)^3a^5 + \frac{7}{2}(dx^3 + cx^2 + bx)^2a^6 + (dx^3 + cx^2 + bx)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")

[Out] $1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 + (d*x^3 + c*x^2 + b*x)*a^7$

maple [B] time = 0.00, size = 25686, normalized size = 1223.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)

[Out] result too large to display

maxima [A] time = 0.74, size = 19, normalized size = 0.90

$$\frac{1}{8}(dx^3 + cx^2 + bx + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8

mupad [B] time = 2.98, size = 1576, normalized size = 75.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)

[Out] $x^{12} * ((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d^3) + x^{11} * (7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 210*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^{13} * (7*b^3*c^5 + 35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 + 70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5 * (7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^{19} * (7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8 * (b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9 * (b^7*c + 7*a^5*d^3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2) + x^{16} * (c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^3 + 105*a*b^2*c*d^4) + x^{10} * (b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b*c^3*d) + x^{15} * (b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^{14} * (a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 + 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 + (105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d + 140*a*b^3*c*d^3) + x^4 * ((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7*a^6*b*d) + x^{20} * ((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6) + x^6 * ((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7 * (a*b^7 + 21*a^2*b^5*c + 35*a^4*b^3*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 + 105*a^4*b^2*c*d) + x^{18} * ((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^{17} * (c^7*d + 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4 + 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3 * (a^7*d + 7*a^5*b^3 + 7*a^6*b*c) + (d^8*x^24)/8 + x^2 * (a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2$

+ 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x

sympy [B] time = 0.42, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)

[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 + 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5 + 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 + 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b**2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d**3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b**4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 + 105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d + 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x**10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 105*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 + 105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10

$$5a^{4}b^{2}c^{2}/2 + 35a^{3}b^{4}c + 7a^{2}b^{6}/2) + x^{5}(7a^{6}cd + 21a^{5}b^{2}d + 21a^{5}b^{2}c^{2} + 35a^{4}b^{3}c + 7a^{3}b^{5}) + x^{4}(7a^{6}bd + 7a^{6}c^{2}/2 + 21a^{5}b^{2}c + 35a^{4}b^{4}/4) + x^{3}(a^{7}d + 7a^{6}bc + 7a^{5}b^{3}) + x^{2}(a^{7}c + 7a^{6}b^{2}/2)$$

$$3.189 \quad \int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=20

$$\frac{1}{8} (bx + cx^2 + dx^3)^8$$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1588}

$$\frac{1}{8} (bx + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (b*x + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 2cx + 3dx^2) (bx + cx^2 + dx^3)^7 dx = \frac{1}{8} (bx + cx^2 + dx^3)^8$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\frac{1}{8} x^8 (b + x(c + dx))^8$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] (x^8*(b + x*(c + d*x))^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 2cx + 3dx^2)(bx + cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7, x]

fricas [B] time = 0.62, size = 496, normalized size = 24.80

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + x^{22}d^7b + 7x^{21}d^5c^3 + 7x^{21}d^6c^2b + \frac{35}{4}x^{20}d^4c^4 + 21x^{20}d^5c^2b + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^3c^5 + 35x^{19}d^4c^3b + 21x^{19}d^5c^2b^2 + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d^3c^4b + \frac{105}{2}x^{18}d^4c^2b^2 + 7x^{18}d^5b^3 + x^{17}d^7c^7 + 21x^{17}d^2c^5b + 70x^{17}d^3c^3b^2 + 35x^{17}d^4c^2b^3 + \frac{1}{8}x^{16}c^8 + 7x^{16}d^6c^6b + \frac{105}{2}x^{16}d^2c^4b^2 + 70x^{16}d^3c^2b^3 + \frac{35}{4}x^{16}d^4b^4 + x^{15}c^7b + 21x^{15}d^5c^5b^2 + 70x^{15}d^2c^3b^3 + 35x^{15}d^3c^2b^4 + \frac{7}{2}x^{14}c^6b^2 + 35x^{14}d^4c^4b^3 + \frac{105}{2}x^{14}d^2c^2b^4 + 7x^{14}d^3b^5 + 7x^{13}c^5b^3 + 35x^{13}d^3c^3b^4 + 21x^{13}d^2c^2b^5 + \frac{35}{4}x^{12}c^4b^4 + 21x^{12}d^2c^2b^5 + \frac{7}{2}x^{12}d^2b^6 + 7x^{11}c^3b^5 + 7x^{11}d^4c^2b^6 + \frac{7}{2}x^{10}c^2b^6 + x^{10}d^4b^7 + x^9c^2b^7 + \frac{1}{8}x^8b^8$

giac [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{1}{8}(dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")

[Out] $\frac{1}{8}(d*x^3 + c*x^2 + b*x)^8$

maple [B] time = 0.00, size = 5596, normalized size = 279.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)

[Out] result too large to display

maxima [A] time = 0.63, size = 18, normalized size = 0.90

$$\frac{1}{8} (dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8

mupad [B] time = 2.32, size = 418, normalized size = 20.90

$\frac{1}{8} (dx^3 + cx^2 + bx)^8$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)

[Out] $x^{14} \left(\frac{7b^2c^6}{2} + 7b^5d^3 + 35b^3c^4d + \frac{105b^4c^2d^2}{2} \right) + x^{12} \left(8(7b^3d^5 + \frac{7c^6d^2}{2} + 35b^2c^4d^3 + \frac{105b^2c^2d^4}{2}) + \frac{35b^4c^4}{4} + \frac{7b^6d^2}{2} + 21b^5c^2d \right) + x^{20} \left(\frac{7b^2d^6}{2} + \frac{35c^4d^4}{4} + 21b^2c^2d^5 \right) + x^{16} \left(\frac{c^8}{8} + \frac{35b^4d^4}{4} + \frac{105b^2c^4d^2}{2} + 70b^3c^2d^3 + 7b^2c^6d \right) + \frac{b^8x^8}{8} + \frac{d^8x^{24}}{8} + x^{10} \left(b^7d + \frac{7b^6c^2}{2} \right) + b^7c^2x^9 + c^7d^7x^{23} + \frac{d^6x^{22}(2bd + 7c^2)}{2} + 7b^3c^2x^{13}(c^4 + 3b^2d^2 + 5b^2c^2d) + 7c^2d^3x^{19}(c^4 + 3b^2d^2 + 5b^2c^2d) + b^2c^2x^{15}(c^6 + 35b^3d^3 + 70b^2c^2d^2 + 21b^2c^4d) + c^2d^2x^{17}(c^6 + 35b^3d^3 + 70b^2c^2d^2 + 21b^2c^4d) + 7b^5c^2x^{11}(bd + c^2) + 7c^2d^5x^{21}(bd + c^2)$

sympy [B] time = 0.18, size = 469, normalized size = 23.45

$\frac{1}{8} (dx^3 + cx^2 + bx)^8$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)

[Out] $b^8x^8/8 + b^7c^2x^9 + c^7d^7x^{23} + d^8x^{24}/8 + x^{22}(bd^7 + 7c^2d^6/2) + x^{21}(7b^2cd^6 + 7c^3d^5) + x^{20}(7b^2d^6/2 + 21b^2c^2d^5 + 35c^4d^4/4) + x^{19}(21b^2c^2d^5 + 35b^2c^3d^4 + 7c^5d^3) + x^{18}(7b^3d^5 + 105b^2c^2d^4/2 + 35b^2c^4d^3 + 7c^6d^2/2) + x^{17}(35b^3c^2d^4 + 70b^2c^3d^3 + 21b^2c^5d^2 + c^7d) + x^{16}(35b^4d^4/4 + 70b^3c^2d^3 + 105b^2c^4d^3$

$$\begin{aligned}
& \left(\frac{2}{2} + 7bc^6d + \frac{c^8}{8} \right) + x^{15} (35b^4cd^3 + 70b^3c^3d^2 + 21 \\
& b^2c^5d + b^7c) + x^{14} (7b^5d^3 + 105b^4c^2d^2 + 35b^3 \\
& c^4d + 7b^2c^6/2) + x^{13} (21b^5cd^2 + 35b^4c^3d + 7b^3 \\
& c^5) + x^{12} (7b^6d^2/2 + 21b^5c^2d + 35b^4c^4/4) + x^{11} (7 \\
& b^6cd + 7b^5c^3) + x^{10} (b^7d + 7b^6c^2/2)
\end{aligned}$$

$$3.190 \quad \int x^7 (b + cx + dx^2)^7 (b + 2cx + 3dx^2) dx$$

Optimal. Leaf size=19

$$\frac{1}{8}x^8 (b + cx + dx^2)^8$$

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1588}

$$\frac{1}{8}x^8 (b + cx + dx^2)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^8*(b + c*x + d*x^2)^8)/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x^7 (b + cx + dx^2)^7 (b + 2cx + 3dx^2) dx = \frac{1}{8}x^8 (b + cx + dx^2)^8$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8 (b + x(c + dx))^8$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]

[Out] (x^8*(b + x*(c + d*x))^8)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 (b + cx + dx^2)^7 (b + 2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2), x]

[Out] IntegrateAlgebraic[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2), x]

fricas [B] time = 0.80, size = 496, normalized size = 26.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b), x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + x^{22}d^7b + 7x^{21}d^5c^3 + 7x^{21}d^6c^2b + \frac{35}{4}x^{20}d^4c^4 + 21x^{20}d^5c^2b + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^3c^5 + 35x^{19}d^4c^3b + 21x^{19}d^5c^2b^2 + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d^3c^4b + \frac{105}{2}x^{18}d^4c^2b^2 + 7x^{18}d^5b^3 + x^{17}d^7c^7 + 21x^{17}d^2c^5b + 70x^{17}d^3c^3b^2 + 35x^{17}d^4c^2b^3 + \frac{1}{8}x^{16}c^8 + 7x^{16}d^6c^2b + \frac{105}{2}x^{16}d^2c^4b^2 + 70x^{16}d^3c^2b^3 + \frac{35}{4}x^{16}d^4b^4 + x^{15}d^7c^7b + 21x^{15}d^2c^5b^2 + 70x^{15}d^3c^3b^3 + 35x^{15}d^4c^2b^4 + \frac{7}{2}x^{14}d^6c^2b^2 + 35x^{14}d^2c^4b^3 + \frac{105}{2}x^{14}d^3c^2b^4 + 7x^{14}d^4b^5 + 7x^{13}d^5c^2b^3 + 35x^{13}d^2c^3b^4 + 21x^{13}d^3c^2b^5 + \frac{35}{4}x^{12}d^4c^2b^4 + 21x^{12}d^2c^2b^5 + \frac{7}{2}x^{12}d^3b^6 + 7x^{11}d^4c^3b^5 + 7x^{11}d^5c^2b^6 + \frac{7}{2}x^{10}d^2c^2b^6 + x^{10}d^3b^7 + x^9d^4c^2b^7 + \frac{1}{8}x^8d^5b^8$

giac [A] time = 0.30, size = 18, normalized size = 0.95

$$\frac{1}{8} (dx^3 + cx^2 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b), x, algorithm="giac")

[Out] $\frac{1}{8}(d*x^3 + c*x^2 + b*x)^8$

maple [B] time = 0.00, size = 5596, normalized size = 294.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)`

[Out] result too large to display

maxima [B] time = 0.60, size = 441, normalized size = 23.21

$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}(7c^2d^6 + 2bd^7)x^{22} + 7(c^3d^5 + b^2cd^6)x^{21} + \frac{7}{4}(5c^4d^4 + 12b^2cd^5 + 2b^3d^6)x^{20} + 7(c^5d^3 + 5b^2c^3d^4 + 3b^2cd^5)x^{19} + \frac{7}{2}(c^6d^2 + 10b^2c^4d^3 + 15b^2c^2d^4 + 2b^3d^5)x^{18} + (c^7d + 21b^2c^5d^2 + 70b^2c^3d^3 + 35b^3c^2d^4)x^{17} + b^7c^7x^9 + \frac{1}{8}(c^8 + 56b^2c^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4)x^{16} + \frac{1}{8}b^8x^8 + (b^2c^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4c^2d^3)x^{15} + \frac{7}{2}(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3)x^{14} + 7(b^3c^5 + 5b^4c^3d + 3b^5c^2d^2)x^{13} + \frac{7}{4}(5b^4c^4 + 12b^5c^2d + 2b^6d^2)x^{12} + 7(b^5c^3 + b^6cd)x^{11} + \frac{1}{2}(7b^6c^2 + 2b^7d)x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")`

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}(7c^2d^6 + 2bd^7)x^{22} + 7(c^3d^5 + b^2cd^6)x^{21} + \frac{7}{4}(5c^4d^4 + 12b^2cd^5 + 2b^3d^6)x^{20} + 7(c^5d^3 + 5b^2c^3d^4 + 3b^2cd^5)x^{19} + \frac{7}{2}(c^6d^2 + 10b^2c^4d^3 + 15b^2c^2d^4 + 2b^3d^5)x^{18} + (c^7d + 21b^2c^5d^2 + 70b^2c^3d^3 + 35b^3c^2d^4)x^{17} + b^7c^7x^9 + \frac{1}{8}(c^8 + 56b^2c^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4)x^{16} + \frac{1}{8}b^8x^8 + (b^2c^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4c^2d^3)x^{15} + \frac{7}{2}(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3)x^{14} + 7(b^3c^5 + 5b^4c^3d + 3b^5c^2d^2)x^{13} + \frac{7}{4}(5b^4c^4 + 12b^5c^2d + 2b^6d^2)x^{12} + 7(b^5c^3 + b^6cd)x^{11} + \frac{1}{2}(7b^6c^2 + 2b^7d)x^{10}$

mupad [B] time = 2.27, size = 418, normalized size = 22.00

$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}(7c^2d^6 + 2bd^7)x^{22} + 7(c^3d^5 + b^2cd^6)x^{21} + \frac{7}{4}(5c^4d^4 + 12b^2cd^5 + 2b^3d^6)x^{20} + 7(c^5d^3 + 5b^2c^3d^4 + 3b^2cd^5)x^{19} + \frac{7}{2}(c^6d^2 + 10b^2c^4d^3 + 15b^2c^2d^4 + 2b^3d^5)x^{18} + (c^7d + 21b^2c^5d^2 + 70b^2c^3d^3 + 35b^3c^2d^4)x^{17} + b^7c^7x^9 + \frac{1}{8}(c^8 + 56b^2c^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4)x^{16} + \frac{1}{8}b^8x^8 + (b^2c^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4c^2d^3)x^{15} + \frac{7}{2}(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3)x^{14} + 7(b^3c^5 + 5b^4c^3d + 3b^5c^2d^2)x^{13} + \frac{7}{4}(5b^4c^4 + 12b^5c^2d + 2b^6d^2)x^{12} + 7(b^5c^3 + b^6cd)x^{11} + \frac{1}{2}(7b^6c^2 + 2b^7d)x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)`

[Out] $x^{14}((7b^2c^6)/2 + 7b^5d^3 + 35b^3c^4d + (105b^4c^2d^2)/2) + x^{12}8*(7b^3d^5 + (7c^6d^2)/2 + 35b^2c^4d^3 + (105b^2c^2d^4)/2) + x^{10}((35b^4c^4)/4 + (7b^6d^2)/2 + 21b^5c^2d) + x^{20}((7b^2d^6)/2 + (35c^4d^4)/4 + 21b^2c^2d^5) + x^{16}(c^8/8 + (35b^4d^4)/4 + (105b^2c^4d^2)/2 + 70b^3c^2d^3 + 7b^2c^6d) + (b^8x^8)/8 + (d^8x^24)/8 + x^{10}(b^7d + (7b^6c^2)/2) + b^7c^7x^9 + cd^7x^{23} + (d^6x^{22}(2bd + 7c^2))/2 + 7b^3c^2x^{13}(c^4 + 3b^2d^2 + 5b^2cd) + 7cd^3x^{19}(c^4 + 3b^2d^2 + 5b^2cd) + b^2c^2x^{15}(c^6 + 35b^3d^3 + 70b^2c^2d^2 + 21b^2c^4d) + cd^2x^{17}(c^6 + 35b^3d^3 + 70b^2c^2d^2 + 21b^2c^4d) + 7b^5c^2x^{11}(bd + c^2) + 7cd^5x^{21}(bd + c^2)$

sympy [B] time = 0.17, size = 469, normalized size = 24.68

$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{1}{2}(7c^2d^6 + 2bd^7)x^{22} + 7(c^3d^5 + b^2cd^6)x^{21} + \frac{7}{4}(5c^4d^4 + 12b^2cd^5 + 2b^3d^6)x^{20} + 7(c^5d^3 + 5b^2c^3d^4 + 3b^2cd^5)x^{19} + \frac{7}{2}(c^6d^2 + 10b^2c^4d^3 + 15b^2c^2d^4 + 2b^3d^5)x^{18} + (c^7d + 21b^2c^5d^2 + 70b^2c^3d^3 + 35b^3c^2d^4)x^{17} + b^7c^7x^9 + \frac{1}{8}(c^8 + 56b^2c^6d + 420b^2c^4d^2 + 560b^3c^2d^3 + 70b^4d^4)x^{16} + \frac{1}{8}b^8x^8 + (b^2c^7 + 21b^2c^5d + 70b^3c^3d^2 + 35b^4c^2d^3)x^{15} + \frac{7}{2}(b^2c^6 + 10b^3c^4d + 15b^4c^2d^2 + 2b^5d^3)x^{14} + 7(b^3c^5 + 5b^4c^3d + 3b^5c^2d^2)x^{13} + \frac{7}{4}(5b^4c^4 + 12b^5c^2d + 2b^6d^2)x^{12} + 7(b^5c^3 + b^6cd)x^{11} + \frac{1}{2}(7b^6c^2 + 2b^7d)x^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)

[Out] $b^{**8}x^{**8}/8 + b^{**7}c*x^{**9} + c*d^{**7}x^{**23} + d^{**8}x^{**24}/8 + x^{**22}*(b*d^{**7} + 7*c^{**2}d^{**6}/2) + x^{**21}*(7*b*c*d^{**6} + 7*c^{**3}d^{**5}) + x^{**20}*(7*b^{**2}d^{**6}/2 + 21*b*c^{**2}d^{**5} + 35*c^{**4}d^{**4}/4) + x^{**19}*(21*b^{**2}c*d^{**5} + 35*b*c^{**3}d^{**4} + 7*c^{**5}d^{**3}) + x^{**18}*(7*b^{**3}d^{**5} + 105*b^{**2}c^{**2}d^{**4}/2 + 35*b*c^{**4}d^{**3} + 7*c^{**6}d^{**2}/2) + x^{**17}*(35*b^{**3}c*d^{**4} + 70*b^{**2}c^{**3}d^{**3} + 21*b*c^{**5}d^{**2} + c^{**7}d) + x^{**16}*(35*b^{**4}d^{**4}/4 + 70*b^{**3}c^{**2}d^{**3} + 105*b^{**2}c^{**4}d^{**2}/2 + 7*b*c^{**6}d + c^{**8}/8) + x^{**15}*(35*b^{**4}c*d^{**3} + 70*b^{**3}c^{**3}d^{**2} + 21*b^{**2}c^{**5}d + b*c^{**7}) + x^{**14}*(7*b^{**5}d^{**3} + 105*b^{**4}c^{**2}d^{**2}/2 + 35*b^{**3}c^{**4}d + 7*b^{**2}c^{**6}/2) + x^{**13}*(21*b^{**5}c*d^{**2} + 35*b^{**4}c^{**3}d + 7*b^{**3}c^{**5}) + x^{**12}*(7*b^{**6}d^{**2}/2 + 21*b^{**5}c^{**2}d + 35*b^{**4}c^{**4}/4) + x^{**11}*(7*b^{**6}c*d + 7*b^{**5}c^{**3}) + x^{**10}*(b^{**7}d + 7*b^{**6}c^{**2}/2)$

$$3.191 \quad \int (b + 3dx^2) (a + bx + dx^3)^7 dx$$

Optimal. Leaf size=16

$$\frac{1}{8} (a + bx + dx^3)^8$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\frac{1}{8} (a + bx + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (a + b*x + d*x^3)^8/8

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (b + 3dx^2) (a + bx + dx^3)^7 dx = \frac{1}{8} (a + bx + dx^3)^8$$

Mathematica [B] time = 0.06, size = 127, normalized size = 7.94

$$\frac{1}{8}x(b + dx^2)(8a^7 + 28a^6x(b + dx^2) + 56a^5x^2(b + dx^2)^2 + 70a^4x^3(b + dx^2)^3 + 56a^3x^4(b + dx^2)^4 + 28a^2x^5(b + dx^2)^5 + 8ax^6(b + dx^2)^6 + x^7(b + dx^2)^7)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 70*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 3dx^2)(a + bx + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^7, x]

fricas [B] time = 0.94, size = 486, normalized size = 30.38

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{22}d^7b + x^{21}d^7a + \frac{7}{2}x^{20}d^6b^2 + 7x^{19}d^6b^2a + 7x^{18}d^5b^3 + \frac{7}{2}x^{18}d^6a^2 + 21x^{17}d^5b^2a + \frac{35}{4}x^{16}d^4b^4 + 21x^{16}d^5b^2a^2 + 35x^{15}d^4b^3a + 7x^{15}d^5a^3 + 7x^{14}d^3b^5 + 105/2x^{14}d^4b^2a^2 + 35x^{13}d^3b^4a + 35x^{13}d^4b^2a^3 + 7/2x^{12}d^2b^6 + 70x^{12}d^3b^3a^2 + 35/4x^{12}d^4a^4 + 21x^{11}d^2b^5a + 70x^{11}d^3b^2a^3 + x^{10}db^7 + 105/2x^{10}d^2b^4a^2 + 35x^{10}d^3b^2a^4 + 7x^9d^2b^6a + 70x^9d^2b^3a^3 + 7x^9d^3a^5 + 1/8x^8b^8 + 21x^8d^2b^5a^2 + 105/2x^8d^2b^2a^4 + x^7b^7a + 35x^7d^2b^4a^3 + 21x^7d^2b^2a^5 + 7/2x^6b^6a^2 + 35x^6d^2b^3a^4 + 7/2x^6d^2a^6 + 7x^5b^5a^3 + 21x^5d^2b^2a^5 + 35/4x^4b^4a^4 + 7x^4d^2b^2a^6 + 7x^3b^3a^5 + x^3da^7 + 7/2x^2b^2a^6 + xba^7$

giac [B] time = 0.41, size = 120, normalized size = 7.50

$$\frac{1}{8}(dx^3 + bx)^8 + (dx^3 + bx)^7a + \frac{7}{2}(dx^3 + bx)^6a^2 + 7(dx^3 + bx)^5a^3 + \frac{35}{4}(dx^3 + bx)^4a^4 + 7(dx^3 + bx)^3a^5 + \frac{7}{2}(dx^3 + bx)^2a^6 + (dx^3 + bx)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")

[Out] $\frac{1}{8}(d^3x^3 + b^3x)^8 + (d^3x^3 + b^3x)^7a + \frac{7}{2}(d^3x^3 + b^3x)^6a^2 + 7(d^3x^3 + b^3x)^5a^3 + \frac{35}{4}(d^3x^3 + b^3x)^4a^4 + 7(d^3x^3 + b^3x)^3a^5 + \frac{7}{2}(d^3x^3 + b^3x)^2a^6 + (d^3x^3 + b^3x)a^7$

maple [B] time = 0.00, size = 2185, normalized size = 136.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)$

[Out] $\frac{1}{8}d^8x^{24}+bd^7x^{22}+d^7a^2x^{21}+\frac{7}{2}b^2d^6x^{20}+7b^2ad^6x^{19}+\frac{1}{18}(21b^3d^5+3d^5(6a^2d^5+15b^3d^4+d^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4)))*x^{18}+21b^2ad^5x^{17}+\frac{1}{16}(b^2(6a^2d^5+15b^3d^4+d^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4))+3d^2(30a^2d^4b+b^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4))+d^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4)))*x^{16}+\frac{1}{15}(105b^3ad^4+3d^4(a^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4)+60b^3ad^3+d^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2)))*x^{15}+\frac{1}{14}(b^2(30a^2d^4b+b^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4))+d^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4))+3d^2(60a^2b^2d^3+b^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4))+d^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2)))*x^{14}+\frac{1}{13}(b^2(a^2(2(3a^2d+b^3)d^3+18b^3d^3+9a^2d^4)+60b^3ad^3+d^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2))+3d^2(a^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4))+b^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2)+d^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab)))*x^{13}+\frac{1}{12}(b^2(60a^2b^2d^3+b^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4))+d^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2))+3d^2(a^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2)+b^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2))+d^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2)))*x^{12}+\frac{1}{11}(b^2(a^2(42b^2a^2d^3+6(3a^2d+b^3)b^2d^2+9d^2b^4))+b^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2)+d^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab))+3d^2(a^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2))+b^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab))+d^2(42a^3d^2b^2+6b^2a^2(3a^2d+b^3))))*x^{11}+\frac{1}{10}(b^2(a^2(2a^3d^3+54b^3ad^2+6(3a^2d+b^3)ad^2))+b^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2))+d^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2))+3d^2(a^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab))+b^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2))+d^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2)))*x^{10}+\frac{1}{9}(b^2(a^2(72b^2a^2d^2+6(3a^2d+b^3)d^2b^2))+b^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab))+d^2(42a^3d^2b^2+6b^2a^2(3a^2d+b^3))) + 3d^2(a^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2))+b^2(42a^3d^2b^2+6b^2a^2(3a^2d+b^3))+d^2(2a^3(3a^2d+b^3)+18b^3a^3)))*x^9+\frac{1}{8}(b^2(a^2(24a^3b^2d^2+18b^4ad+12(3a^2d+b^3)d^2ab))+b^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2))+d^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+3d^2(a^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+b^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+15d^2b^2a^4)*x^8+\frac{1}{7}(b^2(a^2(6a^4d^2+54b^3a^2d+(3a^2d+b^3)^2))+b^2(42a^3d^2b^2+6b^2a^2(3a^2d+b^3))+d^2(2a^3(3a^2d+b^3)+18b^3a^3))+3d^2(a^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+b^2(2a^3(3a^2d+b^3)+18b^3a^3))+6d^2a^5b)*x^7+\frac{1}{6}(b^2(a^2(42a^3d^2b^2+6b^2a^2(3a^2d+b^3))+b^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+15d^2b^2a^4)+3d^2(a^2(2a^3(3a^2d+b^3)+18b^3a^3)+15b^3a^4+d^2a^6))*x^6+\frac{1}{5}(b^2(a^2(12a^4d^2b+6b^2a^2(3a^2d+b^3)+9b^4a^2))+b^2(2a^3(3a^2d+b^3)+18b^3a^3))+6d^2a^5b)+63d^2b^2a^5)*x^5+\frac{1}{4}(b^2(a^2(2a^3(3a^2d+b^3)+18b^3a^3)+15b^3a^4+d^2a^6))+21d^2a^6b)*x^4+\frac{1}{3}(3a^7d+21a^5b^3)*x^3+\frac{7}{2}b^2a^6x^2+ba^7x$

maxima [A] time = 0.55, size = 14, normalized size = 0.88

$$\frac{1}{8} (dx^3 + bx + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + b*x + a)^8

mupad [B] time = 2.63, size = 438, normalized size = 27.38

int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x) = 1/8*(d*x^3+b*x+a)^8

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)

[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)

sympy [B] time = 0.17, size = 483, normalized size = 30.19

integrate((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x) = 1/8*(d*x^3+b*x+a)^8

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)

[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**15*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2) + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d**3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*

$$(7a^{6}d^{2}/2 + 35a^{4}b^{3}d + 7a^{2}b^{6}/2) + x^{5}(21a^{5}b^{2}d + 7a^{3}b^{5}) + x^{4}(7a^{6}bd + 35a^{4}b^{4}/4) + x^{3}(a^{7}d + 7a^{5}b^{3})$$

$$3.192 \quad \int (b + 3dx^2)(bx + dx^3)^7 dx$$

Optimal. Leaf size=15

$$\frac{1}{8}(bx + dx^3)^8$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1588}

$$\frac{1}{8}(bx + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b*x + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (b + 3dx^2)(bx + dx^3)^7 dx = \frac{1}{8}(bx + dx^3)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.53

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7}{2}b^6d^2x^{12} + 7b^5d^3x^{14} + \frac{35}{4}b^4d^4x^{16} + 7b^3d^5x^{18} + \frac{7}{2}b^2d^6x^{20} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b + 3dx^2)(bx + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^7, x]

fricas [B] time = 0.71, size = 88, normalized size = 5.87

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{8}(dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + b*x)^8

maple [B] time = 0.00, size = 89, normalized size = 5.93

$$\frac{1}{8}d^8x^{24} + b d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)

[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$\frac{1}{8}(dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + b*x)^8

mupad [B] time = 0.05, size = 88, normalized size = 5.87

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7 b^6 d^2 x^{12}}{2} + 7 b^5 d^3 x^{14} + \frac{35 b^4 d^4 x^{16}}{4} + 7 b^3 d^5 x^{18} + \frac{7 b^2 d^6 x^{20}}{2} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)

[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2

sympy [B] time = 0.10, size = 97, normalized size = 6.47

$$\frac{b^8 x^8}{8} + b^7 d x^{10} + \frac{7 b^6 d^2 x^{12}}{2} + 7 b^5 d^3 x^{14} + \frac{35 b^4 d^4 x^{16}}{4} + 7 b^3 d^5 x^{18} + \frac{7 b^2 d^6 x^{20}}{2} + b d^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)

[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8

$$3.193 \quad \int x^7 (b + dx^2)^7 (b + 3dx^2) dx$$

Optimal. Leaf size=16

$$\frac{1}{8}x^8 (b + dx^2)^8$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$\frac{1}{8}x^8 (b + dx^2)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2), x]

[Out] (x^8*(b + d*x^2)^8)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^7 (b + dx^2)^7 (b + 3dx^2) dx &= \frac{1}{2} \text{Subst} \left(\int x^3 (b + dx)^7 (b + 3dx) dx, x, x^2 \right) \\ &= \frac{1}{8} x^8 (b + dx^2)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.12

$$\frac{b^8 x^8}{8} + b^7 dx^{10} + \frac{7}{2} b^6 d^2 x^{12} + 7b^5 d^3 x^{14} + \frac{35}{4} b^4 d^4 x^{16} + 7b^3 d^5 x^{18} + \frac{7}{2} b^2 d^6 x^{20} + bd^7 x^{22} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2), x]

[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 (b + dx^2)^7 (b + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7*(b + d*x^2)^7*(b + 3*d*x^2), x]

[Out] IntegrateAlgebraic[x^7*(b + d*x^2)^7*(b + 3*d*x^2), x]

fricas [B] time = 0.80, size = 88, normalized size = 5.50

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b), x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8

giac [A] time = 0.27, size = 13, normalized size = 0.81

$$\frac{1}{8} (dx^3 + bx)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b), x, algorithm="giac")

[Out] 1/8*(d*x^3 + b*x)^8

maple [B] time = 0.00, size = 89, normalized size = 5.56

$$\frac{1}{8}d^8x^{24} + b d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b), x)

[Out] $\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$

maxima [B] time = 0.64, size = 88, normalized size = 5.50

$$\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")`

[Out] $\frac{1}{8}d^8x^{24} + bd^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}b^8x^8$

mupad [B] time = 0.04, size = 88, normalized size = 5.50

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)`

[Out] $\frac{b^8x^8}{8} + \frac{d^8x^{24}}{8} + b^7dx^{10} + bd^7x^{22} + \frac{(7b^6d^2x^{12})}{2} + 7b^5d^3x^{14} + \frac{(35b^4d^4x^{16})}{4} + 7b^3d^5x^{18} + \frac{(7b^2d^6x^{20})}{2}$

sympy [B] time = 0.09, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)`

[Out] $b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8$

$$3.194 \quad \int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=18

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1588}

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2) (a + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.05, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c + dx)(8a^7 + 28a^6x^2(c + dx) + 56a^5x^4(c + dx)^2 + 70a^4x^6(c + dx)^3 + 56a^3x^8(c + dx)^4 + 28a^2x^{10}(c + dx)^5 + 8ax^{12}(c + dx)^6 + x^{14}(c + dx)^7)$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx + 3dx^2)(a + cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7, x]

fricas [B] time = 0.64, size = 488, normalized size = 27.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + x^{21}d^7a + \frac{35}{4}x^{20}d^4c^4 + 7x^{20}d^6c^3a + 7x^{19}d^3c^5 + 21x^{19}d^5c^2a + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d^4c^3a + \frac{7}{2}x^{18}d^6a^2 + x^{17}d^7c^7 + 35x^{17}d^3c^4a + 21x^{17}d^5c^3a^2 + \frac{1}{8}x^{16}c^8 + 21x^{16}d^2c^5a + \frac{105}{2}x^{16}d^4c^2a^2 + 7x^{15}d^6c^3a + 70x^{15}d^3c^3a^2 + 7x^{15}d^5a^3 + x^{14}c^7a + \frac{105}{2}x^{14}d^2c^4a^2 + 35x^{14}d^4c^3a^3 + 21x^{13}d^7c^5a^2 + 70x^{13}d^3c^2a^3 + \frac{7}{2}x^{12}c^6a^2 + 70x^{12}d^2c^3a^3 + \frac{35}{4}x^{12}d^4a^4 + 35x^{11}d^6c^4a^3 + 35x^{11}d^3c^3a^4 + 7x^{10}c^5a^3 + \frac{105}{2}x^{10}d^2c^2a^4 + 35x^9d^6c^3a^4 + 7x^9d^3a^5 + \frac{35}{4}x^8c^4a^4 + 21x^8d^2c^3a^5 + 21x^7d^7c^2a^5 + 7x^6c^3a^5 + \frac{7}{2}x^6d^2a^6 + 7x^5d^6c^3a^6 + \frac{7}{2}x^4c^2a^6 + x^3d^7a^7 + x^2c^7a^7$

giac [B] time = 0.43, size = 136, normalized size = 7.56

$$\frac{1}{8}(dx^3 + cx^2)^8 + (dx^3 + cx^2)^7a + \frac{7}{2}(dx^3 + cx^2)^6a^2 + 7(dx^3 + cx^2)^5a^3 + \frac{35}{4}(dx^3 + cx^2)^4a^4 + 7(dx^3 + cx^2)^3a^5 + \frac{7}{2}(dx^3 + cx^2)^2a^6 + (dx^3 + cx^2)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")

[Out] $\frac{1}{8}(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + \frac{7}{2}(d*x^3 + c*x^2)^6*a^2 + 7*(d*x^3 + c*x^2)^5*a^3 + \frac{35}{4}(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a^5 + \frac{7}{2}(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7$

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)$

[Out] $\frac{1}{8}d^8x^{24}+c^7d^7x^{23}+\frac{7}{2}c^2d^6x^{22}+\frac{1}{21}(42c^3d^5+3d(a^6d+15c^3d^4+d(2(3ad^2+c^3)d^3+18c^3d^3)))*x^{21}+\frac{1}{20}(2c(a^6d+15c^3d^4+d(2(3ad^2+c^3)d^3+18c^3d^3))+3d(6acd^5+c(2(3ad^2+c^3)d^3+18c^3d^3))+d(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2))*x^{20}+\frac{1}{19}(2c(6acd^5+c(2(3ad^2+c^3)d^3+18c^3d^3))+d(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2))+3d(15a^2d^4+c(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2))+d(42a^2d^3+6(3ad^2+c^3)c^2d))*x^{19}+\frac{1}{18}(2c(15a^2d^4+c(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2))+d(42a^2d^3+6(3ad^2+c^3)c^2d))+3d(a(2(3ad^2+c^3)d^3+18c^3d^3)+c(42a^2d^3+6(3ad^2+c^3)c^2d))+d(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2))*x^{18}+\frac{1}{17}(2c(a(2(3ad^2+c^3)d^3+18c^3d^3)+c(42a^2d^3+6(3ad^2+c^3)c^2d))+d(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2))+3d(a(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2)+c(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2))+d(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))*x^{17}+\frac{1}{16}(2c(a(12acd^4+6(3ad^2+c^3)cd^2+9c^4d^2)+c(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2))+d(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))+3d(a(42a^2d^3+6(3ad^2+c^3)c^2d)+c(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))+d(72a^2c^2d^2+6ac^2(3ad^2+c^3)))*x^{16}+\frac{1}{15}(2c(a(42a^2d^3+6(3ad^2+c^3)c^2d)+c(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))+d(72a^2c^2d^2+6ac^2(3ad^2+c^3)))+3d(a(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2)+c(72a^2c^2d^2+6ac^2(3ad^2+c^3)))+d(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3)))*x^{15}+\frac{1}{14}(2c(a(6a^2d^4+54acd^3d^2+(3ad^2+c^3)^2)+c(72a^2c^2d^2+6ac^2(3ad^2+c^3)))+d(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3)))+3d(a(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))+c(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3)))+d(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))*x^{14}+\frac{1}{13}(2c(a(24c^2a^2d^3+18acd^4d+12acd(3ad^2+c^3)))+c(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3)))+d(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))+3d(a(72a^2c^2d^2+6ac^2(3ad^2+c^3))+c(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))+60d^2a^3c^2)*x^{13}+\frac{1}{12}(2c(a(72a^2c^2d^2+6ac^2(3ad^2+c^3))+c(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))+60d^2a^3c^2))+3d(a(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3))+60c^3a^3d+d(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)))*x^{12}+\frac{1}{11}(2c(a(2a^3d^3+54c^3a^2d+6d^2a^2(3ad^2+c^3))+60c^3a^3d+d(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2))+3d(a(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))+c(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2))+30d^2a^4c)*x^{11}+\frac{1}{10}(2c(a(42a^3cd^2+6c^2a^2(3ad^2+c^3)+9a^2c^4))+c(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2))+30d^2a^4c)+315d^2a^4c^2)*x^{10}+\frac{1}{9}(210c^3a^4d+3d(a(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+15c^3a^4+6d^2a^5))*x^9+\frac{1}{8}(2c(a(2a^3(3ad^2+c^3)+18c^3a^3+9a^4d^2)+15c^3a^4+6d^2a^5))+126d^2a^5c)*x^8+21c^2a^5d*x^7+\frac{1}{6}(21a^6d^2+42a^5c^3)*x^6+7c^2a^6d*x^5+\frac{7}{2}c^2a^6*x^4+d^7a^7*x^3+c^7a^7*x^2$

maxima [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{1}{8} (dx^3 + cx^2 + a)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2 + a)^8

mupad [B] time = 2.63, size = 440, normalized size = 24.44

int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7, x) = 1/8*(d*x^3+c*x^2+a)^8

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)

[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3) + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)

sympy [B] time = 0.17, size = 484, normalized size = 26.89

int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7, x) = 1/8*(d*x^3+c*x^2+a)^8

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)

[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c**2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d**3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) + x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a**3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x

$$\begin{aligned} & **10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c** \\ & 3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a** \\ & 5*c**3) \end{aligned}$$

$$3.195 \quad \int (2cx + 3dx^2)(cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=17

$$\frac{1}{8}(cx^2 + dx^3)^8$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1588}

$$\frac{1}{8}(cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]

[Out] (c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :=> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2cx + 3dx^2)(cx^2 + dx^3)^7 dx = \frac{1}{8}(cx^2 + dx^3)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx + 3dx^2)(cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7, x]

fricas [B] time = 0.95, size = 88, normalized size = 5.18

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [A] time = 0.37, size = 15, normalized size = 0.88

$$\frac{1}{8}(dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2)^8

maple [B] time = 0.00, size = 89, normalized size = 5.24

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)

[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$\frac{1}{8}(dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")

[Out] 1/8*(d*x^3 + c*x^2)^8

mupad [B] time = 2.07, size = 88, normalized size = 5.18

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)

[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2

sympy [B] time = 0.10, size = 97, normalized size = 5.71

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

$$3.196 \quad \int x^7 (cx + dx^2)^7 (2cx + 3dx^2) dx$$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

Rubi [A] time = 0.23, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1584, 845}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]

[Out] (x^16*(c + d*x)^8)/8

Rule 845

Int[(x_)^(m_)*((f_) + (g_)*(x_)^(n_))*((b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n + 3, 0]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^7 (cx + dx^2)^7 (2cx + 3dx^2) dx &= \int x^{14} (c + dx)^7 (2cx + 3dx^2) dx \\ &= \frac{1}{8}x^{16}(c + dx)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 (cx + dx^2)^7 (2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]

[Out] IntegrateAlgebraic[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]

fricas [B] time = 0.55, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x), x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [A] time = 0.23, size = 15, normalized size = 1.07

$$\frac{1}{8}(dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x), x, algorithm="giac")

[Out] 1/8*(d*x^3 + c*x^2)^8

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)`

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

maxima [B] time = 0.52, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")`

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

mupad [B] time = 0.05, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)`

[Out] $\frac{(c^8x^{16})}{8} + \frac{(d^8x^{24})}{8} + c^7d^7x^{17} + cd^7x^{23} + \frac{(7c^6d^2x^{18})}{2} + 7c^5d^3x^{19} + \frac{(35c^4d^4x^{20})}{4} + 7c^3d^5x^{21} + \frac{(7c^2d^6x^{22})}{2}$

sympy [B] time = 0.10, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)`

[Out] $c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8$

$$3.197 \quad \int x^{14}(c + dx)^7 (2cx + 3dx^2) dx$$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {845}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2), x]

[Out] (x^16*(c + d*x)^8)/8

Rule 845

Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n + 3, 0]

Rubi steps

$$\int x^{14}(c + dx)^7 (2cx + 3dx^2) dx = \frac{1}{8}x^{16}(c + dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2), x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}(c + dx)^7(2cx + 3dx^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹⁴*(c + d*x)⁷*(2*c*x + 3*d*x²), x]

[Out] IntegrateAlgebraic[x¹⁴*(c + d*x)⁷*(2*c*x + 3*d*x²), x]

fricas [B] time = 0.73, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x), x, algorithm="fricas")

[Out] 1/8*x²⁴*d⁸ + x²³*d⁷*c + 7/2*x²²*d⁶*c² + 7*x²¹*d⁵*c³ + 35/4*x²⁰*d⁴*c⁴ + 7*x¹⁹*d³*c⁵ + 7/2*x¹⁸*d²*c⁶ + x¹⁷*d*c⁷ + 1/8*x¹⁶*c⁸

giac [A] time = 0.26, size = 15, normalized size = 1.07

$$\frac{1}{8}(dx^3 + cx^2)^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x), x, algorithm="giac")

[Out] 1/8*(d*x³ + c*x²)⁸

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁴*(d*x+c)⁷*(3*d*x²+2*c*x), x)

[Out] 1/8*d⁸*x²⁴+c*d⁷*x²³+7/2*c²*d⁶*x²²+7*c³*d⁵*x²¹+35/4*c⁴*d⁴*x²⁰+7*c⁵*d³*x¹⁹+7/2*c⁶*d²*x¹⁸+c⁷*d*x¹⁷+1/8*c⁸*x¹⁶

maxima [B] time = 0.66, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")`

[Out] $1/8*d^8*x^{24} + c*d^7*x^{23} + 7/2*c^2*d^6*x^{22} + 7*c^3*d^5*x^{21} + 35/4*c^4*d^4*x^{20} + 7*c^5*d^3*x^{19} + 7/2*c^6*d^2*x^{18} + c^7*d*x^{17} + 1/8*c^8*x^{16}$

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)`

[Out] $(c^8*x^{16})/8 + (d^8*x^{24})/8 + c^7*d*x^{17} + c*d^7*x^{23} + (7*c^6*d^2*x^{18})/2 + 7*c^5*d^3*x^{19} + (35*c^4*d^4*x^{20})/4 + 7*c^3*d^5*x^{21} + (7*c^2*d^6*x^{22})/2$

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)`

[Out] $c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8$

$$3.198 \quad \int x(2c + 3dx) (a + cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=18

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1588}

$$\frac{1}{8} (a + cx^2 + dx^3)^8$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (a + c*x^2 + d*x^3)^8/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x(2c + 3dx) (a + cx^2 + dx^3)^7 dx = \frac{1}{8} (a + cx^2 + dx^3)^8$$

Mathematica [B] time = 0.01, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c + dx)(8a^7 + 28a^6x^2(c + dx) + 56a^5x^4(c + dx)^2 + 70a^4x^6(c + dx)^3 + 56a^3x^8(c + dx)^4 + 28a^2x^{10}(c + dx)^5 + 8ax^{12}(c + dx)^6 + x^{14}(c + dx)^7)$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(2c + 3dx)(a + cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7, x]

fricas [B] time = 0.63, size = 488, normalized size = 27.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")

[Out] $\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + x^{21}d^7ca + \frac{35}{4}x^{20}d^4c^4 + 7x^{20}d^6c^2a + 7x^{19}d^3c^5 + 21x^{19}d^5c^2a + \frac{7}{2}x^{18}d^2c^6 + 35x^{18}d^4c^3a + \frac{7}{2}x^{18}d^6a^2 + x^{17}d^7c^2 + 35x^{17}d^3c^4a + 21x^{17}d^5c^2a^2 + \frac{1}{8}x^{16}c^8 + 21x^{16}d^2c^5a + \frac{105}{2}x^{16}d^4c^2a^2 + 7x^{15}d^6c^3a + 70x^{15}d^3c^3a^2 + 7x^{15}d^5a^3 + x^{14}c^7a + \frac{105}{2}x^{14}d^2c^4a^2 + 35x^{14}d^4c^2a^3 + 21x^{13}d^6c^5a^2 + 70x^{13}d^3c^2a^3 + \frac{7}{2}x^{12}c^6a^2 + 70x^{12}d^2c^3a^3 + \frac{35}{4}x^{12}d^4a^4 + 35x^{11}d^6c^4a^3 + 35x^{11}d^3c^2a^4 + 7x^{10}c^5a^3 + \frac{105}{2}x^{10}d^2c^2a^4 + 35x^9d^6c^3a^4 + 7x^9d^3a^5 + \frac{35}{4}x^8c^4a^4 + 21x^8d^2c^2a^5 + 21x^7d^6c^2a^5 + 7x^6c^3a^5 + \frac{7}{2}x^6d^2a^6 + 7x^5d^4c^2a^6 + \frac{7}{2}x^4c^2a^6 + x^3d^7a^7 + x^2c^7a^7$

giac [B] time = 0.32, size = 488, normalized size = 27.11

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + ad^7x^{21} + \frac{35}{4}c^4d^4x^{20} + 7ac^3d^6x^{20} + 7c^5d^3x^{19} + 21ac^2d^5x^{19} + \frac{7}{2}c^6d^2x^{18} + 35ac^3d^4x^{18} + \frac{7}{2}a^2d^6x^{18} + c^7d^7x^{17} + 35ac^4d^3x^{17} + 21a^2c^5d^5x^{17} + \frac{1}{8}c^8x^{16} + 21ac^5d^2x^{16} + \frac{105}{2}a^2c^2d^4x^{16} + 7ac^6d^6x^{15} + 70a^2c^3d^3x^{15} + 7a^3d^5x^{15} + ac^7x^{14} + \frac{105}{2}a^2c^4d^2x^{14} + 35a^3c^3d^4x^{14} + 21a^2c^5d^6x^{13} + 70a^3c^2d^3x^{13} + \frac{7}{2}a^2c^6x^{12} + 70a^3c^3d^2x^{12} + \frac{35}{4}a^4d^4x^{12} + 35a^3c^4d^6x^{11} + 35a^4c^3d^3x^{11} + 7a^3c^5x^{10} + \frac{105}{2}a^4d^2x^{10} + \frac{35}{4}a^5c^2x^9 + 35a^4d^4x^9 + 21a^5c^3x^8 + \frac{7}{2}a^4d^6x^8 + 7a^5c^4x^7 + \frac{7}{2}a^4d^3x^7 + 7a^5c^5x^6 + \frac{7}{2}a^4d^2x^6 + 7a^5c^6x^5 + \frac{7}{2}a^4d^7x^5 + 7a^5c^7x^4 + \frac{7}{2}a^4d^4x^4 + 7a^5c^8x^3 + \frac{7}{2}a^4d^5x^3 + 7a^5c^9x^2 + \frac{7}{2}a^4d^6x^2 + 7a^5c^{10}x + \frac{7}{2}a^4d^7x + 7a^5c^{11}$

$$2*a^4*c^2*d^2*x^{10} + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 + 21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2$$

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + \frac{1}{21}(42c^3d^5 + 3(a^6d + 15c^3d^4 + 18c^3d^3 + 2(3ad^2 + c^3)d^3)d)x^{21} + \frac{1}{20}(2(a^6d + 15c^3d^4 + 18c^3d^3 + 2(3ad^2 + c^3)d^3)d)^2 + 3(6a^5cd^5 + (18c^3d^3 + 2(3ad^2 + c^3)d^3)d)x^{20} + \frac{1}{19}(2(6a^5cd^5 + (18c^3d^3 + 2(3ad^2 + c^3)d^3)d)^2 + 3(15a^5c^2d^4 + (12a^5cd^4 + 9c^4d^2 + 6(3ad^2 + c^3)cd^2)d)x^{19} + \frac{1}{18}(2(15a^5c^2d^4 + (12a^5cd^4 + 9c^4d^2 + 6(3ad^2 + c^3)cd^2)d)^2 + 3(42a^5c^2d^3 + 6(3ad^2 + c^3)c^2d)d)x^{18} + \frac{1}{17}(2((18c^3d^3 + 2(3ad^2 + c^3)d^3)d)^2 + 3(42a^5c^2d^3 + 6(3ad^2 + c^3)c^2d)d)^2 + 3(6a^5c^2d^4 + 54a^5c^3d^2 + (3ad^2 + c^3)^2)d)x^{17} + \frac{1}{16}(2((12a^5cd^4 + 9c^4d^2 + 6(3ad^2 + c^3)cd^2)d)^2 + 3(24a^5c^2d^3 + 18a^5c^4d + 12(3ad^2 + c^3)a^5cd)d)^2 + 3((42a^5c^2d^3 + 6(3ad^2 + c^3)c^2d)d)^2 + 3(72a^5c^2d^2 + 6(3ad^2 + c^3)a^5c^2)d)x^{16} + 21a^5c^2d^7 + \frac{1}{15}(2((42a^5c^2d^3 + 6(3ad^2 + c^3)c^2d)d)^2 + 3(24a^5c^2d^3 + 18a^5c^4d + 12(3ad^2 + c^3)a^5cd)d)^2 + 3(72a^5c^2d^2 + 6(3ad^2 + c^3)a^5c^2)d)x^{15} + \frac{1}{14}(2((6a^5c^3d^2 + (3ad^2 + c^3)^2)d)^2 + 3(72a^5c^2d^2 + 6(3ad^2 + c^3)a^5c^2)d)^2 + 3(2a^5c^3d^3 + 54a^5c^3d + 6(3ad^2 + c^3)a^5c^2d)d)x^{14} + 7a^6c^2d^7 + \frac{1}{13}(2((24a^5c^2d^3 + 18a^5c^4d + 12(3ad^2 + c^3)a^5cd)d)^2 + 3(2a^5c^3d^3 + 54a^5c^3d + 6(3ad^2 + c^3)a^5c^2d)d)^2 + 3(42a^5c^3cd^2 + 9a^5c^4 + 6(3ad^2 + c^3)a^5c^2c)d)x^{13} + \frac{7}{2}a^6c^2d^7 + \frac{1}{12}(2((60a^5c^3cd^2 + (72a^5c^2d^2 + 6(3ad^2 + c^3)a^5c^2)d)^2 + 3(60a^5c^3cd^2 + 9a^5c^4 + 6(3ad^2 + c^3)a^5c^2c)d)^2 + 3(60a^5c^3cd + (2a^5c^3d^3 + 54a^5c^3d + 6(3ad^2 + c^3)a^5c^2d)d)^2 + 3(9a^5c^4d^2 + 18a^5c^3cd + 2(3ad^2 + c^3)a^5c^3)d)d)x^{12} + a^7d^7x^3 + \frac{1}{11}(2(60a^5c^3cd + (2a^5c^3d^3 + 54a^5c^3d + 6(3ad^2 + c^3)a^5c^2d)d)^2 + 3(9a^5c^4d^2 + 18a^5c^3cd + 2(3ad^2 + c^3)a^5c^3)d)d)x^{11} + \dots$

$$a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*c+3*(30*a^4*c*d^2+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*c)*d)*x^{11}+a^7*c*x^2+1/10*(315*a^4*c^2*d^2+2*(30*a^4*c*d^2+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*c)*c)*x^{10}+1/9*(210*a^4*c^3*d+3*(6*a^5*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*d)*x^9+1/8*(126*a^5*c*d^2+2*(6*a^5*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*c)*x^8+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6$$

maxima [B] time = 0.59, size = 458, normalized size = 25.44

$\frac{1}{10} (315 a^4 c^2 d^2 + 2 (30 a^4 c d^2 + (42 a^3 c d^2 + 9 a^2 c^4 + 6 (3 a d^2 + c^3) a^2 c) a + (9 a^4 d^2 + 18 a^3 c^3 + 2 (3 a d^2 + c^3) a^3) c) d) x^{10} + \frac{1}{9} (210 a^4 c^3 d + 3 (6 a^5 d^2 + 15 a^4 c^3 + (9 a^4 d^2 + 18 a^3 c^3 + 2 (3 a d^2 + c^3) a^3) a) d) x^9 + \frac{1}{8} (126 a^5 c d^2 + 2 (6 a^5 d^2 + 15 a^4 c^3 + (9 a^4 d^2 + 18 a^3 c^3 + 2 (3 a d^2 + c^3) a^3) a) c) x^8 + \frac{1}{6} (21 a^6 d^2 + 42 a^5 c^3) x^6$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + (7c^3d^5 + ad^7)x^{21} + \frac{7}{4}(5c^4d^4 + 4aac^3d^6)x^{20} + 7(c^5d^3 + 3aac^2d^5)x^{19} + \frac{7}{2}(c^6d^2 + 10aac^3d^4 + a^2d^6)x^{18} + (c^7d + 35aac^4d^3 + 21a^2c^2d^5)x^{17} + \frac{1}{8}(c^8 + 168aac^5d^2 + 420a^2c^2d^4)x^{16} + 7(aac^6d + 10a^2c^3d^3 + a^3d^5)x^{15} + 21a^5c^2d^7x^7 + \frac{1}{2}(2aac^7 + 105a^2c^4d^2 + 70a^3c^3d^4)x^{14} + 7(3a^2c^5d + 10a^3c^2d^3)x^{13} + 7a^6cd^5x^5 + \frac{7}{4}(2a^2c^6 + 40a^3c^3d^2 + 5a^4d^4)x^{12} + \frac{7}{2}a^6c^2x^4 + 35(a^3c^4d + a^4c^3d^3)x^{11} + a^7d^3x^3 + \frac{7}{2}(2a^3c^5 + 15a^4c^2d^2)x^{10} + a^7c^2x^2 + 7(5a^4c^3d + a^5d^3)x^9 + \frac{7}{4}(5a^4c^4 + 12a^5cd^2)x^8 + \frac{7}{2}(2a^5c^3 + a^6d^2)x^6$

mupad [B] time = 0.57, size = 440, normalized size = 24.44

$\frac{1}{8} (d^8 x^{24} + c d^7 x^{23} + \frac{7}{2} c^2 d^6 x^{22} + (7 c^3 d^5 + a d^7) x^{21} + \frac{7}{4} (5 c^4 d^4 + 4 a a c^3 d^6) x^{20} + 7 (c^5 d^3 + 3 a a c^2 d^5) x^{19} + \frac{7}{2} (c^6 d^2 + 10 a a c^3 d^4 + a^2 d^6) x^{18} + (c^7 d + 35 a a c^4 d^3 + 21 a^2 c^2 d^5) x^{17} + \frac{1}{8} (c^8 + 168 a a c^5 d^2 + 420 a^2 c^2 d^4) x^{16} + 7 (a a c^6 d + 10 a^2 c^3 d^3 + a^3 d^5) x^{15} + 21 a^5 c^2 d^7 x^7 + \frac{1}{2} (2 a a c^7 + 105 a^2 c^4 d^2 + 70 a^3 c^3 d^4) x^{14} + 7 (3 a^2 c^5 d + 10 a^3 c^2 d^3) x^{13} + 7 a^6 c d^5 x^5 + \frac{7}{4} (2 a^2 c^6 + 40 a^3 c^3 d^2 + 5 a^4 d^4) x^{12} + \frac{7}{2} a^6 c^2 x^4 + 35 (a^3 c^4 d + a^4 c^3 d^3) x^{11} + a^7 d^3 x^3 + \frac{7}{2} (2 a^3 c^5 + 15 a^4 c^2 d^2) x^{10} + a^7 c^2 x^2 + 7 (5 a^4 c^3 d + a^5 d^3) x^9 + \frac{7}{4} (5 a^4 c^4 + 12 a^5 c d^2) x^8 + \frac{7}{2} (2 a^5 c^3 + a^6 d^2) x^6$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)

[Out] $x^{12}((7a^2c^6)/2 + (35a^4d^4)/4 + 70a^3c^3d^2) + x^6(7a^5c^3 + (7a^6d^2)/2) + x^{20}((35c^4d^4)/4 + 7aac^3d^6) + x^{16}(c^8/8 + 21aac^5d^2 + (105a^2c^2d^4)/2) + x^{18}((7a^2d^6)/2 + (7c^6d^2)/2 + 35aac^3d^4) + (d^8x^{24})/8 + x^{21}(ad^7 + 7c^3d^5) + a^7c^2x^2 + a^7d^3x^3 + cd^7x^{23} + (7a^6c^2x^4)/2 + (7c^2d^6x^{22})/2 + 21a^5c^2d^7x^7 + 7aad^7x^{15}(c^6 + a^2d^4 + 10aac^3d^2) + cd^7x^{17}(c^6 + 21a^2d^4 + 35aac^3d^2) + (7a^4c^2x^8(12ad^2 + 5c^3))/4 + 7a^4d^7x^9(ad^2 + 5c^3) + 7c^2d^3x^{19}(3ad^2 + c^3) + (aac^7x^{14}(2c^6 + 70a^2d^4 + 105aac^3d^2))/2 + 7a^6cd^5x^5 + (7a^3c^2x^{10}(15ad^2 + 2c^3))/2 + 7a^2c^2d^3x^{13}(10ad^2 + 3c^3) + 35a^3cd^5x^{11}(ad^2 + c^3)$

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$\frac{d^{21}}{dx^{21}} \left(\frac{a^7 c x^2 + a^7 d x^3 + 7 a^6 c^2 x^4}{2} + 7 a^6 c d x^5 + 21 a^5 c^2 d x^7 + 7 c^2 d^2 x^6 + \frac{7 c^2 d^2 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8} + x^{20} (7 a c d^6 + 35 c^4 d^4) + x^{19} (21 a c^2 d^5 + 7 c^5 d^3) + x^{18} (7 a^2 d^6 + 35 a c^3 d^4 + 7 c^6 d^2) + x^{17} (21 a^2 c d^5 + 35 a c^4 d^3 + c^7 d) + x^{16} (105 a^2 c^2 d^4 + 21 a c^5 d^2 + c^8) + x^{15} (7 a^3 d^5 + 70 a^2 c^3 d^3 + 7 a c^6 d) + x^{14} (35 a^3 c d^4 + 105 a^2 c^4 d^2 + a c^7) + x^{13} (70 a^3 c^2 d^3 + 21 a^2 c^5 d) + x^{12} (35 a^4 d^4 + 70 a^3 c^3 d^2 + 7 a^2 c^6) + x^{11} (35 a^4 c d^3 + 35 a^3 c^4 d) + x^{10} (105 a^4 c^2 d^2 + 7 a^3 c^5) + x^9 (7 a^5 d^3 + 35 a^4 c^3 d) + x^8 (21 a^5 c d^2 + 35 a^4 c^4) + x^6 (7 a^6 d^2 + 7 a^5 c^3) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)

[Out] $a^7 c x^2 + a^7 d x^3 + 7 a^6 c^2 x^4 / 2 + 7 a^6 c d x^5 + 21 a^5 c^2 d x^7 + 7 c^2 d^2 x^6 + 7 c^2 d^2 x^{22} / 2 + c d^7 x^{23} + d^8 x^{24} / 8 + x^{20} (7 a c d^6 + 35 c^4 d^4) + x^{19} (21 a c^2 d^5 + 7 c^5 d^3) + x^{18} (7 a^2 d^6 + 35 a c^3 d^4 + 7 c^6 d^2 / 2) + x^{17} (21 a^2 c d^5 + 35 a c^4 d^3 + c^7 d) + x^{16} (105 a^2 c^2 d^4 / 2 + 21 a c^5 d^2 + c^8 / 8) + x^{15} (7 a^3 d^5 + 70 a^2 c^3 d^3 + 7 a c^6 d) + x^{14} (35 a^3 c d^4 + 105 a^2 c^4 d^2 / 2 + a c^7) + x^{13} (70 a^3 c^2 d^3 + 21 a^2 c^5 d) + x^{12} (35 a^4 d^4 + 70 a^3 c^3 d^2 + 7 a^2 c^6 / 2) + x^{11} (35 a^4 c d^3 + 35 a^3 c^4 d) + x^{10} (105 a^4 c^2 d^2 / 2 + 7 a^3 c^5) + x^9 (7 a^5 d^3 + 35 a^4 c^3 d) + x^8 (21 a^5 c d^2 + 35 a^4 c^4 / 4) + x^6 (7 a^6 d^2 / 2 + 7 a^5 c^3)$

$$3.199 \quad \int x(2c + 3dx) (cx^2 + dx^3)^7 dx$$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1584, 74}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] (x^16*(c + d*x)^8)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x(2c + 3dx) (cx^2 + dx^3)^7 dx &= \int x^{15}(c + dx)^7(2c + 3dx) dx \\ &= \frac{1}{8}x^{16}(c + dx)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 dx^{17} + \frac{7}{2}c^6 d^2 x^{18} + 7c^5 d^3 x^{19} + \frac{35}{4}c^4 d^4 x^{20} + 7c^3 d^5 x^{21} + \frac{7}{2}c^2 d^6 x^{22} + cd^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(2c + 3dx)(cx^2 + dx^3)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]

[Out] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7, x]

fricas [B] time = 1.04, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [B] time = 0.31, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)`

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

maxima [B] time = 0.60, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")`

[Out] $\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)`

[Out] $\frac{c^8x^{16}}{8} + \frac{d^8x^{24}}{8} + c^7d^7x^{23} + \frac{(7c^6d^2x^{18})}{2} + 7c^5d^3x^{19} + \frac{(35c^4d^4x^{20})}{4} + 7c^3d^5x^{21} + \frac{(7c^2d^6x^{22})}{2}$

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)`

[Out] $c^{**8}x^{**16}/8 + c^{**7}d^{**7}x^{**23} + 7c^{**6}d^{**2}x^{**18}/2 + 7c^{**5}d^{**3}x^{**19} + 35c^{**4}d^{**4}x^{**20}/4 + 7c^{**3}d^{**5}x^{**21} + 7c^{**2}d^{**6}x^{**22}/2 + c^{**7}d^{**7}x^{**23} + d^{**8}x^{**24}/8$

$$3.200 \quad \int x^8(2c + 3dx)(cx + dx^2)^7 dx$$

Optimal. Leaf size=18

$$\frac{1}{8}x^8(cx + dx^2)^8$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {763}

$$\frac{1}{8}x^8(cx + dx^2)^8$$

Antiderivative was successfully verified.

[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (x^8*(c*x + d*x^2)^8)/8

Rule 763

Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p + 2), 0] && NeQ[m + 2*p + 2, 0]

Rubi steps

$$\int x^8(2c + 3dx)(cx + dx^2)^7 dx = \frac{1}{8}x^8(cx + dx^2)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7}{2}c^6d^2x^{18} + 7c^5d^3x^{19} + \frac{35}{4}c^4d^4x^{20} + 7c^3d^5x^{21} + \frac{7}{2}c^2d^6x^{22} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]

[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(2c + 3dx)(cx + dx^2)^7 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7, x]

[Out] IntegrateAlgebraic[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7, x]

fricas [B] time = 1.01, size = 88, normalized size = 4.89

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")

[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8

giac [B] time = 0.29, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16

maple [B] time = 0.00, size = 89, normalized size = 4.94

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)

[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16

maxima [B] time = 0.57, size = 88, normalized size = 4.89

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16

mupad [B] time = 0.04, size = 88, normalized size = 4.89

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)

[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2

sympy [B] time = 0.10, size = 97, normalized size = 5.39

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

$$3.201 \quad \int x^{15}(c + dx)^7(2c + 3dx) dx$$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c + dx)^8$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {74}

$$\frac{1}{8}x^{16}(c + dx)^8$$

Antiderivative was successfully verified.

[In] Int[x¹⁵*(c + d*x)⁷*(2*c + 3*d*x), x]

[Out] (x¹⁶*(c + d*x)⁸)/8

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rubi steps

$$\int x^{15}(c + dx)^7(2c + 3dx) dx = \frac{1}{8}x^{16}(c + dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7}{2} c^6 d^2 x^{18} + 7 c^5 d^3 x^{19} + \frac{35}{4} c^4 d^4 x^{20} + 7 c^3 d^5 x^{21} + \frac{7}{2} c^2 d^6 x^{22} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x¹⁵*(c + d*x)⁷*(2*c + 3*d*x), x]

[Out] (c⁸*x¹⁶)/8 + c⁷*d*x¹⁷ + (7*c⁶*d²*x¹⁸)/2 + 7*c⁵*d³*x¹⁹ + (35*c⁴*d⁴*x²⁰)/4 + 7*c³*d⁵*x²¹ + (7*c²*d⁶*x²²)/2 + c*d⁷*x²³ + (d⁸*x²⁴)/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{15}(c + dx)^7(2c + 3dx) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x¹⁵*(c + d*x)⁷*(2*c + 3*d*x), x]

[Out] IntegrateAlgebraic[x¹⁵*(c + d*x)⁷*(2*c + 3*d*x), x]

fricas [B] time = 1.07, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵*(d*x+c)⁷*(3*d*x+2*c), x, algorithm="fricas")

[Out] 1/8*x²⁴*d⁸ + x²³*d⁷*c + 7/2*x²²*d⁶*c² + 7*x²¹*d⁵*c³ + 35/4*x²⁰*d⁴*c⁴ + 7*x¹⁹*d³*c⁵ + 7/2*x¹⁸*d²*c⁶ + x¹⁷*d*c⁷ + 1/8*x¹⁶*c⁸

giac [B] time = 0.24, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵*(d*x+c)⁷*(3*d*x+2*c), x, algorithm="giac")

[Out] 1/8*d⁸*x²⁴ + c*d⁷*x²³ + 7/2*c²*d⁶*x²² + 7*c³*d⁵*x²¹ + 35/4*c⁴*d⁴*x²⁰ + 7*c⁵*d³*x¹⁹ + 7/2*c⁶*d²*x¹⁸ + c⁷*d*x¹⁷ + 1/8*c⁸*x¹⁶

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁵*(d*x+c)⁷*(3*d*x+2*c), x)

[Out] 1/8*d⁸*x²⁴+c*d⁷*x²³+7/2*c²*d⁶*x²²+7*c³*d⁵*x²¹+35/4*c⁴*d⁴*x²⁰+7*c⁵*d³*x¹⁹+7/2*c⁶*d²*x¹⁸+c⁷*d*x¹⁷+1/8*c⁸*x¹⁶

maxima [B] time = 0.67, size = 88, normalized size = 6.29

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹⁵*(d*x+c)⁷*(3*d*x+2*c),x, algorithm="maxima")

[Out] 1/8*d⁸*x²⁴ + c*d⁷*x²³ + 7/2*c²*d⁶*x²² + 7*c³*d⁵*x²¹ + 35/4*c⁴*d⁴*x²⁰ + 7*c⁵*d³*x¹⁹ + 7/2*c⁶*d²*x¹⁸ + c⁷*d*x¹⁷ + 1/8*c⁸*x¹⁶

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x¹⁵*(2*c + 3*d*x)*(c + d*x)⁷,x)

[Out] (c⁸*x¹⁶)/8 + (d⁸*x²⁴)/8 + c⁷*d*x¹⁷ + c*d⁷*x²³ + (7*c⁶*d²*x¹⁸)/2 + 7*c⁵*d³*x¹⁹ + (35*c⁴*d⁴*x²⁰)/4 + 7*c³*d⁵*x²¹ + (7*c²*d⁶*x²²)/2

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8 x^{16}}{8} + c^7 d x^{17} + \frac{7 c^6 d^2 x^{18}}{2} + 7 c^5 d^3 x^{19} + \frac{35 c^4 d^4 x^{20}}{4} + 7 c^3 d^5 x^{21} + \frac{7 c^2 d^6 x^{22}}{2} + c d^7 x^{23} + \frac{d^8 x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)

[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8

$$3.202 \quad \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a + bx)^5 + ax + \frac{bx^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1591}

$$\frac{1}{160}x^5(2a + bx)^5 + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]

[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx &= \text{Subst} \left(\int (1 + x^4) dx, x, ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{1}{160}x^5(2a + bx)^5 \end{aligned}$$

Mathematica [B] time = 0.01, size = 80, normalized size = 2.86

$$\frac{a^5x^5}{5} + \frac{1}{2}a^4bx^6 + \frac{1}{2}a^3b^2x^7 + \frac{1}{4}a^2b^3x^8 + \frac{1}{16}ab^4x^9 + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]

[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]

[Out] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]

fricas [B] time = 1.52, size = 66, normalized size = 2.36

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7b^2a^3 + \frac{1}{2}x^6ba^4 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4), x, algorithm="fricas")

[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/4*x^8*b^3*a^2 + 1/2*x^7*b^2*a^3 + 1/2*x^6*b*a^4 + 1/5*x^5*a^5 + 1/2*x^2*b + x*a

giac [A] time = 0.37, size = 24, normalized size = 0.86

$$\frac{1}{160} (bx^2 + 2ax)^5 + \frac{1}{2} bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4), x, algorithm="giac")

[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x

maple [B] time = 0.00, size = 67, normalized size = 2.39

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4), x)

[Out] $\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$

maxima [B] time = 0.61, size = 66, normalized size = 2.36

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")`

[Out] $\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4bx^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}bx^2 + ax$

mupad [B] time = 0.05, size = 66, normalized size = 2.36

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)`

[Out] $ax + \frac{(bx^2)}{2} + \frac{(a^5x^5)}{5} + \frac{(b^5x^{10})}{160} + \frac{(a^4bx^6)}{2} + \frac{(ab^4x^9)}{16} + \frac{(a^3b^2x^7)}{2} + \frac{(a^2b^3x^8)}{4}$

sympy [B] time = 0.09, size = 70, normalized size = 2.50

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)`

[Out] $a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*x**9/16 + a*x + b**5*x**10/160 + b*x**2/2$

$$3.203 \quad \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Optimal. Leaf size=31

$$\frac{1}{5} \left(ax + \frac{bx^2}{2} + c \right)^5 + ax + \frac{bx^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1591}

$$\frac{1}{5} \left(ax + \frac{bx^2}{2} + c \right)^5 + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^4 \right) dx &= \text{Subst} \left(\int (1 + x^4) dx, x, c + ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{1}{5} \left(c + ax + \frac{bx^2}{2} \right)^5 \end{aligned}$$

Mathematica [B] time = 0.04, size = 108, normalized size = 3.48

$$\frac{1}{160} x(2a + bx) (16a^4x^4 + 32a^3bx^5 + 24a^2b^2x^6 + 8ab^3x^7 + 80c^3x(2a + bx) + 40c^2x^2(2a + bx)^2 + 10cx^3(2a + bx)^3 + b^4x^8 + 80c^4 + 80)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 + 8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 10*c*x^3*(2*a + b*x)^3))/160

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^4 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

[Out] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]

fricas [B] time = 0.95, size = 208, normalized size = 6.71

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{16}x^8cb^4 + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7cb^3a + \frac{1}{2}x^7b^2a^3 + \frac{1}{4}x^6c^2b^3 + \frac{3}{2}x^6cb^2a^2 + \frac{1}{2}x^6ba^4 + \frac{3}{2}x^5c^2b^2a + 2x^5cba^3 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^4c^3b^2 + 3x^4c^2ba^2 + x^4ca^4 + 2x^3c^3ba + 2x^3c^2a^3 + \frac{1}{2}x^2c^4b + 2x^2c^3a^2 + xc^4a + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4), x, algorithm="fricas")

[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/16*x^8*c*b^4 + 1/4*x^8*b^3*a^2 + 1/2*x^7*c*b^3*a + 1/2*x^7*b^2*a^3 + 1/4*x^6*c^2*b^3 + 3/2*x^6*c*b^2*a^2 + 1/2*x^6*b*a^4 + 3/2*x^5*c^2*b^2*a + 2*x^5*c*b*a^3 + 1/5*x^5*a^5 + 1/2*x^4*c^3*b^2 + 3*x^4*c^2*b*a^2 + x^4*c*a^4 + 2*x^3*c^3*b*a + 2*x^3*c^2*a^3 + 1/2*x^2*c^4*b + 2*x^2*c^3*a^2 + x*c^4*a + 1/2*x^2*b + x*a

giac [B] time = 0.37, size = 88, normalized size = 2.84

$$\frac{1}{160} (bx^2 + 2ax)^5 + \frac{1}{16} (bx^2 + 2ax)^4 c + \frac{1}{4} (bx^2 + 2ax)^3 c^2 + \frac{1}{2} (bx^2 + 2ax)^2 c^3 + \frac{1}{2} (bx^2 + 2ax) c^4 + \frac{1}{2} bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4), x, algorithm="giac")

[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x

maple [B] time = 0.00, size = 325, normalized size = 10.48

$$\frac{b^5x^{10}}{160} + \frac{b^4ax^9}{16} + \frac{cb^4x^8}{8} + \frac{a^2b^3x^8}{8} + \frac{cb^3ax^7}{7} + \frac{a^3b^2x^7}{7} + \frac{c^2b^3x^6}{6} + \frac{3cb^2a^2x^6}{6} + \frac{ba^4x^6}{6} + \frac{3c^2b^2ax^5}{5} + \frac{2cba^3x^5}{5} + \frac{a^5x^5}{5} + \frac{c^3b^2x^4}{4} + \frac{3c^2ba^2x^4}{4} + \frac{ca^4x^4}{4} + \frac{2c^3bax^3}{3} + \frac{2c^2a^3x^3}{3} + \frac{1}{2}c^4bx^2 + \frac{1}{2}c^4ax + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)`

[Out] $1/160*b^5*x^{10}+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b^2+a^2*b^2)) *x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+a^2*b^2)+b*(a*c*b^2+2*(a^2+b*c)*a*b)) *x^7+1/6*(a*(a*c*b^2+2*(a^2+b*c)*a*b)+b*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2)) *x^6+1/5*(a*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2)+b*(2*c^2*a*b+4*a*c*(a^2+b*c))) *x^5+1/4*(a*(2*c^2*a*b+4*a*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2)) *x^4+1/3*(a*(2*c^2*(a^2+b*c)+4*a^2*c^2)+4*a*b*c^3) *x^3+1/2*(4*a^2*c^3+b*(c^4+1)) *x^2+a*(c^4+1) *x$

maxima [B] time = 0.44, size = 187, normalized size = 6.03

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}ab^4x^9 + \frac{1}{16}(4a^2b^3 + b^4c)x^8 + \frac{1}{2}(a^3b^2 + ab^3c)x^7 + \frac{1}{4}(2a^4b + 6a^2b^2c + b^3c^2)x^6 + \frac{1}{10}(2a^5 + 20a^3bc + 15ab^2c^2)x^5 + \frac{1}{2}(2a^4c + 6a^2bc^2 + b^2c^3)x^4 + 2(a^3c^2 + abc^3)x^3 + \frac{1}{2}(4a^2c^3 + bc^4 + b)x^2 + (ac^4 + a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")`

[Out] $1/160*b^5*x^{10} + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 + a)*x$

mupad [B] time = 0.10, size = 180, normalized size = 5.81

$$x^6 \left(\frac{a^4 b}{2} + \frac{3a^2 b^2 c}{2} + \frac{b^3 c^2}{4} \right) + x^4 \left(a^4 c + 3a^2 b c^2 + \frac{b^2 c^3}{2} \right) + x^2 \left(2a^2 c^3 + \frac{b c^4}{2} + \frac{b}{2} \right) + x^5 \left(\frac{a^5}{5} + 2a^3 b c + \frac{3a b^2 c^2}{2} \right) + \frac{b^5 x^{10}}{160} + x^8 \left(\frac{a^2 b^3}{4} + \frac{c b^4}{16} \right) + \frac{a b^4 x^9}{16} + a x (c^4 + 1) + \frac{a b^2 x^7 (a^2 + b c)}{2} + 2a c^2 x^3 (a^2 + b c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)`

[Out] $x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2 + 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*c^2)/2 + 2*a^3*b*c) + (b^5*x^{10})/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) + (a*b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(b*c + a^2)$

sympy [B] time = 0.11, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16} + \frac{b^5x^{10}}{160} + x^8 \left(\frac{a^2b^3}{4} + \frac{b^4c}{16} \right) + x^7 \left(\frac{a^3b^2}{2} + \frac{ab^3c}{2} \right) + x^6 \left(\frac{a^4b}{2} + \frac{3a^2b^2c}{2} + \frac{b^3c^2}{4} \right) + x^5 \left(\frac{a^5}{5} + 2a^3bc + \frac{3ab^2c^2}{2} \right) + x^4 \left(a^4c + 3a^2bc^2 + \frac{b^2c^3}{2} \right) + x^3 (2a^3c^2 + 2abc^3) + x^2 \left(2a^2c^3 + \frac{bc^4}{2} + \frac{b}{2} \right) + x(ac^4 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)`

[Out] $a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a**3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +$

$$x^{**5}*(a^{**5}/5 + 2*a^{**3}*b*c + 3*a*b^{**2}*c^{**2}/2) + x^{**4}*(a^{**4}*c + 3*a^{**2}*b*c^{**2} + b^{**2}*c^{**3}/2) + x^{**3}*(2*a^{**3}*c^{**2} + 2*a*b*c^{**3}) + x^{**2}*(2*a^{**2}*c^{**3} + b*c^{**4}/2 + b/2) + x*(a*c^{**4} + a)$$

$$3.204 \quad \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^n \right) dx$$

Optimal. Leaf size=34

$$\frac{\left(ax + \frac{bx^2}{2} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{\left(ax + \frac{bx^2}{2} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.00

$$\frac{x(2a + bx) \left(\left(ax + \frac{bx^2}{2} \right)^n + n + 1 \right)}{2(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n), x]

[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))

IntegrateAlgebraic [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (a + bx) \left(1 + \left(ax + \frac{bx^2}{2} \right)^n \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n), x]

[Out] (2*a*x + b*x^2)/2 + Defer[IntegrateAlgebraic] [(a + b*x)*(a*x + (b*x^2)/2)^n, x]

fricas [A] time = 0.63, size = 48, normalized size = 1.41

$$\frac{(bn + b)x^2 + (bx^2 + 2ax) \left(\frac{1}{2}bx^2 + ax \right)^n + 2(an + a)x}{2(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n), x, algorithm="fricas")

[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(n + 1)

giac [A] time = 0.40, size = 30, normalized size = 0.88

$$\frac{1}{2}bx^2 + ax + \frac{\left(\frac{1}{2}bx^2 + ax \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n), x, algorithm="giac")

[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{bx^2}{2} + ax + \frac{\left(\frac{1}{2}bx^2 + ax \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*(1+(a*x+1/2*b*x^2)^n), x)`

[Out] $a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^{(n+1)}/(n+1)$

maxima [A] time = 1.13, size = 52, normalized size = 1.53

$$\frac{1}{2}bx^2 + ax + \frac{(bx^2 + 2ax)e^{(n \log(bx+2a)+n \log(x))}}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n), x, algorithm="maxima")`

[Out] $1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^{(n*\log(b*x + 2*a) + n*\log(x))}/(2^{(n + 1)}*n + 2^{(n + 1)})$

mupad [B] time = 2.12, size = 31, normalized size = 0.91

$$\frac{x(2a + bx) \left(n + \left(\frac{bx^2}{2} + ax \right)^n + 1 \right)}{2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x), x)`

[Out] $(x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))$

sympy [A] time = 50.75, size = 230, normalized size = 6.76

$$\left\{ \begin{array}{ll} a \left(x + \frac{\log(x)}{a} \right) & \text{for } b = 0 \wedge n = -1 \\ a \left(\frac{a^n x x^n}{n+1} + \frac{nx}{n+1} + \frac{x}{n+1} \right) & \text{for } b = 0 \\ ax + \frac{bx^2}{2} + \log(x) + \log\left(\frac{2a}{b} + x\right) & \text{for } n = -1 \\ \frac{2 \cdot 2^n abnx}{2 \cdot 2^n bn + 2 \cdot 2^n b} + \frac{2 \cdot 2^n abx}{2 \cdot 2^n bn + 2 \cdot 2^n b} + \frac{2^n b^2 nx^2}{2 \cdot 2^n bn + 2 \cdot 2^n b} + \frac{2^n b^2 x^2}{2 \cdot 2^n bn + 2 \cdot 2^n b} + \frac{2abx(2ax+bx^2)^n}{2 \cdot 2^n bn + 2 \cdot 2^n b} + \frac{b^2 x^2 (2ax+bx^2)^n}{2 \cdot 2^n bn + 2 \cdot 2^n b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n), x)`

[Out] `Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(a**n*x*x**n)/(n + 1) + n*x/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/`

```

b + x), Eq(n, -1)), (2**n*a*b*n*x/(2**n*b*n + 2**n*b) + 2**n*a*b*x/
(2**n*b*n + 2**n*b) + 2**n*b**2*n*x**2/(2**n*b*n + 2**n*b) + 2**n*b
**2*x**2/(2**n*b*n + 2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2**n*b*n
+ 2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2**n*b*n + 2**n*b), True))

```


$$3.205 \quad \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^n \right) dx$$

Optimal. Leaf size=35

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, c + ax + \frac{bx^2}{2} \right) \\ &= ax + \frac{bx^2}{2} + \frac{\left(c + ax + \frac{bx^2}{2} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\left(ax + \frac{bx^2}{2} + c \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n), x]

[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)

IntegrateAlgebraic [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (a + bx) \left(1 + \left(c + ax + \frac{bx^2}{2} \right)^n \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n), x]

[Out] (2*a*x + b*x^2)/2 + Defer[IntegrateAlgebraic] [(a + b*x)*(c + a*x + (b*x^2)/2)^n, x]

fricas [A] time = 1.05, size = 52, normalized size = 1.49

$$\frac{(bn + b)x^2 + (bx^2 + 2ax + 2c) \left(\frac{1}{2}bx^2 + ax + c \right)^n + 2(an + a)x}{2(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n), x, algorithm="fricas")

[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n + a)*x)/(n + 1)

giac [A] time = 0.25, size = 32, normalized size = 0.91

$$\frac{1}{2}bx^2 + ax + c + \frac{\left(\frac{1}{2}bx^2 + ax + c \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n), x, algorithm="giac")

[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 33, normalized size = 0.94

$$\frac{bx^2}{2} + ax + c + \frac{\left(\frac{1}{2}bx^2 + ax + c \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)`

[Out] $c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^{(n+1)}/(n+1)$

maxima [A] time = 1.16, size = 54, normalized size = 1.54

$$\frac{1}{2}bx^2 + ax + \frac{(bx^2 + 2ax + 2c)(bx^2 + 2ax + 2c)^n}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")`

[Out] $1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^{(n + 1)*n} + 2^{(n + 1)})$

mupad [B] time = 2.11, size = 58, normalized size = 1.66

$$ax + \left(\frac{bx^2}{2} + ax + c\right)^n \left(\frac{2c}{2n+2} + \frac{bx^2}{2n+2} + \frac{2ax}{2n+2}\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)`

[Out] $a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x)/(2*n + 2)) + (b*x^2)/2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)`

[Out] Timed out

$$3.206 \quad \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=30

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} \right)^6 + ax + \frac{cx^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} \right)^6 + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{1}{6} \left(ax + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.01, size = 93, normalized size = 3.10

$$\frac{a^6 x^6}{6} + \frac{1}{3} a^5 c x^8 + \frac{5}{18} a^4 c^2 x^{10} + \frac{10}{81} a^3 c^3 x^{12} + \frac{5}{162} a^2 c^4 x^{14} + \frac{1}{243} a c^5 x^{16} + ax + \frac{c^6 x^{18}}{4374} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5), x]

[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5), x]

[Out] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5), x]

fricas [B] time = 0.97, size = 77, normalized size = 2.57

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{5}{162}x^{14}c^4a^2 + \frac{10}{81}x^{12}c^3a^3 + \frac{5}{18}x^{10}c^2a^4 + \frac{1}{3}x^8ca^5 + \frac{1}{6}x^6a^6 + \frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5), x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 5/162*x^14*c^4*a^2 + 10/81*x^12*c^3*a^3 + 5/18*x^10*c^2*a^4 + 1/3*x^8*c*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + x*a

giac [A] time = 0.22, size = 24, normalized size = 0.80

$$\frac{1}{4374} (cx^3 + 3ax)^6 + \frac{1}{3} cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5), x, algorithm="giac")

[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x

maple [B] time = 0.00, size = 78, normalized size = 2.60

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5), x)

[Out] $1/4374*c^6*x^18+1/243*a*c^5*x^16+5/162*a^2*c^4*x^14+10/81*a^3*c^3*x^12+5/18*a^4*c^2*x^10+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+ax$

maxima [B] time = 0.44, size = 77, normalized size = 2.57

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5cx^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")`

[Out] $1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + ax$

mupad [B] time = 0.05, size = 77, normalized size = 2.57

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)`

[Out] $ax + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x^16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162$

sympy [B] time = 0.09, size = 87, normalized size = 2.90

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)`

[Out] $a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81 + 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3/3$

$$3.207 \quad \int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=31

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} + d \right)^6 + ax + \frac{cx^3}{3}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{cx^3}{3} + d \right)^6 + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]

[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{1}{6} \left(d + ax + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.05, size = 140, normalized size = 4.52

$$\frac{x(3a + cx^2) \left(243a^5x^5 + 405a^4cx^7 + 270a^3c^2x^9 + 90a^2c^3x^{11} + 15ac^4x^{13} + 1215d^4(3ax + cx^3) + 540d^3(3ax + cx^3)^2 + 135d^2(3ax + cx^3)^3 + 18d(3ax + cx^3)^4 + c^5x^{15} + 1458d^5 + 1458 \right)}{4374}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]

[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x + c*x^3)^4))/4374

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + cx^2) \left(1 + \left(d + ax + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]

[Out] IntegrateAlgebraic[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5), x]

fricas [B] time = 0.82, size = 291, normalized size = 9.39

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{1}{243}x^{15}d^5c^5 + \frac{5}{162}x^{14}c^4a^2 + \frac{5}{81}x^{13}d^4c^4a + \frac{5}{162}x^{12}d^2c^4 + \frac{10}{81}x^{12}c^3a^3 + \frac{10}{27}x^{11}d^3c^3a^2 + \frac{10}{27}x^{10}d^2c^3a + \frac{5}{18}x^{10}c^2a^4 + \frac{10}{81}x^9d^3c^3 + \frac{10}{9}x^9d^2c^2a^3 + \frac{5}{3}x^8d^2c^2a^2 + \frac{1}{3}x^8c^2a^5 + \frac{10}{9}x^7d^3c^2a + \frac{5}{3}x^7d^2c^2a^2 + \frac{1}{3}x^7c^2a^5 + \frac{10}{3}x^6d^3c^2a^3 + \frac{1}{6}x^6d^2c^2a^4 + \frac{10}{3}x^5d^3c^2a^2 + x^5d^2c^2a^5 + \frac{5}{3}x^4d^4c^2a + \frac{5}{2}x^4d^2a^4 + \frac{1}{3}x^3d^5c + \frac{10}{3}x^3d^3a^3 + \frac{5}{2}x^2d^4a^2 + xd^5a + \frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/162*x^14*c^4*a^2 + 5/81*x^13*d*c^4*a + 5/162*x^12*d^2*c^4 + 10/81*x^12*c^3*a^3 + 10/27*x^11*d*c^3*a^2 + 10/27*x^10*d^2*c^3*a + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 10/9*x^9*d^2*c^2*a^3 + 5/3*x^8*d^2*c^2*a^2 + 1/3*x^8*c^2*a^5 + 10/9*x^7*d^3*c^2*a + 5/3*x^7*d^2*c^2*a^2 + 5/18*x^6*d^4*c^2 + 10/3*x^6*d^2*c^2*a^3 + 1/6*x^6*a^6 + 10/3*x^5*d^3*c^2*a^2 + x^5*d^2c^2a^5 + 5/3*x^4*d^4c^2a + 5/2*x^4*d^2a^4 + 1/3*x^3*d^5c + 10/3*x^3*d^3a^3 + 5/2*x^2*d^4a^2 + xd^5a + 1/3*x^3c + xa

giac [B] time = 0.34, size = 105, normalized size = 3.39

$$\frac{1}{4374}(cx^3 + 3ax)^6 + \frac{1}{243}(cx^3 + 3ax)^5d + \frac{5}{162}(cx^3 + 3ax)^4d^2 + \frac{10}{81}(cx^3 + 3ax)^3d^3 + \frac{5}{18}(cx^3 + 3ax)^2d^4 + \frac{1}{3}(cx^3 + 3ax)d^5 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")

[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x

maple [B] time = 0.00, size = 618, normalized size = 19.94

$$\frac{1}{4374}(cx^3 + 3ax)^6 + \frac{1}{243}(cx^3 + 3ax)^5d + \frac{5}{162}(cx^3 + 3ax)^4d^2 + \frac{10}{81}(cx^3 + 3ax)^3d^3 + \frac{5}{18}(cx^3 + 3ax)^2d^4 + \frac{1}{3}(cx^3 + 3ax)d^5 + \frac{1}{3}cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5), x)$

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{243}a^5c^5x^{16} + \frac{1}{243}c^5d^5x^{15} + \frac{5}{162}a^2c^4x^{14} + \frac{5}{81}a^4c^4d^4x^{13} + \frac{1}{12}(10/27a^3c^3 + c^4/27d^2c^3 + 2/3a^3c^2 + 1/3c^2(2/3d^2c^2 + 4/3a^3c))x^{12} + 10/27a^2c^3d^3x^{11} + 1/10(a(4/27d^2c^3 + 2/3a^3c^2 + 1/3c^2(2/3d^2c^2 + 4/3a^3c)) + c(4/3d^2a^2c^2 + a(2/3d^2c^2 + 4/3a^3c) + 1/3c^2(a^4 + 4a^2cd^2)))x^{10} + 1/9(10/3a^3d^2c^2 + c(d(2/3d^2c^2 + 4/3a^3c) + 4a^3d^2c + 1/3c(4/3d^3c + 4a^3d)))x^9 + 1/8(a(4/3d^2a^2c^2 + a(2/3d^2c^2 + 4/3a^3c) + 1/3c^2(a^4 + 4a^2cd^2)) + c(6d^2a^2c + a(a^4 + 4a^2cd^2)))x^8 + 1/7(a(d(2/3d^2c^2 + 4/3a^3c) + 4a^3d^2c + 1/3c(4/3d^3c + 4a^3d)) + c(d(a^4 + 4a^2cd^2) + a(4/3d^3c + 4a^3d) + 4/3cd^3a))x^7 + 1/6(a(6d^2a^2c + a(a^4 + 4a^2cd^2)) + c(d(4/3d^3c + 4a^3d) + 6a^3d^2 + 1/3cd^4))x^6 + 1/5(a(d(a^4 + 4a^2cd^2) + a(4/3d^3c + 4a^3d) + 4/3cd^3a) + 10a^2cd^3)x^5 + 1/4(a(d(4/3d^3c + 4a^3d) + 6a^3d^2 + 1/3cd^4) + 5a^2cd^4)x^4 + 1/3(10a^3d^3 + c(d^5 + 1))x^3 + 5/2a^2d^4x^2 + a(d^5 + 1)x$

maxima [B] time = 0.46, size = 280, normalized size = 9.03

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}a^5c^5x^{16} + \frac{1}{243}c^5d^5x^{15} + \frac{5}{162}a^2c^4x^{14} + \frac{5}{81}a^4c^4d^4x^{13} + \frac{10}{27}a^2c^3d^3x^{11} + \frac{5}{162}(4a^3c^3 + c^4d^2)x^{12} + \frac{5}{54}(10a^3c^2 + 4a^2cd^2)x^{10} + \frac{10}{81}(10a^2c^2d + c^3d^3)x^9 + \frac{1}{3}(a^2c + 5a^2c^2d^2)x^8 + \frac{5}{2}a^2d^4x^2 + \frac{5}{9}(3a^4cd + 2a^2c^2d^3)x^7 + \frac{1}{18}(3a^6 + 60a^3cd^2 + 5c^2d^4)x^6 + \frac{1}{3}(3a^5d + 10a^2cd^3)x^5 + \frac{5}{6}(3a^4d^2 + 2a^2cd^4)x^4 + \frac{1}{3}(10a^3d^3 + c^4d^5 + c)x^3 + (ad^5 + a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{243}a^5c^5x^{16} + \frac{1}{243}c^5d^5x^{15} + \frac{5}{162}a^2c^4x^{14} + \frac{5}{81}a^4c^4d^4x^{13} + \frac{10}{27}a^2c^3d^3x^{11} + \frac{5}{162}(4a^3c^3 + c^4d^2)x^{12} + \frac{5}{54}(3a^4c^2 + 4a^2cd^2)x^{10} + \frac{10}{81}(9a^3c^2d + c^3d^3)x^9 + \frac{1}{3}(a^5c + 5a^2c^2d^2)x^8 + \frac{5}{2}a^2d^4x^2 + \frac{5}{9}(3a^4cd + 2a^2c^2d^3)x^7 + \frac{1}{18}(3a^6 + 60a^3cd^2 + 5c^2d^4)x^6 + \frac{1}{3}(3a^5d + 10a^2cd^3)x^5 + \frac{5}{6}(3a^4d^2 + 2a^2cd^4)x^4 + \frac{1}{3}(10a^3d^3 + c^4d^5 + c)x^3 + (ad^5 + a)x$

mupad [B] time = 2.27, size = 266, normalized size = 8.58

$$x^5 \left(\frac{a^5 d^5}{3} + \frac{10 a^2 c^4 d^3}{3} \right) + x^4 \left(\frac{5 a^4 d^4}{2} + \frac{5 c d^4}{3} \right) + x^3 \left(\frac{10 a^3 d^3}{3} + \frac{c d^3}{3} + \frac{c}{3} \right) + x^2 \left(\frac{a^6}{6} + \frac{10 a^2 c d^2}{3} + \frac{5 c^2 d^2}{18} \right) + \frac{c^5 x^{18}}{4374} + \frac{a^5 c^5 x^{16}}{243} + a x (d^5 + 1) + \frac{c^5 d^5 x^{15}}{243} + \frac{5 a^2 c^4 x^{14}}{162} + \frac{5 a^2 d^4 x^2}{2} + \frac{5 c^3 x^{12} (4 a^3 + c d^2)}{162} + \frac{a^2 c^4 (a^3 + 5 c d^2)}{3} + \frac{10 a^2 c^3 d^3 x^{11}}{27} + \frac{5 a c^4 d^4 x^{13}}{54} + \frac{10 c^2 d^3 (9 a^3 + c d^2)}{81} + \frac{5 a c^4 d^4 x^{13}}{81} + \frac{5 a c d^3 (3 a^5 + 2 c d^3)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2), x)$

[Out] $x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^15)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c$

$$\frac{1}{54} (4cd^2 + 3a^3)x^{10} + \frac{1}{81} (10c^2dx^9 + (cd^2 + 9a^3)x^9) + \frac{1}{81} (5a^4c^4d^2x^{13} + (5a^4cd^2x^7 + (2cd^2 + 3a^3)x^7))$$

sympy [B] time = 0.14, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162} + \frac{10a^2c^3dx^{11}}{27} + \frac{5a^2d^4x^2}{2} + \frac{5a^2c^3x^{16}}{243} + \frac{5a^2c^4dx^{13}}{81} + \frac{c^5x^{18}}{4374} + \frac{c^5dx^{15}}{243} + x^{12} \left(\frac{10a^3c^3}{81} + \frac{5c^4d^2}{162} \right) + x^{10} \left(\frac{5a^4c^2}{18} + \frac{10a^3d^2}{27} \right) + x^8 \left(\frac{10a^3c^2d}{9} + \frac{10c^3d^3}{81} \right) + x^7 \left(\frac{d^5c}{3} + \frac{5a^2c^2d^2}{3} \right) + x^7 \left(\frac{5a^4cd}{3} + \frac{10a^3d^2}{9} \right) + x^6 \left(\frac{d^6}{6} + \frac{10a^3cd^2}{3} + \frac{5c^2d^4}{18} \right) + x^5 \left(d^5d + \frac{10a^2cd^2}{3} \right) + x^4 \left(\frac{5a^4d^2}{2} + \frac{5cd^4}{3} \right) + x^3 \left(\frac{10a^3d^3}{3} + \frac{c^2d^5}{3} + \frac{c^2}{3} \right) + x(ad^5 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)

[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c**5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6 + 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3) + x*(a*d**5 + a)

$$3.208 \quad \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=34

$$\frac{x^{12}(3b + 2cx)^6}{279936} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1591}

$$\frac{x^{12}(3b + 2cx)^6}{279936} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{x^{12}(3b + 2cx)^6}{279936} \end{aligned}$$

Mathematica [B] time = 0.01, size = 98, normalized size = 2.88

$$\frac{b^6 x^{12}}{384} + \frac{1}{96} b^5 c x^{13} + \frac{5}{288} b^4 c^2 x^{14} + \frac{5}{324} b^3 c^3 x^{15} + \frac{5}{648} b^2 c^4 x^{16} + \frac{1}{486} b c^5 x^{17} + \frac{bx^2}{2} + \frac{c^6 x^{18}}{4374} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^6*x^18)/4374

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]

fricas [B] time = 0.64, size = 80, normalized size = 2.35

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{288}x^{14}c^2b^4 + \frac{1}{96}x^{13}cb^5 + \frac{1}{384}x^{12}b^6 + \frac{1}{3}x^3c + \frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5), x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 5/324*x^15*c^3*b^3 + 5/288*x^14*c^2*b^4 + 1/96*x^13*c*b^5 + 1/384*x^12*b^6 + 1/3*x^3*c + 1/2*x^2*b

giac [A] time = 0.29, size = 30, normalized size = 0.88

$$\frac{1}{279936} (2cx^3 + 3bx^2)^6 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5), x, algorithm="giac")

[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2

maple [B] time = 0.00, size = 81, normalized size = 2.38

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5), x)

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$

maxima [B] time = 0.44, size = 80, normalized size = 2.35

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")`

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$

mupad [B] time = 0.07, size = 80, normalized size = 2.35

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)`

[Out] $\frac{(b*x^2)/2 + (c*x^3)/3 + (b^6*x^{12})/384 + (c^6*x^{18})/4374 + (b^5*c*x^{13})/96 + (b*c^5*x^{17})/486 + (5*b^4*c^2*x^{14})/288 + (5*b^3*c^3*x^{15})/324 + (5*b^2*c^4*x^{16})/648$

sympy [B] time = 0.10, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)`

[Out] $b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**15/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/4374 + c*x**3/3$

$$3.209 \quad \int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=41

$$\frac{1}{6} \left(\frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {1591}

$$\frac{1}{6} \left(\frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.05, size = 146, normalized size = 3.56

$$\frac{x^2(3b + 2cx) (243b^5x^{10} + 810b^4cx^{11} + 1080b^3c^2x^{12} + 720b^2c^3x^{13} + 240bc^4x^{14} + 19440d^4x^2(3b + 2cx) + 4320d^3x^4(3b + 2cx)^2 + 540d^2x^6(3b + 2cx)^3 + 36dx^8(3b + 2cx)^4 + 32c^5x^{15} + 46656d^5 + 46656)}{279936}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + cx^2) \left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] IntegrateAlgebraic[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]

fricas [B] time = 0.92, size = 298, normalized size = 7.27

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{486}x^{17}c^5b + \frac{5}{648}x^{16}c^4b^2 + \frac{1}{243}x^{15}d^5c^5 + \frac{5}{324}x^{15}c^3b^3 + \frac{5}{162}x^{14}d^4c^4b + \frac{5}{288}x^{14}c^2b^4 + \frac{5}{54}x^{13}d^3c^3b^2 + \frac{1}{96}x^{13}c^1b^5 + \frac{5}{162}x^{12}d^2c^4 + \frac{5}{36}x^{12}d^2c^2b^3 + \frac{1}{384}x^{12}b^6 + \frac{5}{27}x^{11}d^2c^3b + \frac{5}{48}x^{11}d^2c^1b^4 + \frac{5}{12}x^{10}d^2c^2b^2 + \frac{1}{32}x^{10}d^1b^5 + \frac{10}{81}x^9d^3c^3 + \frac{5}{12}x^9d^2c^1b^3 + \frac{5}{9}x^8d^3c^2b + \frac{5}{32}x^8d^2b^4 + \frac{5}{6}x^7d^3c^1b^2 + \frac{5}{18}x^6d^4c^2 + \frac{5}{12}x^6d^3b^3 + \frac{5}{6}x^5d^4c^1b + \frac{5}{8}x^4d^4b^2 + \frac{1}{3}x^3d^5c + \frac{1}{2}x^2d^5b + \frac{1}{3}x^3c + \frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5), x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^15*d^5*c^5 + 5/324*x^15*c^3*b^3 + 5/162*x^14*d^4*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^13*d^3*c^3*b^2 + 1/96*x^13*c^1*b^5 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d^2*c^2*b^3 + 1/384*x^12*b^6 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d^2*c^1*b^4 + 5/12*x^10*d^2*c^2*b^2 + 1/32*x^10*d^1*b^5 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^2*c^1*b^3 + 5/9*x^8*d^3*c^2*b + 5/32*x^8*d^2*b^4 + 5/6*x^7*d^3*c^1*b^2 + 5/18*x^6*d^4*c^2 + 5/12*x^6*d^3*b^3 + 5/6*x^5*d^4*c^1*b + 5/8*x^4*d^4*b^2 + 1/3*x^3*d^5*c + 1/2*x^2*d^5*b + 1/3*x^3*c + 1/2*x^2*b

giac [B] time = 0.31, size = 126, normalized size = 3.07

$$\frac{1}{279936}(2cx^3 + 3bx^2)^6 + \frac{1}{7776}(2cx^3 + 3bx^2)^5d + \frac{5}{2592}(2cx^3 + 3bx^2)^4d^2 + \frac{5}{324}(2cx^3 + 3bx^2)^3d^3 + \frac{5}{72}(2cx^3 + 3bx^2)^2d^4 + \frac{1}{6}(2cx^3 + 3bx^2)d^5 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5), x, algorithm="giac")

[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(2*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2

maple [B] time = 0.00, size = 646, normalized size = 15.76

```

[1] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
[2] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+1/15*(5/54*b^3*c^3+c*(1
/81*d*c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)))*x^15+1/14*(b*(1/81*d*
c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c))+c*(2/27*d*b*c^3+1/2*b*(4/27*
c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4)))*x^14+1/13*(b*(2/27*d*b*c^3+
1/2*b*(4/27*c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4))+c*(1/2*d*b^2*c^2
+1/2*b*(2/3*d*b*c^2+1/16*b^4)))*x^13+1/12*(b*(1/2*d*b^2*c^2+1/2*b*(2/3*d*b*
c^2+1/16*b^4))+c*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c^2*d^2+1
/2*d*b^3)))*x^12+1/11*(b*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c
^2*d^2+1/2*d*b^3))+c*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2+1/2*d*b^3
)+2/3*c^2*d^2*b))*x^11+1/10*(b*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2
+1/2*d*b^3)+2/3*c^2*d^2*b)+5/2*b^2*c^2*d^2)*x^10+1/9*(5/2*b^3*c*d^2+c*(d*(2
/3*c^2*d^2+1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3))*x^9+1/8*(b*(d*(2/3*c^2*d^2+
1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3)+10/3*c^2*d^3*b)*x^8+5/6*b^2*c*d^3*x^7+1
/6*(5/2*b^3*d^3+5/3*c^2*d^4)*x^6+5/6*b*c*d^4*x^5+5/8*b^2*d^4*x^4+1/3*(d^5+1
)*c*x^3+1/2*b*(d^5+1)*x^2

```

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5), x)$

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{15}(5/54b^3c^3 + c(1/81d*c^4 + 1/12b^3c^2 + 1/3c(4/27c^3d + 1/6b^3c)))x^{15} + 1/14(b(1/81d*c^4 + 1/12b^3c^2 + 1/3c(4/27c^3d + 1/6b^3c)) + c(2/27d*b*c^3 + 1/2b(4/27c^3d + 1/6b^3c) + 1/3c(2/3d*b*c^2 + 1/16b^4)))x^{14} + 1/13(b(2/27d*b*c^3 + 1/2b(4/27c^3d + 1/6b^3c) + 1/3c(2/3d*b*c^2 + 1/16b^4)) + c(1/2d*b^2*c^2 + 1/2b(2/3d*b*c^2 + 1/16b^4)))x^{13} + 1/12(b(1/2d*b^2*c^2 + 1/2b(2/3d*b*c^2 + 1/16b^4)) + c(d(4/27c^3d + 1/6b^3c) + 1/2b^3*d*c + 1/3c(2/3c^2*d^2 + 1/2d*b^3)))x^{12} + 1/11(b(d(4/27c^3d + 1/6b^3c) + 1/2b^3*d*c + 1/3c(2/3c^2*d^2 + 1/2d*b^3)) + c(d(2/3d*b*c^2 + 1/16b^4) + 1/2b(2/3c^2*d^2 + 1/2d*b^3) + 2/3c^2*d^2*b))x^{11} + 1/10(b(d(2/3d*b*c^2 + 1/16b^4) + 1/2b(2/3c^2*d^2 + 1/2d*b^3) + 2/3c^2*d^2*b) + 5/2b^2*c^2*d^2)x^{10} + 1/9(5/2b^3*c*d^2 + c(d(2/3c^2*d^2 + 1/2d*b^3) + 3/4b^3*d^2 + 4/9c^2*d^3))x^9 + 1/8(b(d(2/3c^2*d^2 + 1/2d*b^3) + 3/4b^3*d^2 + 4/9c^2*d^3) + 10/3c^2*d^3*b)x^8 + 5/6b^2*c*d^3x^7 + 1/6(5/2b^3*d^3 + 5/3c^2*d^4)x^6 + 5/6b*c*d^4x^5 + 5/8b^2*d^4x^4 + 1/3(d^5 + 1)c*x^3 + 1/2b(d^5 + 1)x^2$

maxima [B] time = 0.45, size = 289, normalized size = 7.05

```

1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/15*(5/54*b^3*c^3 + c*(1/81*d*c^4 + 1/12*b^3*c^2 + 1/3*c*(4/27*c^3*d + 1/6*b^3*c)))*x^15 + 1/14*(b*(1/81*d*c^4 + 1/12*b^3*c^2 + 1/3*c*(4/27*c^3*d + 1/6*b^3*c)) + c*(2/27*d*b*c^3 + 1/2*b*(4/27*c^3*d + 1/6*b^3*c) + 1/3*c*(2/3*d*b*c^2 + 1/16*b^4)))*x^14 + 1/13*(b*(2/27*d*b*c^3 + 1/2*b*(4/27*c^3*d + 1/6*b^3*c) + 1/3*c*(2/3*d*b*c^2 + 1/16*b^4)) + c*(1/2*d*b^2*c^2 + 1/2*b*(2/3*d*b*c^2 + 1/16*b^4)))*x^13 + 1/12*(b*(1/2*d*b^2*c^2 + 1/2*b*(2/3*d*b*c^2 + 1/16*b^4)) + c*(d*(4/27*c^3*d + 1/6*b^3*c) + 1/2*b^3*d*c + 1/3*c*(2/3*c^2*d^2 + 1/2*d*b^3)))*x^12 + 1/11*(b*(d*(4/27*c^3*d + 1/6*b^3*c) + 1/2*b^3*d*c + 1/3*c*(2/3*c^2*d^2 + 1/2*d*b^3)) + c*(d*(2/3*d*b*c^2 + 1/16*b^4) + 1/2*b*(2/3*c^2*d^2 + 1/2*d*b^3) + 2/3*c^2*d^2*b))*x^11 + 1/10*(b*(d*(2/3*d*b*c^2 + 1/16*b^4) + 1/2*b*(2/3*c^2*d^2 + 1/2*d*b^3) + 2/3*c^2*d^2*b) + 5/2*b^2*c^2*d^2)*x^10 + 1/9*(5/2*b^3*c*d^2 + c*(d*(2/3*c^2*d^2 + 1/2*d*b^3) + 3/4*b^3*d^2 + 4/9*c^2*d^3))*x^9 + 1/8*(b*(d*(2/3*c^2*d^2 + 1/2*d*b^3) + 3/4*b^3*d^2 + 4/9*c^2*d^3) + 10/3*c^2*d^3*b)*x^8 + 5/6*b^2*c*d^3*x^7 + 1/6*(5/2*b^3*d^3 + 5/3*c^2*d^4)*x^6 + 5/6*b*c*d^4*x^5 + 5/8*b^2*d^4*x^4 + 1/3*(d^5 + 1)*c*x^3 + 1/2*b*(d^5 + 1)*x^2

```

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{1}{972}(15b^3c^3 + 4c^5d)x^{15} + \frac{5}{2592}(9b^4c^2 + 16b^2c^4d)x^{14} + \frac{1}{864}(9b^5c + 80b^2c^3d)x^{13} + \frac{5}{6}b^2c^2d^3x^7 + \frac{1}{10368}(27b^6 + 1440b^3c^2d + 320c^4d^2)x^{12} + \frac{5}{432}(9b^4c^2d + 16b^2c^3d^2)x^{11} + \frac{5}{6}b^2c^2d^4x^5 + \frac{1}{96}(3b^5d + 40b^2c^2d^2)x^{10} + \frac{5}{8}b^2d^4x^4 + \frac{5}{324}(27b^3c^2d^2 + 8c^3d^3)x^9 + \frac{5}{288}(9b^4d^2 + 32b^2c^2d^3)x^8 + \frac{5}{36}(3b^3d^3 + 2c^2d^4)x^6 + \frac{1}{3}(c^2d^5 + c)x^3 + \frac{1}{2}(bd^5 + b)x^2$

mapad [B] time = 2.28, size = 273, normalized size = 6.66

```

1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/15*(5/54*b^3*c^3 + c*(1/81*d*c^4 + 1/12*b^3*c^2 + 1/3*c*(4/27*c^3*d + 1/6*b^3*c)))*x^15 + 1/14*(b*(1/81*d*c^4 + 1/12*b^3*c^2 + 1/3*c*(4/27*c^3*d + 1/6*b^3*c)) + c*(2/27*d*b*c^3 + 1/2*b*(4/27*c^3*d + 1/6*b^3*c) + 1/3*c*(2/3*d*b*c^2 + 1/16*b^4)))*x^14 + 1/13*(b*(2/27*d*b*c^3 + 1/2*b*(4/27*c^3*d + 1/6*b^3*c) + 1/3*c*(2/3*d*b*c^2 + 1/16*b^4)) + c*(1/2*d*b^2*c^2 + 1/2*b*(2/3*d*b*c^2 + 1/16*b^4)))*x^13 + 1/12*(b*(1/2*d*b^2*c^2 + 1/2*b*(2/3*d*b*c^2 + 1/16*b^4)) + c*(d*(4/27*c^3*d + 1/6*b^3*c) + 1/2*b^3*d*c + 1/3*c*(2/3*c^2*d^2 + 1/2*d*b^3)))*x^12 + 1/11*(b*(d*(4/27*c^3*d + 1/6*b^3*c) + 1/2*b^3*d*c + 1/3*c*(2/3*c^2*d^2 + 1/2*d*b^3)) + c*(d*(2/3*d*b*c^2 + 1/16*b^4) + 1/2*b*(2/3*c^2*d^2 + 1/2*d*b^3) + 2/3*c^2*d^2*b))*x^11 + 1/10*(b*(d*(2/3*d*b*c^2 + 1/16*b^4) + 1/2*b*(2/3*c^2*d^2 + 1/2*d*b^3) + 2/3*c^2*d^2*b) + 5/2*b^2*c^2*d^2)*x^10 + 1/9*(5/2*b^3*c*d^2 + c*(d*(2/3*c^2*d^2 + 1/2*d*b^3) + 3/4*b^3*d^2 + 4/9*c^2*d^3))*x^9 + 1/8*(b*(d*(2/3*c^2*d^2 + 1/2*d*b^3) + 3/4*b^3*d^2 + 4/9*c^2*d^3) + 10/3*c^2*d^3*b)*x^8 + 5/6*b^2*c*d^3*x^7 + 1/6*(5/2*b^3*d^3 + 5/3*c^2*d^4)*x^6 + 5/6*b*c*d^4*x^5 + 5/8*b^2*d^4*x^4 + 1/3*(d^5 + 1)*c*x^3 + 1/2*b*(d^5 + 1)*x^2

```

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1), x)$

[Out] $x^{13}((b^5c)/96 + (5b^2c^3d)/54) + x^{14}((5b^4c^2)/288 + (5b^2c^4d)/162) + x^{12}(b^6/384 + (5c^4d^2)/162 + (5b^3c^2d)/36) + (c^6x^{18})/4374 + x^{15}((c^5d)/243 + (5b^3c^3)/324) + (5d^3x^6(2c^2d + 3b^3))/36 + (b^5c^5x^{17})/486 + (5b^2c^4x^{16})/648 + (bx^2(d^5 + 1))/2 + (5b^2d^4x^4)/8 + (cx^3(d^5 + 1))/3 + (5b^2cd^3x^7)/6 + (5bd^2x^8(32c^2d + 9b^3))/288 + (b^2dx^{10}(40c^2d + 3b^3))/96 + (5cd^2x^9(8c^2d + 27b^3))/324 + (5b^2cd^4x^5)/6 + (5b^2cd^4x^{11}(16c^2d + 9b^3))/432$

sympy [B] time = 0.15, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648} + \frac{5b^2cd^4x^7}{6} + \frac{5b^2d^4x^4}{8} + \frac{b^5c^5x^{17}}{486} + \frac{5b^2cd^4x^5}{6} + \frac{c^6x^{18}}{4374} + x^{15}\left(\frac{5b^3c^3}{324} + \frac{c^5d}{243}\right) + x^{14}\left(\frac{5b^4c^2}{288} + \frac{5b^2c^4d}{162}\right) + x^{13}\left(\frac{b^6}{384} + \frac{5b^2c^2d}{36} + \frac{5c^4d^2}{162}\right) + x^{12}\left(\frac{5b^3c^2d}{48} + \frac{5b^2cd^2}{27}\right) + x^{11}\left(\frac{b^5d}{32} + \frac{5b^2cd^2}{12}\right) + x^9\left(\frac{5b^3cd^2}{12} + \frac{10c^3d^3}{81}\right) + x^8\left(\frac{5b^4d^2}{32} + \frac{5b^2cd^2}{9}\right) + x^6\left(\frac{5b^3d^2}{12} + \frac{5c^2d^3}{18}\right) + x^5\left(\frac{cd^5}{3} + \frac{c}{3}\right) + x^2\left(\frac{bd^5}{2} + \frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5), x)

[Out] $5b^2c^4x^{16}/648 + 5b^2c^4d^3x^7/6 + 5b^2d^4x^4/8 + b^5c^5x^{17}/486 + 5b^2cd^4x^5/6 + c^6x^{18}/4374 + x^{15}(5b^3c^3/324 + c^5d/243) + x^{14}(5b^4c^2/288 + 5b^2c^4d/162) + x^{13}(b^6/384 + 5b^2c^2d/36 + 5c^4d^2/162) + x^{12}(5b^3c^2d/48 + 5b^2cd^2/27) + x^{11}(b^5d/32 + 5b^2cd^2/12) + x^{10}(5b^3cd^2/12 + 10c^3d^3/81) + x^8(5b^4d^2/32 + 5b^2cd^2/9) + x^6(5b^3cd^2/12 + 5c^2d^3/18) + x^3(cd^5/3 + c/3) + x^2(bd^5/2 + b/2)$

$$3.210 \quad \int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=46

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.06, size = 244, normalized size = 5.30

$$\frac{a^6 c^6}{6} + \frac{1}{6} a^5 x^2 (3b + 2cx) + \frac{5}{72} a^4 x^3 (3b + 2cx)^2 + \frac{5}{324} a^3 x^4 (3b + 2cx)^3 + \frac{5a^2 x^{10} (3b + 2cx)^4}{2592} + d \left(\frac{b^2 x^{11}}{32} + \frac{5}{48} b^2 c x^{12} + \frac{5}{36} b^3 c^2 x^{13} + \frac{5}{54} b^2 c^2 x^{14} + \frac{5}{162} b^2 c^2 x^{15} + \frac{c^2 x^{16}}{243} + x \right) + \frac{x^2 (729b^6 x^{10} + 2916b^5 c x^{11} + 4860b^4 c^2 x^{12} + 4320b^3 c^3 x^{13} + 2160b^2 c^4 x^{14} + 576b c^5 x^{15} + 243) + 64cx (c^5 x^{15} + 1458)}{279936}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]

[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 + (5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x + (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]

[Out] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]

fricas [B] time = 0.93, size = 309, normalized size = 6.72

$\frac{1}{1296}a^6x^6 + \frac{1}{486}a^5x^7 + \frac{5}{648}a^4x^8 + \frac{5}{324}a^3x^9 + \frac{5}{162}a^2x^{10} + \frac{5}{324}a^2x^{10} + \frac{1}{96}a^2x^{10} + \frac{5}{36}a^2x^{10} + \frac{5}{27}a^2x^{10} + \frac{1}{36}a^2x^{10} + \frac{5}{48}a^2x^{10} + \frac{5}{12}a^2x^{10} + \frac{10}{81}a^2x^{10} + \frac{1}{32}a^2x^{10} + \frac{5}{12}a^2x^{10} + \frac{5}{32}a^2x^{10} + \frac{5}{6}a^2x^{10} + \frac{5}{6}a^2x^{10} + \frac{5}{12}a^2x^{10} + \frac{5}{6}a^2x^{10} + \frac{1}{3}a^2x^{10} + \frac{1}{2}a^2x^{10} + \frac{1}{6}a^2x^{10} + \frac{1}{3}a^2x^{10} + \frac{1}{2}a^2x^{10} + xa$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a + 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/288*x^14*c^2*b^4 + 5/54*x^14*c^3*b^2*a + 5/162*x^14*c^4*a^2 + 1/96*x^13*c*b^5 + 5/36*x^13*c^2*b^3*a + 5/27*x^13*c^3*b*a^2 + 1/384*x^12*b^6 + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*a^2 + 10/81*x^12*c^3*a^3 + 1/32*x^11*b^5*a + 5/12*x^11*c*b^3*a^2 + 5/9*x^11*c^2*b*a^3 + 5/32*x^10*b^4*a^2 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 5/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 1/2*x^7*b*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + 1/2*x^2*b + x*a

giac [A] time = 0.31, size = 37, normalized size = 0.80

$$\frac{1}{279936} (2cx^3 + 3bx^2 + 6ax)^6 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")

[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x

maple [B] time = 0.00, size = 1523, normalized size = 33.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)

[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+c*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)))*x^16+1/15*(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+c*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))*x^15+1/14*(a*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+b*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+c*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))*x^14+1/13*(a*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+b*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+c*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))*x^13+1/12*(a*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+b*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+c*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))*x^12+1/11*(a*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+b*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+c*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b))*x^11+1/10*(a*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+b*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+c*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4))*x^10+1/9*(a*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+b*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*c*a^4*b))*x^9+1/8*(a*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*b^2*a^4+c*a^5))*x^8+1/2*a^5*b*x^7+1/6*a^6*x^6+1/3*c*x^3+1/2*b*x^2+a*x

maxima [B] time = 0.45, size = 289, normalized size = 6.28

$$\frac{1}{4374}c^6x^{18} + \frac{1}{486}b^5c^5x^{17} + \frac{1}{1944}(15b^2c^2 + 8ac^5)x^{16} + \frac{5}{324}(b^3c^3 + 2ab^4c^4)x^{15} + \frac{5}{2592}(9b^4c^2 + 48ab^2c^3 + 16a^2b^2c^4)x^{14} + \frac{1}{864}(9b^5c + 120a^2b^3c^2 + 160a^2b^2c^3)x^{13} + \frac{1}{2}a^5b^5x^7 + \frac{1}{10368}(27b^6 + 1080ab^4c + 4320a^2b^2c^2 + 1280a^3c^3)x^{12} + \frac{1}{6}a^6x^6 + \frac{1}{288}(9ab^5 + 120a^2b^3c + 160a^3b^2c^2)x^{11} + \frac{5}{288}(9a^2b^4 + 48a^3b^2c + 16a^4c^2)x^{10} + \frac{5}{12}(a^3b^3 + 2a^4b^2c)x^9 + \frac{1}{24}(15a^4b^2 + 8a^5c)x^8 + \frac{1}{3}c^3x^3 + \frac{1}{2}b^2x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b^2*c^2)*x^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4*b^2*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x

mupad [B] time = 2.24, size = 270, normalized size = 5.87

$$\frac{1}{2} \left(\frac{10a^6c^3}{81} + \frac{5a^2b^2c^2}{12} + \frac{5ab^4c}{48} + \frac{b^6}{384} \right) + a^{11} \left(\frac{b^2}{2} + \frac{c^3}{3} + \frac{a^5}{6} + \frac{a^4b}{3} + \frac{a^3b^2}{6} + \frac{a^2b^3}{6} + \frac{5a^2b^4}{4374} + \frac{5a^2c^4}{288} + \frac{5a^2b^2c^2 + 48ab^2c + 9b^4}{2592} + \frac{5a^2b^4}{2} + \frac{b^5c^5}{486} + \frac{a^4b^5(15b^2 + 8ac)}{24} + \frac{c^4a^{10}(15b^2 + 8ac)}{1944} + \frac{ab^{11}(160a^2c^2 + 120ab^2c + 9b^4)}{288} + \frac{bca^{11}(160a^2c^2 + 120ab^2c + 9b^4)}{864} + \frac{5a^3b^4(b^2 + 2ac)}{12} + \frac{5bca^5(b^2 + 2ac)}{324} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)

[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 + (5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324

sympy [B] time = 0.16, size = 323, normalized size = 7.02

$$\frac{a^6x^6}{6} + \frac{a^5bx^7}{2} + ax + \frac{b^5c^5x^{17}}{486} + \frac{b^2x^2}{2} + \frac{c^3x^3}{3} + \frac{a^5}{6} + \frac{a^4b}{3} + \frac{a^3b^2}{6} + \frac{a^2b^3}{6} + \frac{5a^2b^4}{4374} + \frac{5a^2c^4}{648} + x^{15} \left(\frac{5ab^2c^2}{162} + \frac{5b^4c^2}{324} \right) + x^{14} \left(\frac{5a^2c^4}{162} + \frac{5ab^2c^2}{54} + \frac{5b^4c^2}{288} \right) + x^{13} \left(\frac{5a^2bc^2}{27} + \frac{5ab^3c^2}{36} + \frac{b^5c}{96} \right) + x^{12} \left(\frac{10b^6c^3}{81} + \frac{5a^2b^2c^2}{12} + \frac{5ab^4c}{48} + \frac{b^6}{384} \right) + x^{11} \left(\frac{5a^3bc^2}{9} + \frac{5a^2b^2c}{12} + \frac{ab^5}{32} \right) + x^{10} \left(\frac{5a^4c^2}{18} + \frac{5a^2b^2c}{6} + \frac{5a^2b^4}{32} \right) + x^9 \left(\frac{5a^5bc}{6} + \frac{5a^3b^3}{12} \right) + x^8 \left(\frac{a^5c}{3} + \frac{5a^4b^2}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)

[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12) + x**8*(a**5*c/3 + 5*a**4*b**2/8)

$$3.211 \quad \int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Optimal. Leaf size=47

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {1591}

$$\frac{1}{6} \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} + d \right)^6 + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx &= \text{Subst} \left(\int (1 + x^5) dx, x, d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{1}{6} \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^6 \end{aligned}$$

Mathematica [B] time = 0.12, size = 248, normalized size = 5.28

$\frac{x(6x^5 + x^3(3b + 2cx) + 7776a^2x^4 + 6480a^2x^3(3b + 2cx) + 2160a^2x^2(3b + 2cx)^2 + 360a^2x(3b + 2cx)^3 + 19440a^2x(6a + x(3b + 2cx)) + 4320a^2x^2(6a + x(3b + 2cx))^2 + 540a^2x^3(6a + x(3b + 2cx))^3 + 36a^2x^4(6a + x(3b + 2cx))^4 + 30a^2x^5(3b + 2cx)^4 + 243b^5x^{10} + 810b^4cx^{11} + 1080b^3c^2x^{12} + 720b^2c^3x^{13} + 240bc^4x^{14} + 32c^5x^{15} + 46656d^6 + 46656d^5x + 46656d^4x^2 + 46656d^3x^3 + 46656d^2x^4 + 46656d^2x^5 + 46656d^2x^6 + 46656d^2x^7 + 46656d^2x^8 + 46656d^2x^9 + 46656d^2x^{10} + 46656d^2x^{11} + 46656d^2x^{12} + 46656d^2x^{13} + 46656d^2x^{14} + 46656d^2x^{15} + 46656d^2x^{16} + 46656d^2x^{17} + 46656d^2x^{18} + 46656d^2x^{19} + 46656d^2x^{20} + 46656d^2x^{21} + 46656d^2x^{22} + 46656d^2x^{23} + 46656d^2x^{24} + 46656d^2x^{25} + 46656d^2x^{26} + 46656d^2x^{27} + 46656d^2x^{28} + 46656d^2x^{29} + 46656d^2x^{30} + 46656d^2x^{31} + 46656d^2x^{32} + 46656d^2x^{33} + 46656d^2x^{34} + 46656d^2x^{35} + 46656d^2x^{36} + 46656d^2x^{37} + 46656d^2x^{38} + 46656d^2x^{39} + 46656d^2x^{40} + 46656d^2x^{41} + 46656d^2x^{42} + 46656d^2x^{43} + 46656d^2x^{44} + 46656d^2x^{45} + 46656d^2x^{46} + 46656d^2x^{47} + 46656d^2x^{48} + 46656d^2x^{49} + 46656d^2x^{50} + 46656d^2x^{51} + 46656d^2x^{52} + 46656d^2x^{53} + 46656d^2x^{54} + 46656d^2x^{55} + 46656d^2x^{56} + 46656d^2x^{57} + 46656d^2x^{58} + 46656d^2x^{59} + 46656d^2x^{60} + 46656d^2x^{61} + 46656d^2x^{62} + 46656d^2x^{63} + 46656d^2x^{64} + 46656d^2x^{65} + 46656d^2x^{66} + 46656d^2x^{67} + 46656d^2x^{68} + 46656d^2x^{69} + 46656d^2x^{70} + 46656d^2x^{71} + 46656d^2x^{72} + 46656d^2x^{73} + 46656d^2x^{74} + 46656d^2x^{75} + 46656d^2x^{76} + 46656d^2x^{77} + 46656d^2x^{78} + 46656d^2x^{79} + 46656d^2x^{80} + 46656d^2x^{81} + 46656d^2x^{82} + 46656d^2x^{83} + 46656d^2x^{84} + 46656d^2x^{85} + 46656d^2x^{86} + 46656d^2x^{87} + 46656d^2x^{88} + 46656d^2x^{89} + 46656d^2x^{90} + 46656d^2x^{91} + 46656d^2x^{92} + 46656d^2x^{93} + 46656d^2x^{94} + 46656d^2x^{95} + 46656d^2x^{96} + 46656d^2x^{97} + 46656d^2x^{98} + 46656d^2x^{99} + 46656d^2x^{100})}{279936}$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]

[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 + 360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a + x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4)/279936

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2) \left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^5 \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]

[Out] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5), x]

fricas [B] time = 0.96, size = 928, normalized size = 19.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")

[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/162*x^14*d*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^14*c^3*b^2*a + 5/162*x^14*c^4*a^2 + 5/54*x^13*d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/81*x^13*d*c^4*a + 5/36*x^13*c^2*b^3*a + 5/27*x^13*c^3*b*a^2 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/384*x^12*b^6 + 10/27*x^12*d*c^3*b*a + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*a^2 + 10/81*x^12*c^3*a^3 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/6*x^11*d*c^2*b^2*a + 1/32*x^11*b^5*a + 10/27*x^11*d*c^3*a^2 + 5/12*x^11*c*b^3*a^2 + 5/9*x^11*c^2*b*a^3 + 5/12*x^10*d^2*c^2*b^2 + 1/32*x^10*d*b^5 + 10/27*x^10*d^2*c^3*a + 5/6*x^10*d*c*b^3*a + 5/3*x^10*d*c^2*b*a^2 + 5/32*x^10*b^4*a^2 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^2*c*b^3 + 5/3*x^9*d^2*c^2*b*a + 5/16*x^9*d*b^4*a + 5/2*x^9*d*c*b^2*a^2 + 10/9*x^9*d*c^2*a^3 + 5/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/9*x^8*d^3*c^2*b + 5/32*x^8*d^2*b^4 + 5/2*x^8*d^2*c*b^2*a + 5/3*x^8*d^2*c^2*a^2 + 5/4*x^8*d*b^3*a^2 + 10/3*x^8*d*c*b*a^3 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 5/6*x^7*d^3

$$\begin{aligned}
& *c*b^2 + 10/9*x^7*d^3*c^2*a + 5/4*x^7*d^2*b^3*a + 5*x^7*d^2*c*b*a^2 + 5/2*x \\
& ^7*d*b^2*a^3 + 5/3*x^7*d*c*a^4 + 1/2*x^7*b*a^5 + 5/18*x^6*d^4*c^2 + 5/12*x^ \\
& 6*d^3*b^3 + 10/3*x^6*d^3*c*b*a + 15/4*x^6*d^2*b^2*a^2 + 10/3*x^6*d^2*c*a^3 \\
& + 5/2*x^6*d*b*a^4 + 1/6*x^6*a^6 + 5/6*x^5*d^4*c*b + 5/2*x^5*d^3*b^2*a + 10/ \\
& 3*x^5*d^3*c*a^2 + 5*x^5*d^2*b*a^3 + x^5*d*a^5 + 5/8*x^4*d^4*b^2 + 5/3*x^4*d \\
& ^4*c*a + 5*x^4*d^3*b*a^2 + 5/2*x^4*d^2*a^4 + 1/3*x^3*d^5*c + 5/2*x^3*d^4*b* \\
& a + 10/3*x^3*d^3*a^3 + 1/2*x^2*d^5*b + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3* \\
& c + 1/2*x^2*b + x*a
\end{aligned}$$

giac [B] time = 0.44, size = 153, normalized size = 3.26

$$\frac{1}{279936}(2cx^3 + 3bx^2 + 6ax)^6 + \frac{1}{7776}(2cx^3 + 3bx^2 + 6ax)^5 d + \frac{5}{2592}(2cx^3 + 3bx^2 + 6ax)^4 d^2 + \frac{5}{324}(2cx^3 + 3bx^2 + 6ax)^3 d^3 + \frac{5}{72}(2cx^3 + 3bx^2 + 6ax)^2 d^4 + \frac{1}{6}(2cx^3 + 3bx^2 + 6ax)d^5 + \frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")

[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x)
^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2
+ 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 + 3*
b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x

maple [B] time = 0.00, size = 4284, normalized size = 91.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)

[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+(1/81*a*c^4
+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*c)*x^16+1/15*
(5/162*a*b*c^4+(1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b
^2)*c^2)*c)*b+c*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*
b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+
1/14*((1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*
c)*a+b*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)
*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+c*(2/27*b*c^3*d
+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b)*c^2+2/3
*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3
*a*c+1/4*b^2)^2))*x^14+1/13*(a*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2
/9*(2/3*a*c+1/4*b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b
^2)*b*c))+b*(2/27*b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(
2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2
/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+c*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1

$$\begin{aligned}
& /4*b^2)*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9 \\
& *(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^ \\
& 2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))) *x^{13}+1/12*(a*(2/27 \\
& *b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b) \\
& *c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)* \\
& b*c+(2/3*a*c+1/4*b^2)^2))+b*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2 \\
& /9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/ \\
& 3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b \\
& *c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+c*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3 \\
& *a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4 \\
& b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b \\
& ^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a \\
& b)^2))) *x^{12}+1/11*(a*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2/9*(2 \\
& /3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/ \\
& 3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2* \\
& (2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+b*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+ \\
& 1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^ \\
& 2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+ \\
& 1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^ \\
& 2))+c*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4 \\
& /9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9* \\
& c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2 \\
& /3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))) *x^{11}+1/10*(\\
& a*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2 \\
& +2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d) \\
&)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(\\
& a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2))+b*(d*(2/9*(a^2+b*d)*c^2+2/3*(2 \\
& /3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/ \\
& 3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2 \\
& /3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2) \\
& +2*(a^2+b*d)*(2/3*c*d+a*b)))+c*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d \\
& +a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/ \\
& 4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b \\
& *d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+ \\
& b*d)^2))) *x^{10}+1/9*(a*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+ \\
& 1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b \\
& ^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+ \\
& a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b) \\
&)))+b*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+ \\
& a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1 \\
& /2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c* \\
& (2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+c*(d*(2/9*c^2*d^ \\
& 2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c \\
& +4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1 \\
& /4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(
\end{aligned}$$

$$\begin{aligned}
& a^2+bd)))*x^9+1/8*(a*(d*(4/9*a*d*c^2+2/3*(a^2+bd)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+bd)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2))+b*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+bd)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd)))+c*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/3*c*(2*d^2*(a^2+bd)+4*a^2*d^2)))*x^8+1/7*(a*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+bd)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd)))+b*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/3*c*(2*d^2*(a^2+bd)+4*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/2*b*(2*d^2*(a^2+bd)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+bd)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/3*c*(2*d^2*(a^2+bd)+4*a^2*d^2))+b*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/2*b*(2*d^2*(a^2+bd)+4*a^2*d^2)+4/3*a*c*d^3)+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+a*(2*d^2*(a^2+bd)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+bd)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+1/2*b*(2*d^2*(a^2+bd)+4*a^2*d^2)+4/3*a*c*d^3)+b*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+a*(2*d^2*(a^2+bd)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4)+c*(d*(2*d^2*(a^2+bd)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+bd))+a*(2*d^2*(a^2+bd)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4)+b*(d*(2*d^2*(a^2+bd)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4)*x^4+1/3*(a*(d*(2*d^2*(a^2+bd)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4*a+(d^5+1)*c)*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+(d^5+1)*a*x
\end{aligned}$$

maxima [B] time = 0.48, size = 773, normalized size = 16.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")

[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +

$$480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^{13} + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8*a*b*c^3)*d)*x^{12} + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^{11} + 1/864*(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^{10} + 5/1296*(108*a^3*b^3 + 216*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x$$

mupad [B] time = 2.45, size = 753, normalized size = 16.02

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2), x)$

[Out] $x^{10}*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3 + x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2 + x^9*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 + (5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2 + x^{14}*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162) + x^{12}*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27 + x^6*(a^6/6 + (5*b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*a*b*d^4)/2) + x^{11}*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6 + x^7*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*d^4)/2) + x^{13}*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 + (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))/1944$

sympy [B] time = 0.27, size = 930, normalized size = 19.79

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5), x)

[Out] $b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/54) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d**2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b**2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x**4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(10*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d**5/2 + b/2) + x*(a*d**5 + a)$

$$3.212 \quad \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3}\right)^n\right) dx$$

Optimal. Leaf size=34

$$\frac{\left(ax + \frac{cx^3}{3}\right)^{n+1}}{n+1} + ax + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1591}

$$\frac{\left(ax + \frac{cx^3}{3}\right)^{n+1}}{n+1} + ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n), x]

[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3}\right)^n\right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{cx^3}{3} \right) \\ &= ax + \frac{cx^3}{3} + \frac{\left(ax + \frac{cx^3}{3}\right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.06

$$\frac{x(3a + cx^2) \left(\left(ax + \frac{cx^3}{3}\right)^n + n + 1 \right)}{3(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n), x]

[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))

IntegrateAlgebraic [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (a + cx^2) \left(1 + \left(ax + \frac{cx^3}{3} \right)^n \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n), x]

[Out] (3*a*x + c*x^3)/3 + Defer[IntegrateAlgebraic] [(a + c*x^2)*(a*x + (c*x^3)/3)^n, x]

fricas [A] time = 1.03, size = 48, normalized size = 1.41

$$\frac{(cn + c)x^3 + (cx^3 + 3ax) \left(\frac{1}{3} cx^3 + ax \right)^n + 3(an + a)x}{3(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n), x, algorithm="fricas")

[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(n + 1)

giac [A] time = 0.42, size = 30, normalized size = 0.88

$$\frac{1}{3} cx^3 + ax + \frac{\left(\frac{1}{3} cx^3 + ax \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n), x, algorithm="giac")

[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{cx^3}{3} + ax + \frac{\left(\frac{1}{3} cx^3 + ax \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)`

[Out] $a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^{(n+1)}/(n+1)$

maxima [A] time = 1.14, size = 54, normalized size = 1.59

$$\frac{1}{3}cx^3 + ax + \frac{(cx^3 + 3ax)e^{(n\log(cx^2+3a)+n\log(x))}}{3^{n+1}n + 3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")`

[Out] $1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^{(n*\log(c*x^2 + 3*a) + n*\log(x))}/(3^{(n + 1)*n} + 3^{(n + 1)})$

mupad [B] time = 2.11, size = 33, normalized size = 0.97

$$\frac{x (cx^2 + 3a) \left(n + \left(\frac{cx^3}{3} + ax \right)^n + 1 \right)}{3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)`

[Out] $(x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))$

sympy [B] time = 112.00, size = 201, normalized size = 5.91

$$\begin{cases} \frac{3 \cdot 3^n a n x}{3 \cdot 3^{n+1} + 3 \cdot 3^n} + \frac{3 \cdot 3^n a x}{3 \cdot 3^{n+1} + 3 \cdot 3^n} + \frac{3^n c n x^3}{3 \cdot 3^{n+1} + 3 \cdot 3^n} + \frac{3^n c x^3}{3 \cdot 3^{n+1} + 3 \cdot 3^n} + \frac{3 a x (3 a x + c x^3)^n}{3 \cdot 3^{n+1} + 3 \cdot 3^n} + \frac{c x^3 (3 a x + c x^3)^n}{3 \cdot 3^{n+1} + 3 \cdot 3^n} & \text{for } n \neq -1 \\ a x + \frac{c x^3}{3} + \log(x) + \log\left(-\sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x\right) + \log\left(\sqrt{3} i \sqrt{a} \sqrt{\frac{1}{c}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)`

[Out] `Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(-sqrt(3)*I*sqrt(a)*sqrt(1/c) + x) + log(sqrt(3)*I*sqrt(a)*sqrt(1/c) + x), True))`

$$3.213 \quad \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx$$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {1591}

$$\frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{n+1}}{n+1} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^n\right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.95

$$\frac{x^2(3b + 2cx) \left(\left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^n + n + 1 \right)}{6(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))

IntegrateAlgebraic [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (bx + cx^2) \left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (x^2*(3*b + 2*c*x))/6 + Defer[IntegrateAlgebraic][x*(b + c*x)*((b*x^2)/2 + (c*x^3)/3)^n, x]

fricas [A] time = 1.05, size = 57, normalized size = 1.30

$$\frac{2(cn + c)x^3 + 3(bn + b)x^2 + (2cx^3 + 3bx^2) \left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 \right)^n}{6(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n), x, algorithm="fricas")

[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1/2*b*x^2)^n)/(n + 1)

giac [A] time = 0.39, size = 36, normalized size = 0.82

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n), x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 37, normalized size = 0.84

$$\frac{cx^3}{3} + \frac{bx^2}{2} + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)`

[Out] $1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^{(n+1)}/(n+1)$

maxima [A] time = 1.17, size = 71, normalized size = 1.61

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + \frac{(2cx^3 + 3bx^2)e^{(n\log(2cx+3b)+2n\log(x))}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")`

[Out] $1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^{(n*\log(2*c*x + 3*b) + 2*n*\log(x))}/(3^{(n + 1)}*2^{(n + 1)}*n + 3^{(n + 1)}*2^{(n + 1)})$

mupad [B] time = 2.21, size = 37, normalized size = 0.84

$$\frac{x^2 (3b + 2cx) \left(n + \left(\frac{cx^3}{3} + \frac{bx^2}{2} \right)^n + 1 \right)}{6(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)`

[Out] $(x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)`

[Out] Timed out

$$3.214 \quad \int \left(a + bx + cx^2 \right) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx$$

Optimal. Leaf size=50

$$\frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1591}

$$\frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{n+1}}{n+1} + ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx &= \text{Subst} \left(\int (1 + x^n) dx, x, ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right) \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} + \frac{\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.19, size = 49, normalized size = 0.98

$$\frac{x(6a + x(3b + 2cx)) \left(\left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n + n + 1 \right)}{6(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n)/(6*(1 + n))

IntegrateAlgebraic [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2) \left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right)^n \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n), x]

[Out] (x*(6*a + 3*b*x + 2*c*x^2))/6 + Defer[IntegrateAlgebraic] [(a + b*x + c*x^2)*(a*x + (b*x^2)/2 + (c*x^3)/3)^n, x]

fricas [A] time = 1.16, size = 72, normalized size = 1.44

$$\frac{2(cn + c)x^3 + 3(bn + b)x^2 + (2cx^3 + 3bx^2 + 6ax) \left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax \right)^n + 6(an + a)x}{6(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")

[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)

giac [A] time = 0.39, size = 42, normalized size = 0.84

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax \right)^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 43, normalized size = 0.86

$$\frac{cx^3}{3} + \frac{bx^2}{2} + ax + \frac{\left(\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)

maxima [A] time = 1.22, size = 83, normalized size = 1.66

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax + \frac{(2cx^3 + 3bx^2 + 6ax)e^{(n \log(2cx^2 + 3bx + 6a) + n \log(x))}}{3^{n+1}2^{n+1}n + 3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2 + 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))

mupad [B] time = 2.17, size = 73, normalized size = 1.46

$$ax + \left(\frac{3bx^2}{6n+6} + \frac{2cx^3}{6n+6} + \frac{6ax}{6n+6}\right) \left(\frac{cx^3}{3} + \frac{bx^2}{2} + ax\right)^n + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)

[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x + (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)

[Out] Timed out

$$3.215 \quad \int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx$$

Optimal. Leaf size=19

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1588}

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

Antiderivative was successfully verified.

[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3), x]

[Out] (5 - 12*x + 6*x^2 + x^3)^2/6

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx = \frac{1}{6}(5 - 12x + 6x^2 + x^3)^2$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.74

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3), x]

[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-4 + 4x + x^2)(5 - 12x + 6x^2 + x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3), x]

fricas [A] time = 0.74, size = 29, normalized size = 1.53

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5), x, algorithm="fricas")

[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x

giac [A] time = 0.35, size = 30, normalized size = 1.58

$$\frac{5}{3}x^3 + \frac{1}{6}(x^3 + 6x^2 - 12x)^2 + 10x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5), x, algorithm="giac")

[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x

maple [A] time = 0.00, size = 30, normalized size = 1.58

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5), x)

[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{6}(x^3 + 6x^2 - 12x + 5)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")

[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2

mupad [B] time = 0.03, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)

[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6

sympy [A] time = 0.06, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)

[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x

$$3.216 \quad \int (2x + x^3)(1 + 4x^2 + x^4) dx$$

Optimal. Leaf size=16

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1588}

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] (1 + 4*x^2 + x^4)^2/8

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2x + x^3)(1 + 4x^2 + x^4) dx = \frac{1}{8}(1 + 4x^2 + x^4)^2$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.31

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + x^3)(1 + 4x^2 + x^4) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*x + x^3)*(1 + 4*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2*x + x^3)*(1 + 4*x^2 + x^4), x]

fricas [A] time = 0.94, size = 17, normalized size = 1.06

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1), x, algorithm="fricas")

[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2

giac [A] time = 0.23, size = 22, normalized size = 1.38

$$\frac{1}{4}x^4 + \frac{1}{8}(x^4 + 4x^2)^2 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1), x, algorithm="giac")

[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+2*x)*(x^4+4*x^2+1), x)

[Out] 1/8*x^8+x^6+9/4*x^4+x^2

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$\frac{1}{8}(x^4 + 4x^2 + 1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")

[Out] 1/8*(x^4 + 4*x^2 + 1)^2

mupad [B] time = 0.03, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)

[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)

[Out] x**8/8 + x**6 + 9*x**4/4 + x**2

$$3.217 \quad \int (1 + 2x) (x + x^2)^3 \left(-18 + 7(x + x^2)^3 \right)^2 dx$$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10} - 36x^7(x+1)^7 + 81x^4(x+1)^4$$

Rubi [B] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1593, 1612}

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int (1 + 2x) (x + x^2)^3 \left(-18 + 7(x + x^2)^3 \right)^2 dx &= \int x^3(1 + x)^3(1 + 2x) \left(-18 + 7(x + x^2)^3 \right)^2 dx \\ &= \int (324x^3 + 1620x^4 + 2916x^5 + 2016x^6 - 1368x^7 - 6804x^8 - 12551x^9 - 756x^{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10}) dx \\ &= 81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + 336x^{13} + 993x^{14} + \frac{6174x^{15}}{5} + 1029x^{16} + 588x^{17} + \frac{441x^{18}}{2} + 49x^{19} + \frac{49x^{20}}{10} \end{aligned}$$

Mathematica [B] time = 0.01, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2, x]

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 2x)(x + x^2)^3 \left(-18 + 7(x + x^2)^3\right)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2, x]

[Out] IntegrateAlgebraic[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2, x]

fricas [B] time = 0.73, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")

[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4

giac [A] time = 0.29, size = 28, normalized size = 0.85

$$\frac{49}{10}(x^2 + x)^{10} - 36(x^2 + x)^7 + 81(x^2 + x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")

[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)`

[Out] $49/10*x^{20}+49*x^{19}+441/2*x^{18}+588*x^{17}+1029*x^{16}+6174/5*x^{15}+993*x^{14}+336*x^{13}-1071/2*x^{12}-1211*x^{11}-12551/10*x^{10}-756*x^9-171*x^8+288*x^7+486*x^6+324*x^5+81*x^4$

maxima [B] time = 0.44, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")`

[Out] $49/10*x^{20} + 49*x^{19} + 441/2*x^{18} + 588*x^{17} + 1029*x^{16} + 6174/5*x^{15} + 993*x^{14} + 336*x^{13} - 1071/2*x^{12} - 1211*x^{11} - 12551/10*x^{10} - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

mupad [B] time = 0.22, size = 86, normalized size = 2.61

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)`

[Out] $81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^{10})/10 - 1211*x^{11} - (1071*x^{12})/2 + 336*x^{13} + 993*x^{14} + (6174*x^{15})/5 + 1029*x^{16} + 588*x^{17} + (441*x^{18})/2 + 49*x^{19} + (49*x^{20})/10$

sympy [B] time = 0.08, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)`

[Out] $49*x^{20}/10 + 49*x^{19} + 441*x^{18}/2 + 588*x^{17} + 1029*x^{16} + 6174*x^{15}/5 + 993*x^{14} + 336*x^{13} - 1071*x^{12}/2 - 1211*x^{11} - 12551*x^{10}/10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4$

$$3.218 \quad \int x^3(1+x)^3(1+2x)(-18+7x^3(1+x)^3)^2 dx$$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10} - 36x^7(x+1)^7 + 81x^4(x+1)^4$$

Rubi [B] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1612}

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Antiderivative was successfully verified.

