Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.1-Rational-functions

Nasser M. Abbasi

September 20, 2021 Compiled on September 20, 2021 at 4:51pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [490]. This is test number [37].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (490)	0.00 (0)
Maple	99.80 (489)	0.20 (1)
Rubi	99.39 (487)	0.61 (3)
Mupad	98.37 (482)	1.63 (8)
Fricas	87.96 (431)	12.04 (59)
Sympy	87.76 (430)	% 12.24 (60)
Giac	85.92 (421)	14.08 (69)
Maxima	83.47 (409)	16.53 (81)
IntegrateAlgebraic	2.45 (12)	97.55 (478)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

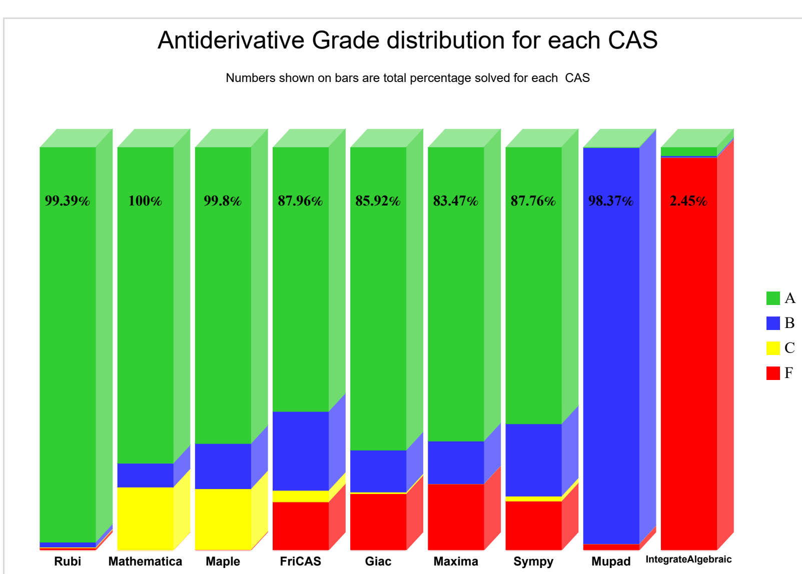
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

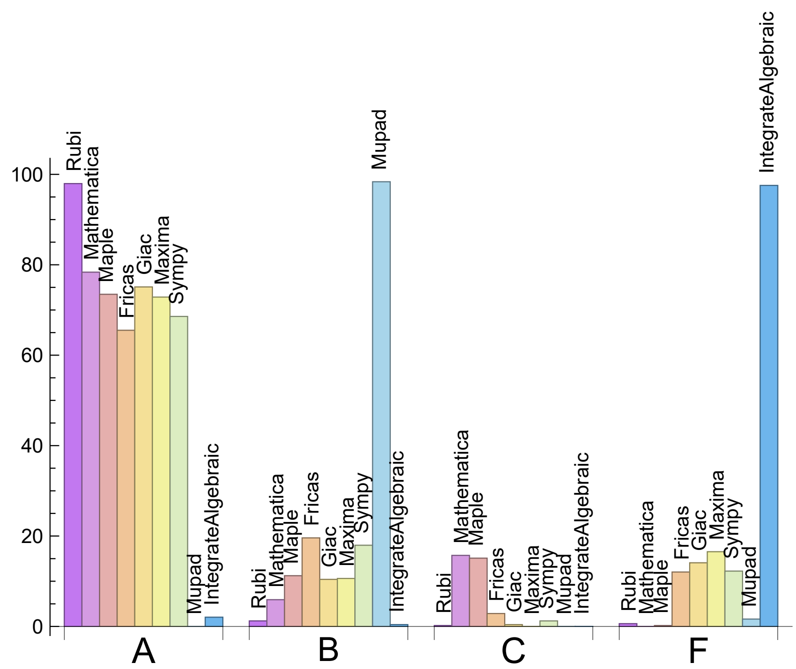
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.96	1.22	0.20	0.61
Mathematica	78.37	5.92	15.71	0.00
Giac	75.10	10.41	0.41	14.08
Maple	73.47	11.22	15.10	0.20
Maxima	72.86	10.61	0.00	16.53
Sympy	68.57	17.96	1.22	12.24
Fricas	65.51	19.59	2.86	12.04
IntegrateAlgebraic	2.04	0.41	0.00	97.55
Mupad	N/A	98.37	0.00	1.63

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	1	100.00 %	0.00 %	0.00 %
Fricas	59	0.00 %	98.31 %	1.69 %
IntegrateAlgebraic	478	100.00 %	0.00 %	0.00 %
Giac	69	91.30 %	2.90 %	5.80 %
Maxima	81	93.83 %	3.70 %	2.47 %
Sympy	60	8.33 %	91.67 %	0.00 %
Mupad	8	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

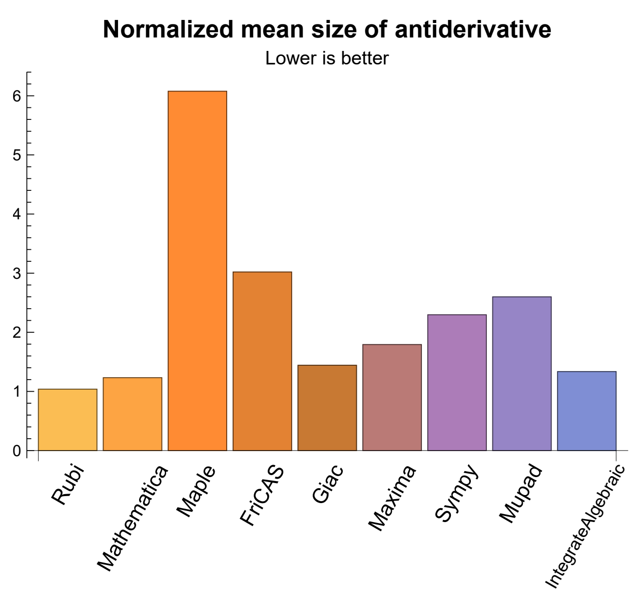
1.3 Performance

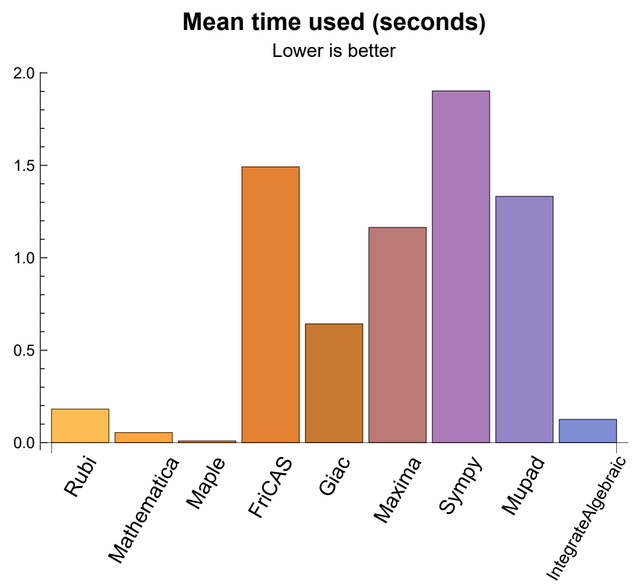
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	119.77	1.04	32.00	1.00
Mathematica	0.05	80.41	1.23	37.00	1.00
Maple	0.01	208.48	6.08	34.00	0.91
Maxima	1.16	115.91	1.79	28.00	0.88
Fricas	1.49	297.22	3.02	36.00	1.10
Sympy	1.90	142.16	2.30	39.00	0.91
Giac	0.64	110.16	1.44	28.00	0.92
Mupad	1.33	452.85	2.60	40.00	0.94
IntegrateAlgebraic	0.13	47.17	1.34	22.50	1.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {147, 148, 149, 150, 151, 152, 153}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

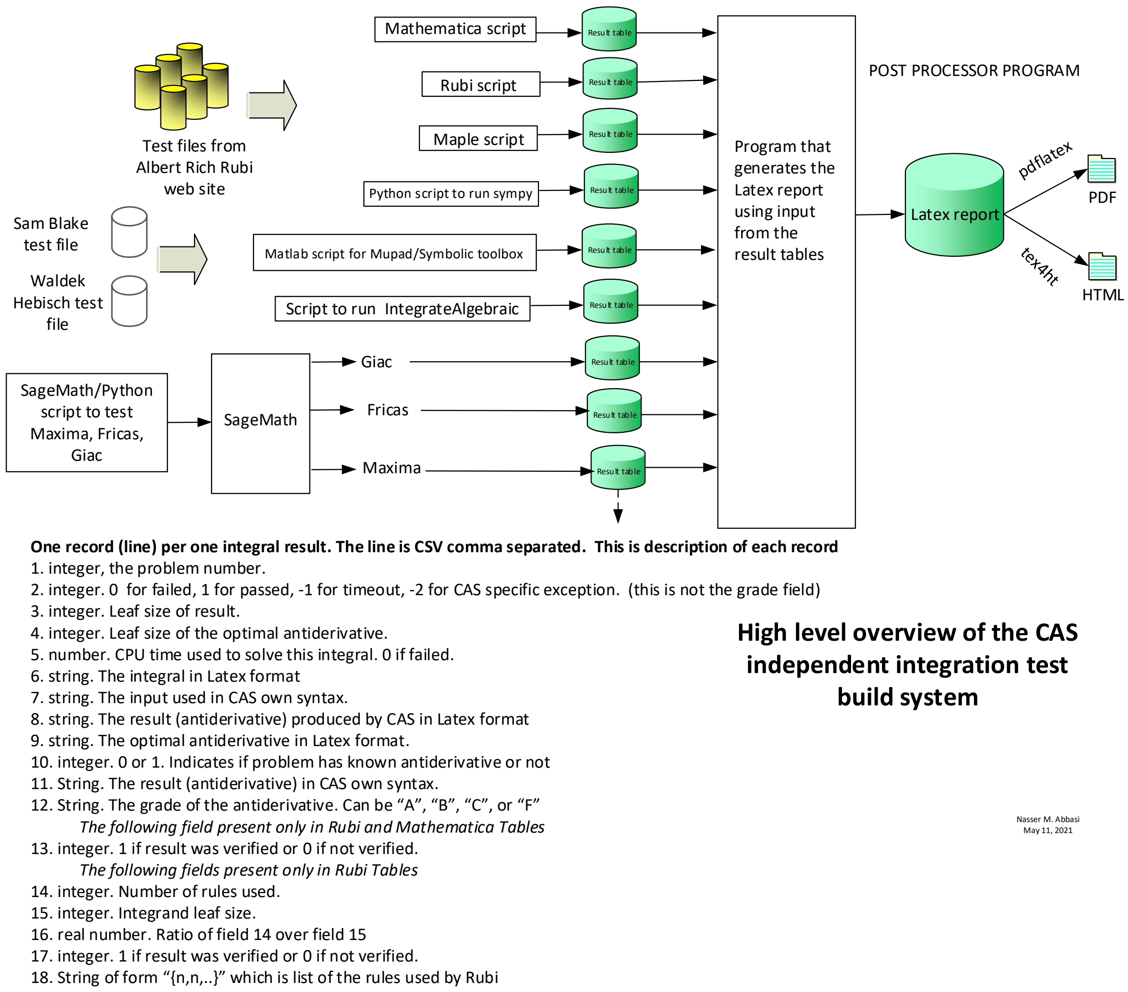
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 61, 73, 217, 218, 229, 420 }

C grade: { 170 }

F grade: { 389, 489, 490 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98, 102, 109, 112, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 189, 190, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490 }

B grade: { 61, 88, 91, 156, 157, 158, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 417 }

C grade: { 12, 13, 14, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 103, 104, 105, 106, 107, 108, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 169, 170, 171, 172, 180, 181, 223, 246, 247, 248, 249, 250, 251, 252, 253, 383, 384, 385, 386, 387, 388, 389, 487 }

F grade: { }

2.1.3 Maple

A grade: { 2, 5, 6, 7, 8, 10, 11, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 160, 161, 162, 163, 168, 169, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488 }

B grade: { 3, 4, 9, 15, 20, 59, 60, 61, 88, 91, 97, 98, 112, 156, 157, 158, 159, 164, 165, 166, 167, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 223, 228, 333, 334, 396, 417, 420, 439, 487, 490 }

C grade: { 1, 12, 13, 14, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 170, 246, 247, 248, 249, 250, 251, 252, 253, 383, 384, 385, 386, 387, 388, 389, 489 }

F grade: { 172 }

2.1.4 Maxima

A grade: { 2, 5, 6, 10, 11, 15, 16, 17, 18, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 89, 92, 93, 94, 95, 96, 112, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 156, 160, 161, 162, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 194, 195, 204, 205, 212, 213, 214, 215, 216, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 488 }

B grade: { 3, 4, 7, 8, 9, 19, 20, 59, 60, 61, 62, 63, 64, 70, 83, 84, 87, 88, 90, 91, 97, 98, 157, 158, 159, 163, 165, 166, 167, 190, 193, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 417, 420, 473, 489, 490 }

C grade: { }

F grade: { 1, 12, 13, 14, 26, 33, 34, 39, 40, 45, 46, 51, 52, 57, 58, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 223, 224, 246, 247, 248, 249, 250, 251, 252, 253, 331, 332, 333, 334, 383, 384, 385, 386, 387, 388, 389, 486, 487 }

2.1.5 FriCAS

A grade: { 1, 2, 5, 6, 10, 11, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 92, 94, 95, 96, 97, 98, 102, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 155, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 204, 205, 212, 213, 214, 215, 216, 220, 221, 222, 224, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 332, 335, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 393, 400, 407, 411, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 481, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 3, 4, 7, 8, 9, 12, 13, 14, 15, 33, 34, 39, 40, 59, 60, 61, 62, 63, 64, 70, 71, 72, 73, 83, 87, 88, 89, 90, 91, 93, 112, 116, 117, 118, 142, 150, 156, 157, 158, 159, 164, 165, 166, 167, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 217, 218, 219, 225, 229, 246, 247, 248, 249, 250, 251, 252, 253, 264, 273, 339, 340, 383, 384, 385, 386, 417, 419, 420, 449, 451, 473, 480, 489, 490 }

C grade: { 45, 46, 51, 52, 99, 100, 101, 103, 104, 105, 330, 337, 364, 389 }

F grade: { 19, 20, 57, 58, 106, 107, 108, 110, 111, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 223, 333, 334, 338, 387, 388, 390, 391, 392, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 408, 409, 410, 479 }

2.1.6 Sympy

A grade: { 1, 5, 10, 11, 12, 13, 14, 16, 17, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 47, 48, 49, 50, 53, 54, 55, 56, 57, 65, 66, 67, 68, 69, 71, 72, 89, 92, 93, 94, 95, 99, 100, 101, 102, 106, 107, 108, 109, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 126, 127, 128, 129, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 168, 173, 174, 183, 184, 185, 187, 204, 215, 216, 220, 221, 222, 224, 225, 226, 227, 228, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 252, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 397, 398, 399, 400, 404, 405, 406, 407, 411, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 487, 488 }

B grade: { 3, 4, 6, 7, 8, 9, 15, 26, 46, 51, 52, 58, 59, 60, 61, 62, 63, 64, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 88, 90, 91, 117, 118, 124, 125, 130, 131, 156, 157, 158, 164, 165, 166, 175, 180, 181, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 217, 218, 219, 229, 247, 264, 335, 417, 419, 420, 455, 473, 485, 486, 489, 490 }

C grade: { 85, 86, 87, 253, 275, 454 }

F grade: { 2, 18, 19, 20, 28, 40, 96, 97, 98, 103, 104, 105, 110, 111, 132, 133, 134, 135, 136, 137, 138, 159, 167, 169, 170, 171, 172, 176, 177, 178, 179, 182, 186, 205, 213, 214, 223, 230, 231, 232, 233, 234, 235, 236, 237, 251, 333, 334, 336, 337, 338, 394, 395, 396, 401, 402, 403, 408, 409, 410 }

2.1.7 Giac

A grade: { 1, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 47, 48, 49, 50, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 92, 93, 94, 95, 96, 97, 98, 102, 109, 113, 114, 115, 119, 120, 121, 122, 126, 127, 128, 129, 154, 156, 160, 161, 162, 164, 165, 166, 168, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 185, 186, 187, 189, 190, 192, 193, 195, 196, 197, 202, 204, 205, 206, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 238, 239, 240, 241, 242, 243, 244, 245, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 416, 418, 419, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488 }

B grade: { 2, 3, 4, 9, 15, 19, 20, 59, 60, 61, 70, 88, 91, 112, 155, 157, 158, 159, 169, 170, 171, 178, 184, 188, 191, 194, 198, 199, 200, 201, 203, 207, 209, 211, 230, 231, 232, 233, 234, 253, 264, 278, 320, 333, 334, 355, 417, 420, 481, 489, 490 }

C grade: { 51, 52 }

F grade: { 39, 40, 45, 46, 57, 58, 99, 100, 101, 103, 104, 105, 106, 107, 108, 110, 111, 116, 117, 118, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 163, 167, 172, 223, 235, 236, 237, 246, 247, 248, 249, 250, 251, 252, 383, 384, 385, 386, 387, 388, 389, 409 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490 }

C grade: { }

F grade: { 96, 97, 98, 163, 167, 170, 172, 223 }

2.1.9 IntegrateAlgebraic

A grade: { 96, 154, 159, 163, 167, 173, 174, 359, 462, 483 }

B grade: { 97, 98 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 160, 161, 162, 164, 165, 166, 168, 169, 170, 171, 172, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 488, 489, 490 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	143	43	0	76	60	55	51	0
N.S.	1	1.00	1.86	0.56	0.00	0.99	0.78	0.71	0.66	0.00
time (sec)	N/A	0.134	0.074	0.054	0.000	0.719	0.334	0.221	0.196	0.001

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	23	46	25	43	0	73	52	0
N.S.	1	1.00	0.77	1.53	0.83	1.43	0.00	2.43	1.73	0.00
time (sec)	N/A	0.018	0.105	0.012	0.489	1.030	0.000	0.196	2.132	0.076

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	98	216	97	107	97	97	0
N.S.	1	1.00	1.00	7.00	15.43	6.93	7.64	6.93	6.93	0.00
time (sec)	N/A	0.007	0.001	0.003	0.686	1.289	0.088	0.254	2.073	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	65	99	64	66	64	64	0
N.S.	1	1.00	1.00	4.64	7.07	4.57	4.71	4.57	4.57	0.00
time (sec)	N/A	0.007	0.001	0.001	0.657	0.750	0.079	0.375	0.028	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	32	31	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	0.89	0.00
time (sec)	N/A	0.007	0.000	0.001	0.735	0.513	0.066	0.283	0.037	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	24	24	26	12	26	0
N.S.	1	1.00	1.00	0.93	1.71	1.71	1.86	0.86	1.86	0.00
time (sec)	N/A	0.009	0.005	0.023	0.649	0.795	0.183	0.264	0.034	0.001

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	57	57	61	12	59	0
N.S.	1	1.00	1.00	0.93	4.07	4.07	4.36	0.86	4.21	0.00
time (sec)	N/A	0.009	0.004	0.003	0.714	0.926	0.349	0.295	2.048	0.001

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	90	90	97	12	92	0
N.S.	1	1.00	1.00	0.93	6.43	6.43	6.93	0.86	6.57	0.00
time (sec)	N/A	0.008	0.003	0.005	0.538	1.110	0.543	0.392	2.069	0.001

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	159	295	204	166	175	166	149	0
N.S.	1	1.00	1.89	3.51	2.43	1.98	2.08	1.98	1.77	0.00
time (sec)	N/A	0.125	0.025	0.002	0.676	0.951	0.101	0.273	2.077	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	82	84	93	83	87	83	79	0
N.S.	1	1.00	1.46	1.50	1.66	1.48	1.55	1.48	1.41	0.00
time (sec)	N/A	0.068	0.010	0.002	0.636	0.626	0.083	0.344	0.039	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	28	31	28	28	0
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88	0.00
time (sec)	N/A	0.006	0.000	0.001	0.550	0.760	0.064	0.270	0.038	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	63	57	0	387	53	212	174	0
N.S.	1	1.00	0.34	0.30	0.00	2.06	0.28	1.13	0.93	0.00
time (sec)	N/A	0.312	0.021	0.005	0.000	1.108	0.406	0.358	0.494	0.001

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	112	136	0	704	192	289	247	0
N.S.	1	1.00	0.46	0.56	0.00	2.87	0.78	1.18	1.01	0.00
time (sec)	N/A	0.249	0.066	0.020	0.000	1.104	1.225	0.357	2.645	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	305	305	149	276	0	1268	474	366	483	0
N.S.	1	1.00	0.49	0.90	0.00	4.16	1.55	1.20	1.58	0.00
time (sec)	N/A	0.302	0.090	0.018	0.000	1.173	2.542	0.397	2.983	0.001

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	653	861	461	987	1018	971	787	0
N.S.	1	1.00	1.81	2.39	1.28	2.73	2.82	2.69	2.18	0.00
time (sec)	N/A	0.658	0.224	0.002	0.680	0.865	0.246	0.268	2.226	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	241	188	180	346	345	346	270	0
N.S.	1	1.00	1.25	0.97	0.93	1.79	1.79	1.79	1.40	0.00
time (sec)	N/A	0.231	0.086	0.002	0.625	0.832	0.135	0.415	0.084	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	76	51	50	62	63	54	54	0
N.S.	1	1.00	1.36	0.91	0.89	1.11	1.12	0.96	0.96	0.00
time (sec)	N/A	0.016	0.000	0.000	0.827	0.951	0.073	0.232	0.045	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	80	87	112	112	0	137	106	0
N.S.	1	1.00	0.93	1.01	1.30	1.30	0.00	1.59	1.23	0.00
time (sec)	N/A	0.073	0.057	0.011	0.647	12.865	0.000	0.307	2.335	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	232	398	2096	0	0	1414	1940	0
N.S.	1	1.00	0.99	1.70	8.96	0.00	0.00	6.04	8.29	0.00
time (sec)	N/A	0.405	0.552	0.026	1.539	0.000	0.000	0.392	8.176	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	495	495	490	1076	11005	0	0	6908	82532	0
N.S.	1	1.00	0.99	2.17	22.23	0.00	0.00	13.96	166.73	0.00
time (sec)	N/A	1.462	1.192	0.034	5.723	0.000	0.000	1.103	20.456	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.015	0.009	0.013	1.453	1.116	0.131	0.305	2.201	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	25	25	24	26	25	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.77	0.84	0.81	0.00
time (sec)	N/A	0.020	0.007	0.007	1.280	1.245	0.146	0.352	0.053	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	8	8	8	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.001	0.000	0.001	0.626	0.815	0.064	0.274	0.034	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	26	19	30	25	0
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89	0.00
time (sec)	N/A	0.015	0.006	0.009	0.587	0.569	0.180	0.362	0.060	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	18	15	24	18	0
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82	0.00
time (sec)	N/A	0.012	0.007	0.005	0.793	0.869	0.205	0.296	2.134	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	61	62	0	211	564	62	213	0
N.S.	1	1.00	0.98	1.00	0.00	3.40	9.10	1.00	3.44	0.00
time (sec)	N/A	0.055	0.082	0.009	0.000	1.126	4.193	0.294	0.465	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	89	91	98	299	20	112	99	0
N.S.	1	1.00	0.77	0.79	0.85	2.60	0.17	0.97	0.86	0.00
time (sec)	N/A	0.063	0.036	0.008	1.281	0.544	0.157	0.264	0.233	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	17	17	16	0	16	16	0
N.S.	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.004	0.002	0.002	0.574	1.168	0.000	0.309	2.500	0.052

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	270	270	285	392	372	277	299	277	261	0
N.S.	1	1.00	1.06	1.45	1.38	1.03	1.11	1.03	0.97	0.00
time (sec)	N/A	0.539	0.040	0.002	0.628	0.760	0.127	0.235	2.298	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	171	231	205	166	180	166	160	0
N.S.	1	1.00	1.00	1.35	1.20	0.97	1.05	0.97	0.94	0.00
time (sec)	N/A	0.092	0.028	0.002	0.802	0.744	0.102	0.243	2.160	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	92	84	94	83	95	83	82	0
N.S.	1	1.00	1.00	0.91	1.02	0.90	1.03	0.90	0.89	0.00
time (sec)	N/A	0.043	0.021	0.001	0.570	0.537	0.085	0.252	0.036	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	28	31	28	28	0
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.88	0.00
time (sec)	N/A	0.006	0.000	0.002	0.651	0.463	0.067	0.268	0.037	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	529	529	71	64	0	905	88	603	1551	0
N.S.	1	1.00	0.13	0.12	0.00	1.71	0.17	1.14	2.93	0.00
time (sec)	N/A	0.896	0.027	0.059	0.000	0.735	1.149	0.336	4.541	0.001

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	746	746	182	232	0	3222	427	1057	5844	0
N.S.	1	1.00	0.24	0.31	0.00	4.32	0.57	1.42	7.83	0.00
time (sec)	N/A	1.328	0.116	0.018	0.000	1.088	109.971	0.367	4.304	0.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	295	295	345	500	383	353	366	323	331	0
N.S.	1	1.00	1.17	1.69	1.30	1.20	1.24	1.09	1.12	0.00
time (sec)	N/A	0.532	0.055	0.002	0.745	0.659	0.137	0.308	0.268	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	207	288	214	205	218	187	201	0
N.S.	1	1.00	1.02	1.42	1.05	1.01	1.07	0.92	0.99	0.00
time (sec)	N/A	0.123	0.029	0.001	0.598	0.703	0.109	0.238	2.243	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	109	100	101	99	112	90	98	0
N.S.	1	1.00	1.02	0.93	0.94	0.93	1.05	0.84	0.92	0.00
time (sec)	N/A	0.051	0.015	0.000	0.607	0.638	0.087	0.293	0.044	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	34	33	33	36	30	33	0
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.97	0.81	0.89	0.00
time (sec)	N/A	0.007	0.000	0.000	0.667	0.560	0.068	0.380	0.043	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	153	153	71	67	0	1115	122	0	1264	0
N.S.	1	1.00	0.46	0.44	0.00	7.29	0.80	0.00	8.26	0.00
time (sec)	N/A	0.254	0.033	0.066	0.000	0.888	1.835	0.000	3.731	0.001

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	342	342	234	288	0	4285	0	0	10351	0
N.S.	1	1.00	0.68	0.84	0.00	12.53	0.00	0.00	30.27	0.00
time (sec)	N/A	0.532	0.193	0.023	0.000	1.294	0.000	0.000	7.109	0.001

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	96	85	84	84	94	84	84	0
N.S.	1	1.00	1.00	0.89	0.88	0.88	0.98	0.88	0.88	0.00
time (sec)	N/A	0.030	0.003	0.002	1.248	0.681	0.075	0.307	0.185	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	65	64	64	71	64	64	0
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.96	0.86	0.86	0.00
time (sec)	N/A	0.023	0.001	0.001	0.939	0.479	0.070	0.334	0.076	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	54	45	44	44	49	44	44	0
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.017	0.001	0.000	0.884	0.758	0.062	0.332	0.032	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	19	19	19	0
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.83	0.83	0.00
time (sec)	N/A	0.003	0.000	0.000	0.911	0.731	0.057	0.315	0.031	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	268	268	45	41	0	1015	41	0	123	0
N.S.	1	1.00	0.17	0.15	0.00	3.79	0.15	0.00	0.46	0.00
time (sec)	N/A	0.396	0.010	0.015	0.000	3.625	0.955	0.000	2.445	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	357	357	113	83	0	1201	3834	0	176	0
N.S.	1	1.00	0.32	0.23	0.00	3.36	10.74	0.00	0.49	0.00
time (sec)	N/A	0.397	0.018	0.013	0.000	5.199	3.224	0.000	0.207	0.004

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	78	77	77	94	77	77	0
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.79	0.00
time (sec)	N/A	0.028	0.002	0.003	0.570	0.721	0.071	0.364	0.150	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.020	0.001	0.001	0.455	0.710	0.066	0.362	0.063	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	38	37	37	42	37	37	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.82	0.00
time (sec)	N/A	0.015	0.001	0.001	0.568	0.736	0.062	0.255	0.026	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	19	17	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.81	0.00
time (sec)	N/A	0.003	0.000	0.000	0.569	1.184	0.056	0.361	0.028	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	47	41	0	499	3432	265	87	0
N.S.	1	1.00	0.20	0.18	0.00	2.13	14.67	1.13	0.37	0.00
time (sec)	N/A	0.315	0.016	0.007	0.000	4.088	2.572	0.520	2.359	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	108	79	0	704	3834	315	174	0
N.S.	1	1.00	0.34	0.25	0.00	2.22	12.09	0.99	0.55	0.00
time (sec)	N/A	0.335	0.024	0.013	0.000	3.888	3.665	0.724	2.207	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	104	85	84	84	100	84	84	0
N.S.	1	1.00	1.00	0.82	0.81	0.81	0.96	0.81	0.81	0.00
time (sec)	N/A	0.034	0.002	0.002	0.661	0.614	0.081	0.364	2.230	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	65	64	64	73	64	64	0
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.84	0.00
time (sec)	N/A	0.024	0.001	0.000	0.646	0.393	0.076	0.384	0.078	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	52	45	44	44	49	44	44	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.94	0.85	0.85	0.00
time (sec)	N/A	0.016	0.001	0.002	0.610	0.857	0.066	0.357	0.033	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	27	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.90	0.80	0.80	0.00
time (sec)	N/A	0.004	0.000	0.001	0.462	0.732	0.058	0.300	0.019	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	263	263	55	49	0	0	41	0	123	0
N.S.	1	1.00	0.21	0.19	0.00	0.00	0.16	0.00	0.47	0.00
time (sec)	N/A	0.493	0.015	0.008	0.000	0.000	2.372	0.000	0.411	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	366	366	128	96	0	0	3839	0	181	0
N.S.	1	1.00	0.35	0.26	0.00	0.00	10.49	0.00	0.49	0.00
time (sec)	N/A	0.508	0.029	0.011	0.000	0.000	3.946	0.000	0.205	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	164	592	163	185	163	163	0
N.S.	1	1.00	1.00	11.71	42.29	11.64	13.21	11.64	11.64	0.00
time (sec)	N/A	0.017	0.002	0.003	0.683	0.923	0.114	0.283	0.167	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	109	228	108	114	108	108	0
N.S.	1	1.00	1.00	7.79	16.29	7.71	8.14	7.71	7.71	0.00
time (sec)	N/A	0.018	0.001	0.002	0.491	1.015	0.101	0.242	0.061	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	61	61	54	53	53	60	53	53	0
N.S.	1	4.36	4.36	3.86	3.79	3.79	4.29	3.79	3.79	0.00
time (sec)	N/A	0.014	0.000	0.001	0.458	0.845	0.074	0.292	0.024	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	46	46	49	12	48	0
N.S.	1	1.00	1.00	0.93	3.29	3.29	3.50	0.86	3.43	0.00
time (sec)	N/A	0.018	0.004	0.006	0.690	0.773	0.288	0.354	0.047	0.001

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	101	101	109	12	103	0
N.S.	1	1.00	1.00	0.93	7.21	7.21	7.79	0.86	7.36	0.00
time (sec)	N/A	0.017	0.004	0.005	0.602	1.169	0.593	0.276	2.105	0.001

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	156	156	168	12	158	0
N.S.	1	1.00	1.00	0.93	11.14	11.14	12.00	0.86	11.29	0.00
time (sec)	N/A	0.018	0.005	0.005	0.544	1.128	0.936	0.355	3.033	0.001

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	31	30	30	29	31	36	0
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.76	0.82	0.95	0.00
time (sec)	N/A	0.025	0.008	0.009	1.113	1.004	0.154	0.295	2.165	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	65	64	64	80	64	64	0
N.S.	1	1.00	1.00	0.77	0.76	0.76	0.95	0.76	0.76	0.00
time (sec)	N/A	0.086	0.002	0.000	0.488	1.068	0.077	0.380	2.166	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	63	50	49	49	60	49	49	0
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78	0.00
time (sec)	N/A	0.076	0.002	0.002	0.727	1.102	0.070	0.365	0.053	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	35	34	34	41	34	34	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77	0.00
time (sec)	N/A	0.069	0.001	0.003	0.749	0.995	0.064	0.362	0.024	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	22	19	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76	0.00
time (sec)	N/A	0.003	0.000	0.002	0.639	1.003	0.057	0.279	0.032	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	62	42	54	56	63	62	27	0
N.S.	1	1.00	2.00	1.35	1.74	1.81	2.03	2.00	0.87	0.00
time (sec)	N/A	0.023	0.018	0.011	1.055	1.251	0.154	0.282	0.066	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	103	84	89	177	104	97	64	0
N.S.	1	1.00	1.16	0.94	1.00	1.99	1.17	1.09	0.72	0.00
time (sec)	N/A	0.061	0.063	0.023	1.289	1.190	1.359	0.278	0.084	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	137	126	119	282	134	112	93	0
N.S.	1	1.00	0.85	0.78	0.74	1.75	0.83	0.70	0.58	0.00
time (sec)	N/A	0.119	0.092	0.023	1.320	1.301	1.453	0.308	0.089	0.001

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	205	132	116	114	223	296	134	126	0
N.S.	1	2.25	1.45	1.27	1.25	2.45	3.25	1.47	1.38	0.00
time (sec)	N/A	0.130	0.103	0.022	1.271	1.316	1.426	0.435	2.184	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	73	127	81	198	209	77	87	0
N.S.	1	1.00	0.94	1.63	1.04	2.54	2.68	0.99	1.12	0.00
time (sec)	N/A	0.063	0.067	0.012	1.458	1.199	0.694	0.341	2.269	0.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	54	89	61	157	153	54	206	0
N.S.	1	1.00	1.08	1.78	1.22	3.14	3.06	1.08	4.12	0.00
time (sec)	N/A	0.039	0.033	0.007	1.648	1.226	0.462	0.308	0.090	0.001

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	38	54	50	136	124	43	46	0
N.S.	1	1.00	0.93	1.32	1.22	3.32	3.02	1.05	1.12	0.00
time (sec)	N/A	0.021	0.016	0.003	1.362	1.155	0.250	0.356	2.076	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	28	24	83	54	17	17	0
N.S.	1	1.00	1.00	1.33	1.14	3.95	2.57	0.81	0.81	0.00
time (sec)	N/A	0.008	0.004	0.003	1.520	0.881	0.186	0.377	0.043	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	48	72	68	154	738	62	173	0
N.S.	1	1.00	0.81	1.22	1.15	2.61	12.51	1.05	2.93	0.00
time (sec)	N/A	0.035	0.041	0.007	1.518	1.244	3.403	0.377	2.588	0.001

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	81	123	123	229	1620	117	425	0
N.S.	1	1.00	1.03	1.56	1.56	2.90	20.51	1.48	5.38	0.00
time (sec)	N/A	0.086	0.053	0.011	1.591	1.138	11.116	0.367	2.583	0.001

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	106	198	197	371	3284	195	573	0
N.S.	1	1.00	0.88	1.64	1.63	3.07	27.14	1.61	4.74	0.00
time (sec)	N/A	0.125	0.166	0.010	1.612	1.438	38.258	0.415	2.771	0.001

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	34	30	109	61	24	27	0
N.S.	1	1.00	1.00	1.10	0.97	3.52	1.97	0.77	0.87	0.00
time (sec)	N/A	0.024	0.012	0.006	1.561	1.273	0.207	0.453	0.057	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	60	86	75	253	117	65	76	0
N.S.	1	1.00	0.95	1.37	1.19	4.02	1.86	1.03	1.21	0.00
time (sec)	N/A	0.033	0.035	0.004	1.560	1.118	0.580	0.359	0.098	0.001

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	75	147	184	595	257	103	181	0
N.S.	1	1.00	0.82	1.62	2.02	6.54	2.82	1.13	1.99	0.00
time (sec)	N/A	0.048	0.071	0.005	1.585	1.257	1.248	0.409	2.221	0.001

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	42	66	279	92	30	31	0
N.S.	1	1.00	1.00	1.20	1.89	7.97	2.63	0.86	0.89	0.00
time (sec)	N/A	0.035	0.019	0.010	1.404	1.464	0.218	0.446	0.103	0.001

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	18	10	24	10	10	0
N.S.	1	1.00	1.00	1.10	1.80	1.00	2.40	1.00	1.00	0.00
time (sec)	N/A	0.003	0.005	0.003	1.569	1.447	0.168	0.318	0.040	0.001

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	31	59	51	55	56	41	42	0
N.S.	1	1.00	0.84	1.59	1.38	1.49	1.51	1.11	1.14	0.00
time (sec)	N/A	0.010	0.014	0.006	1.315	1.148	0.437	0.377	2.066	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	52	94	115	153	146	73	111	0
N.S.	1	1.00	0.87	1.57	1.92	2.55	2.43	1.22	1.85	0.00
time (sec)	N/A	0.016	0.020	0.006	1.505	0.900	0.931	0.304	0.121	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	32	26	25	22	22	27	10	0
N.S.	1	1.00	3.20	2.60	2.50	2.20	2.20	2.70	1.00	0.00
time (sec)	N/A	0.003	0.006	0.010	0.490	0.798	0.177	0.384	2.050	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	45	52	56	85	54	56	43	0
N.S.	1	1.00	1.15	1.33	1.44	2.18	1.38	1.44	1.10	0.00
time (sec)	N/A	0.013	0.026	0.010	0.624	0.929	0.469	0.351	2.064	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	65	78	122	220	141	88	114	0
N.S.	1	1.00	1.02	1.22	1.91	3.44	2.20	1.38	1.78	0.00
time (sec)	N/A	0.022	0.032	0.010	0.641	1.212	1.030	0.357	2.118	0.001

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	15	12	11	11	10	13	4	0
N.S.	1	1.00	3.75	3.00	2.75	2.75	2.50	3.25	1.00	0.00
time (sec)	N/A	0.002	0.002	0.005	0.571	1.282	0.099	0.355	0.149	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	24	25	39	24	27	23	0
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85	0.00
time (sec)	N/A	0.007	0.018	0.008	0.649	1.082	0.117	0.325	0.066	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	37	36	44	71	44	39	36	0
N.S.	1	1.00	0.82	0.80	0.98	1.58	0.98	0.87	0.80	0.00
time (sec)	N/A	0.012	0.018	0.010	0.644	0.822	0.144	0.339	2.090	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	64	64	54	54	58	62	55	0
N.S.	1	1.00	1.08	1.08	0.92	0.92	0.98	1.05	0.93	0.00
time (sec)	N/A	0.055	0.028	0.004	0.542	1.196	0.171	0.310	0.050	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	11	12	11	11	10	11	11	0
N.S.	1	1.00	1.10	1.20	1.10	1.10	1.00	1.10	1.10	0.00
time (sec)	N/A	0.011	0.007	0.001	0.678	1.221	0.086	0.353	0.032	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	51	35	36	35	0	23	-1	45
N.S.	1	1.00	1.16	0.80	0.82	0.80	0.00	0.52	-0.02	1.02
time (sec)	N/A	0.026	0.032	0.006	1.289	1.259	0.000	0.386	0.000	0.132

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	55	152	139	92	0	55	-1	171
N.S.	1	1.00	0.82	2.27	2.07	1.37	0.00	0.82	-0.01	2.55
time (sec)	N/A	0.049	0.086	0.020	1.462	0.933	0.000	0.486	0.000	0.454

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	51	146	135	70	0	70	-1	176
N.S.	1	1.00	0.81	2.32	2.14	1.11	0.00	1.11	-0.02	2.79
time (sec)	N/A	0.040	0.088	0.011	0.587	1.883	0.000	0.417	0.000	0.547

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	132	108	0	6315	238	0	374	0
N.S.	1	1.00	0.56	0.46	0.00	26.99	1.02	0.00	1.60	0.00
time (sec)	N/A	0.372	0.055	0.006	0.000	47.200	2.901	0.000	2.460	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	81	74	0	4759	158	0	437	0
N.S.	1	1.00	0.39	0.35	0.00	22.66	0.75	0.00	2.08	0.00
time (sec)	N/A	0.228	0.035	0.004	0.000	4.037	0.992	0.000	2.300	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	79	72	0	1950	83	0	145	0
N.S.	1	1.00	0.44	0.40	0.00	10.83	0.46	0.00	0.81	0.00
time (sec)	N/A	0.156	0.022	0.003	0.000	3.828	0.700	0.000	0.257	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	116	71	0	442	26	160	144	0
N.S.	1	1.00	0.83	0.51	0.00	3.16	0.19	1.14	1.03	0.00
time (sec)	N/A	0.107	0.033	0.002	0.000	1.576	0.261	0.452	2.311	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	238	119	105	0	4370	0	0	553	0
N.S.	1	1.06	0.53	0.47	0.00	19.51	0.00	0.00	2.47	0.00
time (sec)	N/A	0.482	0.055	0.010	0.000	3.965	0.000	0.000	0.123	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	314	312	173	144	0	8919	0	0	1588	0
N.S.	1	0.99	0.55	0.46	0.00	28.40	0.00	0.00	5.06	0.00
time (sec)	N/A	0.545	0.105	0.010	0.000	3.925	0.000	0.000	2.329	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	C	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	393	393	244	217	0	14765	0	0	1328	0
N.S.	1	1.00	0.62	0.55	0.00	37.57	0.00	0.00	3.38	0.00
time (sec)	N/A	0.604	0.170	0.012	0.000	12.243	0.000	0.000	2.489	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	356	356	106	97	0	0	374	0	1003	0
N.S.	1	1.00	0.30	0.27	0.00	0.00	1.05	0.00	2.82	0.00
time (sec)	N/A	0.426	0.055	0.016	0.000	0.000	3.736	0.000	2.692	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	106	97	0	0	274	0	625	0
N.S.	1	1.00	0.33	0.31	0.00	0.00	0.86	0.00	1.97	0.00
time (sec)	N/A	0.312	0.036	0.005	0.000	0.000	2.671	0.000	2.588	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	104	95	0	0	131	0	205	0
N.S.	1	1.00	0.40	0.36	0.00	0.00	0.50	0.00	0.79	0.00
time (sec)	N/A	0.264	0.034	0.007	0.000	0.000	0.879	0.000	2.357	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	161	94	0	189	26	103	60	0
N.S.	1	1.00	0.73	0.43	0.00	0.86	0.12	0.47	0.27	0.00
time (sec)	N/A	0.185	0.099	0.005	0.000	1.000	0.301	0.501	0.116	0.001

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	393	393	163	139	0	0	0	0	882	0
N.S.	1	1.00	0.41	0.35	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.466	0.073	0.008	0.000	0.000	0.000	0.000	2.183	0.001

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	496	496	238	188	0	0	0	0	2440	0
N.S.	1	1.00	0.48	0.38	0.00	0.00	0.00	0.00	4.92	0.00
time (sec)	N/A	0.894	0.132	0.013	0.000	0.000	0.000	0.000	2.479	0.001

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	195	264	192	219	199	219	175	0
N.S.	1	1.00	1.59	2.15	1.56	1.78	1.62	1.78	1.42	0.00
time (sec)	N/A	0.240	0.037	0.003	0.692	1.033	0.118	0.328	0.215	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	114	138	119	128	114	128	108	0
N.S.	1	1.00	0.95	1.15	0.99	1.07	0.95	1.07	0.90	0.00
time (sec)	N/A	0.063	0.020	0.000	0.667	1.075	0.090	0.296	0.095	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	66	63	65	65	65	65	61	0
N.S.	1	1.00	0.92	0.88	0.90	0.90	0.90	0.90	0.85	0.00
time (sec)	N/A	0.030	0.008	0.000	0.707	1.097	0.076	0.313	0.039	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	23	22	22	22	22	22	0
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.85	0.00
time (sec)	N/A	0.004	0.000	0.000	0.627	0.675	0.061	0.303	0.018	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	57	49	0	457	66	0	571	0
N.S.	1	1.00	0.64	0.55	0.00	5.13	0.74	0.00	6.42	0.00
time (sec)	N/A	0.087	0.023	0.016	0.000	0.883	0.933	0.000	2.576	0.001

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	150	158	0	1948	294	0	4591	0
N.S.	1	1.00	0.89	0.93	0.00	11.53	1.74	0.00	27.17	0.00
time (sec)	N/A	0.291	0.075	0.013	0.000	0.863	6.317	0.000	5.352	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	254	398	0	3971	697	0	8242	0
N.S.	1	1.00	1.01	1.58	0.00	15.76	2.77	0.00	32.71	0.00
time (sec)	N/A	0.532	0.150	0.023	0.000	1.667	15.616	0.000	6.405	0.001

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	204	267	182	222	212	222	178	0
N.S.	1	1.00	0.97	1.27	0.87	1.06	1.01	1.06	0.85	0.00
time (sec)	N/A	0.226	0.035	0.003	0.621	0.783	0.123	0.390	0.217	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	130	143	113	133	128	133	113	0
N.S.	1	1.00	0.97	1.07	0.84	0.99	0.96	0.99	0.84	0.00
time (sec)	N/A	0.140	0.020	0.002	0.624	0.982	0.098	0.351	2.120	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	75	66	59	68	70	68	64	0
N.S.	1	1.00	0.95	0.84	0.75	0.86	0.89	0.86	0.81	0.00
time (sec)	N/A	0.076	0.008	0.000	0.616	0.861	0.077	0.348	0.038	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	27	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.00
time (sec)	N/A	0.010	0.002	0.000	0.611	1.019	0.061	0.360	0.022	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	59	50	0	0	155	0	275	0
N.S.	1	1.00	0.51	0.43	0.00	0.00	1.34	0.00	2.37	0.00
time (sec)	N/A	0.083	0.017	0.004	0.000	0.000	4.430	0.000	2.582	0.001

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	166	162	0	0	539	0	1167	0
N.S.	1	1.00	0.72	0.70	0.00	0.00	2.33	0.00	5.05	0.00
time (sec)	N/A	0.241	0.072	0.012	0.000	0.000	31.420	0.000	2.817	0.001

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	284	405	0	0	1102	0	2200	0
N.S.	1	1.00	0.81	1.16	0.00	0.00	3.16	0.00	6.30	0.00
time (sec)	N/A	0.369	0.164	0.023	0.000	0.000	88.090	0.000	3.394	0.001

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	210	210	204	267	182	222	219	222	178	0
N.S.	1	1.00	0.97	1.27	0.87	1.06	1.04	1.06	0.85	0.00
time (sec)	N/A	0.164	0.036	0.001	0.639	1.246	0.145	0.243	2.294	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	132	143	113	133	134	133	113	0
N.S.	1	1.00	0.96	1.04	0.82	0.96	0.97	0.96	0.82	0.00
time (sec)	N/A	0.121	0.019	0.003	0.563	0.913	0.097	0.285	0.092	0.001

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	73	66	59	68	73	68	64	0
N.S.	1	1.00	0.92	0.84	0.75	0.86	0.92	0.86	0.81	0.00
time (sec)	N/A	0.077	0.010	0.001	0.546	0.935	0.077	0.287	0.042	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	29	27	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.00
time (sec)	N/A	0.009	0.004	0.000	0.609	1.060	0.062	0.313	0.021	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	61	52	0	0	172	0	878	0
N.S.	1	1.00	0.62	0.53	0.00	0.00	1.74	0.00	8.87	0.00
time (sec)	N/A	0.087	0.016	0.004	0.000	0.000	7.607	0.000	2.783	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	182	160	0	0	561	0	1218	0
N.S.	1	1.00	0.81	0.71	0.00	0.00	2.49	0.00	5.41	0.00
time (sec)	N/A	0.213	0.076	0.014	0.000	0.000	43.733	0.000	2.846	0.001