[In] Int[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x]

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int x^3(1+x)^3(1+2x)(-18+7x^3(1+x)^3)^2 dx &= \int (324x^3 + 1620x^4 + 2916x^5 + 2016x^6 - 1368x^7 - 6804x^8 - 12551x^9 - 1211x^{10} - \frac{1071x^{11}}{2} + \frac{336x^{12}}{5} + 993x^{13} + 1029x^{14} + 588x^{15} + 49x^{16} + \frac{441x^{17}}{2} + 49x^{18} + \frac{49x^{19}}{10} + \frac{49x^{20}}{10}) dx \\ &= 81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - \frac{12551x^{10}}{10} - 1211x^{11} - \frac{1071x^{12}}{2} + \frac{336x^{13}}{5} + 993x^{14} + 1029x^{15} + 588x^{16} + \frac{441x^{17}}{2} + 49x^{18} + \frac{49x^{19}}{10} + \frac{49x^{20}}{10} \end{aligned}$$

Mathematica [B] time = 0.00, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x]

[Out] $81x^4 + 324x^5 + 486x^6 + 288x^7 - 171x^8 - 756x^9 - (12551x^{10})/10 - 1211x^{11} - (1071x^{12})/2 + 336x^{13} + 993x^{14} + (6174x^{15})/5 + 1029x^{16} + 588x^{17} + (441x^{18})/2 + 49x^{19} + (49x^{20})/10$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(1+x)^3(1+2x)(-18+7x^3(1+x)^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x]

[Out] IntegrateAlgebraic[x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2, x]

fricas [B] time = 0.66, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")

[Out] $49/10x^{20} + 49x^{19} + 441/2x^{18} + 588x^{17} + 1029x^{16} + 6174/5x^{15} + 993x^{14} + 336x^{13} - 1071/2x^{12} - 1211x^{11} - 12551/10x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$

giac [A] time = 0.36, size = 28, normalized size = 0.85

$$\frac{49}{10}(x^2+x)^{10} - 36(x^2+x)^7 + 81(x^2+x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")

[Out] $49/10*(x^2+x)^{10} - 36*(x^2+x)^7 + 81*(x^2+x)^4$

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x+1)^3*(2*x+1)*(-18+7*x^3*(x+1)^3)^2,x)

[Out] $49/10x^{20} + 49x^{19} + 441/2x^{18} + 588x^{17} + 1029x^{16} + 6174/5x^{15} + 993x^{14} + 336x^{13} - 1071/2x^{12} - 1211x^{11} - 12551/10x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$

maxima [B] time = 0.45, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")

[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 993*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4

mupad [B] time = 0.19, size = 86, normalized size = 2.61

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)

[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10

sympy [B] time = 0.09, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)

[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4

$$3.219 \quad \int \frac{2-x^2}{(1-6x+x^3)^5} dx$$

Optimal. Leaf size=14

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1588}

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Antiderivative was successfully verified.

[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] 1/(12*(1 - 6*x + x^3)^4)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{2-x^2}{(1-6x+x^3)^5} dx = \frac{1}{12(1-6x+x^3)^4}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]

[Out] $1/(12*(1 - 6*x + x^3)^4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 - x^2}{(1 - 6x + x^3)^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - x^2)/(1 - 6*x + x^3)^5, x]

[Out] IntegrateAlgebraic[(2 - x^2)/(1 - 6*x + x^3)^5, x]

fricas [B] time = 0.84, size = 57, normalized size = 4.07

$$\frac{1}{12(x^{12} - 24x^{10} + 4x^9 + 216x^8 - 72x^7 - 858x^6 + 432x^5 + 1224x^4 - 860x^3 + 216x^2 - 24x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5, x, algorithm="fricas")

[Out] $1/12/(x^{12} - 24*x^{10} + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*x^4 - 860*x^3 + 216*x^2 - 24*x + 1)$

giac [A] time = 0.32, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5, x, algorithm="giac")

[Out] $1/12/(x^3 - 6*x + 1)^4$

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2)/(x^3-6*x+1)^5, x)

[Out] $1/12/(x^3-6*x+1)^4$

maxima [A] time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")

[Out] 1/12/(x^3 - 6*x + 1)^4

mupad [B] time = 2.10, size = 12, normalized size = 0.86

$$\frac{1}{12(x^3 - 6x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)

[Out] 1/(12*(x^3 - 6*x + 1)^4)

sympy [B] time = 0.19, size = 56, normalized size = 4.00

$$\frac{1}{12x^{12} - 288x^{10} + 48x^9 + 2592x^8 - 864x^7 - 10296x^6 + 5184x^5 + 14688x^4 - 10320x^3 + 2592x^2 - 288x + 12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)

[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 5184*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)

$$3.220 \quad \int \frac{2x+x^2}{4+3x^2+x^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1587}

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]

[Out] Log[4 + 3*x^2 + x^3]/3

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} dx = \frac{1}{3} \log(4 + 3x^2 + x^3)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]

[Out] Log[4 + 3*x^2 + x^3]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*x + x^2)/(4 + 3*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(2*x + x^2)/(4 + 3*x^2 + x^3), x]

fricas [A] time = 1.02, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(x^3+3*x^2+4), x, algorithm="fricas")

[Out] 1/3*log(x^3 + 3*x^2 + 4)

giac [A] time = 0.37, size = 14, normalized size = 0.93

$$\frac{1}{3} \log(|x^3 + 3x^2 + 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(x^3+3*x^2+4), x, algorithm="giac")

[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x)/(x^3+3*x^2+4), x)

[Out] 1/3*ln(x^3+3*x^2+4)

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{3} \log(x^3 + 3x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")`

[Out] `1/3*log(x^3 + 3*x^2 + 4)`

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + x^2)/(3*x^2 + x^3 + 4),x)`

[Out] `log(3*x^2 + x^3 + 4)/3`

sympy [A] time = 0.09, size = 12, normalized size = 0.80

$$\frac{\log(x^3 + 3x^2 + 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+2*x)/(x**3+3*x**2+4),x)`

[Out] `log(x**3 + 3*x**2 + 4)/3`

$$3.221 \quad \int \frac{1+x+x^3}{4x+2x^2+x^4} dx$$

Optimal. Leaf size=17

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1587}

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

[Out] Log[4*x + 2*x^2 + x^4]/4

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} dx = \frac{1}{4} \log(4x+2x^2+x^4)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{1}{4} \log(x^3 + 2x + 4) + \frac{\log(x)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x + x^3}{4x + 2x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]

fricas [A] time = 0.77, size = 15, normalized size = 0.88

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x), x, algorithm="fricas")

[Out] 1/4*log(x^4 + 2*x^2 + 4*x)

giac [A] time = 0.27, size = 18, normalized size = 1.06

$$\frac{1}{4} \log\left(4 \left|\frac{1}{4}x^4 + \frac{1}{2}x^2 + x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x), x, algorithm="giac")

[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln((x^3 + 2x + 4)x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+1)/(x^4+2*x^2+4*x), x)

[Out] 1/4*ln(x*(x^3+2*x+4))

maxima [A] time = 0.45, size = 15, normalized size = 0.88

$$\frac{1}{4} \log(x^4 + 2x^2 + 4x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")

[Out] 1/4*log(x^4 + 2*x^2 + 4*x)

mupad [B] time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\left(x^3 + 2x + 4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)

[Out] log(x*(2*x + x^3 + 4))/4

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4 + 2x^2 + 4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)

[Out] log(x**4 + 2*x**2 + 4*x)/4

$$3.222 \quad \int \frac{bc-ad-2aex-bex^2-3afx^2-2bf x^3}{(c+dx+ex^2+fx^3)^2} dx$$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {6, 2102, 1588}

$$\frac{a}{c+dx+ex^2+fx^3} + \frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]

[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_)^(p_.)), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 2102

Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m + n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn + (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bf x^3}{(c + dx + ex^2 + fx^3)^2} dx &= \int \frac{bc - ad - 2aex + (-be - 3af)x^2 - 2bf x^3}{(c + dx + ex^2 + fx^3)^2} dx \\
&= \frac{bx}{c + dx + ex^2 + fx^3} - \frac{\int \frac{2adf + 4aefx + 6af^2x^2}{(c + dx + ex^2 + fx^3)^2} dx}{2f} \\
&= \frac{a}{c + dx + ex^2 + fx^3} + \frac{bx}{c + dx + ex^2 + fx^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.58

$$\frac{a + bx}{c + dx + ex^2 + fx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]

[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bf x^3}{(c + dx + ex^2 + fx^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x]

[Out] IntegrateAlgebraic[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2, x]

fricas [A] time = 1.00, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="fricas")

[Out] $(b*x + a)/(f*x^3 + e*x^2 + d*x + c)$

giac [A] time = 0.54, size = 24, normalized size = 0.60

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="giac")

[Out] $(b*x + a)/(f*x^3 + x^2*e + d*x + c)$

maple [A] time = 0.01, size = 28, normalized size = 0.70

$$-\frac{-bx - a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)

[Out] $-(-b*x-a)/(f*x^3+e*x^2+d*x+c)$

maxima [A] time = 0.50, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x, algorithm="maxima")

[Out] $(b*x + a)/(f*x^3 + e*x^2 + d*x + c)$

mupad [B] time = 0.11, size = 23, normalized size = 0.58

$$\frac{a + bx}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x^2 + f*x^3)^2,x)

[Out] $(a + b*x)/(c + d*x + e*x^2 + f*x^3)$

sympy [A] time = 32.56, size = 22, normalized size = 0.55

$$\frac{-a - bx}{c + dx + ex^2 + fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

$$3.223 \quad \int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} dx$$

Optimal. Leaf size=605

$$\tan^{-1} \left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2} \sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}} \right) \left(-a \left(A \left(b - \sqrt{8a^2-4ac+b^2} \right) - C \sqrt{8a^2-4ac+b^2} + bC + 2cD \right) + bD \right) \sqrt{2} a \sqrt{8a^2-4ac+b^2} \sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}$$

Rubi [A] time = 4.54, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2086, 634, 618, 204, 628}

$$\frac{\tan^{-1} \left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2} \sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}} \right) \left(-a \left(A \left(b - \sqrt{8a^2-4ac+b^2} \right) - C \sqrt{8a^2-4ac+b^2} + bC + 2cD \right) + bD \right) \sqrt{2} a \sqrt{8a^2-4ac+b^2} \sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}}{\sqrt{2} a \sqrt{8a^2-4ac+b^2} \sqrt{-b(b-\sqrt{8a^2-4ac+b^2})+4a^2+2ac}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]

[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])])])]/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])]/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2086

Int[(P3_)/((a_) + (b_)*(x_) + (c_)*(x_)^2 + (d_)*(x_)^3 + (e_)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{A + Bx + Cx^2 + Dx^3}{a + bx + cx^2 + bx^3 + ax^4} dx &= -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}D)x}{2a+(b-\sqrt{8a^2+b^2-4ac})x+2ax^2} dx}{\sqrt{8a^2+b^2-4ac}} + \frac{\int \frac{Ab-2aB+A\sqrt{8a^2+b^2-4ac}}{2a+(b+\sqrt{8a^2+b^2-4ac})x+2ax^2} dx}{\sqrt{8a^2+b^2-4ac}} \\
&= -\frac{(2a(A-C) + (b - \sqrt{8a^2+b^2-4ac})D) \int \frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{2a+(b-\sqrt{8a^2+b^2-4ac})x+2ax^2} dx}{4a\sqrt{8a^2+b^2-4ac}} + \frac{(2a(A+C) + (b + \sqrt{8a^2+b^2-4ac})D) \int \frac{b+\sqrt{8a^2+b^2-4ac}+4ax}{2a+(b+\sqrt{8a^2+b^2-4ac})x+2ax^2} dx}{4a\sqrt{8a^2+b^2-4ac}} \\
&= -\frac{(2a(A-C) + (b - \sqrt{8a^2+b^2-4ac})D) \log(2a + (b - \sqrt{8a^2+b^2-4ac})x)}{4a\sqrt{8a^2+b^2-4ac}} + \frac{(2a(A+C) + (b + \sqrt{8a^2+b^2-4ac})D) \log(2a + (b + \sqrt{8a^2+b^2-4ac})x)}{4a\sqrt{8a^2+b^2-4ac}} \\
&= \frac{(4a^2B + b(b - \sqrt{8a^2+b^2-4ac})D - a(A(b - \sqrt{8a^2+b^2-4ac}) + bC - \sqrt{8a^2+b^2-4ac})) \log(2a + (b - \sqrt{8a^2+b^2-4ac})x) - (4a^2B + b(b + \sqrt{8a^2+b^2-4ac})D - a(A(b + \sqrt{8a^2+b^2-4ac}) + bC + \sqrt{8a^2+b^2-4ac})) \log(2a + (b + \sqrt{8a^2+b^2-4ac})x)}{4a\sqrt{8a^2+b^2-4ac}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 98, normalized size = 0.16

$$\text{RootSum}\left[\#1^4a + \#1^3b + \#1^2c + \#1b + a\&, \frac{\#1^3D \log(x - \#1) + \#1^2C \log(x - \#1) + A \log(x - \#1) + \#1B \log(x - \#1)}{4\#1^3a + 3\#1^2b + 2\#1c + b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]

[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 + 4*a*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{a + bx + cx^2 + bx^3 + ax^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]

[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Evaluation time: 2.16Not invertible Error: Bad
Argument Value
```

maple [B] time = 0.09, size = 2105, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a), x)
```

```
[Out] -1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)
*A+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*
a)*C+1/4/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D-1/4/a/(8*a^2-4*
a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D*b-1/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+
b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A+1/(8
*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)
*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*
a*c+b^2)^(1/2))^(1/2))*b*A-1/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2)
)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8
*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^
2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^2-4*a
*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan(
(-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)
^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^
```

$$\begin{aligned}
& (2-4ac+b^2)^{(1/2)} \arctan\left(\frac{-4ax+(8a^2-4ac+b^2)^{(1/2)}-b}{(8a^2+4ac-2b^2+2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D b^2 + 1/a \cdot (8a^2+4ac-2b^2+2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{-4ax+(8a^2-4ac+b^2)^{(1/2)}-b}{(8a^2+4ac-2b^2+2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D b - 4a \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2+2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{-4ax+(8a^2-4ac+b^2)^{(1/2)}-b}{(8a^2+4ac-2b^2+2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + B + 1/2 \cdot (8a^2-4ac+b^2)^{(1/2)} \ln(2ax^2+(8a^2-4ac+b^2)^{(1/2)}x+b^2+2a) \\
& + A - 1/2 \cdot (8a^2-4ac+b^2)^{(1/2)} \ln(2ax^2+(8a^2-4ac+b^2)^{(1/2)}x+b^2+2a) \\
& + C + 1/4 \cdot a \ln(2ax^2+(8a^2-4ac+b^2)^{(1/2)}x+b^2+2a) \\
& + D + 1/4 \cdot a \cdot (8a^2-4ac+b^2)^{(1/2)} \ln(2ax^2+(8a^2-4ac+b^2)^{(1/2)}x+b^2+2a) \\
& + D b + 1 \cdot (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + A + 1 \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + B \cdot A + 1 \cdot (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + C + 1 \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D \cdot C + 2 \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D \cdot C - 1/a \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D b^2 - 1/a \cdot (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + D b - 4a \cdot (8a^2-4ac+b^2)^{(1/2)} / (8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)} \arctan\left(\frac{(b+4ax+(8a^2-4ac+b^2)^{(1/2)})}{(8a^2+4ac-2b^2-2b(8a^2-4ac+b^2)^{(1/2)})^{(1/2)}}\right) \\
& + B
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxima")

[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + Bx + Cx^2 + x^3 D}{ax^4 + bx^3 + cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a), x)
```

```
[Out] Timed out
```

$$3.224 \quad \int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx$$

Optimal. Leaf size=63

$$-\frac{2 \log(2x^2 - (1 - \sqrt{5})x + 2)}{1 - \sqrt{5}} - \frac{2 \log(2x^2 - (1 + \sqrt{5})x + 2)}{1 + \sqrt{5}}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2086, 628}

$$-\frac{2 \log(2x^2 - (1 - \sqrt{5})x + 2)}{1 - \sqrt{5}} - \frac{2 \log(2x^2 - (1 + \sqrt{5})x + 2)}{1 + \sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[5])*x + 2*x^2])/(1 + Sqrt[5])

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 2086

Int[(P3_)/((a_) + (b_)*(x_) + (c_)*(x_)^2 + (d_)*(x_)^3 + (e_)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx = -\frac{\int \frac{-2\sqrt{5}+(10-2\sqrt{5})x}{2+(-1-\sqrt{5})x+2x^2} dx}{\sqrt{5}} + \frac{\int \frac{2\sqrt{5}+(10+2\sqrt{5})x}{2+(-1+\sqrt{5})x+2x^2} dx}{\sqrt{5}}$$

$$= -\frac{2 \log(2 - (1 - \sqrt{5})x + 2x^2)}{1 - \sqrt{5}} - \frac{2 \log(2 - (1 + \sqrt{5})x + 2x^2)}{1 + \sqrt{5}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.87

$$\frac{1}{2} \left((1 + \sqrt{5}) \log(2x^2 + (\sqrt{5} - 1)x + 2) - (\sqrt{5} - 1) \log(-2x^2 + \sqrt{5}x + x - 2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 + (-1 + Sqrt[5])*x + 2*x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

[Out] IntegrateAlgebraic[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]

fricas [A] time = 1.87, size = 83, normalized size = 1.32

$$\frac{1}{2} \sqrt{5} \log\left(\frac{2x^4 - 2x^3 + 7x^2 + \sqrt{5}(2x^3 - x^2 + 2x) - 2x + 2}{x^4 - x^3 + x^2 - x + 1}\right) + \frac{1}{2} \log(x^4 - x^3 + x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1), x, algorithm="fricas")

[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x + 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)

giac [A] time = 0.27, size = 58, normalized size = 0.92

$$-\frac{1}{2} \sqrt{5} \log\left(x^2 - \frac{1}{2}x(\sqrt{5} + 1) + 1\right) + \frac{1}{2} \sqrt{5} \log\left(x^2 + \frac{1}{2}x(\sqrt{5} - 1) + 1\right) + \frac{1}{2} \log(x^4 - x^3 + x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")

[Out] $-1/2*\sqrt{5}*\log(x^2 - 1/2*x*(\sqrt{5} + 1) + 1) + 1/2*\sqrt{5}*\log(x^2 + 1/2*x*(\sqrt{5} - 1) + 1) + 1/2*\log(x^4 - x^3 + x^2 - x + 1)$

maple [A] time = 0.05, size = 82, normalized size = 1.30

$$\frac{\ln(2x^2 - \sqrt{5}x - x + 2)}{2} - \frac{\sqrt{5} \ln(2x^2 - \sqrt{5}x - x + 2)}{2} + \frac{\ln(2x^2 + \sqrt{5}x - x + 2)}{2} + \frac{\sqrt{5} \ln(2x^2 + \sqrt{5}x - x + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)

[Out] $1/2*\ln(x*5^{(1/2)}+2*x^2-x+2)+1/2*\ln(x*5^{(1/2)}+2*x^2-x+2)*5^{(1/2)}+1/2*\ln(-x*5^{(1/2)}+2*x^2-x+2)-1/2*\ln(-x*5^{(1/2)}+2*x^2-x+2)*5^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 - 4x^2 + x + 2}{x^4 - x^3 + x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")

[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)

mupad [B] time = 0.18, size = 75, normalized size = 1.19

$$\frac{\ln\left(x^2 - \frac{\sqrt{5}x}{2} - \frac{x}{2} + 1\right)}{2} + \frac{\ln\left(\frac{\sqrt{5}x}{2} - \frac{x}{2} + x^2 + 1\right)}{2} - \frac{\sqrt{5} \ln\left(x^2 - \frac{\sqrt{5}x}{2} - \frac{x}{2} + 1\right)}{2} + \frac{\sqrt{5} \ln\left(\frac{\sqrt{5}x}{2} - \frac{x}{2} + x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)

[Out] $\log(x^2 - (5^{(1/2)}*x)/2 - x/2 + 1)/2 + \log((5^{(1/2)}*x)/2 - x/2 + x^2 + 1)/2 - (5^{(1/2)}*\log(x^2 - (5^{(1/2)}*x)/2 - x/2 + 1))/2 + (5^{(1/2)}*\log((5^{(1/2)}*x)/2 - x/2 + x^2 + 1))/2$

sympy [A] time = 0.13, size = 58, normalized size = 0.92

$$\left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) \log\left(x^2 + x\left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right) + 1\right) + \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \log\left(x^2 + x\left(-\frac{\sqrt{5}}{2} - \frac{1}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*  
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```


$$3.225 \quad \int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1594, 1680, 14}

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]

Rule 14

Int[(u_)*((c_)*(x_)^(m_)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} dx &= \int \frac{x(3 + 3x + x^2)}{1 + 4x + 6x^2 + 4x^3 + x^4} dx \\
&= \text{Subst} \left(\int \frac{-1 + x^3}{x^4} dx, x, 1 + x \right) \\
&= \text{Subst} \left(\int \left(-\frac{1}{x^4} + \frac{1}{x} \right) dx, x, 1 + x \right) \\
&= \frac{1}{3(1+x)^3} + \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] 1/(3*(1 + x)^3) + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] IntegrateAlgebraic[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

fricas [B] time = 1.17, size = 38, normalized size = 2.71

$$\frac{3(x^3 + 3x^2 + 3x + 1)\log(x + 1) + 1}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1), x, algorithm="fricas")

[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)

giac [A] time = 0.29, size = 13, normalized size = 0.93

$$\frac{1}{3(x+1)^3} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")

[Out] 1/3/(x + 1)^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$\ln(x+1) + \frac{1}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)

[Out] 1/3/(x+1)^3+ln(x+1)

maxima [A] time = 0.44, size = 22, normalized size = 1.57

$$\frac{1}{3(x^3 + 3x^2 + 3x + 1)} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")

[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$\ln(x+1) + \frac{1}{3(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)

[Out] log(x + 1) + 1/(3*(x + 1)^3)

sympy [A] time = 0.10, size = 20, normalized size = 1.43

$$\log(x+1) + \frac{1}{3x^3 + 9x^2 + 9x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

$$3.226 \quad \int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} dx$$

Optimal. Leaf size=28

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1680, 43}

$$\frac{6}{x+1} - \frac{6}{(x+1)^2} + \frac{8}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1 + 3x - 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} dx &= \text{Subst} \left(\int \frac{(-2 + x)^3}{x^4} dx, x, 1 + x \right) \\ &= \text{Subst} \left(\int \left(-\frac{8}{x^4} + \frac{12}{x^3} - \frac{6}{x^2} + \frac{1}{x} \right) dx, x, 1 + x \right) \\ &= \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.86

$$\frac{2(9x^2 + 9x + 4)}{3(x+1)^3} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 3x - 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

[Out] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]

fricas [A] time = 1.10, size = 46, normalized size = 1.64

$$\frac{18x^2 + 3(x^3 + 3x^2 + 3x + 1)\log(x+1) + 18x + 8}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1), x, algorithm="fricas")

[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2 + 3*x + 1)

giac [A] time = 0.39, size = 23, normalized size = 0.82

$$\frac{2(9x^2 + 9x + 4)}{3(x+1)^3} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")

[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\ln(x+1) + \frac{8}{3(x+1)^3} - \frac{6}{(x+1)^2} + \frac{6}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)

[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)

maxima [A] time = 0.45, size = 32, normalized size = 1.14

$$\frac{2(9x^2 + 9x + 4)}{3(x^3 + 3x^2 + 3x + 1)} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")

[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)

mupad [B] time = 0.04, size = 21, normalized size = 0.75

$$\ln(x+1) + \frac{6x^2 + 6x + \frac{8}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)

[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3

sympy [A] time = 0.11, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

```
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```


$$3.227 \quad \int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3} dx$$

Optimal. Leaf size=59

$$\frac{13x}{x^4 + 2x^2 + 3} - \frac{2(13x^2 + 18)x}{(x^4 + 2x^2 + 3)^2} + \frac{2(1 - 2x^2)}{(x^4 + 2x^2 + 3)^2}$$

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {1673, 1678, 1588, 1663, 1660, 8}

$$\frac{13x}{x^4 + 2x^2 + 3} - \frac{2(13x^2 + 18)x}{(x^4 + 2x^2 + 3)^2} + \frac{2(1 - 2x^2)}{(x^4 + 2x^2 + 3)^2}$$

Antiderivative was successfully verified.

[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3,x]

[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)^2 + (13*x)/(3 + 2*x^2 + x^4)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2

- 4*a*c, 0] && LtQ[p, -1]

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :=> With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{9 - 40x - 18x^2 + 174x^4 + 24x^5 + 26x^6 - 39x^8}{(3 + 2x^2 + x^4)^3} dx &= \int \frac{x(-40 + 24x^4)}{(3 + 2x^2 + x^4)^3} dx + \int \frac{9 - 18x^2 + 174x^4 + 26x^6 - 39x^8}{(3 + 2x^2 + x^4)^3} dx \\
&= -\frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{1}{96} \int \frac{3744 - 2496x^2 - 3744x^4}{(3 + 2x^2 + x^4)^2} dx \\
&= \frac{2(1 - 2x^2)}{(3 + 2x^2 + x^4)^2} - \frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{13x}{3 + 2x^2 + x^4} + \frac{1}{32} \\
&= \frac{2(1 - 2x^2)}{(3 + 2x^2 + x^4)^2} - \frac{2x(18 + 13x^2)}{(3 + 2x^2 + x^4)^2} + \frac{13x}{3 + 2x^2 + x^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3,x]

[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9 - 40x - 18x^2 + 174x^4 + 24x^5 + 26x^6 - 39x^8}{(3 + 2x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x^4)^3, x]

fricas [A] time = 1.87, size = 38, normalized size = 0.64

$$\frac{13x^5 - 4x^2 + 3x + 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")

[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

giac [A] time = 1.44, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")

[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2

maple [A] time = 0.01, size = 30, normalized size = 0.51

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)

[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2

maxima [A] time = 0.46, size = 38, normalized size = 0.64

$$\frac{13x^5 - 4x^2 + 3x + 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")

[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)

mupad [B] time = 0.05, size = 28, normalized size = 0.47

$$\frac{13x^5 - 4x^2 + 3x + 2}{(x^4 + 2x^2 + 3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

sympy [A] time = 0.18, size = 36, normalized size = 0.61

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3)**3,x)
```

```
[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

$$3.228 \quad \int \frac{-1+4x^5}{(1+x+x^5)^2} dx$$

Optimal. Leaf size=11

$$-\frac{x}{x^5 + x + 1}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1588}

$$-\frac{x}{x^5 + x + 1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]*Pp, x, q)], x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{-1 + 4x^5}{(1 + x + x^5)^2} dx = -\frac{x}{1 + x + x^5}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 4x^5}{(1 + x + x^5)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] IntegrateAlgebraic[(-1 + 4*x^5)/(1 + x + x^5)^2, x]

fricas [A] time = 1.22, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")

[Out] -x/(x^5 + x + 1)

giac [A] time = 0.33, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")

[Out] -x/(x^5 + x + 1)

maple [B] time = 0.01, size = 41, normalized size = 3.73

$$-\frac{-3x^2 + 5x - 1}{7(x^3 - x^2 + 1)} + \frac{-3x - 1}{7x^2 + 7x + 7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5-1)/(x^5+x+1)^2,x)

[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)

maxima [A] time = 0.44, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")

[Out] -x/(x^5 + x + 1)

mupad [B] time = 2.30, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)

[Out] -x/(x + x^5 + 1)

sympy [A] time = 0.12, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)

[Out] -x/(x**5 + x + 1)

$$3.229 \quad \int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx$$

Optimal. Leaf size=91

$$\frac{x}{16(1-x^2)} + \frac{(29-5x^2)x}{32(x^4-6x^2+1)} + \frac{1}{4} \tanh^{-1}(x) + \frac{1}{64} \left((3-2\sqrt{2}) \tanh^{-1}\left(\frac{(\sqrt{2}-1)x}{1+x}\right) - (3+2\sqrt{2}) \tanh^{-1}\left(\frac{(\sqrt{2}+1)x}{1+x}\right) \right)$$

Rubi [B] time = 0.15, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2073, 207, 638, 618, 206, 632, 31}

$$-\frac{12-5x}{64(-x^2+2x+1)} + \frac{5x+12}{64(-x^2-2x+1)} + \frac{1}{32(1-x)} - \frac{1}{32(x+1)} - \frac{3}{256}(2+3\sqrt{2})\log(-x-\sqrt{2}+1) - \frac{3}{256}(2-3\sqrt{2})\log(-x+\sqrt{2}+1) + \frac{3}{256}(2+3\sqrt{2})\log(x-\sqrt{2}+1) + \frac{3}{256}(2-3\sqrt{2})\log(x+\sqrt{2}+1) - \frac{5 \tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} + \frac{1}{4} \tanh^{-1}(x) + \frac{5 \tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcTanh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] + x])/256

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx &= \int \left(\frac{1}{32(-1+x)^2} + \frac{1}{32(1+x)^2} - \frac{1}{4(-1+x^2)} + \frac{17-7x}{32(-1-2x+x^2)^2} - \frac{3(-4+x)}{64(-1-2x+x^2)} \right) dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{1}{32} \int \frac{17-7x}{(-1-2x+x^2)^2} dx + \frac{1}{32} \int \frac{17+7x}{(-1+2x+x^2)^2} dx - \frac{3}{64} \int \frac{1}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} + \frac{1}{4} \tanh^{-1}(x) - \frac{5}{64} \int \frac{1}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} + \frac{1}{4} \tanh^{-1}(x) - \frac{3}{256} \int \frac{1}{-1-2x+x^2} dx \\ &= \frac{1}{32(1-x)} - \frac{1}{32(1+x)} + \frac{12+5x}{64(1-2x-x^2)} - \frac{12-5x}{64(1+2x-x^2)} - \frac{5 \tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} + \frac{1}{4} \int \frac{1}{-1-2x+x^2} dx \end{aligned}$$

Mathematica [A] time = 0.09, size = 132, normalized size = 1.45

$$\frac{1}{128} \left(-\frac{4x(7x^4 - 46x^2 + 31)}{x^6 - 7x^4 + 7x^2 - 1} - 16 \log(1-x) + (3+2\sqrt{2}) \log(-x+\sqrt{2}-1) + (2\sqrt{2}-3) \log(-x+\sqrt{2}+1) + 16 \log(x+1) - (3+2\sqrt{2}) \log(x+\sqrt{2}-1) + (3-2\sqrt{2}) \log(x+\sqrt{2}+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] + (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] - x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2])*Log[1 + Sqrt[2] + x])/128

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]

fricas [B] time = 2.23, size = 223, normalized size = 2.45

$$\frac{28x^5 - 184x^3 - 2\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x + 3}{x^2 + 2x - 1}\right) - 2\sqrt{2}(x^6 - 7x^4 + 7x^2 - 1) \log\left(\frac{x^2 - 2\sqrt{2}(x-1) - 2x + 3}{x^2 - 2x - 1}\right) - 3(x^6 - 7x^4 + 7x^2 - 1) \log(x^2 + 2x - 1) + 3(x^6 - 7x^4 + 7x^2 - 1) \log(x^2 - 2x - 1) - 16(x^6 - 7x^4 + 7x^2 - 1) \log(x+1) + 16(x^6 - 7x^4 + 7x^2 - 1) \log(x-1) + 124x}{128(x^6 - 7x^4 + 7x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2, x, algorithm="fricas")

[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)

giac [A] time = 0.37, size = 134, normalized size = 1.47

$$\frac{1}{64} \sqrt{2} \log\left(\frac{2x - 2\sqrt{2} + 2}{2x + 2\sqrt{2} + 2}\right) + \frac{1}{64} \sqrt{2} \log\left(\frac{2x - 2\sqrt{2} - 2}{2x + 2\sqrt{2} - 2}\right) - \frac{7x^5 - 46x^3 + 31x}{32(x^6 - 7x^4 + 7x^2 - 1)} + \frac{3}{128} \log(|x^2 + 2x - 1|) - \frac{3}{128} \log(|x^2 - 2x - 1|) + \frac{1}{8} \log(|x + 1|) - \frac{1}{8} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2, x, algorithm="giac")

[Out] $1/64*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} + 2)/\text{abs}(2*x + 2*\sqrt{2} + 2)) + 1/64*\sqrt{2}*\log(\text{abs}(2*x - 2*\sqrt{2} - 2)/\text{abs}(2*x + 2*\sqrt{2} - 2)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*\log(\text{abs}(x^2 + 2*x - 1)) - 3/128*\log(\text{abs}(x^2 - 2*x - 1)) + 1/8*\log(\text{abs}(x + 1)) - 1/8*\log(\text{abs}(x - 1))$

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{32} - \frac{\ln(x-1)}{8} + \frac{\ln(x+1)}{8} - \frac{3 \ln(x^2-2x-1)}{128} + \frac{3 \ln(x^2+2x-1)}{128} - \frac{1}{32(x-1)} + \frac{-5x-12}{64x^2+128x-64} - \frac{1}{32(x+1)} - \frac{5x-12}{64(x^2-2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2, x)$

[Out] $-1/32/(x-1) - 1/8*\ln(x-1) + 1/64*(-5*x-12)/(x^2+2*x-1) + 3/128*\ln(x^2+2*x-1) - 1/32*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*x+2)*2^{(1/2)}) - 1/32/(x+1) + 1/8*\ln(x+1) - 1/64*(-12+5*x)/(x^2-2*x-1) - 3/128*\ln(x^2-2*x-1) - 1/32*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*x-2)*2^{(1/2)})$

maxima [A] time = 0.99, size = 114, normalized size = 1.25

$$\frac{1}{64} \sqrt{2} \log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) + \frac{1}{64} \sqrt{2} \log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{7x^5-46x^3+31x}{32(x^6-7x^4+7x^2-1)} + \frac{3}{128} \log(x^2+2x-1) - \frac{3}{128} \log(x^2-2x-1) + \frac{1}{8} \log(x+1) - \frac{1}{8} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2, x, \text{algorithm}="maxima")$

[Out] $1/64*\sqrt{2}*\log((x - \sqrt{2} + 1)/(x + \sqrt{2} + 1)) + 1/64*\sqrt{2}*\log((x - \sqrt{2} - 1)/(x + \sqrt{2} - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*\log(x^2 + 2*x - 1) - 3/128*\log(x^2 - 2*x - 1) + 1/8*\log(x + 1) - 1/8*\log(x - 1)$

mupad [B] time = 2.37, size = 124, normalized size = 1.36

$$\frac{\operatorname{atan}(x \operatorname{I}) \operatorname{I}}{4} - \frac{7x^5 - 23x^3 + 31x}{x^6 - 7x^4 + 7x^2 - 1} + \operatorname{atan}\left(\frac{x 23313i}{8192 \left(\frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}\right)} - \frac{\sqrt{2} x 65943i}{32768 \left(\frac{27309\sqrt{2}}{32768} - \frac{19317}{16384}\right)}\right) \left(\frac{\sqrt{2} \operatorname{I}}{32} - \frac{3}{64} \operatorname{I}\right) + \operatorname{atan}\left(\frac{x 23313i}{8192 \left(\frac{27309\sqrt{2}}{32768} + \frac{19317}{16384}\right)} + \frac{\sqrt{2} x 65943i}{32768 \left(\frac{27309\sqrt{2}}{32768} + \frac{19317}{16384}\right)}\right) \left(\frac{\sqrt{2} \operatorname{I}}{32} + \frac{3}{64} \operatorname{I}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2, x)$

[Out] $\operatorname{atan}\left(\frac{x*23313i}{8192*((27309*2^{(1/2)})/32768 - 19317/16384)}\right) - (2^{(1/2)}*x*65943i)/(32768*((27309*2^{(1/2)})/32768 - 19317/16384)) * ((2^{(1/2)}*i)/32 - 3i/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (\operatorname{atan}(x*i)*i)/4 + \operatorname{atan}\left(\frac{x*23313i}{8192*((27309*2^{(1/2)})/32768 + 19317/16384)}\right) + (2^{(1/2)}*x*65943i)/(32768*((27309*2^{(1/2)})/32768 + 19317/16384)) * ((2^{(1/2)}*i)/32 + 3i/64)$

sympy [B] time = 1.36, size = 272, normalized size = 2.99

$$\frac{-7x^5 + 46x^3 - 31x}{32x^6 - 224x^4 + 224x^2 - 32} - \frac{\log(x - 1)}{8} + \frac{\log(x + 1)}{8} + \frac{(-3/128 - \sqrt{2}/64) \log(x - 38423555/909328 - 38423555\sqrt{2}/1363992) + 9549859782656(-3/128 - \sqrt{2}/64)^5/170499 - 56267374592(-3/128 - \sqrt{2}/64)^3/56833}{1363992} + \frac{(-3/128 + \sqrt{2}/64) \log(x - 38423555/909328 + 38423555\sqrt{2}/1363992) + 9549859782656(-3/128 + \sqrt{2}/64)^5/170499 - 56267374592(-3/128 + \sqrt{2}/64)^3/56833 + 38423555\sqrt{2}/1363992}{1363992} + \frac{(3/128 - \sqrt{2}/64) \log(x - 56267374592(\sqrt{2}/64 + 3/128)^3/56833 + 9549859782656(\sqrt{2}/64 + 3/128)^5/170499 + 38423555\sqrt{2}/1363992) + (3/128 + \sqrt{2}/64) \log(x - 56267374592(\sqrt{2}/64 - 3/128)^3/56833 + 9549859782656(\sqrt{2}/64 - 3/128)^5/170499 + 38423555\sqrt{2}/1363992)}{1363992}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)

[Out] $(-7x^5 + 46x^3 - 31x)/(32x^6 - 224x^4 + 224x^2 - 32) - \log(x - 1)/8 + \log(x + 1)/8 + (-3/128 - \sqrt{2}/64) \log(x - 38423555/909328 - 38423555\sqrt{2}/1363992) + 9549859782656(-3/128 - \sqrt{2}/64)^5/170499 - 56267374592(-3/128 - \sqrt{2}/64)^3/56833 + (-3/128 + \sqrt{2}/64) \log(x - 38423555/909328 + 38423555\sqrt{2}/1363992) + 9549859782656(-3/128 + \sqrt{2}/64)^5/170499 - 56267374592(-3/128 + \sqrt{2}/64)^3/56833 + 38423555\sqrt{2}/1363992 + (3/128 - \sqrt{2}/64) \log(x - 56267374592(\sqrt{2}/64 + 3/128)^3/56833 + 9549859782656(\sqrt{2}/64 + 3/128)^5/170499 + 38423555\sqrt{2}/1363992) + (3/128 + \sqrt{2}/64) \log(x - 56267374592(\sqrt{2}/64 - 3/128)^3/56833 + 9549859782656(\sqrt{2}/64 - 3/128)^5/170499 + 38423555\sqrt{2}/1363992)$

$$3.230 \quad \int x^m (a + bx + cx^2 + dx^3)^p (a(1 + m) + x(b(2 + m + p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) dx$$

Optimal. Leaf size=25

$$x^{m+1} (a + bx + cx^2 + dx^3)^{p+1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {1590}

$$x^{m+1} (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]

[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1 + m) + x(b(2 + m + p) + x(c(3 + m + 2p) + d(4 + m + 3p)x))) dx = x^{1+m} (a + bx + cx^2 + dx^3)^{p+1}$$

Mathematica [A] time = 0.37, size = 23, normalized size = 0.92

$$x^{m+1}(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]

[Out] $x^{(1+m)*(a+x*(b+x*(c+d*x)))^{(1+p)}}$

IntegrateAlgebraic [F] time = 1.13, size = 0, normalized size = 0.00

$$\int x^m (a + bx + cx^2 + dx^3)^p (a(1+m) + x(b(2+m+p) + x(c(3+m+2p) + d(4+m+3p)x))) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))), x]

[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))), x]

fricas [A] time = 2.02, size = 40, normalized size = 1.60

$$(dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))), x, algorithm="fricas")

[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m

giac [B] time = 2.09, size = 99, normalized size = 3.96

$$(dx^3 + cx^2 + bx + a)^p dx^4 x^m + (dx^3 + cx^2 + bx + a)^p cx^3 x^m + (dx^3 + cx^2 + bx + a)^p bx^2 x^m + (dx^3 + cx^2 + bx + a)^p ax x^m$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))), x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*x*x^m

maple [A] time = 0.01, size = 26, normalized size = 1.04

$$x^{m+1} (dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3*p)*x))), x)

[Out] $x^{(1+m)} \cdot (d \cdot x^3 + c \cdot x^2 + b \cdot x + a)^{(p+1)}$

maxima [A] time = 0.70, size = 44, normalized size = 1.76

$$(dx^4 + cx^3 + bx^2 + ax)e^{(p \log(dx^3 + cx^2 + bx + a) + m \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^m \cdot (d \cdot x^3 + c \cdot x^2 + b \cdot x + a)^p \cdot (a \cdot (1+m) + x \cdot (b \cdot (2+m+p) + x \cdot (c \cdot (3+m+2 \cdot p) + d \cdot (4+m+3 \cdot p) \cdot x)))$), x, algorithm="maxima")

[Out] $(d \cdot x^4 + c \cdot x^3 + b \cdot x^2 + a \cdot x) \cdot e^{(p \cdot \log(d \cdot x^3 + c \cdot x^2 + b \cdot x + a) + m \cdot \log(x))}$

mupad [B] time = 2.66, size = 49, normalized size = 1.96

$$(dx^3 + cx^2 + bx + a)^p (axx^m + bx^m x^2 + cx^m x^3 + dx^m x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int($x^m \cdot (a \cdot (m + 1) + x \cdot (x \cdot (c \cdot (m + 2 \cdot p + 3) + d \cdot x \cdot (m + 3 \cdot p + 4)) + b \cdot (m + p + 2))) \cdot (a + b \cdot x + c \cdot x^2 + d \cdot x^3)^p$), x)

[Out] $(a + b \cdot x + c \cdot x^2 + d \cdot x^3)^p \cdot (a \cdot x \cdot x^m + b \cdot x^m \cdot x^2 + c \cdot x^m \cdot x^3 + d \cdot x^m \cdot x^4)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate($x^m \cdot (d \cdot x^3 + c \cdot x^2 + b \cdot x + a)^p \cdot (a \cdot (1+m) + x \cdot (b \cdot (2+m+p) + x \cdot (c \cdot (3+m+2 \cdot p) + d \cdot (4+m+3 \cdot p) \cdot x)))$), x)

[Out] Timed out

3.231

$$\int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)x^3) dx$$

Optimal. Leaf size=23

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1588}

$$x^3 (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]

[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)x^3) dx = x^3 (a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.23, size = 21, normalized size = 0.91

$$x^3 (a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]

[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)

IntegrateAlgebraic [F] time = 0.96, size = 0, normalized size = 0.00

$$\int x^2 (a + bx + cx^2 + dx^3)^p (3a + b(4 + p)x + c(5 + 2p)x^2 + d(6 + 3p)x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3), x]

[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3), x]

fricas [A] time = 1.81, size = 39, normalized size = 1.70

$$(dx^6 + cx^5 + bx^4 + ax^3)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3), x, algorithm="fricas")

[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p

giac [B] time = 1.89, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^6 + (dx^3 + cx^2 + bx + a)^p cx^5 + (dx^3 + cx^2 + bx + a)^p bx^4 + (dx^3 + cx^2 + bx + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3), x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^3 (d x^3 + c x^2 + b x + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3), x)

[Out] x^3*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.66, size = 39, normalized size = 1.70

$$(dx^6 + cx^5 + bx^4 + ax^3)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x, algorithm="maxima")

[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p

mupad [B] time = 2.55, size = 39, normalized size = 1.70

$$(dx^3 + cx^2 + bx + a)^p (dx^6 + cx^5 + bx^4 + ax^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) + d*x^3*(3*p + 6)),x)

[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+3*p)*x**3),x)

[Out] Timed out

3.232

$$\int x (a + bx + cx^2 + dx^3)^p (2a + b(3 + p)x + c(4 + 2p)x^2 + d(5 + 3p)x^3)$$

Optimal. Leaf size=23

$$x^2 (a + bx + cx^2 + dx^3)^{p+1}$$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1588}

$$x^2 (a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3),x]

[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]},
Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x]
]; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]
]; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x (a + bx + cx^2 + dx^3)^p (2a + b(3 + p)x + c(4 + 2p)x^2 + d(5 + 3p)x^3) dx = x^2 (a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$x^2(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3),x]

[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)

IntegrateAlgebraic [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x (a + bx + cx^2 + dx^3)^p (2a + b(3 + p)x + c(4 + 2p)x^2 + d(5 + 3p)x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3), x]

[Out] Defer[IntegrateAlgebraic][x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3), x]

fricas [A] time = 0.80, size = 39, normalized size = 1.70

$$(dx^5 + cx^4 + bx^3 + ax^2)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3), x, algorithm="fricas")

[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p

giac [B] time = 0.73, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^5 + (dx^3 + cx^2 + bx + a)^p cx^4 + (dx^3 + cx^2 + bx + a)^p bx^3 + (dx^3 + cx^2 + bx + a)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3), x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^2 (dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3), x)

[Out] x^2*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.63, size = 39, normalized size = 1.70

$$(dx^5 + cx^4 + bx^3 + ax^2)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.32, size = 39, normalized size = 1.70

$$(dx^3 + cx^2 + bx + a)^p (dx^5 + cx^4 + bx^3 + ax^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*x^3*(3*p + 5)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p)*x**3),x)
```

```
[Out] Timed out
```

3.233

$$\int (a + bx + cx^2 + dx^3)^p (a + b(2 + p)x + c(3 + 2p)x^2 + d(4 + 3p)x^3) dx$$

Optimal. Leaf size=21

$$x(a + bx + cx^2 + dx^3)^{p+1}$$

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {1588}

$$x(a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (a + bx + cx^2 + dx^3)^p (a + b(2 + p)x + c(3 + 2p)x^2 + d(4 + 3p)x^3) dx = x(a + bx + cx^2 + dx^3)^{1+p}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 0.90

$$x(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)

IntegrateAlgebraic [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2 + dx^3)^p (a + b(2 + p)x + c(3 + 2p)x^2 + d(4 + 3p)x^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

[Out] Defer[IntegrateAlgebraic] [(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]

fricas [A] time = 1.26, size = 37, normalized size = 1.76

$$(dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3), x, algorithm="fricas")

[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p

giac [B] time = 1.08, size = 87, normalized size = 4.14

$$(dx^3 + cx^2 + bx + a)^p dx^4 + (dx^3 + cx^2 + bx + a)^p cx^3 + (dx^3 + cx^2 + bx + a)^p bx^2 + (dx^3 + cx^2 + bx + a)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3), x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$x(dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(p+2)*x+c*(2*p+3)*x^2+d*(4+3*p)*x^3), x)

[Out] x*(d*x^3+c*x^2+b*x+a)^(p+1)

maxima [A] time = 0.61, size = 37, normalized size = 1.76

$$(dx^4 + cx^3 + bx^2 + ax)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.27, size = 37, normalized size = 1.76

$$(dx^3 + cx^2 + bx + a)^p (dx^4 + cx^3 + bx^2 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*(3*p + 4)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x**3),x)
```

```
[Out] Timed out
```

$$3.234 \quad \int \frac{(a+bx+cx^2+dx^3)^p (b(1+p)x+c(2+2p)x^2+d(3+3p)x^3)}{x} dx$$

Optimal. Leaf size=19

$$(a + bx + cx^2 + dx^3)^{p+1}$$

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1585, 1588}

$$(a + bx + cx^2 + dx^3)^{p+1}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1588

Int[(Pp_)*(Qq_)^(m_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{(a + bx + cx^2 + dx^3)^p (b(1+p)x + c(2+2p)x^2 + d(3+3p)x^3)}{x} dx = \int (b(1+p) + c(2+2p)x + d(3+3p)x^2) (a + bx + cx^2 + dx^3)^{1+p} dx$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$(a + x(b + x(c + dx)))^{p+1}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2 + dx^3)^p (b(1 + p)x + c(2 + 2p)x^2 + d(3 + 3p)x^3)}{x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x,x]

[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*p)*x^3))/x, x]

fricas [A] time = 0.82, size = 33, normalized size = 1.74

$$(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p

giac [B] time = 0.30, size = 52, normalized size = 2.74

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1} p}{p + 1} + \frac{(dx^3 + cx^2 + bx + a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x, algorithm="giac")

[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p + 1)/(p + 1)

maple [A] time = 0.01, size = 20, normalized size = 1.05

$$(dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)`

[Out] $(d*x^3+c*x^2+b*x+a)^{(p+1)}$

maxima [A] time = 0.62, size = 33, normalized size = 1.74

$$(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p$

mupad [B] time = 2.19, size = 19, normalized size = 1.00

$$(dx^3 + cx^2 + bx + a)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d*x^3)^p)/x,x)`

[Out] $(a + b*x + c*x^2 + d*x^3)^{(p + 1)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3)/x,x)`

[Out] Timed out

$$3.235 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-a+bp x+c(1+2p)x^2+d(2+3p)x^3)}{x^2} dx$$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p (-a+bp x+c(1+2p)x^2+d(2+3p)x^3)}{x^2} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2, x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x

IntegrateAlgebraic [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2 + dx^3)^p (-a + bpx + c(1 + 2p)x^2 + d(2 + 3p)x^3)}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2, x]

[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2, x]

fricas [A] time = 1.28, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(3p + 2)x^3 + c(2p + 1)x^2 + bpx - a)(dx^3 + cx^2 + bx + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2, x, algorithm="giac")

[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)`

[Out] $(d*x^3+c*x^2+b*x+a)^{(p+1)}/x$

maxima [A] time = 0.62, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x$

mupad [B] time = 3.20, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p + 2)))/x^2,x)`

[Out] $(a + b*x + c*x^2 + d*x^3)^{(p + 1)}/x$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)/x**2,x)`

[Out] Timed out

$$3.236 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx$$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3,x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p (-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3} dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] time = 0.20, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2

IntegrateAlgebraic [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2 + dx^3)^p (-2a + b(-1 + p)x + 2cpx^2 + d(1 + 3p)x^3)}{x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]

[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]

fricas [A] time = 1.08, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d(3p + 1)x^3 + 2cpx^2 + b(p - 1)x - 2a)(dx^3 + cx^2 + bx + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="giac")

[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(3*p+1)*x^3)/x^3,x)`

[Out] $(d*x^3+c*x^2+b*x+a)^{(p+1)}/x^2$

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x^3,x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2$

mupad [B] time = 3.32, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*p + 1)))/x^3,x)`

[Out] $(a + b*x + c*x^2 + d*x^3)^{(p + 1)}/x^2$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x**3)/x**3,x)`

[Out] Timed out

$$3.237 \quad \int \frac{(a+bx+cx^2+dx^3)^p (-3a+b(-2+p)x+c(-1+2p)x^2+3dp x^3)}{x^4} dx$$

Optimal. Leaf size=23

$$\frac{(a + bx + cx^2 + dx^3)^{p+1}}{x^3}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {1590}

$$\frac{(a + bx + cx^2 + dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4, x]

[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3

Rule 1590

Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]

Rubi steps

$$\int \frac{(a + bx + cx^2 + dx^3)^p (-3a + b(-2 + p)x + c(-1 + 2p)x^2 + 3dp x^3)}{x^4} dx = \frac{(a + bx + cx^2 + dx^3)^{1+p}}{x^3}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 0.91

$$\frac{(a + x(b + x(c + dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]

[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3

IntegrateAlgebraic [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2 + dx^3)^p (-3a + b(-2 + p)x + c(-1 + 2p)x^2 + 3dp x^3)}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]

[Out] Defer[IntegrateAlgebraic][[(a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4, x]

fricas [A] time = 1.50, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="fricas")

[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3 dp x^3 + c(2p - 1)x^2 + b(p - 2)x - 3a)(dx^3 + cx^2 + bx + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="giac")

[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2 + b*x + a)^p/x^4, x)

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)`

[Out] $(d*x^3+c*x^2+b*x+a)^{(p+1)}/x^3$

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{(dx^3 + cx^2 + bx + a)(dx^3 + cx^2 + bx + a)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x, algorithm="maxima")`

[Out] $(d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3$

mupad [B] time = 3.35, size = 23, normalized size = 1.00

$$\frac{(dx^3 + cx^2 + bx + a)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*p - 1)))/x^4,x)`

[Out] $(a + b*x + c*x^2 + d*x^3)^{(p + 1)}/x^3$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x**3)/x**4,x)`

[Out] Timed out

$$3.238 \quad \int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=97

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{1}{3} \log(x^2 + x + 1) - \frac{13}{48} \log(2x^2 - x + 2) + \frac{5x}{4} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{1}{3} \log(x^2 + x + 1) - \frac{13}{48} \log(2x^2 - x + 2) + \frac{5x}{4} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (13*Log[2 - x + 2*x^2])/48

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2075

$\text{Int}[(P_)^p*(Qm_), x_Symbol] \rightarrow \text{With}\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Qm, x], x] /; \text{QuadraticProductQ}[PP, x] /; \text{PolyQ}[Qm, x] \&\& \text{PolyQ}[P, x] \&\& \text{ILtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(\frac{5}{4} - \frac{3x}{2} + x^2 + x^3 + \frac{2(-2+x)}{3(1+x+x^2)} + \frac{2-13x}{12(2-x+2x^2)} \right) dx \\ &= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{12} \int \frac{2-13x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{-2+x}{1+x+x^2} dx \\ &= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} - \frac{5}{48} \int \frac{1}{2-x+2x^2} dx - \frac{13}{48} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1}{1+x+x^2} dx \\ &= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{3} \log(1+x+x^2) - \frac{13}{48} \log(2-x+2x^2) + \frac{5}{24} \text{Subst} \left(\frac{1}{1+x+x^2}, \frac{1+2x}{\sqrt{3}} \right) \\ &= \frac{5x}{4} - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) - \frac{10 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{3} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.86

$$\frac{1}{144} \left(36x^4 + 48x^3 - 108x^2 + 48 \log(x^2 + x + 1) - 39 \log(2x^2 - x + 2) + 180x - 160\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 - x + 2*x^2])/144

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

fricas [A] time = 1.34, size = 79, normalized size = 0.81

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2-x+2) + \frac{1}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

giac [A] time = 0.31, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2-x+2) + \frac{1}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{\ln(x^2+x+1)}{3} - \frac{13\ln(2x^2-x+2)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)

maxima [A] time = 1.52, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2-x+2) + \frac{1}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] $\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{3}{4}x^2 - \frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + \frac{5}{4}x - \frac{13}{48}\log(2x^2 - x + 2) + \frac{1}{3}\log(x^2 + x + 1)$

mupad [B] time = 0.19, size = 97, normalized size = 1.00

$$\frac{5x}{4} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right)\left(-\frac{13}{48} + \frac{\sqrt{15}i}{144}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}i}{4}\right)\left(\frac{13}{48} + \frac{\sqrt{15}i}{144}\right) - \frac{3x^2}{4} + \frac{x^3}{3} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)

[Out] $\frac{5x}{4} + \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*5i)/9 + 1/3) - \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*5i)/9 - 1/3) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/144 - 13/48) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4$

sympy [A] time = 0.25, size = 97, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^3}{3} - \frac{3x^2}{4} + \frac{5x}{4} - \frac{13\log\left(x^2 - \frac{x}{2} + 1\right)}{48} + \frac{\log(x^2 + x + 1)}{3} - \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $x^{**4}/4 + x^{**3}/3 - 3*x^{**2}/4 + 5*x/4 - 13*\log(x^{**2} - x/2 + 1)/48 + \log(x^{**2} + x + 1)/3 - \sqrt{15}*atan(4*\sqrt{15}*x/15 - \sqrt{15}/15)/72 - 10*\sqrt{3}*atan(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.239 \quad \int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=90

$$\frac{x^3}{3} + \frac{x^2}{2} + \frac{2}{3} \log(x^2 + x + 1) - \frac{1}{24} \log(2x^2 - x + 2) - \frac{3x}{2} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^3}{3} + \frac{x^2}{2} + \frac{2}{3} \log(x^2 + x + 1) - \frac{1}{24} \log(2x^2 - x + 2) - \frac{3x}{2} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8 *ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/24

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2075

$\text{Int}[(P_)^p*(Qm_), x_Symbol] \rightarrow \text{With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Qm, x], x] /; \text{QuadraticProductQ}[PP, x] /; \text{PolyQ}[Qm, x] \ \&\& \ \text{PolyQ}[P, x] \ \&\& \ \text{ILtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(-\frac{3}{2} + x + x^2 + \frac{2(3+2x)}{3(1+x+x^2)} + \frac{-6-x}{6(2-x+2x^2)} \right) dx \\ &= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{1}{6} \int \frac{-6-x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{3+2x}{1+x+x^2} dx \\ &= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} - \frac{1}{24} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{2}{3} \int \frac{1+2x}{1+x+x^2} dx - \frac{25}{24} \int \frac{1}{2-x+2x^2} dx \\ &= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{2}{3} \log(1+x+x^2) - \frac{1}{24} \log(2-x+2x^2) + \frac{25}{12} \text{Subst} \left(\int \frac{1}{-1} \right) \\ &= -\frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{5\sqrt{5}}{12} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) + \frac{8 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{2}{3} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.87

$$\frac{1}{72} \left(24x^3 + 36x^2 + 48 \log(x^2 + x + 1) - 3 \log(2x^2 - x + 2) - 108x + 64\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 10\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/72

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

fricas [A] time = 1.22, size = 74, normalized size = 0.82

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2-x+2) + \frac{2}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

giac [A] time = 0.38, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2-x+2) + \frac{2}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 69, normalized size = 0.77

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{3x}{2} - \frac{5\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36} + \frac{8\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{2\ln(x^2+x+1)}{3} - \frac{\ln(2x^2-x+2)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] 1/3*x^3+1/2*x^2-3/2*x-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.88, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2-x+2) + \frac{2}{3}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - \frac{3}{2}x - \frac{1}{24}\log(2x^2 - x + 2) + \frac{2}{3}\log(x^2 + x + 1)$

mupad [B] time = 0.18, size = 92, normalized size = 1.02

$$\frac{x^2}{2} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}1i}{4}\right)\left(-\frac{1}{24} + \frac{\sqrt{15}5i}{72}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}1i}{4}\right)\left(\frac{1}{24} + \frac{\sqrt{15}5i}{72}\right) - \frac{3x}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)

[Out] $\log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 + 2/3) - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*4i)/9 - 2/3) - (3*x)/2 + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*5i)/72 - 1/24) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*5i)/72 + 1/24) + x^2/2 + x^3/3$

sympy [A] time = 0.25, size = 92, normalized size = 1.02

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{3x}{2} - \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{24} + \frac{2\log\left(x^2 + x + 1\right)}{3} - \frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{36} + \frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $x**3/3 + x**2/2 - 3*x/2 - \log(x**2 - x/2 + 1)/24 + 2*\log(x**2 + x + 1)/3 - 5*\sqrt{15}*\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/36 + 8*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.240 \quad \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=77

$$\frac{x^2}{2} - \log(x^2 + x + 1) + \frac{1}{4} \log(2x^2 - x + 2) + x + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^2}{2} - \log(x^2 + x + 1) + \frac{1}{4} \log(2x^2 - x + 2) + x + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2075

$\text{Int}[(P_)^p*(Qm_), x_Symbol] \rightarrow \text{With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Qm, x], x] /; \text{QuadraticProductQ}[PP, x] /; \text{PolyQ}[Qm, x] \ \&\& \ \text{PolyQ}[P, x] \ \&\& \ \text{ILtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx &= \int \left(1+x - \frac{2(1+3x)}{3(1+x+x^2)} + \frac{-2+3x}{3(2-x+2x^2)} \right) dx \\ &= x + \frac{x^2}{2} + \frac{1}{3} \int \frac{-2+3x}{2-x+2x^2} dx - \frac{2}{3} \int \frac{1+3x}{1+x+x^2} dx \\ &= x + \frac{x^2}{2} + \frac{1}{4} \int \frac{-1+4x}{2-x+2x^2} dx + \frac{1}{3} \int \frac{1}{1+x+x^2} dx - \frac{5}{12} \int \frac{1}{2-x+2x^2} dx - \int \frac{1}{-3-x^2} dx \\ &= x + \frac{x^2}{2} - \log(1+x+x^2) + \frac{1}{4} \log(2-x+2x^2) - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 \right) \\ &= x + \frac{x^2}{2} + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1-4x}{\sqrt{15}} \right) + \frac{2 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{3\sqrt{3}} - \log(1+x+x^2) + \frac{1}{4} \log(2-x+2x^2) \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.94

$$\frac{1}{36} \left(9(-4 \log(x^2+x+1) + \log(2x^2-x+2) + 2x(x+2)) + 8\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

fricas [A] time = 0.83, size = 67, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)

giac [A] time = 0.39, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)

maple [A] time = 0.00, size = 62, normalized size = 0.81

$$\frac{x^2}{2} + x - \frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \ln(x^2+x+1) + \frac{\ln(2x^2-x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] 1/2*x^2+x+1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-ln(x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.44, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{4}\log(2x^2-x+2) - \log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 - \frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x - 1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) + x + \frac{1}{4}\log(2x^2 - x + 2) - \log(x^2 + x + 1)$

mupad [B] time = 2.29, size = 85, normalized size = 1.10

$$x - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(1 + \frac{\sqrt{3}1i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(-1 + \frac{\sqrt{3}1i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}1i}{4}\right)\left(\frac{1}{4} + \frac{\sqrt{15}1i}{36}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}1i}{4}\right)\left(-\frac{1}{4} + \frac{\sqrt{15}1i}{36}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)

[Out] $x - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 + 1) + \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 - 1) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 + 1/4) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 - 1/4) + x^2/2$

sympy [A] time = 0.23, size = 78, normalized size = 1.01

$$\frac{x^2}{2} + x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log(x^2 + x + 1) - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $x**2/2 + x + \log(x**2 - x/2 + 1)/4 - \log(x**2 + x + 1) - \sqrt{15}\operatorname{atan}(4*\sqrt{15}*x/15 - \sqrt{15}/15)/18 + 2*\sqrt{3}\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/9$

$$3.241 \quad \int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=72

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{6} \log(2x^2 - x + 2) + x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2075, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{6} \log(2x^2 - x + 2) + x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 2075

$\text{Int}[(P_)^p*(Qm_), x_Symbol] \text{ :> With}[\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Qm, x], x] /; \text{QuadraticProductQ}[PP, x]] /; \text{PolyQ}[Qm, x] \ \&\& \ \text{PolyQ}[P, x] \ \&\& \ \text{ILtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x(5 + x + 3x^2 + 2x^3)}{2 + x + 3x^2 + x^3 + 2x^4} dx &= \int \left(1 + \frac{2(-2 + x)}{3(1 + x + x^2)} + \frac{2(1 + x)}{3(2 - x + 2x^2)} \right) dx \\ &= x + \frac{2}{3} \int \frac{-2 + x}{1 + x + x^2} dx + \frac{2}{3} \int \frac{1 + x}{2 - x + 2x^2} dx \\ &= x + \frac{1}{6} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{1}{3} \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{5}{6} \int \frac{1}{2 - x + 2x^2} dx - \frac{5}{3} \int \frac{1}{1 + x + x^2} dx \\ &= x + \frac{1}{3} \log(1 + x + x^2) + \frac{1}{6} \log(2 - x + 2x^2) - \frac{5}{3} \text{Subst} \left(\int \frac{1}{-15 - x^2} dx, x, -1 - 2x \right) \\ &= x - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1 - 4x}{\sqrt{15}} \right) - \frac{10 \tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{3} \log(1 + x + x^2) + \frac{1}{6} \log(2 - x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.96

$$\frac{1}{18} \left(3(2 \log(x^2 + x + 1) + \log(2x^2 - x + 2) + 6x) - 20\sqrt{3} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) + 2\sqrt{15} \tan^{-1} \left(\frac{4x - 1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(5 + x + 3x^2 + 2x^3)}{2 + x + 3x^2 + x^3 + 2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

fricas [A] time = 1.29, size = 62, normalized size = 0.86

$$\frac{1}{9} \sqrt{5} \sqrt{3} \arctan\left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="fricas")

[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

giac [A] time = 0.31, size = 56, normalized size = 0.78

$$\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x-1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x, algorithm="giac")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

maple [A] time = 0.00, size = 57, normalized size = 0.79

$$x + \frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9} - \frac{10\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{\ln(x^2 + x + 1)}{3} + \frac{\ln(2x^2 - x + 2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2), x)

[Out] x+1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+1/3*ln(x^2+x+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.59, size = 56, normalized size = 0.78

$$\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x-1)\right) - \frac{10}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) + x + \frac{1}{6} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)

mupad [B] time = 2.29, size = 80, normalized size = 1.11

$$x + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right)\left(-\frac{1}{6} + \frac{\sqrt{15}i}{18}\right) + \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}i}{4}\right)\left(\frac{1}{6} + \frac{\sqrt{15}i}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)

[Out] x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/6)

sympy [A] time = 0.23, size = 75, normalized size = 1.04

$$x + \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{6} + \frac{\log\left(x^2 + x + 1\right)}{3} + \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{9} - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)

[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9

$$3.242 \quad \int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} dx$$

Optimal. Leaf size=71

$$\frac{2}{3} \log(x^2 + x + 1) - \frac{1}{6} \log(2x^2 - x + 2) - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2074, 634, 618, 204, 628}

$$\frac{2}{3} \log(x^2 + x + 1) - \frac{1}{6} \log(2x^2 - x + 2) - \frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 2074

$\text{Int}[(P_)^p(Q_)^q, x_Symbol] \text{ :> With}\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^pQ^q, x], x] /; \text{!SumQ}[\text{NonfreeFactors}[PP, x]] /; \text{FreeQ}[q, x] \&\& \text{PolyQ}[P, x] \&\& \text{PolyQ}[Q, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{2 + x + 3x^2 + x^3 + 2x^4} dx &= \int \left(\frac{2(3 + 2x)}{3(1 + x + x^2)} + \frac{3 - 2x}{3(2 - x + 2x^2)} \right) dx \\ &= \frac{1}{3} \int \frac{3 - 2x}{2 - x + 2x^2} dx + \frac{2}{3} \int \frac{3 + 2x}{1 + x + x^2} dx \\ &= -\left(\frac{1}{6} \int \frac{-1 + 4x}{2 - x + 2x^2} dx \right) + \frac{2}{3} \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{5}{6} \int \frac{1}{2 - x + 2x^2} dx + \frac{4}{3} \int \frac{1}{1 + x + x^2} dx \\ &= \frac{2}{3} \log(1 + x + x^2) - \frac{1}{6} \log(2 - x + 2x^2) - \frac{5}{3} \text{Subst} \left(\int \frac{1}{-15 - x^2} dx, x, -1 + 4x \right) \\ &= -\frac{1}{3} \sqrt{\frac{5}{3}} \tan^{-1} \left(\frac{1 - 4x}{\sqrt{15}} \right) + \frac{8 \tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{2}{3} \log(1 + x + x^2) - \frac{1}{6} \log(2 - x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.92

$$\frac{1}{18} \left(12 \log(x^2 + x + 1) - 3 \log(2x^2 - x + 2) + 16\sqrt{3} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) + 2\sqrt{15} \tan^{-1} \left(\frac{4x - 1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

[Out] (16*sqrt(3)*ArcTan[(1 + 2*x)/sqrt(3)] + 2*sqrt(15)*ArcTan[(-1 + 4*x)/sqrt(15)] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{2 + x + 3x^2 + x^3 + 2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x]

fricas [A] time = 1.33, size = 61, normalized size = 0.86

$$\frac{1}{9} \sqrt{5} \sqrt{3} \arctan\left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1)\right) + \frac{8}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{6} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

giac [A] time = 0.29, size = 55, normalized size = 0.77

$$\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x-1)\right) + \frac{8}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{6} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

maple [A] time = 0.01, size = 56, normalized size = 0.79

$$\frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9} + \frac{8\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{2 \ln(x^2 + x + 1)}{3} - \frac{\ln(2x^2 - x + 2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)

[Out] -1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.32, size = 55, normalized size = 0.77

$$\frac{1}{9} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x-1)\right) + \frac{8}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{6} \log(2x^2 - x + 2) + \frac{2}{3} \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")

[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)

mupad [B] time = 0.15, size = 79, normalized size = 1.11

$$-\ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(-\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) - \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}1i}{4}\right)\left(\frac{1}{6} + \frac{\sqrt{15}1i}{18}\right) + \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}1i}{4}\right)\left(-\frac{1}{6} + \frac{\sqrt{15}1i}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)

[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6)

sympy [A] time = 0.23, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{6} + \frac{2\log\left(x^2 + x + 1\right)}{3} + \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{9} + \frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)

[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9

$$3.243 \quad \int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=75

$$-\log(x^2 + x + 1) - \frac{1}{4} \log(2x^2 - x + 2) + \frac{5 \log(x)}{2} + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\log(x^2 + x + 1) - \frac{1}{4} \log(2x^2 - x + 2) + \frac{5 \log(x)}{2} + \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 2087

`Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]`

Rubi steps

$$\begin{aligned}
 \int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x(4 + 4x + 4x^2)} dx \\
 &= \frac{1}{3} \int \left(\frac{6}{x} - \frac{2(1 + 3x)}{1 + x + x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x} + \frac{1 + 6x}{2(2 - x + 2x^2)}\right) dx \\
 &= \frac{5 \log(x)}{2} - \frac{1}{6} \int \frac{1 + 6x}{2 - x + 2x^2} dx - \frac{2}{3} \int \frac{1 + 3x}{1 + x + x^2} dx \\
 &= \frac{5 \log(x)}{2} - \frac{1}{4} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{1}{3} \int \frac{1}{1 + x + x^2} dx - \frac{5}{12} \int \frac{1}{2 - x + 2x^2} dx \\
 &= \frac{5 \log(x)}{2} - \log(1 + x + x^2) - \frac{1}{4} \log(2 - x + 2x^2) - \frac{2}{3} \text{Subst}\left(\int \frac{1}{-3 - x^2} dx\right) \\
 &= \frac{1}{6} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1 - 4x}{\sqrt{15}}\right) + \frac{2 \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5 \log(x)}{2} - \log(1 + x + x^2) - \frac{1}{4} \log(2 - x + 2x^2)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.92

$$\frac{1}{36} \left(-36 \log(x^2 + x + 1) - 9 \log(2x^2 - x + 2) + 90 \log(x) + 8\sqrt{3} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) - 2\sqrt{15} \tan^{-1} \left(\frac{4x-1}{\sqrt{15}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 3x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)), x]

fricas [A] time = 1.16, size = 65, normalized size = 0.87

$$-\frac{1}{18} \sqrt{5} \sqrt{3} \arctan\left(\frac{1}{15} \sqrt{5} \sqrt{3} (4x-1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)

giac [A] time = 0.31, size = 60, normalized size = 0.80

$$-\frac{1}{18} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x-1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] $-1/18*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 2/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/4*\log(2*x^2 - x + 2) - \log(x^2 + x + 1) + 5/2*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 60, normalized size = 0.80

$$-\frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} + \frac{5 \ln(x)}{2} - \ln(x^2 + x + 1) - \frac{\ln(2x^2 - x + 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2), x)$

[Out] $-1/4*\ln(2*x^2-x+2)-1/18*15^{(1/2)}*\arctan(1/15*(4*x-1)*15^{(1/2)})+5/2*\ln(x)-\ln(x^2+x+1)+2/9*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 1.57, size = 59, normalized size = 0.79

$$-\frac{1}{18} \sqrt{15} \arctan\left(\frac{1}{15} \sqrt{15} (4x - 1)\right) + \frac{2}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x + 1)\right) - \frac{1}{4} \log(2x^2 - x + 2) - \log(x^2 + x + 1) + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2), x, \text{algorithm}="maxima")$

[Out] $-1/18*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) + 2/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/4*\log(2*x^2 - x + 2) - \log(x^2 + x + 1) + 5/2*\log(x)$

mupad [B] time = 0.15, size = 83, normalized size = 1.11

$$\frac{5 \ln(x)}{2} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} \text{li}}{2}\right) \left(1 + \frac{\sqrt{3} \text{li}}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} \text{li}}{2}\right) \left(-1 + \frac{\sqrt{3} \text{li}}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15} \text{li}}{4}\right) \left(-\frac{1}{4} + \frac{\sqrt{15} \text{li}}{36}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15} \text{li}}{4}\right) \left(\frac{1}{4} + \frac{\sqrt{15} \text{li}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)), x)$

[Out] $(5*\log(x))/2 - \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 + 1) + \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/9 - 1) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 - 1/4) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*1i)/36 + 1/4)$

sympy [A] time = 0.29, size = 78, normalized size = 1.04

$$\frac{5 \log(x)}{2} - \frac{\log\left(x^2 - \frac{x}{2} + 1\right)}{4} - \log(x^2 + x + 1) - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)

[Out] $5 \cdot \log(x)/2 - \log(x^2 - x/2 + 1)/4 - \log(x^2 + x + 1) - \sqrt{15} \cdot \operatorname{atan}(4 \cdot \sqrt{15} \cdot x/15 - \sqrt{15}/15)/18 + 2 \cdot \sqrt{3} \cdot \operatorname{atan}(2 \cdot \sqrt{3} \cdot x/3 + \sqrt{3}/3)/9$

$$3.244 \quad \int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=84

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{24} \log(2x^2 - x + 2) - \frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{3} \log(x^2 + x + 1) + \frac{1}{24} \log(2x^2 - x + 2) - \frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] -5/(2*x) + (5*sqrt[5/3]*ArcTan[(1 - 4*x)/sqrt[15]])/12 - (10*ArcTan[(1 + 2*x)/sqrt[3]])/(3*sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/24

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 800

$\text{Int}[(((d_.) + (e_.)(x_))^{(m_.)}((f_.) + (g_.)(x_)))/((a_.) + (b_.)(x_) + (c_.)(x_)^2), x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rule 2087

$\text{Int}[((P3_)(x_)^{(m_.)})/((a_) + (b_.)(x_) + (c_.)(x_)^2 + (d_.)(x_)^3 + (e_.)(x_)^4), x_Symbol] :> \text{With}[\{q = \text{Sqrt}[8a^2 + b^2 - 4ac], A = \text{Coeff}[P3, x, 0], B = \text{Coeff}[P3, x, 1], C = \text{Coeff}[P3, x, 2], D = \text{Coeff}[P3, x, 3]\}, \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2a*B + 2a*D + A*q + (2a*A - 2a*C + b*D + D*q)*x))/(2a + (b + q)*x + 2a*x^2), x], x] - \text{Dist}[1/q, \text{Int}[(x^m*(b*A - 2a*B + 2a*D - A*q + (2a*A - 2a*C + b*D - D*q)*x))/(2a + (b - q)*x + 2a*x^2), x], x]] /; \text{FreeQ}[\{a, b, c, m\}, x] \&\& \text{PolyQ}[P3, x, 3] \&\& \text{EqQ}[a, e] \&\& \text{EqQ}[b, d]$

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x^2(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x^2(4 + 4x + 4x^2)} dx \\ &= \frac{1}{3} \int \left(\frac{6}{x^2} - \frac{2}{x} + \frac{2(-2 + x)}{1 + x + x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x^2} + \frac{1}{4x} + \frac{13 - 2x}{4(2 - x + 2x^2)}\right) dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} - \frac{1}{12} \int \frac{13 - 2x}{2 - x + 2x^2} dx + \frac{2}{3} \int \frac{-2 + x}{1 + x + x^2} dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{1}{24} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{1}{3} \int \frac{1 + 2x}{1 + x + x^2} dx - \frac{25}{24} \int \frac{1}{2 - x} dx \\ &= -\frac{5}{2x} - \frac{3 \log(x)}{4} + \frac{1}{3} \log(1 + x + x^2) + \frac{1}{24} \log(2 - x + 2x^2) + \frac{25}{12} \text{Subst}\left(\int \frac{1}{2 - x} dx, x, 2 - x\right) \\ &= -\frac{5}{2x} + \frac{5}{12} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1 - 4x}{\sqrt{15}}\right) - \frac{10 \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{3 \log(x)}{4} + \frac{1}{3} \log(1 + x + x^2) \end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.93

$$\frac{-24x \log(x^2 + x + 1) - 3x \log(2x^2 - x + 2) + 54x \log(x) + 80\sqrt{3}x \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + 10\sqrt{15}x \tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right) + 180}{72x}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x + 2*x^2])/x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 3x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)), x]

fricas [A] time = 1.16, size = 76, normalized size = 0.90

$$\frac{10\sqrt{5}\sqrt{3}x \arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) + 80\sqrt{3}x \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 3x \log(2x^2 - x + 2) - 24x \log(x^2 + x + 1) + 54x \log(x) + 180}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^2 + x + 1) + 54*x*log(x) + 180)/x

giac [A] time = 0.26, size = 65, normalized size = 0.77

$$-\frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24} \log(2x^2 - x + 2) + \frac{1}{3} \log(x^2 + x + 1) - \frac{3}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] $-5/36*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) - 10/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 5/2/x + 1/24*\log(2*x^2 - x + 2) + 1/3*\log(x^2 + x + 1) - 3/4*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 65, normalized size = 0.77

$$-\frac{5\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36} - \frac{10\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \frac{3\ln(x)}{4} + \frac{\ln(x^2 + x + 1)}{3} + \frac{\ln(2x^2 - x + 2)}{24} - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2), x)$

[Out] $1/24*\ln(2*x^2-x+2)-5/36*15^{(1/2)}*\arctan(1/15*(4*x-1)*15^{(1/2)})-5/2/x-3/4*\ln(x)+1/3*\ln(x^2+x+1)-10/9*3^{(1/2)}*\arctan(1/3*(2*x+1)*3^{(1/2)})$

maxima [A] time = 1.44, size = 64, normalized size = 0.76

$$-\frac{5}{36}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log(2x^2-x+2) + \frac{1}{3}\log(x^2+x+1) - \frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2), x, \text{algorithm}=\text{"maxima"})$

[Out] $-5/36*\sqrt{15}*\arctan(1/15*\sqrt{15}*(4*x - 1)) - 10/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 5/2/x + 1/24*\log(2*x^2 - x + 2) + 1/3*\log(x^2 + x + 1) - 3/4*\log(x)$

mupad [B] time = 2.28, size = 88, normalized size = 1.05

$$-\frac{3\ln(x)}{4} + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}5i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right)\left(\frac{1}{24} + \frac{\sqrt{15}5i}{72}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}i}{4}\right)\left(-\frac{1}{24} + \frac{\sqrt{15}5i}{72}\right) - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)), x)$

[Out] $\log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*5i)/9 + 1/3) - (3*\log(x))/4 - \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*5i)/9 - 1/3) + \log(x - (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*5i)/72 + 1/24) - \log(x + (15^{(1/2)}*1i)/4 - 1/4)*((15^{(1/2)}*5i)/72 - 1/24) - 5/(2*x)$

sympy [A] time = 0.31, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4} + \frac{\log(x^2 - \frac{x}{2} + 1)}{24} + \frac{\log(x^2 + x + 1)}{3} - \frac{5\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x}{15} - \frac{\sqrt{15}}{15}\right)}{36} - \frac{10\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{9} - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9 - 5/(2*x)
```

$$3.245 \quad \int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=91

$$-\frac{5}{4x^2} + \frac{2}{3} \log(x^2 + x + 1) + \frac{13}{48} \log(2x^2 - x + 2) + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\frac{5}{4x^2} + \frac{2}{3} \log(x^2 + x + 1) + \frac{13}{48} \log(2x^2 - x + 2) + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3 + (13*Log[2 - x + 2*x^2])/48

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 2087

`Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]`

Rubi steps

$$\begin{aligned}
 \int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 3x^2 + x^3 + 2x^4)} dx &= -\left(\frac{1}{3} \int \frac{-6 + 4x}{x^3(4 - 2x + 4x^2)} dx\right) + \frac{1}{3} \int \frac{24 + 16x}{x^3(4 + 4x + 4x^2)} dx \\
 &= \frac{1}{3} \int \left(\frac{6}{x^3} - \frac{2}{x^2} - \frac{4}{x} + \frac{2(3 + 2x)}{1 + x + x^2}\right) dx - \frac{1}{3} \int \left(-\frac{3}{2x^3} + \frac{1}{4x^2} + \frac{13}{8x} + \frac{9 - 2x}{8(2 - x + 2x^2)}\right) dx \\
 &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} - \frac{1}{24} \int \frac{9 - 26x}{2 - x + 2x^2} dx + \frac{2}{3} \int \frac{3 + 2x}{1 + x + x^2} dx \\
 &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} - \frac{5}{48} \int \frac{1}{2 - x + 2x^2} dx + \frac{13}{48} \int \frac{-1 + 4x}{2 - x + 2x^2} dx + \frac{2}{3} \int \frac{3 + 2x}{1 + x + x^2} dx \\
 &= -\frac{5}{4x^2} + \frac{3}{4x} - \frac{15 \log(x)}{8} + \frac{2}{3} \log(1 + x + x^2) + \frac{13}{48} \log(2 - x + 2x^2) + \frac{5}{24} \log(1 + x + x^2) \\
 &= -\frac{5}{4x^2} + \frac{3}{4x} + \frac{1}{24} \sqrt{\frac{5}{3}} \tan^{-1}\left(\frac{1 - 4x}{\sqrt{15}}\right) + \frac{8 \tan^{-1}\left(\frac{1 + 2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{15 \log(x)}{8} + \frac{2}{3} \log(1 + x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.90

$$\frac{1}{144} \left(3 \left(-\frac{60}{x^2} + 32 \log(x^2 + x + 1) + 13 \log(2x^2 - x + 2) + \frac{36}{x} - 90 \log(x) \right) + 128\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) - 2\sqrt{15} \tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] (128*sqrt(3)*ArcTan[(1 + 2*x)/sqrt(3)] - 2*sqrt(15)*ArcTan[(-1 + 4*x)/sqrt(15)] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x + 2*x^2]))/144

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 3x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)), x]

fricas [A] time = 1.13, size = 89, normalized size = 0.98

$$\frac{2\sqrt{5}\sqrt{3}x^2 \arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right) - 128\sqrt{3}x^2 \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 39x^2 \log(2x^2-x+2) - 96x^2 \log(x^2+x+1) + 270x^2 \log(x) - 108x + 180}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")

[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2

giac [A] time = 0.29, size = 70, normalized size = 0.77

$$-\frac{1}{72}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{3x-5}{4x^2} + \frac{13}{48} \log(2x^2-x+2) + \frac{2}{3} \log(x^2+x+1) - \frac{15}{8} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")

[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1) - 15/8*log(abs(x))

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{\sqrt{15} \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9} - \frac{15\ln(x)}{8} + \frac{2\ln(x^2+x+1)}{3} + \frac{13\ln(2x^2-x+2)}{48} + \frac{3}{4x} - \frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2), x)

[Out] 13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/4/x^2+3/4/x-15/8*ln(x)+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 1.26, size = 69, normalized size = 0.76

$$-\frac{1}{72}\sqrt{15} \arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{3x-5}{4x^2} + \frac{13}{48}\log(2x^2-x+2) + \frac{2}{3}\log(x^2+x+1) - \frac{15}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2), x, algorithm="maxima")

[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1) - 15/8*log(x)

mupad [B] time = 0.15, size = 92, normalized size = 1.01

$$\frac{\frac{3x-5}{4x^2} - \frac{15\ln(x)}{8} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}i}{2}\right)\left(-\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{2}{3} + \frac{\sqrt{3}4i}{9}\right) + \ln\left(x - \frac{1}{4} - \frac{\sqrt{15}i}{4}\right)\left(\frac{13}{48} + \frac{\sqrt{15}i}{144}\right) - \ln\left(x - \frac{1}{4} + \frac{\sqrt{15}i}{4}\right)\left(-\frac{13}{48} + \frac{\sqrt{15}i}{144}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)), x)

[Out] ((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)

sympy [A] time = 0.32, size = 94, normalized size = 1.03

$$-\frac{15\log(x)}{8} + \frac{13\log(x^2 - \frac{x}{2} + 1)}{48} + \frac{2\log(x^2 + x + 1)}{3} - \frac{\sqrt{15} \operatorname{atan}\left(\frac{4\sqrt{15}x - \sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x + \sqrt{3}}{3}\right)}{9} + \frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2), x)

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```


$$3.246 \quad \int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=307

$$\frac{1}{42} (7 + 5i\sqrt{7}) x^3 + \frac{1}{42} (7 - 5i\sqrt{7}) x^3 + \frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4x^2 + (1 - i$$

Rubi [A] time = 0.58, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{42} (7 + 5i\sqrt{7}) x^3 + \frac{1}{42} (7 - 5i\sqrt{7}) x^3 + \frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{3}{112} (7 + 11i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) - \frac{1}{28} (35 + 9i\sqrt{7}) x - \frac{1}{28} (35 - 9i\sqrt{7}) x + \frac{11(5\sqrt{7} + 9) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35 + i\sqrt{7})}}\right)}{4\sqrt{14(35 + i\sqrt{7})}} - \frac{11(-5\sqrt{7} + 9) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35 - i\sqrt{7})}}\right)}{4\sqrt{14(35 - i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)/42 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/112

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx &= \frac{i \int \frac{x^3(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x^3(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\
&= \frac{i \int \left(\frac{1}{4}(-9+5i\sqrt{7}) + \frac{1}{2}(5-i\sqrt{7})x + \frac{1}{2}(5-i\sqrt{7})x^2 + \frac{2(9-5i\sqrt{7})-3(11+i\sqrt{7})x}{2(4+(1-i\sqrt{7})x+4x^2)} \right) dx}{\sqrt{7}} \\
&= -\frac{1}{28}(35-9i\sqrt{7})x - \frac{1}{28}(35+9i\sqrt{7})x + \frac{1}{28}(7-5i\sqrt{7})x^2 + \frac{1}{28}(7+5i\sqrt{7})x \\
&= -\frac{1}{28}(35-9i\sqrt{7})x - \frac{1}{28}(35+9i\sqrt{7})x + \frac{1}{28}(7-5i\sqrt{7})x^2 + \frac{1}{28}(7+5i\sqrt{7})x \\
&= -\frac{1}{28}(35-9i\sqrt{7})x - \frac{1}{28}(35+9i\sqrt{7})x + \frac{1}{28}(7-5i\sqrt{7})x^2 + \frac{1}{28}(7+5i\sqrt{7})x \\
&= -\frac{1}{28}(35-9i\sqrt{7})x - \frac{1}{28}(35+9i\sqrt{7})x + \frac{1}{28}(7-5i\sqrt{7})x^2 + \frac{1}{28}(7+5i\sqrt{7})x
\end{aligned}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.36

$$\frac{1}{6} \left(3\text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{3\#1^3 \log(x - \#1) + 19\#1^2 \log(x - \#1) + \#1 \log(x - \#1) + 10 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \& \right] + x(2x^2 + 3x - 15) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 &, (10*Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

fricas [B] time = 3.80, size = 1202, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")

[Out] $\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{112}(-33I\sqrt{7} + 56\sqrt{2101/1568}I\sqrt{7}) - \frac{55}{32} - 21) \log(23324(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^3 - 23765(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 + 7744x + 19470I\sqrt{7} - 33040\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + 38950) - \frac{1}{112}(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21) \log(-23324(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^3 + 49/4(-33/112I\sqrt{7} - \frac{1}{2}\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2(-561I\sqrt{7} + 952\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 869) + \frac{1}{256}(53312(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - 11781I\sqrt{7} + 19992\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 36681)(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21) + 17493(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 + 7744x - 15708I\sqrt{7} + 26656\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 29132) + \frac{1}{112}(2\sqrt{7})\sqrt{-336(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - 336(-33/112I\sqrt{7} - \frac{1}{2}\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - \frac{1}{56}(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21)(-33I\sqrt{7} + 56\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + 63) + 99/2I\sqrt{7} - 84\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 1859/2) + 28\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + 28\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} + 21) \log(-49/4(-33/112I\sqrt{7} - \frac{1}{2}\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2(-561I\sqrt{7} + 952\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 869) - \frac{1}{256}(53312(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - 11781I\sqrt{7} + 19992\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 36681)(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21) + 6272(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 + \frac{1}{256}((17\sqrt{7})(-33I\sqrt{7} + 56\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 21) - 512\sqrt{7})(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21) - 512\sqrt{7})(-33I\sqrt{7} + 56\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 21) + 73728\sqrt{7})\sqrt{-336(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - 336(-33/112I\sqrt{7} - \frac{1}{2}\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - \frac{1}{56}(33I\sqrt{7} + 56\sqrt{-2101/1568}I\sqrt{7} - \frac{55}{32} - 21)(-33I\sqrt{7} + 56\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + 63) + 99/2I\sqrt{7} - 84\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 1859/2) + 15488x - 3762I\sqrt{7} + 6384\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} - 5946) - \frac{1}{112}(2\sqrt{7})\sqrt{-336(33/112I\sqrt{7} - \frac{1}{2}\sqrt{2101/1568}I\sqrt{7} - \frac{55}{32} + \frac{3}{16})^2 - 336$

$(-33/112*I*\sqrt{7} - 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 1/5$
 $6*(33*I*\sqrt{7} + 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21)*(-33*I*\sqrt{7}$
 $) + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 63) + 99/2*I*\sqrt{7} - 84*\sqrt{2}$
 $101/1568*I*\sqrt{7} - 55/32) - 1859/2) - 28*\sqrt{2101/1568*I*\sqrt{7} - 55/32}$
 $) - 28*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21)*\log(-49/4*(-33/112*I*\sqrt{7}$
 $) - 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2*(-561*I*\sqrt{7} + 952*$
 $\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 869) - 1/256*(53312*(33/112*I*\sqrt{7} -$
 $1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 11781*I*\sqrt{7} + 19992*$
 $\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 36681)*(33*I*\sqrt{7} + 56*\sqrt{-2101/15$
 $68*I*\sqrt{7} - 55/32} - 21) + 6272*(33/112*I*\sqrt{7} - 1/2*\sqrt{2101/1568*I$
 $*\sqrt{7} - 55/32} + 3/16)^2 - 1/256*((17*\sqrt{7})*(-33*I*\sqrt{7} + 56*\sqrt{2}$
 $101/1568*I*\sqrt{7} - 55/32} - 21) - 512*\sqrt{7}))* (33*I*\sqrt{7} + 56*\sqrt{-2$
 $101/1568*I*\sqrt{7} - 55/32} - 21) - 512*\sqrt{7})*(-33*I*\sqrt{7} + 56*\sqrt{21$
 $01/1568*I*\sqrt{7} - 55/32} - 21) + 73728*\sqrt{7})*\sqrt{-336*(33/112*I*\sqrt{7}$
 $- 1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 336*(-33/112*I*\sqrt{7}$
 $- 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 1/56*(33*I*\sqrt{7}$
 $+ 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21)*(-33*I*\sqrt{7} + 56*\sqrt{2101$
 $/1568*I*\sqrt{7} - 55/32} + 63) + 99/2*I*\sqrt{7} - 84*\sqrt{2101/1568*I*\sqrt{7}$
 $- 55/32} - 1859/2) + 15488*x - 3762*I*\sqrt{7} + 6384*\sqrt{2101/1568*I*\sqrt{7}$
 $- 55/32} - 5946) - 5/2*x$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^3}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 74, normalized size = 0.24

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \frac{(3\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 19\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 10)\ln(-\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + x)}{16\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 6\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 20\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{5}{2}x + \frac{1}{2} \int \frac{3x^3 + 19x^2 + x + 10}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 2.17, size = 128, normalized size = 0.42

$$\left(\sum_{k=1}^4 \ln\left(-29x + \sqrt{z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}z, k}\right) \left(-\frac{289x}{4} + \sqrt{z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}z, k}\right) \left(\frac{581x}{16} - \sqrt{z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}z, k}\right) \left(\frac{147x}{4} - \frac{49}{16} + \frac{1141}{64} + \frac{47}{4}\right) + 7\right) \sqrt{z^4 - \frac{3z^3}{4} + \frac{16z^2}{7} + \frac{96z}{49} + \frac{128}{343}z, k} - \frac{5x}{2} + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] symsum(log(sqrt(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k))*
sqrt(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/16
- sqrt(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k))*((147*x)/4
- 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*sqrt(z^4 - (3*z^3)/4 +
(16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/3

sympy [A] time = 0.99, size = 61, normalized size = 0.20

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \text{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t \log\left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x + \frac{415}{121}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2688*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t/242 + x + 415/121)))

$$3.247 \quad \int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=269

$$\frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) - \frac{1}{56} (35 - 9i\sqrt{7}) \log(4x^2 +$$

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7}) x^2 + \frac{1}{28} (7 - 5i\sqrt{7}) x^2 - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) - \frac{1}{56} (35 - 9i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{1}{14} (7 + 5i\sqrt{7}) x + \frac{1}{14} (7 - 5i\sqrt{7}) x - \frac{(\sqrt{7} + 53i) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35 + i\sqrt{7})}}\right)}{2\sqrt{14(35 + i\sqrt{7})}} + \frac{(-\sqrt{7} + 53i) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35 - i\sqrt{7})}}\right)}{2\sqrt{14(35 - i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/56

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_)^m)*(f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx &= \frac{i \int \frac{x^2(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x^2(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\
&= \frac{i \int \left(\frac{1}{2}(5-i\sqrt{7}) + \frac{1}{2}(5-i\sqrt{7})x + \frac{i(2(5i+\sqrt{7})+(9i+5\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} \right) dx}{\sqrt{7}} - \frac{i \int \left(\frac{1}{2}(5+i\sqrt{7}) + \frac{1}{2}(5+i\sqrt{7})x + \frac{i(2(5i-\sqrt{7})+(9i-5\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} \right) dx}{\sqrt{7}} \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 - \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 - \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 - \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7-5i\sqrt{7})x^2 + \frac{1}{28} (7+5i\sqrt{7})x^2 -
\end{aligned}$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.38

$$-\text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{5\#1^3 \log(x - \#1) + \#1^2 \log(x - \#1) + 3\#1 \log(x - \#1) + 2 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1}\&\right] + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]

[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]

/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(-37/392*I*sqrt(7) + 79/56) + 13793/2) + x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x^2}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 67, normalized size = 0.25

$$\frac{x^2}{2+x} + \frac{(-5\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2)^3 - \text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2)^2 - 3\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2) - 2)\ln(-\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2)+x)}{8\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2)^3 + 3\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2)^2 + 10\text{RootOf}(2_Z^4+_Z^3+5_Z^2+_Z+2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}x^2 + x - \int \frac{5x^3 + x^2 + 3x + 2}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] $1/2*x^2 + x - \text{integrate}((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)$

mupad [B] time = 0.13, size = 188, normalized size = 0.70

$$x + \frac{x^2}{2} + \left(\frac{179 \sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8} \ln \left(\frac{179 \sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8} \cdot \frac{\sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8} + 459}{\sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}} \right) \frac{1}{7} - \frac{\sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8} \cdot 665 \cdot \frac{\sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8} \cdot 147}{4} - \frac{35 \sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{32} \cdot \frac{49 \sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{16} \cdot 15 \right) \frac{\sqrt{\frac{z^4 + 5z^3}{2} + 2z^2 + \frac{32z}{49} + \frac{128}{343}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2), x)$

[Out] $x + x^2/2 + \text{symsum}(\log((49*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3)/16 - 7*x - (459*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)*x)/8 - (665*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2*x)/8 - (147*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3*x)/4 - (35*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2)/32 - (179*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 - 15)*\text{root}(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)$

sympy [B] time = 2.73, size = 3662, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2), x)$

[Out] $x**2/2 + x + (-5/8 + \sqrt{79/448 + \sqrt{77}/49})*\log(x**2 + x*(-1459*\sqrt{14})*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/536576 - 15*\sqrt{77}*\sqrt{553 + 64*\sqrt{77}}/2096 - 10391*\sqrt{553 + 64*\sqrt{77}}/268288 + 1459*\sqrt{77}/8384 + 522933/268288 + 45*\sqrt{14}*\sqrt{553 + 64*\sqrt{77}})*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/536576) - 510895297*\sqrt{14}*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/71978450944 - 6009493*\sqrt{22}*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/1124663296 - 38714551*\sqrt{77}*\sqrt{553 + 64*\sqrt{77}}/2249326592 - 4417610843*\sqrt{553 + 64*\sqrt{77}}/35989225472 + 153195*\sqrt{22}*\sqrt{553 + 64*\sqrt{77}})*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/2249326592 + 8313499*\sqrt{14}*\sqrt{553 + 64*\sqrt{77}})*\sqrt{-333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/71978450944 + 290832444193/35989225472 + 2303470247*\sqrt{77}/2249326592) + (-5/8 - \sqrt{79/448 + \sqrt{77}/49})*\log(x**2 + x*(-45*\sqrt{14}*\sqrt{553 + 64*\sqrt{77}})*\sqrt{333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/536576 - 1459*\sqrt{14}*\sqrt{333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/536576 + 10391*\sqrt{553 + 64*\sqrt{77}}/268288 + 1459*\sqrt{77}/8384 + 522933/268288 + 15*\sqrt{77}*\sqrt{553 + 64*\sqrt{77}}/2096) - 510895297*\sqrt{14}*\sqrt{333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/71978450944 - 6009493*\sqrt{22}*\sqrt{333*\sqrt{553 + 64*\sqrt{77}} + 21975 + 7648*\sqrt{77}}/1124663296 - 8313499*\sqrt{14}*\sqrt{553 + 64*\sqrt{77}}/2249326592$

$$\begin{aligned}
& 648\sqrt{77}) + 560 + 96\sqrt{77})\sqrt{333\sqrt{553 + 64\sqrt{77})} + 21975 \\
& + 7648\sqrt{77}) + 1459\sqrt{2})\sqrt{553 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{ \\
& t(333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) \\
&)) + 2\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{ \\
& (77))/1568 + 5/14 + 3\sqrt{77)/49})\operatorname{atan}(1073152*x/(-1459\sqrt{2})\sqrt{553 + \\
& 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 76 \\
& 48\sqrt{77}) + 560 + 96\sqrt{77}) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{ \\
& (553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 30\sqrt{ \\
& (7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{ \\
& (77)) + 560 + 96\sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648 \\
& * \sqrt{77})) + 45\sqrt{14})\sqrt{553 + 64\sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{ \\
& (77)) + 21975 + 7648\sqrt{77})/(-1459\sqrt{2})\sqrt{553 + 64\sqrt{77})\sqrt{ \\
& (-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 5 \\
& 60 + 96\sqrt{77}) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{ \\
& (77))} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 30\sqrt{7)\sqrt{-\sqrt{ \\
& (14)\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96 \\
& * \sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77})) + 10 \\
& 45866/(-1459\sqrt{2})\sqrt{553 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{ \\
& (553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 4313\sqrt{ \\
& (2)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{7 \\
& (7)) + 560 + 96\sqrt{77}) + 30\sqrt{7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 6 \\
& 4\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77})\sqrt{-333\sqrt{55 \\
& 3 + 64\sqrt{77})} + 21975 + 7648\sqrt{77})) + 93376\sqrt{77)/(-1459\sqrt{2}) \\
& * \sqrt{553 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + \\
& 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{ \\
& (-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{7 \\
& (7)) + 30\sqrt{7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + \\
& 7648\sqrt{77}) + 560 + 96\sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21 \\
& 975 + 7648\sqrt{77})) - 20782\sqrt{553 + 64\sqrt{77})/(-1459\sqrt{2})\sqrt{(5 \\
& 53 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 \\
& + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{-3 \\
& 33\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + \\
& 30\sqrt{7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{ \\
& (77)) + 560 + 96\sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + \\
& 7648\sqrt{77})) - 3840\sqrt{77)\sqrt{553 + 64\sqrt{77})/(-1459\sqrt{2})\sqrt{ \\
& (553 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 2197 \\
& 5 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{ \\
& (-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) \\
& + 30\sqrt{7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + 764 \\
& 8\sqrt{77}) + 560 + 96\sqrt{77})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 \\
& + 7648\sqrt{77})) - 1459\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 \\
& + 7648\sqrt{77})/(-1459\sqrt{2})\sqrt{553 + 64\sqrt{77})\sqrt{-\sqrt{14})\sqrt{ \\
& (-333\sqrt{553 + 64\sqrt{77})} + 21975 + 7648\sqrt{77}) + 560 + 96\sqrt{77}) \\
&) + 4313\sqrt{2})\sqrt{-\sqrt{14})\sqrt{-333\sqrt{553 + 64\sqrt{77})} + 21975 + \\
& 7648\sqrt{77}) + 560 + 96\sqrt{77}) + 30\sqrt{7)\sqrt{-\sqrt{14})\sqrt{-333\sqrt{
\end{aligned}$$

$$\sqrt{553 + 64\sqrt{77}} + 21975 + 7648\sqrt{77} + 560 + 96\sqrt{77} \cdot \sqrt{-333\sqrt{553 + 64\sqrt{77}} + 21975 + 7648\sqrt{77}}$$

$$3.248 \quad \int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=230

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{1}{14} (7 + 5i\sqrt{7})x + \frac{1}{14} (7 - 5i\sqrt{7})x$$

Rubi [A] time = 0.36, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2087, 773, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{1}{14} (7 + 5i\sqrt{7})x + \frac{1}{14} (7 - 5i\sqrt{7})x - \frac{(7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8x - i\sqrt{7} + 1}{\sqrt{2(35 + i\sqrt{7})}}\right)}{\sqrt{14(35 + i\sqrt{7})}} + \frac{(-7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35 - i\sqrt{7})}}\right)}{\sqrt{14(35 - i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx &= \frac{i \int \frac{x(9-5i\sqrt{7}+(10-2i\sqrt{7})x)}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{x(9+5i\sqrt{7}+(10+2i\sqrt{7})x)}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{i \int \frac{-4(10-2i\sqrt{7})+(-(1-i\sqrt{7})(10-2i\sqrt{7}))+4(9-5i\sqrt{7})}{4+(1-i\sqrt{7})x+4x^2} dx}{4\sqrt{7}} \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x - \frac{1}{28} (-7+5i\sqrt{7}) \int \frac{1+i\sqrt{7}+8x}{4+(1+i\sqrt{7})x+4x^2} dx \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x + \frac{1}{28} (7+5i\sqrt{7}) \log(4+(1-i\sqrt{7})x+4x^2) \\
&= \frac{1}{14} (7-5i\sqrt{7})x + \frac{1}{14} (7+5i\sqrt{7})x - \frac{(19i+7\sqrt{7}) \tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} + \frac{(19i-7\sqrt{7}) \tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{\#1^3 \log(x - \#1) - 2\#1^2 \log(x - \#1) + 2\#1 \log(x - \#1) - \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1}\&\right] + x$$

Antiderivative was successfully verified.

[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x - #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

`fricas` [B] time = 3.81, size = 1190, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*sqrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) - 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
```

$$\begin{aligned}
 & - 1/14) + 1/4)^2 - 1/56*(5*I*\sqrt{7} + 14*\sqrt{53/98}*I*\sqrt{7} - 1/14) + 21 \\
 &)*(-5*I*\sqrt{7} + 14*\sqrt{-53/98}*I*\sqrt{7} - 1/14) - 7) - 5/2*I*\sqrt{7} - 7 \\
 & *\sqrt{53/98}*I*\sqrt{7} - 1/14) - 27/2)*((11*\sqrt{7})*(5*I*\sqrt{7} + 14*\sqrt{53/98}*I \\
 & *\sqrt{7} - 1/14) - 7) + 224*\sqrt{7}))*(-5*I*\sqrt{7} + 14*\sqrt{-53/98}*I \\
 & *\sqrt{7} - 1/14) - 7) + 224*\sqrt{7})*(5*I*\sqrt{7} + 14*\sqrt{53/98}*I*\sqrt{7} \\
 & - 1/14) - 7) + 3456*\sqrt{7})) + 608*x - 220*I*\sqrt{7} - 616*\sqrt{53/98}*I*\sqrt{7} \\
 & - 1/14) + 636) - 1/28*(5*I*\sqrt{7} + 14*\sqrt{53/98}*I*\sqrt{7} - 1/14) - \\
 & 7)*\log(3773*(-5/28*I*\sqrt{7} - 1/2*\sqrt{53/98}*I*\sqrt{7} - 1/14) + 1/4)^3 - \\
 & 1029*(-5/28*I*\sqrt{7} - 1/2*\sqrt{53/98}*I*\sqrt{7} - 1/14) + 1/4)^2 + 304*x \\
 & - 715/2*I*\sqrt{7} - 1001*\sqrt{53/98}*I*\sqrt{7} - 1/14) - 2871/2) + x
 \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^3 + 3x^2 + x + 5)x}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 62, normalized size = 0.27

$$x + \frac{2 \left(\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 - 2 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 2 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) - 1 \right) \ln(-\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + x)}{8 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 3 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 10 \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x + 2 \int \frac{x^3 - 2x^2 + 2x - 1}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 0.19, size = 183, normalized size = 0.80

$$x + \left(\sum_{i=1}^4 \ln \left(\frac{115 \text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)}{8} + 15x - \frac{\text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)}{8} \right) + 137 \frac{\text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)}{8} + 133 \frac{\text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)}{4} + 147 \frac{189 \text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)^2}{16} + 49 \frac{\text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)^3}{16} \right) \frac{\text{root}(x^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}z, k)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)`

[Out] `x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
)/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x
)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 -
(147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z
^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3
+ (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)`

sympy [A] time = 0.95, size = 48, normalized size = 0.21

$$x + \text{RootSum}\left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t \log\left(\frac{3773t^3}{304} - \frac{1029t^2}{304} + \frac{1001t}{152} + x - \frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)`

[Out] `x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19))`

$$3.249 \quad \int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4} dx$$

Optimal. Leaf size=198

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{(7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}}$$

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2086, 634, 618, 204, 628}

$$\frac{1}{28} (7 + 5i\sqrt{7}) \log(4x^2 + (1 - i\sqrt{7})x + 4) + \frac{1}{28} (7 - 5i\sqrt{7}) \log(4x^2 + (1 + i\sqrt{7})x + 4) + \frac{(7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8x - i\sqrt{7} + 1}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} - \frac{(-7\sqrt{7} + 19i) \tan^{-1}\left(\frac{8x + i\sqrt{7} + 1}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2])/28

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2086

Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned} \int \frac{5 + x + 3x^2 + 2x^3}{2 + x + 5x^2 + x^3 + 2x^4} dx &= \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{4+(1-i\sqrt{7})x+4x^2} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{4+(1+i\sqrt{7})x+4x^2} dx}{\sqrt{7}} \\ &= -\left(\frac{1}{28}(-7+5i\sqrt{7}) \int \frac{1+i\sqrt{7}+8x}{4+(1+i\sqrt{7})x+4x^2} dx\right) + \frac{1}{28}(7+5i\sqrt{7}) \int \frac{1-i\sqrt{7}-8x}{4+(1-i\sqrt{7})x+4x^2} dx \\ &= \frac{1}{28}(7+5i\sqrt{7}) \log\left(4+(1-i\sqrt{7})x+4x^2\right) + \frac{1}{28}(7-5i\sqrt{7}) \log\left(4+(1+i\sqrt{7})x+4x^2\right) \\ &= \frac{(19i+7\sqrt{7}) \tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}} - \frac{(19i-7\sqrt{7}) \tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}} + \frac{1}{28}\left(7+\frac{5\sqrt{7}}{2}\right) \log\left(4+x+5x^2+x^3+2x^4\right) \end{aligned}$$

Mathematica [C] time = 0.01, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{2\#1^3 \log(x - \#1) + 3\#1^2 \log(x - \#1) + \#1 \log(x - \#1) + 5 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4), x]

```
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
+ 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{2 + x + 5x^2 + x^3 + 2x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4), x
]
```

fricas [B] time = 5.34, size = 1189, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14)
+ 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
```


*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(53/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823/2)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

maple [C] time = 0.01, size = 58, normalized size = 0.29

$$\frac{(2\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 3\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + \text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 5) \ln(-\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + x)}{8\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 3\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 10\text{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)

[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{2x^4 + x^3 + 5x^2 + x + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 2.34, size = 181, normalized size = 0.91

$$\sum_{k=1}^4 \ln \left(\frac{193 \operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)}{8} - \frac{\operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k) \cdot 137}{8} + \frac{\operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)^2 \cdot 651}{16} - \frac{\operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)^3 \cdot 147}{4} + \frac{273 \operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)^2}{16} - \frac{49 \operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)}{16} + 7 \right) \operatorname{root}(z^4 - z^3 + \frac{6z^2}{7} - \frac{48z}{49} + \frac{128}{343}, z, k)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 + (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 0.91, size = 46, normalized size = 0.23

$$\operatorname{RootSum} \left(343t^4 - 343t^3 + 294t^2 - 336t + 128, \left(t \mapsto t \log \left(-\frac{7203t^3}{304} + \frac{2303t^2}{304} - \frac{2177t}{152} + x + \frac{250}{19} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))

$$3.250 \quad \int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=245

$$-\frac{1}{56} (35 - 9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) + \frac{1}{28} (35 + 9i\sqrt{7})$$

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{1}{56} (35 - 9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) - \frac{1}{56} (35 + 9i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) - \frac{(53 + i\sqrt{7}) \tanh^{-1}\left(\frac{8ix - \sqrt{7} + i}{\sqrt{2(35 - i\sqrt{7})}}\right)}{2\sqrt{14(35 - i\sqrt{7})}} + \frac{(53 - i\sqrt{7}) \tanh^{-1}\left(\frac{8ix + \sqrt{7} + i}{\sqrt{2(35 + i\sqrt{7})}}\right)}{2\sqrt{14(35 + i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 - (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^m)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] := With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\begin{aligned}
\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 5x^2 + x^3 + 2x^4)} dx &= \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} \\
&= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x} + \frac{3(11i+\sqrt{7})-2(9i-5\sqrt{7})x}{2(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \frac{i \int \left(\frac{9-5i\sqrt{7}}{4x} + \frac{3(11i-\sqrt{7})-2(9i+5\sqrt{7})x}{2(4i+(i+\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} \\
&= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{i \int \frac{3(11i+\sqrt{7})-2(9i-5\sqrt{7})x}{4i+(i-\sqrt{7})x+4ix^2} dx}{2\sqrt{7}} \\
&= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{1}{56} (35 - 9i\sqrt{7}) \int \frac{1}{4i +} \\
&= \frac{1}{28} (35 - 9i\sqrt{7}) \log(x) + \frac{1}{28} (35 + 9i\sqrt{7}) \log(x) - \frac{1}{56} (35 - 9i\sqrt{7}) \log(4i + \\
&= -\frac{(53 + i\sqrt{7}) \tanh^{-1} \left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}} \right)}{2\sqrt{14(35-i\sqrt{7})}} + \frac{(53 - i\sqrt{7}) \tanh^{-1} \left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}} \right)}{2\sqrt{14(35+i\sqrt{7})}} + \frac{1}{28}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.41

$$\frac{5 \log(x)}{2} - \frac{1}{2} \text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{10\#1^3 \log(x - \#1) + \#1^2 \log(x - \#1) + 19\#1 \log(x - \#1) + 3 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] + 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

fricas [B] time = 6.12, size = 1143, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/56*(-9*I*\sqrt{7} + 28*\sqrt{37/392*I*\sqrt{7} + 79/56} + 35)*\log(49/4*(27* \\ & I*\sqrt{7} + 84*\sqrt{-37/392*I*\sqrt{7} + 79/56} + 1385)*(9/56*I*\sqrt{7} - 1/ \\ & 2*\sqrt{37/392*I*\sqrt{7} + 79/56} - 5/8)^2 - 2058*(-9/56*I*\sqrt{7} - 1/2*\sqrt{ \\ & (-37/392*I*\sqrt{7} + 79/56) - 5/8})^3 - 5145*(-9/56*I*\sqrt{7} - 1/2*\sqrt{(-3 \\ & 7/392*I*\sqrt{7} + 79/56) - 5/8})^2 + 1/64*(2352*(-9/56*I*\sqrt{7} - 1/2*\sqrt{ \\ & (-37/392*I*\sqrt{7} + 79/56) - 5/8})^2 - 945*I*\sqrt{7} - 2940*\sqrt{(-37/392*I*s \\ & qrt(7) + 79/56) - 28507})*(-9*I*\sqrt{7} + 28*\sqrt{37/392*I*\sqrt{7} + 79/56} \\ & + 35) + 8384*x + 1323/2*I*\sqrt{7} + 2058*\sqrt{-37/392*I*\sqrt{7} + 79/56} + \\ & 16089/2) + 1/8*(2*\sqrt{-12*(9/56*I*\sqrt{7} - 1/2*\sqrt{37/392*I*\sqrt{7} + 79 \\ & /56} - 5/8)}^2 - 12*(-9/56*I*\sqrt{7} - 1/2*\sqrt{-37/392*I*\sqrt{7} + 79/56} - \\ & 5/8)^2 - 1/392*(9*I*\sqrt{7} + 28*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 105)*(- \\ & 9*I*\sqrt{7} + 28*\sqrt{37/392*I*\sqrt{7} + 79/56} + 35) + 45/14*I*\sqrt{7} + 1 \\ & 0*\sqrt{-37/392*I*\sqrt{7} + 79/56} + 11/2) + 2*\sqrt{37/392*I*\sqrt{7} + 79/56} \\ &) + 2*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 5)*\log(-49/4*(27*I*\sqrt{7} + 84*\sqrt{ \\ & (-37/392*I*\sqrt{7} + 79/56) + 1385)*(9/56*I*\sqrt{7} - 1/2*\sqrt{37/392*I*s \\ & qrt(7) + 79/56} - 5/8)}^2 - 15680*(-9/56*I*\sqrt{7} - 1/2*\sqrt{-37/392*I*\sqrt{ \\ & 7} + 79/56} - 5/8)}^2 - 1/64*(2352*(-9/56*I*\sqrt{7} - 1/2*\sqrt{-37/392*I*s \\ & qrt(7) + 79/56} - 5/8)}^2 - 945*I*\sqrt{7} - 2940*\sqrt{-37/392*I*\sqrt{7} + 79/5 \\ & 6} - 28507)*(-9*I*\sqrt{7} + 28*\sqrt{37/392*I*\sqrt{7} + 79/56} + 35) + 7/64* \\ & \sqrt{-12*(9/56*I*\sqrt{7} - 1/2*\sqrt{37/392*I*\sqrt{7} + 79/56} - 5/8)}^2 - 12 \\ & *(-9/56*I*\sqrt{7} - 1/2*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 5/8)}^2 - 1/392*(9 \\ & *I*\sqrt{7} + 28*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 105)*(-9*I*\sqrt{7} + 28*s \\ & qrt(37/392*I*\sqrt{7} + 79/56) + 35) + 45/14*I*\sqrt{7} + 10*\sqrt{-37/392*I*s \\ & qrt(7) + 79/56} + 11/2)*((27*I*\sqrt{7} + 84*\sqrt{-37/392*I*\sqrt{7} + 79/56} \\ & + 1385)*(-9*I*\sqrt{7} + 28*\sqrt{37/392*I*\sqrt{7} + 79/56} + 35) + 11520*I* \\ & \sqrt{7} + 35840*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 35072) + 16768*x + 3492*I \\ & *\sqrt{7} + 10864*\sqrt{-37/392*I*\sqrt{7} + 79/56} + 5484) - 1/8*(2*\sqrt{-12* \\ & (9/56*I*\sqrt{7} - 1/2*\sqrt{37/392*I*\sqrt{7} + 79/56} - 5/8)}^2 - 12*(-9/56*I \\ & *\sqrt{7} - 1/2*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 5/8)}^2 - 1/392*(9*I*\sqrt{7} \\ &) + 28*\sqrt{-37/392*I*\sqrt{7} + 79/56} - 105)*(-9*I*\sqrt{7} + 28*\sqrt{37/39 \\ & 2*I*\sqrt{7} + 79/56} + 35) + 45/14*I*\sqrt{7} + 10*\sqrt{-37/392*I*\sqrt{7} + \\ & 79/56} + 11/2) - 2*\sqrt{37/392*I*\sqrt{7} + 79/56} - 2*\sqrt{-37/392*I*\sqrt{7} \\ &) + 79/56} + 5)*\log(-49/4*(27*I*\sqrt{7} + 84*\sqrt{-37/392*I*\sqrt{7} + 79/56} \end{aligned}$$

) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105))*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 79/56) - 18673/2) + 5/2*log(x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)

maple [C] time = 0.01, size = 67, normalized size = 0.27

$$\frac{5 \ln(x)}{2} + \frac{(-10 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^3 - \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^2 - 19 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) - 3) \ln(-\operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) + x)}{16 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^3 + 6 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^2 + 20 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)

[Out] 1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))+5/2*ln(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \int \frac{10x^3 + x^2 + 19x + 3}{2x^4 + x^3 + 5x^2 + x + 2} dx + \frac{5}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x) + 5/2*log(x)

mupad [B] time = 2.34, size = 237, normalized size = 0.97

$$\frac{5 \log(x)}{2} \left(\sum_{k=1}^4 \frac{223 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)}{8} - \frac{31x}{2} + \frac{71 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k) * x}{16} - \frac{4463 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^2 * x}{64} + \frac{1449 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^3 * x}{16} + \frac{3675 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^4 * x}{32} + \frac{257 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^2}{32} + \frac{1673 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^3}{64} - \frac{441 \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k)^4}{32} + 10 \right) \operatorname{root}(z^4 + (5z^3)/2 + 2z^2 + (32z)/49 + 128/343, z, k), k$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 12.66, size = 60, normalized size = 0.24

$$\frac{5 \log(x)}{2} + \operatorname{RootSum}\left(686t^4 + 1715t^3 + 1372t^2 + 448t + 256, \left(t \mapsto t \log\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{4010520787t^2}{2131036736} + \frac{1537535671t}{532759184} + x + \frac{46660495}{66594898}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)

[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, Lambda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))

$$3.251 \quad \int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=281

$$\frac{3}{112} (7 + 11i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) - \frac{35 + 9i\sqrt{7}}{28x} - \frac{35 - 9i\sqrt{7}}{28x}$$

Rubi [A] time = 0.47, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$\frac{3}{112} (7 + 11i\sqrt{7}) \log(4ix^2 + (-\sqrt{7} + i)x + 4i) + \frac{3}{112} (7 - 11i\sqrt{7}) \log(4ix^2 + (\sqrt{7} + i)x + 4i) - \frac{35 + 9i\sqrt{7}}{28x} - \frac{35 - 9i\sqrt{7}}{28x} - \frac{3}{56} (7 + 11i\sqrt{7}) \log(x) - \frac{3}{56} (7 - 11i\sqrt{7}) \log(x) + \frac{11(9 + 5i\sqrt{7}) \tanh^{-1}\left(\frac{8ix - \sqrt{7} + i}{\sqrt{2(35 - i\sqrt{7})}}\right)}{4\sqrt{14(35 - i\sqrt{7})}} - \frac{11(9 - 5i\sqrt{7}) \tanh^{-1}\left(\frac{8ix + \sqrt{7} + i}{\sqrt{2(35 + i\sqrt{7})}}\right)}{4\sqrt{14(35 + i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] $-(35 - (9*I)*\text{Sqrt}[7])/(28*x) - (35 + (9*I)*\text{Sqrt}[7])/(28*x) + (11*(9 + (5*I)*\text{Sqrt}[7])*\text{ArcTanh}[(I - \text{Sqrt}[7] + (8*I)*x)/\text{Sqrt}[2*(35 - I*\text{Sqrt}[7])]])/(4*\text{Sqrt}[14*(35 - I*\text{Sqrt}[7])]) - (11*(9 - (5*I)*\text{Sqrt}[7])*\text{ArcTanh}[(I + \text{Sqrt}[7] + (8*I)*x)/\text{Sqrt}[2*(35 + I*\text{Sqrt}[7])]])/(4*\text{Sqrt}[14*(35 + I*\text{Sqrt}[7])]) - (3*(7 - (11*I)*\text{Sqrt}[7])*\text{Log}[x])/56 - (3*(7 + (11*I)*\text{Sqrt}[7])*\text{Log}[x])/56 + (3*(7 + (11*I)*\text{Sqrt}[7])*\text{Log}[4*I + (I - \text{Sqrt}[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*\text{Sqrt}[7])*\text{Log}[4*I + (I + \text{Sqrt}[7])*x + (4*I)*x^2])/112$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 5x^2 + x^3 + 2x^4)} dx &= \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x^2(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x^2(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} \\
&= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x^2} + \frac{3(11-i\sqrt{7})}{8x} + \frac{-7(9i-5\sqrt{7})-6(11i+\sqrt{7})x}{4(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \frac{i \int \left(\frac{9-5i\sqrt{7}}{4x^2} + \frac{3(11+5i\sqrt{7})}{8x} + \frac{7(9i+5\sqrt{7})+6(11i-\sqrt{7})x}{4(4i+(i+\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} \\
&= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56} (7-11i\sqrt{7}) \log(x) - \frac{3}{56} (7+11i\sqrt{7}) \log(x) \\
&= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56} (7-11i\sqrt{7}) \log(x) - \frac{3}{56} (7+11i\sqrt{7}) \log(x) \\
&= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56} (7-11i\sqrt{7}) \log(x) - \frac{3}{56} (7+11i\sqrt{7}) \log(x) \\
&= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} + \frac{11(9+5i\sqrt{7}) \tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{4\sqrt{14(35-i\sqrt{7})}} - \frac{11(9-5i\sqrt{7}) \tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{4\sqrt{14(35+i\sqrt{7})}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.39

$$\frac{1}{4} \text{RootSum}\left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{6\#1^3 \log(x - \#1) - 17\#1^2 \log(x - \#1) + 13\#1 \log(x - \#1) - 35 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \&\right] - \frac{5}{2x} - \frac{3 \log(x)}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

$2101/1568*I*\sqrt{7} - 55/32) + 63) + 99/2*I*\sqrt{7} - 84*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 1859/2) + 30976*x + 22671/2*I*\sqrt{7} - 19236*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 53979/2) - (4*\sqrt{7})*\sqrt{-336*(33/112*I*\sqrt{7} - 1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 336*(-33/112*I*\sqrt{7} - 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 1/56*(33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 21)*(-33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 63) + 99/2*I*\sqrt{7} - 84*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 1859/2)*x + x*(33*I*\sqrt{7} + 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21) + x*(-33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 21) + 84*x)*\log(49/4*(-33/112*I*\sqrt{7} - 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2*(-2211*I*\sqrt{7} + 3752*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 3839) + 1/256*(210112*(33/112*I*\sqrt{7} - 1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 46431*I*\sqrt{7} + 78792*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 117483)*(33*I*\sqrt{7} + 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21) - 29792*(33/112*I*\sqrt{7} - 1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 1/256*((67*\sqrt{7}*(-33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 21) - 2432*\sqrt{7}))* (33*I*\sqrt{7} + 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21) - 2432*\sqrt{7}*(-33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 21) + 147456*\sqrt{7}))*\sqrt{-336*(33/112*I*\sqrt{7} - 1/2*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 336*(-33/112*I*\sqrt{7} - 1/2*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} + 3/16)^2 - 1/56*(33*I*\sqrt{7} + 56*\sqrt{-2101/1568*I*\sqrt{7} - 55/32} - 21)*(-33*I*\sqrt{7} + 56*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 63) + 99/2*I*\sqrt{7} - 84*\sqrt{2101/1568*I*\sqrt{7} - 55/32} - 1859/2) + 30976*x + 22671/2*I*\sqrt{7} - 19236*\sqrt{2101/1568*I*\sqrt{7} - 55/32} + 53979/2) + 168*x*\log(x) + 560)/x$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)

maple [C] time = 0.01, size = 72, normalized size = 0.26

$$-\frac{3\ln(x)}{4} + \frac{(6\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 - 17\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 13\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) - 35)\ln(-\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + x)}{32\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^3 + 12\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2)^2 + 40\operatorname{RootOf}(2_Z^4 + _Z^3 + 5_Z^2 + _Z + 2) + 4} - \frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)

[Out] $\frac{1}{4} \sum \left(\frac{6R^3 - 17R^2 + 13R - 35}{(8R^3 + 3R^2 + 10R + 1) \ln(-R+x)}, R = \text{Root Of } (2Z^4 + Z^3 + 5Z^2 + Z + 2) \right) - \frac{5}{2x} - \frac{3}{4} \ln(x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5}{2x} + \frac{1}{4} \int \frac{6x^3 - 17x^2 + 13x - 35}{2x^4 + x^3 + 5x^2 + x + 2} dx - \frac{3}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] $-\frac{5}{2x} + \frac{1}{4} \int \frac{6x^3 - 17x^2 + 13x - 35}{2x^4 + x^3 + 5x^2 + x + 2} dx - \frac{3}{4} \log(x)$

mupad [B] time = 2.30, size = 242, normalized size = 0.86

$$\frac{1}{2} \left(\frac{1199 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{4169 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{43993 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{28 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{3675 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{11647 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{7273 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{441 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}}{\sqrt{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}} + \frac{21}{4} \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} \right) - \frac{3 \log(z)}{4} - \frac{5}{2z}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)

[Out] $\text{symsum}(\log((1199 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 25x + (4169 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + (43993 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 28 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 3675 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 11647 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 7273 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 441 \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}} + 21/4) \sqrt{\frac{z^4 - (3z^3)/4 + (16z^2)/7 + (96z)/49 + 128/343}{32}}), z, k) - (3 \log(x))/4 - 5/(2x)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)

[Out] Timed out

$$3.252 \quad \int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)} dx$$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2} - \frac{35-9i\sqrt{7}}{56x^2} + \frac{1}{32} \left(35-9i\sqrt{7} \right) \log \left(4ix^2 + \left(-\sqrt{7} + i \right) x + 4i \right) + \frac{1}{32} \left(35+9i\sqrt{7} \right) \log \left(4ix^2 + \left(\sqrt{7} + i \right) x + 4i \right)$$

Rubi [A] time = 0.54, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, number of rules / integrand size = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{35+9i\sqrt{7}}{56x^2} - \frac{35-9i\sqrt{7}}{56x^2} + \frac{1}{32} (35-9i\sqrt{7}) \log(4ix^2 + (-\sqrt{7}+i)x + 4i) + \frac{1}{32} (35+9i\sqrt{7}) \log(4ix^2 + (\sqrt{7}+i)x + 4i) + \frac{3(7+11i\sqrt{7})}{56x} + \frac{3(7-11i\sqrt{7})}{56x} - \frac{1}{16} (35+9i\sqrt{7}) \log(x) - \frac{1}{16} (35-9i\sqrt{7}) \log(x) + \frac{(355-73i\sqrt{7}) \tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{8\sqrt{14(35-i\sqrt{7})}} - \frac{(355+73i\sqrt{7}) \tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{8\sqrt{14(35+i\sqrt{7})}}$$

Antiderivative was successfully verified.

[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (11*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d}, x]

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 2087

Int[((P3_)*(x_)^(m_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]

Rubi steps

$$\begin{aligned}
\int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 5x^2 + x^3 + 2x^4)} dx &= \frac{i \int \frac{9-5i\sqrt{7}+(10-2i\sqrt{7})x}{x^3(4+(1-i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} - \frac{i \int \frac{9+5i\sqrt{7}+(10+2i\sqrt{7})x}{x^3(4+(1+i\sqrt{7})x+4x^2)} dx}{\sqrt{7}} \\
&= -\frac{i \int \left(\frac{9+5i\sqrt{7}}{4x^3} + \frac{3(11-i\sqrt{7})}{8x^2} - \frac{7i(-9i+5\sqrt{7})}{16x} + \frac{-223i-61\sqrt{7}+14(9i-5\sqrt{7})x}{8(4i+(i-\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} + \frac{i \int \left(\frac{9-5i\sqrt{7}}{4x^3} + \frac{3(11+i\sqrt{7})}{8x^2} - \frac{7i(-9i-5\sqrt{7})}{16x} + \frac{-223i+61\sqrt{7}+14(9i+5\sqrt{7})x}{8(4i+(i+\sqrt{7})x+4ix^2)} \right) dx}{\sqrt{7}} \\
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} (35-9i\sqrt{7}) \\
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} (35-9i\sqrt{7}) \\
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} - \frac{1}{16} (35-9i\sqrt{7}) \\
&= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3(7-11i\sqrt{7})}{56x} + \frac{3(7+11i\sqrt{7})}{56x} + \frac{(355-73i\sqrt{7})}{8}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 116, normalized size = 0.37

$$\frac{1}{8} \text{RootSum} \left[2\#1^4 + \#1^3 + 5\#1^2 + \#1 + 2\&, \frac{70\#1^3 \log(x - \#1) + 47\#1^2 \log(x - \#1) + 141\#1 \log(x - \#1) + 61 \log(x - \#1)}{8\#1^3 + 3\#1^2 + 10\#1 + 1} \& \right] - \frac{5}{4x^2} + \frac{3}{4x} - \frac{35 \log(x)}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^3(2 + x + 5x^2 + x^3 + 2x^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]

[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)), x]

fricas [B] time = 5.63, size = 1274, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")

[Out]
$$-1/448*(14*x^2*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35)*\log(-49/4*(207711*I*\sqrt{7} + 369264*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 957269)*(9/32*I*\sqrt{7} - 1/2*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 + 9046968*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^3 - 39580485*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 + 16963065*I*\sqrt{7} + 30156560*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 68488563*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35) + 9662336*x - 68336919/4*I*\sqrt{7} - 30371964*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 257023549/4) + 14*x^2*(9*I*\sqrt{7} + 16*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 35)*\log(-9046968*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^3 + 41411909*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 + 9662336*x + 70198191/4*I*\sqrt{7} + 31199196*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 240366533/4) + 1960*x^2*\log(x) + (4*\sqrt{7})*\sqrt{-1344*(9/32*I*\sqrt{7} - 1/2*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 1344*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 7/8*(9*I*\sqrt{7} + 16*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 105)*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35) - 2205/2*I*\sqrt{7} - 1960*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 1661/2)*x^2 - 7*x^2*(9*I*\sqrt{7} + 16*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 35) - 7*x^2*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35) - 980*x^2)*\log(49/4*(207711*I*\sqrt{7} + 369264*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 957269)*(9/32*I*\sqrt{7} - 1/2*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 1831424*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 + 21/1024*(13785856*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 + 16963065*I*\sqrt{7} + 30156560*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) - 68488563*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35) + 1/1024*(4*\sqrt{-1344*(9/32*I*\sqrt{7} - 1/2*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 1344*(-9/32*I*\sqrt{7} - 1/2*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 35/32)^2 - 7/8*(9*I*\sqrt{7} + 16*\sqrt{-9803/6272*I*\sqrt{7} + 2815/896}) + 105)*(-9*I*\sqrt{7} + 16*\sqrt{9803/6272*I*\sqrt{7} + 2815/896}) - 35) - 2205/$$

$2\sqrt{7} - 1960\sqrt{-9803/6272\sqrt{7} + 2815/896} + 1661/2 \cdot (7 \cdot (23079\sqrt{7} \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 35) - 149504\sqrt{7}) \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) - 1046528\sqrt{7} \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 35) - 116260864\sqrt{7}) + 19324672x - 465318\sqrt{7} - 827232\sqrt{-9803/6272\sqrt{7} + 2815/896} + 666914) - (4\sqrt{7} \cdot \sqrt{-1344 \cdot (9/32\sqrt{7} - 1/2\sqrt{9803/6272\sqrt{7} + 2815/896}) + 35/32})^2 - 1344 \cdot (-9/32\sqrt{7} - 1/2\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 35/32)^2 - 7/8 \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 105) \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) - 2205/2\sqrt{7} - 1960\sqrt{-9803/6272\sqrt{7} + 2815/896} + 1661/2) \cdot x^2 + 7x^2 \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 35) + 7x^2 \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) + 980x^2 \cdot \log(49/4 \cdot (207711\sqrt{7} + 369264\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 957269) \cdot (9/32\sqrt{7} - 1/2\sqrt{9803/6272\sqrt{7} + 2815/896}) + 35/32)^2 - 1831424 \cdot (-9/32\sqrt{7} - 1/2\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 35/32)^2 + 21/1024 \cdot (13785856 \cdot (-9/32\sqrt{7} - 1/2\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 35/32)^2 + 16963065\sqrt{7} + 30156560\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 68488563) \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) - 1/1024\sqrt{-1344 \cdot (9/32\sqrt{7} - 1/2\sqrt{9803/6272\sqrt{7} + 2815/896}) + 35/32})^2 - 1344 \cdot (-9/32\sqrt{7} - 1/2\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 35/32)^2 - 7/8 \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) + 105) \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) - 2205/2\sqrt{7} - 1960\sqrt{-9803/6272\sqrt{7} + 2815/896} + 1661/2) \cdot (7 \cdot (23079\sqrt{7} \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 35) - 149504\sqrt{7}) \cdot (-9\sqrt{7} + 16\sqrt{9803/6272\sqrt{7} + 2815/896}) - 35) - 1046528\sqrt{7} \cdot (9\sqrt{7} + 16\sqrt{-9803/6272\sqrt{7} + 2815/896}) - 35) - 116260864\sqrt{7}) + 19324672x - 465318\sqrt{7} - 827232\sqrt{-9803/6272\sqrt{7} + 2815/896} + 666914) - 336x + 560) / x^2$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3 + 3x^2 + x + 5}{(2x^4 + x^3 + 5x^2 + x + 2)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)

maple [C] time = 0.01, size = 77, normalized size = 0.24

$$-\frac{35 \ln(x)}{8} + \frac{(70 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^3 + 47 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^2 + 141 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) + 61) \ln(-\operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) + x)}{64 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^3 + 24 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2)^2 + 80 \operatorname{RootOf}(2Z^4 + Z^3 + 5Z^2 + Z + 2) + 8} + \frac{3}{4x} - \frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2), x)

[Out] 1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x), _R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/4/x^2+3/4/x-35/8*ln(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3x-5}{4x^2} + \frac{1}{8} \int \frac{70x^3 + 47x^2 + 141x + 61}{2x^4 + x^3 + 5x^2 + x + 2} dx - \frac{35}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2), x, algorithm="maxima")

[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x^3 + 5*x^2 + x + 2), x) - 35/8*log(x)

mupad [B] time = 2.25, size = 246, normalized size = 0.78

$$\sum_{k=1}^4 \left(\frac{8939 \operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{128} \cdot \frac{69x}{8} - \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{128} \cdot 14945 \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{1024} \cdot 269991 \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{1393} \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{512} \cdot 18487 \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{256} \cdot 441 \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{32} \cdot 245 \cdot \frac{\operatorname{root}\left(x^4 - \frac{35x^3}{8} + \frac{47x^2}{7} - \frac{8x}{7} + \frac{128}{343}\right)}{32} \cdot \frac{35 \log(x)}{8} \cdot \frac{3x-5}{4x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)), x)

[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 - 5/4)/x^2

sympy [A] time = 2.70, size = 70, normalized size = 0.22

$$-\frac{35 \log(x)}{8} + \operatorname{RootSum}\left(2744t^4 - 12005t^3 + 18424t^2 - 3136t + 1024, \left(t \mapsto t \log\left(-\frac{20101387287723t^4}{91907904361586} + \frac{944515214496t^3}{45953952180793} + \frac{16572327093911939t^2}{5882105879141504} - \frac{4564471749800865t}{735263234892688} + x + \frac{70084064010625}{91907904361586}\right)\right) + \frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2), x)

[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*

$$\frac{_t^{**3}/45953952180793 + 16572327093911939*_t^{**2}/5882105879141504 - 4564471749800865*_t/735263234892688 + x + 70084064010625/91907904361586)}{(3*x - 5)/(4*x^{**2})}$$

$$3.253 \quad \int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx$$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Rubi [A] time = 0.10, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6), x]

[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6} dx = (3a^2) \text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2} dx, x, \frac{x^3}{3a+3bx^2}\right) = \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 4.58

$$\frac{1}{2} \text{RootSum} \left[\#1^6 c^2 + \#1^4 b^2 + 2\#1^2 ab + a^2 \&, \frac{\#1^3 b \log(x - \#1) + 3\#1 a \log(x - \#1)}{3\#1^4 c^2 + 2\#1^2 b^2 + 2ab} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6), x]

[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 &, (3*a*Log[x - #1]*#1 + b*Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) &]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (3a + bx^2)}{a^2 + 2abx^2 + b^2x^4 + c^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6), x]

[Out] IntegrateAlgebraic[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6), x]

fricas [B] time = 2.11, size = 83, normalized size = 4.37

$$\frac{\arctan\left(\frac{cx}{b}\right) - \arctan\left(\frac{bc^2x^5 + ab^2x + (b^3 - ac^2)x^3}{a^2c}\right) + \arctan\left(\frac{bc^2x^3 + (b^3 - ac^2)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2), x, algorithm="fricas")

[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c)) + arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c

giac [B] time = 4.08, size = 87, normalized size = 4.58

$$\frac{\arctan\left(\frac{cx}{b}\right) + \arctan\left(-\frac{bc^2x^5 + b^3x^3 - ac^2x^3 + ab^2x}{a^2c}\right) - \arctan\left(-\frac{bc^2x^3 + b^3x - ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="giac")

[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c

maple [C] time = 0.10, size = 75, normalized size = 3.95

$$\frac{\left(\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^4 b + 3\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^2 a\right) \ln\left(-\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right) + x\right)}{6\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^5 c^2 + 4\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right)^3 b^2 + 4\text{RootOf}\left(c^2_Z^6 + b^2_Z^4 + 2ab_Z^2 + a^2\right) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(-_R+x), _R=RootOf(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + 3a)x^2}{c^2x^6 + b^2x^4 + 2abx^2 + a^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="maxima")

[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)

mupad [B] time = 2.27, size = 252, normalized size = 13.26

$$\text{atan}\left(\frac{27ac^5x^3}{27a^2c^4-4ab^3c^2} - \frac{27b^5x^5}{27a^2c^4-4ab^3c^2} - \frac{31b^3c^3x^3}{27a^2c^4-4ab^3c^2} + \frac{4b^6cx^3}{27a^3c^4-4a^2b^3c^2} + \frac{4b^5cx}{27a^2c^4-4ab^3c^2} + \frac{4b^4c^3x^5}{27a^3c^4-4a^2b^3c^2} - \frac{27ab^2c^3x}{27a^2c^4-4ab^3c^2}\right) + \text{atan}\left(\frac{cx^3}{a} - \frac{cx}{b} + \frac{b^2x}{ac}\right) + \text{atan}\left(\frac{cx}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)

[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/b))/c

sympy [C] time = 1.05, size = 44, normalized size = 2.32

$$\frac{-\frac{i \log\left(-\frac{ia}{c} - \frac{ibx^2}{c} + x^3\right)}{2} + \frac{i \log\left(\frac{ia}{c} + \frac{ibx^2}{c} + x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)

[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)/c

$$3.254 \quad \int \frac{1-3x^4}{(-2+x)(1+x^2)^2} dx$$

Optimal. Leaf size=43

$$-\frac{1-2x}{5(x^2+1)} - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(2-x) - \frac{46}{25} \tan^{-1}(x)$$

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1647, 1629, 635, 203, 260}

$$-\frac{1-2x}{5(x^2+1)} - \frac{14}{25} \log(x^2+1) - \frac{47}{25} \log(2-x) - \frac{46}{25} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log[1 + x^2])/25

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1 - 3x^4}{(-2 + x)(1 + x^2)^2} dx &= -\frac{1 - 2x}{5(1 + x^2)} - \frac{1}{2} \int \frac{-\frac{18}{5} - \frac{4x}{5} + 6x^2}{(-2 + x)(1 + x^2)} dx \\ &= -\frac{1 - 2x}{5(1 + x^2)} - \frac{1}{2} \int \left(\frac{94}{25(-2 + x)} + \frac{4(23 + 14x)}{25(1 + x^2)} \right) dx \\ &= -\frac{1 - 2x}{5(1 + x^2)} - \frac{47}{25} \log(2 - x) - \frac{2}{25} \int \frac{23 + 14x}{1 + x^2} dx \\ &= -\frac{1 - 2x}{5(1 + x^2)} - \frac{47}{25} \log(2 - x) - \frac{28}{25} \int \frac{x}{1 + x^2} dx - \frac{46}{25} \int \frac{1}{1 + x^2} dx \\ &= -\frac{1 - 2x}{5(1 + x^2)} - \frac{46}{25} \tan^{-1}(x) - \frac{47}{25} \log(2 - x) - \frac{14}{25} \log(1 + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.33

$$\frac{2(x - 2) + 3}{5((x - 2)^2 + 4(x - 2) + 5)} - \frac{14}{25} \log((x - 2)^2 + 4(x - 2) + 5) - \frac{47}{25} \log(x - 2) - \frac{46}{25} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (14*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 3x^4}{(-2 + x)(1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]

fricas [A] time = 0.88, size = 47, normalized size = 1.09

$$\frac{46(x^2 + 1) \arctan(x) + 14(x^2 + 1) \log(x^2 + 1) + 47(x^2 + 1) \log(x - 2) - 10x + 5}{25(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*log(x - 2) - 10*x + 5)/(x^2 + 1)

giac [A] time = 0.25, size = 34, normalized size = 0.79

$$\frac{2x - 1}{5(x^2 + 1)} - \frac{46}{25} \arctan(x) - \frac{14}{25} \log(x^2 + 1) - \frac{47}{25} \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")

[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(abs(x - 2))

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{46 \arctan(x)}{25} - \frac{47 \ln(x - 2)}{25} - \frac{14 \ln(x^2 + 1)}{25} - \frac{2 \left(-5x + \frac{5}{2}\right)}{25(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x^4+1)/(-2+x)/(x^2+1)^2,x)

[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(-2+x)

maxima [A] time = 2.10, size = 33, normalized size = 0.77

$$\frac{2x - 1}{5(x^2 + 1)} - \frac{46}{25} \arctan(x) - \frac{14}{25} \log(x^2 + 1) - \frac{47}{25} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")

[Out] $\frac{1}{5} \cdot \frac{2x - 1}{x^2 + 1} - \frac{46}{25} \arctan(x) - \frac{14}{25} \log(x^2 + 1) - \frac{47}{25} \log(x - 2)$

mupad [B] time = 0.05, size = 38, normalized size = 0.88

$$\frac{\frac{2x}{5} - \frac{1}{5}}{x^2 + 1} - \frac{47 \ln(x - 2)}{25} + \ln(x - i) \left(-\frac{14}{25} + \frac{23}{25}i \right) + \ln(x + i) \left(-\frac{14}{25} - \frac{23}{25}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)

[Out] $\left(\frac{2x}{5} - \frac{1}{5} \right) / (x^2 + 1) - \log(x - i) \cdot \left(\frac{14}{25} - \frac{23i}{25} \right) - \log(x + i) \cdot \left(\frac{14}{25} + \frac{23i}{25} \right) - \frac{47 \log(x - 2)}{25}$

sympy [A] time = 0.17, size = 37, normalized size = 0.86

$$-\frac{1 - 2x}{5x^2 + 5} - \frac{47 \log(x - 2)}{25} - \frac{14 \log(x^2 + 1)}{25} - \frac{46 \operatorname{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)

[Out] $-(1 - 2x) / (5x^2 + 5) - 47 \log(x - 2) / 25 - 14 \log(x^2 + 1) / 25 - 46 \operatorname{atan}(x) / 25$

$$3.255 \quad \int \frac{-9-9x+2x^2}{-9x+x^3} dx$$

Optimal. Leaf size=17

$$-\log(3-x) + \log(x) + 2 \log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1593, 1802}

$$-\log(3-x) + \log(x) + 2 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-9-9x+2x^2}{-9x+x^3} dx &= \int \frac{-9-9x+2x^2}{x(-9+x^2)} dx \\ &= \int \left(\frac{1}{3-x} + \frac{1}{x} + \frac{2}{3+x} \right) dx \\ &= -\log(3-x) + \log(x) + 2 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\log(3-x) + \log(x) + 2 \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9 - 9x + 2x^2}{-9x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

[Out] IntegrateAlgebraic[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]

fricas [A] time = 1.15, size = 15, normalized size = 0.88

$$2 \log(x + 3) - \log(x - 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-9*x-9)/(x^3-9*x), x, algorithm="fricas")

[Out] 2*log(x + 3) - log(x - 3) + log(x)

giac [A] time = 0.31, size = 18, normalized size = 1.06

$$2 \log(|x + 3|) - \log(|x - 3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-9*x-9)/(x^3-9*x), x, algorithm="giac")

[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) - \ln(x - 3) + 2 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-9*x-9)/(x^3-9*x), x)

[Out] 2*ln(3+x)-ln(-3+x)+ln(x)

maxima [A] time = 1.12, size = 15, normalized size = 0.88

$$2 \log(x + 3) - \log(x - 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")

[Out] 2*log(x + 3) - log(x - 3) + log(x)

mupad [B] time = 2.20, size = 21, normalized size = 1.24

$$2 \ln(x + 3) - 2 \operatorname{atanh}\left(\frac{1296}{18x + 162} - 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((9*x - 2*x^2 + 9)/(9*x - x^3),x)

[Out] 2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\log(x) - \log(x - 3) + 2 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)

[Out] log(x) - log(x - 3) + 2*log(x + 3)

$$3.256 \quad \int \frac{1+2x^2+x^5}{-x+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^3}{3} + x + 2 \log(1-x) - \log(x) + \log(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + 2 \log(1-x) - \log(x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2+x^5}{-x+x^3} dx &= \int \frac{1+2x^2+x^5}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x} + x^2 + \frac{1}{1+x} \right) dx \\ &= x + \frac{x^3}{3} + 2 \log(1-x) - \log(x) + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3} + x + 2 \log(1 - x) - \log(x) + \log(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x^2 + x^5}{-x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2*x^2 + x^5)/(-x + x^3), x]

[Out] IntegrateAlgebraic[(1 + 2*x^2 + x^5)/(-x + x^3), x]

fricas [A] time = 1.38, size = 21, normalized size = 0.84

$$\frac{1}{3} x^3 + x + \log(x + 1) + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+2*x^2+1)/(x^3-x), x, algorithm="fricas")

[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)

giac [A] time = 0.25, size = 24, normalized size = 0.96

$$\frac{1}{3} x^3 + x + \log(|x + 1|) + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+2*x^2+1)/(x^3-x), x, algorithm="giac")

[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^3}{3} + x - \ln(x) + 2 \ln(x - 1) + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5+2*x^2+1)/(x^3-x),x)`

[Out] $1/3*x^3+x+2*\ln(x-1)+\ln(x+1)-\ln(x)$

maxima [A] time = 1.15, size = 21, normalized size = 0.84

$$\frac{1}{3}x^3 + x + \log(x + 1) + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")`

[Out] $1/3*x^3 + x + \log(x + 1) + 2*\log(x - 1) - \log(x)$

mupad [B] time = 0.05, size = 30, normalized size = 1.20

$$x + 2 \ln(x - 1) + \frac{x^3}{3} + \operatorname{atan}\left(\frac{48i}{11(22x - 2)} + \frac{13i}{11}\right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(2*x^2 + x^5 + 1)/(x - x^3),x)`

[Out] $x + 2*\log(x - 1) + \operatorname{atan}(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3$

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$\frac{x^3}{3} + x - \log(x) + 2 \log(x - 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**5+2*x**2+1)/(x**3-x),x)`

[Out] $x**3/3 + x - \log(x) + 2*\log(x - 1) + \log(x + 1)$

$$3.257 \quad \int \frac{3+2x^2}{(-1+x)^2x} dx$$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {894}

$$\frac{5}{1-x} - \log(1-x) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)/((-1 + x)^2*x), x]

[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{3+2x^2}{(-1+x)^2x} dx &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x} \right) dx \\ &= \frac{5}{1-x} - \log(1-x) + 3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x), x]

[Out] $-5/(-1 + x) - \text{Log}[1 - x] + 3*\text{Log}[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 + 2x^2}{(-1 + x)^2 x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 2*x^2)/((-1 + x)^2*x), x]

[Out] IntegrateAlgebraic[(3 + 2*x^2)/((-1 + x)^2*x), x]

fricas [A] time = 1.43, size = 24, normalized size = 1.09

$$\frac{(x - 1) \log(x - 1) - 3(x - 1) \log(x) + 5}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")

[Out] $-\frac{(x - 1) \log(x - 1) - 3(x - 1) \log(x) + 5}{x - 1}$

giac [A] time = 0.37, size = 28, normalized size = 1.27

$$-\frac{5}{x - 1} + 2 \log(|x - 1|) + 3 \log\left(\left|-\frac{1}{x - 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")

[Out] $-5/(x - 1) + 2*\log(\text{abs}(x - 1)) + 3*\log(\text{abs}(-1/(x - 1) - 1))$

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$3 \ln(x) - \ln(x - 1) - \frac{5}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+3)/(x-1)^2/x,x)

[Out] $-5/(x-1) - \ln(x-1) + 3*\ln(x)$

maxima [A] time = 1.09, size = 18, normalized size = 0.82

$$-\frac{5}{x - 1} - \log(x - 1) + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")

[Out] -5/(x - 1) - log(x - 1) + 3*log(x)

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$3 \ln(x) - \ln(x - 1) - \frac{5}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)

[Out] 3*log(x) - log(x - 1) - 5/(x - 1)

sympy [A] time = 0.11, size = 14, normalized size = 0.64

$$3 \log(x) - \log(x - 1) - \frac{5}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+3)/(-1+x)**2/x,x)

[Out] 3*log(x) - log(x - 1) - 5/(x - 1)

$$3.258 \quad \int \frac{-1+2x^2}{(-1+4x)(1+x^2)} dx$$

Optimal. Leaf size=27

$$\frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(1 - 4x) + \frac{3}{17} \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1629, 635, 203, 260}

$$\frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(1 - 4x) + \frac{3}{17} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]

[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-1 + 2x^2}{(-1 + 4x)(1 + x^2)} dx &= \int \left(-\frac{14}{17(-1 + 4x)} + \frac{3(1 + 4x)}{17(1 + x^2)} \right) dx \\
&= -\frac{7}{34} \log(1 - 4x) + \frac{3}{17} \int \frac{1 + 4x}{1 + x^2} dx \\
&= -\frac{7}{34} \log(1 - 4x) + \frac{3}{17} \int \frac{1}{1 + x^2} dx + \frac{12}{17} \int \frac{x}{1 + x^2} dx \\
&= \frac{3}{17} \tan^{-1}(x) - \frac{7}{34} \log(1 - 4x) + \frac{6}{17} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.41

$$-\frac{7}{34} \log(4x - 1) + \frac{6}{17} \log((4x - 1)^2 + 2(4x - 1) + 17) + \frac{3}{17} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]

[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 + 4*x)^2])/17

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 2x^2}{(-1 + 4x)(1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]

fricas [A] time = 1.52, size = 21, normalized size = 0.78

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

giac [A] time = 0.29, size = 22, normalized size = 0.81

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(|4x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3 \arctan(x)}{17} - \frac{7 \ln(4x - 1)}{34} + \frac{6 \ln(x^2 + 1)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-1)/(4*x-1)/(x^2+1),x)

[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(4*x-1)

maxima [A] time = 2.31, size = 21, normalized size = 0.78

$$\frac{3}{17} \arctan(x) + \frac{6}{17} \log(x^2 + 1) - \frac{7}{34} \log(4x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")

[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)

mupad [B] time = 2.26, size = 25, normalized size = 0.93

$$-\frac{7 \ln\left(x - \frac{1}{4}\right)}{34} + \ln(x - i) \left(\frac{6}{17} - \frac{3}{34}i\right) + \ln(x + i) \left(\frac{6}{17} + \frac{3}{34}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)

[Out] log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/34)

sympy [A] time = 0.14, size = 26, normalized size = 0.96

$$-\frac{7 \log\left(x - \frac{1}{4}\right)}{34} + \frac{6 \log(x^2 + 1)}{17} + \frac{3 \operatorname{atan}(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

```
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

$$3.259 \quad \int \frac{-3+2x-3x^2+x^3}{1+x^2} dx$$

Optimal. Leaf size=21

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1810, 260}

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

Antiderivative was successfully verified.