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	545	545	99	93	0	0	0	0	1563	0
N.S.	1	1.00	0.18	0.17	0.00	0.00	0.00	0.00	2.87	0.00
time (sec)	N/A	1.476	0.069	0.015	0.000	0.000	0.000	0.000	3.175	0.001

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	487	487	99	93	0	0	0	0	1354	0
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.756	0.049	0.003	0.000	0.000	0.000	0.000	3.104	0.001

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	334	97	93	0	0	0	0	825	0
N.S.	1	1.00	0.29	0.28	0.00	0.00	0.00	0.00	2.47	0.00
time (sec)	N/A	0.470	0.059	0.005	0.000	0.000	0.000	0.000	3.342	0.001

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	469	469	95	91	0	0	0	0	1057	0
N.S.	1	1.00	0.20	0.19	0.00	0.00	0.00	0.00	2.25	0.00
time (sec)	N/A	0.684	0.047	0.005	0.000	0.000	0.000	0.000	2.903	0.001

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	522	522	99	90	0	0	0	0	1394	0
N.S.	1	1.00	0.19	0.17	0.00	0.00	0.00	0.00	2.67	0.00
time (sec)	N/A	0.861	0.066	0.005	0.000	0.000	0.000	0.000	0.711	0.001

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	563	563	157	134	0	0	0	0	4002	0
N.S.	1	1.00	0.28	0.24	0.00	0.00	0.00	0.00	7.11	0.00
time (sec)	N/A	1.157	0.115	0.009	0.000	0.000	0.000	0.000	2.546	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	645	640	163	133	0	0	0	0	2663	0
N.S.	1	0.99	0.25	0.21	0.00	0.00	0.00	0.00	4.13	0.00
time (sec)	N/A	1.380	0.139	0.008	0.000	0.000	0.000	0.000	2.721	0.001

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	395	395	61	56	0	0	70	0	427	0
N.S.	1	1.00	0.15	0.14	0.00	0.00	0.18	0.00	1.08	0.00
time (sec)	N/A	1.438	0.023	0.008	0.000	0.000	0.257	0.000	0.651	0.001

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	61	56	0	0	65	0	390	0
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	1.03	0.00
time (sec)	N/A	0.910	0.014	0.009	0.000	0.000	0.275	0.000	2.704	0.001

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	61	56	0	0	61	0	276	0
N.S.	1	1.00	0.17	0.16	0.00	0.00	0.17	0.00	0.76	0.00
time (sec)	N/A	0.515	0.015	0.009	0.000	0.000	0.247	0.000	0.522	0.001

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	59	56	0	1277	48	0	247	0
N.S.	1	1.00	0.24	0.23	0.00	5.15	0.19	0.00	1.00	0.00
time (sec)	N/A	0.323	0.013	0.008	0.000	4.126	0.205	0.000	2.679	0.001

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	361	361	57	54	0	0	61	0	176	0
N.S.	1	1.00	0.16	0.15	0.00	0.00	0.17	0.00	0.49	0.00
time (sec)	N/A	0.548	0.016	0.008	0.000	0.000	0.257	0.000	2.423	0.001

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	377	377	62	53	0	0	65	0	306	0
N.S.	1	1.00	0.16	0.14	0.00	0.00	0.17	0.00	0.81	0.00
time (sec)	N/A	0.721	0.013	0.007	0.000	0.000	0.274	0.000	2.675	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	415	415	103	75	0	0	82	0	432	0
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.20	0.00	1.04	0.00
time (sec)	N/A	0.902	0.019	0.013	0.000	0.000	0.417	0.000	2.328	0.001

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	448	448	109	74	0	0	70	0	340	0
N.S.	1	1.00	0.24	0.17	0.00	0.00	0.16	0.00	0.76	0.00
time (sec)	N/A	1.102	0.021	0.011	0.000	0.000	0.319	0.000	0.292	0.001

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	1064	1064	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.16	0.11	0.00	0.00	0.11	0.00	0.36	0.00
time (sec)	N/A	2.504	0.040	0.015	0.000	0.000	0.397	0.000	2.341	0.001

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	1005	1005	167	122	0	0	112	0	387	0
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39	0.00
time (sec)	N/A	2.403	0.030	0.016	0.000	0.000	0.403	0.000	2.304	0.001

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	677	677	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.25	0.18	0.00	0.00	0.17	0.00	0.57	0.00
time (sec)	N/A	1.553	0.045	0.015	0.000	0.000	0.378	0.000	2.328	0.001

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	682	682	167	122	0	1445	104	0	299	0
N.S.	1	1.00	0.24	0.18	0.00	2.12	0.15	0.00	0.44	0.00
time (sec)	N/A	1.235	0.026	0.012	0.000	3.889	0.307	0.000	0.314	0.001

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	850	850	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.20	0.14	0.00	0.00	0.13	0.00	0.46	0.00
time (sec)	N/A	1.470	0.036	0.013	0.000	0.000	0.388	0.000	2.420	0.001

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	873	873	167	122	0	0	112	0	387	0
N.S.	1	1.00	0.19	0.14	0.00	0.00	0.13	0.00	0.44	0.00
time (sec)	N/A	1.916	0.028	0.013	0.000	0.000	0.397	0.000	2.416	0.001

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	986	986	167	122	0	0	112	0	388	0
N.S.	1	1.00	0.17	0.12	0.00	0.00	0.11	0.00	0.39	0.00
time (sec)	N/A	1.927	0.035	0.014	0.000	0.000	0.387	0.000	2.478	0.001

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	22	21	21	26
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84	1.04
time (sec)	N/A	0.055	0.002	0.000	0.443	0.794	0.094	0.370	0.037	0.030

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	79	114	105	105	88	365	106	0
N.S.	1	1.00	0.84	1.21	1.12	1.12	0.94	3.88	1.13	0.00
time (sec)	N/A	0.127	0.037	0.005	0.439	0.935	0.308	0.263	0.059	0.001

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	172	155	13	154	175	13	154	0
N.S.	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27	0.00
time (sec)	N/A	0.018	0.006	0.003	0.436	0.714	0.131	0.300	2.224	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	182	157	156	156	182	156	156	0
N.S.	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75	0.00
time (sec)	N/A	0.055	0.006	0.002	0.608	1.241	0.136	0.313	0.143	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	186	157	156	156	185	156	156	0
N.S.	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75	0.00
time (sec)	N/A	0.050	0.006	0.002	0.552	1.173	0.139	0.332	2.171	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	230	229	262	0	189	229	21
N.S.	1	1.00	1.00	10.95	10.90	12.48	0.00	9.00	10.90	1.00
time (sec)	N/A	0.031	0.118	0.052	0.645	0.979	0.000	1.857	4.019	0.051

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	9	9	10	10	8	11	8	0
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80	0.00
time (sec)	N/A	0.004	0.004	0.000	0.555	0.726	0.127	0.230	0.050	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	18	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.87	0.00
time (sec)	N/A	0.026	0.006	0.004	0.644	0.767	0.179	0.334	2.081	0.001

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	15	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.00	0.87	0.00
time (sec)	N/A	0.027	0.005	0.006	0.525	0.956	0.197	0.314	0.060	0.001

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	47	23	29	0	-1	24
N.S.	1	1.00	1.00	1.20	3.13	1.53	1.93	0.00	-0.07	1.60
time (sec)	N/A	0.025	0.011	0.018	0.577	0.889	1.483	0.000	0.000	0.042

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	14	177	13	81	87	13	12	0
N.S.	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80	0.00
time (sec)	N/A	0.004	0.021	0.018	0.528	0.943	0.933	0.270	4.377	0.001

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.023	0.029	0.019	0.670	0.911	1.428	0.406	2.225	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	197	81	81	87	15	14	0
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88	0.00
time (sec)	N/A	0.023	0.036	0.016	0.800	1.125	2.061	0.304	7.220	0.001

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	203	612	143	0	0	-1	21
N.S.	1	1.00	1.00	9.67	29.14	6.81	0.00	0.00	-0.05	1.00
time (sec)	N/A	0.032	0.178	0.064	1.034	1.732	0.000	0.000	0.000	0.115

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	17	24	19	26	46	19	23	0
N.S.	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21	0.00
time (sec)	N/A	0.005	0.009	0.005	0.559	1.067	0.710	0.374	2.095	0.001

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	97	31	35	32	0	54	45	0
N.S.	1	1.00	3.59	1.15	1.30	1.19	0.00	2.00	1.67	0.00
time (sec)	N/A	0.023	0.075	0.004	0.990	1.231	0.000	0.334	2.213	0.073

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	A	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	116	97	142	35	33	0	54	-1	0
N.S.	1	4.30	3.59	5.26	1.30	1.22	0.00	2.00	-0.04	0.00
time (sec)	N/A	0.102	0.027	0.259	1.073	0.827	0.000	0.494	0.000	0.265

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	99	33	35	34	0	58	49	0
N.S.	1	1.00	3.41	1.14	1.21	1.17	0.00	2.00	1.69	0.00
time (sec)	N/A	0.024	0.077	0.005	1.013	1.078	0.000	0.346	2.205	0.077

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	108	0	39	42	0	0	-1	0
N.S.	1	1.00	3.00	0.00	1.08	1.17	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.086	0.166	0.135	1.072	0.850	0.000	0.000	0.000	0.178

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	26	29	26	26	29
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.91
time (sec)	N/A	0.034	0.002	0.001	0.781	0.934	0.095	0.313	2.142	0.041

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	14
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	1.17
time (sec)	N/A	0.049	0.001	0.001	0.472	0.928	0.095	0.396	0.019	0.041

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	32	31	98	124	31	32	0
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.76	0.00
time (sec)	N/A	0.057	0.015	0.005	1.489	0.807	0.297	0.404	2.130	0.001

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	26	25	38	0	25	54	0
N.S.	1	1.00	0.92	1.04	1.00	1.52	0.00	1.00	2.16	0.00
time (sec)	N/A	0.024	0.015	0.003	0.672	1.018	0.000	0.294	2.191	0.032

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	21	34	24	36	0	24	46	0
N.S.	1	1.00	0.88	1.42	1.00	1.50	0.00	1.00	1.92	0.00
time (sec)	N/A	0.018	0.047	0.005	0.629	0.859	0.000	0.307	2.124	0.088

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	26	39	35	0	65	51	0
N.S.	1	1.00	0.96	1.04	1.56	1.40	0.00	2.60	2.04	0.00
time (sec)	N/A	0.021	0.022	0.006	0.862	1.018	0.000	0.395	2.155	0.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	20	28	0	20	39	0
N.S.	1	1.00	1.00	1.05	1.00	1.40	0.00	1.00	1.95	0.00
time (sec)	N/A	0.010	0.008	0.004	0.651	0.954	0.000	0.282	2.143	0.027

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	106	26	19	26	73	19	25	0
N.S.	1	1.00	5.58	1.37	1.00	1.37	3.84	1.00	1.32	0.00
time (sec)	N/A	0.009	0.066	0.005	0.534	1.379	11.344	0.341	2.127	0.069

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	108	23	31	27	76	39	26	0
N.S.	1	1.00	4.91	1.05	1.41	1.23	3.45	1.77	1.18	0.00
time (sec)	N/A	0.008	0.036	0.003	1.209	1.030	52.838	0.338	2.161	0.001

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	23	22	32	0	22	43	0
N.S.	1	1.00	0.95	1.05	1.00	1.45	0.00	1.00	1.95	0.00
time (sec)	N/A	0.018	0.012	0.004	0.661	0.987	0.000	0.345	2.138	0.030

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	28	21	30	53	21	27	0
N.S.	1	1.00	0.90	1.33	1.00	1.43	2.52	1.00	1.29	0.00
time (sec)	N/A	0.012	0.011	0.004	0.594	0.912	1.160	0.310	2.162	0.084

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	28	32	31	56	51	28	0
N.S.	1	1.00	0.92	1.17	1.33	1.29	2.33	2.12	1.17	0.00
time (sec)	N/A	0.023	0.014	0.003	0.826	0.792	6.170	0.460	2.199	0.098

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	23	32	29	53	41	26	0
N.S.	1	1.00	1.00	1.05	1.45	1.32	2.41	1.86	1.18	0.00
time (sec)	N/A	0.009	0.010	0.005	0.895	0.970	6.154	0.314	2.188	0.064

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	23	32	32	0	22	43	0
N.S.	1	1.00	0.95	1.05	1.45	1.45	0.00	1.00	1.95	0.00
time (sec)	N/A	0.013	0.006	0.004	0.751	1.072	0.000	0.323	2.142	0.036

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	28	32	30	53	21	27	0
N.S.	1	1.00	0.90	1.33	1.52	1.43	2.52	1.00	1.29	0.00
time (sec)	N/A	0.012	0.005	0.003	0.930	0.663	1.142	0.398	2.148	0.041

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	143	25686	19	1956	1771	160	1576	0
N.S.	1	1.00	6.81	1223.14	0.90	93.14	84.33	7.62	75.05	0.00
time (sec)	N/A	0.125	0.151	0.004	0.740	0.570	0.420	0.299	2.979	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	5596	18	496	469	18	418	0
N.S.	1	1.00	0.90	279.80	0.90	24.80	23.45	0.90	20.90	0.00
time (sec)	N/A	0.052	0.036	0.002	0.633	0.618	0.182	0.314	2.320	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	18	5596	441	496	469	18	418	0
N.S.	1	1.00	0.95	294.53	23.21	26.11	24.68	0.95	22.00	0.00
time (sec)	N/A	0.069	0.017	0.002	0.601	0.802	0.170	0.297	2.267	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	127	2185	14	486	483	120	438	0
N.S.	1	1.00	7.94	136.56	0.88	30.38	30.19	7.50	27.38	0.00
time (sec)	N/A	0.024	0.056	0.001	0.549	0.944	0.170	0.413	2.628	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	98	89	13	88	97	13	88	0
N.S.	1	1.00	6.53	5.93	0.87	5.87	6.47	0.87	5.87	0.00
time (sec)	N/A	0.013	0.003	0.000	0.511	0.713	0.096	0.301	0.050	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	98	89	88	88	97	13	88	0
N.S.	1	1.00	6.12	5.56	5.50	5.50	6.06	0.81	5.50	0.00
time (sec)	N/A	0.028	0.003	0.000	0.645	0.801	0.088	0.268	0.040	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	115	2205	16	488	484	136	440	0
N.S.	1	1.00	6.39	122.50	0.89	27.11	26.89	7.56	24.44	0.00
time (sec)	N/A	0.044	0.053	0.003	0.623	0.639	0.173	0.430	2.632	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	98	89	15	88	97	15	88	0
N.S.	1	1.00	5.76	5.24	0.88	5.18	5.71	0.88	5.18	0.00
time (sec)	N/A	0.025	0.003	0.001	0.661	0.946	0.098	0.373	2.072	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	15	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29	0.00
time (sec)	N/A	0.227	0.003	0.001	0.525	0.545	0.095	0.234	0.047	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	15	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	1.07	6.29	0.00
time (sec)	N/A	0.011	0.003	0.002	0.659	0.731	0.088	0.256	0.041	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	115	2205	458	488	484	488	440	0
N.S.	1	1.00	6.39	122.50	25.44	27.11	26.89	27.11	24.44	0.00
time (sec)	N/A	0.060	0.011	0.002	0.595	0.628	0.169	0.317	0.568	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	88	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29	0.00
time (sec)	N/A	0.022	0.003	0.001	0.598	1.037	0.094	0.306	0.040	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	98	89	88	88	97	88	88	0
N.S.	1	1.00	5.44	4.94	4.89	4.89	5.39	4.89	4.89	0.00
time (sec)	N/A	0.019	0.003	0.002	0.570	1.011	0.096	0.294	0.040	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	98	89	88	88	97	88	88	0
N.S.	1	1.00	7.00	6.36	6.29	6.29	6.93	6.29	6.29	0.00
time (sec)	N/A	0.002	0.002	0.000	0.670	1.071	0.088	0.243	0.039	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	80	67	66	66	70	24	66	0
N.S.	1	1.00	2.86	2.39	2.36	2.36	2.50	0.86	2.36	0.00
time (sec)	N/A	0.027	0.005	0.002	0.610	1.523	0.086	0.366	0.051	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	108	325	187	208	194	88	180	0
N.S.	1	1.00	3.48	10.48	6.03	6.71	6.26	2.84	5.81	0.00
time (sec)	N/A	0.031	0.035	0.001	0.438	0.946	0.114	0.366	0.105	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	31	52	48	230	30	31	0
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.76	0.88	0.91	0.00
time (sec)	N/A	0.008	0.059	0.004	1.133	0.633	50.746	0.403	2.121	0.283

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	33	54	52	0	32	58	0
N.S.	1	1.00	1.00	0.94	1.54	1.49	0.00	0.91	1.66	0.00
time (sec)	N/A	0.009	0.056	0.003	1.159	1.048	0.000	0.251	2.110	0.296

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	93	78	77	77	87	24	77	0
N.S.	1	1.00	3.10	2.60	2.57	2.57	2.90	0.80	2.57	0.00
time (sec)	N/A	0.020	0.006	0.002	0.438	0.967	0.095	0.220	0.055	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	140	618	280	291	314	105	266	0
N.S.	1	1.00	4.52	19.94	9.03	9.39	10.13	3.39	8.58	0.00
time (sec)	N/A	0.038	0.046	0.002	0.455	0.822	0.143	0.336	2.266	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	98	81	80	80	90	30	80	0
N.S.	1	1.00	2.88	2.38	2.35	2.35	2.65	0.88	2.35	0.00
time (sec)	N/A	0.033	0.008	0.002	0.439	0.644	0.098	0.294	0.065	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	146	646	289	298	321	126	273	0
N.S.	1	1.00	3.56	15.76	7.05	7.27	7.83	3.07	6.66	0.00
time (sec)	N/A	0.045	0.051	0.001	0.446	0.922	0.150	0.312	2.283	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	244	1523	289	309	323	37	270	0
N.S.	1	1.00	5.30	33.11	6.28	6.72	7.02	0.80	5.87	0.00
time (sec)	N/A	0.047	0.063	0.002	0.453	0.934	0.157	0.307	2.237	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	248	4284	773	928	930	153	753	0
N.S.	1	1.00	5.28	91.15	16.45	19.74	19.79	3.26	16.02	0.00
time (sec)	N/A	0.091	0.123	0.002	0.480	0.956	0.269	0.442	2.450	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	36	31	54	48	201	30	33	0
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.91	0.88	0.97	0.00
time (sec)	N/A	0.009	0.068	0.004	1.143	1.030	112.002	0.417	2.111	0.299

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	42	37	71	57	0	36	37	0
N.S.	1	1.00	0.95	0.84	1.61	1.30	0.00	0.82	0.84	0.00
time (sec)	N/A	0.010	0.086	0.003	1.173	1.053	0.000	0.392	2.210	0.347

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	49	43	83	72	0	42	73	0
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	1.46	0.00
time (sec)	N/A	0.009	0.193	0.003	1.218	1.161	0.000	0.391	2.170	0.375

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	33	30	17	29	29	30	29	0
N.S.	1	1.00	1.74	1.58	0.89	1.53	1.53	1.58	1.53	0.00
time (sec)	N/A	0.009	0.002	0.001	0.434	0.744	0.061	0.346	0.025	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	21	18	14	17	17	22	17	0
N.S.	1	1.00	1.31	1.12	0.88	1.06	1.06	1.38	1.06	0.00
time (sec)	N/A	0.007	0.002	0.000	0.439	0.936	0.057	0.233	0.034	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	96	96	87	86	86	94	28	86	0
N.S.	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61	0.00
time (sec)	N/A	0.198	0.006	0.001	0.437	0.728	0.082	0.288	0.218	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	96	96	87	86	86	94	28	86	0
N.S.	1	2.91	2.91	2.64	2.61	2.61	2.85	0.85	2.61	0.00
time (sec)	N/A	0.154	0.005	0.001	0.449	0.662	0.091	0.360	0.191	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	57	56	12	12	0
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	0.86	0.00
time (sec)	N/A	0.008	0.006	0.001	0.436	0.837	0.189	0.316	2.100	0.001

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87	0.00
time (sec)	N/A	0.009	0.004	0.002	0.442	1.020	0.089	0.371	0.049	0.001

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	20	14	15	15	14	18	13	0
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.76	0.00
time (sec)	N/A	0.010	0.006	0.003	0.448	0.770	0.099	0.270	0.066	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	23	28	23	23	22	24	23	0
N.S.	1	1.00	0.58	0.70	0.58	0.58	0.55	0.60	0.58	0.00
time (sec)	N/A	0.097	0.059	0.013	0.503	0.997	32.564	0.536	0.108	0.001

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	605	605	98	2105	0	0	0	0	-1	0
N.S.	1	1.00	0.16	3.48	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	4.535	0.069	0.088	0.000	0.000	0.000	0.000	0.000	0.001

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	55	82	0	83	58	58	75	0
N.S.	1	1.00	0.87	1.30	0.00	1.32	0.92	0.92	1.19	0.00
time (sec)	N/A	0.066	0.029	0.049	0.000	1.870	0.126	0.274	0.185	0.001

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	22	38	20	13	12	0
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	0.86	0.00
time (sec)	N/A	0.050	0.008	0.007	0.436	1.166	0.096	0.289	0.040	0.001

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	24	27	32	46	29	23	21	0
N.S.	1	1.00	0.86	0.96	1.14	1.64	1.04	0.82	0.75	0.00
time (sec)	N/A	0.028	0.013	0.006	0.446	1.104	0.109	0.392	0.037	0.001

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	28	30	38	38	36	28	28	0
N.S.	1	1.00	0.47	0.51	0.64	0.64	0.61	0.47	0.47	0.00
time (sec)	N/A	0.091	0.012	0.006	0.463	1.867	0.181	1.436	0.054	0.001

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	41	11	11	8	11	11	0
N.S.	1	1.00	1.00	3.73	1.00	1.00	0.73	1.00	1.00	0.00
time (sec)	N/A	0.007	0.007	0.009	0.438	1.221	0.117	0.331	2.301	0.001

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	205	132	116	114	223	272	134	124	0
N.S.	1	2.25	1.45	1.27	1.25	2.45	2.99	1.47	1.36	0.00
time (sec)	N/A	0.149	0.090	0.019	0.987	2.227	1.362	0.372	2.369	0.001

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	23	26	44	40	0	99	49	0
N.S.	1	1.00	0.92	1.04	1.76	1.60	0.00	3.96	1.96	0.00
time (sec)	N/A	0.025	0.375	0.010	0.699	2.024	0.000	2.090	2.656	1.127

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	39	39	0	89	39	0
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70	0.00
time (sec)	N/A	0.082	0.233	0.010	0.656	1.806	0.000	1.887	2.549	0.959

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	39	39	0	89	39	0
N.S.	1	1.00	0.91	1.04	1.70	1.70	0.00	3.87	1.70	0.00
time (sec)	N/A	0.059	0.177	0.008	0.629	0.803	0.000	0.728	2.319	0.738

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	19	22	37	37	0	87	37	0
N.S.	1	1.00	0.90	1.05	1.76	1.76	0.00	4.14	1.76	0.00
time (sec)	N/A	0.044	0.122	0.005	0.608	1.255	0.000	1.083	2.273	0.557

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	17	20	33	33	0	52	19	0
N.S.	1	1.00	0.89	1.05	1.74	1.74	0.00	2.74	1.00	0.00
time (sec)	N/A	0.043	0.010	0.006	0.621	0.816	0.000	0.296	2.193	0.077

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.034	0.176	0.009	0.619	1.277	0.000	0.000	3.202	0.464

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.035	0.199	0.008	0.636	1.083	0.000	0.000	3.319	0.532

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	21	24	36	36	0	0	23	0
N.S.	1	1.00	0.91	1.04	1.57	1.57	0.00	0.00	1.00	0.00
time (sec)	N/A	0.036	0.188	0.008	0.642	1.502	0.000	0.000	3.355	0.663

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	83	74	73	79	97	73	97	0
N.S.	1	1.00	0.86	0.76	0.75	0.81	1.00	0.75	1.00	0.00
time (sec)	N/A	0.138	0.036	0.007	1.520	1.341	0.245	0.309	0.194	0.001

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	78	69	68	74	92	68	92	0
N.S.	1	1.00	0.87	0.77	0.76	0.82	1.02	0.76	1.02	0.00
time (sec)	N/A	0.123	0.023	0.006	1.880	1.225	0.247	0.378	0.178	0.001

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	72	62	61	67	78	61	85	0
N.S.	1	1.00	0.94	0.81	0.79	0.87	1.01	0.79	1.10	0.00
time (sec)	N/A	0.121	0.031	0.005	1.441	0.828	0.232	0.391	2.290	0.001

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	69	57	56	62	75	56	80	0
N.S.	1	1.00	0.96	0.79	0.78	0.86	1.04	0.78	1.11	0.00
time (sec)	N/A	0.093	0.023	0.005	1.586	1.290	0.232	0.307	2.292	0.001

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	65	56	55	61	75	55	79	0
N.S.	1	1.00	0.92	0.79	0.77	0.86	1.06	0.77	1.11	0.00
time (sec)	N/A	0.078	0.017	0.006	1.316	1.331	0.225	0.287	0.147	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	69	60	59	65	78	60	83	0
N.S.	1	1.00	0.92	0.80	0.79	0.87	1.04	0.80	1.11	0.00
time (sec)	N/A	0.135	0.019	0.007	1.571	1.164	0.288	0.307	0.149	0.001

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	78	65	64	76	87	65	88	0
N.S.	1	1.00	0.93	0.77	0.76	0.90	1.04	0.77	1.05	0.00
time (sec)	N/A	0.152	0.033	0.009	1.441	1.160	0.315	0.257	2.280	0.001

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	82	70	69	89	94	70	92	0
N.S.	1	1.00	0.90	0.77	0.76	0.98	1.03	0.77	1.01	0.00
time (sec)	N/A	0.157	0.055	0.010	1.264	1.130	0.321	0.286	0.152	0.001

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	307	307	109	74	0	1202	61	0	128	0
N.S.	1	1.00	0.36	0.24	0.00	3.92	0.20	0.00	0.42	0.00
time (sec)	N/A	0.578	0.021	0.008	0.000	3.804	0.985	0.000	2.169	0.001

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	101	67	0	1145	3662	0	188	0
N.S.	1	1.00	0.38	0.25	0.00	4.26	13.61	0.00	0.70	0.00
time (sec)	N/A	0.392	0.016	0.007	0.000	4.347	2.732	0.000	0.134	0.001

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	94	62	0	1190	48	0	183	0
N.S.	1	1.00	0.41	0.27	0.00	5.17	0.21	0.00	0.80	0.00
time (sec)	N/A	0.357	0.015	0.008	0.000	3.808	0.950	0.000	0.193	0.001

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	90	58	0	1189	46	0	181	0
N.S.	1	1.00	0.45	0.29	0.00	6.01	0.23	0.00	0.91	0.00
time (sec)	N/A	0.194	0.013	0.007	0.000	5.343	0.914	0.000	2.342	0.001

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	101	67	0	1143	60	0	237	0
N.S.	1	1.00	0.41	0.27	0.00	4.67	0.24	0.00	0.97	0.00
time (sec)	N/A	0.473	0.020	0.011	0.000	6.124	12.657	0.000	2.342	0.001

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	109	72	0	1245	0	0	242	0
N.S.	1	1.00	0.39	0.26	0.00	4.43	0.00	0.00	0.86	0.00
time (sec)	N/A	0.467	0.018	0.013	0.000	6.702	0.000	0.000	2.302	0.001

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	116	77	0	1274	70	0	246	0
N.S.	1	1.00	0.37	0.24	0.00	4.02	0.22	0.00	0.78	0.00
time (sec)	N/A	0.539	0.019	0.012	0.000	5.633	2.705	0.000	2.248	0.001

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	87	75	0	83	44	87	252	0
N.S.	1	1.00	4.58	3.95	0.00	4.37	2.32	4.58	13.26	0.00
time (sec)	N/A	0.105	0.047	0.095	0.000	2.113	1.045	4.080	2.270	0.001

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	57	34	33	47	37	34	38	0
N.S.	1	1.00	1.33	0.79	0.77	1.09	0.86	0.79	0.88	0.00
time (sec)	N/A	0.067	0.023	0.009	2.104	0.878	0.170	0.254	0.052	0.001

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	18	21	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.06	1.24	0.00
time (sec)	N/A	0.037	0.005	0.008	1.116	1.152	0.131	0.311	2.200	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	21	20	24	30	0
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.96	1.20	0.00
time (sec)	N/A	0.037	0.006	0.006	1.147	1.376	0.132	0.254	0.054	0.001

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	20	19	18	24	14	28	18	0
N.S.	1	1.00	0.91	0.86	0.82	1.09	0.64	1.27	0.82	0.00
time (sec)	N/A	0.013	0.010	0.007	1.095	1.427	0.111	0.366	0.036	0.001

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	38	22	21	21	26	22	25	0
N.S.	1	1.00	1.41	0.81	0.78	0.78	0.96	0.81	0.93	0.00
time (sec)	N/A	0.039	0.011	0.007	2.309	1.516	0.144	0.286	2.260	0.001

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	17	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.81	0.00
time (sec)	N/A	0.015	0.004	0.001	2.281	1.489	0.082	0.270	0.031	0.001

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	24	23	23	22	23	23	0
N.S.	1	1.00	1.00	0.89	0.85	0.85	0.81	0.85	0.85	0.00
time (sec)	N/A	0.026	0.007	0.005	2.077	1.445	0.109	0.278	2.140	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	26	25	25	26	29	25	0
N.S.	1	1.00	1.00	0.67	0.64	0.64	0.67	0.74	0.64	0.00
time (sec)	N/A	0.020	0.018	0.009	1.273	1.128	0.241	0.317	2.141	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	23	19	18	18	17	19	18	0
N.S.	1	1.00	1.05	0.86	0.82	0.82	0.77	0.86	0.82	0.00
time (sec)	N/A	0.015	0.004	0.002	1.133	1.015	0.075	0.281	0.027	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.87	0.00
time (sec)	N/A	0.029	0.005	0.003	2.301	0.998	0.091	0.341	2.131	0.001

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	24	21	20	26	17	22	10	0
N.S.	1	1.00	2.00	1.75	1.67	2.17	1.42	1.83	0.83	0.00
time (sec)	N/A	0.021	0.011	0.008	1.078	1.078	0.098	0.297	0.072	0.001

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	22	21	22	19	23	19	0
N.S.	1	1.00	1.00	0.88	0.84	0.88	0.76	0.92	0.76	0.00
time (sec)	N/A	0.036	0.006	0.007	1.129	1.193	0.102	0.269	2.139	0.001

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.00
time (sec)	N/A	0.028	0.006	0.005	2.157	1.216	0.114	0.309	0.038	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	31	35	55	36	30	35	0
N.S.	1	1.00	1.00	0.89	1.00	1.57	1.03	0.86	1.00	0.00
time (sec)	N/A	0.034	0.017	0.007	2.223	1.613	0.147	0.329	2.125	0.001

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	20	17	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.87	0.74	0.00
time (sec)	N/A	0.037	0.005	0.009	1.405	2.088	0.138	0.239	2.188	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	19	20	19	0
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.83	0.87	0.83	0.00
time (sec)	N/A	0.054	0.007	0.006	1.115	1.490	0.109	0.295	0.055	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	25	24	23	32	20	23	25	0
N.S.	1	1.00	0.86	0.83	0.79	1.10	0.69	0.79	0.86	0.00
time (sec)	N/A	0.014	0.010	0.006	2.335	1.160	0.119	0.374	2.125	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	42	41	58	49	43	55	0
N.S.	1	1.00	1.00	0.95	0.93	1.32	1.11	0.98	1.25	0.00
time (sec)	N/A	0.243	0.024	0.010	2.248	1.413	0.212	0.363	0.129	0.001

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	41	38	38	51	38	88	0
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91	0.00
time (sec)	N/A	0.133	0.020	0.005	2.287	1.702	0.213	0.304	0.160	0.001

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	34	44	72	41	44	56	0
N.S.	1	1.00	1.00	1.03	1.33	2.18	1.24	1.33	1.70	0.00
time (sec)	N/A	0.166	0.022	0.012	1.156	2.043	0.186	0.301	2.162	0.001

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.59	0.65	0.65	0.00
time (sec)	N/A	0.007	0.006	0.007	2.200	1.770	0.141	0.288	0.032	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	21	20	20	34	20	20	0
N.S.	1	1.00	0.92	0.88	0.83	0.83	1.42	0.83	0.83	0.00
time (sec)	N/A	0.018	0.009	0.003	2.255	1.296	0.166	0.288	2.134	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	23	18	17	17	17	20	19	0
N.S.	1	1.00	1.53	1.20	1.13	1.13	1.13	1.33	1.27	0.00
time (sec)	N/A	0.060	0.006	0.008	2.313	1.415	0.137	0.319	0.056	0.001

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	18	17	17	19	17	17	0
N.S.	1	1.00	1.00	0.90	0.85	0.85	0.95	0.85	0.85	0.00
time (sec)	N/A	0.012	0.009	0.007	2.398	1.819	0.148	0.375	0.050	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	27	44	29	60	35	0
N.S.	1	1.00	0.89	0.76	0.73	1.19	0.78	1.62	0.95	0.00
time (sec)	N/A	0.040	0.023	0.007	2.224	1.390	0.164	0.287	2.144	0.001

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	22	21	21	26	21	21	0
N.S.	1	1.00	1.00	0.85	0.81	0.81	1.00	0.81	0.81	0.00
time (sec)	N/A	0.011	0.008	0.002	2.216	1.494	0.094	0.309	0.030	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	16	10	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.33	0.83	0.92	0.83	0.00
time (sec)	N/A	0.032	0.004	0.006	1.105	1.643	0.095	0.287	2.125	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	20	21	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.95	1.00	0.00
time (sec)	N/A	0.030	0.007	0.008	1.092	1.418	0.132	0.282	2.144	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	34	17	20	20	0
N.S.	1	1.00	1.00	0.95	0.91	1.55	0.77	0.91	0.91	0.00
time (sec)	N/A	0.015	0.008	0.005	2.231	1.671	0.112	0.282	2.124	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	21	20	32	19	20	22	0
N.S.	1	1.00	1.00	0.88	0.83	1.33	0.79	0.83	0.92	0.00
time (sec)	N/A	0.016	0.011	0.007	2.176	1.436	0.116	0.405	0.029	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	34	33	33	39	33	56	0
N.S.	1	1.00	1.00	0.94	0.92	0.92	1.08	0.92	1.56	0.00
time (sec)	N/A	0.026	0.015	0.005	2.057	1.427	0.188	0.245	0.103	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	28	27	27	29	27	37	0
N.S.	1	1.00	1.00	0.76	0.73	0.73	0.78	0.73	1.00	0.00
time (sec)	N/A	0.024	0.008	0.003	2.003	1.033	0.174	0.285	2.136	0.001

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	21	22	23	21	0
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72	0.00
time (sec)	N/A	0.017	0.006	0.006	1.083	1.190	0.109	0.367	2.214	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	14	16	15	0
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.84	0.79	0.00
time (sec)	N/A	0.013	0.004	0.002	1.066	1.232	0.081	0.266	0.034	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	35	34	34	46	34	36	0
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.12	0.83	0.88	0.00
time (sec)	N/A	0.028	0.014	0.005	2.178	1.527	0.118	0.288	0.043	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	39	32	31	31	34	31	31	0
N.S.	1	1.00	0.95	0.78	0.76	0.76	0.83	0.76	0.76	0.00
time (sec)	N/A	0.028	0.007	0.004	2.074	1.759	0.118	0.278	2.116	0.001

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	24	21	20	20	24	23	20	0
N.S.	1	1.00	0.80	0.70	0.67	0.67	0.80	0.77	0.67	0.00
time (sec)	N/A	0.058	0.011	0.007	1.038	0.690	0.152	0.250	2.125	0.001

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	27	27	31	30	27	0
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.89	0.86	0.77	0.00
time (sec)	N/A	0.040	0.008	0.008	1.118	1.243	0.149	0.324	0.044	0.001

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	32	25	24	34	27	34	26	0
N.S.	1	1.00	0.94	0.74	0.71	1.00	0.79	1.00	0.76	0.00
time (sec)	N/A	0.057	0.014	0.008	1.133	1.573	0.145	0.387	2.109	0.001

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	35	33	44	36	33	39	0
N.S.	1	1.00	1.00	0.83	0.79	1.05	0.86	0.79	0.93	0.00
time (sec)	N/A	0.021	0.021	0.006	2.355	0.915	0.131	0.297	2.187	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	38	37	37	46	37	41	0
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.94	0.76	0.84	0.00
time (sec)	N/A	0.157	0.013	0.006	2.803	1.424	0.227	0.315	0.072	0.001

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	20	19	19	24	22	19	0
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.83	0.76	0.66	0.00
time (sec)	N/A	0.053	0.007	0.008	1.166	1.608	0.142	0.275	2.182	0.001

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	43	34	33	33	41	34	50	0
N.S.	1	1.00	0.93	0.74	0.72	0.72	0.89	0.74	1.09	0.00
time (sec)	N/A	0.044	0.016	0.006	2.161	1.557	0.144	0.317	0.086	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	14	15	14	17	10	15	14	0
N.S.	1	1.00	0.88	0.94	0.88	1.06	0.62	0.94	0.88	0.00
time (sec)	N/A	0.025	0.007	0.004	1.085	0.800	0.093	0.397	0.043	0.001

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	14	16	17	17	14	12	12	0
N.S.	1	1.00	0.67	0.76	0.81	0.81	0.67	0.57	0.57	0.00
time (sec)	N/A	0.019	0.003	0.004	1.088	1.289	0.087	0.301	2.085	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	23	23	17	30	15	0
N.S.	1	1.00	1.00	1.07	1.53	1.53	1.13	2.00	1.00	0.00
time (sec)	N/A	0.027	0.011	0.005	1.268	1.298	0.114	0.262	2.089	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	29	28	28	3	29	57	0
N.S.	1	1.00	1.00	0.94	0.90	0.90	0.10	0.94	1.84	0.00
time (sec)	N/A	0.042	0.012	0.005	2.357	1.257	0.130	0.278	0.112	0.001

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	22	19	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.88	0.76	0.00
time (sec)	N/A	0.042	0.006	0.007	1.088	1.071	0.141	0.287	0.060	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	25	24	36	20	26	22	0
N.S.	1	1.00	0.97	0.83	0.80	1.20	0.67	0.87	0.73	0.00
time (sec)	N/A	0.030	0.015	0.006	1.212	1.138	0.098	0.276	2.108	0.001

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	18	21	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91	0.00
time (sec)	N/A	0.038	0.005	0.005	2.075	1.037	0.137	0.331	2.121	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	93	73	77	136	88	74	96	0
N.S.	1	1.00	0.90	0.71	0.75	1.32	0.85	0.72	0.93	0.00
time (sec)	N/A	0.476	0.043	0.013	2.248	0.979	0.515	0.344	2.200	0.001

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	30	28	25	36	24	25	32	0
N.S.	1	1.00	0.91	0.85	0.76	1.09	0.73	0.76	0.97	0.00
time (sec)	N/A	0.017	0.011	0.005	2.149	1.682	0.127	0.317	0.035	0.001

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	28	29	44	27	30	33	0
N.S.	1	1.00	1.00	0.85	0.88	1.33	0.82	0.91	1.00	0.00
time (sec)	N/A	0.041	0.019	0.008	2.254	1.541	0.149	0.379	2.113	0.001

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	23	19	17	26	19	0
N.S.	1	1.00	1.00	0.96	0.92	0.76	0.68	1.04	0.76	0.00
time (sec)	N/A	0.041	0.005	0.007	1.046	1.278	0.096	0.291	0.043	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	32	31	31	36	31	56	0
N.S.	1	1.00	1.00	0.89	0.86	0.86	1.00	0.86	1.56	0.00
time (sec)	N/A	0.115	0.015	0.004	2.096	1.585	0.202	0.336	0.106	0.001

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	33	22	21	23	0
N.S.	1	1.00	1.00	0.76	0.72	1.14	0.76	0.72	0.79	0.00
time (sec)	N/A	0.107	0.017	0.009	2.089	1.057	0.174	0.227	0.042	0.001

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	36	35	39	46	36	49	0
N.S.	1	1.00	1.00	0.78	0.76	0.85	1.00	0.78	1.07	0.00
time (sec)	N/A	0.058	0.026	0.008	2.093	1.520	0.162	0.296	2.169	0.001

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	26	26	19	18	18	17	20	14	0
N.S.	1	1.18	1.18	0.86	0.82	0.82	0.77	0.91	0.64	0.00
time (sec)	N/A	0.015	0.006	0.005	0.974	1.608	0.098	0.284	0.039	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	18	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.039	0.006	0.007	0.850	1.265	0.136	0.381	0.065	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	18	14	16	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.29	1.00	1.14	1.00	0.00
time (sec)	N/A	0.025	0.004	0.007	0.999	0.958	0.115	0.373	2.120	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	25	18	17	17	17	20	17	0
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89	0.00
time (sec)	N/A	0.027	0.007	0.007	1.029	1.342	0.131	0.292	2.116	0.001

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	19	17	33	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.83	0.74	1.43	0.00
time (sec)	N/A	0.029	0.009	0.006	2.122	1.086	0.183	0.364	0.054	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	57	51	50	50	68	53	58	0
N.S.	1	1.00	0.90	0.81	0.79	0.79	1.08	0.84	0.92	0.00
time (sec)	N/A	0.088	0.027	0.010	2.112	0.903	0.346	0.306	2.213	0.001

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	86	67	54	59	103	65	59	71	0
N.S.	1	1.25	0.97	0.78	0.86	1.49	0.94	0.86	1.03	0.00
time (sec)	N/A	0.099	0.047	0.014	2.070	1.242	0.213	0.236	2.177	0.001

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	19	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	1.12	0.71	0.76	0.76	0.00
time (sec)	N/A	0.013	0.006	0.006	2.043	1.624	0.106	0.302	0.033	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	18	17	17	15	17	17	0
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.79	0.89	0.89	0.00
time (sec)	N/A	0.027	0.005	0.004	2.030	0.809	0.101	0.269	2.098	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	7	7	28	19	0
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	3.11	2.11	0.00
time (sec)	N/A	0.075	0.006	0.005	2.166	0.971	0.138	0.292	2.104	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	12	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.016	0.005	0.002	1.375	1.055	0.087	0.295	0.028	0.001

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	41	51	66	99	55	48	0
N.S.	1	1.00	0.89	0.63	0.78	1.02	1.52	0.85	0.74	0.00
time (sec)	N/A	0.062	0.036	0.008	1.771	1.513	0.448	0.295	0.099	0.001

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	46	24	28	28	0
N.S.	1	1.00	1.00	1.04	1.00	1.64	0.86	1.00	1.00	0.00
time (sec)	N/A	0.022	0.011	0.006	1.656	0.994	0.131	0.285	0.038	0.001

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	26	29	26	26	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.040	0.023	0.004	1.515	0.613	0.103	0.374	0.025	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	41	39	24	23	23	26	27	23	0
N.S.	1	1.32	1.26	0.77	0.74	0.74	0.84	0.87	0.74	0.00
time (sec)	N/A	0.049	0.007	0.008	0.797	1.403	0.194	0.308	0.050	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	22	26	19	23	30	0
N.S.	1	1.00	1.00	0.96	0.92	1.08	0.79	0.96	1.25	0.00
time (sec)	N/A	0.167	0.009	0.006	1.223	1.266	0.155	0.292	2.129	0.001

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	24	35	20	19	23	0
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00	0.00
time (sec)	N/A	0.025	0.011	0.006	1.431	1.095	0.130	0.391	0.033	0.001

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	19	24	35	20	19	23	0
N.S.	1	1.00	1.00	0.83	1.04	1.52	0.87	0.83	1.00	0.00
time (sec)	N/A	0.039	0.006	0.005	1.416	1.877	0.127	0.274	0.030	0.001

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	15	10	13	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.15	0.77	1.00	1.00	0.00
time (sec)	N/A	0.049	0.005	0.005	0.708	2.057	0.100	0.283	2.144	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	193	236	199	4545	138	208	357	0
N.S.	1	1.00	0.94	1.15	0.97	22.06	0.67	1.01	1.73	0.00
time (sec)	N/A	0.274	0.107	0.008	1.652	5.144	1.238	0.442	0.231	0.001

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	41	0	47	44	38	42	0
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.93	0.00
time (sec)	N/A	0.067	0.025	0.009	0.000	1.121	0.154	1.109	2.191	0.001

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	44	41	0	75	46	42	47	0
N.S.	1	1.00	0.75	0.69	0.00	1.27	0.78	0.71	0.80	0.00
time (sec)	N/A	0.064	0.021	0.010	0.000	0.892	0.184	1.803	0.049	0.001

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	234	616	0	0	0	1587	3942	0
N.S.	1	1.00	1.12	2.95	0.00	0.00	0.00	7.59	18.86	0.00
time (sec)	N/A	0.371	0.255	0.061	0.000	0.000	0.000	4.697	3.437	0.001

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	245	633	0	0	0	1625	3046	0
N.S.	1	1.00	1.09	2.83	0.00	0.00	0.00	7.25	13.60	0.00
time (sec)	N/A	0.389	0.248	0.060	0.000	0.000	0.000	4.563	3.220	0.001

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	57	60	65	190	62	61	0
N.S.	1	1.00	1.00	1.02	1.07	1.16	3.39	1.11	1.09	0.00
time (sec)	N/A	0.047	0.027	0.007	0.685	1.196	1.065	0.313	0.213	0.001

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	73	87	84	162	0	85	347	0
N.S.	1	1.00	0.76	0.91	0.88	1.69	0.00	0.89	3.61	0.00
time (sec)	N/A	0.108	0.035	0.008	1.222	1.231	0.000	0.257	1.131	0.001

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	228	336	279	5975	0	320	570	0
N.S.	1	1.00	0.86	1.27	1.06	22.63	0.00	1.21	2.16	0.00
time (sec)	N/A	0.472	0.090	0.007	1.706	4.868	0.000	0.302	2.496	0.001

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	417	417	370	422	349	0	0	401	823	0
N.S.	1	1.00	0.89	1.01	0.84	0.00	0.00	0.96	1.97	0.00
time (sec)	N/A	0.547	0.230	0.014	1.481	0.000	0.000	0.567	2.449	0.001

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	24	21	20	26	19	21	12	0
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	0.75	0.00
time (sec)	N/A	0.008	0.009	0.007	0.753	1.298	0.105	0.239	0.039	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	27	24	23	34	20	30	17	0
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	0.89	0.00
time (sec)	N/A	0.013	0.011	0.009	0.651	0.652	0.118	0.303	2.138	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	85	90	75	106	92	77	103	0
N.S.	1	1.00	0.88	0.93	0.77	1.09	0.95	0.79	1.06	0.00
time (sec)	N/A	0.071	0.050	0.012	1.934	1.189	0.361	0.314	0.177	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.87	0.87	0.00
time (sec)	N/A	0.106	0.008	0.005	1.400	1.063	0.120	0.289	2.120	0.001

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	11	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.85	0.85	0.00
time (sec)	N/A	0.094	0.007	0.004	1.327	1.053	0.117	0.298	0.035	0.001

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	25	24	24	29	24	51	0
N.S.	1	1.00	1.00	0.86	0.83	0.83	1.00	0.83	1.76	0.00
time (sec)	N/A	0.118	0.014	0.004	1.479	1.234	0.193	0.259	2.153	0.001

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	17	10	14	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.21	0.71	1.00	1.00	0.00
time (sec)	N/A	0.011	0.008	0.004	2.074	1.148	0.138	0.253	0.042	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	12	7	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.00	0.58	0.92	0.83	0.00
time (sec)	N/A	0.011	0.003	0.004	0.752	1.226	0.091	0.418	0.037	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	10	17	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	1.55	0.00
time (sec)	N/A	0.034	0.005	0.006	1.608	1.130	0.131	0.292	2.166	0.001

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	13	8	11	10	0
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	0.83	0.00
time (sec)	N/A	0.021	0.003	0.005	0.714	1.136	0.091	0.384	0.039	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	12	12	14	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00	0.00
time (sec)	N/A	0.018	0.003	0.005	0.596	0.932	0.105	0.229	0.047	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	16	23	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	1.35	0.00
time (sec)	N/A	0.037	0.005	0.005	1.532	1.257	0.133	0.243	2.284	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	10	14	19	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.77	1.08	1.46	0.00
time (sec)	N/A	0.035	0.008	0.006	1.410	1.177	0.132	0.343	0.060	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	22	21	20	26	20	22	20	0
N.S.	1	1.00	0.79	0.75	0.71	0.93	0.71	0.79	0.71	0.00
time (sec)	N/A	0.011	0.015	0.007	0.662	1.102	0.112	0.297	0.069	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	26	37	26	43	22	0
N.S.	1	1.00	1.00	0.84	0.81	1.16	0.81	1.34	0.69	0.00
time (sec)	N/A	0.026	0.017	0.009	0.746	1.157	0.134	0.298	2.226	0.001

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	28	20	19	19	19	20	25	0
N.S.	1	1.00	1.22	0.87	0.83	0.83	0.83	0.87	1.09	0.00
time (sec)	N/A	0.034	0.008	0.005	1.558	1.191	0.136	0.310	0.049	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	21	20	36	20	47	28	0
N.S.	1	1.00	0.92	0.88	0.83	1.50	0.83	1.96	1.17	0.00
time (sec)	N/A	0.035	0.014	0.007	1.600	1.352	0.141	0.303	2.111	0.001

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	49	40	39	37	44	39	41	0
N.S.	1	1.00	1.00	0.82	0.80	0.76	0.90	0.80	0.84	0.00
time (sec)	N/A	0.142	0.014	0.007	1.708	1.303	0.224	0.283	2.156	0.001

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	19	20	19	19	19	22	19	0
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76	0.00
time (sec)	N/A	0.055	0.008	0.006	1.067	1.135	0.145	0.392	2.135	0.001

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	54	48	47	60	63	60	61	0
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.02	0.00
time (sec)	N/A	0.246	0.053	0.009	2.798	1.599	0.252	0.343	0.130	0.001

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	5	9	6	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.55	0.82
time (sec)	N/A	0.007	0.000	0.000	0.845	1.165	0.060	0.227	0.016	0.018

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	22	21	21	22	23	21	0
N.S.	1	1.00	1.00	0.76	0.72	0.72	0.76	0.79	0.72	0.00
time (sec)	N/A	0.016	0.005	0.005	1.096	1.136	0.108	0.355	2.124	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	37	37	52	37	37	49	0
N.S.	1	1.00	1.00	0.82	0.82	1.16	0.82	0.82	1.09	0.00
time (sec)	N/A	0.025	0.013	0.007	1.839	0.745	0.137	0.299	0.041	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	29	28	52	29	29	44	0
N.S.	1	1.00	1.00	0.91	0.88	1.62	0.91	0.91	1.38	0.00
time (sec)	N/A	0.254	0.019	0.010	2.025	1.220	0.263	0.325	0.069	0.001

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	159	110	132	178	146	122	124	0
N.S.	1	1.00	1.07	0.74	0.89	1.20	0.99	0.82	0.84	0.00
time (sec)	N/A	0.136	0.068	0.011	2.009	1.287	0.429	0.388	2.189	0.001

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	101	79	99	515	61	92	170	0
N.S.	1	1.00	0.90	0.71	0.88	4.60	0.54	0.82	1.52	0.00
time (sec)	N/A	0.114	0.049	0.007	2.058	4.324	0.965	0.390	2.228	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	14	12	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	1.00	0.86	0.00
time (sec)	N/A	0.016	0.003	0.005	0.907	1.348	0.103	0.262	0.044	0.001

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	12	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83	0.00
time (sec)	N/A	0.029	0.005	0.004	1.005	1.563	0.098	0.372	2.138	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	21	14	17	15	0
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	0.88	0.00
time (sec)	N/A	0.030	0.004	0.007	0.892	1.183	0.108	0.319	0.027	0.001

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	14	17	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88	0.00
time (sec)	N/A	0.029	0.004	0.005	0.900	1.005	0.100	0.260	0.033	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	25	15	23	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.39	0.83	1.28	0.89	0.00
time (sec)	N/A	0.030	0.004	0.006	1.921	1.017	0.105	0.267	0.050	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	14	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.00	0.00
time (sec)	N/A	0.019	0.005	0.005	0.892	1.029	0.086	0.361	0.034	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	31	30	30	36	33	30	0
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71	0.00
time (sec)	N/A	0.040	0.007	0.008	0.878	0.928	0.151	0.287	0.053	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	41	38	38	51	38	88	0
N.S.	1	1.00	1.00	0.89	0.83	0.83	1.11	0.83	1.91	0.00
time (sec)	N/A	0.125	0.017	0.002	1.972	1.154	0.212	0.355	0.002	0.001

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	14	17	8	0
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.89	0.42	0.00
time (sec)	N/A	0.062	0.003	0.007	1.026	1.083	0.109	0.281	0.084	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	31	30	30	34	32	26	0
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65	0.00
time (sec)	N/A	0.021	0.005	0.007	0.995	1.087	0.119	0.284	2.093	0.001

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	26	25	25	27	27	21	0
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64	0.00
time (sec)	N/A	0.020	0.004	0.006	1.020	0.929	0.117	0.343	0.033	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	20	20	20	22	16	0
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62	0.00
time (sec)	N/A	0.015	0.005	0.006	1.027	1.465	0.117	0.270	2.117	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	17	19	13	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62	0.00
time (sec)	N/A	0.007	0.003	0.003	1.173	0.935	0.109	0.286	2.110	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	15	19	8	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.38	0.00
time (sec)	N/A	0.010	0.003	0.005	0.926	1.104	0.110	0.275	0.078	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	22	21	21	24	24	17	0
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63	0.00
time (sec)	N/A	0.016	0.004	0.008	0.916	0.826	0.148	0.373	0.095	0.001

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	27	26	30	31	29	22	0
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65	0.00
time (sec)	N/A	0.030	0.004	0.008	1.062	0.967	0.162	0.266	0.035	0.001

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	32	31	39	36	34	26	0
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63	0.00
time (sec)	N/A	0.034	0.005	0.008	1.015	1.124	0.170	0.276	0.036	0.001

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	37	36	44	41	39	32	0
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67	0.00
time (sec)	N/A	0.041	0.005	0.009	1.071	1.018	0.182	0.238	0.037	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	61	54	0	1506	41	0	144	0
N.S.	1	1.00	0.39	0.34	0.00	9.59	0.26	0.00	0.92	0.00
time (sec)	N/A	0.168	0.016	0.014	0.000	3.453	0.225	0.000	2.745	0.001

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	61	56	0	1546	41	0	142	0
N.S.	1	1.00	0.39	0.36	0.00	9.85	0.26	0.00	0.90	0.00
time (sec)	N/A	0.112	0.017	0.015	0.000	4.054	0.227	0.000	2.805	0.001

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	61	54	0	2271	39	0	142	0
N.S.	1	1.00	0.32	0.29	0.00	12.08	0.21	0.00	0.76	0.00
time (sec)	N/A	0.276	0.013	0.008	0.000	4.129	0.211	0.000	2.780	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	61	56	0	2259	39	0	142	0
N.S.	1	1.00	0.32	0.30	0.00	12.02	0.21	0.00	0.76	0.00
time (sec)	N/A	0.156	0.013	0.010	0.000	4.000	0.212	0.000	2.740	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	663	663	63	69	0	0	133	0	328	0
N.S.	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49	0.00
time (sec)	N/A	1.108	0.037	0.074	0.000	0.000	3.924	0.000	2.699	0.001

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	F(-1)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	663	663	63	69	0	0	133	0	328	0
N.S.	1	1.00	0.10	0.10	0.00	0.00	0.20	0.00	0.49	0.00
time (sec)	N/A	0.648	0.031	0.002	0.000	0.000	3.920	0.000	0.002	0.001

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	C	C	F	C	A	F	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	0	95	67	0	5653	42	0	504	0
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	3.00	0.00
time (sec)	N/A	0.380	0.061	0.278	0.000	4.553	1.866	0.000	3.082	0.001

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	322	314	310	0	384	311	894	0
N.S.	1	1.00	1.01	0.98	0.97	0.00	1.20	0.97	2.79	0.00
time (sec)	N/A	0.256	0.249	0.011	1.956	0.000	5.289	0.361	2.843	0.001

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	243	292	275	0	277	285	556	0
N.S.	1	1.00	0.84	1.00	0.95	0.00	0.95	0.98	1.91	0.00
time (sec)	N/A	0.198	0.100	0.003	2.047	0.000	2.678	0.336	2.657	0.001

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	184	151	207	0	124	215	160	0
N.S.	1	1.00	0.84	0.69	0.95	0.00	0.57	0.98	0.73	0.00
time (sec)	N/A	0.174	0.059	0.002	2.666	0.000	0.823	0.316	2.323	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	134	128	169	121	20	179	33	0
N.S.	1	1.00	0.72	0.69	0.91	0.65	0.11	0.97	0.18	0.00
time (sec)	N/A	0.098	0.018	0.003	2.367	1.073	0.161	0.325	0.083	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	416	416	404	433	345	0	0	371	874	0
N.S.	1	1.00	0.97	1.04	0.83	0.00	0.00	0.89	2.10	0.00
time (sec)	N/A	0.428	0.149	0.007	2.727	0.000	0.000	0.422	0.418	0.001

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	552	552	524	866	561	0	0	646	2436	0
N.S.	1	1.00	0.95	1.57	1.02	0.00	0.00	1.17	4.41	0.00
time (sec)	N/A	0.808	0.606	0.010	2.109	0.000	0.000	2.780	2.779	0.001

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	680	680	738	1201	817	0	0	901	1955	0
N.S.	1	1.00	1.09	1.77	1.20	0.00	0.00	1.32	2.88	0.00
time (sec)	N/A	0.950	0.929	0.012	2.342	0.000	0.000	0.892	3.674	0.001

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	347	390	332	0	350	342	670	0
N.S.	1	1.00	0.99	1.12	0.95	0.00	1.00	0.98	1.92	0.00
time (sec)	N/A	0.298	0.374	0.004	2.200	0.000	8.334	0.342	0.433	0.001

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	321	362	318	0	318	323	391	0
N.S.	1	1.00	1.00	1.12	0.99	0.00	0.99	1.00	1.21	0.00
time (sec)	N/A	0.269	0.316	0.007	2.630	0.000	3.521	0.364	2.483	0.001

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	224	188	238	0	155	241	282	0
N.S.	1	1.00	0.93	0.78	0.99	0.00	0.64	1.00	1.17	0.00
time (sec)	N/A	0.189	0.189	0.005	2.433	0.000	1.092	0.339	0.274	0.001

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	183	143	189	173	39	194	58	0
N.S.	1	1.00	0.91	0.71	0.94	0.86	0.19	0.96	0.29	0.00
time (sec)	N/A	0.116	0.107	0.005	2.108	1.162	0.297	0.246	0.089	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	855	855	558	1122	601	0	0	771	1591	0
N.S.	1	1.00	0.65	1.31	0.70	0.00	0.00	0.90	1.86	0.00
time (sec)	N/A	0.849	0.391	0.019	2.291	0.000	0.000	0.504	2.995	0.001

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1141	1141	807	1636	961	0	0	1104	2246	0
N.S.	1	1.00	0.71	1.43	0.84	0.00	0.00	0.97	1.97	0.00
time (sec)	N/A	1.659	0.865	0.023	2.541	0.000	0.000	103.636	4.104	0.001

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1384	1384	996	2121	1394	0	0	1488	3256	0
N.S.	1	1.00	0.72	1.53	1.01	0.00	0.00	1.08	2.35	0.00
time (sec)	N/A	1.963	1.357	0.026	2.602	0.000	0.000	1.148	5.038	0.001

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	394	394	388	470	392	0	413	389	721	0
N.S.	1	1.00	0.98	1.19	0.99	0.00	1.05	0.99	1.83	0.00
time (sec)	N/A	0.346	0.351	0.007	2.373	0.000	8.013	0.354	0.478	0.001

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	360	360	358	419	364	0	374	356	676	0
N.S.	1	1.00	0.99	1.16	1.01	0.00	1.04	0.99	1.88	0.00
time (sec)	N/A	0.326	0.294	0.007	2.181	0.000	4.878	0.433	0.471	0.001

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	249	222	269	0	192	260	315	0
N.S.	1	1.00	0.94	0.83	1.01	0.00	0.72	0.98	1.18	0.00
time (sec)	N/A	0.249	0.206	0.007	2.204	0.000	1.483	0.320	0.303	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	200	158	212	232	63	204	80	0
N.S.	1	1.00	0.91	0.72	0.97	1.06	0.29	0.93	0.37	0.00
time (sec)	N/A	0.142	0.083	0.005	2.704	1.459	0.461	0.358	0.096	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1352	1352	835	2098	1015	0	0	1259	2720	0
N.S.	1	1.00	0.62	1.55	0.75	0.00	0.00	0.93	2.01	0.00
time (sec)	N/A	1.412	0.738	0.025	2.408	0.000	0.000	0.910	4.406	0.001

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1830	1830	1115	2769	1564	0	0	0	3572	0
N.S.	1	1.00	0.61	1.51	0.85	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	2.781	1.551	0.033	3.097	0.000	0.000	0.000	5.754	0.001

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2204	2204	1338	3334	2198	0	0	2119	6280	0
N.S.	1	1.00	0.61	1.51	1.00	0.00	0.00	0.96	2.85	0.00
time (sec)	N/A	3.165	2.873	0.038	3.988	0.000	0.000	1.534	7.785	0.001

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	33	29	28	28	34	28	30	0
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94	0.00
time (sec)	N/A	0.019	0.010	0.002	2.532	1.534	0.112	0.295	0.041	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	33	29	28	28	34	28	30	0
N.S.	1	1.00	1.03	0.91	0.88	0.88	1.06	0.88	0.94	0.00
time (sec)	N/A	0.029	0.006	0.002	2.530	1.387	0.115	0.380	0.035	0.001

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	29	26	26	36	26	30	0
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94	0.00
time (sec)	N/A	0.017	0.009	0.003	2.156	1.652	0.116	0.388	0.043	0.001

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	29	26	26	36	26	30	0
N.S.	1	1.00	0.97	0.91	0.81	0.81	1.12	0.81	0.94	0.00
time (sec)	N/A	0.035	0.005	0.003	2.329	1.658	0.120	0.335	0.032	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	29	35	45	39	44	34	0
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76	0.00
time (sec)	N/A	0.012	0.022	0.003	2.621	1.613	0.111	0.415	2.319	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	42	29	35	45	39	44	34	0
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.76	0.00
time (sec)	N/A	0.021	0.005	0.003	2.332	1.406	0.118	0.315	0.047	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	23	18	17	17	15	15	6	0
N.S.	1	1.00	3.83	3.00	2.83	2.83	2.50	2.50	1.00	0.00
time (sec)	N/A	0.002	0.003	0.005	1.061	1.393	0.094	0.310	2.270	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	23	18	17	17	15	19	6	0
N.S.	1	1.00	1.10	0.86	0.81	0.81	0.71	0.90	0.29	0.00
time (sec)	N/A	0.004	0.003	0.002	0.942	1.162	0.098	0.416	0.146	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	10	9	24	22	9	9	0
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	0.69	0.00
time (sec)	N/A	0.002	0.003	0.001	0.882	1.346	0.125	0.335	2.319	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	81	11	58	57	24	22	57	9	0
N.S.	1	6.23	0.85	4.46	4.38	1.85	1.69	4.38	0.69	0.00
time (sec)	N/A	0.012	0.002	0.002	0.937	1.104	0.314	0.312	0.025	0.001

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	78	67	66	66	85	68	94	0
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36	0.00
time (sec)	N/A	0.111	0.014	0.009	2.014	1.257	0.250	0.293	0.102	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	78	67	66	66	85	68	94	0
N.S.	1	1.00	1.13	0.97	0.96	0.96	1.23	0.99	1.36	0.00
time (sec)	N/A	0.124	0.006	0.003	1.990	1.359	0.254	0.307	0.039	0.001

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	31	21	20	20	20	21	20	0
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83	0.00
time (sec)	N/A	0.019	0.006	0.003	0.631	1.482	0.078	0.307	0.037	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	21	20	20	19	21	20	0
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77	0.00
time (sec)	N/A	0.016	0.004	0.002	0.686	1.351	0.073	0.295	0.033	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	22	16	15	18	15	15	17	0
N.S.	1	1.00	1.29	0.94	0.88	1.06	0.88	0.88	1.00	0.00
time (sec)	N/A	0.006	0.000	0.000	0.664	0.992	0.057	0.388	0.024	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	22	20	20	18	15	20	17	0
N.S.	1	1.00	0.92	0.83	0.83	0.75	0.62	0.83	0.71	0.00
time (sec)	N/A	0.005	0.001	0.001	0.568	1.000	0.058	0.368	0.021	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	24	23	22	25	20	23	28	0
N.S.	1	1.00	1.09	1.05	1.00	1.14	0.91	1.05	1.27	0.00
time (sec)	N/A	0.019	0.006	0.007	1.378	0.958	0.132	0.393	0.045	0.001

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	27	16	15	15	15	18	15	0
N.S.	1	1.00	1.59	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.013	0.004	0.007	0.755	1.408	0.130	0.327	0.057	0.001

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	13	12	16	13	0
N.S.	1	1.00	1.00	0.74	0.68	0.68	0.63	0.84	0.68	0.00
time (sec)	N/A	0.021	0.004	0.006	0.659	1.279	0.096	0.305	2.336	0.001

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	17	11	10	10	8	13	10	0
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	0.83	0.00
time (sec)	N/A	0.006	0.004	0.001	0.621	1.396	0.089	0.398	2.269	0.001

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	11	11	10	10	8	11	10	0
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.00	0.00
time (sec)	N/A	0.009	0.006	0.002	0.610	1.160	0.132	0.332	0.056	0.001

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	15	15	15	18	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88	0.00
time (sec)	N/A	0.018	0.006	0.005	0.614	0.994	0.125	0.356	0.056	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	17	17	18	21	0
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.91	0.00
time (sec)	N/A	0.021	0.004	0.005	1.640	1.391	0.128	0.376	0.049	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	13	10	13	13	0
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	0.87	0.00
time (sec)	N/A	0.009	0.004	0.000	0.655	1.272	0.092	0.278	0.042	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	17	17	20	20	17	0
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	0.95	0.81	0.00
time (sec)	N/A	0.021	0.005	0.006	0.586	1.534	0.131	0.369	0.067	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	9	14	9	9	7	9	10	0
N.S.	1	1.00	0.90	1.40	0.90	0.90	0.70	0.90	1.00	0.00
time (sec)	N/A	0.010	0.005	0.003	0.781	1.190	0.083	0.261	2.204	0.001

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	18	17	17	20	20	17	0
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.80	0.80	0.68	0.00
time (sec)	N/A	0.021	0.005	0.006	0.646	1.421	0.132	0.362	0.107	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	14	12	14	14	0
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00	0.00
time (sec)	N/A	0.025	0.004	0.001	1.557	1.728	0.105	0.252	2.219	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	14	30	11	16	17	11	20	0
N.S.	1	1.00	1.08	2.31	0.85	1.23	1.31	0.85	1.54	0.00
time (sec)	N/A	0.007	0.007	0.010	0.582	1.448	0.130	0.371	2.245	0.001

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	32	26	25	20	27	25	0
N.S.	1	1.00	0.96	1.23	1.00	0.96	0.77	1.04	0.96	0.00
time (sec)	N/A	0.045	0.008	0.003	0.717	1.513	0.154	0.283	2.234	0.001

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	4	5	4	4	3	5	4	0
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67	0.00
time (sec)	N/A	0.008	0.001	0.001	0.570	1.491	0.064	0.280	0.018	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	20	25	19	27	18	0
N.S.	1	1.00	1.00	0.95	1.00	1.25	0.95	1.35	0.90	0.00
time (sec)	N/A	0.015	0.005	0.007	0.718	1.653	0.105	0.381	0.039	0.001

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	40	28	27	27	31	28	44	0
N.S.	1	1.00	1.05	0.74	0.71	0.71	0.82	0.74	1.16	0.00
time (sec)	N/A	0.025	0.010	0.005	1.973	1.270	0.141	0.389	0.158	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	18	16	15	15	14	16	15	0
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.82	0.94	0.88	0.00
time (sec)	N/A	0.008	0.004	0.002	0.440	1.158	0.073	0.367	0.031	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	36	22	21	21	22	22	25	0
N.S.	1	1.00	1.16	0.71	0.68	0.68	0.71	0.71	0.81	0.00
time (sec)	N/A	0.012	0.005	0.005	0.995	1.455	0.141	0.274	0.053	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	17	15	17	17	13	0
N.S.	1	1.00	1.00	0.84	0.89	0.79	0.89	0.89	0.68	0.00
time (sec)	N/A	0.017	0.006	0.006	0.474	1.301	0.114	0.408	0.090	0.001

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	9	9	9	9	8	11	9	0
N.S.	1	1.00	0.82	0.82	0.82	0.82	0.73	1.00	0.82	0.00
time (sec)	N/A	0.005	0.004	0.000	0.447	1.330	0.088	0.287	2.217	0.001

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	13	12	12	14	12	12	0
N.S.	1	1.00	1.00	0.72	0.67	0.67	0.78	0.67	0.67	0.00
time (sec)	N/A	0.014	0.009	0.006	1.008	1.476	0.143	0.371	2.216	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	44	39	46	83	46	52	45	0
N.S.	1	1.00	0.96	0.85	1.00	1.80	1.00	1.13	0.98	0.00
time (sec)	N/A	0.023	0.019	0.010	0.448	1.321	0.195	0.363	0.042	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	10	14	8	8	7	9	8	0
N.S.	1	1.00	0.83	1.17	0.67	0.67	0.58	0.75	0.67	0.00
time (sec)	N/A	0.002	0.001	0.002	0.438	1.394	0.076	0.275	0.042	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	27	28	23	34	20	25	17	0
N.S.	1	1.00	1.29	1.33	1.10	1.62	0.95	1.19	0.81	0.00
time (sec)	N/A	0.003	0.008	0.008	0.451	1.292	0.108	0.271	2.216	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	21	12	15	17	0
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	0.89	0.00
time (sec)	N/A	0.004	0.008	0.006	0.996	1.045	0.101	0.290	0.027	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	7	9	6	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.90	0.60	0.00
time (sec)	N/A	0.001	0.001	0.001	0.453	1.393	0.058	0.382	0.068	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	20	10	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00	0.00
time (sec)	N/A	0.003	0.002	0.003	0.995	1.350	0.112	0.261	2.245	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	16	15	67	53	15	16	0
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67	0.00
time (sec)	N/A	0.007	0.004	0.002	0.982	1.205	0.134	0.379	2.243	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	17	16	16	26	16	16	0
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.84	0.00
time (sec)	N/A	0.011	0.006	0.002	0.955	1.440	0.108	0.380	0.032	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	17	16	17	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77	0.00
time (sec)	N/A	0.007	0.001	0.000	0.460	1.130	0.057	0.310	0.024	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	15	16	17	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77	0.00
time (sec)	N/A	0.005	0.001	0.000	0.456	1.119	0.056	0.287	0.029	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	15	10	14	15	0
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94	0.00
time (sec)	N/A	0.005	0.001	0.003	0.456	1.303	0.069	0.306	0.028	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	39	30	30	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.05	0.81	0.81	0.00
time (sec)	N/A	0.021	0.010	0.005	0.976	1.259	0.118	0.266	2.219	0.001

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	13	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93	0.00
time (sec)	N/A	0.004	0.000	0.001	0.466	0.969	0.054	0.364	0.024	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	7	9	6	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.64	0.82	0.55	0.82
time (sec)	N/A	0.004	0.000	0.001	0.450	1.281	0.058	0.286	0.016	0.014

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	20	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.80	0.80	1.00	0.00
time (sec)	N/A	0.010	0.006	0.005	0.994	1.444	0.136	0.365	0.049	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.010	0.005	0.003	1.359	2.123	0.129	0.292	2.228	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	19	20	25	0
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.80	1.00	0.00
time (sec)	N/A	0.017	0.006	0.005	1.432	1.268	0.123	0.249	2.209	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	7	7	8	7	12	5	8	7	0
N.S.	1	0.78	0.78	0.89	0.78	1.33	0.56	0.89	0.78	0.00
time (sec)	N/A	0.006	0.003	0.003	0.951	1.179	0.073	0.298	0.023	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	16	20	16	16	0
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73	0.00
time (sec)	N/A	0.011	0.013	0.007	1.579	1.192	0.149	0.292	0.055	0.001

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	31	30	30	44	30	30	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.19	0.81	0.81	0.00
time (sec)	N/A	0.022	0.013	0.003	1.393	1.169	0.122	0.384	0.045	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	15	19	14	16	13	0
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	0.87	0.00
time (sec)	N/A	0.004	0.002	0.004	0.969	1.342	0.086	0.269	0.026	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	15	10	14	13	0
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	0.87	0.00
time (sec)	N/A	0.004	0.001	0.005	0.948	0.903	0.076	0.280	0.023	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	13	11	8	13	11	0
N.S.	1	1.00	1.00	1.09	1.18	1.00	0.73	1.18	1.00	0.00
time (sec)	N/A	0.011	0.003	0.005	0.692	1.303	0.095	0.287	0.056	0.001

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	19	18	17	17	17	17	17	0
N.S.	1	1.00	0.86	0.82	0.77	0.77	0.77	0.77	0.77	0.00
time (sec)	N/A	0.004	0.001	0.000	0.876	0.524	0.056	0.356	0.030	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	19	19	19	9	9	0
N.S.	1	1.00	1.00	0.91	1.73	1.73	1.73	0.82	0.82	0.00
time (sec)	N/A	0.001	0.001	0.000	0.862	1.463	0.057	0.216	0.142	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	16	14	13	13	12	14	11	0
N.S.	1	1.00	1.23	1.08	1.00	1.00	0.92	1.08	0.85	0.00
time (sec)	N/A	0.010	0.004	0.002	0.643	1.003	0.076	0.361	2.211	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	15	14	18	14	16	12	0
N.S.	1	1.00	1.00	0.94	0.88	1.12	0.88	1.00	0.75	0.00
time (sec)	N/A	0.006	0.003	0.007	0.588	1.316	0.100	0.360	0.034	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	24	25	39	24	27	23	0
N.S.	1	1.00	0.96	0.89	0.93	1.44	0.89	1.00	0.85	0.00
time (sec)	N/A	0.011	0.016	0.008	0.815	1.210	0.111	0.379	2.228	0.001

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	18	16	15	15	12	16	15	0
N.S.	1	1.00	1.06	0.94	0.88	0.88	0.71	0.94	0.88	0.00
time (sec)	N/A	0.008	0.003	0.002	0.740	1.578	0.073	0.357	0.026	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	20	15	17	16	0
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	0.89	0.00
time (sec)	N/A	0.006	0.001	0.005	0.728	1.031	0.078	0.371	0.029	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-2)	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	28	14	0	14	14	13	0
N.S.	1	1.00	1.00	1.56	0.78	0.00	0.78	0.78	0.72	0.00
time (sec)	N/A	0.010	0.001	0.001	1.708	0.000	0.057	0.351	0.024	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	34	46	31	25	23	0
N.S.	1	1.00	1.00	0.96	1.48	2.00	1.35	1.09	1.00	0.00
time (sec)	N/A	0.018	0.010	0.006	0.738	1.137	0.109	0.369	0.032	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	12	13	12	15	10	32	12	0
N.S.	1	1.00	0.75	0.81	0.75	0.94	0.62	2.00	0.75	0.00
time (sec)	N/A	0.012	0.006	0.004	1.789	0.870	0.107	0.262	2.219	0.001

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	30	26	25	25	24	26	25	0
N.S.	1	1.00	1.03	0.90	0.86	0.86	0.83	0.90	0.86	0.00
time (sec)	N/A	0.020	0.008	0.001	0.765	1.409	0.080	0.283	0.026	0.001

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	10	12	13	21
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	1.31
time (sec)	N/A	0.006	0.000	0.000	0.891	1.242	0.059	0.348	0.021	0.021

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	17	25	14	12	17	0
N.S.	1	1.00	1.00	0.94	1.00	1.47	0.82	0.71	1.00	0.00
time (sec)	N/A	0.015	0.012	0.006	1.745	1.241	0.124	0.237	0.031	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	43	68	190	151	48	42	0
N.S.	1	1.00	1.00	0.91	1.45	4.04	3.21	1.02	0.89	0.00
time (sec)	N/A	0.066	0.025	0.006	1.021	1.325	0.296	0.281	0.097	0.001

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	61	61	0	240	294	60	82	0
N.S.	1	1.00	1.07	1.07	0.00	4.21	5.16	1.05	1.44	0.00
time (sec)	N/A	0.082	0.027	0.004	0.000	0.773	0.350	0.358	2.227	0.001

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	39	308	0	247	24	147	101	0
N.S.	1	1.00	0.21	1.64	0.00	1.31	0.13	0.78	0.54	0.00
time (sec)	N/A	0.189	0.031	0.102	0.000	1.509	0.517	1.488	2.300	0.001

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	27	28	65	65	60	30	27	0
N.S.	1	1.00	0.45	0.47	1.08	1.08	1.00	0.50	0.45	0.00
time (sec)	N/A	0.136	0.015	0.007	1.249	1.287	0.229	0.397	2.337	0.001

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	C	B	B	B	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	0	27	250	65	65	60	197	27	0
N.S.	1	0.00	1.00	9.26	2.41	2.41	2.22	7.30	1.00	0.00
time (sec)	N/A	0.311	0.010	0.030	1.199	1.293	0.324	0.397	0.045	0.003

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	B	B	B	B	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	0	27	112	65	65	60	111	27	0
N.S.	1	0.00	1.00	4.15	2.41	2.41	2.22	4.11	1.00	0.00
time (sec)	N/A	0.433	0.010	0.019	0.975	1.357	0.288	0.447	0.043	0.002

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [408] had the largest ratio of [.8824]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	24	0.083
2	A	3	3	1.00	29	0.103
3	A	2	2	1.00	29	0.069
4	A	2	2	1.00	29	0.069
5	A	1	0	1.00	27	0.000
6	A	2	2	1.00	29	0.069
7	A	2	2	1.00	29	0.069
8	A	2	2	1.00	29	0.069
9	A	3	2	1.00	27	0.074
10	A	3	2	1.00	27	0.074
11	A	1	0	1.00	25	0.000
12	A	7	7	1.00	27	0.259
13	A	8	8	1.00	27	0.296
14	A	9	8	1.00	27	0.296
15	A	3	2	1.00	46	0.043
16	A	3	2	1.00	46	0.043
17	A	1	0	1.00	44	0.000
18	A	2	1	1.00	46	0.022
19	A	2	1	1.00	46	0.022
20	A	2	1	1.00	46	0.022
21	A	5	4	1.00	11	0.364
22	A	5	4	1.00	17	0.235
23	A	2	2	1.00	7	0.286
24	A	3	2	1.00	13	0.154
25	A	5	5	1.00	11	0.454
26	A	7	7	1.00	16	0.438
27	A	6	6	1.00	9	0.667
28	A	2	2	1.00	7	0.286
29	A	3	2	1.00	29	0.069
30	A	2	1	1.00	29	0.034
31	A	2	1	1.00	29	0.034
32	A	1	0	1.00	27	0.000
33	A	10	6	1.00	29	0.207
34	A	11	7	1.00	29	0.241
35	A	3	2	1.00	32	0.062
36	A	2	1	1.00	32	0.031
37	A	2	1	1.00	32	0.031
38	A	1	0	1.00	30	0.000
39	A	4	3	1.00	32	0.094

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	5	4	1.00	32	0.125
41	A	2	1	1.00	17	0.059
42	A	2	1	1.00	17	0.059
43	A	2	1	1.00	17	0.059
44	A	1	0	1.00	15	0.000
45	A	16	9	1.00	17	0.529
46	A	18	11	1.00	17	0.647
47	A	2	1	1.00	17	0.059
48	A	2	1	1.00	17	0.059
49	A	2	1	1.00	17	0.059
50	A	1	0	1.00	15	0.000
51	A	15	9	1.00	17	0.529
52	A	17	11	1.00	17	0.647
53	A	2	1	1.00	22	0.045
54	A	2	1	1.00	22	0.045
55	A	2	1	1.00	22	0.045
56	A	1	0	1.00	20	0.000
57	A	16	9	1.00	22	0.409
58	A	18	11	1.00	22	0.500
59	A	2	2	1.00	51	0.039
60	A	2	2	1.00	51	0.039
61	B	1	0	4.36	49	0.000
62	A	2	2	1.00	51	0.039
63	A	2	2	1.00	51	0.039
64	A	2	2	1.00	51	0.039
65	A	6	5	1.00	13	0.385
66	A	5	3	1.00	19	0.158
67	A	5	3	1.00	19	0.158
68	A	5	3	1.00	19	0.158
69	A	1	0	1.00	17	0.000
70	A	5	2	1.00	19	0.105
71	A	7	3	1.00	19	0.158
72	A	10	3	1.00	19	0.158
73	B	15	7	2.25	17	0.412
74	A	6	5	1.00	15	0.333
75	A	6	5	1.00	15	0.333
76	A	4	4	1.00	13	0.308
77	A	2	2	1.00	11	0.182
78	A	6	6	1.00	15	0.400
79	A	7	6	1.00	15	0.400
80	A	7	6	1.00	15	0.400
81	A	2	2	1.00	13	0.154
82	A	3	3	1.00	13	0.231
83	A	4	3	1.00	13	0.231
84	A	2	2	1.00	19	0.105
85	A	2	2	1.00	11	0.182
86	A	3	3	1.00	11	0.273
87	A	4	3	1.00	11	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	2	2	1.00	13	0.154
89	A	3	3	1.00	13	0.231
90	A	4	3	1.00	13	0.231
91	A	2	2	1.00	11	0.182
92	A	3	3	1.00	11	0.273
93	A	4	3	1.00	11	0.273
94	A	3	2	1.00	15	0.133
95	A	4	3	1.00	13	0.231
96	A	4	4	1.00	17	0.235
97	A	4	4	1.00	19	0.210
98	A	4	4	1.00	17	0.235
99	A	11	10	1.00	17	0.588
100	A	9	9	1.00	17	0.529
101	A	7	7	1.00	15	0.467
102	A	7	7	1.00	13	0.538
103	A	11	10	1.06	17	0.588
104	A	11	10	0.99	17	0.588
105	A	11	10	1.00	17	0.588
106	A	16	12	1.00	17	0.706
107	A	14	10	1.00	17	0.588
108	A	14	10	1.00	15	0.667
109	A	10	7	1.00	13	0.538
110	A	18	13	1.00	17	0.765
111	A	18	13	1.00	17	0.765
112	A	3	2	1.00	22	0.091
113	A	2	1	1.00	22	0.045
114	A	2	1	1.00	22	0.045
115	A	1	0	1.00	20	0.000
116	A	4	3	1.00	22	0.136
117	A	5	4	1.00	22	0.182
118	A	6	5	1.00	22	0.227
119	A	2	1	1.00	24	0.042
120	A	2	1	1.00	24	0.042
121	A	2	1	1.00	24	0.042
122	A	2	1	1.00	22	0.045
123	A	8	7	1.00	24	0.292
124	A	10	9	1.00	24	0.375
125	A	12	10	1.00	24	0.417
126	A	2	1	1.00	26	0.038
127	A	2	1	1.00	26	0.038
128	A	2	1	1.00	26	0.038
129	A	2	1	1.00	24	0.042
130	A	9	8	1.00	26	0.308
131	A	11	10	1.00	26	0.385
132	A	14	5	1.00	46	0.109
133	A	14	5	1.00	46	0.109
134	A	8	3	1.00	46	0.065
135	A	14	5	1.00	44	0.114

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	14	5	1.00	42	0.119
137	A	14	5	1.00	46	0.109
138	A	14	5	0.99	46	0.109
139	A	14	6	1.00	26	0.231
140	A	14	6	1.00	26	0.231
141	A	14	6	1.00	26	0.231
142	A	8	4	1.00	26	0.154
143	A	14	6	1.00	24	0.250
144	A	14	6	1.00	22	0.273
145	A	14	6	1.00	26	0.231
146	A	14	6	1.00	26	0.231
147	A	23	7	1.00	26	0.269
148	A	23	7	1.00	26	0.269
149	A	14	6	1.00	26	0.231
150	A	17	5	1.00	26	0.192
151	A	23	7	1.00	26	0.269
152	A	23	7	1.00	26	0.269
153	A	23	7	1.00	26	0.269
154	A	2	1	1.00	52	0.019
155	A	4	3	1.00	52	0.058
156	A	1	1	1.00	18	0.056
157	A	3	3	1.00	23	0.130
158	A	3	3	1.00	23	0.130
159	A	3	3	1.00	29	0.103
160	A	1	1	1.00	18	0.056
161	A	4	3	1.00	20	0.150
162	A	4	3	1.00	20	0.150
163	A	4	3	1.00	22	0.136
164	A	1	1	1.00	18	0.056
165	A	3	3	1.00	23	0.130
166	A	3	3	1.00	23	0.130
167	A	3	3	1.00	29	0.103
168	A	1	1	1.00	18	0.056
169	A	1	1	1.00	25	0.040
170	C	7	3	4.30	38	0.079
171	A	1	1	1.00	27	0.037
172	A	1	1	1.00	31	0.032
173	A	2	1	1.00	54	0.019
174	A	3	1	1.00	54	0.019
175	A	5	4	1.00	54	0.074
176	A	1	1	1.00	30	0.033
177	A	1	1	1.00	29	0.034
178	A	1	1	1.00	28	0.036
179	A	1	1	1.00	21	0.048
180	A	1	1	1.00	20	0.050
181	A	1	1	1.00	21	0.048
182	A	1	1	1.00	26	0.038
183	A	1	1	1.00	25	0.040

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	2	2	1.00	26	0.077
185	A	1	1	1.00	24	0.042
186	A	1	1	1.00	24	0.042
187	A	1	1	1.00	23	0.043
188	A	1	1	1.00	30	0.033
189	A	1	1	1.00	29	0.034
190	A	1	1	1.00	28	0.036
191	A	1	1	1.00	21	0.048
192	A	1	1	1.00	20	0.050
193	A	2	2	1.00	21	0.095
194	A	1	1	1.00	26	0.038
195	A	1	1	1.00	25	0.040
196	A	2	2	1.00	26	0.077
197	A	1	1	1.00	22	0.045
198	A	1	1	1.00	24	0.042
199	A	2	2	1.00	23	0.087
200	A	1	1	1.00	23	0.043
201	A	1	1	1.00	19	0.053
202	A	2	1	1.00	22	0.045
203	A	2	1	1.00	23	0.043
204	A	2	1	1.00	22	0.045
205	A	2	1	1.00	23	0.043
206	A	2	1	1.00	24	0.042
207	A	2	1	1.00	25	0.040
208	A	2	1	1.00	31	0.032
209	A	2	1	1.00	32	0.031
210	A	2	1	1.00	35	0.029
211	A	2	1	1.00	36	0.028
212	A	2	1	1.00	24	0.042
213	A	2	1	1.00	31	0.032
214	A	2	1	1.00	35	0.029
215	A	1	1	1.00	22	0.045
216	A	1	1	1.00	18	0.056
217	B	3	2	2.91	26	0.077
218	B	2	1	2.91	28	0.036
219	A	1	1	1.00	18	0.056
220	A	1	1	1.00	20	0.050
221	A	1	1	1.00	21	0.048
222	A	3	3	1.00	52	0.058
223	A	9	5	1.00	38	0.132
224	A	3	2	1.00	32	0.062
225	A	4	3	1.00	33	0.091
226	A	3	2	1.00	34	0.059
227	A	6	6	1.00	43	0.140
228	A	1	1	1.00	16	0.062
229	B	15	7	2.25	25	0.280
230	A	1	1	1.00	56	0.018
231	A	1	1	1.00	51	0.020

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
232	A	1	1	1.00	49	0.020
233	A	1	1	1.00	46	0.022
234	A	2	2	1.00	48	0.042
235	A	1	1	1.00	49	0.020
236	A	1	1	1.00	48	0.021
237	A	1	1	1.00	48	0.021
238	A	10	5	1.00	35	0.143
239	A	10	5	1.00	35	0.143
240	A	10	5	1.00	35	0.143
241	A	10	5	1.00	33	0.152
242	A	10	5	1.00	32	0.156
243	A	13	6	1.00	35	0.171
244	A	13	6	1.00	35	0.171
245	A	13	6	1.00	35	0.171
246	A	13	6	1.00	35	0.171
247	A	13	6	1.00	35	0.171
248	A	11	6	1.00	33	0.182
249	A	9	5	1.00	32	0.156
250	A	13	6	1.00	35	0.171
251	A	13	6	1.00	35	0.171
252	A	13	6	1.00	35	0.171
253	A	2	2	1.00	40	0.050
254	A	6	5	1.00	20	0.250
255	A	3	2	1.00	20	0.100
256	A	3	2	1.00	20	0.100
257	A	2	1	1.00	16	0.062
258	A	5	4	1.00	22	0.182
259	A	3	2	1.00	21	0.095
260	A	6	5	1.00	26	0.192
261	A	2	1	1.00	20	0.050
262	A	2	1	1.00	11	0.091
263	A	4	3	1.00	22	0.136
264	A	3	2	1.00	21	0.095
265	A	3	2	1.00	25	0.080
266	A	6	6	1.00	22	0.273
267	A	5	5	1.00	31	0.161
268	A	3	2	1.00	21	0.095
269	A	4	3	1.00	33	0.091
270	A	4	4	1.00	14	0.286
271	A	7	6	1.00	33	0.182
272	A	7	6	1.00	29	0.207
273	A	6	5	1.00	44	0.114
274	A	3	2	1.00	15	0.133
275	A	5	4	1.00	15	0.267
276	A	4	3	1.00	18	0.167
277	A	3	2	1.00	20	0.100
278	A	5	4	1.00	26	0.154
279	A	3	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
280	A	3	2	1.00	18	0.111
281	A	2	1	1.00	26	0.038
282	A	5	5	1.00	19	0.263
283	A	5	5	1.00	24	0.208
284	A	8	6	1.00	20	0.300
285	A	8	6	1.00	18	0.333
286	A	5	3	1.00	19	0.158
287	A	4	3	1.00	13	0.231
288	A	6	5	1.00	22	0.227
289	A	6	5	1.00	24	0.208
290	A	2	1	1.00	29	0.034
291	A	2	1	1.00	30	0.033
292	A	2	1	1.00	19	0.053
293	A	4	4	1.00	16	0.250
294	A	10	5	1.00	36	0.139
295	A	2	1	1.00	21	0.048
296	A	5	4	1.00	16	0.250
297	A	2	1	1.00	24	0.042
298	A	2	1	1.00	21	0.048
299	A	2	1	1.00	24	0.042
300	A	6	5	1.00	26	0.192
301	A	3	2	1.00	25	0.080
302	A	2	1	1.00	29	0.034
303	A	6	5	1.00	20	0.250
304	A	14	10	1.00	32	0.312
305	A	4	4	1.00	23	0.174
306	A	6	5	1.00	26	0.192
307	A	4	3	1.00	26	0.115
308	A	8	4	1.00	25	0.160
309	A	6	3	1.00	23	0.130
310	A	7	6	1.00	23	0.261
311	A	5	3	1.18	20	0.150
312	A	3	2	1.00	25	0.080
313	A	3	2	1.00	22	0.091
314	A	2	1	1.00	24	0.042
315	A	7	6	1.00	24	0.250
316	A	6	5	1.00	43	0.116
317	A	7	5	1.25	50	0.100
318	A	3	2	1.00	16	0.125
319	A	6	5	1.00	15	0.333
320	A	5	4	1.00	20	0.200
321	A	3	2	1.00	24	0.083
322	A	5	3	1.00	27	0.111
323	A	5	5	1.00	26	0.192
324	A	3	2	1.00	16	0.125
325	A	11	8	1.32	22	0.364
326	A	5	4	1.00	24	0.167
327	A	4	3	1.00	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
328	A	5	3	1.00	36	0.083
329	A	4	3	1.00	26	0.115
330	A	10	9	1.00	20	0.450
331	A	6	6	1.00	27	0.222
332	A	7	7	1.00	20	0.350
333	A	8	7	1.00	25	0.280
334	A	8	7	1.00	22	0.318
335	A	2	1	1.00	18	0.056
336	A	5	4	1.00	20	0.200
337	A	10	9	1.00	20	0.450
338	A	16	12	1.00	20	0.600
339	A	3	2	1.00	14	0.143
340	A	2	2	1.00	20	0.100
341	A	14	8	1.00	20	0.400
342	A	4	3	1.00	26	0.115
343	A	4	3	1.00	24	0.125
344	A	6	4	1.00	30	0.133
345	A	4	4	1.00	21	0.190
346	A	2	1	1.00	15	0.067
347	A	4	3	1.00	18	0.167
348	A	3	2	1.00	22	0.091
349	A	3	2	1.00	16	0.125
350	A	6	5	1.00	16	0.312
351	A	3	2	1.00	25	0.080
352	A	3	2	1.00	19	0.105
353	A	2	1	1.00	23	0.043
354	A	5	4	1.00	23	0.174
355	A	5	4	1.00	21	0.190
356	A	10	6	1.00	28	0.214
357	A	2	1	1.00	24	0.042
358	A	6	5	1.00	26	0.192
359	A	2	1	1.00	14	0.071
360	A	5	3	1.00	16	0.188
361	A	5	5	1.00	16	0.312
362	A	7	5	1.00	43	0.116
363	A	17	13	1.00	26	0.500
364	A	18	13	1.00	16	0.812
365	A	3	2	1.00	15	0.133
366	A	3	2	1.00	15	0.133
367	A	3	2	1.00	17	0.118
368	A	3	2	1.00	15	0.133
369	A	4	3	1.00	15	0.200
370	A	4	3	1.00	18	0.167
371	A	3	2	1.00	20	0.100
372	A	7	6	1.00	29	0.207
373	A	4	3	1.00	22	0.136
374	A	6	4	1.00	16	0.250
375	A	6	4	1.00	16	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
376	A	5	4	1.00	14	0.286
377	A	4	3	1.00	12	0.250
378	A	4	3	1.00	16	0.188
379	A	6	5	1.00	16	0.312
380	A	4	3	1.00	16	0.188
381	A	4	3	1.00	16	0.188
382	A	4	3	1.00	16	0.188
383	A	8	5	1.00	17	0.294
384	A	8	5	1.00	19	0.263
385	A	8	5	1.00	15	0.333
386	A	8	5	1.00	17	0.294
387	A	16	10	1.00	23	0.435
388	A	17	10	1.00	21	0.476
389	F	0	0	N/A	0	N/A
390	A	15	11	1.00	17	0.647
391	A	13	9	1.00	17	0.529
392	A	13	9	1.00	15	0.600
393	A	9	6	1.00	9	0.667
394	A	17	12	1.00	17	0.706
395	A	17	12	1.00	17	0.706
396	A	17	12	1.00	17	0.706
397	A	16	12	1.00	17	0.706
398	A	14	10	1.00	17	0.588
399	A	14	10	1.00	15	0.667
400	A	10	7	1.00	9	0.778
401	A	31	14	1.00	17	0.824
402	A	31	14	1.00	17	0.824
403	A	31	14	1.00	17	0.824
404	A	15	11	1.00	17	0.647
405	A	15	10	1.00	17	0.588
406	A	15	10	1.00	15	0.667
407	A	11	7	1.00	9	0.778
408	A	46	15	1.00	17	0.882
409	A	46	15	1.00	17	0.882
410	A	46	15	1.00	17	0.882
411	A	4	4	1.00	14	0.286
412	A	5	5	1.00	13	0.385
413	A	4	4	1.00	16	0.250
414	A	5	5	1.00	18	0.278
415	A	3	2	1.00	14	0.143
416	A	4	3	1.00	16	0.188
417	A	2	2	1.00	11	0.182
418	A	1	0	1.00	17	0.000
419	A	1	1	1.00	11	0.091
420	B	1	0	6.23	73	0.000
421	A	11	7	1.00	13	0.538
422	A	13	9	1.00	19	0.474
423	A	3	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
424	A	3	2	1.00	11	0.182
425	A	1	0	1.00	11	0.000
426	A	1	0	1.00	11	0.000
427	A	5	4	1.00	16	0.250
428	A	2	1	1.00	16	0.062
429	A	4	3	1.00	15	0.200
430	A	1	1	1.00	15	0.067
431	A	1	1	1.00	20	0.050
432	A	3	2	1.00	15	0.133
433	A	6	5	1.00	13	0.385
434	A	1	1	1.00	22	0.045
435	A	3	2	1.00	18	0.111
436	A	3	3	1.00	20	0.150
437	A	3	2	1.00	16	0.125
438	A	6	6	1.00	17	0.353
439	A	1	1	1.00	17	0.059
440	A	4	3	1.00	25	0.120
441	A	2	2	1.00	20	0.100
442	A	3	2	1.00	18	0.111
443	A	6	5	1.00	15	0.333
444	A	2	1	1.00	11	0.091
445	A	5	5	1.00	13	0.385
446	A	3	2	1.00	20	0.100
447	A	2	1	1.00	16	0.062
448	A	4	3	1.00	16	0.188
449	A	2	1	1.00	16	0.062
450	A	1	1	1.00	9	0.111
451	A	2	2	1.00	7	0.286
452	A	2	2	1.00	11	0.182
453	A	1	1	1.00	7	0.143
454	A	1	1	1.00	9	0.111
455	A	1	1	1.00	9	0.111
456	A	2	2	1.00	10	0.200
457	A	2	1	1.00	13	0.077
458	A	2	1	1.00	11	0.091
459	A	2	1	1.00	14	0.071
460	A	4	4	1.00	16	0.250
461	A	2	1	1.00	7	0.143
462	A	2	1	1.00	11	0.091
463	A	5	5	1.00	13	0.385
464	A	5	5	1.00	13	0.385
465	A	5	4	1.00	14	0.286
466	A	2	1	0.78	13	0.077
467	A	3	2	1.00	20	0.100
468	A	4	4	1.00	18	0.222
469	A	2	1	1.00	12	0.083
470	A	2	1	1.00	10	0.100
471	A	3	2	1.00	16	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	2	1	1.00	11	0.091
473	A	1	1	1.00	7	0.143
474	A	2	1	1.00	15	0.067
475	A	2	1	1.00	12	0.083
476	A	3	3	1.00	16	0.188
477	A	2	1	1.00	11	0.091
478	A	2	1	1.00	17	0.059
479	A	2	1	1.00	29	0.034
480	A	2	1	1.00	18	0.056
481	A	3	2	1.00	14	0.143
482	A	2	1	1.00	24	0.042
483	A	2	1	1.00	11	0.091
484	A	3	2	1.00	18	0.111
485	A	3	3	1.00	15	0.200
486	A	3	3	1.00	16	0.188
487	A	10	7	1.00	15	0.467
488	A	5	2	1.00	50	0.040
489	F	0	0	N/A	0	N/A
490	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{1}{2\sqrt{3}\, b^{3/2}-9bx+9x^3} \, dx$