[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]

[Out] -3*x + x^2/2 + Log[1 + x^2]/2

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-3 + 2x - 3x^2 + x^3}{1 + x^2} dx &= \int \left(-3 + x + \frac{x}{1 + x^2} \right) dx \\ &= -3x + \frac{x^2}{2} + \int \frac{x}{1 + x^2} dx \\ &= -3x + \frac{x^2}{2} + \frac{1}{2} \log(1 + x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) - 3x$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]

[Out] -3*x + x^2/2 + Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + 2x - 3x^2 + x^3}{1 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]

[Out] IntegrateAlgebraic[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]

fricas [A] time = 1.49, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1), x, algorithm="fricas")

[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)

giac [A] time = 0.27, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1), x, algorithm="giac")

[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^2}{2} - 3x + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3*x^2+2*x-3)/(x^2+1), x)

[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)

maxima [A] time = 2.28, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\ln(x^2 + 1)}{2} - 3x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)

[Out] log(x^2 + 1)/2 - 3*x + x^2/2

sympy [A] time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^2}{2} - 3x + \frac{\log(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)

[Out] x**2/2 - 3*x + log(x**2 + 1)/2

$$3.260 \quad \int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} dx$$

Optimal. Leaf size=27

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

Antiderivative was successfully verified.

[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

`Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}
 \int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} dx &= \int \left(x^2 + \frac{x}{10 + 6x + x^2} \right) dx \\
 &= \frac{x^3}{3} + \int \frac{x}{10 + 6x + x^2} dx \\
 &= \frac{x^3}{3} + \frac{1}{2} \int \frac{6 + 2x}{10 + 6x + x^2} dx - 3 \int \frac{1}{10 + 6x + x^2} dx \\
 &= \frac{x^3}{3} + \frac{1}{2} \log(10 + 6x + x^2) + 6 \operatorname{Subst} \left(\int \frac{1}{-4 - x^2} dx, x, 6 + 2x \right) \\
 &= \frac{x^3}{3} - 3 \tan^{-1}(3 + x) + \frac{1}{2} \log(10 + 6x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^3}{3} + \frac{1}{2} \log(x^2 + 6x + 10) - 3 \tan^{-1}(x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 10x^2 + 6x^3 + x^4}{10 + 6x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

[Out] IntegrateAlgebraic[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]

fricas [A] time = 1.45, size = 23, normalized size = 0.85

$$\frac{1}{3} x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

giac [A] time = 0.28, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3} - 3 \arctan(x + 3) + \frac{\ln(x^2 + 6x + 10)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x)

[Out] 1/3*x^3-3*arctan(x+3)+1/2*ln(x^2+6*x+10)

maxima [A] time = 2.08, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3 - 3 \arctan(x + 3) + \frac{1}{2} \log(x^2 + 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")

[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)

mupad [B] time = 2.14, size = 23, normalized size = 0.85

$$\frac{\ln(x^2 + 6x + 10)}{2} - 3 \operatorname{atan}(x + 3) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)

[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3

sympy [A] time = 0.11, size = 22, normalized size = 0.81

$$\frac{x^3}{3} + \frac{\log(x^2 + 6x + 10)}{2} - 3 \operatorname{atan}(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)

[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)

$$3.261 \quad \int \frac{1}{-18+27x-7x^2-3x^3+x^4} dx$$

Optimal. Leaf size=39

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2058}

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120

Rule 2058

Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p, x], x] /; !SumQ[NonfreeFactors[u, x]] /; PolyQ[P, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{-18+27x-7x^2-3x^3+x^4} dx &= \int \left(\frac{1}{12(-3+x)} - \frac{1}{5(-2+x)} + \frac{1}{8(-1+x)} - \frac{1}{120(3+x)} \right) dx \\ &= \frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{1}{8} \log(1-x) - \frac{1}{5} \log(2-x) + \frac{1}{12} \log(3-x) - \frac{1}{120} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-18 + 27x - 7x^2 - 3x^3 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

[Out] IntegrateAlgebraic[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]

fricas [A] time = 1.13, size = 25, normalized size = 0.64

$$-\frac{1}{120} \log(x + 3) + \frac{1}{8} \log(x - 1) - \frac{1}{5} \log(x - 2) + \frac{1}{12} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18), x, algorithm="fricas")

[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)

giac [A] time = 0.32, size = 29, normalized size = 0.74

$$-\frac{1}{120} \log(|x + 3|) + \frac{1}{8} \log(|x - 1|) - \frac{1}{5} \log(|x - 2|) + \frac{1}{12} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18), x, algorithm="giac")

[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*log(abs(x - 3))

maple [A] time = 0.01, size = 26, normalized size = 0.67

$$\frac{\ln(x - 3)}{12} - \frac{\ln(x - 2)}{5} + \frac{\ln(x - 1)}{8} - \frac{\ln(x + 3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4-3*x^3-7*x^2+27*x-18), x)

[Out] 1/8*ln(x-1)-1/5*ln(x-2)-1/120*ln(x+3)+1/12*ln(x-3)

maxima [A] time = 1.27, size = 25, normalized size = 0.64

$$-\frac{1}{120} \log(x + 3) + \frac{1}{8} \log(x - 1) - \frac{1}{5} \log(x - 2) + \frac{1}{12} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")

[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)

mupad [B] time = 2.14, size = 25, normalized size = 0.64

$$\frac{\ln(x-1)}{8} - \frac{\ln(x-2)}{5} + \frac{\ln(x-3)}{12} - \frac{\ln(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)

[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120

sympy [A] time = 0.24, size = 26, normalized size = 0.67

$$\frac{\log(x-3)}{12} - \frac{\log(x-2)}{5} + \frac{\log(x-1)}{8} - \frac{\log(x+3)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)

[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120

$$3.262 \quad \int \frac{1+x^3}{-2+x} dx$$

Optimal. Leaf size=22

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1850}

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-2 + x), x]

[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]

Rule 1850

Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand [Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-2+x} dx &= \int \left(4 + \frac{9}{-2+x} + 2x + x^2 \right) dx \\ &= 4x + x^2 + \frac{x^3}{3} + 9 \log(2-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(x-2) - \frac{44}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-2 + x), x]

[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{-2+x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^3)/(-2 + x), x]

[Out] IntegrateAlgebraic[(1 + x^3)/(-2 + x), x]

fricas [A] time = 1.01, size = 18, normalized size = 0.82

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(-2+x), x, algorithm="fricas")

[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)

giac [A] time = 0.28, size = 19, normalized size = 0.86

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(-2+x), x, algorithm="giac")

[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^3}{3} + x^2 + 4x + 9 \ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x-2), x)

[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)

maxima [A] time = 1.13, size = 18, normalized size = 0.82

$$\frac{1}{3}x^3 + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)/(-2+x),x, algorithm="maxima")`

[Out] `1/3*x^3 + x^2 + 4*x + 9*log(x - 2)`

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$4x + 9 \ln(x - 2) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 + 1)/(x - 2),x)`

[Out] `4*x + 9*log(x - 2) + x^2 + x^3/3`

sympy [A] time = 0.07, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+1)/(-2+x),x)`

[Out] `x**3/3 + x**2 + 4*x + 9*log(x - 2)`

$$3.263 \quad \int \frac{3x-4x^2+3x^3}{1+x^2} dx$$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1594, 1802, 203}

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(-n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{3x - 4x^2 + 3x^3}{1 + x^2} dx &= \int \frac{x(3 - 4x + 3x^2)}{1 + x^2} dx \\
&= \int \left(-4 + 3x + \frac{4}{1 + x^2} \right) dx \\
&= -4x + \frac{3x^2}{2} + 4 \int \frac{1}{1 + x^2} dx \\
&= -4x + \frac{3x^2}{2} + 4 \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{3x^2}{2} - 4x + 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]

[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x - 4x^2 + 3x^3}{1 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]

[Out] IntegrateAlgebraic[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]

fricas [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{3}{2} x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1), x, algorithm="fricas")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

giac [A] time = 0.34, size = 13, normalized size = 0.87

$$\frac{3}{2} x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x)

[Out] -4*x+3/2*x^2+4*arctan(x)

maxima [A] time = 2.30, size = 13, normalized size = 0.87

$$\frac{3}{2}x^2 - 4x + 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")

[Out] 3/2*x^2 - 4*x + 4*arctan(x)

mupad [B] time = 2.13, size = 13, normalized size = 0.87

$$4 \operatorname{atan}(x) - 4x + \frac{3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)

[Out] 4*atan(x) - 4*x + (3*x^2)/2

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)

[Out] 3*x**2/2 - 4*x + 4*atan(x)

$$3.264 \quad \int \frac{5+3x}{1-x-x^2+x^3} dx$$

Optimal. Leaf size=12

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2074, 206}

$$\frac{4}{1-x} + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3), x]

[Out] 4/(1 - x) + ArcTanh[x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{5+3x}{1-x-x^2+x^3} dx &= \int \left(\frac{4}{(-1+x)^2} + \frac{1}{1-x^2} \right) dx \\ &= \frac{4}{1-x} + \int \frac{1}{1-x^2} dx \\ &= \frac{4}{1-x} + \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 2.00

$$-\frac{4}{x-1} - \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3), x]

[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + 3x}{1 - x - x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + 3*x)/(1 - x - x^2 + x^3), x]

[Out] IntegrateAlgebraic[(5 + 3*x)/(1 - x - x^2 + x^3), x]

fricas [B] time = 1.08, size = 26, normalized size = 2.17

$$\frac{(x - 1) \log(x + 1) - (x - 1) \log(x - 1) - 8}{2(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1), x, algorithm="fricas")

[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)

giac [B] time = 0.30, size = 22, normalized size = 1.83

$$-\frac{4}{x-1} + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1), x, algorithm="giac")

[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))

maple [A] time = 0.01, size = 21, normalized size = 1.75

$$-\frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+3*x)/(x^3-x^2-x+1), x)

[Out] -4/(x-1)-1/2*ln(x-1)+1/2*ln(x+1)

maxima [A] time = 1.08, size = 20, normalized size = 1.67

$$-\frac{4}{x-1} + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")

[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)

mupad [B] time = 0.07, size = 10, normalized size = 0.83

$$\operatorname{atanh}(x) - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)

[Out] atanh(x) - 4/(x - 1)

sympy [B] time = 0.10, size = 17, normalized size = 1.42

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+3*x)/(x**3-x**2-x+1),x)

[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)

$$3.265 \quad \int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1593, 1620}

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]

[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{-1-x-x^3+x^4}{-x^2+x^3} dx &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2} dx \\ &= \int \left(-\frac{2}{-1+x} + \frac{1}{x^2} + \frac{2}{x} + x \right) dx \\ &= -\frac{1}{x} + \frac{x^2}{2} - 2 \log(1-x) + 2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} - 2 \log(1-x) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]

[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 - x - x^3 + x^4}{-x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]

fricas [A] time = 1.19, size = 22, normalized size = 0.88

$$\frac{x^3 - 4x \log(x-1) + 4x \log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2), x, algorithm="fricas")

[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x

giac [A] time = 0.27, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 - \frac{1}{x} - 2 \log(|x-1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3-x-1)/(x^3-x^2), x, algorithm="giac")

[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^2}{2} + 2 \ln(x) - 2 \ln(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4-x^3-x-1)/(x^3-x^2),x)`

[Out] `1/2*x^2-2*ln(x-1)-1/x+2*ln(x)`

maxima [A] time = 1.13, size = 21, normalized size = 0.84

$$\frac{1}{2}x^2 - \frac{1}{x} - 2 \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")`

[Out] `1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)`

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \operatorname{atanh}(2x-1) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)`

[Out] `4*atanh(2*x - 1) - 1/x + x^2/2`

sympy [A] time = 0.10, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2 \log(x) - 2 \log(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4-x**3-x-1)/(x**3-x**2),x)`

[Out] `x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x`

$$3.266 \quad \int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$$

Optimal. Leaf size=13

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1673, 1149, 203, 1247, 626, 31}

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]

[Out] ArcTan[x] + Log[2 + x^2]/2

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(n_), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 626

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m+p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1149

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[(d + e*x^2)^(p+q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx &= \int \frac{x(1+x^2)}{2+3x^2+x^4} dx + \int \frac{2+x^2}{2+3x^2+x^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{2+3x+x^2} dx, x, x^2 \right) + \int \frac{1}{1+x^2} dx \\ &= \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) \\ &= \tan^{-1}(x) + \frac{1}{2} \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]
```

[Out] IntegrateAlgebraic[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]

fricas [A] time = 1.22, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")

[Out] arctan(x) + 1/2*log(x^2 + 2)

giac [A] time = 0.31, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")

[Out] arctan(x) + 1/2*log(x^2 + 2)

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln(x^2 + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x)

[Out] arctan(x)+1/2*ln(x^2+2)

maxima [A] time = 2.16, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")

[Out] arctan(x) + 1/2*log(x^2 + 2)

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
```

```
[Out] log(x^2 + 2)/2 + atan(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 2)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
```

```
[Out] log(x**2 + 2)/2 + atan(x)
```

$$3.267 \quad \int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} dx$$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2} + \frac{1}{2} \log(x^2+2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1814, 1586, 635, 203, 260}

$$-\frac{1}{(x^2+2)^2} + \frac{1}{2} \log(x^2+2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&

EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{-4 + 8x - 4x^2 + 4x^3 - x^4 + x^5}{(2 + x^2)^3} dx &= -\frac{1}{(2 + x^2)^2} - \frac{1}{8} \int \frac{16 - 16x + 8x^2 - 8x^3}{(2 + x^2)^2} dx \\ &= -\frac{1}{(2 + x^2)^2} - \frac{1}{8} \int \frac{8 - 8x}{2 + x^2} dx \\ &= -\frac{1}{(2 + x^2)^2} - \int \frac{1}{2 + x^2} dx + \int \frac{x}{2 + x^2} dx \\ &= -\frac{1}{(2 + x^2)^2} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{2} \log(2 + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$-\frac{1}{(x^2 + 2)^2} + \frac{1}{2} \log(x^2 + 2) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]

[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 8x - 4x^2 + 4x^3 - x^4 + x^5}{(2 + x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]

[Out] IntegrateAlgebraic[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]

fricas [A] time = 1.61, size = 55, normalized size = 1.57

$$\frac{\sqrt{2}(x^4 + 4x^2 + 4) \arctan\left(\frac{1}{2}\sqrt{2}x\right) - (x^4 + 4x^2 + 4) \log(x^2 + 2) + 2}{2(x^4 + 4x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*log(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)

giac [A] time = 0.33, size = 30, normalized size = 0.86

$$-\frac{1}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{(x^2 + 2)^2} + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)

maple [A] time = 0.01, size = 31, normalized size = 0.89

$$-\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{2} + \frac{\ln(x^2 + 2)}{2} - \frac{1}{(x^2 + 2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)

[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)

maxima [A] time = 2.22, size = 35, normalized size = 1.00

$$-\frac{1}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - \frac{1}{x^4 + 4x^2 + 4} + \frac{1}{2} \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")

[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)

mupad [B] time = 2.12, size = 35, normalized size = 1.00

$$\frac{\ln(x^2 + 2)}{2} - \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4 + 4x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)

[Out] log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)

sympy [A] time = 0.15, size = 36, normalized size = 1.03

$$\frac{\log(x^2 + 2)}{2} - \frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{1}{x^4 + 4x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)

[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)

$$3.268 \quad \int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx$$

Optimal. Leaf size=23

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2} \log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1594, 1628}

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1-3x+x^2}{-2x+x^2+x^3} dx &= \int \frac{-1-3x+x^2}{x(-2+x+x^2)} dx \\ &= \int \left(\frac{1}{1-x} + \frac{1}{2x} + \frac{3}{2(2+x)} \right) dx \\ &= -\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\log(1-x) + \frac{\log(x)}{2} + \frac{3}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 - 3x + x^2}{-2x + x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]

fricas [A] time = 2.09, size = 17, normalized size = 0.74

$$\frac{3}{2} \log(x+2) - \log(x-1) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x), x, algorithm="fricas")

[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)

giac [A] time = 0.24, size = 20, normalized size = 0.87

$$\frac{3}{2} \log(|x+2|) - \log(|x-1|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x), x, algorithm="giac")

[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(x)}{2} - \ln(x-1) + \frac{3 \ln(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-3*x-1)/(x^3+x^2-2*x),x)`

[Out] `-ln(x-1)+3/2*ln(x+2)+1/2*ln(x)`

maxima [A] time = 1.40, size = 17, normalized size = 0.74

$$\frac{3}{2} \log(x+2) - \log(x-1) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")`

[Out] `3/2*log(x + 2) - log(x - 1) + 1/2*log(x)`

mupad [B] time = 2.19, size = 17, normalized size = 0.74

$$\frac{3 \ln(x+2)}{2} - \ln(x-1) + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)`

[Out] `(3*log(x + 2))/2 - log(x - 1) + log(x)/2`

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2} - \log(x-1) + \frac{3 \log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)`

[Out] `log(x)/2 - log(x - 1) + 3*log(x + 2)/2`

$$3.269 \quad \int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1594, 1628, 628}

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1594

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a,
b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx &= \int \frac{3-x+3x^2-2x^3+x^4}{x(3-2x+x^2)} dx \\
&= \int \left(\frac{1}{x} + x + \frac{1-x}{3-2x+x^2} \right) dx \\
&= \frac{x^2}{2} + \log(x) + \int \frac{1-x}{3-2x+x^2} dx \\
&= \frac{x^2}{2} + \log(x) - \frac{1}{2} \log(3-2x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]

fricas [A] time = 1.49, size = 19, normalized size = 0.83

$$\frac{1}{2} x^2 - \frac{1}{2} \log(x^2 - 2x + 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

giac [A] time = 0.29, size = 20, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 2x + 3) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{x^2}{2} + \ln(x) - \frac{\ln(x^2 - 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x)

[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)

maxima [A] time = 1.12, size = 19, normalized size = 0.83

$$\frac{1}{2}x^2 - \frac{1}{2}\log(x^2 - 2x + 3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\ln(x) - \frac{\ln(x^2 - 2x + 3)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)

[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2

sympy [A] time = 0.11, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log(x) - \frac{\log(x^2 - 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
```

```
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

$$3.270 \quad \int \frac{-1+x+x^3}{(1+x^2)^2} dx$$

Optimal. Leaf size=29

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) - \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1814, 635, 203, 260}

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + x^3)/(1 + x^2)^2, x]

[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /

; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^3}{(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{1-2x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} - \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x^2)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{2} \left(-\frac{x}{x^2+1} + \log(x^2+1) - \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2, x]

[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x+x^3}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x + x^3)/(1 + x^2)^2, x]

[Out] IntegrateAlgebraic[(-1 + x + x^3)/(1 + x^2)^2, x]

fricas [A] time = 1.16, size = 32, normalized size = 1.10

$$\frac{(x^2+1) \arctan(x) - (x^2+1) \log(x^2+1) + x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")

[Out] $-1/2*((x^2 + 1)*\arctan(x) - (x^2 + 1)*\log(x^2 + 1) + x)/(x^2 + 1)$

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2 + 1)} - \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")`

[Out] $-1/2*x/(x^2 + 1) - 1/2*\arctan(x) + 1/2*\log(x^2 + 1)$

maple [A] time = 0.01, size = 24, normalized size = 0.83

$$-\frac{x}{2(x^2 + 1)} - \frac{\arctan(x)}{2} + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x-1)/(x^2+1)^2,x)`

[Out] $-1/2*x/(x^2+1)-1/2*\arctan(x)+1/2*\ln(x^2+1)$

maxima [A] time = 2.33, size = 23, normalized size = 0.79

$$-\frac{x}{2(x^2 + 1)} - \frac{1}{2} \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")`

[Out] $-1/2*x/(x^2 + 1) - 1/2*\arctan(x) + 1/2*\log(x^2 + 1)$

mupad [B] time = 2.13, size = 25, normalized size = 0.86

$$\frac{\ln(x^2 + 1)}{2} - \frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^3 - 1)/(x^2 + 1)^2,x)`

[Out] $\log(x^2 + 1)/2 - \operatorname{atan}(x)/2 - x/(2*(x^2 + 1))$

sympy [A] time = 0.12, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2 + 2} + \frac{\log(x^2 + 1)}{2} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x-1)/(x**2+1)**2,x)

[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2

$$3.271 \quad \int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx$$

Optimal. Leaf size=44

$$\log(x^2 - x + 1) - \frac{3}{x+1} + \log(x) - 2 \log(x+1) - \frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.24, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 6725, 634, 618, 204, 628}

$$\log(x^2 - x + 1) - \frac{3}{x+1} + \log(x) - 2 \log(x+1) - \frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]

[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x] + Log[1 - x + x^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \ :> \ \text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)})^n, x] /; \text{FreeQ}[\{a, b, p, q\}, x] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p]$

Rule 6725

$\text{Int}[(u_)/((a_) + (b_.)*(x_)^{(n_.)}), x_Symbol] \ :> \ \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1 + 2x - x^2 + 8x^3 + x^4}{(x + x^2)(1 + x^3)} dx &= \int \frac{1 + 2x - x^2 + 8x^3 + x^4}{x(1 + x)(1 + x^3)} dx \\ &= \int \left(\frac{1}{x} + \frac{3}{(1 + x)^2} - \frac{2}{1 + x} + \frac{2x}{1 - x + x^2} \right) dx \\ &= -\frac{3}{1 + x} + \log(x) - 2\log(1 + x) + 2 \int \frac{x}{1 - x + x^2} dx \\ &= -\frac{3}{1 + x} + \log(x) - 2\log(1 + x) + \int \frac{1}{1 - x + x^2} dx + \int \frac{-1 + 2x}{1 - x + x^2} dx \\ &= -\frac{3}{1 + x} + \log(x) - 2\log(1 + x) + \log(1 - x + x^2) - 2 \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, \right. \\ &= -\frac{3}{1 + x} - \frac{2 \tan^{-1} \left(\frac{1 - 2x}{\sqrt{3}} \right)}{\sqrt{3}} + \log(x) - 2\log(1 + x) + \log(1 - x + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\log(x^2 - x + 1) - \frac{3}{x + 1} + \log(x) - 2\log(x + 1) + \frac{2 \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]

[Out] $-3/(1+x) + (2 \operatorname{ArcTan}[-1+2x]/\sqrt{3})/\sqrt{3} + \operatorname{Log}[x] - 2 \operatorname{Log}[1+x] + \operatorname{Log}[1-x+x^2]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(1+2*x-x^2+8*x^3+x^4)/((x+x^2)*(1+x^3)),x]`

[Out] `IntegrateAlgebraic[(1+2*x-x^2+8*x^3+x^4)/((x+x^2)*(1+x^3)),x]`

fricas [A] time = 1.41, size = 58, normalized size = 1.32

$$\frac{2\sqrt{3}(x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3(x+1)\log(x^2-x+1) - 6(x+1)\log(x+1) + 3(x+1)\log(x) - 9}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")`

[Out] $\frac{1}{3} * (2 * \sqrt{3}) * (x + 1) * \arctan(1/3 * \sqrt{3} * (2 * x - 1)) + 3 * (x + 1) * \log(x^2 - x + 1) - 6 * (x + 1) * \log(x + 1) + 3 * (x + 1) * \log(x) - 9) / (x + 1)$

giac [A] time = 0.36, size = 43, normalized size = 0.98

$$\frac{2}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{3}{x+1} + \log(x^2-x+1) - 2 \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")`

[Out] $\frac{2}{3} * \sqrt{3} * \arctan(1/3 * \sqrt{3} * (2 * x - 1)) - 3 / (x + 1) + \log(x^2 - x + 1) - 2 * \log(\operatorname{abs}(x + 1)) + \log(\operatorname{abs}(x))$

maple [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{2\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} + \ln(x) - 2 \ln(x+1) + \ln(x^2-x+1) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)`

[Out] $\ln(x^2-x+1)+2/3*3^{(1/2)}*\arctan(1/3*(2*x-1)*3^{(1/2)})-3/(x+1)-2*\ln(x+1)+\ln(x)$

maxima [A] time = 2.25, size = 41, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1}+\log(x^2-x+1)-2\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")`

[Out] $2/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 3/(x + 1) + \log(x^2 - x + 1) - 2*\log(x + 1) + \log(x)$

mupad [B] time = 0.13, size = 55, normalized size = 1.25

$$\ln(x) - 2 \ln(x + 1) - \frac{3}{x + 1} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) \left(-1 + \frac{\sqrt{3} 1i}{3}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(1 + \frac{\sqrt{3} 1i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)`

[Out] $\log(x) - 2*\log(x + 1) - 3/(x + 1) - \log(x - (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/3 - 1) + \log(x + (3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/3 + 1)$

sympy [A] time = 0.21, size = 49, normalized size = 1.11

$$\log(x) - 2\log(x + 1) + \log(x^2 - x + 1) + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)`

[Out] $\log(x) - 2*\log(x + 1) + \log(x**2 - x + 1) + 2*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/3 - 3/(x + 1)$

$$3.272 \quad \int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx &= \int \left(-\frac{5}{5 + x^2} + \frac{6 + x}{3 + 2x + x^2} \right) dx \\
 &= -\left(5 \int \frac{1}{5 + x^2} dx \right) + \int \frac{6 + x}{3 + 2x + x^2} dx \\
 &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \int \frac{2 + 2x}{3 + 2x + x^2} dx + 5 \int \frac{1}{3 + 2x + x^2} dx \\
 &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \log(3 + 2x + x^2) - 10 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2 + 2x \right) \\
 &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{1+x}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{2} \log(3 + 2x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]

fricas [A] time = 1.70, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

giac [A] time = 0.30, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5} \arctan\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2} \arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)

[Out] -arctan(1/5*5^(1/2)*x)*5^(1/2)+1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))

maxima [A] time = 2.29, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

mupad [B] time = 0.16, size = 88, normalized size = 1.91

$$\frac{\ln(x+1-\sqrt{2}i)}{2} + \frac{\ln(x+1+\sqrt{2}i)}{2} + \sqrt{5} \operatorname{atan}\left(\frac{2000\sqrt{5}}{2000x+1120} - \frac{224\sqrt{5}x}{2000x+1120}\right) - \frac{\sqrt{2} \ln(x+1-\sqrt{2}i)}{4} + \frac{\sqrt{2} \ln(x+1+\sqrt{2}i)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)

[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log(x^2 + 2x + 3)}{2} - \sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)

[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/2

$$3.273 \quad \int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2} dx$$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

Rubi [A] time = 0.17, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {6742, 261, 260, 629, 628}

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^2), x]

[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned} \int \frac{-3 + 25x + 23x^2 + 32x^3 + 15x^4 + 7x^5 + x^6}{(1+x^2)^2(2+x+x^2)^2} dx &= \int \left(\frac{6x}{(1+x^2)^2} + \frac{2x}{1+x^2} + \frac{-1-2x}{(2+x+x^2)^2} + \frac{-1-2x}{2+x+x^2} \right) dx \\ &= 2 \int \frac{x}{1+x^2} dx + 6 \int \frac{x}{(1+x^2)^2} dx + \int \frac{-1-2x}{(2+x+x^2)^2} dx + \\ &= -\frac{3}{1+x^2} + \frac{1}{2+x+x^2} + \log(1+x^2) - \log(2+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{x^2+1} + \frac{1}{x^2+x+2} + \log(x^2+1) - \log(x^2+x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2 * (2 + x + x^2)^2), x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + 25x + 23x^2 + 32x^3 + 15x^4 + 7x^5 + x^6}{(1+x^2)^2(2+x+x^2)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^2), x]
```

```
[Out] IntegrateAlgebraic[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 + x + x^2)^2), x]
```

fricas [B] time = 2.04, size = 72, normalized size = 2.18

$$\frac{2x^2 + (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + x + 2) - (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + 1) + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")

[Out] $-(2x^2 + (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + x + 2) - (x^4 + x^3 + 3x^2 + x + 2) \log(x^2 + 1) + 3x + 5)/(x^4 + x^3 + 3x^2 + x + 2)$

giac [A] time = 0.30, size = 44, normalized size = 1.33

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} - \log(x^2 + x + 2) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")

[Out] $-(2x^2 + 3x + 5)/(x^4 + x^3 + 3x^2 + x + 2) - \log(x^2 + x + 2) + \log(x^2 + 1)$

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$\ln(x^2 + 1) - \ln(x^2 + x + 2) - \frac{3}{x^2 + 1} + \frac{1}{x^2 + x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x)

[Out] $-3/(x^2+1)+1/(x^2+x+2)+\ln(x^2+1)-\ln(x^2+x+2)$

maxima [A] time = 1.16, size = 44, normalized size = 1.33

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} - \log(x^2 + x + 2) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")

[Out] $-(2x^2 + 3x + 5)/(x^4 + x^3 + 3x^2 + x + 2) - \log(x^2 + x + 2) + \log(x^2 + 1)$

mupad [B] time = 2.16, size = 56, normalized size = 1.70

$$-\frac{2x^2 + 3x + 5}{x^4 + x^3 + 3x^2 + x + 2} + \operatorname{atan}\left(\frac{\frac{x^{224i}}{11} + \frac{224}{11}i}{44x^2 + 16x + 60} - \frac{3}{11}i\right) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x^2 + 2)^2),x)
```

```
[Out] atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x^2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

sympy [A] time = 0.19, size = 41, normalized size = 1.24

$$\frac{-2x^2 - 3x - 5}{x^4 + x^3 + 3x^2 + x + 2} + \log(x^2 + 1) - \log(x^2 + x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+2)**2,x)
```

```
[Out] (-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x**2 + x + 2)
```

$$3.274 \quad \int \frac{1}{(1+x^2)(4+x^2)} dx$$

Optimal. Leaf size=17

$$\frac{1}{3} \tan^{-1}(x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {391, 203}

$$\frac{1}{3} \tan^{-1}(x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(4 + x^2)),x]

[Out] -ArcTan[x/2]/6 + ArcTan[x]/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)(4+x^2)} dx &= \frac{1}{3} \int \frac{1}{1+x^2} dx - \frac{1}{3} \int \frac{1}{4+x^2} dx \\ &= -\frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6} \tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]

[Out] ArcTan[2/x]/6 + ArcTan[x]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x^2)(4+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1 + x^2)*(4 + x^2)),x]

[Out] IntegrateAlgebraic[1/((1 + x^2)*(4 + x^2)), x]

fricas [A] time = 1.77, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)

giac [A] time = 0.29, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")

[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{\arctan(x)}{3} - \frac{\arctan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(x^2+4),x)

[Out] $-1/6*\arctan(1/2*x)+1/3*\arctan(x)$

maxima [A] time = 2.20, size = 11, normalized size = 0.65

$$-\frac{1}{6} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")`

[Out] $-1/6*\arctan(1/2*x) + 1/3*\arctan(x)$

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{\operatorname{atan}(x)}{3} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^2 + 1)*(x^2 + 4)),x)`

[Out] $\operatorname{atan}(x)/3 - \operatorname{atan}(x/2)/6$

sympy [A] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{6} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2+1)/(x**2+4),x)`

[Out] $-\operatorname{atan}(x/2)/6 + \operatorname{atan}(x)/3$

$$3.275 \quad \int \frac{a+bx^3}{1+x^2} dx$$

Optimal. Leaf size=24

$$a \tan^{-1}(x) + \frac{bx^2}{2} - \frac{1}{2}b \log(x^2 + 1)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1810, 635, 203, 260}

$$a \tan^{-1}(x) + \frac{bx^2}{2} - \frac{1}{2}b \log(x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3)/(1 + x^2), x]

[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3}{1 + x^2} dx &= \int \left(bx + \frac{a - bx}{1 + x^2} \right) dx \\
&= \frac{bx^2}{2} + \int \frac{a - bx}{1 + x^2} dx \\
&= \frac{bx^2}{2} + a \int \frac{1}{1 + x^2} dx - b \int \frac{x}{1 + x^2} dx \\
&= \frac{bx^2}{2} + a \tan^{-1}(x) - \frac{1}{2} b \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$a \tan^{-1}(x) + \frac{1}{2} b (x^2 - \log(x^2 + 1))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3)/(1 + x^2), x]

[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^3}{1 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^3)/(1 + x^2), x]

[Out] IntegrateAlgebraic[(a + b*x^3)/(1 + x^2), x]

fricas [A] time = 1.30, size = 20, normalized size = 0.83

$$\frac{1}{2} bx^2 + a \arctan(x) - \frac{1}{2} b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1), x, algorithm="fricas")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

giac [A] time = 0.29, size = 20, normalized size = 0.83

$$\frac{1}{2} bx^2 + a \arctan(x) - \frac{1}{2} b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{bx^2}{2} + a \arctan(x) - \frac{b \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a)/(x^2+1),x)

[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)

maxima [A] time = 2.25, size = 20, normalized size = 0.83

$$\frac{1}{2}bx^2 + a \arctan(x) - \frac{1}{2}b \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)

mupad [B] time = 2.13, size = 20, normalized size = 0.83

$$\frac{bx^2}{2} - \frac{b \ln(x^2 + 1)}{2} + a \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)/(x^2 + 1),x)

[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)

sympy [C] time = 0.17, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right) \log(x - i) + \left(\frac{ia}{2} - \frac{b}{2}\right) \log(x + i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a)/(x**2+1),x)

[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)

$$3.276 \quad \int \frac{x+x^2}{(4+x)(-4+x^2)} dx$$

Optimal. Leaf size=15

$$\log(x+4) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1593, 1629, 207}

$$\log(x+4) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)), x]

[Out] -ArcTanh[x/2]/2 + Log[4 + x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x + x^2}{(4 + x)(-4 + x^2)} dx &= \int \frac{x(1 + x)}{(4 + x)(-4 + x^2)} dx \\
&= \int \left(\frac{1}{4 + x} + \frac{1}{-4 + x^2} \right) dx \\
&= \log(4 + x) + \int \frac{1}{-4 + x^2} dx \\
&= -\frac{1}{2} \tanh^{-1} \left(\frac{x}{2} \right) + \log(4 + x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{1}{4} \log(2 - x) - \frac{1}{4} \log(x + 2) + \log(x + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)), x]

[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + x^2}{(4 + x)(-4 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + x^2)/((4 + x)*(-4 + x^2)), x]

[Out] IntegrateAlgebraic[(x + x^2)/((4 + x)*(-4 + x^2)), x]

fricas [A] time = 1.42, size = 17, normalized size = 1.13

$$\log(x + 4) - \frac{1}{4} \log(x + 2) + \frac{1}{4} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4), x, algorithm="fricas")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)

giac [A] time = 0.32, size = 20, normalized size = 1.33

$$\log(|x + 4|) - \frac{1}{4} \log(|x + 2|) + \frac{1}{4} \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")

[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{\ln(x - 2)}{4} - \frac{\ln(x + 2)}{4} + \ln(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x)/(4+x)/(x^2-4),x)

[Out] ln(4+x)+1/4*ln(x-2)-1/4*ln(x+2)

maxima [A] time = 2.31, size = 17, normalized size = 1.13

$$\log(x + 4) - \frac{1}{4} \log(x + 2) + \frac{1}{4} \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")

[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)

mupad [B] time = 0.06, size = 19, normalized size = 1.27

$$\ln(x + 4) + \frac{\operatorname{atanh}\left(\frac{90}{7(21x+48)} - \frac{8}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2)/((x^2 - 4)*(x + 4)),x)

[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2

sympy [A] time = 0.14, size = 17, normalized size = 1.13

$$\frac{\log(x - 2)}{4} - \frac{\log(x + 2)}{4} + \log(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
```

```
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

$$3.277 \quad \int \frac{4+x^2}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=20

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {522, 203}

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rubi steps

$$\begin{aligned} \int \frac{4+x^2}{(1+x^2)(2+x^2)} dx &= -\left(2 \int \frac{1}{2+x^2} dx\right) + 3 \int \frac{1}{1+x^2} dx \\ &= 3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$3 \tan^{-1}(x) - \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]

[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + x^2}{(1 + x^2)(2 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]

[Out] IntegrateAlgebraic[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]

fricas [A] time = 1.82, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4)/(x^2+1)/(x^2+2), x, algorithm="fricas")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)

giac [A] time = 0.38, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+4)/(x^2+1)/(x^2+2), x, algorithm="giac")

[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)

maple [A] time = 0.01, size = 18, normalized size = 0.90

$$3 \arctan(x) - \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+4)/(x^2+1)/(x^2+2),x)`

[Out] `3*arctan(x)-2^(1/2)*arctan(1/2*2^(1/2)*x)`

maxima [A] time = 2.40, size = 17, normalized size = 0.85

$$-\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")`

[Out] `-sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)`

mupad [B] time = 0.05, size = 17, normalized size = 0.85

$$3 \operatorname{atan}(x) - \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)`

[Out] `3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)`

sympy [A] time = 0.15, size = 19, normalized size = 0.95

$$3 \operatorname{atan}(x) - \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+4)/(x**2+1)/(x**2+2),x)`

[Out] `3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)`

$$3.278 \quad \int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=37

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2(1-x)} + \frac{1}{2} \log(1-x) + 2 \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2(1-x)} + \frac{1}{2} \log(1-x) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]

[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_)*(x_)^(m_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{5 - 4x + 3x^2 + x^4}{(-1 + x)^2(1 + x^2)} dx &= \int \left(1 + \frac{5}{2(-1 + x)^2} + \frac{1}{2(-1 + x)} + \frac{4 + 3x}{2(1 + x^2)} \right) dx \\
&= \frac{5}{2(1 - x)} + x + \frac{1}{2} \log(1 - x) + \frac{1}{2} \int \frac{4 + 3x}{1 + x^2} dx \\
&= \frac{5}{2(1 - x)} + x + \frac{1}{2} \log(1 - x) + \frac{3}{2} \int \frac{x}{1 + x^2} dx + 2 \int \frac{1}{1 + x^2} dx \\
&= \frac{5}{2(1 - x)} + x + 2 \tan^{-1}(x) + \frac{1}{2} \log(1 - x) + \frac{3}{4} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{3}{4} \log(x^2 + 1) + x + \frac{5}{2 - 2x} + \frac{1}{2} \log(x - 1) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]

[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 - 4x + 3x^2 + x^4}{(-1 + x)^2(1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]

fricas [A] time = 1.39, size = 44, normalized size = 1.19

$$\frac{4x^2 + 8(x - 1) \arctan(x) + 3(x - 1) \log(x^2 + 1) + 2(x - 1) \log(x - 1) - 4x - 10}{4(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1), x, algorithm="fricas")

[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) - 4*x - 10)/(x - 1)

giac [B] time = 0.29, size = 60, normalized size = 1.62

$$\frac{1}{2}\pi - 2\pi \left[\frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right] + x - \frac{5}{2(x-1)} + 2 \arctan(x) + \frac{3}{4} \log \left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1 \right) + 2 \log(|x-1|) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")

[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$x + 2 \arctan(x) + \frac{\ln(x-1)}{2} + \frac{3 \ln(x^2+1)}{4} - \frac{5}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x)

[Out] x-5/2/(x-1)+1/2*ln(x-1)+3/4*ln(x^2+1)+2*arctan(x)

maxima [A] time = 2.22, size = 27, normalized size = 0.73

$$x - \frac{5}{2(x-1)} + 2 \arctan(x) + \frac{3}{4} \log(x^2+1) + \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)

mupad [B] time = 2.14, size = 35, normalized size = 0.95

$$x + \frac{\ln(x-1)}{2} - \frac{5}{2(x-1)} + \ln(x-i) \left(\frac{3}{4} - i \right) + \ln(x+1i) \left(\frac{3}{4} + 1i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)

[Out] x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(x - 1))

sympy [A] time = 0.16, size = 29, normalized size = 0.78

$$x + \frac{\log(x-1)}{2} + \frac{3 \log(x^2+1)}{4} + 2 \operatorname{atan}(x) - \frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1), x)
```

```
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```


$$3.279 \quad \int \frac{1+x^4}{2+x^2} dx$$

Optimal. Leaf size=26

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1154, 203}

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(2 + x^2), x]

[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1154

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^4}{2+x^2} dx &= \int \left(-2 + x^2 + \frac{5}{2+x^2} \right) dx \\ &= -2x + \frac{x^3}{3} + 5 \int \frac{1}{2+x^2} dx \\ &= -2x + \frac{x^3}{3} + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(2 + x^2), x]

[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{2+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^4)/(2 + x^2), x]

[Out] IntegrateAlgebraic[(1 + x^4)/(2 + x^2), x]

fricas [A] time = 1.49, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+2), x, algorithm="fricas")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x

giac [A] time = 0.31, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^2+2), x, algorithm="giac")

[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x

maple [A] time = 0.00, size = 22, normalized size = 0.85

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+1)/(x^2+2),x)`

[Out] `-2*x+1/3*x^3+5/2*2^(1/2)*arctan(1/2*2^(1/2)*x)`

maxima [A] time = 2.22, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+1)/(x^2+2),x, algorithm="maxima")`

[Out] `1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x`

mupad [B] time = 0.03, size = 21, normalized size = 0.81

$$\frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - 2x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4 + 1)/(x^2 + 2),x)`

[Out] `(5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3`

sympy [A] time = 0.09, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+1)/(x**2+2),x)`

[Out] `x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2`

$$3.280 \quad \int \frac{2+2x+x^4}{x^4+x^5} dx$$

Optimal. Leaf size=12

$$\log(x+1) - \frac{2}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1593, 1620}

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x + x^4)/(x^4 + x^5), x]

[Out] -2/(3*x^3) + Log[1 + x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{2+2x+x^4}{x^4+x^5} dx &= \int \frac{2+2x+x^4}{x^4(1+x)} dx \\ &= \int \left(\frac{2}{x^4} + \frac{1}{1+x} \right) dx \\ &= -\frac{2}{3x^3} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(x+1) - \frac{2}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5), x]

[Out] -2/(3*x^3) + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 2x + x^4}{x^4 + x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 2*x + x^4)/(x^4 + x^5), x]

[Out] IntegrateAlgebraic[(2 + 2*x + x^4)/(x^4 + x^5), x]

fricas [A] time = 1.64, size = 16, normalized size = 1.33

$$\frac{3x^3 \log(x+1) - 2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x+2)/(x^5+x^4), x, algorithm="fricas")

[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3

giac [A] time = 0.29, size = 11, normalized size = 0.92

$$-\frac{2}{3x^3} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x+2)/(x^5+x^4), x, algorithm="giac")

[Out] -2/3/x^3 + log(abs(x + 1))

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$\ln(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+2*x+2)/(x^5+x^4),x)`

[Out] `-2/3/x^3+ln(x+1)`

maxima [A] time = 1.11, size = 10, normalized size = 0.83

$$-\frac{2}{3x^3} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")`

[Out] `-2/3/x^3 + log(x + 1)`

mupad [B] time = 2.12, size = 10, normalized size = 0.83

$$\ln(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + x^4 + 2)/(x^4 + x^5),x)`

[Out] `log(x + 1) - 2/(3*x^3)`

sympy [A] time = 0.09, size = 10, normalized size = 0.83

$$\log(x+1) - \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+2*x+2)/(x**5+x**4),x)`

[Out] `log(x + 1) - 2/(3*x**3)`

$$3.281 \quad \int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx$$

Optimal. Leaf size=21

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2074}

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx &= \int \left(\frac{1}{2-x} + \frac{2}{-1+x} + \frac{1}{1+x} \right) dx \\ &= 2 \log(1-x) - \log(2-x) + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$2 \log(1-x) - \log(2-x) + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]

fricas [A] time = 1.42, size = 17, normalized size = 0.81

$$\log(x + 1) + 2 \log(x - 1) - \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, algorithm="fricas")

[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)

giac [A] time = 0.28, size = 20, normalized size = 0.95

$$\log(|x + 1|) + 2 \log(|x - 1|) - \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, algorithm="giac")

[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\ln(x - 2) + 2 \ln(x - 1) + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x)

[Out] 2*ln(x-1)-ln(x-2)+ln(x+1)

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$\log(x + 1) + 2 \log(x - 1) - \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2), x, algorithm="maxima")

[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)

mupad [B] time = 2.14, size = 21, normalized size = 1.00

$$2 \ln(x - 1) - 2 \operatorname{atanh}\left(\frac{144}{11(22x - 50)} + \frac{13}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2), x)`

[Out] `2*log(x - 1) - 2*atanh(144/(11*(22*x - 50))) + 13/11)`

sympy [A] time = 0.13, size = 15, normalized size = 0.71

$$-\log(x - 2) + 2 \log(x - 1) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2), x)`

[Out] `-log(x - 2) + 2*log(x - 1) + log(x + 1)`

$$3.282 \quad \int \frac{2+x+x^3}{1+2x^2+x^4} dx$$

Optimal. Leaf size=22

$$\frac{x}{x^2+1} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {28, 1814, 635, 203, 260}

$$\frac{x}{x^2+1} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :=
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,

```

0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

```

Rubi steps

$$\begin{aligned}
\int \frac{2 + x + x^3}{1 + 2x^2 + x^4} dx &= \int \frac{2 + x + x^3}{(1 + x^2)^2} dx \\
&= \frac{x}{1 + x^2} - \frac{1}{2} \int \frac{-2 - 2x}{1 + x^2} dx \\
&= \frac{x}{1 + x^2} + \int \frac{1}{1 + x^2} dx + \int \frac{x}{1 + x^2} dx \\
&= \frac{x}{1 + x^2} + \tan^{-1}(x) + \frac{1}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x}{x^2 + 1} + \frac{1}{2} \log(x^2 + 1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x + x^3}{1 + 2x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]

fricas [A] time = 1.67, size = 34, normalized size = 1.55

$$\frac{2(x^2 + 1) \arctan(x) + (x^2 + 1) \log(x^2 + 1) + 2x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")

[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{x}{x^2 + 1} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")

[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{x}{x^2 + 1} + \arctan(x) + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+2)/(x^4+2*x^2+1),x)

[Out] 1/(x^2+1)*x+arctan(x)+1/2*ln(x^2+1)

maxima [A] time = 2.23, size = 20, normalized size = 0.91

$$\frac{x}{x^2 + 1} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")

[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

mupad [B] time = 2.12, size = 20, normalized size = 0.91

$$\frac{\ln(x^2 + 1)}{2} + \operatorname{atan}(x) + \frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)

[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$\frac{x}{x^2 + 1} + \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

$$3.283 \quad \int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} dx$$

Optimal. Leaf size=24

$$-\frac{1}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {28, 1814, 635, 203, 260}

$$-\frac{1}{2(x^2+1)} + \frac{1}{2} \log(x^2+1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2

Rule 28

Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_)]^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] / ; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1 + 2x + x^2 + x^3}{1 + 2x^2 + x^4} dx &= \int \frac{1 + 2x + x^2 + x^3}{(1 + x^2)^2} dx \\ &= -\frac{1}{2(1 + x^2)} - \frac{1}{2} \int \frac{-2 - 2x}{1 + x^2} dx \\ &= -\frac{1}{2(1 + x^2)} + \int \frac{1}{1 + x^2} dx + \int \frac{x}{1 + x^2} dx \\ &= -\frac{1}{2(1 + x^2)} + \tan^{-1}(x) + \frac{1}{2} \log(1 + x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{2(x^2 + 1)} + \frac{1}{2} \log(x^2 + 1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x + x^2 + x^3}{1 + 2x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]

fricas [A] time = 1.44, size = 32, normalized size = 1.33

$$\frac{2(x^2 + 1) \arctan(x) + (x^2 + 1) \log(x^2 + 1) - 1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")

[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)

giac [A] time = 0.40, size = 20, normalized size = 0.83

$$-\frac{1}{2(x^2 + 1)} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")

[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \frac{\ln(x^2 + 1)}{2} - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x)

[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)

maxima [A] time = 2.18, size = 20, normalized size = 0.83

$$-\frac{1}{2(x^2 + 1)} + \arctan(x) + \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")

[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{\ln(x^2 + 1)}{2} + \operatorname{atan}(x) - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1), x)`

[Out] `log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))`

sympy [A] time = 0.12, size = 19, normalized size = 0.79

$$\frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x) - \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1), x)`

[Out] `log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)`

$$3.284 \quad \int \frac{3+4x}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=36

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1010, 391, 203, 444, 36, 31}

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]

[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^2]

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +

$d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 444

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] \text{:> Dist}[1/n, \text{Subst}[\text{Int}[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[m - n + 1, 0]$

Rule 1010

$\text{Int}[(g_) + (h_.)*(x_)]*((a_) + (c_.)*(x_)^2)^{(p_.)}*((d_) + (f_.)*(x_)^2)^{(q_.)}, x_Symbol] \text{:> Dist}[g, \text{Int}[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + \text{Dist}[h, \text{Int}[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; \text{FreeQ}[\{a, c, d, f, g, h, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{3+4x}{(1+x^2)(2+x^2)} dx &= 3 \int \frac{1}{(1+x^2)(2+x^2)} dx + 4 \int \frac{x}{(1+x^2)(2+x^2)} dx \\ &= 2 \text{Subst} \left(\int \frac{1}{(1+x)(2+x)} dx, x, x^2 \right) + 3 \int \frac{1}{1+x^2} dx - 3 \int \frac{1}{2+x^2} dx \\ &= 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}} + 2 \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - 2 \text{Subst} \left(\int \frac{1}{2+x} dx, x, x^2 \right) \\ &= 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}} + 2 \log(1+x^2) - 2 \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$2 \log(x^2 + 1) - 2 \log(x^2 + 2) + 3 \tan^{-1}(x) - \frac{3 \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]

[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 + 4x}{(1 + x^2)(2 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]

[Out] IntegrateAlgebraic[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]

fricas [A] time = 1.43, size = 33, normalized size = 0.92

$$-\frac{3}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x) - 2 \log(x^2 + 2) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x^2+1)/(x^2+2), x, algorithm="fricas")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)

giac [A] time = 0.25, size = 33, normalized size = 0.92

$$-\frac{3}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} x\right) + 3 \arctan(x) - 2 \log(x^2 + 2) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x^2+1)/(x^2+2), x, algorithm="giac")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)

maple [A] time = 0.00, size = 34, normalized size = 0.94

$$3 \arctan(x) - \frac{3\sqrt{2} \arctan\left(\frac{\sqrt{2} x}{2}\right)}{2} + 2 \ln(x^2 + 1) - 2 \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3+4*x)/(x^2+1)/(x^2+2), x)

[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.06, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log(x^2 + 2) + 2\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x^2 + 1)

mupad [B] time = 0.10, size = 56, normalized size = 1.56

$$\ln(x - i)\left(2 - \frac{3}{2}i\right) + \ln(x + 1i)\left(2 + \frac{3}{2}i\right) + \ln\left(x - \sqrt{2}1i\right)\left(-2 + \frac{\sqrt{2}3i}{4}\right) - \ln\left(x + \sqrt{2}1i\right)\left(2 + \frac{\sqrt{2}3i}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)

[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)

sympy [A] time = 0.19, size = 39, normalized size = 1.08

$$2\log(x^2 + 1) - 2\log(x^2 + 2) + 3\operatorname{atan}(x) - \frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)

[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)/2

$$3.285 \quad \int \frac{2+x}{(1+x^2)(4+x^2)} dx$$

Optimal. Leaf size=37

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3} \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1010, 391, 203, 444, 36, 31}

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]

[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.)*((d_) + (f_.)*(x_)^2)^(q_.), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h, Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{2+x}{(1+x^2)(4+x^2)} dx &= 2 \int \frac{1}{(1+x^2)(4+x^2)} dx + \int \frac{x}{(1+x^2)(4+x^2)} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)(4+x)} dx, x, x^2 \right) + \frac{2}{3} \int \frac{1}{1+x^2} dx - \frac{2}{3} \int \frac{1}{4+x^2} dx \\ &= -\frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^2 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{4+x} dx, x, x^2 \right) \\ &= -\frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x) + \frac{1}{6} \log(1+x^2) - \frac{1}{6} \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6} \log(x^2 + 1) - \frac{1}{6} \log(x^2 + 4) - \frac{1}{3} \tan^{-1} \left(\frac{x}{2} \right) + \frac{2}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)), x]
```

```
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x}{(1+x^2)(4+x^2)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x)/((1 + x^2)*(4 + x^2)), x]
```

[Out] IntegrateAlgebraic[(2 + x)/((1 + x^2)*(4 + x^2)), x]

fricas [A] time = 1.03, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan\left(\frac{1}{2}x\right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

giac [A] time = 0.29, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan\left(\frac{1}{2}x\right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2 \arctan(x)}{3} - \frac{\arctan\left(\frac{x}{2}\right)}{3} + \frac{\ln(x^2 + 1)}{6} - \frac{\ln(x^2 + 4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/(x^2+1)/(x^2+4),x)

[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)

maxima [A] time = 2.00, size = 27, normalized size = 0.73

$$-\frac{1}{3} \arctan\left(\frac{1}{2}x\right) + \frac{2}{3} \arctan(x) - \frac{1}{6} \log(x^2 + 4) + \frac{1}{6} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")

[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)

mupad [B] time = 2.14, size = 37, normalized size = 1.00

$$\ln(x - i) \left(\frac{1}{6} - \frac{1}{3}i\right) + \ln(x + i) \left(\frac{1}{6} + \frac{1}{3}i\right) + \ln(x - 2i) \left(-\frac{1}{6} + \frac{1}{6}i\right) + \ln(x + 2i) \left(-\frac{1}{6} - \frac{1}{6}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)`

[Out] $\log(x - 1i)*(1/6 - 1i/3) + \log(x + 1i)*(1/6 + 1i/3) - \log(x - 2i)*(1/6 - 1i/6) - \log(x + 2i)*(1/6 + 1i/6)$

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{\log(x^2 + 1)}{6} - \frac{\log(x^2 + 4)}{6} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{2 \operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(x**2+1)/(x**2+4),x)`

[Out] $\log(x^2 + 1)/6 - \log(x^2 + 4)/6 - \operatorname{atan}(x/2)/3 + 2*\operatorname{atan}(x)/3$

$$3.286 \quad \int \frac{2-x+x^3}{-7-6x+x^2} dx$$

Optimal. Leaf size=29

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1657, 632, 31}

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{2-x+x^3}{-7-6x+x^2} dx &= \int \left(6+x + \frac{2(22+21x)}{-7-6x+x^2} \right) dx \\
&= 6x + \frac{x^2}{2} + 2 \int \frac{22+21x}{-7-6x+x^2} dx \\
&= 6x + \frac{x^2}{2} - \frac{1}{4} \int \frac{1}{1+x} dx + \frac{169}{4} \int \frac{1}{-7+x} dx \\
&= 6x + \frac{x^2}{2} + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2} + 6x + \frac{169}{4} \log(7-x) - \frac{1}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-x+x^3}{-7-6x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - x + x^3)/(-7 - 6*x + x^2), x]

[Out] IntegrateAlgebraic[(2 - x + x^3)/(-7 - 6*x + x^2), x]

fricas [A] time = 1.19, size = 21, normalized size = 0.72

$$\frac{1}{2} x^2 + 6x - \frac{1}{4} \log(x+1) + \frac{169}{4} \log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7), x, algorithm="fricas")

[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$\frac{1}{2} x^2 + 6x - \frac{1}{4} \log(|x+1|) + \frac{169}{4} \log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")

[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x - \frac{\ln(x+1)}{4} + \frac{169 \ln(x-7)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x+2)/(x^2-6*x-7),x)

[Out] 1/2*x^2+6*x-1/4*ln(x+1)+169/4*ln(x-7)

maxima [A] time = 1.08, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x - \frac{1}{4} \log(x+1) + \frac{169}{4} \log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")

[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)

mupad [B] time = 2.21, size = 21, normalized size = 0.72

$$6x - \frac{\ln(x+1)}{4} + \frac{169 \ln(x-7)}{4} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)

[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169 \log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x+2)/(x**2-6*x-7),x)

[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4

$$3.287 \quad \int \frac{-1+x^5}{-1+x^2} dx$$

Optimal. Leaf size=19

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1810, 627, 31}

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-1 + x^2), x]

[Out] x^2/2 + x^4/4 + Log[1 + x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^5}{-1+x^2} dx &= \int \left(x + x^3 - \frac{1-x}{-1+x^2} \right) dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1-x}{-1+x^2} dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1}{-1-x} dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^5)/(-1 + x^2), x]

[Out] x^2/2 + x^4/4 + Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^5}{-1+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^5)/(-1 + x^2), x]

[Out] IntegrateAlgebraic[(-1 + x^5)/(-1 + x^2), x]

fricas [A] time = 1.23, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")

[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)

giac [A] time = 0.27, size = 16, normalized size = 0.84

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")

[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^4}{4} + \frac{x^2}{2} + \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5-1)/(x^2-1),x)

[Out] 1/2*x^2+1/4*x^4+ln(x+1)

maxima [A] time = 1.07, size = 15, normalized size = 0.79

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")

[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$\ln(x + 1) + \frac{x^2}{2} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5 - 1)/(x^2 - 1),x)

[Out] log(x + 1) + x^2/2 + x^4/4

sympy [A] time = 0.08, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5-1)/(x**2-1),x)

[Out] x**4/4 + x**2/2 + log(x + 1)

$$3.288 \quad \int \frac{5+2x-x^2+x^3}{1+x+x^2} dx$$

Optimal. Leaf size=41

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]

[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1657

`Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}
 \int \frac{5 + 2x - x^2 + x^3}{1 + x + x^2} dx &= \int \left(-2 + x + \frac{7 + 3x}{1 + x + x^2} \right) dx \\
 &= -2x + \frac{x^2}{2} + \int \frac{7 + 3x}{1 + x + x^2} dx \\
 &= -2x + \frac{x^2}{2} + \frac{3}{2} \int \frac{1 + 2x}{1 + x + x^2} dx + \frac{11}{2} \int \frac{1}{1 + x + x^2} dx \\
 &= -2x + \frac{x^2}{2} + \frac{3}{2} \log(1 + x + x^2) - 11 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2x \right) \\
 &= -2x + \frac{x^2}{2} + \frac{11 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(1 + x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{x^2}{2} + \frac{3}{2} \log(x^2 + x + 1) - 2x + \frac{11 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]`

[Out] `-2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2])/2`

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + 2x - x^2 + x^3}{1 + x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]

[Out] IntegrateAlgebraic[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]

fricas [A] time = 1.53, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")

[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 + x + 1)

giac [A] time = 0.29, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")

[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 + x + 1)

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$\frac{x^2}{2} - 2x + \frac{11\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{3\ln(x^2+x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x)

[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.18, size = 34, normalized size = 0.83

$$\frac{1}{2}x^2 + \frac{11}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2x + \frac{3}{2}\log(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2 + \frac{11\sqrt{3}}{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x + 1)\right) - 2x + \frac{3}{2}\log(x^2 + x + 1)$

mupad [B] time = 0.04, size = 36, normalized size = 0.88

$$\frac{3 \ln(x^2 + x + 1)}{2} - 2x + \frac{11\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1), x)`

[Out] $(3\log(x + x^2 + 1))/2 - 2x + (11\sqrt{3}\operatorname{atan}((2\sqrt{3}x)/3 + \sqrt{3}/3))/3 + x^2/2$

sympy [A] time = 0.12, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3 \log(x^2 + x + 1)}{2} + \frac{11\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-x**2+2*x+5)/(x**2+x+1), x)`

[Out] $x^2/2 - 2x + 3\log(x^2 + x + 1)/2 + 11\sqrt{3}\operatorname{atan}(2\sqrt{3}x/3 + \sqrt{3}/3)/3$

$$3.289 \quad \int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} dx$$

Optimal. Leaf size=41

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{4} \log(x^2 - 4x + 5) + \frac{3x}{2} + 6 \tan^{-1}(2 - x)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3}{4} \log(x^2 - 4x + 5) + \frac{3x}{2} + 6 \tan^{-1}(2 - x)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

`Int[(Pq_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}
 \int \frac{-3 + x - 2x^3 + x^4}{10 - 8x + 2x^2} dx &= \int \left(\frac{3}{2} + x + \frac{x^2}{2} - \frac{3(6-x)}{10-8x+2x^2} \right) dx \\
 &= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - 3 \int \frac{6-x}{10-8x+2x^2} dx \\
 &= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{3}{4} \int \frac{-8+4x}{10-8x+2x^2} dx - 12 \int \frac{1}{10-8x+2x^2} dx \\
 &= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{3}{4} \log(5-4x+x^2) + 24 \operatorname{Subst} \left(\int \frac{1}{-16-x^2} dx, x, -8+4x \right) \\
 &= \frac{3x}{2} + \frac{x^2}{2} + \frac{x^3}{6} + 6 \tan^{-1}(2-x) + \frac{3}{4} \log(5-4x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{1}{2} \left(\frac{x^3}{3} + x^2 + \frac{3}{2} \log(x^2 - 4x + 5) + 3x + 12 \tan^{-1}(2-x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2]))/2/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + x - 2x^3 + x^4}{10 - 8x + 2x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

[Out] IntegrateAlgebraic[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]

fricas [A] time = 1.76, size = 31, normalized size = 0.76

$$\frac{1}{6} x^3 + \frac{1}{2} x^2 + \frac{3}{2} x - 6 \arctan(x-2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

giac [A] time = 0.28, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6 \arctan(x - 2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

maple [A] time = 0.00, size = 32, normalized size = 0.78

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} - 6 \arctan(x - 2) + \frac{3 \ln(x^2 - 4x + 5)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x)

[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)

maxima [A] time = 2.07, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6 \arctan(x - 2) + \frac{3}{4} \log(x^2 - 4x + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")

[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)

mupad [B] time = 2.12, size = 31, normalized size = 0.76

$$\frac{3x}{2} - 6 \operatorname{atan}(x - 2) + \frac{3 \ln(x^2 - 4x + 5)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)

[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6

sympy [A] time = 0.12, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3 \log(x^2 - 4x + 5)}{4} - 6 \operatorname{atan}(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)

[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)

$$3.290 \quad \int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx$$

Optimal. Leaf size=30

$$x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x)$$

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1612}

$$x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)} dx &= \int \left(1 + \frac{61}{2(-3+x)} - \frac{25}{-2+x} + \frac{7}{2(-1+x)} \right) dx \\ &= x + \frac{7}{2} \log(1-x) - 25 \log(2-x) + \frac{61}{2} \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$x + \frac{61}{2} \log(x-3) - 25 \log(x-2) + \frac{7}{2} \log(x-1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] $x + (61*\text{Log}[-3 + x])/2 - 25*\text{Log}[-2 + x] + (7*\text{Log}[-1 + x])/2$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x + 3x^2 + x^3}{(-3 + x)(-2 + x)(-1 + x)} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]`

[Out] `IntegrateAlgebraic[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)), x]`

fricas [A] time = 0.69, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x - 1) - 25 \log(x - 2) + \frac{61}{2} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")`

[Out] `x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)`

giac [A] time = 0.25, size = 23, normalized size = 0.77

$$x + \frac{7}{2} \log(|x - 1|) - 25 \log(|x - 2|) + \frac{61}{2} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")`

[Out] `x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))`

maple [A] time = 0.01, size = 21, normalized size = 0.70

$$x + \frac{61 \ln(x - 3)}{2} - 25 \ln(x - 2) + \frac{7 \ln(x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+3*x^2+2*x+1)/(x-3)/(x-2)/(x-1),x)`

[Out] `x+7/2*ln(x-1)-25*ln(x-2)+61/2*ln(x-3)`

maxima [A] time = 1.04, size = 20, normalized size = 0.67

$$x + \frac{7}{2} \log(x - 1) - 25 \log(x - 2) + \frac{61}{2} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")

[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)

mupad [B] time = 2.13, size = 20, normalized size = 0.67

$$x + \frac{7 \ln(x-1)}{2} - 25 \ln(x-2) + \frac{61 \ln(x-3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)

[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2

sympy [A] time = 0.15, size = 24, normalized size = 0.80

$$x + \frac{61 \log(x-3)}{2} - 25 \log(x-2) + \frac{7 \log(x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)

[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2

$$3.291 \quad \int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx$$

Optimal. Leaf size=35

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2074}

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]

[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3} dx &= \int \left(-2 + \frac{13}{3(-4+x)} + x - \frac{22}{3(2+x)} + \frac{20}{3+x} \right) dx \\ &= -2x + \frac{x^2}{2} + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(2+x) + 20 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^2}{2} - 2x + \frac{13}{3} \log(4-x) - \frac{22}{3} \log(x+2) + 20 \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]

[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 - 7x + x^2 - x^3 + x^4}{-24 - 14x + x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]

[Out] IntegrateAlgebraic[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]

fricas [A] time = 1.24, size = 27, normalized size = 0.77

$$\frac{1}{2}x^2 - 2x + 20 \log(x + 3) - \frac{22}{3} \log(x + 2) + \frac{13}{3} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24), x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)

giac [A] time = 0.32, size = 30, normalized size = 0.86

$$\frac{1}{2}x^2 - 2x + 20 \log(|x + 3|) - \frac{22}{3} \log(|x + 2|) + \frac{13}{3} \log(|x - 4|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24), x, algorithm="giac")

[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x - 4))

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{x^2}{2} - 2x - \frac{22 \ln(x + 2)}{3} + 20 \ln(x + 3) + \frac{13 \ln(x - 4)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24), x)

[Out] -2*x+1/2*x^2-22/3*ln(x+2)+13/3*ln(x-4)+20*ln(x+3)

maxima [A] time = 1.12, size = 27, normalized size = 0.77

$$\frac{1}{2}x^2 - 2x + 20 \log(x + 3) - \frac{22}{3} \log(x + 2) + \frac{13}{3} \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")`

[Out] $1/2*x^2 - 2*x + 20*\log(x + 3) - 22/3*\log(x + 2) + 13/3*\log(x - 4)$

mupad [B] time = 0.04, size = 27, normalized size = 0.77

$$20 \ln(x + 3) - \frac{22 \ln(x + 2)}{3} - 2x + \frac{13 \ln(x - 4)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)`

[Out] $20*\log(x + 3) - (22*\log(x + 2))/3 - 2*x + (13*\log(x - 4))/3 + x^2/2$

sympy [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13 \log(x - 4)}{3} - \frac{22 \log(x + 2)}{3} + 20 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)`

[Out] $x**2/2 - 2*x + 13*\log(x - 4)/3 - 22*\log(x + 2)/3 + 20*\log(x + 3)$

$$3.292 \quad \int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx$$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1612}

$$\frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{(-1+x)^2 x(1+x)} dx &= \int \left(\frac{3}{2(-1+x)^2} - \frac{5}{4(-1+x)} + \frac{2}{x} - \frac{3}{4(1+x)} \right) dx \\ &= \frac{3}{2(1-x)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)} - \frac{5}{4} \log(1-x) + 2 \log(x) - \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]

[Out] $-3/(2*(-1 + x)) - (5*\text{Log}[1 - x])/4 + 2*\text{Log}[x] - (3*\text{Log}[1 + x])/4$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x^2}{(-1 + x)^2 x (1 + x)} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]`

[Out] `IntegrateAlgebraic[(2 + x^2)/((-1 + x)^2*x*(1 + x)), x]`

fricas [A] time = 1.57, size = 34, normalized size = 1.00

$$-\frac{3(x-1)\log(x+1) + 5(x-1)\log(x-1) - 8(x-1)\log(x) + 6}{4(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")`

[Out] $-1/4*(3*(x - 1)*\log(x + 1) + 5*(x - 1)*\log(x - 1) - 8*(x - 1)*\log(x) + 6)/(x - 1)$

giac [A] time = 0.39, size = 34, normalized size = 1.00

$$-\frac{3}{2(x-1)} + 2 \log\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4} \log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")`

[Out] $-3/2/(x - 1) + 2*\log(\text{abs}(-1/(x - 1) - 1)) - 3/4*\log(\text{abs}(-2/(x - 1) - 1))$

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$2\ln(x) - \frac{5\ln(x-1)}{4} - \frac{3\ln(x+1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+2)/(x-1)^2/x/(x+1),x)`

[Out] $-3/2/(x-1)-5/4*\ln(x-1)-3/4*\ln(x+1)+2*\ln(x)$

maxima [A] time = 1.13, size = 24, normalized size = 0.71

$$-\frac{3}{2(x-1)} - \frac{3}{4} \log(x+1) - \frac{5}{4} \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")

[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)

mupad [B] time = 2.11, size = 26, normalized size = 0.76

$$2 \ln(x) - \frac{3 \ln(x+1)}{4} - \frac{5 \ln(x-1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)

[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))

sympy [A] time = 0.14, size = 27, normalized size = 0.79

$$2 \log(x) - \frac{5 \log(x-1)}{4} - \frac{3 \log(x+1)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)

[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)

$$3.293 \quad \int \frac{3+x^2+x^3}{(2+x^2)^2} dx$$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1814, 635, 203, 260}

$$\frac{x+4}{4(x^2+2)} + \frac{1}{2} \log(x^2+2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]

[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{3 + x^2 + x^3}{(2 + x^2)^2} dx &= \frac{4 + x}{4(2 + x^2)} - \frac{1}{4} \int \frac{-5 - 4x}{2 + x^2} dx \\ &= \frac{4 + x}{4(2 + x^2)} + \frac{5}{4} \int \frac{1}{2 + x^2} dx + \int \frac{x}{2 + x^2} dx \\ &= \frac{4 + x}{4(2 + x^2)} + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2} \log(2 + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{x + 4}{4(x^2 + 2)} + \frac{1}{2} \log(x^2 + 2) + \frac{5 \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2, x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3 + x^2 + x^3}{(2 + x^2)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + x^2 + x^3)/(2 + x^2)^2, x]
```

```
[Out] IntegrateAlgebraic[(3 + x^2 + x^3)/(2 + x^2)^2, x]
```

fricas [A] time = 0.91, size = 44, normalized size = 1.05

$$\frac{5\sqrt{2}(x^2 + 2) \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 4(x^2 + 2) \log(x^2 + 2) + 2x + 8}{8(x^2 + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")

[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) + 2*x + 8)/(x^2 + 2)

giac [A] time = 0.30, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2}\log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")

[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2)

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$\frac{5\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{8} + \frac{\ln(x^2+2)}{2} + \frac{\frac{x}{4}+1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+3)/(x^2+2)^2,x)

[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.36, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + \frac{x+4}{4(x^2+2)} + \frac{1}{2}\log(x^2+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")

[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2)

mupad [B] time = 2.19, size = 39, normalized size = 0.93

$$\frac{\ln(x^2+2)}{2} + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8} + \frac{x}{4(x^2+2)} + \frac{1}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)`

[Out] `log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x^2 + 2)`

sympy [A] time = 0.13, size = 36, normalized size = 0.86

$$\frac{x + 4}{4x^2 + 8} + \frac{\log(x^2 + 2)}{2} + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+3)/(x**2+2)**2,x)`

[Out] `(x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8`

$$3.294 \quad \int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)} dx$$

Optimal. Leaf size=49

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {6728, 634, 618, 204, 628}

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]

[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 6728

`Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]`

Rubi steps

$$\begin{aligned} \int \frac{-35 + 70x - 4x^2 + 2x^3}{(26 - 10x + x^2)(17 - 2x + x^2)} dx &= \int \left(\frac{5003 + 2006x}{1025(26 - 10x + x^2)} + \frac{-4651 + 44x}{1025(17 - 2x + x^2)} \right) dx \\ &= \frac{\int \frac{5003 + 2006x}{26 - 10x + x^2} dx}{1025} + \frac{\int \frac{-4651 + 44x}{17 - 2x + x^2} dx}{1025} \\ &= \frac{22 \int \frac{-2 + 2x}{17 - 2x + x^2} dx}{1025} + \frac{1003 \int \frac{-10 + 2x}{26 - 10x + x^2} dx}{1025} - \frac{4607 \int \frac{1}{17 - 2x + x^2} dx}{1025} + \frac{15033 \int \frac{1}{17 - 2x + x^2} dx}{1025} \\ &= \frac{1003 \log(26 - 10x + x^2)}{1025} + \frac{22 \log(17 - 2x + x^2)}{1025} + \frac{9214 \operatorname{Subst}\left(\int \frac{1}{-64 - x} dx\right)}{1025} \\ &= -\frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{1}{4}(-1 + x)\right)}{4100} + \frac{1003 \log(26 - 10x + x^2)}{1025} \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{15033 \tan^{-1}(5 - x)}{1025} - \frac{4607 \tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

[In] `Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)), x]`

[Out] `(-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025`

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-35 + 70x - 4x^2 + 2x^3}{(26 - 10x + x^2)(17 - 2x + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)), x]

[Out] IntegrateAlgebraic[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)), x]

fricas [A] time = 1.42, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x - 5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17), x, algorithm="fricas")

[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)

giac [A] time = 0.32, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x - 5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17), x, algorithm="giac")

[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{15033 \arctan(x - 5)}{1025} - \frac{4607 \arctan\left(\frac{x}{4} - \frac{1}{4}\right)}{4100} + \frac{1003 \ln(x^2 - 10x + 26)}{1025} + \frac{22 \ln(x^2 - 2x + 17)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17), x)

[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+26)+22/1025*ln(x^2-2*x+17)

maxima [A] time = 2.80, size = 37, normalized size = 0.76

$$\frac{15033}{1025} \arctan(x-5) - \frac{4607}{4100} \arctan\left(\frac{1}{4}x - \frac{1}{4}\right) + \frac{22}{1025} \log(x^2 - 2x + 17) + \frac{1003}{1025} \log(x^2 - 10x + 26)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="maxima")

[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2 - 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)

mupad [B] time = 0.07, size = 41, normalized size = 0.84

$$\ln(x-1-4i)\left(\frac{22}{1025} + \frac{4607i}{8200}\right) + \ln(x-1+4i)\left(\frac{22}{1025} - \frac{4607i}{8200}\right) + \ln(x-5-i)\left(\frac{1003}{1025} - \frac{15033i}{2050}\right) + \ln(x-5+i)\left(\frac{1003}{1025} + \frac{15033i}{2050}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)

[Out] log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 4607i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i))*(1003/1025 + 15033i/2050)

sympy [A] time = 0.23, size = 46, normalized size = 0.94

$$\frac{1003 \log(x^2 - 10x + 26)}{1025} + \frac{22 \log(x^2 - 2x + 17)}{1025} - \frac{4607 \operatorname{atan}\left(\frac{x}{4} - \frac{1}{4}\right)}{4100} + \frac{15033 \operatorname{atan}(x - 5)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)

[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025

$$3.295 \quad \int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx$$

Optimal. Leaf size=29

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1612}

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]

[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx &= \int \left(\frac{3}{2(-5+x)} - \frac{11}{14(-3+x)} + \frac{2}{7(4+x)} \right) dx \\ &= -\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(4+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{11}{14} \log(3-x) + \frac{3}{2} \log(5-x) + \frac{2}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]

[Out] $(-11 \cdot \text{Log}[3 - x])/14 + (3 \cdot \text{Log}[5 - x])/2 + (2 \cdot \text{Log}[4 + x])/7$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x^2}{(-5 + x)(-3 + x)(4 + x)} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]`

[Out] `IntegrateAlgebraic[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]`

fricas [A] time = 1.61, size = 19, normalized size = 0.66

$$\frac{2}{7} \log(x + 4) - \frac{11}{14} \log(x - 3) + \frac{3}{2} \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2)/(-5+x)/(-3+x)/(4+x), x, algorithm="fricas")`

[Out] `2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)`

giac [A] time = 0.27, size = 22, normalized size = 0.76

$$\frac{2}{7} \log(|x + 4|) - \frac{11}{14} \log(|x - 3|) + \frac{3}{2} \log(|x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2)/(-5+x)/(-3+x)/(4+x), x, algorithm="giac")`

[Out] `2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))`

maple [A] time = 0.01, size = 20, normalized size = 0.69

$$\frac{3 \ln(x - 5)}{2} - \frac{11 \ln(x - 3)}{14} + \frac{2 \ln(x + 4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+2)/(x-5)/(x-3)/(x+4), x)`

[Out] `2/7*ln(x+4)-11/14*ln(x-3)+3/2*ln(x-5)`

maxima [A] time = 1.17, size = 19, normalized size = 0.66

$$\frac{2}{7} \log(x + 4) - \frac{11}{14} \log(x - 3) + \frac{3}{2} \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")`

[Out] $2/7*\log(x + 4) - 11/14*\log(x - 3) + 3/2*\log(x - 5)$

mupad [B] time = 2.18, size = 19, normalized size = 0.66

$$\frac{2 \ln(x + 4)}{7} - \frac{11 \ln(x - 3)}{14} + \frac{3 \ln(x - 5)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)`

[Out] $(2*\log(x + 4))/7 - (11*\log(x - 3))/14 + (3*\log(x - 5))/2$

sympy [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{3 \log(x - 5)}{2} - \frac{11 \log(x - 3)}{14} + \frac{2 \log(x + 4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)`

[Out] $3*\log(x - 5)/2 - 11*\log(x - 3)/14 + 2*\log(x + 4)/7$

$$3.296 \quad \int \frac{x^4}{(-1+x)(2+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{x^2}{2} - \frac{2}{3} \log(x^2 + 2) + x + \frac{1}{3} \log(1 - x) - \frac{2}{3} \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1629, 635, 203, 260}

$$\frac{x^2}{2} - \frac{2}{3} \log(x^2 + 2) + x + \frac{1}{3} \log(1 - x) - \frac{2}{3} \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^4/((-1 + x)*(2 + x^2)), x]

[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2])/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(-1+x)(2+x^2)} dx &= \int \left(1 + \frac{1}{3(-1+x)} + x - \frac{4(1+x)}{3(2+x^2)} \right) dx \\
&= x + \frac{x^2}{2} + \frac{1}{3} \log(1-x) - \frac{4}{3} \int \frac{1+x}{2+x^2} dx \\
&= x + \frac{x^2}{2} + \frac{1}{3} \log(1-x) - \frac{4}{3} \int \frac{1}{2+x^2} dx - \frac{4}{3} \int \frac{x}{2+x^2} dx \\
&= x + \frac{x^2}{2} - \frac{2}{3} \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + \frac{1}{3} \log(1-x) - \frac{2}{3} \log(2+x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{1}{6} \left(3x^2 - 4 \log(x^2 + 2) + 6x + 2 \log(x - 1) - 4\sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - 9 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]

[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 + x^2])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-1+x)(2+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/((-1 + x)*(2 + x^2)),x]

[Out] IntegrateAlgebraic[x^4/((-1 + x)*(2 + x^2)), x]

fricas [A] time = 1.56, size = 33, normalized size = 0.72

$$\frac{1}{2} x^2 - \frac{2}{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) + x - \frac{2}{3} \log(x^2 + 2) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")

[Out] $\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log(x^2 + 2) + \frac{1}{3}\log(x - 1)$

giac [A] time = 0.32, size = 34, normalized size = 0.74

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log(x^2 + 2) + \frac{1}{3}\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")`

[Out] $\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log(x^2 + 2) + \frac{1}{3}\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{x^2}{2} + x - \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{3} + \frac{\ln(x - 1)}{3} - \frac{2\ln(x^2 + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(x-1)/(x^2+2),x)`

[Out] $\frac{1}{2}x^2 + x + \frac{1}{3}\ln(x - 1) - \frac{2}{3}\ln(x^2 + 2) - \frac{2}{3}2^{(1/2)}\arctan\left(\frac{1}{2}2^{(1/2)}x\right)$

maxima [A] time = 2.16, size = 33, normalized size = 0.72

$$\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log(x^2 + 2) + \frac{1}{3}\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 - \frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + x - \frac{2}{3}\log(x^2 + 2) + \frac{1}{3}\log(x - 1)$

mupad [B] time = 0.09, size = 50, normalized size = 1.09

$$x + \frac{\ln(x - 1)}{3} + \ln\left(x - \sqrt{2}i\right)\left(-\frac{2}{3} + \frac{\sqrt{2}i}{3}\right) - \ln\left(x + \sqrt{2}i\right)\left(\frac{2}{3} + \frac{\sqrt{2}i}{3}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/((x^2 + 2)*(x - 1)),x)`

[Out] $x + \log(x - 1)/3 + \log(x - 2^{(1/2)*1i})*((2^{(1/2)*1i})/3 - 2/3) - \log(x + 2^{(1/2)*1i})*((2^{(1/2)*1i})/3 + 2/3) + x^{2/2}$

sympy [A] time = 0.14, size = 41, normalized size = 0.89

$$\frac{x^2}{2} + x + \frac{\log(x-1)}{3} - \frac{2\log(x^2+2)}{3} - \frac{2\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(-1+x)/(x**2+2),x)`

[Out] $x^{**2}/2 + x + \log(x - 1)/3 - 2*\log(x^{**2} + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)/3$

$$3.297 \quad \int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx$$

Optimal. Leaf size=16

$$2 \log(1-x) - \frac{3}{x+1}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2074}

$$2 \log(1-x) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

[Out] -3/(1 + x) + 2*Log[1 - x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{-1+7x+2x^2}{-1-x+x^2+x^3} dx &= \int \left(\frac{2}{-1+x} + \frac{3}{(1+x)^2} \right) dx \\ &= -\frac{3}{1+x} + 2 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$2 \log(x-1) - \frac{3}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

[Out] -3/(1 + x) + 2*Log[-1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 7x + 2x^2}{-1 - x + x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]

fricas [A] time = 0.80, size = 17, normalized size = 1.06

$$\frac{2(x+1)\log(x-1) - 3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1), x, algorithm="fricas")

[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)

giac [A] time = 0.40, size = 15, normalized size = 0.94

$$-\frac{3}{x+1} + 2 \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1), x, algorithm="giac")

[Out] -3/(x + 1) + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$2 \ln(x-1) - \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1), x)

[Out] 2*ln(x-1)-3/(x+1)

maxima [A] time = 1.09, size = 14, normalized size = 0.88

$$-\frac{3}{x+1} + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")

[Out] -3/(x + 1) + 2*log(x - 1)

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$2 \ln(x - 1) - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)

[Out] 2*log(x - 1) - 3/(x + 1)

sympy [A] time = 0.09, size = 10, normalized size = 0.62

$$2 \log(x - 1) - \frac{3}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)

[Out] 2*log(x - 1) - 3/(x + 1)

$$3.298 \quad \int \frac{1+2x}{-1+3x-3x^2+x^3} dx$$

Optimal. Leaf size=21

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2074}

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

[Out] -3/(2*(1 - x)^2) + 2/(1 - x)

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{-1+3x-3x^2+x^3} dx &= \int \left(\frac{3}{(-1+x)^3} + \frac{2}{(-1+x)^2} \right) dx \\ &= -\frac{3}{2(1-x)^2} + \frac{2}{1-x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

[Out] (1 - 4*x)/(2*(-1 + x)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 2x}{-1 + 3x - 3x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]

fricas [A] time = 1.29, size = 17, normalized size = 0.81

$$-\frac{4x - 1}{2(x^2 - 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1), x, algorithm="fricas")

[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)

giac [A] time = 0.30, size = 12, normalized size = 0.57

$$-\frac{4x - 1}{2(x - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1), x, algorithm="giac")

[Out] -1/2*(4*x - 1)/(x - 1)^2

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{3}{2(x - 1)^2} - \frac{2}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)/(x^3-3*x^2+3*x-1), x)

[Out] -3/2/(x-1)^2-2/(x-1)

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$-\frac{4x - 1}{2(x^2 - 2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")`

[Out] `-1/2*(4*x - 1)/(x^2 - 2*x + 1)`

mupad [B] time = 2.09, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)`

[Out] `-(4*x - 1)/(2*(x - 1)^2)`

sympy [A] time = 0.09, size = 14, normalized size = 0.67

$$\frac{1-4x}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)`

[Out] `(1 - 4*x)/(2*x**2 - 4*x + 2)`

$$3.299 \quad \int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx$$

Optimal. Leaf size=15

$$\frac{1}{1-x} - \frac{2}{(x+1)^2}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1620}

$$\frac{1}{1-x} - \frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]

[Out] (1 - x)^(-1) - 2/(1 + x)^2

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3} dx &= \int \left(\frac{1}{(-1+x)^2} + \frac{4}{(1+x)^3} \right) dx \\ &= \frac{1}{1-x} - \frac{2}{(1+x)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{(x+1)^2} - \frac{1}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]

[Out] $-(-1 + x)^{-1} - 2/(1 + x)^2$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 - 5x + 7x^2 + x^3}{(-1 + x)^2(1 + x)^3} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]`

[Out] `IntegrateAlgebraic[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]`

fricas [A] time = 1.30, size = 23, normalized size = 1.53

$$-\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")`

[Out] `-(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)`

giac [A] time = 0.26, size = 30, normalized size = 2.00

$$-\frac{1}{x-1} + \frac{\frac{4}{x-1} + 1}{2\left(\frac{2}{x-1} + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")`

[Out] `-1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2`

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{2}{(x+1)^2} - \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x)`

[Out] `-2/(x+1)^2-1/(x-1)`

maxima [A] time = 1.27, size = 23, normalized size = 1.53

$$-\frac{x^2 + 4x - 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")

[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)

mupad [B] time = 2.09, size = 15, normalized size = 1.00

$$-\frac{1}{x-1} - \frac{2}{(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)

[Out] - 1/(x - 1) - 2/(x + 1)^2

sympy [A] time = 0.11, size = 17, normalized size = 1.13

$$\frac{-x^2 - 4x + 1}{x^3 + x^2 - x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)

[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)

$$3.300 \quad \int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$$

Optimal. Leaf size=31

$$\log(x^2 + x + 1) + \log(x + 1) - \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2074, 634, 618, 204, 628}

$$\log(x^2 + x + 1) + \log(x + 1) - \frac{2 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]

[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 2074

`Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx &= \int \left(\frac{1}{1+x} + \frac{2x}{1+x+x^2} \right) dx \\
 &= \log(1+x) + 2 \int \frac{x}{1+x+x^2} dx \\
 &= \log(1+x) - \int \frac{1}{1+x+x^2} dx + \int \frac{1+2x}{1+x+x^2} dx \\
 &= \log(1+x) + \log(1+x+x^2) + 2 \operatorname{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1+2x \right) \\
 &= -\frac{2 \tan^{-1} \left(\frac{1+2x}{\sqrt{3}} \right)}{\sqrt{3}} + \log(1+x) + \log(1+x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\log(x^2+x+1) + \log(x+1) - \frac{2 \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]

[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]

fricas [A] time = 1.26, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log(x^2+x+1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)

giac [A] time = 0.28, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log(x^2+x+1) + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x + 1))

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \ln(x+1) + \ln(x^2+x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x)

[Out] ln(x+1)+ln(x^2+x+1)-2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.36, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log(x^2+x+1) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)

mupad [B] time = 0.11, size = 57, normalized size = 1.84

$$\ln\left(x + \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) + \ln(x+1) + \frac{\sqrt{3} \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} 1i}{2}\right) 1i}{3} - \frac{\sqrt{3} \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) 1i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1), x)`

[Out] `log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1) + (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3`

sympy [A] time = 0.13, size = 3, normalized size = 0.10

$$\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1), x)`

[Out] `log(x + 1)`

$$3.301 \quad \int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$$

Optimal. Leaf size=25

$$\frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1594, 1628}

$$\frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx &= \int \frac{-1+2x+x^2}{x(-2+3x+2x^2)} dx \\ &= \int \left(\frac{1}{2x} - \frac{1}{10(2+x)} + \frac{1}{5(-1+2x)} \right) dx \\ &= \frac{1}{10} \log(1-2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10} \log(1 - 2x) + \frac{\log(x)}{2} - \frac{1}{10} \log(x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 2x + x^2}{-2x + 3x^2 + 2x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

[Out] IntegrateAlgebraic[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]

fricas [A] time = 1.07, size = 19, normalized size = 0.76

$$\frac{1}{10} \log(2x - 1) - \frac{1}{10} \log(x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x), x, algorithm="fricas")

[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)

giac [A] time = 0.29, size = 22, normalized size = 0.88

$$\frac{1}{10} \log(|2x - 1|) - \frac{1}{10} \log(|x + 2|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x), x, algorithm="giac")

[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\ln(x)}{2} + \frac{\ln(2x - 1)}{10} - \frac{\ln(x + 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)`

[Out] `1/10*ln(2*x-1)-1/10*ln(x+2)+1/2*ln(x)`

maxima [A] time = 1.09, size = 19, normalized size = 0.76

$$\frac{1}{10} \log(2x - 1) - \frac{1}{10} \log(x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")`

[Out] `1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)`

mupad [B] time = 0.06, size = 19, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{24}{145\left(\frac{29x}{100} - \frac{11}{50}\right)} + \frac{35}{29}\right)}{5} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)`

[Out] `atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2`

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$\frac{\log(x)}{2} + \frac{\log\left(x - \frac{1}{2}\right)}{10} - \frac{\log(x + 2)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)`

[Out] `log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10`

$$3.302 \quad \int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx$$

Optimal. Leaf size=30

$$\frac{x^2}{2} + x + \frac{2}{1-x} + \log(1-x) - \log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2074}

$$\frac{x^2}{2} + x + \frac{2}{1-x} + \log(1-x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] :=> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3} dx &= \int \left(1 + \frac{1}{-1-x} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x} + x \right) dx \\ &= \frac{2}{1-x} + x + \frac{x^2}{2} + \log(1-x) - \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2 - \frac{2}{x-1} + \log(1-x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + 4x - 2x^2 + x^4}{1 - x - x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]

fricas [A] time = 1.14, size = 36, normalized size = 1.20

$$\frac{x^3 + x^2 - 2(x - 1)\log(x + 1) + 2(x - 1)\log(x - 1) - 2x - 4}{2(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1), x, algorithm="fricas")

[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x - 1)

giac [A] time = 0.28, size = 26, normalized size = 0.87

$$\frac{1}{2}x^2 + x - \frac{2}{x - 1} - \log(|x + 1|) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1), x, algorithm="giac")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + x + \ln(x - 1) - \ln(x + 1) - \frac{2}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1), x)

[Out] 1/2*x^2+x+ln(x-1)-2/(x-1)-ln(x+1)

maxima [A] time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + x - \frac{2}{x - 1} - \log(x + 1) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")

[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)

mupad [B] time = 2.11, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \operatorname{atan}(x1i) 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)

[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2

sympy [A] time = 0.10, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x - 1) - \log(x + 1) - \frac{2}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)

[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)

$$3.303 \quad \int \frac{4-x+2x^2}{4x+x^3} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 - x + 2*x^2)/(4*x + x^3), x]

[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3} dx &= \int \frac{4-x+2x^2}{x(4+x^2)} dx \\
&= \int \left(\frac{1}{x} + \frac{-1+x}{4+x^2} \right) dx \\
&= \log(x) + \int \frac{-1+x}{4+x^2} dx \\
&= \log(x) - \int \frac{1}{4+x^2} dx + \int \frac{x}{4+x^2} dx \\
&= -\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \log(x) + \frac{1}{2} \log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 4) + \log(x) - \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3), x]

[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4-x+2x^2}{4x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 - x + 2*x^2)/(4*x + x^3), x]

[Out] IntegrateAlgebraic[(4 - x + 2*x^2)/(4*x + x^3), x]

fricas [A] time = 1.04, size = 17, normalized size = 0.74

$$-\frac{1}{2} \arctan\left(\frac{1}{2} x\right) + \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$-\frac{1}{2} \arctan\left(\frac{1}{2}x\right) + \frac{1}{2} \log(x^2 + 4) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{\arctan\left(\frac{x}{2}\right)}{2} + \ln(x) + \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2-x+4)/(x^3+4*x),x)

[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)

maxima [A] time = 2.08, size = 17, normalized size = 0.74

$$-\frac{1}{2} \arctan\left(\frac{1}{2}x\right) + \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")

[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)

mupad [B] time = 2.12, size = 21, normalized size = 0.91

$$\ln(x) + \ln(x - 2i) \left(\frac{1}{2} + \frac{1}{4}i\right) + \ln(x + 2i) \left(\frac{1}{2} - \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 - x + 4)/(4*x + x^3),x)

[Out] $\log(x - 2i)*(1/2 + 1i/4) + \log(x + 2i)*(1/2 - 1i/4) + \log(x)$

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\log(x) + \frac{\log(x^2 + 4)}{2} - \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**2-x+4)/(x**3+4*x),x)`

[Out] $\log(x) + \log(x^2 + 4)/2 - \operatorname{atan}(x/2)/2$

$$3.304 \quad \int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx$$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8(x^2+1)} + \frac{3x}{16(x^2+1)} + \frac{x+1}{8(x^2+1)^2} + \frac{15}{16} \log(x^2+1) - \frac{1}{2} \log(x^2+x+1) + \frac{1}{8} \log(1-x) - \log(x) + \frac{7}{16} \tan^{-1}(x) - \frac{7}{16} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.48, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8(x^2+1)} + \frac{3x}{16(x^2+1)} + \frac{x+1}{8(x^2+1)^2} + \frac{15}{16} \log(x^2+1) - \frac{1}{2} \log(x^2+x+1) + \frac{1}{8} \log(1-x) - \log(x) + \frac{7}{16} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)), x]

[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2)) + (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

$\text{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 618

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e) / (2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e / (2*c), \text{Int}[(b + 2*c*x) / (a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 635

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

Rule 639

$\text{Int}[(d_) + (e_)*(x_)] * ((a_) + (c_)*(x_)^2)^{p_}, x_Symbol] \rightarrow \text{Simp}[(a*e - c*d*x) * (a + c*x^2)^{p+1} / (2*a*c*(p+1)), x] + \text{Dist}[(d*(2*p+3)) / (2*a*(p+1)), \text{Int}[(a + c*x^2)^{p+1}, x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

Rule 6728

$\text{Int}[u_ / ((a_) + (b_)*(x_)^{n_}) + (c_)*(x_)^{n2_}], x_Symbol] \rightarrow \text{With}[\{v = \text{RationalFunctionExpand}[u / (a + b*x^n + c*x^{2*n}), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx &= \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2(1+x^2)^3} + \frac{3(1+x)}{4(1+x^2)^2} + \frac{-1+15x}{8(1+x^2)} + \frac{-1-x}{1+x} \right) dx \\
&= \frac{1}{8} \log(1-x) - \log(x) + \frac{1}{8} \int \frac{-1+15x}{1+x^2} dx + \frac{1}{2} \int \frac{1-x}{(1+x^2)^3} dx + \frac{3}{4} \int \frac{1+x}{(1+x^2)^2} dx \\
&= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{1}{8} \log(1-x) - \log(x) - \frac{1}{8} \int \frac{1}{1+x^2} dx + \frac{3}{8} \int \frac{1+x}{1+x^2} dx \\
&= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{3x}{16(1+x^2)} + \frac{1}{4} \tan^{-1}(x) + \frac{1}{8} \log(1-x) - \frac{1}{8} \log(x) \\
&= \frac{1+x}{8(1+x^2)^2} - \frac{3(1-x)}{8(1+x^2)} + \frac{3x}{16(1+x^2)} + \frac{7}{16} \tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.90

$$\frac{1}{48} \left(-14 \log(1-x^3) + \frac{6(x+1)}{(x^2+1)^2} + \frac{9(3x-2)}{x^2+1} + 45 \log(x^2+1) - 10 \log(x^2+x+1) + 20 \log(1-x) - 48 \log(x) + 21 \tan^{-1}(x) - 16\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)), x]

[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*sqrt[3]*ArcTan[(1 + 2*x)/sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)), x]

[Out] IntegrateAlgebraic[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)), x]

fricas [A] time = 0.98, size = 136, normalized size = 1.32

$$\frac{27x^3 - 16\sqrt{3}(x^4 + 2x^2 + 1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 18x^2 + 21(x^4 + 2x^2 + 1)\arctan(x) - 24(x^4 + 2x^2 + 1)\log(x^2 + x + 1) + 45(x^4 + 2x^2 + 1)\log(x^2 + 1) + 6(x^4 + 2x^2 + 1)\log(x-1) - 48(x^4 + 2x^2 + 1)\log(x) + 33x - 12}{48(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")

[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1) - 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)

giac [A] time = 0.34, size = 74, normalized size = 0.72

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3 - 6x^2 + 11x - 4}{16(x^2+1)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2+x+1) + \frac{15}{16}\log(x^2+1) + \frac{1}{8}\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1) + 1/8*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 73, normalized size = 0.71

$$\frac{7\arctan(x)}{16} - \frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} - \ln(x) + \frac{\ln(x-1)}{8} + \frac{15\ln(x^2+1)}{16} - \frac{\ln(x^2+x+1)}{2} + \frac{\frac{9}{2}x^3 - 3x^2 + \frac{11}{2}x - 2}{8(x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)

[Out] 1/8*ln(x-1)+1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)-ln(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.25, size = 77, normalized size = 0.75

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3 - 6x^2 + 11x - 4}{16(x^4 + 2x^2 + 1)} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2 + x + 1) + \frac{15}{16}\log(x^2 + 1) + \frac{1}{8}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4)/(x^4 + 2*x^2 + 1) + 7/16*\arctan(x) - 1/2*\log(x^2 + x + 1) + 15/16*\log(x^2 + 1) + 1/8*\log(x - 1) - \log(x)$

mupad [B] time = 2.20, size = 96, normalized size = 0.93

$$\frac{\ln(x-1)}{8} - \ln(x) + \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}1i}{6}\right) - \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{6}\right) + \frac{9x^3 - 3x^2 + 11x - 4}{16x^4 + 2x^2 + 1} + \ln(x-i)\left(\frac{15}{16} - \frac{7}{32}i\right) + \ln(x+i)\left(\frac{15}{16} + \frac{7}{32}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)`

[Out] $\log(x - 1)/8 + \log(x - 1i)*(15/16 - 7i/32) + \log(x + 1i)*(15/16 + 7i/32) - \log(x) + \log(x - (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/6 - 1/2) - \log(x + (3^{(1/2)}*1i)/2 + 1/2)*((3^{(1/2)}*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3)/16 - 1/4)/(2*x^2 + x^4 + 1)$

sympy [A] time = 0.51, size = 88, normalized size = 0.85

$$-\log(x) + \frac{\log(x-1)}{8} + \frac{15\log(x^2+1)}{16} - \frac{\log(x^2+x+1)}{2} + \frac{7\operatorname{atan}(x)}{16} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{9x^3 - 6x^2 + 11x - 4}{16x^4 + 32x^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)`

[Out] $-\log(x) + \log(x - 1)/8 + 15*\log(x**2 + 1)/16 - \log(x**2 + x + 1)/2 + 7*\operatorname{atan}(x)/16 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/3 + (9*x**3 - 6*x**2 + 11*x - 4)/(16*x**4 + 32*x**2 + 16)$

$$3.305 \quad \int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx$$

Optimal. Leaf size=33

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \frac{3}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1814, 635, 203, 260}

$$\frac{2-x}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \frac{3}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /

; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx &= \frac{2-x}{2(1+x^2)} - \frac{1}{2} \int \frac{-3+2x}{1+x^2} dx \\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2} \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\ &= \frac{2-x}{2(1+x^2)} + \frac{3}{2} \tan^{-1}(x) - \frac{1}{2} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{2} \left(\frac{2-x}{x^2+1} - \log(x^2+1) + 3 \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]

fricas [A] time = 1.68, size = 36, normalized size = 1.09

$$\frac{3(x^2+1) \arctan(x) - (x^2+1) \log(x^2+1) - x + 2}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2, x, algorithm="fricas")

[Out] $1/2*(3*(x^2 + 1)*\arctan(x) - (x^2 + 1)*\log(x^2 + 1) - x + 2)/(x^2 + 1)$

giac [A] time = 0.32, size = 25, normalized size = 0.76

$$-\frac{x-2}{2(x^2+1)} + \frac{3}{2} \arctan(x) - \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")`

[Out] $-1/2*(x - 2)/(x^2 + 1) + 3/2*\arctan(x) - 1/2*\log(x^2 + 1)$

maple [A] time = 0.00, size = 28, normalized size = 0.85

$$\frac{3 \arctan(x)}{2} - \frac{\ln(x^2+1)}{2} - \frac{\frac{x}{2}-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x)`

[Out] $-(1/2*x-1)/(x^2+1)-1/2*\ln(x^2+1)+3/2*\arctan(x)$

maxima [A] time = 2.15, size = 25, normalized size = 0.76

$$-\frac{x-2}{2(x^2+1)} + \frac{3}{2} \arctan(x) - \frac{1}{2} \log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")`

[Out] $-1/2*(x - 2)/(x^2 + 1) + 3/2*\arctan(x) - 1/2*\log(x^2 + 1)$

mupad [B] time = 0.03, size = 32, normalized size = 0.97

$$\frac{3 \operatorname{atan}(x)}{2} - \frac{\ln(x^2+1)}{2} - \frac{x}{2(x^2+1)} + \frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)`

[Out] $(3*\operatorname{atan}(x))/2 - \log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)$

sympy [A] time = 0.13, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2} - \frac{\log(x^2+1)}{2} + \frac{3\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)

[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2

$$3.306 \quad \int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} dx$$

Optimal. Leaf size=33

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) - 2 \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2(x^2+1)} - \frac{1}{2} \log(x^2+1) + \log(x) - 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805


```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 - 3x + 2x^2 - x^3}{x(1 + x^2)^2} dx &= -\frac{1 + 2x}{2(1 + x^2)} - \frac{1}{2} \int \frac{-2 + 4x}{x(1 + x^2)} dx \\
&= -\frac{1 + 2x}{2(1 + x^2)} - \frac{1}{2} \int \left(-\frac{2}{x} + \frac{2(2 + x)}{1 + x^2} \right) dx \\
&= -\frac{1 + 2x}{2(1 + x^2)} + \log(x) - \int \frac{2 + x}{1 + x^2} dx \\
&= -\frac{1 + 2x}{2(1 + x^2)} + \log(x) - 2 \int \frac{1}{1 + x^2} dx - \int \frac{x}{1 + x^2} dx \\
&= -\frac{1 + 2x}{2(1 + x^2)} - 2 \tan^{-1}(x) + \log(x) - \frac{1}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{-2x - 1}{2(x^2 + 1)} - \frac{1}{2} \log(x^2 + 1) + \log(x) - 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]

[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 3x + 2x^2 - x^3}{x(1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]

fricas [A] time = 1.54, size = 44, normalized size = 1.33

$$\frac{4(x^2 + 1)\arctan(x) + (x^2 + 1)\log(x^2 + 1) - 2(x^2 + 1)\log(x) + 2x + 1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")

[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) + 2*x + 1)/(x^2 + 1)

giac [A] time = 0.38, size = 30, normalized size = 0.91

$$-\frac{2x + 1}{2(x^2 + 1)} - 2\arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$-2\arctan(x) + \ln(x) - \frac{\ln(x^2 + 1)}{2} - \frac{x + \frac{1}{2}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)

[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)

maxima [A] time = 2.25, size = 29, normalized size = 0.88

$$-\frac{2x + 1}{2(x^2 + 1)} - 2\arctan(x) - \frac{1}{2}\log(x^2 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")

[Out] $-1/2*(2*x + 1)/(x^2 + 1) - 2*\arctan(x) - 1/2*\log(x^2 + 1) + \log(x)$

mupad [B] time = 2.11, size = 33, normalized size = 1.00

$$\ln(x) - \frac{x + \frac{1}{2}}{x^2 + 1} + \ln(x - i) \left(-\frac{1}{2} + i\right) + \ln(x + i) \left(-\frac{1}{2} - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2), x)`

[Out] $\log(x) - \log(x + 1i)*(1/2 + 1i) - \log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 + 1)$

sympy [A] time = 0.15, size = 27, normalized size = 0.82

$$-\frac{2x + 1}{2x^2 + 2} + \log(x) - \frac{\log(x^2 + 1)}{2} - 2 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2, x)`

[Out] $-(2*x + 1)/(2*x**2 + 2) + \log(x) - \log(x**2 + 1)/2 - 2*\operatorname{atan}(x)$

$$3.307 \quad \int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$$

Optimal. Leaf size=25

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx &= \int \frac{1-x-x^2+x^3+x^4}{x(-1+x^2)} dx \\
&= \int \left(1 - \frac{1}{x} + x + \frac{x}{-1+x^2}\right) dx \\
&= x + \frac{x^2}{2} - \log(x) + \int \frac{x}{-1+x^2} dx \\
&= x + \frac{x^2}{2} - \log(x) + \frac{1}{2} \log(1-x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2} \log(1-x^2) + x - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

[Out] IntegrateAlgebraic[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]

fricas [A] time = 1.28, size = 19, normalized size = 0.76

$$\frac{1}{2}x^2 + x + \frac{1}{2} \log(x^2 - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x), x, algorithm="fricas")

[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)

giac [A] time = 0.29, size = 26, normalized size = 1.04

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")

[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{x^2}{2} + x - \ln(x) + \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)

[Out] 1/2*x^2+x+1/2*ln(x-1)+1/2*ln(x+1)-ln(x)

maxima [A] time = 1.05, size = 23, normalized size = 0.92

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")

[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)

mupad [B] time = 0.04, size = 19, normalized size = 0.76

$$x + \frac{\ln(x^2-1)}{2} - \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)

[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2

sympy [A] time = 0.10, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log(x^2-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

$$3.308 \quad \int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=36

$$-\frac{1}{2} \log(x^2 + 1) + \log(x^2 + 2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 1) + \log(x^2 + 2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{2 - 4x^2 + x^3}{(1 + x^2)(2 + x^2)} dx &= \int \left(\frac{6 - x}{1 + x^2} + \frac{2(-5 + x)}{2 + x^2} \right) dx \\
&= 2 \int \frac{-5 + x}{2 + x^2} dx + \int \frac{6 - x}{1 + x^2} dx \\
&= 2 \int \frac{x}{2 + x^2} dx + 6 \int \frac{1}{1 + x^2} dx - 10 \int \frac{1}{2 + x^2} dx - \int \frac{x}{1 + x^2} dx \\
&= 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{1}{2} \log(1 + x^2) + \log(2 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 1) + \log(x^2 + 2) + 6 \tan^{-1}(x) - 5\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 - 4x^2 + x^3}{(1 + x^2)(2 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] IntegrateAlgebraic[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)), x]

fricas [A] time = 1.59, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

giac [A] time = 0.34, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$6 \arctan(x) - 5\sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right) - \frac{\ln(x^2 + 1)}{2} + \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)

[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 2.10, size = 31, normalized size = 0.86

$$-5\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) + 6 \arctan(x) + \log(x^2 + 2) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")

[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2 + 1)

mupad [B] time = 0.11, size = 56, normalized size = 1.56

$$\ln(x - i) \left(-\frac{1}{2} - 3i\right) + \ln(x + i) \left(-\frac{1}{2} + 3i\right) + \ln\left(x - \sqrt{2}i\right) \left(1 + \frac{\sqrt{2}5i}{2}\right) - \ln\left(x + \sqrt{2}i\right) \left(-1 + \frac{\sqrt{2}5i}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)

[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x - 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)

sympy [A] time = 0.20, size = 36, normalized size = 1.00

$$-\frac{\log(x^2 + 1)}{2} + \log(x^2 + 2) + 6 \operatorname{atan}(x) - 5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2), x)

[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)

$$3.309 \quad \int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx &= \int \left(\frac{1}{9(1+x^2)} - \frac{13}{3(4+x^2)^2} + \frac{8}{9(4+x^2)} \right) dx \\
&= \frac{1}{9} \int \frac{1}{1+x^2} dx + \frac{8}{9} \int \frac{1}{4+x^2} dx - \frac{13}{3} \int \frac{1}{(4+x^2)^2} dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{4}{9} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x) - \frac{13}{24} \int \frac{1}{4+x^2} dx \\
&= -\frac{13x}{24(4+x^2)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

[Out] IntegrateAlgebraic[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]

fricas [A] time = 1.06, size = 33, normalized size = 1.14

$$\frac{25(x^2+4) \arctan\left(\frac{1}{2}x\right) + 16(x^2+4) \arctan(x) - 78x}{144(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")

[Out] $1/144*(25*(x^2 + 4)*\arctan(1/2*x) + 16*(x^2 + 4)*\arctan(x) - 78*x)/(x^2 + 4)$

giac [A] time = 0.23, size = 21, normalized size = 0.72

$$-\frac{13x}{24(x^2 + 4)} + \frac{25}{144} \arctan\left(\frac{1}{2}x\right) + \frac{1}{9} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")`

[Out] $-13/24*x/(x^2 + 4) + 25/144*\arctan(1/2*x) + 1/9*\arctan(x)$

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$-\frac{13x}{24(x^2 + 4)} + \frac{\arctan(x)}{9} + \frac{25 \arctan\left(\frac{x}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)`

[Out] $-13/24*x/(x^2+4)+25/144*\arctan(1/2*x)+1/9*\arctan(x)$

maxima [A] time = 2.09, size = 21, normalized size = 0.72

$$-\frac{13x}{24(x^2 + 4)} + \frac{25}{144} \arctan\left(\frac{1}{2}x\right) + \frac{1}{9} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")`

[Out] $-13/24*x/(x^2 + 4) + 25/144*\arctan(1/2*x) + 1/9*\arctan(x)$

mupad [B] time = 0.04, size = 23, normalized size = 0.79

$$\frac{25 \operatorname{atan}\left(\frac{x}{2}\right)}{144} + \frac{\operatorname{atan}(x)}{9} - \frac{13x}{24(x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)`

[Out] $(25*\operatorname{atan}(x/2))/144 + \operatorname{atan}(x)/9 - (13*x)/(24*(x^2 + 4))$

sympy [A] time = 0.17, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2 + 96} + \frac{25 \operatorname{atan}\left(\frac{x}{2}\right)}{144} + \frac{\operatorname{atan}(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)

[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9

$$3.310 \quad \int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$$

Optimal. Leaf size=46

$$\frac{5}{8} \log(x^2 + x + 2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8} \log(x^2 + x + 2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]

[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x + x^2])/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1594

`Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]`

Rule 1628

`Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}
 \int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx &= \int \frac{1+x^2+x^3}{x^2(2+x+x^2)} dx \\
 &= \int \left(\frac{1}{2x^2} - \frac{1}{4x} + \frac{3+5x}{4(2+x+x^2)} \right) dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{1}{4} \int \frac{3+5x}{2+x+x^2} dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{1}{8} \int \frac{1}{2+x+x^2} dx + \frac{5}{8} \int \frac{1+2x}{2+x+x^2} dx \\
 &= -\frac{1}{2x} - \frac{\log(x)}{4} + \frac{5}{8} \log(2+x+x^2) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-7-x^2} dx, x, 1+2x \right) \\
 &= -\frac{1}{2x} + \frac{\tan^{-1} \left(\frac{1+2x}{\sqrt{7}} \right)}{4\sqrt{7}} - \frac{\log(x)}{4} + \frac{5}{8} \log(2+x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{5}{8} \log(x^2 + x + 2) - \frac{1}{2x} - \frac{\log(x)}{4} + \frac{\tan^{-1} \left(\frac{2x+1}{\sqrt{7}} \right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]

[Out] $-1/2*1/x + \text{ArcTan}[(1 + 2*x)/\text{Sqrt}[7]]/(4*\text{Sqrt}[7]) - \text{Log}[x]/4 + (5*\text{Log}[2 + x + x^2])/8$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^2 + x^3}{2x^2 + x^3 + x^4} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]`

[Out] `IntegrateAlgebraic[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]`

fricas [A] time = 1.52, size = 39, normalized size = 0.85

$$\frac{2\sqrt{7}x \arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) + 35x \log(x^2 + x + 2) - 14x \log(x) - 28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+1)/(x^4+x^3+2*x^2), x, algorithm="fricas")`

[Out] $1/56*(2*\text{sqrt}(7)*x*\arctan(1/7*\text{sqrt}(7)*(2*x + 1)) + 35*x*\log(x^2 + x + 2) - 14*x*\log(x) - 28)/x$

giac [A] time = 0.30, size = 36, normalized size = 0.78

$$\frac{1}{28}\sqrt{7} \arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right) - \frac{1}{2x} + \frac{5}{8} \log(x^2 + x + 2) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+1)/(x^4+x^3+2*x^2), x, algorithm="giac")`

[Out] $1/28*\text{sqrt}(7)*\arctan(1/7*\text{sqrt}(7)*(2*x + 1)) - 1/2/x + 5/8*\log(x^2 + x + 2) - 1/4*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{\sqrt{7} \arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{28} - \frac{\ln(x)}{4} + \frac{5 \ln(x^2 + x + 2)}{8} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+x^2+1)/(x^4+x^3+2*x^2), x)`

[Out] $-1/2/x - 1/4 \ln(x) + 5/8 \ln(x^2 + x + 2) + 1/28 \arctan(1/7 \sqrt{7} (2x + 1)) \sqrt{7}$

maxima [A] time = 2.09, size = 35, normalized size = 0.76

$$\frac{1}{28} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x + 1)\right) - \frac{1}{2x} + \frac{5}{8} \log(x^2 + x + 2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")`

[Out] $1/28 \sqrt{7} \arctan(1/7 \sqrt{7} (2x + 1)) - 1/2/x + 5/8 \log(x^2 + x + 2) - 1/4 \log(x)$

mupad [B] time = 2.17, size = 49, normalized size = 1.07

$$-\frac{\ln(x)}{4} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{7} 1i}{2}\right) \left(-\frac{5}{8} + \frac{\sqrt{7} 1i}{56}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{7} 1i}{2}\right) \left(\frac{5}{8} + \frac{\sqrt{7} 1i}{56}\right) - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)`

[Out] $\log(x + (\sqrt{7} 1i)/2 + 1/2) * ((\sqrt{7} 1i)/56 + 5/8) - \log(x - (\sqrt{7} 1i)/2 + 1/2) * ((\sqrt{7} 1i)/56 - 5/8) - \log(x)/4 - 1/(2*x)$

sympy [A] time = 0.16, size = 46, normalized size = 1.00

$$-\frac{\log(x)}{4} + \frac{5 \log(x^2 + x + 2)}{8} + \frac{\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x}{7} + \frac{\sqrt{7}}{7}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)`

[Out] $-\log(x)/4 + 5 \log(x^2 + x + 2)/8 + \sqrt{7} \operatorname{atan}(2 \sqrt{7} x / 7 + \sqrt{7} / 7) / 28 - 1 / (2 * x)$

$$3.311 \quad \int \frac{1-12x+x^2+x^3}{-12+x+x^2} dx$$

Optimal. Leaf size=22

$$\frac{x^2}{2} - \frac{2}{7} \tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1657, 616, 31}

$$\frac{x^2}{2} + \frac{1}{7} \log(3-x) - \frac{1}{7} \log(x+4)$$

Antiderivative was successfully verified.

[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[Expand Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{1 - 12x + x^2 + x^3}{-12 + x + x^2} dx &= \int \left(x + \frac{1}{-12 + x + x^2} \right) dx \\
&= \frac{x^2}{2} + \int \frac{1}{-12 + x + x^2} dx \\
&= \frac{x^2}{2} + \frac{1}{7} \int \frac{1}{-3 + x} dx - \frac{1}{7} \int \frac{1}{4 + x} dx \\
&= \frac{x^2}{2} + \frac{1}{7} \log(3 - x) - \frac{1}{7} \log(4 + x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{x^2}{2} + \frac{1}{7} \log(3 - x) - \frac{1}{7} \log(x + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 12x + x^2 + x^3}{-12 + x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

[Out] IntegrateAlgebraic[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]

fricas [A] time = 1.61, size = 18, normalized size = 0.82

$$\frac{1}{2} x^2 - \frac{1}{7} \log(x + 4) + \frac{1}{7} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12), x, algorithm="fricas")

[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{1}{2} x^2 - \frac{1}{7} \log(|x + 4|) + \frac{1}{7} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")

[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^2}{2} + \frac{\ln(x-3)}{7} - \frac{\ln(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)

[Out] 1/2*x^2-1/7*ln(x+4)+1/7*ln(x-3)

maxima [A] time = 0.97, size = 18, normalized size = 0.82

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)

mupad [B] time = 0.04, size = 14, normalized size = 0.64

$$\frac{x^2}{2} - \frac{2 \operatorname{atanh}\left(\frac{2x}{7} + \frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)

[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7

sympy [A] time = 0.10, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)

[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7

$$3.312 \quad \int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx$$

Optimal. Leaf size=17

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1594, 1628}

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-3+5x+6x^2}{-3x+2x^2+x^3} dx &= \int \frac{-3+5x+6x^2}{x(-3+2x+x^2)} dx \\ &= \int \left(\frac{2}{-1+x} + \frac{1}{x} + \frac{3}{3+x} \right) dx \\ &= 2 \log(1-x) + \log(x) + 3 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2 \log(1-x) + \log(x) + 3 \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + 5x + 6x^2}{-3x + 2x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]

fricas [A] time = 1.27, size = 15, normalized size = 0.88

$$3 \log(x + 3) + 2 \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x), x, algorithm="fricas")

[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)

giac [A] time = 0.38, size = 18, normalized size = 1.06

$$3 \log(|x + 3|) + 2 \log(|x - 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x), x, algorithm="giac")

[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) + 2 \ln(x - 1) + 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x), x)

[Out] 2*ln(x-1)+3*ln(x+3)+ln(x)

maxima [A] time = 0.85, size = 15, normalized size = 0.88

$$3 \log(x + 3) + 2 \log(x - 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x - 1) + 3 \ln(x + 3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\log(x) + 2 \log(x - 1) + 3 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

$$3.313 \quad \int \frac{-2+3x+5x^2}{2x^2+x^3} dx$$

Optimal. Leaf size=14

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 893}

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]

Rule 893

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{-2+3x+5x^2}{2x^2+x^3} dx &= \int \frac{-2+3x+5x^2}{x^2(2+x)} dx \\ &= \int \left(-\frac{1}{x^2} + \frac{2}{x} + \frac{3}{2+x} \right) dx \\ &= \frac{1}{x} + 2 \log(x) + 3 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 2 \log(x) + 3 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2 + 3x + 5x^2}{2x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]

fricas [A] time = 0.96, size = 18, normalized size = 1.29

$$\frac{3x \log(x + 2) + 2x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2), x, algorithm="fricas")

[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x

giac [A] time = 0.37, size = 16, normalized size = 1.14

$$\frac{1}{x} + 3 \log(|x + 2|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2), x, algorithm="giac")

[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2 \ln(x) + 3 \ln(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x^2+3*x-2)/(x^3+2*x^2),x)`

[Out] `1/x+2*ln(x)+3*ln(x+2)`

maxima [A] time = 1.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 3 \log(x + 2) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")`

[Out] `1/x + 3*log(x + 2) + 2*log(x)`

mupad [B] time = 2.12, size = 14, normalized size = 1.00

$$3 \ln(x + 2) + 2 \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)`

[Out] `3*log(x + 2) + 2*log(x) + 1/x`

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$2 \log(x) + 3 \log(x + 2) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)`

[Out] `2*log(x) + 3*log(x + 2) + 1/x`

$$3.314 \quad \int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx$$

Optimal. Leaf size=19

$$\log(1-x) - 2\log(x+2) - 3\log(x+3)$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2074}

$$\log(1-x) - 2\log(x+2) - 3\log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]

[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rubi steps

$$\begin{aligned} \int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} dx &= \int \left(\frac{1}{-1+x} - \frac{2}{2+x} - \frac{3}{3+x} \right) dx \\ &= \log(1-x) - 2\log(2+x) - 3\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.32

$$-2 \left(-\frac{1}{2} \log(1-x) + \log(x+2) + \frac{3}{2} \log(x+3) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]

[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{18 - 2x - 4x^2}{-6 + x + 4x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]

fricas [A] time = 1.34, size = 17, normalized size = 0.89

$$-3 \log(x + 3) - 2 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6), x, algorithm="fricas")

[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)

giac [A] time = 0.29, size = 20, normalized size = 1.05

$$-3 \log(|x + 3|) - 2 \log(|x + 2|) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6), x, algorithm="giac")

[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))

maple [A] time = 0.01, size = 18, normalized size = 0.95

$$\ln(x - 1) - 2 \ln(x + 2) - 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6), x)

[Out] ln(x-1)-2*ln(x+2)-3*ln(x+3)

maxima [A] time = 1.03, size = 17, normalized size = 0.89

$$-3 \log(x + 3) - 2 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6), x, algorithm="maxima")

[Out] $-3*\log(x + 3) - 2*\log(x + 2) + \log(x - 1)$

mupad [B] time = 2.12, size = 17, normalized size = 0.89

$$\ln(x - 1) - 2 \ln(x + 2) - 3 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6), x)$

[Out] $\log(x - 1) - 2*\log(x + 2) - 3*\log(x + 3)$

sympy [A] time = 0.13, size = 17, normalized size = 0.89

$$\log(x - 1) - 2 \log(x + 2) - 3 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-4*x**2-2*x+18)/(x**3+4*x**2+x-6), x)$

[Out] $\log(x - 1) - 2*\log(x + 2) - 3*\log(x + 3)$

$$3.315 \quad \int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 626

Int[((d_) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1247

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(
p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q
= Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
&& !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx &= \int \frac{1-2x^2}{4+5x^2+x^4} dx + \int \frac{x(1+x^2)}{4+5x^2+x^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1+x}{4+5x+x^2} dx, x, x^2 \right) - 3 \int \frac{1}{4+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{4+x} dx, x, x^2 \right) \\ &= -\frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x) + \frac{1}{2} \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 4) - \frac{3}{2} \tan^{-1} \left(\frac{x}{2} \right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]

fricas [A] time = 1.09, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4), x, algorithm="fricas")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4), x, algorithm="giac")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\arctan(x) - \frac{3 \arctan\left(\frac{x}{2}\right)}{2} + \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4), x)

[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)

maxima [A] time = 2.12, size = 17, normalized size = 0.74

$$-\frac{3}{2} \arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2} \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4), x, algorithm="maxima")

[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)

mupad [B] time = 0.05, size = 33, normalized size = 1.43

$$-\operatorname{atan}\left(\frac{1305}{4(144x-162)} + \frac{9}{8}\right) + \ln(x-2i)\left(\frac{1}{2} + \frac{3}{4}i\right) + \ln(x+2i)\left(\frac{1}{2} - \frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4), x)`

[Out] `log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x - 162)) + 9/8)`

sympy [A] time = 0.18, size = 19, normalized size = 0.83

$$\frac{\log(x^2 + 4)}{2} - \frac{3 \operatorname{atan}\left(\frac{x}{2}\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4), x)`

[Out] `log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)`

$$3.316 \quad \int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} dx$$

Optimal. Leaf size=63

$$\frac{11049 \log(x^2 + x + 5)}{260015} - \frac{3146 \log(7 - 3x)}{80155} - \frac{334}{323} \log(2x+1) + \frac{4822 \log(5x + 2)}{4879} + \frac{3988 \tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049 \log(x^2 + x + 5)}{260015} - \frac{3146 \log(7 - 3x)}{80155} - \frac{334}{323} \log(2x + 1) + \frac{4822 \log(5x + 2)}{4879} + \frac{3988 \tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5), x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x + x^2])/260015
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] &&
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-32 + 5x - 27x^2 + 4x^3}{-70 - 299x - 286x^2 + 50x^3 - 13x^4 + 30x^5} dx = \int \left(-\frac{668}{323(1+2x)} - \frac{9438}{80155(-7+3x)} + \frac{24110}{4879(2+5x)} + \frac{4}{260} \right) dx$$

$$= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} + \frac{4x}{260}$$

$$= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} + \frac{4x}{260}$$

$$= -\frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4822 \log(2+5x)}{4879} + \frac{4x}{260}$$

$$= \frac{3988 \tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146 \log(7-3x)}{80155} - \frac{334}{323} \log(1+2x) + \frac{4x}{260}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{453009 \log(x^2 + x + 5) - 418418 \log(7 - 3x) - 11023670 \log(2x + 1) + 10536070 \log(5x + 2) + 163508\sqrt{19} \tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5), x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-32 + 5x - 27x^2 + 4x^3}{-70 - 299x - 286x^2 + 50x^3 - 13x^4 + 30x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5), x]

[Out] IntegrateAlgebraic[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 + 30*x^5), x]

fricas [A] time = 0.90, size = 50, normalized size = 0.79

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19}(2x+1)\right) + \frac{11049}{260015} \log(x^2 + x + 5) + \frac{4822}{4879} \log(5x + 2) - \frac{3146}{80155} \log(3x - 7) - \frac{334}{323} \log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70), x, algorithm="fricas")

[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(2*x + 1)

giac [A] time = 0.31, size = 53, normalized size = 0.84

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19}(2x+1)\right) + \frac{11049}{260015} \log(x^2 + x + 5) + \frac{4822}{4879} \log(5x + 2) - \frac{3146}{80155} \log(3x - 7) - \frac{334}{323} \log(2x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70), x, algorithm="giac")

[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 334/323*log(abs(2*x + 1))

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{3988\sqrt{19} \arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)}{260015} - \frac{334 \ln(2x + 1)}{323} + \frac{4822 \ln(5x + 2)}{4879} - \frac{3146 \ln(3x - 7)}{80155} + \frac{11049 \ln(x^2 + x + 5)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)

[Out] 4822/4879*ln(2+5*x)-3146/80155*ln(3*x-7)-334/323*ln(2*x+1)+11049/260015*ln(x^2+x+5)+3988/260015*arctan(1/19*(2*x+1)*19^(1/2))*19^(1/2)

maxima [A] time = 2.11, size = 50, normalized size = 0.79

$$\frac{3988}{260015} \sqrt{19} \arctan\left(\frac{1}{19} \sqrt{19} (2x+1)\right) + \frac{11049}{260015} \log(x^2+x+5) + \frac{4822}{4879} \log(5x+2) - \frac{3146}{80155} \log(3x-7) - \frac{334}{323} \log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x, algorithm="maxima")

[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(2*x + 1)

mupad [B] time = 2.21, size = 58, normalized size = 0.92

$$\frac{4822 \ln\left(x + \frac{2}{5}\right)}{4879} - \frac{334 \ln\left(x + \frac{1}{2}\right)}{323} - \frac{3146 \ln\left(x - \frac{7}{3}\right)}{80155} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{19} 1i}{2}\right) \left(-\frac{11049}{260015} + \frac{\sqrt{19} 1994i}{260015}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{19} 1i}{2}\right) \left(\frac{11049}{260015} + \frac{\sqrt{19} 1994i}{260015}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^5 + 70),x)

[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/80155 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)

sympy [A] time = 0.35, size = 68, normalized size = 1.08

$$-\frac{3146 \log\left(x - \frac{7}{3}\right)}{80155} + \frac{4822 \log\left(x + \frac{2}{5}\right)}{4879} - \frac{334 \log\left(x + \frac{1}{2}\right)}{323} + \frac{11049 \log\left(x^2 + x + 5\right)}{260015} + \frac{3988 \sqrt{19} \operatorname{atan}\left(\frac{2\sqrt{19}x}{19} + \frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-70),x)

[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 + 11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(19)/19)/260015

$$3.317 \quad \int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} dx$$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452(2x^2+1)} + \frac{2843 \log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096 \log(2-5x)}{99825} + \frac{503 \tan^{-1}(\sqrt{2}x)}{7986\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452(2x^2+1)} + \frac{2843 \log(2x^2+1)}{7986} + \frac{5828}{9075(2-5x)} - \frac{59096 \log(2-5x)}{99825} + \frac{272\sqrt{2} \tan^{-1}(\sqrt{2}x)}{1331} - \frac{251 \tan^{-1}(\sqrt{2}x)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 639

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*

$a*(p + 1)), \text{Int}[(a + c*x^2)^(p + 1), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{Lt} Q[p, -1] \&\& \text{NeQ}[p, -3/2]$

Rule 2074

$\text{Int}[(P_)^(p_)*(Q_)^(q_.), x_Symbol] := \text{With}\{\{PP = \text{Factor}[P]\}, \text{Int}[\text{ExpandIntegrand}[PP^p*Q^q, x], x] /; \text{!SumQ}[\text{NonfreeFactors}[PP, x]]] /; \text{FreeQ}[q, x] \&\& \text{PolyQ}[P, x] \&\& \text{PolyQ}[Q, x] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[P, x]$

Rubi steps

$$\begin{aligned} \int \frac{8 - 13x^2 - 7x^3 + 12x^5}{4 - 20x + 41x^2 - 80x^3 + 116x^4 - 80x^5 + 100x^6} dx &= \int \left(\frac{5828}{1815(-2 + 5x)^2} - \frac{59096}{19965(-2 + 5x)} + \frac{-251 + 313x}{363(1 + 2x^2)} \right) dx \\ &= \frac{5828}{9075(2 - 5x)} - \frac{59096 \log(2 - 5x)}{99825} + \frac{2 \int \frac{816 + 2843x}{1 + 2x^2} dx}{3993} \\ &= \frac{5828}{9075(2 - 5x)} - \frac{313 + 502x}{1452(1 + 2x^2)} - \frac{59096 \log(2 - 5x)}{99825} \\ &= \frac{5828}{9075(2 - 5x)} - \frac{313 + 502x}{1452(1 + 2x^2)} - \frac{251 \tan^{-1}(\sqrt{2}x)}{726\sqrt{2}} + \dots \end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.97

$$\frac{142150 \log(2x^2 + 1) - \frac{33(36458x^2 + 4675x + 2554)}{10x^3 - 4x^2 + 5x - 2} - 236384 \log(2 - 5x) + 12575\sqrt{2} \tan^{-1}(\sqrt{2}x)}{399300}$$

Antiderivative was successfully verified.

[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*sqrt(2)*ArcTan[sqrt(2)*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8 - 13x^2 - 7x^3 + 12x^5}{4 - 20x + 41x^2 - 80x^3 + 116x^4 - 80x^5 + 100x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

[Out] IntegrateAlgebraic[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6), x]

fricas [A] time = 1.24, size = 103, normalized size = 1.49

$$\frac{12575\sqrt{2}(10x^3 - 4x^2 + 5x - 2)\arctan(\sqrt{2}x) - 1203114x^2 + 142150(10x^3 - 4x^2 + 5x - 2)\log(2x^2 + 1) - 236384(10x^3 - 4x^2 + 5x - 2)\log(5x - 2) - 154275x - 84282}{399300(10x^3 - 4x^2 + 5x - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4), x, algorithm="fricas")

[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x - 2)

giac [A] time = 0.24, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan(\sqrt{2}x) - \frac{36458x^2 + 4675x + 2554}{12100(2x^2 + 1)(5x - 2)} + \frac{2843}{7986}\log(2x^2 + 1) - \frac{59096}{99825}\log(|5x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4), x, algorithm="giac")

[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/((2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x - 2))

maple [A] time = 0.01, size = 54, normalized size = 0.78

$$\frac{503\sqrt{2}\arctan(\sqrt{2}x)}{15972} - \frac{59096\ln(5x - 2)}{99825} + \frac{2843\ln(2x^2 + 1)}{7986} - \frac{5828}{9075(5x - 2)} + \frac{-\frac{2761x}{4} - \frac{3443}{8}}{3993x^2 + \frac{3993}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4), x)

[Out] $-5828/9075/(5*x-2)-59096/99825*\ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2)+2843/7986*\ln(2*x^2+1)+503/15972*\arctan(2^{(1/2)*x})*2^{(1/2)}$

maxima [A] time = 2.07, size = 59, normalized size = 0.86

$$\frac{503}{15972} \sqrt{2} \arctan(\sqrt{2}x) - \frac{36458x^2 + 4675x + 2554}{12100(10x^3 - 4x^2 + 5x - 2)} + \frac{2843}{7986} \log(2x^2 + 1) - \frac{59096}{99825} \log(5x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x, algorithm="maxima")`

[Out] $503/15972*\sqrt{2}*\arctan(\sqrt{2}*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*\log(2*x^2 + 1) - 59096/99825*\log(5*x - 2)$

mupad [B] time = 2.18, size = 71, normalized size = 1.03

$$-\frac{59096 \ln\left(x - \frac{2}{5}\right)}{99825} - \frac{\frac{18229x^2}{60500} + \frac{17x}{440} + \frac{1277}{60500}}{x^3 - \frac{2x^2}{5} + \frac{x}{2} - \frac{1}{5}} - \ln\left(x - \frac{\sqrt{2} \operatorname{Im}(i)}{2}\right) \left(-\frac{2843}{7986} + \frac{\sqrt{2} \operatorname{Im}(503i)}{31944}\right) + \ln\left(x + \frac{\sqrt{2} \operatorname{Im}(i)}{2}\right) \left(\frac{2843}{7986} + \frac{\sqrt{2} \operatorname{Im}(503i)}{31944}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x^5 + 100*x^6 + 4),x)`

[Out] $\log(x + (2^{(1/2)}*1i)/2)*((2^{(1/2)}*503i)/31944 + 2843/7986) - ((17*x)/440 + (18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - \log(x - (2^{(1/2)}*1i)/2)*((2^{(1/2)}*503i)/31944 - 2843/7986) - (59096*\log(x - 2/5))/99825$

sympy [A] time = 0.21, size = 65, normalized size = 0.94

$$\frac{-36458x^2 - 4675x - 2554}{121000x^3 - 48400x^2 + 60500x - 24200} - \frac{59096 \log\left(x - \frac{2}{5}\right)}{99825} + \frac{2843 \log\left(x^2 + \frac{1}{2}\right)}{7986} + \frac{503\sqrt{2} \operatorname{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*x**2-20*x+4),x)`

[Out] $(-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200) - 59096*\log(x - 2/5)/99825 + 2843*\log(x**2 + 1/2)/7986 + 503*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x)/15972$

$$3.318 \quad \int \frac{9+x^4}{x^2(9+x^2)} dx$$

Optimal. Leaf size=17

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1262, 203}

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]

[Out] -x^(-1) + x - (10*ArcTan[x/3])/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1262

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \frac{9+x^4}{x^2(9+x^2)} dx &= \int \left(1 + \frac{1}{x^2} - \frac{10}{9+x^2}\right) dx \\ &= -\frac{1}{x} + x - 10 \int \frac{1}{9+x^2} dx \\ &= -\frac{1}{x} + x - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x - \frac{1}{x} - \frac{10}{3} \tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]

[Out] -x^(-1) + x - (10*ArcTan[x/3])/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9 + x^4}{x^2(9 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(9 + x^4)/(x^2*(9 + x^2)),x]

[Out] IntegrateAlgebraic[(9 + x^4)/(x^2*(9 + x^2)), x]

fricas [A] time = 1.62, size = 19, normalized size = 1.12

$$\frac{3x^2 - 10x \arctan\left(\frac{1}{3}x\right) - 3}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")

[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x

giac [A] time = 0.30, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")

[Out] x - 1/x - 10/3*arctan(1/3*x)

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$x - \frac{10 \arctan\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+9)/x^2/(x^2+9),x)

[Out] $-1/x + x - 10/3 \cdot \arctan(1/3 \cdot x)$

maxima [A] time = 2.04, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3} \arctan\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")`

[Out] $x - 1/x - 10/3 \cdot \arctan(1/3 \cdot x)$

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$x - \frac{10 \operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4 + 9)/(x^2*(x^2 + 9)),x)`

[Out] $x - (10 \cdot \operatorname{atan}(x/3))/3 - 1/x$

sympy [A] time = 0.11, size = 12, normalized size = 0.71

$$x - \frac{10 \operatorname{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**4+9)/x**2/(x**2+9),x)`

[Out] $x - 10 \cdot \operatorname{atan}(x/3)/3 - 1/x$

$$3.319 \quad \int \frac{2x+x^4}{1+x^2} dx$$

Optimal. Leaf size=19

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^4)/(1 + x^2), x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(-n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^(-n), x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]

&& PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{2x + x^4}{1 + x^2} dx &= \int \frac{x(2 + x^3)}{1 + x^2} dx \\
 &= \int \left(-1 + x^2 + \frac{1 + 2x}{1 + x^2} \right) dx \\
 &= -x + \frac{x^3}{3} + \int \frac{1 + 2x}{1 + x^2} dx \\
 &= -x + \frac{x^3}{3} + 2 \int \frac{x}{1 + x^2} dx + \int \frac{1}{1 + x^2} dx \\
 &= -x + \frac{x^3}{3} + \tan^{-1}(x) + \log(1 + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^3}{3} + \log(x^2 + 1) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^4)/(1 + x^2), x]

[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + x^4}{1 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*x + x^4)/(1 + x^2), x]

[Out] IntegrateAlgebraic[(2*x + x^4)/(1 + x^2), x]

fricas [A] time = 0.81, size = 17, normalized size = 0.89

$$\frac{1}{3} x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

giac [A] time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^3}{3} - x + \arctan(x) + \ln(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+2*x)/(x^2+1),x)

[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)

maxima [A] time = 2.03, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")

[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)

mupad [B] time = 2.10, size = 17, normalized size = 0.89

$$\ln(x^2 + 1) - x + \operatorname{atan}(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^4)/(x^2 + 1),x)

[Out] log(x^2 + 1) - x + atan(x) + x^3/3

sympy [A] time = 0.10, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log(x^2 + 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+2*x)/(x**2+1),x)

[Out] x**3/3 - x + log(x**2 + 1) + atan(x)

$$3.320 \quad \int \frac{-x+x^3}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=9

$$\log(1-x) + \tan^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1586, 1593, 1629, 203}

$$\log(1-x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

[Out] ArcTan[x] + Log[1 - x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-x + x^3}{(-1 + x)^2 (1 + x^2)} dx &= \int \frac{x + x^2}{(-1 + x)(1 + x^2)} dx \\
&= \int \frac{x(1 + x)}{(-1 + x)(1 + x^2)} dx \\
&= \int \left(\frac{1}{-1 + x} + \frac{1}{1 + x^2} \right) dx \\
&= \log(1 - x) + \int \frac{1}{1 + x^2} dx \\
&= \tan^{-1}(x) + \log(1 - x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

[Out] ArcTan[x] + Log[1 - x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x + x^3}{(-1 + x)^2 (1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]

fricas [A] time = 0.97, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1), x, algorithm="fricas")

[Out] arctan(x) + log(x - 1)

giac [B] time = 0.29, size = 28, normalized size = 3.11

$$\frac{1}{4}\pi - \pi \left\lfloor \frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right\rfloor + \arctan(x) + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")

[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x - 1))

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\arctan(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-x)/(x-1)^2/(x^2+1),x)

[Out] ln(x-1)+arctan(x)

maxima [A] time = 2.17, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] arctan(x) + log(x - 1)

mupad [B] time = 2.10, size = 19, normalized size = 2.11

$$\ln(x - 1) - \operatorname{atan}\left(\frac{5}{4x + 2} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)

[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)

sympy [A] time = 0.14, size = 7, normalized size = 0.78

$$\log(x - 1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)

[Out] log(x - 1) + atan(x)

$$3.321 \quad \int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx$$

Optimal. Leaf size=12

$$x^2 + \log(x^2 + x + 1) + x$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1657, 628}

$$x^2 + \log(x^2 + x + 1) + x$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]

[Out] x + x^2 + Log[1 + x + x^2]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq,
x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{2+5x+3x^2+2x^3}{1+x+x^2} dx &= \int \left(1 + 2x + \frac{1+2x}{1+x+x^2} \right) dx \\ &= x + x^2 + \int \frac{1+2x}{1+x+x^2} dx \\ &= x + x^2 + \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x^2 + \log(x^2 + x + 1) + x$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]

[Out] x + x^2 + Log[1 + x + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 5x + 3x^2 + 2x^3}{1 + x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]

[Out] IntegrateAlgebraic[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]

fricas [A] time = 1.06, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1), x, algorithm="fricas")

[Out] x^2 + x + log(x^2 + x + 1)

giac [A] time = 0.29, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1), x, algorithm="giac")

[Out] x^2 + x + log(x^2 + x + 1)

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x^2 + x + \ln(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1), x)

[Out] x+x^2+ln(x^2+x+1)

maxima [A] time = 1.37, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")

[Out] x^2 + x + log(x^2 + x + 1)

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$x + \ln(x^2 + x + 1) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)

[Out] x + log(x + x^2 + 1) + x^2

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$x^2 + x + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)

[Out] x**2 + x + log(x**2 + x + 1)

$$3.322 \quad \int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx$$

Optimal. Leaf size=65

$$\frac{3}{2x^2} - \frac{1}{x} + 3 \log(x) - \frac{1}{10} (15 - \sqrt{5}) \log(2x - \sqrt{5} + 1) - \frac{1}{10} (15 + \sqrt{5}) \log(2x + \sqrt{5} + 1)$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1628, 632, 31}

$$\frac{3}{2x^2} - \frac{1}{x} + 3 \log(x) - \frac{1}{10} (15 - \sqrt{5}) \log(2x - \sqrt{5} + 1) - \frac{1}{10} (15 + \sqrt{5}) \log(2x + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]

[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 - ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx &= \int \left(-\frac{3}{x^3} + \frac{1}{x^2} + \frac{3}{x} + \frac{-1-3x}{-1+x+x^2} \right) dx \\
&= \frac{3}{2x^2} - \frac{1}{x} + 3 \log(x) + \int \frac{-1-3x}{-1+x+x^2} dx \\
&= \frac{3}{2x^2} - \frac{1}{x} + 3 \log(x) + \frac{1}{10} (-15 + \sqrt{5}) \int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx - \frac{1}{10} (15 + \sqrt{5}) \int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx \\
&= \frac{3}{2x^2} - \frac{1}{x} + 3 \log(x) - \frac{1}{10} (15 - \sqrt{5}) \log(1 - \sqrt{5} + 2x) - \frac{1}{10} (15 + \sqrt{5}) \log(1 + \sqrt{5} + 2x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{10} \left(\frac{15}{x^2} - \frac{10}{x} + (\sqrt{5} - 15) \log(-2x + \sqrt{5} - 1) + 30 \log(x) - (15 + \sqrt{5}) \log(2x + \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]

[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-4x-5x^2+3x^3}{x^3(-1+x+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]

[Out] IntegrateAlgebraic[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]

fricas [A] time = 1.51, size = 66, normalized size = 1.02

$$\frac{\sqrt{5} x^2 \log\left(\frac{2x^2 - \sqrt{5}(2x+1) + 2x+3}{x^2+x-1}\right) - 15 x^2 \log(x^2 + x - 1) + 30 x^2 \log(x) - 10 x + 15}{10 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1), x, algorithm="fricas")

[Out] $\frac{1}{10} \sqrt{5} \log\left(\frac{(2x^2 - \sqrt{5})(2x + 1) + 2x + 3}{x^2 + x - 1}\right) - 15x^2 \log(x^2 + x - 1) + 30x^2 \log(x) - 10x + 15)/x^2$

giac [A] time = 0.30, size = 55, normalized size = 0.85

$$\frac{1}{10} \sqrt{5} \log\left(\frac{|2x - \sqrt{5} + 1|}{|2x + \sqrt{5} + 1|}\right) - \frac{2x - 3}{2x^2} - \frac{3}{2} \log(|x^2 + x - 1|) + 3 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")`

[Out] $\frac{1}{10} \sqrt{5} \log(\text{abs}(2x - \sqrt{5} + 1)/\text{abs}(2x + \sqrt{5} + 1)) - 1/2*(2x - 3)/x^2 - 3/2*\log(\text{abs}(x^2 + x - 1)) + 3*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 41, normalized size = 0.63

$$-\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5} + 3 \ln(x) - \frac{3 \ln(x^2 + x - 1)}{2} - \frac{1}{x} + \frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)`

[Out] $-3/2*\ln(x^2+x-1)-1/5*5^{(1/2)}*\operatorname{arctanh}(1/5*(2*x+1)*5^{(1/2)})-1/x+3/2/x^2+3*\ln(x)$

maxima [A] time = 1.77, size = 51, normalized size = 0.78

$$\frac{1}{10} \sqrt{5} \log\left(\frac{2x - \sqrt{5} + 1}{2x + \sqrt{5} + 1}\right) - \frac{2x - 3}{2x^2} - \frac{3}{2} \log(x^2 + x - 1) + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")`

[Out] $\frac{1}{10} \sqrt{5} \log((2x - \sqrt{5} + 1)/(2x + \sqrt{5} + 1)) - 1/2*(2x - 3)/x^2 - 3/2*\log(x^2 + x - 1) + 3*\log(x)$

mupad [B] time = 0.10, size = 48, normalized size = 0.74

$$3 \ln(x) - \frac{x - \frac{3}{2}}{x^2} + \ln\left(x - \frac{\sqrt{5}}{2} + \frac{1}{2}\right) \left(\frac{\sqrt{5}}{10} - \frac{3}{2}\right) - \ln\left(x + \frac{\sqrt{5}}{2} + \frac{1}{2}\right) \left(\frac{\sqrt{5}}{10} + \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)`

[Out] $3\log(x) - (x - 3/2)/x^2 + \log(x - 5^{1/2}/2 + 1/2)*(5^{1/2}/10 - 3/2) - \log(x + 5^{1/2}/2 + 1/2)*(5^{1/2}/10 + 3/2)$

sympy [A] time = 0.45, size = 99, normalized size = 1.52

$$3\log(x) + \left(-\frac{3}{2} + \frac{\sqrt{5}}{10}\right)\log\left(x - \frac{405}{202} - \frac{35\sqrt{5}}{202} + \frac{110\left(-\frac{3}{2} + \frac{\sqrt{5}}{10}\right)^2}{101}\right) + \left(-\frac{3}{2} - \frac{\sqrt{5}}{10}\right)\log\left(x - \frac{405}{202} + \frac{35\sqrt{5}}{202} + \frac{110\left(-\frac{3}{2} - \frac{\sqrt{5}}{10}\right)^2}{101}\right) + \frac{3-2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)`

[Out] $3\log(x) + (-3/2 + \sqrt{5}/10)*\log(x - 405/202 - 35*\sqrt{5}/202 + 110*(-3/2 + \sqrt{5}/10)**2/101) + (-3/2 - \sqrt{5}/10)*\log(x - 405/202 + 35*\sqrt{5}/202 + 110*(-3/2 - \sqrt{5}/10)**2/101) + (3 - 2*x)/(2*x**2)$

$$3.323 \quad \int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2} dx$$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2} + \log(x^2+2x+2) - \tan^{-1}(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1660, 634, 617, 204, 628}

$$-\frac{1}{x^2+2x+2} + \log(x^2+2x+2) - \tan^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{4 + 8x + 5x^2 + 2x^3}{(2 + 2x + x^2)^2} dx &= -\frac{1}{2 + 2x + x^2} + \frac{1}{4} \int \frac{4 + 8x}{2 + 2x + x^2} dx \\ &= -\frac{1}{2 + 2x + x^2} - \int \frac{1}{2 + 2x + x^2} dx + \int \frac{2 + 2x}{2 + 2x + x^2} dx \\ &= -\frac{1}{2 + 2x + x^2} + \log(2 + 2x + x^2) + \text{Subst}\left(\int \frac{1}{-1 - x^2} dx, x, 1 + x\right) \\ &= -\frac{1}{2 + 2x + x^2} - \tan^{-1}(1 + x) + \log(2 + 2x + x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} + \log(x^2 + 2x + 2) - \tan^{-1}(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + 8x + 5x^2 + 2x^3}{(2 + 2x + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]

[Out] IntegrateAlgebraic[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2, x]

fricas [A] time = 0.99, size = 46, normalized size = 1.64

$$\frac{(x^2 + 2x + 2) \arctan(x + 1) - (x^2 + 2x + 2) \log(x^2 + 2x + 2) + 1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")

[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(x^2 + 2*x + 2)

giac [A] time = 0.28, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \log(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")

[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\arctan(x + 1) + \ln(x^2 + 2x + 2) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)

[Out] -1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)

maxima [A] time = 1.66, size = 28, normalized size = 1.00

$$-\frac{1}{x^2 + 2x + 2} - \arctan(x + 1) + \log(x^2 + 2x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")

[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)

mupad [B] time = 0.04, size = 28, normalized size = 1.00

$$\ln(x^2 + 2x + 2) - \operatorname{atan}(x + 1) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2, x)`

[Out] `log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)`

sympy [A] time = 0.13, size = 24, normalized size = 0.86

$$\log(x^2 + 2x + 2) - \operatorname{atan}(x + 1) - \frac{1}{x^2 + 2x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2, x)`

[Out] `log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)`

$$3.324 \quad \int \frac{(-1+x)^4 x^4}{1+x^2} dx$$

Optimal. Leaf size=32

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1629, 203}

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[((-1 + x)^4*x^4)/(1 + x^2), x]

[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :=> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{(-1+x)^4 x^4}{1+x^2} dx &= \int \left(4 - 4x^2 + 5x^4 - 4x^5 + x^6 - \frac{4}{1+x^2} \right) dx \\ &= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4 \int \frac{1}{1+x^2} dx \\ &= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4 \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2), x]

[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-1 + x)^4 x^4}{1 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((-1 + x)^4*x^4)/(1 + x^2), x]

[Out] IntegrateAlgebraic[((-1 + x)^4*x^4)/(1 + x^2), x]

fricas [A] time = 0.61, size = 26, normalized size = 0.81

$$\frac{1}{7} x^7 - \frac{2}{3} x^6 + x^5 - \frac{4}{3} x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^4*x^4/(x^2+1), x, algorithm="fricas")

[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)

giac [A] time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{7} x^7 - \frac{2}{3} x^6 + x^5 - \frac{4}{3} x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^4*x^4/(x^2+1), x, algorithm="giac")

[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x-1)^4*x^4/(x^2+1),x)`

[Out] `4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)`

maxima [A] time = 1.51, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4 \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")`

[Out] `1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)`

mupad [B] time = 0.02, size = 26, normalized size = 0.81

$$4x - 4 \operatorname{atan}(x) - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(x-1)^4)/(x^2+1),x)`

[Out] `4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7`

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)**4*x**4/(x**2+1),x)`

[Out] `x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)`

$$3.325 \quad \int \frac{-20x+4x^2}{9-10x^2+x^4} dx$$

Optimal. Leaf size=31

$$\log(1-x) - \frac{1}{2} \log(3-x) + \frac{3}{2} \log(x+1) - 2 \log(x+3)$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1593, 1662, 12, 1107, 616, 31, 1130, 207}

$$\frac{5}{4} \log(1-x^2) - \frac{5}{4} \log(9-x^2) - \frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]

[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1130

Int[((d_)*(x_))^(m_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1662

Int[(Pq_)*((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2 + c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]

Rubi steps

$$\begin{aligned}
 \int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} dx &= \int \frac{x(-20 + 4x)}{9 - 10x^2 + x^4} dx \\
 &= \int -\frac{20x}{9 - 10x^2 + x^4} dx + \int \frac{4x^2}{9 - 10x^2 + x^4} dx \\
 &= 4 \int \frac{x^2}{9 - 10x^2 + x^4} dx - 20 \int \frac{x}{9 - 10x^2 + x^4} dx \\
 &= -\left(\frac{1}{2} \int \frac{1}{-1 + x^2} dx\right) + \frac{9}{2} \int \frac{1}{-9 + x^2} dx - 10 \operatorname{Subst}\left(\int \frac{1}{9 - 10x + x^2} dx, x, x^2\right) \\
 &= -\frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x) - \frac{5}{4} \operatorname{Subst}\left(\int \frac{1}{-9 + x} dx, x, x^2\right) + \frac{5}{4} \operatorname{Subst}\left(\int \frac{1}{-1 + x} dx, x, x^2\right) \\
 &= -\frac{3}{2} \tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2} \tanh^{-1}(x) + \frac{5}{4} \log(1 - x^2) - \frac{5}{4} \log(9 - x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x) - \frac{1}{8}\log(3-x) + \frac{3}{8}\log(x+1) - \frac{1}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]

[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-20x + 4x^2}{9 - 10x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]

[Out] IntegrateAlgebraic[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]

fricas [A] time = 1.40, size = 23, normalized size = 0.74

$$-2 \log(x+3) + \frac{3}{2} \log(x+1) + \log(x-1) - \frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9), x, algorithm="fricas")

[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)

giac [A] time = 0.31, size = 27, normalized size = 0.87

$$-2 \log(|x+3|) + \frac{3}{2} \log(|x+1|) + \log(|x-1|) - \frac{1}{2} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9), x, algorithm="giac")

[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x - 3))

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$-\frac{\ln(x-3)}{2} + \ln(x-1) + \frac{3\ln(x+1)}{2} - 2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2-20*x)/(x^4-10*x^2+9),x)`

[Out] `ln(x-1)+3/2*ln(x+1)-2*ln(x+3)-1/2*ln(x-3)`

maxima [A] time = 0.80, size = 23, normalized size = 0.74

$$-2 \log(x+3) + \frac{3}{2} \log(x+1) + \log(x-1) - \frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")`

[Out] `-2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)`

mupad [B] time = 0.05, size = 23, normalized size = 0.74

$$\ln(x-1) + \frac{3 \ln(x+1)}{2} - \frac{\ln(x-3)}{2} - 2 \ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)`

[Out] `log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)`

sympy [A] time = 0.19, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2} + \log(x-1) + \frac{3 \log(x+1)}{2} - 2 \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)`

[Out] `-log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)`

$$3.326 \quad \int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx$$

Optimal. Leaf size=24

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

Rubi [A] time = 0.17, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6725, 635, 203, 260}

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx &= \int \left(\frac{2}{-1+x} + \frac{1}{x^2} + \frac{1-2x}{1+x^2} \right) dx \\
&= -\frac{1}{x} + 2 \log(1-x) + \int \frac{1-2x}{1+x^2} dx \\
&= -\frac{1}{x} + 2 \log(1-x) - 2 \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\
&= -\frac{1}{x} + \tan^{-1}(x) + 2 \log(1-x) - \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\log(x^2 + 1) - \frac{1}{x} + 2 \log(1 - x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]

[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]

fricas [A] time = 1.27, size = 26, normalized size = 1.08

$$\frac{x \arctan(x) - x \log(x^2 + 1) + 2x \log(x - 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1), x, algorithm="fricas")

[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x

giac [A] time = 0.29, size = 23, normalized size = 0.96

$$-\frac{1}{x} + \arctan(x) - \log(x^2 + 1) + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")

[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$\arctan(x) + 2 \ln(x - 1) - \ln(x^2 + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x)

[Out] 2*ln(x-1)-1/x-ln(x^2+1)+arctan(x)

maxima [A] time = 1.22, size = 22, normalized size = 0.92

$$-\frac{1}{x} + \arctan(x) - \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")

[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)

mupad [B] time = 2.13, size = 30, normalized size = 1.25

$$2 \ln(x - 1) - \frac{1}{x} + \ln(x - i) \left(-1 - \frac{1}{2}i\right) + \ln(x + i) \left(-1 + \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)

[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x

sympy [A] time = 0.15, size = 19, normalized size = 0.79

$$2 \log(x - 1) - \log(x^2 + 1) + \operatorname{atan}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
```

```
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

$$3.327 \quad \int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3} dx$$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1814, 12, 203}

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3, x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1814

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{1 - 3x + 2x^2 - 4x^3 + x^4}{(1 + x^2)^3} dx &= -\frac{1}{4(1 + x^2)^2} - \frac{1}{4} \int \frac{-4 + 16x - 4x^2}{(1 + x^2)^2} dx \\
&= -\frac{1}{4(1 + x^2)^2} + \frac{2}{1 + x^2} + \frac{1}{8} \int \frac{8}{1 + x^2} dx \\
&= -\frac{1}{4(1 + x^2)^2} + \frac{2}{1 + x^2} + \int \frac{1}{1 + x^2} dx \\
&= -\frac{1}{4(1 + x^2)^2} + \frac{2}{1 + x^2} + \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2 + 1} - \frac{1}{4(x^2 + 1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 3x + 2x^2 - 4x^3 + x^4}{(1 + x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]

[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3, x]

fricas [A] time = 1.10, size = 35, normalized size = 1.52

$$\frac{8x^2 + 4(x^4 + 2x^2 + 1) \arctan(x) + 7}{4(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")

[Out] $1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*\arctan(x) + 7)/(x^4 + 2*x^2 + 1)$

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{8x^2 + 7}{4(x^2 + 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")`

[Out] $1/4*(8*x^2 + 7)/(x^2 + 1)^2 + \arctan(x)$

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$\arctan(x) + \frac{2x^2 + \frac{7}{4}}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)`

[Out] $(2*x^2+7/4)/(x^2+1)^2+\arctan(x)$

maxima [A] time = 1.43, size = 24, normalized size = 1.04

$$\frac{8x^2 + 7}{4(x^4 + 2x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")`

[Out] $1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + \arctan(x)$

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2x^2 + \frac{7}{4}}{x^4 + 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)`

[Out] $\operatorname{atan}(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)$

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)

[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)

$$3.328 \quad \int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} dx$$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2073, 261, 203}

$$\frac{2}{x^2+1} - \frac{1}{4(x^2+1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6), x]

[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 2073

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P /. x -> Sqrt[x]]}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1 - 3x + 2x^2 - 4x^3 + x^4}{1 + 3x^2 + 3x^4 + x^6} dx &= \int \left(\frac{x}{(1+x^2)^3} - \frac{4x}{(1+x^2)^2} + \frac{1}{1+x^2} \right) dx \\
&= - \left(4 \int \frac{x}{(1+x^2)^2} dx \right) + \int \frac{x}{(1+x^2)^3} dx + \int \frac{1}{1+x^2} dx \\
&= -\frac{1}{4(1+x^2)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2 + 1} - \frac{1}{4(x^2 + 1)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6), x]

[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 3x + 2x^2 - 4x^3 + x^4}{1 + 3x^2 + 3x^4 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6), x]

[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6), x]

fricas [A] time = 1.88, size = 35, normalized size = 1.52

$$\frac{8x^2 + 4(x^4 + 2x^2 + 1) \arctan(x) + 7}{4(x^4 + 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1), x, algorithm="fricas")

[Out] $1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*\arctan(x) + 7)/(x^4 + 2*x^2 + 1)$

giac [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{8x^2 + 7}{4(x^2 + 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")`

[Out] $1/4*(8*x^2 + 7)/(x^2 + 1)^2 + \arctan(x)$

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\arctan(x) + \frac{2x^2 + \frac{7}{4}}{(x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)`

[Out] $\arctan(x) + (2*x^2 + 7/4)/(x^2 + 1)^2$

maxima [A] time = 1.42, size = 24, normalized size = 1.04

$$\frac{8x^2 + 7}{4(x^4 + 2x^2 + 1)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima")`

[Out] $1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + \arctan(x)$

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{2x^2 + \frac{7}{4}}{x^4 + 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)`

[Out] $\operatorname{atan}(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)$

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)

[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)

$$3.329 \quad \int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$$

Optimal. Leaf size=13

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1594, 1628, 628}

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^(m*Pq)*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx &= \int \frac{1+x+2x^2+2x^3}{x^2(1+x+x^2)} dx \\
&= \int \left(\frac{1}{x^2} + \frac{1+2x}{1+x+x^2} \right) dx \\
&= -\frac{1}{x} + \int \frac{1+2x}{1+x+x^2} dx \\
&= -\frac{1}{x} + \log(1+x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]

[Out] -x^(-1) + Log[1 + x + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]

[Out] IntegrateAlgebraic[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]

fricas [A] time = 2.06, size = 15, normalized size = 1.15

$$\frac{x \log(x^2 + x + 1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2), x, algorithm="fricas")

[Out] (x*log(x^2 + x + 1) - 1)/x

giac [A] time = 0.28, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")

[Out] -1/x + log(x^2 + x + 1)

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\ln(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x)

[Out] -1/x+ln(x^2+x+1)

maxima [A] time = 0.71, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log(x^2 + x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")

[Out] -1/x + log(x^2 + x + 1)

mupad [B] time = 2.14, size = 13, normalized size = 1.00

$$\ln(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)

[Out] log(x + x^2 + 1) - 1/x

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log(x^2 + x + 1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
```

```
[Out] log(x**2 + x + 1) - 1/x
```

$$3.330 \quad \int \frac{x^2(c+dx)^2}{a+bx^3} dx$$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}} + \frac{\sqrt[3]{a}d(\sqrt[3]{a}d + 2\sqrt[3]{b}c)}{\sqrt{3}b^2}$$