Optimal. Leaf size=77

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2063, 44}

$$\frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] 1/(3*Sqrt[3]*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) - Log[Sqrt[b] - Sqrt[3]*x]/(2
7*b) + Log[2*Sqrt[b] + Sqrt[3]*x]/(27*b)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2063

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[1/(3^(3*p)*
a^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d},
 x] && EqQ[4*b^3 + 27*a^2*d, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx &= \left(324b^3\right)\int \frac{1}{\left(6\sqrt{3}\,b^{3/2}-18bx\right)^2\left(6\sqrt{3}\,b^{3/2}+9bx\right)}\,dx\\ &= \left(324b^3\right)\int \left(\frac{1}{324\sqrt{3}\,b^{7/2}\left(\sqrt{3}\,\sqrt{b}-3x\right)^2}+\frac{1}{2916b^4\left(\sqrt{3}\,\sqrt{b}-3x\right)}+\frac{1}{2916b^4\left(2\sqrt{3}\,\sqrt{\right.}\right.\\ &= \frac{1}{3\sqrt{3}\,\sqrt{b}\,\left(\sqrt{3}\,\sqrt{b}-3x\right)}-\frac{\log\left(\sqrt{b}-\sqrt{3}\,x\right)}{27b}+\frac{\log\left(2\sqrt{b}+\sqrt{3}\,x\right)}{27b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 143, normalized size = 1.86

$$-\frac{\left(3x-\sqrt{3}\,\sqrt{b}\right)\left(2\sqrt{3}\,\sqrt{b}+3x\right)\left(\left(3x-\sqrt{3}\,\sqrt{b}\right)\log\left(3x-\sqrt{3}\,\sqrt{b}\right)+\left(\sqrt{3}\,\sqrt{b}-3x\right)\log\left(2\sqrt{3}\,\sqrt{b}+3x\right)+3\sqrt{3}\,\sqrt{b}\right)}{81b\left(2\sqrt{3}\,b^{3/2}-9bx+9x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] -1/81*((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(3*Sqrt[3]*Sqrt
[b] + (-(Sqrt[3]*Sqrt[b]) + 3*x)*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (Sqrt[3]*S
qrt[b] - 3*x)*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(b*(2*Sqrt[3]*b^(3/2) - 9*b*x
+ 9*x^3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2\sqrt{3}\,b^{3/2}-9bx+9x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3)^(-1), x]
```

fricas [A] time = 0.72, size = 76, normalized size = 0.99

$$-\frac{3\,\sqrt{3}\,\sqrt{b}\,x-\left(3\,x^2-b\right)\log\left(2\,\sqrt{3}\,\sqrt{b}\,+3\,x\right)+\left(3\,x^2-b\right)\log\left(-\sqrt{3}\,\sqrt{b}\,+3\,x\right)+3\,b}{27\left(3\,bx^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/27*(3*sqrt(3)*sqrt(b)*x - (3*x^2 - b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (3*
x^2 - b)*log(-sqrt(3)*sqrt(b) + 3*x) + 3*b)/(3*b*x^2 - b^2)
```

giac [A] time = 0.22, size = 55, normalized size = 0.71

$$\frac{\log\left(\left|9\,\sqrt{3}\,x+18\,\sqrt{b}\right|\right)}{27\,b}-\frac{\log\left(\left|-\sqrt{3}\,x+\sqrt{b}\right|\right)}{27\,b}-\frac{1}{9\left(\sqrt{3}\,x-\sqrt{b}\right)\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/27*log(abs(9*sqrt(3)*x + 18*sqrt(b)))/b - 1/27*log(abs(-sqrt(3)*x + sqrt(
b)))/b - 1/9/((sqrt(3)*x - sqrt(b))*sqrt(b))
```

maple [C] time = 0.05, size = 43, normalized size = 0.56

$$\frac{\ln\left(-\mathrm{RootOf}\left(9\_Z^3-9\_Zb+2\sqrt{3}\,b^{\frac{3}{2}}\right)+x\right)}{27\,\mathrm{RootOf}\left(9\_Z^3-9\_Zb+2\sqrt{3}\,b^{\frac{3}{2}}\right)^2-9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x)
```

```
[Out] 1/9*sum(1/(3*_R^2-b)*ln(x-_R),_R=RootOf(-9*b*_Z+9*_Z^3+2*b^(3/2)*3^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{9\,x^3+2\,\sqrt{3}\,b^{\frac{3}{2}}-9\,bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x^3+2*b^(3/2)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

mupad [B] time = 0.20, size = 51, normalized size = 0.66

$$\frac{2\sqrt{3}\,\sqrt{27}\,\operatorname{atanh}\left(\frac{\sqrt{3}\,\sqrt{27}}{27}+\frac{2\sqrt{27}\,x}{9\sqrt{b}}\right)}{243\,b}-\frac{\sqrt{3}}{27\sqrt{b}\,\left(x-\frac{\sqrt{3}\,\sqrt{b}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
```

```
[Out] (2*3^(1/2)*27^(1/2)*atanh((3^(1/2)*27^(1/2))/27 + (2*27^(1/2)*x)/(9*b^(1/2)
)))/(243*b) - 3^(1/2)/(27*b^(1/2)*(x - (3^(1/2)*b^(1/2))/3))
```

sympy [A] time = 0.33, size = 60, normalized size = 0.78

$$-\frac{3\sqrt{3}}{81\sqrt{b}\,x-27\sqrt{3}\,b}+\frac{-\frac{\log\left(-\frac{\sqrt{3}\,\sqrt{b}}{3}+x\right)}{27}+\frac{\log\left(\frac{2\sqrt{3}\,\sqrt{b}}{3}+x\right)}{27}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*b*x+9*x**3+2*b**(3/2)*3**(1/2)),x)
```

```
[Out] -3*sqrt(3)/(81*sqrt(b)*x - 27*sqrt(3)*b) + (-log(-sqrt(3)*sqrt(b)/3 + x)/27
 + log(2*sqrt(3)*sqrt(b)/3 + x)/27)/b
```

3.2 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$

Optimal. Leaf size=30

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2067, 15, 30}

$$\frac{\left(\frac{a}{b}+x\right)\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p}{3p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a/b + x)*(b^3*(a/b + x)^3)^p)/(1 + 3*p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\begin{aligned}
\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx &= \text{Subst}\left(\int \left(b^3x^3\right)^p\,dx, x, \frac{a}{b}+x\right)\\
&= \left(\left(\frac{a}{b}+x\right)^{-3p}\left(b^3\left(\frac{a}{b}+x\right)^3\right)^p\right)\text{Subst}\left(\int x^{3p}\,dx, x, \frac{a}{b}+x\right)\\
&= \frac{(a+bx)\left((a+bx)^3\right)^p}{b(1+3p)}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 23, normalized size = 0.77

$$\frac{(a+bx)\left((a+bx)^3\right)^p}{3bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

```
[Out] ((a + b*x)*((a + b*x)^3)^p)/(b + 3*b*p)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
[Out] Defer[IntegrateAlgebraic][(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p, x]
```

fricas [A] time = 1.03, size = 43, normalized size = 1.43

$$\frac{(bx+a)\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="fricas")
[Out] (b*x + a)*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p/(3*b*p + b)
```

giac [B] time = 0.20, size = 73, normalized size = 2.43

$$\frac{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p bx+\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)^p a}{3\,bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="giac")
[Out] ((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*b*x + (b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3)^p*a)/(3*b*p + b)
```

maple [A] time = 0.01, size = 46, normalized size = 1.53

$$\frac{(bx+a)\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)^p}{\left(3p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)
[Out] (b*x+a)/b/(1+3*p)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p
```

maxima [A] time = 0.49, size = 25, normalized size = 0.83

$$\frac{(bx+a)(bx+a)^{3\,p}}{b\left(3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm="maxima")
[Out] (b*x + a)*(b*x + a)^(3*p)/(b*(3*p + 1))
```

mupad [B] time = 2.13, size = 52, normalized size = 1.73

$$\left(\frac{x}{3\,p+1}+\frac{a}{b\,\left(3\,p+1\right)}\right)\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
```

```
[Out] (x/(3*p + 1) + a/(b*(3*p + 1)))*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x}{\sqrt[3]{a^3}} & \text{for } b = 0 \wedge p = -\frac{1}{3} \\ x\left(a^3\right)^p & \text{for } b = 0 \\ \int \frac{1}{\sqrt[3]{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\, dx & \text{for } p = -\frac{1}{3} \\ \frac{a\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b} + \frac{bx\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p}{3bp+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

```
[Out] Piecewise((x/(a**3)**(1/3), Eq(b, 0) & Eq(p, -1/3)), (x*(a**3)**p, Eq(b, 0)
), (Integral((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**(-1/3), x), E
q(p, -1/3)), (a*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p +
 b) + b*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(3*b*p + b), T
rue))
```

3.3 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3 dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{10}}{10b}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
[Out] (a + b*x)^10/(10*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3 dx &= \int (a+bx)^9\,dx\\ &= \frac{(a+bx)^{10}}{10b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
[Out] (a + b*x)^10/(10*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3, x]
```

fricas [B] time = 1.29, size = 97, normalized size = 6.93

$$\frac{1}{10}x^{10}b^9 + x^9b^8a + \frac{9}{2}x^8b^7a^2 + 12x^7b^6a^3 + 21x^6b^5a^4 + \frac{126}{5}x^5b^4a^5 + 21x^4b^3a^6 + 12x^3b^2a^7 + \frac{9}{2}x^2ba^8 + xa^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^9 + x^9*b^8*a + 9/2*x^8*b^7*a^2 + 12*x^7*b^6*a^3 + 21*x^6*b^5*a
^4 + 126/5*x^5*b^4*a^5 + 21*x^4*b^3*a^6 + 12*x^3*b^2*a^7 + 9/2*x^2*b*a^8 +
x*a^9
```

giac [B] time = 0.25, size = 97, normalized size = 6.93

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8bx^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 9/2*a^2*b^7*x^8 + 12*a^3*b^6*x^7 + 21*a^4*b^5*x
^6 + 126/5*a^5*b^4*x^5 + 21*a^6*b^3*x^4 + 12*a^7*b^2*x^3 + 9/2*a^8*b*x^2 +
a^9*x
```

maple [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{1}{10}b^9x^{10} + a\,b^8x^9 + \frac{9}{2}a^2b^7x^8 + 12a^3b^6x^7 + 21a^4b^5x^6 + \frac{126}{5}a^5b^4x^5 + 21a^6b^3x^4 + 12a^7b^2x^3 + \frac{9}{2}a^8b\,x^2 + a^9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] 1/10*b^9*x^10+a*b^8*x^9+9/2*a^2*b^7*x^8+12*a^3*b^6*x^7+21*a^4*b^5*x^6+126/5
*a^5*b^4*x^5+21*a^6*b^3*x^4+12*a^7*b^2*x^3+9/2*a^8*b*x^2+a^9*x
```

maxima [B] time = 0.69, size = 216, normalized size = 15.43

$$\frac{1}{10}b^9x^{10} + ab^8x^9 + \frac{27}{8}a^2b^7x^8 + \frac{27}{7}a^3b^6x^7 + \frac{27}{4}a^6b^3x^4 + a^9x + \frac{3}{4}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2\right)a^6 + \frac{9}{10}\left(5b^3x^6 + 18ab^2x^5\right)a^4b^2 + \frac{3}{70}\left(10b^6x^7 + 70ab^5x^6 + 126a^2b^4x^5 + 210a^4b^2x^3 + 21\left(4b^3x^5 + 15ab^2x^4\right)a^2b\right)a^3 + \frac{9}{56}\left(7b^6x^8 + 48ab^5x^7 + 84a^2b^4x^6\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^9*x^10 + a*b^8*x^9 + 27/8*a^2*b^7*x^8 + 27/7*a^3*b^6*x^7 + 27/4*a^6*
b^3*x^4 + a^9*x + 3/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^6 + 9/10*(5*b
^3*x^6 + 18*a*b^2*x^5)*a^4*b^2 + 3/70*(10*b^6*x^7 + 70*a*b^5*x^6 + 126*a^2*
b^4*x^5 + 210*a^4*b^2*x^3 + 21*(4*b^3*x^5 + 15*a*b^2*x^4)*a^2*b)*a^3 + 9/56
*(7*b^6*x^8 + 48*a*b^5*x^7 + 84*a^2*b^4*x^6)*a^2*b
```

mupad [B] time = 2.07, size = 97, normalized size = 6.93

$$a^9\,x + \frac{9\,a^8\,b\,x^2}{2} + 12\,a^7\,b^2\,x^3 + 21\,a^6\,b^3\,x^4 + \frac{126\,a^5\,b^4\,x^5}{5} + 21\,a^4\,b^5\,x^6 + 12\,a^3\,b^6\,x^7 + \frac{9\,a^2\,b^7\,x^8}{2} + a\,b^8\,x^9 + \frac{b^9\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] a^9*x + (b^9*x^10)/10 + (9*a^8*b*x^2)/2 + a*b^8*x^9 + 12*a^7*b^2*x^3 + 21*a
^6*b^3*x^4 + (126*a^5*b^4*x^5)/5 + 21*a^4*b^5*x^6 + 12*a^3*b^6*x^7 + (9*a^2
*b^7*x^8)/2
```

sympy [B] time = 0.09, size = 107, normalized size = 7.64

$$a^9x + \frac{9a^8bx^2}{2} + 12a^7b^2x^3 + 21a^6b^3x^4 + \frac{126a^5b^4x^5}{5} + 21a^4b^5x^6 + 12a^3b^6x^7 + \frac{9a^2b^7x^8}{2} + ab^8x^9 + \frac{b^9x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] a**9*x + 9*a**8*b*x**2/2 + 12*a**7*b**2*x**3 + 21*a**6*b**3*x**4 + 126*a**5
*b**4*x**5/5 + 21*a**4*b**5*x**6 + 12*a**3*b**6*x**7 + 9*a**2*b**7*x**8/2 +
 a*b**8*x**9 + b**9*x**10/10
```

3.4 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^7}{7b}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2059, 32}

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx &= \int (a+bx)^6\,dx\\ &= \frac{(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] (a + b*x)^7/(7*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2, x]
```

fricas [B] time = 0.75, size = 64, normalized size = 4.57

$$\frac{1}{7}x^7b^6 + x^6b^5a + 3x^5b^4a^2 + 5x^4b^3a^3 + 5x^3b^2a^4 + 3x^2ba^5 + xa^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^6 + x^6*b^5*a + 3*x^5*b^4*a^2 + 5*x^4*b^3*a^3 + 5*x^3*b^2*a^4 + 3
*x^2*b*a^5 + x*a^6
```

giac [B] time = 0.37, size = 64, normalized size = 4.57

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 3*a^2*b^4*x^5 + 5*a^3*b^3*x^4 + 5*a^4*b^2*x^3 + 3
*a^5*b*x^2 + a^6*x
```

maple [B] time = 0.00, size = 65, normalized size = 4.64

$$\frac{1}{7}b^6x^7 + ab^5x^6 + 3a^2b^4x^5 + 5a^3b^3x^4 + 5a^4b^2x^3 + 3a^5bx^2 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] 1/7*b^6*x^7+a*b^5*x^6+3*a^2*b^4*x^5+5*a^3*b^3*x^4+5*a^4*b^2*x^3+3*a^5*b*x^2
+a^6*x
```

maxima [B] time = 0.66, size = 99, normalized size = 7.07

$$\frac{1}{7}b^6x^7 + ab^5x^6 + \frac{9}{5}a^2b^4x^5 + 3a^4b^2x^3 + a^6x + \frac{1}{2}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2\right)a^3 + \frac{3}{10}\left(4b^3x^5 + 15ab^2x^4\right)a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + a*b^5*x^6 + 9/5*a^2*b^4*x^5 + 3*a^4*b^2*x^3 + a^6*x + 1/2*(b^
3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2)*a^3 + 3/10*(4*b^3*x^5 + 15*a*b^2*x^4)*a^
2*b
```

mupad [B] time = 0.03, size = 64, normalized size = 4.57

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] a^6*x + (b^6*x^7)/7 + 3*a^5*b*x^2 + a*b^5*x^6 + 5*a^4*b^2*x^3 + 5*a^3*b^3*x
^4 + 3*a^2*b^4*x^5
```

sympy [B] time = 0.08, size = 66, normalized size = 4.71

$$a^6x + 3a^5bx^2 + 5a^4b^2x^3 + 5a^3b^3x^4 + 3a^2b^4x^5 + ab^5x^6 + \frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] a**6*x + 3*a**5*b*x**2 + 5*a**4*b**2*x**3 + 5*a**3*b**3*x**4 + 3*a**2*b**4*
x**5 + a*b**5*x**6 + b**6*x**7/7
```

3.5 $\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal. Leaf size=35

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2}a^2bx^2+a^3x+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

Rubi steps

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx=a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3x+\frac{3}{2}a^2bx^2+ab^2x^3+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] a^3*x + (3*a^2*b*x^2)/2 + a*b^2*x^3 + (b^3*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3,x]
[Out] IntegrateAlgebraic[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3, x]
```

fricas [A] time = 0.51, size = 31, normalized size = 0.89

$$\frac{1}{4}x^4b^3+x^3b^2a+\frac{3}{2}x^2ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="fricas")
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

giac [A] time = 0.28, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x)
[Out] a^3*x+3/2*a^2*b*x^2+a*b^2*x^3+1/4*b^3*x^4
```

maxima [A] time = 0.74, size = 31, normalized size = 0.89

$$\frac{1}{4}\,b^3x^4 + ab^2x^3 + \frac{3}{2}\,a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.89

$$a^3\,x + \frac{3\,a^2\,b\,x^2}{2} + a\,b^2\,x^3 + \frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x,x)
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

sympy [A] time = 0.07, size = 32, normalized size = 0.91

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3,x)
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

3.6 $\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx &= \int \frac{1}{(a+bx)^3}\,dx\\ &= -\frac{1}{2b(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-1), x]
```

fricas [A] time = 0.79, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="fricas")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [A] time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx+a)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

maple [A] time = 0.02, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(bx+a)^{2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
[Out] -1/2/b/(b*x+a)^2
```

maxima [A] time = 0.65, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="maxima")
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

mupad [B] time = 0.03, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^{2}\,b+4\,a\,b^{2}\,x+2\,b^{3}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2a^{2}b+4ab^{2}x+2b^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

3.7 $\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{5b(a+bx)^5}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
[Out] -1/(5*b*(a + b*x)^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx &= \int \frac{1}{(a+bx)^6}\,dx\\ &= -\frac{1}{5b(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
[Out] -1/5*1/(b*(a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2),x]
[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-2), x]
```

fricas [B] time = 0.93, size = 57, normalized size = 4.07

$$-\frac{1}{5\left(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="fricas")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

giac [A] time = 0.29, size = 12, normalized size = 0.86

$$-\frac{1}{5\,(bx+a)^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="giac")
```

```
[Out] -1/5/((b*x + a)^5*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{5\,(bx+a)^5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

```
[Out] -1/5/b/(b*x+a)^5
```

maxima [B] time = 0.71, size = 57, normalized size = 4.07

$$-\frac{1}{5\left(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm="maxima")
```

```
[Out] -1/5/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x
 + a^5*b)
```

mupad [B] time = 2.05, size = 59, normalized size = 4.21

$$-\frac{1}{5\,a^5\,b+25\,a^4\,b^2\,x+50\,a^3\,b^3\,x^2+50\,a^2\,b^4\,x^3+25\,a\,b^5\,x^4+5\,b^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

```
[Out] -1/(5*a^5*b + 5*b^6*x^5 + 25*a^4*b^2*x + 25*a*b^5*x^4 + 50*a^3*b^3*x^2 + 50
*a^2*b^4*x^3)
```

sympy [B] time = 0.35, size = 61, normalized size = 4.36

$$-\frac{1}{5a^5b+25a^4b^2x+50a^3b^3x^2+50a^2b^4x^3+25ab^5x^4+5b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

```
[Out] -1/(5*a**5*b + 25*a**4*b**2*x + 50*a**3*b**3*x**2 + 50*a**2*b**4*x**3 + 25*
a*b**5*x**4 + 5*b**6*x**5)
```

3.8 $\int \frac{1}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{8b(a+bx)^8}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2058, 32}

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/(8*b*(a + b*x)^8)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3} \, dx &= \int \frac{1}{(a+bx)^9} \, dx \\ &= -\frac{1}{8b(a+bx)^8} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{8b(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] -1/8*1/(b*(a + b*x)^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(-3), x]
```

fricas [B] time = 1.11, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

giac [A] time = 0.39, size = 12, normalized size = 0.86

$$-\frac{1}{8\,(bx+a)^8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="giac")
```

```
[Out] -1/8/((b*x + a)^8*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{8\,(bx+a)^8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

```
[Out] -1/8/b/(b*x+a)^8
```

maxima [B] time = 0.54, size = 90, normalized size = 6.43

$$-\frac{1}{8\left(b^9x^8+8\,ab^8x^7+28\,a^2b^7x^6+56\,a^3b^6x^5+70\,a^4b^5x^4+56\,a^5b^4x^3+28\,a^6b^3x^2+8\,a^7b^2x+a^8b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(b^9*x^8 + 8*a*b^8*x^7 + 28*a^2*b^7*x^6 + 56*a^3*b^6*x^5 + 70*a^4*b^5*
x^4 + 56*a^5*b^4*x^3 + 28*a^6*b^3*x^2 + 8*a^7*b^2*x + a^8*b)
```

mupad [B] time = 2.07, size = 92, normalized size = 6.57

$$-\frac{1}{8\,a^8\,b+64\,a^7\,b^2\,x+224\,a^6\,b^3\,x^2+448\,a^5\,b^4\,x^3+560\,a^4\,b^5\,x^4+448\,a^3\,b^6\,x^5+224\,a^2\,b^7\,x^6+64\,a\,b^8\,x^7+8\,b^9\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

```
[Out] -1/(8*a^8*b + 8*b^9*x^8 + 64*a^7*b^2*x + 64*a*b^8*x^7 + 224*a^6*b^3*x^2 + 4
48*a^5*b^4*x^3 + 560*a^4*b^5*x^4 + 448*a^3*b^6*x^5 + 224*a^2*b^7*x^6)
```

sympy [B] time = 0.54, size = 97, normalized size = 6.93

$$-\frac{1}{8a^8b+64a^7b^2x+224a^6b^3x^2+448a^5b^4x^3+560a^4b^5x^4+448a^3b^6x^5+224a^2b^7x^6+64ab^8x^7+8b^9x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

```
[Out] -1/(8*a**8*b + 64*a**7*b**2*x + 224*a**6*b**3*x**2 + 448*a**5*b**4*x**3 + 5
60*a**4*b**5*x**4 + 448*a**3*b**6*x**5 + 224*a**2*b**7*x**6 + 64*a*b**8*x**
7 + 8*b**9*x**8)
```

3.9 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$

Optimal. Leaf size=84

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{b^3x\left(b^2-3ac\right)^3}{c^3}+\frac{(b+cx)^{10}}{10c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] -((b^3*(b^2 - 3*a*c)^3*x)/c^3) + (3*b^2*(b^2 - 3*a*c)^2*(b + c*x)^4)/(4*c^4
) - (3*b*(b^2 - 3*a*c)*(b + c*x)^7)/(7*c^4) + (b + c*x)^10/(10*c^4)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx=\frac{1}{27}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^3\,dx,x,\frac{b}{c}+x\right)$$

$$=\frac{1}{27}\text{Subst}\left(\int \left(\frac{27\left(-b^3+3abc\right)^3}{c^3}+81\left(b^3-3abc\right)^2x^3-81bc^3\left(b^2-3ac\right.\right.\right.$$

$$=-\frac{b^3\left(b^2-3ac\right)^3x}{c^3}+\frac{3b^2\left(b^2-3ac\right)^2(b+cx)^4}{4c^4}-\frac{3b\left(b^2-3ac\right)(b+cx)^7}{7c^4}$$

Mathematica [A] time = 0.02, size = 159, normalized size = 1.89

$$27a^3b^3x+\frac{81}{2}a^2b^4x^2+\frac{27}{4}b^2x^4\left(a^2c^2+6ab^2c+b^4\right)+\frac{9}{7}bc^3x^7\left(ac+9b^2\right)+9b^2c^2x^6\left(ac+2b^2\right)+\frac{27}{5}b^3cx^5\left(5ac+3b^2\right)+27ab^3x^3\left(ac+b^2\right)+\frac{9}{2}b^2c^4x^8+bc^5x^9+\frac{c^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] 27*a^3*b^3*x + (81*a^2*b^4*x^2)/2 + 27*a*b^3*(b^2 + a*c)*x^3 + (27*b^2*(b^4
 + 6*a*b^2*c + a^2*c^2)*x^4)/4 + (27*b^3*c*(3*b^2 + 5*a*c)*x^5)/5 + 9*b^2*c
^2*(2*b^2 + a*c)*x^6 + (9*b*c^3*(9*b^2 + a*c)*x^7)/7 + (9*b^2*c^4*x^8)/2 +
b*c^5*x^9 + (c^6*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3, x]
```

fricas [B] time = 0.95, size = 166, normalized size = 1.98

$$\frac{1}{10}x^{10}c^6 + x^9c^5b + \frac{9}{2}x^8c^4b^2 + \frac{81}{7}x^7c^3b^3 + \frac{9}{7}x^7c^4ba + 18x^6c^2b^4 + 9x^6c^3b^2a + \frac{81}{5}x^5cb^5 + 27x^5c^2b^3a + \frac{27}{4}x^4b^6 + \frac{81}{2}x^4cb^4a + \frac{27}{4}x^4c^2b^2a^2 + 27x^3b^5a + 27x^3cb^3a^2 + \frac{81}{2}x^2b^4a^2 + 27xb^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^6 + x^9*c^5*b + 9/2*x^8*c^4*b^2 + 81/7*x^7*c^3*b^3 + 9/7*x^7*c^
4*b*a + 18*x^6*c^2*b^4 + 9*x^6*c^3*b^2*a + 81/5*x^5*c*b^5 + 27*x^5*c^2*b^3*
a + 27/4*x^4*b^6 + 81/2*x^4*c*b^4*a + 27/4*x^4*c^2*b^2*a^2 + 27*x^3*b^5*a +
 27*x^3*c*b^3*a^2 + 81/2*x^2*b^4*a^2 + 27*x*b^3*a^3
```

giac [B] time = 0.27, size = 166, normalized size = 1.98

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{9}{2}b^2c^4x^8 + \frac{81}{7}b^3c^3x^7 + \frac{9}{7}abc^4x^7 + 18b^4c^2x^6 + 9ab^2c^3x^6 + \frac{81}{5}b^5cx^5 + 27ab^3c^2x^5 + \frac{27}{4}b^6x^4 + \frac{81}{2}ab^4cx^4 + \frac{27}{4}a^2b^2c^2x^4 + 27ab^5x^3 + 27a^2b^3cx^3 + \frac{81}{2}a^2b^4x^2 + 27a^3b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 9/2*b^2*c^4*x^8 + 81/7*b^3*c^3*x^7 + 9/7*a*b*c^
4*x^7 + 18*b^4*c^2*x^6 + 9*a*b^2*c^3*x^6 + 81/5*b^5*c*x^5 + 27*a*b^3*c^2*x^
5 + 27/4*b^6*x^4 + 81/2*a*b^4*c*x^4 + 27/4*a^2*b^2*c^2*x^4 + 27*a*b^5*x^3 +
 27*a^2*b^3*c*x^3 + 81/2*a^2*b^4*x^2 + 27*a^3*b^3*x
```

maple [B] time = 0.00, size = 295, normalized size = 3.51

$$\frac{c^6x^{10}}{10} + bc^5x^9 + \frac{9b^2c^4x^8}{2} + \frac{81a^2b^4x^2}{2} + 27a^3b^3x + \frac{\left(3abc^4 + 63b^3c^3 + \left(6abc^2 + 18b^3c\right)c^2\right)x^7}{7} + \frac{\left(18ab^2c^3 + 45b^4c^2 + 3\left(6abc^2 + 18b^3c\right)bc + \left(18ab^2c + 9b^4\right)c^2\right)x^6}{6} + \frac{\left(63ab^3c^2 + 3\left(6abc^2 + 18b^3c\right)b^2 + 3\left(18ab^2c + 9b^4\right)bc\right)x^5}{5} + \frac{\left(9a^2b^2c^2 + 54ab^4c + 3\left(6abc^2 + 18b^3c\right)ab + 3\left(18ab^2c + 9b^4\right)b^2\right)x^4}{4} + \frac{\left(27a^2b^3c + 54ab^5 + 3\left(18ab^2c + 9b^4\right)ab\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] 1/10*c^6*x^10+b*c^5*x^9+9/2*b^2*c^4*x^8+1/7*(3*a*b*c^4+63*b^3*c^3+c^2*(6*a*
b*c^2+18*b^3*c))*x^7+1/6*(18*a*b^2*c^3+45*b^4*c^2+3*b*c*(6*a*b*c^2+18*b^3*c
)+c^2*(18*a*b^2*c+9*b^4))*x^6+1/5*(63*a*b^3*c^2+3*b^2*(6*a*b*c^2+18*b^3*c)+
3*b*c*(18*a*b^2*c+9*b^4))*x^5+1/4*(3*a*b*(6*a*b*c^2+18*b^3*c)+3*b^2*(18*a*b
^2*c+9*b^4)+54*b^4*c*a+9*c^2*a^2*b^2)*x^4+1/3*(3*a*b*(18*a*b^2*c+9*b^4)+54*
b^5*a+27*b^3*c*a^2)*x^3+81/2*a^2*b^4*x^2+27*a^3*b^3*x
```

maxima [B] time = 0.68, size = 204, normalized size = 2.43

$$\frac{1}{10}c^6x^{10} + bc^5x^9 + \frac{27}{8}b^2c^4x^8 + \frac{27}{7}b^3c^3x^7 + \frac{27}{4}b^6x^4 + 27a^3b^3x + \frac{27}{4}\left(c^2x^4 + 4bcx^3 + 6b^2x^2\right)a^2b^2 + \frac{9}{10}\left(5c^2x^6 + 18bcx^5\right)b^4 + \frac{9}{70}\left(10c^4x^7 + 70bc^3x^6 + 126b^2c^2x^5 + 210b^4x^3 + 21\left(4c^2x^5 + 15bcx^4\right)b^2\right)ab + \frac{9}{56}\left(7c^4x^8 + 48bc^3x^7 + 84b^2c^2x^6\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^6*x^10 + b*c^5*x^9 + 27/8*b^2*c^4*x^8 + 27/7*b^3*c^3*x^7 + 27/4*b^6*
x^4 + 27*a^3*b^3*x + 27/4*(c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a^2*b^2 + 9/10*
(5*c^2*x^6 + 18*b*c*x^5)*b^4 + 9/70*(10*c^4*x^7 + 70*b*c^3*x^6 + 126*b^2*c^
```

```
2*x^5 + 210*b^4*x^3 + 21*(4*c^2*x^5 + 15*b*c*x^4)*b^2)*a*b + 9/56*(7*c^4*x^
8 + 48*b*c^3*x^7 + 84*b^2*c^2*x^6)*b^2
```

mupad [B] time = 2.08, size = 149, normalized size = 1.77

$$x^4\left(\frac{27\,a^2\,b^2\,c^2}{4}+\frac{81\,a\,b^4\,c}{2}+\frac{27\,b^6}{4}\right)+\frac{c^6\,x^{10}}{10}+27\,a^3\,b^3\,x+b\,c^5\,x^9+\frac{81\,a^2\,b^4\,x^2}{2}+\frac{9\,b^2\,c^4\,x^8}{2}+9\,b^2\,c^2\,x^6\,\left(2\,b^2+a\,c\right)+27\,a\,b^3\,x^3\,\left(b^2+a\,c\right)+\frac{27\,b^3\,c\,x^5\,\left(3\,b^2+5\,a\,c\right)}{5}+\frac{9\,b\,c^3\,x^7\,\left(9\,b^2+a\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] x^4*((27*b^6)/4 + (27*a^2*b^2*c^2)/4 + (81*a*b^4*c)/2) + (c^6*x^10)/10 + 27
*a^3*b^3*x + b*c^5*x^9 + (81*a^2*b^4*x^2)/2 + (9*b^2*c^4*x^8)/2 + 9*b^2*c^2
*x^6*(a*c + 2*b^2) + 27*a*b^3*x^3*(a*c + b^2) + (27*b^3*c*x^5*(5*a*c + 3*b^
2))/5 + (9*b*c^3*x^7*(a*c + 9*b^2))/7
```

sympy [B] time = 0.10, size = 175, normalized size = 2.08

$$27a^3b^3x+\frac{81a^2b^4x^2}{2}+\frac{9b^2c^4x^8}{2}+bc^5x^9+\frac{c^6x^{10}}{10}+x^7\left(\frac{9abc^4}{7}+\frac{81b^3c^3}{7}\right)+x^6\left(9ab^2c^3+18b^4c^2\right)+x^5\left(27ab^3c^2+\frac{81b^5c}{5}\right)+x^4\left(\frac{27a^2b^2c^2}{4}+\frac{81ab^4c}{2}+\frac{27b^6}{4}\right)+x^3\left(27a^2b^3c+27ab^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] 27*a**3*b**3*x + 81*a**2*b**4*x**2/2 + 9*b**2*c**4*x**8/2 + b*c**5*x**9 + c
**6*x**10/10 + x**7*(9*a*b*c**4/7 + 81*b**3*c**3/7) + x**6*(9*a*b**2*c**3 +
 18*b**4*c**2) + x**5*(27*a*b**3*c**2 + 81*b**5*c/5) + x**4*(27*a**2*b**2*c
**2/4 + 81*a*b**4*c/2 + 27*b**6/4) + x**3*(27*a**2*b**3*c + 27*a*b**5)
```

3.10 $\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx$

Optimal. Leaf size=56

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2060, 194}

$$-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{b^2x\left(b^2-3ac\right)^2}{c^2}+\frac{(b+cx)^7}{7c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] (b^2*(b^2 - 3*a*c)^2*x)/c^2 - (b*(b^2 - 3*a*c)*(b + c*x)^4)/(2*c^3) + (b +
c*x)^7/(7*c^3)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2060

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3)^(p_), x_Symbol] :> D
ist[1/3^p, Subst[Int[Simp[(3*a*c - b^2)/c + (c^2*x^3)/b, x]^p, x], x, c/(3*
d) + x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[p, 0] && EqQ[c^2 - 3*b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2 \, dx &= \frac{1}{9}\text{Subst}\left(\int \left(3b\left(3a-\frac{b^2}{c}\right)+3c^2x^3\right)^2 \, dx, x, \frac{b}{c}+x\right)\\ &= \frac{1}{9}\text{Subst}\left(\int \left(\frac{9\left(-b^3+3abc\right)^2}{c^2}-18bc\left(b^2-3ac\right)x^3+9c^4x^6\right) dx, x, \frac{b}{c}+x\right)\\ &= \frac{b^2\left(b^2-3ac\right)^2 x}{c^2}-\frac{b\left(b^2-3ac\right)(b+cx)^4}{2c^3}+\frac{(b+cx)^7}{7c^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.46

$$9a^2b^2x+9ab^3x^2+\frac{3}{2}bcx^4\left(ac+3b^2\right)+3b^2x^3\left(2ac+b^2\right)+3b^2c^2x^5+bc^3x^6+\frac{c^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] 9*a^2*b^2*x + 9*a*b^3*x^2 + 3*b^2*(b^2 + 2*a*c)*x^3 + (3*b*c*(3*b^2 + a*c)*
x^4)/2 + 3*b^2*c^2*x^5 + b*c^3*x^6 + (c^4*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2, x]
```

fricas [A] time = 0.63, size = 83, normalized size = 1.48

$$\frac{1}{7}x^7c^4 + x^6c^3b + 3x^5c^2b^2 + \frac{9}{2}x^4cb^3 + \frac{3}{2}x^4c^2ba + 3x^3b^4 + 6x^3cb^2a + 9x^2b^3a + 9xb^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^4 + x^6*c^3*b + 3*x^5*c^2*b^2 + 9/2*x^4*c*b^3 + 3/2*x^4*c^2*b*a +
 3*x^3*b^4 + 6*x^3*c*b^2*a + 9*x^2*b^3*a + 9*x*b^2*a^2
```

giac [A] time = 0.34, size = 83, normalized size = 1.48

$$\frac{1}{7}c^4x^7 + bc^3x^6 + 3\,b^2c^2x^5 + \frac{9}{2}\,b^3cx^4 + \frac{3}{2}\,abc^2x^4 + 3\,b^4x^3 + 6\,ab^2cx^3 + 9\,ab^3x^2 + 9\,a^2b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 3*b^2*c^2*x^5 + 9/2*b^3*c*x^4 + 3/2*a*b*c^2*x^4 +
 3*b^4*x^3 + 6*a*b^2*c*x^3 + 9*a*b^3*x^2 + 9*a^2*b^2*x
```

maple [A] time = 0.00, size = 84, normalized size = 1.50

$$\frac{c^4x^7}{7} + b\,c^3x^6 + 3b^2c^2x^5 + 9a\,b^3x^2 + 9a^2b^2x + \frac{\left(6ab\,c^2 + 18b^3c\right)x^4}{4} + \frac{\left(18a\,b^2c + 9b^4\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] 1/7*c^4*x^7+b*c^3*x^6+3*b^2*c^2*x^5+1/4*(6*a*b*c^2+18*b^3*c)*x^4+1/3*(18*a*
b^2*c+9*b^4)*x^3+9*a*b^3*x^2+9*a^2*b^2*x
```

maxima [A] time = 0.64, size = 93, normalized size = 1.66

$$\frac{1}{7}\,c^4x^7 + bc^3x^6 + \frac{9}{5}\,b^2c^2x^5 + 3\,b^4x^3 + 9\,a^2b^2x + \frac{3}{2}\left(c^2x^4 + 4\,bcx^3 + 6\,b^2x^2\right)ab + \frac{3}{10}\left(4\,c^2x^5 + 15\,bcx^4\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^4*x^7 + b*c^3*x^6 + 9/5*b^2*c^2*x^5 + 3*b^4*x^3 + 9*a^2*b^2*x + 3/2*(
c^2*x^4 + 4*b*c*x^3 + 6*b^2*x^2)*a*b + 3/10*(4*c^2*x^5 + 15*b*c*x^4)*b^2
```

mupad [B] time = 0.04, size = 79, normalized size = 1.41

$$x^3\,\left(3\,b^4 + 6\,a\,c\,b^2\right) + \frac{c^4\,x^7}{7} + 9\,a^2\,b^2\,x + 9\,a\,b^3\,x^2 + b\,c^3\,x^6 + 3\,b^2\,c^2\,x^5 + \frac{3\,b\,c\,x^4\,\left(3\,b^2 + a\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] x^3*(3*b^4 + 6*a*b^2*c) + (c^4*x^7)/7 + 9*a^2*b^2*x + 9*a*b^3*x^2 + b*c^3*x
^6 + 3*b^2*c^2*x^5 + (3*b*c*x^4*(a*c + 3*b^2))/2
```

sympy [**A**] time = 0.08, size = 87, normalized size = 1.55

$$9a^2b^2x + 9ab^3x^2 + 3b^2c^2x^5 + bc^3x^6 + \frac{c^4x^7}{7} + x^4\left(\frac{3abc^2}{2} + \frac{9b^3c}{2}\right) + x^3\left(6ab^2c + 3b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**2*c**2*x**5 + b*c**3*x**6 + c**4*x**7/
7 + x**4*(3*a*b*c**2/2 + 9*b**3*c/2) + x**3*(6*a*b**2*c + 3*b**4)
```

3.11 $\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx$

Optimal. Leaf size=32

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

Rubi steps

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx = 3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] 3*a*b*x + (3*b^2*x^2)/2 + b*c*x^3 + (c^2*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3,x]
```

```
[Out] IntegrateAlgebraic[3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3, x]
```

fricas [A] time = 0.76, size = 28, normalized size = 0.88

$$\frac{1}{4}x^4c^2 + x^3cb + \frac{3}{2}x^2b^2 + 3xba$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c^2 + x^3*c*b + 3/2*x^2*b^2 + 3*x*b*a
```

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{4}c^2x^4 + bcx^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{4}c^2x^4 + bc\,x^3 + \frac{3}{2}b^2x^2 + 3abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x)
```

```
[Out] 3*a*b*x+3/2*b^2*x^2+b*c*x^3+1/4*c^2*x^4
```

maxima [A] time = 0.55, size = 28, normalized size = 0.88

$$\frac{1}{4}\,c^2x^4 + bcx^3 + \frac{3}{2}\,b^2x^2 + 3\,abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^3 + 3/2*b^2*x^2 + 3*a*b*x
```

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{3\,b^2\,x^2}{2} + b\,c\,x^3 + 3\,a\,b\,x + \frac{c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2,x)
```

```
[Out] (3*b^2*x^2)/2 + (c^2*x^4)/4 + 3*a*b*x + b*c*x^3
```

sympy [A] time = 0.06, size = 31, normalized size = 0.97

$$3abx + \frac{3b^2x^2}{2} + bcx^3 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b,x)
```

```
[Out] 3*a*b*x + 3*b**2*x**2/2 + b*c*x**3 + c**2*x**4/4
```

3.12 $\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Rubi [A] time = 0.31, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2067, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}+\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3))]/(
Sqrt[3]*b^(2/3)*(b^2 - 3*a*c)^(2/3))) + Log[b - b^(1/3)*(b^2 - 3*a*c)^(1/3)
 + c*x]/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) +
 b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]/(6*b^(2/3)*(b^2
 - 3*a*c)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,dx, x, \frac{b}{c}+x\right)$$

$$= \frac{c^{2/3}\,\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}+c^{2/3}x}\,dx, x, \frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{c^{2/3}\,\text{Subst}\left(\int \frac{-\frac{2\sqrt[3]{b}\,\sqrt[3]{b^2-3a}}{\sqrt[3]{c}}}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\,\sqrt[3]{c}}\right.}{3b^{2/3}\left(b^2-3\right.}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}\,\sqrt[3]{c}\,\sqrt[3]{b^2-3ac}+2c^{4/3}x}{\frac{b^{2/3}\left(b^2-3ac\right)^{2/3}}{c^{2/3}}+\sqrt[3]{b}\,\sqrt[3]{c}\,\sqrt[3]{b^2-3ac}\,x+c^{4/3}}\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-3ac\right)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}\,(\right.}{6b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\left(b^2-3ac\right)^{2/3}} + \frac{\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}} - \frac{\log\left(b^{2/3}\left(b^2-\right.\right.}{}$$

Mathematica [C] time = 0.02, size = 63, normalized size = 0.34

$$\frac{1}{3}\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b
*c*#1 + c^2*#1^2) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-1), x]
```

fricas [B] time = 1.11, size = 387, normalized size = 2.06

$$-\frac{2\sqrt{3}\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{6}}\left(b^{3}-3abc\right)\arctan\left(\frac{2\sqrt{3}\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{3}}\left(b^{3}-3abc\right)}{3\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{5}{6}}}\right)+\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{2}{3}}\log\left(-b^{5}+3ab^{3}c-\left(b^{3}c^{2}-3abc^{3}\right)x^{2}-2\left(b^{4}c-3ab^{2}c^{2}\right)x-\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{2}{3}}(cx+b)-\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{3}}\left(b^{3}-3abc\right)\right)-2\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{2}{3}}\log\left(-b^{4}+3ab^{2}c-\left(b^{3}c-3abc^{2}\right)x+\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{2}{3}}\right)}{6\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*(b^3 - 3*a*b*c)*arc
tan(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt
(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4
*c + 9*a^2*b^2*c^2)^(5/6)) + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b
^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b
^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*
b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) - 2*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)
*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^
2*c^2)^(2/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

giac [A] time = 0.36, size = 212, normalized size = 1.13

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^{3}+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^{3}+3abc\right)^{\frac{1}{3}}}\right)}{3\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^{3}+3abc\right)^{\frac{1}{3}}\right)^{2}+4\left(cx+b+\left(-b^{3}+3abc\right)^{\frac{1}{3}}\right)^{2}\right)}{6\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{3}}}+\frac{\log\left(\left|cx+b+\left(-b^{3}+3abc\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{6}-6ab^{4}c+9a^{2}b^{2}c^{2}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan((sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3
))/(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1
/3) - 1/6*log(4*(sqrt(3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^
2 + 4*(c*x + b + (-b^3 + 3*a*b*c)^(1/3))^2)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^
2)^(1/3) + 1/3*log(abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3)))/(b^6 - 6*a*b^4*c
+ 9*a^2*b^2*c^2)^(1/3)
```

maple [C] time = 0.00, size = 57, normalized size = 0.30

$$\frac{\ln\left(-\mathrm{RootOf}\left(c^{2}\_Z^{3}+3bc\_Z^{2}+3b^{2}\_Z+3ab\right)+x\right)}{3\,\mathrm{RootOf}\left(c^{2}\_Z^{3}+3bc\_Z^{2}+3b^{2}\_Z+3ab\right)^{2}c^{2}+6\,\mathrm{RootOf}\left(c^{2}\_Z^{3}+3bc\_Z^{2}+3b^{2}\_Z+3ab\right)bc+3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

```
[Out] 1/3*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3
*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^{2}x^{3}+3\,bcx^{2}+3\,b^{2}x+3\,ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="maxima")
```

```
[Out] integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

mupad [B] time = 0.49, size = 174, normalized size = 0.93

$$\frac{\ln\left(b+b^{1/3}\left(3ac-b^2\right)^{1/3}+cx\right)}{3b^{2/3}\left(3ac-b^2\right)^{2/3}}+\frac{\ln\left(3bc^3+3c^4x+\frac{3b^{1/3}c^3\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(3ac-b^2\right)^{1/3}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6b^{2/3}\left(3ac-b^2\right)^{2/3}}-\frac{\ln\left(3bc^3+3c^4x-\frac{3b^{1/3}c^3\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(3ac-b^2\right)^{1/3}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6b^{2/3}\left(3ac-b^2\right)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

```
[Out] log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x)/(3*b^(2/3)*(3*a*c - b^2)^(2/3))
+ (log(3*b*c^3 + 3*c^4*x + (3*b^(1/3)*c^3*(3^(1/2)*1i - 1)*(3*a*c - b^2)^(1
/3))/2)*(3^(1/2)*1i - 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3)) - (log(3*b*c^3 +
3*c^4*x - (3*b^(1/3)*c^3*(3^(1/2)*1i + 1)*(3*a*c - b^2)^(1/3))/2)*(3^(1/2)*
1i + 1))/(6*b^(2/3)*(3*a*c - b^2)^(2/3))
```

sympy [A] time = 0.41, size = 53, normalized size = 0.28

$$\operatorname{RootSum}\left(t^3\left(243a^2b^2c^2-162ab^4c+27b^6\right)-1,\left(t\mapsto t\log\left(x+\frac{9tabc-3tb^3+b}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

```
[Out] RootSum(_t**3*(243*a**2*b**2*c**2 - 162*a*b**4*c + 27*b**6) - 1, Lambda(_t,
 _t*log(x + (9*_t*a*b*c - 3*_t*b**3 + b)/c)))
```

3.13 $\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\,\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

Rubi [A] time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}+\frac{c\log\left(\sqrt[3]{b}\,c\sqrt[3]{b^2-3ac}\,\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}-\frac{2c\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}+\frac{2c\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -(c*(b/c + x))/(3*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3))
+ (2*c*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(Sqrt[3]*b^(1/3
))])/(3*Sqrt[3]*b^(5/3)*(b^2 - 3*a*c)^(5/3)) - (2*c*Log[b - b^(1/3)*(b^2 -
3*a*c)^(1/3) + c*x])/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3)) + (c*Log[b^(2/3)*(b^2
- 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c + x)^2]
)/(9*b^(5/3)*(b^2 - 3*a*c)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^2}\,dx, x, \frac{b}{c}+x\right)$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b\left(3a-\frac{b^2}{c}\right)+c^2x^3}\,\ldots\right.}{3b\left(b^2-3ac\right)}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{\left(2c^{5/3}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}{\sqrt[3]{c}}\ldots}\right.}{9b^{5/3}\left(b^2-3a\ldots\right.}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3a\ldots}\right.\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} - \frac{2c\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3a\ldots}\right.\right.}{9b^{5/3}\left(b^2-3ac\right)^{5/3}}$$

$$= -\frac{c\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)} + \frac{2c\tan^{-1}\left(\frac{1+\frac{2(b+cx)}{\sqrt[3]{b}\,\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}\left(b^2-3ac\right)^{5/3}} - \frac{2\ldots}{\ldots}$$

Mathematica [C]   time = 0.07, size = 112, normalized size = 0.46

$$-\frac{2c\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]+\frac{3(b+cx)}{3ab+x\left(3b^2+3bcx+c^2x^2\right)}}{9\left(b^3-3abc\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] -1/9*((3*(b + c*x))/(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2)) + 2*c*RootSum[3
*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c
^2*#1^2) & ])/(b^3 - 3*a*b*c)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-2), x]
```

fricas [B]   time = 1.10, size = 704, normalized size = 2.87

$$-\frac{3b^7-18ab^5c+27a^2b^3c^2-2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{6}}\left(3ab^4c-9a^2b^2c^2+\left(b^3c^3-3abc^4\right)x^3+3\left(b^4c^2-3ab^2c^3\right)x^2+3\left(b^5c-3ab^3c^2\right)x\right)\arctan\left(\frac{2\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)+\sqrt{3}\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)}{3\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{5}{6}}}\right)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\left(c^3x^3+3bc^2x^2+3b^2cx+3abc\right)\log\left(-b^5+3ab^3c-\left(b^3c^2-3abc^3\right)x^2-2\left(b^4c-3ab^2c^2\right)x-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}(cx+b)-\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}\left(b^3-3abc\right)\right)+2\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\left(c^3x^3+3bc^2x^2+3b^2cx+3abc\right)\log\left(-b^4+3ab^2c-\left(b^3c-3abc^2\right)x+\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{2}{3}}\right)+3\left(b^6c-6ab^4c^2+9a^2b^2c^3\right)x}{9\left(3ab^{10}-27a^2b^8c+81a^3b^6c^2-81a^4b^4c^3+\left(b^9c^2-9ab^7c^3+27a^2b^5c^4-27a^3b^3c^5\right)x^3+3\left(b^{10}c-9ab^8c^2+27a^2b^6c^3-27a^3b^4c^4\right)x^2+3\left(b^{11}-9ab^9c+27a^2b^7c^2-27a^3b^5c^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b^7 - 18*a*b^5*c + 27*a^2*b^3*c^2 - 2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*
a^2*b^2*c^2)^(1/6)*(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 +
 3*(b^4*c^2 - 3*a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x)*arctan(1/3*(2*s
qrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(3)*(b^6 - 6
*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*c + 9*a^2*b
^2*c^2)^(5/6)) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2
*x^2 + 3*b^2*c*x + 3*a*b*c)*log(-b^5 + 3*a*b^3*c - (b^3*c^2 - 3*a*b*c^3)*x^
2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*
x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c)) + 2*(b^6
- 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + 3*a
*b*c)*log(-b^4 + 3*a*b^2*c - (b^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a
^2*b^2*c^2)^(2/3)) + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3)*x)/(3*a*b^10 -
 27*a^2*b^8*c + 81*a^3*b^6*c^2 - 81*a^4*b^4*c^3 + (b^9*c^2 - 9*a*b^7*c^3 +
27*a^2*b^5*c^4 - 27*a^3*b^3*c^5)*x^3 + 3*(b^10*c - 9*a*b^8*c^2 + 27*a^2*b^6
*c^3 - 27*a^3*b^4*c^4)*x^2 + 3*(b^11 - 9*a*b^9*c + 27*a^2*b^7*c^2 - 27*a^3*
b^5*c^3)*x)
```

giac [A]   time = 0.36, size = 289, normalized size = 1.18

$$-\frac{2\sqrt{3}\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)-\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^3}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(\left|cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right|\right)}{9\left(b^3-3abc\right)}-\frac{cx+b}{3\left(c^2x^3+3bcx^2+3b^2x+3ab\right)\left(b^3-3abc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
```

```
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^3/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^3 - 3*a*b*c) - 1/3*(c*x + b)/((c
^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)*(b^3 - 3*a*b*c))
```

maple [C] time = 0.02, size = 136, normalized size = 0.56

$$\frac{2c\ln\left(-\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)+x\right)}{9\left(3ac-b^2\right)b\left(\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)^2c^2+2\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)bc+b^2\right)}+\frac{\frac{cx}{3(3ac-b^2)b}+\frac{1}{9ac-3b^2}}{c^2x^3+3bc\,x^2+3b^2x+3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

```
[Out] (1/3*c/b/(3*a*c-b^2)*x+1/3/(3*a*c-b^2))/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)+2
/9*c/b/(3*a*c-b^2)*sum(1/(_R^2*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c
^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\frac{1}{3}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right|\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}\right)c}{3\left(b^3-3abc\right)}-\frac{cx+b}{3\left(3ab^4-9a^2b^2c+\left(b^3c^2-3abc^3\right)x^3+3\left(b^4c-3ab^2c^2\right)x^2+3\left(b^5-3ab^3c\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="maxima")
```

```
[Out] -2/3*c*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*b
*c) - 1/3*(c*x + b)/(3*a*b^4 - 9*a^2*b^2*c + (b^3*c^2 - 3*a*b*c^3)*x^3 + 3*
(b^4*c - 3*a*b^2*c^2)*x^2 + 3*(b^5 - 3*a*b^3*c)*x)
```

mupad [B] time = 2.65, size = 247, normalized size = 1.01

$$\frac{\frac{1}{3(3ac-b^2)}+\frac{cx}{3b(3ac-b^2)}}{3b^2x+3bcx^2+3ab+c^2x^3}+\frac{2c\ln\left(b+b^{1/3}\left(3ac-b^2\right)^{1/3}+cx\right)}{9b^{5/3}\left(3ac-b^2\right)^{5/3}}-\frac{\ln\left(2b-b^{1/3}\left(3ac-b^2\right)^{1/3}+2cx-\sqrt{3}\,b^{1/3}\left(3ac-b^2\right)^{1/3}1\mathrm{i}\right)\left(c+\sqrt{3}\,c\,1\mathrm{i}\right)}{9b^{5/3}\left(3ac-b^2\right)^{5/3}}-\frac{\ln\left(2b-b^{1/3}\left(3ac-b^2\right)^{1/3}+2cx+\sqrt{3}\,b^{1/3}\left(3ac-b^2\right)^{1/3}1\mathrm{i}\right)\left(c-\sqrt{3}\,c\,1\mathrm{i}\right)}{9b^{5/3}\left(3ac-b^2\right)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

```
[Out] (1/(3*(3*a*c - b^2)) + (c*x)/(3*b*(3*a*c - b^2)))/(3*a*b + 3*b^2*x + c^2*x^
3 + 3*b*c*x^2) + (2*c*log(b + b^(1/3)*(3*a*c - b^2)^(1/3) + c*x))/(9*b^(5/3
)*(3*a*c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x - 3
^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c + 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*
c - b^2)^(5/3)) - (log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2*c*x + 3^(1/2)*
b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*(c - 3^(1/2)*c*1i))/(9*b^(5/3)*(3*a*c - b^2
)^(5/3))
```

sympy [A] time = 1.22, size = 192, normalized size = 0.78

$$\frac{b+cx}{27a^2b^2c-9ab^4+x^3\left(9abc^3-3b^3c^2\right)+x^2\left(27ab^2c^2-9b^4c\right)+x\left(27ab^3c-9b^5\right)}+\mathrm{RootSum}\left(t^3\left(177147a^5b^5c^5-295245a^4b^7c^4+196830a^3b^9c^3-65610a^2b^{11}c^2+10935ab^{13}c-729b^{15}\right)-8c^3,\left(t\mapsto t\log\left(x+\frac{81ta^2b^2c^2-54tab^4c+9tb^6+2bc}{2c^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

```
[Out] (b + c*x)/(27*a**2*b**2*c - 9*a*b**4 + x**3*(9*a*b*c**3 - 3*b**3*c**2) + x*
*2*(27*a*b**2*c**2 - 9*b**4*c) + x*(27*a*b**3*c - 9*b**5)) + RootSum(_t**3*
(177147*a**5*b**5*c**5 - 295245*a**4*b**7*c**4 + 196830*a**3*b**9*c**3 - 65
610*a**2*b**11*c**2 + 10935*a*b**13*c - 729*b**15) - 8*c**3, Lambda(_t, _t*
log(x + (81*_t*a**2*b**2*c**2 - 54*_t*a*b**4*c + 9*_t*b**6 + 2*b*c)/(2*c**2
))))
```

3.14 $\int \frac{1}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

Optimal. Leaf size=305

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2\right.}}{27b^{8/3}\left(b^2\right.}$$

Rubi [A] time = 0.30, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2067, 199, 200, 31, 634, 617, 204, 628}

$$\frac{5c^2\left(\frac{b}{c}+x\right)}{18b^2\left(b^2-3ac\right)^2\left(3ab+3b^2x+3bcx^2+c^2x^3\right)}-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c^2\log\left(-\sqrt[3]{b}\sqrt[3]{b^2-3ac}+b+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}}-\frac{5c^2\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}+c^2\left(\frac{b}{c}+x\right)^2\right)}{54b^{8/3}\left(b^2-3ac\right)^{8/3}}-\frac{5c^2\tan^{-1}\left(\frac{\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] -(c*(b/c + x))/(6*b*(b^2 - 3*a*c)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2
) + (5*c^2*(b/c + x))/(18*b^2*(b^2 - 3*a*c)^2*(3*a*b + 3*b^2*x + 3*b*c*x^2
+ c^2*x^3)) - (5*c^2*ArcTan[(b^(1/3) + (2*(b + c*x))/(b^2 - 3*a*c)^(1/3))/(
Sqrt[3]*b^(1/3))])/(9*Sqrt[3]*b^(8/3)*(b^2 - 3*a*c)^(8/3)) + (5*c^2*Log[b -
 b^(1/3)*(b^2 - 3*a*c)^(1/3) + c*x])/(27*b^(8/3)*(b^2 - 3*a*c)^(8/3)) - (5*
c^2*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c +
x) + c^2*(b/c + x)^2])/(54*b^(8/3)*(b^2 - 3*a*c)^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx=\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2x^3\right)^3}\,dx,x,\frac{b}{c}+x\right)$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}-\frac{(5c)\,\text{Subst}\left(\int \frac{1}{\left(b\left(3a-\frac{b^2}{c}\right)+\cdots\right.}\right.}{6b\left(b^2-\cdots\right.}$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c(b\cdots}{18b^2\left(b^2-3ac\right)^2\left(3ab+\cdots\right.}$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c(b\cdots}{18b^2\left(b^2-3ac\right)^2\left(3ab+\cdots\right.}$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c(b\cdots}{18b^2\left(b^2-3ac\right)^2\left(3ab+\cdots\right.}$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c(b\cdots}{18b^2\left(b^2-3ac\right)^2\left(3ab+\cdots\right.}$$

$$=-\frac{c\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}+\frac{5c(b\cdots}{18b^2\left(b^2-3ac\right)^2\left(3ab+\cdots\right.}$$

Mathematica [C] time = 0.09, size = 149, normalized size = 0.49

$$\frac{10c^2\text{RootSum}\left[\#1^3c^2+3\#1^2bc+3\#1b^2+3ab\&,\frac{\log(x-\#1)}{\#1^2c^2+2\#1bc+b^2}\&\right]-\frac{3(b+cx)\left(-3bc\left(8a+5cx^2\right)+3b^3-15b^2cx-5c^3x^3\right)}{\left(3ab+x\left(3b^2+3bcx+c^2x^2\right)\right)^2}}{54\left(b^3-3abc\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] ((-3*(b + c*x)*(3*b^3 - 15*b^2*c*x - 5*c^3*x^3 - 3*b*c*(8*a + 5*c*x^2)))/(3
*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2 + 10*c^2*RootSum[3*a*b + 3*b^2*#1 +
 3*b*c*#1^2 + c^2*#1^3 & , Log[x - #1]/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(54
*(b^3 - 3*a*b*c)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^(-3), x]
```

fricas [B]   time = 1.17, size = 1268, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(9*b^10 - 126*a*b^8*c + 513*a^2*b^6*c^2 - 648*a^3*b^4*c^3 - 15*(b^6*c
^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^4 - 60*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b
^3*c^5)*x^3 - 90*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^2 + 10*sqrt(3)*(
9*a^2*b^5*c^2 - 27*a^3*b^3*c^3 + (b^3*c^6 - 3*a*b*c^7)*x^6 + 6*(b^4*c^5 - 3
*a*b^2*c^6)*x^5 + 15*(b^5*c^4 - 3*a*b^3*c^5)*x^4 + 6*(3*b^6*c^3 - 8*a*b^4*c
^4 - 3*a^2*b^2*c^5)*x^3 + 9*(b^7*c^2 - a*b^5*c^3 - 6*a^2*b^3*c^4)*x^2 + 18*
(a*b^6*c^2 - 3*a^2*b^4*c^3)*x)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/6)*arct
an(1/3*(2*sqrt(3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) + sqrt(
3)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3)*(b^3 - 3*a*b*c))/(b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(5/6)) + 5*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*
a*b^3*c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*
a*b^2*c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^5 + 3*a*b^3*
c - (b^3*c^2 - 3*a*b*c^3)*x^2 - 2*(b^4*c - 3*a*b^2*c^2)*x - (b^6 - 6*a*b^4*
c + 9*a^2*b^2*c^2)^(2/3)*(c*x + b) - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(1/3
)*(b^3 - 3*a*b*c)) - 10*(c^6*x^6 + 6*b*c^5*x^5 + 15*b^2*c^4*x^4 + 18*a*b^3*
c^2*x + 9*a^2*b^2*c^2 + 6*(3*b^3*c^3 + a*b*c^4)*x^3 + 9*(b^4*c^2 + 2*a*b^2*
c^3)*x^2)*(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)*log(-b^4 + 3*a*b^2*c - (b
^3*c - 3*a*b*c^2)*x + (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)^(2/3)) - 36*(b^9*c
- 4*a*b^7*c^2 - 3*a^2*b^5*c^3 + 18*a^3*b^3*c^4)*x)/(9*a^2*b^14 - 108*a^3*b^
12*c + 486*a^4*b^10*c^2 - 972*a^5*b^8*c^3 + 729*a^6*b^6*c^4 + (b^12*c^4 - 1
2*a*b^10*c^5 + 54*a^2*b^8*c^6 - 108*a^3*b^6*c^7 + 81*a^4*b^4*c^8)*x^6 + 6*(
b^13*c^3 - 12*a*b^11*c^4 + 54*a^2*b^9*c^5 - 108*a^3*b^7*c^6 + 81*a^4*b^5*c^
7)*x^5 + 15*(b^14*c^2 - 12*a*b^12*c^3 + 54*a^2*b^10*c^4 - 108*a^3*b^8*c^5 +
 81*a^4*b^6*c^6)*x^4 + 6*(3*b^15*c - 35*a*b^13*c^2 + 150*a^2*b^11*c^3 - 270
*a^3*b^9*c^4 + 135*a^4*b^7*c^5 + 81*a^5*b^5*c^6)*x^3 + 9*(b^16 - 10*a*b^14*
c + 30*a^2*b^12*c^2 - 135*a^4*b^8*c^4 + 162*a^5*b^6*c^5)*x^2 + 18*(a*b^15 -
 12*a^2*b^13*c + 54*a^3*b^11*c^2 - 108*a^4*b^9*c^3 + 81*a^5*b^7*c^4)*x)
```

giac [A]   time = 0.40, size = 366, normalized size = 1.20

$$\frac{5\left(2\sqrt{3}\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)-\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)+2\left(\frac{c^6}{b^6-6ab^4c+9a^2b^2c^2}\right)^{\frac{1}{3}}\log\left(\left|cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right|\right)\right)}{54\left(b^6-6ab^4c+9a^2b^2c^2\right)}+\frac{5c^4x^4+20bc^3x^3+30b^2c^2x^2+12b^3cx+24abc^2x-3b^4+24ab^2c}{18\left(b^6-6ab^4c+9a^2b^2c^2\right)\left(c^2x^3+3bcx^2+3b^2x+3ab\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="giac")
```

```
[Out] 5/54*(2*sqrt(3)*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*arctan((sqrt(
3)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))/(c*x + b + (-b^3 + 3*a
*b*c)^(1/3))) - (c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(4*(sqrt(3
)*c*x + sqrt(3)*b - sqrt(3)*(-b^3 + 3*a*b*c)^(1/3))^2 + 4*(c*x + b + (-b^3
+ 3*a*b*c)^(1/3))^2) + 2*(c^6/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2))^(1/3)*log(
abs(c*x + b + (-b^3 + 3*a*b*c)^(1/3))))/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2) +
 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 + 12*b^3*c*x + 24*a*b*c^2*
x - 3*b^4 + 24*a*b^2*c)/((b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)*(c^2*x^3 + 3*b*c
*x^2 + 3*b^2*x + 3*a*b)^2)
```

maple [C]   time = 0.02, size = 276, normalized size = 0.90

$$\frac{5c^2\ln\left(-\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)+x\right)}{27\left(9a^2c^2-6ab^2c+b^4\right)b^2\left(\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)^2c^2+2\,\mathrm{RootOf}\left(c^2\_Z^3+3bc\_Z^2+3b^2\_Z+3ab\right)bc+b^2\right)}+\frac{\frac{5c^4x^4}{18\left(9a^2c^2-6ab^2c+b^4\right)b^2}+\frac{10c^3x^3}{9\left(9a^2c^2-6ab^2c+b^4\right)b}+\frac{5c^2x^2}{3\left(9a^2c^2-6ab^2c+b^4\right)}+\frac{2\left(2ac+b^2\right)cx}{3\left(9a^2c^2-6ab^2c+b^4\right)b}+\frac{8ac-b^2}{54a^2c^2-36ab^2c+6b^4}}{\left(c^2x^3+3bcx^2+3b^2x+3ab\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

```
[Out] (5/18*c^4/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^4+10/9/b*c^3/(9*a^2*c^2-6*a*b^2*c
+b^4)*x^3+5/3*c^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+2/3/b*(2*a*c+b^2)*c/(9*a^2*
c^2-6*a*b^2*c+b^4)*x+1/6*(8*a*c-b^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2*x^3+3*
b*c*x^2+3*b^2*x+3*a*b)^2+5/27*c^2/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*sum(1/(_R^2
*c^2+2*_R*b*c+b^2)*ln(-_R+x),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\frac{5}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}}{cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}}\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}-\frac{\log\left(4\left(\sqrt{3}cx+\sqrt{3}b-\sqrt{3}\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2+4\left(cx+b+\left(-b^3+3abc\right)^{\frac{1}{3}}\right)^2\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}+\frac{2\log\left(\left|cx+b-\left(-b^3+3abc\right)^{\frac{1}{3}}\right|\right)}{\left(b^6-6ab^4c+9a^2b^2c^2\right)^{\frac{1}{3}}}\right)c^2}{9\left(b^6-6ab^4c+9a^2b^2c^2\right)}+\frac{5c^4x^4+20bc^3x^3+30b^2c^2x^2-3b^4+24ab^2c+12\left(b^3c+2abc^2\right)x}{18\left(9a^2b^8-54a^3b^6c+81a^4b^4c^2+\left(b^6c^4-6ab^4c^5+9a^2b^2c^6\right)x^6+6\left(b^7c^3-6ab^5c^4+9a^2b^3c^5\right)x^5+15\left(b^8c^2-6ab^6c^3+9a^2b^4c^4\right)x^4+6\left(3b^9c-17ab^7c^2+21a^2b^5c^3+9a^3b^3c^4\right)x^3+9\left(b^{10}-4ab^8c-3a^2b^6c^2+18a^3b^4c^3\right)x^2+18\left(ab^9-6a^2b^7c+9a^3b^5c^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="maxima")
```

```
[Out] 5/9*c^2*integrate(1/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*
b^4*c + 9*a^2*b^2*c^2) + 1/18*(5*c^4*x^4 + 20*b*c^3*x^3 + 30*b^2*c^2*x^2 -
3*b^4 + 24*a*b^2*c + 12*(b^3*c + 2*a*b*c^2)*x)/(9*a^2*b^8 - 54*a^3*b^6*c +
81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 -
 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c
^4)*x^4 + 6*(3*b^9*c - 17*a*b^7*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 +
 9*(b^10 - 4*a*b^8*c - 3*a^2*b^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*
a^2*b^7*c + 9*a^3*b^5*c^2)*x)
```

mupad [B] time = 2.98, size = 483, normalized size = 1.58

$$\frac{\frac{8ac-b^2}{6\left(9a^2c^2-6ab^2c+b^4\right)}+\frac{5c^2x^2}{3\left(9a^2c^2-6ab^2c+b^4\right)}+\frac{10c^3x^3}{9b\left(9a^2c^2-6ab^2c+b^4\right)}+\frac{5c^4x^4}{18b^2\left(9a^2c^2-6ab^2c+b^4\right)}+\frac{2cx\left(b^2+2ac\right)}{3b\left(9a^2c^2-6ab^2c+b^4\right)}}{x^2\left(9b^4+18acb^2\right)+9a^2b^2+c^4x^6+x^3\left(18b^3c+6abc^2\right)+6bc^3x^5+15b^2c^2x^4+18ab^3x}+\frac{5c^2\ln\left(b\left(3ac-b^2\right)^{8/3}-b^{19/3}+cx\left(3ac-b^2\right)^{8/3}+27a^3b^{1/3}c^3-27a^2b^{7/3}c^2+9ab^{13/3}c\right)}{27b^{8/3}\left(3ac-b^2\right)^{8/3}}-\frac{5c^2\ln\left(2b-b^{1/3}\left(3ac-b^2\right)^{1/3}+2cx-\sqrt{3}b^{1/3}\left(3ac-b^2\right)^{1/3}1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)}{27b^{8/3}\left(3ac-b^2\right)^{8/3}}+\frac{5c^2\ln\left(2b-b^{1/3}\left(3ac-b^2\right)^{1/3}+2cx+\sqrt{3}b^{1/3}\left(3ac-b^2\right)^{1/3}1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}1\mathrm{i}}{2}\right)}{27b^{8/3}\left(3ac-b^2\right)^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

```
[Out] ((8*a*c - b^2)/(6*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (5*c^2*x^2)/(3*(b^4 + 9*
a^2*c^2 - 6*a*b^2*c)) + (10*c^3*x^3)/(9*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) +
(5*c^4*x^4)/(18*b^2*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/
(3*b*(b^4 + 9*a^2*c^2 - 6*a*b^2*c)))/(x^2*(9*b^4 + 18*a*b^2*c) + 9*a^2*b^2
+ c^4*x^6 + x^3*(18*b^3*c + 6*a*b*c^2) + 6*b*c^3*x^5 + 15*b^2*c^2*x^4 + 18*
a*b^3*x) + (5*c^2*log(b*(3*a*c - b^2)^(8/3) - b^(19/3) + c*x*(3*a*c - b^2)^
(8/3) + 27*a^3*b^(1/3)*c^3 - 27*a^2*b^(7/3)*c^2 + 9*a*b^(13/3)*c))/(27*b^(8
/3)*(3*a*c - b^2)^(8/3)) - (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3) + 2
*c*x - 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2))/(27*
b^(8/3)*(3*a*c - b^2)^(8/3)) + (5*c^2*log(2*b - b^(1/3)*(3*a*c - b^2)^(1/3)
 + 2*c*x + 3^(1/2)*b^(1/3)*(3*a*c - b^2)^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2))/
(27*b^(8/3)*(3*a*c - b^2)^(8/3))
```

sympy [A] time = 2.54, size = 474, normalized size = 1.55

$$\frac{24ab^2c-3b^4+30b^2c^2x^2+20bc^3x^3+5c^4x^4+x\left(24abc^2+12b^3c\right)}{1458a^4b^4c^2-972a^3b^6c+162a^2b^8+x^6\left(162a^2b^2c^6-108ab^4c^5+18b^6c^4\right)+x^5\left(972a^2b^3c^5-648ab^5c^4+108b^7c^3\right)+x^4\left(2430a^2b^4c^4-1620ab^6c^3+270b^8c^2\right)+x^3\left(972a^3b^3c^4+2268a^2b^5c^3-1836ab^7c^2+324b^9c\right)+x^2\left(2916a^3b^4c^3-486a^2b^6c^2-648ab^8c+162b^{10}\right)+x\left(2916a^3b^5c^2-1944a^2b^7c+324ab^9\right)}+\operatorname{RootSum}\left(t^3\left(129140163a^8b^8c^8-344373768a^7b^{10}c^7+401769396a^6b^{12}c^6-267846264a^5b^{14}c^5+111602610a^4b^{16}c^4-29760696a^3b^{18}c^3+4960116a^2b^{20}c^2-472392ab^{22}c+19683b^{24}\right)-125c^6,\left(t\mapsto t\log\left(x+\frac{729ta^3b^3c^3-729ta^2b^5c^2+243tab^7c-27tb^9+5bc^2}{5c^3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

```
[Out] (24*a*b**2*c - 3*b**4 + 30*b**2*c**2*x**2 + 20*b*c**3*x**3 + 5*c**4*x**4 +
x*(24*a*b*c**2 + 12*b**3*c))/(1458*a**4*b**4*c**2 - 972*a**3*b**6*c + 162*a
**2*b**8 + x**6*(162*a**2*b**2*c**6 - 108*a*b**4*c**5 + 18*b**6*c**4) + x**
5*(972*a**2*b**3*c**5 - 648*a*b**5*c**4 + 108*b**7*c**3) + x**4*(2430*a**2*
b**4*c**4 - 1620*a*b**6*c**3 + 270*b**8*c**2) + x**3*(972*a**3*b**3*c**4 +
2268*a**2*b**5*c**3 - 1836*a*b**7*c**2 + 324*b**9*c) + x**2*(2916*a**3*b**4
```

```
*c**3 - 486*a**2*b**6*c**2 - 648*a*b**8*c + 162*b**10) + x*(2916*a**3*b**5*
c**2 - 1944*a**2*b**7*c + 324*a*b**9)) + RootSum(_t**3*(129140163*a**8*b**8
*c**8 - 344373768*a**7*b**10*c**7 + 401769396*a**6*b**12*c**6 - 267846264*a
**5*b**14*c**5 + 111602610*a**4*b**16*c**4 - 29760696*a**3*b**18*c**3 + 496
0116*a**2*b**20*c**2 - 472392*a*b**22*c + 19683*b**24) - 125*c**6, Lambda(_
t, _t*log(x + (729*_t*a**3*b**3*c**3 - 729*_t*a**2*b**5*c**2 + 243*_t*a*b**
7*c - 27*_t*b**9 + 5*b*c**2)/(5*c**3))))
```

3.15

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 \, dx$$

Optimal. Leaf size=361

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2 - 5abdf(cf+de) + b^2\left(c^2f^2 + 3cdef + d^2e^2\right)\right)}{8b^7} + \frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2\right.}{}$$

Rubi [A] time = 0.66, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{3df(a+bx)^8\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{8b^7}+\frac{(a+bx)^7(-2adf+bcf+bde)\left(10a^2d^2f^2-10abdf(cf+de)+b^2\left(c^2f^2+8cdef+d^2e^2\right)\right)}{7b^7}+\frac{(a+bx)^6(bc-ad)(be-af)\left(5a^2d^2f^2-5abdf(cf+de)+b^2\left(c^2f^2+3cdef+d^2e^2\right)\right)}{2b^7}+\frac{d^2f^2(a+bx)^9(-2adf+bcf+bde)}{3b^7}+\frac{3(a+bx)^5(bc-ad)^2(be-af)^2(-2adf+bcf+bde)}{5b^7}+\frac{(a+bx)^4(bc-ad)^3(be-af)^3}{4b^7}+\frac{d^3f^3(a+bx)^{10}}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^3,x]
```

```
[Out] ((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^4)/(4*b^7) + (3*(b*c - a*d)^2*(b*e -
 a*f)^2*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^5)/(5*b^7) + ((b*c - a*d)*(b*e
- a*f)*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2))*(a + b*x)^6)/(2*b^7) + ((b*d*e + b*c*f - 2*a*d*f)*(10*a^2*d^2*f^2
 - 10*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 8*c*d*e*f + c^2*f^2))*(a + b*x)^
7)/(7*b^7) + (3*d*f*(5*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*(a + b*x)^8)/(8*b^7) + (d^2*f^2*(b*d*e + b*c*f - 2*a
*d*f)*(a + b*x)^9)/(3*b^7) + (d^3*f^3*(a + b*x)^10)/(10*b^7)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 dx = \int (a+bx)^3(c+dx)^3(e+fx)^3 \, dx$$

$$= \int \left(\frac{(bc-ad)^3(be-af)^3(a+bx)^3}{b^6} + \frac{3(bc}{}\right.$$

$$= \frac{(bc-ad)^3(be-af)^3(a+bx)^4}{4b^7} + \frac{3(bc-a}{}$$

Mathematica [A] time = 0.22, size = 653, normalized size = 1.81

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^3,x]
```

```
[Out] a^3*c^3*e^3*x + (3*a^2*c^2*e^2*(b*c*e + a*d*e + a*c*f)*x^2)/2 + a*c*e*(b^2*
c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^3
+ ((b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*e^2 + 3*c*
d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))
*x^4)/4 + (3*(b^3*c^2*e^2*(d*e + c*f) + 3*a*b^2*c*e*(d^2*e^2 + 3*c*d*e*f +
c^2*f^2) + a^3*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a^2*b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^5)/5 + ((a^3*d^2*f^2*(d*e + c*f) + b
^3*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + 3*a^2*b*d*f*(d^2*e^2 + 3*c*d*e*f +
 c^2*f^2) + a*b^2*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^6)
/2 + ((a^3*d^3*f^3 + 9*a^2*b*d^2*f^2*(d*e + c*f) + 9*a*b^2*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + (3*b*d*f*(a^2*d^2*f^2 + 3*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2))*x^8)/8 + (b^2*d^2*f^2*(b*d*e + b*c*f + a*d*f)*x^9)/3
 + (b^3*d^3*f^3*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^3, x]
```

fricas [B] time = 0.87, size = 987, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*f^3*d^3*b^3 + 1/3*x^9*f^2*e*d^3*b^3 + 1/3*x^9*f^3*d^2*c*b^3 + 1/3
*x^9*f^3*d^3*b^2*a + 3/8*x^8*f*e^2*d^3*b^3 + 9/8*x^8*f^2*e*d^2*c*b^3 + 3/8*
x^8*f^3*d*c^2*b^3 + 9/8*x^8*f^2*e*d^3*b^2*a + 9/8*x^8*f^3*d^2*c*b^2*a + 3/8
*x^8*f^3*d^3*b*a^2 + 1/7*x^7*e^3*d^3*b^3 + 9/7*x^7*f*e^2*d^2*c*b^3 + 9/7*x^
7*f^2*e*d*c^2*b^3 + 1/7*x^7*f^3*c^3*b^3 + 9/7*x^7*f*e^2*d^3*b^2*a + 27/7*x^
7*f^2*e*d^2*c*b^2*a + 9/7*x^7*f^3*d*c^2*b^2*a + 9/7*x^7*f^2*e*d^3*b*a^2 + 9
/7*x^7*f^3*d^2*c*b*a^2 + 1/7*x^7*f^3*d^3*a^3 + 1/2*x^6*e^3*d^2*c*b^3 + 3/2*
x^6*f*e^2*d*c^2*b^3 + 1/2*x^6*f^2*e*c^3*b^3 + 1/2*x^6*e^3*d^3*b^2*a + 9/2*x
^6*f*e^2*d^2*c*b^2*a + 9/2*x^6*f^2*e*d*c^2*b^2*a + 1/2*x^6*f^3*c^3*b^2*a +
3/2*x^6*f*e^2*d^3*b*a^2 + 9/2*x^6*f^2*e*d^2*c*b*a^2 + 3/2*x^6*f^3*d*c^2*b*a
^2 + 1/2*x^6*f^2*e*d^3*a^3 + 1/2*x^6*f^3*d^2*c*a^3 + 3/5*x^5*e^3*d*c^2*b^3
+ 3/5*x^5*f*e^2*c^3*b^3 + 9/5*x^5*e^3*d^2*c*b^2*a + 27/5*x^5*f*e^2*d*c^2*b^
2*a + 9/5*x^5*f^2*e*c^3*b^2*a + 3/5*x^5*e^3*d^3*b*a^2 + 27/5*x^5*f*e^2*d^2*
c*b*a^2 + 27/5*x^5*f^2*e*d*c^2*b*a^2 + 3/5*x^5*f^3*c^3*b*a^2 + 3/5*x^5*f*e^
2*d^3*a^3 + 9/5*x^5*f^2*e*d^2*c*a^3 + 3/5*x^5*f^3*d*c^2*a^3 + 1/4*x^4*e^3*c
^3*b^3 + 9/4*x^4*e^3*d*c^2*b^2*a + 9/4*x^4*f*e^2*c^3*b^2*a + 9/4*x^4*e^3*d^
2*c*b*a^2 + 27/4*x^4*f*e^2*d*c^2*b*a^2 + 9/4*x^4*f^2*e*c^3*b*a^2 + 1/4*x^4*
e^3*d^3*a^3 + 9/4*x^4*f*e^2*d^2*c*a^3 + 9/4*x^4*f^2*e*d*c^2*a^3 + 1/4*x^4*f
^3*c^3*a^3 + x^3*e^3*c^3*b^2*a + 3*x^3*e^3*d*c^2*b*a^2 + 3*x^3*f*e^2*c^3*b*
a^2 + x^3*e^3*d^2*c*a^3 + 3*x^3*f*e^2*d*c^2*a^3 + x^3*f^2*e*c^3*a^3 + 3/2*x
^2*e^3*c^3*b*a^2 + 3/2*x^2*e^3*d*c^2*a^3 + 3/2*x^2*f*e^2*c^3*a^3 + x*e^3*c^
3*a^3
```

giac [B] time = 0.27, size = 971, normalized size = 2.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="giac")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*b^3*c*d^2*f^3*x^9 + 1/3*a*b^2*d^3*f^3*x^9 + 1/3
*b^3*d^3*f^2*x^9*e + 3/8*b^3*c^2*d*f^3*x^8 + 9/8*a*b^2*c*d^2*f^3*x^8 + 3/8*
a^2*b*d^3*f^3*x^8 + 9/8*b^3*c*d^2*f^2*x^8*e + 9/8*a*b^2*d^3*f^2*x^8*e + 1/7
*b^3*c^3*f^3*x^7 + 9/7*a*b^2*c^2*d*f^3*x^7 + 9/7*a^2*b*c*d^2*f^3*x^7 + 1/7*
a^3*d^3*f^3*x^7 + 3/8*b^3*d^3*f*x^8*e^2 + 9/7*b^3*c^2*d*f^2*x^7*e + 27/7*a*
b^2*c*d^2*f^2*x^7*e + 9/7*a^2*b*d^3*f^2*x^7*e + 1/2*a*b^2*c^3*f^3*x^6 + 3/2
*a^2*b*c^2*d*f^3*x^6 + 1/2*a^3*c*d^2*f^3*x^6 + 9/7*b^3*c*d^2*f*x^7*e^2 + 9/
7*a*b^2*d^3*f*x^7*e^2 + 1/2*b^3*c^3*f^2*x^6*e + 9/2*a*b^2*c^2*d*f^2*x^6*e +
 9/2*a^2*b*c*d^2*f^2*x^6*e + 1/2*a^3*d^3*f^2*x^6*e + 3/5*a^2*b*c^3*f^3*x^5
+ 3/5*a^3*c^2*d*f^3*x^5 + 1/7*b^3*d^3*x^7*e^3 + 3/2*b^3*c^2*d*f*x^6*e^2 + 9
/2*a*b^2*c*d^2*f*x^6*e^2 + 3/2*a^2*b*d^3*f*x^6*e^2 + 9/5*a*b^2*c^3*f^2*x^5*
e + 27/5*a^2*b*c^2*d*f^2*x^5*e + 9/5*a^3*c*d^2*f^2*x^5*e + 1/4*a^3*c^3*f^3*
x^4 + 1/2*b^3*c*d^2*x^6*e^3 + 1/2*a*b^2*d^3*x^6*e^3 + 3/5*b^3*c^3*f*x^5*e^2
 + 27/5*a*b^2*c^2*d*f*x^5*e^2 + 27/5*a^2*b*c*d^2*f*x^5*e^2 + 3/5*a^3*d^3*f*
x^5*e^2 + 9/4*a^2*b*c^3*f^2*x^4*e + 9/4*a^3*c^2*d*f^2*x^4*e + 3/5*b^3*c^2*d
*x^5*e^3 + 9/5*a*b^2*c*d^2*x^5*e^3 + 3/5*a^2*b*d^3*x^5*e^3 + 9/4*a*b^2*c^3*
f*x^4*e^2 + 27/4*a^2*b*c^2*d*f*x^4*e^2 + 9/4*a^3*c*d^2*f*x^4*e^2 + a^3*c^3*
f^2*x^3*e + 1/4*b^3*c^3*x^4*e^3 + 9/4*a*b^2*c^2*d*x^4*e^3 + 9/4*a^2*b*c*d^2
*x^4*e^3 + 1/4*a^3*d^3*x^4*e^3 + 3*a^2*b*c^3*f*x^3*e^2 + 3*a^3*c^2*d*f*x^3*
e^2 + a*b^2*c^3*x^3*e^3 + 3*a^2*b*c^2*d*x^3*e^3 + a^3*c*d^2*x^3*e^3 + 3/2*a
^3*c^3*f*x^2*e^2 + 3/2*a^2*b*c^3*x^2*e^3 + 3/2*a^3*c^2*d*x^2*e^3 + a^3*c^3*
x*e^3
```

maple [B] time = 0.00, size = 861, normalized size = 2.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 1/10*b^3*d^3*f^3*x^10+1/3*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2*x^9+1/8*((a*c*f+a
*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e
+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^8+1/7*(a*c*e*b^2*d^2*f^2+2*(a*c*f+a
*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*
e)*(a*d*f+b*c*f+b*d*e)))*x^7+1/6*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+
a*d*e+b*c*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c
*f+b*d*e)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(
2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2))*x^6+1/5*(a*c*e*(2*(a*c*
f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*b*
d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+(a*d*f+b*c*f+b*d*e)*(2*a*c*e
*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*b*d*f*a*c*e*(a*c*f+a*d*e+b*c*
e))*x^5+1/4*(a*c*e*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)
)+(a*c*f+a*d*e+b*c*e)*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2
*(a*d*f+b*c*f+b*d*e)*a*c*e*(a*c*f+a*d*e+b*c*e)+b*d*f*a^2*c^2*e^2)*x^4+1/3*(
a*c*e*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*(a*c*f+a*d*e+b*
c*e)^2*a*c*e+(a*d*f+b*c*f+b*d*e)*a^2*c^2*e^2)*x^3+3/2*a^2*c^2*e^2*(a*c*f+a*
d*e+b*c*e)*x^2+a^3*c^3*e^3*x
```

maxima [A] time = 0.68, size = 461, normalized size = 1.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3
,x, algorithm="maxima")
```

```
[Out] 1/10*b^3*d^3*f^3*x^10 + 1/3*(b*d*e + b*c*f + a*d*f)*b^2*d^2*f^2*x^9 + 3/8*(
b*d*e + b*c*f + a*d*f)^2*b*d*f*x^8 + a^3*c^3*e^3*x + 1/7*(b*d*e + b*c*f + a
*d*f)^3*x^7 + 1/4*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e +
 a*d*e + a*c*f)*x^2)*a^2*c^2*e^2 + 1/4*(b*c*e + a*d*e + a*c*f)^3*x^4 + 1/70
*(30*b^2*d^2*f^2*x^7 + 70*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + 42*(b*d*e + b
*c*f + a*d*f)^2*x^5 + 70*(b*c*e + a*d*e + a*c*f)^2*x^3 + 21*(4*b*d*f*x^5 +
5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e + a*d*e + a*c*f))*a*c*e + 1/10*(5*b*d
*f*x^6 + 6*(b*d*e + (b*c + a*d)*f)*x^5)*(b*c*e + a*d*e + a*c*f)^2 + 1/56*(2
1*b^2*d^2*f^2*x^8 + 48*(b^2*d^2*e*f + (b^2*c*d + a*b*d^2)*f^2)*x^7 + 28*(b^
2*d^2*e^2 + 2*(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*f^2
)*x^6)*(b*c*e + a*d*e + a*c*f)
```

mupad [B] time = 2.23, size = 787, normalized size = 2.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^3,x)
```

```
[Out] x^7*((a^3*d^3*f^3)/7 + (b^3*c^3*f^3)/7 + (b^3*d^3*e^3)/7 + (9*a*b^2*c^2*d*f
^3)/7 + (9*a^2*b*c*d^2*f^3)/7 + (9*a*b^2*d^3*e^2*f)/7 + (9*a^2*b*d^3*e*f^2)
/7 + (9*b^3*c*d^2*e^2*f)/7 + (9*b^3*c^2*d*e*f^2)/7 + (27*a*b^2*c*d^2*e*f^2)
/7) + x^5*((3*a^2*b*c^3*f^3)/5 + (3*a^2*b*d^3*e^3)/5 + (3*a^3*c^2*d*f^3)/5
+ (3*b^3*c^2*d*e^3)/5 + (3*a^3*d^3*e^2*f)/5 + (3*b^3*c^3*e^2*f)/5 + (9*a*b^
2*c*d^2*e^3)/5 + (9*a*b^2*c^3*e*f^2)/5 + (9*a^3*c*d^2*e*f^2)/5 + (27*a*b^2*
c^2*d*e^2*f)/5 + (27*a^2*b*c*d^2*e^2*f)/5 + (27*a^2*b*c^2*d*e*f^2)/5) + x^6
*((a*b^2*c^3*f^3)/2 + (a*b^2*d^3*e^3)/2 + (a^3*c*d^2*f^3)/2 + (b^3*c*d^2*e^
3)/2 + (a^3*d^3*e*f^2)/2 + (b^3*c^3*e*f^2)/2 + (3*a^2*b*c^2*d*f^3)/2 + (3*a
^2*b*d^3*e^2*f)/2 + (3*b^3*c^2*d*e^2*f)/2 + (9*a*b^2*c*d^2*e^2*f)/2 + (9*a*
b^2*c^2*d*e*f^2)/2 + (9*a^2*b*c*d^2*e*f^2)/2) + x^4*((a^3*c^3*f^3)/4 + (a^3
*d^3*e^3)/4 + (b^3*c^3*e^3)/4 + (9*a*b^2*c^2*d*e^3)/4 + (9*a^2*b*c*d^2*e^3)
/4 + (9*a*b^2*c^3*e^2*f)/4 + (9*a^2*b*c^3*e*f^2)/4 + (9*a^3*c*d^2*e^2*f)/4
+ (9*a^3*c^2*d*e*f^2)/4 + (27*a^2*b*c^2*d*e^2*f)/4) + a^3*c^3*e^3*x + (b^3*
d^3*f^3*x^10)/10 + (3*a^2*c^2*e^2*x^2*(a*c*f + a*d*e + b*c*e))/2 + (b^2*d^2
*f^2*x^9*(a*d*f + b*c*f + b*d*e))/3 + a*c*e*x^3*(a^2*c^2*f^2 + a^2*d^2*e^2
+ b^2*c^2*e^2 + 3*a*b*c*d*e^2 + 3*a*b*c^2*e*f + 3*a^2*c*d*e*f) + (3*b*d*f*x
^8*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 3*a*b*c*d*f^2 + 3*a*b*d^2*e*f
 + 3*b^2*c*d*e*f))/8
```

sympy [B] time = 0.25, size = 1018, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**3,x)
```