Rubi [A] time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{5/3}} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c - \sqrt[3]{a}d)\log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{5/3}} + \frac{\sqrt[3]{a}d(\sqrt[3]{a}d + 2\sqrt[3]{b}c)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} + \frac{c^2\log(a+bx^3)}{3b} + \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3), x]

[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(5/3)) + (a^(1/3)*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b


```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(c+dx)^2}{a+bx^3} dx &= \int \left(\frac{2cd}{b} + \frac{d^2x}{b} - \frac{2acd+ad^2x-bc^2x^2}{b(a+bx^3)} \right) dx \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3} dx}{b} \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\int \frac{2acd+ad^2x}{a+bx^3} dx}{b} + c^2 \int \frac{x^2}{a+bx^3} dx \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{c^2 \log(a+bx^3)}{3b} - \frac{\int \frac{\sqrt[3]{a}(4a\sqrt[3]{b}cd+a^{4/3}d^2)+\sqrt[3]{b}(-2a\sqrt[3]{b}cd+a^{4/3}d^2)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{3a^{2/3}b^{4/3}} - \frac{(\sqrt[3]{a}d(2c-))}{\dots} \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c-\sqrt[3]{a}d)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{5/3}} + \frac{c^2 \log(a+bx^3)}{3b} + \frac{(\sqrt[3]{a}d(2\sqrt[3]{b}c-))}{\dots} \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c-\sqrt[3]{a}d)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{5/3}} + \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c-\sqrt[3]{a}d)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x)}{6b^{5/3}} \\
&= \frac{2cdx}{b} + \frac{d^2x^2}{2b} + \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c+\sqrt[3]{a}d)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}} - \frac{\sqrt[3]{a}d(2\sqrt[3]{b}c-\sqrt[3]{a}d)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{3b^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 193, normalized size = 0.94

$$\frac{-\sqrt[3]{a}d(\sqrt[3]{a}d-2\sqrt[3]{b}c)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)+2b^{2/3}c^2\log(a+bx^3)+2\sqrt[3]{a}d(\sqrt[3]{a}d-2\sqrt[3]{b}c)\log(\sqrt[3]{a}+\sqrt[3]{b}x)+2\sqrt{3}\sqrt[3]{a}d(\sqrt[3]{a}d+2\sqrt[3]{b}c)\tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)+12b^{2/3}cdx+3b^{2/3}d^2x^2}{6b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3), x]

[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 2*a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3]/(6*b^(5/3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx)^2}{a+bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(c + d*x)^2)/(a + b*x^3), x]

[Out] IntegrateAlgebraic[(x^2*(c + d*x)^2)/(a + b*x^3), x]

fricas [C] time = 5.14, size = 4545, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a), x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)/b^3) \\ & + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)* \\ & c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b*\log(1/4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)* \\ & c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)^2*b \\ & ^3 + 3*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b + 6*c^2 + 3*\sqrt{1/3}*b*\sqrt{-((2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*\log(-1/4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*\log(-1/4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*\sqrt{3} + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))$$

$$\begin{aligned}
& b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3) \\
&)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2 \\
& 2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) + 1) - 2c^2/b)^2*b^3 - 3*(2*(1/2)^{(2/3)}*(c^4/ \\
& b^2 - (b^4c^4 + 2a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3) \\
&)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + \\
& a^2d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 \\
& - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} \\
& (1/3)*(I*\text{sqrt}(3) + 1) - 2c^2/b)*b^2c^2 - 5b^3c^4 - 4a^2c^3d^3 + 2*(8b^3c^3 \\
& *d + a^2d^4)*x + 3/4*\text{sqrt}(1/3)*((2*(1/2)^{(2/3)}*(c^4/b^2 - (b^4c^4 + 2a^2c^3d^3) \\
&)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 \\
& + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} + (1 \\
& /2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)* \\
& c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) + 1) - \\
& 2c^2/b)*b^3 - 6b^2c^2)*\text{sqrt}(-((2*(1/2)^{(2/3)}*(c^4/b^2 - (b^4c^4 + 2a^2c^3d^3) \\
& ^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 \\
& + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} + \\
& (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3) \\
&)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) + 1) \\
& - 2c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b^4c^4 + 2a^2c^3d^3)/b^3)*(-I \\
& *\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3) \\
& ^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}* \\
& (2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + \\
& (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) + 1) - 2c^2/b)*b \\
& ^2c^2 + 4b^3c^4 + 32a^2c^3d^3)/b^3)) + ((2*(1/2)^{(2/3)}*(c^4/b^2 - (b^4c^4 + \\
& 2a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - \\
& 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} \\
& + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3) \\
&)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) \\
&) + 1) - 2c^2/b)*b + 6c^2 - 3*\text{sqrt}(1/3)*b*\text{sqrt}(-((2*(1/2)^{(2/3)}*(c^4/b^2 \\
& - (b^4c^4 + 2a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)* \\
& a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2* \\
& d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3* \\
& (b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)} \\
&)*(I*\text{sqrt}(3) + 1) - 2c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b^4c^4 + 2 \\
& a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - \\
& 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1 \\
& /3)} + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2 \\
& c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) \\
& + 1) - 2c^2/b)*b^2c^2 + 4b^3c^4 + 32a^2c^3d^3)/b^3))*\text{log}(-1/4*(2*(1/2)^{(2 \\
& /3)}*(c^4/b^2 - (b^4c^4 + 2a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3 \\
& c^3 + a^2d^3)a^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3 \\
& ^3d^3 + a^2d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2c^6/b^3 + (8b^3c^3 + a^2d^3)a \\
& ^2d^3/b^5 - 3*(b^4c^4 + 2a^2c^3d^3)*c^2/b^4 + (b^2c^6 - 2a^2b^3c^3d^3 + a^2d \\
& ^6)/b^5)^{(1/3)}*(I*\text{sqrt}(3) + 1) - 2c^2/b)^2*b^3 - 3*(2*(1/2)^{(2/3)}*(c^4/b^2 \\
& - (b^4c^4 + 2a^2c^3d^3)/b^3)*(-I*\text{sqrt}(3) + 1)/(2c^6/b^3 + (8b^3c^3 + a^2d^3)
\end{aligned}$$

$a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/b*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3*d + a*d^4)*x - 3/4*sqrt(1/3)*((2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^{(2/3)}*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/b)*b^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))))/b$

giac [A] time = 0.44, size = 208, normalized size = 1.01

$$\frac{c^2 \log(|bx^3 + a|)}{3b} - \frac{\sqrt{3} \left(2(-ab^2)^{\frac{1}{3}} bcd - (-ab^2)^{\frac{2}{3}} d^2 \right) \arctan \left(\frac{\sqrt{3} \left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{3b^3} + \frac{bd^2x^2 + 4bcdx}{2b^2} - \frac{\left(2(-ab^2)^{\frac{1}{3}} bcd + (-ab^2)^{\frac{2}{3}} d^2 \right) \log \left(x^2 + x \left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}} \right)}{6b^3} + \frac{\left(ab^4d^2 \left(\frac{a}{b}\right)^{\frac{1}{3}} + 2ab^4cd \right) \left(\frac{a}{b}\right)^{\frac{1}{3}} \log \left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")

[Out] $1/3*c^2*\log(\text{abs}(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^{(1/3)}*b*c*d - (-a*b^2)^{(2/3)}*d^2)*\arctan(1/3*sqrt(3)*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/b^3 + 1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^{(1/3)}*b*c*d + (-a*b^2)^{(2/3)}*d^2)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/b^3 + 1/3*(a*b^4*d^2*(-a/b)^{(1/3)} + 2*a*b^4*c*d)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b^5)$

maple [A] time = 0.01, size = 236, normalized size = 1.15

$$\frac{d^2x^2}{2b} - \frac{2\sqrt{3}acd \arctan \left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}} - 1} \right)}{3} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{2acd \ln \left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} + \frac{acd \ln \left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^2} - \frac{\sqrt{3} a d^2 \arctan \left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}} - 1} \right)}{3} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{a d^2 \ln \left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} - \frac{a d^2 \ln \left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}} \right)}{6 \left(\frac{a}{b}\right)^{\frac{1}{3}} b^2} + \frac{c^2 \ln(bx^3 + a)}{3b} + \frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d*x+c)^2/(b*x^3+a),x)

[Out] $\frac{1}{2}d^2x^2/b + 2c*d*x/b - 2/3/b^2*a*c*d/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)}) + 1/3/b^2*a*c*d/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - 2/3/b^2*a*c*d/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) + 1/3/b^2*a*d^2/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)}) - 1/6/b^2*a*d^2/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)}) - 1/3/b^2*a*d^2*3^{(1/2)}/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1)) + 1/3*c^2*\ln(b*x^3+a)/b$

maxima [A] time = 1.65, size = 199, normalized size = 0.97

$$\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + 2acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab} + \frac{d^2x^2 + 4cdx}{2b} + \frac{\left(2bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} + 2acd\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}} + ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2acd\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")

[Out] $-1/3*\sqrt{3}*(a*d^2*(a/b)^{(2/3)} + 2*a*c*d*(a/b)^{(1/3)})*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*b*c^2*(a/b)^{(2/3)} - a*d^2*(a/b)^{(1/3)} + 2*a*c*d)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*(a/b)^{(2/3)}) + 1/3*(b*c^2*(a/b)^{(2/3)} + a*d^2*(a/b)^{(1/3)} - 2*a*c*d)*\log(x + (a/b)^{(1/3)})/(b^2*(a/b)^{(2/3)})$

mupad [B] time = 0.23, size = 357, normalized size = 1.73

$$\sum_{k=0}^{\infty} \left(\frac{(b^k + \text{root}(27b^{5k}z^3 - 27b^4c^2z^2 + 18ab^2cd^3z + 9b^3c^4z + 2ab^3c^4z + 2a^2b^3c^4z - b^2c^6 - a^2d^6, z, k))^2 * (d^2x^2 + 4cdx)}{b^{2k} * \text{root}(27b^{5k}z^3 - 27b^4c^2z^2 + 18ab^2cd^3z + 9b^3c^4z + 2ab^3c^4z + 2a^2b^3c^4z - b^2c^6 - a^2d^6, z, k)} \right) + \frac{d^2x^2 + 4cdx}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(c + d*x)^2)/(a + b*x^3),x)

[Out] $\text{symsum}(\log((a*(b*c^4 + 9*\text{root}(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k))^2*b^3 - 6*\text{root}(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k))*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6*\text{root}(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k))*b^2*c*d*x))/b*\text{root}(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b$

sympy [A] time = 1.24, size = 138, normalized size = 0.67

$$\text{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t(18ab^2cd^3 + 9b^3c^4) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t \log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4acd^3 + 5bc^4}{ad^4 + 8bc^3d}\right)\right)\right) + \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))) + 2*c*d*x/b + d**2*x**2/(2*b))
```

$$3.331 \quad \int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2} dx$$

Optimal. Leaf size=45

$$\frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5-7x^2}{8(x^4+2x^2+3)}$$

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1594, 1663, 1660, 12, 618, 204}

$$\frac{5-7x^2}{8(x^4+2x^2+3)} + \frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]

[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a

, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^(
p, x), x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
 \int \frac{-x + 2x^3 + 4x^5}{(3 + 2x^2 + x^4)^2} dx &= \int \frac{x(-1 + 2x^2 + 4x^4)}{(3 + 2x^2 + x^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{-1 + 2x + 4x^2}{(3 + 2x + x^2)^2} dx, x, x^2 \right) \\
 &= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{1}{16} \text{Subst} \left(\int \frac{18}{3 + 2x + x^2} dx, x, x^2 \right) \\
 &= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{9}{8} \text{Subst} \left(\int \frac{1}{3 + 2x + x^2} dx, x, x^2 \right) \\
 &= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} - \frac{9}{4} \text{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2(1 + x^2) \right) \\
 &= \frac{5 - 7x^2}{8(3 + 2x^2 + x^4)} + \frac{9 \tan^{-1} \left(\frac{1+x^2}{\sqrt{2}} \right)}{8\sqrt{2}}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{9 \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5-7x^2}{8(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]

[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x + 2x^3 + 4x^5}{(3 + 2x^2 + x^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]

[Out] IntegrateAlgebraic[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2, x]

fricas [A] time = 1.12, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-14x^2+10}{16(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")

[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 + 10)/(x^4 + 2*x^2 + 3)

giac [A] time = 1.11, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-\frac{7x^2-5}{8(x^4+2x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")

[Out] $9/16*\sqrt{2}*\arctan(1/2*\sqrt{2}*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3)$

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2} \arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} + \frac{-\frac{7x^2}{4} + \frac{5}{4}}{2x^4 + 4x^2 + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2, x)$

[Out] $1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^{(1/2)}*\arctan(1/4*(2*x^2+2)*2^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7x^2 - 5}{8(x^4 + 2x^2 + 3)} + \frac{9}{4} \int \frac{x}{x^4 + 2x^2 + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2, x, \text{algorithm}="maxima")$

[Out] $-1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*\text{integrate}(x/(x^4 + 2*x^2 + 3), x)$

mupad [B] time = 2.19, size = 42, normalized size = 0.93

$$\frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{7x^2}{8} - \frac{5}{8}}{x^4 + 2x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2, x)$

[Out] $(9*2^{(1/2)}*\operatorname{atan}(2^{(1/2)}/2 + (2^{(1/2)}*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2 + x^4 + 3)$

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{5 - 7x^2}{8x^4 + 16x^2 + 24} + \frac{9\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x^2}{2} + \frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2, x)$

[Out] $(5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*\sqrt{2}*\operatorname{atan}(\sqrt{2}*x**2/2 + \sqrt{2}(2)/2)/16$

$$3.332 \quad \int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx$$

Optimal. Leaf size=59

$$\tan^{-1}(2x^2 + 1) + \frac{2x^2 + 1}{2(2x^4 + 2x^2 + 1)} + \frac{4x^2 + 3}{16(2x^4 + 2x^2 + 1)^2}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1593, 1663, 1660, 12, 614, 617, 204}

$$\frac{2x^2 + 1}{2(2x^4 + 2x^2 + 1)} + \frac{4x^2 + 3}{16(2x^4 + 2x^2 + 1)^2} + \tan^{-1}(2x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]

[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)) + ArcTan[1 + 2*x^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x + x^5}{(1 + 2x^2 + 2x^4)^3} dx &= \int \frac{x(1 + x^4)}{(1 + 2x^2 + 2x^4)^3} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1 + x^2}{(1 + 2x + 2x^2)^3} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1}{16} \text{Subst} \left(\int \frac{16}{(1 + 2x + 2x^2)^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \text{Subst} \left(\int \frac{1}{(1 + 2x + 2x^2)^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} + \text{Subst} \left(\int \frac{1}{1 + 2x + 2x^2} dx, x, x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} - \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, 1 + 2x^2 \right) \\
&= \frac{3 + 4x^2}{16(1 + 2x^2 + 2x^4)^2} + \frac{1 + 2x^2}{2(1 + 2x^2 + 2x^4)} + \tan^{-1}(1 + 2x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.75

$$\tan^{-1}(2x^2 + 1) + \frac{32x^6 + 48x^4 + 36x^2 + 11}{16(2x^4 + 2x^2 + 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3, x]

[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + x^5}{(1 + 2x^2 + 2x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]

[Out] IntegrateAlgebraic[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3, x]

fricas [A] time = 0.89, size = 75, normalized size = 1.27

$$\frac{32x^6 + 48x^4 + 36x^2 + 16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1) \arctan(2x^2 + 1) + 11}{16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arctan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)

giac [A] time = 1.80, size = 42, normalized size = 0.71

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{16(2x^4 + 2x^2 + 1)^2} + \arctan(2x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 + 1)

maple [A] time = 0.01, size = 41, normalized size = 0.69

$$\arctan(2x^2 + 1) + \frac{2x^6 + 3x^4 + \frac{9}{4}x^2 + \frac{11}{16}}{(2x^4 + 2x^2 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x)

[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{16(4x^8 + 8x^6 + 8x^4 + 4x^2 + 1)} + 2 \int \frac{x}{2x^4 + 2x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")

[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) + 2*integrate(x/(2*x^4 + 2*x^2 + 1), x)

mupad [B] time = 0.05, size = 47, normalized size = 0.80

$$\operatorname{atan}\left(2x^2 + 1\right) + \frac{\frac{x^6}{2} + \frac{3x^4}{4} + \frac{9x^2}{16} + \frac{11}{64}}{x^8 + 2x^6 + 2x^4 + x^2 + \frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)

[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2*x^6 + x^8 + 1/4)

sympy [A] time = 0.18, size = 46, normalized size = 0.78

$$\frac{32x^6 + 48x^4 + 36x^2 + 11}{64x^8 + 128x^6 + 128x^4 + 64x^2 + 16} + \operatorname{atan}\left(2x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)

[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2 + 16) + atan(2*x**2 + 1)

$$3.333 \quad \int \frac{a+bx+cx^2}{d+ex^2+fx^4} dx$$

Optimal. Leaf size=209

$$\frac{\left(c - \frac{ce-2af}{\sqrt{e^2-4df}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}} + c\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} - \frac{b \tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Rubi [A] time = 0.37, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(c - \frac{ce-2af}{\sqrt{e^2-4df}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}} + c\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}} - \frac{b \tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]

[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e - Sqrt[e^2 - 4*d*f]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c + (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx + cx^2}{d + ex^2 + fx^4} dx &= \int \frac{bx}{d + ex^2 + fx^4} dx + \int \frac{a + cx^2}{d + ex^2 + fx^4} dx \\
&= b \int \frac{x}{d + ex^2 + fx^4} dx + \frac{1}{2} \left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx + \frac{1}{2} \left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \int \frac{1}{\frac{e}{2} + \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \\
&= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e + \sqrt{e^2 - 4df}}} + \frac{1}{2} b \text{Subst} \left(\int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \right) \\
&= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e + \sqrt{e^2 - 4df}}} - b \text{Subst} \left(\int \frac{1}{\frac{e}{2} - \frac{1}{2}\sqrt{e^2 - 4df} + fx^2} dx \right) \\
&= \frac{\left(c - \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\left(c + \frac{ce - 2af}{\sqrt{e^2 - 4df}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{2} \sqrt{f} \sqrt{e + \sqrt{e^2 - 4df}}} - \frac{b \tanh^{-1} \left(\frac{e + 2fx^2}{\sqrt{e^2 - 4df}} \right)}{\sqrt{e^2 - 4df}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}(2af + c(\sqrt{e^2 - 4df} - e)) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e - \sqrt{e^2 - 4df}}} \right)}{\sqrt{f} \sqrt{e - \sqrt{e^2 - 4df}}} + \frac{\sqrt{2}(c(\sqrt{e^2 - 4df} + e) - 2af) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{f} x}{\sqrt{e + \sqrt{e^2 - 4df}}} \right)}{\sqrt{f} \sqrt{e + \sqrt{e^2 - 4df}}} + b \log(\sqrt{e^2 - 4df} - e - 2fx^2) - b \log(\sqrt{e^2 - 4df} + e + 2fx^2)}{2\sqrt{e^2 - 4df}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]

[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sqrt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqrt[e^2 - 4*d*f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{d + ex^2 + fx^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 4.70, size = 1587, normalized size = 7.59
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*sqrt(-4*d*f + e^2)*b*log(x^2 + 1/2*
(sqrt(-4*d*f + e^2) + e)/f)/((16*d^2*f^2 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2
+ f^2*e^2 - 2*f*e^3 + e^4)*f^2) - 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*s
qrt(-4*d*f + e^2)*b*log(x^2 - 1/2*(sqrt(-4*d*f + e^2) - e)/f)/((16*d^2*f^2
- 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) + 1/4*((1
6*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2))*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e + s
qrt(-4*d*f + e^2))*f)*d*f^3 - 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*f
+ e^2))*f)*d*f^2*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqr
t(-4*d*f + e^2))*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*
f + e^2))*f)*f^2*e + 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*
f + e^2))*f)*d*f*e^2 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2))*f)*f^2*e^2 + 16
*d*f^2*e^2 + 2*(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
+ sqrt(-4*d*f + e^2))*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2))*f)*
f*e^3 + 2*f^2*e^3 - 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt
(f*e + sqrt(-4*d*f + e^2))*f)*e^3 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2))*f
)*e^4 - 2*f*e^4)*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e +
sqrt(-4*d*f + e^2))*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4
*d*f + e^2))*f)*d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*f
+ e^2))*f)*d*f*e - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f
+ e^2)*sqrt(f*e + sqrt(-4*d*f + e^2))*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqr
t((sqrt(-4*d*f + e^2) + e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d
^2*f*e^2 + d*f^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f)) + 1/4*((16*sqrt(2)*sqrt(f
*e - sqrt(-4*d*f + e^2))*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2
))*f)*d*f^3 + 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2))*f)*d*f^2*
```

$$\begin{aligned}
& e + 8*d*f^3*e - 4*\sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}* \\
& f)*d*f*e + \sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)^2* \\
& e - 8*(4*d*f - e^2)*d*f^2 - 8*\sqrt{2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)*d*f* \\
& e^2 + \sqrt{2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)^2*e^2 - 16*d*f^2*e^2 - 2*(\\
& 4*d*f - e^2)*f^2*e - 2*\sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + \\
& e^2}}*f)*f*e^2 - 2*\sqrt{2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)*f*e^3 - 2*f^2*e^ \\
& 3 + 2*(4*d*f - e^2)*f*e^2 + \sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d \\
& *f + e^2}}*f)*e^3 + \sqrt{2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)*e^4 + 2*f*e^4)* \\
& a - 2*(8*d^2*f^3 - 4*\sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + e^ \\
& 2}}*f)*d^2*f + \sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)*d \\
& *f^2 - 2*\sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e - \sqrt{-4*d*f + e^2}}*f)*d*f*e \\
& - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + \sqrt{2}*\sqrt{-4*d*f + e^2}*\sqrt{f*e \\
& - \sqrt{-4*d*f + e^2}}*f)*d*e^2)*c*\arctan(2*\sqrt{1/2}*x/\sqrt{-(\sqrt{-4*d*f \\
& + e^2} - e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d^2*f*e^2 + d*f^ \\
& 2*e^2 - 2*d*f*e^3 + d*e^4)*\text{abs}(f))
\end{aligned}$$

maple [B] time = 0.06, size = 616, normalized size = 2.95

$$\frac{2\sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{(4d-f)\sqrt{-4df+e^2}} + \frac{2\sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{2} e^2 \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{2(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{2} e^2 \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{2(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{4d+f^2} \sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{4d+f^2} \sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{4d+f^2} \sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{2(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{4d+f^2} \sqrt{2} \operatorname{arctanh}\left(\frac{e}{\sqrt{-4df+e^2}}\right)}{2(4d-f)\sqrt{-4df+e^2}} + \frac{\sqrt{4d+f^2} \operatorname{atan}\left(\frac{2x^2+e-\sqrt{-4d+f^2}}{2(4d-f)}\right)}{2(4d-f)} + \frac{\sqrt{4d+f^2} \operatorname{atan}\left(\frac{2x^2+e-\sqrt{-4d+f^2}}{2(4d-f)}\right)}{8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(f*x^4+e*x^2+d), x)`

[Out]
$$\begin{aligned}
& -1/2*(-4*d*f+e^2)^{(1/2)}/(4*d*f-e^2)*b*\ln(-2*f*x^2+(-4*d*f+e^2)^{(1/2)}-e)-2*f \\
& /((4*d*f-e^2)*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(\\
& ((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}))*c*d+1/2/(4*d*f-e^2)*2^{(1/2)}/(((4*d*f+e^2)^ \\
& ^{(1/2)}-e)*f)^{(1/2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}))*c* \\
& e^2+f*(-4*d*f+e^2)^{(1/2)}/(4*d*f-e^2)*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/ \\
& 2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}))*a-1/2*(-4*d*f+e^2) \\
& ^{(1/2)}/(4*d*f-e^2)*2^{(1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}*\operatorname{arctanh}(f*x*2^{(\\
& 1/2)}/(((4*d*f+e^2)^{(1/2)}-e)*f)^{(1/2)}))*c*e+1/2*(-4*d*f+e^2)^{(1/2)}/(4*d*f-e \\
& ^2)*b*\ln(2*f*x^2+(-4*d*f+e^2)^{(1/2)}+e)+2*f/(4*d*f-e^2)*2^{(1/2)}/((e+(-4*d*f+e \\
& ^2)^{(1/2)})*f)^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/((e+(-4*d*f+e^2)^{(1/2)})*f)^{(1/2)}))*c* \\
& d-1/2/(4*d*f-e^2)*2^{(1/2)}/((e+(-4*d*f+e^2)^{(1/2)})*f)^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/ \\
& 2)}/((e+(-4*d*f+e^2)^{(1/2)})*f)^{(1/2)}))*c*e^2+f*(-4*d*f+e^2)^{(1/2)}/(4*d*f-e^2) \\
& *2^{(1/2)}/((e+(-4*d*f+e^2)^{(1/2)})*f)^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/((e+(-4*d*f+e^ \\
& 2)^{(1/2)})*f)^{(1/2)}))*a-1/2*(-4*d*f+e^2)^{(1/2)}/(4*d*f-e^2)*2^{(1/2)}/((e+(-4*d* \\
& f+e^2)^{(1/2)})*f)^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/((e+(-4*d*f+e^2)^{(1/2)})*f)^{(1/2)})) \\
& *c*e
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{fx^4 + ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

mupad [B] time = 3.44, size = 3942, normalized size = 18.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

```
[Out] symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f*x
z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d
^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 +
32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z +
4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a
^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f
+ a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f - 16*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*f^3 - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^
2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c
^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^2*f^3*x + 4*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
+ b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*e^2*f^2 + 16*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
+ b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*d*f^3*x + 2*root(16
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
```

$$\begin{aligned}
& *d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b^2*e*f^2*x + 4*root(1 \\
& 6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 \\
& - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2* \\
& f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2 \\
& *d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c \\
& *d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*d*f^2*x - 2*root(1 \\
& 6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 \\
& - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2* \\
& f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2 \\
& *d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c \\
& *d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*e^2*f*x + 32*root(\\
& 16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 \\
& - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2 \\
& *f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^ \\
& 2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2* \\
& c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& *f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*d*e*f^3*x - 4*root(\\
& 16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 \\
& - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2 \\
& *f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^ \\
& 2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2* \\
& c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& *f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*e^2*f^2*x + 4*roo \\
& t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z \\
& ^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d \\
& ^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b* \\
& c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^ \\
& 2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2 \\
& *e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a*b*e*f^2 - 8*root(\\
& 16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 \\
& - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2 \\
& *f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^ \\
& 2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2* \\
& c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e* \\
& *f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b*c*d*f^2 - 2*a*b*c*f \\
& ^2*x + b*c^2*e*f*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3* \\
& f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16* \\
& a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 \\
& + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - \\
& 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d \\
& *e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, \\
& z, k)*a*c*e*f^2*x)*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3
\end{aligned}$$

```
*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2
*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 +
4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16
*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e
+ b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z,
k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
```

```
[Out] Timed out
```


$$3.334 \quad \int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}} + e^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{2de \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.39, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}} + e^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{2de \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4), x]

[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Module[{q = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx &= \int \frac{2dex}{a+bx^2+cx^4} dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4} dx \\
&= (2de) \int \frac{x}{a+bx^2+cx^4} dx + \frac{1}{2} \left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx + \frac{1}{2} \left(e^2 + \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx \\
&= \frac{\left(e^2 + \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + (de) \text{Subst} \left(\frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2}, x, \frac{b+\sqrt{b^2-4ac}}{2c} \right) \\
&= \frac{\left(e^2 + \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - (2de) \text{Subst} \left(\frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2}, x, \frac{b-\sqrt{b^2-4ac}}{2c} \right) \\
&= \frac{\left(e^2 + \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2 - \frac{2cd^2-be^2}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de \tanh^{-1} \left(\frac{\sqrt{b^2-4ac}-b-2cx^2}{\sqrt{b^2-4ac}+b+2cx^2} \right)}{\sqrt{b^2-4ac}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 245, normalized size = 1.09

$$\frac{\frac{\sqrt{2} \left(e^2 (\sqrt{b^2-4ac}-b) + 2cd^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2} \left(e^2 (\sqrt{b^2-4ac}+b) - 2cd^2 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}} \right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + 2de \log \left(\frac{\sqrt{b^2-4ac}-b-2cx^2}{\sqrt{b^2-4ac}+b+2cx^2} \right) - 2de \log \left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}-b-2cx^2} \right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4), x]

[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*x^2 + c*x^4),x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*x^2 + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

giac [B] time = 4.56, size = 1625, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out]
$$-(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\sqrt{b^2 - 4*a*c}*d*e*\log(x^2 + 1/2*(b + \sqrt{b^2 - 4*a*c}))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*\sqrt{b^2 - 4*a*c}*d*e*\log(x^2 + 1/2*(b - \sqrt{b^2 - 4*a*c}))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^4 - 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2*c - 2*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3*c - 2*b^4*c + 16*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c^2 + 8*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 + \sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^3 - 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c - 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b^2*c + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a^2*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*b*c - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}*c})*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b + \sqrt{b^2 - 4*a*c}))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c)) + 1/4*((\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^4 - 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c - 2*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3*c + 2*b^4*c + 16*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c^2 + 8*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c^2 + \sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^3 - 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c - 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b^2*c + \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2 + 4*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*c + 2*\sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b*c - \sqrt{2})*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*\arctan(2*\sqrt{1/2}*x/\sqrt{(b - \sqrt{b^2 - 4*a*c}))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*\text{abs}(c))$$

$$\begin{aligned} & \text{rt}(2) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * c^3 + 32 * a^2 * c^3 + 8 * a * b * c^3 + \text{sqrt} \\ & (2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^3 - 4 * \text{sqrt}(2) * \text{sqrt} \\ & (b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b * c - 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 \\ & * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b^2 * c + \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt} \\ & (b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * b * c^2 - 2 * (b^2 - 4 * a * c) * b^2 * c + 8 * (b^2 - 4 * a * c) \\ &) * a * c^2 + 2 * (b^2 - 4 * a * c) * b * c^2) * d^2 + 2 * (2 * a * b^2 * c^2 - 8 * a^2 * c^3 - \text{sqrt}(2) \\ & * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b^2 + 4 * \text{sqrt}(2) * \text{sqrt}(b \\ & ^2 - 4 * a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a^2 * c + 2 * \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * \\ & a * c) * \text{sqrt}(b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * b * c - \text{sqrt}(2) * \text{sqrt}(b^2 - 4 * a * c) * \text{sqrt} \\ & (b * c - \text{sqrt}(b^2 - 4 * a * c) * c) * a * c^2 - 2 * (b^2 - 4 * a * c) * a * c^2) * e^2) * \arctan(2 * \text{sq} \\ & \text{rt}(1/2) * x / \text{sqrt}((b - \text{sqrt}(b^2 - 4 * a * c)) / c)) / ((a * b^4 - 8 * a^2 * b^2 * c - 2 * a * b^3 * \\ & c + 16 * a^3 * c^2 + 8 * a^2 * b * c^2 + a * b^2 * c^2 - 4 * a^2 * c^3) * \text{abs}(c)) \end{aligned}$$

maple [B] time = 0.06, size = 633, normalized size = 2.83

$$\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{2(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{2(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{2(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{2(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}} - \frac{\sqrt{4ac+d^2}\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}a}{\sqrt{b+\sqrt{4ac+2d^2}}}\right)}{(4ac-d^2)\sqrt{b+\sqrt{4ac+2d^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x)

$$\begin{aligned} \text{[Out]} & -(-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * d * e * \ln(-2 * c * x^2 + (-4 * a * c + b^2)^{(1/2)} - b) - 2 * c / (\\ & 4 * a * c - b^2) * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)} * \operatorname{arctanh}(c * x * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)}) * e^2 * a + 1/2 / (4 * a * c - b^2) * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)} * \operatorname{arctanh}(c * x * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)}) * e^2 * a + 1/2 * (-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)} * \operatorname{arctanh}(c * x * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)}) * b * e^2 + c * (-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)} * \operatorname{arctanh}(c * x * 2^{(1/2)} / (((-4 * a * c + b^2)^{(1/2)} - b) * c)^{(1/2)}) * d^2 + (-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * d * e * \ln(2 * c * x^2 + (-4 * a * c + b^2)^{(1/2)} + b) + 2 * c / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(c * x * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)}) * e^2 * a - 1/2 / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(c * x * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)}) * e^2 * b^2 - 1/2 * (-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(c * x * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)}) * b * e^2 + c * (-4 * a * c + b^2)^{(1/2)} / (4 * a * c - b^2) * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)} * \arctan(c * x * 2^{(1/2)} / ((b + (-4 * a * c + b^2)^{(1/2)}) * c)^{(1/2)}) * d^2 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex + d)^2}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

$$\begin{aligned}
& c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k) \\
& *ac^2d^3e + 8\text{root}(16ab^4cz^4 - 128a^2b^2c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k) \\
& *b^2cd^3e + 32\text{root}(16ab^4cz^4 - 128a^2b^2c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k) \\
& *ac^3d^2e^2x + 12\text{root}(16ab^4cz^4 - 128a^2b^2c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k) \\
& *b^2cd^2e^2x - 8\text{root}(16ab^4cz^4 - 128a^2b^2c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k) \\
& *b^2cd^2e^2x) * \text{root}(16ab^4cz^4 - 128a^2b^2c^2z^4 + 256a^3c^3z^4 - 48ab^2cd^2e^2z^2 - 16a^2b^3ce^4z^2 - 16ab^2c^2d^4z^2 + 192a^2c^2d^2e^2z^2 + 4b^3cd^4z^2 + 4ab^3e^4z^2 + 8b^2cd^5ez + 32a^2cd^5e^5z - 32ac^2d^5e^5z - 8ab^2d^5e^5z + 2b^2cd^6e^2 + 2ac^2d^4e^4 + 2ab^2d^2e^6 + b^2d^4e^4 + c^2d^8 + a^2e^8, z, k), k, 1, 4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)

[Out] Timed out

$$3.335 \quad \int \frac{x^2}{(a+bx)(c+dx)} dx$$

Optimal. Leaf size=56

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {72}

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x)*(c + d*x)),x]

[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b*c - a*d))

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a+bx)(c+dx)} dx &= \int \left(\frac{1}{bd} + \frac{a^2}{b(bc-ad)(a+bx)} + \frac{c^2}{d(-bc+ad)(c+dx)} \right) dx \\ &= \frac{x}{bd} + \frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{a^2 \log(a+bx)}{b^2(bc-ad)} - \frac{c^2 \log(c+dx)}{d^2(bc-ad)} + \frac{x}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]

[Out] $x/(b*d) + (a^2*\text{Log}[a + b*x])/(b^2*(b*c - a*d)) - (c^2*\text{Log}[c + d*x])/(d^2*(b*c - a*d))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx)(c + dx)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((a + b*x)*(c + d*x)),x]

[Out] IntegrateAlgebraic[x^2/((a + b*x)*(c + d*x)), x]

fricas [A] time = 1.20, size = 65, normalized size = 1.16

$$\frac{a^2 d^2 \log(bx + a) - b^2 c^2 \log(dx + c) + (b^2 cd - abd^2)x}{b^3 cd^2 - ab^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] $(a^2*d^2*\log(b*x + a) - b^2*c^2*\log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*c*d^2 - a*b^2*d^3)$

giac [A] time = 0.31, size = 62, normalized size = 1.11

$$\frac{a^2 \log(|bx + a|)}{b^3 c - ab^2 d} - \frac{c^2 \log(|dx + c|)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] $a^2*\log(\text{abs}(b*x + a))/(b^3*c - a*b^2*d) - c^2*\log(\text{abs}(d*x + c))/(b*c*d^2 - a*d^3) + x/(b*d)$

maple [A] time = 0.01, size = 57, normalized size = 1.02

$$-\frac{a^2 \ln(bx + a)}{(ad - bc)b^2} + \frac{c^2 \ln(dx + c)}{(ad - bc)d^2} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x+a)/(d*x+c),x)

[Out] $x/b/d+1/d^2*c^2/(a*d-b*c)*\ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*\ln(b*x+a)$

maxima [A] time = 0.68, size = 60, normalized size = 1.07

$$\frac{a^2 \log(bx + a)}{b^3c - ab^2d} - \frac{c^2 \log(dx + c)}{bcd^2 - ad^3} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x/(b*d)

mupad [B] time = 0.21, size = 61, normalized size = 1.09

$$\frac{a^2 d^2 \ln(a + bx) - b^2 c^2 \ln(c + dx) - a b d^2 x + b^2 c d x}{b^2 d^2 (a d - b c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)*(c + d*x)),x)

[Out] -(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^2*d^2*(a*d - b*c))

sympy [B] time = 1.06, size = 190, normalized size = 3.39

$$-\frac{a^2 \log\left(x + \frac{\frac{a^4 d^3}{b(ad-bc)} - \frac{2a^3 cd^2}{ad-bc} + \frac{a^2 bc^2 d}{ad-bc} + a^2 cd + abc^2}{a^2 d^2 + b^2 c^2}\right)}{b^2 (ad - bc)} + \frac{c^2 \log\left(x + \frac{-\frac{a^2 bc^2 d}{ad-bc} + a^2 cd + \frac{2ab^2 c^3}{ad-bc} + abc^2 - \frac{b^3 c^4}{d(ad-bc)}}{a^2 d^2 + b^2 c^2}\right)}{d^2 (ad - bc)} + \frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x+a)/(d*x+c),x)

[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)

$$3.336 \quad \int \frac{x^2}{(c+dx)(a+bx^2)} dx$$

Optimal. Leaf size=96

$$\frac{ad \log(a + bx^2)}{2b(ad^2 + bc^2)} + \frac{c^2 \log(c + dx)}{d(ad^2 + bc^2)} - \frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2 + bc^2)}$$

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1629, 635, 205, 260}

$$\frac{ad \log(a + bx^2)}{2b(ad^2 + bc^2)} + \frac{c^2 \log(c + dx)}{d(ad^2 + bc^2)} - \frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(ad^2 + bc^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]

[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d^2)))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,

d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(c+dx)(a+bx^2)} dx &= \int \left(\frac{c^2}{(bc^2+ad^2)(c+dx)} - \frac{a(c-dx)}{(bc^2+ad^2)(a+bx^2)} \right) dx \\
 &= \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} - \frac{a \int \frac{c-dx}{a+bx^2} dx}{bc^2+ad^2} \\
 &= \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} - \frac{(ac) \int \frac{1}{a+bx^2} dx}{bc^2+ad^2} + \frac{(ad) \int \frac{x}{a+bx^2} dx}{bc^2+ad^2} \\
 &= -\frac{\sqrt{a} c \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc^2+ad^2)} + \frac{c^2 \log(c+dx)}{d(bc^2+ad^2)} + \frac{ad \log(a+bx^2)}{2b(bc^2+ad^2)}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.76

$$\frac{-2\sqrt{a}\sqrt{b}cd \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + ad^2 \log(a+bx^2) + 2bc^2 \log(c+dx)}{2abd^3 + 2b^2c^2d}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c+d*x)*(a+b*x^2)),x]

[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c+d*x] + a*d^2*Log[a+b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+dx)(a+bx^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((c+d*x)*(a+b*x^2)),x]

[Out] IntegrateAlgebraic[x^2/((c+d*x)*(a+b*x^2)), x]

fricas [A] time = 1.23, size = 162, normalized size = 1.69

$$\left[\frac{bcd\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + ad^2 \log(bx^2 + a) + 2bc^2 \log(dx + c)}{2(b^2c^2d + abd^3)}, -\frac{2bcd\sqrt{\frac{a}{b}} \arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - ad^2 \log(bx^2 + a) - 2bc^2 \log(dx + c)}{2(b^2c^2d + abd^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")

[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3)]

giac [A] time = 0.26, size = 85, normalized size = 0.89

$$\frac{ad \log(bx^2 + a)}{2(b^2c^2 + abd^2)} + \frac{c^2 \log(|dx + c|)}{bc^2d + ad^3} - \frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2 + ad^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")

[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))

maple [A] time = 0.01, size = 87, normalized size = 0.91

$$-\frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad^2 + bc^2)\sqrt{ab}} + \frac{ad \ln(bx^2 + a)}{2(ad^2 + bc^2)b} + \frac{c^2 \ln(dx + c)}{(ad^2 + bc^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(d*x+c)/(b*x^2+a),x)

[Out] 1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*c/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)

maxima [A] time = 1.22, size = 84, normalized size = 0.88

$$\frac{ad \log(bx^2 + a)}{2(b^2c^2 + abd^2)} + \frac{c^2 \log(dx + c)}{bc^2d + ad^3} - \frac{ac \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bc^2 + ad^2)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")

[Out] $\frac{1}{2}a*d*\log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*\log(d*x + c)/(b*c^2*d + a*d^3) - a*c*\arctan(b*x/\sqrt{a*b})/((b*c^2 + a*d^2)*\sqrt{a*b})$

mupad [B] time = 1.13, size = 347, normalized size = 3.61

$$\frac{\ln\left(a c + a d x + \frac{(c\sqrt{-ab^3+abd})\left(x(2b^2c^2-5abd^2)-5abcd+\frac{2b^2d(c\sqrt{-ab^3+abd})(-bx^2+4acd+3ax^2)}{2b^3c^2+2ab^2d^2}\right)}{2b^3c^2+2ab^2d^2}\right)(c\sqrt{-ab^3+abd})}{2b^3c^2+2ab^2d^2} - \frac{\ln\left(a c + a d x + \frac{(c\sqrt{-ab^3-abd})\left(bx(5ad^2-2b^2c^2)+5abcd+\frac{d(c\sqrt{-ab^3-abd})(-bx^2+4acd+3ax^2)}{b^2c^2+ad^2}\right)}{2b^2(b^2+ad^2)}\right)(c\sqrt{-ab^3-abd})}{2(b^3c^2+ab^2d^2)} + \frac{c^2 \ln(c+dx)}{bc^2d+ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x^2)*(c + d*x)),x)

[Out] $(\log(a*c + a*d*x + ((c*(-a*b^3)^{(1/2)} + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) - 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^{(1/2)} + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x)))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3)^{(1/2)} + a*b*d)/(2*b^3*c^2 + 2*a*b^2*d^2) - (\log(a*c + a*d*x + ((c*(-a*b^3)^{(1/2)} - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^{(1/2)} - a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 + b*c^2)))*(c*(-a*b^3)^{(1/2)} - a*b*d)/(2*(b^3*c^2 + a*b^2*d^2)) + (c^2*\log(c + d*x))/(a*d^3 + b*c^2*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)

[Out] Timed out

$$3.337 \quad \int \frac{x^2}{(c+dx)(a+bx^3)} dx$$

Optimal. Leaf size=264

$$\frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3} (bc^3 - ad^3)} - \frac{\sqrt[3]{a} d \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} b^{2/3} (a^{2/3} d^2 + \sqrt[3]{a} \sqrt[3]{b} cd + b^{2/3} c^2)} + \frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c)}{3b^{2/3} (bc^3 - ad^3)}$$

Rubi [A] time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, number of rules / integrand size = 0.450, Rules used = {6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6b^{2/3} (bc^3 - ad^3)} - \frac{\sqrt[3]{a} d \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} b^{2/3} (a^{2/3} d^2 + \sqrt[3]{a} \sqrt[3]{b} cd + b^{2/3} c^2)} + \frac{\sqrt[3]{a} d (\sqrt[3]{a} d + \sqrt[3]{b} c) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3b^{2/3} (bc^3 - ad^3)} + \frac{c^2 \log(a + bx^3)}{3(bc^3 - ad^3)} - \frac{c^2 \log(c + dx)}{bc^3 - ad^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]

[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2)) + (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x]/(3*b^(2/3)*(b*c^3 - a*d^3)) - (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(6*b^(2/3)*(b*c^3 - a*d^3)) + (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
 implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(c+dx)(a+bx^3)} dx &= \int \left(-\frac{c^2 d}{(bc^3 - ad^3)(c+dx)} + \frac{acd - ad^2 x + bc^2 x^2}{(bc^3 - ad^3)(a+bx^3)} \right) dx \\
&= -\frac{c^2 \log(c+dx)}{bc^3 - ad^3} + \frac{\int \frac{acd - ad^2 x + bc^2 x^2}{a+bx^3} dx}{bc^3 - ad^3} \\
&= -\frac{c^2 \log(c+dx)}{bc^3 - ad^3} + \frac{\int \frac{acd - ad^2 x}{a+bx^3} dx}{bc^3 - ad^3} + \frac{(bc^2) \int \frac{x^2}{a+bx^3} dx}{bc^3 - ad^3} \\
&= -\frac{c^2 \log(c+dx)}{bc^3 - ad^3} + \frac{c^2 \log(a+bx^3)}{3(bc^3 - ad^3)} + \frac{\int \frac{\sqrt[3]{a}(2a\sqrt[3]{b}cd - a^{4/3}d^2) + \sqrt[3]{b}(-a\sqrt[3]{b}cd - a^{4/3}d^2)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}\sqrt[3]{b}(bc^3 - ad^3)} + \frac{\int \frac{x^2}{a+bx^3} dx}{3} \\
&= \frac{\sqrt[3]{a}d(\sqrt[3]{b}c + \sqrt[3]{a}d) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc^3 - ad^3)} - \frac{c^2 \log(c+dx)}{bc^3 - ad^3} + \frac{c^2 \log(a+bx^3)}{3(bc^3 - ad^3)} + \frac{(a^{2/3}d)}{2\sqrt[3]{b}(b^{2/3}c^2 + \sqrt[3]{a}\sqrt[3]{b}cd + a^{2/3}d^2)} \\
&= \frac{\sqrt[3]{a}d(\sqrt[3]{b}c + \sqrt[3]{a}d) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc^3 - ad^3)} - \frac{c^2 \log(c+dx)}{bc^3 - ad^3} - \frac{\sqrt[3]{a}d(\sqrt[3]{b}c + \sqrt[3]{a}d) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6b^{2/3}(bc^3 - ad^3)} \\
&= -\frac{\sqrt[3]{a}d \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(b^{2/3}c^2 + \sqrt[3]{a}\sqrt[3]{b}cd + a^{2/3}d^2)} + \frac{\sqrt[3]{a}d(\sqrt[3]{b}c + \sqrt[3]{a}d) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3b^{2/3}(bc^3 - ad^3)} - \frac{c^2 \log(c+dx)}{bc^3 - ad^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 228, normalized size = 0.86

$$\frac{-\sqrt[3]{a}\sqrt[3]{b}cd \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - a^{2/3}d^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 2b^{2/3}c^2 \log(a+bx^3) + 2\sqrt[3]{a}d(\sqrt[3]{a}d + \sqrt[3]{b}c) \log(\sqrt[3]{a} + \sqrt[3]{b}x) + 2\sqrt{3}\sqrt[3]{a}d\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}}\right) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}}\right) - 6b^{2/3}c^2 \log(c+dx)}{6b^{2/3}(bc^3 - ad^3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c + d*x)*(a + b*x^3)), x]

[Out] (2*sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3]/(6*b^(2/3)*(b*c^3 - a*d^3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+dx)(a+bx^3)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^3)),x]

[Out] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^3)), x]

fricas [C] time = 4.87, size = 5975, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out]
$$-1/12*(2*(b*c^3 - a*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/(b*c^3 - a*d^3)*\log(-3/2*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/(b*c^3 - a*d^3)*b*c^2 - 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/(b*c^3 - a*d^3)) + d*x - 2*c) + 12*c^2*\log(d*x + c) - ((b*c^3 - a*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 - 3*\sqrt{1/3}*(b*c^3 - a*d^3)*\sqrt{-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*\sqrt{3} + 1) - 2*c^2/(b*c^3 - a*d^3)) + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3)))*(-I*\sqrt{3} + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2$$

$$\begin{aligned}
& 2) + 1/(b^3c^3 - a^2b^2d^3)^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2d^3)^3 \\
& - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) \\
& 2) + 1/(b^3c^3 - a^2b^2d^3)^{1/3} * (I\sqrt{3} + 1) - 2c^2/(b^3c^3 - a^2d^3) \\
&))/(b^3c^6 - 2a^2b^2c^3d^3 + a^2b^2d^6)) * \log(3/2 * (2^{1/2})^{2/3} * (c^4/(b \\
& *c^3 - a^2d^3)^2 - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + 1)/(2c^6/(b^3c^3 - a \\
& *d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2 \\
& d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a \\
& *d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2 \\
& d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} * (I\sqrt{3} + 1) - 2c^2/(b^3c^3 \\
& - a^2d^3) * b^2c^2 + 1/4 * (b^2c^3 - a^2bd^3) * (2^{1/2})^{2/3} * (c^4/(b^3c^3 - a^2d^ \\
& 3)^2 - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + 1)/(2c^6/(b^3c^3 - a^2d^3)^3 - 3 \\
& *c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) \\
& + 1/(b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2d^3)^3 - 3 \\
& *c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) \\
& + 1/(b^3c^3 - a^2b^2d^3))^{1/3} * (I\sqrt{3} + 1) - 2c^2/(b^3c^3 - a^2d^3))^{1/3} \\
& + 3/4 * \sqrt{1/3} * (b^2c^3 - a^2bd^3) * (2^{1/2})^{2/3} * (c^4/(b^3c^3 - a^2d^3)^2 \\
& - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + 1)/(2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/ \\
& ((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) + 1/(\\
& b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/ \\
& ((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) + 1/(\\
& b^3c^3 - a^2b^2d^3))^{1/3} * (I\sqrt{3} + 1) - 2c^2/(b^3c^3 - a^2d^3) * \sqrt{- \\
& (4b^4c^4 - 16a^2c^2d^3 + (b^3c^6 - 2a^2b^2c^3d^3 + a^2b^2d^6) * (2^{1/2})^{2/3} * (c^4/(b^3c^3 - a^2d^3)^2 \\
& - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + 1)/(2c^6 \\
& / (b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((\\
& b^3c^3 - a^2d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6 \\
& / (b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((\\
& b^3c^3 - a^2d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} * (I\sqrt{3} + 1) - 2c \\
& ^2/(b^3c^3 - a^2d^3))^{1/3} + 4 * (b^2c^5 - a^2b^2c^2d^3) * (2^{1/2})^{2/3} * (c^4/(b^3c \\
& ^3 - a^2d^3)^2 - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + 1)/(2c^6/(b^3c^3 - a^2 \\
& d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3 \\
&)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2 \\
& d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3 \\
&)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} * (I\sqrt{3} + 1) - 2c^2/(b^3c^3 - \\
& a^2d^3)))/(b^3c^6 - 2a^2b^2c^3d^3 + a^2b^2d^6)) + 2*d*x + 2*c) - ((b^3c^3 \\
& - a^2d^3) * (2^{1/2})^{2/3} * (c^4/(b^3c^3 - a^2d^3)^2 - c/(b^2c^3 - a^2bd^3)) * (-I \\
& *sqrt{3} + 1)/(2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 \\
& - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} \\
& + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 \\
& - a^2d^3)) + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} * \\
& (I\sqrt{3} + 1) - 2c^2/(b^3c^3 - a^2d^3)) + 6c^2 + 3*sqrt{1/3} * (b^3c^3 - a^2 \\
& d^3) * \sqrt{-(4b^4c^4 - 16a^2c^2d^3 + (b^3c^6 - 2a^2b^2c^3d^3 + a^2b^2d^6) * (\\
& 2^{1/2})^{2/3} * (c^4/(b^3c^3 - a^2d^3)^2 - c/(b^2c^3 - a^2bd^3)) * (-I\sqrt{3} + \\
& 1)/(2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3)) \\
& + a^2d^3/((b^3c^3 - a^2d^3)^2b^2) + 1/(b^3c^3 - a^2b^2d^3))^{1/3} + (1/2)^{1/3} * (2c^6/(b^3c^3 - a^2d^3)^3 - 3c^3/((b^2c^3 - a^2bd^3)*(b^3c^3 - a^2d^3))
\end{aligned}$$

$$\begin{aligned}
& + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) \\
& + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^{(2/3)} \\
& *(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b \\
& *c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c \\
& ^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b \\
& *c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c \\
& ^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2 \\
& /(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6))*log(3/2*(2*(1/ \\
& 2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(\\
& 2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d \\
& ^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(\\
& 2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d \\
& ^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) \\
& - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^{(2/3)}*(c \\
& ^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^ \\
& 3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 \\
& - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^ \\
& 3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 \\
& - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/(b \\
& *c^3 - a*d^3))^2 - 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b \\
& *c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a \\
& *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
& ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a \\
& *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
& ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 \\
& - a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6) \\
& *(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a \\
& *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
& ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a \\
& *d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d \\
& ^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 \\
& - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 \\
& - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 \\
& - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} \\
& + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) \\
& + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 \\
& - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^{(2/3)}*(c^4/(b*c^3 - a*d^3)^2 \\
& - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 \\
& - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)} \\
& + (1/2)^{(1/3)}*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) \\
& + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^{(1/3)}*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 \\
& - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x + 2*c))/(b*c^3 - a*d^3)
\end{aligned}$$

giac [A] time = 0.30, size = 320, normalized size = 1.21

$$\frac{-\frac{c^2 d \log(dx + c)}{bc^3 d - ad^4} + \frac{c^2 \log(bx^3 + a)}{3(bc^3 - ad^3)} + \frac{(-ab^2)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^2 - \sqrt{3}(-ab^2)^{\frac{1}{3}}bcd + \sqrt{3}(-ab^2)^{\frac{2}{3}}d^2} + \frac{\left(ab^2c^2d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - a^2bd^5\left(-\frac{a}{b}\right)^{\frac{1}{3}} - ab^2c^4d + a^2bcd^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ab^3c^6 - 2a^2b^2c^3d^3 + a^3bd^6)} + \frac{\left((-ab^2)^{\frac{1}{3}}bcd - (-ab^2)^{\frac{2}{3}}d^2\right) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(b^3c^3 - ab^2d^3)}}{b^3c^3 - a*d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] $-c^2*d*\log(\text{abs}(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*\log(\text{abs}(b*x^3 + a))/(b*c^3 - a*d^3) + (-a*b^2)^{(1/3)}*d*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*b^2*c^2 - \sqrt{3}*(-a*b^2)^{(1/3)}*b*c*d + \sqrt{3}*(-a*b^2)^{(2/3)}*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^{(1/3)} - a^2*b*d^5*(-a/b)^{(1/3)} - a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)})/(a*b^3*c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^{(1/3)}*b*c*d - (-a*b^2)^{(2/3)}*d^2)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(b^3*c^3 - a*b^2*d^3)$

maple [A] time = 0.01, size = 336, normalized size = 1.27

$$\frac{\sqrt{3} a c d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{a c d \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{a c d \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3} a d^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{a d^2 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{a d^2 \ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a d^3-b c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{c^2 \ln\left(b x^3+a\right)}{3\left(a d^3-b c^3\right)}+\frac{c^2 \ln(dx+c)}{a d^3-b c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(d*x+c)/(b*x^3+a),x)

[Out] $-1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+1/6/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/3/(a*d^3-b*c^3)*a*d^2/b/(a/b)^{(1/3)}*\ln(x+(a/b)^{(1/3)})+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/b)^{(1/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})+1/3/(a*d^3-b*c^3)*a*d^2*3^{(1/2)}/b/(a/b)^{(1/3)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))-1/3/(a*d^3-b*c^3)*c^2*\ln(b*x^3+a)+c^2/(a*d^3-b*c^3)*\ln(d*x+c)$

maxima [A] time = 1.71, size = 279, normalized size = 1.06

$$\frac{c^2 \log(dx+c)}{b c^3-a d^3}-\frac{\sqrt{3}\left(a d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a c d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2 c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-a b d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(2 b c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a c d\right) \log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2 c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-a b d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\left(b c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+a d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+a c d\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2 c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-a b d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")

[Out] $-c^2*\log(d*x + c)/(b*c^3 - a*d^3) - 1/3*\sqrt{3}*(a*d^2*(a/b)^{(2/3)} - a*c*d*(a/b)^{(1/3)})*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((b^2*c^3*(a/b)^{(2/3)} - a*b*d^3*(a/b)^{(2/3)))*(a/b)^{(1/3)}) + 1/6*(2*b*c^2*(a/b)^{(2/3)} - a*d^2*(a/b)^{(1/3)} - a*c*d)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*c^3*(a/b)^{(2/3)} - a*b*d^3*(a/b)^{(2/3)}) + 1/3*(b*c^2*(a/b)^{(2/3)} + a*d^2*(a/b)$

$$\sqrt[3]{1} + a*c*d)*\log(x + (a/b)^{1/3})/(b^2*c^3*(a/b)^{2/3} - a*b*d^3*(a/b)^{2/3})$$

mupad [B] time = 2.50, size = 570, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^3)*(c + d*x)),x)`

[Out] `symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3 - b*c^3)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(d*x+c)/(b*x**3+a),x)`

[Out] Timed out

$$3.338 \quad \int \frac{x^2}{(c+dx)(a+bx^4)} dx$$

Optimal. Leaf size=417

$$\frac{\sqrt{a} d^3 \tan^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{2\sqrt{b} (ad^4 + bc^4)} - \frac{c^2 d \log(a + bx^4)}{4(ad^4 + bc^4)} + \frac{c^2 d \log(c + dx)}{ad^4 + bc^4} + \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)}$$

Rubi [A] time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6725, 1461, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{c^2 d \log(a + bx^4)}{4(ad^4 + bc^4)} + \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)} - \frac{c(\sqrt{a} d^2 + \sqrt{b} c^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)} + \frac{\sqrt{a} d^3 \tan^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)}{2\sqrt{b} (ad^4 + bc^4)} + \frac{c^2 d \log(c + dx)}{ad^4 + bc^4} - \frac{c(\sqrt{b} c^2 - \sqrt{a} d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt{a}}\right)}{2\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)} + \frac{c(\sqrt{b} c^2 - \sqrt{a} d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt{a}} + 1\right)}{2\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} (ad^4 + bc^4)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((c + d*x)*(a + b*x^4)), x]

[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```


[{a, c, d, e, p, q}, x]

Rule 1461

Int[((A_) + (B_)*(x_)^(m_.))*((d_) + (e_)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] := Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE x p a n d[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{(c+dx)(a+bx^4)} dx &= \int \left(\frac{c^2 d^2}{(bc^4+ad^4)(c+dx)} + \frac{(c-dx)(-ad^2+bc^2x^2)}{(bc^4+ad^4)(a+bx^4)} \right) dx \\
 &= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)(-ad^2+bc^2x^2)}{a+bx^4} dx}{bc^4+ad^4} \\
 &= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{c \int \frac{-ad^2+bc^2x^2}{a+bx^4} dx}{bc^4+ad^4} - \frac{d \int \frac{x(-ad^2+bc^2x^2)}{a+bx^4} dx}{bc^4+ad^4} \\
 &= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} - \frac{d \operatorname{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2} dx, x, x^2\right)}{2(bc^4+ad^4)} + \frac{\left(c\left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}}\right)\right) \int \frac{\sqrt{a}\sqrt{b+bx^2}}{a+bx^4} dx}{2(bc^4+ad^4)} \\
 &= \frac{c^2 d \log(c+dx)}{bc^4+ad^4} - \frac{(bc^2 d) \operatorname{Subst}\left(\int \frac{x}{a+bx^2} dx, x, x^2\right)}{2(bc^4+ad^4)} + \frac{(ad^3) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^2\right)}{2(bc^4+ad^4)} \\
 &= \frac{\sqrt{a}d^3 \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc^4+ad^4)} + \frac{c^2 d \log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}c\left(c^2 + \frac{\sqrt{a}d^2}{\sqrt{b}}\right) \log\left(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x^2 - \sqrt{a}\right)}{4\sqrt{2} \sqrt[4]{a}(bc^4+ad^4)} \\
 &= \frac{\sqrt{a}d^3 \tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc^4+ad^4)} - \frac{\sqrt[4]{b}c\left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}}\right) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} \sqrt[4]{a}(bc^4+ad^4)} + \frac{\sqrt[4]{b}c\left(c^2 - \frac{\sqrt{a}d^2}{\sqrt{b}}\right) \tan^{-1}\left(\frac{\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x^2 - \sqrt{a}}{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x - \sqrt{a}}\right)}{2\sqrt{2} \sqrt[4]{a}(bc^4+ad^4)}
 \end{aligned}$$

Mathematica [A] time = 0.23, size = 370, normalized size = 0.89

$$\frac{-2(2a^{3/4}b^3 - \sqrt{2}\sqrt{b}ab^2 + \sqrt{2}b^{3/4}c^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}}{\sqrt{c}}\right) + 2(-2a^{3/4}b^3 - \sqrt{2}\sqrt{b}ab^2 + \sqrt{2}b^{3/4}c^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}}{\sqrt{c}} + 1\right) + \sqrt{b}c(\sqrt{2}(\sqrt{a}d^2 + \sqrt{b}c^2)\log(-\sqrt{2}\sqrt{b}\sqrt{c}x + \sqrt{a} + \sqrt{b}x^2) - \sqrt{2}\sqrt{b}c^2\log(\sqrt{2}\sqrt{b}\sqrt{c}x + \sqrt{a} + \sqrt{b}x^2) - 2\sqrt{b}\sqrt{c}d\log(c + dx) + 8\sqrt{b}\sqrt{c}d\log(c + dx) - \sqrt{2}\sqrt{a}d^2\log(\sqrt{2}\sqrt{b}\sqrt{c}x + \sqrt{a} + \sqrt{b}x^2))}{8\sqrt{b}\sqrt{b}(ab^3 + bc^4)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]

[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - 2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4])/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c + dx)(a + bx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^4)),x]

[Out] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^4)),x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.57, size = 401, normalized size = 0.96

$$\frac{c^2 d^2 \log(dx + c) - c^2 d \log(bx^4 + a)}{bc^4 d + ad^5} + \frac{(\sqrt{2}\sqrt{ab}bd + (ab^3)^{\frac{1}{2}}bc)\arctan\left(\frac{\sqrt{2}(2x - \sqrt{2}(\frac{1}{2})^{\frac{1}{2}})}{2(\frac{1}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}ab^2d^2 + \sqrt{2}\sqrt{ab}b^2c^2 - 2(ab^3)^{\frac{1}{2}}cd)} + \frac{(\sqrt{2}\sqrt{ab}bd + (ab^3)^{\frac{1}{2}}bc)\arctan\left(\frac{\sqrt{2}(2x - \sqrt{2}(\frac{1}{2})^{\frac{1}{2}})}{2(\frac{1}{2})^{\frac{1}{2}}}\right)}{2(\sqrt{2}ab^2d^2 + \sqrt{2}\sqrt{ab}b^2c^2 + 2(ab^3)^{\frac{1}{2}}cd)} - \frac{((ab^3)^{\frac{1}{2}}abcd^2 + (ab^3)^{\frac{1}{2}}c^2)\log\left(x^2 + \sqrt{2}x(\frac{1}{2})^{\frac{1}{2}} + \sqrt{\frac{1}{2}}\right)}{4(\sqrt{2}ab^3c^4 + \sqrt{2}a^2b^2d^4)} + \frac{((ab^3)^{\frac{1}{2}}abcd^2 + (ab^3)^{\frac{1}{2}}c^2)\log\left(x^2 - \sqrt{2}x(\frac{1}{2})^{\frac{1}{2}} + \sqrt{\frac{1}{2}}\right)}{4(\sqrt{2}ab^3c^4 + \sqrt{2}a^2b^2d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")

[Out] $c^2 d^2 \log(\text{abs}(d x + c)) / (b c^4 d + a d^5) - 1/4 c^2 d \log(\text{abs}(b x^4 + a)) / (b c^4 + a d^4) + 1/2 (\sqrt{2} \sqrt{a b}) b^3 d + (a b^3)^{1/4} b^3 c \arctan(1/2 \sqrt{2} (2 x + \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}) / (\sqrt{2} a b^2 d^2 + \sqrt{2} \sqrt{a b} b^2 c^2 - 2 (a b^3)^{3/4} c d) + 1/2 (\sqrt{2} \sqrt{a b}) b^3 d + (a b^3)^{1/4} b^3 c \arctan(1/2 \sqrt{2} (2 x - \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}) / (\sqrt{2} a b^2 d^2 + \sqrt{2} \sqrt{a b} b^2 c^2 + 2 (a b^3)^{3/4} c d) - 1/4 ((a b^3)^{1/4} a b^3 c d^2 + (a b^3)^{3/4} c^3) \log(x^2 + \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2} a b^3 c^4 + \sqrt{2} a^2 b^2 d^4) + 1/4 ((a b^3)^{1/4} a b^3 c d^2 + (a b^3)^{3/4} c^3) \log(x^2 - \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2} a b^3 c^4 + \sqrt{2} a^2 b^2 d^4)$

maple [A] time = 0.01, size = 422, normalized size = 1.01

$$\frac{a d^3 \arctan\left(\frac{\sqrt{2} x}{(\frac{a}{b})^{1/4}}\right) + \frac{\sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x - 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x + 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \ln\left(\frac{x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right) - c^2 d \ln(b x^4 + a) + c^2 d \ln(d x + c) + \frac{c^2 d \ln(d x + c)}{a d^4 + b c^4} - \frac{(\frac{a}{b})^{1/4} \sqrt{2} c d^2 \arctan\left(\frac{\sqrt{2} x - 1}{(\frac{a}{b})^{1/4}}\right) - (\frac{a}{b})^{1/4} \sqrt{2} c d^2 \arctan\left(\frac{\sqrt{2} x + 1}{(\frac{a}{b})^{1/4}}\right) - (\frac{a}{b})^{1/4} \sqrt{2} c d^2 \ln\left(\frac{x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right)}{4(a d^4 + b c^4)}}{2(a d^4 + b c^4) \sqrt{a b}} + \frac{\sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x - 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x + 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \ln\left(\frac{x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right) - c^2 d \ln(b x^4 + a) + c^2 d \ln(d x + c)}{4(a d^4 + b c^4) \left(\frac{a}{b}\right)^{1/4}} + \frac{\sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x - 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \arctan\left(\frac{\sqrt{2} x + 1}{(\frac{a}{b})^{1/4}}\right) + \sqrt{2} c^3 \ln\left(\frac{x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right) - c^2 d \ln(b x^4 + a) + c^2 d \ln(d x + c)}{8(a d^4 + b c^4) \left(\frac{a}{b}\right)^{1/4}} - \frac{c^2 d \ln(d x + c)}{4(a d^4 + b c^4)} + \frac{c^2 d \ln(d x + c)}{a d^4 + b c^4} - \frac{(\frac{a}{b})^{1/4} \sqrt{2} c d^2 \arctan\left(\frac{\sqrt{2} x - 1}{(\frac{a}{b})^{1/4}}\right) - (\frac{a}{b})^{1/4} \sqrt{2} c d^2 \arctan\left(\frac{\sqrt{2} x + 1}{(\frac{a}{b})^{1/4}}\right) - (\frac{a}{b})^{1/4} \sqrt{2} c d^2 \ln\left(\frac{x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right)}{4(a d^4 + b c^4)} - \frac{(\frac{a}{b})^{1/4} \sqrt{2} c d^2 \ln\left(\frac{x + (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}}}{(x - (\frac{a}{b})^{1/4} \sqrt{2} x + \sqrt{\frac{a}{b}})\sqrt{2}}\right)}{8(a d^4 + b c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2/(d x + c)/(b x^4 + a), x)$

[Out] $c^2 d \ln(d x + c) / (a d^4 + b c^4) - 1/4 / (a d^4 + b c^4) c^2 d^2 (a/b)^{1/4} 2^{1/2} a \arctan(2^{1/2} / (a/b)^{1/4} x - 1) - 1/8 / (a d^4 + b c^4) c^2 d^2 (a/b)^{1/4} 2^{1/2} \ln((x^2 + (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2})) - 1/4 / (a d^4 + b c^4) c^2 d^2 (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) + 1/2 / (a d^4 + b c^4) a d^3 / (a b)^{1/2} \arctan(x^2 (b/a)^{1/2}) + 1/8 / (a d^4 + b c^4) c^3 / (a/b)^{1/4} 2^{1/2} \ln((x^2 - (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2}) / (x^2 + (a/b)^{1/4} x 2^{1/2} + (a/b)^{1/2})) + 1/4 / (a d^4 + b c^4) c^3 / (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) + 1/4 / (a d^4 + b c^4) c^3 / (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1) - 1/4 c^2 d \ln(b x^4 + a) / (a d^4 + b c^4)$

maxima [A] time = 1.48, size = 349, normalized size = 0.84

$$\frac{c^2 d \log(dx + c)}{b c^4 + a d^4} - \frac{\sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{3}{4}} c^2 d + \sqrt{a} b^2 c^3 + a b c d^2 \right) \log\left(\sqrt{b} x^2 + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right) + \sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{3}{4}} c^2 d - \sqrt{a} b^2 c^3 - a b c d^2 \right) \log\left(\sqrt{b} x^2 - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{\frac{3}{a^{\frac{3}{4}} b^{\frac{3}{4}}}} - \frac{2 \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{3}{4}} c^2 - \sqrt{2} a^{\frac{5}{4}} b^{\frac{5}{4}} c d^2 - 2 a^{\frac{3}{2}} b d^2 \right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{b} x + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{b}}\right)}{\frac{3}{a^{\frac{3}{4}} b^{\frac{3}{4}}}} - \frac{2 \left(\sqrt{2} a^{\frac{3}{4}} b^{\frac{3}{4}} c^2 - \sqrt{2} a^{\frac{5}{4}} b^{\frac{5}{4}} c d^2 + 2 a^{\frac{3}{2}} b d^2 \right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{b} x - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{b}}\right)}{\frac{3}{a^{\frac{3}{4}} b^{\frac{3}{4}}}}}{8(b c^4 + a d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2/(d x + c)/(b x^4 + a), x, \text{algorithm}="maxima")$

[Out] $c^2 d \log(d x + c) / (b c^4 + a d^4) - 1/8 (\sqrt{2} (\sqrt{2} a^{3/4} b^{5/4} c^2 d + \sqrt{2} a^{5/4} b^{5/4} c^2 d^2 + \sqrt{2} a^{3/2} b^{5/4} c^2 d^2) \log(\sqrt{2} x^2 + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a}) / (a^{3/4} b^{5/4}) + \sqrt{2} (\sqrt{2} a^{3/4} b^{5/4} c^2 d - \sqrt{2} a^{5/4} b^{5/4} c^2 d^2 - \sqrt{2} a^{3/2} b^{5/4} c^2 d^2) \log(\sqrt{2} x^2 - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a}) / (a^{3/4} b^{5/4}) - 2 (\sqrt{2} a^{3/4} b^{5/4} c^2 d^2 - \sqrt{2} a^{5/4} b^{5/4} c^2 d^2 + 2 a^{3/2} b^{5/4} c^2 d^2) \arctan(1/2 \sqrt{2} (2 x + \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}) / (\sqrt{2} a b^2 d^2 + \sqrt{2} \sqrt{a b} b^2 c^2 - 2 (a b^3)^{3/4} c d) + 2 (\sqrt{2} a^{3/4} b^{5/4} c^2 d - \sqrt{2} a^{5/4} b^{5/4} c^2 d^2 - \sqrt{2} a^{3/2} b^{5/4} c^2 d^2) \arctan(1/2 \sqrt{2} (2 x - \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}) / (\sqrt{2} a b^2 d^2 + \sqrt{2} \sqrt{a b} b^2 c^2 + 2 (a b^3)^{3/4} c d) - 1/4 ((a b^3)^{1/4} a b^3 c d^2 + (a b^3)^{3/4} c^3) \log(x^2 + \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2} a b^3 c^4 + \sqrt{2} a^2 b^2 d^4) + 1/4 ((a b^3)^{1/4} a b^3 c d^2 + (a b^3)^{3/4} c^3) \log(x^2 - \sqrt{2} x (a/b)^{1/4} + \sqrt{a/b}) / (\sqrt{2} a b^3 c^4 + \sqrt{2} a^2 b^2 d^4)$

$$\begin{aligned} &) * x + \sqrt{2} * a^{(1/4)} * b^{(1/4)} / \sqrt{\sqrt{a} * \sqrt{b}} / (a^{(3/4)} * \sqrt{\sqrt{a}} \\ & * \sqrt{b}) * b^{(5/4)} - 2 * (\sqrt{2} * a^{(3/4)} * b^{(7/4)} * c^3 - \sqrt{2} * a^{(5/4)} * b^{(5/4)} \\ & * c * d^2 + 2 * a^{(3/2)} * b * d^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{b}) * x - \sqrt{2} * a^{(1/4)} \\ & * b^{(1/4)}) / \sqrt{\sqrt{a} * \sqrt{b}} / (a^{(3/4)} * \sqrt{\sqrt{a} * \sqrt{b}} * b^{(5/4)}) \\ & / (b * c^4 + a * d^4) \end{aligned}$$

mupad [B] time = 2.45, size = 823, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x^4)*(c + d*x)),x)`

[Out] `symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k))*b*c^3 + 4*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5 + 320*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*c*d^3 + 160*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^4*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c + d*x))/(a*d^4 + b*c^4)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(d*x+c)/(b*x**4+a),x)`

[Out] Timed out

$$3.339 \quad \int \frac{x}{(1-x)(1+x)^2} dx$$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {77, 207}

$$\frac{1}{2(x+1)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 - x)*(1 + x)^2),x]

[Out] 1/(2*(1 + x)) + ArcTanh[x]/2

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{(1-x)(1+x)^2} dx &= \int \left(-\frac{1}{2(1+x)^2} - \frac{1}{2(-1+x^2)} \right) dx \\ &= \frac{1}{2(1+x)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{1}{4} \left(\frac{2}{x+1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1-x)*(1+x)^2),x]

[Out] (2/(1+x) - Log[1-x] + Log[1+x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-x)(1+x)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((1-x)*(1+x)^2),x]

[Out] IntegrateAlgebraic[x/((1-x)*(1+x)^2),x]

fricas [B] time = 1.30, size = 26, normalized size = 1.62

$$\frac{(x+1)\log(x+1) - (x+1)\log(x-1) + 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")

[Out] 1/4*((x+1)*log(x+1) - (x+1)*log(x-1) + 2)/(x+1)

giac [A] time = 0.24, size = 21, normalized size = 1.31

$$\frac{1}{2(x+1)} - \frac{1}{4} \log \left(\left| -\frac{2}{x+1} + 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")

[Out] 1/2/(x+1) - 1/4*log(abs(-2/(x+1) + 1))

maple [A] time = 0.01, size = 21, normalized size = 1.31

$$-\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1-x)/(x+1)^2,x)`

[Out] `-1/4*ln(x-1)+1/2/(x+1)+1/4*ln(x+1)`

maxima [A] time = 0.75, size = 20, normalized size = 1.25

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")`

[Out] `1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)`

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}(x)}{2} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((x - 1)*(x + 1)^2),x)`

[Out] `atanh(x)/2 + 1/(2*(x + 1))`

sympy [A] time = 0.10, size = 19, normalized size = 1.19

$$-\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1-x)/(1+x)**2,x)`

[Out] `-log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)`

$$3.340 \quad \int \frac{x^2}{(1-x^2)(1+x^2)^2} dx$$

Optimal. Leaf size=19

$$\frac{1}{4} \tanh^{-1}(x) - \frac{x}{4(x^2 + 1)}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {471, 206}

$$\frac{1}{4} \tanh^{-1}(x) - \frac{x}{4(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - x^2)*(1 + x^2)^2), x]

[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1-x^2)(1+x^2)^2} dx &= -\frac{x}{4(1+x^2)} + \frac{1}{4} \int \frac{1}{1-x^2} dx \\ &= -\frac{x}{4(1+x^2)} + \frac{1}{4} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.42

$$\frac{1}{8} \left(-\frac{2x}{x^2 + 1} - \log(1 - x) + \log(x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2), x]

[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1 - x^2)(1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((1 - x^2)*(1 + x^2)^2), x]

[Out] IntegrateAlgebraic[x^2/((1 - x^2)*(1 + x^2)^2), x]

fricas [B] time = 0.65, size = 34, normalized size = 1.79

$$\frac{(x^2 + 1) \log(x + 1) - (x^2 + 1) \log(x - 1) - 2x}{8(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)

giac [A] time = 0.30, size = 30, normalized size = 1.58

$$-\frac{1}{4\left(x + \frac{1}{x}\right)} + \frac{1}{16} \log\left(\left|x + \frac{1}{x} + 2\right|\right) - \frac{1}{16} \log\left(\left|x + \frac{1}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")

[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))

maple [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{x}{4(x^2+1)} - \frac{\ln(x-1)}{8} + \frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-x^2+1)/(x^2+1)^2,x)`

[Out] `-1/8*ln(x-1)-1/4/(x^2+1)*x+1/8*ln(x+1)`

maxima [A] time = 0.65, size = 23, normalized size = 1.21

$$-\frac{x}{4(x^2+1)} + \frac{1}{8} \log(x+1) - \frac{1}{8} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")`

[Out] `-1/4*x/(x^2+1)+1/8*log(x+1)-1/8*log(x-1)`

mupad [B] time = 2.14, size = 17, normalized size = 0.89

$$\frac{\operatorname{atanh}(x)}{4} - \frac{x}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^2/((x^2-1)*(x^2+1)^2),x)`

[Out] `atanh(x)/4-x/(4*(x^2+1))`

sympy [A] time = 0.12, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2+4} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-x**2+1)/(x**2+1)**2,x)`

[Out] `-x/(4*x**2+4)-log(x-1)/8+log(x+1)/8`

$$3.341 \quad \int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$$

Optimal. Leaf size=97

$$-\frac{x}{6(x^3+1)} + \frac{1}{72} \log(x^2-x+1) + \frac{1}{24} \log(x^2+x+1) - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(x+1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {471, 522, 200, 31, 634, 618, 204, 628}

$$-\frac{x}{6(x^3+1)} + \frac{1}{72} \log(x^2-x+1) + \frac{1}{24} \log(x^2+x+1) - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(x+1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1 - x^3)*(1 + x^3)^2), x]

[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/72 + Log[1 + x + x^2]/24

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx &= -\frac{x}{6(1+x^3)} + \frac{1}{6} \int \frac{1+2x^3}{(1-x^3)(1+x^3)} dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12} \int \frac{1}{1+x^3} dx + \frac{1}{4} \int \frac{1}{1-x^3} dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{36} \int \frac{1}{1+x} dx - \frac{1}{36} \int \frac{2-x}{1-x+x^2} dx + \frac{1}{12} \int \frac{1}{1-x} dx + \frac{1}{12} \int \frac{2+x}{1-x+x^2} dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{24} \int \frac{1}{1-x+x^2} dx \\
&= -\frac{x}{6(1+x^3)} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \log(1-x+x^2) + \frac{1}{24} \log(1+x+x^2) \\
&= -\frac{x}{6(1+x^3)} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12} \log(1-x) - \frac{1}{36} \log(1+x) + \frac{1}{72} \log(1-x+x^2) + \frac{1}{24} \log(1+x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.88

$$\frac{1}{72} \left(-\frac{12x}{x^3+1} + \log(x^2-x+1) + 3\log(x^2+x+1) - 6\log(1-x) - 2\log(x+1) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + 6\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((1-x^3)*(1+x^3)^2),x]

[Out] ((-12*x)/(1+x^3) - 2*Sqrt[3]*ArcTan[(-1+2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTan[(1+2*x)/Sqrt[3]] - 6*Log[1-x] - 2*Log[1+x] + Log[1-x+x^2] + 3*Log[1+x+x^2])/72

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/((1-x^3)*(1+x^3)^2),x]

[Out] IntegrateAlgebraic[x^3/((1-x^3)*(1+x^3)^2),x]

fricas [A] time = 1.19, size = 106, normalized size = 1.09

$$\frac{6\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 3(x^3+1)\log(x^2+x+1) + (x^3+1)\log(x^2-x+1) - 2(x^3+1)\log(x+1) - 6(x^3+1)\log(x-1) - 12x}{72(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")

[Out] $\frac{1}{72} \cdot (6 \cdot \sqrt{3} \cdot (x^3 + 1) \cdot \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x + 1)\right) - 2 \cdot \sqrt{3} \cdot (x^3 + 1) \cdot \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x - 1)\right) + 3 \cdot (x^3 + 1) \cdot \log(x^2 + x + 1) + (x^3 + 1) \cdot \log(x^2 - x + 1) - 2 \cdot (x^3 + 1) \cdot \log(x + 1) - 6 \cdot (x^3 + 1) \cdot \log(x - 1) - 12 \cdot x) / (x^3 + 1)$

giac [A] time = 0.31, size = 77, normalized size = 0.79

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24} \log(x^2+x+1) + \frac{1}{72} \log(x^2-x+1) - \frac{1}{36} \log(|x+1|) - \frac{1}{12} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")

[Out] $\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x + 1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x - 1)\right) - \frac{1}{6} \frac{x}{x^3 + 1} + \frac{1}{24} \log(x^2 + x + 1) + \frac{1}{72} \log(x^2 - x + 1) - \frac{1}{36} \log(\text{abs}(x + 1)) - \frac{1}{12} \log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3} \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} + \frac{\ln(x^2-x+1)}{72} + \frac{\ln(x^2+x+1)}{24} + \frac{-2x-2}{36x^2-36x+36} + \frac{1}{18x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(-x^3+1)/(x^3+1)^2,x)

[Out] $-1/12 \cdot \ln(x-1) + 1/36 \cdot (-2x-2)/(x^2-x+1) + 1/72 \cdot \ln(x^2-x+1) - 1/36 \cdot 3^{(1/2)} \cdot \arctan\left(\frac{1}{3} \cdot (2x-1) \cdot 3^{(1/2)}\right) + 1/18 \cdot (x+1) - 1/36 \cdot \ln(x+1) + 1/24 \cdot \ln(x^2+x+1) + 1/12 \cdot 3^{(1/2)} \cdot \arctan\left(\frac{1}{3} \cdot (2x+1) \cdot 3^{(1/2)}\right)$

maxima [A] time = 1.93, size = 75, normalized size = 0.77

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x+1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24} \log(x^2+x+1) + \frac{1}{72} \log(x^2-x+1) - \frac{1}{36} \log(x+1) - \frac{1}{12} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")

[Out] $\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x + 1)\right) - \frac{1}{36} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \cdot (2x - 1)\right) - \frac{1}{6} \frac{x}{x^3 + 1} + \frac{1}{24} \log(x^2 + x + 1) + \frac{1}{72} \log(x^2 - x + 1) - \frac{1}{36} \log(x + 1) - \frac{1}{12} \log(x - 1)$

mupad [B] time = 0.18, size = 103, normalized size = 1.06

$$-\frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} - \frac{x}{6(x^3+1)} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(-\frac{1}{24} + \frac{\sqrt{3}1i}{24}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(\frac{1}{24} + \frac{\sqrt{3}1i}{24}\right) + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)\left(\frac{1}{72} + \frac{\sqrt{3}1i}{72}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)\left(-\frac{1}{72} + \frac{\sqrt{3}1i}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)

[Out] $\log(x + (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/24 + 1/24) - \log(x + 1)/36 - x/(6*(x^3 + 1)) - \log(x - (3^{1/2}*1i)/2 + 1/2)*((3^{1/2}*1i)/24 - 1/24) - \log(x - 1)/12 + \log(x - (3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/72 + 1/72) - \log(x + (3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/72 - 1/72)$

sympy [A] time = 0.36, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6} - \frac{\log(x-1)}{12} - \frac{\log(x+1)}{36} + \frac{\log(x^2-x+1)}{72} + \frac{\log(x^2+x+1)}{24} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)

[Out] $-x/(6*x**3 + 6) - \log(x - 1)/12 - \log(x + 1)/36 + \log(x**2 - x + 1)/72 + \log(x**2 + x + 1)/24 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/36 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/12$

$$3.342 \quad \int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {6725, 203, 260}

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx &= \int \left(\frac{3}{1+x^2} + \frac{x}{3+x^2} \right) dx \\ &= 3 \int \frac{1}{1+x^2} dx + \int \frac{x}{3+x^2} dx \\ &= 3 \tan^{-1}(x) + \frac{1}{2} \log(3+x^2)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 3) + 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

[Out] 3*ArcTan[x] + Log[3 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

[Out] IntegrateAlgebraic[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

fricas [A] time = 1.06, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3), x, algorithm="fricas")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

giac [A] time = 0.29, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$3 \arctan(x) + \frac{\ln(x^2 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)

[Out] 3*arctan(x)+1/2*ln(x^2+3)

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$3 \arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")

[Out] 3*arctan(x) + 1/2*log(x^2 + 3)

mupad [B] time = 2.12, size = 13, normalized size = 0.87

$$\frac{\ln(x^2 + 3)}{2} + 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)

[Out] log(x^2 + 3)/2 + 3*atan(x)

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$\frac{\log(x^2 + 3)}{2} + 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)

[Out] log(x**2 + 3)/2 + 3*atan(x)

$$3.343 \quad \int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Optimal. Leaf size=13

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6725, 203, 260}

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]

[Out] ArcTan[x] + Log[3 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx &= \int \left(\frac{1}{1+x^2} + \frac{x}{3+x^2} \right) dx \\ &= \int \frac{1}{1+x^2} dx + \int \frac{x}{3+x^2} dx \\ &= \tan^{-1}(x) + \frac{1}{2} \log(3+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 3) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

[Out] ArcTan[x] + Log[3 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

[Out] IntegrateAlgebraic[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]

fricas [A] time = 1.05, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3), x, algorithm="fricas")

[Out] arctan(x) + 1/2*log(x^2 + 3)

giac [A] time = 0.30, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")

[Out] arctan(x) + 1/2*log(x^2 + 3)

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln(x^2 + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)

[Out] arctan(x)+1/2*ln(x^2+3)

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log(x^2 + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")

[Out] arctan(x) + 1/2*log(x^2 + 3)

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln(x^2 + 3)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)

[Out] log(x^2 + 3)/2 + atan(x)

sympy [A] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\log(x^2 + 3)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)

[Out] log(x**2 + 3)/2 + atan(x)

$$3.344 \quad \int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=29

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{-4 + 6x - x^2 + 3x^3}{(1 + x^2)(2 + x^2)} dx &= \int \left(\frac{3(-1 + x)}{1 + x^2} + \frac{2}{2 + x^2} \right) dx \\
&= 2 \int \frac{1}{2 + x^2} dx + 3 \int \frac{-1 + x}{1 + x^2} dx \\
&= \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) - 3 \int \frac{1}{1 + x^2} dx + 3 \int \frac{x}{1 + x^2} dx \\
&= -3 \tan^{-1}(x) + \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + \frac{3}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{3}{2} \log(x^2 + 1) - 3 \tan^{-1}(x) + \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 6x - x^2 + 3x^3}{(1 + x^2)(2 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

[Out] IntegrateAlgebraic[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]

fricas [A] time = 1.23, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} x \right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2), x, algorithm="fricas")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

giac [A] time = 0.26, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

maple [A] time = 0.00, size = 25, normalized size = 0.86

$$-3 \arctan(x) + \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{2}\right) + \frac{3 \ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)

[Out] -3*arctan(x)+3/2*ln(x^2+1)+2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.48, size = 24, normalized size = 0.83

$$\sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3 \arctan(x) + \frac{3}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)

mupad [B] time = 2.15, size = 51, normalized size = 1.76

$$-\sqrt{2} \operatorname{atan}\left(\frac{24\sqrt{2}}{24x-64} + \frac{32\sqrt{2}x}{24x-64}\right) + \ln(x-i)\left(\frac{3}{2} + \frac{3}{2}i\right) + \ln(x+1i)\left(\frac{3}{2} - \frac{3}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)

[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))

sympy [A] time = 0.19, size = 29, normalized size = 1.00

$$\frac{3 \log(x^2 + 1)}{2} - 3 \operatorname{atan}(x) + \sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2), x)

[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)

$$3.345 \quad \int \frac{1}{(4-4x+x^2)(5-4x+x^2)} dx$$

Optimal. Leaf size=14

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]

[Out] (2 - x)^(-1) + ArcTan[2 - x]

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 693

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p])) ||

IntegerQ[(m + 2*p + 3)/2])

Rubi steps

$$\begin{aligned} \int \frac{1}{(4 - 4x + x^2)(5 - 4x + x^2)} dx &= \int \frac{1}{(-2 + x)^2 (5 - 4x + x^2)} dx \\ &= \frac{1}{2 - x} - \int \frac{1}{5 - 4x + x^2} dx \\ &= \frac{1}{2 - x} + 2 \operatorname{Subst} \left(\int \frac{1}{-4 - x^2} dx, x, -4 + 2x \right) \\ &= \frac{1}{2 - x} + \tan^{-1}(2 - x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(2 - x) - \frac{1}{x - 2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)), x]

[Out] -(-2 + x)^(-1) + ArcTan[2 - x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(4 - 4x + x^2)(5 - 4x + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)), x]

[Out] IntegrateAlgebraic[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)), x]

fricas [A] time = 1.15, size = 17, normalized size = 1.21

$$-\frac{(x - 2) \arctan(x - 2) + 1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5), x, algorithm="fricas")

[Out] $-\frac{((x - 2) \cdot \arctan(x - 2) + 1)}{(x - 2)}$

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{1}{x - 2} - \arctan(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")`

[Out] $-1/(x - 2) - \arctan(x - 2)$

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$-\arctan(x - 2) - \frac{1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-4*x+4)/(x^2-4*x+5),x)`

[Out] $-\arctan(x-2)-1/(x-2)$

maxima [A] time = 2.07, size = 14, normalized size = 1.00

$$-\frac{1}{x - 2} - \arctan(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")`

[Out] $-1/(x - 2) - \arctan(x - 2)$

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$-\operatorname{atan}(x - 2) - \frac{1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)`

[Out] $-\operatorname{atan}(x - 2) - 1/(x - 2)$

sympy [A] time = 0.14, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x - 2) - \frac{1}{x - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

$$3.346 \quad \int \frac{-3+x+x^2}{(-3+x)x^2} dx$$

Optimal. Leaf size=12

$$\log(3-x) - \frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {893}

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[(-3 + x + x^2)/((-3 + x)*x^2), x]

[Out] -x^(-1) + Log[3 - x]

Rule 893

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned} \int \frac{-3+x+x^2}{(-3+x)x^2} dx &= \int \left(\frac{1}{-3+x} + \frac{1}{x^2} \right) dx \\ &= -\frac{1}{x} + \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(3-x) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2), x]

[Out] $-x^{-1} + \text{Log}[3 - x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + x + x^2}{(-3 + x)x^2} dx$$

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[(-3 + x + x^2)/((-3 + x)*x^2), x]`

[Out] `IntegrateAlgebraic[(-3 + x + x^2)/((-3 + x)*x^2), x]`

fricas [A] time = 1.23, size = 12, normalized size = 1.00

$$\frac{x \log(x - 3) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")`

[Out] `(x*log(x - 3) - 1)/x`

giac [A] time = 0.42, size = 11, normalized size = 0.92

$$-\frac{1}{x} + \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")`

[Out] `-1/x + log(abs(x - 3))`

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\ln(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x-3)/(x-3)/x^2,x)`

[Out] `-1/x+ln(x-3)`

maxima [A] time = 0.75, size = 10, normalized size = 0.83

$$-\frac{1}{x} + \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")

[Out] -1/x + log(x - 3)

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$\ln(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)

[Out] log(x - 3) - 1/x

sympy [A] time = 0.09, size = 7, normalized size = 0.58

$$\log(x - 3) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x-3)/(-3+x)/x**2,x)

[Out] log(x - 3) - 1/x

$$3.347 \quad \int \frac{1+x+4x^2}{x+4x^3} dx$$

Optimal. Leaf size=11

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1593, 1802, 203}

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + 4*x^2)/(x + 4*x^3), x]

[Out] ArcTan[2*x]/2 + Log[x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
 \int \frac{1+x+4x^2}{x+4x^3} dx &= \int \frac{1+x+4x^2}{x(1+4x^2)} dx \\
 &= \int \left(\frac{1}{x} + \frac{1}{1+4x^2} \right) dx \\
 &= \log(x) + \int \frac{1}{1+4x^2} dx \\
 &= \frac{1}{2} \tan^{-1}(2x) + \log(x)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x) + \frac{1}{2} \tan^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3), x]

[Out] ArcTan[2*x]/2 + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+4x^2}{x+4x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + 4*x^2)/(x + 4*x^3), x]

[Out] IntegrateAlgebraic[(1 + x + 4*x^2)/(x + 4*x^3), x]

fricas [A] time = 1.13, size = 9, normalized size = 0.82

$$\frac{1}{2} \arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x), x, algorithm="fricas")

[Out] 1/2*arctan(2*x) + log(x)

giac [A] time = 0.29, size = 10, normalized size = 0.91

$$\frac{1}{2} \arctan(2x) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")

[Out] 1/2*arctan(2*x) + log(abs(x))

maple [A] time = 0.01, size = 10, normalized size = 0.91

$$\frac{\arctan(2x)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2+x+1)/(4*x^3+x),x)

[Out] 1/2*arctan(2*x)+ln(x)

maxima [A] time = 1.61, size = 9, normalized size = 0.82

$$\frac{1}{2} \arctan(2x) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")

[Out] 1/2*arctan(2*x) + log(x)

mupad [B] time = 2.17, size = 17, normalized size = 1.55

$$\ln(x) - \frac{\operatorname{atan}\left(\frac{17}{32\left(\frac{x}{16} - \frac{1}{8}\right)} + 4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)

[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2

sympy [A] time = 0.13, size = 8, normalized size = 0.73

$$\log(x) + \frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2+x+1)/(4*x**3+x),x)

[Out] log(x) + atan(2*x)/2

$$3.348 \quad \int \frac{1-x+3x^2}{-x^2+x^3} dx$$

Optimal. Leaf size=12

$$\frac{1}{x} + 3 \log(1-x)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 893}

$$\frac{1}{x} + 3 \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3), x]

[Out] x^(-1) + 3*Log[1 - x]

Rule 893

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{1-x+3x^2}{-x^2+x^3} dx &= \int \frac{1-x+3x^2}{(-1+x)x^2} dx \\ &= \int \left(\frac{3}{-1+x} - \frac{1}{x^2} \right) dx \\ &= \frac{1}{x} + 3 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{x} + 3 \log(1 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3), x]

[Out] x^(-1) + 3*Log[1 - x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x + 3x^2}{-x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x + 3*x^2)/(-x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 - x + 3*x^2)/(-x^2 + x^3), x]

fricas [A] time = 1.14, size = 13, normalized size = 1.08

$$\frac{3x \log(x - 1) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+1)/(x^3-x^2), x, algorithm="fricas")

[Out] (3*x*log(x - 1) + 1)/x

giac [A] time = 0.38, size = 11, normalized size = 0.92

$$\frac{1}{x} + 3 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-x+1)/(x^3-x^2), x, algorithm="giac")

[Out] 1/x + 3*log(abs(x - 1))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2-x+1)/(x^3-x^2),x)`

[Out] `3*ln(x-1)+1/x`

maxima [A] time = 0.71, size = 10, normalized size = 0.83

$$\frac{1}{x} + 3 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")`

[Out] `1/x + 3*log(x - 1)`

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$3 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(3*x^2 - x + 1)/(x^2 - x^3),x)`

[Out] `3*log(x - 1) + 1/x`

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$3 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2-x+1)/(x**3-x**2),x)`

[Out] `3*log(x - 1) + 1/x`

$$3.349 \quad \int \frac{4+3x+x^2}{x+x^2} dx$$

Optimal. Leaf size=12

$$x + 4 \log(x) - 2 \log(x + 1)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1593, 893}

$$x + 4 \log(x) - 2 \log(x + 1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/(x + x^2), x]

[Out] x + 4*Log[x] - 2*Log[1 + x]

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{4+3x+x^2}{x+x^2} dx &= \int \frac{4+3x+x^2}{x(1+x)} dx \\ &= \int \left(1 + \frac{4}{x} - \frac{2}{1+x} \right) dx \\ &= x + 4 \log(x) - 2 \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + 4 \log(x) - 2 \log(x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]

[Out] x + 4*Log[x] - 2*Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + 3x + x^2}{x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 3*x + x^2)/(x + x^2),x]

[Out] IntegrateAlgebraic[(4 + 3*x + x^2)/(x + x^2), x]

fricas [A] time = 0.93, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")

[Out] x - 2*log(x + 1) + 4*log(x)

giac [A] time = 0.23, size = 14, normalized size = 1.17

$$x - 2 \log(|x + 1|) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")

[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x + 4 \ln(x) - 2 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3*x+4)/(x^2+x),x)

[Out] x+4*ln(x)-2*ln(x+1)

maxima [A] time = 0.60, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
```

```
[Out] x - 2*log(x + 1) + 4*log(x)
```

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$x - 2 \ln(x + 1) + 4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/(x + x^2),x)
```

```
[Out] x - 2*log(x + 1) + 4*log(x)
```

sympy [A] time = 0.10, size = 12, normalized size = 1.00

$$x + 4 \log(x) - 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
```

```
[Out] x + 4*log(x) - 2*log(x + 1)
```

$$3.350 \quad \int \frac{4+x+3x^2}{x+x^3} dx$$

Optimal. Leaf size=17

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1593, 1802, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + x + 3*x^2)/(x + x^3), x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x+3x^2}{x+x^3} dx &= \int \frac{4+x+3x^2}{x(1+x^2)} dx \\
&= \int \left(\frac{4}{x} + \frac{1-x}{1+x^2} \right) dx \\
&= 4 \log(x) + \int \frac{1-x}{1+x^2} dx \\
&= 4 \log(x) + \int \frac{1}{1+x^2} dx - \int \frac{x}{1+x^2} dx \\
&= \tan^{-1}(x) + 4 \log(x) - \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 1) + 4 \log(x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x + 3*x^2)/(x + x^3), x]

[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+x+3x^2}{x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x + 3*x^2)/(x + x^3), x]

[Out] IntegrateAlgebraic[(4 + x + 3*x^2)/(x + x^3), x]

fricas [A] time = 1.26, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)

giac [A] time = 0.24, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\arctan(x) + 4 \ln(x) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+x+4)/(x^3+x),x)

[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)

maxima [A] time = 1.53, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2} \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")

[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)

mupad [B] time = 2.28, size = 23, normalized size = 1.35

$$4 \ln(x) + \ln(x - i) \left(-\frac{1}{2} - \frac{1}{2}i \right) + \ln(x + i) \left(-\frac{1}{2} + \frac{1}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 4)/(x + x^3),x)

[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$4 \log(x) - \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+x+4)/(x**3+x),x)

[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)

$$3.351 \quad \int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx$$

Optimal. Leaf size=13

$$2 \log(4x + 1) - \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1629, 204}

$$2 \log(4x + 1) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)), x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{7-4x+8x^2}{(1+4x)(1+x^2)} dx &= \int \left(\frac{8}{1+4x} + \frac{1}{-1-x^2} \right) dx \\ &= 2 \log(1+4x) + \int \frac{1}{-1-x^2} dx \\ &= -\tan^{-1}(x) + 2 \log(1+4x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$2 \log(4x + 1) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)), x]

[Out] -ArcTan[x] + 2*Log[1 + 4*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7 - 4x + 8x^2}{(1 + 4x)(1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)), x]

fricas [A] time = 1.18, size = 13, normalized size = 1.00

$$- \arctan(x) + 2 \log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1), x, algorithm="fricas")

[Out] -arctan(x) + 2*log(4*x + 1)

giac [A] time = 0.34, size = 14, normalized size = 1.08

$$- \arctan(x) + 2 \log(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1), x, algorithm="giac")

[Out] -arctan(x) + 2*log(abs(4*x + 1))

maple [A] time = 0.01, size = 14, normalized size = 1.08

$$- \arctan(x) + 2 \ln(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1), x)

[Out] -arctan(x)+2*ln(1+4*x)

maxima [A] time = 1.41, size = 13, normalized size = 1.00

$$- \arctan(x) + 2 \log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")

[Out] -arctan(x) + 2*log(4*x + 1)

mupad [B] time = 0.06, size = 19, normalized size = 1.46

$$\operatorname{atan}\left(\frac{4x+1}{x-4}\right) + 2 \ln\left(x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)

[Out] atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)

sympy [A] time = 0.13, size = 10, normalized size = 0.77

$$2 \log\left(x + \frac{1}{4}\right) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)

[Out] 2*log(x + 1/4) - atan(x)

$$3.352 \quad \int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {27, 88}

$$\frac{1}{2(x+1)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(-1+x)(1+2x+x^2)} dx &= \int \frac{x^2}{(-1+x)(1+x)^2} dx \\ &= \int \left(\frac{1}{4(-1+x)} - \frac{1}{2(1+x)^2} + \frac{3}{4(1+x)} \right) dx \\ &= \frac{1}{2(1+x)} + \frac{1}{4} \log(1-x) + \frac{3}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{4} \left(\frac{2}{x+1} + \log(x-1) + 3 \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-1+x)(1+2x+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((-1 + x)*(1 + 2*x + x^2)),x]

[Out] IntegrateAlgebraic[x^2/((-1 + x)*(1 + 2*x + x^2)), x]

fricas [A] time = 1.10, size = 26, normalized size = 0.93

$$\frac{3(x+1)\log(x+1) + (x+1)\log(x-1) + 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")

[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)

giac [A] time = 0.30, size = 22, normalized size = 0.79

$$\frac{1}{2(x+1)} + \frac{3}{4} \log(|x+1|) + \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")

[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{\ln(x-1)}{4} + \frac{3 \ln(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x-1)/(x^2+2*x+1),x)`

[Out] `1/4*ln(x-1)+1/2/(x+1)+3/4*ln(x+1)`

maxima [A] time = 0.66, size = 20, normalized size = 0.71

$$\frac{1}{2(x+1)} + \frac{3}{4} \log(x+1) + \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")`

[Out] `1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)`

mupad [B] time = 0.07, size = 20, normalized size = 0.71

$$\frac{\ln(x-1)}{4} + \frac{3 \ln(x+1)}{4} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((x - 1)*(2*x + x^2 + 1)),x)`

[Out] `log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))`

sympy [A] time = 0.11, size = 20, normalized size = 0.71

$$\frac{\log(x-1)}{4} + \frac{3 \log(x+1)}{4} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-1+x)/(x**2+2*x+1),x)`

[Out] `log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)`

$$3.353 \quad \int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx$$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {893}

$$-\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)), x]

[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx &= \int \left(-\frac{9}{16(-1+2x)^2} + \frac{41}{64(-1+2x)} - \frac{25}{64(3+2x)} \right) dx \\ &= -\frac{9}{32(1-2x)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{9}{32(2x-1)} + \frac{41}{128} \log(1-2x) - \frac{25}{128} \log(2x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]

[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 3x + x^2}{(-1 + 2x)^2(3 + 2x)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]

[Out] IntegrateAlgebraic[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)), x]

fricas [A] time = 1.16, size = 37, normalized size = 1.16

$$-\frac{25(2x-1)\log(2x+3) - 41(2x-1)\log(2x-1) - 36}{128(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")

[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x - 1)

giac [A] time = 0.30, size = 43, normalized size = 1.34

$$\frac{9}{32(2x-1)} - \frac{1}{8} \log\left(\frac{|2x-1|}{2(2x-1)^2}\right) - \frac{25}{128} \log\left(\left|-\frac{4}{2x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")

[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/(2*x - 1) - 1))

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{41 \ln(2x-1)}{128} - \frac{25 \ln(2x+3)}{128} + \frac{9}{32(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)

[Out] $9/32/(2*x-1)+41/128*\ln(2*x-1)-25/128*\ln(3+2*x)$

maxima [A] time = 0.75, size = 26, normalized size = 0.81

$$\frac{9}{32(2x-1)} - \frac{25}{128} \log(2x+3) + \frac{41}{128} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")`

[Out] $9/32/(2*x - 1) - 25/128*\log(2*x + 3) + 41/128*\log(2*x - 1)$

mupad [B] time = 2.23, size = 22, normalized size = 0.69

$$\frac{41 \ln\left(x - \frac{1}{2}\right)}{128} - \frac{25 \ln\left(x + \frac{3}{2}\right)}{128} + \frac{9}{64\left(x - \frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)`

[Out] $(41*\log(x - 1/2))/128 - (25*\log(x + 3/2))/128 + 9/(64*(x - 1/2))$

sympy [A] time = 0.13, size = 26, normalized size = 0.81

$$\frac{41 \log\left(x - \frac{1}{2}\right)}{128} - \frac{25 \log\left(x + \frac{3}{2}\right)}{128} + \frac{9}{64x - 32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)`

[Out] $41*\log(x - 1/2)/128 - 25*\log(x + 3/2)/128 + 9/(64*x - 32)$

$$3.354 \quad \int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$$

Optimal. Leaf size=23

$$\frac{1}{2} \log(x^2 + 1) + 2 \log(1 - x) - 3 \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2} \log(x^2 + 1) + 2 \log(1 - x) - 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]

[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_)^(m_.))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx &= \int \left(\frac{2}{-1+x} + \frac{-3+x}{1+x^2} \right) dx \\
&= 2 \log(1-x) + \int \frac{-3+x}{1+x^2} dx \\
&= 2 \log(1-x) - 3 \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\
&= -3 \tan^{-1}(x) + 2 \log(1-x) + \frac{1}{2} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.22

$$\frac{1}{2} \log((x-1)^2 + 2(x-1) + 2) + 2 \log(x-1) - 3 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]

[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5-4x+3x^2}{(-1+x)(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]

fricas [A] time = 1.19, size = 19, normalized size = 0.83

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1), x, algorithm="fricas")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)

giac [A] time = 0.31, size = 20, normalized size = 0.87

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-3 \arctan(x) + 2 \ln(x - 1) + \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)

[Out] 2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)

maxima [A] time = 1.56, size = 19, normalized size = 0.83

$$-3 \arctan(x) + \frac{1}{2} \log(x^2 + 1) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")

[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)

mupad [B] time = 0.05, size = 25, normalized size = 1.09

$$2 \ln(x - 1) + \ln(x - i) \left(\frac{1}{2} + \frac{3i}{2} \right) + \ln(x + 1i) \left(\frac{1}{2} - \frac{3i}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)

[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)

sympy [A] time = 0.14, size = 19, normalized size = 0.83

$$2 \log(x - 1) + \frac{\log(x^2 + 1)}{2} - 3 \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)

[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)

$$3.355 \quad \int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=24

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x-1} + \log(1-x) + \tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x-1} + \log(1-x) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1629

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-1 - 2x + x^2}{(-1 + x)^2 (1 + x^2)} dx &= \int \left(-\frac{1}{(-1 + x)^2} + \frac{1}{-1 + x} + \frac{1 - x}{1 + x^2} \right) dx \\
&= \frac{1}{-1 + x} + \log(1 - x) + \int \frac{1 - x}{1 + x^2} dx \\
&= \frac{1}{-1 + x} + \log(1 - x) + \int \frac{1}{1 + x^2} dx - \int \frac{x}{1 + x^2} dx \\
&= \frac{1}{-1 + x} + \tan^{-1}(x) + \log(1 - x) - \frac{1}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$-\frac{1}{2} \log(x^2 + 1) + \frac{1}{x - 1} + \log(x - 1) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 - 2x + x^2}{(-1 + x)^2 (1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]

fricas [A] time = 1.35, size = 36, normalized size = 1.50

$$\frac{2(x - 1) \arctan(x) - (x - 1) \log(x^2 + 1) + 2(x - 1) \log(x - 1) + 2}{2(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1), x, algorithm="fricas")

[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)/(x - 1)

giac [B] time = 0.30, size = 47, normalized size = 1.96

$$\frac{1}{4}\pi - \pi \left[\frac{\pi + 4 \arctan(x)}{4\pi} + \frac{1}{2} \right] + \frac{1}{x-1} + \arctan(x) - \frac{1}{2} \log \left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")

[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x) - 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \ln(x-1) - \frac{\ln(x^2+1)}{2} + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)

[Out] ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)

maxima [A] time = 1.60, size = 20, normalized size = 0.83

$$\frac{1}{x-1} + \arctan(x) - \frac{1}{2} \log(x^2+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")

[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)

mupad [B] time = 2.11, size = 28, normalized size = 1.17

$$\ln(x-1) + \frac{1}{x-1} + \ln(x-i) \left(-\frac{1}{2} - \frac{1}{2}i \right) + \ln(x+1i) \left(-\frac{1}{2} + \frac{1}{2}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)

[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1)

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)

[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)

$$3.356 \quad \int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)} dx$$

Optimal. Leaf size=49

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

Rubi [A] time = 0.14, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6728, 634, 618, 204, 628, 617}

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

Antiderivative was successfully verified.

[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)), x]

[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{5 + x^3}{(10 - 6x + x^2) \left(\frac{1}{2} - x + x^2\right)} dx &= \int \left(\frac{2(345 + 56x)}{221(10 - 6x + x^2)} + \frac{2(76 + 109x)}{221(1 - 2x + 2x^2)} \right) dx \\ &= \frac{2}{221} \int \frac{345 + 56x}{10 - 6x + x^2} dx + \frac{2}{221} \int \frac{76 + 109x}{1 - 2x + 2x^2} dx \\ &= \frac{109}{442} \int \frac{-2 + 4x}{1 - 2x + 2x^2} dx + \frac{56}{221} \int \frac{-6 + 2x}{10 - 6x + x^2} dx + \frac{261}{221} \int \frac{1}{1 - 2x + 2x^2} dx \\ &= \frac{56}{221} \log(10 - 6x + x^2) + \frac{109}{442} \log(1 - 2x + 2x^2) + \frac{261}{221} \operatorname{Subst} \left(\int \frac{1}{-1 - x^2} dx \right) \\ &= -\frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x) + \frac{56}{221} \log(10 - 6x + x^2) + \frac{109}{442} \log(1 - 2x + 2x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{56}{221} \log(x^2 - 6x + 10) + \frac{109}{442} \log(2x^2 - 2x + 1) - \frac{261}{221} \tan^{-1}(1 - 2x) - \frac{1026}{221} \tan^{-1}(3 - x)$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]

[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x + x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5 + x^3}{(10 - 6x + x^2)\left(\frac{1}{2} - x + x^2\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)), x]

[Out] IntegrateAlgebraic[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)), x]

fricas [A] time = 1.30, size = 37, normalized size = 0.76

$$\frac{261}{221} \arctan(2x - 1) + \frac{1026}{221} \arctan(x - 3) + \frac{109}{442} \log\left(x^2 - x + \frac{1}{2}\right) + \frac{56}{221} \log(x^2 - 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2), x, algorithm="fricas")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^2 - x + 1/2) + 56/221*log(x^2 - 6*x + 10)

giac [A] time = 0.28, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x - 1) + \frac{1026}{221} \arctan(x - 3) + \frac{109}{442} \log(2x^2 - 2x + 1) + \frac{56}{221} \log(x^2 - 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2), x, algorithm="giac")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x + 1) + 56/221*log(x^2 - 6*x + 10)

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{1026 \arctan(x - 3)}{221} + \frac{261 \arctan(2x - 1)}{221} + \frac{56 \ln(x^2 - 6x + 10)}{221} + \frac{109 \ln(2x^2 - 2x + 1)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2), x)

[Out] 261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*ln(2*x^2-2*x+1)

maxima [A] time = 1.71, size = 39, normalized size = 0.80

$$\frac{261}{221} \arctan(2x - 1) + \frac{1026}{221} \arctan(x - 3) + \frac{109}{442} \log(2x^2 - 2x + 1) + \frac{56}{221} \log(x^2 - 6x + 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")

[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x + 1) + 56/221*log(x^2 - 6*x + 10)

mupad [B] time = 2.16, size = 41, normalized size = 0.84

$$\ln(x - 3 - i) \left(\frac{56}{221} - \frac{513i}{221} \right) + \ln(x - 3 + i) \left(\frac{56}{221} + \frac{513i}{221} \right) + \ln\left(x - \frac{1}{2} - \frac{1}{2}i\right) \left(\frac{109}{442} - \frac{261i}{442} \right) + \ln\left(x - \frac{1}{2} + \frac{1}{2}i\right) \left(\frac{109}{442} + \frac{261i}{442} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)

[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/221) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(109/442 + 261i/442)

sympy [A] time = 0.22, size = 44, normalized size = 0.90

$$\frac{56 \log(x^2 - 6x + 10)}{221} + \frac{109 \log\left(x^2 - x + \frac{1}{2}\right)}{442} + \frac{1026 \operatorname{atan}(x - 3)}{221} + \frac{261 \operatorname{atan}(2x - 1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)

[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3)/221 + 261*atan(2*x - 1)/221

$$3.357 \quad \int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} dx$$

Optimal. Leaf size=25

$$4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x)$$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1612}

$$4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)), x]

[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]

Rule 1612

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[m, n]

Rubi steps

$$\begin{aligned} \int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} dx &= \int \left(\frac{11}{-3+x} - \frac{14}{-2+x} + \frac{4}{-1+x} \right) dx \\ &= 4 \log(1-x) - 14 \log(2-x) + 11 \log(3-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.76

$$11 \log(x-3) - 14 \log(x-2) + 4 \log(x-1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)), x]

[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + 3x + x^2}{(-3 + x)(-2 + x)(-1 + x)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] IntegrateAlgebraic[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)), x]

fricas [A] time = 1.13, size = 19, normalized size = 0.76

$$4 \log(x - 1) - 14 \log(x - 2) + 11 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")

[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)

giac [A] time = 0.39, size = 22, normalized size = 0.88

$$4 \log(|x - 1|) - 14 \log(|x - 2|) + 11 \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")

[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$11 \ln(x - 3) - 14 \ln(x - 2) + 4 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+3*x+4)/(x-3)/(x-2)/(x-1),x)

[Out] 4*ln(x-1)-14*ln(x-2)+11*ln(x-3)

maxima [A] time = 1.07, size = 19, normalized size = 0.76

$$4 \log(x - 1) - 14 \log(x - 2) + 11 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")

[Out] $4 \cdot \log(x - 1) - 14 \cdot \log(x - 2) + 11 \cdot \log(x - 3)$

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \ln(x - 1) - 14 \ln(x - 2) + 11 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)`

[Out] $4 \cdot \log(x - 1) - 14 \cdot \log(x - 2) + 11 \cdot \log(x - 3)$

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$11 \log(x - 3) - 14 \log(x - 2) + 4 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)`

[Out] $11 \cdot \log(x - 3) - 14 \cdot \log(x - 2) + 4 \cdot \log(x - 1)$

$$3.358 \quad \int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx$$

Optimal. Leaf size=60

$$-\frac{481 \log(x^2 + x + 1)}{5586} - \frac{79}{273(x + 5)} + \frac{200 \log(3 - 2x)}{3211} + \frac{2731 \log(x + 5)}{24843} + \frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Rubi [A] time = 0.25, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481 \log(x^2 + x + 1)}{5586} - \frac{79}{273(x + 5)} + \frac{200 \log(3 - 2x)}{3211} + \frac{2731 \log(x + 5)}{24843} + \frac{451 \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 6728

$\text{Int}[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))], x_Symbol] \ :> \ \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n + c*x^(2*n)), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx &= \int \left(\frac{79}{273(5+x)^2} + \frac{2731}{24843(5+x)} + \frac{400}{3211(-3+2x)} + \frac{-15-481x}{2793(1+x+x^2)} \right) dx \\ &= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} + \frac{\int \frac{-15-481x}{1+x+x^2} dx}{2793} \\ &= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} + \frac{451 \int \frac{1}{1+x+x^2} dx}{5586} - \frac{481 \log(1+x+x^2)}{5586} \\ &= -\frac{79}{273(5+x)} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} - \frac{481 \log(1+x+x^2)}{5586} \\ &= -\frac{79}{273(5+x)} + \frac{451 \tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}} + \frac{200 \log(3-2x)}{3211} + \frac{2731 \log(5+x)}{24843} - \end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{-243867 \log(x^2 + x + 1) - \frac{819546}{x+5} + 176400 \log(3-2x) + 311334 \log(x+5) + 152438\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

[Out] IntegrateAlgebraic[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]

fricas [A] time = 1.60, size = 60, normalized size = 1.00

$$\frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - 243867(x+5)\log(x^2+x+1) + 176400(x+5)\log(2x-3) + 311334(x+5)\log(x+5) - 819546}{2832102(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1), x, algorithm="fricas")

[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x + 5) - 819546)/(x + 5)

giac [A] time = 0.34, size = 60, normalized size = 1.00

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log\left(-\frac{9}{x+5} + \frac{21}{(x+5)^2} + 1\right) + \frac{200}{3211}\log\left(\left|-\frac{13}{x+5} + 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1), x, algorithm="giac")

[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{451\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{8379} + \frac{200\ln(2x-3)}{3211} + \frac{2731\ln(x+5)}{24843} - \frac{481\ln(x^2+x+1)}{5586} - \frac{79}{273(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1), x)

[Out] -79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)-481/5586*ln(x^2+x+1)+451/8379*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))

maxima [A] time = 2.80, size = 47, normalized size = 0.78

$$\frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{79}{273(x+5)} - \frac{481}{5586}\log(x^2+x+1) + \frac{200}{3211}\log(2x-3) + \frac{2731}{24843}\log(x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")

[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)

mupad [B] time = 0.13, size = 61, normalized size = 1.02

$$\frac{200 \ln\left(x - \frac{3}{2}\right)}{3211} + \frac{2731 \ln(x+5)}{24843} - \frac{79}{273(x+5)} - \ln\left(x + \frac{1}{2} - \frac{\sqrt{3} i}{2}\right) \left(\frac{481}{5586} + \frac{\sqrt{3} 451 i}{16758}\right) + \ln\left(x + \frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(-\frac{481}{5586} + \frac{\sqrt{3} 451 i}{16758}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)

[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)

sympy [A] time = 0.25, size = 63, normalized size = 1.05

$$\frac{200 \log\left(x - \frac{3}{2}\right)}{3211} + \frac{2731 \log(x+5)}{24843} - \frac{481 \log(x^2 + x + 1)}{5586} + \frac{451 \sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{8379} - \frac{79}{273x + 1365}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)

[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586 + 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)

$$3.359 \quad \int \frac{-1+x^3}{1+x+x^2} dx$$

Optimal. Leaf size=11

$$\frac{x^2}{2} - x$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1586}

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(1 + x + x^2), x]

[Out] -x + x^2/2

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{1+x+x^2} dx &= \int (-1+x) dx \\ &= -x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(1 + x + x^2), x]

[Out] -x + x^2/2

IntegrateAlgebraic [A] time = 0.02, size = 9, normalized size = 0.82

$$\frac{1}{2}(x-1)^2$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^3)/(1 + x + x^2), x]

[Out] (-1 + x)^2/2

fricas [A] time = 1.17, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1), x, algorithm="fricas")

[Out] 1/2*x^2 - x

giac [A] time = 0.23, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1), x, algorithm="giac")

[Out] 1/2*x^2 - x

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x^2+x+1), x)

[Out] -x+1/2*x^2

maxima [A] time = 0.84, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")

[Out] 1/2*x^2 - x

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x + x^2 + 1),x)

[Out] (x*(x - 2))/2

sympy [A] time = 0.06, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(x**2+x+1),x)

[Out] x**2/2 - x

$$3.360 \quad \int \frac{-3+x^3}{-7-6x+x^2} dx$$

Optimal. Leaf size=29

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1657, 632, 31}

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1657

Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand[Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-3 + x^3}{-7 - 6x + x^2} dx &= \int \left(6 + x + \frac{39 + 43x}{-7 - 6x + x^2} \right) dx \\
&= 6x + \frac{x^2}{2} + \int \frac{39 + 43x}{-7 - 6x + x^2} dx \\
&= 6x + \frac{x^2}{2} + \frac{1}{2} \int \frac{1}{1+x} dx + \frac{85}{2} \int \frac{1}{-7+x} dx \\
&= 6x + \frac{x^2}{2} + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2} + 6x + \frac{85}{2} \log(7-x) + \frac{1}{2} \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2), x]

[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + x^3}{-7 - 6x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + x^3)/(-7 - 6*x + x^2), x]

[Out] IntegrateAlgebraic[(-3 + x^3)/(-7 - 6*x + x^2), x]

fricas [A] time = 1.14, size = 21, normalized size = 0.72

$$\frac{1}{2} x^2 + 6x + \frac{1}{2} \log(x+1) + \frac{85}{2} \log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7), x, algorithm="fricas")

[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)

giac [A] time = 0.35, size = 23, normalized size = 0.79

$$\frac{1}{2} x^2 + 6x + \frac{1}{2} \log(|x+1|) + \frac{85}{2} \log(|x-7|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")

[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{\ln(x+1)}{2} + \frac{85\ln(x-7)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3)/(x^2-6*x-7),x)

[Out] 1/2*x^2+6*x+1/2*ln(x+1)+85/2*ln(x-7)

maxima [A] time = 1.10, size = 21, normalized size = 0.72

$$\frac{1}{2}x^2 + 6x + \frac{1}{2}\log(x+1) + \frac{85}{2}\log(x-7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")

[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)

mupad [B] time = 2.12, size = 21, normalized size = 0.72

$$6x + \frac{\ln(x+1)}{2} + \frac{85\ln(x-7)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - 3)/(6*x - x^2 + 7),x)

[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{85\log(x-7)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3)/(x**2-6*x-7),x)

[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2

$$3.361 \quad \int \frac{1+x^3}{(13+4x+x^2)^2} dx$$

Optimal. Leaf size=45

$$\frac{47x + 67}{18(x^2 + 4x + 13)} + \frac{1}{2} \log(x^2 + 4x + 13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1660, 634, 618, 204, 628}

$$\frac{47x + 67}{18(x^2 + 4x + 13)} + \frac{1}{2} \log(x^2 + 4x + 13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]

[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

`[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{(13+4x+x^2)^2} dx &= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{36} \int \frac{-50+36x}{13+4x+x^2} dx \\ &= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{2} \int \frac{4+2x}{13+4x+x^2} dx - \frac{61}{18} \int \frac{1}{13+4x+x^2} dx \\ &= \frac{67+47x}{18(13+4x+x^2)} + \frac{1}{2} \log(13+4x+x^2) + \frac{61}{9} \text{Subst}\left(\int \frac{1}{-36-x^2} dx, x, 4+2x\right) \\ &= \frac{67+47x}{18(13+4x+x^2)} - \frac{61}{54} \tan^{-1}\left(\frac{2+x}{3}\right) + \frac{1}{2} \log(13+4x+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$\frac{47x+67}{18(x^2+4x+13)} + \frac{1}{2} \log(x^2+4x+13) - \frac{61}{54} \tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2, x]

[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{(13+4x+x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^3)/(13 + 4*x + x^2)^2, x]

[Out] IntegrateAlgebraic[(1 + x^3)/(13 + 4*x + x^2)^2, x]

fricas [A] time = 0.74, size = 52, normalized size = 1.16

$$\frac{61(x^2 + 4x + 13) \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) - 27(x^2 + 4x + 13) \log(x^2 + 4x + 13) - 141x - 201}{54(x^2 + 4x + 13)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")

[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)

giac [A] time = 0.30, size = 37, normalized size = 0.82

$$\frac{47x + 67}{18(x^2 + 4x + 13)} - \frac{61}{54} \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) + \frac{1}{2} \log(x^2 + 4x + 13)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")

[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2 + 4*x + 13)

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{61 \arctan\left(\frac{x}{3} + \frac{2}{3}\right)}{54} + \frac{\ln(x^2 + 4x + 13)}{2} + \frac{\frac{47x}{18} + \frac{67}{18}}{x^2 + 4x + 13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^2+4*x+13)^2,x)

[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)

maxima [A] time = 1.84, size = 37, normalized size = 0.82

$$\frac{47x + 67}{18(x^2 + 4x + 13)} - \frac{61}{54} \arctan\left(\frac{1}{3}x + \frac{2}{3}\right) + \frac{1}{2} \log(x^2 + 4x + 13)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")

[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2 + 4*x + 13)

mupad [B] time = 0.04, size = 49, normalized size = 1.09

$$\frac{\ln(x^2 + 4x + 13)}{2} - \frac{61 \operatorname{atan}\left(\frac{x}{3} + \frac{2}{3}\right)}{54} + \frac{47x}{18(x^2 + 4x + 13)} + \frac{67}{18(x^2 + 4x + 13)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)/(4*x + x^2 + 13)^2,x)

[Out] log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 13)) + 67/(18*(4*x + x^2 + 13))

sympy [A] time = 0.14, size = 37, normalized size = 0.82

$$\frac{47x + 67}{18x^2 + 72x + 234} + \frac{\log(x^2 + 4x + 13)}{2} - \frac{61 \operatorname{atan}\left(\frac{x}{3} + \frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)

[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 + 2/3)/54

$$3.362 \quad \int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2} dx$$

Optimal. Leaf size=32

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Rubi [A] time = 0.25, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {6725, 203, 261, 635, 260}

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^2), x]

[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-32 + 36x - 42x^2 + 21x^3 - 10x^4 + 3x^5}{x(1+x^2)(4+x^2)^2} dx &= \int \left(-\frac{2}{x} + \frac{2}{1+x^2} - \frac{2x}{(4+x^2)^2} + \frac{1+2x}{4+x^2} \right) dx \\ &= -2 \log(x) + 2 \int \frac{1}{1+x^2} dx - 2 \int \frac{x}{(4+x^2)^2} dx + \int \frac{1+2x}{4+x^2} dx \\ &= \frac{1}{4+x^2} + 2 \tan^{-1}(x) - 2 \log(x) + 2 \int \frac{x}{4+x^2} dx + \int \frac{1}{4+x^2} dx \\ &= \frac{1}{4+x^2} + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + 2 \tan^{-1}(x) - 2 \log(x) + \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{1}{x^2+4} + \log(x^2+4) - 2\log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + 2 \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
x^2)^2), x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-32 + 36x - 42x^2 + 21x^3 - 10x^4 + 3x^5}{x(1+x^2)(4+x^2)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 +
x^2)*(4 + x^2)^2), x]
```

```
[Out] IntegrateAlgebraic[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 +
x^2)*(4 + x^2)^2), x]
```

fricas [A] time = 1.22, size = 52, normalized size = 1.62

$$\frac{(x^2 + 4) \arctan\left(\frac{1}{2}x\right) + 4(x^2 + 4) \arctan(x) + 2(x^2 + 4) \log(x^2 + 4) - 4(x^2 + 4) \log(x) + 2}{2(x^2 + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")

[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2 + 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)

giac [A] time = 0.32, size = 29, normalized size = 0.91

$$\frac{1}{x^2 + 4} + \frac{1}{2} \arctan\left(\frac{1}{2}x\right) + 2 \arctan(x) + \log(x^2 + 4) - 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="giac")

[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x))

maple [A] time = 0.01, size = 29, normalized size = 0.91

$$2 \arctan(x) + \frac{\arctan\left(\frac{x}{2}\right)}{2} - 2 \ln(x) + \ln(x^2 + 4) + \frac{1}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)

[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)

maxima [A] time = 2.02, size = 28, normalized size = 0.88

$$\frac{1}{x^2 + 4} + \frac{1}{2} \arctan\left(\frac{1}{2}x\right) + 2 \arctan(x) + \log(x^2 + 4) - 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")

[Out] $1/(x^2 + 4) + 1/2 \arctan(1/2x) + 2 \arctan(x) + \log(x^2 + 4) - 2 \log(x)$

mupad [B] time = 0.07, size = 44, normalized size = 1.38

$$\frac{1}{x^2 + 4} - 2 \ln(x) - 2 \operatorname{atan}\left(\frac{328000}{7(36288x - 19584)} + \frac{34}{63}\right) + \ln(x - 2i) \left(1 - \frac{1}{4}i\right) + \ln(x + 2i) \left(1 + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2), x)`

[Out] $\log(x - 2i) * (1 - 1i/4) + \log(x + 2i) * (1 + 1i/4) - 2 * \operatorname{atan}(328000 / (7 * (36288 * x - 19584))) + 34/63 - 2 * \log(x) + 1 / (x^2 + 4)$

sympy [A] time = 0.26, size = 29, normalized size = 0.91

$$-2 \log(x) + \log(x^2 + 4) + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2} + 2 \operatorname{atan}(x) + \frac{1}{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2, x)`

[Out] $-2 * \log(x) + \log(x^2 + 4) + \operatorname{atan}(x/2)/2 + 2 * \operatorname{atan}(x) + 1 / (x^2 + 4)$

$$3.363 \quad \int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8} dx$$

Optimal. Leaf size=148

$$\frac{x^2}{2} - \frac{\log(x^2 - \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} + \frac{\log(x^2 + \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}} + 1\right)}{2\sqrt{2} 7^{3/4}}$$

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1790, 1403, 211, 1165, 628, 1162, 617, 204, 1584, 1478, 275, 321, 207}

$$\frac{x^2}{2} - \frac{\log(x^2 - \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} + \frac{\log(x^2 + \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{4\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}} + 1\right)}{2\sqrt{2} 7^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1403

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2
_))^(p_), x_Symbol] := Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
```


FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1478

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[(f*x)^m*(d + e*x^n)^(q + p)*(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1790

Int[(Pq_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n)], {k, 0, (q - j)/n + 1}*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && !PolyQ[Pq, x^n]

Rubi steps

$$\begin{aligned}
\int \frac{-1 + x^4 + 7x^5 + x^9}{-7 + 6x^4 + x^8} dx &= \int \left(\frac{-1 + x^4}{-7 + 6x^4 + x^8} + \frac{x(7x^4 + x^8)}{-7 + 6x^4 + x^8} \right) dx \\
&= \int \frac{-1 + x^4}{-7 + 6x^4 + x^8} dx + \int \frac{x(7x^4 + x^8)}{-7 + 6x^4 + x^8} dx \\
&= \int \frac{1}{7 + x^4} dx + \int \frac{x^5(7 + x^4)}{-7 + 6x^4 + x^8} dx \\
&= \frac{\int \frac{\sqrt{7-x^2}}{7+x^4} dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7+x^2}}{7+x^4} dx}{2\sqrt{7}} + \int \frac{x^5}{-1 + x^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{-1 + x^2} dx, x, x^2 \right) - \frac{\int \frac{\sqrt{2} \sqrt[4]{7} + 2x}{-\sqrt{7} - \sqrt{2} \sqrt[4]{7} x - x^2} dx}{4\sqrt{2} 7^{3/4}} - \frac{\int \frac{\sqrt{2} \sqrt[4]{7} - 2x}{-\sqrt{7} + \sqrt{2} \sqrt[4]{7} x - x^2} dx}{4\sqrt{2} 7^{3/4}} + \frac{\int \frac{1}{\sqrt{7}}}{4\sqrt{2} 7^{3/4}} \\
&= \frac{x^2}{2} - \frac{\log(\sqrt{7} - \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}} + \frac{\log(\sqrt{7} + \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1 + x^2} \right) \\
&= \frac{x^2}{2} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2} 7^{3/4}} - \frac{1}{2} \tanh^{-1}(x^2) - \frac{\log(\sqrt{7} - \sqrt{2} \sqrt[4]{7} x + x^2)}{4\sqrt{2} 7^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 159, normalized size = 1.07

$$\frac{1}{56} \left(28x^2 - 14 \log(x^2 + 1) - \sqrt{2} \sqrt[4]{7} \log(\sqrt{7}x^2 - \sqrt{2} 7^{3/4}x + 7) + \sqrt{2} \sqrt[4]{7} \log(\sqrt{7}x^2 + \sqrt{2} 7^{3/4}x + 7) + 14 \log(1 - x) + 14 \log(x + 1) - 2\sqrt{2} \sqrt[4]{7} \tan^{-1}\left(1 - \frac{\sqrt{2}x}{\sqrt[4]{7}}\right) + 2\sqrt{2} \sqrt[4]{7} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*Log[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sqrt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + x^4 + 7x^5 + x^9}{-7 + 6x^4 + x^8} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

[Out] IntegrateAlgebraic[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]

fricas [A] time = 1.29, size = 178, normalized size = 1.20

$$-\frac{1}{686} \cdot 343^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{7} \cdot 343^{\frac{1}{4}} \sqrt{2} x + \frac{1}{49} \cdot 343^{\frac{3}{4}} \sqrt{2} \sqrt{343^{\frac{1}{4}} \sqrt{2} x + 49 x^2 + 49 \sqrt{7}} - 1\right) - \frac{1}{686} \cdot 343^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{7} \cdot 343^{\frac{1}{4}} \sqrt{2} x + \frac{1}{49} \cdot 343^{\frac{3}{4}} \sqrt{2} \sqrt{-343^{\frac{1}{4}} \sqrt{2} x + 49 x^2 + 49 \sqrt{7}} + 1\right) + \frac{1}{2744} \cdot 343^{\frac{3}{4}} \sqrt{2} \log(343^{\frac{1}{4}} \sqrt{2} x + 49 x^2 + 49 \sqrt{7}) - \frac{1}{2744} \cdot 343^{\frac{3}{4}} \sqrt{2} \log(-343^{\frac{1}{4}} \sqrt{2} x + 49 x^2 + 49 \sqrt{7}) + \frac{1}{2} x^2 - \frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")

[Out] $-1/686 \cdot 343^{(3/4)} \cdot \text{sqrt}(2) \cdot \arctan(-1/7 \cdot 343^{(1/4)} \cdot \text{sqrt}(2) \cdot x + 1/49 \cdot 343^{(1/4)} \cdot \text{sqrt}(2) \cdot \text{sqrt}(343^{(3/4)} \cdot \text{sqrt}(2) \cdot x + 49 \cdot x^2 + 49 \cdot \text{sqrt}(7)) - 1) - 1/686 \cdot 343^{(3/4)} \cdot \text{sqrt}(2) \cdot \arctan(-1/7 \cdot 343^{(1/4)} \cdot \text{sqrt}(2) \cdot x + 1/49 \cdot 343^{(1/4)} \cdot \text{sqrt}(2) \cdot \text{sqrt}(-343^{(3/4)} \cdot \text{sqrt}(2) \cdot x + 49 \cdot x^2 + 49 \cdot \text{sqrt}(7)) + 1) + 1/2744 \cdot 343^{(3/4)} \cdot \text{sqrt}(2) \cdot \log(343^{(3/4)} \cdot \text{sqrt}(2) \cdot x + 49 \cdot x^2 + 49 \cdot \text{sqrt}(7)) - 1/2744 \cdot 343^{(3/4)} \cdot \text{sqrt}(2) \cdot \log(-343^{(3/4)} \cdot \text{sqrt}(2) \cdot x + 49 \cdot x^2 + 49 \cdot \text{sqrt}(7)) + 1/2 \cdot x^2 - 1/4 \cdot \log(x^2 + 1) + 1/4 \cdot \log(x^2 - 1)$

giac [A] time = 0.39, size = 122, normalized size = 0.82

$$\frac{1}{2} x^2 + \frac{1}{28} \cdot 28^{\frac{1}{4}} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} (2x + 7^{\frac{1}{4}} \sqrt{2})\right) + \frac{1}{28} \cdot 28^{\frac{1}{4}} \arctan\left(\frac{1}{14} \cdot 7^{\frac{3}{4}} \sqrt{2} (2x - 7^{\frac{1}{4}} \sqrt{2})\right) + \frac{1}{56} \cdot 28^{\frac{1}{4}} \log(x^2 + 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}) - \frac{1}{56} \cdot 28^{\frac{1}{4}} \log(x^2 - 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(x + 1) + \frac{1}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")

[Out] $1/2 \cdot x^2 + 1/28 \cdot 28^{(1/4)} \cdot \arctan(1/14 \cdot 7^{(3/4)} \cdot \text{sqrt}(2) \cdot (2 \cdot x + 7^{(1/4)} \cdot \text{sqrt}(2))) + 1/28 \cdot 28^{(1/4)} \cdot \arctan(1/14 \cdot 7^{(3/4)} \cdot \text{sqrt}(2) \cdot (2 \cdot x - 7^{(1/4)} \cdot \text{sqrt}(2))) + 1/56 \cdot 28^{(1/4)} \cdot \log(x^2 + 7^{(1/4)} \cdot \text{sqrt}(2) \cdot x + \text{sqrt}(7)) - 1/56 \cdot 28^{(1/4)} \cdot \log(x^2 - 7^{(1/4)} \cdot \text{sqrt}(2) \cdot x + \text{sqrt}(7)) - 1/4 \cdot \log(x^2 + 1) + 1/4 \cdot \log(\text{abs}(x + 1)) + 1/4 \cdot \log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 110, normalized size = 0.74

$$\frac{x^2}{2} + \frac{7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} 7^{\frac{3}{4}} x}{7} - 1\right)}{28} + \frac{7^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} 7^{\frac{3}{4}} x}{7} + 1\right)}{28} + \frac{7^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}}{x^2 - 7^{\frac{1}{4}} \sqrt{2} x + \sqrt{7}}\right)}{56} + \frac{\ln(x - 1)}{4} + \frac{\ln(x + 1)}{4} - \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)

[Out] $1/2 \cdot x^2 + 1/4 \cdot \ln(x - 1) - 1/4 \cdot \ln(x^2 + 1) + 1/56 \cdot 7^{(1/4)} \cdot 2^{(1/2)} \cdot \ln((x^2 + 7^{(1/4)} \cdot x \cdot 2^{(1/2)} + 7^{(1/2)}) / (x^2 - 7^{(1/4)} \cdot x \cdot 2^{(1/2)} + 7^{(1/2)})) + 1/28 \cdot \arctan(1 + 1/7 \cdot x \cdot 2^{(1/2)} \cdot 7^{(3/4)}) \cdot 7^{(1/4)} \cdot 2^{(1/2)} + 1/28 \cdot \arctan(-1 + 1/7 \cdot x \cdot 2^{(1/2)} \cdot 7^{(3/4)}) \cdot 7^{(1/4)} \cdot 2^{(1/2)} + 1/4 \cdot \ln(x + 1)$

maxima [A] time = 2.01, size = 132, normalized size = 0.89

$$\frac{1}{2}x^2 + \frac{1}{28} \cdot 7^{1/4} \sqrt{2} \arctan\left(\frac{1}{14} \cdot 7^{1/4} \sqrt{2}(2x + 7^{1/4} \sqrt{2})\right) + \frac{1}{28} \cdot 7^{1/4} \sqrt{2} \arctan\left(\frac{1}{14} \cdot 7^{1/4} \sqrt{2}(2x - 7^{1/4} \sqrt{2})\right) + \frac{1}{56} \cdot 7^{1/4} \sqrt{2} \log(x^2 + 7^{1/4} \sqrt{2}x + \sqrt{7}) - \frac{1}{56} \cdot 7^{1/4} \sqrt{2} \log(x^2 - 7^{1/4} \sqrt{2}x + \sqrt{7}) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{4} \log(x + 1) + \frac{1}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")

[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(x + 1) + 1/4*log(x - 1)

mupad [B] time = 2.19, size = 124, normalized size = 0.84

$$\frac{\operatorname{atan}(x^2 1i) 1i}{2} + \frac{x^2}{2} + \sqrt{2} 7^{1/4} \operatorname{atan}\left(\frac{\sqrt{2} 7^{1/4} x \left(\frac{89653248}{2401} + \frac{89653248i}{2401}\right)}{\frac{1048576}{49} + \frac{\sqrt{7} 179306496i}{2401}} + \frac{\sqrt{2} 7^{3/4} x \left(-\frac{524288}{343} + \frac{524288i}{343}\right)}{\frac{1048576}{49} + \frac{\sqrt{7} 179306496i}{2401}}\right) \left(\frac{1}{28} + \frac{1}{28}i\right) + \sqrt{2} 7^{1/4} \operatorname{atan}\left(\frac{\sqrt{2} 7^{1/4} x \left(\frac{89653248}{2401} - \frac{89653248i}{2401}\right)}{\frac{1048576}{49} + \frac{\sqrt{7} 179306496i}{2401}} + \frac{\sqrt{2} 7^{3/4} x \left(-\frac{524288}{343} - \frac{524288i}{343}\right)}{\frac{1048576}{49} + \frac{\sqrt{7} 179306496i}{2401}}\right) \left(-\frac{1}{28} + \frac{1}{28}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)

[Out] (atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(89653248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 1048576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(89653248/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048576/49))*(1/28 - 1i/28)

sympy [A] time = 0.43, size = 146, normalized size = 0.99

$$\frac{x^2}{2} + \frac{\log(x^2 - 1)}{4} - \frac{\log(x^2 + 1)}{4} - \frac{\sqrt{2} \sqrt[4]{7} \log(x^2 - \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{56} + \frac{\sqrt{2} \sqrt[4]{7} \log(x^2 + \sqrt{2} \sqrt[4]{7} x + \sqrt{7})}{56} + \frac{\sqrt{2} \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot 7^{3/4} x}{7} - 1\right)}{28} + \frac{\sqrt{2} \sqrt[4]{7} \operatorname{atan}\left(\frac{\sqrt{2} \cdot 7^{3/4} x}{7} + 1\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)

[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28

$$3.364 \quad \int \frac{1+x^3+x^6}{x+x^5} dx$$

Optimal. Leaf size=112

$$-\frac{1}{4} \log(x^4+1) + \frac{x^2}{2} + \frac{\log(x^2 - \sqrt{2}x+1)}{4\sqrt{2}} - \frac{\log(x^2 + \sqrt{2}x+1)}{4\sqrt{2}} - \frac{1}{2} \tan^{-1}(x^2) + \log(x) - \frac{\tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x+1)}{2\sqrt{2}}$$

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$, Rules used = {1593, 1833, 297, 1162, 617, 204, 1165, 628, 1834, 1248, 635, 203, 260}

$$\frac{x^2}{2} + \frac{\log(x^2 - \sqrt{2}x+1)}{4\sqrt{2}} - \frac{\log(x^2 + \sqrt{2}x+1)}{4\sqrt{2}} - \frac{1}{4} \log(x^4+1) - \frac{1}{2} \tan^{-1}(x^2) + \log(x) - \frac{\tan^{-1}(1 - \sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(\sqrt{2}x+1)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3 + x^6)/(x + x^5), x]

[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a,

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
, x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0,
n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0]
] && !PolyQ[Pq, x^(n/2)]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3+x^6}{x+x^5} dx &= \int \frac{1+x^3+x^6}{x(1+x^4)} dx \\
&= \int \left(\frac{x^2}{1+x^4} + \frac{1+x^6}{x(1+x^4)} \right) dx \\
&= \int \frac{x^2}{1+x^4} dx + \int \frac{1+x^6}{x(1+x^4)} dx \\
&= -\left(\frac{1}{2} \int \frac{1-x^2}{1+x^4} dx \right) + \frac{1}{2} \int \frac{1+x^2}{1+x^4} dx + \int \left(\frac{1}{x} + x + \frac{x(-1-x^2)}{1+x^4} \right) dx \\
&= \frac{x^2}{2} + \log(x) + \frac{1}{4} \int \frac{1}{1-\sqrt{2}x+x^2} dx + \frac{1}{4} \int \frac{1}{1+\sqrt{2}x+x^2} dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2} dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2} dx}{4\sqrt{2}} \\
&= \frac{x^2}{2} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{-1-x}{1+x^2} dx, x, x^2 \right) + \\
&= \frac{x^2}{2} - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}} \\
&= \frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2) - \frac{\tan^{-1}(1-\sqrt{2}x)}{2\sqrt{2}} + \frac{\tan^{-1}(1+\sqrt{2}x)}{2\sqrt{2}} + \log(x) + \frac{\log(1-\sqrt{2}x+x^2)}{4\sqrt{2}} - \frac{\log(1+\sqrt{2}x+x^2)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.90

$$\frac{1}{8}(-2\log(x^4+1) + 4x^2 + \sqrt{2} \log(x^2 - \sqrt{2}x + 1) - \sqrt{2} \log(x^2 + \sqrt{2}x + 1) + 8\log(x) - 2(\sqrt{2} - 2)\tan^{-1}(1 - \sqrt{2}x) + 2(2 + \sqrt{2})\tan^{-1}(\sqrt{2}x + 1))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3 + x^6)/(x + x^5), x]

[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1 + Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3+x^6}{x+x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^3 + x^6)/(x + x^5),x]

[Out] IntegrateAlgebraic[(1 + x^3 + x^6)/(x + x^5), x]

fricas [C] time = 4.32, size = 515, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")

[Out] $\frac{1}{2}x^2 - \frac{1}{4}(2\sqrt{\frac{1}{4}I} + I + 1)\log((2\sqrt{\frac{1}{4}I} + I + 1)^3 - 5(2\sqrt{\frac{1}{4}I} + I + 1)^2 + 3x + 20\sqrt{\frac{1}{4}I} + 10I + 5) - \frac{1}{4}(2\sqrt{-\frac{1}{4}I} - I + 1)\log(-(2\sqrt{\frac{1}{4}I} + I + 1)^3 - (2\sqrt{\frac{1}{4}I} + I + 2)(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + 4(2\sqrt{\frac{1}{4}I} + I + 1)^2 - ((2\sqrt{\frac{1}{4}I} + I + 1)^2 - 8\sqrt{\frac{1}{4}I} - 4I - 6)(2\sqrt{-\frac{1}{4}I} - I + 1) + 3x - 16\sqrt{\frac{1}{4}I} - 8I - 9) + \frac{1}{4}(\sqrt{\frac{1}{4}I} + \sqrt{-\frac{1}{4}I} - 2\sqrt{-\frac{3}{16}(2\sqrt{\frac{1}{4}I} + I + 1)^2 - \frac{1}{8}(2\sqrt{\frac{1}{4}I} + I - 3)(2\sqrt{-\frac{1}{4}I} - I + 1) - \frac{3}{16}(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \sqrt{\frac{1}{4}I} + \frac{1}{2}I - \frac{1}{2}) - 1)\log(\frac{1}{2}(2\sqrt{\frac{1}{4}I} + I + 2)(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \frac{1}{2}(2\sqrt{\frac{1}{4}I} + I + 1)^2 + \frac{1}{2}((2\sqrt{\frac{1}{4}I} + I + 1)^2 - 8\sqrt{\frac{1}{4}I} - 4I - 6)(2\sqrt{-\frac{1}{4}I} - I + 1) + 2\sqrt{-\frac{3}{16}(2\sqrt{\frac{1}{4}I} + I + 1)^2 - \frac{1}{8}(2\sqrt{\frac{1}{4}I} + I - 3)(2\sqrt{-\frac{1}{4}I} - I + 1) - \frac{3}{16}(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \sqrt{\frac{1}{4}I} + \frac{1}{2}I - \frac{1}{2})((2\sqrt{\frac{1}{4}I} + I + 2)(2\sqrt{-\frac{1}{4}I} - I + 1) + 2\sqrt{\frac{1}{4}I} + I - 1) + 3x - 2\sqrt{\frac{1}{4}I} - I + 2) + \frac{1}{4}(\sqrt{\frac{1}{4}I} + \sqrt{-\frac{1}{4}I} + 2\sqrt{-\frac{3}{16}(2\sqrt{\frac{1}{4}I} + I + 1)^2 - \frac{1}{8}(2\sqrt{\frac{1}{4}I} + I - 3)(2\sqrt{-\frac{1}{4}I} - I + 1) - \frac{3}{16}(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \sqrt{\frac{1}{4}I} + \frac{1}{2}I - \frac{1}{2}) - 1)\log(\frac{1}{2}(2\sqrt{\frac{1}{4}I} + I + 2)(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \frac{1}{2}(2\sqrt{\frac{1}{4}I} + I + 1)^2 + \frac{1}{2}((2\sqrt{\frac{1}{4}I} + I + 1)^2 - 8\sqrt{\frac{1}{4}I} - 4I - 6)(2\sqrt{-\frac{1}{4}I} - I + 1) - 2\sqrt{-\frac{3}{16}(2\sqrt{\frac{1}{4}I} + I + 1)^2 - \frac{1}{8}(2\sqrt{\frac{1}{4}I} + I - 3)(2\sqrt{-\frac{1}{4}I} - I + 1) - \frac{3}{16}(2\sqrt{-\frac{1}{4}I} - I + 1)^2 + \sqrt{\frac{1}{4}I} + \frac{1}{2}I - \frac{1}{2})((2\sqrt{\frac{1}{4}I} + I + 2)(2\sqrt{-\frac{1}{4}I} - I + 1) + 2\sqrt{\frac{1}{4}I} + I - 1) + 3x - 2\sqrt{\frac{1}{4}I} - I + 2) + \log(x)$

giac [A] time = 0.39, size = 92, normalized size = 0.82

$\frac{1}{2}x^2 + \frac{1}{4}(\sqrt{2} + 2)\arctan\left(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})\right) + \frac{1}{4}(\sqrt{2} - 2)\arctan\left(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})\right) - \frac{1}{8}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{1}{8}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{1}{4}\log(x^4 + 1) + \log(|x|)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")

[Out] $\frac{1}{2}x^2 + \frac{1}{4}(\sqrt{2} + 2)\arctan(\frac{1}{2}\sqrt{2}(2x + \sqrt{2})) + \frac{1}{4}(\sqrt{2} - 2)\arctan(\frac{1}{2}\sqrt{2}(2x - \sqrt{2})) - \frac{1}{8}\sqrt{2}\log(x^2 + \sqrt{2}x + 1) + \frac{1}{8}\sqrt{2}\log(x^2 - \sqrt{2}x + 1) - \frac{1}{4}\log(x^4 + 1) + \log(|x|)$

maple [A] time = 0.01, size = 79, normalized size = 0.71

$$\frac{x^2}{2} - \frac{\arctan(x^2)}{2} + \frac{\sqrt{2} \arctan(\sqrt{2}x - 1)}{4} + \frac{\sqrt{2} \arctan(\sqrt{2}x + 1)}{4} + \ln(x) + \frac{\sqrt{2} \ln\left(\frac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right)}{8} - \frac{\ln(x^4 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6+x^3+1)/(x^5+x),x)

[Out] 1/2*x^2-1/2*arctan(x^2)+1/4*arctan(-1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+1/4*arctan(1+2^(1/2)*x)*2^(1/2)-1/4*ln(x^4+1)+ln(x)

maxima [A] time = 2.06, size = 99, normalized size = 0.88

$$\frac{1}{4}\sqrt{2}(\sqrt{2}+1)\arctan\left(\frac{1}{2}\sqrt{2}(2x+\sqrt{2})\right)-\frac{1}{4}\sqrt{2}(\sqrt{2}-1)\arctan\left(\frac{1}{2}\sqrt{2}(2x-\sqrt{2})\right)-\frac{1}{8}\sqrt{2}(\sqrt{2}+1)\log(x^2+\sqrt{2}x+1)-\frac{1}{8}\sqrt{2}(\sqrt{2}-1)\log(x^2-\sqrt{2}x+1)+\frac{1}{2}x^2+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*(sqrt(2)+1)*arctan(1/2*sqrt(2)*(2*x+sqrt(2)))-1/4*sqrt(2)*(sqrt(2)-1)*arctan(1/2*sqrt(2)*(2*x-sqrt(2)))-1/8*sqrt(2)*(sqrt(2)+1)*log(x^2+sqrt(2)*x+1)-1/8*sqrt(2)*(sqrt(2)-1)*log(x^2-sqrt(2)*x+1)+1/2*x^2+log(x)

mupad [B] time = 2.23, size = 170, normalized size = 1.52

$$\ln(x) + \sum_{k=1}^4 \ln\left(\text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\right) \left(8 \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right) + x + \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right) x^9 + \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^2 x^{240} + \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^3 x^{320} - 16 \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^2 + 8\right) \text{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + x^6 + 1)/(x + x^5),x)

[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k))*(8*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x + 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k), k, 1, 4) + x^2/2

sympy [A] time = 0.96, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \text{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t \log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, Lambda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/219 + x - 344/219)))
```

$$3.365 \quad \int \frac{1+x^2}{-x+x^2} dx$$

Optimal. Leaf size=14

$$x + 2 \log(1 - x) - \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 894}

$$x + 2 \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(-x + x^2), x]

[Out] x + 2*Log[1 - x] - Log[x]

Rule 894

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{-x+x^2} dx &= \int \frac{1+x^2}{(-1+x)x} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x} \right) dx \\ &= x + 2 \log(1 - x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x + 2 \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(-x + x^2), x]

[Out] x + 2*Log[1 - x] - Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^2}{-x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(-x + x^2), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(-x + x^2), x]

fricas [A] time = 1.35, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2-x), x, algorithm="fricas")

[Out] x + 2*log(x - 1) - log(x)

giac [A] time = 0.26, size = 14, normalized size = 1.00

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2-x), x, algorithm="giac")

[Out] x + 2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$x - \ln(x) + 2 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^2-x), x)

[Out] x+2*ln(x-1)-ln(x)

maxima [A] time = 0.91, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")

[Out] x + 2*log(x - 1) - log(x)

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$x + 2 \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 + 1)/(x - x^2),x)

[Out] x + 2*log(x - 1) - log(x)

sympy [A] time = 0.10, size = 10, normalized size = 0.71

$$x - \log(x) + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)/(x**2-x),x)

[Out] x - log(x) + 2*log(x - 1)

$$3.366 \quad \int \frac{1+x^3}{-x+x^3} dx$$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 1802}

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-x + x^3), x]

[Out] x + Log[1 - x] - Log[x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x+x^3} dx &= \int \frac{1+x^3}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{1}{-1+x} - \frac{1}{x}\right) dx \\ &= x + \log(1-x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-x + x^3), x]

[Out] x + Log[1 - x] - Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^3}{-x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^3)/(-x + x^3), x]

[Out] IntegrateAlgebraic[(1 + x^3)/(-x + x^3), x]

fricas [A] time = 1.56, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x), x, algorithm="fricas")

[Out] x + log(x - 1) - log(x)

giac [A] time = 0.37, size = 12, normalized size = 1.00

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x), x, algorithm="giac")

[Out] x + log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$x - \ln(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^3-x), x)

[Out] x+ln(x-1)-ln(x)

maxima [A] time = 1.01, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)/(x^3-x),x, algorithm="maxima")`

[Out] $x + \log(x - 1) - \log(x)$

mupad [B] time = 2.14, size = 10, normalized size = 0.83

$$x - 2 \operatorname{atanh}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3 + 1)/(x - x^3),x)`

[Out] $x - 2 \operatorname{atanh}(2x - 1)$

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+1)/(x**3-x),x)`

[Out] $x - \log(x) + \log(x - 1)$

$$3.367 \quad \int \frac{1+x^3}{-x^2+x^3} dx$$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2 \log(1-x) - \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 1620}

$$x + \frac{1}{x} + 2 \log(1-x) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^3)/(-x^2 + x^3), x]

[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegerQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x^2+x^3} dx &= \int \frac{1+x^3}{(-1+x)x^2} dx \\ &= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x^2} - \frac{1}{x} \right) dx \\ &= \frac{1}{x} + x + 2 \log(1-x) - \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2 \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(-x^2 + x^3), x]

[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^3}{-x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^3)/(-x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + x^3)/(-x^2 + x^3), x]

fricas [A] time = 1.18, size = 21, normalized size = 1.24

$$\frac{x^2 + 2x \log(x - 1) - x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x^2), x, algorithm="fricas")

[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x

giac [A] time = 0.32, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^3-x^2), x, algorithm="giac")

[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$x - \ln(x) + 2 \ln(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3+1)/(x^3-x^2),x)`

[Out] `x+2*ln(x-1)+1/x-ln(x)`

maxima [A] time = 0.89, size = 15, normalized size = 0.88

$$x + \frac{1}{x} + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")`

[Out] `x + 1/x + 2*log(x - 1) - log(x)`

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x + 2 \ln(x - 1) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3 + 1)/(x^2 - x^3),x)`

[Out] `x + 2*log(x - 1) - log(x) + 1/x`

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$x - \log(x) + 2 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+1)/(x**3-x**2),x)`

[Out] `x - log(x) + 2*log(x - 1) + 1/x`

$$3.368 \quad \int \frac{-1+x^5}{-x+x^3} dx$$

Optimal. Leaf size=17

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 1802}

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^(m*Pq)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^5}{-x+x^3} dx &= \int \frac{-1+x^5}{x(-1+x^2)} dx \\ &= \int \left(1 + \frac{1}{-1-x} + \frac{1}{x} + x^2 \right) dx \\ &= x + \frac{x^3}{3} + \log(x) - \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^5)/(-x + x^3), x]

[Out] x + x^3/3 + Log[x] - Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + x^5}{-x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^5)/(-x + x^3), x]

[Out] IntegrateAlgebraic[(-1 + x^5)/(-x + x^3), x]

fricas [A] time = 1.01, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^3-x), x, algorithm="fricas")

[Out] 1/3*x^3 + x - log(x + 1) + log(x)

giac [A] time = 0.26, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 + x - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5-1)/(x^3-x), x, algorithm="giac")

[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^3}{3} + x + \ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^5-1)/(x^3-x),x)`

[Out] `x+1/3*x^3+ln(x)-ln(x+1)`

maxima [A] time = 0.90, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^5-1)/(x^3-x),x, algorithm="maxima")`

[Out] `1/3*x^3 + x - log(x + 1) + log(x)`

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x - 2 \operatorname{atanh}(2x + 1) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^5 - 1)/(x - x^3),x)`

[Out] `x - 2*atanh(2*x + 1) + x^3/3`

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**5-1)/(x**3-x),x)`

[Out] `x**3/3 + x + log(x) - log(x + 1)`

$$3.369 \quad \int \frac{1+x^4}{x^3+x^5} dx$$

Optimal. Leaf size=18

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 1252, 894}

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)/(x^3 + x^5), x]

[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]

Rule 894

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)^(n_.))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x^3+x^5} dx &= \int \frac{1+x^4}{x^3(1+x^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1+x^2}{x^2(1+x)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{2}{1+x} \right) dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} - \log(x) + \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)/(x^3 + x^5), x]

[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{x^3+x^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^4)/(x^3 + x^5), x]

[Out] IntegrateAlgebraic[(1 + x^4)/(x^3 + x^5), x]

fricas [A] time = 1.02, size = 25, normalized size = 1.39

$$\frac{2x^2 \log(x^2 + 1) - 2x^2 \log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3), x, algorithm="fricas")

[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2

giac [A] time = 0.27, size = 23, normalized size = 1.28

$$\frac{x^2 - 1}{2x^2} + \log(x^2 + 1) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")

[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$-\ln(x) + \ln(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)/(x^5+x^3),x)

[Out] -1/2/x^2-ln(x)+ln(x^2+1)

maxima [A] time = 1.92, size = 16, normalized size = 0.89

$$-\frac{1}{2x^2} + \log(x^2 + 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")

[Out] -1/2/x^2 + log(x^2 + 1) - log(x)

mupad [B] time = 0.05, size = 16, normalized size = 0.89

$$\ln(x^2 + 1) - \ln(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 + 1)/(x^3 + x^5),x)

[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)

sympy [A] time = 0.11, size = 15, normalized size = 0.83

$$-\log(x) + \log(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**5+x**3),x)
```

```
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

$$3.370 \quad \int \frac{1+x^2}{x+2x^2+x^3} dx$$

Optimal. Leaf size=10

$$\frac{2}{x+1} + \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1594, 27, 894}

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(x + 2*x^2 + x^3), x]

[Out] 2/(1 + x) + Log[x]

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p], x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 894

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)^n)*((a_.) + (c_.)*(x_)^2)^p], x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1594

Int[(u_.)*((a_.)*(x_)^p + (b_.)*(x_)^q + (c_.)*(x_)^r)^n], x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rubi steps

$$\begin{aligned}
 \int \frac{1+x^2}{x+2x^2+x^3} dx &= \int \frac{1+x^2}{x(1+2x+x^2)} dx \\
 &= \int \frac{1+x^2}{x(1+x)^2} dx \\
 &= \int \left(\frac{1}{x} - \frac{2}{(1+x)^2} \right) dx \\
 &= \frac{2}{1+x} + \log(x)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x+1} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3), x]

[Out] 2/(1 + x) + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{x+2x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(x + 2*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(x + 2*x^2 + x^3), x]

fricas [A] time = 1.03, size = 14, normalized size = 1.40

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x), x, algorithm="fricas")

[Out] ((x + 1)*log(x) + 2)/(x + 1)

giac [A] time = 0.36, size = 11, normalized size = 1.10

$$\frac{2}{x+1} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")

[Out] 2/(x + 1) + log(abs(x))

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^3+2*x^2+x),x)

[Out] 2/(x+1)+ln(x)

maxima [A] time = 0.89, size = 10, normalized size = 1.00

$$\frac{2}{x+1} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")

[Out] 2/(x + 1) + log(x)

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x + 2*x^2 + x^3),x)

[Out] log(x) + 2/(x + 1)

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\log(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
```

```
[Out] log(x) + 2/(x + 1)
```

$$3.371 \quad \int \frac{1+x^5}{-10x-3x^2+x^3} dx$$

Optimal. Leaf size=42

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1594, 1628}

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x])/14

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned} \int \frac{1+x^5}{-10x-3x^2+x^3} dx &= \int \frac{1+x^5}{x(-10-3x+x^2)} dx \\ &= \int \left(19 + \frac{3126}{35(-5+x)} - \frac{1}{10x} + 3x + x^2 - \frac{31}{14(2+x)} \right) dx \\ &= 19x + \frac{3x^2}{2} + \frac{x^3}{3} + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x + \frac{3126}{35} \log(5-x) - \frac{\log(x)}{10} - \frac{31}{14} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x])/14

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]

fricas [A] time = 0.93, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14} \log(x+2) + \frac{3126}{35} \log(x-5) - \frac{1}{10} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x), x, algorithm="fricas")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log(x)

giac [A] time = 0.29, size = 33, normalized size = 0.79

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14} \log(|x+2|) + \frac{3126}{35} \log(|x-5|) - \frac{1}{10} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x), x, algorithm="giac")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5)) - 1/10*log(abs(x))

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\ln(x)}{10} + \frac{3126 \ln(x-5)}{35} - \frac{31 \ln(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5+1)/(x^3-3*x^2-10*x),x)

[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(x+2)-1/10*ln(x)+3126/35*ln(x-5)

maxima [A] time = 0.88, size = 30, normalized size = 0.71

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14} \log(x+2) + \frac{3126}{35} \log(x-5) - \frac{1}{10} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")

[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log(x)

mupad [B] time = 0.05, size = 30, normalized size = 0.71

$$19x - \frac{31 \ln(x+2)}{14} + \frac{3126 \ln(x-5)}{35} - \frac{\ln(x)}{10} + \frac{3x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)

[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 + x^3/3

sympy [A] time = 0.15, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126 \log(x-5)}{35} - \frac{31 \log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)

[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/14

$$3.372 \quad \int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)} dx$$

Optimal. Leaf size=46

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + \frac{5 \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]

[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx &= \int \left(-\frac{5}{5 + x^2} + \frac{6 + x}{3 + 2x + x^2} \right) dx \\ &= -\left(5 \int \frac{1}{5 + x^2} dx \right) + \int \frac{6 + x}{3 + 2x + x^2} dx \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \int \frac{2 + 2x}{3 + 2x + x^2} dx + 5 \int \frac{1}{3 + 2x + x^2} dx \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{2} \log(3 + 2x + x^2) - 10 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, 2 + 2x \right) \\ &= -\sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{1+x}{\sqrt{2}} \right)}{\sqrt{2}} + \frac{1}{2} \log(3 + 2x + x^2) \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + 2x + 3) - \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{5 \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3 + 2*x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15 - 5x + x^2 + x^3}{(5 + x^2)(3 + 2x + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]

[Out] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]

fricas [A] time = 1.15, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

giac [A] time = 0.36, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5} \arctan\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2} \arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)

[Out] -5^(1/2)*arctan(1/5*5^(1/2)*x)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))+1/2*ln(x^2+2*x+3)

maxima [A] time = 1.97, size = 38, normalized size = 0.83

$$\frac{5}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}(x+1)\right) - \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5}x\right) + \frac{1}{2} \log(x^2 + 2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")

[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1/2*log(x^2 + 2*x + 3)

mupad [B] time = 0.00, size = 88, normalized size = 1.91

$$\frac{\ln(x+1-\sqrt{2}i)}{2} + \frac{\ln(x+1+\sqrt{2}i)}{2} + \sqrt{5} \operatorname{atan}\left(\frac{2000\sqrt{5}}{2000x+1120} - \frac{224\sqrt{5}x}{2000x+1120}\right) - \frac{\sqrt{2} \ln(x+1-\sqrt{2}i) 5i}{4} + \frac{\sqrt{2} \ln(x+1+\sqrt{2}i) 5i}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)

[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log(x^2 + 2x + 3)}{2} - \sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x}{2} + \frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)

[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/2

$$3.373 \quad \int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$$

Optimal. Leaf size=19

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6688, 616, 31}

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))), x]

[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx &= \int \frac{1}{3+10x+3x^2} dx \\ &= \frac{3}{8} \int \frac{1}{1+3x} dx - \frac{3}{8} \int \frac{1}{9+3x} dx \\ &= -\frac{1}{8} \log(3+x) + \frac{1}{8} \log(1+3x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1+x^2)*(3+(10*x)/(1+x^2))),x]

[Out] -1/8*Log[3+x]+Log[1+3*x]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1+x^2)*(3+(10*x)/(1+x^2))),x]

[Out] IntegrateAlgebraic[1/((1+x^2)*(3+(10*x)/(1+x^2))),x]

fricas [A] time = 1.08, size = 15, normalized size = 0.79

$$\frac{1}{8} \log(3x+1) - \frac{1}{8} \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")

[Out] 1/8*log(3*x+1)-1/8*log(x+3)

giac [A] time = 0.28, size = 17, normalized size = 0.89

$$\frac{1}{8} \log(|3x+1|) - \frac{1}{8} \log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")

[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{\ln(3x + 1)}{8} - \frac{\ln(x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)/(3+10/(x^2+1)*x),x)

[Out] -1/8*ln(x+3)+1/8*ln(1+3*x)

maxima [A] time = 1.03, size = 15, normalized size = 0.79

$$\frac{1}{8} \log(3x + 1) - \frac{1}{8} \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")

[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)

mupad [B] time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{3x}{4} + \frac{5}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)

[Out] -atanh((3*x)/4 + 5/4)/4

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{\log\left(x + \frac{1}{3}\right)}{8} - \frac{\log(x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)

[Out] log(x + 1/3)/8 - log(x + 3)/8

$$3.374 \quad \int \frac{x^3}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=40

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1386, 701, 632, 31}

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Int[x^3/(13 + 2/x + 15*x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}

, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^4}{2 + 13x + 15x^2} dx \\
 &= \int \left(\frac{139}{3375} - \frac{13x}{225} + \frac{x^2}{15} - \frac{278 + 1417x}{3375(2 + 13x + 15x^2)} \right) dx \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{\int \frac{278+1417x}{2+13x+15x^2} dx}{3375} \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} + \frac{3}{875} \int \frac{1}{3 + 15x} dx - \frac{80}{189} \int \frac{1}{10 + 15x} dx \\
 &= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567} \log(2 + 3x) + \frac{\log(1 + 5x)}{4375}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567} \log(3x + 2) + \frac{\log(5x + 1)}{4375}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(13 + 2/x + 15*x), x]

[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]/4375

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{13 + \frac{2}{x} + 15x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3/(13 + 2/x + 15*x), x]

[Out] IntegrateAlgebraic[x^3/(13 + 2/x + 15*x), x]

fricas [A] time = 1.09, size = 30, normalized size = 0.75

$$\frac{1}{45} x^3 - \frac{13}{450} x^2 + \frac{139}{3375} x + \frac{1}{4375} \log(5x + 1) - \frac{16}{567} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x + 2)

giac [A] time = 0.28, size = 32, normalized size = 0.80

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x+1|) - \frac{16}{567}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(abs(3*x + 2))

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\ln(5x+1)}{4375} - \frac{16\ln(3x+2)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(13+2/x+15*x),x)

[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)

maxima [A] time = 0.99, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x+1) - \frac{16}{567}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")

[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x + 2)

mupad [B] time = 2.09, size = 26, normalized size = 0.65

$$\frac{139x}{3375} - \frac{16\ln\left(x + \frac{2}{3}\right)}{567} + \frac{\ln\left(x + \frac{1}{5}\right)}{4375} - \frac{13x^2}{450} + \frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(15*x + 2/x + 13),x)`

[Out] $(139*x)/3375 - (16*\log(x + 2/3))/567 + \log(x + 1/5)/4375 - (13*x^2)/450 + x^3/45$

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x + \frac{1}{5}\right)}{4375} - \frac{16\log\left(x + \frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(13+2/x+15*x),x)`

[Out] $x^{**3}/45 - 13*x^{**2}/450 + 139*x/3375 + \log(x + 1/5)/4375 - 16*\log(x + 2/3)/567$

$$3.375 \quad \int \frac{x^2}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=33

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1386, 701, 632, 31}

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[x^2/(13 + 2/x + 15*x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 701

Int[((d_) + (e_)*(x_))^(m_)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1386

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}

, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^3}{2 + 13x + 15x^2} dx \\
 &= \int \left(-\frac{13}{225} + \frac{x}{15} + \frac{26 + 139x}{225(2 + 13x + 15x^2)} \right) dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} + \frac{1}{225} \int \frac{26 + 139x}{2 + 13x + 15x^2} dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} - \frac{3}{175} \int \frac{1}{3 + 15x} dx + \frac{40}{63} \int \frac{1}{10 + 15x} dx \\
 &= -\frac{13x}{225} + \frac{x^2}{30} + \frac{8}{189} \log(2 + 3x) - \frac{1}{875} \log(1 + 5x)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30} - \frac{13x}{225} + \frac{8}{189} \log(3x + 2) - \frac{1}{875} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(13 + 2/x + 15*x), x]

[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{13 + \frac{2}{x} + 15x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(13 + 2/x + 15*x), x]

[Out] IntegrateAlgebraic[x^2/(13 + 2/x + 15*x), x]

fricas [A] time = 0.93, size = 25, normalized size = 0.76

$$\frac{1}{30} x^2 - \frac{13}{225} x - \frac{1}{875} \log(5x + 1) + \frac{8}{189} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

giac [A] time = 0.34, size = 27, normalized size = 0.82

$$\frac{1}{30}x^2 - \frac{13}{225}x - \frac{1}{875}\log(|5x + 1|) + \frac{8}{189}\log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\ln(5x + 1)}{875} + \frac{8\ln(3x + 2)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(13+2/x+15*x),x)

[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)

maxima [A] time = 1.02, size = 25, normalized size = 0.76

$$\frac{1}{30}x^2 - \frac{13}{225}x - \frac{1}{875}\log(5x + 1) + \frac{8}{189}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")

[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)

mupad [B] time = 0.03, size = 21, normalized size = 0.64

$$\frac{8\ln\left(x + \frac{2}{3}\right)}{189} - \frac{13x}{225} - \frac{\ln\left(x + \frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(15*x + 2/x + 13),x)

[Out] $(8 \log(x + 2/3))/189 - (13x)/225 - \log(x + 1/5)/875 + x^2/30$

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x + \frac{1}{5}\right)}{875} + \frac{8 \log\left(x + \frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(13+2/x+15*x),x)`

[Out] $x^2/30 - 13x/225 - \log(x + 1/5)/875 + 8 \log(x + 2/3)/189$

$$3.376 \quad \int \frac{x}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=26

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1386, 703, 632, 31}

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 703

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]

Rule 1386

Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^{(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*xⁿ + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}}

, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x^2}{2 + 13x + 15x^2} dx \\ &= \frac{x}{15} + \frac{1}{15} \int \frac{-2 - 13x}{2 + 13x + 15x^2} dx \\ &= \frac{x}{15} + \frac{3}{35} \int \frac{1}{3 + 15x} dx - \frac{20}{21} \int \frac{1}{10 + 15x} dx \\ &= \frac{x}{15} - \frac{4}{63} \log(2 + 3x) + \frac{1}{175} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15} - \frac{4}{63} \log(3x + 2) + \frac{1}{175} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(13 + 2/x + 15*x),x]

[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{13 + \frac{2}{x} + 15x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(13 + 2/x + 15*x),x]

[Out] IntegrateAlgebraic[x/(13 + 2/x + 15*x), x]

fricas [A] time = 1.47, size = 20, normalized size = 0.77

$$\frac{1}{15} x + \frac{1}{175} \log(5x + 1) - \frac{4}{63} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")

[Out] $1/15*x + 1/175*\log(5*x + 1) - 4/63*\log(3*x + 2)$

giac [A] time = 0.27, size = 22, normalized size = 0.85

$$\frac{1}{15}x + \frac{1}{175}\log(|5x + 1|) - \frac{4}{63}\log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(13+2/x+15*x),x, algorithm="giac")`

[Out] $1/15*x + 1/175*\log(\text{abs}(5*x + 1)) - 4/63*\log(\text{abs}(3*x + 2))$

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15} + \frac{\ln(5x + 1)}{175} - \frac{4 \ln(3x + 2)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(13+2/x+15*x),x)`

[Out] $1/15*x - 4/63*\ln(3*x+2) + 1/175*\ln(5*x+1)$

maxima [A] time = 1.03, size = 20, normalized size = 0.77

$$\frac{1}{15}x + \frac{1}{175}\log(5x + 1) - \frac{4}{63}\log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(13+2/x+15*x),x, algorithm="maxima")`

[Out] $1/15*x + 1/175*\log(5*x + 1) - 4/63*\log(3*x + 2)$

mupad [B] time = 2.12, size = 16, normalized size = 0.62

$$\frac{x}{15} - \frac{4 \ln\left(x + \frac{2}{3}\right)}{63} + \frac{\ln\left(x + \frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(15*x + 2/x + 13),x)`

[Out] $x/15 - (4*\log(x + 2/3))/63 + \log(x + 1/5)/175$

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x + \frac{1}{5}\right)}{175} - \frac{4 \log\left(x + \frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

$$3.377 \quad \int \frac{1}{13 + \frac{2}{x} + 15x} dx$$

Optimal. Leaf size=21

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1350, 632, 31}

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[(13 + 2/x + 15*x)^(-1), x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 1350

Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{13 + \frac{2}{x} + 15x} dx &= \int \frac{x}{2 + 13x + 15x^2} dx \\
&= -\left(\frac{3}{7} \int \frac{1}{3 + 15x} dx\right) + \frac{10}{7} \int \frac{1}{10 + 15x} dx \\
&= \frac{2}{21} \log(2 + 3x) - \frac{1}{35} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21} \log(3x + 2) - \frac{1}{35} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(13 + 2/x + 15*x)^(-1), x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{13 + \frac{2}{x} + 15x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(13 + 2/x + 15*x)^(-1), x]

[Out] IntegrateAlgebraic[(13 + 2/x + 15*x)^(-1), x]

fricas [A] time = 0.93, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x + 1) + \frac{2}{21} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(13+2/x+15*x), x, algorithm="fricas")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)

giac [A] time = 0.29, size = 19, normalized size = 0.90

$$-\frac{1}{35} \log(|5x + 1|) + \frac{2}{21} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")

[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln(5x+1)}{35} + \frac{2\ln(3x+2)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(13+2/x+15*x),x)

[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)

maxima [A] time = 1.17, size = 17, normalized size = 0.81

$$-\frac{1}{35} \log(5x+1) + \frac{2}{21} \log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")

[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)

mupad [B] time = 2.11, size = 13, normalized size = 0.62

$$\frac{2 \ln\left(x + \frac{2}{3}\right)}{21} - \frac{\ln\left(x + \frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15*x + 2/x + 13),x)

[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x + \frac{1}{5}\right)}{35} + \frac{2\log\left(x + \frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(13+2/x+15*x),x)

[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21

$$3.378 \quad \int \frac{1}{x\left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=21

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 616, 31}

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(13 + 2/x + 15*x)),x]

[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{2 + 13x + 15x^2} dx \\ &= \frac{15}{7} \int \frac{1}{3 + 15x} dx - \frac{15}{7} \int \frac{1}{10 + 15x} dx \\ &= -\frac{1}{7} \log(2 + 3x) + \frac{1}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]

[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(13 + \frac{2}{x} + 15x\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x*(13 + 2/x + 15*x)),x]

[Out] IntegrateAlgebraic[1/(x*(13 + 2/x + 15*x)), x]

fricas [A] time = 1.10, size = 17, normalized size = 0.81

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)

giac [A] time = 0.27, size = 19, normalized size = 0.90

$$\frac{1}{7} \log(|5x + 1|) - \frac{1}{7} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")

[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(5x + 1)}{7} - \frac{\ln(3x + 2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(13+2/x+15*x),x)

[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)

maxima [A] time = 0.93, size = 17, normalized size = 0.81

$$\frac{1}{7} \log(5x + 1) - \frac{1}{7} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")

[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)

mupad [B] time = 0.08, size = 8, normalized size = 0.38

$$-\frac{2 \operatorname{atanh}\left(\frac{30x}{7} + \frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(15*x + 2/x + 13)),x)

[Out] -(2*atanh((30*x)/7 + 13/7))/7

sympy [A] time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x + \frac{1}{5}\right)}{7} - \frac{\log\left(x + \frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(13+2/x+15*x),x)

[Out] log(x + 1/5)/7 - log(x + 2/3)/7

$$3.379 \quad \int \frac{1}{x^2 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=27

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1386, 705, 29, 632, 31}

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] := Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol]
  ] :-> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x]
  && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x(2 + 13x + 15x^2)} dx \\ &= \frac{1}{2} \int \frac{1}{x} dx + \frac{1}{2} \int \frac{-13 - 15x}{2 + 13x + 15x^2} dx \\ &= \frac{\log(x)}{2} + \frac{45}{14} \int \frac{1}{10 + 15x} dx - \frac{75}{7} \int \frac{1}{3 + 15x} dx \\ &= \frac{\log(x)}{2} + \frac{3}{14} \log(2 + 3x) - \frac{5}{7} \log(1 + 5x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2} + \frac{3}{14} \log(3x + 2) - \frac{5}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(13 + 2/x + 15*x)),x]

[Out] IntegrateAlgebraic[1/(x^2*(13 + 2/x + 15*x)), x]

fricas [A] time = 0.83, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

giac [A] time = 0.37, size = 24, normalized size = 0.89

$$-\frac{5}{7} \log(|5x + 1|) + \frac{3}{14} \log(|3x + 2|) + \frac{1}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")

[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2} - \frac{5 \ln(5x + 1)}{7} + \frac{3 \ln(3x + 2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(13+2/x+15*x),x)

[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)

maxima [A] time = 0.92, size = 21, normalized size = 0.78

$$-\frac{5}{7} \log(5x + 1) + \frac{3}{14} \log(3x + 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")

[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)

mupad [B] time = 0.09, size = 17, normalized size = 0.63

$$\frac{3 \ln\left(x + \frac{2}{3}\right)}{14} - \frac{5 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(15*x + 2/x + 13)),x)

[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2} - \frac{5 \log\left(x + \frac{1}{5}\right)}{7} + \frac{3 \log\left(x + \frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(13+2/x+15*x),x)

[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14

$$3.380 \quad \int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=34

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
  := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
  t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
  x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
  4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
  , -1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
  (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
  + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
  c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol]
  := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n},
  x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^2 (2 + 13x + 15x^2)} dx \\
&= -\frac{1}{2x} + \frac{1}{2} \int \frac{-13 - 15x}{x(2 + 13x + 15x^2)} dx \\
&= -\frac{1}{2x} + \frac{1}{2} \int \left(-\frac{13}{2x} - \frac{27}{14(2 + 3x)} + \frac{250}{7(1 + 5x)} \right) dx \\
&= -\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(2 + 3x) + \frac{25}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x} - \frac{13 \log(x)}{4} - \frac{9}{28} \log(3x + 2) + \frac{25}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^3*(13 + 2/x + 15*x)),x]

[Out] IntegrateAlgebraic[1/(x^3*(13 + 2/x + 15*x)), x]

fricas [A] time = 0.97, size = 30, normalized size = 0.88

$$\frac{100x \log(5x + 1) - 9x \log(3x + 2) - 91x \log(x) - 14}{28x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")

[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x

giac [A] time = 0.27, size = 29, normalized size = 0.85

$$-\frac{1}{2x} + \frac{25}{7} \log(|5x + 1|) - \frac{9}{28} \log(|3x + 2|) - \frac{13}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")

[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13 \ln(x)}{4} + \frac{25 \ln(5x + 1)}{7} - \frac{9 \ln(3x + 2)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(13+2/x+15*x),x)

[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)

maxima [A] time = 1.06, size = 26, normalized size = 0.76

$$-\frac{1}{2x} + \frac{25}{7} \log(5x + 1) - \frac{9}{28} \log(3x + 2) - \frac{13}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")

[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)

mupad [B] time = 0.03, size = 22, normalized size = 0.65

$$\frac{25 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{9 \ln\left(x + \frac{2}{3}\right)}{28} - \frac{13 \ln(x)}{4} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(15*x + 2/x + 13)),x)

[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13 \log(x)}{4} + \frac{25 \log\left(x + \frac{1}{5}\right)}{7} - \frac{9 \log\left(x + \frac{2}{3}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(13+2/x+15*x),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

$$3.381 \quad \int \frac{1}{x^4 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=41

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x+2) - \frac{125}{7} \log(5x+1)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x+2) - \frac{125}{7} \log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]

[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1 + 5*x])/7

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^3 (2 + 13x + 15x^2)} dx \\
&= -\frac{1}{4x^2} + \frac{1}{2} \int \frac{-13 - 15x}{x^2 (2 + 13x + 15x^2)} dx \\
&= -\frac{1}{4x^2} + \frac{1}{2} \int \left(-\frac{13}{2x^2} + \frac{139}{4x} + \frac{81}{28(2 + 3x)} - \frac{1250}{7(1 + 5x)} \right) dx \\
&= -\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(2 + 3x) - \frac{125}{7} \log(1 + 5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2} + \frac{13}{4x} + \frac{139 \log(x)}{8} + \frac{27}{56} \log(3x + 2) - \frac{125}{7} \log(5x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]

[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(13 + 2/x + 15*x)),x]

[Out] IntegrateAlgebraic[1/(x^4*(13 + 2/x + 15*x)), x]

fricas [A] time = 1.12, size = 39, normalized size = 0.95

$$\frac{1000 x^2 \log(5x + 1) - 27 x^2 \log(3x + 2) - 973 x^2 \log(x) - 182x + 14}{56 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")

[Out] $-1/56*(1000*x^2*\log(5*x + 1) - 27*x^2*\log(3*x + 2) - 973*x^2*\log(x) - 182*x + 14)/x^2$

giac [A] time = 0.28, size = 34, normalized size = 0.83

$$\frac{13x-1}{4x^2} - \frac{125}{7} \log(|5x+1|) + \frac{27}{56} \log(|3x+2|) + \frac{139}{8} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")`

[Out] $1/4*(13*x - 1)/x^2 - 125/7*\log(\text{abs}(5*x + 1)) + 27/56*\log(\text{abs}(3*x + 2)) + 139/8*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139 \ln(x)}{8} - \frac{125 \ln(5x+1)}{7} + \frac{27 \ln(3x+2)}{56} + \frac{13}{4x} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(13+2/x+15*x),x)`

[Out] $-1/4/x^2+13/4/x+139/8*\ln(x)+27/56*\ln(3*x+2)-125/7*\ln(5*x+1)$

maxima [A] time = 1.02, size = 31, normalized size = 0.76

$$\frac{13x-1}{4x^2} - \frac{125}{7} \log(5x+1) + \frac{27}{56} \log(3x+2) + \frac{139}{8} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")`

[Out] $1/4*(13*x - 1)/x^2 - 125/7*\log(5*x + 1) + 27/56*\log(3*x + 2) + 139/8*\log(x)$

mupad [B] time = 0.04, size = 26, normalized size = 0.63

$$\frac{27 \ln\left(x + \frac{2}{3}\right)}{56} - \frac{125 \ln\left(x + \frac{1}{5}\right)}{7} + \frac{139 \ln(x)}{8} + \frac{\frac{13x}{4} - \frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(15*x + 2/x + 13)),x)`

[Out] $(27*\log(x + 2/3))/56 - (125*\log(x + 1/5))/7 + (139*\log(x))/8 + ((13*x)/4 - 1/4)/x^2$

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139 \log(x)}{8} - \frac{125 \log\left(x + \frac{1}{5}\right)}{7} + \frac{27 \log\left(x + \frac{2}{3}\right)}{56} + \frac{13x - 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(13+2/x+15*x),x)

[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)

$$3.382 \quad \int \frac{1}{x^5 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Optimal. Leaf size=48

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1386, 709, 800}

$$\frac{13}{8x^2} - \frac{1}{6x^3} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]

[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/112 + (625*Log[1 + 5*x])/7

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1386

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] := Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5 \left(13 + \frac{2}{x} + 15x\right)} dx &= \int \frac{1}{x^4 (2 + 13x + 15x^2)} dx \\
&= -\frac{1}{6x^3} + \frac{1}{2} \int \frac{-13 - 15x}{x^3 (2 + 13x + 15x^2)} dx \\
&= -\frac{1}{6x^3} + \frac{1}{2} \int \left(-\frac{13}{2x^3} + \frac{139}{4x^2} - \frac{1417}{8x} - \frac{243}{56(2+3x)} + \frac{6250}{7(1+5x)} \right) dx \\
&= -\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(2+3x) + \frac{625}{7} \log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3} + \frac{13}{8x^2} - \frac{139}{8x} - \frac{1417 \log(x)}{16} - \frac{81}{112} \log(3x+2) + \frac{625}{7} \log(5x+1)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]

[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/112 + (625*Log[1 + 5*x])/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(13 + \frac{2}{x} + 15x\right)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^5*(13 + 2/x + 15*x)),x]

[Out] IntegrateAlgebraic[1/(x^5*(13 + 2/x + 15*x)), x]

fricas [A] time = 1.02, size = 44, normalized size = 0.92

$$\frac{30000 x^3 \log(5x+1) - 243 x^3 \log(3x+2) - 29757 x^3 \log(x) - 5838 x^2 + 546 x - 56}{336 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")

[Out] $1/336*(30000*x^3*\log(5*x + 1) - 243*x^3*\log(3*x + 2) - 29757*x^3*\log(x) - 5838*x^2 + 546*x - 56)/x^3$

giac [A] time = 0.24, size = 39, normalized size = 0.81

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(|5x + 1|) - \frac{81}{112} \log(|3x + 2|) - \frac{1417}{16} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")`

[Out] $-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*\log(\text{abs}(5*x + 1)) - 81/112*\log(\text{abs}(3*x + 2)) - 1417/16*\log(\text{abs}(x))$

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417 \ln(x)}{16} + \frac{625 \ln(5x + 1)}{7} - \frac{81 \ln(3x + 2)}{112} - \frac{139}{8x} + \frac{13}{8x^2} - \frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^5/(13+2/x+15*x),x)`

[Out] $-1/6/x^3+13/8/x^2-139/8/x-1417/16*\ln(x)-81/112*\ln(3*x+2)+625/7*\ln(5*x+1)$

maxima [A] time = 1.07, size = 36, normalized size = 0.75

$$-\frac{417x^2 - 39x + 4}{24x^3} + \frac{625}{7} \log(5x + 1) - \frac{81}{112} \log(3x + 2) - \frac{1417}{16} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")`

[Out] $-1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*\log(5*x + 1) - 81/112*\log(3*x + 2) - 1417/16*\log(x)$

mupad [B] time = 0.04, size = 32, normalized size = 0.67

$$\frac{625 \ln\left(x + \frac{1}{5}\right)}{7} - \frac{81 \ln\left(x + \frac{2}{3}\right)}{112} - \frac{1417 \ln(x)}{16} - \frac{\frac{139x^2}{8} - \frac{13x}{8} + \frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(15*x + 2/x + 13)),x)`

[Out] $(625 \cdot \log(x + 1/5))/7 - (81 \cdot \log(x + 2/3))/112 - (1417 \cdot \log(x))/16 - ((139 \cdot x^2)/8 - (13 \cdot x)/8 + 1/6)/x^3$

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417 \log(x)}{16} + \frac{625 \log\left(x + \frac{1}{5}\right)}{7} - \frac{81 \log\left(x + \frac{2}{3}\right)}{112} + \frac{-417x^2 + 39x - 4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(13+2/x+15*x),x)`

[Out] $-1417 \cdot \log(x)/16 + 625 \cdot \log(x + 1/5)/7 - 81 \cdot \log(x + 2/3)/112 + (-417 \cdot x^2 + 39 \cdot x - 4)/(24 \cdot x^3)$

$$3.383 \quad \int \frac{x^2}{2-(1+x^2)^4} dx$$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{\sqrt[4]{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}}$$

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6740, 206, 203, 1972, 205}

$$\frac{i\sqrt{1-i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{\sqrt[4]{2}-1} \tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 - (1 + x^2)^4),x]

[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]]/2^(3/4) - ((I/4)*Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]]/2^(3/4) - (Sqrt[1 + 2^(1/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2 - (1 + x^2)^4} dx &= \int \left(\frac{-\sqrt[4]{2} + \sqrt{2}}{8(-1 + \sqrt[4]{2} - x^2)} + \frac{-\sqrt[4]{2} - \sqrt{2}}{8(1 + \sqrt[4]{2} + x^2)} + \frac{-\sqrt[4]{2} - i\sqrt{2}}{8(\sqrt[4]{2} - i(1 + x^2))} + \frac{-\sqrt[4]{2} + i\sqrt{2}}{8(\sqrt[4]{2} + i(1 + x^2))} \right) dx \\ &= -\frac{(1 - \sqrt[4]{2}) \int \frac{1}{-1 + \sqrt[4]{2} - x^2} dx}{4 \cdot 2^{3/4}} - \frac{(1 - i\sqrt[4]{2}) \int \frac{1}{\sqrt[4]{2} + i(1 + x^2)} dx}{4 \cdot 2^{3/4}} - \frac{(1 + i\sqrt[4]{2}) \int \frac{1}{\sqrt[4]{2} - i(1 + x^2)} dx}{4 \cdot 2^{3/4}} - \frac{(1 + \sqrt[4]{2}) \int \frac{1}{1 + \sqrt[4]{2} + x^2} dx}{4 \cdot 2^{3/4}} \\ &= -\frac{\sqrt{1 + \sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{-1 + \sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{-1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{(1 - i\sqrt[4]{2}) \int \frac{1}{i + \sqrt[4]{2} + ix^2} dx}{4 \cdot 2^{3/4}} \\ &= \frac{i\sqrt{1 - i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1 - i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1 + i\sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt{1 + \sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8} \text{RootSum}\left[\#1^8 + 4\#1^6 + 6\#1^4 + 4\#1^2 - 1 \&, \frac{\#1 \log(x - \#1)}{\#1^6 + 3\#1^4 + 3\#1^2 + 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 - (1 + x^2)^4), x]

[Out] -1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 &, (Log[x - #1]*#1)/(1 + 3*#1^2 + 3*#1^4 + #1^6) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2 - (1 + x^2)^4} dx$$

$$\begin{aligned}
& - \sqrt{2}) - 4\sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*} \\
& (2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1)*((\sqrt{2}*(2^{3/4} + \\
& \sqrt{2}) + \sqrt{2})*(2^{3/4} - \sqrt{2}) - \sqrt{2}*(2^{3/4} + \sqrt{2}) - 4*\sqrt{2}) \\
& - 4*\sqrt{2}*(2^{3/4} + \sqrt{2}) + 4*\sqrt{2})*\sqrt{1/2*\sqrt{2} - \sqrt{2}} \\
& t(-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) \\
& - 3/16*(2^{3/4} - \sqrt{2})^2 + 1)) + 6*x) + 1/16*\sqrt{2^{3/4} - \sqrt{2}}* \\
& \log(1/4*((2^{3/4} + \sqrt{2})^3 + (2^{3/4} + \sqrt{2} + 1)*(2^{3/4} - \sqrt{2}) \\
& ^2 - ((2^{3/4} + \sqrt{2})^2 - 4)*(2^{3/4} - \sqrt{2}) - 4*2^{3/4} - 4*\sqrt{2} \\
&) - 6)*\sqrt{2^{3/4} - \sqrt{2}} + 3*x) - 1/16*\sqrt{2^{3/4} - \sqrt{2}}*\log(-1 \\
& /4*((2^{3/4} + \sqrt{2})^3 + (2^{3/4} + \sqrt{2} + 1)*(2^{3/4} - \sqrt{2})^2 - \\
& ((2^{3/4} + \sqrt{2})^2 - 4)*(2^{3/4} - \sqrt{2}) - 4*2^{3/4} - 4*\sqrt{2} - \\
& 6)*\sqrt{2^{3/4} - \sqrt{2}} + 3*x) - \sqrt{-1/256*2^{3/4} - 1/256*\sqrt{2}}*\log \\
& (4*((2^{3/4} + \sqrt{2})^3 - (2^{3/4} + \sqrt{2})^2 - 10)*\sqrt{-1/256*2^{3/4} \\
&) - 1/256*\sqrt{2}} + 3*x) + \sqrt{-1/256*2^{3/4} - 1/256*\sqrt{2}}*\log(-4*((2 \\
& ^{3/4} + \sqrt{2})^3 - (2^{3/4} + \sqrt{2})^2 - 10)*\sqrt{-1/256*2^{3/4} - 1/2 \\
& 56*\sqrt{2}} + 3*x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{(x^2 + 1)^4 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")

[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)

maple [C] time = 0.01, size = 54, normalized size = 0.34

$$\frac{\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^2 \ln(-\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1) + x)}{8(\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^7 + 3\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^5 + 3\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1)^3 + \text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 - 1))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2-(x^2+1)^4),x)

[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(-_R+x),_R=RootOf(-Z^8+4*_Z^6+6*_Z^4+4*_Z^2-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{(x^2 + 1)^4 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")

[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)

mupad [B] time = 2.75, size = 144, normalized size = 0.92

$$\sum_{i=1}^2 \ln\left(-\operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\left(86x - \operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\left(\operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\left(4096x - \operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\left(262144x + \operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\left(67108864\right) + 256\right) - 1\right)\operatorname{root}\left(x^8 - \frac{x^4}{16384} + \frac{x^2}{1048576} - \frac{1}{1073741824}, x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 + 1)^4 - 2),x)

[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.22, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8 - 65536t^4 + 1024t^2 - 1, \left(t \mapsto t \log\left(-\frac{67108864t^7}{3} - \frac{262144t^5}{3} - \frac{40t}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2-(x**2+1)**4),x)

[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x))

$$3.384 \quad \int \frac{x^2}{2-(1-x^2)^4} dx$$

Optimal. Leaf size=157

$$\frac{\sqrt{\sqrt[4]{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}}$$

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6740, 206, 203, 1972, 208}

$$\frac{\sqrt{\sqrt[4]{2}-1} \tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1-i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{i\sqrt{1+i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{1+\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 - (1 - x^2)^4), x]

[Out] $-\frac{(\text{Sqrt}[-1 + 2^{(1/4)}] \cdot \text{ArcTan}[x/\text{Sqrt}[-1 + 2^{(1/4)}]])}{(4 \cdot 2^{(3/4)})} - \frac{((I/4) \cdot \text{Sqrt}[1 - I \cdot 2^{(1/4)}] \cdot \text{ArcTanh}[x/\text{Sqrt}[1 - I \cdot 2^{(1/4)}]])}{2^{(3/4)}} + \frac{((I/4) \cdot \text{Sqrt}[1 + I \cdot 2^{(1/4)}] \cdot \text{ArcTanh}[x/\text{Sqrt}[1 + I \cdot 2^{(1/4)}]])}{2^{(3/4)}} + \frac{(\text{Sqrt}[1 + 2^{(1/4)}] \cdot \text{ArcTanh}[x/\text{Sqrt}[1 + 2^{(1/4)}]])}{(4 \cdot 2^{(3/4)})}$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_)*(u_)^(n_)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2 - (1 - x^2)^4} dx &= \int \left(\frac{\sqrt[4]{2} + \sqrt{2}}{8(1 + \sqrt[4]{2} - x^2)} + \frac{\sqrt[4]{2} - \sqrt{2}}{8(-1 + \sqrt[4]{2} + x^2)} + \frac{\sqrt[4]{2} + i\sqrt{2}}{8(\sqrt[4]{2} - i(1 - x^2))} + \frac{\sqrt[4]{2} - i\sqrt{2}}{8(\sqrt[4]{2} + i(1 - x^2))} \right) dx \\ &= \frac{(1 - \sqrt[4]{2}) \int \frac{1}{-1 + \sqrt[4]{2} + x^2} dx}{4 \cdot 2^{3/4}} + \frac{(1 - i\sqrt[4]{2}) \int \frac{1}{\sqrt[4]{2} + i(1 - x^2)} dx}{4 \cdot 2^{3/4}} + \frac{(1 + i\sqrt[4]{2}) \int \frac{1}{\sqrt[4]{2} - i(1 - x^2)} dx}{4 \cdot 2^{3/4}} + \frac{(1 + \sqrt[4]{2}) \int \frac{1}{1 + \sqrt[4]{2} - x^2} dx}{4 \cdot 2^{3/4}} \\ &= -\frac{\sqrt{-1 + \sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{-1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt{1 + \sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(1 - i\sqrt[4]{2}) \int \frac{1}{i + \sqrt[4]{2} - ix^2} dx}{4 \cdot 2^{3/4}} + \frac{(1 + \sqrt[4]{2}) \int \frac{1}{1 + \sqrt[4]{2} - x^2} dx}{4 \cdot 2^{3/4}} \\ &= -\frac{\sqrt{-1 + \sqrt[4]{2}} \tan^{-1}\left(\frac{x}{\sqrt{-1 + \sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} - \frac{i\sqrt{1 - i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{i\sqrt{1 + i\sqrt[4]{2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i\sqrt[4]{2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(1 + \sqrt[4]{2}) \int \frac{1}{1 + \sqrt[4]{2} - x^2} dx}{4 \cdot 2^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8} \text{RootSum}\left[\#1^8 - 4\#1^6 + 6\#1^4 - 4\#1^2 - 1 \&, \frac{\#1 \log(x - \#1)}{\#1^6 - 3\#1^4 + 3\#1^2 - 1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 - (1 - x^2)^4), x]

[Out] -1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 &, (Log[x - #1]*#1)/(-1 + 3*#1^2 - 3*#1^4 + #1^6) &]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2 - (1 - x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(2 - (1 - x^2)^4), x]

[Out] IntegrateAlgebraic[x^2/(2 - (1 - x^2)^4), x]

fricas [B] time = 4.05, size = 1546, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(-x^2+1)^4), x, algorithm="fricas")

[Out]
$$-1/16*\sqrt{2}*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1)}}*\log(1/4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})^2 - \sqrt{2}*(2^{3/4} - \sqrt{2})^2 - (\sqrt{2}*(2^{3/4} - \sqrt{2})^2 - 4*\sqrt{2})*(2^{3/4} + \sqrt{2})) + 4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})) + \sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1} - 4*\sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}) + 6*x) + 1/16*\sqrt{2}*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}*\log(-1/4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})^2 - \sqrt{2}*(2^{3/4} - \sqrt{2})^2 - (\sqrt{2}*(2^{3/4} - \sqrt{2})^2 - 4*\sqrt{2})*(2^{3/4} + \sqrt{2})) + 4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})) + \sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1} - 4*\sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}) + 6*x) - 1/16*\sqrt{2}*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}*\log(1/4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})^2 - \sqrt{2}*(2^{3/4} - \sqrt{2})^2 - (\sqrt{2}*(2^{3/4} - \sqrt{2})^2 - 4*\sqrt{2})*(2^{3/4} + \sqrt{2})) - \sqrt{2})*(2^{3/4} + \sqrt{2})) + \sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1} - 4*\sqrt{2}*(2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}) + 6*x) + 1/16*\sqrt{2}*\sqrt{-1/2*\sqrt{2} + \sqrt{-3/16*(2^{3/4} + \sqrt{2})^2 + 1/8*(2^{3/4} + \sqrt{2})*(2^{3/4} - \sqrt{2}) - 3/16*(2^{3/4} - \sqrt{2})^2 + 1}}*\log(-1/4*((\sqrt{2}*(2^{3/4} - \sqrt{2}) - \sqrt{2})*(2^{3/4} + \sqrt{2})^2 - \sqrt{2}*(2^{3/4} - \sqrt{2})^2 - (\sqrt{2}*(2^{3/4} - \sqrt{2})^2 - 4*\sqrt{2})*(2^{3/4} + \sqrt{2}))*$$

$2^{3/4} + \sqrt{2}) - 4*((\sqrt{2})*2^{3/4} - \sqrt{2}) - \sqrt{2})*2^{3/4} + \sqrt{2}) + \sqrt{2}*2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-3/16*2^{3/4} + \sqrt{2})^2 + 1/8*2^{3/4} + \sqrt{2})*2^{3/4} - \sqrt{2}) - 3/16*2^{3/4} - \sqrt{2})^2 + 1) - 4*\sqrt{2}*2^{3/4} - \sqrt{2}) - 4*\sqrt{2})*\sqrt{-1/2*\sqrt{2} - \sqrt{-3/16*2^{3/4} + \sqrt{2})^2 + 1/8*2^{3/4} + \sqrt{2})*2^{3/4} - \sqrt{2}) - 3/16*2^{3/4} - \sqrt{2})^2 + 1)) + 6*x) + 1/16*\sqrt{2^{3/4} + \sqrt{2})*\log(1/4*((2^{3/4} - \sqrt{2})^3 + (2^{3/4} + \sqrt{2})^2*(2^{3/4} - \sqrt{2}) - 1) - ((2^{3/4} - \sqrt{2})^2 - 4)*(2^{3/4} + \sqrt{2}) - 4*2^{3/4} + 4*\sqrt{2} + 6)*\sqrt{2^{3/4} + \sqrt{2}}) + 3*x) - 1/16*\sqrt{2^{3/4} + \sqrt{2})*\log(-1/4*((2^{3/4} - \sqrt{2})^3 + (2^{3/4} + \sqrt{2})^2*(2^{3/4} - \sqrt{2}) - 1) - ((2^{3/4} - \sqrt{2})^2 - 4)*(2^{3/4} + \sqrt{2}) - 4*2^{3/4} + 4*\sqrt{2} + 6)*\sqrt{2^{3/4} + \sqrt{2}}) + 3*x) - \sqrt{-1/256*2^{3/4} + 1/256*\sqrt{2})*\log(4*((2^{3/4} - \sqrt{2})^3 + (2^{3/4} - \sqrt{2})^2 + 10)*\sqrt{-1/256*2^{3/4} + 1/256*\sqrt{2}}) + 3*x) + \sqrt{-1/256*2^{3/4} + 1/256*\sqrt{2})*\log(-4*((2^{3/4} - \sqrt{2})^3 + (2^{3/4} - \sqrt{2})^2 + 10)*\sqrt{-1/256*2^{3/4} + 1/256*\sqrt{2}}) + 3*x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{(x^2 - 1)^4 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")

[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)

maple [C] time = 0.02, size = 56, normalized size = 0.36

$$\frac{\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^2 \ln(-\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1) + x)}{8(\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^7 - 3\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^5 + 3\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1)^3 - \text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 - 1))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2-(-x^2+1)^4),x)

[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(-Z^8-4*_Z^6+6*_Z^4-4*_Z^2-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{(x^2 - 1)^4 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")

[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)

mupad [B] time = 2.80, size = 142, normalized size = 0.90

$$\sum_{k=1}^8 \ln \left(-\text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \left(56x + \text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \left(\text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \left(4096x + \text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \left(262144x - \text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \left(67108864 \right) + 256 \right) \right) - 1 \right) \text{root}\left(z^8 - \frac{z^4}{16384} - \frac{z^2}{1048576} - \frac{1}{1073741824}, z, k\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^2/((x^2 - 1)^4 - 2),x)

[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)

sympy [A] time = 0.23, size = 41, normalized size = 0.26

$$-\text{RootSum}\left(1073741824t^8 - 65536t^4 - 1024t^2 - 1, \left(t \mapsto t \log\left(-\frac{67108864t^7}{3} + \frac{262144t^5}{3} + \frac{40t}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(2-(-x**2+1)**4),x)

[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x))

$$3.385 \quad \int \frac{x^2}{2+(1+x^2)^4} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{1}{8}i$$

Rubi [A] time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6740, 204, 203, 1972, 205}

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{1}{8}i(\sqrt[4]{-2} + \sqrt{2}) \sqrt{\frac{1+i}{2^{3/4} + (1+i)}} \tan^{-1}\left(\sqrt{\frac{1+i}{2^{3/4} + (1+i)}} x\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + (1 + x^2)^4), x]

[Out] $((-1)^{(1/4)} \cdot \text{Sqrt}[1 - (-2)^{(1/4)}] \cdot \text{ArcTan}[x/\text{Sqrt}[1 - (-2)^{(1/4)}]])/(4 \cdot 2^{(3/4)}) - ((-1)^{(3/4)} \cdot \text{Sqrt}[1 + I \cdot (-2)^{(1/4)}] \cdot \text{ArcTan}[x/\text{Sqrt}[1 + I \cdot (-2)^{(1/4)}]])/(4 \cdot 2^{(3/4)}) - ((-1)^{(1/4)} \cdot \text{Sqrt}[1 + (-2)^{(1/4)}] \cdot \text{ArcTan}[x/\text{Sqrt}[1 + (-2)^{(1/4)}]])/(4 \cdot 2^{(3/4)}) + (I/8) \cdot ((-2)^{(1/4)} + \text{Sqrt}[2]) \cdot \text{Sqrt}[(1 + I)/((1 + I) + 2^{(3/4)})] \cdot \text{ArcTan}[\text{Sqrt}[(1 + I)/((1 + I) + 2^{(3/4)})] \cdot x]$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2 + (1 + x^2)^4} dx &= \int \left(\frac{-\sqrt[4]{-2} + i\sqrt{2}}{8(-1 + \sqrt[4]{-2} - x^2)} + \frac{-\sqrt[4]{-2} - i\sqrt{2}}{8(1 + \sqrt[4]{-2} + x^2)} + \frac{-\sqrt[4]{-2} + \sqrt{2}}{8(\sqrt[4]{-2} - i(1 + x^2))} + \frac{-\sqrt[4]{-2} - \sqrt{2}}{8(\sqrt[4]{-2} + i(1 + x^2))} \right) dx \\ &= \frac{1}{8} \left(-\sqrt[4]{-2} - \sqrt{2} \right) \int \frac{1}{\sqrt[4]{-2} + i(1 + x^2)} dx + \frac{1}{8} \left(-\sqrt[4]{-2} - i\sqrt{2} \right) \int \frac{1}{1 + \sqrt[4]{-2} + x^2} dx + \frac{1}{8} \left(-\sqrt[4]{-2} + \sqrt{2} \right) \int \frac{1}{\sqrt[4]{-2} - i(1 + x^2)} dx + \frac{1}{8} \left(-\sqrt[4]{-2} + i\sqrt{2} \right) \int \frac{1}{1 - \sqrt[4]{-2} - x^2} dx \\ &= \frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1} \left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}} \right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1} \left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}} \right)}{4 \cdot 2^{3/4}} + \frac{1}{8} \left(-\sqrt[4]{-2} - \sqrt{2} \right) \int \frac{1}{1 + \sqrt[4]{-2} + x^2} dx + \frac{1}{8} \left(-\sqrt[4]{-2} + \sqrt{2} \right) \int \frac{1}{1 - \sqrt[4]{-2} - x^2} dx \\ &= \frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tan^{-1} \left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}} \right)}{4 \cdot 2^{3/4}} - \frac{(-1)^{3/4} \sqrt{1 + i\sqrt[4]{-2}} \tan^{-1} \left(\frac{x}{\sqrt{1 + i\sqrt[4]{-2}}} \right)}{4 \cdot 2^{3/4}} - \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tan^{-1} \left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}} \right)}{4 \cdot 2^{3/4}} + \frac{1}{8} \left(-\sqrt[4]{-2} + \sqrt{2} \right) \int \frac{1}{1 - \sqrt[4]{-2} - x^2} dx \end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8} \text{RootSum} \left[\#1^8 + 4\#1^6 + 6\#1^4 + 4\#1^2 + 3\&, \frac{\#1 \log(x - \#1)}{\#1^6 + 3\#1^4 + 3\#1^2 + 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 + (1 + x^2)^4), x]

[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 &, (Log[x - #1]*#1)/(1 + 3*#1^2 + 2 + 3*#1^4 + #1^6) &]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2 + (1 + x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(2 + (1 + x^2)^4),x]

[Out] IntegrateAlgebraic[x^2/(2 + (1 + x^2)^4), x]

fricas [B] time = 4.13, size = 2271, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*\sqrt{2}*\sqrt{\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1} + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log((16384*\sqrt{2})*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - \sqrt{2})*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 16384*\sqrt{2})*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1)*((\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - \sqrt{2})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})) + \sqrt{2})*\sqrt{\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1} + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}}) + 2*x) + 1/16*\sqrt{2}*\sqrt{\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1} + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log(-(16384*\sqrt{2})*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - \sqrt{2})*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 16384*\sqrt{2})*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1)*((\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - \sqrt{2})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})) + \sqrt{2})*\sqrt{\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{1/8192*I*\sqrt{2}}})^2 - 12288*(-1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}}) - 1} + 32*\sqrt{1/8192} \end{aligned}$$

+ 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 + 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")

[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)

maple [C] time = 0.01, size = 54, normalized size = 0.29

$$\frac{\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^2 \ln(-\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3) + x)}{8\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^7 + 24\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^5 + 24\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)^3 + 8\text{RootOf}(-Z^8 + 4Z^6 + 6Z^4 + 4Z^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+(x^2+1)^4),x)

[Out] 1/8*sum(1/(_R^7+3*_R^5+3*_R^3+_R)*_R^2*ln(-_R+x),_R=RootOf(-Z^8+4*_Z^6+6*_Z^4+4*_Z^2+3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 + 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)

mupad [B] time = 2.78, size = 142, normalized size = 0.76

$$\sum_{i=1}^4 \ln\left(\text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)\right) \left(40x + \text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)\right) \left(\text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)\right) \left(4096x - \text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)\right) \left(786432x - \text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)\right) \left(768\right) - 3 \cdot \text{root}\left(x^8 + \frac{x^4}{16384} + \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x +
root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/1638
4 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.21, size = 39, normalized size = 0.21

$$\text{RootSum}\left(1073741824t^8 + 65536t^4 + 1024t^2 + 3, \left(t \mapsto t \log\left(67108864t^7 - 262144t^5 + 4096t^3 + 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

$$3.386 \quad \int \frac{x^2}{2+(1-x^2)^4} dx$$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}}$$

Rubi [A] time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {6740, 206, 207, 1972, 208}

$$\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} - \frac{1}{8} i (\sqrt[4]{-2} + \sqrt{2}) \sqrt{\frac{1+i}{2^{3/4} + (1+i)}} \tanh^{-1}\left(\sqrt{\frac{1+i}{2^{3/4} + (1+i)}} x\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(2 + (1 - x^2)^4), x]

[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1972

`Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]`

Rule 6740

`Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IntegerQ[n, 0] && PolynomialInQ[v, u, x]`

Rubi steps

$$\begin{aligned} \int \frac{x^2}{2 + (1 - x^2)^4} dx &= \int \left(\frac{\sqrt[4]{-2} + i\sqrt{2}}{8(1 + \sqrt[4]{-2} - x^2)} + \frac{\sqrt[4]{-2} - i\sqrt{2}}{8(-1 + \sqrt[4]{-2} + x^2)} + \frac{\sqrt[4]{-2} - \sqrt{2}}{8(\sqrt[4]{-2} - i(1 - x^2))} + \frac{\sqrt[4]{-2} + \sqrt{2}}{8(\sqrt[4]{-2} + i(1 - x^2))} \right) dx \\ &= \frac{1}{8} (\sqrt[4]{-2} - \sqrt{2}) \int \frac{1}{\sqrt[4]{-2} - i(1 - x^2)} dx + \frac{1}{8} (\sqrt[4]{-2} + i\sqrt{2}) \int \frac{1}{-1 + \sqrt[4]{-2} + x^2} dx + \frac{1}{8} (\sqrt[4]{-2} + \sqrt{2}) \int \frac{1}{\sqrt[4]{-2} + i(1 - x^2)} dx \\ &= -\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{1}{8} (\sqrt[4]{-2} - \sqrt{2}) \int \frac{1}{-1 + \sqrt[4]{-2} + x^2} dx \\ &= -\frac{\sqrt[4]{-1} \sqrt{1 - \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 - \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{(-1)^{3/4} \sqrt{1 + i\sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + i\sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} + \frac{\sqrt[4]{-1} \sqrt{1 + \sqrt[4]{-2}} \tanh^{-1}\left(\frac{x}{\sqrt{1 + \sqrt[4]{-2}}}\right)}{4 \cdot 2^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8} \text{RootSum} \left[\#1^8 - 4\#1^6 + 6\#1^4 - 4\#1^2 + 3\&, \frac{\#1 \log(x - \#1)}{\#1^6 - 3\#1^4 + 3\#1^2 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(2 + (1 - x^2)^4), x]

[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 &, (Log[x - #1]*#1)/(-1 + 3*#1^2 - 3*#1^4 + #1^6) &]/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2 + (1 - x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(2 + (1 - x^2)^4),x]

[Out] IntegrateAlgebraic[x^2/(2 + (1 - x^2)^4), x]

fricas [B] time = 4.00, size = 2259, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*\sqrt{2}*\sqrt{\left(\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}}}\right)^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1} \\ & + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log\left(\frac{16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 + 16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1}{(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})}\right) \\ & - \sqrt{2}*\sqrt{\left(\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}}}\right)^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1} \\ & + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log\left(\frac{16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 + 16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1}{(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})}\right) \\ & - \sqrt{2}*\sqrt{\left(\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}}}\right)^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1} \\ & + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log\left(\frac{16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 + 16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1}{(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})}\right) \\ & - \sqrt{2}*\sqrt{\left(\sqrt{-12288*(1/256*I*\sqrt{2}) - 1/2*\sqrt{-1/8192*I*\sqrt{2}}}\right)^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1} \\ & + 32*\sqrt{1/8192*I*\sqrt{2}} + 32*\sqrt{-1/8192*I*\sqrt{2}})*\log\left(\frac{16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + 16384*(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 + 16384*\sqrt{2}*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - \sqrt{-12288*(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}})^2 - 12288*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})^2 - 1/8*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) - 1}{(\sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})) + \sqrt{2}*(-I*\sqrt{2} + 128*\sqrt{-1/8192*I*\sqrt{2}}) + \sqrt{2}*(I*\sqrt{2} + 128*\sqrt{1/8192*I*\sqrt{2}})}\right) \end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2)) + 32*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) + 2*x) - 1/16*\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(-122 \\
& 88*(1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt} \\
& (2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2 - 1/8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sq} \\
& \text{rt}(2)))*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) - 1) + 32*\text{sqrt}(1/8192*I* \\
& \text{sqrt}(2)) + 32*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))*\log((16384*\text{sqrt}(2)*(-1/256*I*\text{sqrt}(2) \\
& - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) \\
& + 16384*(\text{sqrt}(2)*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2))) + \text{sqrt}(2))*(1/25 \\
& 6*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 + 16384*\text{sqrt}(2)*(-1/256*I*\text{sqrt} \\
& (2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2 + \text{sqrt}(-12288*(1/256*I*\text{sqrt}(2) - 1/2*\text{sq} \\
& \text{rt}(-1/8192*I*\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt} \\
& (2)))^2 - 1/8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))*(-I*\text{sqrt}(2) + 128*\text{sq} \\
& \text{rt}(-1/8192*I*\text{sqrt}(2))) - 1)*((\text{sqrt}(2)*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2) \\
&))) + \text{sqrt}(2))*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) + \text{sqrt}(2)*(I*\text{sqrt} \\
& (2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))) - \text{sqrt}(2))*\text{sqrt}(-\text{sqrt}(-12288*(1/256*I*\text{sq} \\
& \text{rt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt} \\
& (1/8192*I*\text{sqrt}(2)))^2 - 1/8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))*(-I*\text{sq} \\
& \text{rt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) - 1) + 32*\text{sqrt}(1/8192*I*\text{sqrt}(2)) + 32* \\
& \text{sqrt}(-1/8192*I*\text{sqrt}(2))) + 2*x) + 1/16*\text{sqrt}(2)*\text{sqrt}(-\text{sqrt}(-12288*(1/256*I*s \\
& \text{qrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sq} \\
& \text{rt}(1/8192*I*\text{sqrt}(2)))^2 - 1/8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))*(-I*s \\
& \text{qrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) - 1) + 32*\text{sqrt}(1/8192*I*\text{sqrt}(2)) + 32 \\
& *\text{sqrt}(-1/8192*I*\text{sqrt}(2)))*\log(-(16384*\text{sqrt}(2)*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(\\
& 1/8192*I*\text{sqrt}(2)))^2*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) + 16384*(\text{sq} \\
& \text{rt}(2)*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2))) + \text{sqrt}(2))*(1/256*I*\text{sqrt}(2) \\
& - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 + 16384*\text{sqrt}(2)*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sq} \\
& \text{rt}(1/8192*I*\text{sqrt}(2)))^2 + \text{sqrt}(-12288*(1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I \\
& *\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2 - 1/ \\
& 8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I \\
& *\text{sqrt}(2))) - 1)*((\text{sqrt}(2)*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2))) + \text{sqrt}(2) \\
&))*(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) + \text{sqrt}(2)*(I*\text{sqrt}(2) + 128*\text{sq} \\
& \text{rt}(1/8192*I*\text{sqrt}(2)))) - \text{sqrt}(2))*\text{sqrt}(-\text{sqrt}(-12288*(1/256*I*\text{sqrt}(2) - 1/2* \\
& \text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2 - 12288*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sq} \\
& \text{rt}(2)))^2 - 1/8*(I*\text{sqrt}(2) + 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)))*(-I*\text{sqrt}(2) + 128* \\
& \text{sqrt}(-1/8192*I*\text{sqrt}(2))) - 1) + 32*\text{sqrt}(1/8192*I*\text{sqrt}(2)) + 32*\text{sqrt}(-1/8192 \\
& *I*\text{sqrt}(2))) + 2*x) - \text{sqrt}(1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))*\text{l} \\
& \text{og}(8*(8388608*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^3 + 32768*(1/ \\
& 256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2*(-I*\text{sqrt}(2) - 128*\text{sqrt}(1/819 \\
& 2*I*\text{sqrt}(2)) - 1) - 32768*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2 \\
& *(-I*\text{sqrt}(2) + 128*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) - 2*I*\text{sqrt}(2) - 256*\text{sqrt}(1/8192 \\
& *I*\text{sqrt}(2)) - 3)*\text{sqrt}(1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2))) + x) + \\
& \text{sqrt}(1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(-1/8192*I*\text{sqrt}(2)))*\log(-8*(8388608*(-1/25 \\
& 6*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^3 + 32768*(1/256*I*\text{sqrt}(2) - 1/2* \\
& \text{sqrt}(-1/8192*I*\text{sqrt}(2)))^2*(-I*\text{sqrt}(2) - 128*\text{sqrt}(1/8192*I*\text{sqrt}(2)) - 1) - \\
& 32768*(-1/256*I*\text{sqrt}(2) - 1/2*\text{sqrt}(1/8192*I*\text{sqrt}(2)))^2*(-I*\text{sqrt}(2) + 128*s \\
& \text{qrt}(-1/8192*I*\text{sqrt}(2))) - 2*I*\text{sqrt}(2) - 256*\text{sqrt}(1/8192*I*\text{sqrt}(2)) - 3)*\text{sq}
\end{aligned}$$

$t(1/256*I*\sqrt{2} - 1/2*\sqrt{-1/8192*I*\sqrt{2}}) + x) + \sqrt{-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})*\log(8*(8388608*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}))^3 + 32768*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}))^2 - 2*I*\sqrt{2} - 256*\sqrt{1/8192*I*\sqrt{2}} - 5)*\sqrt{-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}) + x) - \sqrt{-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}})*\log(-8*(8388608*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}))^3 + 32768*(-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}))^2 - 2*I*\sqrt{2} - 256*\sqrt{1/8192*I*\sqrt{2}} - 5)*\sqrt{-1/256*I*\sqrt{2} - 1/2*\sqrt{1/8192*I*\sqrt{2}}) + x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")

[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)

maple [C] time = 0.01, size = 56, normalized size = 0.30

$$\frac{\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^2 \ln(-\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3) + x)}{8\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^7 - 24\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^5 + 24\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)^3 - 8\text{RootOf}(-Z^8 - 4Z^6 + 6Z^4 - 4Z^2 + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2+(-x^2+1)^4),x)

[Out] 1/8*sum(1/(_R^7-3*_R^5+3*_R^3-_R)*_R^2*ln(-_R+x),_R=RootOf(-Z^8-4*_Z^6+6*_Z^4-4*_Z^2+3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 1)^4 + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)

mupad [B] time = 2.74, size = 142, normalized size = 0.76

$$\sum_{i=1}^8 \ln\left(\text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)\right) \left(40x - \text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)\right) \left(\text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)\right) \left(4096x + \text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)\right) \left(786432x + \text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)\right) \left(768\right) - 3) \text{root}\left(x^4 + \frac{x^4}{16384} - \frac{x^2}{1048576} + \frac{3}{1073741824} \cdot x, -i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x -
root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384 - z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/1638
4 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.21, size = 39, normalized size = 0.21

$$\text{RootSum}\left(1073741824t^8 + 65536t^4 - 1024t^2 + 3, \left(t \mapsto t \log\left(67108864t^7 + 262144t^5 + 4096t^3 - 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

$$3.387 \quad \int \frac{1-x^2}{a+b(1-x^2)^4} dx$$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Rubi [A] time = 1.11, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{-a} + \sqrt{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right)}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{-a} + \sqrt{b}}\right)}{4\sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} - \frac{\sqrt{\sqrt{-a} + \sqrt{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{-a} + \sqrt{b}} \sqrt[4]{b}}{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt[4]{b}}\right)}{4\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} + \frac{\sqrt{\sqrt{-a} + \sqrt{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{-a} + \sqrt{b}} \sqrt[4]{b}}{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}\right)}{4\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{-a} + \sqrt{b}}\right)}{4\sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4), x]

[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x]/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)])]/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x]/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)])]/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{a, x}]^{-1} \rightarrow \text{Simp}[\text{Rt}[a/b, 2] * \text{ArcTan}[x/\text{Rt}[a/b, 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 208

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{a, x}]^{-1} \rightarrow \text{Simp}[\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

Rule 618

$\text{Int}[\frac{(a_.) + (b_.)(x_) + (c_.)(x_)^2}{b^2 - 4ac - x^2, x}]^{-1} \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Dist}[(2cd - be)/(2c), \text{Int}[1/(a + bx + cx^2), x], x] + \text{Dist}[e/(2c), \text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

Rule 1166

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - be)/(2q), \text{Int}[1/(b/2 - q/2 + cx^2), x], x] + \text{Dist}[e/2 - (2cd - be)/(2q), \text{Int}[1/(b/2 + q/2 + cx^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$

Rule 1169

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2q - b/c, 2]\}, \text{Dist}[1/(2cq*r), \text{Int}[(d*r - (d - eq)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2cq*r), \text{Int}[(d*r + (d - eq)*x)/(q + r*x + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - b*d*e + ae^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{a+b(1-x^2)^4} dx &= \int \left(\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}-b(1-x^2)^2)} - \frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}+b(1-x^2)^2)} \right) dx \\
&= -\frac{\sqrt{b} \int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b(1-x^2)^2} dx}{2\sqrt{-a}} - \frac{\sqrt{b} \int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b(1-x^2)^2} dx}{2\sqrt{-a}} \\
&= -\frac{\sqrt{b} \int \frac{1-x^2}{(\sqrt{-a}-\sqrt{b})\sqrt{b}+2bx^2-bx^4} dx}{2\sqrt{-a}} - \frac{\sqrt{b} \int \frac{1-x^2}{(\sqrt{-a}+\sqrt{b})\sqrt{b}-2bx^2+bx^4} dx}{2\sqrt{-a}} \\
&= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}{\sqrt[8]{b}} \left(1+\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x}{\sqrt[8]{b}} + x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} + \sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}{\sqrt[8]{b}} \left(1+\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} + \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x}{\sqrt[8]{b}} + x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} + \sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right) \int \frac{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x}{\sqrt[8]{b}}}{8\sqrt{-a}\sqrt{b}} dx}{8\sqrt{-a}\sqrt{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\right)}{8\sqrt{-a}\sqrt{b}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.10

$$\frac{\text{RootSum}\left[\#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + a + b\&, \frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4), x]

[Out] $-\frac{1}{8}\text{RootSum}[a + b - 4*b*\#1^2 + 6*b*\#1^4 - 4*b*\#1^6 + b*\#1^8 \& , \text{Log}[x - \#1] / (\#1 - 2*\#1^3 + \#1^5) \&] / b$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x^2}{a + b(1 - x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(a + b*(1 - x^2)^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(a + b*(1 - x^2)^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{(x^2 - 1)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4), x, algorithm="giac")

[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)

maple [C] time = 0.07, size = 69, normalized size = 0.10

$$\frac{(-\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^2 + 1) \ln(-\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b) + x)}{8b(\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^7 - 3\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^5 + 3\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^3 - \text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(a+b*(-x^2+1)^4), x)

[Out] $1/8/b*\text{sum}((-R^2+1)/(R^7-3*R^5+3*R^3-R)*\ln(-R+x), R=\text{RootOf}(Z^8*b-4*Z^6*b+6*Z^4*b-4*Z^2*b+a+b))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{(x^2 - 1)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")`

[Out] `-integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)`

mupad [B] time = 2.70, size = 328, normalized size = 0.49

$\sum_{k=0}^{\infty} \left(\frac{(-1)^k (64 \sqrt{16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1} (z, k)^{2 a b + 1} (4096 \sqrt{16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1} (z, k)^4 a^2 b^2 + 128 \sqrt{16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1} (z, k)^2 a b - 32768 \sqrt{16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1} (z, k)^5 a^3 b^2 x + 1) \sqrt{16777216 a^5 b^3 z^8 + 16777216 a^4 b^4 z^8 + 1048576 a^3 b^3 z^6 + 24576 a^2 b^2 z^4 + 256 a b z^2 + 1} (z, k), k, 1, 8) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)`

[Out] `symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)`

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$-\text{RootSum}(t^8 (16777216 a^5 b^3 + 16777216 a^4 b^4) + 1048576 t^6 a^3 b^3 + 24576 t^4 a^2 b^2 + 256 t^2 a b + 1, (t \mapsto t \log(-6291456 t^7 a^3 b^3 - 6291456 t^7 a^3 b^4 + 65536 t^5 a^3 b^2 - 327680 t^5 a^2 b^3 - 512 t^3 a^2 b - 5632 t^3 a b^2 - 32 t b + x)))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)`

[Out] `-RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-6291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x)))`

$$3.388 \quad \int \frac{1-x^2}{a+b(-1+x^2)^4} dx$$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Rubi [A] time = 0.65, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{-a} + \sqrt{b}} \log\left(-\sqrt{2} \sqrt[8]{b} x \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}} + \sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b} x^2\right) \sqrt{\sqrt{\sqrt{-a} + \sqrt{b}} + \sqrt[4]{b}}}{8\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{-a} + \sqrt{b}}\right)}{4\sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} - \frac{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b} \tan^{-1}\left(\frac{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b}}{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b}}\right)}{4\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} + \frac{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b} \tan^{-1}\left(\frac{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b}}{\sqrt{\sqrt{-a} + \sqrt{b}} - \sqrt{b}}\right)}{4\sqrt{2} \sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{-a} + \sqrt{b}}\right)}{4\sqrt{-a} b^{3/8} \sqrt{\sqrt{-a} + \sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x]/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x]/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{a, x}]^{-1} \rightarrow \text{Simp}[\text{Rt}[a/b, 2] * \text{ArcTan}[x/\text{Rt}[a/b, 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 208

$\text{Int}[\frac{(a_.) + (b_.)(x_)^2}{a, x}]^{-1} \rightarrow \text{Simp}[\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

Rule 618

$\text{Int}[\frac{(a_.) + (b_.)(x_) + (c_.)(x_)^2}{b^2 - 4ac - x^2, x}]^{-1} \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2cd - be, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)(x_)}{(a_.) + (b_.)(x_) + (c_.)(x_)^2}, x_Symbol] \rightarrow \text{Dist}[(2cd - be)/(2c), \text{Int}[1/(a + bx + cx^2), x], x] + \text{Dist}[e/(2c), \text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[2cd - be, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4ac]$

Rule 1166

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - be)/(2q), \text{Int}[1/(b/2 - q/2 + cx^2), x], x] + \text{Dist}[e/2 - (2cd - be)/(2q), \text{Int}[1/(b/2 + q/2 + cx^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - ae^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$

Rule 1169

$\text{Int}[\frac{(d_.) + (e_.)(x_)^2}{(a_.) + (b_.)(x_)^2 + (c_.)(x_)^4}, x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[a/c, 2]\}, \text{With}\{r = \text{Rt}[2q - b/c, 2]\}, \text{Dist}[1/(2cq*r), \text{Int}[(d*r - (d - eq)*x)/(q - r*x + x^2), x], x] + \text{Dist}[1/(2cq*r), \text{Int}[(d*r + (d - eq)*x)/(q + r*x + x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - b*d*e + ae^2, 0] \ \&\& \ \text{NegQ}[b^2 - 4ac]$

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{a+b(-1+x^2)^4} dx &= - \int \frac{-1+x^2}{a+b(-1+x^2)^4} dx \\
&= - \int \left(\frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}-b(-1+x^2)^2)} - \frac{\sqrt{b}(-1+x^2)}{2\sqrt{-a}(\sqrt{-a}\sqrt{b}+b(-1+x^2)^2)} \right) dx \\
&= \frac{\sqrt{b} \int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b(-1+x^2)^2} dx}{2\sqrt{-a}} + \frac{\sqrt{b} \int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b(-1+x^2)^2} dx}{2\sqrt{-a}} \\
&= \frac{\sqrt{b} \int \frac{-1+x^2}{(\sqrt{-a}-\sqrt{b})\sqrt{b}+2bx^2-bx^4} dx}{2\sqrt{-a}} + \frac{\sqrt{b} \int \frac{-1+x^2}{(\sqrt{-a}+\sqrt{b})\sqrt{b}-2bx^2+bx^4} dx}{2\sqrt{-a}} \\
&= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} - \left(-1-\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} - \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}x}{\sqrt[8]{b}} + x^2} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}} + \frac{\sqrt[4]{b}\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[8]{b}}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}}{\sqrt[8]{b}} + \left(-1-\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{-a}+\sqrt{b}}{\sqrt[4]{b}} + \frac{\sqrt{2}\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}x}{\sqrt[8]{b}}} dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right) \int \frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}} dx}{8\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}} \log\left(\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}}\right)}{8\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}+\sqrt[4]{b}}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}-\sqrt[4]{b}} \tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}-\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{-a}+\sqrt{b}+\sqrt[4]{b}}-\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}b^{3/8}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.10

$$\frac{\text{RootSum}\left[\#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + a + b \&, \frac{\log(x-\#1)}{\#1^5 - 2\#1^3 + \#1}\right]}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1]/(#1 - 2*#1^3 + #1^5) &]/b

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - x^2}{a + b(-1 + x^2)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

[Out] IntegrateAlgebraic[(1 - x^2)/(a + b*(-1 + x^2)^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(x^2-1)^4), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{(x^2 - 1)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(a+b*(x^2-1)^4), x, algorithm="giac")

[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)

maple [C] time = 0.00, size = 69, normalized size = 0.10

$$\frac{(-\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^2 + 1) \ln(-\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b) + x)}{8b(\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^7 - 3\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^5 + 3\text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b)^3 - \text{RootOf}(b_Z^8 - 4b_Z^6 + 6b_Z^4 - 4b_Z^2 + a + b))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)/(a+b*(x^2-1)^4),x)`

[Out] `1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z^6*b+6*_Z^4*b-4*_Z^2*b+a+b))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{(x^2 - 1)^4 b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")`

[Out] `-integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)`

mupad [B] time = 0.00, size = 328, normalized size = 0.49

$\sum_{k=0}^{\infty} \left(\frac{1}{k!} \left(\frac{1}{(16777216a^5b^3z^8 + 16777216a^4b^4z^8 + 1048576a^3b^3z^6 + 24576a^2b^2z^4 + 256abz^2 + 1, z, k)^2} \right) \left(\frac{1}{(16777216a^5b^3z^8 + 16777216a^4b^4z^8 + 1048576a^3b^3z^6 + 24576a^2b^2z^4 + 256abz^2 + 1, z, k)^4} \right) \right) \left(\frac{1}{(16777216a^5b^3z^8 + 16777216a^4b^4z^8 + 1048576a^3b^3z^6 + 24576a^2b^2z^4 + 256abz^2 + 1, z, k)^5} \right) \left(\frac{1}{(16777216a^5b^3z^8 + 16777216a^4b^4z^8 + 1048576a^3b^3z^6 + 24576a^2b^2z^4 + 256abz^2 + 1, z, k)^8} \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)`

[Out] `symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)`

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$-\text{RootSum}\left(t^8\left(16777216a^5b^3 + 16777216a^4b^4\right) + 1048576t^6a^3b^3 + 24576t^4a^2b^2 + 256t^2ab + 1, (t \mapsto t \log(-6291456t^7a^4b^3 - 6291456t^7a^3b^4 + 65536t^5a^2b^2 - 327680t^5a^2b^3 - 512t^3a^2b - 5632t^3ab^2 - 32tb + x))\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)/(a+b*(x**2-1)**4),x)`

[Out] `-RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-6291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))`

$$3.389 \quad \int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}} + \frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx$$

Verification is not applicable to the result.

[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3), x]

[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]

Rubi steps

$$\begin{aligned} \int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3} dx &= \int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} + \frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} \right) dx \\ &= 2 \int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx + \int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx + \int \frac{x^4}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6} dx \\ &= 2 \int \frac{x^2}{ax^6+b(1+x^2)^3} dx + \int \frac{1}{ax^6+b(1+x^2)^3} dx + \int \frac{x^4}{ax^6+b(1+x^2)^3} dx \end{aligned}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.57

$$\frac{1}{6} \text{RootSum} \left[\#1^6 a + \#1^6 b + 3\#1^4 b + 3\#1^2 b + b \&, \frac{\#1^4 \log(x - \#1) + 2\#1^2 \log(x - \#1) + \log(x - \#1)}{\#1^5 a + \#1^5 b + 2\#1^3 b + \#1 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]

[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) &]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)^2}{ax^6 + b(1+x^2)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]

[Out] IntegrateAlgebraic[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3), x]

fricas [C] time = 4.55, size = 5653, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")

[Out] 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*log(1/6*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) - 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) + 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)

$$\begin{aligned}
& *(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - \\
& 1296*(I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b \\
& + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a* \\
& b + b^2)) + 3*\sqrt{1/3}*(a*b + b^2)*\sqrt{-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*\sqrt{3} \\
& + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1 \\
& /31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a \\
& + b)^2*b^5))^{(1/3)} - 1296*(I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104 \\
& /((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2 \\
& *b^5))^{(1/3)} - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*\sqrt{3} + 1)*(1/(\\
& a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + \\
& b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/ \\
& 3)} - 1296*(I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)* \\
& (a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 7 \\
& 2/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + \\
& b^2))*\log(1/12*b*\sqrt{-((a*b + b^2)*((-I*\sqrt{3} + 1)*(1/(a*b^3 + b^4) - 1 \\
& /((a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2 \\
&)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 1296*(I*\sqrt{ \\
& t(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1 \\
& /46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a*b + b^2)) + \\
& 3*\sqrt{1/3}*(a*b + b^2)*\sqrt{-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*\sqrt{3} + 1) \\
& *(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a* \\
& b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5) \\
&)^{(1/3)} - 1296*(I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + \\
& b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3 \\
&) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*\sqrt{3} + 1)*(1/(a*b^3 + b^4 \\
&) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b \\
& + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 1296*(\\
& I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2) \\
&) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a*b + b^ \\
& 2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x) \\
& - 1/72*\sqrt{-((a*b + b^2)*((-I*\sqrt{3} + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^ \\
& 2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/466 \\
& 56/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 1296*(I*\sqrt{3} + 1)* \\
& (-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a* \\
& b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a*b + b^2)) + 3*\sqrt{1/ \\
& 3}*(a*b + b^2)*\sqrt{-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*\sqrt{3} + 1)*(1/(a*b^3 \\
& + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4) \\
& *(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - \\
& 1296*(I*\sqrt{3} + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b \\
& + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a* \\
& b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*\sqrt{3} + 1)*(1/(a*b^3 + b^4) - 1/(a*b \\
& + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - \\
& 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 1296*(I*\sqrt{3} \\
& + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/4665 \\
& 6/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^{(1/3)} - 72/(a*b + b^2)) + 2073
\end{aligned}$$


```

*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) +
1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
+ b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^
4 + b^5)) + 216)/(a*b + b^2)) + x) - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) +
1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/(
(a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b
^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3
+ b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(
1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4
+ b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt
(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((
a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^
5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5))
+ 216)/(a*b + b^2))*log(-1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*
b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b
^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
- 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a
*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/
(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-
I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(
1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^
3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(
1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^
4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
- 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*
b + b^2)) + x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 1)^2}{ax^6 + (x^2 + 1)^3 b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")

[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)

maple [C] time = 0.28, size = 67, normalized size = 0.40

$$\frac{\left(\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)^4+2\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)^2+1\right)\ln\left(-\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)+x\right)}{6\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)^5a+6\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)^5b+12\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)^3b+6\text{RootOf}\left((a+b)_Z^6+3_Z^4b+3_Z^2b+b\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)

[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(-_R+x),_R=RootOf((a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2 + 1)^2}{ax^6 + (x^2 + 1)^3 b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")

[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)

mupad [B] time = 3.08, size = 504, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)

[Out] symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2

$+ 1, z, k)^5 * a * b^4 * x - 1)) * \text{root}(46656 * a * b^5 * z^6 + 46656 * b^6 * z^6 + 3888 * b^4 * z^4 + 108 * b^2 * z^2 + 1, z, k), k, 1, 6)$

sympy [A] time = 1.87, size = 42, normalized size = 0.25

$$\text{RootSum}\left(t^6 (46656ab^5 + 46656b^6) + 3888t^4b^4 + 108t^2b^2 + 1, (t \mapsto t \log(6tb + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)

[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**2 + 1, Lambda(_t, _t*log(6*_t*b + x)))

$$3.390 \quad \int \frac{(d+ex)^3}{a+cx^4} dx$$

Optimal. Leaf size=320

$$\frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}}$$

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2} a^{3/4} c^{3/4}} - \frac{d(3\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} c^{3/4}} + \frac{d(3\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} c^{3/4}} + \frac{3d^2 e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{c}} + \frac{e^3 \log(a + cx^4)}{4c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4), x]

[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^4])/(4*c)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
 \int \frac{(d+ex)^3}{a+cx^4} dx &= \int \left(\frac{d^3+3de^2x^2}{a+cx^4} + \frac{x(3d^2e+e^3x^2)}{a+cx^4} \right) dx \\
 &= \int \frac{d^3+3de^2x^2}{a+cx^4} dx + \int \frac{x(3d^2e+e^3x^2)}{a+cx^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{3d^2e+e^3x}{a+cx^2} dx, x, x^2 \right) + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} - 3e^2 \right) \right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2c} + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + 3e^2 \right) \right) \int \frac{\sqrt{a}}{a+cx^4} dx}{2c} \\
 &= \frac{1}{2} (3d^2e) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right) + \frac{1}{2} e^3 \text{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right) + \frac{\left(d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + 3e^2 \right) \right) \int \frac{\sqrt{a}}{a+cx^4} dx}{4c} \\
 &= \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{cx} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{d(\sqrt{c}d^2 - 3\sqrt{a}e^2)}{2\sqrt{2}a^{3/4}c^{3/4}} \\
 &= \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d(\sqrt{c}d^2 + 3\sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d(\sqrt{c}d^2 + 3\sqrt{a}e^2) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.25, size = 322, normalized size = 1.01

$$\frac{-\sqrt{2}\sqrt{c}(\sqrt{a}\sqrt{c}d^3 - 3a^{3/4}d^2) \log(-\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) + \sqrt{2}\sqrt{c}(\sqrt{a}\sqrt{c}d^3 - 3a^{3/4}d^2) \log(\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) - 2\sqrt{a}\sqrt{c}d(6\sqrt{a}\sqrt{c}de + 3\sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}}\right) + 2\sqrt{a}\sqrt{c}d(-6\sqrt{a}\sqrt{c}de + 3\sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx}}{\sqrt{a}} + 1\right) + 2a^3 \log(a+cx^4)}{8ac}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4), x]

[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2

] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4]/(8*a*c)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{a + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4), x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.36, size = 311, normalized size = 0.97

$$\frac{e^3 \log\left(\frac{cx^4 + a}{4c}\right) + \frac{\sqrt{2}\left(3\sqrt{3}\sqrt{ac}c^2d^2e + (ac)^{\frac{3}{2}}c^2d^2 + 3(ac)^{\frac{3}{2}}d^2e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{d}{e}\right)^{\frac{1}{2}}\right)}{2\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4ac^3} + \frac{\sqrt{2}\left(3\sqrt{3}\sqrt{ac}c^2d^2e + (ac)^{\frac{3}{2}}c^2d^2 + 3(ac)^{\frac{3}{2}}d^2e\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{d}{e}\right)^{\frac{1}{2}}\right)}{2\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4ac^3} + \frac{\sqrt{2}\left((ac)^{\frac{3}{2}}c^2d^2 - 3(ac)^{\frac{3}{2}}d^2e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{d}{e}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{c}}\right) - \sqrt{2}\left((ac)^{\frac{3}{2}}c^2d^2 - 3(ac)^{\frac{3}{2}}d^2e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{d}{e}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{c}}\right)}{8ac^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a), x, algorithm="giac")

[Out] $\frac{1}{4}e^3 \log(\text{abs}(cx^4 + a))/c + \frac{1}{4}\sqrt{2} \cdot (3\sqrt{2}\sqrt{ac})c^2d^2e + (ac^3)^{\frac{1}{4}}c^2d^3 + 3(ac^3)^{\frac{3}{4}}d^2e^2 \cdot \arctan\left(\frac{1}{2}\sqrt{2} \cdot (2x + \sqrt{2}) \cdot (a/c)^{\frac{1}{4}}\right) / (a/c)^{\frac{1}{4}} / (ac^3) + \frac{1}{4}\sqrt{2} \cdot (3\sqrt{2}\sqrt{ac})c^2d^2e + (ac^3)^{\frac{1}{4}}c^2d^3 + 3(ac^3)^{\frac{3}{4}}d^2e^2 \cdot \arctan\left(\frac{1}{2}\sqrt{2} \cdot (2x - \sqrt{2}) \cdot (a/c)^{\frac{1}{4}}\right) / (a/c)^{\frac{1}{4}} / (ac^3) + \frac{1}{8}\sqrt{2} \cdot ((ac^3)^{\frac{1}{4}}c^2d^3 - 3(ac^3)^{\frac{3}{4}}d^2e^2) \cdot \log(x^2 + \sqrt{2}x \cdot (a/c)^{\frac{1}{4}} + \sqrt{a/c}) / (ac^3) - \frac{1}{8}\sqrt{2} \cdot ((ac^3)^{\frac{1}{4}}c^2d^3 - 3(ac^3)^{\frac{3}{4}}d^2e^2) \cdot \log(x^2 - \sqrt{2}x \cdot (a/c)^{\frac{1}{4}} + \sqrt{a/c}) / (ac^3)$

maple [A] time = 0.01, size = 314, normalized size = 0.98

$$\frac{3d^2e \arctan\left(\sqrt{\frac{c}{a}}x\right)}{2\sqrt{ac}} + \frac{\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}d^3 \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4a} + \frac{\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}d^3 \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4a} + \frac{\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}d^3 \ln\left(\frac{x^2+\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{8a} + \frac{3\sqrt{2}d^2e^2 \arctan\left(\frac{\sqrt{2}x-1}{\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4\left(\frac{d}{e}\right)^{\frac{1}{2}}c} + \frac{3\sqrt{2}d^2e^2 \arctan\left(\frac{\sqrt{2}x+1}{\left(\frac{d}{e}\right)^{\frac{1}{2}}}\right)}{4\left(\frac{d}{e}\right)^{\frac{1}{2}}c} + \frac{3\sqrt{2}d^2e \ln\left(\frac{x^2+\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{d}{e}\right)^{\frac{1}{2}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{d}{e}\right)^{\frac{1}{2}}c} + \frac{e^3 \ln(cx^4 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(c*x^4+a),x)`

[Out] $\frac{1}{8}d^3\left(\frac{a}{c}\right)^{1/4}/a^{1/2}\ln\left(\frac{x^2+(a/c)^{1/4}x^{1/2}+(a/c)^{1/2}}{x^2-(a/c)^{1/4}x^{1/2}+(a/c)^{1/2}}\right)+\frac{1}{4}d^3\left(\frac{a}{c}\right)^{1/4}/a^{1/2}\arctan\left(\frac{2^{1/2}/(a/c)^{1/4}x+1}{2^{1/2}/(a/c)^{1/4}x-1}\right)+\frac{3}{2}e^2d^2/(a^2c)^{1/2}\arctan\left(\frac{x^2(c/a)^{1/2}}{x^2+(a/c)^{1/4}x^{1/2}+(a/c)^{1/2}}\right)+\frac{3}{8}d^2e^2/c\left(\frac{a}{c}\right)^{1/4}2^{1/2}\ln\left(\frac{x^2-(a/c)^{1/4}x^{1/2}+(a/c)^{1/2}}{x^2+(a/c)^{1/4}x^{1/2}+(a/c)^{1/2}}\right)+\frac{3}{4}d^2e^2/c\left(\frac{a}{c}\right)^{1/4}2^{1/2}\arctan\left(\frac{2^{1/2}/(a/c)^{1/4}x+1}{2^{1/2}/(a/c)^{1/4}x-1}\right)+\frac{1}{4}e^3\ln(c*x^4+a)/c$

maxima [A] time = 1.96, size = 310, normalized size = 0.97

$$\frac{\sqrt{2}\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}+cd^3-3\sqrt{a}\sqrt{c}d^2}\right)\log\left(\sqrt{c}x^2+\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}x+\sqrt{a}}\right)}{8a^{\frac{3}{2}}c^{\frac{1}{2}}}+\frac{\sqrt{2}\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}-cd^3+3\sqrt{a}\sqrt{c}d^2}\right)\log\left(\sqrt{c}x^2-\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}x+\sqrt{a}}\right)}{8a^{\frac{3}{2}}c^{\frac{1}{2}}}+\frac{\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}d^3+3\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}d^2-6\sqrt{a}cd^2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{2}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{2}}}+\frac{\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}d^3+3\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}d^2+6\sqrt{a}cd^2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{2}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")`

[Out] $\frac{1}{8}\sqrt{2}\left(\sqrt{2}\right)a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+c^{\frac{3}{4}}d^3-3\sqrt{a}\sqrt{c}d^2e^2\log\left(\frac{\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}}{a^{\frac{3}{4}}c^{\frac{5}{4}}}\right)+\frac{1}{8}\sqrt{2}\left(\sqrt{2}\right)a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-c^{\frac{3}{4}}d^3+3\sqrt{a}\sqrt{c}d^2e^2\log\left(\frac{\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}}{a^{\frac{3}{4}}c^{\frac{5}{4}}}\right)+\frac{1}{4}\left(\sqrt{2}\right)a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}d^2e^2-6\sqrt{a}\sqrt{c}d^2e^2\arctan\left(\frac{1/2\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{\sqrt{a}\sqrt{c}}\right)/\sqrt{a}\sqrt{c}+1/4\left(\sqrt{2}\right)a^{\frac{1}{4}}c^{\frac{5}{4}}d^3+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}d^2e^2+6\sqrt{a}\sqrt{c}d^2e^2\arctan\left(\frac{1/2\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{\sqrt{a}\sqrt{c}}\right)/\sqrt{a}\sqrt{c}$

mupad [B] time = 2.84, size = 894, normalized size = 2.79

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)^3/(a + c*x^4),x)`

[Out] $\text{symsum}\left(\log\left(-2cd^2(5ad^6e^6-3cd^5e^2+3a^7e^7x+8\sqrt{256a^3c^4z^4-256a^3c^3e^3z^3+480a^2c^3d^4e^2z^2+96a^3c^2e^6z^2+192a^2c^2d^4e^5z-48a^3c^3d^8e^ez-16a^3c^3e^9z+3a^2c^2d^4e^8+3ac^2d^8e^4+c^3d^12+a^3e^12},z,k\right)^2ac^2d+2\sqrt{256a^3c^4z^4-256a^3c^3e^3z^3+480a^2c^3d^4e^2z^2+96a^3c^2e^6z^2+192a^2c^2d^4e^5z-48a^3c^3d^8e^ez-16a^3c^3e^9z+3a^2c^2d^4e^8+3ac^2d^8e^4+c^3d^12+a^3e^12},z,k\right)cd^4x-5cd^4e^3x-24\sqrt{256a^3c^4z^4-256a^3c^3e^3z^3+480a^2c^3d^4e^2z^2+96a^3c^2e^6z^2+192a^2c^2d^4e^5z-48a^3c^3d^8e^ez-16a^3c^3e^9z+3a^2c^2d^4e^8+3ac^2d^8e^4+c^3d^12+a^3e^12}$

```

*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a
^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)
^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*
d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z
- 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12
, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*
c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8
*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*
e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^
2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d
^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^
3*e^12, z, k), k, 1, 4)

```

sympy [A] time = 5.29, size = 384, normalized size = 1.20

RootSum(256*t^4*a**3*c**4 - 256*t**3*a**3*c**3*e**3 + 480*t**2*c**3*d**4*e**2) + (-16*a**3*c*e**9 + 192*a**2*c**2*d**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**8*e**4 + c**3*d**12, Lambda(t, t*log(x + (1728*t**3*a**4*c**3*e**6 + 960*t**2*a**4*c**2*e**9 - 2016*t**2*a**3*c**3*d**4*e**5 + 48*t**2*a**2*c**4*d**8*e + 324*t*a**4*c*e**12 + 4716*t*a**3*c**2*d**4*e**8 + 1452*t*a**2*c**3*d**8*e**4 + 4*t*a*c**4*d**12 - 27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**11*e**4 + c**4*d**15))))

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**3/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c**3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a**3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**11*e**4 + c**4*d**15))))

$$3.391 \quad \int \frac{(d+ex)^2}{a+cx^4} dx$$

Optimal. Leaf size=291

$$\frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{a}e^2)}{\sqrt{a}}$$

Rubi [A] time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}c^{3/4}} - \frac{(\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{(\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{de \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4), x]

[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{a+cx^4} dx &= \int \left(\frac{2dex}{a+cx^4} + \frac{d^2+e^2x^2}{a+cx^4} \right) dx \\
&= (2de) \int \frac{x}{a+cx^4} dx + \int \frac{d^2+e^2x^2}{a+cx^4} dx \\
&= (de) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right) + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4} dx}{2c} \\
&= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} + \frac{\left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4c} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2 \right) \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} \\
&= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2 \right) \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}c^{3/4}} \\
&= \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2 + \sqrt{a}e^2 \right) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2 - \sqrt{a}e^2 \right) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 243, normalized size = 0.84

$$\frac{-\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2) \left(\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) \right) - 2(4\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right) + 2(-4\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1 \right)}{8a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4), x]

[Out] $(-2*(\text{Sqrt}[2]*\text{Sqrt}[c]*d^2 + 4*a^{(1/4)}*c^{(1/4)}*d*e + \text{Sqrt}[2]*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 2*(\text{Sqrt}[2]*\text{Sqrt}[c]*d^2 - 4*a^{(1/4)}*c^{(1/4)}*d*e + \text{Sqrt}[2]*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] - \text{Sqrt}[2]*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*(\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] - \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2]))/(8*a^{(3/4)}*c^{(3/4)})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4), x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 285, normalized size = 0.98

$$\frac{\sqrt{2} \left(2\sqrt{2}\sqrt{ac}c^2de + (ac)^{\frac{1}{2}}c^2d^2 + (ac)^{\frac{3}{2}}c^2 \right) \arctan\left(\frac{\sqrt{2}(2x+\sqrt{2}(\frac{c}{a})^{\frac{1}{4}})}{2(\frac{c}{a})^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2} \left(2\sqrt{2}\sqrt{ac}c^2de + (ac)^{\frac{1}{2}}c^2d^2 + (ac)^{\frac{3}{2}}c^2 \right) \arctan\left(\frac{\sqrt{2}(2x-\sqrt{2}(\frac{c}{a})^{\frac{1}{4}})}{2(\frac{c}{a})^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2} \left((ac)^{\frac{1}{2}}c^2d^2 - (ac)^{\frac{3}{2}}c^2 \right) \log\left(x^2 + \sqrt{2}x(\frac{c}{a})^{\frac{1}{4}} + \sqrt{\frac{c}{a}}\right)}{8ac^3} - \frac{\sqrt{2} \left((ac)^{\frac{1}{2}}c^2d^2 - (ac)^{\frac{3}{2}}c^2 \right) \log\left(x^2 - \sqrt{2}x(\frac{c}{a})^{\frac{1}{4}} + \sqrt{\frac{c}{a}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{4}\sqrt{2}*(2*\sqrt{2}*\sqrt{ac})*c^2*d*e + (a*c^3)^{\frac{1}{4}}*c^2*d^2 + (a*c^3)^{\frac{3}{4}}*e^2*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{\frac{1}{4}})/(a/c)^{\frac{1}{4}})/(a*c^3) + 1/4*\sqrt{2}*(2*\sqrt{2}*\sqrt{ac})*c^2*d*e + (a*c^3)^{\frac{1}{4}}*c^2*d^2 + (a*c^3)^{\frac{3}{4}}*e^2*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{\frac{1}{4}})/(a/c)^{\frac{1}{4}})/(a*c^3) + 1/8*\sqrt{2}*((a*c^3)^{\frac{1}{4}}*c^2*d^2 - (a*c^3)^{\frac{3}{4}}*e^2)*\log(x^2 + \sqrt{2}x*(a/c)^{\frac{1}{4}} + \sqrt{a/c})/(a*c^3) - 1/8*\sqrt{2}*((a*c^3)^{\frac{1}{4}}*c^2*d^2 - (a*c^3)^{\frac{3}{4}}*e^2)*\log(x^2 - \sqrt{2}x*(a/c)^{\frac{1}{4}} + \sqrt{a/c})/(a*c^3)$

maple [A] time = 0.00, size = 292, normalized size = 1.00

$$\frac{de \arctan\left(\frac{\sqrt{\frac{c}{a}}x}{\sqrt{ac}}\right)}{\sqrt{ac}} + \frac{\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}-1\right)}{4a} + \frac{\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}+1\right)}{4a} + \frac{\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}d^2 \ln\left(\frac{x^2+\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{a}}}{x^2-\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{a}}}\right)}{8a} + \frac{\sqrt{2}e^2 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{c}{a}\right)^{\frac{1}{4}}c} + \frac{\sqrt{2}e^2 \arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{c}{a}\right)^{\frac{1}{4}}c} + \frac{\sqrt{2}e^2 \ln\left(\frac{x^2-\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{a}}}{x^2+\left(\frac{c}{a}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{a}}}\right)}{8\left(\frac{c}{a}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a),x)

[Out] $\frac{1}{8}d^2*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\ln((x^2+(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}})/(x^2-(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}}))+1/4*d^2*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x+1)+1/4*d^2*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x-1)+d*e/(a*c)^{\frac{1}{2}}*\arctan((1/a*c)^{\frac{1}{2}}*x^2)+1/8*e^2/c/(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*\ln((x^2-(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}})/(x^2+(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}}))+1/4*e^2/c/(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x+1)+1/4*e^2/c/(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x-1)$

maxima [A] time = 2.05, size = 275, normalized size = 0.95

$$\frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}x} + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}x} + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{3}{4}}d^2 + \sqrt{2a^{\frac{3}{4}}c^{\frac{1}{2}}e^2} - 4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{4a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2a^{\frac{1}{2}}c^{\frac{3}{4}}d^2 + \sqrt{2a^{\frac{3}{4}}c^{\frac{1}{2}}e^2} + 4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2a^{\frac{1}{2}}c^{\frac{1}{2}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{4a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))

mupad [B] time = 2.66, size = 556, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + c*x^4),x)

[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4)

sympy [A] time = 2.68, size = 277, normalized size = 0.95

$$\text{RootSum}\left(256t^4a^3c^3 + 192t^2a^2c^2d^2e^2 + t(32a^2cde^5 - 32ac^2d^5e) + a^2e^8 + 2acd^4e^4 + c^2d^8, \left(t \mapsto t \log\left(x + \frac{64t^3a^4c^2e^6 + 448t^3a^3c^3d^4e^2 - 160t^2a^3c^2d^5e^2 + 32t^2a^2c^3d^7e + 60ta^3cd^2e^6 + 256ta^2c^2d^4e^4 + 4ta^3d^{10} + 6a^3de^{11} - 24a^2cd^6e^7 - 30ac^2d^7e^3}{a^3c^{12} - 33a^2cd^4e^6 - 33ac^2d^8e^4 + c^3d^{12}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lambda(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2 - 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))

$$3.392 \quad \int \frac{d+ex}{a+cx^4} dx$$

Optimal. Leaf size=219

$$\frac{d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{2\sqrt{a} \sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4), x]

[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{a+cx^4} dx &= \int \left(\frac{d}{a+cx^4} + \frac{ex}{a+cx^4} \right) dx \\
&= d \int \frac{1}{a+cx^4} dx + e \int \frac{x}{a+cx^4} dx \\
&= \frac{d \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{d \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{1}{2} e \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right) \\
&= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} + \frac{d \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{d \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{d \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
&= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\
&= \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}\sqrt{c}} - \frac{d \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d \log \left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d \log \left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2 \right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 184, normalized size = 0.84

$$\frac{-2(\sqrt[4]{a}e + \sqrt{2}\sqrt[4]{c}d) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right) + 2(\sqrt{2}\sqrt[4]{c}d - 2\sqrt[4]{a}e) \tan^{-1} \left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1 \right) + \sqrt{2}\sqrt[4]{c}d (\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2) - \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2))}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4), x]

[Out] $(-2*(\text{Sqrt}[2]*c^{(1/4)}*d + 2*a^{(1/4)}*e)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}]) + 2*(\text{Sqrt}[2]*c^{(1/4)}*d - 2*a^{(1/4)}*e)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + \text{Sqrt}[2]*c^{(1/4)}*d*(-\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2]))/(8*a^{(3/4)}*\text{Sqrt}[c])$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4), x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.32, size = 215, normalized size = 0.98

$$\frac{\sqrt{2} (ac^3)^{\frac{1}{4}} d \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2} (ac^3)^{\frac{1}{4}} d \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - (ac^3)^{\frac{1}{4}}cd\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce - (ac^3)^{\frac{1}{4}}cd\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{8}\sqrt{2}(a*c^3)^{\frac{1}{4}}*d*\log(x^2 + \sqrt{2}*x*(a/c)^{\frac{1}{4}} + \sqrt{a/c})/(a*c) - \frac{1}{8}\sqrt{2}(a*c^3)^{\frac{1}{4}}*d*\log(x^2 - \sqrt{2}*x*(a/c)^{\frac{1}{4}} + \sqrt{a/c})/(a*c) - \frac{1}{4}\sqrt{2}*(\sqrt{2}*\sqrt{a*c}*c*e - (a*c^3)^{\frac{1}{4}}*c*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{\frac{1}{4}})/(a/c)^{\frac{1}{4}})/(a*c^2) - \frac{1}{4}\sqrt{2}*(\sqrt{2}*\sqrt{a*c}*c*e - (a*c^3)^{\frac{1}{4}}*c*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{\frac{1}{4}})/(a/c)^{\frac{1}{4}})/(a*c^2)$

maple [A] time = 0.00, size = 151, normalized size = 0.69

$$\frac{e \arctan\left(\sqrt{\frac{c}{a}} x^2\right)}{2\sqrt{ac}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(c*x^4+a),x)

[Out] $\frac{1}{8}d*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\ln((x^2+(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}})/(x^2-(a/c)^{\frac{1}{4}}*2^{\frac{1}{2}}*x+(a/c)^{\frac{1}{2}}))+\frac{1}{4}d*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x+1)+\frac{1}{4}d*(a/c)^{\frac{1}{4}}/a*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}/(a/c)^{\frac{1}{4}}*x-1)+\frac{1}{2}e/(a*c)^{\frac{1}{2}}*\arctan((1/a*c)^{\frac{1}{2}}*x^2)$

maxima [A] time = 2.67, size = 207, normalized size = 0.95

$$\frac{\sqrt{2} d \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{8 a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{\sqrt{2} d \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{8 a^{\frac{3}{4}} c^{\frac{1}{4}}} + \frac{\left(\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d - 2 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{4 a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}} + \frac{\left(\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d + 2 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{4 a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")

[Out] $\frac{1}{8}\sqrt{2}d\log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})/(a^{3/4}c^{1/4}) - \frac{1}{8}\sqrt{2}d\log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a})/(a^{3/4}c^{1/4}) + \frac{1}{4}(\sqrt{2}a^{1/4}c^{1/4}d - 2\sqrt{a}e)\operatorname{arctan}(\frac{1}{2}\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4}))/\sqrt{a}\sqrt{c})/(a^{3/4}\sqrt{a}\sqrt{c})c^{1/4} + \frac{1}{4}(\sqrt{2}a^{1/4}c^{1/4}d + 2\sqrt{a}e)\operatorname{arctan}(\frac{1}{2}\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4}))/\sqrt{a}\sqrt{c})/(a^{3/4}\sqrt{a}\sqrt{c})c^{1/4}$

mupad [B] time = 2.32, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2d+3ex}{6cx^3} & \text{if } a = 0 \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}c^{1/4}x}{a^{1/4}}-1\right)(2a^{1/4}e+\sqrt{2}c^{1/4}d)}{4a^{3/4}\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\sqrt{2}c^{1/4}x}{a^{1/4}}+1\right)(4a^{1/4}e-2\sqrt{2}c^{1/4}d)}{8a^{3/4}\sqrt{c}} + \frac{\sqrt{2}d\ln\left(\frac{\sqrt{a}+\sqrt{c}x^2+\sqrt{2}a^{1/4}c^{1/4}x}{\sqrt{a}+\sqrt{c}x^2-\sqrt{2}a^{1/4}c^{1/4}x}\right)}{8a^{3/4}c^{1/4}} & \text{if } a \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)/(a + c*x^4),x)

[Out] $\operatorname{piecewise}(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a \neq 0, (\operatorname{atan}((2^{1/2}*c^{1/4}*x)/a^{1/4} - 1)*(2*a^{1/4}*e + 2^{1/2}*c^{1/4}*d))/(4*a^{3/4}*c^{1/2}) - (\operatorname{atan}((2^{1/2}*c^{1/4}*x)/a^{1/4} + 1)*(4*a^{1/4}*e - 2*2^{1/2}*c^{1/4}*d))/(8*a^{3/4}*c^{1/2}) + (2^{1/2}*d*\log((a^{1/2} + c^{1/2}*x^2 + 2^{1/2}*a^{1/4}*c^{1/4}*x)/(a^{1/2} + c^{1/2}*x^2 - 2^{1/2}*a^{1/4}*c^{1/4}*x)))/(8*a^{3/4}*c^{1/2}))$

sympy [A] time = 0.82, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2 + 32t^2a^2ce^2 - 16tacd^2e + ae^4 + cd^4, \left(t \mapsto t \log\left(x + \frac{-128t^3a^3ce^2 - 16t^2a^2cd^2e - 8ta^2e^4 - 4tacd^4 + 5ad^2e^3}{4ade^4 - cd^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x**4+a),x)

[Out] $\operatorname{RootSum}(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e**4 + c*d**4, \operatorname{Lambda}(_t, _t*\log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d**4 - c*d**5))))$

$$3.393 \quad \int \frac{1}{a+cx^4} dx$$

Optimal. Leaf size=185

$$\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{4\sqrt{2} a^{3/4} \sqrt[4]{c}} - \frac{\tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}} + \frac{\tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} \sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-1), x]

[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \ :> \ \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^2}{(a_.) + (c_.)*(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rubi steps

$$\begin{aligned} \int \frac{1}{a + cx^4} dx &= \frac{\int \frac{\sqrt{a} - \sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a} + \sqrt{c}x^2}{a+cx^4} dx}{2\sqrt{a}} \\ &= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{-\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, 1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} \\ &= -\frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) + \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2\right) - 2\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-1), x]

[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + cx^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^4)^(-1), x]

[Out] IntegrateAlgebraic[(a + c*x^4)^(-1), x]

fricas [A] time = 1.07, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}} + \sqrt{a^2\sqrt{-\frac{1}{a^3c}} + x^2}a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} \log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a), x, algorithm="fricas")

[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)

giac [A] time = 0.33, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a), x, algorithm="giac")

[Out] $\frac{1}{4}\sqrt{2}*(a*c^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{(1/4)))/(a/c)^{(1/4)})/(a*c) + 1/4*\sqrt{2}*(a*c^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/c)^{(1/4)))/(a/c)^{(1/4)})/(a*c) + 1/8*\sqrt{2}*(a*c^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c))/(a*c) - 1/8*\sqrt{2}*(a*c^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c))/(a*c)$

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{4a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+a),x)`

[Out] $\frac{1}{8}*(a/c)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)))/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2))})+1/4*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+1/4*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)$

maxima [A] time = 2.37, size = 169, normalized size = 0.91

$$\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}})}{2\sqrt{a}\sqrt{c}}\right)}{4\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+a),x, algorithm="maxima")`

[Out] $\frac{1}{4}\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{c})/\sqrt{a}*\sqrt{c})/(a*c) + 1/4*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{c})/\sqrt{a}*\sqrt{c})/(a*c) + 1/8*\sqrt{2}*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(1/4)}) - 1/8*\sqrt{2}*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(1/4)})$

mupad [B] time = 0.08, size = 33, normalized size = 0.18

$$\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right) + \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2(-a)^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x^4),x)`

[Out] $-(\operatorname{atan}((c^{1/4}x)/(-a)^{1/4}) + \operatorname{atanh}((c^{1/4}x)/(-a)^{1/4}))/2(-a)^{3/4}c^{1/4}$

sympy [A] time = 0.16, size = 20, normalized size = 0.11

$$\operatorname{RootSum}(256t^4a^3c + 1, (t \mapsto t \log(4ta + x)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x**4+a),x)`

[Out] `RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))`

$$3.394 \quad \int \frac{1}{(d+ex)(a+cx^4)} dx$$

Optimal. Leaf size=416

$$\frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)} + \frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)}$$

Rubi [A] time = 0.43, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$\frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)} + \frac{\sqrt[4]{c} d (\sqrt{c} d^2 - \sqrt{a} e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)} - \frac{\sqrt[4]{c} d (\sqrt{a} e^2 + \sqrt{c} d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4} (ae^4 + cd^4)} + \frac{\sqrt[4]{c} d (\sqrt{a} e^2 + \sqrt{c} d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{2\sqrt{2} a^{3/4} (ae^4 + cd^4)} - \frac{e^3 \log(a + cx^4)}{4(ae^4 + cd^4)} - \frac{\sqrt{c} d^2 e \tan^{-1}\left(\frac{\sqrt{c} x}{\sqrt{a}}\right)}{2\sqrt{a} (ae^4 + cd^4)} + \frac{e^3 \log(d + ex)}{ae^4 + cd^4}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + c*x^4)), x]

[Out] $-(\text{Sqrt}[c] * d^2 * e * \text{ArcTan}[(\text{Sqrt}[c] * x^2) / \text{Sqrt}[a]]) / (2 * \text{Sqrt}[a] * (c * d^4 + a * e^4)) - (c^{(1/4)} * d * (\text{Sqrt}[c] * d^2 + \text{Sqrt}[a] * e^2) * \text{ArcTan}[1 - (\text{Sqrt}[2] * c^{(1/4)} * x) / a^{(1/4)}]) / (2 * \text{Sqrt}[2] * a^{(3/4)} * (c * d^4 + a * e^4)) + (c^{(1/4)} * d * (\text{Sqrt}[c] * d^2 + \text{Sqrt}[a] * e^2) * \text{ArcTan}[1 + (\text{Sqrt}[2] * c^{(1/4)} * x) / a^{(1/4)}]) / (2 * \text{Sqrt}[2] * a^{(3/4)} * (c * d^4 + a * e^4)) + (e^3 * \text{Log}[d + e * x]) / (c * d^4 + a * e^4) - (c^{(1/4)} * d * (\text{Sqrt}[c] * d^2 - \text{Sqrt}[a] * e^2) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2]) / (4 * \text{Sqrt}[2] * a^{(3/4)} * (c * d^4 + a * e^4)) + (c^{(1/4)} * d * (\text{Sqrt}[c] * d^2 - \text{Sqrt}[a] * e^2) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2]) / (4 * \text{Sqrt}[2] * a^{(3/4)} * (c * d^4 + a * e^4)) - (e^3 * \text{Log}[a + c * x^4]) / (4 * (c * d^4 + a * e^4))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

[{a, c, d, e, p, q}, x]

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)(a+cx^4)} dx = \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)} \right) dx$$

$$= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4} dx}{cd^4+ae^4}$$

$$= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \left(\frac{d^3+de^2x^2}{a+cx^4} + \frac{x(-d^2e-e^3x^2)}{a+cx^4} \right) dx}{cd^4+ae^4}$$

$$= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \int \frac{d^3+de^2x^2}{a+cx^4} dx}{cd^4+ae^4} + \frac{c \int \frac{x(-d^2e-e^3x^2)}{a+cx^4} dx}{cd^4+ae^4}$$

$$= \frac{e^3 \log(d+ex)}{cd^4+ae^4} + \frac{c \text{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2} dx, x, x^2\right)}{2(cd^4+ae^4)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^4+ae^4)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^4+ae^4)}$$

$$= \frac{e^3 \log(d+ex)}{cd^4+ae^4} - \frac{(cd^2e) \text{Subst}\left(\int \frac{1}{a+cx^2} dx, x, x^2\right)}{2(cd^4+ae^4)} - \frac{(ce^3) \text{Subst}\left(\int \frac{x}{a+cx^2} dx, x, x^2\right)}{2(cd^4+ae^4)} + \frac{\left(d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}-e^2\right)\right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{2(cd^4+ae^4)}$$

$$= -\frac{\sqrt{c}d^2e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)} + \frac{e^3 \log(d+ex)}{cd^4+ae^4} - \frac{4\sqrt{c}d(\sqrt{c}d^2-\sqrt{a}e^2) \log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2})}{4\sqrt{2}a^{3/4}(cd^4+ae^4)}$$

$$= -\frac{\sqrt{c}d^2e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)} - \frac{4\sqrt{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{cx^2}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)} + \frac{4\sqrt{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{cx^2}}{\sqrt{a}}\right)}{2\sqrt{2}a^{3/4}(cd^4+ae^4)}$$

Mathematica [A] time = 0.15, size = 404, normalized size = 0.97

$$\frac{-2a^{3/4}e^3 \log(a+cx^4) + 8a^{3/4}e^3 \log(d+ex) - \sqrt{2}e^3 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2} + \sqrt{a} + \sqrt{c}x^2) + \sqrt{2}e^3 \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2} + \sqrt{a} + \sqrt{c}x^2) - 2\sqrt{c}d(-2\sqrt{c}\sqrt[4]{a}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{cx^2}}{\sqrt{a}}\right) + 2\sqrt{c}d(2\sqrt{c}\sqrt[4]{a}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{cx^2}}{\sqrt{a}} + 1\right) + \sqrt{2}\sqrt{a}\sqrt{c}d^2 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2} + \sqrt{a} + \sqrt{c}x^2) - \sqrt{2}\sqrt{a}\sqrt{c}d^2 \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{cx^2} + \sqrt{a} + \sqrt{c}x^2)}{8a^{3/4}(a^4+cd^4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]

[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(1/4)*d*(Sqrt[2]*Sqrt[a]*e^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[c]*d^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[a]*e^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[c]*d^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]

$\frac{3}{4}d^3 \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2] - \text{Sqrt}[2] * \text{Sqrt}[a] * c^{(1/4)} * d * e^2 * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2] - 2 * a^{(3/4)} * e^3 * \text{Log}[a + c * x^4]) / (8 * a^{(3/4)} * (c * d^4 + a * e^4))$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)(a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)),x]

[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.42, size = 371, normalized size = 0.89

$$\frac{(ac^3)^{\frac{1}{2}} c d \arctan\left(\frac{\sqrt{2} \sqrt{x + \sqrt{2}} \left(\frac{z}{c}\right)^{\frac{1}{2}}}{z \left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{2 \sqrt{2} ac^2 d^2 - 2 (ac^3)^{\frac{1}{2}} acde + \sqrt{2} \sqrt{ac} ac^2} + \frac{(ac^3)^{\frac{1}{2}} c d \arctan\left(\frac{\sqrt{2} \sqrt{x - \sqrt{2}} \left(\frac{z}{c}\right)^{\frac{1}{2}}}{z \left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{2 \sqrt{2} ac^2 d^2 + 2 (ac^3)^{\frac{1}{2}} acde + \sqrt{2} \sqrt{ac} ac^2} + \frac{\left((ac^3)^{\frac{1}{2}} c^2 d^3 - (ac^3)^{\frac{1}{2}} d^2\right) \log\left(x^2 + \sqrt{2} x \left(\frac{z}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{z}{c}}\right)}{4 \sqrt{2} ac^3 d^4 + \sqrt{2} a^2 c^2 e^4} - \frac{\left((ac^3)^{\frac{1}{2}} c^2 d^3 - (ac^3)^{\frac{1}{2}} d^2\right) \log\left(x^2 - \sqrt{2} x \left(\frac{z}{c}\right)^{\frac{1}{2}} + \sqrt{\frac{z}{c}}\right)}{4 \sqrt{2} ac^3 d^4 + \sqrt{2} a^2 c^2 e^4} - \frac{e^3 \log(|cx^4 + d|)}{4 (cd^4 + ac^4)} + \frac{e^4 \log(|xe + d|)}{cd^4 e + ac^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{2} * (a * c^3)^{(1/4)} * c * d * \arctan(1/2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2) * (a/c)^{(1/4)})) / (a/c)^{(1/4)} / (\text{sqrt}(2) * a * c^2 * d^2 - 2 * (a * c^3)^{(1/4)} * a * c * d * e + \text{sqrt}(2) * \text{sqrt}(a * c) * a * c * e^2) + 1/2 * (a * c^3)^{(1/4)} * c * d * \arctan(1/2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2) * (a/c)^{(1/4)})) / (a/c)^{(1/4)} / (\text{sqrt}(2) * a * c^2 * d^2 + 2 * (a * c^3)^{(1/4)} * a * c * d * e + \text{sqrt}(2) * \text{sqrt}(a * c) * a * c * e^2) + 1/4 * ((a * c^3)^{(1/4)} * c^2 * d^3 - (a * c^3)^{(3/4)} * d * e^2) * \log(x^2 + \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * a * c^3 * d^4 + \text{sqrt}(2) * a^2 * c^2 * e^4) - 1/4 * ((a * c^3)^{(1/4)} * c^2 * d^3 - (a * c^3)^{(3/4)} * d * e^2) * \log(x^2 - \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (\text{sqrt}(2) * a * c^3 * d^4 + \text{sqrt}(2) * a^2 * c^2 * e^4) - 1/4 * e^3 * \log(\text{abs}(c * x^4 + a)) / (c * d^4 + a * e^4) + e^4 * \log(\text{abs}(x * e + d)) / (c * d^4 * e + a * e^5)$

maple [A] time = 0.01, size = 433, normalized size = 1.04

$$\frac{c d^2 e \arctan\left(\sqrt{\frac{z}{c}} x^2\right)}{2 (a e^4 + c d^4) \sqrt{ac}} + \frac{\left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} c d^3 \arctan\left(\frac{\sqrt{2} x - 1}{\left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{4 (a e^4 + c d^4) a} + \frac{\left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} c d^3 \arctan\left(\frac{\sqrt{2} x + 1}{\left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{4 (a e^4 + c d^4) a} + \frac{\left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} c d^3 \ln\left(\frac{x^2 + \left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{z}{c}}}{x^2 - \left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{z}{c}}}\right)}{8 (a e^4 + c d^4) a} + \frac{\sqrt{2} d^2 e^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{4 (a e^4 + c d^4) \left(\frac{z}{c}\right)^{\frac{1}{2}}} + \frac{\sqrt{2} d^2 e^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{z}{c}\right)^{\frac{1}{2}}}\right)}{4 (a e^4 + c d^4) \left(\frac{z}{c}\right)^{\frac{1}{2}}} + \frac{\sqrt{2} d e^2 \ln\left(\frac{x^2 - \left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{z}{c}}}{x^2 + \left(\frac{z}{c}\right)^{\frac{1}{2}} \sqrt{2} x + \sqrt{\frac{z}{c}}}\right)}{8 (a e^4 + c d^4) \left(\frac{z}{c}\right)^{\frac{1}{2}}} - \frac{e^3 \ln(c x^4 + a)}{4 (a e^4 + c d^4)} + \frac{e^3 \ln(ex + d)}{a e^4 + c d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(c*x^4+a),x)`

[Out]
$$e^3 \ln(e*x+d)/(a*e^4+c*d^4) + 1/8*c/(a*e^4+c*d^4)*d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) + 1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) - 1/2*c/(a*e^4+c*d^4)*e*d^2/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2) + 1/8/(a*e^4+c*d^4)*d*e^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) + 1/4/(a*e^4+c*d^4)*d*e^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1) + 1/4/(a*e^4+c*d^4)*d*e^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) - 1/4*e^3*\ln(c*x^4+a)/(a*e^4+c*d^4)$$

maxima [A] time = 2.73, size = 345, normalized size = 0.83

$$\frac{e^3 \log(ex+d)}{cd^4 + ae^4} - \frac{\left(\frac{\sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^3 - cd^3 + \sqrt{a} \sqrt{cd^2} \right) \log \left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{a^{\frac{3}{4}} c^{\frac{1}{4}} d^{\frac{3}{4}}} + \frac{\sqrt{2} \left(\sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^3 + cd^3 - \sqrt{a} \sqrt{cd^2} \right) \log \left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a} \right)}{a^{\frac{3}{4}} c^{\frac{1}{4}} d^{\frac{3}{4}}} - \frac{2 \left(\sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^3 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 + 2 \sqrt{a} cd^2 \right) \arctan \left(\frac{\sqrt{2} \left(2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{c}} \right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} d^{\frac{3}{4}}} - \frac{2 \left(\sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^3 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 - 2 \sqrt{a} cd^2 \right) \arctan \left(\frac{\sqrt{2} \left(2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} \right)}{2 \sqrt{a} \sqrt{c}} \right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} d^{\frac{3}{4}}} \right)}{8(cd^4 + ae^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")`

[Out]
$$e^3 * \log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(\text{sqrt}(2)*(\text{sqrt}(2)*a^{(3/4)}*c^{(1/4)}*e^3 - c*d^3 + \text{sqrt}(a)*\text{sqrt}(c)*d*e^2)*\log(\text{sqrt}(c)*x^2 + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(5/4)}) + \text{sqrt}(2)*(\text{sqrt}(2)*a^{(3/4)}*c^{(1/4)}*e^3 + c*d^3 - \text{sqrt}(a)*\text{sqrt}(c)*d*e^2)*\log(\text{sqrt}(c)*x^2 - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(5/4)}) - 2*(\text{sqrt}(2)*a^{(1/4)}*c^{(5/4)}*d^3 + \text{sqrt}(2)*a^{(3/4)}*c^{(3/4)}*d*e^2 + 2*\text{sqrt}(a)*c*d^2*e)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2)*\text{sqrt}(c)*x + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))*c^{(5/4)}) - 2*(\text{sqrt}(2)*a^{(1/4)}*c^{(5/4)}*d^3 + \text{sqrt}(2)*a^{(3/4)}*c^{(3/4)}*d*e^2 - 2*\text{sqrt}(a)*c*d^2*e)*\arctan(1/2*\text{sqrt}(2)*(\text{sqrt}(2)*\text{sqrt}(c)*x - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))*c^{(5/4)})/(c*d^4 + a*e^4)$$

mapad [B] time = 0.42, size = 874, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + c*x^4)*(d + e*x)),x)`

[Out] `symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1), z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3`