```
[Out] a**3*c**3*e**3*x + b**3*d**3*f**3*x**10/10 + x**9*(a*b**2*d**3*f**3/3 + b**
3*c*d**2*f**3/3 + b**3*d**3*e*f**2/3) + x**8*(3*a**2*b*d**3*f**3/8 + 9*a*b*
*2*c*d**2*f**3/8 + 9*a*b**2*d**3*e*f**2/8 + 3*b**3*c**2*d*f**3/8 + 9*b**3*c
*d**2*e*f**2/8 + 3*b**3*d**3*e**2*f/8) + x**7*(a**3*d**3*f**3/7 + 9*a**2*b*
c*d**2*f**3/7 + 9*a**2*b*d**3*e*f**2/7 + 9*a*b**2*c**2*d*f**3/7 + 27*a*b**2
*c*d**2*e*f**2/7 + 9*a*b**2*d**3*e**2*f/7 + b**3*c**3*f**3/7 + 9*b**3*c**2*
d*e*f**2/7 + 9*b**3*c*d**2*e**2*f/7 + b**3*d**3*e**3/7) + x**6*(a**3*c*d**2
```

```
*f**3/2 + a**3*d**3*e*f**2/2 + 3*a**2*b*c**2*d*f**3/2 + 9*a**2*b*c*d**2*e*f
**2/2 + 3*a**2*b*d**3*e**2*f/2 + a*b**2*c**3*f**3/2 + 9*a*b**2*c**2*d*e*f**
2/2 + 9*a*b**2*c*d**2*e**2*f/2 + a*b**2*d**3*e**3/2 + b**3*c**3*e*f**2/2 +
3*b**3*c**2*d*e**2*f/2 + b**3*c*d**2*e**3/2) + x**5*(3*a**3*c**2*d*f**3/5 +
 9*a**3*c*d**2*e*f**2/5 + 3*a**3*d**3*e**2*f/5 + 3*a**2*b*c**3*f**3/5 + 27*
a**2*b*c**2*d*e*f**2/5 + 27*a**2*b*c*d**2*e**2*f/5 + 3*a**2*b*d**3*e**3/5 +
 9*a*b**2*c**3*e*f**2/5 + 27*a*b**2*c**2*d*e**2*f/5 + 9*a*b**2*c*d**2*e**3/
5 + 3*b**3*c**3*e**2*f/5 + 3*b**3*c**2*d*e**3/5) + x**4*(a**3*c**3*f**3/4 +
 9*a**3*c**2*d*e*f**2/4 + 9*a**3*c*d**2*e**2*f/4 + a**3*d**3*e**3/4 + 9*a**
2*b*c**3*e*f**2/4 + 27*a**2*b*c**2*d*e**2*f/4 + 9*a**2*b*c*d**2*e**3/4 + 9*
a*b**2*c**3*e**2*f/4 + 9*a*b**2*c**2*d*e**3/4 + b**3*c**3*e**3/4) + x**3*(a
**3*c**3*e*f**2 + 3*a**3*c**2*d*e**2*f + a**3*c*d**2*e**3 + 3*a**2*b*c**3*e
**2*f + 3*a**2*b*c**2*d*e**3 + a*b**2*c**3*e**3) + x**2*(3*a**3*c**3*e**2*f
/2 + 3*a**3*c**2*d*e**3/2 + 3*a**2*b*c**3*e**3/2)
```

3.16

$$\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$

Optimal. Leaf size=193

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(b$$

Rubi [A] time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2059, 88}

$$\frac{(a+bx)^5\left(6a^2d^2f^2-6abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)}{5b^5}+\frac{df(a+bx)^6(-2adf+bcf+bde)}{3b^5}+\frac{(a+bx)^4(bc-ad)(be-af)(-2adf+bcf+bde)}{2b^5}+\frac{(a+bx)^3(bc-ad)^2(be-af)^2}{3b^5}+\frac{d^2f^2(a+bx)^7}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^2,x]
```

```
[Out] ((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^3)/(3*b^5) + ((b*c - a*d)*(b*e - a*f
)*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^4)/(2*b^5) + ((6*a^2*d^2*f^2 - 6*a*b*
d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*(a + b*x)^5)/(5*b^5)
 + (d*f*(b*d*e + b*c*f - 2*a*d*f)*(a + b*x)^6)/(3*b^5) + (d^2*f^2*(a + b*x)
^7)/(7*b^5)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx &= \int (a+bx)^2(c+dx)^2(e+fx)^2\,dx\\ &= \int \left(\frac{(bc-ad)^2(be-af)^2(a+bx)^2}{b^4}+\frac{2(bc-a\\ &= \frac{(bc-ad)^2(be-af)^2(a+bx)^3}{3b^5}+\frac{(bc-ad)(be\end{aligned}$$

Mathematica [A] time = 0.09, size = 241, normalized size = 1.25

$$\frac{1}{5}x^5\left(a^2d^2f^2+4abdf(cf+de)+b^2\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{2}x^4\left(a^2df(cf+de)+ab\left(c^2f^2+4cdef+d^2e^2\right)+b^2ce(cf+de)\right)+\frac{1}{3}x^3\left(a^2\left(c^2f^2+4cdef+d^2e^2\right)+4abce(cf+de)+b^2c^2e^2\right)+a^2c^2e^2x+\frac{1}{3}bdfx^6(adf+bcf+bde)+acex^2(acf+ade+bce)+\frac{1}{7}b^2d^2f^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^2,x]
```

```
[Out] a^2*c^2*e^2*x + a*c*e*(b*c*e + a*d*e + a*c*f)*x^2 + ((b^2*c^2*e^2 + 4*a*b*c
*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^3)/3 + ((b^2*c*e*(d
*e + c*f) + a^2*d*f*(d*e + c*f) + a*b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^4)
/2 + ((a^2*d^2*f^2 + 4*a*b*d*f*(d*e + c*f) + b^2*(d^2*e^2 + 4*c*d*e*f + c^2
*f^2))*x^5)/5 + (b*d*f*(b*d*e + b*c*f + a*d*f)*x^6)/3 + (b^2*d^2*f^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^2, x]
```

fricas [A] time = 0.83, size = 346, normalized size = 1.79

$$\frac{1}{7}x^7f^2d^2b^2 + \frac{1}{3}x^6fed^2b^2 + \frac{1}{3}x^6f^2dcb^2 + \frac{1}{3}x^6f^2d^2ba + \frac{1}{5}x^5e^2d^2b^2 + \frac{4}{5}x^5fedcb^2 + \frac{1}{5}x^5f^2c^2b^2 + \frac{4}{5}x^5fed^2ba + \frac{4}{5}x^5f^2dcba + \frac{1}{5}x^5f^2d^2a^2 + \frac{1}{2}x^4e^2dcb^2 + \frac{1}{2}x^4fec^2b^2 + \frac{1}{2}x^4e^2d^2ba + 2x^4fedcba + \frac{1}{2}x^4f^2c^2ba + \frac{1}{2}x^4fed^2a^2 + \frac{1}{2}x^4f^2dca^2 + \frac{1}{3}x^3e^2c^2b^2 + \frac{4}{3}x^3e^2dcba + \frac{4}{3}x^3fec^2ba + \frac{1}{3}x^3e^2d^2a^2 + \frac{4}{3}x^3fedca^2 + \frac{1}{3}x^3f^2c^2a^2 + x^2e^2c^2ba + x^2e^2dca^2 + x^2fec^2a^2 + xe^2c^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f^2*d^2*b^2 + 1/3*x^6*f*e*d^2*b^2 + 1/3*x^6*f^2*d*c*b^2 + 1/3*x^6*f
^2*d^2*b*a + 1/5*x^5*e^2*d^2*b^2 + 4/5*x^5*f*e*d*c*b^2 + 1/5*x^5*f^2*c^2*b^
2 + 4/5*x^5*f*e*d^2*b*a + 4/5*x^5*f^2*d*c*b*a + 1/5*x^5*f^2*d^2*a^2 + 1/2*x
^4*e^2*d*c*b^2 + 1/2*x^4*f*e*c^2*b^2 + 1/2*x^4*e^2*d^2*b*a + 2*x^4*f*e*d*c*
b*a + 1/2*x^4*f^2*c^2*b*a + 1/2*x^4*f*e*d^2*a^2 + 1/2*x^4*f^2*d*c*a^2 + 1/3
*x^3*e^2*c^2*b^2 + 4/3*x^3*e^2*d*c*b*a + 4/3*x^3*f*e*c^2*b*a + 1/3*x^3*e^2*
d^2*a^2 + 4/3*x^3*f*e*d*c*a^2 + 1/3*x^3*f^2*c^2*a^2 + x^2*e^2*c^2*b*a + x^2
*e^2*d*c*a^2 + x^2*f*e*c^2*a^2 + x*e^2*c^2*a^2
```

giac [A] time = 0.42, size = 346, normalized size = 1.79

$$\frac{1}{7}b^2d^2f^2x^7 + \frac{1}{3}b^2cdf^2x^6 + \frac{1}{3}abd^2f^2x^6 + \frac{1}{3}b^2d^2fx^6e + \frac{1}{5}b^2c^2f^2x^5 + \frac{4}{5}abcdf^2x^5 + \frac{1}{5}a^2d^2f^2x^5 + \frac{4}{5}b^2cdfx^5e + \frac{4}{5}abd^2fx^5e + \frac{1}{2}abc^2f^2x^4 + \frac{1}{2}a^2cdf^2x^4 + \frac{1}{5}b^2d^2x^5e^2 + \frac{1}{2}b^2c^2fx^4e + 2abcdfx^4e + \frac{1}{2}a^2d^2fx^4e + \frac{1}{3}a^2c^2f^2x^3 + \frac{1}{2}b^2cdx^4e^2 + \frac{1}{2}abd^2x^4e^2 + \frac{4}{3}abc^2fx^3e + \frac{4}{3}a^2cdfx^3e + \frac{1}{3}b^2c^2x^3e^2 + \frac{4}{3}abcdx^3e^2 + \frac{1}{3}a^2d^2x^3e^2 + a^2c^2fx^2e + abc^2x^2e^2 + a^2cdx^2e^2 + a^2c^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="giac")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*b^2*c*d*f^2*x^6 + 1/3*a*b*d^2*f^2*x^6 + 1/3*b^2*d
^2*f*x^6*e + 1/5*b^2*c^2*f^2*x^5 + 4/5*a*b*c*d*f^2*x^5 + 1/5*a^2*d^2*f^2*x^
5 + 4/5*b^2*c*d*f*x^5*e + 4/5*a*b*d^2*f*x^5*e + 1/2*a*b*c^2*f^2*x^4 + 1/2*a
^2*c*d*f^2*x^4 + 1/5*b^2*d^2*x^5*e^2 + 1/2*b^2*c^2*f*x^4*e + 2*a*b*c*d*f*x^
4*e + 1/2*a^2*d^2*f*x^4*e + 1/3*a^2*c^2*f^2*x^3 + 1/2*b^2*c*d*x^4*e^2 + 1/2
*a*b*d^2*x^4*e^2 + 4/3*a*b*c^2*f*x^3*e + 4/3*a^2*c*d*f*x^3*e + 1/3*b^2*c^2*
x^3*e^2 + 4/3*a*b*c*d*x^3*e^2 + 1/3*a^2*d^2*x^3*e^2 + a^2*c^2*f*x^2*e + a*b
*c^2*x^2*e^2 + a^2*c*d*x^2*e^2 + a^2*c^2*x*e^2
```

maple [A] time = 0.00, size = 188, normalized size = 0.97

$$\frac{b^2d^2f^2x^7}{7} + \frac{\left(adf + bcf + bde\right)bdf\,x^6}{3} + a^2c^2e^2x + \left(acf + ade + bce\right)ace\,x^2 + \frac{\left(2\left(acf + ade + bce\right)bdf + \left(adf + bcf + bde\right)^2\right)x^5}{5} + \frac{\left(2abcdef + 2\left(acf + ade + bce\right)\left(adf + bcf + bde\right)\right)x^4}{4} + \frac{\left(2\left(adf + bcf + bde\right)ace + \left(acf + ade + bce\right)^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] 1/7*b^2*d^2*f^2*x^7+1/3*(a*d*f+b*c*f+b*d*e)*b*d*f*x^6+1/5*(2*(a*c*f+a*d*e+b
*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)*x^5+1/4*(2*a*b*c*d*e*f+2*(a*c*f+a*d*e+b*
c*e)*(a*d*f+b*c*f+b*d*e))*x^4+1/3*(2*(a*d*f+b*c*f+b*d*e)*a*c*e+(a*c*f+a*d*e
+b*c*e)^2)*x^3+a*c*e*(a*c*f+a*d*e+b*c*e)*x^2+a^2*c^2*e^2*x
```

maxima [A] time = 0.62, size = 180, normalized size = 0.93

$$\frac{1}{7} b^2 d^2 f^2 x^7 + \frac{1}{3} \left(bde + bcf + adf\right) bdf x^6 + a^2 c^2 e^2 x + \frac{1}{5} \left(bde + bcf + adf\right)^2 x^5 + \frac{1}{3} \left(bce + ade + acf\right)^2 x^3 + \frac{1}{6} \left(3\, bdf x^4 + 4 \left(bde + bcf + adf\right) x^3 + 6 \left(bce + ade + acf\right) x^2\right) ace + \frac{1}{10} \left(4\, bdf x^5 + 5 \left(bde + (bc + ad) f\right) x^4\right) \left(bce + ade + acf\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2
,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d^2*f^2*x^7 + 1/3*(b*d*e + b*c*f + a*d*f)*b*d*f*x^6 + a^2*c^2*e^2*x
 + 1/5*(b*d*e + b*c*f + a*d*f)^2*x^5 + 1/3*(b*c*e + a*d*e + a*c*f)^2*x^3 +
1/6*(3*b*d*f*x^4 + 4*(b*d*e + b*c*f + a*d*f)*x^3 + 6*(b*c*e + a*d*e + a*c*f
)*x^2)*a*c*e + 1/10*(4*b*d*f*x^5 + 5*(b*d*e + (b*c + a*d)*f)*x^4)*(b*c*e +
a*d*e + a*c*f)
```

mupad [B] time = 0.08, size = 270, normalized size = 1.40

$$x^4 \left(\frac{a^2 c d f^2}{2} + \frac{a^2 d^2 e f}{2} + \frac{a b c^2 f^2}{2} + 2 a b c d e f + \frac{a b d^2 e^2}{2} + \frac{b^2 c^2 e f}{2} + \frac{b^2 c d e^2}{2}\right) + x^3 \left(\frac{a^2 c^2 f^2}{3} + \frac{4 a^2 c d e f}{3} + \frac{a^2 d^2 e^2}{3} + \frac{4 a b c^2 e f}{3} + \frac{4 a b c d e^2}{3} + \frac{b^2 c^2 e^2}{3}\right) + x^5 \left(\frac{a^2 d^2 f^2}{5} + \frac{4 a b c d f^2}{5} + \frac{4 a b d^2 e f}{5} + \frac{b^2 c^2 f^2}{5} + \frac{4 b^2 c d e f}{5} + \frac{b^2 d^2 e^2}{5}\right) + a^2 c^2 e^2 x + \frac{b^2 d^2 f^2 x^7}{7} + a c e x^2 \left(a c f + a d e + b c e\right) + \frac{b d f x^6 \left(a d f + b c f + b d e\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*
f*x^3)^2,x)
```

```
[Out] x^4*((a*b*c^2*f^2)/2 + (a*b*d^2*e^2)/2 + (a^2*c*d*f^2)/2 + (b^2*c*d*e^2)/2
+ (a^2*d^2*e*f)/2 + (b^2*c^2*e*f)/2 + 2*a*b*c*d*e*f) + x^3*((a^2*c^2*f^2)/3
 + (a^2*d^2*e^2)/3 + (b^2*c^2*e^2)/3 + (4*a*b*c*d*e^2)/3 + (4*a*b*c^2*e*f)/
3 + (4*a^2*c*d*e*f)/3) + x^5*((a^2*d^2*f^2)/5 + (b^2*c^2*f^2)/5 + (b^2*d^2*
e^2)/5 + (4*a*b*c*d*f^2)/5 + (4*a*b*d^2*e*f)/5 + (4*b^2*c*d*e*f)/5) + a^2*c
^2*e^2*x + (b^2*d^2*f^2*x^7)/7 + a*c*e*x^2*(a*c*f + a*d*e + b*c*e) + (b*d*f
*x^6*(a*d*f + b*c*f + b*d*e))/3
```

sympy [A] time = 0.14, size = 345, normalized size = 1.79

$$a^2 c^2 e^2 x + \frac{b^2 d^2 f^2 x^7}{7} + x^6 \left(\frac{a b d^2 f^2}{3} + \frac{b^2 c d f^2}{3} + \frac{b^2 d^2 e f}{3}\right) + x^5 \left(\frac{a^2 d^2 f^2}{5} + \frac{4 a b c d f^2}{5} + \frac{4 a b d^2 e f}{5} + \frac{b^2 c^2 f^2}{5} + \frac{4 b^2 c d e f}{5} + \frac{b^2 d^2 e^2}{5}\right) + x^4 \left(\frac{a^2 c d f^2}{2} + \frac{a^2 d^2 e f}{2} + \frac{a b c^2 f^2}{2} + 2 a b c d e f + \frac{a b d^2 e^2}{2} + \frac{b^2 c^2 e f}{2} + \frac{b^2 c d e^2}{2}\right) + x^3 \left(\frac{a^2 c^2 f^2}{3} + \frac{4 a^2 c d e f}{3} + \frac{a^2 d^2 e^2}{3} + \frac{4 a b c^2 e f}{3} + \frac{4 a b c d e^2}{3} + \frac{b^2 c^2 e^2}{3}\right) + x^2 \left(a^2 c^2 e f + a^2 c d e^2 + a b c^2 e^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3)
**2,x)
```

```
[Out] a**2*c**2*e**2*x + b**2*d**2*f**2*x**7/7 + x**6*(a*b*d**2*f**2/3 + b**2*c*d
*f**2/3 + b**2*d**2*e*f/3) + x**5*(a**2*d**2*f**2/5 + 4*a*b*c*d*f**2/5 + 4*
a*b*d**2*e*f/5 + b**2*c**2*f**2/5 + 4*b**2*c*d*e*f/5 + b**2*d**2*e**2/5) +
x**4*(a**2*c*d*f**2/2 + a**2*d**2*e*f/2 + a*b*c**2*f**2/2 + 2*a*b*c*d*e*f +
 a*b*d**2*e**2/2 + b**2*c**2*e*f/2 + b**2*c*d*e**2/2) + x**3*(a**2*c**2*f**
2/3 + 4*a**2*c*d*e*f/3 + a**2*d**2*e**2/3 + 4*a*b*c**2*e*f/3 + 4*a*b*c*d*e*
*2/3 + b**2*c**2*e**2/3) + x**2*(a**2*c**2*e*f + a**2*c*d*e**2 + a*b*c**2*e
**2)
```

3.17

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx$$

Optimal. Leaf size=56

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{1}{3}x^3(adf + bcf + bde) + \frac{1}{2}x^2(acf + ade + bce) + acex + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Int[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*f
*x^3,x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^2)/2 + ((b*d*e + b*c*f + a*d*f)*x^3)/3
 + (b*d*f*x^4)/4
```

Rubi steps

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx = acex + \frac{1}{2}(bce + ade + acf)x^2 + \frac{1}{3}(bde + bc$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.36

$$acex + \frac{1}{2}acfx^2 + \frac{1}{2}adex^2 + \frac{1}{3}adfx^3 + \frac{1}{2}bcex^2 + \frac{1}{3}bcfx^3 + \frac{1}{3}bdex^3 + \frac{1}{4}bdfx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 +
 b*d*f*x^3,x]
```

```
[Out] a*c*e*x + (b*c*e*x^2)/2 + (a*d*e*x^2)/2 + (a*c*f*x^2)/2 + (b*d*e*x^3)/3 + (
b*c*f*x^3)/3 + (a*d*f*x^3)/3 + (b*d*f*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d
*f)*x^2 + b*d*f*x^3,x]
```

```
[Out] IntegrateAlgebraic[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d
*f)*x^2 + b*d*f*x^3, x]
```

fricas [A] time = 0.95, size = 62, normalized size = 1.11

$$\frac{1}{4}x^4fdb + \frac{1}{3}x^3edb + \frac{1}{3}x^3fcb + \frac{1}{3}x^3fda + \frac{1}{2}x^2ecb + \frac{1}{2}x^2eda + \frac{1}{2}x^2fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="fricas")
```

```
[Out] 1/4*x^4*f*d*b + 1/3*x^3*e*d*b + 1/3*x^3*f*c*b + 1/3*x^3*f*d*a + 1/2*x^2*e*c
*b + 1/2*x^2*e*d*a + 1/2*x^2*f*c*a + x*e*c*a
```

giac [A] time = 0.23, size = 54, normalized size = 0.96

$$\frac{1}{4}bdfx^4 + \frac{1}{3}\left(bcf + adf + bde\right)x^3 + acxe + \frac{1}{2}\left(acf + bce + ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="giac")
```

```
[Out] 1/4*b*d*f*x^4 + 1/3*(b*c*f + a*d*f + b*d*e)*x^3 + a*c*x*e + 1/2*(a*c*f + b*
c*e + a*d*e)*x^2
```

maple [A] time = 0.00, size = 51, normalized size = 0.91

$$\frac{bdf\,x^4}{4} + acex + \frac{\left(adf + bcf + bde\right)x^3}{3} + \frac{\left(acf + ade + bce\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x)
```

```
[Out] a*c*e*x+1/2*(a*c*f+a*d*e+b*c*e)*x^2+1/3*(a*d*f+b*c*f+b*d*e)*x^3+1/4*b*d*f*x
^4
```

maxima [A] time = 0.83, size = 50, normalized size = 0.89

$$\frac{1}{4}bdfx^4 + acex + \frac{1}{3}\left(bde + bcf + adf\right)x^3 + \frac{1}{2}\left(bce + ade + acf\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3,x,
algorithm="maxima")
```

```
[Out] 1/4*b*d*f*x^4 + a*c*e*x + 1/3*(b*d*e + b*c*f + a*d*f)*x^3 + 1/2*(b*c*e + a*
d*e + a*c*f)*x^2
```

mupad [B] time = 0.04, size = 54, normalized size = 0.96

$$\frac{b\,d\,f\,x^4}{4} + \left(\frac{a\,d\,f}{3} + \frac{b\,c\,f}{3} + \frac{b\,d\,e}{3}\right)x^3 + \left(\frac{a\,c\,f}{2} + \frac{a\,d\,e}{2} + \frac{b\,c\,e}{2}\right)x^2 + a\,c\,e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f
*x^3,x)
```

```
[Out] x^2*((a*c*f)/2 + (a*d*e)/2 + (b*c*e)/2) + x^3*((a*d*f)/3 + (b*c*f)/3 + (b*d
*e)/3) + a*c*e*x + (b*d*f*x^4)/4
```

sympy [A] time = 0.07, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^4}{4} + x^3\left(\frac{adf}{3} + \frac{bcf}{3} + \frac{bde}{3}\right) + x^2\left(\frac{acf}{2} + \frac{ade}{2} + \frac{bce}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**3,x
)
```

```
[Out] a*c*e*x + b*d*f*x**4/4 + x**3*(a*d*f/3 + b*c*f/3 + b*d*e/3) + x**2*(a*c*f/2
 + a*d*e/2 + b*c*e/2)
```

3.18 $\displaystyle \int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal. Leaf size=86

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\log(e+fx)}{(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-1),x]
```

```
[Out] (b*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)) - (d*Log[c + d*x])/((b*c - a*d)*
(d*e - c*f)) + (f*Log[e + f*x])/((b*e - a*f)*(d*e - c*f))
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx=\int \left(\frac{b^2}{(bc-ad)(be-af)(a+bx)}+\frac{a}{(bc-ad)(-de}\right.$$
$$=\frac{b\log(a+bx)}{(bc-ad)(be-af)}-\frac{d\log(c+dx)}{(bc-ad)(de-cf)}+\frac{f\,l}{(be-}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 0.93

$$\frac{b\log(a+bx)(cf-de)+d(be-af)\log(c+dx)+f(ad-bc)\log(e+fx)}{(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-1),x]
```

```
[Out] (b*(-(d*e) + c*f)*Log[a + b*x] + d*(b*e - a*f)*Log[c + d*x] + (-(b*c) + a*d
)*f*Log[e + f*x])/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-1), x]
```

fricas [A] time = 12.86, size = 112, normalized size = 1.30

$$\frac{(bc-ad)f\log\left(fx+e\right)+\left(bde-bcf\right)\log\left(bx+a\right)-\left(bde-adf\right)\log\left(dx+c\right)}{\left(b^2cd-abd^2\right)e^2-\left(b^2c^2-a^2d^2\right)ef+\left(abc^2-a^2cd\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*f*log(f*x + e) + (b*d*e - b*c*f)*log(b*x + a) - (b*d*e - a*d*f
)*log(d*x + c))/((b^2*c*d - a*b*d^2)*e^2 - (b^2*c^2 - a^2*d^2)*e*f + (a*b*c
^2 - a^2*c*d)*f^2)
```

giac [A] time = 0.31, size = 137, normalized size = 1.59

$$-\frac{b^2\log\left(|bx+a|\right)}{ab^2cf-a^2bdf-b^3ce+ab^2de}+\frac{d^2\log\left(|dx+c|\right)}{bc^2df-acd^2f-bcd^2e+ad^3e}+\frac{f^2\log\left(\left|fx+e\right|\right)}{acf^3-bcf^2e-adf^2e+bdfe^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/(a*b^2*c*f - a^2*b*d*f - b^3*c*e + a*b^2*d*e) + d^2*
log(abs(d*x + c))/(b*c^2*d*f - a*c*d^2*f - b*c*d^2*e + a*d^3*e) + f^2*log(a
bs(f*x + e))/(a*c*f^3 - b*c*f^2*e - a*d*f^2*e + b*d*f*e^2)
```

maple [A] time = 0.01, size = 87, normalized size = 1.01

$$\frac{b\ln\left(bx+a\right)}{\left(af-be\right)\left(ad-bc\right)}-\frac{d\ln\left(dx+c\right)}{\left(cf-de\right)\left(ad-bc\right)}+\frac{f\ln\left(fx+e\right)}{\left(cf-de\right)\left(af-be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3),x)
```

```
[Out] f/(c*f-d*e)/(a*f-b*e)*ln(f*x+e)-d/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)+b/(a*f-b*e)
/(a*d-b*c)*ln(b*x+a)
```

maxima [A] time = 0.65, size = 112, normalized size = 1.30

$$\frac{b\log\left(bx+a\right)}{\left(b^2c-abd\right)e-\left(abc-a^2d\right)f}-\frac{d\log\left(dx+c\right)}{\left(bcd-ad^2\right)e-\left(bc^2-acd\right)f}+\frac{f\log\left(fx+e\right)}{bde^2+acf^2-(bc+ad)ef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/((b^2*c - a*b*d)*e - (a*b*c - a^2*d)*f) - d*log(d*x + c)/((b
*c*d - a*d^2)*e - (b*c^2 - a*c*d)*f) + f*log(f*x + e)/(b*d*e^2 + a*c*f^2 -
(b*c + a*d)*e*f)
```

mupad [B] time = 2.33, size = 106, normalized size = 1.23

$$\frac{b\,\ln\left(a+b\,x\right)}{b^2\,c\,e+a^2\,d\,f-a\,b\,c\,f-a\,b\,d\,e}+\frac{d\,\ln\left(c+d\,x\right)}{a\,d^2\,e+b\,c^2\,f-a\,c\,d\,f-b\,c\,d\,e}+\frac{f\,\ln\left(e+f\,x\right)}{a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3),x)
```

```
[Out] (b*log(a + b*x))/(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e) + (d*log(c + d*x))
/(a*d^2*e + b*c^2*f - a*c*d*f - b*c*d*e) + (f*log(e + f*x))/(a*c*f^2 + b*d*
e^2 - a*d*e*f - b*c*e*f)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3),x)
```

```
[Out] Timed out
```

3.19 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c}{(bc}$$

Rubi [A] time = 0.41, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = $\{2058\}$

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(de-cf)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}+\frac{2f^3\log(e+fx)(-adf-bcf+2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^3) + (
2*f^3*(2*b*d*e - b*c*f - a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f)^3)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx = \int \left(\frac{b^4}{(bc-ad)^2(be-af)^2(a+bx)^2}-\frac{2b}{(bc-}\right.$$

$$= -\frac{b^3}{(bc-ad)^2(be-af)^2(a+bx)}-\frac{}{(bc-ad}$$

Mathematica [A] time = 0.55, size = 232, normalized size = 0.99

$$-\frac{b^3}{(a+bx)(bc-ad)^2(be-af)^2}-\frac{2b^3\log(a+bx)(-2adf+bcf+bde)}{(bc-ad)^3(be-af)^3}-\frac{d^3}{(c+dx)(bc-ad)^2(de-cf)^2}-\frac{2d^3\log(c+dx)(adf-2bcf+bde)}{(bc-ad)^3(cf-de)^3}-\frac{f^3}{(e+fx)(be-af)^2(de-cf)^2}-\frac{2f^3\log(e+fx)(adf+bcf-2bde)}{(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
+ b*d*f*x^3)^(-2),x]
```

```
[Out] -(b^3/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - d^3/((b*c - a*d)^2*(d*e -
c*f)^2*(c + d*x)) - f^3/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (2*b^3*(b
*d*e + b*c*f - 2*a*d*f)*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) - (2*d^
3*(b*d*e - 2*b*c*f + a*d*f)*Log[c + d*x])/((b*c - a*d)^3*(-(d*e) + c*f)^3)
- (2*f^3*(-2*b*d*e + b*c*f + a*d*f)*Log[e + f*x])/((b*e - a*f)^3*(d*e - c*f
)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.39, size = 1414, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="giac")
```

```
[Out] 2*(b^5*c*f - 2*a*b^4*d*f + b^5*d*e)*log(abs(b*x + a))/(a^3*b^4*c^3*f^3 - 3*
a^4*b^3*c^2*d*f^3 + 3*a^5*b^2*c*d^2*f^3 - a^6*b*d^3*f^3 - 3*a^2*b^5*c^3*f^2
*e + 9*a^3*b^4*c^2*d*f^2*e - 9*a^4*b^3*c*d^2*f^2*e + 3*a^5*b^2*d^3*f^2*e +
3*a*b^6*c^3*f*e^2 - 9*a^2*b^5*c^2*d*f*e^2 + 9*a^3*b^4*c*d^2*f*e^2 - 3*a^4*b
^3*d^3*f*e^2 - b^7*c^3*e^3 + 3*a*b^6*c^2*d*e^3 - 3*a^2*b^5*c*d^2*e^3 + a^3*
b^4*d^3*e^3) + 2*(2*b*c*d^4*f - a*d^5*f - b*d^5*e)*log(abs(d*x + c))/(b^3*c
^6*d*f^3 - 3*a*b^2*c^5*d^2*f^3 + 3*a^2*b*c^4*d^3*f^3 - a^3*c^3*d^4*f^3 - 3*
b^3*c^5*d^2*f^2*e + 9*a*b^2*c^4*d^3*f^2*e - 9*a^2*b*c^3*d^4*f^2*e + 3*a^3*c
^2*d^5*f^2*e + 3*b^3*c^4*d^3*f*e^2 - 9*a*b^2*c^3*d^4*f*e^2 + 9*a^2*b*c^2*d^
5*f*e^2 - 3*a^3*c*d^6*f*e^2 - b^3*c^3*d^4*e^3 + 3*a*b^2*c^2*d^5*e^3 - 3*a^2
*b*c*d^6*e^3 + a^3*d^7*e^3) - 2*(b*c*f^5 + a*d*f^5 - 2*b*d*f^4*e)*log(abs(f
*x + e))/(a^3*c^3*f^7 - 3*a^2*b*c^3*f^6*e - 3*a^3*c^2*d*f^6*e + 3*a*b^2*c^3
*f^5*e^2 + 9*a^2*b*c^2*d*f^5*e^2 + 3*a^3*c*d^2*f^5*e^2 - b^3*c^3*f^4*e^3 -
9*a*b^2*c^2*d*f^4*e^3 - 9*a^2*b*c*d^2*f^4*e^3 - a^3*d^3*f^4*e^3 + 3*b^3*c^2
*d*f^3*e^4 + 9*a*b^2*c*d^2*f^3*e^4 + 3*a^2*b*d^3*f^3*e^4 - 3*b^3*c*d^2*f^2*
e^5 - 3*a*b^2*d^3*f^2*e^5 + b^3*d^3*f*e^6) - (2*b^3*c^2*d*f^3*x^2 - 2*a*b^2
*c*d^2*f^3*x^2 + 2*a^2*b*d^3*f^3*x^2 - 2*b^3*c*d^2*f^2*x^2*e - 2*a*b^2*d^3*
f^2*x^2*e + 2*b^3*c^3*f^3*x - a*b^2*c^2*d*f^3*x - a^2*b*c*d^2*f^3*x + 2*a^3
*d^3*f^3*x + 2*b^3*d^3*f*x^2*e^2 - b^3*c^2*d*f^2*x*e - a^2*b*d^3*f^2*x*e +
a*b^2*c^3*f^3 - 2*a^2*b*c^2*d*f^3 + a^3*c*d^2*f^3 - b^3*c*d^2*f*x*e^2 - a*b
^2*d^3*f*x*e^2 + b^3*c^3*f^2*e + a^3*d^3*f^2*e + 2*b^3*d^3*x*e^3 - 2*b^3*c^
2*d*f*e^2 - 2*a^2*b*d^3*f*e^2 + b^3*c*d^2*e^3 + a*b^2*d^3*e^3)/((a^2*b^2*c^
4*f^4 - 2*a^3*b*c^3*d*f^4 + a^4*c^2*d^2*f^4 - 2*a*b^3*c^4*f^3*e + 2*a^2*b^2
*c^3*d*f^3*e + 2*a^3*b*c^2*d^2*f^3*e - 2*a^4*c*d^3*f^3*e + b^4*c^4*f^2*e^2
+ 2*a*b^3*c^3*d*f^2*e^2 - 6*a^2*b^2*c^2*d^2*f^2*e^2 + 2*a^3*b*c*d^3*f^2*e^2
 + a^4*d^4*f^2*e^2 - 2*b^4*c^3*d*f*e^3 + 2*a*b^3*c^2*d^2*f*e^3 + 2*a^2*b^2*
c*d^3*f*e^3 - 2*a^3*b*d^4*f*e^3 + b^4*c^2*d^2*e^4 - 2*a*b^3*c*d^3*e^4 + a^2
*b^2*d^4*e^4)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f*x^2 + b*d*x^2*e + a*c*f*x + b*
c*x*e + a*d*x*e + a*c*e))
```

maple [A] time = 0.03, size = 398, normalized size = 1.70

$$\frac{4 a b^{3} d f \ln \left(b x +a \right)}{\left(a f -b e \right)^{3} \left(a d -b c \right)^{3}}+\frac{2 a d^{4} f \ln \left(d x +c \right)}{\left(c f -d e \right)^{3} \left(a d -b c \right)^{3}}-\frac{2 a d f^{4} \ln \left(f x +e \right)}{\left(c f -d e \right)^{3} \left(a f -b e \right)^{3}}-\frac{2 b^{4} c f \ln \left(b x +a \right)}{\left(a f -b e \right)^{3} \left(a d -b c \right)^{3}}-\frac{2 b^{4} d e \ln \left(b x +a \right)}{\left(a f -b e \right)^{3} \left(a d -b c \right)^{3}}-\frac{4 b c d^{3} f \ln \left(d x +c \right)}{\left(c f -d e \right)^{3} \left(a d -b c \right)^{3}}-\frac{2 b c f^{4} \ln \left(f x +e \right)}{\left(c f -d e \right)^{3} \left(a f -b e \right)^{3}}+\frac{2 b d^{4} e \ln \left(d x +c \right)}{\left(c f -d e \right)^{3} \left(a d -b c \right)^{3}}+\frac{4 b d e f^{3} \ln \left(f x +e \right)}{\left(c f -d e \right)^{3} \left(a f -b e \right)^{3}}-\frac{b^{3}}{\left(a f -b e \right)^{2} \left(a d -b c \right)^{2} \left(b x +a \right)}-\frac{d^{3}}{\left(c f -d e \right)^{2} \left(a d -b c \right)^{2} \left(d x +c \right)}-\frac{f^{3}}{\left(c f -d e \right)^{2} \left(a f -b e \right)^{2} \left(f x +e \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^2,x)
```

```
[Out] -f^3/(c*f-d*e)^2/(a*f-b*e)^2/(f*x+e)-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e
)*a*d-2*f^4/(c*f-d*e)^3/(a*f-b*e)^3*ln(f*x+e)*b*c+4*f^3/(c*f-d*e)^3/(a*f-b*
e)^3*ln(f*x+e)*b*d*e-d^3/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+2*d^4/(c*f-d*e)^3/
(a*d-b*c)^3*ln(d*x+c)*a*f-4*d^3/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*c*f+2*d
^4/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c)*b*e-b^3/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a
)+4*b^3/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*a*d*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)
^3*ln(b*x+a)*c*f-2*b^4/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)*d*e
```

maxima [B] time = 1.54, size = 2096, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^2,x, algorithm="maxima")
```

```
[Out] -2*(b^4*d*e + (b^4*c - 2*a*b^3*d)*f)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d
 + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*
a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*
c*d^2 - a^6*d^3)*f^3) + 2*(b*d^4*e - (2*b*c*d^3 - a*d^4)*f)*log(d*x + c)/((
b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*e^3 - 3*(b^3*c^4*d
^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*e^2*f + 3*(b^3*c^5*d -
3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*f^2 - (b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + 2*(2*b*d*e*f^3 - (b*c + a*d)
*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*e
^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b
^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d
+ a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*e*f^5) - ((b^3*c*d^2 + a*b
^2*d^3)*e^3 - 2*(b^3*c^2*d + a^2*b*d^3)*e^2*f + (b^3*c^3 + a^3*d^3)*e*f^2 +
 (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*f^3 + 2*(b^3*d^3*e^2*f - (b^3*c*d^
2 + a*b^2*d^3)*e*f^2 + (b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*f^3)*x^2 + (2*
b^3*d^3*e^3 - (b^3*c*d^2 + a*b^2*d^3)*e^2*f - (b^3*c^2*d + a^2*b*d^3)*e*f^2
 + (2*b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*f^3)*x)/((a*b^4*c^3*
d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4)*e^5 - 2*(a*b^4*c^4*d - a^2*b^3*c^3
*d^2 - a^3*b^2*c^2*d^3 + a^4*b*c*d^4)*e^4*f + (a*b^4*c^5 + 2*a^2*b^3*c^4*d
- 6*a^3*b^2*c^3*d^2 + 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^3*f^2 - 2*(a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e^2*f^3 + (a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2)*e*f^4 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*
d^5)*e^4*f - 2*(b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*
e^3*f^2 + (b^5*c^4*d + 2*a*b^4*c^3*d^2 - 6*a^2*b^3*c^2*d^3 + 2*a^3*b^2*c*d^
4 + a^4*b*d^5)*e^2*f^3 - 2*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3
 + a^4*b*c*d^4)*e*f^4 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)
*f^5)*x^3 + ((b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*e^5 - (b^5*c^3*d^2
 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 + a^3*b^2*d^5)*e^4*f - (b^5*c^4*d - 2*a*b^
4*c^3*d^2 + 2*a^2*b^3*c^2*d^3 - 2*a^3*b^2*c*d^4 + a^4*b*d^5)*e^3*f^2 + (b^5
*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 +
a^5*d^5)*e^2*f^3 - (2*a*b^4*c^5 - a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 - a^4*b
*c^2*d^3 + 2*a^5*c*d^4)*e*f^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^
2 + a^5*c^2*d^3)*f^5)*x^2 + ((b^5*c^3*d^2 - a*b^4*c^2*d^3 - a^2*b^3*c*d^4 +
 a^3*b^2*d^5)*e^5 - (2*b^5*c^4*d - a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 - a^3*
b^2*c*d^4 + 2*a^4*b*d^5)*e^4*f + (b^5*c^5 + a*b^4*c^4*d - 2*a^2*b^3*c^3*d^2
 - 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 + a^5*d^5)*e^3*f^2 - (a*b^4*c^5 - 2*a^2*
b^3*c^4*d + 2*a^3*b^2*c^3*d^2 - 2*a^4*b*c^2*d^3 + a^5*c*d^4)*e^2*f^3 - (a^2
*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*e*f^4 + (a^3*b^2*c^
5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*f^5)*x)
```

mupad [B] time = 8.18, size = 1940, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^2,x)
```

```
[Out] - ((a*b^2*c^3*f^3 + a*b^2*d^3*e^3 + a^3*c*d^2*f^3 + b^3*c*d^2*e^3 + a^3*d^3
*e*f^2 + b^3*c^3*e*f^2 - 2*a^2*b*c^2*d*f^3 - 2*a^2*b*d^3*e^2*f - 2*b^3*c^2*
d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2
*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*
d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c
^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^
3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e
*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2) + (2*x^2*(a^2*b*d^3*f^3 + b^3*c^2*d*f^3 +
 b^3*d^3*e^2*f - a*b^2*c*d^2*f^3 - a*b^2*d^3*e*f^2 - b^3*c*d^2*e*f^2))/(a^2
*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^
4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b
^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f +
2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a
^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*
b^2*c^2*d^2*e^2*f^2) - (x*(a*b^2*c^2*d*f^3 - 2*b^3*c^3*f^3 - 2*b^3*d^3*e^3
- 2*a^3*d^3*f^3 + a^2*b*c*d^2*f^3 + a*b^2*d^3*e^2*f + a^2*b*d^3*e*f^2 + b^3
*c*d^2*e^2*f + b^3*c^2*d*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c
^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*
c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a
^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f + 2*a*b^3*c^3*d*
e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 + 2*a^3*b*c*d^3*e^2
*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2))/(x^2*(a*d*f + b*
c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*d*f*x^3) - (log(a + b*
x)*(b^4*(2*c*f + 2*d*e) - 4*a*b^3*d*f))/(b^6*c^3*e^3 + a^6*d^3*f^3 - a^3*b^
3*c^3*f^3 - a^3*b^3*d^3*e^3 - 3*a*b^5*c^2*d*e^3 - 3*a^5*b*c*d^2*f^3 - 3*a*b
^5*c^3*e^2*f - 3*a^5*b*d^3*e*f^2 + 3*a^2*b^4*c*d^2*e^3 + 3*a^4*b^2*c^2*d*f^
3 + 3*a^2*b^4*c^3*e*f^2 + 3*a^4*b^2*d^3*e^2*f + 9*a^2*b^4*c^2*d*e^2*f - 9*a
^3*b^3*c*d^2*e^2*f - 9*a^3*b^3*c^2*d*e*f^2 + 9*a^4*b^2*c*d^2*e*f^2) - (log(
c + d*x)*(d^4*(2*a*f + 2*b*e) - 4*b*c*d^3*f))/(a^3*d^6*e^3 + b^3*c^6*f^3 -
a^3*c^3*d^3*f^3 - b^3*c^3*d^3*e^3 - 3*a^2*b*c*d^5*e^3 - 3*a*b^2*c^5*d*f^3 -
 3*a^3*c*d^5*e^2*f - 3*b^3*c^5*d*e*f^2 + 3*a*b^2*c^2*d^4*e^3 + 3*a^2*b*c^4*
d^2*f^3 + 3*a^3*c^2*d^4*e*f^2 + 3*b^3*c^4*d^2*e^2*f - 9*a*b^2*c^3*d^3*e^2*f
 + 9*a*b^2*c^4*d^2*e*f^2 + 9*a^2*b*c^2*d^4*e^2*f - 9*a^2*b*c^3*d^3*e*f^2) -
 (log(e + f*x)*(f^4*(2*a*d + 2*b*c) - 4*b*d*e*f^3))/(a^3*c^3*f^6 + b^3*d^3*
e^6 - a^3*d^3*e^3*f^3 - b^3*c^3*e^3*f^3 - 3*a^2*b*c^3*e*f^5 - 3*a*b^2*d^3*e
^5*f - 3*a^3*c^2*d*e*f^5 - 3*b^3*c*d^2*e^5*f + 3*a*b^2*c^3*e^2*f^4 + 3*a^2*
b*d^3*e^4*f^2 + 3*a^3*c*d^2*e^2*f^4 + 3*b^3*c^2*d*e^4*f^2 + 9*a*b^2*c*d^2*e
^4*f^2 - 9*a*b^2*c^2*d*e^3*f^3 - 9*a^2*b*c*d^2*e^3*f^3 + 9*a^2*b*c^2*d*e^2*
f^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**2,x)
```

```
[Out] Timed out
```

3.20 $\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx$

Optimal. Leaf size=495

$$\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf\right.}{(bc-}$$

Rubi [A] time = 1.46, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2058}

$$\frac{3b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5}-\frac{3d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(de-cf)^5}+\frac{3f^5\log(e+fx)\left(2a^2d^2f^2-abdf(7de-3cf)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}+\frac{3b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{2(a+bx)^2(bc-ad)^3(be-af)^3}+\frac{3d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}+\frac{d^5}{2(c+dx)^2(bc-ad)^3(de-cf)^3}-\frac{3f^5(-adf-bcf+2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{2(e+fx)^2(be-af)^3(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2 + b*d*
f*x^3)^(-3),x]
```