```

*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x**4+a),x)

[Out] Timed out

$$3.395 \quad \int \frac{1}{(d+ex)^2(a+cx^4)} dx$$

Optimal. Leaf size=552

$$\frac{\sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right) \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2 \right) + \sqrt[4]{c} \left(\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4) \right)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} + \dots$$

Rubi [A] time = 0.81, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{c} (\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4)) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} + \frac{\sqrt[4]{c} (\sqrt{c} d^2 (cd^4 - 3ae^4) - \sqrt{a} e^2 (3cd^4 - ae^4)) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2)}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} - \frac{\sqrt[4]{c} (\sqrt{c} d^2 (cd^4 - 3ae^4) + \sqrt{a} e^2 (3cd^4 - ae^4)) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x}{\sqrt{a} + \sqrt{c} x^2}\right)}{2\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} - \frac{\sqrt[4]{c} (\sqrt{c} d^2 (cd^4 - 3ae^4) + \sqrt{a} e^2 (3cd^4 - ae^4)) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x}{\sqrt{a} + \sqrt{c} x^2} + 1\right)}{2\sqrt{2} a^{3/4} (ae^4 + cd^4)^2} - \frac{c^{3/4} \log(a+cx)}{(a^2+cd^2)} - \frac{\sqrt[4]{c} (a^2 - a^2) \tan^{-1}\left(\frac{d+cx}{\sqrt{a} + \sqrt{c} x^2}\right)}{\sqrt{c} (a^2+cd^2)} - \frac{d^2}{(d+cx)(a^2+cd^2)} + \frac{4a^2 \log(d+cx)}{(a^2+cd^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]

[Out] $-(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (\text{Sqrt}[c]*d*e*(c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^2 - (c^{(1/4)}*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*(\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^2) - (c*d^3*e^3*\text{Log}[a + c*x^4])/(c*d^4 + a*e^4)^2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

$\text{Int}[(x_)^m / ((a_) + (b_)*(x_)^n), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 617

$\text{Int}[(a_) + (b_)*(x_) + (c_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 635

$\text{Int}[(d_) + (e_)*(x_)] / ((a_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 1162

$\text{Int}[(d_) + (e_)*(x_)^2] / ((a_) + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}[(d_) + (e_)*(x_)^2] / ((a_) + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1168

$\text{Int}[(d_) + (e_)*(x_)^2] / ((a_) + (c_)*(x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
  := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
  [{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
  [Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
  }]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
  0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
  xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
  [n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^2(a+cx^4)} dx &= \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^2} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)} + \frac{c(d^2(cd^4-3ae^4)-2de(cd^4-ae^4))}{(cd^4+ae^4)^2} \right) dx \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} + \frac{c \int \frac{d^2(cd^4-3ae^4)-2de(cd^4-ae^4)+e^2(3cd^4-ae^4)}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} + \frac{c \int \left(\frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4} + \frac{d^2(cd^4-3ae^4)}{a+cx^4} \right) dx}{(cd^4+ae^4)^2} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} + \frac{c \int \frac{x(-2de(cd^4-ae^4)-4cd^3e^3x^2)}{a+cx^4} dx}{(cd^4+ae^4)^2} + \frac{c \int \frac{d^2(cd^4-3ae^4)}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} + \frac{c \operatorname{Subst} \left(\int \frac{-2de(cd^4-ae^4)-4cd^3e^3x}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)^2} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(2c^2d^3e^3) \operatorname{Subst} \left(\int \frac{x}{a+cx^2} dx, x, x^2 \right)}{(cd^4+ae^4)^2} - \frac{\sqrt{c} d^2}{2\sqrt{2} \sqrt{a}} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c} de (cd^4-ae^4) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{\sqrt{a} (cd^4+ae^4)^2} + \frac{4cd^3e^3 \log(d+ex)}{(cd^4+ae^4)^2} + \frac{\sqrt{c}}{2\sqrt{2} \sqrt{a}} \\
&= -\frac{e^3}{(cd^4+ae^4)(d+ex)} - \frac{\sqrt{c} de (cd^4-ae^4) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right)}{\sqrt{a} (cd^4+ae^4)^2} - \frac{\sqrt{c} \left(3cd^4e^2 - ae^6 + \frac{\sqrt{c} d^2}{2\sqrt{2} \sqrt{a}} \right)}{2\sqrt{2} \sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.61, size = 524, normalized size = 0.95

$$\frac{\sqrt{2} \sqrt{c} (a^2 d^2 + c^2 d^2 + c^2 e^2) \sqrt{a} \sqrt{a+cx^4} \log\left(\frac{d+ex}{\sqrt{a+cx^4}}\right) + \sqrt{2} \sqrt{c} (a^2 d^2 + c^2 d^2 + c^2 e^2) \sqrt{a} \sqrt{a+cx^4} \log\left(\frac{d+ex}{\sqrt{a+cx^4}}\right) + \frac{2 \sqrt{c} (d^2 - c^2 e^2) (a^2 d^2 + c^2 d^2 + c^2 e^2) \sqrt{a} \sqrt{a+cx^4} \log\left(\frac{d+ex}{\sqrt{a+cx^4}}\right) + \frac{\sqrt{2} \sqrt{c} d^2}{2\sqrt{2} \sqrt{a}}}{8 (a^4 + c^4 e^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]

[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-Sqrt[c]*d^2) + Sqrt[a]*e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d

$$\begin{aligned} & \sqrt{2}e^2 - 4a^{3/4}c^{1/4}d^3e^3 + \sqrt{2}ae^4 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}c^{1/4}(4x/a^{1/4})}{a^{3/4}}\right] + (2c^{1/4}(\sqrt{c}d^2 - \sqrt{a}e^2)(\sqrt{2}cd^4 + 4a^{1/4}c^{3/4}d^3e + 4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2 + 4a^{3/4}c^{1/4}d^3e^3 + \sqrt{2}ae^4) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}c^{1/4}(4x/a^{1/4})}{a^{3/4}}\right]) \\ & / a^{3/4} + 32cd^3e^3 \operatorname{Log}[d + ex] - (\sqrt{2}c^{1/4}(c^{3/2}d^6 - 3\sqrt{a}cd^4e^2 - 3a\sqrt{c}d^2e^4 + a^{3/2}e^6) \operatorname{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / a^{3/4} + (\sqrt{2}c^{1/4}(c^{3/2}d^6 - 3\sqrt{a}cd^4e^2 - 3a\sqrt{c}d^2e^4 + a^{3/2}e^6) \operatorname{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2]) / a^{3/4} - 8cd^3e^3 \operatorname{Log}[a + cx^4]) \\ & / (8(c^4d^4 + a^4e^4)^2) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 2.78, size = 646, normalized size = 1.17

$$\frac{\frac{\sqrt{2} \operatorname{Log}\left(\frac{d+ex}{a}\right) - \frac{4cd^3e^3 \operatorname{Log}(d+ex)}{2\sqrt{2}cd^3e^3} + \frac{\sqrt{2}cd^3e^3 \operatorname{Log}(d+ex)}{2\sqrt{2}cd^3e^3}}{2\sqrt{2}cd^3e^3} + \frac{\left(\frac{c^2d^6 - (ex)^2\sqrt{c}}{c^2d^6}\right) \operatorname{arctan}\left(\frac{\sqrt{2}cd^3e^3}{c^2d^6 - (ex)^2\sqrt{c}}\right)}{2\sqrt{2}cd^3e^3} + \frac{\left(\frac{c^2d^6 - (ex)^2\sqrt{c}}{c^2d^6}\right) \operatorname{arctan}\left(\frac{\sqrt{2}cd^3e^3}{c^2d^6 - (ex)^2\sqrt{c}}\right)}{2\sqrt{2}cd^3e^3} + \frac{\left(\sqrt{2}cd^3e^3 - 3\sqrt{2}cd^3e^3 - 3\sqrt{2}cd^3e^3 + \sqrt{2}cd^3e^3\right) \operatorname{Log}\left(\frac{d+ex}{a}\right) + \sqrt{2}cd^3e^3 \operatorname{Log}\left(\frac{d+ex}{a}\right)}{8\sqrt{2}cd^3e^3} + \frac{\left(\sqrt{2}cd^3e^3 - 3\sqrt{2}cd^3e^3 - 3\sqrt{2}cd^3e^3 + \sqrt{2}cd^3e^3\right) \operatorname{Log}\left(\frac{d+ex}{a}\right) + \sqrt{2}cd^3e^3 \operatorname{Log}\left(\frac{d+ex}{a}\right)}{8\sqrt{2}cd^3e^3} + \frac{\sqrt{2}cd^3e^3}{8\sqrt{2}cd^3e^3}}{8\sqrt{2}cd^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a), x, algorithm="giac")

[Out] $-cd^3e^3 \log(\operatorname{abs}(cx^4 + a)) / (c^2d^8 + 2ac^2d^4e^4 + a^2e^8) + 4cd^3e^4 \log(\operatorname{abs}(xe + d)) / (c^2d^8e + 2ac^2d^4e^5 + a^2e^9) + 1/2 * ((ac^3)^{1/4} c^2d^2 - (ac^3)^{3/4} e^2) \operatorname{arctan}(1/2 \sqrt{2} * (2x + \sqrt{2})(a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * ac^3d^4 - 4(ac^3)^{1/4} ac^2d^3e + 4\sqrt{2} * \sqrt{ac} * ac^2d^2e^2 + \sqrt{2} * a^2c^2e^4 - 4(ac^3)^{3/4} ad^3e^3) + 1/2 * ((ac^3)^{1/4} c^2d^2 - (ac^3)^{3/4} e^2) \operatorname{arctan}(1/2 \sqrt{2} * (2x - \sqrt{2})(a/c)^{1/4}) / (a/c)^{1/4} / (\sqrt{2} * ac^3d^4 + 4(ac^3)^{1/4} ac^2d^3e - 4\sqrt{2} * \sqrt{ac} * ac^2d^2e^2 + \sqrt{2} * a^2c^2e^4 - 4(ac^3)^{3/4} ad^3e^3)$

$$4) * a * c^2 * d^3 * e + 4 * \sqrt{2} * \sqrt{a * c} * a * c^2 * d^2 * e^2 + \sqrt{2} * a^2 * c^2 * e^4 + 4 * (a * c^3)^{3/4} * a * d * e^3 + 1/8 * (\sqrt{2} * (a * c^3)^{1/4} * c^3 * d^6 - 3 * \sqrt{2} * (a * c^3)^{3/4} * c * d^4 * e^2 - 3 * \sqrt{2} * (a * c^3)^{1/4} * a * c^2 * d^2 * e^4 + \sqrt{2} * (a * c^3)^{3/4} * a * e^6) * \log(x^2 + \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) / (a * c^4 * d^8 + 2 * a^2 * c^3 * d^4 * e^4 + a^3 * c^2 * e^8) - 1/8 * (\sqrt{2} * (a * c^3)^{1/4} * c^3 * d^6 - 3 * \sqrt{2} * (a * c^3)^{3/4} * c * d^4 * e^2 - 3 * \sqrt{2} * (a * c^3)^{1/4} * a * c^2 * d^2 * e^4 + \sqrt{2} * (a * c^3)^{3/4} * a * e^6) * \log(x^2 - \sqrt{2} * x * (a/c)^{1/4} + \sqrt{a/c}) / (a * c^4 * d^8 + 2 * a^2 * c^3 * d^4 * e^4 + a^3 * c^2 * e^8) - (c * d^4 * e^3 + a * e^7) / ((c * d^4 + a * e^4)^2 * (x * e + d))$$

maple [A] time = 0.01, size = 866, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(c*x^4+a),x)

[Out]
$$-e^3/(a * e^4 + c * d^4) / (e * x + d) + 4 * c * d^3 * e^3 * \ln(e * x + d) / (a * e^4 + c * d^4)^{2-3/4} * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^2 * e^4 + 1/4 * c^2 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} / a * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^6 - 3/8 * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^2 * e^4 + 1/8 * c^2 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} / a * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^6 - 3/4 * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^2 * e^4 + 1/4 * c^2 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} / a * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^6 + c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} / a * 2^{1/2} * \arctan((1/a * c)^{1/2} * x^2) * e^5 * d * a - c^2 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan((1/a * c)^{1/2} * x^2) * e * d^5 - 1/8 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * a * e^6 + 3/8 * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^4 * e^2 - 1/4 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * a * e^6 + 3/4 * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^4 * e^2 - 1/4 / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * a * e^6 + 3/4 * c / (a * e^4 + c * d^4)^{2-3/4} * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^4 * e^2 - c * d^3 * e^3 * \ln(c * x^4 + a) / (a * e^4 + c * d^4)^2$$

maxima [A] time = 2.11, size = 561, normalized size = 1.02

$$\frac{4cd^3e^3 \log(ex+d)}{c^2d^4 + 2acd^3 + a^2e^2} - \frac{c^2}{cd^4 + a} - \frac{c^2}{8(c^2d^4 + 2acd^3 + a^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")

```
[Out] 4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 +
a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)
*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)*sq
rt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*
c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sqrt(a)
*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e + 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))/(c^2*d^8 + 2*a*c*d^4
*e^4 + a^2*e^8)
```

mupad [B] time = 2.78, size = 2436, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^2), x)
```

```
[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3
*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2
*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*
z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*
a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(51
2*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^
3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e
^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4
+ 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 +
256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*
d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^4*z^4 + 256*
a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*
e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^5*e^10 + 32*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d^2
*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^
4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a*c^7*d^12*e + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 25
```


$$\begin{aligned}
&6*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e \\
&*z + c, z, k)*c^6*d^5*e^4*x - 4*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^ \\
&8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + \\
&32*a*c*d*e*z + c, z, k)^2*c^7*d^10*e*x + 64*root(512*a^4*c*d^4*e^4*z^4 + 25 \\
&6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^ \\
&2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^7*e^4 + 384*root(512*a^4*c*d^ \\
&4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 \\
&+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*d*e^14 + 320*ro \\
&ot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3 \\
&*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4* \\
&e^15*x + 248*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8 \\
&*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z \\
&, k)^2*a*c^6*d^6*e^5*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^ \\
&4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a \\
&*c*d*e*z + c, z, k)^3*a*c^7*d^11*e^2*x + 32*root(512*a^4*c*d^4*e^4*z^4 + 25 \\
&6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^ \\
&2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d*e^8*x + 316*root(512*a^4*c*d^4* \\
&e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + \\
&320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^2*e^9*x + 640*r \\
&oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^ \\
&3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6 \\
&*d^7*e^6*x + 704*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5 \\
&*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + \\
&c, z, k)^3*a^3*c^5*d^3*e^10*x - 192*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^ \\
&2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^ \\
&2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^12*e^3*x - 64*root(512*a^4*c*d^4*e^ \\
&4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 32 \\
&0*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^8*e^7*x + 448*roo \\
&t(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3* \\
&c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d \\
&^4*e^11*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))*root(512*a^4*c*d^4*e^4*z^4 \\
&+ 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2* \\
&c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 + \\
&a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a* \\
&c*d^4*e^4)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)

[Out] Timed out

$$3.396 \quad \int \frac{1}{(d+ex)^3(a+cx^4)} dx$$

Optimal. Leaf size=680

$$\frac{\sqrt{c} e (a^2 e^8 - 12acd^4 e^4 + 3c^2 d^8) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right) c^{3/4} d (3a^2 e^8 - 12acd^4 e^4 - 2\sqrt{a} \sqrt{c} d^2 e^2 (3cd^4 - 5ae^4) + c^2 d^8) \log \left(\frac{2\sqrt{a} (ae^4 + cd^4)^3}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^3} \right)}{2\sqrt{a} (ae^4 + cd^4)^3}$$

Rubi [A] time = 0.95, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$\frac{1}{2\sqrt{a} (ae^4 + cd^4)^3} \left(\sqrt{c} e (a^2 e^8 - 12acd^4 e^4 + 3c^2 d^8) \tan^{-1} \left(\frac{\sqrt{c} x^2}{\sqrt{a}} \right) c^{3/4} d (3a^2 e^8 - 12acd^4 e^4 - 2\sqrt{a} \sqrt{c} d^2 e^2 (3cd^4 - 5ae^4) + c^2 d^8) \log \left(\frac{2\sqrt{a} (ae^4 + cd^4)^3}{4\sqrt{2} a^{3/4} (ae^4 + cd^4)^3} \right) \right)$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]

[Out] $-e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d + e*x)) - (\text{Sqrt}[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)^3) - (c^{(3/4)}*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (c^{(3/4)}*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^3 - (c^{(3/4)}*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (c^{(3/4)}*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*\text{Log}[a + c*x^4])/(2*(c*d^4 + a*e^4)^3)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*

c)]

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
  :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
  [{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
  [Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))]/(a + b*x^n), {ii, 0, n/2 - 1
  }]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
  0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
  xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
  [n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3(a+cx^4)} dx &= \int \left(\frac{e^4}{(cd^4+ae^4)(d+ex)^3} + \frac{4cd^3e^4}{(cd^4+ae^4)^2(d+ex)^2} + \frac{2cd^2e^4(5cd^4-3ae^4)}{(cd^4+ae^4)^3(d+ex)} + \frac{c(d(e^2+e^4x^2+e^6x^4))}{(cd^4+ae^4)^3(d+ex)^3} \right) dx \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} + \frac{2cd^2e^3(5cd^4-3ae^4)\log(d+ex)}{(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\operatorname{atan}\left(\frac{d+ex}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3} \\
&= -\frac{e^3}{2(cd^4+ae^4)(d+ex)^2} - \frac{4cd^3e^3}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}e(3c^2d^8-12acd^4e^4+a^2e^8)\operatorname{atan}\left(\frac{d+ex}{\sqrt{a}}\right)}{2\sqrt{a}(cd^4+ae^4)^3}
\end{aligned}$$

Mathematica [A] time = 0.93, size = 738, normalized size = 1.09

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]

[Out] $(-4*a^{(3/4)}*e^3*(c*d^4 + a*e^4)^2 - 32*a^{(3/4)}*c*d^3*e^3*(c*d^4 + a*e^4)*(d + e*x) - 2*\operatorname{Sqrt}[c]*(\operatorname{Sqrt}[2]*c^{(9/4)}*d^9 - 6*a^{(1/4)}*c^2*d^8*e + 6*\operatorname{Sqrt}[2]*$

$\text{Sqrt}[a]*c^{(7/4)}*d^7*e^2 - 12*\text{Sqrt}[2]*a*c^{(5/4)}*d^5*e^4 + 24*a^{(5/4)}*c*d^4*e^5 - 10*\text{Sqrt}[2]*a^{(3/2)}*c^{(3/4)}*d^3*e^6 + 3*\text{Sqrt}[2]*a^2*c^{(1/4)}*d*e^8 - 2*a^{(9/4)}*e^9*(d + e*x)^2*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 2*\text{Sqrt}[c]*(\text{Sqrt}[2]*c^{(9/4)}*d^9 + 6*a^{(1/4)}*c^2*d^8*e + 6*\text{Sqrt}[2]*\text{Sqrt}[a]*c^{(7/4)}*d^7*e^2 - 12*\text{Sqrt}[2]*a*c^{(5/4)}*d^5*e^4 - 24*a^{(5/4)}*c*d^4*e^5 - 10*\text{Sqrt}[2]*a^{(3/2)}*c^{(3/4)}*d^3*e^6 + 3*\text{Sqrt}[2]*a^2*c^{(1/4)}*d*e^8 + 2*a^{(9/4)}*e^9)*(d + e*x)^2*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}] + 16*a^{(3/4)}*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*(d + e*x)^2*\text{Log}[d + e*x] - \text{Sqrt}[2]*c^{(3/4)}*d*(c^2*d^8 - 6*\text{Sqrt}[a]*c^{(3/2)}*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^{(3/2)}*\text{Sqrt}[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + \text{Sqrt}[2]*c^{(3/4)}*d*(c^2*d^8 - 6*\text{Sqrt}[a]*c^{(3/2)}*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^{(3/2)}*\text{Sqrt}[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] + 4*a^{(3/4)}*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d + e*x)^2*\text{Log}[a + c*x^4]/(8*a^{(3/4)}*(c*d^4 + a*e^4)^3*(d + e*x)^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + cx^4)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)), x]

[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.89, size = 901, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a), x, algorithm="giac")

[Out] $\frac{1}{4}*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^2*d^3 + 2*a*c^2*e^3 - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)}*d*e^2)*\text{arctan}(1/2*\text{sqrt}(2)*(2*x + \text{sqrt}(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a*c^3*d^6 - 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^5*e + 9*\text{sqrt}(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 - 8*\text{sqrt}(2)*(a*c^3)^{(3/4)}*a*d^3*e^3 - 3*\text{sqrt}(2)*(a*c^3)^{(1/4)})$

$$\begin{aligned}
& *a^2*c*d*e^5 + \text{sqrt}(a*c)*a^2*c*e^6) + 1/4*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^2*d^3 - \\
& 2*a*c^2*e^3 - 3*\text{sqrt}(2)*(a*c^3)^{(3/4)}*d*e^2)*\arctan(1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(\\
& (2)*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(a*c^3*d^6 + 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^ \\
& 5*e - 9*\text{sqrt}(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 + 8*\text{sqrt}(2)*(a*c^3)^{(3/ \\
& 4)*a*d^3*e^3 + 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a^2*c*d*e^5 + \text{sqrt}(a*c)*a^2*c*e^6) + \\
& 1/8*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^3*d^9 - 6*\text{sqrt}(2)*(a*c^3)^{(3/4)}*c*d^7*e^2 - 1 \\
& 2*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^5*e^4 + 10*\text{sqrt}(2)*(a*c^3)^{(3/4)}*a*d^3*e^6 \\
& + 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a^2*c*d*e^8)*\log(x^2 + \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{sq} \\
& \text{rt}(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) \\
& - 1/8*(\text{sqrt}(2)*(a*c^3)^{(1/4)}*c^3*d^9 - 6*\text{sqrt}(2)*(a*c^3)^{(3/4)}*c*d^7*e^2 - \\
& 12*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a*c^2*d^5*e^4 + 10*\text{sqrt}(2)*(a*c^3)^{(3/4)}*a*d^3*e^6 \\
& + 3*\text{sqrt}(2)*(a*c^3)^{(1/4)}*a^2*c*d*e^8)*\log(x^2 - \text{sqrt}(2)*x*(a/c)^{(1/4)} + \text{s} \\
& \text{qrt}(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12) \\
& - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*\log(\text{abs}(c*x^4 + a))/(c^3*d^12 + 3*a* \\
& c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^ \\
& 8)*\log(\text{abs}(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3* \\
& e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a \\
& *c*d^3*e^8)*x)/(c*d^4 + a*e^4)^3*(x*e + d)^2)
\end{aligned}$$

maple [B] time = 0.01, size = 1201, normalized size = 1.77

result too large to display

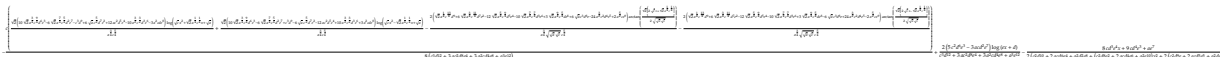
Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^3/(c*x^4+a), x)$

[Out]
$$\begin{aligned}
& -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-6*e^7* \\
& d^2*c/(a*e^4+c*d^4)^3*\ln(e*x+d)*a+10*e^3*d^6*c^2/(a*e^4+c*d^4)^3*\ln(e*x+d)+ \\
& 3/4*c/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \\
& *d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)} \\
& *x-1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/ \\
& (a/c)^{(1/4)}*x-1)*d^9+3/8*c/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*a^2^{(1/2)}*\ln((x^2+(a \\
& /c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) *d \\
& *e^8-3/2*c^2/(a*e^4+c*d^4)^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)} \\
&) *x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) *d^5*e^4+1/8*c^3/(\\
& a*e^4+c*d^4)^3*(a/c)^{(1/4)}/a^2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1 \\
& /2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) *d^9+3/4*c/(a*e^4+c*d^4)^3*(a/ \\
& c)^{(1/4)}*a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^8-3*c^2/(a*e^4+c*d^4) \\
&)^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^5*e^4+1/4*c^3/(a* \\
& e^4+c*d^4)^3*(a/c)^{(1/4)}/a^2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^9-1/2* \\
& c/(a*e^4+c*d^4)^3/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^9*a^2+6*c^2/(a*e^ \\
& 4+c*d^4)^3/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*d^4*e^5*a-3/2*c^3/(a*e^4+c \\
& *d^4)^3/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e*d^8-5/4*c/(a*e^4+c*d^4)^3/(\\
& a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1 \\
& /4)}*2^{(1/2)}*x+(a/c)^{(1/2)})) *a*d^3*e^6+3/4*c^2/(a*e^4+c*d^4)^3/(a/c)^{(1/4)}*2
\end{aligned}$$

$$\begin{aligned} & \int \frac{1}{(x^2 - (a/c)^{1/4} x + (a/c)^{1/2}) \sqrt{x^2 + (a/c)^{1/4} x + (a/c)^{1/2}}} dx \\ & = \frac{1}{\sqrt{a}} \left[\frac{\sqrt{2} (10\sqrt{2} a^{3/4} c^{9/4} d^6 e^3 - 6\sqrt{2} a^{7/4} c^{5/4} d^2 e^7 + c^3 d^9 - 6\sqrt{2} a^{3/2} c^{3/2} d^3 e^6 - 3a^2 c d e^8) \log(\sqrt{c} x^2 + \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{a})}{(a^{3/4} c^{5/4})} + \frac{\sqrt{2} (10\sqrt{2} a^{3/4} c^{9/4} d^6 e^3 - 6\sqrt{2} a^{7/4} c^{5/4} d^2 e^7 + c^3 d^9 - 6\sqrt{2} a^{3/2} c^{3/2} d^3 e^6 + 3a^2 c d e^8) \log(\sqrt{c} x^2 - \sqrt{2} a^{1/4} c^{1/4} x + \sqrt{a})}{(a^{3/4} c^{5/4})} - 2 \frac{(\sqrt{2} a^{1/4} c^{13/4} d^9 + 6\sqrt{2} a^{3/4} c^{11/4} d^7 e^2 - 12\sqrt{2} a^{5/4} c^{9/4} d^5 e^4 - 10\sqrt{2} a^{7/4} c^{7/4} d^3 e^6 + 3\sqrt{2} a^{9/4} c^{5/4} d e^8 + 6\sqrt{2} a c^3 d^8 e - 24 a^{3/2} c^2 d^4 e^5 + 2 a^{5/2} c e^9) \arctan(1/2 \sqrt{2} (2\sqrt{2} c x + \sqrt{2} a^{1/4} c^{1/4}) / \sqrt{\sqrt{a} \sqrt{c}})}{(a^{3/4} \sqrt{\sqrt{a} \sqrt{c}}) c^{5/4}} - 2 \frac{(\sqrt{2} a^{1/4} c^{13/4} d^9 + 6\sqrt{2} a^{3/4} c^{11/4} d^7 e^2 - 12\sqrt{2} a^{5/4} c^{9/4} d^5 e^4 - 10\sqrt{2} a^{7/4} c^{7/4} d^3 e^6 + 3\sqrt{2} a^{9/4} c^{5/4} d e^8 - 6\sqrt{2} a c^3 d^8 e + 24 a^{3/2} c^2 d^4 e^5 - 2 a^{5/2} c e^9) \arctan(1/2 \sqrt{2} (2\sqrt{2} c x - \sqrt{2} a^{1/4} c^{1/4}) / \sqrt{\sqrt{a} \sqrt{c}})}{(a^{3/4} \sqrt{\sqrt{a} \sqrt{c}}) c^{5/4}} \right] / (c^3 d^{12} + 3 a c^2 d^8 e^4 + 3 a^2 c d^4 e^8 + a^3 e^{12}) + 2 \frac{(5 c^2 d^6 e^3 - 3 a c d^2 e^7) \log(e x + d)}{(c^3 d^{12} + 3 a c^2 d^8 e^4 + 3 a^2 c d^4 e^8 + a^3 e^{12})} - 1/2 \frac{(8 c d^3 e^4 x + 9 c d^4 e^3 + a e^7) / (c^2 d^{10} + 2 a c d^6 e^4 + a^2 d^2 e^8 + (c^2 d^8 e^2 + 2 a c d^4 e^6 + a^2 e^{10}) x^2 + 2(c^2 d^9 e + 2 a c d^5 e^5 + a^2 d e^9) x)}{(c^2 d^{10} + 2 a c d^6 e^4 + a^2 d^2 e^8 + (c^2 d^8 e^2 + 2 a c d^4 e^6 + a^2 e^{10}) x^2 + 2(c^2 d^9 e + 2 a c d^5 e^5 + a^2 d e^9) x)} \end{aligned}$$

maxima [A] time = 2.34, size = 817, normalized size = 1.20



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/8*c*(\sqrt{2}*(10*\sqrt{2}*a^{3/4}*c^{9/4}*d^6*e^3 - 6*\sqrt{2}*a^{7/4}*c^{5/4}*d^2*e^7 - c^3*d^9 + 6*\sqrt{2}*a^{3/2}*c^{3/2}*d^3*e^6 - 3*a^2*c*d*e^8)*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4}*c^{5/4} + \sqrt{2}*(10*\sqrt{2}*a^{3/4}*c^{9/4} \\ & *d^6*e^3 - 6*\sqrt{2}*a^{7/4}*c^{5/4}*d^2*e^7 + c^3*d^9 - 6*\sqrt{2}*a^{3/2}*c^{3/2}*d^3*e^6 + 3*a^2*c*d*e^8)*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4}*c^{5/4} - 2 \\ & *(\sqrt{2}*a^{1/4}*c^{13/4}*d^9 + 6*\sqrt{2}*a^{3/4}*c^{11/4}*d^7*e^2 - 12*\sqrt{2}*a^{5/4}*c^{9/4}*d^5*e^4 - 10*\sqrt{2}*a^{7/4}*c^{7/4}*d^3*e^6 + 3*\sqrt{2}*a^{9/4}*c^{5/4}*d*e^8 + 6*\sqrt{2}*a*c^3*d^8*e - 24*a^{3/2}*c^2*d^4*e^5 + \\ & 2*a^{5/2}*c*e^9)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*c*x + \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{\sqrt{a}*\sqrt{c}})/a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}}*c^{5/4} - 2*(\sqrt{2}*a^{1/4}*c^{13/4}*d^9 + 6*\sqrt{2}*a^{3/4}*c^{11/4}*d^7*e^2 - 12*\sqrt{2}*a^{5/4}*c^{9/4}*d^5*e^4 - 10*\sqrt{2}*a^{7/4}*c^{7/4}*d^3*e^6 + 3*\sqrt{2}*a^{9/4}*c^{5/4}*d*e^8 - 6*\sqrt{2}*a*c^3*d^8*e + 24*a^{3/2}*c^2*d^4*e^5 - 2*a^{5/2}*c*e^9)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*c*x - \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{\sqrt{a}*\sqrt{c}})/a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}}*c^{5/4}) / (c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7) * \log(e*x + d) / (c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) - 1/2*(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7) / (c^2*d^{10} + 2*a*c*d^6*e^4 + a^2*d^2*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^{10})*x^2 + 2*(c^2*d^9*e + 2*a*c*d^5*e^5 + a^2*d*e^9)*x) \end{aligned}$$

mupad [B] time = 3.67, size = 1955, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)*(d + e*x)^3),x)


```
[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12
*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 +
768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a
^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*
a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*
c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 +
6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 +
256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^
3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d
^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7
*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 +
4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^
4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^
2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e
^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^
7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e
)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^
8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3
*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^
3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2
, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6
+ 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a
^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^
8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 1
28*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4
*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10
+ 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a
*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(36*a^4*c^5*e^1
7 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*
d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a
^2*c^2*d^8*e^8)) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*
e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c
^2*d^8*e^8)) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^
3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*
z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1
536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2
+ 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*
c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e
^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(1
0*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*
a^2*c*d^4*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)

[Out] Timed out

$$3.397 \quad \int \frac{(d+ex)^3}{(a+cx^4)^2} dx$$

Optimal. Leaf size=349

$$\frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}}$$

Rubi [A] time = 0.30, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1854, 27, 12, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} - \frac{3d(\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d(\sqrt{a}e^2 + \sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{3d^2 e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{c}} - \frac{ae^3 - cx(3d^2 ex + d^3 + 3d^2 x^2)}{4ac(a + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]

[Out] $-(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^{(3/2)}*Sqrt[c]) - (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*Sqrt[2]*a^{(7/4)}*c^{(3/4)}) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*Sqrt[2]*a^{(7/4)}*c^{(3/4)}) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{(7/4)}*c^{(3/4)}) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*c^{(1/4)}*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^{(7/4)}*c^{(3/4)})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 27

Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,

c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
  x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]* (a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{(a+cx^4)^2} dx &= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} - \frac{\int \frac{-3d^3 - 6d^2ex - 3de^2x^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} - \frac{\int -\frac{3d(d+ex)^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \frac{(d+ex)^2}{a+cx^4} dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \left(\frac{2dex}{a+cx^4} + \frac{d^2+e^2x^2}{a+cx^4} \right) dx}{4a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d) \int \frac{d^2+e^2x^2}{a+cx^4} dx}{4a} + \frac{(3d^2e) \int \frac{x}{a+cx^4} dx}{2a} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{(3d^2e) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{4a} + \frac{\left(3d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \right) \int \frac{\sqrt{a} \sqrt{c} x}{a+cx^4} dx}{8ac} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} + \frac{\left(3d \left(\frac{\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16ac} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d(\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
&= -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{4ac(a+cx^4)} + \frac{3d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d(\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}c^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 347, normalized size = 0.99

$$\frac{3\sqrt{2}\sqrt{c}(a^{3/4}de^2 - \sqrt{a}\sqrt{c}d^3)\log(-\sqrt{2}\sqrt[4]{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) + 3\sqrt{2}\sqrt{c}(\sqrt[4]{a}\sqrt{c}d^3 - a^{3/4}de^2)\log(\sqrt{2}\sqrt[4]{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2) - 6\sqrt[4]{a}\sqrt{c}d(4\sqrt[4]{a}\sqrt{c}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 6\sqrt[4]{a}\sqrt{c}d(-4\sqrt[4]{a}\sqrt{c}de + \sqrt{2}\sqrt{a}e^2 + \sqrt{2}\sqrt{c}d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right) - \frac{8d(\sqrt{c}d^2 - \sqrt{a}e^2)\log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x)}{16\sqrt{2}a^{7/4}c^{3/4}}}{32a^2c}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]

[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e

$$\begin{aligned} &^2) * \text{ArcTan}[1 - (\text{Sqrt}[2] * c^{(1/4)} * x) / a^{(1/4)}] + 6 * a^{(1/4)} * c^{(1/4)} * d * (\text{Sqrt}[2] * \\ &\text{Sqrt}[c] * d^2 - 4 * a^{(1/4)} * c^{(1/4)} * d * e + \text{Sqrt}[2] * \text{Sqrt}[a] * e^2) * \text{ArcTan}[1 + (\text{Sqrt}[\\ &2] * c^{(1/4)} * x) / a^{(1/4)}] + 3 * \text{Sqrt}[2] * c^{(1/4)} * (-a^{(1/4)} * \text{Sqrt}[c] * d^3 + a^{(3/ \\ &4)} * d * e^2) * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] * a^{(1/4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2] + 3 * \text{Sqrt}[2] \\ &] * c^{(1/4)} * (a^{(1/4)} * \text{Sqrt}[c] * d^3 - a^{(3/4)} * d * e^2) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] * a^{(1/ \\ &4)} * c^{(1/4)} * x + \text{Sqrt}[c] * x^2)] / (32 * a^2 * c) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 342, normalized size = 0.98

$$\frac{3cdx^2 + 3cd^2x + cd^3 - ad^3}{4(c^4 + a)ac} + \frac{3\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2d^2e + (ac)^{\frac{1}{2}}c^2d^3 + (ac)^{\frac{3}{2}}d^2e\right)\arctan\left(\frac{\sqrt{2}\left(2 + \sqrt{2}\sqrt{2}\right)^{\frac{1}{2}}}{2(2)^{\frac{1}{2}}}\right)}{16a^2c^3} + \frac{3\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2d^2e + (ac)^{\frac{1}{2}}c^2d^3 + (ac)^{\frac{3}{2}}d^2e\right)\arctan\left(\frac{\sqrt{2}\left(2 - \sqrt{2}\sqrt{2}\right)^{\frac{1}{2}}}{2(2)^{\frac{1}{2}}}\right)}{16a^2c^3} + \frac{3\sqrt{2}\left((ac)^{\frac{1}{2}}c^2d^3 - (ac)^{\frac{3}{2}}d^2e\right)\log\left(x^2 + \sqrt{2}x\left(\frac{1}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{c}}\right)}{32a^2c^3} - \frac{3\sqrt{2}\left((ac)^{\frac{1}{2}}c^2d^3 - (ac)^{\frac{3}{2}}d^2e\right)\log\left(x^2 - \sqrt{2}x\left(\frac{1}{2}\right)^{\frac{1}{2}} + \sqrt{\frac{a}{c}}\right)}{32a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} &1/4 * (3 * c * d * x^3 * e^2 + 3 * c * d^2 * x^2 * e + c * d^3 * x - a * e^3) / ((c * x^4 + a) * a * c) + 3 \\ &/ 16 * \text{sqrt}(2) * (2 * \text{sqrt}(2) * \text{sqrt}(a * c) * c^2 * d^2 * e + (a * c^3)^{(1/4)} * c^2 * d^3 + (a * c^3 \\ &)^{(3/4)} * d * e^2) * \arctan(1/2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2)) * (a/c)^{(1/4)}) / (a/c)^{(1/4)} / \\ &(a^2 * c^3) + 3/16 * \text{sqrt}(2) * (2 * \text{sqrt}(2) * \text{sqrt}(a * c) * c^2 * d^2 * e + (a * c^3)^{(1/4)} * c^2 \\ &* d^3 + (a * c^3)^{(3/4)} * d * e^2) * \arctan(1/2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2)) * (a/c)^{(1/4)}) / \\ &(a/c)^{(1/4)} / (a^2 * c^3) + 3/32 * \text{sqrt}(2) * ((a * c^3)^{(1/4)} * c^2 * d^3 - (a * c^3)^{(3/4)} \\ &) * d * e^2 * \log(x^2 + \text{sqrt}(2) * x * (a/c)^{(1/4)} + \text{sqrt}(a/c)) / (a^2 * c^3) - 3/32 * \text{sqrt}(\\ &2) * ((a * c^3)^{(1/4)} * c^2 * d^3 - (a * c^3)^{(3/4)} * d * e^2) * \log(x^2 - \text{sqrt}(2) * x * (a/c) \\ &^{\wedge}(1/4) + \text{sqrt}(a/c)) / (a^2 * c^3) \end{aligned}$$

maple [A] time = 0.00, size = 390, normalized size = 1.12

$$\frac{c^3 x^4}{4(c^2 x^4 + a)^2} + \frac{3cd^2 x^3}{4(c^2 x^4 + a)^2} + \frac{3d^2 e x^2}{4(c^2 x^4 + a)^2} + \frac{d^3 x}{4(c^2 x^4 + a)^2} + \frac{3d^2 e \arctan\left(\sqrt{\frac{c}{a}} x\right)}{4\sqrt{ac} a} + \frac{3\sqrt{2} d e^2 \arctan\left(\frac{\sqrt{2} x}{c} - 1\right)}{16\left(\frac{c}{a}\right)^{\frac{1}{4}} ac} + \frac{3\sqrt{2} d e^2 \arctan\left(\frac{\sqrt{2} x}{c} + 1\right)}{16\left(\frac{c}{a}\right)^{\frac{1}{4}} ac} + \frac{3\sqrt{2} d e^2 \ln\left(\frac{x - \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} + \sqrt{c}}{x + \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} + \sqrt{c}}\right)}{32\left(\frac{c}{a}\right)^{\frac{1}{4}} ac} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^3 \arctan\left(\frac{\sqrt{2} x}{c} - 1\right)}{16a^2} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^3 \arctan\left(\frac{\sqrt{2} x}{c} + 1\right)}{16a^2} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^3 \ln\left(\frac{x - \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} + \sqrt{c}}{x + \left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} + \sqrt{c}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a)^2,x)

[Out] $\frac{1}{4} d^3 x/a / (c x^4 + a) + \frac{3}{32} d^3/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) + \frac{3}{16} d^3/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) + \frac{3}{16} d^3/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) + \frac{3}{4} e * d^2 * x^2/a / (c x^4 + a) + \frac{3}{4} e * d^2/a / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) + \frac{3}{4} d * e^2 * x^3/a / (c x^4 + a) + \frac{3}{32} d * e^2/a/c / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) + \frac{3}{16} d * e^2/a/c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) + \frac{3}{16} d * e^2/a/c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) + \frac{1}{4} e^3 * x^4/a / (c x^4 + a)$

maxima [A] time = 2.20, size = 332, normalized size = 0.95

$$3d \left[\frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}d^2) \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2 - \sqrt{a}d^2) \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d^2 - 4\sqrt{a}\sqrt{cd}\right) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{c}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d^2 + 4\sqrt{a}\sqrt{cd}\right) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{c}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{3}{4}}} \right] + \frac{3cd^2x^3 + 3cd^2ex^2 + cd^3x - ae^3}{4(ac^2x^4 + a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $\frac{3}{32} d * (\sqrt{2} * (\sqrt{c} * d^2 - \sqrt{a} * d^2) * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * x + \sqrt{a}) * a^{\frac{1}{4}} * c^{\frac{1}{4}} * x + \sqrt{a}) / (a^{\frac{3}{4}} * c^{\frac{3}{4}}) - \sqrt{2} * (\sqrt{c} * d^2 - \sqrt{a} * d^2) * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * x + \sqrt{a}) / (a^{\frac{3}{4}} * c^{\frac{3}{4}}) + 2 * (\sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * d^2 + \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * d^2 - 4 * \sqrt{a} * \sqrt{cd}) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x + \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}})) / \sqrt{a} * \sqrt{c}} / (a^{\frac{3}{4}} * \sqrt{a} * \sqrt{c} * c^{\frac{3}{4}}) + 2 * (\sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * d^2 + \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}} * d^2 + 4 * \sqrt{a} * \sqrt{cd}) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{\frac{1}{4}} * c^{\frac{1}{4}})) / \sqrt{a} * \sqrt{c}} / (a^{\frac{3}{4}} * \sqrt{a} * \sqrt{c} * c^{\frac{3}{4}}) / a + 1/4 * (3 * c * d * e^2 * x^3 + 3 * c * d^2 * e * x^2 + c * d^3 * x - a * e^3) / (a * c^2 * x^4 + a^2 * c)$

mupad [B] time = 0.43, size = 670, normalized size = 1.92

$$\frac{3cd^2x^3 + 3cd^2ex^2 + cd^3x - ae^3}{4(ac^2x^4 + a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x)^3/(a + c*x^4)^2, x)$

[Out] $\text{symsum}(\log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*d - 48*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a*c^2*d^4*x + 48*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e*x - 192*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*d*e^3))/(64*a^3))*\text{root}(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a) - e^3/(4*c) + (3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)$

sympy [A] time = 8.33, size = 350, normalized size = 1.00

$\text{RootSum}\left(\left(65536*d^7*z^4 + 27648*d^4*c^2*d^4*e^2*z^2 + 3456*d^3*c*d^4*e^5*z - 3456*d^2*c^2*d^8*e*z + 162*d*c*d^8*e^4 + 81*d^2*d^4*e^8 + 81*c^2*d^12\right)\left(1 + \log\left(x + \frac{4096*d^3*c^2*d^6 + 28672*d^2*c^3*d^4*e^2 - 7680*d^2*d^5*c^2*d^4*e^5 + 1536*d^2*d^4*c^3*d^8*e + 2160*d^4*c*d^4*e^8 + 9216*d^3*c^2*d^8*e^4 + 144*d^2*d^3*d^12 + 162*d^3*d^4*e^11 - 648*d^2*c*d^8*e^7 - 810*d^2*d^12*e^3}{27*d^3*d^12 - 891*d^2*c*d^7*e^8 - 891*d*c^2*d^11*e^4 + 27*c^3*d^15}\right)\right) + \frac{-d^3 + c*d^3*x + 3*c*d^2*e*x^2 + 3*c*d^2*e^2*x^3}{4*d^2 + 4*c*d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)**3/(c*x**4+a)**2, x)$

[Out] $\text{RootSum}(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**8*e**4 + 81*c**2*d**12, \text{Lambda}(_t, _t*\log(x + (4096*_t**3*a**7*c**2*e**6 + 28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t**2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 810*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c**2*d**11*e**4 + 27*c**3*d**15)))) + (-a**3 + c*d**3*x + 3*c*d**2*e*x**2 + 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)$

$$3.398 \quad \int \frac{(d+ex)^2}{(a+cx^4)^2} dx$$

Optimal. Leaf size=322

$$\frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} - \frac{(\sqrt{a}e^2)}{4a(a+cx^4)}$$

Rubi [A] time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} + \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{16\sqrt{2} a^{7/4} c^{3/4}} - \frac{(\sqrt{a}e^2 + 3\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{(\sqrt{a}e^2 + 3\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} c^{3/4}} + \frac{de \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2} \sqrt{c}} + \frac{x(d+ex)^2}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]

[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1855

```
Int[(Pq)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
```

[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n), {ii, 0, n/2 - 1 }], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
 \int \frac{(d+ex)^2}{(a+cx^4)^2} dx &= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4} dx}{4a} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \left(-\frac{4dex}{a+cx^4} + \frac{-3d^2-e^2x^2}{a+cx^4} \right) dx}{4a} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4} dx}{4a} + \frac{(de) \int \frac{x}{a+cx^4} dx}{a} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{(de) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2a} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} - e^2 \right) \int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4} dx}{8ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{a+cx^4} dx}{8ac} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16ac} + \frac{\left(\frac{3\sqrt{c}d^2}{\sqrt{a}} + e^2 \right) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}} dx}{16ac} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2 - \sqrt{a}e^2) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}c^{3/4}} \\
 &= \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2 + \sqrt{a}e^2) \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}c^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.32, size = 321, normalized size = 1.00

$$\frac{\sqrt{2}(a^{3/4}d^2 - 3\sqrt[4]{a}\sqrt{c}d^2) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} + \frac{\sqrt{2}(3\sqrt[4]{a}\sqrt{c}d^2 - a^{3/4}d^2) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} - \frac{2\sqrt[4]{a}(8\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + 3\sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{32a^2} + \frac{2\sqrt[4]{a}(-8\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2 + 3\sqrt{2}\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{32a^2} + \frac{8ax(dx)^2}{a+cx^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]

[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*a^(3/4)*c^(3/4))

$$\frac{1}{4}])/c^{3/4} + (2*a^{1/4}*(3*\text{Sqrt}[2]*\text{Sqrt}[c]*d^2 - 8*a^{1/4}*c^{1/4}*d*e + \text{Sqrt}[2]*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/c^{3/4} + (\text{Sqrt}[2]*(-3*a^{1/4}*\text{Sqrt}[c]*d^2 + a^{3/4}*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4})*c^{1/4}*x + \text{Sqrt}[c]*x^2])/c^{3/4} + (\text{Sqrt}[2]*(3*a^{1/4}*\text{Sqrt}[c]*d^2 - a^{3/4}*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/c^{3/4})/(32*a^2)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.36, size = 323, normalized size = 1.00

$$\frac{x^2 e^2 + 2 d x^2 e + d^2 x}{4(c x^4 + a)^2} + \frac{\sqrt{2} \left(4 \sqrt{2} \sqrt{a c} c^2 d e + 3 (a c^3)^{\frac{1}{2}} c^2 d^2 + (a c^3)^{\frac{3}{2}} e^2 \right) \arctan \left(\frac{\sqrt{2} (2 x + \sqrt{2} (c)^{\frac{1}{2}})}{2 (c)^{\frac{1}{2}}} \right)}{16 a^2 c^3} + \frac{\sqrt{2} \left(4 \sqrt{2} \sqrt{a c} c^2 d e + 3 (a c^3)^{\frac{1}{2}} c^2 d^2 + (a c^3)^{\frac{3}{2}} e^2 \right) \arctan \left(\frac{\sqrt{2} (2 x - \sqrt{2} (c)^{\frac{1}{2}})}{2 (c)^{\frac{1}{2}}} \right)}{16 a^2 c^3} + \frac{\sqrt{2} \left(3 (a c^3)^{\frac{1}{2}} c^2 d e - (a c^3)^{\frac{3}{2}} e^2 \right) \log \left(x^2 + \sqrt{2} x (c)^{\frac{1}{2}} + \sqrt{c} \right)}{32 a^2 c^3} - \frac{\sqrt{2} \left(3 (a c^3)^{\frac{1}{2}} c^2 d e - (a c^3)^{\frac{3}{2}} e^2 \right) \log \left(x^2 - \sqrt{2} x (c)^{\frac{1}{2}} + \sqrt{c} \right)}{32 a^2 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\frac{1}{4} * (x^3 * e^2 + 2 * d * x^2 * e + d^2 * x) / ((c * x^4 + a) * a) + \frac{1}{16} * \text{sqrt}(2) * (4 * \text{sqrt}(2) * \text{sqrt}(a * c) * c^2 * d * e + 3 * (a * c^3)^{\frac{1}{4}} * c^2 * d^2 + (a * c^3)^{\frac{3}{4}} * e^2) * \arctan(1 / (2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2) * (a / c)^{\frac{1}{4}})) / (a / c)^{\frac{1}{4}}) / (a^2 * c^3) + \frac{1}{16} * \text{sqrt}(2) * (4 * \text{sqrt}(2) * \text{sqrt}(a * c) * c^2 * d * e + 3 * (a * c^3)^{\frac{1}{4}} * c^2 * d^2 + (a * c^3)^{\frac{3}{4}} * e^2) * \arctan(1 / (2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2) * (a / c)^{\frac{1}{4}})) / (a / c)^{\frac{1}{4}}) / (a^2 * c^3) + \frac{1}{32} * \text{sqrt}(2) * (3 * (a * c^3)^{\frac{1}{4}} * c^2 * d^2 - (a * c^3)^{\frac{3}{4}} * e^2) * \log(x^2 + \text{sqrt}(2) * x * (a / c)^{\frac{1}{4}} + \text{sqrt}(a / c)) / (a^2 * c^3) - \frac{1}{32} * \text{sqrt}(2) * (3 * (a * c^3)^{\frac{1}{4}} * c^2 * d^2 - (a * c^3)^{\frac{3}{4}} * e^2) * \log(x^2 - \text{sqrt}(2) * x * (a / c)^{\frac{1}{4}} + \text{sqrt}(a / c)) / (a^2 * c^3)$$

maple [A] time = 0.01, size = 362, normalized size = 1.12

$$\frac{e^2 x^3}{4(c x^4 + a)a} + \frac{d e x^2}{2(c x^4 + a)a} + \frac{d^2 x}{4(c x^4 + a)a} + \frac{d e \arctan\left(\sqrt{\frac{c}{a}} x\right)}{2\sqrt{ac} a} + \frac{\sqrt{2} e^2 \arctan\left(\frac{\sqrt{2} x}{(\frac{c}{a})^{\frac{1}{4}}}\right)}{16\left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{\sqrt{2} e^2 \arctan\left(\frac{\sqrt{2} x}{(\frac{c}{a})^{\frac{1}{4}}}\right)}{16\left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{\sqrt{2} e^2 \ln\left(\frac{(\frac{c}{a})^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}{(\frac{c}{a})^{\frac{1}{4}} \sqrt{2} x - \sqrt{\frac{c}{a}}}\right)}{32\left(\frac{c}{a}\right)^{\frac{1}{4}} a c} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(\frac{c}{a})^{\frac{1}{4}}}\right)}{16a^2} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^2 \arctan\left(\frac{\sqrt{2} x}{(\frac{c}{a})^{\frac{1}{4}}}\right)}{16a^2} + \frac{3\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} d^2 \ln\left(\frac{(\frac{c}{a})^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{a}}}{(\frac{c}{a})^{\frac{1}{4}} \sqrt{2} x - \sqrt{\frac{c}{a}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+a)^2,x)

[Out] $\frac{1}{4} d^2 x/a / (c x^4 + a) + 3/32 d^2/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)}) * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) + 3/16 d^2/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) + 3/16 d^2/a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) + 1/2 d * e * x^2/a / (c x^4 + a) + 1/2 d * e/a / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) + 1/4 * e^2 * x^3/a / (c x^4 + a) + 1/32 * e^2/a / c / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) + 1/16 * e^2/a / c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) + 1/16 * e^2/a / c / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1)$

maxima [A] time = 2.63, size = 318, normalized size = 0.99

$$\frac{e^2 x^3 + 2 d e x^2 + d^2 x}{4(a c x^4 + a^2)} + \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} d^2) \log(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a})}{a^{\frac{3}{4}} c^{\frac{3}{4}}} - \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} d^2) \log(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a})}{a^{\frac{3}{4}} c^{\frac{3}{4}}} + \frac{2 (3 \sqrt{2} a^{\frac{1}{4}} c^{\frac{3}{4}} d^2 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 - 8 \sqrt{a} \sqrt{c} d e) \arctan\left(\frac{\sqrt{2} (2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2 \sqrt{a} \sqrt{c}}\right)}{32 a} + \frac{2 (3 \sqrt{2} a^{\frac{1}{4}} c^{\frac{3}{4}} d^2 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 + 8 \sqrt{a} \sqrt{c} d e) \arctan\left(\frac{\sqrt{2} (2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2 \sqrt{a} \sqrt{c}}\right)}{32 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{4} * (e^2 * x^3 + 2 * d * e * x^2 + d^2 * x) / (a * c * x^4 + a^2) + \frac{1}{32} * (\sqrt{2}) * (3 * \sqrt{c}) * d^2 - \sqrt{c} * e^2 * \log(\sqrt{c} * x^2 + \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) - \sqrt{2} * (3 * \sqrt{c}) * d^2 - \sqrt{c} * e^2 * \log(\sqrt{c} * x^2 - \sqrt{2} * a^{(1/4)} * c^{(1/4)} * x + \sqrt{a}) / (a^{(3/4)} * c^{(3/4)}) + 2 * (3 * \sqrt{c}) * a^{(1/4)} * c^{(3/4)} * d^2 + \sqrt{2} * a^{(3/4)} * c^{(1/4)} * e^2 - 8 * \sqrt{c} * a * \sqrt{c} * d * e * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x + \sqrt{2} * a^{(1/4)} * c^{(1/4)})) / \sqrt{c} * \sqrt{a} * \sqrt{c} * (c)) / (a^{(3/4)} * \sqrt{c} * \sqrt{a} * \sqrt{c} * c^{(3/4)}) + 2 * (3 * \sqrt{c}) * a^{(1/4)} * c^{(3/4)} * d^2 + \sqrt{2} * a^{(3/4)} * c^{(1/4)} * e^2 + 8 * \sqrt{c} * a * \sqrt{c} * d * e * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{c} * x - \sqrt{2} * a^{(1/4)} * c^{(1/4)})) / \sqrt{c} * \sqrt{a} * \sqrt{c} * (c)) / (a^{(3/4)} * \sqrt{c} * \sqrt{a} * \sqrt{c} * c^{(3/4)}) / a$

mupad [B] time = 2.48, size = 391, normalized size = 1.21

$$\frac{d^2 x^3 + 2 d e x^2 + d^2 x}{4(a c x^4 + a^2)} + \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} d^2) \log(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a})}{a^{\frac{3}{4}} c^{\frac{3}{4}}} - \frac{\sqrt{2} (3 \sqrt{c} d^2 - \sqrt{a} d^2) \log(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a})}{a^{\frac{3}{4}} c^{\frac{3}{4}}} + \frac{2 (3 \sqrt{2} a^{\frac{1}{4}} c^{\frac{3}{4}} d^2 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 - 8 \sqrt{a} \sqrt{c} d e) \arctan\left(\frac{\sqrt{2} (2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2 \sqrt{a} \sqrt{c}}\right)}{32 a} + \frac{2 (3 \sqrt{2} a^{\frac{1}{4}} c^{\frac{3}{4}} d^2 + \sqrt{2} a^{\frac{3}{4}} c^{\frac{1}{4}} d^2 + 8 \sqrt{a} \sqrt{c} d e) \arctan\left(\frac{\sqrt{2} (2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}})}{2 \sqrt{a} \sqrt{c}}\right)}{32 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + c*x^4)^2,x)

```
[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(log((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), 1, 4)
```

sympy [A] time = 3.52, size = 318, normalized size = 0.99

RootSum(65536*a^7*c^3 + 11264*a^4*c^2*d^2*e^2 + (256*a^2*d^5 - 2304*a^2*d^5*e) + a^2*e^8 + 82*a*c*d^4*e^4 + 81*c^2*d^8, (t -> t*log(x + (4096*d^2*e^2 + 356352*d^2*e^2 - 23552*d^2*e^2 + 27648*d^2*e^2 + 912*a^4*c^2*d^2 + 43584*d^2*c^2*d^2 + 3888*d^2*c^2*d^2 + 12*a^4*d^11 - 1088*d^2*c^2*d^2 - 7020*a^2*d^2))) + (d^2*x + 2*d*e*x^2 + e^2*x^3)/(4*a^2 + 4*a*c*x^4))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d**9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

$$3.399 \quad \int \frac{d+ex}{(a+cx^4)^2} dx$$

Optimal. Leaf size=241

$$\frac{3d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{c}} + \frac{x(d+ex)}{4a(a+cx^4)}$$

Rubi [A] time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{3d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{4a^{3/2} \sqrt{c}} + \frac{x(d+ex)}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4)^2, x]

[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
 \int \frac{d+ex}{(a+cx^4)^2} dx &= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \frac{-3d-2ex}{a+cx^4} dx}{4a} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4} \right) dx}{4a} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d) \int \frac{1}{a+cx^4} dx}{4a} + \frac{e \int \frac{x}{a+cx^4} dx}{2a} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{(3d) \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{(3d) \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{e \operatorname{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{4a} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} + \frac{(3d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{(3d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} - \frac{(3d) \int \frac{1}{\sqrt{a} + \sqrt{2}\sqrt[4]{a}x + \sqrt{c}x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\
 &= \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}\sqrt{c}} - \frac{3d \tan^{-1} \left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \tan^{-1} \left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} \right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3d \log \left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d \log \left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2 \right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}
 \end{aligned}$$

Mathematica [A] time = 0.19, size = 224, normalized size = 0.93

$$\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2(4\sqrt[4]{a}e+3\sqrt{2}\sqrt[4]{c}d)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{3\sqrt{2}d\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{3\sqrt{2}d\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4)^2, x]

[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)

*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/Sqrt[c] - (3*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex}{(a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^2,x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^2, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 241, normalized size = 1.00

$$\frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c} + \frac{x^2e+dx}{4(cx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3(ac^3)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3(ac^3)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] $\frac{3}{32}\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)/(a^2c) - \frac{3}{32}\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)/(a^2c) + \frac{1}{4}\frac{x^2e+dx}{(cx^4+a)a} + \frac{1}{16}\sqrt{2}(2\sqrt{2}\sqrt{ac}ce+3(ac^3)^{\frac{1}{4}}cd)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)/(a^2c^2) + \frac{1}{16}\sqrt{2}(2\sqrt{2}\sqrt{ac}ce+3(ac^3)^{\frac{1}{4}}cd)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)/(a^2c^2)$

maple [A] time = 0.00, size = 188, normalized size = 0.78

$$\frac{ex^2}{4(cx^4+a)a} + \frac{dx}{4(cx^4+a)a} + \frac{e\arctan\left(\sqrt{\frac{c}{a}}x\right)}{4\sqrt{ac}a} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}d\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)/(c*x^4+a)^2,x)`

[Out] $\frac{1}{4} \frac{d*x}{a} \frac{1}{(c*x^4+a)} + \frac{3}{32} \frac{d}{a^2} \frac{1}{(a/c)^{1/4}} * 2^{1/2} * \ln\left(\frac{(x^2+(a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/4}}{(x^2-(a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/4}}\right) + \frac{3}{16} \frac{d}{a^2} \frac{1}{(a/c)^{1/4}} * 2^{1/2} * \arctan\left(\frac{2^{1/2}}{(a/c)^{1/4} * x + 1}\right) + \frac{3}{16} \frac{d}{a^2} \frac{1}{(a/c)^{1/4}} * 2^{1/2} * \arctan\left(\frac{2^{1/2}}{(a/c)^{1/4} * x - 1}\right) + \frac{1}{4} \frac{e*x^2}{a} \frac{1}{(c*x^4+a)} + \frac{1}{4} \frac{e}{a} \frac{1}{(a*c)^{1/2}} * \arctan\left(\frac{1}{a*c}\right)^{1/2} * x^2$

maxima [A] time = 2.43, size = 238, normalized size = 0.99

$$\frac{ex^2 + dx}{4(acx^4 + a^2)} + \frac{\frac{3\sqrt{2}d \log\left(\frac{\sqrt{c}x^2 + \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}}}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right) - 3\sqrt{2}d \log\left(\frac{\sqrt{c}x^2 - \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}}}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right) + \frac{2\left(3\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}d - 4\sqrt{a}e\right) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{1}{4}}} + \frac{2\left(3\sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}d + 4\sqrt{a}e\right) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2a^{\frac{1}{4}}c^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a}\sqrt{c}c^{\frac{1}{4}}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")`

[Out] $\frac{1}{4} \frac{(e*x^2 + d*x)}{(a*c*x^4 + a^2)} + \frac{1}{32} \frac{(3*\sqrt{2}*d*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4} - 3*\sqrt{2}*d*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4} + 2*(3*\sqrt{2}*a^{1/4}*c^{1/4}*d - 4*\sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{a} + 2*(3*\sqrt{2}*a^{1/4}*c^{1/4}*d + 4*\sqrt{a}*e)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{a}}{\sqrt{a}*\sqrt{c}}$

mupad [B] time = 0.27, size = 282, normalized size = 1.17

$$\frac{\int \frac{e^{2x} + 2e^x - \sqrt{65536t^7c^2z^4 + 2048t^4c^4e^2z^2 - 1152t^2c^2d^2ez + 81c^4d^4 + 16a^4e^4}}{a^{16}} + \frac{e^{2x} - \frac{dx}{c^2}}{c^2 + a}}{\sqrt{65536t^7c^2z^4 + 2048t^4c^4e^2z^2 - 1152t^2c^2d^2ez + 81c^4d^4 + 16a^4e^4}} + \frac{e^{2x} - \frac{dx}{c^2}}{c^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)/(a + c*x^4)^2,x)`

[Out] $\text{symsum}\left(\log\left(\frac{(c^2*(3*d*e^2 + 2*e^3*x - 192*\text{root}(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128*\text{root}(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*\text{root}(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x)}{(16*a^3)*\text{root}(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k), k, 1, 4)} + \frac{(e*x^2)}{(4*a)} + \frac{(d*x)}{(4*a)}\right) \frac{1}{(a + c*x^4)}$

sympy [A] time = 1.09, size = 155, normalized size = 0.64

$$\text{RootSum}\left(\frac{-32768t^3a^6ce^2 - 4608t^2a^4cd^2e - 512ta^3e^4 - 1296ta^2cd^4 + 360ad^2e^3}{192ade^4 - 243cd^5}\right) \frac{dx + ex^2}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**  
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**  
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360  
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a  
*c*x**4)
```

$$3.400 \quad \int \frac{1}{(a+cx^4)^2} dx$$

Optimal. Leaf size=202

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}}$$

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{16\sqrt{2} a^{7/4} \sqrt[4]{c}} - \frac{3 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{8\sqrt{2} a^{7/4} \sqrt[4]{c}} + \frac{x}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-2), x]

[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b}

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^4)^2} dx &= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{a+cx^4} dx}{4a} \\
&= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} + \frac{3 \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{8a^{3/2}} \\
&= \frac{x}{4a(a+cx^4)} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} + \frac{3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{16a^{3/2}\sqrt{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} + 2x}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}} - 2x}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} - x^2} dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} \\
&= \frac{x}{4a(a+cx^4)} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3 \operatorname{Arctan}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \operatorname{Arctan}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} - \frac{3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{3\sqrt{2} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{3\sqrt{2} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{\sqrt[4]{c}} - \frac{6\sqrt{2} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-2), x]

[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(32*a^(7/4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^4)^(-2), x]

[Out] IntegrateAlgebraic[(a + c*x^4)^(-2), x]

fricas [A] time = 1.16, size = 173, normalized size = 0.86

$$\frac{12(acx^4 + a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} \arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}} + \sqrt{a^4\sqrt{-\frac{1}{a^7c}} + x^2}a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right) + 3(acx^4 + a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} \log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} + x\right) - 3(acx^4 + a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} \log\left(-a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}} + x\right) + 4x}{16(acx^4 + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")

[Out] $\frac{1}{16} * (12 * (a * c * x^4 + a^2) * (-1 / (a^7 * c))^{(1/4)} * \arctan(-a^5 * c * x * (-1 / (a^7 * c))^{(3/4)} + \sqrt{a^4 * \sqrt{-1 / (a^7 * c)} + x^2} * a^5 * c * (-1 / (a^7 * c))^{(3/4)}) + 3 * (a * c * x^4 + a^2) * (-1 / (a^7 * c))^{(1/4)} * \log(a^2 * (-1 / (a^7 * c))^{(1/4)} + x) - 3 * (a * c * x^4 + a^2) * (-1 / (a^7 * c))^{(1/4)} * \log(-a^2 * (-1 / (a^7 * c))^{(1/4)} + x) + 4 * x) / (a * c * x^4 + a^2)$

giac [A] time = 0.25, size = 194, normalized size = 0.96

$$\frac{x}{4(cx^4 + a)a} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}(2x + \sqrt{2}(\frac{a}{c})^{\frac{1}{4}})}{2(\frac{a}{c})^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}(2x - \sqrt{2}(\frac{a}{c})^{\frac{1}{4}})}{2(\frac{a}{c})^{\frac{1}{4}}}\right)}{16a^2c} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x(\frac{a}{c})^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x(\frac{a}{c})^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")

[Out] $\frac{1}{4} * x / ((c * x^4 + a) * a) + \frac{3}{16} * \sqrt{2} * (a * c^3)^{\frac{1}{4}} * \arctan(1/2 * \sqrt{2} * (2 * x + \sqrt{2} * (a/c)^{\frac{1}{4}}) / (a/c)^{\frac{1}{4}}) / (a^2 * c) + \frac{3}{16} * \sqrt{2} * (a * c^3)^{\frac{1}{4}} * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (a/c)^{\frac{1}{4}}) / (a/c)^{\frac{1}{4}}) / (a^2 * c) + \frac{3}{32} * \sqrt{2} * (a * c^3)^{\frac{1}{4}} * \log(x^2 + \sqrt{2} * x * (a/c)^{\frac{1}{4}} + \sqrt{a/c}) / (a^2 * c) - \frac{3}{32} * \sqrt{2} * (a * c^3)^{\frac{1}{4}} * \log(x^2 - \sqrt{2} * x * (a/c)^{\frac{1}{4}} + \sqrt{a/c}) / (a^2 * c)$

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4(cx^4 + a)a} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{16a^2} + \frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}x + \sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+a)^2,x)

[Out] $\frac{1}{4} \frac{x}{a/(c x^4 + a)} + \frac{3}{32} \frac{1}{a^2} \left(\frac{a}{c} \right)^{\frac{1}{4}} 2^{\frac{1}{2}} \ln \left(\frac{x^2 + \left(\frac{a}{c} \right)^{\frac{1}{4}} 2^{\frac{1}{2}} \sqrt{x + \left(\frac{a}{c} \right)^{\frac{1}{2}}}}{x^2 - \left(\frac{a}{c} \right)^{\frac{1}{4}} 2^{\frac{1}{2}} \sqrt{x + \left(\frac{a}{c} \right)^{\frac{1}{2}}}} \right) + \frac{3}{16} \frac{1}{a^2} \left(\frac{a}{c} \right)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan \left(\frac{2^{\frac{1}{2}}}{\left(\frac{a}{c} \right)^{\frac{1}{4}} x + 1} \right) + \frac{3}{16} \frac{1}{a^2} \left(\frac{a}{c} \right)^{\frac{1}{4}} 2^{\frac{1}{2}} \arctan \left(\frac{2^{\frac{1}{2}}}{\left(\frac{a}{c} \right)^{\frac{1}{4}} x - 1} \right)$

maxima [A] time = 2.11, size = 189, normalized size = 0.94

$$\frac{x}{4(acx^4 + a^2)} + \frac{3}{32a} \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{cx} + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{cx} - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{cx} + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{cx} - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $\frac{1}{4} \frac{x}{a c x^4 + a^2} + \frac{3}{32} \frac{2 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c}} + \frac{3}{32} \frac{2 \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{\sqrt{a} \sqrt{a} \sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}}$

mapad [B] time = 0.09, size = 58, normalized size = 0.29

$$\frac{x}{4a(c x^4 + a)} + \frac{3 \operatorname{atan}\left(\frac{c^{\frac{1}{4}} x}{(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{7}{4}} c^{\frac{1}{4}}} + \frac{3 \operatorname{atanh}\left(\frac{c^{\frac{1}{4}} x}{(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{7}{4}} c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + c*x^4)^2,x)

[Out] $\frac{x}{4a(a + c x^4)} + \frac{3 \operatorname{atan}\left(\frac{c^{\frac{1}{4}} x}{(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{7}{4}} c^{\frac{1}{4}}} + \frac{3 \operatorname{atanh}\left(\frac{c^{\frac{1}{4}} x}{(-a)^{\frac{1}{4}}}\right)}{8(-a)^{\frac{7}{4}} c^{\frac{1}{4}}}$

sympy [A] time = 0.30, size = 39, normalized size = 0.19

$$\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}\left(65536t^4 a^7 c + 81, \left(t \mapsto t \log\left(\frac{16ta^2}{3} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+a)**2,x)

[Out] $\frac{x}{4a^2 + 4acx^4} + \operatorname{RootSum}(65536t^4 a^7 c + 81, \operatorname{Lambda}(t, t \log(16t a^2 / 3 + x)))$

$$3.401 \quad \int \frac{1}{(d+ex)(a+cx^4)^2} dx$$

Optimal. Leaf size=855

$$\frac{\log(d+ex)e^7}{(cd^4+ae^4)^2} - \frac{\log(cx^4+a)e^7}{4(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2 \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^2} + \dots$$

Rubi [A] time = 0.85, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

Antiderivative was successfully verified.

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + c*x^4)^2), x]

[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4)) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^2)

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] \text{ ; FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_)^{(m_)}/((a_) + (b_ \cdot)(x_)^{(n_)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x^n, x]]/(b \cdot n), x] \text{ ; FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 275

$\text{Int}[(x_)^{(m_)} \cdot ((a_) + (b_ \cdot)(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} \cdot (a + b \cdot x^{(n/k)})^p, x], x, x^k], x] \text{ ; } k \neq 1 \text{ ; FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_) + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] \text{ ; RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ \|\ \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) \text{ ; FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d_) + (e_ \cdot)(x_)]/((a_) + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] \text{ ; FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 635

$\text{Int}[(d_) + (e_ \cdot)(x_)]/((a_) + (c_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c \cdot x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c \cdot x^2), x], x] \text{ ; FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \ !\text{NiceSqrtQ}[-(a \cdot c)]$

Rule 1162

$\text{Int}[(d_) + (e_ \cdot)(x_)^2]/((a_) + (c_ \cdot)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] \text{ ; FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+cx^4)^2} dx &= \int \left(\frac{e^8}{(cd^4+ae^4)^2(d+ex)} + \frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(cd^4+ae^4)(a+cx^4)^2} - \frac{ce^4(-d^3+d^2ex-de^2x^2+e^3x^3)}{(cd^4+ae^4)^2(a+cx^4)} \right) dx \\
&= \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4} dx}{(cd^4+ae^4)^2} + \frac{c \int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{(a+cx^4)^2} dx}{cd^4+ae^4} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \left(\frac{-d^3-de^2x^2}{a+cx^4} + \frac{x(d^2e+e^3x^2)}{a+cx^4} \right) dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{-d^3-de^2x^2}{a+cx^4} dx}{(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{x(d^2e+e^3x^2)}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(ce^4) \text{Subst} \left(\int \frac{d^2e+e^3x}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)^2} + \frac{(ce^4) \int \frac{-d^3-de^2x^2}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} - \frac{(cd^2e^5) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{2(cd^4+ae^4)^2} - \frac{(ce^4) \int \frac{-d^3-de^2x^2}{a+cx^4} dx}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c}d^2e^5 \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}(cd^4+ae^4)} + \frac{e^7 \log(d+ex)}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c}d^2e^5 \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}(cd^4+ae^4)} - \frac{4\sqrt{c}de^4}{(cd^4+ae^4)^2} \\
&= \frac{ae^3+cx(d^3-d^2ex+de^2x^2)}{4a(cd^4+ae^4)(a+cx^4)} - \frac{\sqrt{c}d^2e^5 \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{2\sqrt{a}(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2e \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{4a^{3/2}(cd^4+ae^4)} - \frac{4\sqrt{c}de^4}{(cd^4+ae^4)^2}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 558, normalized size = 0.65

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2), x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
+ 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 +
5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/a^(7/4) +
32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2
- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c
*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 +
a*e^4)^2)
```

```
IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(d + ex)(a + cx^4)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^2), x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^2), x]
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [A] time = 0.50, size = 771, normalized size = 0.90
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 - 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + e^8*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

maple [A] time = 0.02, size = 1122, normalized size = 1.31

result too large to display

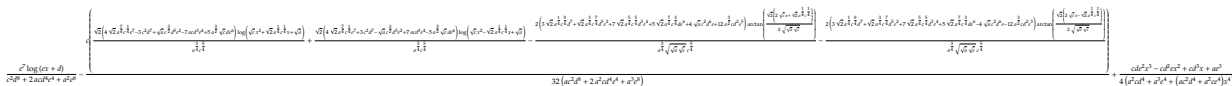
Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d*e^6*x^3+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^5*e^2/a*x^3-1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^5*d^2*x^2-1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*e*d^6/a*x^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d^3*x*e^4+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^7/a*x+1/4/(a*e^4+c*d^4)^2/(c*x^4+a)*e^7*a+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^3*d^4+7/16*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7+7/32*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^4+3/32*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7+7/16*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7-3/4*c/(a*e^4+c*d^4)^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^2-1/4*c^2/(a*e^4+c*d^4)^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^6+5/32/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^6+1/32*c/(a*e^4+c*d^4)^2/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^
```


$$(1/2)*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^5*e^2-1/4*e^7*\ln(c*x^4+a)/(a*e^4+c*d^4)^2$$

maxima [A] time = 2.29, size = 601, normalized size = 0.70



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")

[Out]
$$e^7*\log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(\sqrt{2})*(4*\sqrt{2}*a^{(7/4)}*c^{(1/4)}*e^7 - 3*c^2*d^7 + \sqrt{a}*c^{(3/2)}*d^5*e^2 - 7*a*c*d^3*e^4 + 5*a^{(3/2)}*\sqrt{c}*d*e^6)*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \sqrt{2}*(4*\sqrt{2}*a^{(7/4)}*c^{(1/4)}*e^7 + 3*c^2*d^7 - \sqrt{a}*c^{(3/2)}*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^{(3/2)}*\sqrt{c}*d*e^6)*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) - 2*(3*\sqrt{2}*a^{(1/4)}*c^{(9/4)}*d^7 + \sqrt{2}*a^{(3/4)}*c^{(7/4)}*d^5*e^2 + 7*\sqrt{2}*a^{(5/4)}*c^{(5/4)}*d^3*e^4 + 5*\sqrt{2}*a^{(7/4)}*c^{(3/4)}*d*e^6 + 4*\sqrt{a}*c^2*d^6*e + 12*a^{(3/2)}*c*d^2*e^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{c})})/(a^{(3/4)}*\sqrt{(\sqrt{a}*\sqrt{c})})*c^{(5/4)} - 2*(3*\sqrt{2}*a^{(1/4)}*c^{(9/4)}*d^7 + \sqrt{2}*a^{(3/4)}*c^{(7/4)}*d^5*e^2 + 7*\sqrt{2}*a^{(5/4)}*c^{(5/4)}*d^3*e^4 + 5*\sqrt{2}*a^{(7/4)}*c^{(3/4)}*d*e^6 - 4*\sqrt{a}*c^2*d^6*e - 12*a^{(3/2)}*c*d^2*e^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{(\sqrt{a}*\sqrt{c})})/(a^{(3/4)}*\sqrt{(\sqrt{a}*\sqrt{c})})*c^{(5/4)})/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/4*(c*d*e^2*x^3 - c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*c^2*d^4 + a^2*c*e^4)*x^4)$$

mupad [B] time = 2.99, size = 1591, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^2*(d + e*x)),x)

[Out]
$$e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + \text{symsum}(\log((81*c^5*d^5*e^6 + 64*a*c^4*d*e^{10})/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + \text{root}(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(\text{root}(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)$$

$$\begin{aligned}
& 7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k) * (\text{root}(131072*a^8*c*d^4*e^4*z^4 \\
& + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4* \\
& c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k) * (\text{root}(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k) * ((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 + 65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (81*c^5*d^4*e^7*x)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) * \text{root}(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*x^4))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)

[Out] Timed out

$$3.402 \quad \int \frac{1}{(d+ex)^2(a+cx^4)^2} dx$$

Optimal. Leaf size=1141

$$\frac{8cd^3 \log(d+ex)e^7}{(cd^4+ae^4)^3} - \frac{2cd^3 \log(cx^4+a)e^7}{(cd^4+ae^4)^3} - \frac{e^7}{(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt{c}d(3cd^4-ae^4) \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}(\sqrt{c}(5$$

Rubi [A] time = 1.66, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)^2), x]

[Out] $-(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (\text{Sqrt}[c]*d*e^5*(3*c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(c*d^4 + a*e^4)^3) - (\text{Sqrt}[c]*d*e*(c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)*(3*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (2*c*d^3*e^7*\text{Log}[a + c*x^4])/(c*d^4 + a*e^4)^3$

Rule 204

$\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 205

$\text{Int}[(a_.) + (b_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 260

$\text{Int}[(x_)^{(m_.)}/((a_.) + (b_.)(x_)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x \ \&\& \ \text{EqQ}[m, n - 1]$

Rule 275

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m + 1)/k - 1}*(a + b*x^{(n/k)})^p, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_.) + (b_.)(x_) + (c_.)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*c\}, \text{Simplify}[(a*c)/b^2], \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)(x_))/((a_.) + (b_.)(x_) + (c_.)(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 635

$\text{Int}[(d_.) + (e_.)(x_))/((a_.) + (c_.)(x_)^2), x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

Rule 1162

$\text{Int}[(d_.) + (e_.)(x_)^2)/((a_.) + (c_.)(x_)^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e$

$$\frac{1}{(2*c)} \int \frac{1}{\text{Simp}[d/e - q*x + x^2, x]} dx$$
 /; FreeQ[{a, c, d, e}, x] & EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

$$\int \frac{(d_+) + (e_+)(x_+)^2}{(a_+) + (c_+)(x_+)^4} dx$$
 := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

$$\int \frac{(d_+) + (e_+)(x_+)^2}{(a_+) + (c_+)(x_+)^4} dx$$
 := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

$$\int (x_+)^{(d_+) + (e_+)(x_+)^2} (q_+)((a_+) + (c_+)(x_+)^4)^{(p_+)} dx$$
 := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

$$\int (Pq_+)((a_+) + (b_+)(x_+)^n)^{(p_+)} dx$$
 := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

$$\int \frac{Pq_+}{(a_+) + (b_+)(x_+)^n} dx$$
 := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n]

Rule 6742

$$\int u_+ dx$$
 := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Mathematica [A] time = 0.86, size = 807, normalized size = 0.71

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2), x]

[Out]
$$\frac{((-32e^7(c^4d + ae^4))/(d + ex) + (8c(c^4d + ae^4)(c^4x(d^2 - 2de^2x + 3e^2x^2) + ae^3(4d^3 - 3d^2ex + 2de^2x^2 - e^3x^3)))/(a(a + cx^4)) + (2c^{1/4}(-3\sqrt{2}c^{5/2}d^{10} + 8a^{1/4}c^{9/4}d^9e - 3\sqrt{2}\sqrt{a}c^2d^8e^2 - 14\sqrt{2}a^{3/2}d^6e^4 + 48a^{5/4}c^{5/4}d^5e^5 - 30\sqrt{2}a^{3/2}c^4d^4e^6 + 21\sqrt{2}a^2\sqrt{c}d^2e^8 - 24a^{9/4}c^{1/4}de^9 + 5\sqrt{2}a^{5/2}e^{10})\text{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}])/a^{7/4} + (2c^{1/4}(3\sqrt{2}c^{5/2}d^{10} + 8a^{1/4}c^{9/4}d^9e + 3\sqrt{2}\sqrt{a}c^2d^8e^2 + 14\sqrt{2}a^{3/2}d^6e^4 + 48a^{5/4}c^{5/4}d^5e^5 + 30\sqrt{2}a^{3/2}c^4d^4e^6 - 21\sqrt{2}a^2\sqrt{c}d^2e^8 - 24a^{9/4}c^{1/4}de^9 - 5\sqrt{2}a^{5/2}e^{10})\text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}])/a^{7/4} + 256c^3d^3e^7 \text{Log}[d + ex] - (\sqrt{2}c^{1/4}(3c^{5/2}d^{10} - 3\sqrt{a}c^2d^8e^2 + 14a^{3/2}d^6e^4 - 30a^{3/2}c^4d^4e^6 - 21a^2\sqrt{c}d^2e^8 + 5a^{5/2}e^{10})\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{7/4} + (\sqrt{2}c^{1/4}(3c^{5/2}d^{10} - 3\sqrt{a}c^2d^8e^2 + 14a^{3/2}d^6e^4 - 30a^{3/2}c^4d^4e^6 - 21a^2\sqrt{c}d^2e^8 + 5a^{5/2}e^{10})\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{7/4} - 64c^3d^3e^7 \text{Log}[a + cx^4])/(32(c^4d + ae^4)^3}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 103.64, size = 1104, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -2*c*d^3*e^7*\log(\text{abs}(c*x^4 + a))/(c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4* \\ & e^8 + a^3*e^{12}) + 8*c*d^3*e^8*\log(\text{abs}(x*e + d))/(c^3*d^{12}*e + 3*a*c^2*d^8*e \\ & ^5 + 3*a^2*c*d^4*e^9 + a^3*e^{13}) + 1/8*(5*\sqrt{2}*\sqrt{a*c}*c^2*d^3*e + 3*(\\ & a*c^3)^{(1/4)}*c^2*d^4 + 3*\sqrt{2}*a*c^2*d*e^3 + 6*(a*c^3)^{(3/4)}*d^2*e^2 - 5* \\ & (a*c^3)^{(1/4)}*a*c*e^4)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/c)^{(1/4)})/(a/c) \\ & ^{(1/4)})/(\sqrt{2}*a^2*c^3*d^6 - 6*(a*c^3)^{(1/4)}*a^2*c^2*d^5*e + 9*\sqrt{2}*\sqrt{ \\ & a*c}*a^2*c^2*d^4*e^2 + 9*\sqrt{2}*a^3*c^2*d^2*e^4 - 16*(a*c^3)^{(3/4)}*a^2* \\ & d^3*e^3 - 6*(a*c^3)^{(1/4)}*a^3*c*d*e^5 + \sqrt{2}*\sqrt{a*c}*a^3*c*e^6) + 1/8* \\ & (5*\sqrt{2}*\sqrt{a*c}*c^2*d^3*e + 3*(a*c^3)^{(1/4)}*c^2*d^4 - 3*\sqrt{2}*a*c^2* \\ & d*e^3 + 6*(a*c^3)^{(3/4)}*d^2*e^2 - 5*(a*c^3)^{(1/4)}*a*c*e^4)*\arctan(1/2*\sqrt{2}*(\\ & 2*x - \sqrt{2}*(a/c)^{(1/4)})/(a/c)^{(1/4)})/(\sqrt{2}*a^2*c^3*d^6 + 6*(a*c^3) \\ & ^{(1/4)}*a^2*c^2*d^5*e + 9*\sqrt{2}*\sqrt{a*c}*a^2*c^2*d^4*e^2 + 9*\sqrt{2}*a^3* \\ & c^2*d^2*e^4 + 16*(a*c^3)^{(3/4)}*a^2*d^3*e^3 + 6*(a*c^3)^{(1/4)}*a^3*c*d*e^5 + \\ & \sqrt{2}*\sqrt{a*c}*a^3*c*e^6) + 1/32*(3*\sqrt{2}*(a*c^3)^{(1/4)}*c^4*d^{10} - 3* \\ & \sqrt{2}*(a*c^3)^{(3/4)}*c^2*d^8*e^2 + 14*\sqrt{2}*(a*c^3)^{(1/4)}*a*c^3*d^6*e^4 \\ & - 30*\sqrt{2}*(a*c^3)^{(3/4)}*a*c*d^4*e^6 - 21*\sqrt{2}*(a*c^3)^{(1/4)}*a^2*c^2*d \\ & ^2*e^8 + 5*\sqrt{2}*(a*c^3)^{(3/4)}*a^2*e^{10})*\log(x^2 + \sqrt{2}*x*(a/c)^{(1/4)} \\ & + \sqrt{a/c})/(a^2*c^5*d^{12} + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2* \\ & e^{12}) - 1/32*(3*\sqrt{2}*(a*c^3)^{(1/4)}*c^4*d^{10} - 3*\sqrt{2}*(a*c^3)^{(3/4)}* \\ & c^2*d^8*e^2 + 14*\sqrt{2}*(a*c^3)^{(1/4)}*a*c^3*d^6*e^4 - 30*\sqrt{2}*(a*c^3)^{(\\ & 3/4)}*a*c*d^4*e^6 - 21*\sqrt{2}*(a*c^3)^{(1/4)}*a^2*c^2*d^2*e^8 + 5*\sqrt{2}*(a* \\ & c^3)^{(3/4)}*a^2*e^{10})*\log(x^2 - \sqrt{2}*x*(a/c)^{(1/4)} + \sqrt{a/c})/(a^2*c^5* \\ & d^{12} + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^{12}) + 1/4*(3*c^2*d \\ & ^4*x^4*e^3 + c^2*d^5*x^3*e^2 - c^2*d^6*x^2*e + c^2*d^7*x - 5*a*c*x^4*e^7 + \\ & a*c*d*x^3*e^6 - a*c*d^2*x^2*e^5 + a*c*d^3*x*e^4 + 4*a*c*d^4*e^3 - 4*a^2*e^7 \\ &)/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*x^5*e + c*d*x^4 + a*x*e + a*d \\ &)) \end{aligned}$$

maple [A] time = 0.02, size = 1636, normalized size = 1.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x)

[Out]
$$\begin{aligned} & 8*c*d^3*e^7*\ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*\ln(c*x^4+a)/(a*e^4+c*d^4) \\ & ^3-e^7/(a*e^4+c*d^4)^2/(e*x+d)-1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^6*x*e^4- \\ & 3*c^2/(a*e^4+c*d^4)^3/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^5-5/32/(a \\ & *e^4+c*d^4)^3*a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/} \end{aligned}$$

$$\begin{aligned}
& 2)) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * e^{-10-5/16} / (a * e^4 + c * d^4)^3 * a / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * e^{-10-5/16} / (a * e^4 + c * d^4)^3 \\
& * a / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * e^{10+1/4} * c^3 / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * d^{10} / a * x^{-1/4} * c / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * e^{10} * a * x^3 + c / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * e^{7 * d^3 * a + 1/2 * c^2} / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * e^{6 * x^3 * d^4 + c^2} / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * d^7 * e^{3-21/16} * c / (a * e^4 + c * d^4)^3 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^2 * e^{8+3/16} * c^3 / (a * e^4 + c * d^4)^3 \\
& / a^2 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^{10-1/2} * c^3 / (a * e^4 + c * d^4)^3 / a * (a/c)^{1/2} * \arctan((1/a * c)^{1/2} * x^2) * e^{d^9+15/16} * c / (a * e^4 + c * d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^4 * e^{6+15/8} * c / (a * e^4 + c * d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^4 * e^{6+15/8} * c / (a * e^4 + c * d^4)^3 / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^4 * e^{6+3/2} * c / (a * e^4 + c * d^4)^3 * a / (a * c)^{1/2} * \arctan((1/a * c)^{1/2} * x^2) * e^{9 * d + 3/4} * c^3 / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * e^{2/a * x^3 * d^8 - 1/2 * c^3} / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * d^9 * e / a * x^2 + 1/2 * c / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * d * e^{9 * a * x^2 - 3/4} * c / (a * e^4 + c * d^4)^3 / (c * x^4 + a) * d^2 * a * x * e^{8-21/16} * c / (a * e^4 + c * d^4)^3 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^2 * e^{8+3/16} * c^3 / (a * e^4 + c * d^4)^3 / a^2 * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^{10-21/32} * c / (a * e^4 + c * d^4)^3 * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^2 * e^{8+3/32} * c^3 / (a * e^4 + c * d^4)^3 / a^2 * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^{10+7/16} * c^2 / (a * e^4 + c * d^4)^3 / a * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^6 * e^{4+7/8} * c^2 / (a * e^4 + c * d^4)^3 / a * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^6 * e^{4+3/16} * c^2 / (a * e^4 + c * d^4)^3 / a / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x + 1) * d^8 * e^{2+3/16} * c^2 / (a * e^4 + c * d^4)^3 / a / (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^8 * e^{2+7/8} * c^2 / (a * e^4 + c * d^4)^3 / a * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/c)^{1/4} * x - 1) * d^6 * e^{4+3/32} * c^2 / (a * e^4 + c * d^4)^3 / a / (a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^8 * e^2
\end{aligned}$$

maxima [A] time = 2.54, size = 961, normalized size = 0.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")

[Out] $8 * c * d^3 * e^7 * \log(e * x + d) / (c^3 * d^{12} + 3 * a * c^2 * d^8 * e^4 + 3 * a^2 * c * d^4 * e^8 + a^3 * e^{12}) - 1/32 * c * (\sqrt{2}) * (32 * \sqrt{2}) * a^{7/4} * c^{5/4} * d^3 * e^7 - 3 * c^3 * d^{10} + 3 * \sqrt{a} * c^{5/2} * d^8 * e^2 - 14 * a * c^2 * d^6 * e^4 + 30 * a^{3/2} * c^{3/2} * d^4 * e^6 + 21 * a^2 * c * d^2 * e^8 - 5 * a^{5/2} * \sqrt{c} * e^{10}) * \log(\sqrt{c} * x^2 + \sqrt{2}) * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{5/4}) + \sqrt{2} * (32 * \sqrt{2}) * a^{7/4} * c^{5/4} * d^3 * e^7 + 3 * c^3 * d^{10} - 3 * \sqrt{a} * c^{5/2} * d^8 * e^2 + 14 * a * c^2 * d^6 * e^4 - 30 * a^{3/2} * c^{3/2} * d^4 * e^6 - 21 * a^2 * c * d^2 * e^8 + 5 * a^{5/2} * \sqrt{c} * e^{10}) * 1$

```

og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14
*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*
sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 + 8*sqrt(
a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt
(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sq
rt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sq
rt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt
(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/2)*c^2*d^5*e^5 +
24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^3*
d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^12) + 1/4*(4*a*c*d^4*e^3
- 4*a^2*e^7 + (3*c^2*d^4*e^3 - 5*a*c*e^7)*x^4 + (c^2*d^5*e^2 + a*c*d*e^6)*
x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/(a^2*c^2*d
^9 + 2*a^3*c*d^5*e^4 + a^4*d*e^8 + (a*c^3*d^8*e + 2*a^2*c^2*d^4*e^5 + a^3*c
*e^9)*x^5 + (a*c^3*d^9 + 2*a^2*c^2*d^5*e^4 + a^3*c*d*e^8)*x^4 + (a^2*c^2*d^
8*e + 2*a^3*c*d^4*e^5 + a^4*e^9)*x)

```

mupad [B] time = 4.10, size = 2246, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

```

[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 655
36*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 1812
48*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 1
9200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 +
2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(
256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^
6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^
4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3
+ 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*
e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*
d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*
d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12
*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2
*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*
d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a
^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(roo
t(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^1
2*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*
e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*

```

$$\begin{aligned}
&^5z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^{11}*c^4*d*e^{22} - 32768*a^6* \\
&c^9*d^{21}*e^2 - 32768*a^7*c^8*d^{17}*e^6 + 196608*a^8*c^7*d^{13}*e^{10} + 458752*a \\
&^9*c^6*d^9*e^{14} + 360448*a^{10}*c^5*d^5*e^{18})/(256*(a^8*e^{16} + a^4*c^4*d^{16} + \\
&4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^ \\
&11*c^4*e^{23} - 49152*a^6*c^9*d^{20}*e^3 - 114688*a^7*c^8*d^{16}*e^7 + 32768*a^8* \\
&c^7*d^{12}*e^{11} + 294912*a^9*c^6*d^8*e^{15} + 278528*a^{10}*c^5*d^4*e^{19})/(256*(\\
&a^8*e^{16} + a^4*c^4*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^4 + 6*a^6*c^2 \\
&*d^8*e^8))) + (5120*a^9*c^4*e^{21} - 3072*a^4*c^9*d^{20}*e + 17408*a^5*c^8*d^{16} \\
&*e^5 + 337920*a^6*c^7*d^{12}*e^9 + 616448*a^7*c^6*d^8*e^{13} + 304128*a^8*c^5*d \\
&^4*e^{17})/(256*(a^8*e^{16} + a^4*c^4*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}* \\
&e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^{11}*e^{10} - 32768*a^5*c^8*d^ \\
&15*e^6 - 10240*a^4*c^9*d^{19}*e^2 + 770048*a^7*c^6*d^7*e^{14} + 391168*a^8*c^5* \\
&d^3*e^{18})/(256*(a^8*e^{16} + a^4*c^4*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^1 \\
&2*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(768*a^3*c^8*d^{14}*e^5 - 576*a^2*c^9*d^{18}* \\
&e + 105088*a^4*c^7*d^{10}*e^9 + 221952*a^5*c^6*d^6*e^{13} + 183744*a^6*c^5*d^2* \\
&e^{17})/(256*(a^8*e^{16} + a^4*c^4*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^ \\
&4 + 6*a^6*c^2*d^8*e^8))) + (x*(200*a*c^8*d^{13}*e^4 + 19400*a^4*c^5*d*e^{16} + \\
&7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^{12})/(256*(a^8*e^{16} + a^4*c^4*d^1 \\
&6 + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^4 + 6*a^6*c^2*d^8*e^8))) + (81*c^7* \\
&d^9*e^6 - 254*a*c^6*d^5*e^{10} + 625*a^2*c^5*d*e^{14})/(256*(a^8*e^{16} + a^4*c^4 \\
&*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6 \\
&25*a^2*c^5*e^{15} + 81*c^7*d^8*e^7 - 894*a*c^6*d^4*e^{11})/(256*(a^8*e^{16} + a^ \\
&4*c^4*d^{16} + 4*a^7*c*d^4*e^{12} + 4*a^5*c^3*d^{12}*e^4 + 6*a^6*c^2*d^8*e^8)))r \\
&oot(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d \\
&^12*z^4 + 65536*a^{10}*e^{12}*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2 \\
&*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d \\
&*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k), k, 1, 4) + ((x^4*(3*c^2*d^4*e^3 - \\
&5*a*c*e^7))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (a*e^7 - c*d^4*e^3 \\
&))/(a*e^4 + c*d^4)^2 + (c*d^3*x)/(4*a*(a*e^4 + c*d^4)) - (c*d^2*e*x^2)/(4*a* \\
&(a*e^4 + c*d^4)) + (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)))/(a*d + a*e*x + c*d* \\
&x^4 + c*e*x^5) + (8*c*d^3*e^7*log(d + e*x))/(a^3*e^{12} + c^3*d^{12} + 3*a*c^2* \\
&d^8*e^4 + 3*a^2*c*d^4*e^8)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)

[Out] Timed out

$$3.403 \quad \int \frac{1}{(d+ex)^3(a+cx^4)^2} dx$$

Optimal. Leaf size=1384

$$\frac{12cd^2(3cd^4 - ae^4)\log(d+ex)e^7}{(cd^4 + ae^4)^4} - \frac{3cd^2(3cd^4 - ae^4)\log(cx^4 + a)e^7}{(cd^4 + ae^4)^4} - \frac{8cd^3e^7}{(cd^4 + ae^4)^3(d+ex)} - \frac{e^7}{2(cd^4 + ae^4)^2(d+ex)}$$

Rubi [A] time = 1.96, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)^2), x]

[Out] $-e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (\text{Sqrt}[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)^4) - (\text{Sqrt}[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/ (2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*c*d^4 - a*e^4)*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/ (16*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/ (4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2$

$$2*(3*c*d^4 - 5*a*e^4)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2]/(16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^3 - (c^{(3/4)}*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8)) * \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*\text{Log}[a + c*x^4])/(c*d^4 + a*e^4)^4$$
Rule 204

$$\text{Int}[\frac{(a_ + (b_)*(x_)^2)^{-1}}{x_Symbol}] :> -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}]/\frac{\text{Rt}[-a, 2]*\text{Rt}[-b, 2]}{x}], x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$$
Rule 205

$$\text{Int}[\frac{(a_ + (b_)*(x_)^2)^{-1}}{x_Symbol}] :> \text{Simp}[\frac{\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]]}{a}, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b]$$
Rule 260

$$\text{Int}[\frac{(x_)^{(m_)}}{(a_ + (b_)*(x_)^{(n_)})}, x_Symbol] :> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \&\& \text{EqQ}[m, n - 1]$$
Rule 275

$$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)})], x_Symbol] :> \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)}*(a + b*x^{(n/k)})^p], x], x, x^k], x] /; k \neq 1 /; \text{FreeQ}[\{a, b, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[m]$$
Rule 617

$$\text{Int}[\frac{(a_ + (b_)*(x_) + (c_)*(x_)^2)^{-1}}{x_Symbol}] :> \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2)], x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$$
Rule 628

$$\text{Int}[\frac{(d_ + (e_)*(x_))}{(a_ + (b_)*(x_) + (c_)*(x_)^2)}, x_Symbol] :> \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$$
Rule 635

$$\text{Int}[\frac{(d_ + (e_)*(x_))}{(a_ + (c_)*(x_)^2)}, x_Symbol] :> \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x]$$

}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,

0] && Expon[Pq, x] < n

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)^3 (a+cx^4)^2} dx &= \int \left(\frac{e^8}{(cd^4+ae^4)^2 (d+ex)^3} + \frac{8cd^3e^8}{(cd^4+ae^4)^3 (d+ex)^2} + \frac{12cd^2e^8(3cd^4-ae^4)}{(cd^4+ae^4)^4 (d+ex)} + \frac{c(d+ex)}{(cd^4+ae^4)^4} \right) dx \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{12cd^2e^7(3cd^4-ae^4) \log(d+ex)}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4} \\
&= -\frac{e^7}{2(cd^4+ae^4)^2 (d+ex)^2} - \frac{8cd^3e^7}{(cd^4+ae^4)^3 (d+ex)} + \frac{c(2ad^2e^3(5cd^4-3ae^4) + x(d+ex))}{(cd^4+ae^4)^4}
\end{aligned}$$

Mathematica [A] time = 1.36, size = 996, normalized size = 0.72

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2), x]

[Out]
$$\frac{(-16e^7(c^4d + ae^4)^2)/(d + ex)^2 - (256c^3d^3e^7(c^4d + ae^4))/(d + ex) + (8c(c^4d + ae^4)*(-a^2e^7(6d^2 - 3d*ex + e^2x^2)) + c^2d^7*x*(d^2 - 3d*ex + 6e^2x^2) + 2a*c*d^3e^3(5d^3 - 6d^2*ex + 6d*ex^2 - 5e^3x^3)))/(a*(a + cx^4)) - (6\sqrt{c}*(\sqrt{2}*c^{13/4}*d^{13} - 4a^{1/4}*c^3*d^{12}*e + 2\sqrt{2}*\sqrt{a}*c^{11/4}*d^{11}*e^2 + 9\sqrt{2}*\sqrt{a}*c^{9/4}*d^9*e^4 - 44a^{5/4}*c^2*d^8*e^5 + 36\sqrt{2}*\sqrt{a}^{3/2}*c^{7/4}*d^7*e^6 - 49\sqrt{2}*\sqrt{a}^2*c^{5/4}*d^5*e^8 + 84a^{9/4}*c*d^4*e^9 - 30\sqrt{2}*\sqrt{a}^{5/2}*c^{3/4}*d^3*e^{10} + 7\sqrt{2}*\sqrt{a}^3*c^{1/4}*d*e^{12} - 4a^{13/4}*e^{13})*\text{ArcTan}[1 - (\sqrt{2}*c^{1/4}*x)/a^{1/4}])/a^{7/4} + (6\sqrt{c}*(\sqrt{2}*c^{13/4}*d^{13} + 4a^{1/4}*c^3*d^{12}*e + 2\sqrt{2}*\sqrt{a}*c^{11/4}*d^{11}*e^2 + 9\sqrt{2}*\sqrt{a}*c^{9/4}*d^9*e^4 + 44a^{5/4}*c^2*d^8*e^5 + 36\sqrt{2}*\sqrt{a}^{3/2}*c^{7/4}*d^7*e^6 - 49\sqrt{2}*\sqrt{a}^2*c^{5/4}*d^5*e^8 - 84a^{9/4}*c*d^4*e^9 - 30\sqrt{2}*\sqrt{a}^{5/2}*c^{3/4}*d^3*e^{10} + 7\sqrt{2}*\sqrt{a}^3*c^{1/4}*d*e^{12} + 4a^{13/4}*e^{13})*\text{ArcTan}[1 + (\sqrt{2}*c^{1/4}*x)/a^{1/4}])/a^{7/4} + 384*c*d^2*e^7*(3*c*d^4 - a*e^4)*\text{Log}[d + ex] - (3*\sqrt{2}*c^{3/4}*(c^3*d^{13} - 2*\sqrt{a}*c^{5/2}*d^{11}*e^2 + 9*a*c^2*d^9*e^4 - 36*a^{3/2}*c^{3/2}*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^{5/2}*\sqrt{c}*d^3*e^{10} + 7*a^3*d*e^{12})*\text{Log}[\sqrt{a} - \sqrt{2}*\sqrt{a}^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/a^{7/4} + (3*\sqrt{2}*c^{3/4}*(c^3*d^{13} - 2*\sqrt{a}*c^{5/2}*d^{11}*e^2 + 9*a*c^2*d^9*e^4 - 36*a^{3/2}*c^{3/2}*d^7*e^6 - 49*a^2*c*d^5*e^8 + 30*a^{5/2}*\sqrt{c}*d^3*e^{10} + 7*a^3*d*e^{12})*\text{Log}[\sqrt{a} + \sqrt{2}*\sqrt{a}^{1/4}*c^{1/4}*x + \sqrt{c}*x^2])/a^{7/4} - 96*c*d^2*e^7*(3*c*d^4 - a*e^4)*\text{Log}[a + cx^4)]/(32*(c^4d + ae^4)^4)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + cx^4)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^2), x]

[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^2), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.15, size = 1488, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")

[Out]
$$\frac{3}{8} \sqrt{2} \sqrt{ac} c^3 d^4 e + (ac^3)^{1/4} c^3 d^5 + 4 \sqrt{2} a^3 c^3 d^2 e^3 + 2 (ac^3)^{3/4} c^3 d^3 e^2 - 9 (ac^3)^{1/4} a^2 c^2 d e^4 + 2 \sqrt{2} \sqrt{ac} a^2 c^2 e^5 \arctan\left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2} (a/c)^{1/4}) / (a/c)^{1/4}\right) / (\sqrt{2} a^2 c^4 d^8 - 8 (ac^3)^{1/4} a^2 c^3 d^7 e + 16 \sqrt{2} \sqrt{ac} a^2 c^3 d^6 e^2 + 34 \sqrt{2} a^3 c^3 d^4 e^4 - 40 (ac^3)^{3/4} a^2 c^3 d^5 e^3 - 40 (ac^3)^{1/4} a^3 c^2 d^3 e^5 + 16 \sqrt{2} \sqrt{ac} a^3 c^2 d^2 e^6 + \sqrt{2} a^4 c^2 e^8 - 8 (ac^3)^{3/4} a^3 d e^7) + 3/8 (2 \sqrt{2} \sqrt{ac} c^3 d^4 e + (ac^3)^{1/4} c^3 d^5 - 4 \sqrt{2} a^3 c^3 d^2 e^3 + 2 (ac^3)^{3/4} c^3 d^3 e^2 - 9 (ac^3)^{1/4} a^2 c^2 d e^4 + 2 \sqrt{2} \sqrt{ac} a^2 c^2 e^5) \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2} (a/c)^{1/4}) / (a/c)^{1/4}\right) / (\sqrt{2} a^2 c^4 d^8 + 8 (ac^3)^{1/4} a^2 c^3 d^7 e + 16 \sqrt{2} \sqrt{ac} a^2 c^3 d^6 e^2 + 34 \sqrt{2} a^3 c^3 d^4 e^4 + 40 (ac^3)^{3/4} a^2 c^3 d^5 e^3 + 40 (ac^3)^{1/4} a^3 c^2 d^3 e^5 + 16 \sqrt{2} \sqrt{ac} a^3 c^2 d^2 e^6 + \sqrt{2} a^4 c^2 e^8 + 8 (ac^3)^{3/4} a^3 d e^7) + 3/32 (\sqrt{2} (ac^3)^{1/4} c^4 d^{13} - 2 \sqrt{2} (ac^3)^{3/4} c^2 d^{11} e^2 + 9 \sqrt{2} (ac^3)^{1/4} a^2 c^3 d^9 e^4 - 36 \sqrt{2} (ac^3)^{3/4} a^2 c^3 d^7 e^6 - 49 \sqrt{2} (ac^3)^{1/4} a^2 c^2 d^5 e^8 + 30 \sqrt{2} (ac^3)^{3/4} a^2 d^3 e^{10} + 7 \sqrt{2} (ac^3)^{1/4} a^3 c^3 d e^{12}) \log(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{2} (a/c)) / (a^2 c^5 d^{16} + 4 a^3 c^4 d^{12} e^4 + 6 a^4 c^3 d^8 e^8 + 4 a^5 c^2 d^4 e^{12} + a^6 c e^{16}) - 3/32 (\sqrt{2} (ac^3)^{1/4} c^4 d^{13} - 2 \sqrt{2} (ac^3)^{3/4} c^2 d^{11} e^2 + 9 \sqrt{2} (ac^3)^{1/4} a^2 c^3 d^9 e^4 - 36 \sqrt{2} (ac^3)^{3/4} a^2 c^3 d^7 e^6 - 49 \sqrt{2} (ac^3)^{1/4} a^2 c^2 d^5 e^8 + 30 \sqrt{2} (ac^3)^{3/4} a^2 d^3 e^{10} + 7 \sqrt{2} (ac^3)^{1/4} a^3 c^3 d e^{12}) \log(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{2} (a/c)) / (a^2 c^5 d^{16} + 4 a^3 c^4 d^{12} e^4 + 6 a^4 c^3 d^8 e^8 + 4 a^5 c^2 d^4 e^{12} + a^6 c e^{16}) - 3 (3 c^2 d^6 e^7 - a^2 c^2 d^8 e^8 + 4 a^3 c^2 d^4 e^{12} + a^4 e^{16}) + 12 (3 c^2 d^6 e^8 - a^2 c^2 d^8 e^8 + 4 a^3 c^2 d^4 e^{12} + a^4 e^{16}) + 12 (3 c^2 d^6 e^8 - a^2 c^2 d^8 e^8 + 4 a^3 c^2 d^4 e^{12} + a^4 e^{16}) + 1/4 (10 a^3 c^3 d^{12} e^3 - 30 a^2 c^2 d^8 e^7 - 42 a^3 c^3 d^4 e^{11} + 6 (c^4 d^{11} e^4 - 6 a^2 c^3 d^7 e^8 - 7 a^2 c^2 d^3 e^{12}) x^5 + 3 (3 c^4 d^{12} e^3 - 11 a^2 c^3 d^8 e^7 - 15 a^2 c^2 d^4 e^{11} - a^3 c^2 e^{15}) x^4 - 2 a^4 e^{15} + (c^4 d^{13} e^2 + 3 a^2 c^3 d^9 e^6 + 3 a^2 c^2 d^5 e^{10} + a^3 c^3 d e^{14}) x^3 - (c^4 d^{14} e + 3 a^2 c^3 d^{10} e^5 + 3 a^2 c^2 d^6 e^9 + a^3 c^3 d^2 e^{13}) x^2 + (c^4 d^{15} + 9 a^2 c^3 d^{11} e^4 - 33 a^2 c^2 d^7 e^8 - 41 a^3 c^3 d^3 e^{12}) x) / ((c^4 d^4 + a e^4)^4 (c^4 x^4 + a) (x e + d)^2 a)$$

maple [A] time = 0.03, size = 2121, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(e*x+d)^3/(c*x^4+a)^2, x)$

[Out]
$$\begin{aligned} & -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)-12*e \\ & ^{11}*d^2*c/(a*e^4+c*d^4)^4*\ln(e*x+d)*a-1/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^{13}* \\ & a^2*x^2-3/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^{11}*d^2*a^2-c^3/(a*e^4+c*d^4)^4/(c \\ & *x^4+a)*d^7*e^6*x^3+9/4*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*e^5*x^2*d^8-11/4*c^3/ \\ & (a*e^4+c*d^4)^4/(c*x^4+a)*d^9*x*e^4+1/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^{13}/ \\ & a*x+c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*a*d^6*e^7-33/4*c^3/(a*e^4+c*d^4)^4/(a*c)^ \\ & (1/2)*\arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^8-3/4*c/(a*e^4+c*d^4)^4*a^2/(a*c)^{(1/ \\ & 2)}*\arctan((1/a*c)^{(1/2)}*x^2)*e^{13}+3*c/(a*e^4+c*d^4)^4*a*\ln(c*x^4+a)*e^{11}*d^ \\ & 2+36*e^7*d^6*c^2/(a*e^4+c*d^4)^4*\ln(e*x+d)+5/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a \\ &)*d^{10}*e^3-9*c^2/(a*e^4+c*d^4)^4*\ln(c*x^4+a)*e^7*d^6+27/16*c^3/(a*e^4+c*d^4 \\ &)^4/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^9*e^4+21/32*c/(\\ & a*e^4+c*d^4)^4*a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1 \\ & /2)))/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d*e^{12}+21/16*c/(a*e^4+c*d^4)^ \\ & 4*a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d*e^{12}-45/16*c/(a*e \\ & ^4+c*d^4)^4*a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2) \\ &))/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^3*e^{10}-45/8*c/(a*e^4+c*d^4)^4* \\ & a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^3*e^{10}-45/8*c/(a*e^ \\ & 4+c*d^4)^4*a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^3*e^{10}-1 \\ & 47/16*c^2/(a*e^4+c*d^4)^4*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+ \\ & 1)*d^5*e^8+27/8*c^2/(a*e^4+c*d^4)^4/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4) \\ & }*2^{(1/2)}*x+(a/c)^{(1/2)))/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^7*e^6+27 \\ & /4*c^2/(a*e^4+c*d^4)^4/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)* \\ & d^7*e^6+27/4*c^2/(a*e^4+c*d^4)^4/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(\\ & 1/4)}*x+1)*d^7*e^6+63/4*c^2/(a*e^4+c*d^4)^4*a/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/ \\ & 2)}*x^2)*e^9*d^4+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(\\ & 1/2)}/(a/c)^{(1/4)}*x-1)*d^{13}+3/32*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2) \\ & }*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)))/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c \\ &)^{(1/2)}))*d^{13}+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1 \\ & /2)}/(a/c)^{(1/4)}*x+1)*d^{13}-3/4*c^4/(a*e^4+c*d^4)^4/a/(a*c)^{(1/2)}*\arctan((1/a \\ & *c)^{(1/2)}*x^2)*e*d^{12}-147/16*c^2/(a*e^4+c*d^4)^4*(a/c)^{(1/4)}*2^{(1/2)}*\arctan \\ & (2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^5*e^8-147/32*c^2/(a*e^4+c*d^4)^4*(a/c)^{(1/4)}*2^{ \\ & (1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)))/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x \\ & +(a/c)^{(1/2)}))*d^5*e^8-5/2*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*d^3*e^{10}*a*x^3+3/2 \\ & *c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^{11}*e^2/a*x^3+11/4*c^2/(a*e^4+c*d^4)^4/(c*x \\ & ^4+a)*e^9*a*x^2*d^4-3/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*e/a*x^2*d^{12}-9/4*c^2/ \\ & (a*e^4+c*d^4)^4/(c*x^4+a)*d^5*a*x*e^8+3/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*d*a^2 \\ & *x*e^{12}+3/16*c^3/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}* \\ & 2^{(1/2)}*x+(a/c)^{(1/2)))/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^{11}*e^2+3/ \\ & 8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1) \\ & *d^{11}*e^2+3/8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c \\ &)^{(1/4)}*x+1)*d^{11}*e^2+27/16*c^3/(a*e^4+c*d^4)^4/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan \end{aligned}$$

$$\frac{n(2^{1/2}/(a/c)^{1/4} * x - 1) * d^9 * e^4 + 27/32 * c^3 / (a * e^4 + c * d^4)^4 / a * (a/c)^{1/4} * 2^{1/2} * \ln((x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/2})) * d^9 * e^4 + 21/16 * c / (a * e^4 + c * d^4)^4 * a * (a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x - 1) * d * e^{12}}$$

maxima [A] time = 2.60, size = 1394, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -3/32 * c * (\sqrt{2}) * (48 * \sqrt{2}) * a^{7/4} * c^{9/4} * d^6 * e^7 - 16 * \sqrt{2} * a^{11/4} * c^{5/4} * d^2 * e^{11} - c^4 * d^{13} + 2 * \sqrt{a} * c^{7/2} * d^{11} * e^2 - 9 * a * c^3 * d^9 * e^4 \\ & + 36 * a^{3/2} * c^{5/2} * d^7 * e^6 + 49 * a^2 * c^2 * d^5 * e^8 - 30 * a^{5/2} * c^{3/2} * d^3 * e^{10} - 7 * a^3 * c * d * e^{12} * \log(\sqrt{c} * x^2 + \sqrt{2}) * a^{1/4} * c^{1/4} * x + \sqrt{a}) / (a^{3/4} * c^{5/4}) \\ & + \sqrt{2} * (48 * \sqrt{2}) * a^{7/4} * c^{9/4} * d^6 * e^7 - 16 * \sqrt{2} * a^{11/4} * c^{5/4} * d^2 * e^{11} + c^4 * d^{13} - 2 * \sqrt{a} * c^{7/2} * d^{11} * e^2 + 9 * a * c^3 * d^9 * e^4 \\ & - 36 * a^{3/2} * c^{5/2} * d^7 * e^6 - 49 * a^2 * c^2 * d^5 * e^8 + 30 * a^{5/2} * c^{3/2} * d^3 * e^{10} + 7 * a^3 * c * d * e^{12} * \log(\sqrt{c} * x^2 - \sqrt{2}) * a^{1/4} * c^{1/4} * x \\ & + \sqrt{a}) / (a^{3/4} * c^{5/4}) - 2 * (\sqrt{2}) * a^{1/4} * c^{17/4} * d^{13} + 2 * \sqrt{2} * a^{3/4} * c^{15/4} * d^{11} * e^2 + 9 * \sqrt{2} * a^{5/4} * c^{13/4} * d^9 * e^4 + 36 * \sqrt{2} * a^{7/4} * c^{11/4} * d^7 * e^6 \\ & - 49 * \sqrt{2} * a^{9/4} * c^{9/4} * d^5 * e^8 - 30 * \sqrt{2} * a^{11/4} * c^{7/4} * d^3 * e^{10} + 7 * \sqrt{2} * a^{13/4} * c^{5/4} * d * e^{12} + 4 * \sqrt{a} * c^4 * d^{12} * e \\ & + 44 * a^{3/2} * c^3 * d^8 * e^5 - 84 * a^{5/2} * c^2 * d^4 * e^9 + 4 * a^{7/2} * c * e^{13} * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c}) * x + \sqrt{2}) * a^{1/4} * c^{1/4}) / \sqrt{a * c} \\ & / (a^{3/4} * \sqrt{a * c}) * c^{5/4}) - 2 * (\sqrt{2}) * a^{1/4} * c^{17/4} * d^{13} + 2 * \sqrt{2} * a^{3/4} * c^{15/4} * d^{11} * e^2 + 9 * \sqrt{2} * a^{5/4} * c^{13/4} * d^9 * e^4 + 36 * \sqrt{2} * a^{7/4} * c^{11/4} * d^7 * e^6 \\ & - 49 * \sqrt{2} * a^{9/4} * c^{9/4} * d^5 * e^8 - 30 * \sqrt{2} * a^{11/4} * c^{7/4} * d^3 * e^{10} + 7 * \sqrt{2} * a^{13/4} * c^{5/4} * d * e^{12} - 4 * \sqrt{a} * c^4 * d^{12} * e \\ & - 44 * a^{3/2} * c^3 * d^8 * e^5 + 84 * a^{5/2} * c^2 * d^4 * e^9 - 4 * a^{7/2} * c * e^{13} * \arctan(1/2 * \sqrt{2}) * (2 * \sqrt{c}) * x - \sqrt{2}) * a^{1/4} * c^{1/4}) / \sqrt{a * c} \\ & / (a^{3/4} * \sqrt{a * c}) * c^{5/4})) / (a * c^4 * d^{16} + 4 * a^2 * c^3 * d^{12} * e^4 + 6 * a^3 * c^2 * d^8 * e^8 + 4 * a^4 * c * d^4 * e^{12} + a^5 * e^{16}) \\ & + 12 * (3 * c^2 * d^6 * e^7 - a * c * d^2 * e^{11}) * \log(e * x + d) / (c^4 * d^{16} + 4 * a * c^3 * d^{12} * e^4 + 6 * a^2 * c^2 * d^8 * e^8 + 4 * a^3 * c * d^4 * e^{12} + a^4 * e^{16}) \\ & + 1/4 * (10 * a * c^2 * d^8 * e^3 - 40 * a^2 * c * d^4 * e^7 - 2 * a^3 * e^{11} + 6 * (c^3 * d^7 * e^4 - 7 * a * c^2 * d^3 * e^8) * x^5 \\ & + 3 * (3 * c^3 * d^8 * e^3 - 14 * a * c^2 * d^4 * e^7 - a^2 * c * e^{11}) * x^4 + (c^3 * d^9 * e^2 + 2 * a * c^2 * d^5 * e^6 + a^2 * c * d * e^{10}) * x^3 \\ & - (c^3 * d^{10} * e + 2 * a * c^2 * d^6 * e^5 + a^2 * c * d^2 * e^9) * x^2 + (c^3 * d^{11} + 8 * a * c^2 * d^7 * e^4 - 41 * a^2 * c * d^3 * e^8) * x) / (a^2 * c^3 * d^{14} \\ & + 3 * a^3 * c^2 * d^{10} * e^4 + 3 * a^4 * c * d^6 * e^8 + a^5 * d^2 * e^{12} + (a * c^4 * d^{12} * e^2 + 3 * a^2 * c^3 * d^8 * e^6 + 3 * a^3 * c^2 * d^4 * e^{10} + a^4 * c * e^{14}) * x^6 \\ & + 2 * (a * c^4 * d^{13} * e + 3 * a^2 * c^3 * d^9 * e^5 + 3 * a^3 * c^2 * d^5 * e^9 + a^4 * c * d * e^{13}) * x^5 + (a * c^4 * d^{14} + 3 * a^2 * c^3 * d^{10} * e^4 \\ & + 3 * a^3 * c^2 * d^6 * e^8 + a^4 * c * d^2 * e^{12}) * x^4 + (a^2 * c^3 * d^{12} * e^2 + 3 * a^3 * c^2 * d^8 * e^6 + 3 * a^4 * c * d^4 * e^{10} + a^5 * e^{14}) * x^2 \\ & + 2 * (a^2 * c^3 * d^{13} * e + 3 * a^3 * c^2 * d^9 * e^5 + 3 * a^4 * c * d^5 * e^9 + a^5 * d * e^{13}) * x \end{aligned}$$

*e¹³)*x)

mupad [B] time = 5.04, size = 3256, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((a + c*x^4)^2*(d + e*x)^3), x)$

[Out] $\text{symsum}(\log(\text{root}(262144*a^{10}*c*d^4*e^{12}*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^{12}*e^4*z^4 + 65536*a^7*c^4*d^{16}*z^4 + 65536*a^{11}*e^{16}*z^4 - 786432*a^8*c*d^2*e^{11}*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^{10}*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((108*a*c^{10}*d^{19}*e^3 + 3888*a^2*c^9*d^{15}*e^7 - 99576*a^3*c^8*d^{11}*e^{11} + 591408*a^4*c^7*d^7*e^{15} - 79380*a^5*c^6*d^3*e^{19})/(256*(a^{10}*e^{24} + a^4*c^6*d^{24} + 6*a^9*c*d^4*e^{20} + 6*a^5*c^5*d^{20}*e^4 + 15*a^6*c^4*d^{16}*e^8 + 20*a^7*c^3*d^{12}*e^{12} + 15*a^8*c^2*d^8*e^{16})) + \text{root}(262144*a^{10}*c*d^4*e^{12}*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^{12}*e^4*z^4 + 65536*a^7*c^4*d^{16}*z^4 + 65536*a^{11}*e^{16}*z^4 - 786432*a^8*c*d^2*e^{11}*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^{10}*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^{24} + 4608*a^3*c^{10}*d^{21}*e^4 + 154368*a^4*c^9*d^{17}*e^8 - 331776*a^5*c^8*d^{13}*e^{12} + 5976576*a^6*c^7*d^9*e^{16} - 612864*a^7*c^6*d^5*e^{20})/(256*(a^{10}*e^{24} + a^4*c^6*d^{24} + 6*a^9*c*d^4*e^{20} + 6*a^5*c^5*d^{20}*e^4 + 15*a^6*c^4*d^{16}*e^8 + 20*a^7*c^3*d^{12}*e^{12} + 15*a^8*c^2*d^8*e^{16})) + \text{root}(262144*a^{10}*c*d^4*e^{12}*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^{12}*e^4*z^4 + 65536*a^7*c^4*d^{16}*z^4 + 65536*a^{11}*e^{16}*z^4 - 786432*a^8*c*d^2*e^{11}*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^{10}*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((18432*a^5*c^{10}*d^{23}*e^5 - 3072*a^4*c^{11}*d^{27}*e + 1170432*a^6*c^9*d^{19}*e^9 + 2863104*a^7*c^8*d^{15}*e^{13} + 1797120*a^8*c^7*d^{11}*e^{17} - 423936*a^9*c^6*d^7*e^{21} - 506880*a^{10}*c^5*d^3*e^{25})/(256*(a^{10}*e^{24} + a^4*c^6*d^{24} + 6*a^9*c*d^4*e^{20} + 6*a^5*c^5*d^{20}*e^4 + 15*a^6*c^4*d^{16}*e^8 + 20*a^7*c^3*d^{12}*e^{12} + 15*a^8*c^2*d^8*e^{16})) + \text{root}(262144*a^{10}*c*d^4*e^{12}*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^{12}*e^4*z^4 + 65536*a^7*c^4*d^{16}*z^4 + 65536*a^{11}*e^{16}*z^4 - 786432*a^8*c*d^2*e^{11}*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^{10}*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((98304*a^{13}*c^4*d*e^{30} - 32768*a^6*c^{11}*d^{29}*e^2 - 98304*a^7*c^{10}*d^{25}*e^6 + 98304*a^8*c^9*d^{21}*e^{10} + 819200*a^9*c^8*d^{17}*e^{14} + 1474560*a^{10}*c^7*d^{13}*e^{18} + 1277952*a^{11}*c^6*d^9*e^{22} + 557056*a^{12}*c^5*d^5*e^{26})/(256*(a^{10}*e^{24} + a^4*c^6*d^{24} + 6*a^9*c*d^4*e^{20} + 6*a^5*c^5*d^{20}*e^4 + 15*a^6*c^4*d^{16}*e^8 + 20*a^7*c^3*d^{12}*e^{12} + 15*a^8*c^2*d^8*e^{16}))$

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)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^
24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*
c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*
(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*
c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^
4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 403
0464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^2
2 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^
4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 1
5*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*
a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4
093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c
^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*
a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 2505
6*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 8661
6*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*
a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*
d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18
+ 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20
+ 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*
c^2*d^8*e^16)) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e
^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)))*root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e
^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*
e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712
*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 5
8752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4
, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4
+ c*d^4))*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
^2*d^3*e^8))/(2*a*(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
) + (3*x^4*(a^2*c*e^11 - 3*c^3*d^8*e^3 + 14*a*c^2*d^4*e^7))/(4*a*(a^3*e^12
+ c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (c*d^2*e*x^2)/(4*a*(a*e^
4 + c*d^4)) - (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)) - (d*x*(c^3*d^10 + 8*a*c^
2*d^6*e^4 - 41*a^2*c*d^2*e^8))/(4*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4)))/(a*d^2 + a*e^2*x^2 + c*d^2*x^4 + c*e^2*x^6 + 2*a*d*e*x + 2*c
*d*e*x^5) + (log(d + e*x)*(36*c^2*d^6*e^7 - 12*a*c*d^2*e^11))/(a^4*e^16 + c
^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)

[Out] Timed out

$$3.404 \quad \int \frac{(d+ex)^3}{(a+cx^4)^3} dx$$

Optimal. Leaf size=394

$$\frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

Rubi [A] time = 0.35, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} - \frac{3d(5\sqrt{a}e^2 + 7\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{3d(5\sqrt{a}e^2 + 7\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{a}} + 1\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{x(18d^2cx + 7d^3 + 15d^2x^2)}{32a^2(a + cx^4)} + \frac{9d^2c \tan^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{ae^3 - cx(3d^2cx + d^3 + 3d^2x^2)}{8ac(a + cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]

[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x

$^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 617

$\text{Int}[(a_ + (b_ \cdot x_) + (c_ \cdot x_)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c]) /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d_ + (e_ \cdot x_))/(a_ + (b_ \cdot x_) + (c_ \cdot x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 1162

$\text{Int}[(d_ + (e_ \cdot x_)^2)/(a_ + (c_ \cdot x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

Rule 1165

$\text{Int}[(d_ + (e_ \cdot x_)^2)/(a_ + (c_ \cdot x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

Rule 1168

$\text{Int}[(d_ + (e_ \cdot x_)^2)/(a_ + (c_ \cdot x_)^4), x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a \cdot c, 2]\}, \text{Dist}[(d \cdot q + a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q + c \cdot x^2)/(a + c \cdot x^4), x], x] + \text{Dist}[(d \cdot q - a \cdot e)/(2 \cdot a \cdot c), \text{Int}[(q - c \cdot x^2)/(a + c \cdot x^4), x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[-(a \cdot c)]$

Rule 1854

$\text{Int}[(Pq) \cdot ((a_ + (b_ \cdot x_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Module}[\{q = \text{Expon}[Pq, x], i\}, \text{Simp}[(a \cdot \text{Coeff}[Pq, x, q] - b \cdot x \cdot \text{ExpandToSum}[Pq - \text{Coeff}[Pq, x, q] \cdot x^q, x]) \cdot (a + b \cdot x^n)^{(p + 1)})/(a \cdot b \cdot n \cdot (p + 1)), x] + \text{Dist}[1/(a \cdot n \cdot (p + 1)), \text{Int}[\text{Sum}[(n \cdot (p + 1) + i + 1) \cdot \text{Coeff}[Pq, x, i] \cdot x^i, \{i, 0, q - 1\}] \cdot (a + b \cdot x^n)^{(p + 1)}, x], x] /; q == n - 1] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[n,$

0] && LtQ[p, -1]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\int \frac{(d + ex)^3}{(a + cx^4)^3} dx = -\frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} - \frac{\int \frac{-7d^3 - 18d^2ex - 15de^2x^2}{(a + cx^4)^2} dx}{8a}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{\int \frac{21d^3 + 36d^2ex + 15de^2x^2}{a + cx^4} dx}{32a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{\int \left(\frac{36d^2ex}{a + cx^4} + \frac{21d^3 + 15de^2x^2}{a + cx^4}\right) dx}{32a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{\int \frac{21d^3 + 15de^2x^2}{a + cx^4} dx}{32a^2} + \frac{(9d^2e)}{8a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{(9d^2e) \text{Subst}\left(\int \frac{1}{a + cx^2} dx, x, \sqrt{cx^2}\right)}{16a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{(3d(7d^2e + 3de^2))}{8a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d(7d^2e + 3de^2)}{8a^2}$$

$$= \frac{x(7d^3 + 18d^2ex + 15de^2x^2)}{32a^2(a + cx^4)} - \frac{ae^3 - cx(d^3 + 3d^2ex + 3de^2x^2)}{8ac(a + cx^4)^2} + \frac{9d^2e \tan^{-1}\left(\frac{\sqrt{cx^2}}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d(7d^2e + 3de^2)}{8a^2}$$

Mathematica [A] time = 0.35, size = 388, normalized size = 0.98

$$\frac{3\sqrt{2}(5a^3d^2e^2 - 7\sqrt{a} \sqrt{c} d^3) \log(-\sqrt{2} \sqrt{c} \sqrt{cx + \sqrt{a} + \sqrt{c}x^2}) + 3\sqrt{2}(7\sqrt{c} \sqrt{c} d^3 - 5a^3d^2e^2) \log(\sqrt{2} \sqrt{c} \sqrt{cx + \sqrt{a} + \sqrt{c}x^2}) - 32a^2(ac^3 - cd^4(d^3 + 3d^2ex + 3de^2x^2))}{c^3(a + cx^4)^2} - \frac{6\sqrt{a}d(24\sqrt{c} \sqrt{c} de + 5\sqrt{2} \sqrt{a}e^2 + 7\sqrt{2} \sqrt{c} d^3) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt{cx}}{\sqrt{a}}\right) + 6\sqrt{a}d(-24\sqrt{c} \sqrt{c} de + 5\sqrt{2} \sqrt{a}e^2 + 7\sqrt{2} \sqrt{c} d^3) \tan^{-1}\left(\frac{\sqrt{2} \sqrt{cx}}{\sqrt{a}} + 1\right) + 8ad(7d^2e + 18d^2ex + 15d^2e^2x^2)}{c^3(a + cx^4)^2} + \frac{8ad(7d^2e + 18d^2ex + 15d^2e^2x^2)}{a + cx^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]

[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*sqrt

[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^3}{(a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^3, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 389, normalized size = 0.99

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{a}d^2e+7(a^2)^{\frac{1}{4}}c^{\frac{1}{4}}d^2e+5(a^2)^{\frac{1}{4}}d^2e\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}d^2e}{2(a^2)^{\frac{1}{4}}}\right)+3\sqrt{2}\left(12\sqrt{2}\sqrt{a}d^2e+7(a^2)^{\frac{1}{4}}c^{\frac{1}{4}}d^2e+5(a^2)^{\frac{1}{4}}d^2e\right)\arctan\left(\frac{\sqrt{2}\sqrt{a}d^2e}{2(a^2)^{\frac{1}{4}}}\right)+3\sqrt{2}\left(7(a^2)^{\frac{1}{4}}c^{\frac{1}{4}}d^2e-5(a^2)^{\frac{1}{4}}d^2e\right)\log\left(x^2+\sqrt{2}\sqrt{a}d^2e+\sqrt{a}\right)+3\sqrt{2}\left(7(a^2)^{\frac{1}{4}}c^{\frac{1}{4}}d^2e-5(a^2)^{\frac{1}{4}}d^2e\right)\log\left(x^2-\sqrt{2}\sqrt{a}d^2e+\sqrt{a}\right)+15c^2d^2e^2+18c^2d^2e+7c^2d^2e+27acd^2e^2+30acd^2e+11ad^2e-4d^2e}{32(c^4+a)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")

[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)

maple [A] time = 0.01, size = 470, normalized size = 1.19

$$\frac{e^{3x}d^3}{8(c^4x^4+a)^2} + \frac{3d^2x^2}{8(c^4x^4+a)^2} + \frac{e^{3x}d^2}{8(c^4x^4+a)^2} + \frac{3d^2x^2}{8(c^4x^4+a)^2} + \frac{15d^2x^2}{32(c^4x^4+a)^2} + \frac{d^2x}{8(c^4x^4+a)^2} + \frac{9d^2x^2}{16(c^4x^4+a)^2} + \frac{7d^2x}{32(c^4x^4+a)^2} + \frac{9d^2x \arctan\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}a^2} + \frac{15\sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\sqrt{a}}\right)}{128\left(\frac{c}{a}\right)^{3/4}} + \frac{15\sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\sqrt{a}}\right)}{128\left(\frac{c}{a}\right)^{3/4}} + \frac{15\sqrt{2}d^2 \ln\left(\frac{\sqrt{c}x + \sqrt{c^2x^2+a}}{\sqrt{c}x - \sqrt{c^2x^2+a}}\right)}{256\left(\frac{c}{a}\right)^{3/4}} + \frac{21\left(\frac{c}{a}\right)^{1/4} \sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\sqrt{a}}\right)}{128a^2} + \frac{21\left(\frac{c}{a}\right)^{1/4} \sqrt{2}d^2 \arctan\left(\frac{\sqrt{2}x}{\sqrt{a}}\right)}{128a^2} + \frac{21\left(\frac{c}{a}\right)^{1/4} \sqrt{2}d^2 \ln\left(\frac{\sqrt{c}x + \sqrt{c^2x^2+a}}{\sqrt{c}x - \sqrt{c^2x^2+a}}\right)}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3/(c*x^4+a)^3,x)

[Out] $\frac{1}{8}d^3x/a/(c^4x^4+a)^2 + \frac{7}{32}d^3/a^2x/(c^4x^4+a) + \frac{21}{256}d^3/a^3(a/c)^{1/4} * 2^{1/2} * \ln((x^2+(a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/4}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4}) + \frac{21}{128}d^3/a^3(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x + 1) + \frac{21}{128}d^3/a^3(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x - 1) + \frac{3}{8}e * d^2 * x^2/a/(c^4x^4+a)^2 + \frac{9}{16}e * d^2/a^2 * x^2/(c^4x^4+a) + \frac{9}{16}e * d^2/a^2/(a*c)^{1/2} * \arctan((1/a*c)^{1/2} * x^2) + \frac{3}{8}d * e^2 * x^3/a/(c^4x^4+a)^2 + \frac{15}{32}d * e^2/a^2 * x^3/(c^4x^4+a) + \frac{15}{256}d * e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/4}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4}) + \frac{15}{128}d * e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x + 1) + \frac{15}{128}d * e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x - 1) + \frac{1}{8}e^3 * x^4/a/(c^4x^4+a)^2 + \frac{1}{8}e^3/a^2 * x^4/(c^4x^4+a)$

maxima [A] time = 2.37, size = 392, normalized size = 0.99

$$\frac{15c^2d^2x^7 + 18c^2d^2cx^6 + 7c^2d^2x^5 + 27acd^2x^4 + 30acd^2cx^3 + 11acd^2x^2 - 4d^2x}{32(a^2c^2x^8 + 2a^2c^2x^4 + a^4c)} + \frac{3d \left(\frac{\sqrt{2}(\sqrt{c^2x^2+a} \log(\sqrt{c^2x^2+a} + \sqrt{2a^2x^2+a} + \sqrt{a}))}{a^{3/4}} - \frac{\sqrt{2}(\sqrt{c^2x^2+a} \log(\sqrt{c^2x^2+a} - \sqrt{2a^2x^2+a} + \sqrt{a}))}{a^{3/4}} + \frac{2(\sqrt{2a^2x^2+a} + \sqrt{2a^2x^2-24\sqrt{c}da}) \arctan\left(\frac{\sqrt{c}x + \sqrt{2a^2x^2+a}}{2\sqrt{c}a}\right)}{a^{3/4}\sqrt{c^2x^2+a}} + \frac{2(\sqrt{2a^2x^2+a} - \sqrt{2a^2x^2-24\sqrt{c}da}) \arctan\left(\frac{\sqrt{c}x - \sqrt{2a^2x^2+a}}{2\sqrt{c}a}\right)}{a^{3/4}\sqrt{c^2x^2+a}} \right)}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")

[Out] $\frac{1}{32}(15c^2d^2e^2x^7 + 18c^2d^2e^2cx^6 + 7c^2d^2e^2x^5 + 27a^2c^2d^2e^2x^4 + 30a^2c^2d^2e^2cx^3 - 4a^2e^2x^2)/(a^2c^3x^8 + 2a^3c^2x^4 + a^4c) + \frac{3}{256}d^3(\sqrt{2})(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a}) / (a^{3/4}c^{3/4}) - \sqrt{2}(7\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{a}) / (a^{3/4}c^{3/4}) + \frac{2(7\sqrt{2}a^{1/4}c^{3/4}d^2 + 5\sqrt{2}a^{1/4}c^{3/4}e^2 - 24\sqrt{a}\sqrt{c}d^2e) \arctan(1/2\sqrt{2}(2\sqrt{c}x + \sqrt{2}a^{1/4}c^{1/4})/\sqrt{a}\sqrt{c})}{a^{3/4}\sqrt{a}\sqrt{c}} + \frac{2(7\sqrt{2}a^{1/4}c^{3/4}d^2 + 5\sqrt{2}a^{1/4}c^{3/4}e^2 - 24\sqrt{a}\sqrt{c}d^2e) \arctan(1/2\sqrt{2}(2\sqrt{c}x - \sqrt{2}a^{1/4}c^{1/4})/\sqrt{a}\sqrt{c})}{a^{3/4}\sqrt{a}\sqrt{c}}$

mupad [B] time = 0.48, size = 721, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)^3/(a + c*x^4)^3,x)`

[Out] $((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(16*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + \text{symsum}(\log((3*c*d^2*(6867*c*d^5*e^2 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k)^{2*a^5*c^2*d + 9600*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k)*a^3*c*e^4*x - 18816*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k)*a^2*c^2*d^4*x + 196608*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k)^{2*a^5*c^2*e*x - 46080*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k)*a^3*c*d*e^3)))/(32768*a^6))*\text{root}(268435456*a^{11}*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^{12}, z, k), k, 1, 4)$

sympy [A] time = 8.01, size = 413, normalized size = 1.05

RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (26214400*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 58012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*a**2*c**3*x**8)

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/(c*x**4+a)**3,x)`

[Out] $\text{RootSum}(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, \text{Lambda}(_t, _t*\log(x + (26214400*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 58012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*a**2*c**3*x**8)$

$$3.405 \quad \int \frac{(d+ex)^2}{(a+cx^4)^3} dx$$

Optimal. Leaf size=360

$$\frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}}$$

Rubi [A] time = 0.33, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2 - 5\sqrt{a}e^2) \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c}x + \sqrt{a} + \sqrt{c}x^2)}{128\sqrt{2}a^{11/4}c^{3/4}} - \frac{(5\sqrt{a}e^2 + 21\sqrt{c}d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{(5\sqrt{a}e^2 + 21\sqrt{c}d^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}} + 1\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{x(7d^2 + 12dex + 5e^2x^2)}{32a^2(a + cx^4)} + \frac{3de \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} + \frac{x(d + ex)^2}{8a(a + cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]

[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(32*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(-p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))]^p, x], x, x]

x^k , x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1855

Int[(Pq)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff [Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
 \int \frac{(d+ex)^2}{(a+cx^4)^3} dx &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} - \frac{\int \frac{-7d^2-12dex-5e^2x^2}{(a+cx^4)^2} dx}{8a} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4} dx}{32a^2} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \left(\frac{24dex}{a+cx^4} + \frac{21d^2+5e^2x^2}{a+cx^4} \right) dx}{32a^2} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{\int \frac{21d^2+5e^2x^2}{a+cx^4} dx}{32a^2} + \frac{(3de) \int \frac{x}{a+cx^4} dx}{4a^2} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{(3de) \text{Subst} \left(\int \frac{1}{a+cx^2} dx, x, x^2 \right)}{8a^2} + \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right)}{64a^2} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}} + 2}{-\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{c}}} dx}{128\sqrt{2}a^{9/4}c^{3/4}} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{\left(\frac{21\sqrt{c}d^2}{\sqrt{a}} - 5e^2 \right) \log \left(\sqrt{a} - \sqrt{\frac{a+cx^4}{a}} \right)}{128\sqrt{2}a^{9/4}c^{3/4}} \\
 &= \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de \tan^{-1} \left(\frac{\sqrt{c}x^2}{\sqrt{a}} \right)}{8a^{5/2}\sqrt{c}} - \frac{(21\sqrt{c}d^2 + 5\sqrt{a}e^2) \tan^{-1} \left(1 - \sqrt{\frac{a+cx^4}{a}} \right)}{64\sqrt{2}a^{11/4}c^{3/4}}
 \end{aligned}$$

Mathematica [A] time = 0.29, size = 358, normalized size = 0.99

$$\frac{\sqrt{2}(5a^{3/4}d^2 - 21\sqrt{a}\sqrt{c}d^2)\log(-\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} + \frac{\sqrt{2}(21\sqrt{a}\sqrt{c}d^2 - 5a^{3/4}d^2)\log(\sqrt{2}\sqrt{a}\sqrt{c}x + \sqrt{a} + \sqrt{c}x^2)}{c^{3/4}} + \frac{32d^2(d+ex)^2}{(d+cx^4)^2} - \frac{2\sqrt{a}(48\sqrt{a}\sqrt{c}d+5\sqrt{2}\sqrt{a}d^2+21\sqrt{2}\sqrt{c}d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{a}}\right)}{c^{3/4}} + \frac{2\sqrt{a}(-48\sqrt{a}\sqrt{c}d+5\sqrt{2}\sqrt{a}d^2+21\sqrt{2}\sqrt{c}d^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x+1}{\sqrt{a}}\right)}{c^{3/4}} + \frac{8a(7d^2+12d+5d^2)}{d+cx^4}$$

256a³

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]

[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(a + c*x^4) - (2*a^(1/4)*(21*sqrt[2]*sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d*e + 5*sqrt[2]*sqrt[a]*e^2)*ArcTan[1 - (sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (2*a^(1/4)*(21*sqrt[2]*sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*sqrt[2]*sqrt[a]*e^2)*ArcTan[1 + (sqrt[2]*c^(1/4)*x)/a^(1/4)]/c^(3/4) + (sqrt[2]*(-21*a^(1/4)*sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (sqrt[2]*(21*a^(1/4)*sqrt[c]*d^2 - 5*a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex)^2}{(a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^3, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.43, size = 356, normalized size = 0.99

$$\frac{5cx^7d^2 + 12cd^2e + 7c^2d^2e^2 + 9ax^3d^2 + 20ad^2e^2 + 11ad^2e^3}{32(c^4 + a)^2d^2} + \frac{\sqrt{2}\left(24\sqrt{2}\sqrt{a}cd^2 + 21(a^2)^{\frac{1}{2}}c^2d^2 + 5(a^2)^{\frac{1}{2}}e^2\right)\arctan\left(\frac{\sqrt{2}(2+\sqrt{2})\sqrt{c}x}{2(c)^{\frac{1}{2}}}\right)}{128a^2c^2} + \frac{\sqrt{2}\left(24\sqrt{2}\sqrt{a}cd^2 + 21(a^2)^{\frac{1}{2}}c^2d^2 + 5(a^2)^{\frac{1}{2}}e^2\right)\arctan\left(\frac{\sqrt{2}(2-\sqrt{2})\sqrt{c}x}{2(c)^{\frac{1}{2}}}\right)}{128a^2c^2} + \frac{\sqrt{2}\left(21(a^2)^{\frac{1}{2}}c^2d^2 - 5(a^2)^{\frac{1}{2}}e^2\right)\log\left(x^2 + \sqrt{2}x(c)^{\frac{1}{2}} + \sqrt{c}\right)}{256a^2c^2} + \frac{\sqrt{2}\left(21(a^2)^{\frac{1}{2}}c^2d^2 - 5(a^2)^{\frac{1}{2}}e^2\right)\log\left(x^2 - \sqrt{2}x(c)^{\frac{1}{2}} + \sqrt{c}\right)}{256a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")

[Out] $\frac{1}{32}(5c^2x^7e^2 + 12c^2dx^6e + 7c^2d^2x^5 + 9a^2x^3e^2 + 20a^2dx^2e + 11a^2d^2x)/(c^2x^4 + a)^2 + \frac{1}{128}\sqrt{2}(24\sqrt{2})\sqrt{ac}c^2d^2e + 21(a^2c^3)^{1/4}c^2d^2 + 5(a^2c^3)^{3/4}e^2 \arctan(1/2\sqrt{2})(2x + \sqrt{2}(a/c)^{1/4})/(a/c)^{1/4} + 1/128\sqrt{2}(24\sqrt{2})\sqrt{ac}c^2d^2e + 21(a^2c^3)^{1/4}c^2d^2 + 5(a^2c^3)^{3/4}e^2 \arctan(1/2\sqrt{2})(2x - \sqrt{2}(a/c)^{1/4})/(a/c)^{1/4} + 1/256\sqrt{2}(21(a^2c^3)^{1/4}c^2d^2 - 5(a^2c^3)^{3/4}e^2) \log(x^2 + \sqrt{2}x(a/c)^{1/4} + \sqrt{ac})/(a^3c^3) - 1/256\sqrt{2}(21(a^2c^3)^{1/4}c^2d^2 - 5(a^2c^3)^{3/4}e^2) \log(x^2 - \sqrt{2}x(a/c)^{1/4} + \sqrt{ac})/(a^3c^3)$

maple [A] time = 0.01, size = 419, normalized size = 1.16

$$\frac{c^2x^7}{8(c^2x^4+a)^2} + \frac{dcx^6}{4(c^2x^4+a)a} + \frac{5c^2d^2x^5}{32(c^2x^4+a)^2} + \frac{d^2x^5}{8(c^2x^4+a)a} + \frac{3de^2x^3}{8(c^2x^4+a)a^2} + \frac{7d^2x^3}{32(c^2x^4+a)a^2} + \frac{3d \arctan\left(\frac{\sqrt{2}x}{a}\right)}{8\sqrt{ac}a^2} + \frac{5\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}x}{a}-1\right)}{128\left(\frac{c}{a}\right)^{1/4}c^2} + \frac{5\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{128\left(\frac{c}{a}\right)^{1/4}c^2} + \frac{5\sqrt{2}c^2 \ln\left(\frac{c-\left(\frac{c}{a}\right)^{1/4}\sqrt{2}x+\sqrt{c}}{c+\left(\frac{c}{a}\right)^{1/4}\sqrt{2}x+\sqrt{c}}\right)}{256\left(\frac{c}{a}\right)^{1/4}c^2} + \frac{21\left(\frac{c}{a}\right)^{1/4}\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}x}{a}-1\right)}{128a^3} + \frac{21\left(\frac{c}{a}\right)^{1/4}\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}x}{a}+1\right)}{128a^3} + \frac{21\left(\frac{c}{a}\right)^{1/4}\sqrt{2}c^2 \ln\left(\frac{c-\left(\frac{c}{a}\right)^{1/4}\sqrt{2}x+\sqrt{c}}{c+\left(\frac{c}{a}\right)^{1/4}\sqrt{2}x+\sqrt{c}}\right)}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x+d)^2/(c*x^4+a)^3, x)$

[Out] $\frac{1}{8}d^2x/a/(c^2x^4+a)^2 + 7/32d^2/a^2x/(c^2x^4+a) + 21/256d^2/a^3(a/c)^{1/4} * 2^{1/2} * \ln((x^2+(a/c)^{1/4}) * 2^{1/2} * x + (a/c)^{1/4}) / (x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4}) + 21/128d^2/a^3(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x + 1) + 21/128d^2/a^3(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x - 1) + 1/4d^2e*x^2/a/(c^2x^4+a)^2 + 3/8d^2e/a^2x^2/(c^2x^4+a) + 3/8d^2e/a^2(a/c)^{1/4} * \arctan((1/a*c)^{1/4} * x^2) + 1/8e^2*x^3/a/(c^2x^4+a)^2 + 5/32e^2/a^2*x^3/(c^2x^4+a) + 5/256e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \ln((x^2 - (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4}) / (x^2 + (a/c)^{1/4} * 2^{1/2} * x + (a/c)^{1/4})) + 5/128e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x + 1) + 5/128e^2/a^2/c/(a/c)^{1/4} * 2^{1/2} * \arctan(2^{1/2}/(a/c)^{1/4} * x - 1)$

maxima [A] time = 2.18, size = 364, normalized size = 1.01

$$\frac{5c^2x^7 + 12cdx^6 + 7c^2d^2x^5 + 9a^2dx^3 + 20a^2d^2x + 11a^2d^2}{32(a^2c^2x^4 + 2a^2cx^4 + a^4)} + \frac{\sqrt{2}(21\sqrt{c}d^2 - 5\sqrt{c}d^2) \log\left(\frac{\sqrt{c}x^2 + \sqrt{2}c^{1/4}x + \sqrt{c}}{\sqrt{c}x^2 - \sqrt{2}c^{1/4}x + \sqrt{c}}\right)}{a^{3/4}c^{1/4}} - \frac{\sqrt{2}(21\sqrt{c}d^2 - 5\sqrt{c}d^2) \log\left(\frac{\sqrt{c}x^2 - \sqrt{2}c^{1/4}x + \sqrt{c}}{\sqrt{c}x^2 + \sqrt{2}c^{1/4}x + \sqrt{c}}\right)}{a^{3/4}c^{1/4}} + \frac{2(21\sqrt{2}a^{1/4}c^{1/4}d^2 + 5\sqrt{2}a^{1/4}c^{1/4}d^2 - 48\sqrt{c}de) \arctan\left(\frac{\sqrt{2}\sqrt{c}x + \sqrt{2}c^{1/4}}{2\sqrt{c}\sqrt{c}}\right)}{256a^2\sqrt{c}\sqrt{c}^{1/4}} + \frac{2(21\sqrt{2}a^{1/4}c^{1/4}d^2 + 5\sqrt{2}a^{1/4}c^{1/4}d^2 + 48\sqrt{c}de) \arctan\left(\frac{\sqrt{2}\sqrt{c}x - \sqrt{2}c^{1/4}}{2\sqrt{c}\sqrt{c}}\right)}{256a^2\sqrt{c}\sqrt{c}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)^2/(c*x^4+a)^3, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{32}(5c^2e^2x^7 + 12c^2d^2e^2x^6 + 7c^2d^2x^5 + 9a^2e^2x^3 + 20a^2d^2e^2x^2 + 11a^2d^2x)/(a^2c^2x^8 + 2a^3c^2x^4 + a^4) + \frac{1}{256}(\sqrt{2})(21\sqrt{2}c)d^2 - 5\sqrt{2}a)e^2) \log(\sqrt{c}x^2 + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}a)/(a^{3/4}c^{3/4}) - \sqrt{2}(21\sqrt{2}c)d^2 - 5\sqrt{2}a)e^2) \log(\sqrt{c}x^2 - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}a)/(a^{3/4}c^{3/4}) + 2(21\sqrt{2}c)d^2 + 5\sqrt{2}a)e^2) \arctan(1/2\sqrt{2})(2\sqrt{2}c)x + \sqrt{2}a^{1/4}c^{1/4})/\sqrt{c}a$

$$\frac{(a*\sqrt{c})/(a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{3/4} + 2*(21*\sqrt{2})*a^{1/4}*c^{3/4}*d^2 + 5*\sqrt{2})*a^{3/4}*c^{1/4}*e^2 + 48*\sqrt{a}*\sqrt{c}*d*e*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2})*a^{1/4}*c^{1/4})/\sqrt{\sqrt{a}*\sqrt{c}})}{(a^{3/4}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{3/4}}/a^2$$

mupad [B] time = 0.47, size = 676, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x)^2/(a + c*x^4)^3, x)$

[Out] $((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + \text{symsum}(\log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4*x + 56448*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*e^3 - 393216*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k), k, 1, 4)$

sympy [A] time = 4.88, size = 374, normalized size = 1.04

RootSum(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x + 56448*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*e^3 - 393216*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x)/(32768*a^6))*\text{root}(268435456*a^{11}*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k), k, 1, 4)

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)**2/(c*x**4+a)**3, x)$

[Out] $\text{RootSum}(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 111906*a*c*d**4*e**4 + 194481*c**2*d**8, \text{Lambda}(_t, _t*\log(x + (262144000*_t**3*a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*$

$$\begin{aligned} & c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d* \\ & *10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d* \\ & *9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d* \\ & **8*e**4 + 85766121*c**3*d**12))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e** \\ & 2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3* \\ & c*x**4 + 32*a**2*c**2*x**8) \end{aligned}$$

$$3.406 \quad \int \frac{d+ex}{(a+cx^4)^3} dx$$

Optimal. Leaf size=266

$$\frac{21d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{3e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{16a^{5/2} \sqrt{c}} + \frac{x(d+ex)}{8a(a+cx^4)^2}$$

Rubi [A] time = 0.25, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7d+6ex)}{32a^2(a+cx^4)} - \frac{21d \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21d \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21d \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{3e \tan^{-1}\left(\frac{\sqrt{c} x^2}{\sqrt{a}}\right)}{16a^{5/2} \sqrt{c}} + \frac{x(d+ex)}{8a(a+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)/(a + c*x^4)^3, x]

[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4)) + (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4), x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&

AtomQ[SplitProduct[SumBaseQ, b]])

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1855

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := -Simp[(x*Pq*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]

Rule 1876

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{(a+cx^4)^3} dx &= \frac{x(d+ex)}{8a(a+cx^4)^2} - \frac{\int \frac{-7d-6ex}{(a+cx^4)^2} dx}{8a} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \frac{21d+12ex}{a+cx^4} dx}{32a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4} \right) dx}{32a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d) \int \frac{1}{a+cx^4} dx}{32a^2} + \frac{(3e) \int \frac{x}{a+cx^4} dx}{8a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{(21d) \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{(21d) \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{(3e) \text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{c}x^2} dx\right)}{16a^2} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} + \frac{(21d) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + x^2} dx}{128a^{5/2}\sqrt{c}} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d \log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d \log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x + \sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \\
&= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} + \frac{3e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 249, normalized size = 0.94

$$\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6(8\sqrt[4]{a}e+7\sqrt{2}\sqrt[4]{c}d)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}$$

256a^{11/4}

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]

[Out] $\left(\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{(a+cx^4)} - \frac{6(7\sqrt{2}c^{1/4}d+8a^{1/4}e)\operatorname{ArcTan}\left[1-\frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right]}{\sqrt{c}} + \frac{6(7\sqrt{2}c^{1/4}d-8a^{1/4}e)\operatorname{ArcTan}\left[1+\frac{\sqrt{2}c^{1/4}x}{a^{1/4}}\right]}{\sqrt{c}} - \frac{21\sqrt{2}d\log\left[\sqrt{a}-\sqrt{2}c^{1/4}x+\sqrt{c}x^2\right]}{c^{1/4}} + \frac{21\sqrt{2}d\log\left[\sqrt{a}+\sqrt{2}c^{1/4}x+\sqrt{c}x^2\right]}{c^{1/4}}\right)/(256a^{11/4})$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{(a+cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^3,x]

[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^3, x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.32, size = 260, normalized size = 0.98

$$\frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2+\sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{c}}\right)}{256a^3c} - \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}d\log\left(x^2-\sqrt{2}x\left(\frac{d}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{c}}\right)}{256a^3c} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7(ac^3)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2+\sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7(ac^3)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2-\sqrt{2}\left(\frac{d}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{d}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^2} + \frac{6cx^6e+7cdx^5+10adx^2e+11adx}{32(cx^4+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")

[Out] $\frac{21}{256}\sqrt{2}(ac^3)^{\frac{1}{4}}d\log(x^2+\sqrt{2}x\sqrt{a/c}+\sqrt{a/c})/(a^3c) - \frac{21}{256}\sqrt{2}(ac^3)^{\frac{1}{4}}d\log(x^2-\sqrt{2}x\sqrt{a/c}+\sqrt{a/c})/(a^3c) + \frac{3}{128}\sqrt{2}(4\sqrt{2}\sqrt{ac}ce+7(ac^3)^{\frac{1}{4}}cd)\arctan(1/2\sqrt{2}(2x+\sqrt{2}\sqrt{a/c}))/a^{3/4} + \frac{3}{128}\sqrt{2}(4\sqrt{2}\sqrt{ac}ce+7(ac^3)^{\frac{1}{4}}cd)\arctan(1/2\sqrt{2}(2x-\sqrt{2}\sqrt{a/c}))/a^{3/4}$

$1/2*\text{sqrt}(2)*(2*x - \text{sqrt}(2)*(a/c)^{(1/4)})/(a/c)^{(1/4)}/(a^3*c^2) + 1/32*(6*c*x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/(c*x^4 + a)^2*a^2$

maple [A] time = 0.01, size = 222, normalized size = 0.83

$$\frac{e x^2}{8(c x^4+a)^2 a} + \frac{d x}{8(c x^4+a)^2 a} + \frac{3 e x^2}{16(c x^4+a) a^2} + \frac{7 d x}{32(c x^4+a) a^2} + \frac{3 e \arctan\left(\sqrt{\frac{c}{a}} x^2\right)}{16 \sqrt{a c} a^2} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}-1\right)}{128 a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{a}\right)^{\frac{1}{4}}}+1\right)}{128 a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2+\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{c}{a}}}{x^2-\left(\frac{c}{a}\right)^{\frac{1}{4}} \sqrt{2} x+\sqrt{\frac{c}{a}}}\right)}{256 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)/(c*x^4+a)^3,x)`

[Out] $1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))+21/128*d/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)+21/128*d/a^3*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)+1/8*e*x^2/a/(c*x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^{(1/2)}*\arctan((1/a*c)^{(1/2)}*x^2)$

maxima [A] time = 2.20, size = 269, normalized size = 1.01

$$\frac{6 c e x^6 + 7 c d x^5 + 10 a e x^2 + 11 a d x}{32(a^2 c^2 x^8 + 2 a^3 c x^4 + a^4)} + \frac{3 \left(\frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} + \frac{2 \left(7 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d - 8 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}} + \frac{2 \left(7 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d + 8 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}} \right)}{256 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")`

[Out] $1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 3/256*(7*\text{sqrt}(2)*d*\log(\text{sqrt}(c)*x^2 + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(1/4)}) - 7*\text{sqrt}(2)*d*\log(\text{sqrt}(c)*x^2 - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*x + \text{sqrt}(a)))/(a^{(3/4)}*c^{(1/4)}) + 2*(7*\text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*d - 8*\text{sqrt}(a)*e)*\arctan(1/2*\text{sqrt}(2)*(2*\text{sqrt}(c)*x + \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c))*c^{(1/4)}) + 2*(7*\text{sqrt}(2)*a^{(1/4)}*c^{(1/4)}*d + 8*\text{sqrt}(a)*e)*\arctan(1/2*\text{sqrt}(2)*(2*\text{sqrt}(c)*x - \text{sqrt}(2)*a^{(1/4)}*c^{(1/4)})/\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c)))/(a^{(3/4)}*\text{sqrt}(\text{sqrt}(a)*\text{sqrt}(c))*c^{(1/4)})/a^2$

mupad [B] time = 0.30, size = 315, normalized size = 1.18

$$\frac{6 c^2 e x^6 + 7 c^2 d x^5 + 10 a c e x^2 + 11 a c d x}{32(a^2 c^2 x^8 + 2 a^3 c x^4 + a^4)} + \frac{3 \left(\frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} - \frac{7 \sqrt{2} d \log\left(\sqrt{c} x^2 - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{1}{4}}} + \frac{2 \left(7 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d - 8 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x + \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}} + \frac{2 \left(7 \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} d + 8 \sqrt{a} e\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x - \sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{a} \sqrt{c}}\right)}{a^{\frac{3}{4}} \sqrt{a} \sqrt{c} c^{\frac{1}{4}}} \right)}{256 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x)/(a + c*x^4)^3,x)`

[Out]
$$\frac{(5ex^2)/(16a) + (11dx)/(32a) + (7c^2dx^5)/(32a^2) + (3c^3ex^6)/(16a^2)}{(a^2 + c^2x^8 + 2acx^4)} + \text{symsum}(\log((3c^2(63d^2e^2 + 36e^3x - 7168\text{root}(268435456a^{11}c^2z^4 + 4718592a^6c^2e^2z^2 - 2709504a^3cd^2ez + 194481cd^4 + 20736a^4e^4, z, k)^2a^5cd - 1176\text{root}(268435456a^{11}c^2z^4 + 4718592a^6c^2e^2z^2 - 2709504a^3cd^2ez + 194481cd^4 + 20736a^4e^4, z, k)a^2cd^2ex + 4096\text{root}(268435456a^{11}c^2z^4 + 4718592a^6c^2e^2z^2 - 2709504a^3cd^2ez + 194481cd^4 + 20736a^4e^4, z, k)^2a^5cex)))/(2048a^6))\text{root}(268435456a^{11}c^2z^4 + 4718592a^6c^2e^2z^2 - 2709504a^3cd^2ez + 194481cd^4 + 20736a^4e^4, z, k), k, 1, 4)$$

sympy [A] time = 1.48, size = 192, normalized size = 0.72

$$\text{RootSum}\left(268435456a^{11}c^2 + 4718592a^6c^2e - 2709504a^3cd^2e + 20736a^4e^4 + 194481cd^4, \left(t \mapsto t \log\left(x + \frac{-67108864t^3a^9ce^2 - 9633792t^2a^6cd^2e - 589824ta^4e^4 - 2765952ta^3cd^4 + 423360ad^2e^3}{193536ad^4 - 453789cd^5}\right)\right)\right) + \frac{11adx + 10acx^2 + 7cdx^5 + 6cex^6}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(c*x**4+a)**3,x)`

[Out]
$$\text{RootSum}(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, \text{Lambda}(_t, _t*\log(x + (-67108864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4 - 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8)$$

$$3.407 \quad \int \frac{1}{(a+cx^4)^3} dx$$

Optimal. Leaf size=219

$$\frac{21 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}}$$

Rubi [A] time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7x}{32a^2(a+cx^4)} - \frac{21 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x + \sqrt{a} + \sqrt{c} x^2\right)}{128\sqrt{2} a^{11/4} \sqrt[4]{c}} - \frac{21 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{21 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}} + 1\right)}{64\sqrt{2} a^{11/4} \sqrt[4]{c}} + \frac{x}{8a(a+cx^4)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*x^4)^(-3), x]

[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 211

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),

```

x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))

```

Rule 617

```

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 628

```

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

Rule 1162

```

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

```

Rule 1165

```

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+cx^4)^3} dx &= \frac{x}{8a(a+cx^4)^2} + \frac{7 \int \frac{1}{(a+cx^4)^2} dx}{8a} \\
&= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{1}{a+cx^4} dx}{32a^2} \\
&= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} + \frac{21 \int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4} dx}{64a^{5/2}} \\
&= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} + \frac{21 \int \frac{1}{\frac{\sqrt{a}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}}+x^2} dx}{128a^{5/2}\sqrt{c}} + \frac{21 \int \frac{1}{\frac{\sqrt{a}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{c}}+x^2} dx}{128a^{5/2}\sqrt{c}} - \frac{21 \int \frac{\sqrt{2}}{\sqrt{c}} dx}{128\sqrt{2}} \\
&= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21 \log(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \log(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \\
&= \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21 \tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} - \frac{21 \log(\sqrt{2})}{128\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{(a+cx^4)^2} + \frac{56a^{3/4}x}{a+cx^4} - \frac{21\sqrt{2} \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}} + \frac{21\sqrt{2} \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2)}{\sqrt[4]{c}} - \frac{42\sqrt{2} \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{42\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*x^4)^(-3), x]

[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*sqrt[2]*ArcTan[1 - (sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*sqrt[2]*ArcTan[1 + (sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*sqrt[2]*Log[sqrt[a] - sqrt[2]*a^(1/4)*c^(1/4)*x + sqrt[c]*x^2])/c^(1/4) + (21*sqrt[2]*Log[sqrt[a] + sqrt[2]*a^(1/4)*c^(1/4)*x + sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + c*x^4)^(-3), x]

[Out] IntegrateAlgebraic[(a + c*x^4)^(-3), x]

fricas [A] time = 1.46, size = 232, normalized size = 1.06

$$\frac{28 cx^5 + 84 (a^2 c^2 x^8 + 2 a^3 c x^4 + a^4) \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} \arctan\left(-\frac{a^6 c x \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} + \sqrt{a^6 \sqrt{-\frac{1}{a^{11} c}} + x^2} a^6 c \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}}\right)}{128 (a^2 c^2 x^8 + 2 a^3 c x^4 + a^4)} + 21 (a^2 c^2 x^8 + 2 a^3 c x^4 + a^4) \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} \log\left(a^3 \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} + x\right) - 21 (a^2 c^2 x^8 + 2 a^3 c x^4 + a^4) \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} \log\left(-a^3 \left(-\frac{1}{a^{11} c}\right)^{\frac{1}{4}} + x\right) + 44 a x}{128 (a^2 c^2 x^8 + 2 a^3 c x^4 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")

[Out] $\frac{1}{128} * (28 * c * x^5 + 84 * (a^2 * c^2 * x^8 + 2 * a^3 * c * x^4 + a^4) * (-1 / (a^{11} * c))^{(1/4)} * \arctan(-a^6 * c * x * (-1 / (a^{11} * c))^{(3/4)} + \sqrt{a^6 * \sqrt{-1 / (a^{11} * c)} + x^2} * a^8 * c * (-1 / (a^{11} * c))^{(3/4)}) + 21 * (a^2 * c^2 * x^8 + 2 * a^3 * c * x^4 + a^4) * (-1 / (a^{11} * c))^{(1/4)} * \log(a^3 * (-1 / (a^{11} * c))^{(1/4)} + x) - 21 * (a^2 * c^2 * x^8 + 2 * a^3 * c * x^4 + a^4) * (-1 / (a^{11} * c))^{(1/4)} * \log(-a^3 * (-1 / (a^{11} * c))^{(1/4)} + x) + 44 * a * x) / (a^2 * c^2 * x^8 + 2 * a^3 * c * x^4 + a^4)$

giac [A] time = 0.36, size = 204, normalized size = 0.93

$$\frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}(2x + \sqrt{2}(\frac{a}{c})^{\frac{1}{4}})}{2(\frac{a}{c})^{\frac{1}{4}}}\right)}{128 a^3 c} + \frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}(2x - \sqrt{2}(\frac{a}{c})^{\frac{1}{4}})}{2(\frac{a}{c})^{\frac{1}{4}}}\right)}{128 a^3 c} + \frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x(\frac{a}{c})^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256 a^3 c} - \frac{21 \sqrt{2} (ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x(\frac{a}{c})^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256 a^3 c} + \frac{7cx^5 + 11ax}{32(cx^4 + a)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")

[Out] $\frac{21}{128} * \sqrt{2} * (a * c^3)^{(1/4)} * \arctan(1/2 * \sqrt{2} * (2 * x + \sqrt{2} * (a/c)^{(1/4)}) / (a/c)^{(1/4)}) / (a^3 * c) + 21/128 * \sqrt{2} * (a * c^3)^{(1/4)} * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (a/c)^{(1/4)}) / (a/c)^{(1/4)}) / (a^3 * c) + 21/256 * \sqrt{2} * (a * c^3)^{(1/4)} * \log(x^2 + \sqrt{2} * x * (a/c)^{(1/4)} + \sqrt{a/c}) / (a^3 * c) - 21/256 * \sqrt{2} * (a * c^3)^{(1/4)} * \log(x^2 - \sqrt{2} * x * (a/c)^{(1/4)} + \sqrt{a/c}) / (a^3 * c) + 1/32 * (7 * c * x^5 + 11 * a * x) / ((c * x^4 + a)^2 * a^2)$

maple [A] time = 0.00, size = 158, normalized size = 0.72

$$\frac{x}{8(c x^4 + a)^2 a} + \frac{7x}{32(c x^4 + a) a^2} + \frac{21 \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{128 a^3} + \frac{21 \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{128 a^3} + \frac{21 \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} \ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{c}}}\right)}{256 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+a)^3,x)`

[Out] $\frac{1}{8}x/a/(c*x^4+a)^2 + \frac{7}{32}x/a^2/(c*x^4+a) + \frac{21}{256}a^{-3}*(a/c)^{(1/4)}*2^{(1/2)}*\ln\left(\frac{x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}}{x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}}\right) + \frac{21}{128}a^{-3}*(a/c)^{(1/4)}*2^{(1/2)}*\arctan\left(\frac{2^{(1/2)}}{(a/c)^{(1/4)}*x+1}\right) + \frac{21}{128}a^{-3}*(a/c)^{(1/4)}*2^{(1/2)}*\arctan\left(\frac{2^{(1/2)}}{(a/c)^{(1/4)}*x-1}\right)$

maxima [A] time = 2.70, size = 212, normalized size = 0.97

$$\frac{7cx^5 + 11ax}{32(a^2c^2x^8 + 2a^3cx^4 + a^4)} + \frac{21 \left(\frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{2\sqrt{2} \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{a}\sqrt{c}} + \frac{\sqrt{2} \log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2} \log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} \right)}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+a)^3,x, algorithm="maxima")`

[Out] $\frac{1}{32}*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + \frac{21}{256}*(2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{a}*\sqrt{c}) + 2*\sqrt{2}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2}*a^{1/4}*c^{1/4}))/\sqrt{a}*\sqrt{c}))/(\sqrt{a}*\sqrt{a}*\sqrt{c}) + \sqrt{2}*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4}*c^{1/4} - \sqrt{2}*\log(\sqrt{c}*x^2 - \sqrt{2}*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4}*c^{1/4})/a^2$

mupad [B] time = 0.10, size = 80, normalized size = 0.37

$$\frac{\frac{11x}{32a} + \frac{7cx^5}{32a^2}}{a^2 + 2acx^4 + c^2x^8} - \frac{21 \operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}} - \frac{21 \operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{64(-a)^{11/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*x^4)^3,x)`

[Out] $\left(\frac{11x}{32a} + \frac{7cx^5}{32a^2}\right)/(a^2 + c^2x^8 + 2acx^4) - \left(21*\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)\right)/(64*(-a)^{11/4}*c^{1/4}) - \left(21*\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)\right)/(64*(-a)^{11/4}*c^{1/4})$

sympy [A] time = 0.46, size = 63, normalized size = 0.29

$$\frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t \log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSum(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

$$3.408 \quad \int \frac{1}{(d+ex)(a+cx^4)^3} dx$$

Optimal. Leaf size=1352

$$\frac{\log(d+ex)e^{11}}{(cd^4+ae^4)^3} - \frac{\log(cx^4+a)e^{11}}{4(cd^4+ae^4)^3} - \frac{\sqrt{c}d^2 \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}(cd^4+ae^4)^3} - \frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3} + \frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2) \tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}(cd^4+ae^4)^3}$$

Rubi [A] time = 1.41, antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)*(a + c*x^4)^3), x]

[Out] $(c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^4)^2*(a + c*x^4)) - (\text{Sqrt}[c]*d^2*e^9*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(c*d^4 + a*e^4)^3) - (\text{Sqrt}[c]*d^2*e^5*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(4*a^{(3/2)}*(c*d^4 + a*e^4)^2) - (3*\text{Sqrt}[c]*d^2*e*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(16*a^{(5/2)}*(c*d^4 + a*e^4)) - (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) - (c^{(1/4)}*d*e^4*(3*\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*d*(21*\text{Sqrt}[c]*d^2 + 5*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(64*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*d*e^4*(3*\text{Sqrt}[c]*d^2 + \text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*d*(21*\text{Sqrt}[c]*d^2 + 5*\text{Sqrt}[a]*e^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*x)/a^{(1/4)}])/(64*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (e^{11}*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^3 - (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) - (c^{(1/4)}*d*e^4*(3*\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) - (c^{(1/4)}*d*(21*\text{Sqrt}[c]*d^2 - 5*\text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(128*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)) + (c^{(1/4)}*d*e^8*(\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*d*e^4*(3*\text{Sqrt}[c]*d^2 - \text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^2) + (c^{(1/4)}*d*(21*\text{Sqrt}[c]*d^2 - 5*\text{Sqrt}[a]*e^2)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(128*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4))$

$$e^2) \cdot \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] \cdot a^{1/4} \cdot c^{1/4} \cdot x + \text{Sqrt}[c] \cdot x^2] / (16 \cdot \text{Sqrt}[2] \cdot a^{7/4} \cdot (c \cdot d^4 + a \cdot e^4)^2) + (c^{1/4} \cdot d \cdot (21 \cdot \text{Sqrt}[c] \cdot d^2 - 5 \cdot \text{Sqrt}[a] \cdot e^2) \cdot \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] \cdot a^{1/4} \cdot c^{1/4} \cdot x + \text{Sqrt}[c] \cdot x^2]) / (128 \cdot \text{Sqrt}[2] \cdot a^{11/4} \cdot (c \cdot d^4 + a \cdot e^4)) - (e^{11} \cdot \text{Log}[a + c \cdot x^4]) / (4 \cdot (c \cdot d^4 + a \cdot e^4)^3)$$
Rule 204

$$\text{Int}[\frac{(a_.) + (b_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.)^2}^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2] \cdot x}{\text{Rt}[-a, 2]}] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 205

$$\text{Int}[\frac{(a_.) + (b_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.)^2}^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]]) / a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$$
Rule 260

$$\text{Int}[(x_.)^{(m_.)} / ((a_.) + (b_.) \cdot (x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x^n, x]] / (b \cdot n), x] /; \text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{EqQ}[m, n - 1]$$
Rule 275

$$\text{Int}[(x_.)^{(m_.)} \cdot ((a_.) + (b_.) \cdot (x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{(m+1)/k - 1} \cdot (a + b \cdot x^{(n/k)})^p, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$$
Rule 617

$$\text{Int}[\frac{(a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2}{(a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2}^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c) / b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2 \cdot c \cdot x) / b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c])] /; \text{FreeQ}\{a, b, c\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$$
Rule 628

$$\text{Int}[\frac{(d_.) + (e_.) \cdot (x_.)}{(a_.) + (b_.) \cdot (x_.) + (c_.) \cdot (x_.)^2}, x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$$
Rule 635

$$\text{Int}[\frac{(d_.) + (e_.) \cdot (x_.)}{(a_.) + (c_.) \cdot (x_.)^2}, x_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c \cdot x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c \cdot x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ !\text{NiceSqrtQ}[-(a \cdot c)]$$

Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1 /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

Mathematica [A] time = 0.74, size = 835, normalized size = 0.62

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)*(a + c*x^4)^3), x]

[Out]
$$\frac{((32(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) - (2*c^{1/4}*d*(21*\text{Sqrt}[2]*c^{5/2}*d^{10} - 24*a^{1/4}*c^{9/4}*d^9*e + 5*\text{Sqrt}[2]*\text{Sqrt}[a]*c^2*d^8*e^2 + 66*\text{Sqrt}[2]*a*c^{3/2}*d^6*e^4 - 80*a^{5/4}*c^{5/4}*d^5*e^5 + 18*\text{Sqrt}[2]*a^{3/2}*c*d^4*e^6 + 77*\text{Sqrt}[2]*a^2*\text{Sqrt}[c]*d^2*e^8 - 120*a^{9/4}*c^{1/4}*d*e^9 + 45*\text{Sqrt}[2]*a^{5/2}*e^{10})*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/a^{11/4} + (2*c^{1/4}*d*(21*\text{Sqrt}[2]*c^{5/2}*d^{10} + 24*a^{1/4}*c^{9/4}*d^9*e + 5*\text{Sqrt}[2]*\text{Sqrt}[a]*c^2*d^8*e^2 + 66*\text{Sqrt}[2]*a*c^{3/2}*d^6*e^4 + 80*a^{5/4}*c^{5/4}*d^5*e^5 + 18*\text{Sqrt}[2]*a^{3/2}*c*d^4*e^6 + 77*\text{Sqrt}[2]*a^2*\text{Sqrt}[c]*d^2*e^8 + 120*a^{9/4}*c^{1/4}*d*e^9 + 45*\text{Sqrt}[2]*a^{5/2}*e^{10})*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{1/4}*x)/a^{1/4}])/a^{11/4} + 256*e^{11}*\text{Log}[d + e*x] + (\text{Sqrt}[2]*c^{1/4}*(-21*c^{5/2}*d^{11} + 5*\text{Sqrt}[a]*c^2*d^9*e^2 - 66*a*c^{3/2}*d^7*e^4 + 18*a^{3/2}*c*d^5*e^6 - 77*a^2*\text{Sqrt}[c]*d^3*e^8 + 45*a^{5/2}*d*e^{10})*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/a^{11/4} + (\text{Sqrt}[2]*c^{1/4}*(21*c^{5/2}*d^{11} - 5*\text{Sqrt}[a]*c^2*d^9*e^2 + 66*a*c^{3/2}*d^7*e^4 - 18*a^{3/2}*c*d^5*e^6 + 77*a^2*\text{Sqrt}[c]*d^3*e^8 - 45*a^{5/2}*d*e^{10})*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*c^{1/4}*x + \text{Sqrt}[c]*x^2])/a^{11/4} - 64*e^{11}*\text{Log}[a + c*x^4])/(256*(c*d^4 + a*e^4)^3}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)(a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^3), x]

[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.91, size = 1259, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")

[Out]
$$\frac{1}{64} \cdot (51 \sqrt{2}) \sqrt{ac} \cdot c^2 d^4 e + 21 (ac^3)^{1/4} c^2 d^5 - 75 \sqrt{2} (ac^2 d^2 e^3 + 122 (ac^3)^{3/4} d^3 e^2 + 45 (ac^3)^{1/4} ac d e^4) \arctan\left(\frac{1/2 \sqrt{2} (2x + \sqrt{2} (a/c)^{1/4})}{(a/c)^{1/4} (\sqrt{2} a^3 c^3 d^6 - 6 (ac^3)^{1/4} a^3 c^2 d^5 e + 9 \sqrt{2} \sqrt{ac} a^3 c^2 d^4 e^2 + 9 \sqrt{2} a^4 c^2 d^2 e^4 - 16 (ac^3)^{3/4} a^3 d^3 e^3 - 6 (ac^3)^{1/4} a^4 c d e^5 + \sqrt{2} \sqrt{ac} a^4 c e^6)}\right) + \frac{1}{64} \cdot (51 \sqrt{2}) \sqrt{ac} \cdot c^2 d^4 e + 21 (ac^3)^{1/4} c^2 d^5 + 75 \sqrt{2} (ac^2 d^2 e^3 + 122 (ac^3)^{3/4} d^3 e^2 + 45 (ac^3)^{1/4} ac d e^4) \arctan\left(\frac{1/2 \sqrt{2} (2x - \sqrt{2} (a/c)^{1/4})}{(a/c)^{1/4} (\sqrt{2} a^3 c^3 d^6 + 6 (ac^3)^{1/4} a^3 c^2 d^5 e + 9 \sqrt{2} \sqrt{ac} a^3 c^2 d^4 e^2 + 9 \sqrt{2} a^4 c^2 d^2 e^4 + 16 (ac^3)^{3/4} a^3 d^3 e^3 + 6 (ac^3)^{1/4} a^4 c d e^5 + \sqrt{2} \sqrt{ac} a^4 c e^6)}\right) + \frac{1}{256} \cdot (21 \sqrt{2}) (ac^3)^{1/4} c^4 d^{11} - 5 \sqrt{2} (ac^3)^{3/4} c^2 d^9 e^2 + 66 \sqrt{2} (ac^3)^{1/4} ac^3 d^7 e^4 - 18 \sqrt{2} (ac^3)^{3/4} ac^3 d^5 e^6 + 77 \sqrt{2} (ac^3)^{1/4} a^2 c^2 d^3 e^8 - 45 \sqrt{2} (ac^3)^{3/4} a^2 d e^{10} \log(x^2 + \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (a^3 c^5 d^{12} + 3 a^4 c^4 d^8 e^4 + 3 a^5 c^3 d^4 e^8 + a^6 c^2 e^{12}) - 1/256 \cdot (21 \sqrt{2}) (ac^3)^{1/4} c^4 d^{11} - 5 \sqrt{2} (ac^3)^{3/4} c^2 d^9 e^2 + 66 \sqrt{2} (ac^3)^{1/4} ac^3 d^7 e^4 - 18 \sqrt{2} (ac^3)^{3/4} ac^3 d^5 e^6 + 77 \sqrt{2} (ac^3)^{1/4} a^2 c^2 d^3 e^8 - 45 \sqrt{2} (ac^3)^{3/4} a^2 d e^{10} \log(x^2 - \sqrt{2} x (a/c)^{1/4} + \sqrt{a/c}) / (a^3 c^5 d^{12} + 3 a^4 c^4 d^8 e^4 + 3 a^5 c^3 d^4 e^8 + a^6 c^2 e^{12}) - 1/4 e^{11} \log(\text{abs}(c x^4 + a)) / (c^3 d^{12} e + 3 a c^2 d^8 e^5 + 3 a^2 c d^4 e^9 + a^3 e^{13}) + 1/32 \cdot (4 a^2 c^2 d^8 e^3 + 16 a^3 c d^4 e^7 + (5 c^4 d^9 e^2 + 18 a c^3 d^5 e^6 + 13 a^2 c^2 d e^{10}) x^7 - 2 \cdot (3 c^4 d^{10} e + 10 a c^3 d^6 e^5 + 7 a^2 c^2 d^2 e^9) x^6 + (7 c^4 d^{11} + 22 a c^3 d^7 e^4 + 15 a^2 c^2 d^3 e^8) x^5 + 8 \cdot (a^2 c^2 d^4 e^7 + a^3 c e^{11}) x^4 + 12 a^4 e^{11} + (9 a c^3 d^9 e^2 + 26 a^2 c^2 d^5 e^6 + 17 a^3 c d e^{10}) x^3 - 2 \cdot (5 a c^3 d^{10} e + 14 a^2 c^2 d^6 e^5 + 9 a^3 c d^2 e^9) x^2 + (11 a c^3 d^{11} + 30 a^2 c^2 d^7 e^4 + 19 a^3 c d^3 e^8) x) / ((c d^4 + a e^4)^3 (c x^4 + a)^2 a^2)$$

maple [A] time = 0.02, size = 2098, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)/(c*x^4+a)^3,x)

[Out]
$$e^{11} \ln(e^x + d) / (a^4 + c^4 d^4)^{3-1/4} e^{11} \ln(c^4 x^4 + a) / (a^4 + c^4 d^4)^{3+3/8} / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 e^{11} a^2 + 33/128 c^2 / (a^4 + c^4 d^4)^3 / a^2 (a/c)^{1/4})^{2^{1/2}} \ln((x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d^7 e^4 + 33/64 c^2 / (a^4 + c^4 d^4)^3 / a^2 (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d^7 e^4 + 77/128 c / (a^4 + c^4 d^4)^3 / a (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d^3 e^8 + 5/256 c^2 / (a^4 + c^4 d^4)^3 / a^2 / (a/c)^{1/4})^{2^{1/2}} \ln((x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d^9 e^2 + 5/128 c^2 / (a^4 + c^4 d^4)^3 / a^2 / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d^9 e^2 + 5/128 c^2 / (a^4 + c^4 d^4)^3 / a^2 / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d^9 e^2 + 9/64 c / (a^4 + c^4 d^4)^3 / a / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d^5 e^6 + 9/64 c / (a^4 + c^4 d^4)^3 / a / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d^5 e^6 + 77/128 c / (a^4 + c^4 d^4)^3 / a (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d^3 e^8 + 77/256 c / (a^4 + c^4 d^4)^3 / a (a/c)^{1/4})^{2^{1/2}} \ln((x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d^3 e^8 + 9/128 c / (a^4 + c^4 d^4)^3 / a / (a/c)^{1/4})^{2^{1/2}} \ln((x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d^5 e^6 + 33/64 c^2 / (a^4 + c^4 d^4)^3 / a^2 (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d^7 e^4 + 13/32 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^10 x^7 - 7/16 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^2 e^9 x^6 + 15/32 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^3 x^5 e^8 + 7/32 c^4 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^11 / a^2 x^5 + 1/4 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 x^4 e^7 d^4 + 13/16 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^5 e^6 x^3 + 1/4 c / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 x^4 e^11 a + 1/2 c / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 e^7 d^4 a - 7/8 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^6 e^5 x^2 + 15/16 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^7 x e^4 + 11/32 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^11 / a x - 15/16 c / (a^4 + c^4 d^4)^3 / (a c)^{1/2} \arctan((1/a c)^{1/2} x^2) e^9 d^2 + 45/256 / (a^4 + c^4 d^4)^3 / (a/c)^{1/4})^{2^{1/2}} \ln((x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d e^{10} + 45/128 / (a^4 + c^4 d^4)^3 / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d e^{10} + 45/128 / (a^4 + c^4 d^4)^3 / (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d e^{10} + 9/32 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^9 e^2 / a x^3 - 5/16 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^10 e / a x^2 - 9/16 c / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^2 e^9 a x^2 + 19/32 c / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^3 a x e^8 + 21/128 c^3 / (a^4 + c^4 d^4)^3 / a^3 (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x - 1) d^{11} + 21/256 c^3 / (a^4 + c^4 d^4)^3 / a^3 (a/c)^{1/4})^{2^{1/2}} \ln((x^2 + (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}) / (x^2 - (a/c)^{1/4})^{2^{1/2}} x + (a/c)^{1/2}))^{2^{1/2}} d^{11} + 21/128 c^3 / (a^4 + c^4 d^4)^3 / a^3 (a/c)^{1/4})^{2^{1/2}} \arctan(2^{1/2} / (a/c)^{1/4})^{2^{1/2}} x + 1) d^{11} - 5/8 c^2 / (a^4 + c^4 d^4)^3 / a (a c)^{1/2} \arctan((1/a c)^{1/2} x^2) e^5 d^6 - 3/16 c^3 / (a^4 + c^4 d^4)^3 / a^2 (a c)^{1/2} \arctan((1/a c)^{1/2} x^2) e^5 d^{10} - 3/16 c^4 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^{10} e / a^2 x^6 + 11/16 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^7 / a x^5 e^4 + 17/32 c / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d e^{10} a x^3 + 9/16 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^5 e^6 / a x^7 + 5/32 c^4 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^9 e^2 / a^2 x^7 + 1/8 c^2 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 e^3 d^8 - 5/8 c^3 / (a^4 + c^4 d^4)^3 / (c^4 x^4 + a)^2 d^6 e^5 / a x^6$$

maxima [A] time = 2.41, size = 1015, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & e^{11} \log(e*x + d) / (c^3*d^{12} + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^{12}) \\ & - 1/256*c*(\sqrt{2}*(32*\sqrt{2}*a^{(11/4)}*c^{(1/4)}*e^{11} - 21*c^3*d^{11} + 5*\sqrt{2} \\ & *a*c^{(5/2)}*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^{(3/2)}*c^{(3/2)}*d^5*e^6 - 77*a \\ & ^2*c*d^3*e^8 + 45*a^{(5/2)}*\sqrt{c}*d*e^{10})*\log(\sqrt{c}*x^2 + \sqrt{2}*a^{(1/4)} \\ & *c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \sqrt{2}*(32*\sqrt{2}*a^{(11/4)}*c^{(1/4)} \\ & *e^{11} + 21*c^3*d^{11} - 5*\sqrt{a}*c^{(5/2)}*d^9*e^2 + 66*a*c^2*d^7*e^4 - 18* \\ & a^{(3/2)}*c^{(3/2)}*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^{(5/2)}*\sqrt{c}*d*e^{10})*\log \\ & (\sqrt{c}*x^2 - \sqrt{2}*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) - 2*(\\ & 21*\sqrt{2}*a^{(1/4)}*c^{(13/4)}*d^{11} + 5*\sqrt{2}*a^{(3/4)}*c^{(11/4)}*d^9*e^2 + 66* \\ & \sqrt{2}*a^{(5/4)}*c^{(9/4)}*d^7*e^4 + 18*\sqrt{2}*a^{(7/4)}*c^{(7/4)}*d^5*e^6 + 77* \\ & \sqrt{2}*a^{(9/4)}*c^{(5/4)}*d^3*e^8 + 45*\sqrt{2}*a^{(11/4)}*c^{(3/4)}*d*e^{10} + 24*\sqrt{2} \\ & *a*c^3*d^{10}*e + 80*a^{(3/2)}*c^2*d^6*e^5 + 120*a^{(5/2)}*c*d^2*e^9)*\arctan(1 \\ & /2*\sqrt{2}*(2*\sqrt{c}*x + \sqrt{2}*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(\\ & a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}}*c^{(5/4)}) - 2*(21*\sqrt{2}*a^{(1/4)}*c^{(13/4)}*d^{11} \\ & + 5*\sqrt{2}*a^{(3/4)}*c^{(11/4)}*d^9*e^2 + 66*\sqrt{2}*a^{(5/4)}*c^{(9/4)}*d^7*e^4 \\ & + 18*\sqrt{2}*a^{(7/4)}*c^{(7/4)}*d^5*e^6 + 77*\sqrt{2}*a^{(9/4)}*c^{(5/4)}*d^3*e^8 \\ & + 45*\sqrt{2}*a^{(11/4)}*c^{(3/4)}*d*e^{10} - 24*\sqrt{a}*c^3*d^{10}*e - 80*a^{(3/2)}* \\ & c^2*d^6*e^5 - 120*a^{(5/2)}*c*d^2*e^9)*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}*x - \sqrt{2} \\ & *a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}}*c^{(5/4)}) \\ & / (a^2*c^3*d^{12} + 3*a^3*c^2*d^8*e^4 + 3*a^4*c*d^4*e^8 + a^5*e^{12}) + \\ & 1/32*(8*a^2*c*e^7*x^4 + 4*a^2*c*d^4*e^3 + 12*a^3*e^7 + (5*c^3*d^5*e^2 + 13* \\ & a*c^2*d*e^6)*x^7 - 2*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5)*x^6 + (7*c^3*d^7 + 15* \\ & a*c^2*d^3*e^4)*x^5 + (9*a*c^2*d^5*e^2 + 17*a^2*c*d*e^6)*x^3 - 2*(5*a*c^2*d^6 \\ & *e + 9*a^2*c*d^2*e^5)*x^2 + (11*a*c^2*d^7 + 19*a^2*c*d^3*e^4)*x)/(a^4*c^2* \\ & d^8 + 2*a^5*c*d^4*e^4 + a^6*e^8 + (a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2 \\ & *e^8)*x^8 + 2*(a^3*c^3*d^8 + 2*a^4*c^2*d^4*e^4 + a^5*c*e^8)*x^4) \end{aligned}$$

mupad [B] time = 4.41, size = 2720, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + c*x^4)^3*(d + e*x)),x)

[Out]
$$\begin{aligned} & \text{symsum}(\log((194481*c^7*d^{13}*e^6 + 871362*a*c^6*d^9*e^{10} + 425984*a^3*c^4*d* \\ & e^{18} + 1148881*a^2*c^5*d^5*e^{14})/(1048576*(a^{12}*e^{16} + a^8*c^4*d^{16} + 4*a^1 \\ & 1*c*d^4*e^{12} + 4*a^9*c^3*d^{12}*e^4 + 6*a^{10}*c^2*d^8*e^8)) + \text{root}(805306368*a \\ & ^{12}*c^2*d^8*e^4*z^4 + 805306368*a^{13}*c*d^4*e^8*z^4 + 268435456*a^{11}*c^3*d^1 \\ & 2*z^4 + 268435456*a^{14}*e^{12}*z^4 + 268435456*a^{11}*e^{11}*z^3 + 43057152*a^7*c* \end{aligned}$$

$$\begin{aligned}
& d^4 e^6 z^2 + 11599872 a^6 c^2 d^8 e^2 z^2 + 100663296 a^8 e^{10} z^2 + 96522 \\
& 24 a^4 c d^4 e^5 z + 2709504 a^3 c^2 d^8 e z + 16777216 a^5 e^9 z + 676881 * \\
& a * c d^4 e^4 + 194481 c^2 d^8 + 1048576 a^2 e^8, z, k) * (\text{root}(805306368 a^{12} * \\
& c^2 d^8 e^4 z^4 + 805306368 a^{13} c d^4 e^8 z^4 + 268435456 a^{11} c^3 d^{12} z^4 \\
& + 268435456 a^{14} e^{12} z^4 + 268435456 a^{11} e^{11} z^3 + 43057152 a^7 c d^4 e^6 z^2 \\
& + 11599872 a^6 c^2 d^8 e^2 z^2 + 100663296 a^8 e^{10} z^2 + 9652224 a^4 c \\
& d^4 e^5 z + 2709504 a^3 c^2 d^8 e z + 16777216 a^5 e^9 z + 676881 a * c * \\
& d^4 e^4 + 194481 c^2 d^8 + 1048576 a^2 e^8, z, k) * (\text{root}(805306368 a^{12} c^2 d^8 e^4 z^4 \\
& + 805306368 a^{13} c d^4 e^8 z^4 + 268435456 a^{11} c^3 d^{12} z^4 + 268435456 a^{14} e^{12} z^4 \\
& + 268435456 a^{11} e^{11} z^3 + 43057152 a^7 c d^4 e^6 z^2 + 11599872 a^6 c^2 d^8 e^2 z^2 \\
& + 100663296 a^8 e^{10} z^2 + 9652224 a^4 c d^4 e^5 z + 2709504 a^3 c^2 d^8 e z + 16777216 a^5 e^9 z \\
& + 676881 a * c d^4 e^4 + 194481 c^2 d^8 + 1048576 a^2 e^8, z, k) * (\text{root}(805306368 a^{12} c^2 d^8 e^4 z^4 \\
& + 805306368 a^{13} c d^4 e^8 z^4 + 268435456 a^{11} c^3 d^{12} z^4 + 268435456 a^{14} e^{12} z^4 \\
& + 268435456 a^{11} e^{11} z^3 + 43057152 a^7 c d^4 e^6 z^2 + 11599872 a^6 c^2 d^8 e^2 z^2 \\
& + 100663296 a^8 e^{10} z^2 + 9652224 a^4 c d^4 e^5 z + 2709504 a^3 c^2 d^8 e z + 16777216 a^5 e^9 z \\
& + 676881 a * c d^4 e^4 + 194481 c^2 d^8 + 1048576 a^2 e^8, z, k) * ((402653184 a^{15} c^4 d e^{22} - 134 \\
& 217728 a^{10} c^9 d^{21} e^2 - 134217728 a^{11} c^8 d^{17} e^6 + 805306368 a^{12} c^7 \\
& d^{13} e^{10} + 1879048192 a^{13} c^6 d^9 e^{14} + 1476395008 a^{14} c^5 d^5 e^{18}) / (\\
& 1048576 * (a^{12} e^{16} + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 \\
& + 6 a^{10} c^2 d^8 e^8)) + (x * (335544320 a^{15} c^4 e^{23} - 201326592 a^{10} c^9 d^{20} e^3 \\
& - 469762048 a^{11} c^8 d^{16} e^7 + 134217728 a^{12} c^7 d^{12} e^{11} + 1207 \\
& 959552 a^{13} c^6 d^8 e^{15} + 1140850688 a^{14} c^5 d^4 e^{19})) / (1048576 * (a^{12} e^{16} \\
& + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (211288064 a^{12} c^4 d e^{21} \\
& - 11010048 a^7 c^9 d^{21} e + 20447232 a^8 c^8 d^{17} e^5 + 204472320 a^9 c^7 d^{13} e^9 + 514850816 a^{10} c^6 d^9 e^{13} \\
& + 553123840 a^{11} c^5 d^5 e^{17}) / (1048576 * (a^{12} e^{16} + a^8 c^4 d^{16} + 4 a^{11} c \\
& d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (x * (251658240 a^{12} c^4 e^{22} \\
& - 28311552 a^7 c^9 d^{20} e^2 - 67108864 a^8 c^8 d^{16} e^6 + 18874368 a^9 c^7 d^{12} e^{10} \\
& + 377487360 a^{10} c^6 d^8 e^{14} + 571473920 a^{11} c^5 d^4 e^{18})) / (1048576 * (a^{12} e^{16} \\
& + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (36962304 a^9 c^4 d e^{20} \\
& + 11010048 a^5 c^8 d^{17} e^4 + 57999360 a^6 c^7 d^{13} e^8 + 138805248 a^7 c^6 d^9 e^{12} + 1413 \\
& 61152 a^8 c^5 d^5 e^{16}) / (1048576 * (a^{12} e^{16} + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} \\
& + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (x * (62914560 a^9 c^4 e^{21} - 1806336 a^4 c^9 d^{20} e \\
& + 2670592 a^5 c^8 d^{16} e^5 + 43032576 a^6 c^7 d^{12} e^9 + 143179776 a^7 c^6 d^8 e^{13} \\
& + 171732992 a^8 c^5 d^4 e^{17})) / (1048576 * (a^{12} e^{16} + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} \\
& + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (4030464 a^6 c^4 d e^{19} + 576576 a^2 c^8 d^{17} e^3 \\
& + 5061824 a^3 c^7 d^{13} e^7 + 15959232 a^4 c^6 d^9 e^{11} + 17863744 a^5 c^5 d^5 e^{15}) / (1048576 * (a^{12} e^{16} \\
& + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8)) + (x * (5242880 a^6 c^4 e^{20} \\
& + 755136 a^2 c^8 d^{16} e^4 + 6023488 a^3 c^7 d^{12} e^8 + 19579200 a^4 c^6 d^8 e^{12} + 22240704 a^5 c^5 \\
& d^4 e^{16})) / (1048576 * (a^{12} e^{16} + a^8 c^4 d^{16} + 4 a^{11} c d^4 e^{12} + 4 a^9 c^3 d^{12} e^4 + 6 a^{10} c^2 d^8 e^8))
\end{aligned}$$

$$\begin{aligned}
& 9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(194481*c^7*d^12*e^7 + 871362*a \\
& *c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^1 \\
& 6 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) * \text{root}(805 \\
& 306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11 \\
& *c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305715 \\
& 2*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 \\
& + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + \\
& 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + (\\
& (3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3 \\
& *d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x \\
& ^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4) \\
&) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 + \\
& 19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d \\
& ^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^ \\
& 2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4) \\
&) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a \\
& *c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^11*log(d + e*x))/(a^3*e^12 + \\
& c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)

[Out] Timed out

$$3.409 \quad \int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$$

Optimal. Leaf size=1830

result too large to display

Rubi [A] time = 2.78, antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + c*x^4)^3), x]

[Out] $-(e^{11}/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (\text{Sqrt}[c]*d*e^9*(5*c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(c*d^4 + a*e^4)^4) - (\text{Sqrt}[c]*d*e^5*(3*c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*\text{Sqrt}[c]*d*e*(c*d^4 - a*e^4)*\text{ArcTan}[(\text{Sqrt}[c]*x^2)/\text{Sqrt}[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(21*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + 5*\text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(64*\text{Sqrt}[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*\text{Sqrt}[c]*d^2*(3*c*d^4 - a*e^4) + \text{Sqrt}[a]*e^2*(11*c*d^4 - a*e^4))*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) + 5*\text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(64*\text{Sqrt}[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) + \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*\text{Sqrt}[c]*d^2*(3*c*d^4 - a*e^4) + \text{Sqrt}[a]*e^2*(11*c*d^4 - a*e^4))*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^(1/4)*x)/a^(1/4)])/(2*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*\text{Sqrt}[a]*c*d^4*e^2 - 3*a*\text{Sqrt}[c]*d^2*e^4 + a^(3/2)*e^6)*Log[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*(21*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - 5*\text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*Log[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*c^(1/4)*x + \text{Sqrt}[c]*x^2])/(128*\text{Sqrt}[2]*a^(11/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) - \text{Sqrt}[a]$

$$\begin{aligned} & *e^{2*(7*c*d^4 - a*e^4)} * \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] / (16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^3) + (c^{(1/4)}*e^8*(9*c^{(3/2)}*d^6 - \\ & 11*\text{Sqrt}[a]*c*d^4*e^2 - 3*a*\text{Sqrt}[c]*d^2*e^4 + a^{(3/2)}*e^6)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] / (4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^4) \\ & + (c^{(1/4)}*(21*\text{Sqrt}[c]*d^2*(c*d^4 - 3*a*e^4) - 5*\text{Sqrt}[a]*e^2*(3*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] / (128*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)^2) \\ & + (c^{(1/4)}*e^4*(3*\text{Sqrt}[c]*d^2*(5*c*d^4 - 3*a*e^4) - \text{Sqrt}[a]*e^2*(7*c*d^4 - a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2] / (16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^{11}*\text{Log}[a + c*x^4]) / (c*d^4 + a*e^4)^4 \end{aligned}$$
Rule 204

$$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(x_.)^2}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\frac{\text{Rt}[-b, 2]*x}{\text{Rt}[-a, 2]}] / (\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 205

$$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^2}{(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$
Rule 260

$$\text{Int}[\frac{(x_.)^m}{(a_.) + (b_.)*(x_.)^n}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$$
Rule 275

$$\text{Int}[(x_.)^m * ((a_.) + (b_.)*(x_.)^n)^p, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m + 1)/k - 1)*(a + b*x^{(n/k)})^p}, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$$
Rule 617

$$\text{Int}[\frac{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$
Rule 628

$$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1165

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 1168

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 1854

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq, x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q, x])*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]* (a + b*x^n)^(p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

Mathematica [A] time = 1.55, size = 1115, normalized size = 0.61

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]

[Out]
$$\begin{aligned} &((-256e^{11}(cd^4 + ae^4))/(d + ex) + (8c(cd^4 + ae^4)(c^2d^8x(7d^2 - 12de^x + 15e^2x^2) + 2accd^4e^4x(13d^2 - 24de^x + 33e^2x^2) + a^2e^7(64d^3 - 45d^2e^x + 28de^2x^2 - 13e^3x^3)))/(a^2(a + cx^4)) + (32c(cd^4 + ae^4)^2(cd^4x(d^2 - 2de^x + 3e^2x^2) + ae^3(4d^3 - 3d^2e^x + 2de^2x^2 - e^3x^3)))/(a(a + cx^4)^2) - (6c^{1/4}(7\sqrt{2}c^{7/2}d^{14} - 16a^{1/4}c^{13/4}d^{13}e + 5\sqrt{2}Sqrt[a]c^3d^{12}e^2 + 33\sqrt{2}a^{5/2}d^{10}e^4 - 80a^{5/4}c^{9/4}d^9e^5 + 27\sqrt{2}a^{3/2}c^2d^8e^6 + 77\sqrt{2}a^2c^{3/2}d^6e^8 - 240a^{9/4}c^{5/4}d^5e^9 + 135\sqrt{2}a^{5/2}cd^4e^{10} - 77\sqrt{2}a^3Sqrt[c]d^2e^{12} + 80a^{13/4}c^{1/4}de^{13} - 15\sqrt{2}a^{7/2}e^{14}) * ArcTan[1 - (Sqrt[2]c^{1/4}x)/a^{1/4}])/a^{11/4} + (6c^{1/4}(7\sqrt{2}c^{7/2}d^{14} + 16a^{1/4}c^{13/4}d^{13}e + 5\sqrt{2}Sqrt[a]c^3d^{12}e^2 + 33\sqrt{2}a^{5/2}d^{10}e^4 + 80a^{5/4}c^{9/4}d^9e^5 + 27\sqrt{2}a^{3/2}c^2d^8e^6 + 77\sqrt{2}a^2c^{3/2}d^6e^8 + 240a^{9/4}c^{5/4}d^5e^9 + 135\sqrt{2}a^{5/2}cd^4e^{10} - 77\sqrt{2}a^3Sqrt[c]d^2e^{12} - 80a^{13/4}c^{1/4}de^{13} - 15\sqrt{2}a^{7/2}e^{14}) * ArcTan[1 + (Sqrt[2]c^{1/4}x)/a^{1/4}])/a^{11/4} + 3072cd^3e^{11}Log[d + ex] - (3\sqrt{2}c^{1/4}(7c^{7/2}d^{14} - 5\sqrt{2}Sqrt[a]c^3d^{12}e^2 + 33ac^{5/2}d^{10}e^4 - 27a^{3/2}c^2d^8e^6 + 77a^2c^{3/2}d^6e^8 - 135a^{5/2}cd^4e^{10} - 77a^3Sqrt[c]d^2e^{12} + 15a^{7/2}e^{14}) * Log[Sqrt[a] - Sqrt[2]a^{1/4}c^{1/4}x + Sqrt[c]x^2])/a^{11/4} + (3\sqrt{2}c^{1/4}(7c^{7/2}d^{14} - 5\sqrt{2}Sqrt[a]c^3d^{12}e^2 + 33ac^{5/2}d^{10}e^4 - 27a^{3/2}c^2d^8e^6 + 77a^2c^{3/2}d^6e^8 - 135a^{5/2}cd^4e^{10} - 77a^3Sqrt[c]d^2e^{12} + 15a^{7/2}e^{14}) * Log[Sqrt[a] + Sqrt[2]a^{1/4}c^{1/4}x + Sqrt[c]x^2])/a^{11/4}) - 768cd^3e^{11}Log[a + cx^4])/(256*(cd^4 + ae^4)^4) \end{aligned}$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^2 (a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^3),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.03, size = 2769, normalized size = 1.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x)

[Out]
$$12*c*d^3*e^{11}\ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^{11}\ln(c*x^4+a)/(a*e^4+c*d^4)^4-e^{11}/(a*e^4+c*d^4)^3/(e*x+d)+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^{10}*e^4+231/256*c^2/(a*e^4+c*d^4)^4/a*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^6*e^8+1/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^3*d^{11}+99/256*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^{10}*e^4+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{10}*e^4+81/256*c^2/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^8*e^6+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^8*e^6+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^{12}*e^2+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{12}*e^2+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^8*e^6+15/256*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^{(1/4)}*2^{(1/2)}*\ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))$$

$$\begin{aligned}
&) * x + (a/c)^{(1/2)}) * d^{12} * e^{2+5/2 * c / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{11 * d^3 * a^2 - 1} \\
& 7/32 * c / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{14 * a^2 * x^3 + 11/32 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{14} / a * x^2 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * x^4 * d^7 * e^{7-13/32 * c^2} \\
& / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{14 * a * x^7 + 7/32 * c^5 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{14} / a^2 * x^5 + 3 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{7 * d^7 * a + 101/32 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{6 * x^3 * d^8 - 17/8 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^9} \\
& * e^5 * x^2 + 29/32 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{10} * x * e^4 + 53/32 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{10 * x^7 * d^4 - 5/8 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^5 * e^9 * x^6 - 19/32 * c^3 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^6 * x^5 * e^8 - 45/8 * c^2 / (a * e^4 + c * d^4)^4 / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) * e^9 * d^5 - 45/256 / (a * e^4 + c * d^4)^4 * a / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * e^{14 - 45/128 / (a * e^4 + c * d^4)^4 * a / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * e^{14 - 45/128 / (a * e^4 + c * d^4)^4 * a / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * e^{14 + 9/8 * c / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d * e^{13 * a^2 * x^2 - 57/32 * c / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^2 * a^2 * x * e^{12 - 15/8 * c^3 / (a * e^4 + c * d^4)^4 / a / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) * e^5 * d^9 - 3/8 * c^4 / (a * e^4 + c * d^4)^4 / a^2 / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) * e * d^{13} + 405/256 * c / (a * e^4 + c * d^4)^4 / (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^4 * e^{10} + 405/128 * c / (a * e^4 + c * d^4)^4 / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^4 * e^{10} + 405/128 * c / (a * e^4 + c * d^4)^4 / (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^4 * e^{10} + 27/32 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^2 / a * x^3 * d^{12} - 3/8 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^5 * e^9 * a * x^2 - 5/8 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{13} * e / a * x^2 - 39/32 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^6 * a * x * e^8 + 2 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * x^4 * a * d^3 * e^{11} + 81/32 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^6 / a * x^7 * d^8 + 15/32 * c^5 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^2 / a^2 * x^7 * d^{12} + 7/8 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d * e^{13 * a * x^6 - 15/8 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^9 * e^5 / a * x^6 - 3/8 * c^5 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{13} * e / a^2 * x^6 - 45/32 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^2 * a * x^5 * e^{12} + 33/32 * c^4 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * d^{10} / a * x^5 * e^4 + 57/32 * c^2 / (a * e^4 + c * d^4)^4 / (c * x^4 + a)^2 * e^{10} * a * x^3 * d^4 + 15/8 * c / (a * e^4 + c * d^4)^4 * a / (a * c)^{(1/2)} * \arctan((1/a * c)^{(1/2)} * x^2) * e^{13 * d - 23} \\
& 1/128 * c / (a * e^4 + c * d^4)^4 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^2 * e^{12} + 21/128 * c^4 / (a * e^4 + c * d^4)^4 / a^3 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x - 1) * d^{14} - 231/256 * c / (a * e^4 + c * d^4)^4 * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^2 * e^{12} + 21/256 * c^4 / (a * e^4 + c * d^4)^4 / a^3 * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^14 - 231/128 * c / (a * e^4 + c * d^4)^4 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^2 * e^{12} + 21/128 * c^4 / (a * e^4 + c * d^4)^4 / a^3 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^{14}
\end{aligned}$$

maxima [A] time = 3.10, size = 1564, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")

[Out] $12*c*d^3*e^{11}*\log(e*x + d)/(c^4*d^{16} + 4*a*c^3*d^{12}*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^{12} + a^4*e^{16}) - 3/256*c*(\sqrt{2}*(128*\sqrt{2})*a^{(11/4)}*c^{(5/4)}*d^3*e^{11} - 7*c^4*d^{14} + 5*\sqrt{a}*c^{(7/2)}*d^{12}*e^2 - 33*a*c^3*d^{10}*e^4 + 27*a^{(3/2)}*c^{(5/2)}*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^{(5/2)}*c^{(3/2)}*d^4*e^{10} + 77*a^3*c*d^2*e^{12} - 15*a^{(7/2)}*\sqrt{c}*e^{14})*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) + \sqrt{2}*(128*\sqrt{2})*a^{(11/4)}*c^{(5/4)}*d^3*e^{11} + 7*c^4*d^{14} - 5*\sqrt{a}*c^{(7/2)}*d^{12}*e^2 + 33*a*c^3*d^{10}*e^4 - 27*a^{(3/2)}*c^{(5/2)}*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 135*a^{(5/2)}*c^{(3/2)}*d^4*e^{10} - 77*a^3*c*d^2*e^{12} + 15*a^{(7/2)}*\sqrt{c}*e^{14})*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{(1/4)}*c^{(1/4)}*x + \sqrt{a})/(a^{(3/4)}*c^{(5/4)}) - 2*(7*\sqrt{2})*a^{(1/4)}*c^{(17/4)}*d^{14} + 5*\sqrt{2})*a^{(3/4)}*c^{(15/4)}*d^{12}*e^2 + 33*\sqrt{2})*a^{(5/4)}*c^{(13/4)}*d^{10}*e^4 + 27*\sqrt{2})*a^{(7/4)}*c^{(11/4)}*d^8*e^6 + 77*\sqrt{2})*a^{(9/4)}*c^{(9/4)}*d^6*e^8 + 135*\sqrt{2})*a^{(11/4)}*c^{(7/4)}*d^4*e^{10} - 77*\sqrt{2})*a^{(13/4)}*c^{(5/4)}*d^2*e^{12} - 15*\sqrt{2})*a^{(15/4)}*c^{(3/4)}*e^{14} + 16*\sqrt{a})*c^4*d^{13}*e + 80*a^{(3/2)}*c^3*d^9*e^5 + 240*a^{(5/2)}*c^2*d^5*e^9 - 80*a^{(7/2)}*c*d*e^{13})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c})*x + \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{(5/4)}) - 2*(7*\sqrt{2})*a^{(1/4)}*c^{(17/4)}*d^{14} + 5*\sqrt{2})*a^{(3/4)}*c^{(15/4)}*d^{12}*e^2 + 33*\sqrt{2})*a^{(5/4)}*c^{(13/4)}*d^{10}*e^4 + 27*\sqrt{2})*a^{(7/4)}*c^{(11/4)}*d^8*e^6 + 77*\sqrt{2})*a^{(9/4)}*c^{(9/4)}*d^6*e^8 + 135*\sqrt{2})*a^{(11/4)}*c^{(7/4)}*d^4*e^{10} - 77*\sqrt{2})*a^{(13/4)}*c^{(5/4)}*d^2*e^{12} - 15*\sqrt{2})*a^{(15/4)}*c^{(3/4)}*e^{14} - 16*\sqrt{a})*c^4*d^{13}*e - 80*a^{(3/2)}*c^3*d^9*e^5 - 240*a^{(5/2)}*c^2*d^5*e^9 + 80*a^{(7/2)}*c*d*e^{13})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c})*x - \sqrt{2})*a^{(1/4)}*c^{(1/4)})/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a}*\sqrt{c}})*c^{(5/4)})/(a^2*c^4*d^{16} + 4*a^3*c^3*d^{12}*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^{12} + a^6*e^{16}) + 1/32*(16*a^2*c^2*d^8*e^3 + 80*a^3*c*d^4*e^7 - 32*a^4*e^{11} + 3*(5*c^4*d^8*e^3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2*e^{11})*x^8 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*e^6 + 5*a^2*c^2*d*e^{10})*x^7 - (5*c^4*d^{10}*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^{11} + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 + 3*(9*a*c^3*d^8*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^{11})*x^4 + (7*a*c^3*d^9*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^{10})*x^3 - 3*(3*a*c^3*d^{10}*e + 10*a^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^{11} + 34*a^2*c^2*d^7*e^4 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^{13} + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8 + a^7*d*e^{12} + (a^2*c^5*d^{12}*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a^5*c^2*e^{13})*x^9 + (a^2*c^5*d^{13} + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 + a^5*c^2*d*e^{12})*x^8 + 2*(a^3*c^4*d^{12}*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4*e^9 + a^6*c*d*e^{13})*x^5 + 2*(a^3*c^4*d^{13} + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^5*e^8 + a^6*c*d*e^{12})*x^4 + (a^4*c^3*d^{12}*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d^4*e^9 + a^7*e^{13})*x)$

mupad [B] time = 5.75, size = 3572, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((a + c*x^4)^3*(d + e*x)^2),x)$

[Out] $\text{symsum}(\log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^5*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(a^{14}*e^{24} + a^8*c^6*d^{24} + 6*a^{13}*c*d^4*e^{20} + 6*a^9*c^5*d^{20}*e^4 + 15*a^{10}*c^4*d^{16}*e^8 + 20*a^{11}*c^3*d^{12}*e^{12} + 15*a^{12}*c^2*d^8*e^{16}))) + \text{root}(1610612736*a^{13}*c^2*d^8*e^8*z^4 + 1073741824*a^{12}*c^3*d^{12}*e^4*z^4 + 1073741824*a^{14}*c*d^4*e^{12}*z^4 + 268435456*a^{11}*c^4*d^{16}*z^4 + 268435456*a^{15}*e^{16}*z^4 + 3221225472*a^{11}*c*d^3*e^{11}*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^{10}*e^2*z^2 + 1153105920*a^8*c*d^2*e^{10}*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(\text{root}(1610612736*a^{13}*c^2*d^8*e^8*z^4 + 1073741824*a^{12}*c^3*d^{12}*e^4*z^4 + 1073741824*a^{14}*c*d^4*e^{12}*z^4 + 268435456*a^{11}*c^4*d^{16}*z^4 + 268435456*a^{15}*e^{16}*z^4 + 3221225472*a^{11}*c*d^3*e^{11}*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^{10}*e^2*z^2 + 1153105920*a^8*c*d^2*e^{10}*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(\text{root}(1610612736*a^{13}*c^2*d^8*e^8*z^4 + 1073741824*a^{12}*c^3*d^{12}*e^4*z^4 + 1073741824*a^{14}*c*d^4*e^{12}*z^4 + 268435456*a^{11}*c^4*d^{16}*z^4 + 268435456*a^{15}*e^{16}*z^4 + 3221225472*a^{11}*c*d^3*e^{11}*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^{10}*e^2*z^2 + 1153105920*a^8*c*d^2*e^{10}*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*((23592960*a^{14}*c^4*e^{29} - 11010048*a^7*c^{11}*d^{28}*e + 33030144*a^8*c^{10}*d^{24}*e^5 + 504889344*a^9*c^9*d^{20}*e^9 + 3103260672*a^{10}*c^8*d^{16}*e^{13} + 6799491072*a^{11}*c^7*d^{12}*e^{17} + 6101139456*a^{12}*c^6*d^8*e^{21} + 1967652864*a^{13}*c^5*d^4*e^{25}))/1048576*(a^{14}*e^{24} + a^8*c^6*d^{24} + 6*a^{13}*c*d^4*e^{20} + 6*a^9*c^5*d^{20}*e^4 + 15*a^{10}*c^4*d^{16}*e^8 + 20*a^{11}*c^3*d^{12}*e^{12} + 15*a^{12}*c^2*d^8*e^{16}))) + \text{root}(1610612736*a^{13}*c^2*d^8*e^8*z^4 + 1073741824*a^{12}*c^3*d^{12}*e^4*z^4 + 1073741824*a^{14}*c*d^4*e^{12}*z^4 + 268435456*a^{11}*c^4*d^{16}*z^4 + 268435456*a^{15}*e^{16}*z^4 + 3221225472*a^{11}*c*d^3*e^{11}*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^{10}*e^2*z^2 + 1153105920*a^8*c*d^2*e^{10}*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*((402653184*a^{17}*c^4*d*e^{30} - 134217728*a^{10}*c^{11}*d^{29}*e^2 - 402653184*a^{11}*c^{10}*d^{25}*e^6 + 402653184*a^{12}*c^9*d^{21}*e^{10} + 3355443200*a^{13}*c^8*d^{17}*e^{14} + 6039797760*a^{14}*c^7*d^{13}*e^{18} + 5234491392*a^{15}*c^6*d^9*e^{22} + 2281701376*a^{16}*c^5*d^5*e^{26}))/1048576*(a^{14}*e^{24} + a^8*c^6*d^{24} + 6*a^{13}*c*d^4*e^{20} + 6*a^9*c^5*d^{20}*e^4 + 15*a^{10}*c^4*d^{16}*e^8 + 20*a^{11}*c^3*d^{12}*e^{12} + 15*a^{12}*c^2*d^8*e^{16}))) + (x*(335544320*a^{17}*c^4*e^{31} - 201326592*a^{10}*c^{11}*d^{28}*e^3 - 872415232*a^{11}*c^{10}*d^{24}*e^7 - 1006632960*a^{12}*c^9*d^{20}*e^{11} + 1006632960*a^{13}*c^8*d^{16}*e^{15} + 3690987520*a^{14}*c^7*d^{12}*e^{19} + 3825205248*a^{15}*c^6*d^8*e^{23} + 1811939328*a^{16}*c^5*d^4*e$

$$\begin{aligned}
& \text{^27)})/(1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x*(2554331136a^{10}c^8d^{15}e^{14} - 144703488a^8c^{10}d^{23}e^6 - 154140672a^9c^9d^{19}e^{10} - 34603008a^7c^{11}d^{27}e^2 + 7659847680a^{11}c^7d^{11}e^{18} + 7556038656a^{12}c^6d^7e^{22} + 2494562304a^{13}c^5d^3e^{26}))/ \\
& ((1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16}))) + (12681216a^5c^{10}d^{23}e^4 + 127107072a^6c^9d^{19}e^8 + 674168832a^7c^8d^{15}e^{12} + 1018626048a^8c^7d^{11}e^{16} - 446201856a^9c^6d^7e^{20} + 906854400a^{10}c^5d^3e^{24}))/ \\
& (1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x*(516096a^5c^{10}d^{22}e^5 - 1806336a^4c^{11}d^{26}e + 90427392a^6c^9d^{18}e^9 + 896090112a^7c^8d^{14}e^{13} + 1960906752a^8c^7d^{10}e^{17} + 1732829184a^9c^6d^6e^{21} + 1183887360a^{10}c^5d^2e^{25}))/ \\
& (1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (387072a^2c^{10}d^{22}e^3 + 8004096a^3c^9d^{18}e^7 + 49379328a^4c^8d^{14}e^{11} + 49572864a^5c^7d^{10}e^{15} - 156930048a^6c^6d^6e^{19} + 125452800a^7c^5d^2e^{23}))/ \\
& (1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16})) + (x*(126360000a^7c^5d^5e^{24} + 561600a^2c^{10}d^{21}e^4 + 9609408a^3c^9d^{17}e^8 + 75731328a^4c^8d^{13}e^{12} + 114991488a^5c^7d^9e^{16} - 80136000a^6c^6d^5e^{20}))/ \\
& (1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16}))) + (x*(4100625a^4c^5e^{23} + 194481c^9d^{16}e^7 + 1527012a^2c^8d^{12}e^{11} - 167994a^2c^7d^8e^{15} - 13988700a^3c^6d^4e^{19}))/ \\
& (1048576*(a^{14}e^{24} + a^8c^6d^{24} + 6a^{13}c*d^4e^{20} + 6a^9c^5d^{20}e^4 + 15a^{10}c^4d^{16}e^8 + 20a^{11}c^3d^{12}e^{12} + 15a^{12}c^2d^8e^{16}))) * \text{root}(1610612736a^{13}c^2d^8e^8z^4 + 1073741824a^{12}c^3d^{12}e^4z^4 + 1073741824a^{14}c*d^4e^{12}z^4 + 268435456a^{11}c^4d^{16}z^4 + 268435456a^{15}e^{16}z^4 + 3221225472a^{11}c*d^3e^{11}z^3 + 239468544a^7c^2d^6e^6z^2 + 39518208a^6c^3d^{10}e^2z^2 + 1153105920a^8c*d^2e^{10}z^2 + 32071680a^4c^2d^5e^5z + 5419008a^3c^3d^9e^z + 124416000a^5c*d^9e^z + 1138050a^2c^2d^4e^4 + 4100625a^2c^2e^8 + 194481c^3d^8, z, k), k, 1, 4) + ((c^2d^8e^3 - 2a^2e^{11} + 5a^2c^4e^7)/(2*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) + (3*x^8*(5c^4d^8e^3 - 15a^2c^2e^{11} + 22a^2c^3d^4e^7))/(32a^2*(a^3e^{12} + c^3d^{12} + 3a^2c^2d^8e^4 + 3a^2c^2d^4e^8)) + (x^5*(7c^3d^7 + 19a^2c^2d^3e^4))/(32a^2*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) - (3*x^2*(3c^2d^6e + 7a^2c^2d^2e^5))/(32a*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) + (x^3*(7c^2d^5e^2 + 19a^2c^2d^5e^6))/(32a*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) + (x*(11c^2d^7 + 23a^2c^2d^3e^4))/(32a*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) - (x^6*(5c^3d^6e + 17a^2c^2d^2e^5))/(32a^2*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) + (3e^2*x^7*(c^3d^5 + 5a^2c^2d^2e^4))/(32a^2*(a^2e^8 + c^2d^8 + 2a^2c^4e^4)) + (3e^2*x^4*(9c^3d^8e - 27a^2c^2e^9 + 46a^2c^2d^4e^5))/
\end{aligned}$$

$$(32*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(a^2*d + c^2*d*x^8 + c^2*e*x^9 + a^2*e*x + 2*a*c*d*x^4 + 2*a*c*e*x^5) + (12*c*d^3*e^11*\log(d + e*x))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)

[Out] Timed out

$$3.410 \quad \int \frac{1}{(d+ex)^3(a+cx^4)^3} dx$$

Optimal. Leaf size=2204

result too large to display

Rubi [A] time = 3.16, antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^3*(a + c*x^4)^3), x]

[Out]
$$-e^{11}/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^{11})/((c*d^4 + a*e^4)^4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13$$

$$\begin{aligned} & *c*d^4 - 3*a*e^4)*\text{Log}[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^{(3/4)}*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^5) + (c^{(3/4)}*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^4) + (c^{(3/4)}*d*(10*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(128*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)^3) + (3*c^{(3/4)}*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*(c*d^4 + a*e^4)^5) - (c^{(3/4)}*d*e^4*(4*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*(c*d^4 + a*e^4)^4) - (c^{(3/4)}*d*(10*\text{Sqrt}[a]*\text{Sqrt}[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*c^{(1/4)}*x + \text{Sqrt}[c]*x^2])/(128*\text{Sqrt}[2]*a^{(11/4)}*(c*d^4 + a*e^4)^3) - (3*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*\text{Log}[a + c*x^4])/(2*(c*d^4 + a*e^4)^5) \end{aligned}$$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
```

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_.)}{(a_.) + (b_.)(x_.) + (c_.)(x_.)^2}, x_Symbol] \ :> \ \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 635

$\text{Int}[\frac{(d_.) + (e_.)(x_.)}{(a_.) + (c_.)(x_.)^2}, x_Symbol] \ :> \ \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)(x_.)^2}{(a_.) + (c_.)(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\ \& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)(x_.)^2}{(a_.) + (c_.)(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

Rule 1168

$\text{Int}[\frac{(d_.) + (e_.)(x_.)^2}{(a_.) + (c_.)(x_.)^4}, x_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[a*c, 2]\}, \text{Dist}[(d*q + a*e)/(2*a*c), \text{Int}[(q + c*x^2)/(a + c*x^4), x], x] + \text{Dist}[(d*q - a*e)/(2*a*c), \text{Int}[(q - c*x^2)/(a + c*x^4), x], x]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[-(a*c)]$

Rule 1248

$\text{Int}[(x_.) * ((d_.) + (e_.)(x_.)^2)^{(q_.)} * ((a_.) + (c_.)(x_.)^4)^{(p_.)}, x_Symbol] \ :> \ \text{Dist}[1/2, \text{Subst}[\text{Int}[(d + e*x)^q * (a + c*x^2)^p, x], x, x^2], x] \ /; \ \text{FreeQ}[\{a, c, d, e, p, q\}, x]$

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Module[{q = Expon[Pq,
x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1})*(a + b*x^n)^(p
+ 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
+ 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

Mathematica [A] time = 2.87, size = 1338, normalized size = 0.61

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]

[Out]
$$\frac{(-128e^{11}(cd^4 + ae^4)^2)/(d + ex)^2 - (3072c^3d^3e^{11}(cd^4 + ae^4))/(d + ex) + (8c^3d^3e^{11}(-96d^2 + 45d^2ex - 14e^2x^2) + c^3d^3e^{11}x(7d^2 - 18d^2ex + 30e^2x^2) + ac^2d^7e^4x(43d^2 - 114d^2ex + 204e^2x^2) + a^2c^2d^3e^7(288d^3 - 303d^2ex + 274d^2e^2x^2 - 210e^3x^3)))/(a^2(a + cx^4)) + (32c^3d^3e^{11}(-a^2e^7(6d^2 - 3d^2ex + e^2x^2)) + c^2d^7ex(d^2 - 3d^2ex + 6e^2x^2) + 2ac^2d^3e^3(5d^3 - 6d^2ex + 6d^2e^2x^2 - 5e^3x^3)))/(a(a + cx^4)^2) - (6\sqrt{c}(7\sqrt{2}c^{17/4}d^{17} - 24a^{1/4}c^4d^{16}e + 10\sqrt{2}c^{15/4}d^{15}e^2 + 50\sqrt{2}c^{13/4}d^{13}e^4 - 176a^{5/4}c^3d^{12}e^5 + 78\sqrt{2}c^{11/4}d^{11}e^6 + 220\sqrt{2}c^9d^9e^8 - 960a^{9/4}c^2d^8e^9 + 702\sqrt{2}c^{7/4}d^7e^{10} - 770\sqrt{2}c^{5/4}d^5e^{12} + 1200a^{13/4}c^3d^4e^{13} - 390\sqrt{2}c^{7/2}d^3e^{14} + 77\sqrt{2}c^4d^4e^{16} - 40a^{17/4}e^{17})\text{ArcTan}[1 - (\sqrt{2}c^{1/4}x)/a^{1/4}])/a^{11/4} + (6\sqrt{c}(7\sqrt{2}c^{17/4}d^{17} + 24a^{1/4}c^4d^{16}e + 10\sqrt{2}c^{15/4}d^{15}e^2 + 50\sqrt{2}c^{13/4}d^{13}e^4 + 176a^{5/4}c^3d^{12}e^5 + 78\sqrt{2}c^{11/4}d^{11}e^6 + 220\sqrt{2}c^9d^9e^8 + 960a^{9/4}c^2d^8e^9 + 702\sqrt{2}c^{7/4}d^7e^{10} - 770\sqrt{2}c^{5/4}d^5e^{12} - 1200a^{13/4}c^3d^4e^{13} - 390\sqrt{2}c^{7/2}d^3e^{14} + 77\sqrt{2}c^4d^4e^{16} + 40a^{17/4}e^{17})\text{ArcTan}[1 + (\sqrt{2}c^{1/4}x)/a^{1/4}])/a^{11/4} + 1536c^2d^2e^{11}(13c^3d^4 - 3a^3e^4)\text{Log}[d + ex] - (3\sqrt{2}c^{3/4}(7c^4d^{17} - 10\sqrt{a}c^{7/2}d^{15}e^2 + 50ac^3d^{13}e^4 - 78a^{3/2}c^{5/2}d^{11}e^6 + 220a^2c^2d^9e^8 - 702a^{5/2}c^{3/2}d^7e^{10} - 770a^3c^3d^5e^{12} + 390a^{7/2}\sqrt{c}d^3e^{14} + 77a^4d^4e^{16})\text{Log}[\sqrt{a} - \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{11/4} + (3\sqrt{2}c^{3/4}(7c^4d^{17} - 10\sqrt{a}c^{7/2}d^{15}e^2 + 50ac^3d^{13}e^4 - 78a^{3/2}c^{5/2}d^{11}e^6 + 220a^2c^2d^9e^8 - 702a^{5/2}c^{3/2}d^7e^{10} - 770a^3c^3d^5e^{12} + 390a^{7/2}\sqrt{c}d^3e^{14} + 77a^4d^4e^{16})\text{Log}[\sqrt{a} + \sqrt{2}a^{1/4}c^{1/4}x + \sqrt{c}x^2])/a^{11/4} - 384c^2d^2e^{11}(13c^3d^4 - 3a^3e^4)\text{Log}[a + cx^4))/(256(c^3d^4 + a^3e^4)^5)$$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d + ex)^3 (a + cx^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^3),x]

[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^3), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.53, size = 2119, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & \frac{3}{64} * (23 * \sqrt{2} * a * c^4 * d^6 * e - 65 * (a * c^3)^{(1/4)} * a * c^3 * d^5 * e^2 + 30 * \sqrt{2} * \\ & \sqrt{a * c} * a * c^3 * d^4 * e^3 - 7 * (a * c^3)^{(3/4)} * c^2 * d^7 - 115 * \sqrt{2} * a^2 * c^3 * d^2 * \\ & e^5 + 65 * (a * c^3)^{(3/4)} * a * c * d^3 * e^4 + 123 * (a * c^3)^{(1/4)} * a^2 * c^2 * d * e^6 + 20 * \\ & \sqrt{2} * \sqrt{a * c} * a^2 * c^2 * e^7) * \arctan(1/2 * \sqrt{2} * (2 * x + \sqrt{2} * (a/c)^{(1/4)})) / (a/c)^{(1/4)} / (\sqrt{2} * \sqrt{a * c} * a^3 * c^4 * d^{10} - 25 * \sqrt{2} * a^4 * c^4 * d^8 * e^2 + 10 * (a * c^3)^{(3/4)} * a^3 * c^2 * d^9 * e + 80 * (a * c^3)^{(1/4)} * a^4 * c^3 * d^7 * e^3 + 90 * \sqrt{2} * \sqrt{a * c} * a^4 * c^3 * d^6 * e^4 - 90 * \sqrt{2} * a^5 * c^3 * d^4 * e^6 + 148 * (a * c^3)^{(3/4)} * a^4 * c * d^5 * e^5 + 80 * (a * c^3)^{(1/4)} * a^5 * c^2 * d^3 * e^7 + 25 * \sqrt{2} * \sqrt{a * c} * a^5 * c^2 * d^2 * e^8 - \sqrt{2} * a^6 * c^2 * e^{10} + 10 * (a * c^3)^{(3/4)} * a^5 * d * e^9) - \\ & \frac{3}{64} * (23 * \sqrt{2} * a * c^4 * d^6 * e + 65 * (a * c^3)^{(1/4)} * a * c^3 * d^5 * e^2 - 30 * \sqrt{2} * \\ & \sqrt{a * c} * a * c^3 * d^4 * e^3 + 7 * (a * c^3)^{(3/4)} * c^2 * d^7 - 115 * \sqrt{2} * a^2 * c^3 * d^2 * e^5 - 65 * (a * c^3)^{(3/4)} * a * c * d^3 * e^4 - 123 * (a * c^3)^{(1/4)} * a^2 * c^2 * d * e^6 - 20 * \\ & \sqrt{2} * \sqrt{a * c} * a^2 * c^2 * e^7) * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (a/c)^{(1/4)})) / (a/c)^{(1/4)} / (\sqrt{2} * \sqrt{a * c} * a^3 * c^4 * d^{10} - 25 * \sqrt{2} * a^4 * c^4 * d^8 * e^2 - 10 * (a * c^3)^{(3/4)} * a^3 * c^2 * d^9 * e - 80 * (a * c^3)^{(1/4)} * a^4 * c^3 * d^7 * e^3 + 90 * \sqrt{2} * \sqrt{a * c} * a^4 * c^3 * d^6 * e^4 - 90 * \sqrt{2} * a^5 * c^3 * d^4 * e^6 - 148 * (a * c^3)^{(3/4)} * a^4 * c * d^5 * e^5 - 80 * (a * c^3)^{(1/4)} * a^5 * c^2 * d^3 * e^7 + 25 * \sqrt{2} * \sqrt{a * c} * a^5 * c^2 * d^2 * e^8 - \sqrt{2} * a^6 * c^2 * e^{10} - 10 * (a * c^3)^{(3/4)} * a^5 * d * e^9) \\ & + \frac{3}{256} * (7 * \sqrt{2} * (a * c^3)^{(1/4)} * c^5 * d^{17} - 10 * \sqrt{2} * (a * c^3)^{(3/4)} * c^3 * d^{15} * e^2 + 50 * \sqrt{2} * (a * c^3)^{(1/4)} * a * c^4 * d^{13} * e^4 - 78 * \sqrt{2} * (a * c^3)^{(3/4)} * a * c^2 * d^{11} * e^6 + 220 * \sqrt{2} * (a * c^3)^{(1/4)} * a^2 * c^3 * d^9 * e^8 - 702 * \sqrt{2} * (a * c^3)^{(3/4)} * a^2 * c * d^7 * e^{10} - 770 * \sqrt{2} * (a * c^3)^{(1/4)} * a^3 * c^2 * d^5 * e^{12} + 390 * \sqrt{2} * (a * c^3)^{(3/4)} * a^3 * d^3 * e^{14} + 77 * \sqrt{2} * (a * c^3)^{(1/4)} * a^4 * c * d * e^{16}) * \log(x^2 + \sqrt{2} * x * (a/c)^{(1/4)} + \sqrt{a/c}) / (a^3 * c^6 * d^{20} + 5 * a^4 * c^5 * d^{16} * e^4 + 10 * a^5 * c^4 * d^{12} * e^8 + 10 * a^6 * c^3 * d^8 * e^{12} + 5 * a^7 * c^2 * d^4 * e^{16} + a^8 * c * e^{20}) - \frac{3}{256} * (7 * \sqrt{2} * (a * c^3)^{(1/4)} * c^5 * d^{17} - 10 * \sqrt{2} * (a * c^3)^{(3/4)} * c^3 * d^{15} * e^2 + 50 * \sqrt{2} * (a * c^3)^{(1/4)} * a * c^4 * d^{13} * e^4 - 78 * \sqrt{2} * (a * c^3)^{(3/4)} * a * c^2 * d^{11} * e^6 + 220 * \sqrt{2} * (a * c^3)^{(1/4)} * a^2 * c^3 * d^9 * e^8 - 702 * \sqrt{2} * (a * c^3)^{(3/4)} * a^2 * c * d^7 * e^{10} - 770 * \sqrt{2} * (a * c^3)^{(1/4)} * a^3 * c^2 * d^5 * e^{12} + 390 * \sqrt{2} * (a * c^3)^{(3/4)} * a^3 * d^3 * e^{14} + 77 * \sqrt{2} * (a * c^3)^{(1/4)} * a^4 * c * d * e^{16}) * \log(x^2 + \sqrt{2} * x * (a/c)^{(1/4)} + \sqrt{a/c}) / (a^3 * c^6 * d^{20} + 5 * a^4 * c^5 * d^{16} * e^4 + 10 * a^5 * c^4 * d^{12} * e^8 + 10 * a^6 * c^3 * d^8 * e^{12} + 5 * a^7 * c^2 * d^4 * e^{16} + a^8 * c * e^{20}) \end{aligned}$$

$$\begin{aligned} &)^{(3/4)} * c^3 * d^{15} * e^2 + 50 * \sqrt{2} * (a * c^3)^{(1/4)} * a * c^4 * d^{13} * e^4 - 78 * \sqrt{2} \\ & * (a * c^3)^{(3/4)} * a * c^2 * d^{11} * e^6 + 220 * \sqrt{2} * (a * c^3)^{(1/4)} * a^2 * c^3 * d^9 * e^8 - \\ & 702 * \sqrt{2} * (a * c^3)^{(3/4)} * a^2 * c * d^7 * e^{10} - 770 * \sqrt{2} * (a * c^3)^{(1/4)} * a^3 * c \\ & ^2 * d^5 * e^{12} + 390 * \sqrt{2} * (a * c^3)^{(3/4)} * a^3 * d^3 * e^{14} + 77 * \sqrt{2} * (a * c^3)^{(1/4)} * a^4 * c * d * e^{16} \\ & * \log(x^2 - \sqrt{2} * x * (a/c)^{(1/4)} + \sqrt{a/c}) / (a^3 * c^6 * d^{20} + 5 * a^4 * c^5 * d^{16} * e^4 \\ & + 10 * a^5 * c^4 * d^{12} * e^8 + 10 * a^6 * c^3 * d^8 * e^{12} + 5 * a^7 * c^2 * d^4 * e^{16} + a^8 * c * e^{20}) \\ & - 3/2 * (13 * c^2 * d^6 * e^{11} - 3 * a * c * d^2 * e^{15}) * \log(\text{abs}(c * x^4 + a)) / (c^5 * d^{20} + 5 * a * c^4 * d^{16} * e^4 \\ & + 10 * a^2 * c^3 * d^{12} * e^8 + 10 * a^3 * c^2 * d^8 * e^{12} + 5 * a^4 * c * d^4 * e^{16} + a^5 * e^{20}) \\ & + 6 * (13 * c^2 * d^6 * e^{12} - 3 * a * c * d^2 * e^{16}) * \log(\text{abs}(x * e + d)) / (c^5 * d^{20} * e + 5 * a * c^4 * d^{16} * e^5 \\ & + 10 * a^2 * c^3 * d^{12} * e^9 + 10 * a^3 * c^2 * d^8 * e^{13} + 5 * a^4 * c * d^4 * e^{17} + a^5 * e^{21}) \\ & + 1/32 * (30 * c^5 * d^{11} * x^9 * e^4 + 42 * c^5 * d^{12} * x^8 * e^3 + c^5 * d^{13} * x^7 * e^2 - 4 * c^5 * d^{14} * x^6 * e \\ & + 7 * c^5 * d^{15} * x^5 + 204 * a * c^4 * d^7 * x^9 * e^8 + 294 * a * c^4 * d^8 * x^8 * e^7 + 19 * a * c^4 * d^9 * x^7 * e^6 \\ & - 28 * a * c^4 * d^{10} * x^6 * e^5 + 97 * a * c^4 * d^{11} * x^5 * e^4 + 78 * a * c^4 * d^{12} * x^4 * e^3 + 5 * a * c^4 * d^{13} * x^3 * e^2 \\ & - 8 * a * c^4 * d^{14} * x^2 * e + 11 * a * c^4 * d^{15} * x - 594 * a^2 * c^3 * d^3 * x^9 * e^{12} - 546 * a^2 * c^3 * d^4 * x^8 * e^{11} \\ & + 35 * a^2 * c^3 * d^5 * x^7 * e^{10} - 44 * a^2 * c^3 * d^6 * x^6 * e^9 + 461 * a^2 * c^3 * d^7 * x^5 * e^8 + 586 * a^2 * c^3 * d^8 * x^4 * e^7 \\ & + 31 * a^2 * c^3 * d^9 * x^3 * e^6 - 40 * a^2 * c^3 * d^{10} * x^2 * e^5 + 79 * a^2 * c^3 * d^{11} * x * e^4 + 40 * a^2 * c^3 * d^{12} * e^3 \\ & - 30 * a^3 * c^2 * x^8 * e^{15} + 17 * a^3 * c^2 * d * x^7 * e^{14} - 20 * a^3 * c^2 * d^2 * x^6 * e^{13} - 1165 * a^3 * c^2 * d^3 * x^5 * e^{12} \\ & - 1078 * a^3 * c^2 * d^4 * x^4 * e^{11} + 47 * a^3 * c^2 * d^5 * x^3 * e^{10} - 56 * a^3 * c^2 * d^6 * x^2 * e^9 + 269 * a^3 * c^2 * d^7 * x * e^8 \\ & + 304 * a^3 * c^2 * d^8 * e^7 - 50 * a^4 * c * x^4 * e^{15} + 21 * a^4 * c * d * x^3 * e^{14} - 24 * a^4 * c * d^2 * x^2 * e^{13} \\ & - 567 * a^4 * c * d^3 * x * e^{12} - 520 * a^4 * c * d^4 * e^{11} - 16 * a^5 * e^{15}) / ((a^2 * c^4 * d^{16} + 4 * a^3 * c^3 * d^{12} * e^4 \\ & + 6 * a^4 * c^2 * d^8 * e^8 + 4 * a^5 * c * d^4 * e^{12} + a^6 * e^{16}) * (c * x^5 * e + c * d * x^4 + a * x * e + a * d)^2) \end{aligned}$$

maple [A] time = 0.04, size = 3334, normalized size = 1.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x)

[Out]
$$\begin{aligned} &-1/2 * e^{11} / (a * e^4 + c * d^4)^3 / (e * x + d)^2 - 12 * c * d^3 * e^{11} / (a * e^4 + c * d^4)^4 / (e * x + d) + \\ &65/32 * c^3 / (a * e^4 + c * d^4)^5 / a * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * \\ &x - 1) * d^9 * e^8 + 75/64 * c^4 / (a * e^4 + c * d^4)^5 / a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * \\ &x - 1) * d^{13} * e^4 + 165/64 * c^3 / (a * e^4 + c * d^4)^5 / a * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^9 * e^8 + 75/128 * c^4 / (a * e^4 + c * d^4)^5 / a^2 * (a/c)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)}) / (x^2 - (a/c)^{(1/4)} * 2^{(1/2)} * x + (a/c)^{(1/2)})) * d^{13} * e^4 + 165/32 * c^3 / (a * e^4 + c * d^4)^5 / a * (a/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/c)^{(1/4)} * x + 1) * d^9 * e^8 + 78 * e^{11} * d^6 * c^2 / (a * e^4 + c * d^4)^5 * \ln(e * x + d) + 5/4 * c^4 / (a * e^4 + c * d^4)^5 / (c * x^4 + a)^2 * e^3 * d^{14} - 39/2 * c^2 / (a * e^4 + c * d^4)^5 * \ln(c * x^4 + a) * e^{11} * d^6 - 129/16 * c^3 / (a * e^4 + c * d^4)^5 / (c * x^4 + a)^2 * d^5 * a * x^5 * e^{12} + 25/16 * c^5 / (a * e^4 + c * d^4)^5 / (c * x^4 + a)^2 * d^{13} / a * x^5 * e^4 - 125/16 * c^2 / (a * e^4 + c * d^4)^5 / (c * x^ \end{aligned}$$

$$\begin{aligned}
& 4+a)^2*d^3*e^{14*a^2*x^3-31/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^{10*a*x^3+27/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{15}*e^2/a*x^3+75/8*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{13*a^2*x^2*d^4+15/2*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*a*x^2*d^8-15/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a*x^2*d^{16}-141/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a^2*x*e^{12-85/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*a*x*e^8-3*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a^2*d^2*e^{15+6*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a*d^6*e^{11-105/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^3*e^{14*a*x^7+117/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{11}*e^6/a*x^7+15/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{15}*e^2/a^2*x^7-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^5*e^{12-1155/128*c^2/(a*e^4+c*d^4)^5*(a/c)^{(1/4)}*2^{(1/2)}*ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^5*e^{12-1155/64*c^2/(a*e^4+c*d^4)^5*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^5*e^{12+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^7*e^{10+1053/128*c^2/(a*e^4+c*d^4)^5/(a/c)^{(1/4)}*2^{(1/2)}*ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^7*e^{10+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^7*e^{10-9/16*c^5/(a*e^4+c*d^4)^5/a^2/(a*c)^{(1/2)}*arctan((1/a*c)^{(1/2)}*x^2)*e*d^{16+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^{17+21/256*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^{(1/4)}*2^{(1/2)}*ln((x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^{17+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{17+225/8*c^2/(a*e^4+c*d^4)^5*a/(a*c)^{(1/2)}*arctan((1/a*c)^{(1/2)}*x^2)*e^{13*d^4-33/8*c^4/(a*e^4+c*d^4)^5/a/(a*c)^{(1/2)}*arctan((1/a*c)^{(1/2)}*x^2)*e^5*d^{12+57/32*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^3*x*e^{16+65/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{13*a*x^6*d^4-33/8*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5/a*x^6*d^{12-9/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a^2*x^6*d^{16+45/32*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^2*x^5*e^{16-18*e^{15*d^2*c/(a*e^4+c*d^4)^5*ln(e*x+d)*a+5*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*x^6*d^8-65/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*x^5*e^8+121/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{11}*e^6*x^3-27/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5*x^2*d^{12+5/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{13}*x*e^4+9*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*d^{10}*e^7-7/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{17*a^2*x^6+7/32*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{17}/a^2*x^5+11/32*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^{17}/a*x+43/4*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^7*d^{10}*a+23/4*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{11*d^6*a^2-3/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^{10*x^7-9/16*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{17*a^3*x^2-15/4*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^{15*d^2*a^3-15/16*c/(a*e^4+c*d^4)^5*a^2/(a*c)^{(1/2)}*arctan((1/a*c)^{(1/2)}*x^2)*e^{17+9/2*c/(a*e^4+c*d^4)^5*a*ln(c*x^4+a)*e^{15*d^2-45/2*c^3/(a*e^4+c*d^4)^5/(a*c)^{(1/2)}*arctan((1/a*c)^{(1/2)}*x^2)*e^9*d^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x+1)*d^{13}*e^4+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^{(1/4)}*2^{(1/2)}*arctan(2^{(1/2)}/(a/c)^{(1/4)}*x-1)*d^{11}*e^6+117/128*c^3/(a*e^4+c*d^4)^5/a/(a/c)^{(1/4)}*2^{(1/2)}*ln((x^2-(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)})/(x^2+(a/c)^{(1/4)}*2^{(1/2)}*x+(a/c)^{(1/2)}))*d^{11}*e^6+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^{(1/4)}*2^{(1/2)}*a
\end{aligned}$$

$$\begin{aligned} & \operatorname{rctan}(2^{1/2}/(a/c)^{1/4}*x+1)*d^{11}*e^{6+15/64*c^4}/(a*e^4+c*d^4)^5/a^2/(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^{15}*e^{2+15/128*c^4}/(a*e^4+c*d^4)^5/a^2/(a/c)^{1/4} \\ & *2^{1/2}*ln((x^2-(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2})) \\ & *d^{15}*e^{2+15/64*c^4}/(a*e^4+c*d^4)^5/a^2/(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^{15}*e^{2-585/64*c^4}/(a*e^4+c*d^4)^5 \\ & *a/(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d^3*e^{14-585/128*c^4}/(a*e^4+c*d^4)^5 \\ & *a/(a/c)^{1/4} \\ & *2^{1/2}*ln((x^2-(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2}))/((x^2+(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2})) \\ & *d^3*e^{14-585/64*c^4}/(a*e^4+c*d^4)^5 \\ & *a/(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d^3*e^{14+231/128*c^4}/(a*e^4+c*d^4)^5 \\ & *a*(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x-1)*d*e^{16+231/256*c^4}/(a*e^4+c*d^4)^5 \\ & *a*(a/c)^{1/4} \\ & *2^{1/2}*ln((x^2+(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2}))/((x^2-(a/c)^{1/4}*2^{1/2}*x+(a/c)^{1/2})) \\ & *d*e^{16+231/128*c^4}/(a*e^4+c*d^4)^5 \\ & *a*(a/c)^{1/4} \\ & *2^{1/2}*\arctan(2^{1/2}/(a/c)^{1/4}*x+1)*d*e^{16} \end{aligned}$$

maxima [A] time = 3.99, size = 2198, normalized size = 1.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -3/256*c*(\sqrt{2}*(832*\sqrt{2})*a^{11/4}*c^{9/4}*d^6*e^{11} - 192*\sqrt{2})*a^{15/4} \\ & *c^{5/4}*d^2*e^{15} - 7*c^5*d^{17} + 10*\sqrt{a}*c^{9/2}*d^{15}*e^2 - 50*a*c^4 \\ & *d^{13}*e^4 + 78*a^{3/2}*c^{7/2}*d^{11}*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^{5/2} \\ & *c^{5/2}*d^7*e^{10} + 770*a^3*c^2*d^5*e^{12} - 390*a^{7/2}*c^{3/2}*d^3*e^{14} - 7 \\ & *a^4*c*d*e^{16})*\log(\sqrt{c}*x^2 + \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a}))/a^{3/4} \\ & *c^{5/4}) + \sqrt{2}*(832*\sqrt{2})*a^{11/4}*c^{9/4}*d^6*e^{11} - 192*\sqrt{2} \\ & *a^{15/4}*c^{5/4}*d^2*e^{15} + 7*c^5*d^{17} - 10*\sqrt{a}*c^{9/2}*d^{15}*e^2 + 50 \\ & *a*c^4*d^{13}*e^4 - 78*a^{3/2}*c^{7/2}*d^{11}*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a^{5/2} \\ & *c^{5/2}*d^7*e^{10} - 770*a^3*c^2*d^5*e^{12} + 390*a^{7/2}*c^{3/2}*d^3*e^{14} \\ & + 77*a^4*c*d*e^{16})*\log(\sqrt{c}*x^2 - \sqrt{2})*a^{1/4}*c^{1/4}*x + \sqrt{a} \\ &))/a^{3/4}*c^{5/4}) - 2*(7*\sqrt{2})*a^{1/4}*c^{21/4}*d^{17} + 10*\sqrt{2})*a^{3/4} \\ & *c^{19/4}*d^{15}*e^2 + 50*\sqrt{2})*a^{5/4}*c^{17/4}*d^{13}*e^4 + 78*\sqrt{2})*a^{7/4} \\ & *c^{15/4}*d^{11}*e^6 + 220*\sqrt{2})*a^{9/4}*c^{13/4}*d^9*e^8 + 702*\sqrt{2})*a^{11/4} \\ & *c^{11/4}*d^7*e^{10} - 770*\sqrt{2})*a^{13/4}*c^{9/4}*d^5*e^{12} - 390*\sqrt{2})*a^{15/4} \\ & *c^{7/4}*d^3*e^{14} + 77*\sqrt{2})*a^{17/4}*c^{5/4}*d*e^{16} + 24*\sqrt{a}*c^5*d^{16}*e \\ & + 176*a^{3/2}*c^4*d^{12}*e^5 + 960*a^{5/2}*c^3*d^8*e^9 - 1200*a^{7/2}*c^2*d^4*e^{13} \\ & + 40*a^{9/2}*c*e^{17})*\arctan(1/2*\sqrt{2}*(2*\sqrt{c})*x + \sqrt{2})*a^{1/4}*c^{1/4} \\ &)/\sqrt{\sqrt{a}*\sqrt{c}}))/a^{3/4}*\sqrt{a}*\sqrt{c})*c^{5/4}) - 2*(7*\sqrt{2})*a^{1/4} \\ & *c^{21/4}*d^{17} + 10*\sqrt{2})*a^{3/4}*c^{19/4}*d^{15}*e^2 + 50*\sqrt{2})*a^{5/4} \\ & *c^{17/4}*d^{13}*e^4 + 78*\sqrt{2})*a^{7/4}*c^{15/4}*d^{11}*e^6 + 220*\sqrt{2})*a^{9/4} \\ & *c^{13/4}*d^9*e^8 + 702*\sqrt{2})*a^{11/4}*c^{11/4}*d^7*e^{10} - 770*\sqrt{2})*a^{13/4} \\ & *c^{9/4}*d^5*e^{12} - 390*\sqrt{2})*a^{15/4}*c^{7/4}*d^3*e^{14} + 77*\sqrt{2})*a^{17/4} \\ & *c^{5/4}*d*e^{16} - 24*\sqrt{a}*c^5*d^{16}*e - 176*a^{3/2}*c^4*d^{12}*e^5 - 960*a^{5/2} \\ & *c^3*d^8*e^9 + 1 \end{aligned}$$

$$\begin{aligned}
& 200*a^{(7/2)}*c^2*d^4*e^{13} - 40*a^{(9/2)}*c*e^{17}*\arctan(1/2*\sqrt{2}*(2*\sqrt{c}) \\
& *x - \sqrt{2}*a^{(1/4)}*c^{(1/4)}/\sqrt{\sqrt{a}*\sqrt{c}})/(a^{(3/4)}*\sqrt{\sqrt{a} \\
& *\sqrt{c}})*c^{(5/4)))/(a^2*c^5*d^{20} + 5*a^3*c^4*d^{16}*e^4 + 10*a^4*c^3*d^{12}*e^8 \\
& + 10*a^5*c^2*d^8*e^{12} + 5*a^6*c*d^4*e^{16} + a^7*e^{20}) + 6*(13*c^2*d^6*e^{11} \\
& - 3*a*c*d^2*e^{15})*\log(e*x + d)/(c^5*d^{20} + 5*a*c^4*d^{16}*e^4 + 10*a^2*c^3*d^{12} \\
& *e^8 + 10*a^3*c^2*d^8*e^{12} + 5*a^4*c*d^4*e^{16} + a^5*e^{20}) + 1/32*(40*a^2* \\
& c^3*d^{12}*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d^4*e^{11} - 16*a^5*e^{15} + 6*(\\
& 5*c^5*d^{11}*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^{12})*x^9 + 6*(7*c^5*d^{11} \\
& *e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^{11} - 5*a^3*c^2*e^{15})*x^8 + (c^5 \\
& *d^{13}*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^{10} + 17*a^3*c^2*d*e^{14})*x^7 \\
& - 4*(c^5*d^{14}*e + 7*a*c^4*d^{10}*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^{13} \\
& *x^6 + (7*c^5*d^{15} + 97*a*c^4*d^{11}*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3* \\
& c^2*d^3*e^{12})*x^5 + 2*(39*a*c^4*d^{12}*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^2 \\
& *d^4*e^{11} - 25*a^4*c*e^{15})*x^4 + (5*a*c^4*d^{13}*e^2 + 31*a^2*c^3*d^9*e^6 + \\
& 47*a^3*c^2*d^5*e^{10} + 21*a^4*c*d*e^{14})*x^3 - 8*(a*c^4*d^{14}*e + 5*a^2*c^3*d^{10} \\
& *e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^{13})*x^2 + (11*a*c^4*d^{15} + 79*a^2 \\
& *c^3*d^{11}*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^{12})*x)/(a^4*c^4*d^{18} \\
& + 4*a^5*c^3*d^{14}*e^4 + 6*a^6*c^2*d^{10}*e^8 + 4*a^7*c*d^6*e^{12} + a^8*d^2*e^{16} \\
& + (a^2*c^6*d^{16}*e^2 + 4*a^3*c^5*d^{12}*e^6 + 6*a^4*c^4*d^8*e^{10} + 4*a^5*c^3 \\
& *d^4*e^{14} + a^6*c^2*e^{18})*x^{10} + 2*(a^2*c^6*d^{17}*e + 4*a^3*c^5*d^{13}*e^5 + 6 \\
& *a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^{13} + a^6*c^2*d*e^{17})*x^9 + (a^2*c^6*d^{18} \\
& + 4*a^3*c^5*d^{14}*e^4 + 6*a^4*c^4*d^{10}*e^8 + 4*a^5*c^3*d^6*e^{12} + a^6*c^2*d \\
& ^2*e^{16})*x^8 + 2*(a^3*c^5*d^{16}*e^2 + 4*a^4*c^4*d^{12}*e^6 + 6*a^5*c^3*d^8*e^{10} \\
& + 4*a^6*c^2*d^4*e^{14} + a^7*c*e^{18})*x^6 + 4*(a^3*c^5*d^{17}*e + 4*a^4*c^4*d^{13} \\
& *e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^{13} + a^7*c*d*e^{17})*x^5 + 2*(a^3 \\
& *c^5*d^{18} + 4*a^4*c^4*d^{14}*e^4 + 6*a^5*c^3*d^{10}*e^8 + 4*a^6*c^2*d^6*e^{12} + \\
& a^7*c*d^2*e^{16})*x^4 + (a^4*c^4*d^{16}*e^2 + 4*a^5*c^3*d^{12}*e^6 + 6*a^6*c^2*d \\
& ^8*e^{10} + 4*a^7*c*d^4*e^{14} + a^8*e^{18})*x^2 + 2*(a^4*c^4*d^{17}*e + 4*a^5*c^3 \\
& *d^{13}*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^{13} + a^8*d*e^{17})*x)
\end{aligned}$$

mupad [B] time = 7.78, size = 6280, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/((a + c*x^4)^3*(d + e*x)^3), x)$

[Out] $\text{symsum}(\log(\text{root}(2684354560*a^{12}*c^9*d^{36}*e^4*z^5 + 32212254720*a^{18}*c^3*d^{12} \\
2*e^{28}*z^5 + 32212254720*a^{14}*c^7*d^{28}*e^{12}*z^5 + 2684354560*a^{20}*c*d^4*e^3 \\
6*z^5 + 56371445760*a^{17}*c^4*d^{16}*e^{24}*z^5 + 56371445760*a^{15}*c^6*d^{24}*e^{16} \\
*z^5 + 12079595520*a^{19}*c^2*d^8*e^{32}*z^5 + 12079595520*a^{13}*c^8*d^{32}*e^8*z^5 \\
+ 67645734912*a^{16}*c^5*d^{20}*e^{20}*z^5 + 268435456*a^{11}*c^{10}*d^{40}*z^5 + 268 \\
435456*a^{21}*e^{40}*z^5 + 45339770880*a^9*c^6*d^{20}*e^{14}*z^3 - 79148482560*a^{13} \\
*c^2*d^4*e^{30}*z^3 + 791941349376*a^{12}*c^3*d^8*e^{26}*z^3 + 1239810048*a^7*c^8 \\
*d^{28}*e^6*z^3 - 1555444924416*a^{11}*c^4*d^{12}*e^{22}*z^3 + 83755008*a^6*c^9*d^3$

$$\begin{aligned}
& 2e^{2z^3} + 81566760960a^{10}c^5d^{16}e^{18z^3} + 12177506304a^8c^7d^{24}e^{10z^3} + 117964800a^{14}c^3e^{34z^3} - 2785204224a^6c^6d^{18}e^{13z^2} + 8128512a^3c^9d^{30}e^z z^2 + 2700933120a^{10}c^2d^2e^{29z^2} - 543361222656a^8c^4d^{10}e^{21z^2} + 1048135680a^5c^7d^{22}e^9z^2 + 118499328a^4c^8d^{26}e^5z^2 - 55938263040a^7c^5d^{14}e^{17z^2} + 123990497280a^9c^3d^6e^{25z^2} + 24139215a^2c^7d^{20}e^8z + 2819286a^8c^8d^{24}e^4z + 10462847841a^6c^3d^4e^{24z} - 5777473473a^4c^5d^{12}e^{16z} - 43509753450a^5c^4d^8e^{20z} - 548810316a^3c^6d^{16}e^{12z} + 12960000a^7c^2e^{28z} + 194481c^9d^{28z} - 977636142a^2c^4d^6e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^5c^5d^{10}e^{15} - 15169518c^6d^{14}e^{11}, z, k) \cdot (\text{root}(2684354560a^{12}c^9d^{36}e^4z^5 + 32212254720a^{18}c^3d^{12}e^{28z^5} + 32212254720a^{14}c^7d^{28}e^{12z^5} + 2684354560a^{20}c^1d^4e^{36z^5} + 56371445760a^{17}c^4d^{16}e^{24z^5} + 56371445760a^{15}c^6d^{24}e^{16z^5} + 12079595520a^{19}c^2d^8e^{32z^5} + 12079595520a^{13}c^8d^{32}e^8z^5 + 67645734912a^{16}c^5d^{20}e^{20z^5} + 268435456a^{11}c^{10}d^{40}z^5 + 268435456a^{21}e^{40z^5} + 45339770880a^9c^6d^{20}e^{14z^3} - 79148482560a^{13}c^2d^4e^{30z^3} + 791941349376a^{12}c^3d^8e^{26z^3} + 1239810048a^7c^8d^{28}e^6z^3 - 1555444924416a^{11}c^4d^{12}e^{22z^3} + 83755008a^6c^9d^{32}e^2z^3 + 81566760960a^{10}c^5d^{16}e^{18z^3} + 12177506304a^8c^7d^{24}e^{10z^3} + 117964800a^{14}c^3e^{34z^3} - 2785204224a^6c^6d^{18}e^{13z^2} + 8128512a^3c^9d^{30}e^z z^2 + 2700933120a^{10}c^2d^2e^{29z^2} - 543361222656a^8c^4d^{10}e^{21z^2} + 1048135680a^5c^7d^{22}e^9z^2 + 118499328a^4c^8d^{26}e^5z^2 - 55938263040a^7c^5d^{14}e^{17z^2} + 123990497280a^9c^3d^6e^{25z^2} + 24139215a^2c^7d^{20}e^8z + 2819286a^8c^8d^{24}e^4z + 10462847841a^6c^3d^4e^{24z} - 5777473473a^4c^5d^{12}e^{16z} - 43509753450a^5c^4d^8e^{20z} - 548810316a^3c^6d^{16}e^{12z} + 12960000a^7c^2e^{28z} + 194481c^9d^{28z} - 977636142a^2c^4d^6e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^5c^5d^{10}e^{15} - 15169518c^6d^{14}e^{11}, z, k) \cdot (\text{root}(2684354560a^{12}c^9d^{36}e^4z^5 + 32212254720a^{18}c^3d^{12}e^{28z^5} + 32212254720a^{14}c^7d^{28}e^{12z^5} + 2684354560a^{20}c^1d^4e^{36z^5} + 56371445760a^{17}c^4d^{16}e^{24z^5} + 56371445760a^{15}c^6d^{24}e^{16z^5} + 12079595520a^{19}c^2d^8e^{32z^5} + 12079595520a^{13}c^8d^{32}e^8z^5 + 67645734912a^{16}c^5d^{20}e^{20z^5} + 268435456a^{11}c^{10}d^{40}z^5 + 268435456a^{21}e^{40z^5} + 45339770880a^9c^6d^{20}e^{14z^3} - 79148482560a^{13}c^2d^4e^{30z^3} + 791941349376a^{12}c^3d^8e^{26z^3} + 1239810048a^7c^8d^{28}e^6z^3 - 1555444924416a^{11}c^4d^{12}e^{22z^3} + 83755008a^6c^9d^{32}e^2z^3 + 81566760960a^{10}c^5d^{16}e^{18z^3} + 12177506304a^8c^7d^{24}e^{10z^3} + 117964800a^{14}c^3e^{34z^3} - 2785204224a^6c^6d^{18}e^{13z^2} + 8128512a^3c^9d^{30}e^z z^2 + 2700933120a^{10}c^2d^2e^{29z^2} - 543361222656a^8c^4d^{10}e^{21z^2} + 1048135680a^5c^7d^{22}e^9z^2 + 118499328a^4c^8d^{26}e^5z^2 - 55938263040a^7c^5d^{14}e^{17z^2} + 123990497280a^9c^3d^6e^{25z^2} + 24139215a^2c^7d^{20}e^8z + 2819286a^8c^8d^{24}e^4z + 10462847841a^6c^3d^4e^{24z} - 5777473473a^4c^5d^{12}e^{16z} - 43509753450a^5c^4d^8e^{20z} - 548810316a^3c^6d^{16}e^{12z} + 12960000a^7c^2e^{28z} + 194481c^9d^{28z} - 977636142a^2c^4d^6e^{19} + 233280000a^3c^3d^2e^{23} - 140556060a^5c^5d^{10}e^{15} - 15169518c^6d^{14}e^{11}, z, k)
\end{aligned}$$

$$\begin{aligned}
& 6*d^{14}*e^{11}, z, k)*((44040192*a^8*c^{12}*d^{31}*e^5 - 11010048*a^7*c^{13}*d^{35}*e \\
& + 994050048*a^9*c^{11}*d^{27}*e^9 + 13683916800*a^{10}*c^{10}*d^{23}*e^{13} + 429360414 \\
& 72*a^{11}*c^9*d^{19}*e^{17} + 52628029440*a^{12}*c^8*d^{15}*e^{21} + 23429382144*a^{13}*c \\
& ^7*d^{11}*e^{25} - 2132803584*a^{14}*c^6*d^7*e^{29} - 3125280768*a^{15}*c^5*d^3*e^{33}) \\
& /((1048576*(a^{16}*e^{32} + a^8*c^8*d^{32} + 8*a^{15}*c*d^4*e^{28} + 8*a^9*c^7*d^{28}*e^4 \\
& + 28*a^{10}*c^6*d^{24}*e^8 + 56*a^{11}*c^5*d^{20}*e^{12} + 70*a^{12}*c^4*d^{16}*e^{16} + \\
& 56*a^{13}*c^3*d^{12}*e^{20} + 28*a^{14}*c^2*d^8*e^{24})) + \text{root}(2684354560*a^{12}*c^9*d \\
& ^{36}*e^4*z^5 + 32212254720*a^{18}*c^3*d^{12}*e^{28}*z^5 + 32212254720*a^{14}*c^7*d^{12} \\
& 8*e^{12}*z^5 + 2684354560*a^{20}*c*d^4*e^{36}*z^5 + 56371445760*a^{17}*c^4*d^{16}*e^2 \\
& 4*z^5 + 56371445760*a^{15}*c^6*d^{24}*e^{16}*z^5 + 12079595520*a^{19}*c^2*d^8*e^{32}* \\
& z^5 + 12079595520*a^{13}*c^8*d^{32}*e^8*z^5 + 67645734912*a^{16}*c^5*d^{20}*e^{20}*z^ \\
& 5 + 268435456*a^{11}*c^{10}*d^{40}*z^5 + 268435456*a^{21}*e^{40}*z^5 + 45339770880*a^ \\
& 9*c^6*d^{20}*e^{14}*z^3 - 79148482560*a^{13}*c^2*d^4*e^{30}*z^3 + 791941349376*a^{12} \\
& *c^3*d^8*e^{26}*z^3 + 1239810048*a^7*c^8*d^{28}*e^6*z^3 - 1555444924416*a^{11}*c^ \\
& 4*d^{12}*e^{22}*z^3 + 83755008*a^6*c^9*d^{32}*e^2*z^3 + 81566760960*a^{10}*c^5*d^{16} \\
& *e^{18}*z^3 + 12177506304*a^8*c^7*d^{24}*e^{10}*z^3 + 117964800*a^{14}*c*e^{34}*z^3 - \\
& 2785204224*a^6*c^6*d^{18}*e^{13}*z^2 + 8128512*a^3*c^9*d^{30}*e*z^2 + 2700933120 \\
& *a^{10}*c^2*d^2*e^{29}*z^2 - 543361222656*a^8*c^4*d^{10}*e^{21}*z^2 + 1048135680*a^ \\
& 5*c^7*d^{22}*e^9*z^2 + 118499328*a^4*c^8*d^{26}*e^5*z^2 - 55938263040*a^7*c^5*d \\
& ^{14}*e^{17}*z^2 + 123990497280*a^9*c^3*d^6*e^{25}*z^2 + 24139215*a^2*c^7*d^{20}*e^ \\
& 8*z + 2819286*a*c^8*d^{24}*e^4*z + 10462847841*a^6*c^3*d^4*e^{24}*z - 577747347 \\
& 3*a^4*c^5*d^{12}*e^{16}*z - 43509753450*a^5*c^4*d^8*e^{20}*z - 548810316*a^3*c^6*d \\
& ^{16}*e^{12}*z + 12960000*a^7*c^2*e^{28}*z + 194481*c^9*d^{28}*z - 977636142*a^2*c \\
& ^4*d^6*e^{19} + 233280000*a^3*c^3*d^2*e^{23} - 140556060*a*c^5*d^{10}*e^{15} - 1516 \\
& 9518*c^6*d^{14}*e^{11}, z, k)*((402653184*a^{19}*c^4*d*e^{38} - 134217728*a^{10}*c^{13} \\
& *d^{37}*e^2 - 671088640*a^{11}*c^{12}*d^{33}*e^6 - 536870912*a^{12}*c^{11}*d^{29}*e^{10} + \\
& 3758096384*a^{13}*c^{10}*d^{25}*e^{14} + 13153337344*a^{14}*c^9*d^{21}*e^{18} + 206695301 \\
& 12*a^{15}*c^8*d^{17}*e^{22} + 18790481920*a^{16}*c^7*d^{13}*e^{26} + 10200547328*a^{17}*c \\
& ^6*d^9*e^{30} + 3087007744*a^{18}*c^5*d^5*e^{34})/(1048576*(a^{16}*e^{32} + a^8*c^8*d \\
& ^{32} + 8*a^{15}*c*d^4*e^{28} + 8*a^9*c^7*d^{28}*e^4 + 28*a^{10}*c^6*d^{24}*e^8 + 56*a^ \\
& 11*c^5*d^{20}*e^{12} + 70*a^{12}*c^4*d^{16}*e^{16} + 56*a^{13}*c^3*d^{12}*e^{20} + 28*a^{14}* \\
& c^2*d^8*e^{24})) + (x*(335544320*a^{19}*c^4*e^{39} - 201326592*a^{10}*c^{13}*d^{36}*e^3 \\
& - 1275068416*a^{11}*c^{12}*d^{32}*e^7 - 2952790016*a^{12}*c^{11}*d^{28}*e^{11} - 1879048 \\
& 192*a^{13}*c^{10}*d^{24}*e^{15} + 4697620480*a^{14}*c^9*d^{20}*e^{19} + 12213813248*a^{15}* \\
& c^8*d^{16}*e^{23} + 13153337344*a^{16}*c^7*d^{12}*e^{27} + 7784628224*a^{17}*c^6*d^8*e^ \\
& 31 + 2483027968*a^{18}*c^5*d^4*e^{35})/(1048576*(a^{16}*e^{32} + a^8*c^8*d^{32} + 8* \\
& a^{15}*c*d^4*e^{28} + 8*a^9*c^7*d^{28}*e^4 + 28*a^{10}*c^6*d^{24}*e^8 + 56*a^{11}*c^5*d \\
& ^{20}*e^{12} + 70*a^{12}*c^4*d^{16}*e^{16} + 56*a^{13}*c^3*d^{12}*e^{20} + 28*a^{14}*c^2*d^8* \\
& e^{24})) - (x*(40894464*a^7*c^{13}*d^{34}*e^2 + 276824064*a^8*c^{12}*d^{30}*e^6 + 96 \\
& 8884224*a^9*c^{11}*d^{26}*e^{10} - 13010731008*a^{10}*c^{10}*d^{22}*e^{14} - 53433335808* \\
& a^{11}*c^9*d^{18}*e^{18} - 71647100928*a^{12}*c^8*d^{14}*e^{22} - 34313601024*a^{13}*c^7* \\
& d^{10}*e^{26} + 1837105152*a^{14}*c^6*d^6*e^{30} + 4193255424*a^{15}*c^5*d^2*e^{34}))/ \\
& (1048576*(a^{16}*e^{32} + a^8*c^8*d^{32} + 8*a^{15}*c*d^4*e^{28} + 8*a^9*c^7*d^{28}*e^4 \\
& + 28*a^{10}*c^6*d^{24}*e^8 + 56*a^{11}*c^5*d^{20}*e^{12} + 70*a^{12}*c^4*d^{16}*e^{16} + 56 \\
& *a^{13}*c^3*d^{12}*e^{20} + 28*a^{14}*c^2*d^8*e^{24})) + (33914880*a^{12}*c^5*d*e^{32} +
\end{aligned}$$

$$\begin{aligned}
& 13713408*a^5*c^12*d^29*e^4 + 225902592*a^6*c^11*d^25*e^8 + 2352070656*a^7*c^10*d^21*e^12 + 2474606592*a^8*c^9*d^17*e^16 - 21361803264*a^9*c^8*d^13*e^20 + 88707170304*a^10*c^7*d^9*e^24 - 5526503424*a^11*c^6*d^5*e^28)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(132710400*a^12*c^5*e^33 - 1806336*a^4*c^13*d^32*e - 2027520*a^5*c^12*d^28*e^5 + 162017280*a^6*c^11*d^24*e^9 + 4635316224*a^7*c^10*d^20*e^13 + 15604273152*a^8*c^9*d^16*e^17 + 39318663168*a^9*c^8*d^12*e^21 + 64184389632*a^10*c^7*d^8*e^25 - 2525073408*a^11*c^6*d^4*e^29))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (320544*a^2*c^12*d^27*e^3 + 11448000*a^3*c^11*d^23*e^7 + 114031584*a^4*c^10*d^19*e^11 - 213750144*a^5*c^9*d^15*e^15 - 3499271712*a^6*c^8*d^11*e^19 + 9699804864*a^7*c^7*d^7*e^23 - 933615072*a^8*c^6*d^3*e^27)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(514944*a^2*c^12*d^26*e^4 + 14314752*a^3*c^11*d^22*e^8 + 266343552*a^4*c^10*d^18*e^12 + 297948672*a^5*c^9*d^14*e^16 - 2642613120*a^6*c^8*d^10*e^20 + 1782459648*a^7*c^7*d^6*e^24 + 846599040*a^8*c^6*d^2*e^28))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (194481*c^11*d^21*e^6 + 2430324*a*c^10*d^17*e^10 + 12960000*a^5*c^6*d*e^26 - 5918346*a^2*c^9*d^13*e^14 - 83522988*a^3*c^8*d^9*e^18 + 71628705*a^4*c^7*d^5*e^22)/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(12960000*a^5*c^6*e^27 + 194481*c^11*d^20*e^7 + 2430324*a*c^10*d^16*e^11 - 21081546*a^2*c^9*d^12*e^15 - 227814444*a^3*c^8*d^8*e^19 + 105734241*a^4*c^7*d^4*e^23))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)))*root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280
\end{aligned}$$

$$\begin{aligned}
& *a^9*c^3*d^6*e^{25}z^2 + 24139215*a^2*c^7*d^{20}*e^8*z + 2819286*a*c^8*d^{24}*e^4*z + 10462847841*a^6*c^3*d^4*e^{24}z - 5777473473*a^4*c^5*d^{12}*e^{16}z - 435 \\
& 09753450*a^5*c^4*d^8*e^{20}z - 548810316*a^3*c^6*d^{16}*e^{12}z + 12960000*a^7* \\
& c^2*e^{28}z + 194481*c^9*d^{28}z - 977636142*a^2*c^4*d^6*e^{19} + 233280000*a^3 \\
& *c^3*d^2*e^{23} - 140556060*a*c^5*d^{10}*e^{15} - 15169518*c^6*d^{14}*e^{11}, z, k), \\
& k, 1, 5) - ((2*a^3*e^{15} - 5*c^3*d^{12}*e^3 - 38*a*c^2*d^8*e^7 + 65*a^2*c*d^4* \\
& e^{11})/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (3*x^8*(5*a^3*c^2*e^{15} - \\
& 7*c^5*d^{12}*e^3 - 49*a*c^4*d^8*e^7 + 91*a^2*c^3*d^4*e^{11}))/((16*a^2*(a^4*e^{16} \\
& + c^4*d^{16} + 4*a*c^3*d^{12}*e^4 + 4*a^3*c*d^4*e^{12} + 6*a^2*c^2*d^8*e^8)) - (\\
& x^5*(7*c^5*d^{15} + 97*a*c^4*d^{11}*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^ \\
& 3*e^{12}))/((32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (3*x^9*(5*c^5*d^{1 \\
& 1*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^{12}))/((16*a^2*(a^4*e^{16} + c^4*d^ \\
& 16 + 4*a*c^3*d^{12}*e^4 + 4*a^3*c*d^4*e^{12} + 6*a^2*c^2*d^8*e^8)) + (x^2*(c^2* \\
& d^6*e + 3*a*c*d^2*e^5))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^3*(5 \\
& *c^2*d^5*e^2 + 21*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - \\
& (x*(11*c^4*d^{15} + 79*a*c^3*d^{11}*e^4 - 567*a^3*c*d^3*e^{12} + 269*a^2*c^2*d^7* \\
& e^8))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (x^6*(c^3*d^6*e + 5*a* \\
& c^2*d^2*e^5))/(8*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^4*(25*a^3*c* \\
& e^{15} - 39*c^4*d^{12}*e^3 - 293*a*c^3*d^8*e^7 + 539*a^2*c^2*d^4*e^{11}))/((16*a*(\\
& a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (e^2*x^7*(c^3*d^5 + 17*a*c^2*d*e^4) \\
&))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d^2 + a^2*e^2*x^2 + c^ \\
& 2*d^2*x^8 + c^2*e^2*x^{10} + 2*a^2*d*e*x + 2*a*c*d^2*x^4 + 2*a*c*e^2*x^6 + 2* \\
& c^2*d*e*x^9 + 4*a*c*d*e*x^5)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)

[Out] Timed out

$$3.411 \quad \int \frac{-1+x}{1-x+x^2} dx$$

Optimal. Leaf size=32

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(1 - x + x^2), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{1-x+x^2} dx &= -\left(\frac{1}{2} \int \frac{1}{1-x+x^2} dx\right) + \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx \\
&= \frac{1}{2} \log(1-x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log(1-x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2} \log(x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/(1 - x + x^2), x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{1-x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1+ x)/(1 - x + x^2), x]

[Out] IntegrateAlgebraic[(-1 + x)/(1 - x + x^2), x]

fricas [A] time = 1.53, size = 28, normalized size = 0.88

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x - 1)\right) + \frac{1}{2} \log(x^2 - x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

giac [A] time = 0.29, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x^2-x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(x^2-x+1),x)

[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 1)/(x^2 - x + 1),x)

[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3

sympy [A] time = 0.11, size = 34, normalized size = 1.06

$$\frac{\log(x^2-x+1)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-x+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

$$3.412 \quad \int \frac{-1+x^2}{1+x^3} dx$$

Optimal. Leaf size=32

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1872, 634, 618, 204, 628}

$$\frac{1}{2} \log(x^2 - x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(1 + x^3), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1872

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rubi steps

$$\begin{aligned}
 \int \frac{-1+x^2}{1+x^3} dx &= \int \frac{-1+x}{1-x+x^2} dx \\
 &= -\left(\frac{1}{2} \int \frac{1}{1-x+x^2} dx\right) + \frac{1}{2} \int \frac{-1+2x}{1-x+x^2} dx \\
 &= \frac{1}{2} \log(1-x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\
 &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2} \log(1-x+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2} \log(x^2 - x + 1) - \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(1 + x^3), x]

[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{1+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^2)/(1 + x^3), x]

[Out] IntegrateAlgebraic[(-1 + x^2)/(1 + x^3), x]

fricas [A] time = 1.39, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

giac [A] time = 0.38, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x^2-x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^3+1),x)

[Out] -1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*ln(x^2-x+1)

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log(x^2-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 1)/(x^3 + 1), x)`

[Out] `log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3`

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{\log(x^2 - x + 1)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)/(x**3+1), x)`

[Out] `log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3`

$$3.413 \quad \int \frac{-4+3x}{4-2x+x^2} dx$$

Optimal. Leaf size=32

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {634, 618, 204, 628}

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3*x)/(4 - 2*x + x^2), x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{-4 + 3x}{4 - 2x + x^2} dx &= \frac{3}{2} \int \frac{-2 + 2x}{4 - 2x + x^2} dx - \int \frac{1}{4 - 2x + x^2} dx \\
&= \frac{3}{2} \log(4 - 2x + x^2) + 2 \operatorname{Subst} \left(\int \frac{1}{-12 - x^2} dx, x, -2 + 2x \right) \\
&= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(4 - 2x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2} \log(x^2 - 2x + 4) - \frac{\tan^{-1} \left(\frac{x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2), x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 3x}{4 - 2x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 3*x)/(4 - 2*x + x^2), x]

[Out] IntegrateAlgebraic[(-4 + 3*x)/(4 - 2*x + x^2), x]

fricas [A] time = 1.65, size = 26, normalized size = 0.81

$$-\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (x-1) \right) + \frac{3}{2} \log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4), x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

giac [A] time = 0.39, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3} + \frac{3\ln(x^2 - 2x + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4+3*x)/(x^2-2*x+4),x)

[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))

maxima [A] time = 2.16, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log(x^2 - 2x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{3\ln(x^2 - 2x + 4)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 4)/(x^2 - 2*x + 4),x)

[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log(x^2 - 2x + 4)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

$$3.414 \quad \int \frac{-8+2x+3x^2}{8+x^3} dx$$

Optimal. Leaf size=32

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1872, 634, 618, 204, 628}

$$\frac{3}{2} \log(x^2 - 2x + 4) + \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1872

Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :=> With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]

Rubi steps

$$\begin{aligned}
 \int \frac{-8 + 2x + 3x^2}{8 + x^3} dx &= \frac{1}{2} \int \frac{-8 + 6x}{4 - 2x + x^2} dx \\
 &= \frac{3}{2} \int \frac{-2 + 2x}{4 - 2x + x^2} dx - \int \frac{1}{4 - 2x + x^2} dx \\
 &= \frac{3}{2} \log(4 - 2x + x^2) + 2 \operatorname{Subst} \left(\int \frac{1}{-12 - x^2} dx, x, -2 + 2x \right) \\
 &= \frac{\tan^{-1} \left(\frac{1-x}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{3}{2} \log(4 - 2x + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2} \log(x^2 - 2x + 4) - \frac{\tan^{-1} \left(\frac{x-1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-8 + 2x + 3x^2}{8 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

[Out] IntegrateAlgebraic[(-8 + 2*x + 3*x^2)/(8 + x^3), x]

fricas [A] time = 1.66, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log(x^2-2x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

giac [A] time = 0.33, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log(x^2-2x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}+\frac{3\ln(x^2-2x+4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2*x-8)/(x^3+8),x)

[Out] -1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))+3/2*ln(x^2-2*x+4)

maxima [A] time = 2.33, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log(x^2-2x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{3\ln(x^2-2x+4)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 3*x^2 - 8)/(x^3 + 8), x)`

[Out] $(3 \log(x^2 - 2x + 4))/2 - (3^{1/2} \operatorname{atan}((3^{1/2}x)/3 - 3^{1/2}/3))/3$

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3 \log(x^2 - 2x + 4)}{2} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+2*x-8)/(x**3+8), x)`

[Out] $3 \log(x^2 - 2x + 4)/2 - \operatorname{sqrt}(3) \operatorname{atan}(\operatorname{sqrt}(3)x/3 - \operatorname{sqrt}(3)/3)/3$

$$3.415 \quad \int \frac{2+x}{-1+2x+x^2} dx$$

Optimal. Leaf size=45

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 31}

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/(-1 + 2*x + x^2), x]

[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x])/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{-1+2x+x^2} dx &= -\left(\frac{1}{4}(-2 + \sqrt{2}) \int \frac{1}{1 + \sqrt{2} + x} dx\right) + \frac{1}{4}(2 + \sqrt{2}) \int \frac{1}{1 - \sqrt{2} + x} dx \\ &= \frac{1}{4}(2 + \sqrt{2}) \log(1 - \sqrt{2} + x) + \frac{1}{4}(2 - \sqrt{2}) \log(1 + \sqrt{2} + x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.93

$$\frac{1}{4} \left((2 + \sqrt{2}) \log(-x + \sqrt{2} - 1) - (\sqrt{2} - 2) \log(x + \sqrt{2} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/(-1 + 2*x + x^2), x]

[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x}{-1+2x+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x)/(-1 + 2*x + x^2), x]

[Out] IntegrateAlgebraic[(2 + x)/(-1 + 2*x + x^2), x]

fricas [A] time = 1.61, size = 45, normalized size = 1.00

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x + 3}{x^2 + 2x - 1}\right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+2*x-1), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*log(x^2 + 2*x - 1)

giac [A] time = 0.42, size = 44, normalized size = 0.98

$$\frac{1}{4} \sqrt{2} \log\left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|}\right) + \frac{1}{2} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(x^2+2*x-1), x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*log(abs(x^2 + 2*x - 1))

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+2)/(x^2+2*x-1),x)`

[Out] $1/2*\ln(x^2+2*x-1)-1/2*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*x+2)*2^{(1/2)})$

maxima [A] time = 2.62, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log(x^2+2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")`

[Out] $1/4*\sqrt{2}*\log((x-\sqrt{2}+1)/(x+\sqrt{2}+1))+1/2*\log(x^2+2*x-1)$

mupad [B] time = 2.32, size = 34, normalized size = 0.76

$$\ln(x-\sqrt{2}+1)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln(x+\sqrt{2}+1)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+2)/(2*x+x^2-1),x)`

[Out] $\log(x-2^{(1/2)}+1)*(2^{(1/2)}/4+1/2)-\log(x+2^{(1/2)}+1)*(2^{(1/2)}/4-1/2)$

sympy [A] time = 0.11, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log(x+1+\sqrt{2})+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log(x-\sqrt{2}+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+x)/(x**2+2*x-1),x)`

[Out] $(1/2-\sqrt{2}/4)*\log(x+1+\sqrt{2})+(\sqrt{2}/4+1/2)*\log(x-\sqrt{2}+1)$

$$3.416 \quad \int \frac{-4+x^2}{2-5x+x^3} dx$$

Optimal. Leaf size=45

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2055, 632, 31}

$$\frac{1}{4} (2 + \sqrt{2}) \log(x - \sqrt{2} + 1) + \frac{1}{4} (2 - \sqrt{2}) \log(x + \sqrt{2} + 1)$$

Antiderivative was successfully verified.