```
[Out] -b^5/(2*(b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2) + (3*b^5*(b*d*e + b*c*f -
2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) + d^5/(2*(b*c - a*d)^3*(d
*e - c*f)^3*(c + d*x)^2) + (3*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)^4
*(d*e - c*f)^4*(c + d*x)) - f^5/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
 - (3*f^5*(2*b*d*e - b*c*f - a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x)
) + (3*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d*
e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) - (3*d^5*(2*a
^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*f
^2))*Log[c + d*x])/((b*c - a*d)^5*(d*e - c*f)^5) + (3*f^5*(2*a^2*d^2*f^2 -
a*b*d*f*(7*d*e - 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log[e +
f*x])/((b*e - a*f)^5*(d*e - c*f)^5)
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx = \int \left(\frac{b^6}{(bc-ad)^3(be-af)^3(a+bx)^3}-\frac{3}{(bc-}\right.$$

$$= -\frac{b^5}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{3b^5}{(bc-}$$

Mathematica [A] time = 1.19, size = 490, normalized size = 0.99

$$\frac{1}{2}\left(\frac{6f^5\log(e+fx)\left(2a^2d^2f^2+abdf(3cf-7de)+b^2\left(2c^2f^2-7cdef+7d^2e^2\right)\right)}{(be-af)^5(de-cf)^5}+\frac{6d^5\log(c+dx)\left(2a^2d^2f^2+abdf(3de-7cf)+b^2\left(7c^2f^2-7cdef+2d^2e^2\right)\right)}{(bc-ad)^5(cf-de)^5}+\frac{6b^5\log(a+bx)\left(7a^2d^2f^2-7abdf(cf+de)+b^2\left(2c^2f^2+3cdef+2d^2e^2\right)\right)}{(bc-ad)^5(be-af)^5}+\frac{6b^5(-2adf+bcf+bde)}{(a+bx)(bc-ad)^4(be-af)^4}-\frac{b^5}{(a+bx)^2(bc-ad)^3(be-af)^3}+\frac{6d^5(adf-2bcf+bde)}{(c+dx)(bc-ad)^4(de-cf)^4}-\frac{d^5}{(c+dx)^2(bc-ad)^3(cf-de)^3}+\frac{6f^5(adf+bcf-2bde)}{(e+fx)(be-af)^4(de-cf)^4}-\frac{f^5}{(e+fx)^2(be-af)^3(de-cf)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*d*f)*x^2
 + b*d*f*x^3)^(-3),x]
```

```
[Out] (-(b^5/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^2)) + (6*b^5*(b*d*e + b*c*f -
 2*a*d*f))/((b*c - a*d)^4*(b*e - a*f)^4*(a + b*x)) - d^5/((b*c - a*d)^3*(-(
d*e) + c*f)^3*(c + d*x)^2) + (6*d^5*(b*d*e - 2*b*c*f + a*d*f))/((b*c - a*d)
^4*(d*e - c*f)^4*(c + d*x)) - f^5/((b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2)
```

```
 + (6*f^5*(-2*b*d*e + b*c*f + a*d*f))/((b*e - a*f)^4*(d*e - c*f)^4*(e + f*x
)) + (6*b^5*(7*a^2*d^2*f^2 - 7*a*b*d*f*(d*e + c*f) + b^2*(2*d^2*e^2 + 3*c*d
*e*f + 2*c^2*f^2))*Log[a + b*x])/((b*c - a*d)^5*(b*e - a*f)^5) + (6*d^5*(2*
a^2*d^2*f^2 + a*b*d*f*(3*d*e - 7*c*f) + b^2*(2*d^2*e^2 - 7*c*d*e*f + 7*c^2*
f^2))*Log[c + d*x])/((b*c - a*d)^5*(-(d*e) + c*f)^5) + (6*f^5*(2*a^2*d^2*f^
2 + a*b*d*f*(-7*d*e + 3*c*f) + b^2*(7*d^2*e^2 - 7*c*d*e*f + 2*c^2*f^2))*Log
[e + f*x])/((b*e - a*f)^5*(d*e - c*f)^5))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f + a*
d*f)*x^2 + b*d*f*x^3)^(-3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.10, size = 6908, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="giac")
```

```
[Out] -3*(2*b^8*c^2*f^2 - 7*a*b^7*c*d*f^2 + 7*a^2*b^6*d^2*f^2 + 3*b^8*c*d*f*e - 7
*a*b^7*d^2*f*e + 2*b^8*d^2*e^2)*log(abs(b*x + a))/(a^5*b^6*c^5*f^5 - 5*a^6*
b^5*c^4*d*f^5 + 10*a^7*b^4*c^3*d^2*f^5 - 10*a^8*b^3*c^2*d^3*f^5 + 5*a^9*b^2
*c*d^4*f^5 - a^10*b*d^5*f^5 - 5*a^4*b^7*c^5*f^4*e + 25*a^5*b^6*c^4*d*f^4*e
- 50*a^6*b^5*c^3*d^2*f^4*e + 50*a^7*b^4*c^2*d^3*f^4*e - 25*a^8*b^3*c*d^4*f^
4*e + 5*a^9*b^2*d^5*f^4*e + 10*a^3*b^8*c^5*f^3*e^2 - 50*a^4*b^7*c^4*d*f^3*e
^2 + 100*a^5*b^6*c^3*d^2*f^3*e^2 - 100*a^6*b^5*c^2*d^3*f^3*e^2 + 50*a^7*b^4
*c*d^4*f^3*e^2 - 10*a^8*b^3*d^5*f^3*e^2 - 10*a^2*b^9*c^5*f^2*e^3 + 50*a^3*b
^8*c^4*d*f^2*e^3 - 100*a^4*b^7*c^3*d^2*f^2*e^3 + 100*a^5*b^6*c^2*d^3*f^2*e^
3 - 50*a^6*b^5*c*d^4*f^2*e^3 + 10*a^7*b^4*d^5*f^2*e^3 + 5*a*b^10*c^5*f*e^4
- 25*a^2*b^9*c^4*d*f*e^4 + 50*a^3*b^8*c^3*d^2*f*e^4 - 50*a^4*b^7*c^2*d^3*f*
e^4 + 25*a^5*b^6*c*d^4*f*e^4 - 5*a^6*b^5*d^5*f*e^4 - b^11*c^5*e^5 + 5*a*b^1
0*c^4*d*e^5 - 10*a^2*b^9*c^3*d^2*e^5 + 10*a^3*b^8*c^2*d^3*e^5 - 5*a^4*b^7*c
*d^4*e^5 + a^5*b^6*d^5*e^5) + 3*(7*b^2*c^2*d^6*f^2 - 7*a*b*c*d^7*f^2 + 2*a^
2*d^8*f^2 - 7*b^2*c*d^7*f*e + 3*a*b*d^8*f*e + 2*b^2*d^8*e^2)*log(abs(d*x +
c))/(b^5*c^10*d*f^5 - 5*a*b^4*c^9*d^2*f^5 + 10*a^2*b^3*c^8*d^3*f^5 - 10*a^3
*b^2*c^7*d^4*f^5 + 5*a^4*b*c^6*d^5*f^5 - a^5*c^5*d^6*f^5 - 5*b^5*c^9*d^2*f^
4*e + 25*a*b^4*c^8*d^3*f^4*e - 50*a^2*b^3*c^7*d^4*f^4*e + 50*a^3*b^2*c^6*d^
5*f^4*e - 25*a^4*b*c^5*d^6*f^4*e + 5*a^5*c^4*d^7*f^4*e + 10*b^5*c^8*d^3*f^3
*e^2 - 50*a*b^4*c^7*d^4*f^3*e^2 + 100*a^2*b^3*c^6*d^5*f^3*e^2 - 100*a^3*b^2
*c^5*d^6*f^3*e^2 + 50*a^4*b*c^4*d^7*f^3*e^2 - 10*a^5*c^3*d^8*f^3*e^2 - 10*b
^5*c^7*d^4*f^2*e^3 + 50*a*b^4*c^6*d^5*f^2*e^3 - 100*a^2*b^3*c^5*d^6*f^2*e^3
```

+ 100*a^3*b^2*c^4*d^7*f^2*e^3 - 50*a^4*b*c^3*d^8*f^2*e^3 + 10*a^5*c^2*d^9*
f^2*e^3 + 5*b^5*c^6*d^5*f*e^4 - 25*a*b^4*c^5*d^6*f*e^4 + 50*a^2*b^3*c^4*d^7
*f*e^4 - 50*a^3*b^2*c^3*d^8*f*e^4 + 25*a^4*b*c^2*d^9*f*e^4 - 5*a^5*c*d^10*f
*e^4 - b^5*c^5*d^6*e^5 + 5*a*b^4*c^4*d^7*e^5 - 10*a^2*b^3*c^3*d^8*e^5 + 10*
a^3*b^2*c^2*d^9*e^5 - 5*a^4*b*c*d^10*e^5 + a^5*d^11*e^5) + 3*(2*b^2*c^2*f^8
+ 3*a*b*c*d*f^8 + 2*a^2*d^2*f^8 - 7*b^2*c*d*f^7*e - 7*a*b*d^2*f^7*e + 7*b^
2*d^2*f^6*e^2)*log(abs(f*x + e))/(a^5*c^5*f^11 - 5*a^4*b*c^5*f^10*e - 5*a^5
*c^4*d*f^10*e + 10*a^3*b^2*c^5*f^9*e^2 + 25*a^4*b*c^4*d*f^9*e^2 + 10*a^5*c^
3*d^2*f^9*e^2 - 10*a^2*b^3*c^5*f^8*e^3 - 50*a^3*b^2*c^4*d*f^8*e^3 - 50*a^4*
b*c^3*d^2*f^8*e^3 - 10*a^5*c^2*d^3*f^8*e^3 + 5*a*b^4*c^5*f^7*e^4 + 50*a^2*b
^3*c^4*d*f^7*e^4 + 100*a^3*b^2*c^3*d^2*f^7*e^4 + 50*a^4*b*c^2*d^3*f^7*e^4 +
5*a^5*c*d^4*f^7*e^4 - b^5*c^5*f^6*e^5 - 25*a*b^4*c^4*d*f^6*e^5 - 100*a^2*b
^3*c^3*d^2*f^6*e^5 - 100*a^3*b^2*c^2*d^3*f^6*e^5 - 25*a^4*b*c*d^4*f^6*e^5 -
a^5*d^5*f^6*e^5 + 5*b^5*c^4*d*f^5*e^6 + 50*a*b^4*c^3*d^2*f^5*e^6 + 100*a^2
*b^3*c^2*d^3*f^5*e^6 + 50*a^3*b^2*c*d^4*f^5*e^6 + 5*a^4*b*d^5*f^5*e^6 - 10*
b^5*c^3*d^2*f^4*e^7 - 50*a*b^4*c^2*d^3*f^4*e^7 - 50*a^2*b^3*c*d^4*f^4*e^7 -
10*a^3*b^2*d^5*f^4*e^7 + 10*b^5*c^2*d^3*f^3*e^8 + 25*a*b^4*c*d^4*f^3*e^8 +
10*a^2*b^3*d^5*f^3*e^8 - 5*b^5*c*d^4*f^2*e^9 - 5*a*b^4*d^5*f^2*e^9 + b^5*d
^5*f*e^10) + 1/2*(12*b^7*c^5*d^2*f^7*x^5 - 30*a*b^6*c^4*d^3*f^7*x^5 + 12*a^
2*b^5*c^3*d^4*f^7*x^5 + 12*a^3*b^4*c^2*d^5*f^7*x^5 - 30*a^4*b^3*c*d^6*f^7*x
^5 + 12*a^5*b^2*d^7*f^7*x^5 - 30*b^7*c^4*d^3*f^6*x^5*e + 96*a*b^6*c^3*d^4*f
^6*x^5*e - 72*a^2*b^5*c^2*d^5*f^6*x^5*e + 96*a^3*b^4*c*d^6*f^6*x^5*e - 30*a
^4*b^3*d^7*f^6*x^5*e + 24*b^7*c^6*d*f^7*x^4 - 42*a*b^6*c^5*d^2*f^7*x^4 - 21
*a^2*b^5*c^4*d^3*f^7*x^4 + 42*a^3*b^4*c^3*d^4*f^7*x^4 - 21*a^4*b^3*c^2*d^5*
f^7*x^4 - 42*a^5*b^2*c*d^6*f^7*x^4 + 24*a^6*b*d^7*f^7*x^4 + 12*b^7*c^3*d^4*
f^5*x^5*e^2 - 72*a*b^6*c^2*d^5*f^5*x^5*e^2 - 72*a^2*b^5*c*d^6*f^5*x^5*e^2 +
12*a^3*b^4*d^7*f^5*x^5*e^2 - 42*b^7*c^5*d^2*f^6*x^4*e + 102*a*b^6*c^4*d^3*
f^6*x^4*e + 18*a^2*b^5*c^3*d^4*f^6*x^4*e + 18*a^3*b^4*c^2*d^5*f^6*x^4*e + 1
02*a^4*b^3*c*d^6*f^6*x^4*e - 42*a^5*b^2*d^7*f^6*x^4*e + 12*b^7*c^7*f^7*x^3
+ 6*a*b^6*c^6*d*f^7*x^3 - 74*a^2*b^5*c^5*d^2*f^7*x^3 + 38*a^3*b^4*c^4*d^3*f
^7*x^3 + 38*a^4*b^3*c^3*d^4*f^7*x^3 - 74*a^5*b^2*c^2*d^5*f^7*x^3 + 6*a^6*b*
c*d^6*f^7*x^3 + 12*a^7*d^7*f^7*x^3 + 12*b^7*c^2*d^5*f^4*x^5*e^3 + 96*a*b^6*
c*d^6*f^4*x^5*e^3 + 12*a^2*b^5*d^7*f^4*x^5*e^3 - 21*b^7*c^4*d^3*f^5*x^4*e^2
+ 18*a*b^6*c^3*d^4*f^5*x^4*e^2 - 234*a^2*b^5*c^2*d^5*f^5*x^4*e^2 + 18*a^3*
b^4*c*d^6*f^5*x^4*e^2 - 21*a^4*b^3*d^7*f^5*x^4*e^2 + 6*b^7*c^6*d*f^6*x^3*e
- 56*a*b^6*c^5*d^2*f^6*x^3*e + 172*a^2*b^5*c^4*d^3*f^6*x^3*e - 136*a^3*b^4*
c^3*d^4*f^6*x^3*e + 172*a^4*b^3*c^2*d^5*f^6*x^3*e - 56*a^5*b^2*c*d^6*f^6*x^
3*e + 6*a^6*b*d^7*f^6*x^3*e + 18*a*b^6*c^7*f^7*x^2 - 37*a^2*b^5*c^6*d*f^7*x
^2 - 3*a^3*b^4*c^5*d^2*f^7*x^2 + 32*a^4*b^3*c^4*d^3*f^7*x^2 - 3*a^5*b^2*c^3
*d^4*f^7*x^2 - 37*a^6*b*c^2*d^5*f^7*x^2 + 18*a^7*c*d^6*f^7*x^2 - 30*b^7*c*d
^6*f^3*x^5*e^4 - 30*a*b^6*d^7*f^3*x^5*e^4 + 42*b^7*c^3*d^4*f^4*x^4*e^3 + 18
*a*b^6*c^2*d^5*f^4*x^4*e^3 + 18*a^2*b^5*c*d^6*f^4*x^4*e^3 + 42*a^3*b^4*d^7*
f^4*x^4*e^3 - 74*b^7*c^5*d^2*f^5*x^3*e^2 + 172*a*b^6*c^4*d^3*f^5*x^3*e^2 -
104*a^2*b^5*c^3*d^4*f^5*x^3*e^2 - 104*a^3*b^4*c^2*d^5*f^5*x^3*e^2 + 172*a^4
*b^3*c*d^6*f^5*x^3*e^2 - 74*a^5*b^2*d^7*f^5*x^3*e^2 + 18*b^7*c^7*f^6*x^2*e
- 34*a*b^6*c^6*d*f^6*x^2*e + 9*a^2*b^5*c^5*d^2*f^6*x^2*e + a^3*b^4*c^4*d^3*
f^6*x^2*e + a^4*b^3*c^3*d^4*f^6*x^2*e + 9*a^5*b^2*c^2*d^5*f^6*x^2*e - 34*a^
6*b*c*d^6*f^6*x^2*e + 18*a^7*d^7*f^6*x^2*e + 4*a^2*b^5*c^7*f^7*x - 12*a^3*b
^4*c^6*d*f^7*x + 8*a^4*b^3*c^5*d^2*f^7*x + 8*a^5*b^2*c^4*d^3*f^7*x - 12*a^6
*b*c^3*d^4*f^7*x + 4*a^7*c^2*d^5*f^7*x + 12*b^7*d^7*f^2*x^5*e^5 - 21*b^7*c^
2*d^5*f^3*x^4*e^4 + 102*a*b^6*c*d^6*f^3*x^4*e^4 - 21*a^2*b^5*d^7*f^3*x^4*e^
4 + 38*b^7*c^4*d^3*f^4*x^3*e^3 - 136*a*b^6*c^3*d^4*f^4*x^3*e^3 - 104*a^2*b^
5*c^2*d^5*f^4*x^3*e^3 - 136*a^3*b^4*c*d^6*f^4*x^3*e^3 + 38*a^4*b^3*d^7*f^4*
x^3*e^3 - 37*b^7*c^6*d*f^5*x^2*e^2 + 9*a*b^6*c^5*d^2*f^5*x^2*e^2 + 234*a^2*
b^5*c^4*d^3*f^5*x^2*e^2 - 208*a^3*b^4*c^3*d^4*f^5*x^2*e^2 + 234*a^4*b^3*c^2
*d^5*f^5*x^2*e^2 + 9*a^5*b^2*c*d^6*f^5*x^2*e^2 - 37*a^6*b*d^7*f^5*x^2*e^2 +
28*a*b^6*c^7*f^6*x*e - 66*a^2*b^5*c^6*d*f^6*x*e + 34*a^3*b^4*c^5*d^2*f^6*x
*e - 16*a^4*b^3*c^4*d^3*f^6*x*e + 34*a^5*b^2*c^3*d^4*f^6*x*e - 66*a^6*b*c^2
*d^5*f^6*x*e + 28*a^7*c*d^6*f^6*x*e - a^3*b^4*c^7*f^7 + 4*a^4*b^3*c^6*d*f^7

 - 6*a^5*b^2*c^5*d^2*f^7 + 4*a^6*b*c^4*d^3*f^7 - a^7*c^3*d^4*f^7 - 42*b^7*c
*d^6*f^2*x^4*e^5 - 42*a*b^6*d^7*f^2*x^4*e^5 + 38*b^7*c^3*d^4*f^3*x^3*e^4 +
172*a*b^6*c^2*d^5*f^3*x^3*e^4 + 172*a^2*b^5*c*d^6*f^3*x^3*e^4 + 38*a^3*b^4*
d^7*f^3*x^3*e^4 - 3*b^7*c^5*d^2*f^4*x^2*e^3 + a*b^6*c^4*d^3*f^4*x^2*e^3 - 2
08*a^2*b^5*c^3*d^4*f^4*x^2*e^3 - 208*a^3*b^4*c^2*d^5*f^4*x^2*e^3 + a^4*b^3*
c*d^6*f^4*x^2*e^3 - 3*a^5*b^2*d^7*f^4*x^2*e^3 + 4*b^7*c^7*f^5*x*e^2 - 66*a*
b^6*c^6*d*f^5*x*e^2 + 156*a^2*b^5*c^5*d^2*f^5*x*e^2 - 52*a^3*b^4*c^4*d^3*f^
5*x*e^2 - 52*a^4*b^3*c^3*d^4*f^5*x*e^2 + 156*a^5*b^2*c^2*d^5*f^5*x*e^2 - 66
*a^6*b*c*d^6*f^5*x*e^2 + 4*a^7*d^7*f^5*x*e^2 + 7*a^2*b^5*c^7*f^6*e - 21*a^3
*b^4*c^6*d*f^6*e + 14*a^4*b^3*c^5*d^2*f^6*e + 14*a^5*b^2*c^4*d^3*f^6*e - 21
*a^6*b*c^3*d^4*f^6*e + 7*a^7*c^2*d^5*f^6*e + 24*b^7*d^7*f*x^4*e^6 - 74*b^7*
c^2*d^5*f^2*x^3*e^5 - 56*a*b^6*c*d^6*f^2*x^3*e^5 - 74*a^2*b^5*d^7*f^2*x^3*e
^5 + 32*b^7*c^4*d^3*f^3*x^2*e^4 + a*b^6*c^3*d^4*f^3*x^2*e^4 + 234*a^2*b^5*c
^2*d^5*f^3*x^2*e^4 + a^3*b^4*c*d^6*f^3*x^2*e^4 + 32*a^4*b^3*d^7*f^3*x^2*e^4
 - 12*b^7*c^6*d*f^4*x*e^3 + 34*a*b^6*c^5*d^2*f^4*x*e^3 - 52*a^2*b^5*c^4*d^3
*f^4*x*e^3 - 52*a^4*b^3*c^2*d^5*f^4*x*e^3 + 34*a^5*b^2*c*d^6*f^4*x*e^3 - 12
*a^6*b*d^7*f^4*x*e^3 + 7*a*b^6*c^7*f^5*e^2 - 26*a^2*b^5*c^6*d*f^5*e^2 + 52*
a^3*b^4*c^5*d^2*f^5*e^2 - 78*a^4*b^3*c^4*d^3*f^5*e^2 + 52*a^5*b^2*c^3*d^4*f
^5*e^2 - 26*a^6*b*c^2*d^5*f^5*e^2 + 7*a^7*c*d^6*f^5*e^2 + 6*b^7*c*d^6*f*x^3
*e^6 + 6*a*b^6*d^7*f*x^3*e^6 - 3*b^7*c^3*d^4*f^2*x^2*e^5 + 9*a*b^6*c^2*d^5*
f^2*x^2*e^5 + 9*a^2*b^5*c*d^6*f^2*x^2*e^5 - 3*a^3*b^4*d^7*f^2*x^2*e^5 + 8*b
^7*c^5*d^2*f^3*x*e^4 - 16*a*b^6*c^4*d^3*f^3*x*e^4 - 52*a^2*b^5*c^3*d^4*f^3*
x*e^4 - 52*a^3*b^4*c^2*d^5*f^3*x*e^4 - 16*a^4*b^3*c*d^6*f^3*x*e^4 + 8*a^5*b
^2*d^7*f^3*x*e^4 - b^7*c^7*f^4*e^3 - 21*a*b^6*c^6*d*f^4*e^3 + 52*a^2*b^5*c^
5*d^2*f^4*e^3 + 52*a^5*b^2*c^2*d^5*f^4*e^3 - 21*a^6*b*c*d^6*f^4*e^3 - a^7*d
^7*f^4*e^3 + 12*b^7*d^7*x^3*e^7 - 37*b^7*c^2*d^5*f*x^2*e^6 - 34*a*b^6*c*d^6
*f*x^2*e^6 - 37*a^2*b^5*d^7*f*x^2*e^6 + 8*b^7*c^4*d^3*f^2*x*e^5 + 34*a*b^6*
c^3*d^4*f^2*x*e^5 + 156*a^2*b^5*c^2*d^5*f^2*x*e^5 + 34*a^3*b^4*c*d^6*f^2*x*
e^5 + 8*a^4*b^3*d^7*f^2*x*e^5 + 4*b^7*c^6*d*f^3*e^4 + 14*a*b^6*c^5*d^2*f^3*
e^4 - 78*a^2*b^5*c^4*d^3*f^3*e^4 - 78*a^4*b^3*c^2*d^5*f^3*e^4 + 14*a^5*b^2*
c*d^6*f^3*e^4 + 4*a^6*b*d^7*f^3*e^4 + 18*b^7*c*d^6*x^2*e^7 + 18*a*b^6*d^7*x
^2*e^7 - 12*b^7*c^3*d^4*f*x*e^6 - 66*a*b^6*c^2*d^5*f*x*e^6 - 66*a^2*b^5*c*d
^6*f*x*e^6 - 12*a^3*b^4*d^7*f*x*e^6 - 6*b^7*c^5*d^2*f^2*e^5 + 14*a*b^6*c^4*
d^3*f^2*e^5 + 52*a^2*b^5*c^3*d^4*f^2*e^5 + 52*a^3*b^4*c^2*d^5*f^2*e^5 + 14*
a^4*b^3*c*d^6*f^2*e^5 - 6*a^5*b^2*d^7*f^2*e^5 + 4*b^7*c^2*d^5*x*e^7 + 28*a*
b^6*c*d^6*x*e^7 + 4*a^2*b^5*d^7*x*e^7 + 4*b^7*c^4*d^3*f*e^6 - 21*a*b^6*c^3*
d^4*f*e^6 - 26*a^2*b^5*c^2*d^5*f*e^6 - 21*a^3*b^4*c*d^6*f*e^6 + 4*a^4*b^3*d
^7*f*e^6 - b^7*c^3*d^4*e^7 + 7*a*b^6*c^2*d^5*e^7 + 7*a^2*b^5*c*d^6*e^7 - a^
3*b^4*d^7*e^7)/((a^4*b^4*c^8*f^8 - 4*a^5*b^3*c^7*d*f^8 + 6*a^6*b^2*c^6*d^2*
f^8 - 4*a^7*b*c^5*d^3*f^8 + a^8*c^4*d^4*f^8 - 4*a^3*b^5*c^8*f^7*e + 12*a^4*
b^4*c^7*d*f^7*e - 8*a^5*b^3*c^6*d^2*f^7*e - 8*a^6*b^2*c^5*d^3*f^7*e + 12*a^
7*b*c^4*d^4*f^7*e - 4*a^8*c^3*d^5*f^7*e + 6*a^2*b^6*c^8*f^6*e^2 - 8*a^3*b^5
*c^7*d*f^6*e^2 - 22*a^4*b^4*c^6*d^2*f^6*e^2 + 48*a^5*b^3*c^5*d^3*f^6*e^2 -
22*a^6*b^2*c^4*d^4*f^6*e^2 - 8*a^7*b*c^3*d^5*f^6*e^2 + 6*a^8*c^2*d^6*f^6*e^
2 - 4*a*b^7*c^8*f^5*e^3 - 8*a^2*b^6*c^7*d*f^5*e^3 + 48*a^3*b^5*c^6*d^2*f^5*
e^3 - 36*a^4*b^4*c^5*d^3*f^5*e^3 - 36*a^5*b^3*c^4*d^4*f^5*e^3 + 48*a^6*b^2*
c^3*d^5*f^5*e^3 - 8*a^7*b*c^2*d^6*f^5*e^3 - 4*a^8*c*d^7*f^5*e^3 + b^8*c^8*f
^4*e^4 + 12*a*b^7*c^7*d*f^4*e^4 - 22*a^2*b^6*c^6*d^2*f^4*e^4 - 36*a^3*b^5*c
^5*d^3*f^4*e^4 + 90*a^4*b^4*c^4*d^4*f^4*e^4 - 36*a^5*b^3*c^3*d^5*f^4*e^4 -
22*a^6*b^2*c^2*d^6*f^4*e^4 + 12*a^7*b*c*d^7*f^4*e^4 + a^8*d^8*f^4*e^4 - 4*b
^8*c^7*d*f^3*e^5 - 8*a*b^7*c^6*d^2*f^3*e^5 + 48*a^2*b^6*c^5*d^3*f^3*e^5 - 3
6*a^3*b^5*c^4*d^4*f^3*e^5 - 36*a^4*b^4*c^3*d^5*f^3*e^5 + 48*a^5*b^3*c^2*d^6
*f^3*e^5 - 8*a^6*b^2*c*d^7*f^3*e^5 - 4*a^7*b*d^8*f^3*e^5 + 6*b^8*c^6*d^2*f^
2*e^6 - 8*a*b^7*c^5*d^3*f^2*e^6 - 22*a^2*b^6*c^4*d^4*f^2*e^6 + 48*a^3*b^5*c
^3*d^5*f^2*e^6 - 22*a^4*b^4*c^2*d^6*f^2*e^6 - 8*a^5*b^3*c*d^7*f^2*e^6 + 6*a
^6*b^2*d^8*f^2*e^6 - 4*b^8*c^5*d^3*f*e^7 + 12*a*b^7*c^4*d^4*f*e^7 - 8*a^2*b
^6*c^3*d^5*f*e^7 - 8*a^3*b^5*c^2*d^6*f*e^7 + 12*a^4*b^4*c*d^7*f*e^7 - 4*a^5
*b^3*d^8*f*e^7 + b^8*c^4*d^4*e^8 - 4*a*b^7*c^3*d^5*e^8 + 6*a^2*b^6*c^2*d^6*
e^8 - 4*a^3*b^5*c*d^7*e^8 + a^4*b^4*d^8*e^8)*(b*d*f*x^3 + b*c*f*x^2 + a*d*f

```
*x^2 + b*d*x^2*e + a*c*f*x + b*c*x*e + a*d*x*e + a*c*e)^2)
```

maple [B] time = 0.03, size = 1076, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^3,x)
```

```
[Out] 21*d^6/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*b^2*c*e*f-21*b^6/(a*f-b*e)^5/(a*d-
b*c)^5*ln(b*x+a)*a*c*d*f^2-21*b^6/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a*d^2*e
*f+9*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*c*d*e*f+9*f^7/(c*f-d*e)^5/(a*f-b
*e)^5*ln(f*x+e)*a*b*c*d-21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*a*b*d^2*e-
21*f^6/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c*d*e+21*d^6/(c*f-d*e)^5/(a*d-
b*c)^5*ln(d*x+c)*a*b*c*f^2-9*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a*b*e*f-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*e^2*b^2+3*b^6/(a*f-b*e)^4/(a*d-b*c)
^4/(b*x+a)*c*f+3*b^6/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*d*e+6*b^7/(a*f-b*e)^5/
(a*d-b*c)^5*ln(b*x+a)*c^2*f^2+6*b^7/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*d^2*e
^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*c+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*
ln(f*x+e)*a^2*d^2+6*f^7/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)*b^2*c^2+3*d^6/(c*
f-d*e)^4/(a*d-b*c)^4/(d*x+c)*a*f+3*d^6/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*e-
6*d^7/(c*f-d*e)^5/(a*d-b*c)^5*ln(d*x+c)*a^2*f^2+3*f^6/(c*f-d*e)^4/(a*f-b*e)
^4/(f*x+e)*a*d-1/2*b^5/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2+1/2*d^5/(c*f-d*e)^
3/(a*d-b*c)^3/(d*x+c)^2-1/2*f^5/(c*f-d*e)^3/(a*f-b*e)^3/(f*x+e)^2-6*f^5/(c*
f-d*e)^4/(a*f-b*e)^4/(f*x+e)*b*d*e+21*f^5/(c*f-d*e)^5/(a*f-b*e)^5*ln(f*x+e)
*d^2*e^2*b^2-6*d^5/(c*f-d*e)^4/(a*d-b*c)^4/(d*x+c)*b*c*f-21*d^5/(c*f-d*e)^5
/(a*d-b*c)^5*ln(d*x+c)*b^2*c^2*f^2-6*b^5/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)*a*
d*f+21*b^5/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)*a^2*d^2*f^2
```

maxima [B] time = 5.72, size = 11005, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)
^3,x, algorithm="maxima")
```

```
[Out] 3*(2*b^7*d^2*e^2 + (3*b^7*c*d - 7*a*b^6*d^2)*e*f + (2*b^7*c^2 - 7*a*b^6*c*d
 + 7*a^2*b^5*d^2)*f^2)*log(b*x + a)/((b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8
*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b
^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*
b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4
*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2
- 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d
^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^
4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5
)*e*f^4 - (a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*
c^2*d^3 + 5*a^9*b*c*d^4 - a^10*d^5)*f^5) - 3*(2*b^2*d^7*e^2 - (7*b^2*c*d^6
- 3*a*b*d^7)*e*f + (7*b^2*c^2*d^5 - 7*a*b*c*d^6 + 2*a^2*d^7)*f^2)*log(d*x +
 c)/((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d
^8 + 5*a^4*b*c*d^9 - a^5*d^10)*e^5 - 5*(b^5*c^6*d^4 - 5*a*b^4*c^5*d^5 + 10*
a^2*b^3*c^4*d^6 - 10*a^3*b^2*c^3*d^7 + 5*a^4*b*c^2*d^8 - a^5*c*d^9)*e^4*f +
 10*(b^5*c^7*d^3 - 5*a*b^4*c^6*d^4 + 10*a^2*b^3*c^5*d^5 - 10*a^3*b^2*c^4*d^
6 + 5*a^4*b*c^3*d^7 - a^5*c^2*d^8)*e^3*f^2 - 10*(b^5*c^8*d^2 - 5*a*b^4*c^7*
d^3 + 10*a^2*b^3*c^6*d^4 - 10*a^3*b^2*c^5*d^5 + 5*a^4*b*c^4*d^6 - a^5*c^3*d
^7)*e^2*f^3 + 5*(b^5*c^9*d - 5*a*b^4*c^8*d^2 + 10*a^2*b^3*c^7*d^3 - 10*a^3*
b^2*c^6*d^4 + 5*a^4*b*c^5*d^5 - a^5*c^4*d^6)*e*f^4 - (b^5*c^10 - 5*a*b^4*c^
9*d + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 + 5*a^4*b*c^6*d^4 - a^5*c^5*d
^5)*f^5) + 3*(7*b^2*d^2*e^2*f^5 - 7*(b^2*c*d + a*b*d^2)*e*f^6 + (2*b^2*c^2
+ 3*a*b*c*d + 2*a^2*d^2)*f^7)*log(f*x + e)/(b^5*d^5*e^10 + a^5*c^5*f^10 - 5
*(b^5*c*d^4 + a*b^4*d^5)*e^9*f + 5*(2*b^5*c^2*d^3 + 5*a*b^4*c*d^4 + 2*a^2*b
```

```
^3*d^5)*e^8*f^2 - 10*(b^5*c^3*d^2 + 5*a*b^4*c^2*d^3 + 5*a^2*b^3*c*d^4 + a^3
*b^2*d^5)*e^7*f^3 + 5*(b^5*c^4*d + 10*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 +
10*a^3*b^2*c*d^4 + a^4*b*d^5)*e^6*f^4 - (b^5*c^5 + 25*a*b^4*c^4*d + 100*a^2
*b^3*c^3*d^2 + 100*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*e^5*f^5 + 5*
(a*b^4*c^5 + 10*a^2*b^3*c^4*d + 20*a^3*b^2*c^3*d^2 + 10*a^4*b*c^2*d^3 + a^5
*c*d^4)*e^4*f^6 - 10*(a^2*b^3*c^5 + 5*a^3*b^2*c^4*d + 5*a^4*b*c^3*d^2 + a^5
*c^2*d^3)*e^3*f^7 + 5*(2*a^3*b^2*c^5 + 5*a^4*b*c^4*d + 2*a^5*c^3*d^2)*e^2*f
^8 - 5*(a^4*b*c^5 + a^5*c^4*d)*e*f^9) - 1/2*((b^7*c^3*d^4 - 7*a*b^6*c^2*d^5
 - 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^7 - (4*b^7*c^4*d^3 - 21*a*b^6*c^3*d^4 -
 26*a^2*b^5*c^2*d^5 - 21*a^3*b^4*c*d^6 + 4*a^4*b^3*d^7)*e^6*f + 2*(3*b^7*c^
5*d^2 - 7*a*b^6*c^4*d^3 - 26*a^2*b^5*c^3*d^4 - 26*a^3*b^4*c^2*d^5 - 7*a^4*b
^3*c*d^6 + 3*a^5*b^2*d^7)*e^5*f^2 - 2*(2*b^7*c^6*d + 7*a*b^6*c^5*d^2 - 39*a
^2*b^5*c^4*d^3 - 39*a^4*b^3*c^2*d^5 + 7*a^5*b^2*c*d^6 + 2*a^6*b*d^7)*e^4*f^
3 + (b^7*c^7 + 21*a*b^6*c^6*d - 52*a^2*b^5*c^5*d^2 - 52*a^5*b^2*c^2*d^5 + 2
1*a^6*b*c*d^6 + a^7*d^7)*e^3*f^4 - (7*a*b^6*c^7 - 26*a^2*b^5*c^6*d + 52*a^3
*b^4*c^5*d^2 - 78*a^4*b^3*c^4*d^3 + 52*a^5*b^2*c^3*d^4 - 26*a^6*b*c^2*d^5 +
 7*a^7*c*d^6)*e^2*f^5 - 7*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c^5*d^
2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*e*f^6 + (a^3*b^4*c^7
 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*f^7
 - 6*(2*b^7*d^7*e^5*f^2 - 5*(b^7*c*d^6 + a*b^6*d^7)*e^4*f^3 + 2*(b^7*c^2*d^
5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*e^3*f^4 + 2*(b^7*c^3*d^4 - 6*a*b^6*c^2*d^5
 - 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^2*f^5 - (5*b^7*c^4*d^3 - 16*a*b^6*c^3*d
^4 + 12*a^2*b^5*c^2*d^5 - 16*a^3*b^4*c*d^6 + 5*a^4*b^3*d^7)*e*f^6 + (2*b^7*
c^5*d^2 - 5*a*b^6*c^4*d^3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 5*a^4*b
^3*c*d^6 + 2*a^5*b^2*d^7)*f^7)*x^5 - 3*(8*b^7*d^7*e^6*f - 14*(b^7*c*d^6 + a
*b^6*d^7)*e^5*f^2 - (7*b^7*c^2*d^5 - 34*a*b^6*c*d^6 + 7*a^2*b^5*d^7)*e^4*f^
3 + 2*(7*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 + 7*a^3*b^4*d^7)*e
^3*f^4 - (7*b^7*c^4*d^3 - 6*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 - 6*a^3*b^4*
c*d^6 + 7*a^4*b^3*d^7)*e^2*f^5 - 2*(7*b^7*c^5*d^2 - 17*a*b^6*c^4*d^3 - 3*a^
2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 - 17*a^4*b^3*c*d^6 + 7*a^5*b^2*d^7)*e*f^6
 + (8*b^7*c^6*d - 14*a*b^6*c^5*d^2 - 7*a^2*b^5*c^4*d^3 + 14*a^3*b^4*c^3*d^4
 - 7*a^4*b^3*c^2*d^5 - 14*a^5*b^2*c*d^6 + 8*a^6*b*d^7)*f^7)*x^4 - 2*(6*b^7*
d^7*e^7 + 3*(b^7*c*d^6 + a*b^6*d^7)*e^6*f - (37*b^7*c^2*d^5 + 28*a*b^6*c*d^
6 + 37*a^2*b^5*d^7)*e^5*f^2 + (19*b^7*c^3*d^4 + 86*a*b^6*c^2*d^5 + 86*a^2*b
^5*c*d^6 + 19*a^3*b^4*d^7)*e^4*f^3 + (19*b^7*c^4*d^3 - 68*a*b^6*c^3*d^4 - 5
2*a^2*b^5*c^2*d^5 - 68*a^3*b^4*c*d^6 + 19*a^4*b^3*d^7)*e^3*f^4 - (37*b^7*c^
5*d^2 - 86*a*b^6*c^4*d^3 + 52*a^2*b^5*c^3*d^4 + 52*a^3*b^4*c^2*d^5 - 86*a^4
*b^3*c*d^6 + 37*a^5*b^2*d^7)*e^2*f^5 + (3*b^7*c^6*d - 28*a*b^6*c^5*d^2 + 86
*a^2*b^5*c^4*d^3 - 68*a^3*b^4*c^3*d^4 + 86*a^4*b^3*c^2*d^5 - 28*a^5*b^2*c*d
^6 + 3*a^6*b*d^7)*e*f^6 + (6*b^7*c^7 + 3*a*b^6*c^6*d - 37*a^2*b^5*c^5*d^2 +
 19*a^3*b^4*c^4*d^3 + 19*a^4*b^3*c^3*d^4 - 37*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + 6*a^7*d^7)*f^7)*x^3 - (18*(b^7*c*d^6 + a*b^6*d^7)*e^7 - (37*b^7*c^2*d^
5 + 34*a*b^6*c*d^6 + 37*a^2*b^5*d^7)*e^6*f - 3*(b^7*c^3*d^4 - 3*a*b^6*c^2*d
^5 - 3*a^2*b^5*c*d^6 + a^3*b^4*d^7)*e^5*f^2 + (32*b^7*c^4*d^3 + a*b^6*c^3*d
^4 + 234*a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + 32*a^4*b^3*d^7)*e^4*f^3 - (3*b^7
*c^5*d^2 - a*b^6*c^4*d^3 + 208*a^2*b^5*c^3*d^4 + 208*a^3*b^4*c^2*d^5 - a^4*
b^3*c*d^6 + 3*a^5*b^2*d^7)*e^3*f^4 - (37*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 234*
a^2*b^5*c^4*d^3 + 208*a^3*b^4*c^3*d^4 - 234*a^4*b^3*c^2*d^5 - 9*a^5*b^2*c*d
^6 + 37*a^6*b*d^7)*e^2*f^5 + (18*b^7*c^7 - 34*a*b^6*c^6*d + 9*a^2*b^5*c^5*d
^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 - 34*a^6*b*c*d^6
 + 18*a^7*d^7)*e*f^6 + (18*a*b^6*c^7 - 37*a^2*b^5*c^6*d - 3*a^3*b^4*c^5*d^2
 + 32*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 37*a^6*b*c^2*d^5 + 18*a^7*c*d^6
)*f^7)*x^2 - 2*(2*(b^7*c^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*e^7 - 3*(2*b^
7*c^3*d^4 + 11*a*b^6*c^2*d^5 + 11*a^2*b^5*c*d^6 + 2*a^3*b^4*d^7)*e^6*f + (4
*b^7*c^4*d^3 + 17*a*b^6*c^3*d^4 + 78*a^2*b^5*c^2*d^5 + 17*a^3*b^4*c*d^6 + 4
*a^4*b^3*d^7)*e^5*f^2 + 2*(2*b^7*c^5*d^2 - 4*a*b^6*c^4*d^3 - 13*a^2*b^5*c^3
*d^4 - 13*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + 2*a^5*b^2*d^7)*e^4*f^3 - (6*b
^7*c^6*d - 17*a*b^6*c^5*d^2 + 26*a^2*b^5*c^4*d^3 + 26*a^4*b^3*c^2*d^5 - 17*
a^5*b^2*c*d^6 + 6*a^6*b*d^7)*e^3*f^4 + (2*b^7*c^7 - 33*a*b^6*c^6*d + 78*a^2
```

```
*b^5*c^5*d^2 - 26*a^3*b^4*c^4*d^3 - 26*a^4*b^3*c^3*d^4 + 78*a^5*b^2*c^2*d^5
 - 33*a^6*b*c*d^6 + 2*a^7*d^7)*e^2*f^5 + (14*a*b^6*c^7 - 33*a^2*b^5*c^6*d +
 17*a^3*b^4*c^5*d^2 - 8*a^4*b^3*c^4*d^3 + 17*a^5*b^2*c^3*d^4 - 33*a^6*b*c^2
*d^5 + 14*a^7*c*d^6)*e*f^6 + 2*(a^2*b^5*c^7 - 3*a^3*b^4*c^6*d + 2*a^4*b^3*c
^5*d^2 + 2*a^5*b^2*c^4*d^3 - 3*a^6*b*c^3*d^4 + a^7*c^2*d^5)*f^7)*x)/((a^2*b
^8*c^6*d^4 - 4*a^3*b^7*c^5*d^5 + 6*a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + a^
6*b^4*c^2*d^8)*e^10 - 4*(a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 2*a^4*b^6*c^
5*d^5 + 2*a^5*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 + a^7*b^3*c^2*d^8)*e^9*f + 2*
(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^4 + 24*a^5*b^5*c^
5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8)*e^8*f^2
 - 4*(a^2*b^8*c^9*d + 2*a^3*b^7*c^8*d^2 - 12*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^
6*d^4 + 9*a^6*b^4*c^5*d^5 - 12*a^7*b^3*c^4*d^6 + 2*a^8*b^2*c^3*d^7 + a^9*b*
c^2*d^8)*e^7*f^3 + (a^2*b^8*c^10 + 12*a^3*b^7*c^9*d - 22*a^4*b^6*c^8*d^2 -
36*a^5*b^5*c^7*d^3 + 90*a^6*b^4*c^6*d^4 - 36*a^7*b^3*c^5*d^5 - 22*a^8*b^2*c
^4*d^6 + 12*a^9*b*c^3*d^7 + a^10*c^2*d^8)*e^6*f^4 - 4*(a^3*b^7*c^10 + 2*a^4
*b^6*c^9*d - 12*a^5*b^5*c^8*d^2 + 9*a^6*b^4*c^7*d^3 + 9*a^7*b^3*c^6*d^4 - 1
2*a^8*b^2*c^5*d^5 + 2*a^9*b*c^4*d^6 + a^10*c^3*d^7)*e^5*f^5 + 2*(3*a^4*b^6*
c^10 - 4*a^5*b^5*c^9*d - 11*a^6*b^4*c^8*d^2 + 24*a^7*b^3*c^7*d^3 - 11*a^8*b
^2*c^6*d^4 - 4*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^4*f^6 - 4*(a^5*b^5*c^10 -
3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 +
 a^10*c^5*d^5)*e^3*f^7 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^
2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*e^2*f^8 + ((b^10*c^4*d^6 - 4*a*b^9*c^3*
d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^8*f^2 - 4*(b^10
*c^5*d^5 - 3*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*
b^6*c*d^9 + a^5*b^5*d^10)*e^7*f^3 + 2*(3*b^10*c^6*d^4 - 4*a*b^9*c^5*d^5 - 1
1*a^2*b^8*c^4*d^6 + 24*a^3*b^7*c^3*d^7 - 11*a^4*b^6*c^2*d^8 - 4*a^5*b^5*c*d
^9 + 3*a^6*b^4*d^10)*e^6*f^4 - 4*(b^10*c^7*d^3 + 2*a*b^9*c^6*d^4 - 12*a^2*b
^8*c^5*d^5 + 9*a^3*b^7*c^4*d^6 + 9*a^4*b^6*c^3*d^7 - 12*a^5*b^5*c^2*d^8 + 2
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^5*f^5 + (b^10*c^8*d^2 + 12*a*b^9*c^7*d^3 -
 22*a^2*b^8*c^6*d^4 - 36*a^3*b^7*c^5*d^5 + 90*a^4*b^6*c^4*d^6 - 36*a^5*b^5*
c^3*d^7 - 22*a^6*b^4*c^2*d^8 + 12*a^7*b^3*c*d^9 + a^8*b^2*d^10)*e^4*f^6 - 4
*(a*b^9*c^8*d^2 + 2*a^2*b^8*c^7*d^3 - 12*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^
5 + 9*a^5*b^5*c^4*d^6 - 12*a^6*b^4*c^3*d^7 + 2*a^7*b^3*c^2*d^8 + a^8*b^2*c*
d^9)*e^3*f^7 + 2*(3*a^2*b^8*c^8*d^2 - 4*a^3*b^7*c^7*d^3 - 11*a^4*b^6*c^6*d^
4 + 24*a^5*b^5*c^5*d^5 - 11*a^6*b^4*c^4*d^6 - 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2
*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 2*a^5*b^5*c^6*
d^4 + 2*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 + a^8*b^2*c^3*d^7)*e*f^9 + (a^4
*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 +
a^8*b^2*c^4*d^6)*f^10)*x^6 + 2*((b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8
*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*e^9*f - 3*(b^10*c^5*d^5 - 3*a*b^
9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 + a^5*b
^5*d^10)*e^8*f^2 + 2*(b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7
 - 9*a^4*b^6*c^2*d^8 + a^6*b^4*d^10)*e^7*f^3 + 2*(b^10*c^7*d^3 - 5*a*b^9*c^
6*d^4 + 9*a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 - 5*a^4*b^6*c^3*d^7 + 9*a^5*b
^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*e^6*f^4 - 3*(b^10*c^8*d^2 - 6*
a^2*b^8*c^6*d^4 + 8*a^3*b^7*c^5*d^5 - 6*a^4*b^6*c^4*d^6 + 8*a^5*b^5*c^3*d^7
 - 6*a^6*b^4*c^2*d^8 + a^8*b^2*d^10)*e^5*f^5 + (b^10*c^9*d + 9*a*b^9*c^8*d^
2 - 18*a^2*b^8*c^7*d^3 - 10*a^3*b^7*c^6*d^4 + 18*a^4*b^6*c^5*d^5 + 18*a^5*b
^5*c^4*d^6 - 10*a^6*b^4*c^3*d^7 - 18*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^
9*b*d^10)*e^4*f^6 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d
^3 + 5*a^4*b^6*c^6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^
3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^3*f^7 + 6*(a^2*b^8*c^9*d -
 a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 3*a^5*b^5*c^6*d^4 + 3*a^6*b^4*c^5*d^
5 - 3*a^7*b^3*c^4*d^6 - a^8*b^2*c^3*d^7 + a^9*b*c^2*d^8)*e^2*f^8 - (4*a^3*b
^7*c^9*d - 9*a^4*b^6*c^8*d^2 + 10*a^6*b^4*c^6*d^4 - 9*a^8*b^2*c^4*d^6 + 4*a
^9*b*c^3*d^7)*e*f^9 + (a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^
3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*f^10)*x^5 + ((b^
10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*e^10 - 3*(3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 5*
```