[In] Int[(-4 + x^2)/(2 - 5*x + x^3), x]

[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x])/4

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 2055

Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] := Module[{gcd = PolyGCD[P, Q, x]}, Int[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^q, x] /; NeQ[gcd, 1] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2-5x+x^3} dx &= \int \frac{2+x}{-1+2x+x^2} dx \\ &= -\left(\frac{1}{4}(-2+\sqrt{2}) \int \frac{1}{1+\sqrt{2}+x} dx\right) + \frac{1}{4}(2+\sqrt{2}) \int \frac{1}{1-\sqrt{2}+x} dx \\ &= \frac{1}{4}(2+\sqrt{2}) \log(1-\sqrt{2}+x) + \frac{1}{4}(2-\sqrt{2}) \log(1+\sqrt{2}+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.93

$$\frac{1}{4} \left((2+\sqrt{2}) \log(-x+\sqrt{2}-1) - (\sqrt{2}-2) \log(x+\sqrt{2}+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3), x]

[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+x^2}{2-5x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + x^2)/(2 - 5*x + x^3), x]

[Out] IntegrateAlgebraic[(-4 + x^2)/(2 - 5*x + x^3), x]

fricas [A] time = 1.41, size = 45, normalized size = 1.00

$$\frac{1}{4} \sqrt{2} \log\left(\frac{x^2 - 2\sqrt{2}(x+1) + 2x + 3}{x^2 + 2x - 1}\right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*log(x^2 + 2*x - 1)

giac [A] time = 0.31, size = 44, normalized size = 0.98

$$\frac{1}{4} \sqrt{2} \log \left(\frac{|2x - 2\sqrt{2} + 2|}{|2x + 2\sqrt{2} + 2|} \right) + \frac{1}{2} \log(|x^2 + 2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")

[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*log(abs(x^2 + 2*x - 1))

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2} + \frac{\ln(x^2 + 2x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-4)/(x^3-5*x+2),x)

[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))

maxima [A] time = 2.33, size = 35, normalized size = 0.78

$$\frac{1}{4} \sqrt{2} \log \left(\frac{x - \sqrt{2} + 1}{x + \sqrt{2} + 1} \right) + \frac{1}{2} \log(x^2 + 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x - 1)

mupad [B] time = 0.05, size = 34, normalized size = 0.76

$$\ln(x - \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} + \frac{1}{2} \right) - \ln(x + \sqrt{2} + 1) \left(\frac{\sqrt{2}}{4} - \frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)

[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 - 1/2)

sympy [A] time = 0.12, size = 39, normalized size = 0.87

$$\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right)\log(x + 1 + \sqrt{2}) + \left(\frac{\sqrt{2}}{4} + \frac{1}{2}\right)\log(x - \sqrt{2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-4)/(x**3-5*x+2),x)

[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2) + 1)

$$3.417 \quad \int \frac{2}{-1+4x^2} dx$$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 207}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

[In] Int[2/(-1 + 4*x^2), x]

[Out] -ArcTanh[2*x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{2}{-1+4x^2} dx = 2 \int \frac{1}{-1+4x^2} dx = -\tanh^{-1}(2x)$$

Mathematica [B] time = 0.00, size = 23, normalized size = 3.83

$$2 \left(\frac{1}{4} \log(1 - 2x) - \frac{1}{4} \log(2x + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[2/(-1 + 4*x^2), x]

[Out] $2*(\text{Log}[1 - 2*x]/4 - \text{Log}[1 + 2*x]/4)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2}{-1 + 4x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[2/(-1 + 4*x^2),x]

[Out] IntegrateAlgebraic[2/(-1 + 4*x^2), x]

fricas [B] time = 1.39, size = 17, normalized size = 2.83

$$-\frac{1}{2} \log(2x + 1) + \frac{1}{2} \log(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1),x, algorithm="fricas")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

giac [B] time = 0.31, size = 15, normalized size = 2.50

$$-\frac{1}{2} \log\left(\left|x + \frac{1}{2}\right|\right) + \frac{1}{2} \log\left(\left|x - \frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1),x, algorithm="giac")

[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))

maple [B] time = 0.00, size = 18, normalized size = 3.00

$$\frac{\ln(2x - 1)}{2} - \frac{\ln(2x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(4*x^2-1),x)

[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)

maxima [B] time = 1.06, size = 17, normalized size = 2.83

$$-\frac{1}{2} \log(2x + 1) + \frac{1}{2} \log(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x^2-1),x, algorithm="maxima")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

mupad [B] time = 2.27, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2/(4*x^2 - 1),x)

[Out] -atanh(2*x)

sympy [B] time = 0.09, size = 15, normalized size = 2.50

$$\frac{\log\left(x - \frac{1}{2}\right)}{2} - \frac{\log\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2/(4*x**2-1),x)

[Out] log(x - 1/2)/2 - log(x + 1/2)/2

$$3.418 \quad \int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx$$

Optimal. Leaf size=21

$$\frac{1}{2} \log(1-2x) - \frac{1}{2} \log(2x+1)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{1}{2} \log(1-2x) - \frac{1}{2} \log(2x+1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2

Rubi steps

$$\int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx = \frac{1}{2} \log(1-2x) - \frac{1}{2} \log(1+2x)$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$2 \left(\frac{1}{4} \log(1-2x) - \frac{1}{4} \log(2x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{-1+2x} - \frac{1}{1+2x} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

[Out] IntegrateAlgebraic[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]

fricas [A] time = 1.16, size = 17, normalized size = 0.81

$$-\frac{1}{2} \log(2x + 1) + \frac{1}{2} \log(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

giac [A] time = 0.42, size = 19, normalized size = 0.90

$$-\frac{1}{2} \log(|2x + 1|) + \frac{1}{2} \log(|2x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")

[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(2x - 1)}{2} - \frac{\ln(2x + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x-1)-1/(2*x+1),x)

[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)

maxima [A] time = 0.94, size = 17, normalized size = 0.81

$$-\frac{1}{2} \log(2x + 1) + \frac{1}{2} \log(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")

[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)

mupad [B] time = 0.15, size = 6, normalized size = 0.29

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x - 1) - 1/(2*x + 1), x)
```

```
[Out] -atanh(2*x)
```

sympy [A] time = 0.10, size = 15, normalized size = 0.71

$$\frac{\log\left(x - \frac{1}{2}\right)}{2} - \frac{\log\left(x + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x), x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

$$3.419 \quad \int \frac{x}{(1-x^2)^5} dx$$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {261}

$$\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

[In] Int[x/(1 - x^2)^5,x]

[Out] 1/(8*(1 - x^2)^4)

Rule 261

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int \frac{x}{(1-x^2)^5} dx = \frac{1}{8(1-x^2)^4}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 - x^2)^5,x]

[Out] 1/(8*(-1 + x^2)^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-x^2)^5} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(1 - x^2)^5,x]

[Out] IntegrateAlgebraic[x/(1 - x^2)^5, x]

fricas [B] time = 1.35, size = 24, normalized size = 1.85

$$\frac{1}{8(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")

[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)

giac [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")

[Out] 1/8/(x^2 - 1)^4

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^2+1)^5,x)

[Out] 1/8/(x^2-1)^4

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x^2+1)^5,x, algorithm="maxima")`

[Out] $1/8/(x^2 - 1)^4$

mupad [B] time = 2.32, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2 - 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/(x^2 - 1)^5,x)`

[Out] $1/(8*(x^2 - 1)^4)$

sympy [B] time = 0.12, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-x**2+1)**5,x)`

[Out] $1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)$

3.420

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} + \frac{5}{256(1+x)^2} \right) dx$$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

Rubi [B] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{5}{256(x+1)} + \frac{5}{256(x+1)^2} + \frac{1}{64(x+1)^3} + \frac{1}{128(x+1)^4} + \frac{5}{256(1-x)} + \frac{5}{256(1-x)^2} + \frac{1}{64(1-x)^3} + \frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x)) + 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x))

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} + \frac{5}{256(1+x)^2} \right) dx$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

[Out] 1/(8*(-1 + x^2)^4)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

[Out] IntegrateAlgebraic[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(256*(1 + x)^2), x]

fricas [B] time = 1.10, size = 24, normalized size = 1.85

$$\frac{1}{8(x^8 - 4x^6 + 6x^4 - 4x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")

[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)

giac [B] time = 0.31, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)} - \frac{5}{256(x-1)} + \frac{5}{256(x+1)^2} + \frac{5}{256(x-1)^2} + \frac{1}{64(x+1)^3} - \frac{1}{64(x-1)^3} + \frac{1}{128(x+1)^4} + \frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")

[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4

maple [B] time = 0.00, size = 58, normalized size = 4.46

$$\frac{1}{128(x-1)^4} - \frac{1}{64(x-1)^3} + \frac{5}{256(x-1)^2} - \frac{5}{256(x-1)} + \frac{1}{128(x+1)^4} + \frac{1}{64(x+1)^3} + \frac{5}{256(x+1)^2} + \frac{5}{256(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/64/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x)`

[Out] $\frac{1}{128}(x-1)^4 - \frac{1}{64}(x-1)^3 + \frac{5}{256}(x-1)^2 - \frac{5}{256}(x-1) + \frac{1}{128}(x+1)^4 + \frac{1}{64}(x+1)^3 + \frac{5}{256}(x+1)^2 + \frac{5}{256}(x+1)$

maxima [B] time = 0.94, size = 57, normalized size = 4.38

$$\frac{5}{256(x+1)} - \frac{5}{256(x-1)} + \frac{5}{256(x+1)^2} + \frac{5}{256(x-1)^2} + \frac{1}{64(x+1)^3} - \frac{1}{64(x-1)^3} + \frac{1}{128(x+1)^4} + \frac{1}{128(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")`

[Out] $\frac{5}{256}(x+1) - \frac{5}{256}(x-1) + \frac{5}{256}(x+1)^2 + \frac{5}{256}(x-1)^2 + \frac{1}{64}(x+1)^3 - \frac{1}{64}(x-1)^3 + \frac{1}{128}(x+1)^4 + \frac{1}{128}(x-1)^4$

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{1}{8(x^2-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(5/(256*(x-1)^2) - 5/(256*(x+1)^2) - 5/(128*(x-1)^3) - 5/(128*(x+1)^3) + 3/(64*(x-1)^4) - 3/(64*(x+1)^4) - 1/(32*(x-1)^5) - 1/(32*(x+1)^5),x)`

[Out] $\frac{1}{8(x^2-1)^4}$

sympy [B] time = 0.31, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)`

[Out] $\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$

$$3.421 \quad \int \frac{1+x^6}{-1+x^6} dx$$

Optimal. Leaf size=69

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(-1 + x^6), x]

[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^6}{-1+x^6} dx &= x + 2 \int \frac{1}{-1+x^6} dx \\
&= x - \frac{2}{3} \int \frac{1}{1-x^2} dx - \frac{2}{3} \int \frac{1-\frac{x}{2}}{1-x+x^2} dx - \frac{2}{3} \int \frac{1+\frac{x}{2}}{1+x+x^2} dx \\
&= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{6} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{2} \int \frac{1}{1-x+x^2} dx - \frac{1}{2} \int \frac{1}{1+x+x^2} dx \\
&= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1-x+x^2) - \frac{1}{6} \log(1+x+x^2) + \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) + \dots \\
&= x - \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1-x+x^2) - \frac{1}{6} \log(1+x+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6} \left(\log(x^2 - x + 1) - \log(x^2 + x + 1) + 6x + 2 \log(1 - x) - 2 \log(x + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^6)/(-1 + x^6),x]

[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x^6}{-1 + x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^6)/(-1 + x^6),x]

[Out] IntegrateAlgebraic[(1 + x^6)/(-1 + x^6), x]

fricas [A] time = 1.26, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1) + 1/3*log(x - 1)

giac [A] time = 0.29, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs(x + 1)) + 1/3*log(abs(x - 1))

maple [A] time = 0.01, size = 67, normalized size = 0.97

$$x - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{6} - \frac{\ln(x^2+x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6+1)/(x^6-1),x)`

[Out] $x + 1/3 \ln(x-1) - 1/6 \ln(x^2+x+1) - 1/3 \cdot 3^{1/2} \arctan(1/3 \cdot (2x+1) \cdot 3^{1/2}) + 1/6 \ln(x^2-x+1) - 1/3 \cdot 3^{1/2} \arctan(1/3 \cdot (2x-1) \cdot 3^{1/2}) - 1/3 \ln(x+1)$

maxima [A] time = 2.01, size = 66, normalized size = 0.96

$$-\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + x - \frac{1}{6} \log(x^2+x+1) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1) + \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6+1)/(x^6-1),x, algorithm="maxima")`

[Out] $-1/3 \cdot \sqrt{3} \arctan(1/3 \cdot \sqrt{3} \cdot (2x+1)) - 1/3 \cdot \sqrt{3} \arctan(1/3 \cdot \sqrt{3} \cdot (2x-1)) + x - 1/6 \cdot \log(x^2+x+1) + 1/6 \cdot \log(x^2-x+1) - 1/3 \cdot \log(x+1) + 1/3 \cdot \log(x-1)$

mupad [B] time = 0.10, size = 94, normalized size = 1.36

$$x + \frac{\operatorname{atan}(x \cdot 1i) \cdot 2i}{3} - \operatorname{atan}\left(\frac{x \cdot 32i}{-32 + \sqrt{3} \cdot 32i} - \frac{32 \sqrt{3} x}{-32 + \sqrt{3} \cdot 32i}\right) \left(\frac{\sqrt{3}}{3} - \frac{1}{3}i\right) - \operatorname{atan}\left(\frac{x \cdot 32i}{32 + \sqrt{3} \cdot 32i} + \frac{32 \sqrt{3} x}{32 + \sqrt{3} \cdot 32i}\right) \left(\frac{\sqrt{3}}{3} + \frac{1}{3}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6 + 1)/(x^6 - 1),x)`

[Out] $x + (\operatorname{atan}(x \cdot 1i) \cdot 2i)/3 - \operatorname{atan}((x \cdot 32i)/(3^{1/2} \cdot 32i - 32) - (32 \cdot 3^{1/2} \cdot x)/(3^{1/2} \cdot 32i - 32)) \cdot (3^{1/2}/3 - 1i/3) - \operatorname{atan}((x \cdot 32i)/(3^{1/2} \cdot 32i + 32) + (32 \cdot 3^{1/2} \cdot x)/(3^{1/2} \cdot 32i + 32)) \cdot (3^{1/2}/3 + 1i/3)$

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**6+1)/(x**6-1),x)`

[Out] $x + \log(x-1)/3 - \log(x+1)/3 + \log(x^2-x+1)/6 - \log(x^2+x+1)/6 - \sqrt{3} \cdot \operatorname{atan}(2 \cdot \sqrt{3} \cdot x/3 - \sqrt{3}/3)/3 - \sqrt{3} \cdot \operatorname{atan}(2 \cdot \sqrt{3} \cdot x/3 + \sqrt{3}/3)/3$

$$3.422 \quad \int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx$$

Optimal. Leaf size=69

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1593, 1584, 388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{6} \log(x^2 + x + 1) + x + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 210

Int[((a_) + (b_.)*(x_)^(n_))(-1), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx &= \int \frac{x^3 \left(\frac{1}{x^3} + x^3 \right)}{-1 + x^6} dx \\
&= \int \frac{1 + x^6}{-1 + x^6} dx \\
&= x + 2 \int \frac{1}{-1 + x^6} dx \\
&= x - \frac{2}{3} \int \frac{1}{1 - x^2} dx - \frac{2}{3} \int \frac{1 - \frac{x}{2}}{1 - x + x^2} dx - \frac{2}{3} \int \frac{1 + \frac{x}{2}}{1 + x + x^2} dx \\
&= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \int \frac{-1 + 2x}{1 - x + x^2} dx - \frac{1}{6} \int \frac{1 + 2x}{1 + x + x^2} dx - \frac{1}{2} \int \frac{1}{1 - x + x^2} dx - \frac{1}{2} \int \frac{1}{1 + x + x^2} dx \\
&= x - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1 - x + x^2) - \frac{1}{6} \log(1 + x + x^2) + \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2x \right) \\
&= x - \frac{\tan^{-1} \left(\frac{-1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{1 + 2x}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{2}{3} \tanh^{-1}(x) + \frac{1}{6} \log(1 - x + x^2) - \frac{1}{6} \log(1 + x + x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6} \left(\log(x^2 - x + 1) - \log(x^2 + x + 1) + 6x + 2 \log(1 - x) - 2 \log(x + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{2x - 1}{\sqrt{3}} \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2x + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

[Out] IntegrateAlgebraic[(x^(-3) + x^3)/(-x^(-3) + x^3), x]

fricas [A] time = 1.36, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1) + 1/3*log(x - 1)

giac [A] time = 0.31, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(|x+1|) + \frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs(x + 1)) + 1/3*log(abs(x - 1))

maple [A] time = 0.00, size = 67, normalized size = 0.97

$$x - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} + \frac{\ln(x^2-x+1)}{6} - \frac{\ln(x^2+x+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^3+x^3)/(-1/x^3+x^3),x)

[Out] x-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/3*ln(x-1)-1/3*ln(x+1)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)

maxima [A] time = 1.99, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log(x^2+x+1) + \frac{1}{6}\log(x^2-x+1) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1) + 1/3*log(x - 1)

mupad [B] time = 0.04, size = 94, normalized size = 1.36

$$x + \frac{\operatorname{atan}(x1i) 2i}{3} - \operatorname{atan}\left(\frac{x 32i}{-32 + \sqrt{3} 32i} - \frac{32 \sqrt{3} x}{-32 + \sqrt{3} 32i}\right) \left(\frac{\sqrt{3}}{3} - \frac{1}{3}i\right) - \operatorname{atan}\left(\frac{x 32i}{32 + \sqrt{3} 32i} + \frac{32 \sqrt{3} x}{32 + \sqrt{3} 32i}\right) \left(\frac{\sqrt{3}}{3} + \frac{1}{3}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(1/x^3 + x^3)/(1/x^3 - x^3), x)`

[Out] `x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (32*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)`

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1/x**3+x**3)/(-1/x**3+x**3), x)`

[Out] `x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3`

$$3.423 \quad \int \frac{-x+x^3}{6+2x} dx$$

Optimal. Leaf size=24

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 772}

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(-x + x^3)/(6 + 2*x), x]

[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-x+x^3}{6+2x} dx &= \int \frac{x(-1+x^2)}{6+2x} dx \\ &= \int \left(4 - \frac{3x}{2} + \frac{x^2}{2} - \frac{12}{3+x} \right) dx \\ &= 4x - \frac{3x^2}{4} + \frac{x^3}{6} - 12 \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.29

$$\frac{1}{2} \left(\frac{x^3}{3} - \frac{3x^2}{2} + 8x - 24 \log(x + 3) + \frac{93}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + x^3)/(6 + 2*x), x]

[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x + x^3}{6 + 2x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-x + x^3)/(6 + 2*x), x]

[Out] IntegrateAlgebraic[(-x + x^3)/(6 + 2*x), x]

fricas [A] time = 1.48, size = 20, normalized size = 0.83

$$\frac{1}{6} x^3 - \frac{3}{4} x^2 + 4x - 12 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(6+2*x), x, algorithm="fricas")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)

giac [A] time = 0.31, size = 21, normalized size = 0.88

$$\frac{1}{6} x^3 - \frac{3}{4} x^2 + 4x - 12 \log(|x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-x)/(6+2*x), x, algorithm="giac")

[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \ln(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-x)/(6+2*x),x)`

[Out] `4*x-3/4*x^2+1/6*x^3-12*ln(x+3)`

maxima [A] time = 0.63, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-x)/(6+2*x),x, algorithm="maxima")`

[Out] `1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)`

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$4x - 12 \ln(x + 3) - \frac{3x^2}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x - x^3)/(2*x + 6),x)`

[Out] `4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6`

sympy [A] time = 0.08, size = 20, normalized size = 0.83

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12 \log(x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-x)/(6+2*x),x)`

[Out] `x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)`

$$3.424 \quad \int \frac{x+x^3}{-1+x} dx$$

Optimal. Leaf size=26

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \log(1-x)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 772}

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(x + x^3)/(-1 + x), x]

[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x+x^3}{-1+x} dx &= \int \frac{x(1+x^2)}{-1+x} dx \\ &= \int \left(2 + \frac{2}{-1+x} + x + x^2 \right) dx \\ &= 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2 \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6} (2x^3 + 3x^2 + 12x + 12 \log(x-1) - 17)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^3)/(-1 + x), x]

[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + x^3}{-1 + x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + x^3)/(-1 + x), x]

[Out] IntegrateAlgebraic[(x + x^3)/(-1 + x), x]

fricas [A] time = 1.35, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2 \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x), x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)

giac [A] time = 0.30, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2 \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x), x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x)/(x-1), x)

[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)

maxima [A] time = 0.69, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$2x + 2\ln(x-1) + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^3)/(x - 1),x)

[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3

sympy [A] time = 0.07, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x)/(-1+x),x)

[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)

$$3.425 \quad \int (ac + (bc + d)x) dx$$

Optimal. Leaf size=17

$$acx + \frac{1}{2}x^2(bc + d)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$acx + \frac{1}{2}x^2(bc + d)$$

Antiderivative was successfully verified.

[In] Int[a*c + (b*c + d)*x, x]

[Out] a*c*x + ((b*c + d)*x^2)/2

Rubi steps

$$\int (ac + (bc + d)x) dx = acx + \frac{1}{2}(bc + d)x^2$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.29

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[a*c + (b*c + d)*x, x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ac + (bc + d)x) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a*c + (b*c + d)*x, x]

[Out] IntegrateAlgebraic[a*c + (b*c + d)*x, x]

fricas [A] time = 0.99, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")

[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a

giac [A] time = 0.39, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")

[Out] a*c*x + 1/2*(b*c + d)*x^2

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$acx + \frac{(bc + d)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*c+(b*c+d)*x,x)

[Out] a*c*x+1/2*(b*c+d)*x^2

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")

[Out] a*c*x + 1/2*(b*c + d)*x^2

mupad [B] time = 0.02, size = 17, normalized size = 1.00

$$\left(\frac{d}{2} + \frac{bc}{2}\right)x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c + x*(d + b*c),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

sympy [A] time = 0.06, size = 15, normalized size = 0.88

$$acx + x^2 \left(\frac{bc}{2} + \frac{d}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

$$3.426 \quad \int(dx + c(a + bx)) dx$$

Optimal. Leaf size=24

$$\frac{c(a + bx)^2}{2b} + \frac{dx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{c(a + bx)^2}{2b} + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[d*x + c*(a + b*x), x]

[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)

Rubi steps

$$\int(dx + c(a + bx)) dx = \frac{dx^2}{2} + \frac{c(a + bx)^2}{2b}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[d*x + c*(a + b*x), x]

[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx + c(a + bx)) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[d*x + c*(a + b*x), x]

[Out] IntegrateAlgebraic[d*x + c*(a + b*x), x]

fricas [A] time = 1.00, size = 18, normalized size = 0.75

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")

[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a

giac [A] time = 0.37, size = 20, normalized size = 0.83

$$\frac{1}{2}dx^2 + \frac{1}{2}(bx^2 + 2ax)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")

[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{dx^2}{2} + \left(\frac{1}{2}bx^2 + ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d*x+c*(b*x+a),x)

[Out] 1/2*d*x^2+c*(1/2*b*x^2+a*x)

maxima [A] time = 0.57, size = 20, normalized size = 0.83

$$\frac{1}{2}dx^2 + \frac{1}{2}(bx^2 + 2ax)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")

[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c

mupad [B] time = 0.02, size = 17, normalized size = 0.71

$$\left(\frac{d}{2} + \frac{bc}{2}\right)x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x + c*(a + b*x),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

```
sympy [A] time = 0.06, size = 15, normalized size = 0.62
```

$$acx + x^2 \left(\frac{bc}{2} + \frac{d}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

$$3.427 \quad \int \frac{4+4x}{x^2(1+x^2)} dx$$

Optimal. Leaf size=22

$$-2 \log(x^2 + 1) - \frac{4}{x} + 4 \log(x) - 4 \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {801, 635, 203, 260}

$$-2 \log(x^2 + 1) - \frac{4}{x} + 4 \log(x) - 4 \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]

[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{4+4x}{x^2(1+x^2)} dx &= \int \left(\frac{4}{x^2} + \frac{4}{x} - \frac{4(1+x)}{1+x^2} \right) dx \\
&= -\frac{4}{x} + 4 \log(x) - 4 \int \frac{1+x}{1+x^2} dx \\
&= -\frac{4}{x} + 4 \log(x) - 4 \int \frac{1}{1+x^2} dx - 4 \int \frac{x}{1+x^2} dx \\
&= -\frac{4}{x} - 4 \tan^{-1}(x) + 4 \log(x) - 2 \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.09

$$4 \left(-\frac{1}{2} \log(x^2 + 1) - \frac{1}{x} + \log(x) - \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)), x]

[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+4x}{x^2(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 4*x)/(x^2*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(4 + 4*x)/(x^2*(1 + x^2)), x]

fricas [A] time = 0.96, size = 25, normalized size = 1.14

$$\frac{2(2x \arctan(x) + x \log(x^2 + 1) - 2x \log(x) + 2)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1), x, algorithm="fricas")

[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x

giac [A] time = 0.39, size = 23, normalized size = 1.05

$$-\frac{4}{x} - 4 \arctan(x) - 2 \log(x^2 + 1) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")

[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))

maple [A] time = 0.01, size = 23, normalized size = 1.05

$$-4 \arctan(x) + 4 \ln(x) - 2 \ln(x^2 + 1) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4+4*x)/x^2/(x^2+1),x)

[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)

maxima [A] time = 1.38, size = 22, normalized size = 1.00

$$-\frac{4}{x} - 4 \arctan(x) - 2 \log(x^2 + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")

[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)

mupad [B] time = 0.04, size = 28, normalized size = 1.27

$$4 \ln(x) - \frac{4}{x} + \ln(x - i) (-2 + 2i) + \ln(x + i) (-2 - 2i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)

[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x

sympy [A] time = 0.13, size = 20, normalized size = 0.91

$$4 \log(x) - 2 \log(x^2 + 1) - 4 \operatorname{atan}(x) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
```

```
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

$$3.428 \quad \int \frac{24+8x}{x(-4+x^2)} dx$$

Optimal. Leaf size=17

$$5 \log(2-x) - 6 \log(x) + \log(x+2)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {801}

$$5 \log(2-x) - 6 \log(x) + \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{24+8x}{x(-4+x^2)} dx &= \int \left(\frac{5}{-2+x} - \frac{6}{x} + \frac{1}{2+x} \right) dx \\ &= 5 \log(2-x) - 6 \log(x) + \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.59

$$8 \left(\frac{5}{8} \log(2-x) - \frac{3 \log(x)}{4} + \frac{1}{8} \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{24+8x}{x(-4+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(24 + 8*x)/(x*(-4 + x^2)),x]

[Out] IntegrateAlgebraic[(24 + 8*x)/(x*(-4 + x^2)), x]

fricas [A] time = 1.41, size = 15, normalized size = 0.88

$$\log(x + 2) + 5 \log(x - 2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)

giac [A] time = 0.33, size = 18, normalized size = 1.06

$$\log(|x + 2|) + 5 \log(|x - 2|) - 6 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")

[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$-6 \ln(x) + 5 \ln(x - 2) + \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((24+8*x)/x/(x^2-4),x)

[Out] 5*ln(x-2)+ln(x+2)-6*ln(x)

maxima [A] time = 0.75, size = 15, normalized size = 0.88

$$\log(x + 2) + 5 \log(x - 2) - 6 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")

[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$5 \ln(x - 2) + \ln(x + 2) - 6 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 24)/(x*(x^2 - 4)),x)
```

```
[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$-6 \log(x) + 5 \log(x - 2) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x**2-4),x)
```

```
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

$$3.429 \quad \int \frac{-1+x^2}{-2x+x^3} dx$$

Optimal. Leaf size=19

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 446, 72}

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(-2*x + x^3), x]

[Out] Log[x]/2 + Log[2 - x^2]/4

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
  x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{-2x+x^3} dx &= \int \frac{-1+x^2}{x(-2+x^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{-1+x}{(-2+x)x} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{2(-2+x)} + \frac{1}{2x} \right) dx, x, x^2 \right) \\
&= \frac{\log(x)}{2} + \frac{1}{4} \log(2-x^2)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{4} \log(2-x^2) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(-2*x + x^3), x]

[Out] Log[x]/2 + Log[2 - x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{-2x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^2)/(-2*x + x^3), x]

[Out] IntegrateAlgebraic[(-1 + x^2)/(-2*x + x^3), x]

fricas [A] time = 1.28, size = 13, normalized size = 0.68

$$\frac{1}{4} \log(x^2 - 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x), x, algorithm="fricas")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{1}{4} \log(x^2) + \frac{1}{4} \log(|x^2 - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")

[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{\ln(x)}{2} + \frac{\ln(x^2 - 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^3-2*x),x)

[Out] 1/4*ln(x^2-2)+1/2*ln(x)

maxima [A] time = 0.66, size = 13, normalized size = 0.68

$$\frac{1}{4} \log(x^2 - 2) + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")

[Out] 1/4*log(x^2 - 2) + 1/2*log(x)

mupad [B] time = 2.34, size = 13, normalized size = 0.68

$$\frac{\ln(x^2 - 2)}{4} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/(2*x - x^3),x)

[Out] log(x^2 - 2)/4 + log(x)/2

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\log(x)}{2} + \frac{\log(x^2 - 2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3-2*x),x)
```

```
[Out] log(x)/2 + log(x**2 - 2)/4
```

$$3.430 \quad \int \frac{1+x^2}{3x+x^3} dx$$

Optimal. Leaf size=12

$$\frac{1}{3} \log(x^3 + 3x)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1587}

$$\frac{1}{3} \log(x^3 + 3x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(3*x + x^3), x]

[Out] Log[3*x + x^3]/3

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{1+x^2}{3x+x^3} dx = \frac{1}{3} \log(3x+x^3)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{1}{3} \log(x^2 + 3) + \frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(3*x + x^3), x]

[Out] Log[x]/3 + Log[3 + x^2]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{3x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(3*x + x^3), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(3*x + x^3), x]

fricas [A] time = 1.40, size = 10, normalized size = 0.83

$$\frac{1}{3} \log(x^3 + 3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+3*x), x, algorithm="fricas")

[Out] 1/3*log(x^3 + 3*x)

giac [A] time = 0.40, size = 13, normalized size = 1.08

$$\frac{1}{3} \log\left(3 \left|\frac{1}{3} x^3 + x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^3+3*x), x, algorithm="giac")

[Out] 1/3*log(3*abs(1/3*x^3 + x))

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{\ln\left(\frac{(x^2+3)x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^3+3*x), x)

[Out] 1/3*ln(x*(x^2+3))

maxima [A] time = 0.62, size = 10, normalized size = 0.83

$$\frac{1}{3} \log(x^3 + 3x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")`

[Out] `1/3*log(x^3 + 3*x)`

mupad [B] time = 2.27, size = 10, normalized size = 0.83

$$\frac{\ln(x^3 + 3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(3*x + x^3),x)`

[Out] `log(3*x + x^3)/3`

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\frac{\log(x^3 + 3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(x**3+3*x),x)`

[Out] `log(x**3 + 3*x)/3`

$$3.431 \quad \int \frac{a+3bx^2}{ax+bx^3} dx$$

Optimal. Leaf size=10

$$\log(ax + bx^3)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1587}

$$\log(ax + bx^3)$$

Antiderivative was successfully verified.

[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[a*x + b*x^3]

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{a + 3bx^2}{ax + bx^3} dx = \log(ax + bx^3)$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log(a + bx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]

[Out] Log[x] + Log[a + b*x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + 3bx^2}{ax + bx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + 3*b*x^2)/(a*x + b*x^3), x]

[Out] IntegrateAlgebraic[(a + 3*b*x^2)/(a*x + b*x^3), x]

fricas [A] time = 1.16, size = 10, normalized size = 1.00

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*b*x^2+a)/(b*x^3+a*x), x, algorithm="fricas")

[Out] log(b*x^3 + a*x)

giac [A] time = 0.33, size = 11, normalized size = 1.10

$$\log(|bx^3 + ax|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*b*x^2+a)/(b*x^3+a*x), x, algorithm="giac")

[Out] log(abs(b*x^3 + a*x))

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln((bx^2 + a)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*b*x^2+a)/(b*x^3+a*x), x)

[Out] ln(x*(b*x^2+a))

maxima [A] time = 0.61, size = 10, normalized size = 1.00

$$\log(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*b*x^2+a)/(b*x^3+a*x), x, algorithm="maxima")

[Out] log(b*x^3 + a*x)

mupad [B] time = 0.06, size = 10, normalized size = 1.00

$$\ln(bx^3 + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 3*b*x^2)/(a*x + b*x^3),x)
```

```
[Out] log(a*x + b*x^3)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log(ax + bx^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

```
[Out] log(a*x + b*x**3)
```


$$3.432 \quad \int \frac{-2+4x}{-x+x^3} dx$$

Optimal. Leaf size=17

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 801}

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(-2 + 4*x)/(-x + x^3), x]

[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]

Rule 801

Int[(((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1593

Int[(u_.)*((a_.)*(x_.)^(p_.) + (b_.)*(x_.)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{-2+4x}{-x+x^3} dx &= \int \frac{-2+4x}{x(-1+x^2)} dx \\ &= \int \left(\frac{1}{-1+x} + \frac{2}{x} - \frac{3}{1+x} \right) dx \\ &= \log(1-x) + 2\log(x) - 3\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\log(1-x) + 2\log(x) - 3\log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 4*x)/(-x + x^3), x]

[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2 + 4x}{-x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + 4*x)/(-x + x^3), x]

[Out] IntegrateAlgebraic[(-2 + 4*x)/(-x + x^3), x]

fricas [A] time = 0.99, size = 15, normalized size = 0.88

$$-3 \log(x + 1) + \log(x - 1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+4*x)/(x^3-x), x, algorithm="fricas")

[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)

giac [A] time = 0.36, size = 18, normalized size = 1.06

$$-3 \log(|x + 1|) + \log(|x - 1|) + 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+4*x)/(x^3-x), x, algorithm="giac")

[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$2 \ln(x) + \ln(x - 1) - 3 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2+4*x)/(x^3-x), x)

[Out] ln(x-1)-3*ln(x+1)+2*ln(x)

maxima [A] time = 0.61, size = 15, normalized size = 0.88

$$-3 \log(x + 1) + \log(x - 1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")

[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$\ln(x - 1) - 3 \ln(x + 1) + 2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x - 2)/(x - x^3),x)

[Out] log(x - 1) - 3*log(x + 1) + 2*log(x)

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$2 \log(x) + \log(x - 1) - 3 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+4*x)/(x**3-x),x)

[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)

$$3.433 \quad \int \frac{4+x}{4x+x^3} dx$$

Optimal. Leaf size=23

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/(4*x + x^3), x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1593

`Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]`

Rubi steps

$$\begin{aligned}\int \frac{4+x}{4x+x^3} dx &= \int \frac{4+x}{x(4+x^2)} dx \\ &= \int \left(\frac{1}{x} + \frac{1-x}{4+x^2} \right) dx \\ &= \log(x) + \int \frac{1-x}{4+x^2} dx \\ &= \log(x) + \int \frac{1}{4+x^2} dx - \int \frac{x}{4+x^2} dx \\ &= \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \log(x) - \frac{1}{2} \log(4+x^2)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{1}{2} \log(x^2 + 4) + \log(x) + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x)/(4*x + x^3), x]

[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+x}{4x+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + x)/(4*x + x^3), x]

[Out] IntegrateAlgebraic[(4 + x)/(4*x + x^3), x]

fricas [A] time = 1.39, size = 17, normalized size = 0.74

$$\frac{1}{2} \arctan\left(\frac{1}{2} x\right) - \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)

giac [A] time = 0.38, size = 18, normalized size = 0.78

$$\frac{1}{2} \arctan\left(\frac{1}{2}x\right) - \frac{1}{2} \log(x^2 + 4) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\arctan\left(\frac{x}{2}\right)}{2} + \ln(x) - \frac{\ln(x^2 + 4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+4)/(x^3+4*x),x)

[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)

maxima [A] time = 1.64, size = 17, normalized size = 0.74

$$\frac{1}{2} \arctan\left(\frac{1}{2}x\right) - \frac{1}{2} \log(x^2 + 4) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")

[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)

mupad [B] time = 0.05, size = 21, normalized size = 0.91

$$\ln(x) + \ln(x - 2i) \left(-\frac{1}{2} - \frac{1}{4}i\right) + \ln(x + 2i) \left(-\frac{1}{2} + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4)/(4*x + x^3),x)

[Out] $\log(x) - \log(x + 2i) \cdot (1/2 - 1i/4) - \log(x - 2i) \cdot (1/2 + 1i/4)$

sympy [A] time = 0.13, size = 17, normalized size = 0.74

$$\log(x) - \frac{\log(x^2 + 4)}{2} + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4+x)/(x**3+4*x),x)`

[Out] $\log(x) - \log(x^2 + 4)/2 + \operatorname{atan}(x/2)/2$

$$3.434 \quad \int \frac{-x+2x^3}{1-x^2+x^4} dx$$

Optimal. Leaf size=15

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1587}

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Antiderivative was successfully verified.

[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4), x]

[Out] Log[1 - x^2 + x^4]/2

Rule 1587

Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; EqQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x, q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]

Rubi steps

$$\int \frac{-x + 2x^3}{1 - x^2 + x^4} dx = \frac{1}{2} \log(1 - x^2 + x^4)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4), x]

[Out] Log[1 - x^2 + x^4]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x + 2x^3}{1 - x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-x + 2*x^3)/(1 - x^2 + x^4), x]

[Out] IntegrateAlgebraic[(-x + 2*x^3)/(1 - x^2 + x^4), x]

fricas [A] time = 1.27, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1), x, algorithm="fricas")

[Out] 1/2*log(x^4 - x^2 + 1)

giac [A] time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1), x, algorithm="giac")

[Out] 1/2*log(x^4 - x^2 + 1)

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3-x)/(x^4-x^2+1), x)

[Out] 1/2*ln(x^4-x^2+1)

maxima [A] time = 0.65, size = 13, normalized size = 0.87

$$\frac{1}{2} \log(x^4 - x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")

[Out] 1/2*log(x^4 - x^2 + 1)

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)

[Out] log(x^4 - x^2 + 1)/2

sympy [A] time = 0.09, size = 10, normalized size = 0.67

$$\frac{\log(x^4 - x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3-x)/(x**4-x**2+1),x)

[Out] log(x**4 - x**2 + 1)/2

$$3.435 \quad \int \frac{-3+x}{2x+3x^2+x^3} dx$$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5}{2}\log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2} + 4\log(x+1) - \frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3), x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2

Rule 800

Int[(((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rubi steps

$$\begin{aligned} \int \frac{-3+x}{2x+3x^2+x^3} dx &= \int \frac{-3+x}{x(2+3x+x^2)} dx \\ &= \int \left(-\frac{3}{2x} + \frac{4}{1+x} - \frac{5}{2(2+x)} \right) dx \\ &= -\frac{3\log(x)}{2} + 4\log(1+x) - \frac{5}{2}\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{3 \log(x)}{2} + 4 \log(x+1) - \frac{5}{2} \log(x+2)$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3), x]

[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + x}{2x + 3x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + x)/(2*x + 3*x^2 + x^3), x]

[Out] IntegrateAlgebraic[(-3 + x)/(2*x + 3*x^2 + x^3), x]

fricas [A] time = 1.53, size = 17, normalized size = 0.81

$$-\frac{5}{2} \log(x+2) + 4 \log(x+1) - \frac{3}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x^3+3*x^2+2*x), x, algorithm="fricas")

[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)

giac [A] time = 0.37, size = 20, normalized size = 0.95

$$-\frac{5}{2} \log(|x+2|) + 4 \log(|x+1|) - \frac{3}{2} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)/(x^3+3*x^2+2*x), x, algorithm="giac")

[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{3 \ln(x)}{2} + 4 \ln(x+1) - \frac{5 \ln(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x-3)/(x^3+3*x^2+2*x),x)`

[Out] $-3/2*\ln(x)+4*\ln(x+1)-5/2*\ln(x+2)$

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{5}{2} \log(x+2) + 4 \log(x+1) - \frac{3}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")`

[Out] $-5/2*\log(x+2) + 4*\log(x+1) - 3/2*\log(x)$

mupad [B] time = 0.07, size = 17, normalized size = 0.81

$$4 \ln(x+1) - \frac{5 \ln(x+2)}{2} - \frac{3 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x-3)/(2*x+3*x^2+x^3),x)`

[Out] $4*\log(x+1) - (5*\log(x+2))/2 - (3*\log(x))/2$

sympy [A] time = 0.13, size = 20, normalized size = 0.95

$$-\frac{3 \log(x)}{2} + 4 \log(x+1) - \frac{5 \log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-3+x)/(x**3+3*x**2+2*x),x)`

[Out] $-3*\log(x)/2 + 4*\log(x+1) - 5*\log(x+2)/2$

$$3.436 \quad \int \frac{2+4x}{x^2+2x^3+x^4} dx$$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1594, 27, 74}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

[Out] -2/(x*(1 + x))

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 74

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 1594

Int[(u_.)*((a_.)*(x_.)^(p_.) + (b_.)*(x_.)^(q_.) + (c_.)*(x_.)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned}\int \frac{2+4x}{x^2+2x^3+x^4} dx &= \int \frac{2+4x}{x^2(1+2x+x^2)} dx \\ &= \int \frac{2+4x}{x^2(1+x)^2} dx \\ &= -\frac{2}{x(1+x)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

[Out] -2/(x + x^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+4x}{x^2+2x^3+x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

[Out] IntegrateAlgebraic[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]

fricas [A] time = 1.19, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+4*x)/(x^4+2*x^3+x^2), x, algorithm="fricas")

[Out] -2/(x^2 + x)

giac [A] time = 0.26, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")

[Out] -2/(x^2 + x)

maple [A] time = 0.00, size = 14, normalized size = 1.40

$$-\frac{2}{x} + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+4*x)/(x^4+2*x^3+x^2),x)

[Out] -2/x+2/(x+1)

maxima [A] time = 0.78, size = 9, normalized size = 0.90

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")

[Out] -2/(x^2 + x)

mupad [B] time = 2.20, size = 10, normalized size = 1.00

$$-\frac{2}{x(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)

[Out] -2/(x*(x + 1))

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)

[Out] -2/(x**2 + x)

$$3.437 \quad \int \frac{1+x}{-6x+x^2+x^3} dx$$

Optimal. Leaf size=25

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1594, 800}

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(-6*x + x^2 + x^3), x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15

Rule 800

Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q-p) + c*x^(r-p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{-6x+x^2+x^3} dx &= \int \frac{1+x}{x(-6+x+x^2)} dx \\ &= \int \left(\frac{3}{10(-2+x)} - \frac{1}{6x} - \frac{2}{15(3+x)} \right) dx \\ &= \frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{3}{10} \log(2-x) - \frac{\log(x)}{6} - \frac{2}{15} \log(x+3)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(-6*x + x^2 + x^3), x]

[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{-6x+x^2+x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x)/(-6*x + x^2 + x^3), x]

[Out] IntegrateAlgebraic[(1 + x)/(-6*x + x^2 + x^3), x]

fricas [A] time = 1.42, size = 17, normalized size = 0.68

$$-\frac{2}{15} \log(x+3) + \frac{3}{10} \log(x-2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+x^2-6*x), x, algorithm="fricas")

[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)

giac [A] time = 0.36, size = 20, normalized size = 0.80

$$-\frac{2}{15} \log(|x+3|) + \frac{3}{10} \log(|x-2|) - \frac{1}{6} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^3+x^2-6*x), x, algorithm="giac")

[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$-\frac{\ln(x)}{6} + \frac{3 \ln(x-2)}{10} - \frac{2 \ln(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+1)/(x^3+x^2-6*x),x)`

[Out] $3/10*\ln(x-2)-2/15*\ln(x+3)-1/6*\ln(x)$

maxima [A] time = 0.65, size = 17, normalized size = 0.68

$$-\frac{2}{15} \log(x+3) + \frac{3}{10} \log(x-2) - \frac{1}{6} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")`

[Out] $-2/15*\log(x+3) + 3/10*\log(x-2) - 1/6*\log(x)$

mupad [B] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3 \ln(x-2)}{10} - \frac{2 \ln(x+3)}{15} - \frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+1)/(x^2-6*x+x^3),x)`

[Out] $(3*\log(x-2))/10 - (2*\log(x+3))/15 - \log(x)/6$

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\log(x)}{6} + \frac{3 \log(x-2)}{10} - \frac{2 \log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(x**3+x**2-6*x),x)`

[Out] $-\log(x)/6 + 3*\log(x-2)/10 - 2*\log(x+3)/15$

$$3.438 \quad \int \frac{4x^2+x^3}{x+x^3} dx$$

Optimal. Leaf size=14

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1593, 1584, 774, 635, 203, 260}

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(4*x^2 + x^3)/(x + x^3), x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 774

Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]

&& IntegerQ[n] && PosQ[q - p]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{4x^2 + x^3}{x + x^3} dx &= \int \frac{4x^2 + x^3}{x(1 + x^2)} dx \\
 &= \int \frac{x(4 + x)}{1 + x^2} dx \\
 &= x + \int \frac{-1 + 4x}{1 + x^2} dx \\
 &= x + 4 \int \frac{x}{1 + x^2} dx - \int \frac{1}{1 + x^2} dx \\
 &= x - \tan^{-1}(x) + 2 \log(1 + x^2)
 \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$2 \log(x^2 + 1) + x - \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(4*x^2 + x^3)/(x + x^3), x]

[Out] x - ArcTan[x] + 2*Log[1 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + x^3}{x + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*x^2 + x^3)/(x + x^3), x]

[Out] IntegrateAlgebraic[(4*x^2 + x^3)/(x + x^3), x]

fricas [A] time = 1.73, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")

[Out] x - arctan(x) + 2*log(x^2 + 1)

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")

[Out] x - arctan(x) + 2*log(x^2 + 1)

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$x - \arctan(x) + 2 \ln(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+4*x^2)/(x^3+x),x)

[Out] x-arctan(x)+2*ln(x^2+1)

maxima [A] time = 1.56, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")

[Out] x - arctan(x) + 2*log(x^2 + 1)

mupad [B] time = 2.22, size = 14, normalized size = 1.00

$$x + 2 \ln(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2 + x^3)/(x + x^3),x)

```
[Out] x + 2*log(x^2 + 1) - atan(x)
```

```
sympy [A] time = 0.10, size = 12, normalized size = 0.86
```

$$x + 2 \log(x^2 + 1) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
```

```
[Out] x + 2*log(x**2 + 1) - atan(x)
```

$$3.439 \quad \int \frac{x+2x^3}{(x^2+x^4)^3} dx$$

Optimal. Leaf size=13

$$-\frac{1}{4(x^4+x^2)^2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$-\frac{1}{4(x^4+x^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] -1/(4*(x^2 + x^4)^2)

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{x + 2x^3}{(x^2 + x^4)^3} dx = -\frac{1}{4(x^2 + x^4)^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4(x^2+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] $-1/4*1/(x^4*(1 + x^2)^2)$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 2x^3}{(x^2 + x^4)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + 2*x^3)/(x^2 + x^4)^3,x]

[Out] IntegrateAlgebraic[(x + 2*x^3)/(x^2 + x^4)^3, x]

fricas [A] time = 1.45, size = 16, normalized size = 1.23

$$-\frac{1}{4(x^8 + 2x^6 + x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")

[Out] $-1/4/(x^8 + 2*x^6 + x^4)$

giac [A] time = 0.37, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4 + x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")

[Out] $-1/4/(x^4 + x^2)^2$

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{1}{2x^2} - \frac{1}{4x^4} - \frac{1}{4(x^2 + 1)^2} - \frac{1}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+x)/(x^4+x^2)^3,x)

[Out] $-1/4/(x^2+1)^2 - 1/2/(x^2+1) - 1/4/x^4 + 1/2/x^2$

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{4(x^4 + x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")

[Out] -1/4/(x^4 + x^2)^2

mupad [B] time = 2.24, size = 20, normalized size = 1.54

$$-\frac{1}{4x^8 + 8x^6 + 4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)

[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)

sympy [A] time = 0.13, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8 + 8x^6 + 4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)

[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)

$$3.440 \quad \int \frac{ax^2+bx^3}{cx^2+dx^3} dx$$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc - ad) \log(c + dx)}{d^2}$$

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1593, 1584, 43}

$$\frac{bx}{d} - \frac{(bc - ad) \log(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{ax^2 + bx^3}{cx^2 + dx^3} dx &= \int \frac{x^2(a + bx)}{cx^2 + dx^3} dx \\
&= \int \frac{a + bx}{c + dx} dx \\
&= \int \left(\frac{b}{d} + \frac{-bc + ad}{d(c + dx)} \right) dx \\
&= \frac{bx}{d} - \frac{(bc - ad) \log(c + dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad - bc) \log(c + dx)}{d^2} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]

[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]

[Out] IntegrateAlgebraic[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]

fricas [A] time = 1.51, size = 25, normalized size = 0.96

$$\frac{bdx - (bc - ad) \log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2), x, algorithm="fricas")

[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2

giac [A] time = 0.28, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad) \log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")

[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2

maple [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a \ln(dx + c)}{d} - \frac{bc \ln(dx + c)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)

[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c

maxima [A] time = 0.72, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad) \log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")

[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2

mupad [B] time = 2.23, size = 25, normalized size = 0.96

$$\frac{\ln(c + dx) (ad - bc)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)

[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d

sympy [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc) \log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)

[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2

$$3.441 \quad \int \frac{x+x^2}{-2x-x^2+x^3} dx$$

Optimal. Leaf size=6

$$\log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1586, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]

[Out] Log[2 - x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 1586

Int[(u_.)*(P_x_)^(p_.)*(Q_x_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p+q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\int \frac{x+x^2}{-2x-x^2+x^3} dx = \int \frac{1}{-2+x} dx = \log(2-x)$$

Mathematica [A] time = 0.00, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]

[Out] Log[-2 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + x^2}{-2x - x^2 + x^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + x^2)/(-2*x - x^2 + x^3), x]

[Out] IntegrateAlgebraic[(x + x^2)/(-2*x - x^2 + x^3), x]

fricas [A] time = 1.49, size = 4, normalized size = 0.67

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x), x, algorithm="fricas")

[Out] log(x - 2)

giac [A] time = 0.28, size = 5, normalized size = 0.83

$$\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x), x, algorithm="giac")

[Out] log(abs(x - 2))

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\ln(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x)/(x^3-x^2-2*x), x)

[Out] ln(x-2)

maxima [A] time = 0.57, size = 4, normalized size = 0.67

$$\log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x)/(x^3-x^2-2*x), x, algorithm="maxima")

[Out] $\log(x - 2)$

mupad [B] time = 0.02, size = 4, normalized size = 0.67

$\ln(x - 2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x + x^2)/(2*x + x^2 - x^3), x)`

[Out] $\log(x - 2)$

sympy [A] time = 0.06, size = 3, normalized size = 0.50

$\log(x - 2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x)/(x**3-x**2-2*x), x)`

[Out] $\log(x - 2)$

$$3.442 \quad \int \frac{1-5x^2}{x^3(1+x^2)} dx$$

Optimal. Leaf size=20

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {446, 77}

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]

[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{1-5x^2}{x^3(1+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1-5x}{x^2(1+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x^2} - \frac{6}{x} + \frac{6}{1+x} \right) dx, x, x^2 \right) \\ &= -\frac{1}{2x^2} - 6 \log(x) + 3 \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 6 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)), x]

[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 5x^2}{x^3(1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - 5*x^2)/(x^3*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(1 - 5*x^2)/(x^3*(1 + x^2)), x]

fricas [A] time = 1.65, size = 25, normalized size = 1.25

$$\frac{6x^2 \log(x^2 + 1) - 12x^2 \log(x) - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+1)/x^3/(x^2+1), x, algorithm="fricas")

[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2

giac [A] time = 0.38, size = 27, normalized size = 1.35

$$\frac{6x^2 - 1}{2x^2} + 3 \log(x^2 + 1) - 3 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5*x^2+1)/x^3/(x^2+1), x, algorithm="giac")

[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$-6 \ln(x) + 3 \ln(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-5*x^2+1)/x^3/(x^2+1),x)`

[Out] `-1/2/x^2-6*ln(x)+3*ln(x^2+1)`

maxima [A] time = 0.72, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2} + 3 \log(x^2 + 1) - 3 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")`

[Out] `-1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)`

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$3 \ln(x^2 + 1) - 6 \ln(x) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)`

[Out] `3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)`

sympy [A] time = 0.11, size = 19, normalized size = 0.95

$$-6 \log(x) + 3 \log(x^2 + 1) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-5*x**2+1)/x**3/(x**2+1),x)`

[Out] `-6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)`

$$3.443 \quad \int \frac{2x}{(-1+x)(5+x^2)} dx$$

Optimal. Leaf size=38

$$-\frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(1 - x) + \frac{1}{3} \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {12, 801, 635, 203, 260}

$$-\frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(1 - x) + \frac{1}{3} \sqrt{5} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]

[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
  x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
  x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
 \int \frac{2x}{(-1+x)(5+x^2)} dx &= 2 \int \frac{x}{(-1+x)(5+x^2)} dx \\
 &= 2 \int \left(\frac{1}{6(-1+x)} + \frac{5-x}{6(5+x^2)} \right) dx \\
 &= \frac{1}{3} \log(1-x) + \frac{1}{3} \int \frac{5-x}{5+x^2} dx \\
 &= \frac{1}{3} \log(1-x) - \frac{1}{3} \int \frac{x}{5+x^2} dx + \frac{5}{3} \int \frac{1}{5+x^2} dx \\
 &= \frac{1}{3} \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) + \frac{1}{3} \log(1-x) - \frac{1}{6} \log(5+x^2)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$2 \left(-\frac{1}{12} \log(x^2 + 5) + \frac{1}{6} \log(1-x) + \frac{1}{6} \sqrt{5} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)), x]
```

```
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x}{(-1+x)(5+x^2)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x)/((-1 + x)*(5 + x^2)), x]
```

```
[Out] IntegrateAlgebraic[(2*x)/((-1 + x)*(5 + x^2)), x]
```

fricas [A] time = 1.27, size = 27, normalized size = 0.71

$$\frac{1}{3} \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} x \right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)

giac [A] time = 0.39, size = 28, normalized size = 0.74

$$\frac{1}{3} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} x\right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))

maple [A] time = 0.00, size = 28, normalized size = 0.74

$$\frac{\sqrt{5} \arctan\left(\frac{\sqrt{5} x}{5}\right)}{3} + \frac{\ln(x - 1)}{3} - \frac{\ln(x^2 + 5)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*x/(x-1)/(x^2+5),x)

[Out] 1/3*ln(x-1)-1/6*ln(x^2+5)+1/3*5^(1/2)*arctan(1/5*5^(1/2)*x)

maxima [A] time = 1.97, size = 27, normalized size = 0.71

$$\frac{1}{3} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} x\right) - \frac{1}{6} \log(x^2 + 5) + \frac{1}{3} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")

[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)

mupad [B] time = 0.16, size = 44, normalized size = 1.16

$$\frac{\ln(x - 1)}{3} - \ln(x - \sqrt{5} 1i) \left(\frac{1}{6} + \frac{\sqrt{5} 1i}{6}\right) + \ln(x + \sqrt{5} 1i) \left(-\frac{1}{6} + \frac{\sqrt{5} 1i}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x)/((x^2 + 5)*(x - 1)),x)

[Out] $\log(x - 1)/3 - \log(x - 5^{1/2}i) * ((5^{1/2}i)/6 + 1/6) + \log(x + 5^{1/2}i) * ((5^{1/2}i)/6 - 1/6)$

sympy [A] time = 0.14, size = 31, normalized size = 0.82

$$\frac{\log(x - 1)}{3} - \frac{\log(x^2 + 5)}{6} + \frac{\sqrt{5} \operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*x/(-1+x)/(x**2+5), x)`

[Out] $\log(x - 1)/3 - \log(x^2 + 5)/6 + \sqrt{5} * \operatorname{atan}(\sqrt{5} * x/5)/3$

$$3.444 \quad \int \frac{2+x^2}{2+x} dx$$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {697}

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2 + 6*Log[2 + x]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{2+x^2}{2+x} dx &= \int \left(-2 + x + \frac{6}{2+x} \right) dx \\ &= -2x + \frac{x^2}{2} + 6 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6 \log(x + 2) - 6$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^2)/(2 + x), x]

[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + x^2}{2 + x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x^2)/(2 + x), x]

[Out] IntegrateAlgebraic[(2 + x^2)/(2 + x), x]

fricas [A] time = 1.16, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x), x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x + 6*log(x + 2)

giac [A] time = 0.37, size = 16, normalized size = 0.94

$$\frac{1}{2}x^2 - 2x + 6 \log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x), x, algorithm="giac")

[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - 2x + 6 \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2)/(x+2), x)

[Out] -2*x+1/2*x^2+6*ln(x+2)

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x + 6*log(x + 2)

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$6 \ln(x + 2) - 2x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 2)/(x + 2),x)

[Out] 6*log(x + 2) - 2*x + x^2/2

sympy [A] time = 0.07, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+2)/(2+x),x)

[Out] x**2/2 - 2*x + 6*log(x + 2)

$$3.445 \quad \int \frac{1}{(-3+x)(4+x^2)} dx$$

Optimal. Leaf size=31

$$-\frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(3 - x) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(3 - x) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((-3 + x)*(4 + x^2)),x]

[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*xⁿ, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]}

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -

$c*e*x)/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(-3+x)(4+x^2)} dx &= \frac{1}{13} \int \frac{1}{-3+x} dx + \frac{1}{13} \int \frac{-3-x}{4+x^2} dx \\ &= \frac{1}{13} \log(3-x) - \frac{1}{13} \int \frac{x}{4+x^2} dx - \frac{3}{13} \int \frac{1}{4+x^2} dx \\ &= -\frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{13} \log(3-x) - \frac{1}{26} \log(4+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.16

$$-\frac{1}{26} \log((x-3)^2 + 6(x-3) + 13) + \frac{1}{13} \log(x-3) - \frac{3}{26} \tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((-3 + x)*(4 + x^2)), x]

[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3+x)(4+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((-3 + x)*(4 + x^2)), x]

[Out] IntegrateAlgebraic[1/((-3 + x)*(4 + x^2)), x]

fricas [A] time = 1.46, size = 21, normalized size = 0.68

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3+x)/(x^2+4), x, algorithm="fricas")

[Out] $-3/26*\arctan(1/2*x) - 1/26*\log(x^2 + 4) + 1/13*\log(x - 3)$

giac [A] time = 0.27, size = 22, normalized size = 0.71

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")`

[Out] $-3/26*\arctan(1/2*x) - 1/26*\log(x^2 + 4) + 1/13*\log(\text{abs}(x - 3))$

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$-\frac{3 \arctan\left(\frac{x}{2}\right)}{26} + \frac{\ln(x - 3)}{13} - \frac{\ln(x^2 + 4)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x-3)/(x^2+4),x)`

[Out] $-1/26*\ln(x^2+4)-3/26*\arctan(1/2*x)+1/13*\ln(x-3)$

maxima [A] time = 0.99, size = 21, normalized size = 0.68

$$-\frac{3}{26} \arctan\left(\frac{1}{2}x\right) - \frac{1}{26} \log(x^2 + 4) + \frac{1}{13} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")`

[Out] $-3/26*\arctan(1/2*x) - 1/26*\log(x^2 + 4) + 1/13*\log(x - 3)$

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln(x - 3)}{13} + \ln(x - 2i) \left(-\frac{1}{26} + \frac{3}{52}i\right) + \ln(x + 2i) \left(-\frac{1}{26} - \frac{3}{52}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^2 + 4)*(x - 3)),x)`

[Out] $\log(x - 3)/13 - \log(x - 2i)*(1/26 - 3i/52) - \log(x + 2i)*(1/26 + 3i/52)$

sympy [A] time = 0.14, size = 22, normalized size = 0.71

$$\frac{\log(x-3)}{13} - \frac{\log(x^2+4)}{26} - \frac{3 \operatorname{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-3+x)/(x**2+4),x)`

[Out] `log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26`

$$3.446 \quad \int \frac{-2+3x^6}{x(5+2x^6)} dx$$

Optimal. Leaf size=19

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {446, 72}

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{-2+3x^6}{x(5+2x^6)} dx &= \frac{1}{6} \text{Subst} \left(\int \frac{-2+3x}{x(5+2x)} dx, x, x^6 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \left(-\frac{2}{5x} + \frac{19}{5(5+2x)} \right) dx, x, x^6 \right) \\ &= -\frac{2 \log(x)}{5} + \frac{19}{60} \log(5+2x^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2 \log(x)}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)), x]

[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2 + 3x^6}{x(5 + 2x^6)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + 3*x^6)/(x*(5 + 2*x^6)), x]

[Out] IntegrateAlgebraic[(-2 + 3*x^6)/(x*(5 + 2*x^6)), x]

fricas [A] time = 1.30, size = 15, normalized size = 0.79

$$\frac{19}{60} \log(2x^6 + 5) - \frac{2}{5} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^6-2)/x/(2*x^6+5), x, algorithm="fricas")

[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)

giac [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{19}{60} \log(2x^6 + 5) - \frac{1}{15} \log(x^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^6-2)/x/(2*x^6+5), x, algorithm="giac")

[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{2 \ln(x)}{5} + \frac{19 \ln(2x^6 + 5)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^6-2)/x/(2*x^6+5),x)`

[Out] `-2/5*ln(x)+19/60*ln(2*x^6+5)`

maxima [A] time = 0.47, size = 17, normalized size = 0.89

$$\frac{19}{60} \log(2x^6 + 5) - \frac{1}{15} \log(x^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")`

[Out] `19/60*log(2*x^6 + 5) - 1/15*log(x^6)`

mupad [B] time = 0.09, size = 13, normalized size = 0.68

$$\frac{19 \ln\left(x^6 + \frac{5}{2}\right)}{60} - \frac{2 \ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)`

[Out] `(19*log(x^6 + 5/2))/60 - (2*log(x))/5`

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$-\frac{2 \log(x)}{5} + \frac{19 \log(2x^6 + 5)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**6-2)/x/(2*x**6+5),x)`

[Out] `-2*log(x)/5 + 19*log(2*x**6 + 5)/60`

$$3.447 \quad \int \frac{3+2x}{(-2+x)(5+x)} dx$$

Optimal. Leaf size=11

$$\log(2-x) + \log(x+5)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {72}

$$\log(2-x) + \log(x+5)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[2 - x] + Log[5 + x]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\int \frac{3+2x}{(-2+x)(5+x)} dx = \int \left(\frac{1}{-2+x} + \frac{1}{5+x} \right) dx$$

$$= \log(2-x) + \log(5+x)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\log(x-2) + \log(x+5)$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[-2 + x] + Log[5 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+2x}{(-2+x)(5+x)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3 + 2*x)/((-2 + x)*(5 + x)), x]

[Out] IntegrateAlgebraic[(3 + 2*x)/((-2 + x)*(5 + x)), x]

fricas [A] time = 1.33, size = 9, normalized size = 0.82

$$\log(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-2+x)/(5+x), x, algorithm="fricas")

[Out] log(x^2 + 3*x - 10)

giac [A] time = 0.29, size = 11, normalized size = 1.00

$$\log(|x + 5|) + \log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-2+x)/(5+x), x, algorithm="giac")

[Out] log(abs(x + 5)) + log(abs(x - 2))

maple [A] time = 0.00, size = 9, normalized size = 0.82

$$\ln((x - 2)(x + 5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+3)/(x-2)/(x+5), x)

[Out] ln((x-2)*(x+5))

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\log(x + 5) + \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+2*x)/(-2+x)/(5+x), x, algorithm="maxima")

[Out] log(x + 5) + log(x - 2)

mupad [B] time = 2.22, size = 9, normalized size = 0.82

$$\ln(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
```

```
[Out] log(3*x + x^2 - 10)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$\log(x^2 + 3x - 10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

$$3.448 \quad \int \frac{x^4}{4+5x^2+x^4} dx$$

Optimal. Leaf size=18

$$x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1122, 1166, 203}

$$x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^4/(4 + 5*x^2 + x^4), x]

[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1122

Int[((d_)*(x_)^(m_))*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m-3)*(a + b*x^2 + c*x^4)^(p+1))/(c*(m+4*p+1)), x] - Dist[d^4/(c*(m+4*p+1)), Int[(d*x)^(m-4)*Simp[a*(m-3) + b*(m+2*p-1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m+4*p+1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{4 + 5x^2 + x^4} dx &= x - \int \frac{4 + 5x^2}{4 + 5x^2 + x^4} dx \\ &= x + \frac{1}{3} \int \frac{1}{1 + x^2} dx - \frac{16}{3} \int \frac{1}{4 + x^2} dx \\ &= x - \frac{8}{3} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x + \frac{8}{3} \tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(4 + 5*x^2 + x^4), x]

[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{4 + 5x^2 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^4/(4 + 5*x^2 + x^4), x]

[Out] IntegrateAlgebraic[x^4/(4 + 5*x^2 + x^4), x]

fricas [A] time = 1.48, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4), x, algorithm="fricas")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

giac [A] time = 0.37, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$x + \frac{\arctan(x)}{3} - \frac{8 \arctan\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^4+5*x^2+4),x)

[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)

maxima [A] time = 1.01, size = 12, normalized size = 0.67

$$x - \frac{8}{3} \arctan\left(\frac{1}{2}x\right) + \frac{1}{3} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")

[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)

mupad [B] time = 2.22, size = 12, normalized size = 0.67

$$x - \frac{8 \operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(5*x^2 + x^4 + 4),x)

[Out] x - (8*atan(x/2))/3 + atan(x)/3

sympy [A] time = 0.14, size = 14, normalized size = 0.78

$$x - \frac{8 \operatorname{atan}\left(\frac{x}{2}\right)}{3} + \frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(x**4+5*x**2+4),x)

[Out] x - 8*atan(x/2)/3 + atan(x)/3

$$3.449 \quad \int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$$

Optimal. Leaf size=46

$$\frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8} \log(x+1) + 2 \log(x+2) - \frac{17}{8} \log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {88}

$$\frac{1}{x+2} + \frac{5}{4(x+3)} + \frac{1}{4(x+3)^2} + \frac{1}{8} \log(x+1) + 2 \log(x+2) - \frac{17}{8} \log(x+3)$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)*(2+x)^2*(3+x)^3),x]

[Out] (2+x)^(-1) + 1/(4*(3+x)^2) + 5/(4*(3+x)) + Log[1+x]/8 + 2*Log[2+x] - (17*Log[3+x])/8

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx &= \int \left(\frac{1}{8(1+x)} - \frac{1}{(2+x)^2} + \frac{2}{2+x} - \frac{1}{2(3+x)^3} - \frac{5}{4(3+x)^2} - \frac{17}{8(3+x)} \right) dx \\ &= \frac{1}{2+x} + \frac{1}{4(3+x)^2} + \frac{5}{4(3+x)} + \frac{1}{8} \log(1+x) + 2 \log(2+x) - \frac{17}{8} \log(3+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8} \left(\frac{8}{x+2} + \frac{10}{x+3} + \frac{2}{(x+3)^2} + \log(-x-1) + 16 \log(x+2) - 17 \log(x+3) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3), x]

[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Log[3 + x])/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1 + x)*(2 + x)^2*(3 + x)^3), x]

[Out] IntegrateAlgebraic[1/((1 + x)*(2 + x)^2*(3 + x)^3), x]

fricas [B] time = 1.32, size = 83, normalized size = 1.80

$$\frac{18x^2 - 17(x^3 + 8x^2 + 21x + 18)\log(x + 3) + 16(x^3 + 8x^2 + 21x + 18)\log(x + 2) + (x^3 + 8x^2 + 21x + 18)\log(x + 1) + 100x + 136}{8(x^3 + 8x^2 + 21x + 18)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3, x, algorithm="fricas")

[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 21*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/(x^3 + 8*x^2 + 21*x + 18)

giac [A] time = 0.36, size = 52, normalized size = 1.13

$$\frac{1}{x+2} - \frac{\frac{7}{x+2} + 6}{4\left(\frac{1}{x+2} + 1\right)^2} + \frac{1}{8} \log\left(\left|-\frac{1}{x+2} + 1\right|\right) - \frac{17}{8} \log\left(\left|-\frac{1}{x+2} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3, x, algorithm="giac")

[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2) + 1)) - 17/8*log(abs(-1/(x + 2) - 1))

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\ln(x + 1)}{8} + 2\ln(x + 2) - \frac{17\ln(x + 3)}{8} + \frac{1}{x + 2} + \frac{1}{4(x + 3)^2} + \frac{5}{4(x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)/(x+2)^2/(x+3)^3,x)

[Out] 1/(x+2)+1/4/(x+3)^2+5/4/(x+3)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(x+3)

maxima [A] time = 0.45, size = 46, normalized size = 1.00

$$\frac{9x^2 + 50x + 68}{4(x^3 + 8x^2 + 21x + 18)} - \frac{17}{8} \log(x + 3) + 2 \log(x + 2) + \frac{1}{8} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")

[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log(x + 2) + 1/8*log(x + 1)

mupad [B] time = 0.04, size = 45, normalized size = 0.98

$$\frac{\ln(x + 1)}{8} + 2 \ln(x + 2) - \frac{17 \ln(x + 3)}{8} + \frac{\frac{9x^2}{4} + \frac{25x}{2} + 17}{x^3 + 8x^2 + 21x + 18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)

[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 17)/(21*x + 8*x^2 + x^3 + 18)

sympy [A] time = 0.19, size = 46, normalized size = 1.00

$$\frac{9x^2 + 50x + 68}{4x^3 + 32x^2 + 84x + 72} + \frac{\log(x + 1)}{8} + 2 \log(x + 2) - \frac{17 \log(x + 3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)

[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(x + 2) - 17*log(x + 3)/8

$$3.450 \quad \int \frac{x}{-1+x^2} dx$$

Optimal. Leaf size=12

$$\frac{1}{2} \log(1-x^2)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {260}

$$\frac{1}{2} \log(1-x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(-1 + x^2), x]

[Out] Log[1 - x^2]/2

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2} \log(1-x^2)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2} \log(x^2 - 1)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-1 + x^2), x]

[Out] Log[-1 + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{-1+x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(-1 + x^2),x]

[Out] IntegrateAlgebraic[x/(-1 + x^2), x]

fricas [A] time = 1.39, size = 8, normalized size = 0.67

$$\frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1),x, algorithm="fricas")

[Out] 1/2*log(x^2 - 1)

giac [A] time = 0.27, size = 9, normalized size = 0.75

$$\frac{1}{2} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1),x, algorithm="giac")

[Out] 1/2*log(abs(x^2 - 1))

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$\frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2-1),x)

[Out] 1/2*ln(x-1)+1/2*ln(x+1)

maxima [A] time = 0.44, size = 8, normalized size = 0.67

$$\frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-1),x, algorithm="maxima")

[Out] 1/2*log(x^2 - 1)

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x^2 - 1),x)`

[Out] `log(x^2 - 1)/2`

sympy [A] time = 0.08, size = 7, normalized size = 0.58

$$\frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2-1),x)`

[Out] `log(x**2 - 1)/2`

$$3.451 \quad \int \frac{1}{(-1+x^2)^2} dx$$

Optimal. Leaf size=21

$$\frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {199, 207}

$$\frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)^(-2), x]

[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{(-1+x^2)^2} dx &= \frac{x}{2(1-x^2)} - \frac{1}{2} \int \frac{1}{-1+x^2} dx \\ &= \frac{x}{2(1-x^2)} + \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4} \left(-\frac{2x}{x^2-1} - \log(1-x) + \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)^(-2), x]

[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x^2)^(-2), x]

[Out] IntegrateAlgebraic[(-1 + x^2)^(-2), x]

fricas [B] time = 1.29, size = 34, normalized size = 1.62

$$\frac{(x^2 - 1) \log(x + 1) - (x^2 - 1) \log(x - 1) - 2x}{4(x^2 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)

giac [A] time = 0.27, size = 25, normalized size = 1.19

$$-\frac{x}{2(x^2-1)} + \frac{1}{4} \log(|x+1|) - \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-1)^2,x, algorithm="giac")

[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{1}{4(x-1)} - \frac{1}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-1)^2,x)`

[Out] `-1/4/(x-1)-1/4*ln(x-1)-1/4/(x+1)+1/4*ln(x+1)`

maxima [A] time = 0.45, size = 23, normalized size = 1.10

$$-\frac{x}{2(x^2-1)} + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-1)^2,x, algorithm="maxima")`

[Out] `-1/2*x/(x^2-1) + 1/4*log(x+1) - 1/4*log(x-1)`

mupad [B] time = 2.22, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2} - \frac{x}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-1)^2,x)`

[Out] `atanh(x)/2 - x/(2*(x^2-1))`

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-1)**2,x)`

[Out] `-x/(2*x**2-2) - log(x-1)/4 + log(x+1)/4`

$$3.452 \quad \int \frac{x^2}{(1+x^2)^2} dx$$

Optimal. Leaf size=19

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {288, 203}

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1+x^2)^2} dx &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2} \tan^{-1}(x) - \frac{x}{2(x^2 + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + x^2)^2,x]

[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 + x^2)^2,x]

[Out] IntegrateAlgebraic[x^2/(1 + x^2)^2, x]

fricas [A] time = 1.05, size = 21, normalized size = 1.11

$$\frac{(x^2 + 1) \arctan(x) - x}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")

[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)

giac [A] time = 0.29, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2 + 1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")

[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2(x^2 + 1)} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2+1)^2,x)`

[Out] `-1/2/(x^2+1)*x+1/2*arctan(x)`

maxima [A] time = 1.00, size = 15, normalized size = 0.79

$$-\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(x^2+1)^2,x, algorithm="maxima")`

[Out] `-1/2*x/(x^2+1) + 1/2*arctan(x)`

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2} - \frac{x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2+1)^2,x)`

[Out] `atan(x)/2 - x/(2*(x^2+1))`

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(x**2+1)**2,x)`

[Out] `-x/(2*x**2+2) + atan(x)/2`

$$3.453 \quad \int \frac{1}{2+3x} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \log(3x + 2)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{1}{3} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Int[(2 + 3*x)^(-1), x]

[Out] Log[2 + 3*x]/3

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\int \frac{1}{2+3x} dx = \frac{1}{3} \log(2+3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3} \log(3x + 2)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 3*x)^(-1), x]

[Out] Log[2 + 3*x]/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+3x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 3*x)^(-1), x]

[Out] IntegrateAlgebraic[(2 + 3*x)^(-1), x]

fricas [A] time = 1.39, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*x), x, algorithm="fricas")

[Out] 1/3*log(3*x + 2)

giac [A] time = 0.38, size = 9, normalized size = 0.90

$$\frac{1}{3} \log(|3x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*x), x, algorithm="giac")

[Out] 1/3*log(abs(3*x + 2))

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln(3x + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3*x+2), x)

[Out] 1/3*ln(3*x+2)

maxima [A] time = 0.45, size = 8, normalized size = 0.80

$$\frac{1}{3} \log(3x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+3*x), x, algorithm="maxima")

[Out] 1/3*log(3*x + 2)

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x + \frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3*x + 2),x)`

[Out] `log(x + 2/3)/3`

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(3x + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2+3*x),x)`

[Out] `log(3*x + 2)/3`

$$3.454 \quad \int \frac{1}{a^2+x^2} dx$$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + x^2)^(-1), x]

[Out] ArcTan[x/a]/a

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{a^2+x^2} dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2 + x^2)^(-1), x]

[Out] ArcTan[x/a]/a

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2 + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a^2 + x^2)^(-1), x]

[Out] IntegrateAlgebraic[(a^2 + x^2)^(-1), x]

fricas [A] time = 1.35, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2), x, algorithm="fricas")

[Out] arctan(x/a)/a

giac [A] time = 0.26, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2), x, algorithm="giac")

[Out] arctan(x/a)/a

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+x^2), x)

[Out] arctan(x/a)/a

maxima [A] time = 0.99, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="maxima")

[Out] arctan(x/a)/a

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + x^2),x)

[Out] atan(x/a)/a

sympy [C] time = 0.11, size = 20, normalized size = 2.00

$$\frac{-\frac{i \log(-ia+x)}{2} + \frac{i \log(ia+x)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+x**2),x)

[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a

$$3.455 \quad \int \frac{1}{a+bx^2} dx$$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{1}{a+bx^2} dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2)^(-1), x]

[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^2)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*x^2)^(-1), x]

fricas [A] time = 1.20, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab} \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right)}{2ab}, \frac{\sqrt{ab} \arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a), x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]

giac [A] time = 0.38, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a), x, algorithm="giac")

[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a), x)

[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)

maxima [A] time = 0.98, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a),x, algorithm="maxima")

[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)

mupad [B] time = 2.24, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*x^2),x)

[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}} \log\left(-a\sqrt{-\frac{1}{ab}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}} \log\left(a\sqrt{-\frac{1}{ab}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**2+a),x)

[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1/(a*b)) + x)/2

$$3.456 \quad \int \frac{1}{2-x+x^2} dx$$

Optimal. Leaf size=19

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {618, 204}

$$-\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(2 - x + x^2)^(-1), x]

[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2-x+x^2} dx &= -\left(2 \text{Subst}\left(\int \frac{1}{-7-x^2} dx, x, -1+2x\right)\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{2x-1}{\sqrt{7}} \right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x + x^2)^(-1), x]

[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2 - x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 - x + x^2)^(-1), x]

[Out] IntegrateAlgebraic[(2 - x + x^2)^(-1), x]

fricas [A] time = 1.44, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (2x - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-x+2), x, algorithm="fricas")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))

giac [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan \left(\frac{1}{7} \sqrt{7} (2x - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2-x+2), x, algorithm="giac")

[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{2\sqrt{7} \arctan \left(\frac{(2x-1)\sqrt{7}}{7} \right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2-x+2),x)`

[Out] `2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))`

maxima [A] time = 0.95, size = 16, normalized size = 0.84

$$\frac{2}{7} \sqrt{7} \arctan\left(\frac{1}{7} \sqrt{7} (2x - 1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2-x+2),x, algorithm="maxima")`

[Out] `2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))`

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{\sqrt{7}(2x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2 - x + 2),x)`

[Out] `(2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7`

sympy [A] time = 0.11, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7} \operatorname{atan}\left(\frac{2\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2-x+2),x)`

[Out] `2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7`

$$3.457 \quad \int x^2 (4 - x^2)^2 dx$$

Optimal. Leaf size=22

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {270}

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x^2 (4 - x^2)^2 dx &= \int (16x^2 - 8x^4 + x^6) dx \\ &= \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(4 - x^2)^2,x]

[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (4 - x^2)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(4 - x^2)^2,x]

[Out] IntegrateAlgebraic[x^2*(4 - x^2)^2, x]

fricas [A] time = 1.13, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

giac [A] time = 0.31, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^2+4)^2,x)

[Out] 16/3*x^3-8/5*x^5+1/7*x^7

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")

[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3

mupad [B] time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3 (15x^4 - 168x^2 + 560)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(x^2 - 4)^2,x)

[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**2+4)**2,x)

[Out] x**7/7 - 8*x**5/5 + 16*x**3/3

$$3.458 \quad \int x(1-x^3)^2 dx$$

Optimal. Leaf size=22

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {270}

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int x(1-x^3)^2 dx &= \int (x - 2x^4 + x^7) dx \\ &= \frac{x^2}{2} - \frac{2x^5}{5} + \frac{x^8}{8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 - x^3)^2,x]

[Out] x^2/2 - (2*x^5)/5 + x^8/8

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(1-x^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(1 - x^3)^2,x]

[Out] IntegrateAlgebraic[x*(1 - x^3)^2, x]

fricas [A] time = 1.12, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^3+1)^2,x)

[Out] 1/2*x^2-2/5*x^5+1/8*x^8

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8 - \frac{2}{5}x^5 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")

[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^2 (5x^6 - 16x^3 + 20)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^3 - 1)^2,x)

[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40

sympy [A] time = 0.06, size = 15, normalized size = 0.68

$$\frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**3+1)**2,x)

[Out] x**8/8 - 2*x**5/5 + x**2/2

$$3.459 \quad \int \frac{-4+5x^2+x^3}{x^2} dx$$

Optimal. Leaf size=16

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {14}

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

[In] Int[(-4 + 5*x^2 + x^3)/x^2, x]

[Out] 4/x + 5*x + x^2/2

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-4+5x^2+x^3}{x^2} dx &= \int \left(5 - \frac{4}{x^2} + x \right) dx \\ &= \frac{4}{x} + 5x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + 5*x^2 + x^3)/x^2, x]

[Out] 4/x + 5*x + x^2/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4 + 5x^2 + x^3}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-4 + 5*x^2 + x^3)/x^2,x]

[Out] IntegrateAlgebraic[(-4 + 5*x^2 + x^3)/x^2, x]

fricas [A] time = 1.30, size = 15, normalized size = 0.94

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 + 10*x^2 + 8)/x

giac [A] time = 0.31, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 + 5*x + 4/x

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+5*x^2-4)/x^2,x)

[Out] 4/x+5*x+1/2*x^2

maxima [A] time = 0.46, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 + 5*x + 4/x

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2 + x^3 - 4)/x^2,x)

[Out] (10*x^2 + x^3 + 8)/(2*x)

sympy [A] time = 0.07, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+5*x**2-4)/x**2,x)

[Out] x**2/2 + 5*x + 4/x

$$3.460 \quad \int \frac{-1+x}{3-4x+3x^2} dx$$

Optimal. Leaf size=37

$$\frac{1}{6} \log(3x^2 - 4x + 3) + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {634, 618, 204, 628}

$$\frac{1}{6} \log(3x^2 - 4x + 3) + \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(3 - 4*x + 3*x^2), x]

[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{3-4x+3x^2} dx &= \frac{1}{6} \int \frac{-4+6x}{3-4x+3x^2} dx - \frac{1}{3} \int \frac{1}{3-4x+3x^2} dx \\
&= \frac{1}{6} \log(3-4x+3x^2) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{-20-x^2} dx, x, -4+6x \right) \\
&= \frac{\tan^{-1} \left(\frac{2-3x}{\sqrt{5}} \right)}{3\sqrt{5}} + \frac{1}{6} \log(3-4x+3x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6} \log(3x^2 - 4x + 3) - \frac{\tan^{-1} \left(\frac{3x-2}{\sqrt{5}} \right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2), x]

[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{3-4x+3x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + x)/(3 - 4*x + 3*x^2), x]

[Out] IntegrateAlgebraic[(-1 + x)/(3 - 4*x + 3*x^2), x]

fricas [A] time = 1.26, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan \left(\frac{1}{5} \sqrt{5} (3x-2) \right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3), x, algorithm="fricas")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

giac [A] time = 0.27, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (3x - 2)\right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{5} \arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15} + \frac{\ln(3x^2 - 4x + 3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(3*x^2-4*x+3),x)

[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))

maxima [A] time = 0.98, size = 30, normalized size = 0.81

$$-\frac{1}{15} \sqrt{5} \arctan\left(\frac{1}{5} \sqrt{5} (3x - 2)\right) + \frac{1}{6} \log(3x^2 - 4x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")

[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)

mupad [B] time = 2.22, size = 30, normalized size = 0.81

$$\frac{\ln\left(x^2 - \frac{4x}{3} + 1\right)}{6} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{3\sqrt{5}x}{5} - \frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 1)/(3*x^2 - 4*x + 3),x)

[Out] log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))/15

sympy [A] time = 0.12, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2 - \frac{4x}{3} + 1\right)}{6} - \frac{\sqrt{5} \operatorname{atan}\left(\frac{3\sqrt{5}x}{5} - \frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(3*x**2-4*x+3),x)

[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15

$$3.461 \quad \int (2 + x^3)^2 dx$$

Optimal. Leaf size=14

$$\frac{x^7}{7} + x^4 + 4x$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {194}

$$\frac{x^7}{7} + x^4 + 4x$$

Antiderivative was successfully verified.

[In] Int[(2 + x^3)^2, x]

[Out] 4*x + x^4 + x^7/7

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int (2 + x^3)^2 dx &= \int (4 + 4x^3 + x^6) dx \\ &= 4x + x^4 + \frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x^7}{7} + x^4 + 4x$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x^3)^2, x]

[Out] 4*x + x^4 + x^7/7

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2 + x^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + x^3)^2,x]

[Out] IntegrateAlgebraic[(2 + x^3)^2, x]

fricas [A] time = 0.97, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="fricas")

[Out] 1/7*x^7 + x^4 + 4*x

giac [A] time = 0.36, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="giac")

[Out] 1/7*x^7 + x^4 + 4*x

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+2)^2,x)

[Out] 4*x+x^4+1/7*x^7

maxima [A] time = 0.47, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+2)^2,x, algorithm="maxima")

[Out] 1/7*x^7 + x^4 + 4*x

mupad [B] time = 0.02, size = 13, normalized size = 0.93

$$\frac{x(x^6 + 7x^3 + 28)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 2)^2,x)

[Out] (x*(7*x^3 + x^6 + 28))/7

sympy [A] time = 0.05, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+2)**2,x)

[Out] x**7/7 + x**4 + 4*x

$$3.462 \quad \int \frac{-4+x^2}{2+x} dx$$

Optimal. Leaf size=11

$$\frac{x^2}{2} - 2x$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {627}

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

[In] Int[(-4 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2+x} dx &= \int (-2+x) dx \\ &= -2x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - 2x$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + x^2)/(2 + x), x]

[Out] -2*x + x^2/2

IntegrateAlgebraic [A] time = 0.01, size = 9, normalized size = 0.82

$$\frac{1}{2}(x-2)^2$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-4 + x^2)/(2 + x), x]

[Out] (-2 + x)^2/2

fricas [A] time = 1.28, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x), x, algorithm="fricas")

[Out] 1/2*x^2 - 2*x

giac [A] time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x), x, algorithm="giac")

[Out] 1/2*x^2 - 2*x

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-4)/(x+2), x)

[Out] -2*x+1/2*x^2

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")

[Out] 1/2*x^2 - 2*x

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x(x-4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 4)/(x + 2),x)

[Out] (x*(x - 4))/2

sympy [A] time = 0.06, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-4)/(2+x),x)

[Out] x**2/2 - 2*x

$$3.463 \quad \int \frac{1}{(2+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((2 + x)*(1 + x^2)),x]

[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^{(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*xⁿ, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]}

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -

$c*e*x)/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(2+x)(1+x^2)} dx &= \frac{1}{5} \int \frac{1}{2+x} dx + \frac{1}{5} \int \frac{2-x}{1+x^2} dx \\ &= \frac{1}{5} \log(2+x) - \frac{1}{5} \int \frac{x}{1+x^2} dx + \frac{2}{5} \int \frac{1}{1+x^2} dx \\ &= \frac{2}{5} \tan^{-1}(x) + \frac{1}{5} \log(2+x) - \frac{1}{10} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2) + \frac{2}{5} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((2 + x)*(1 + x^2)), x]

[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2+x)(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((2 + x)*(1 + x^2)), x]

[Out] IntegrateAlgebraic[1/((2 + x)*(1 + x^2)), x]

fricas [A] time = 1.44, size = 19, normalized size = 0.76

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1), x, algorithm="fricas")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)

giac [A] time = 0.37, size = 20, normalized size = 0.80

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{2 \arctan(x)}{5} + \frac{\ln(x + 2)}{5} - \frac{\ln(x^2 + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+2)/(x^2+1),x)

[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)

maxima [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2}{5} \arctan(x) - \frac{1}{10} \log(x^2 + 1) + \frac{1}{5} \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")

[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)

mupad [B] time = 0.05, size = 25, normalized size = 1.00

$$\frac{\ln(x + 2)}{5} + \ln(x - i) \left(-\frac{1}{10} - \frac{1}{5}i \right) + \ln(x + 1i) \left(-\frac{1}{10} + \frac{1}{5}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*(x + 2)),x)

[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)

sympy [A] time = 0.14, size = 20, normalized size = 0.80

$$\frac{\log(x + 2)}{5} - \frac{\log(x^2 + 1)}{10} + \frac{2 \operatorname{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

$$3.464 \quad \int \frac{1}{(1+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*xⁿ, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -

$c*e*x)/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)(1+x^2)} dx &= \frac{1}{2} \int \frac{1}{1+x} dx + \frac{1}{2} \int \frac{1-x}{1+x^2} dx \\ &= \frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx \\ &= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \log(1+x) - \frac{1}{4} \log(1+x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1 + x)*(1 + x^2)),x]

[Out] IntegrateAlgebraic[1/((1 + x)*(1 + x^2)), x]

fricas [A] time = 2.12, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

giac [A] time = 0.29, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln(x + 1)}{2} - \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)/(x^2+1),x)

[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)

maxima [A] time = 1.36, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)

mupad [B] time = 2.23, size = 25, normalized size = 1.00

$$\frac{\ln(x + 1)}{2} + \ln(x - i) \left(-\frac{1}{4} - \frac{1}{4}i \right) + \ln(x + 1i) \left(-\frac{1}{4} + \frac{1}{4}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 + 1)*(x + 1)),x)

[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x + 1)}{2} - \frac{\log(x^2 + 1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

$$3.465 \quad \int \frac{x}{(1+x)(1+x^2)} dx$$

Optimal. Leaf size=25

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {801, 635, 203, 260}

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)(1+x^2)} dx &= \int \left(-\frac{1}{2(1+x)} + \frac{1+x}{2(1+x^2)} \right) dx \\
&= -\frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1+x}{1+x^2} dx \\
&= -\frac{1}{2} \log(1+x) + \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{x}{1+x^2} dx \\
&= \frac{1}{2} \tan^{-1}(x) - \frac{1}{2} \log(1+x) + \frac{1}{4} \log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + x)*(1 + x^2)),x]

[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((1 + x)*(1 + x^2)),x]

[Out] IntegrateAlgebraic[x/((1 + x)*(1 + x^2)), x]

fricas [A] time = 1.27, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)

giac [A] time = 0.25, size = 20, normalized size = 0.80

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} - \frac{\ln(x+1)}{2} + \frac{\ln(x^2+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+1)/(x^2+1),x)

[Out] 1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)

maxima [A] time = 1.43, size = 19, normalized size = 0.76

$$\frac{1}{2} \arctan(x) + \frac{1}{4} \log(x^2 + 1) - \frac{1}{2} \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)

mupad [B] time = 2.21, size = 25, normalized size = 1.00

$$-\frac{\ln(x+1)}{2} + \ln(x-i) \left(\frac{1}{4} - \frac{1}{4}i\right) + \ln(x+1i) \left(\frac{1}{4} + \frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((x^2 + 1)*(x + 1)),x)

[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)

sympy [A] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(x**2+1),x)

[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2

$$3.466 \quad \int \frac{2x+x^2}{(1+x)^2} dx$$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {683}

$$x + \frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^2)/(1 + x)^2, x]

[Out] x + (1 + x)^(-1)

Rule 683

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_ Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])

Rubi steps

$$\begin{aligned} \int \frac{2x+x^2}{(1+x)^2} dx &= \int \left(1 - \frac{1}{(1+x)^2}\right) dx \\ &= x + \frac{1}{1+x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.78

$$x + \frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(2*x + x^2)/(1 + x)^2, x]

[Out] x + (1 + x)^(-1)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x + x^2}{(1 + x)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2*x + x^2)/(1 + x)^2,x]

[Out] IntegrateAlgebraic[(2*x + x^2)/(1 + x)^2, x]

fricas [A] time = 1.18, size = 12, normalized size = 1.33

$$\frac{x^2 + x + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")

[Out] (x^2 + x + 1)/(x + 1)

giac [A] time = 0.30, size = 8, normalized size = 0.89

$$x + \frac{1}{x + 1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")

[Out] x + 1/(x + 1) + 1

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+2*x)/(x+1)^2,x)

[Out] x+1/(x+1)

maxima [A] time = 0.95, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.78

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(x + 1)^2,x)
```

```
[Out] x + 1/(x + 1)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.56

$$x + \frac{1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```


$$3.467 \quad \int \frac{-10+x^2}{4+9x^2+2x^4} dx$$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1166, 203}

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{-10+x^2}{4+9x^2+2x^4} dx &= -\left(3 \int \frac{1}{1+2x^2} dx\right) + 4 \int \frac{1}{8+2x^2} dx \\ &= \tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{2}\right) - \frac{3 \tan^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-10 + x^2}{4 + 9x^2 + 2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

[Out] IntegrateAlgebraic[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]

fricas [A] time = 1.19, size = 16, normalized size = 0.73

$$-\frac{3}{2} \sqrt{2} \arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-10)/(2*x^4+9*x^2+4), x, algorithm="fricas")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$-\frac{3}{2} \sqrt{2} \arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-10)/(2*x^4+9*x^2+4), x, algorithm="giac")

[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)

maple [A] time = 0.01, size = 17, normalized size = 0.77

$$-\frac{3\sqrt{2} \arctan(\sqrt{2}x)}{2} + \arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2-10)/(2*x^4+9*x^2+4),x)`

[Out] `arctan(1/2*x)-3/2*2^(1/2)*arctan(2^(1/2)*x)`

maxima [A] time = 1.58, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan(\sqrt{2}x) + \arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")`

[Out] `-3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)`

mupad [B] time = 0.05, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)`

[Out] `atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2`

sympy [A] time = 0.15, size = 20, normalized size = 0.91

$$\operatorname{atan}\left(\frac{x}{2}\right) - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-10)/(2*x**4+9*x**2+4),x)`

[Out] `atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2`

$$3.468 \quad \int \frac{31+5x}{11-4x+3x^2} dx$$

Optimal. Leaf size=37

$$\frac{5}{6} \log(3x^2 - 4x + 11) - \frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {634, 618, 204, 628}

$$\frac{5}{6} \log(3x^2 - 4x + 11) - \frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ

$[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rubi steps

$$\begin{aligned} \int \frac{31 + 5x}{11 - 4x + 3x^2} dx &= \frac{5}{6} \int \frac{-4 + 6x}{11 - 4x + 3x^2} dx + \frac{103}{3} \int \frac{1}{11 - 4x + 3x^2} dx \\ &= \frac{5}{6} \log(11 - 4x + 3x^2) - \frac{206}{3} \text{Subst}\left(\int \frac{1}{-116 - x^2} dx, x, -4 + 6x\right) \\ &= -\frac{103 \tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}} + \frac{5}{6} \log(11 - 4x + 3x^2) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{5}{6} \log(3x^2 - 4x + 11) + \frac{103 \tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/6

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{31 + 5x}{11 - 4x + 3x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

[Out] IntegrateAlgebraic[(31 + 5*x)/(11 - 4*x + 3*x^2), x]

fricas [A] time = 1.17, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")

[Out] $103/87*\sqrt{29}*\arctan(1/29*\sqrt{29}*(3*x - 2)) + 5/6*\log(3*x^2 - 4*x + 11)$

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")`

[Out] $103/87*\sqrt{29}*\arctan(1/29*\sqrt{29}*(3*x - 2)) + 5/6*\log(3*x^2 - 4*x + 11)$

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{103\sqrt{29} \arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87} + \frac{5 \ln(3x^2 - 4x + 11)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((31+5*x)/(3*x^2-4*x+11),x)`

[Out] $5/6*\ln(3*x^2-4*x+11)+103/87*29^{(1/2)}*\arctan(1/58*(6*x-4)*29^{(1/2)})$

maxima [A] time = 1.39, size = 30, normalized size = 0.81

$$\frac{103}{87} \sqrt{29} \arctan\left(\frac{1}{29} \sqrt{29} (3x - 2)\right) + \frac{5}{6} \log(3x^2 - 4x + 11)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")`

[Out] $103/87*\sqrt{29}*\arctan(1/29*\sqrt{29}*(3*x - 2)) + 5/6*\log(3*x^2 - 4*x + 11)$

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{5 \ln\left(x^2 - \frac{4x}{3} + \frac{11}{3}\right)}{6} + \frac{103 \sqrt{29} \operatorname{atan}\left(\frac{3\sqrt{29}x}{29} - \frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((5*x + 31)/(3*x^2 - 4*x + 11),x)`

[Out] $(5*\log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^{(1/2)}*\operatorname{atan}((3*29^{(1/2)}*x)/29 - (2*29^{(1/2)})/29))/87$

sympy [A] time = 0.12, size = 44, normalized size = 1.19

$$\frac{5 \log\left(x^2 - \frac{4x}{3} + \frac{11}{3}\right)}{6} + \frac{103\sqrt{29} \operatorname{atan}\left(\frac{3\sqrt{29}x}{29} - \frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((31+5*x)/(3*x**2-4*x+11),x)

[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(29)/29)/87

$$3.469 \quad \int \frac{-2+x^2+x^3}{x^4} dx$$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^2 + x^3)/x^4, x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2+x^3}{x^4} dx &= \int \left(-\frac{2}{x^4} + \frac{1}{x^2} + \frac{1}{x} \right) dx \\ &= \frac{2}{3x^3} - \frac{1}{x} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^2 + x^3)/x^4, x]

[Out] 2/(3*x^3) - x^(-1) + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2 + x^2 + x^3}{x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + x^2 + x^3)/x^4, x]

[Out] IntegrateAlgebraic[(-2 + x^2 + x^3)/x^4, x]

fricas [A] time = 1.34, size = 19, normalized size = 1.27

$$\frac{3x^3 \log(x) - 3x^2 + 2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")

[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3

giac [A] time = 0.27, size = 16, normalized size = 1.07

$$-\frac{3x^2 - 2}{3x^3} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")

[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) - \frac{1}{x} + \frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2-2)/x^4,x)

[Out] 2/3/x^3-1/x+ln(x)

maxima [A] time = 0.97, size = 15, normalized size = 1.00

$$-\frac{3x^2 - 2}{3x^3} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")

[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\ln(x) - \frac{x^2 - \frac{2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 - 2)/x^4,x)

[Out] log(x) - (x^2 - 2/3)/x^3

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\log(x) + \frac{2 - 3x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+x**2-2)/x**4,x)

[Out] log(x) + (2 - 3*x**2)/(3*x**3)

$$3.470 \quad \int \frac{1+x+x^3}{x^2} dx$$

Optimal. Leaf size=15

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {14}

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^3}{x^2} dx &= \int \left(\frac{1}{x^2} + \frac{1}{x} + x \right) dx \\ &= -\frac{1}{x} + \frac{x^2}{2} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x + x^3)/x^2,x]

[Out] -x^(-1) + x^2/2 + Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x + x^3}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x + x^3)/x^2,x]

[Out] IntegrateAlgebraic[(1 + x + x^3)/x^2, x]

fricas [A] time = 0.90, size = 15, normalized size = 1.00

$$\frac{x^3 + 2x \log(x) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x

giac [A] time = 0.28, size = 14, normalized size = 0.93

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 - 1/x + log(abs(x))

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2}{2} + \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x+1)/x^2,x)

[Out] -1/x+1/2*x^2+ln(x)

maxima [A] time = 0.95, size = 13, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3+x+1)/x^2,x, algorithm="maxima")`

[Out] `1/2*x^2 - 1/x + log(x)`

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(x) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^3 + 1)/x^2,x)`

[Out] `log(x) - 1/x + x^2/2`

sympy [A] time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3+x+1)/x**2,x)`

[Out] `x**2/2 + log(x) - 1/x`

$$3.471 \quad \int \frac{-2+x^2}{x(2+x^2)} dx$$

Optimal. Leaf size=11

$$\log(x^2 + 2) - \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {446, 72}

$$\log(x^2 + 2) - \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-2 + x^2)/(x*(2 + x^2)), x]

[Out] -Log[x] + Log[2 + x^2]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{-2+x^2}{x(2+x^2)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-2+x}{x(2+x)} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{x} + \frac{2}{2+x} \right) dx, x, x^2 \right) \\ &= -\log(x) + \log(2+x^2) \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x^2 + 2) - \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + x^2)/(x*(2 + x^2)), x]

[Out] -Log[x] + Log[2 + x^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2 + x^2}{x(2 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + x^2)/(x*(2 + x^2)), x]

[Out] IntegrateAlgebraic[(-2 + x^2)/(x*(2 + x^2)), x]

fricas [A] time = 1.30, size = 11, normalized size = 1.00

$$\log(x^2 + 2) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2), x, algorithm="fricas")

[Out] log(x^2 + 2) - log(x)

giac [A] time = 0.29, size = 13, normalized size = 1.18

$$\log(x^2 + 2) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2), x, algorithm="giac")

[Out] log(x^2 + 2) - 1/2*log(x^2)

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\ln(x) + \ln(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-2)/x/(x^2+2), x)

[Out] -ln(x)+ln(x^2+2)

maxima [A] time = 0.69, size = 13, normalized size = 1.18

$$\log(x^2 + 2) - \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")

[Out] log(x^2 + 2) - 1/2*log(x^2)

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\ln(x^2 + 2) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 2)/(x*(x^2 + 2)),x)

[Out] log(x^2 + 2) - log(x)

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\log(x) + \log(x^2 + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-2)/x/(x**2+2),x)

[Out] -log(x) + log(x**2 + 2)

$$3.472 \quad \int (-3 + x)(-7 + 4x^2) dx$$

Optimal. Leaf size=22

$$-4x^3 + \frac{1}{16}(7 - 4x^2)^2 + 21x$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {641}

$$-4x^3 + \frac{1}{16}(7 - 4x^2)^2 + 21x$$

Antiderivative was successfully verified.

[In] Int[(-3 + x)*(-7 + 4*x^2), x]

[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] / ; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int (-3 + x)(-7 + 4x^2) dx &= \frac{1}{16}(7 - 4x^2)^2 - 3 \int (-7 + 4x^2) dx \\ &= 21x - 4x^3 + \frac{1}{16}(7 - 4x^2)^2 \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.86

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + x)*(-7 + 4*x^2), x]

[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3 + x)(-7 + 4x^2) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + x)*(-7 + 4*x^2), x]

[Out] IntegrateAlgebraic[(-3 + x)*(-7 + 4*x^2), x]

fricas [A] time = 0.52, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7), x, algorithm="fricas")

[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x

giac [A] time = 0.36, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7), x, algorithm="giac")

[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x

maple [A] time = 0.00, size = 18, normalized size = 0.82

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-3)*(4*x^2-7), x)

[Out] x^4-4*x^3-7/2*x^2+21*x

maxima [A] time = 0.88, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")

[Out] $x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2 - 7)*(x - 3),x)

[Out] $21x - \frac{(7x^2)}{2} - 4x^3 + x^4$

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-3+x)*(4*x**2-7),x)

[Out] $x**4 - 4*x**3 - 7*x**2/2 + 21*x$

$$3.473 \quad \int (-2 + 7x)^3 dx$$

Optimal. Leaf size=11

$$\frac{1}{28}(2 - 7x)^4$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{1}{28}(2 - 7x)^4$$

Antiderivative was successfully verified.

[In] Int[(-2 + 7*x)^3,x]

[Out] (2 - 7*x)^4/28

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int (-2 + 7x)^3 dx = \frac{1}{28}(2 - 7x)^4$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{28}(7x - 2)^4$$

Antiderivative was successfully verified.

[In] Integrate[(-2 + 7*x)^3,x]

[Out] (-2 + 7*x)^4/28

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-2 + 7x)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-2 + 7*x)^3,x]

[Out] IntegrateAlgebraic[(-2 + 7*x)^3, x]

fricas [B] time = 1.46, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+7*x)^3,x, algorithm="fricas")

[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x

giac [A] time = 0.22, size = 9, normalized size = 0.82

$$\frac{1}{28}(7x - 2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+7*x)^3,x, algorithm="giac")

[Out] 1/28*(7*x - 2)^4

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{(7x - 2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2+7*x)^3,x)

[Out] 1/28*(-2+7*x)^4

maxima [B] time = 0.86, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4 - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2+7*x)^3,x, algorithm="maxima")

[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x

mupad [B] time = 0.14, size = 9, normalized size = 0.82

$$\frac{(7x - 2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((7*x - 2)^3,x)`

[Out] `(7*x - 2)^4/28`

sympy [B] time = 0.06, size = 19, normalized size = 1.73

$$\frac{343x^4}{4} - 98x^3 + 42x^2 - 8x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-2+7*x)**3,x)`

[Out] `343*x**4/4 - 98*x**3 + 42*x**2 - 8*x`

$$3.474 \quad \int \frac{-7+4x^2}{3+2x} dx$$

Optimal. Leaf size=13

$$x^2 - 3x + \log(2x + 3)$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {697}

$$x^2 - 3x + \log(2x + 3)$$

Antiderivative was successfully verified.

[In] Int[(-7 + 4*x^2)/(3 + 2*x), x]

[Out] -3*x + x^2 + Log[3 + 2*x]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{-7+4x^2}{3+2x} dx &= \int \left(-3 + 2x + \frac{2}{3+2x} \right) dx \\ &= -3x + x^2 + \log(3 + 2x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.23

$$x^2 - 3x + \log(2x + 3) - \frac{27}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(-7 + 4*x^2)/(3 + 2*x), x]

[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-7+4x^2}{3+2x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-7 + 4*x^2)/(3 + 2*x),x]

[Out] IntegrateAlgebraic[(-7 + 4*x^2)/(3 + 2*x), x]

fricas [A] time = 1.00, size = 13, normalized size = 1.00

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")

[Out] x^2 - 3*x + log(2*x + 3)

giac [A] time = 0.36, size = 14, normalized size = 1.08

$$x^2 - 3x + \log(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")

[Out] x^2 - 3*x + log(abs(2*x + 3))

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$x^2 - 3x + \ln(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x^2-7)/(2*x+3),x)

[Out] -3*x+x^2+ln(2*x+3)

maxima [A] time = 0.64, size = 13, normalized size = 1.00

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")

[Out] x^2 - 3*x + log(2*x + 3)

mupad [B] time = 2.21, size = 11, normalized size = 0.85

$$\ln\left(x + \frac{3}{2}\right) - 3x + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)/(2*x + 3),x)
```

```
[Out] log(x + 3/2) - 3*x + x^2
```

```
sympy [A] time = 0.08, size = 12, normalized size = 0.92
```

$$x^2 - 3x + \log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-7)/(3+2*x),x)
```

```
[Out] x**2 - 3*x + log(2*x + 3)
```

$$3.475 \quad \int \frac{1+x}{(-1+x)x^2} dx$$

Optimal. Leaf size=16

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {77}

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((-1 + x)*x^2), x]

[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(-1+x)x^2} dx &= \int \left(\frac{2}{-1+x} - \frac{1}{x^2} - \frac{2}{x} \right) dx \\ &= \frac{1}{x} + 2 \log(1-x) - 2 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2 \log(1-x) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((-1 + x)*x^2), x]

[Out] $x^{(-1)} + 2*\text{Log}[1 - x] - 2*\text{Log}[x]$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(-1+x)x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*x^2), x]

[Out] IntegrateAlgebraic[(1 + x)/((-1 + x)*x^2), x]

fricas [A] time = 1.32, size = 18, normalized size = 1.12

$$\frac{2x \log(x-1) - 2x \log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")

[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x

giac [A] time = 0.36, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2 \log(|x-1|) - 2 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")

[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-2 \ln(x) + 2 \ln(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x-1)/x^2,x)

[Out] 2*ln(x-1)+1/x-2*ln(x)

maxima [A] time = 0.59, size = 14, normalized size = 0.88

$$\frac{1}{x} + 2 \log(x-1) - 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")

[Out] 1/x + 2*log(x - 1) - 2*log(x)

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{x} - 4 \operatorname{atanh}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/(x^2*(x - 1)),x)

[Out] 1/x - 4*atanh(2*x - 1)

sympy [A] time = 0.10, size = 14, normalized size = 0.88

$$-2 \log(x) + 2 \log(x - 1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-1+x)/x**2,x)

[Out] -2*log(x) + 2*log(x - 1) + 1/x

$$3.476 \quad \int \frac{1}{4x^2+4x^3+x^4} dx$$

Optimal. Leaf size=27

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1106, 199, 206}

$$\frac{x+1}{2(1-(x+1)^2)} + \frac{1}{2} \tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1), x]

[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 1106

Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1], c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4]^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]

Rubi steps

$$\begin{aligned} \int \frac{1}{4x^2 + 4x^3 + x^4} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, 1+x \right) \\ &= \frac{1+x}{2(1-(1+x)^2)} + \frac{1}{2} \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4} \left(-\frac{2(x+1)}{x(x+2)} - \log(x) + \log(x+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1), x]

[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4x^2 + 4x^3 + x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*x^2 + 4*x^3 + x^4)^(-1), x]

[Out] IntegrateAlgebraic[(4*x^2 + 4*x^3 + x^4)^(-1), x]

fricas [A] time = 1.21, size = 39, normalized size = 1.44

$$\frac{(x^2 + 2x) \log(x+2) - (x^2 + 2x) \log(x) - 2x - 2}{4(x^2 + 2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2), x, algorithm="fricas")

[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)

giac [A] time = 0.38, size = 27, normalized size = 1.00

$$-\frac{x+1}{2(x^2+2x)} + \frac{1}{4} \log(|x+2|) - \frac{1}{4} \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4} + \frac{\ln(x+2)}{4} - \frac{1}{4x} - \frac{1}{4(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+4*x^3+4*x^2),x)

[Out] -1/4*ln(x)+1/4*ln(x+2)-1/4/x-1/4/(x+2)

maxima [A] time = 0.81, size = 25, normalized size = 0.93

$$-\frac{x+1}{2(x^2+2x)} + \frac{1}{4} \log(x+2) - \frac{1}{4} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")

[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)

mupad [B] time = 2.23, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}(x+1)}{2} - \frac{\frac{x}{2} + \frac{1}{2}}{x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^2 + 4*x^3 + x^4),x)

[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)

sympy [A] time = 0.11, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```


$$3.477 \quad \int \frac{1+x^2}{1+x} dx$$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2 \log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {697}

$$\frac{x^2}{2} - x + 2 \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + x), x]

[Out] -x + x^2/2 + 2*Log[1 + x]

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{1+x} dx &= \int \left(-1 + x + \frac{2}{1+x} \right) dx \\ &= -x + \frac{x^2}{2} + 2 \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2} (x^2 - 2x + 4 \log(x+1) - 3)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/(1 + x), x]

[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + x^2)/(1 + x), x]

[Out] IntegrateAlgebraic[(1 + x^2)/(1 + x), x]

fricas [A] time = 1.58, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(1+x), x, algorithm="fricas")

[Out] 1/2*x^2 - x + 2*log(x + 1)

giac [A] time = 0.36, size = 16, normalized size = 0.94

$$\frac{1}{2}x^2 - x + 2 \log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(1+x), x, algorithm="giac")

[Out] 1/2*x^2 - x + 2*log(abs(x + 1))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - x + 2 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x+1), x)

[Out] -x+1/2*x^2+2*ln(x+1)

maxima [A] time = 0.74, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - x + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/(1+x),x, algorithm="maxima")`

[Out] `1/2*x^2 - x + 2*log(x + 1)`

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2 \ln(x + 1) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)/(x + 1),x)`

[Out] `2*log(x + 1) - x + x^2/2`

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/(1+x),x)`

[Out] `x**2/2 - x + 2*log(x + 1)`

$$3.478 \quad \int \frac{-1+3x-3x^2+x^3}{x^2} dx$$

Optimal. Leaf size=18

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {14}

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2, x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]

Rule 14

Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-1 + 3x - 3x^2 + x^3}{x^2} dx &= \int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x \right) dx \\ &= \frac{1}{x} - 3x + \frac{x^2}{2} + 3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2, x]

[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1 + 3x - 3x^2 + x^3}{x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/x^2, x]

[Out] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/x^2, x]

fricas [A] time = 1.03, size = 20, normalized size = 1.11

$$\frac{x^3 - 6x^2 + 6x \log(x) + 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")

[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x

giac [A] time = 0.37, size = 17, normalized size = 0.94

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")

[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x^2}{2} - 3x + 3 \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-3*x^2+3*x-1)/x^2,x)

[Out] 1/x-3*x+1/2*x^2+3*ln(x)

maxima [A] time = 0.73, size = 16, normalized size = 0.89

$$\frac{1}{2}x^2 - 3x + \frac{1}{x} + 3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")

[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$3 \ln(x) - 3x + \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)

[Out] 3*log(x) - 3*x + 1/x + x^2/2

sympy [A] time = 0.08, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3 \log(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)

[Out] x**2/2 - 3*x + 3*log(x) + 1/x

$$3.479 \quad \int \left(\frac{1}{2} (3 - \sqrt{37}) + x \right) \left(\frac{1}{2} (3 + \sqrt{37}) + x \right) dx$$

Optimal. Leaf size=18

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {43}

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Antiderivative was successfully verified.

[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x), x]

[Out] -7*x + (3*x^2)/2 + x^3/3

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \left(\frac{1}{2} (3 - \sqrt{37}) + x \right) \left(\frac{1}{2} (3 + \sqrt{37}) + x \right) dx &= \int (-7 + 3x + x^2) dx \\ &= -7x + \frac{3x^2}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Antiderivative was successfully verified.

[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x), x]

[Out] -7*x + (3*x^2)/2 + x^3/3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{2} (3 - \sqrt{37}) + x \right) \left(\frac{1}{2} (3 + \sqrt{37}) + x \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x), x]

[Out] IntegrateAlgebraic[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)), x, algorithm="fricas")

[Out] Exception raised: SyntaxError >> Malformed expression

giac [A] time = 0.35, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)), x, algorithm="giac")

[Out] 1/3*x^3 + 3/2*x^2 - 7*x

maple [A] time = 0.00, size = 28, normalized size = 1.56

$$\frac{x^3}{3} + \frac{3x^2}{2} + \left(\frac{3}{2} - \frac{\sqrt{37}}{2} \right) \left(\frac{3}{2} + \frac{\sqrt{37}}{2} \right) x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)), x)

[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x

maxima [A] time = 1.71, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")`

[Out] `1/3*x^3 + 3/2*x^2 - 7*x`

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x(2x^2 + 9x - 42)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)`

[Out] `(x*(9*x + 2*x^2 - 42))/6`

sympy [A] time = 0.06, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)`

[Out] `x**3/3 + 3*x**2/2 - 7*x`

$$3.480 \quad \int \frac{4+3x^2+2x^3}{(1+x)^4} dx$$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1850}

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{4+3x^2+2x^3}{(1+x)^4} dx &= \int \left(\frac{5}{(1+x)^4} - \frac{3}{(1+x)^2} + \frac{2}{1+x} \right) dx \\ &= -\frac{5}{3(1+x)^3} + \frac{3}{1+x} + 2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4 + 3x^2 + 2x^3}{(1+x)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] IntegrateAlgebraic[(4 + 3*x^2 + 2*x^3)/(1 + x)^4, x]

fricas [B] time = 1.14, size = 46, normalized size = 2.00

$$\frac{9x^2 + 6(x^3 + 3x^2 + 3x + 1)\log(x + 1) + 18x + 4}{3(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")

[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1)

giac [A] time = 0.37, size = 25, normalized size = 1.09

$$\frac{9x^2 + 18x + 4}{3(x+1)^3} + 2 \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")

[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$2 \ln(x+1) - \frac{5}{3(x+1)^3} + \frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^3+3*x^2+4)/(x+1)^4,x)

[Out] -5/3/(x+1)^3+3/(x+1)+2*ln(x+1)

maxima [A] time = 0.74, size = 34, normalized size = 1.48

$$\frac{9x^2 + 18x + 4}{3(x^3 + 3x^2 + 3x + 1)} + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")

[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$2 \ln(x + 1) + \frac{3x^2 + 6x + \frac{4}{3}}{(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)

[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3

sympy [A] time = 0.11, size = 31, normalized size = 1.35

$$\frac{9x^2 + 18x + 4}{3x^3 + 9x^2 + 9x + 3} + 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)

[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)

$$3.481 \quad \int \frac{x}{(1+x)^2(1+x^2)} dx$$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)} + \frac{1}{2} \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {801, 203}

$$\frac{1}{2(x+1)} + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)^2*(1 + x^2)),x]

[Out] 1/(2*(1 + x)) + ArcTan[x]/2

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 801

Int[(((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2(1+x^2)} dx &= \int \left(-\frac{1}{2(1+x)^2} + \frac{1}{2(1+x^2)} \right) dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= \frac{1}{2(1+x)} + \frac{1}{2} \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.75

$$\frac{1}{2} \left(\frac{1}{x+1} + \tan^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]

[Out] ((1 + x)^(-1) + ArcTan[x])/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)^2(1+x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/((1 + x)^2*(1 + x^2)),x]

[Out] IntegrateAlgebraic[x/((1 + x)^2*(1 + x^2)), x]

fricas [A] time = 0.87, size = 15, normalized size = 0.94

$$\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")

[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)

giac [B] time = 0.26, size = 32, normalized size = 2.00

$$-\frac{1}{8}\pi - \frac{1}{2}\pi \left[-\frac{\pi - 4\arctan(x)}{4\pi} + \frac{1}{2} \right] + \frac{1}{2(x+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")

[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/2*arctan(x)

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\arctan(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x+1)^2/(x^2+1),x)`

[Out] `1/2/(x+1)+1/2*arctan(x)`

maxima [A] time = 1.79, size = 12, normalized size = 0.75

$$\frac{1}{2(x+1)} + \frac{1}{2} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")`

[Out] `1/2/(x + 1) + 1/2*arctan(x)`

mupad [B] time = 2.22, size = 12, normalized size = 0.75

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x^2 + 1)*(x + 1)^2),x)`

[Out] `atan(x)/2 + 1/(2*(x + 1))`

sympy [A] time = 0.11, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}(x)}{2} + \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)**2/(x**2+1),x)`

[Out] `atan(x)/2 + 1/(2*x + 2)`

$$3.482 \quad \int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx$$

Optimal. Leaf size=29

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1850}

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]

Rule 1850

Int[(Pq_)*((a_) + (b_)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int \frac{7-2x+3x^2-x^3+x^4}{2+x} dx &= \int \left(-20 + 9x - 3x^2 + x^3 + \frac{47}{2+x} \right) dx \\ &= -20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4} + 47 \log(2+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x+2) - 70$$

Antiderivative was successfully verified.

[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7 - 2x + 3x^2 - x^3 + x^4}{2 + x} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

[Out] IntegrateAlgebraic[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]

fricas [A] time = 1.41, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x), x, algorithm="fricas")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

giac [A] time = 0.28, size = 26, normalized size = 0.90

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47 \log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x), x, algorithm="giac")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \ln(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2), x)

[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)

maxima [A] time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4 - x^3 + \frac{9}{2}x^2 - 20x + 47 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")

[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)

mupad [B] time = 0.03, size = 25, normalized size = 0.86

$$47 \ln(x + 2) - 20x + \frac{9x^2}{2} - x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)

[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4

sympy [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47 \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)

[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)

$$3.483 \quad \int \frac{-1+x^3}{-1+x} dx$$

Optimal. Leaf size=16

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1586}

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(-1 + x), x]

[Out] x + x^2/2 + x^3/3

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x} dx &= \int (1+x+x^2) dx \\ &= x + \frac{x^2}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(-1 + x), x]

[Out] x + x^2/2 + x^3/3

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.31

$$\frac{1}{6} (2(x-1)^2 + 9(x-1) + 18)(x-1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^3)/(-1 + x), x]

[Out] ((18 + 9*(-1 + x) + 2*(-1 + x)^2)*(-1 + x))/6

fricas [A] time = 1.24, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x), x, algorithm="fricas")

[Out] 1/3*x^3 + 1/2*x^2 + x

giac [A] time = 0.35, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x), x, algorithm="giac")

[Out] 1/3*x^3 + 1/2*x^2 + x

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x-1), x)

[Out] x+1/2*x^2+1/3*x^3

maxima [A] time = 0.89, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")

[Out] 1/3*x^3 + 1/2*x^2 + x

mupad [B] time = 0.02, size = 13, normalized size = 0.81

$$\frac{x(2x^2 + 3x + 6)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x - 1),x)

[Out] (x*(3*x + 2*x^2 + 6))/6

sympy [A] time = 0.06, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(-1+x),x)

[Out] x**3/3 + x**2/2 + x

$$3.484 \quad \int \frac{2+2x}{(-1+x)^3(1+x^2)} dx$$

Optimal. Leaf size=17

$$\frac{1}{x-1} - \frac{1}{(1-x)^2} + \tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {801, 203}

$$\frac{1}{x-1} - \frac{1}{(1-x)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]

[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 801

Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{2+2x}{(-1+x)^3(1+x^2)} dx &= \int \left(\frac{2}{(-1+x)^3} - \frac{1}{(-1+x)^2} + \frac{1}{1+x^2} \right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{(1-x)^2} + \frac{1}{-1+x} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{x + (x - 1)^2 \tan^{-1}(x) - 2}{(x - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]

[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 2x}{(-1 + x)^3 (1 + x^2)} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]

[Out] IntegrateAlgebraic[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]

fricas [A] time = 1.24, size = 25, normalized size = 1.47

$$\frac{(x^2 - 2x + 1) \arctan(x) + x - 2}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)^3/(x^2+1), x, algorithm="fricas")

[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)

giac [A] time = 0.24, size = 12, normalized size = 0.71

$$\frac{x - 2}{(x - 1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+2*x)/(-1+x)^3/(x^2+1), x, algorithm="giac")

[Out] (x - 2)/(x - 1)^2 + arctan(x)

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{(x - 1)^2} + \frac{1}{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+2)/(x-1)^3/(x^2+1),x)`

[Out] `-1/(x-1)^2+1/(x-1)+arctan(x)`

maxima [A] time = 1.74, size = 17, normalized size = 1.00

$$\frac{x-2}{x^2-2x+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")`

[Out] `(x - 2)/(x^2 - 2*x + 1) + arctan(x)`

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{x-2}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)`

[Out] `atan(x) + (x - 2)/(x^2 - 2*x + 1)`

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2-2x+1} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+2*x)/(-1+x)**3/(x**2+1),x)`

[Out] `(x - 2)/(x**2 - 2*x + 1) + atan(x)`

$$3.485 \quad \int \frac{1}{bx+c(d+ex)^2} dx$$

Optimal. Leaf size=47

$$\frac{2 \tanh^{-1} \left(\frac{b+2ce(d+ex)}{\sqrt{b} \sqrt{b+4cde}} \right)}{\sqrt{b} \sqrt{b+4cde}}$$

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1981, 618, 206}

$$\frac{2 \tanh^{-1} \left(\frac{b+2ce(d+ex)}{\sqrt{b} \sqrt{b+4cde}} \right)}{\sqrt{b} \sqrt{b+4cde}}$$

Antiderivative was successfully verified.

[In] Int[(b*x + c*(d + e*x)^2)^(-1), x]

[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sqrt[b + 4*c*d*e])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QuadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{bx + c(d + ex)^2} dx &= \int \frac{1}{cd^2 + (b + 2cde)x + ce^2x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b(b + 4cde) - x^2} dx, x, b + 2cde + 2ce^2x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b} \sqrt{b+4cde}}\right)}{\sqrt{b} \sqrt{b+4cde}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b} \sqrt{b+4cde}}\right)}{\sqrt{b} \sqrt{b+4cde}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]

[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e]])/(Sqrt[b]*Sqrt[b + 4*c*d*e])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx + c(d + ex)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(b*x + c*(d + e*x)^2)^(-1),x]

[Out] IntegrateAlgebraic[(b*x + c*(d + e*x)^2)^(-1), x]

fricas [A] time = 1.33, size = 190, normalized size = 4.04

$$\left[\frac{\log\left(\frac{2c^2e^4x^2 + 2c^2d^2e^2 + 4bcde + b^2 + 2(2c^2de^3 + bce^2)x - \sqrt{4bcde + b^2}(2ce^2x + 2cde + b)}{ce^2x^2 + cd^2 + (2cde + b)x}\right)}{\sqrt{4bcde + b^2}}, \frac{2\sqrt{-4bcde - b^2} \arctan\left(\frac{\sqrt{-4bcde - b^2}(2ce^2x + 2cde + b)}{4bcde + b^2}\right)}{4bcde + b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")

[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c

$d^2 + (2cde + b)x) / \sqrt{4bcd + b^2}$, $2\sqrt{-4bcd - b^2} \arctan(\sqrt{-4bcd - b^2} (2ce^2x + 2cde + b) / (4bcd + b^2)) / (4bcd + b^2)$]

giac [A] time = 0.28, size = 48, normalized size = 1.02

$$\frac{2 \arctan\left(\frac{2cxe^2+2cde+b}{\sqrt{-4bcde-b^2}}\right)}{\sqrt{-4bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")

[Out] $2\arctan((2cxe^2 + 2cde + b) / \sqrt{-4bcd - b^2}) / \sqrt{-4bcd - b^2}$

maple [A] time = 0.01, size = 43, normalized size = 0.91

$$\frac{2 \operatorname{arctanh}\left(\frac{2ce^2x+2cde+b}{\sqrt{4bcde+b^2}}\right)}{\sqrt{4bcde+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+c*(e*x+d)^2),x)

[Out] $-2/(4bcd + b^2)^{1/2} \operatorname{arctanh}((2ce^2x + 2cde + b) / (4bcd + b^2)^{1/2})$

maxima [A] time = 1.02, size = 68, normalized size = 1.45

$$\frac{\log\left(\frac{2ce^2x+2cde+b-\sqrt{(4cde+b)b}}{2ce^2x+2cde+b+\sqrt{(4cde+b)b}}\right)}{\sqrt{(4cde+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")

[Out] $\log((2ce^2x + 2cde + b - \sqrt{(4cde + b)b}) / (2ce^2x + 2cde + b + \sqrt{(4cde + b)b})) / \sqrt{(4cde + b)b}$

mupad [B] time = 0.10, size = 42, normalized size = 0.89

$$\frac{2 \operatorname{atanh}\left(\frac{2cxe^2+2cde+b}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*(d + e*x)^2 + b*x), x)`

[Out] $-(2*\operatorname{atanh}((b + 2*c*d*e + 2*c*e^2*x)/(b^{1/2}*(b + 4*c*d*e)^{1/2}))) / (b^{1/2}*(b + 4*c*d*e)^{1/2})$

sympy [B] time = 0.30, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b(b+4cde)}} \log\left(x + \frac{-b^2\sqrt{\frac{1}{b(b+4cde)}} - 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right) - \sqrt{\frac{1}{b(b+4cde)}} \log\left(x + \frac{b^2\sqrt{\frac{1}{b(b+4cde)}} + 4bcde\sqrt{\frac{1}{b(b+4cde)}} + b + 2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x+c*(e*x+d)**2), x)`

[Out] $\sqrt{1/(b*(b + 4*c*d*e))}*\log(x + (-b**2*\sqrt{1/(b*(b + 4*c*d*e))} - 4*b*c*d*e*\sqrt{1/(b*(b + 4*c*d*e))} + b + 2*c*d*e)/(2*c*e**2)) - \sqrt{1/(b*(b + 4*c*d*e))}*\log(x + (b**2*\sqrt{1/(b*(b + 4*c*d*e))} + 4*b*c*d*e*\sqrt{1/(b*(b + 4*c*d*e))} + b + 2*c*d*e)/(2*c*e**2))$

$$3.486 \quad \int \frac{1}{a+bx+c(d+ex)^2} dx$$

Optimal. Leaf size=57

$$\frac{2 \tanh^{-1} \left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}} \right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1981, 618, 206}

$$\frac{2 \tanh^{-1} \left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}} \right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*(d + e*x)^2)^(-1), x]

[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q uadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a + bx + c(d + ex)^2} dx &= \int \frac{1}{a + cd^2 + (b + 2cde)x + ce^2x^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b^2 + 4bcde - 4ace^2 - x^2} dx, x, b + 2cde + 2ce^2x\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2 + 4bcde - 4ace^2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.07

$$\frac{2 \tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1), x]

[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx + c(d + ex)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x + c*(d + e*x)^2)^(-1), x]

[Out] IntegrateAlgebraic[(a + b*x + c*(d + e*x)^2)^(-1), x]

fricas [A] time = 0.77, size = 240, normalized size = 4.21

$$\left[\frac{\log\left(\frac{2c^2e^4x^2+4bcde+2(c^2d^2-ac)e^2+b^2+2(2c^2de^3+bce^2)x-\sqrt{4bcde-4ace^2+b^2}(2ce^2x+2cde+b)}{ce^2x^2+cd^2+(2cde+b)x+a}\right)}{\sqrt{4bcde-4ace^2+b^2}}, -\frac{2\sqrt{-4bcde+4ace^2-b^2} \arctan\left(-\frac{\sqrt{-4bcde+4ace^2-b^2}(2ce^2x+2cde+b)}{4bcde-4ace^2+b^2}\right)}{4bcde-4ace^2+b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2), x, algorithm="fricas")

[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +

b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b*c*d*e - 4*a*c*e^2 + b^2)]

giac [A] time = 0.36, size = 60, normalized size = 1.05

$$\frac{2 \arctan\left(\frac{2cxe^2+2cde+b}{\sqrt{-4bcde+4ace^2-b^2}}\right)}{\sqrt{-4bcde+4ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")

[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)

maple [A] time = 0.00, size = 61, normalized size = 1.07

$$\frac{2 \arctan\left(\frac{2ce^2x+2cde+b}{\sqrt{4ace^2-4bcde-b^2}}\right)}{\sqrt{4ace^2-4bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*x+c*(e*x+d)^2),x)

[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for more details)Is 4*a*c*e^2 - 4*b*c*d*e-b^2 positive or negative?

mupad [B] time = 2.23, size = 82, normalized size = 1.44

$$\frac{2 \operatorname{atan}\left(\frac{b+2cde}{\sqrt{-b^2-4cdbe+4ace^2}} + \frac{2ce^2x}{\sqrt{-b^2-4cdbe+4ace^2}}\right)}{\sqrt{-b^2-4cdbe+4ace^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + c*(d + e*x)^2 + b*x),x)`

[Out] $(2*\operatorname{atan}((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2)} + (2*c*e^2*x)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2}))/((4*a*c*e^2 - b^2 - 4*b*c*d*e)^{(1/2)})$

sympy [B] time = 0.35, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} \log\left(x + \frac{-4acc^2\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} + b^2\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} + 4bcde\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} + b + 2cde}{2c^2}}\right) + \sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} \log\left(x + \frac{4acc^2\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} - b^2\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} - 4bcde\sqrt{-\frac{1}{4acc^2 - b^2 - 4bcde}} + b + 2cde}{2c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x+c*(e*x+d)**2),x)`

[Out] $-\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*\log(x + (-4*a*c*e**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + 4*b*c*d*e*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*e**2)) + \operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*\log(x + (4*a*c*e**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - 4*b*c*d*e*\operatorname{sqrt}(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*e**2))$

$$3.487 \quad \int \frac{x^2}{1+(-1+x^2)^2} dx$$

Optimal. Leaf size=188

$$\frac{\log\left(x^2 - \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(x^2 + \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})} - 2x}{\sqrt{2(\sqrt{2}-1)}}\right)$$

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1989, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2 - \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{\log\left(x^2 + \sqrt{2(1+\sqrt{2})}x + \sqrt{2}\right)}{4\sqrt{2(1+\sqrt{2})}} - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})} - 2x}{\sqrt{2(\sqrt{2}-1)}}\right) + \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \tan^{-1}\left(\frac{2x + \sqrt{2(1+\sqrt{2})}}{\sqrt{2(\sqrt{2}-1)}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + (-1 + x^2)^2), x]

[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])]) - 2*x]/Sqrt[2*(-1 + Sqrt[2])])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])]) + 2*x]/Sqrt[2*(-1 + Sqrt[2])])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
- 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1 + (-1 + x^2)^2} dx &= \int \frac{x^2}{2 - 2x^2 + x^4} dx \\
&= -\left(\frac{1}{2} \int \frac{\sqrt{2} - x^2}{2 - 2x^2 + x^4} dx\right) + \frac{1}{2} \int \frac{\sqrt{2} + x^2}{2 - 2x^2 + x^4} dx \\
&= \frac{1}{4} \int \frac{1}{\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2} dx + \frac{1}{4} \int \frac{1}{\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2} dx + \frac{\int \frac{\sqrt{2(1 + \sqrt{2})}}{-\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x}}{4\sqrt{2(1 + \sqrt{2})}} dx \\
&= \frac{\log\left(\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{\log\left(\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{2(1 + \sqrt{2})}\right) \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{2(1 + \sqrt{2})} - 2x}{\sqrt{2(-1 + \sqrt{2})}}\right)}{2\sqrt{2(-1 + \sqrt{2})}} + \frac{\tan^{-1}\left(\frac{\sqrt{2(1 + \sqrt{2})} + 2x}{\sqrt{2(-1 + \sqrt{2})}}\right)}{2\sqrt{2(-1 + \sqrt{2})}} + \frac{\log\left(\sqrt{2} - \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}} - \frac{\log\left(\sqrt{2} + \sqrt{2(1 + \sqrt{2})}x + x^2\right)}{4\sqrt{2(1 + \sqrt{2})}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(1 + (-1 + x^2)^2), x]

[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^(3/2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1 + (-1 + x^2)^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(1 + (-1 + x^2)^2), x]

[Out] IntegrateAlgebraic[x^2/(1 + (-1 + x^2)^2), x]

fricas [A] time = 1.51, size = 247, normalized size = 1.31

$$\frac{1}{16} \cdot 2^{\frac{1}{4}} \sqrt{2} + 4(\sqrt{2} - 2) \log(2^{\frac{3}{4}} \sqrt{2} + 2x^2 + 2\sqrt{2}) - \frac{1}{16} \cdot 2^{\frac{1}{4}} \sqrt{2} + 4(\sqrt{2} - 2) \log(-2^{\frac{3}{4}} \sqrt{2} + 2x^2 + 2\sqrt{2}) - \frac{1}{4} \cdot 2^{\frac{3}{4}} \sqrt{2} + 4 \arctan\left(-\frac{1}{2} \cdot 2^{\frac{3}{4}} \sqrt{2} + 4 + \frac{1}{2} \cdot 2^{\frac{1}{4}} \sqrt{2} \sqrt{2} + 4 + 2x^2 + 2\sqrt{2} \sqrt{2} + 4 - \sqrt{2} - 1\right) - \frac{1}{4} \cdot 2^{\frac{3}{4}} \sqrt{2} + 4 \arctan\left(-\frac{1}{2} \cdot 2^{\frac{3}{4}} \sqrt{2} + 4 + \frac{1}{2} \cdot 2^{\frac{1}{4}} \sqrt{2} \sqrt{2} + 4 + 2x^2 + 2\sqrt{2} \sqrt{2} + 4 + \sqrt{2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")

[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) - sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)

giac [A] time = 1.49, size = 147, normalized size = 0.78

$$\frac{1}{4} \sqrt{2} \sqrt{2} + 2 \arctan\left(\frac{2^{\frac{3}{4}}(2x + 2^{\frac{1}{4}}\sqrt{2} + 2)}{2\sqrt{-\sqrt{2} + 2}}\right) + \frac{1}{4} \sqrt{2} \sqrt{2} + 2 \arctan\left(\frac{2^{\frac{3}{4}}(2x - 2^{\frac{1}{4}}\sqrt{2} + 2)}{2\sqrt{-\sqrt{2} + 2}}\right) - \frac{1}{8} \sqrt{2} \sqrt{2} - 2 \log(x^2 + 2^{\frac{1}{4}}x\sqrt{2} + 2 + \sqrt{2}) + \frac{1}{8} \sqrt{2} \sqrt{2} - 2 \log(x^2 - 2^{\frac{1}{4}}x\sqrt{2} + 2 + \sqrt{2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")

[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2} (2 + 2\sqrt{2}) \arctan\left(\frac{2^{3/4}\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right) - (2 + 2\sqrt{2}) \arctan\left(\frac{2^{3/4}\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right) + \sqrt{2} (2 + 2\sqrt{2}) \arctan\left(\frac{2^{3/4}\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right) - (2 + 2\sqrt{2}) \arctan\left(\frac{2^{3/4}\sqrt{2}}{\sqrt{-2+2\sqrt{2}}}\right) + \frac{\sqrt{2+2\sqrt{2}} \sqrt{2} \ln(x^2 - \sqrt{2+2\sqrt{2}} x + \sqrt{2})}{8} - \frac{\sqrt{2+2\sqrt{2}} \ln(x^2 - \sqrt{2+2\sqrt{2}} x + \sqrt{2})}{8} - \frac{\sqrt{2+2\sqrt{2}} \sqrt{2} \ln(x^2 + \sqrt{2+2\sqrt{2}} x + \sqrt{2})}{8} + \frac{\sqrt{2+2\sqrt{2}} \ln(x^2 + \sqrt{2+2\sqrt{2}} x + \sqrt{2})}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+(x^2-1)^2),x)

[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct

$$\frac{\ln\left(\frac{(2x - (2 + 2\sqrt{2})^{1/2})^{1/2}}{(-2 + 2\sqrt{2})^{1/2}} - \frac{1}{8}(2 + 2\sqrt{2})^{1/2}\right) + \frac{1}{4}(2 + 2\sqrt{2})^{1/2}}{(-2 + 2\sqrt{2})^{1/2}}\right) + \arctan\left(\frac{(2x - (2 + 2\sqrt{2})^{1/2})^{1/2}}{(-2 + 2\sqrt{2})^{1/2}}\right)}{x^2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^2 - 1)^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")

[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)

mupad [B] time = 2.30, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32x\left(\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}} + \sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}}\right)^3\right)\left(2\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}} + 2\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}}\right) + \operatorname{atanh}\left(32x\left(\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}} - \sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}}\right)^3\right)\left(2\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}} - 2\sqrt{\frac{\sqrt{2}}{32} - \frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x^2 - 1)^2 + 1),x)

$$\operatorname{atanh}\left(32x\left(-\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2} + \left(\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2}\right)^3\left(2\left(-\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2} + 2\left(\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2}\right) + \operatorname{atanh}\left(32x\left(-\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2} - \left(\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2}\right)^3\left(2\left(-\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2} - 2\left(\frac{2^{1/2}}{32} - \frac{1}{32}\right)^{1/2}\right)$$

sympy [A] time = 0.52, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t \log(64t^3 + 4t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(1+(x**2-1)**2),x)

[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))

$$3.488 \quad \int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4} dx$$

Optimal. Leaf size=60

$$\frac{x^4}{(x^4+x+3)^3} - \frac{3x}{(x^4+x+3)^3} + \frac{2}{(x^4+x+3)^3} - \frac{5x^6}{(x^4+x+3)^3} + \frac{5x^2}{(x^4+x+3)^3}$$

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2102, 1588}

$$-\frac{5x^6}{(x^4+x+3)^3} + \frac{x^4}{(x^4+x+3)^3} + \frac{5x^2}{(x^4+x+3)^3} - \frac{3x}{(x^4+x+3)^3} + \frac{2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4), x]

[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]
}], Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] := With[{m = Expon[Pm, x], n = Expon[Qn, x]
}], Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m + n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn + (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0 /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{15 - 36x + 5x^2 + 12x^3 - 34x^4 + 140x^5 + 15x^6 + 8x^7 - 30x^9}{(3 + x + x^4)^4} dx &= -\frac{5x^6}{(3 + x + x^4)^3} + \frac{1}{6} \int \frac{-90 + 216x - 30x^2}{(3 + x + x^4)^3} dx \\
&= \frac{x^4}{(3 + x + x^4)^3} - \frac{5x^6}{(3 + x + x^4)^3} - \frac{1}{48} \int \frac{72x^2 - 180x + 90}{(3 + x + x^4)^3} dx \\
&= \frac{5x^2}{(3 + x + x^4)^3} + \frac{x^4}{(3 + x + x^4)^3} - \frac{5x^6}{(3 + x + x^4)^3} \\
&= -\frac{3x}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^3} + \frac{x^4}{(3 + x + x^4)^3} \\
&= \frac{2}{(3 + x + x^4)^3} - \frac{3x}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.45

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4),x]

[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15 - 36x + 5x^2 + 12x^3 - 34x^4 + 140x^5 + 15x^6 + 8x^7 - 30x^9}{(3 + x + x^4)^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4),x]

[Out] IntegrateAlgebraic[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4), x]

fricas [A] time = 1.29, size = 65, normalized size = 1.08

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="fricas")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

giac [A] time = 0.40, size = 30, normalized size = 0.50

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="giac")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3

maple [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x)

[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3

maxima [A] time = 1.25, size = 65, normalized size = 1.08

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x, algorithm="maxima")

[Out] $-(5x^6 - x^4 - 5x^2 + 3x - 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$

mupad [B] time = 2.34, size = 27, normalized size = 0.45

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(5x^2 - 36x + 12x^3 - 34x^4 + 140x^5 + 15x^6 + 8x^7 - 30x^9 + 15)/(x + x^4 + 3)^4, x)$

[Out] $(5x^2 - 3x + x^4 - 5x^6 + 2)/(x + x^4 + 3)^3$

sympy [A] time = 0.23, size = 60, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((30x**9-8x**7-15x**6-140x**5+34x**4-12x**3-5x**2+36x-15)/(x**4+x+3)**4, x)$

[Out] $(-5x**6 + x**4 + 5x**2 - 3x + 2)/(x**12 + 3x**9 + 9x**8 + 3x**6 + 18x**5 + 27x**4 + x**3 + 9x**2 + 27x + 27)$

$$3.489 \quad \int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$$

Optimal. Leaf size=27

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx$$

Verification is not applicable to the result.

[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]

[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[Int][x/(3 + x + x^4)^2, x]

Rubi steps

$$\begin{aligned} \int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx &= 3 \int \frac{-47 + 228x + 120x^2}{(3 + x + x^4)^4} dx \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \\ &= -\frac{19}{4(3 + x + x^4)^3} + \frac{19}{(3 + x + x^4)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]

[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{3(-47 + 228x + 120x^2 + 19x^3)}{(3 + x + x^4)^4} + \frac{42 - 320x - 75x^2 - 8x^3}{(3 + x + x^4)^3} + \frac{30x}{(3 + x + x^4)^2} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]

[Out] IntegrateAlgebraic[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2, x]

fricas [B] time = 1.29, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")

[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 27*x^4 + x^3 + 9*x^2 + 27*x + 27)

giac [B] time = 0.40, size = 197, normalized size = 7.30

$$\frac{1}{95025} \left(\frac{377432x^2 - 260856x + 70353}{x^4 + x + 3} - \frac{255032x^2 - 182945x + 68061}{x^4 + x + 3} - \frac{7650(16x^2 - 128x + 3)}{x^4 + x + 3} \right) - \frac{2(16x^2 - 64x^2 + x + 12)}{3(16x^2 - 64x^2 + x + 12)} + \frac{754864x^2 - 260856x^2 + 469374x^2 + 132012x^2 - 417984x^2 - 13339729x^2 + 2696430x + 2183454}{360350(x^4 + x + 3)^2} + \frac{510064x^{11} - 1828456x^{10} + 453734x^9 + 1402676x^8 - 472048x^7 - 1350133x^6 + 4720744x^5 + 3747556x^4 - 1093781x^3 - 30736107x^2 + 11020894x + 4117662}{360350(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")

```
[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 -
1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3
)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 -
2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 26964
30*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 45
3734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*
x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3
```

maple [C] time = 0.03, size = 250, normalized size = 9.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-2
08792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^
2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(-_R+x),_R=Root
Of(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum
((-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))+3*(-255032/
585225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/5
85225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+109357
81/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/1
95075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_
Z^4+_Z+3))
```

maxima [B] time = 1.20, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 0.05, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)`

[Out] $(5x^2 - 3x + x^4 - 5x^6 + 2)/(x + x^4 + 3)^3$

sympy [B] time = 0.32, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)`

[Out] $(-5x^6 + x^4 + 5x^2 - 3x + 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$

$$3.490 \quad \int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3} - \frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4} \right) dx$$

Optimal. Leaf size=27

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} \right) dx$$

Verification is not applicable to the result.

[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4, x]

[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 + x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3), x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]

Rubi steps

$$\begin{aligned}
\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} \right) dx &= - \left(3 \int \frac{(1 + 4x^3)(2 - 3x + 5x^2 + x^4)}{(3 + x + x^4)^4} dx \right) \\
&= - \frac{10x^6}{(3 + x + x^4)^3} + \frac{5x^2}{(3 + x + x^4)^2} - \frac{1}{6} \\
&= \frac{3x^4}{2(3 + x + x^4)^3} - \frac{10x^6}{(3 + x + x^4)^3} - \frac{1}{6} \\
&= - \frac{5x^3}{(3 + x + x^4)^3} + \frac{3x^4}{2(3 + x + x^4)^3} - \frac{1}{6} \\
&= - \frac{12x^2}{(3 + x + x^4)^3} - \frac{5x^3}{(3 + x + x^4)^3} + \frac{1}{6} \\
&= - \frac{63x}{22(3 + x + x^4)^3} - \frac{12x^2}{(3 + x + x^4)^3} - \frac{1}{6} \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{6} \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{6} \\
&= \frac{7}{2(3 + x + x^4)^3} - \frac{63x}{22(3 + x + x^4)^3} - \frac{1}{6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]

[Out] $(2 - 3x + 5x^2 + x^4 - 5x^6)/(3 + x + x^4)^3$

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-3 + 10x + 4x^3 - 30x^5}{(3 + x + x^4)^3} - \frac{3(1 + 4x^3)(2 - 3x + 5x^2 + x^4 - 5x^6)}{(3 + x + x^4)^4} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4, x]

[Out] IntegrateAlgebraic[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4, x]

fricas [B] time = 1.36, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")

[Out] $-(5x^6 - x^4 - 5x^2 + 3x - 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$

giac [B] time = 0.45, size = 111, normalized size = 4.11

$$\frac{69136x^7 - 147344x^6 - 30784x^5 + 120988x^4 + 137584x^3 + 1167854x^2 - 100680x + 18621}{390150(x^4 + x + 3)^2} - \frac{69136x^{11} - 147344x^{10} - 30784x^9 + 190124x^8 + 197648x^7 + 2645788x^6 - 72044x^5 + 129019x^4 + 1580606x^3 + 1452132x^2 + 887031x - 724437}{390150(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="giac")

[Out] $1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 1167854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^{11} - 147344*x^{10} - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 129019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3$

maple [B] time = 0.02, size = 112, normalized size = 4.15

$$\frac{-\frac{34568}{195075}x^7 + \frac{73672}{195075}x^6 + \frac{15392}{195075}x^5 - \frac{60494}{195075}x^4 - \frac{68792}{195075}x^3 - \frac{583927}{195075}x^2 + \frac{3356}{13005}x - \frac{2069}{43350} - \frac{34568}{195075}x^{11} + \frac{73672}{195075}x^{10} + \frac{15392}{195075}x^9 - \frac{95062}{195075}x^8 - \frac{98824}{195075}x^7 - \frac{1322894}{195075}x^6 + \frac{36022}{195075}x^5 - \frac{129019}{390150}x^4 - \frac{790303}{195075}x^3 - \frac{80674}{21675}x^2 - \frac{32853}{14450}x + \frac{26831}{14450}}{(x^4 + x + 3)^2} + \frac{69136x^{11} - 147344x^{10} - 30784x^9 + 190124x^8 + 197648x^7 + 2645788x^6 - 72044x^5 + 129019x^4 + 1580606x^3 + 1452132x^2 + 887031x - 724437}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x)`

[Out]
$$-\left(-\frac{34568}{195075}x^7+\frac{73672}{195075}x^6+\frac{15392}{195075}x^5-\frac{60494}{195075}x^4-\frac{68792}{195075}x^3-\frac{583927}{195075}x^2+\frac{3356}{13005}x-\frac{2069}{43350}\right)/(x^4+x+3)^2+3\left(-\frac{34568}{585225}x^{11}+\frac{73672}{585225}x^{10}+\frac{15392}{585225}x^9-\frac{95062}{585225}x^8-\frac{98824}{585225}x^7-\frac{1322894}{585225}x^6+\frac{36022}{585225}x^5-\frac{129019}{1170450}x^4-\frac{790303}{585225}x^3-\frac{80674}{65025}x^2-\frac{10951}{14450}x+\frac{26831}{43350}\right)/(x^4+x+3)^3$$

maxima [B] time = 0.98, size = 65, normalized size = 2.41

$$\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")`

[Out]
$$-(5x^6 - x^4 - 5x^2 + 3x - 2)/(x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27)$$

mupad [B] time = 0.04, size = 27, normalized size = 1.00

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{(x^4 + x + 3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)`

[Out]
$$(5x^2 - 3x + x^4 - 5x^6 + 2)/(x + x^4 + 3)^3$$

sympy [B] time = 0.29, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4+5*x**2-3*x+2)/(x**4+x+3)**4,x)`

[Out]
$$(-5x^{**6} + x^{**4} + 5x^{**2} - 3x + 2)/(x^{**12} + 3x^{**9} + 9x^{**8} + 3x^{**6} + 18x^{**5} + 27x^{**4} + x^{**3} + 9x^{**2} + 27x + 27)$$

Chapter 4

Appendix

Local contents

- 4.1 Download section2324
- 4.2 Listing of Grading functions2324

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
```

```

If[ExpnType[result]<=ExpnType[optimal],
  If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
    If[LeafCount[result]<=2*LeafCount[optimal],
      "A",
      "B"],
    "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],

```

```

    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

4.2.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```



```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
    ))
    else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or type
(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```



```

        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(6,m1) #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```