```
a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^8*f^2 + 4*(4*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 - 9*a^2*b^8*c^5*d^5 + 10*a^3*b^7*c^4*d^6 + 10*a^4*b^
6*c^3*d^7 - 9*a^5*b^5*c^2*d^8 - 5*a^6*b^4*c*d^9 + 4*a^7*b^3*d^10)*e^7*f^3 -
 (9*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 - 90*a^2*b^8*c^6*d^4 + 36*a^3*b^7*c^5*d
^5 + 50*a^4*b^6*c^4*d^6 + 36*a^5*b^5*c^3*d^7 - 90*a^6*b^4*c^2*d^8 + 20*a^7*
b^3*c*d^9 + 9*a^8*b^2*d^10)*e^6*f^4 + 12*(2*a*b^9*c^8*d^2 - 3*a^2*b^8*c^7*d
^3 - 3*a^3*b^7*c^6*d^4 + 4*a^4*b^6*c^5*d^5 + 4*a^5*b^5*c^4*d^6 - 3*a^6*b^4*
c^3*d^7 - 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9)*e^5*f^5 + (b^10*c^10 - 15*a^
2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^
5 - 50*a^6*b^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^1
0)*e^4*f^6 - 4*(a*b^9*c^10 - 10*a^4*b^6*c^7*d^3 + 9*a^5*b^5*c^6*d^4 + 9*a^6
*b^4*c^5*d^5 - 10*a^7*b^3*c^4*d^6 + a^10*c*d^9)*e^3*f^7 + 3*(2*a^2*b^8*c^10
 - 5*a^4*b^6*c^8*d^2 - 12*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 12*a^7*b^3
*c^5*d^5 - 5*a^8*b^2*c^4*d^6 + 2*a^10*c^2*d^8)*e^2*f^8 - 4*(a^3*b^7*c^10 -
6*a^5*b^5*c^8*d^2 + 5*a^6*b^4*c^7*d^3 + 5*a^7*b^3*c^6*d^4 - 6*a^8*b^2*c^5*d
^5 + a^10*c^3*d^7)*e*f^9 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c
^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*f^10)*x^4 + 2*((b^10*c^5*d^5 - 3
*a*b^9*c^4*d^6 + 2*a^2*b^8*c^3*d^7 + 2*a^3*b^7*c^2*d^8 - 3*a^4*b^6*c*d^9 +
a^5*b^5*d^10)*e^10 - (3*b^10*c^6*d^4 - 8*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
+ 5*a^4*b^6*c^2*d^8 - 8*a^5*b^5*c*d^9 + 3*a^6*b^4*d^10)*e^9*f + (2*b^10*c^7
*d^3 - 5*a*b^9*c^6*d^4 + 3*a^2*b^8*c^5*d^5 + 3*a^5*b^5*c^2*d^8 - 5*a^6*b^4*
c*d^9 + 2*a^7*b^3*d^10)*e^8*f^2 + 2*(b^10*c^8*d^2 - 16*a^3*b^7*c^5*d^5 + 30
*a^4*b^6*c^4*d^6 - 16*a^5*b^5*c^3*d^7 + a^8*b^2*d^10)*e^7*f^3 - (3*b^10*c^9
*d + 5*a*b^9*c^8*d^2 - 60*a^3*b^7*c^6*d^4 + 52*a^4*b^6*c^5*d^5 + 52*a^5*b^5
*c^4*d^6 - 60*a^6*b^4*c^3*d^7 + 5*a^8*b^2*c*d^9 + 3*a^9*b*d^10)*e^6*f^4 + (
b^10*c^10 + 8*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 32*a^3*b^7*c^7*d^3 - 52*a^4
*b^6*c^6*d^4 + 144*a^5*b^5*c^5*d^5 - 52*a^6*b^4*c^4*d^6 - 32*a^7*b^3*c^3*d^
7 + 3*a^8*b^2*c^2*d^8 + 8*a^9*b*c*d^9 + a^10*d^10)*e^5*f^5 - (3*a*b^9*c^10
+ 5*a^2*b^8*c^9*d - 60*a^4*b^6*c^7*d^3 + 52*a^5*b^5*c^6*d^4 + 52*a^6*b^4*c^
5*d^5 - 60*a^7*b^3*c^4*d^6 + 5*a^9*b*c^2*d^8 + 3*a^10*c*d^9)*e^4*f^6 + 2*(a
^2*b^8*c^10 - 16*a^5*b^5*c^7*d^3 + 30*a^6*b^4*c^6*d^4 - 16*a^7*b^3*c^5*d^5
+ a^10*c^2*d^8)*e^3*f^7 + (2*a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 3*a^5*b^5*c^8
*d^2 + 3*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*e^2*f^8 - (3*a
^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8*a
^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e*f^9 + (a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*
a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*f^10)
*x^3 + ((b^10*c^6*d^4 - 9*a^2*b^8*c^4*d^6 + 16*a^3*b^7*c^3*d^7 - 9*a^4*b^6*
c^2*d^8 + a^6*b^4*d^10)*e^10 - 4*(b^10*c^7*d^3 - 6*a^2*b^8*c^5*d^5 + 5*a^3*
b^7*c^4*d^6 + 5*a^4*b^6*c^3*d^7 - 6*a^5*b^5*c^2*d^8 + a^7*b^3*d^10)*e^9*f +
 3*(2*b^10*c^8*d^2 - 5*a^2*b^8*c^6*d^4 - 12*a^3*b^7*c^5*d^5 + 30*a^4*b^6*c^
4*d^6 - 12*a^5*b^5*c^3*d^7 - 5*a^6*b^4*c^2*d^8 + 2*a^8*b^2*d^10)*e^8*f^2 -
4*(b^10*c^9*d - 10*a^3*b^7*c^6*d^4 + 9*a^4*b^6*c^5*d^5 + 9*a^5*b^5*c^4*d^6
- 10*a^6*b^4*c^3*d^7 + a^9*b*d^10)*e^7*f^3 + (b^10*c^10 - 15*a^2*b^8*c^8*d^
2 + 40*a^3*b^7*c^7*d^3 - 50*a^4*b^6*c^6*d^4 + 48*a^5*b^5*c^5*d^5 - 50*a^6*b
^4*c^4*d^6 + 40*a^7*b^3*c^3*d^7 - 15*a^8*b^2*c^2*d^8 + a^10*d^10)*e^6*f^4 +
 12*(2*a^2*b^8*c^9*d - 3*a^3*b^7*c^8*d^2 - 3*a^4*b^6*c^7*d^3 + 4*a^5*b^5*c^
6*d^4 + 4*a^6*b^4*c^5*d^5 - 3*a^7*b^3*c^4*d^6 - 3*a^8*b^2*c^3*d^7 + 2*a^9*b
*c^2*d^8)*e^5*f^5 - (9*a^2*b^8*c^10 + 20*a^3*b^7*c^9*d - 90*a^4*b^6*c^8*d^2
 + 36*a^5*b^5*c^7*d^3 + 50*a^6*b^4*c^6*d^4 + 36*a^7*b^3*c^5*d^5 - 90*a^8*b^
2*c^4*d^6 + 20*a^9*b*c^3*d^7 + 9*a^10*c^2*d^8)*e^4*f^6 + 4*(4*a^3*b^7*c^10
- 5*a^4*b^6*c^9*d - 9*a^5*b^5*c^8*d^2 + 10*a^6*b^4*c^7*d^3 + 10*a^7*b^3*c^6
*d^4 - 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + 4*a^10*c^3*d^7)*e^3*f^7 - 3*(3
*a^4*b^6*c^10 - 8*a^5*b^5*c^9*d + 5*a^6*b^4*c^8*d^2 + 5*a^8*b^2*c^6*d^4 - 8
*a^9*b*c^5*d^5 + 3*a^10*c^4*d^6)*e^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d
+ 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*f^10)*x^2 + 2*((a*b^9
*c^6*d^4 - 3*a^2*b^8*c^5*d^5 + 2*a^3*b^7*c^4*d^6 + 2*a^4*b^6*c^3*d^7 - 3*a^
5*b^5*c^2*d^8 + a^6*b^4*c*d^9)*e^10 - (4*a*b^9*c^7*d^3 - 9*a^2*b^8*c^6*d^4
+ 10*a^4*b^6*c^4*d^6 - 9*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9)*e^9*f + 6*(a*b^
9*c^8*d^2 - a^2*b^8*c^7*d^3 - 3*a^3*b^7*c^6*d^4 + 3*a^4*b^6*c^5*d^5 + 3*a^5
```

```
*b^5*c^4*d^6 - 3*a^6*b^4*c^3*d^7 - a^7*b^3*c^2*d^8 + a^8*b^2*c*d^9)*e^8*f^2
 - 2*(2*a*b^9*c^9*d + 3*a^2*b^8*c^8*d^2 - 16*a^3*b^7*c^7*d^3 + 5*a^4*b^6*c^
6*d^4 + 12*a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 - 16*a^7*b^3*c^3*d^7 + 3*a^8
*b^2*c^2*d^8 + 2*a^9*b*c*d^9)*e^7*f^3 + (a*b^9*c^10 + 9*a^2*b^8*c^9*d - 18*
a^3*b^7*c^8*d^2 - 10*a^4*b^6*c^7*d^3 + 18*a^5*b^5*c^6*d^4 + 18*a^6*b^4*c^5*
d^5 - 10*a^7*b^3*c^4*d^6 - 18*a^8*b^2*c^3*d^7 + 9*a^9*b*c^2*d^8 + a^10*c*d^
9)*e^6*f^4 - 3*(a^2*b^8*c^10 - 6*a^4*b^6*c^8*d^2 + 8*a^5*b^5*c^7*d^3 - 6*a^
6*b^4*c^6*d^4 + 8*a^7*b^3*c^5*d^5 - 6*a^8*b^2*c^4*d^6 + a^10*c^2*d^8)*e^5*f
^5 + 2*(a^3*b^7*c^10 - 5*a^4*b^6*c^9*d + 9*a^5*b^5*c^8*d^2 - 5*a^6*b^4*c^7*
d^3 - 5*a^7*b^3*c^6*d^4 + 9*a^8*b^2*c^5*d^5 - 5*a^9*b*c^4*d^6 + a^10*c^3*d^
7)*e^4*f^6 + 2*(a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a
^8*b^2*c^6*d^4 + a^10*c^4*d^6)*e^3*f^7 - 3*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d
+ 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*e
^2*f^8 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*
d^3 + a^10*c^6*d^4)*e*f^9)*x)
```

mupad [B] time = 20.46, size = 82532, normalized size = 166.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*c*e) + a*c*e + b*
d*f*x^3)^3,x)
```

```
[Out] symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8*
c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^11
*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573300
*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3 - 3
43980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*f^10*
z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d^11*e^
11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^11*c^10*
d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340*a^13*b^
7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3 + 326340*
a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f^10*z^3 +
326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^10*e^8*f^12
*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^10*c^12*d^8*e
^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a^8*b^12*c^12
*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 245700*a^14*b
^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^3 + 245700*a
^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^12*z^3 + 245
700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e^12*f^8*z^3
- 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d^12*e^9*f^11
*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*c^7*d^13*e^1
1*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11*b^9*c^7*d^1
3*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100*a^9*b^11*c^
8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 191100*a^8*b^
12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^3 - 191100*a
^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^13*z^3 - 123
900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e^6*f^14*z^3
- 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*d^7*e^6*f^14
*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*c^14*d^6*e^7
*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*b^13*c^14*d^
6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900*a^6*b^14*c^
13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 101700*a^15*b
^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^3 + 101700*a
^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f^9*z^3 + 101
700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e^14*f^6*z^3
+ 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d^14*e^15*f^5
*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*c^9*d^11*e^1
5*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*b^15*c^11*d^
```

9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*a^14*b^6*c^6
*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 65820*a^10*b^
10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^3 - 65820*a^
6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^13*z^3 - 5670
0*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*f^12*z^3 + 5
6700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e^5*f^15*z^3
- 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^16*e^13*f^7*z
^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*d^15*e^13*f^
7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c^7*d^13*e^15
*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^13*c^15*d^5*e
^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*b^13*c^7*d^1
3*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a^5*b^15*c^12
*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 48252*a^15*b^5*c
^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 - 48252*a^10*
b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*z^3 - 48252*
a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15*f^5*z^3 - 3
2400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e^8*f^12*z^3
- 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^16*e^12*f^8*z
^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*d^16*e^14*f^
6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c^6*d^14*e^16
*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^14*c^8*d^12*e
^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*b^16*c^12*d^
8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*a^16*b^4*c^4
*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 20565*a^10*b^
10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^3 + 20565*a^
4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^15*z^3 + 1566
0*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*f^17*z^3 + 1
5660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e^3*f^17*z^3
+ 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15*z
^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^1
2*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c^15*d^5*e^12
*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3*c^9*d^11*e^
4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^4*c^11*d^9*e
^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b^9*c^16*d^4*
e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b^11*c^17*d^3
*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*b^16*c^17*d^
3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3*b^17*c^16*d
^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^17*b^3*c^7*d
^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a^14*b^6*c^13
*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*a^13*b^7*c^1
4*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100*a^7*b^13*c^
17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 8100*a^6*b^14*c
^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 8100*a^3*b^17*
c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7980*a^16*b^4
*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 - 7980*a^15*b^
5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 - 7980*a^11*b
^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3 - 7980*a^9*b
^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3 - 7980*a^5*
b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^3 - 7980*a^4
*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z^3 + 6300*a^
18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*z^3 + 6300*a
^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14*z^3 + 6300*
a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^6*z^3 - 4260
*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^13*z^3 - 426
0*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*f^7*z^3 - 42
60*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15*f^5*z^3 - 4
260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^18*f^2*z^3 -
4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^18*f^2*z^3 -
4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e^15*f^5*z^3

```
+ 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17*e^10*f^10*z
^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*d^17*e^17*f^
3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^10*d^10*e^17
*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*c^3*d^17*e^9
*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3*c^2*d^18*e^
11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^9*c^2*d^18*e
^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^11*c^3*d^17*
e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b^17*c^9*d^11
*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*b^18*c^11*d^
9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^18*b^2*c^2*d
^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070*a^10*b^10*c
^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1070*a^2*b^18
*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 + 525*a^18*b^2
*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 525*a^16*b^4*
c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 525*a^12*b^8*c
^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525*a^8*b^12*c^
4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*a^4*b^16*c^8
*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a^2*b^18*c^12
*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19*b*c^4*d^16*
e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*c*d^19*e^13*
f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^19*e^16*f^4*z
^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e^19*f*z^3 +
900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f*z^3 + 900*a
*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3 - 750*a^19*b
*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750*a^17*b^3*c^
12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12*b^8*c^17*d^
3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*c^19*d*e^3*f
^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*d*e^8*f^12*z
^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^12*f^8*z^3 -
750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15*z^3 - 420*a
^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3 - 420*a^15*b
^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420*a^14*b^6*c*
d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*b^14*c^5*d^1
5*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*c^6*d^14*e^1
9*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^6*e^15*f^5*z
^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9*f^11*z^3 +
350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9*z^3 + 350*a
^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3 + 350*a^9*b^
11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350*a^2*b^18*c^
19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^19*c^18*d^2*
e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^10*d^10*e*f^1
9*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*f^19*z^3 - 9
0*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3 - 90*a*b^19*
c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^11*d^9*e^19*
f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z^3 + 10*a^19
*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^11*c^20*e*f^1
9*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z^3 + 10*a^11
*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*c^9*d^11*e^2
0*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14*f^6*z^3 - 2
10*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3 + 120*b^20*c
^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*c^12*d^8*e^1
8*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*e^4*f^16*z^3
 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*z^3 + 120*a^
20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a^20*c^2*d^18
*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d^20*e^14*f^6
*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*f^7*z^3 + 12
0*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 - 45*a^12*b^8*
d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^14*c^20*e^4*
f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e^3*f^17*z^3
```

```
+ 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^3 - 45*a^2*b
^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^16*b^4*c^14*
d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^13*d^7*f^20*
z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^20*z^3 - 45*a
^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 210*a^6*b^14*c
^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^13*c^3*d^17*e
^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18*e^20*z^3 -
45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*d^20*e^10*f^
10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^10*b^10*d^20
*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^13*z + 1890*
a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 92610*a^6*b^8*c^
4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a^4*b^10*c^4*
d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*b^7*c^4*d^10
*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^10*c^7*d^7*e^
3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c^8*d^6*e^3*f
^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*d^10*e^7*f^7
*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*e^4*f^10*z -
 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*f^9*z - 3213
0*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11*z - 32130*a
^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z - 32130*a^2*
b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 22680*a^8*b^6
*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 22680*a^4*b^10*c
^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a^2*b^12*c^8*
d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10*b^4*c^2*d^1
2*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b^12*c^2*d^12
*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*c^3*d^11*e^5
*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d^11*e^6*f^8*
z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e^6*f^8*z - 1
3230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^8*z - 13230*
a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z + 3402*a^7*b^7
*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^5*b^9*c^2*d^
12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*c^5*d^9*e^7*
f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*e^3*f^11*z +
 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^4*z + 7938*a
*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5670*a^11*b^3
*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2*b^12*c^11*d
^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^11*d^3*e^2*f^
12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*e*f^13*z - 3
780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z - 3780*a^4*b
^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*b^13*c^4*d^1
0*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^13*e^5*f^9*z
 - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*z - 2268*a*b
^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^7*b^7*c^6*d^
8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d^7*e*f^13*z
+ 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z + 1890*a*b^
13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^13*e^13*f*z
- 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^13*b*c*d^13*f
^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z - 882*b^14*c^
11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^12*d^2*e^2*f^
12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*z + 567*b^14
*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^4*d^10*e^10*
f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z - 918*a^7*b
^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11*d^14*e^11*f
^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^2*z + 567*a^
8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^4*d^14*e^4*f
^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9*z - 126*a^5
*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3*c^3*d^11*f^
14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^14*z + 693*a^
2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b^8*c^8*d^6*f
```

^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^14*z - 126*a
^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^14*e^14*z +
36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d^9*e^2*f^9 +
9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 9018*a^2*b^9*c
^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d^8*e^2*f^9 +
9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 9018*a*b^10*c^4
*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*f^10 + 2268*
b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^11*e^5*f^6 +
2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^11*c^3*d^8*e^
3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 - 1458*a^3*b^
8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*e^4*f^7 - 14
58*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*b^9*c^4*d^7*
f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*b^5*d^11*f^1
1, z, k)*((39*a^5*b^11*c^14*d^2*f^16 - 102*a^6*b^10*c^13*d^3*f^16 + 132*a^7
*b^9*c^12*d^4*f^16 - 84*a^8*b^8*c^11*d^5*f^16 + 21*a^9*b^7*c^10*d^6*f^16 +
21*a^10*b^6*c^9*d^7*f^16 - 84*a^11*b^5*c^8*d^8*f^16 + 132*a^12*b^4*c^7*d^9*
f^16 - 102*a^13*b^3*c^6*d^10*f^16 + 39*a^14*b^2*c^5*d^11*f^16 + 39*a^5*b^11
*d^16*e^14*f^2 - 102*a^6*b^10*d^16*e^13*f^3 + 132*a^7*b^9*d^16*e^12*f^4 - 8
4*a^8*b^8*d^16*e^11*f^5 + 21*a^9*b^7*d^16*e^10*f^6 + 21*a^10*b^6*d^16*e^9*f
^7 - 84*a^11*b^5*d^16*e^8*f^8 + 132*a^12*b^4*d^16*e^7*f^9 - 102*a^13*b^3*d^
16*e^6*f^10 + 39*a^14*b^2*d^16*e^5*f^11 + 39*b^16*c^5*d^11*e^14*f^2 - 102*b
^16*c^6*d^10*e^13*f^3 + 132*b^16*c^7*d^9*e^12*f^4 - 84*b^16*c^8*d^8*e^11*f^
5 + 21*b^16*c^9*d^7*e^10*f^6 + 21*b^16*c^10*d^6*e^9*f^7 - 84*b^16*c^11*d^5*
e^8*f^8 + 132*b^16*c^12*d^4*e^7*f^9 - 102*b^16*c^13*d^3*e^6*f^10 + 39*b^16*
c^14*d^2*e^5*f^11 - 6*a^4*b^12*c^15*d*f^16 - 6*a^15*b*c^4*d^12*f^16 - 6*a^4
*b^12*d^16*e^15*f - 6*a^15*b*d^16*e^4*f^12 - 6*b^16*c^4*d^12*e^15*f - 6*b^1
6*c^15*d*e^4*f^12 + 24*a*b^15*c^3*d^13*e^15*f + 24*a*b^15*c^15*d*e^3*f^13 +
24*a^3*b^13*c*d^15*e^15*f + 24*a^3*b^13*c^15*d*e*f^15 + 24*a^15*b*c*d^15*e
^3*f^13 + 24*a^15*b*c^3*d^13*e*f^15 - 117*a*b^15*c^4*d^12*e^14*f^2 + 150*a*
b^15*c^5*d^11*e^13*f^3 + 159*a*b^15*c^6*d^10*e^12*f^4 - 546*a*b^15*c^7*d^9*
e^11*f^5 + 414*a*b^15*c^8*d^8*e^10*f^6 - 168*a*b^15*c^9*d^7*e^9*f^7 + 414*a
*b^15*c^10*d^6*e^8*f^8 - 546*a*b^15*c^11*d^5*e^7*f^9 + 159*a*b^15*c^12*d^4*
e^6*f^10 + 150*a*b^15*c^13*d^3*e^5*f^11 - 117*a*b^15*c^14*d^2*e^4*f^12 - 36
*a^2*b^14*c^2*d^14*e^15*f - 36*a^2*b^14*c^15*d*e^2*f^14 - 117*a^4*b^12*c*d^
15*e^14*f^2 - 117*a^4*b^12*c^14*d^2*e*f^15 + 150*a^5*b^11*c*d^15*e^13*f^3 +
150*a^5*b^11*c^13*d^3*e*f^15 + 159*a^6*b^10*c*d^15*e^12*f^4 + 159*a^6*b^10
*c^12*d^4*e*f^15 - 546*a^7*b^9*c*d^15*e^11*f^5 - 546*a^7*b^9*c^11*d^5*e*f^1
5 + 414*a^8*b^8*c*d^15*e^10*f^6 + 414*a^8*b^8*c^10*d^6*e*f^15 - 168*a^9*b^7
*c*d^15*e^9*f^7 - 168*a^9*b^7*c^9*d^7*e*f^15 + 414*a^10*b^6*c*d^15*e^8*f^8
+ 414*a^10*b^6*c^8*d^8*e*f^15 - 546*a^11*b^5*c*d^15*e^7*f^9 - 546*a^11*b^5*
c^7*d^9*e*f^15 + 159*a^12*b^4*c*d^15*e^6*f^10 + 159*a^12*b^4*c^6*d^10*e*f^1
5 + 150*a^13*b^3*c*d^15*e^5*f^11 + 150*a^13*b^3*c^5*d^11*e*f^15 - 117*a^14*
b^2*c*d^15*e^4*f^12 - 117*a^14*b^2*c^4*d^12*e*f^15 - 36*a^15*b*c^2*d^14*e^2
*f^14 + 78*a^2*b^14*c^3*d^13*e^14*f^2 + 318*a^2*b^14*c^4*d^12*e^13*f^3 - 12
69*a^2*b^14*c^5*d^11*e^12*f^4 + 1134*a^2*b^14*c^6*d^10*e^11*f^5 + 618*a^2*b
^14*c^7*d^9*e^10*f^6 - 843*a^2*b^14*c^8*d^8*e^9*f^7 - 843*a^2*b^14*c^9*d^7*
e^8*f^8 + 618*a^2*b^14*c^10*d^6*e^7*f^9 + 1134*a^2*b^14*c^11*d^5*e^6*f^10 -
1269*a^2*b^14*c^12*d^4*e^5*f^11 + 318*a^2*b^14*c^13*d^3*e^4*f^12 + 78*a^2*
b^14*c^14*d^2*e^3*f^13 + 78*a^3*b^13*c^2*d^14*e^14*f^2 - 732*a^3*b^13*c^3*d
^13*e^13*f^3 + 978*a^3*b^13*c^4*d^12*e^12*f^4 + 1722*a^3*b^13*c^5*d^11*e^11
*f^5 - 4548*a^3*b^13*c^6*d^10*e^10*f^6 + 1362*a^3*b^13*c^7*d^9*e^9*f^7 + 22
32*a^3*b^13*c^8*d^8*e^8*f^8 + 1362*a^3*b^13*c^9*d^7*e^7*f^9 - 4548*a^3*b^13
*c^10*d^6*e^6*f^10 + 1722*a^3*b^13*c^11*d^5*e^5*f^11 + 978*a^3*b^13*c^12*d^
4*e^4*f^12 - 732*a^3*b^13*c^13*d^3*e^3*f^13 + 78*a^3*b^13*c^14*d^2*e^2*f^14
+ 318*a^4*b^12*c^2*d^14*e^13*f^3 + 978*a^4*b^12*c^3*d^13*e^12*f^4 - 4452*a
^4*b^12*c^4*d^12*e^11*f^5 + 3495*a^4*b^12*c^5*d^11*e^10*f^6 + 4302*a^4*b^12
*c^6*d^10*e^9*f^7 - 4518*a^4*b^12*c^7*d^9*e^8*f^8 - 4518*a^4*b^12*c^8*d^8*e
^7*f^9 + 4302*a^4*b^12*c^9*d^7*e^6*f^10 + 3495*a^4*b^12*c^10*d^6*e^5*f^11 -
4452*a^4*b^12*c^11*d^5*e^4*f^12 + 978*a^4*b^12*c^12*d^4*e^3*f^13 + 318*a^4

*b^12*c^13*d^3*e^2*f^14 - 1269*a^5*b^11*c^2*d^14*e^12*f^4 + 1722*a^5*b^11*c
^3*d^13*e^11*f^5 + 3495*a^5*b^11*c^4*d^12*e^10*f^6 - 9348*a^5*b^11*c^5*d^11
*e^9*f^7 + 2799*a^5*b^11*c^6*d^10*e^8*f^8 + 4824*a^5*b^11*c^7*d^9*e^7*f^9 +
 2799*a^5*b^11*c^8*d^8*e^6*f^10 - 9348*a^5*b^11*c^9*d^7*e^5*f^11 + 3495*a^5
*b^11*c^10*d^6*e^4*f^12 + 1722*a^5*b^11*c^11*d^5*e^3*f^13 - 1269*a^5*b^11*c
^12*d^4*e^2*f^14 + 1134*a^6*b^10*c^2*d^14*e^11*f^5 - 4548*a^6*b^10*c^3*d^13
*e^10*f^6 + 4302*a^6*b^10*c^4*d^12*e^9*f^7 + 2799*a^6*b^10*c^5*d^11*e^8*f^8
 - 3744*a^6*b^10*c^6*d^10*e^7*f^9 - 3744*a^6*b^10*c^7*d^9*e^6*f^10 + 2799*a
^6*b^10*c^8*d^8*e^5*f^11 + 4302*a^6*b^10*c^9*d^7*e^4*f^12 - 4548*a^6*b^10*c
^10*d^6*e^3*f^13 + 1134*a^6*b^10*c^11*d^5*e^2*f^14 + 618*a^7*b^9*c^2*d^14*e
^10*f^6 + 1362*a^7*b^9*c^3*d^13*e^9*f^7 - 4518*a^7*b^9*c^4*d^12*e^8*f^8 + 4
824*a^7*b^9*c^5*d^11*e^7*f^9 - 3744*a^7*b^9*c^6*d^10*e^6*f^10 + 4824*a^7*b^
9*c^7*d^9*e^5*f^11 - 4518*a^7*b^9*c^8*d^8*e^4*f^12 + 1362*a^7*b^9*c^9*d^7*e
^3*f^13 + 618*a^7*b^9*c^10*d^6*e^2*f^14 - 843*a^8*b^8*c^2*d^14*e^9*f^7 + 22
32*a^8*b^8*c^3*d^13*e^8*f^8 - 4518*a^8*b^8*c^4*d^12*e^7*f^9 + 2799*a^8*b^8*
c^5*d^11*e^6*f^10 + 2799*a^8*b^8*c^6*d^10*e^5*f^11 - 4518*a^8*b^8*c^7*d^9*e
^4*f^12 + 2232*a^8*b^8*c^8*d^8*e^3*f^13 - 843*a^8*b^8*c^9*d^7*e^2*f^14 - 84
3*a^9*b^7*c^2*d^14*e^8*f^8 + 1362*a^9*b^7*c^3*d^13*e^7*f^9 + 4302*a^9*b^7*c
^4*d^12*e^6*f^10 - 9348*a^9*b^7*c^5*d^11*e^5*f^11 + 4302*a^9*b^7*c^6*d^10*e
^4*f^12 + 1362*a^9*b^7*c^7*d^9*e^3*f^13 - 843*a^9*b^7*c^8*d^8*e^2*f^14 + 61
8*a^10*b^6*c^2*d^14*e^7*f^9 - 4548*a^10*b^6*c^3*d^13*e^6*f^10 + 3495*a^10*b
^6*c^4*d^12*e^5*f^11 + 3495*a^10*b^6*c^5*d^11*e^4*f^12 - 4548*a^10*b^6*c^6*
d^10*e^3*f^13 + 618*a^10*b^6*c^7*d^9*e^2*f^14 + 1134*a^11*b^5*c^2*d^14*e^6*
f^10 + 1722*a^11*b^5*c^3*d^13*e^5*f^11 - 4452*a^11*b^5*c^4*d^12*e^4*f^12 +
1722*a^11*b^5*c^5*d^11*e^3*f^13 + 1134*a^11*b^5*c^6*d^10*e^2*f^14 - 1269*a^
12*b^4*c^2*d^14*e^5*f^11 + 978*a^12*b^4*c^3*d^13*e^4*f^12 + 978*a^12*b^4*c^
4*d^12*e^3*f^13 - 1269*a^12*b^4*c^5*d^11*e^2*f^14 + 318*a^13*b^3*c^2*d^14*e
^4*f^12 - 732*a^13*b^3*c^3*d^13*e^3*f^13 + 318*a^13*b^3*c^4*d^12*e^2*f^14 +
 78*a^14*b^2*c^2*d^14*e^3*f^13 + 78*a^14*b^2*c^3*d^13*e^2*f^14)/(56*a^3*b^1
3*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16
 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*
b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*
a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f
^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c
^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a
^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^
14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^
16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^
16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^1
2 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^
6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^1
6*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8
*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*
b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*
b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9
*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*
c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a
^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f
^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^
15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e
^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^
3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^
11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 1
12*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c
^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14
+ 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*
c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f
^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12
*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6

*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a
^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12
*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 1
60*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c
^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*
e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 -
 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^
3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c
^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^
13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 -
8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4
*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14
*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^
13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 +
 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^
5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^
13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e
^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4
 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640
*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10
*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^
3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^
3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640
*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c
^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6
*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 3
00*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^
8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d
^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^
8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264
*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*
c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e
^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 -
 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9
*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d
^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^
13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 862
4*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*
b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*
d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*
f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1
344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^1
1*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^
6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^
5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 -
 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^
12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^
5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^
5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 +
840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13
*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6
*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3
*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 13
44*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b
^2*c^8*d^8*e^2*f^14) + root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 5733
00*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 +
 573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*
z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11
*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^1

```
1*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10
*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^
9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 32
6340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z
^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10
*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d
^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^1
0*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*a
^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 + 2
45700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14*z^
3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^8*f^
12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*d^8*e
^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7*c^8*d
^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^12*b^8*
c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - 191100*a^11
*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12*z^3 - 191100
*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9*f^11*z^3 - 1
91100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^8*e^11*f^9*z^
3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^9*d^11*e^7*f^
13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b^7*c^11*d^9*e
^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a^11*b^9*c^13*
d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123900*a^9*b^11*
c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3 - 123900*a^7*
b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6*z^3 - 123900
*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^13*f^7*z^3 + 1
01700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^14*e^9*f^11*z^
3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^5*d^15*e^11*f
^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b^9*c^5*d^15*e
^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a^9*b^11*c^6*d
^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101700*a^6*b^14*
c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3 + 101700*a^5*
b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14*z^3 - 65820*
a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6*f^14*z^3 - 6
5820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6*e^10*f^10*z^
3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7*d^13*e^7*f^1
3*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*c^7*d^13*e^8*
f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b^7*c^12*d^8*e
^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^13*b^7*c^4*d^1
6*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*a^12*b^8*c^5*
d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 56700*a^8*b^12*c
^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 - 56700*a^7*b^1
3*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3 + 56700*a^7*
b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*z^3 - 56700*a
^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f^7*z^3 - 4825
2*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^10*f^10*z^3 -
 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d^15*e^15*f^5*
z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c^10*d^10*e^15
*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b^4*c^6*d^14*e
^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^14*b^6*c^4*d^1
6*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*a^12*b^8*c^4*
d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 32400*a^8*b^12*c
^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 - 32400*a^6*b^1
4*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3 - 32400*a^4*
b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16*z^3 + 20565*
a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4*f^16*z^3 + 2
0565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4*e^10*f^10*z^
3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8*d^12*e^5*f^1
5*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*c^12*d^8*e^3*
f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b^8*c^15*d^5*e
```

^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8*b^12*c^17*d^3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*a^5*b^15*c^17*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 15660*a^3*b^17*c^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 - 9750*a^17*b^3*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 - 9750*a^16*b^4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3 - 9750*a^11*b^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3 - 9750*a^9*b^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^3 - 9750*a^4*b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z^3 - 9750*a^3*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*z^3 - 8100*a^17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13*z^3 - 8100*a^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^7*z^3 - 8100*a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f^6*z^3 - 8100*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*f^3*z^3 - 8100*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17*f^3*z^3 - 8100*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^14*f^6*z^3 - 7980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^9*f^11*z^3 - 7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e^11*f^9*z^3 - 7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*e^15*f^5*z^3 - 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15*e^16*f^4*z^3 - 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^11*e^16*f^4*z^3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d^9*e^15*f^5*z^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*d^6*e^2*f^18*z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18*d^2*e^6*f^14*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^14*d^6*e^14*f^6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^5*d^15*e^7*f^13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c^2*d^18*e^13*f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*c^2*d^18*e^15*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13*c^5*d^15*e^18*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^15*c^7*d^13*e^18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^18*c^13*d^7*e^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17*b^3*c^3*d^17*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a^10*b^10*c^3*d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 1470*a^3*b^17*c^10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1350*a^18*b^2*c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 + 1350*a^17*b^3*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 + 1350*a^11*b^9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3 + 1350*a^9*b^11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3 + 1350*a^3*b^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^3 + 1350*a^2*b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*z^3 - 1070*a^18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^18*z^3 - 1070*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10*f^10*z^3 - 1070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^4*f^16*z^3 + 525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2*f^18*z^3 + 525*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*f^18*z^3 + 525*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f^16*z^3 + 525*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^12*z^3 + 525*a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^8*z^3 + 525*a^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z^3 + 900*a^19*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 + 900*a^16*b^4*c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a^13*b^7*c*d^19*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^13*c^4*d^16*e^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^7*d^13*e^19*f*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*e^16*f^4*z^3 - 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f^12*z^3 - 750*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z^3 - 750*a^12*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 - 750*a^8*b^12*c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a^3*b^17*c^19*

d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19*c^17*d^3*e^1
2*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*d^14*e^5*f^15
*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^6*e*f^19*z^3
- 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f^19*z^3 - 420
*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z^3 - 420*a^6*
b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 - 420*a^5*b^15*
c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a*b^19*c^14*d^
6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b*c^2*d^18*e^9
*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*d^19*e^11*f^9
*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*e^18*f^2*z^3
+ 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^19*f*z^3 + 350
*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z^3 + 350*a*b^
19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 - 90*a^19*b*c^1
0*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b^10*c^19*d*e*
f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e^10*f^10*z^3
- 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3 + 10*b^20*c^
11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^19*e^9*f^11*z
^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3 + 10*a^9*b^1
1*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^11*d^9*f^20*z
^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3 + 10*a*b^19*
c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*c^16*d^4*e^14
*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*e^13*f^7*z^3
+ 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z^3 - 45*b^20*
c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a^20*c^6*d^14*
e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d^13*e^3*f^17*
z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^18*z^3 - 45*a
^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 210*a^16*b^4*d
^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b^3*d^20*e^13*
f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^12*f^8*z^3 -
45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3 - 210*a^6*b^
14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^7*b^13*c^20*e
^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^20*e^2*f^18*z^
3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20*z^3 - 210*a^
16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 120*a^17*b^3*c^
13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2*c^12*d^8*f^2
0*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*e^20*z^3 - 21
0*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3 + 120*a^7*b^1
3*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*b^12*c^2*d^18
*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^10*z^3 - a^20*
d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*f^20*z^3 - a^
10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^2*c*d^13*e*f^
13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12*f^2*z + 9261
0*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^10*z + 92610*a
^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z - 66150*a^7*
b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z - 66150*a^4*b^1
0*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 66150*a^3*b^11*c
^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*a^3*b^11*c^4*
d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5*b^9*c^5*d^9*
e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10*c^5*d^9*e^5*
f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*d^12*e^3*f^11
*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^12*e^9*f^5*z
- 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*e^9*f^5*z + 2
2680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*f^10*z + 2268
0*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^6*z + 22680*a
^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z + 19278*a^10
*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z + 19278*a^2*b
^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 18522*a^6*b^8*
c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*a^5*b^9*c^3*d
^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b^11*c^5*d^9*e

```
^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^2*d^12*e^6*f^
8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*e^2*f^12*z +
3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^12*z + 3402*a^
5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3402*a^2*b^12*
c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^10*b^4*c*d^13*
e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c*d^13*e^10*f^
4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e^10*f^4*z - 5
670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12*z - 5670*a^2
*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 5670*a*b^13*c^1
1*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*b^5*c^4*d^10*
e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*d^5*e*f^13*z
 - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^10*z - 3780*a*
b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268*a^8*b^6*c*d^
13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*d^13*e^8*f^6*
z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^6*z + 1890*a^
7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*a^6*b^8*c^7*d
^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*d^7*e^6*f^8*z
 + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z - 252*b^14*c*d^
13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*f*z - 252*a^1
3*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^7*e^7*f^7*z -
 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z + 693*b^14*c^1
2*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8*d^6*e^6*f^8*
z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z + 441*b^14*c^
4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*d^9*e^9*f^5*z
 - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z - 882*a^3*b^11
*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12*d^14*e^12*f^
2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z + 441*a^10*b^
4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5*d^14*e^5*f^9
*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z - 882*a^11*b^3
*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2*c^2*d^12*f^1
4*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*z + 567*a^6*b
^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^10*c^10*d^4*f^
14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z + 36*b^14*d^
14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*a^2*b^9*c^2*d
^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10*e^2*f^9 + 90
18*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018*a*b^10*c^3*d
^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10*e*f^10 - 901
8*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^11*c^5*d^6*e*
f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2268*a*b^10*d^
11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^11 - 1458*b^1
1*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*d^9*e^4*f^7 -
 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*a^2*b^9*d^11*
e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^11 - 1161*a^2*
b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^11 - 756*a^6*
b^5*d^11*f^11, z, k)*((20*a^11*b^8*c^16*d^3*f^19 - 7*a^10*b^9*c^17*d^2*f^19
 - 28*a^12*b^7*c^15*d^4*f^19 + 14*a^13*b^6*c^14*d^5*f^19 + 14*a^14*b^5*c^13
*d^6*f^19 - 28*a^15*b^4*c^12*d^7*f^19 + 20*a^16*b^3*c^11*d^8*f^19 - 7*a^17*
b^2*c^10*d^9*f^19 - 7*a^10*b^9*d^19*e^17*f^2 + 20*a^11*b^8*d^19*e^16*f^3 -
28*a^12*b^7*d^19*e^15*f^4 + 14*a^13*b^6*d^19*e^14*f^5 + 14*a^14*b^5*d^19*e^
13*f^6 - 28*a^15*b^4*d^19*e^12*f^7 + 20*a^16*b^3*d^19*e^11*f^8 - 7*a^17*b^2
*d^19*e^10*f^9 - 7*b^19*c^10*d^9*e^17*f^2 + 20*b^19*c^11*d^8*e^16*f^3 - 28*
b^19*c^12*d^7*e^15*f^4 + 14*b^19*c^13*d^6*e^14*f^5 + 14*b^19*c^14*d^5*e^13*
f^6 - 28*b^19*c^15*d^4*e^12*f^7 + 20*b^19*c^16*d^3*e^11*f^8 - 7*b^19*c^17*d
^2*e^10*f^9 + a^9*b^10*c^18*d*f^19 + a^18*b*c^9*d^10*f^19 + a^9*b^10*d^19*e
^18*f + a^18*b*d^19*e^9*f^10 + b^19*c^9*d^10*e^18*f + b^19*c^18*d*e^9*f^10
- 7*a*b^18*c^8*d^11*e^18*f - 7*a*b^18*c^18*d*e^8*f^11 - 7*a^8*b^11*c*d^18*e
^18*f - 7*a^8*b^11*c^18*d*e*f^18 - 7*a^18*b*c*d^18*e^8*f^11 - 7*a^18*b*c^8*
d^11*e*f^18 + 34*a*b^18*c^9*d^10*e^17*f^2 - 27*a*b^18*c^10*d^9*e^16*f^3 - 1
68*a*b^18*c^11*d^8*e^15*f^4 + 546*a*b^18*c^12*d^7*e^14*f^5 - 756*a*b^18*c^1
```

3*d^6*e^13*f^6 + 546*a*b^18*c^14*d^5*e^12*f^7 - 168*a*b^18*c^15*d^4*e^11*f^8 - 27*a*b^18*c^16*d^3*e^10*f^9 + 34*a*b^18*c^17*d^2*e^9*f^10 + 20*a^2*b^17*c^7*d^12*e^18*f + 20*a^2*b^17*c^18*d*e^7*f^12 - 28*a^3*b^16*c^6*d^13*e^18*f - 28*a^3*b^16*c^18*d*e^6*f^13 + 14*a^4*b^15*c^5*d^14*e^18*f + 14*a^4*b^15*c^18*d*e^5*f^14 + 14*a^5*b^14*c^4*d^15*e^18*f + 14*a^5*b^14*c^18*d*e^4*f^15 - 28*a^6*b^13*c^3*d^16*e^18*f - 28*a^6*b^13*c^18*d*e^3*f^16 + 20*a^7*b^12*c^2*d^17*e^18*f + 20*a^7*b^12*c^18*d*e^2*f^17 + 34*a^9*b^10*c*d^18*e^17*f^2 + 34*a^9*b^10*c^17*d^2*e*f^18 - 27*a^10*b^9*c*d^18*e^16*f^3 - 27*a^10*b^9*c^16*d^3*e*f^18 - 168*a^11*b^8*c*d^18*e^15*f^4 - 168*a^11*b^8*c^15*d^4*e*f^18 + 546*a^12*b^7*c*d^18*e^14*f^5 + 546*a^12*b^7*c^14*d^5*e*f^18 - 756*a^13*b^6*c*d^18*e^13*f^6 - 756*a^13*b^6*c^13*d^6*e*f^18 + 546*a^14*b^5*c*d^18*e^12*f^7 + 546*a^14*b^5*c^12*d^7*e*f^18 - 168*a^15*b^4*c*d^18*e^11*f^8 - 168*a^15*b^4*c^11*d^8*e*f^18 - 27*a^16*b^3*c*d^18*e^10*f^9 - 27*a^16*b^3*c^10*d^9*e*f^18 + 34*a^17*b^2*c*d^18*e^9*f^10 + 34*a^17*b^2*c^9*d^10*e*f^18 + 20*a^18*b*c^2*d^17*e^7*f^12 - 28*a^18*b*c^3*d^16*e^6*f^13 + 14*a^18*b*c^4*d^15*e^5*f^14 + 14*a^18*b*c^5*d^14*e^4*f^15 - 28*a^18*b*c^6*d^13*e^3*f^16 + 20*a^18*b*c^7*d^12*e^2*f^17 - 27*a^2*b^17*c^8*d^11*e^17*f^2 - 371*a^2*b^17*c^9*d^10*e^16*f^3 + 1560*a^2*b^17*c^10*d^9*e^15*f^4 - 2484*a^2*b^17*c^11*d^8*e^14*f^5 + 1302*a^2*b^17*c^12*d^7*e^13*f^6 + 1302*a^2*b^17*c^13*d^6*e^12*f^7 - 2484*a^2*b^17*c^14*d^5*e^11*f^8 + 1560*a^2*b^17*c^15*d^4*e^10*f^9 - 371*a^2*b^17*c^16*d^3*e^9*f^10 - 27*a^2*b^17*c^17*d^2*e^8*f^11 - 168*a^3*b^16*c^7*d^12*e^17*f^2 + 1560*a^3*b^16*c^8*d^11*e^16*f^3 - 3464*a^3*b^16*c^9*d^10*e^15*f^4 + 924*a^3*b^16*c^10*d^9*e^14*f^5 + 7728*a^3*b^16*c^11*d^8*e^13*f^6 - 13104*a^3*b^16*c^12*d^7*e^12*f^7 + 7728*a^3*b^16*c^13*d^6*e^11*f^8 + 924*a^3*b^16*c^14*d^5*e^10*f^9 - 3464*a^3*b^16*c^15*d^4*e^9*f^10 + 1560*a^3*b^16*c^16*d^3*e^8*f^11 - 168*a^3*b^16*c^17*d^2*e^7*f^12 + 546*a^4*b^15*c^6*d^13*e^17*f^2 - 2484*a^4*b^15*c^7*d^12*e^16*f^3 + 924*a^4*b^15*c^8*d^11*e^15*f^4 + 12550*a^4*b^15*c^9*d^10*e^14*f^5 - 26838*a^4*b^15*c^10*d^9*e^13*f^6 + 15288*a^4*b^15*c^11*d^8*e^12*f^7 + 15288*a^4*b^15*c^12*d^7*e^11*f^8 - 26838*a^4*b^15*c^13*d^6*e^10*f^9 + 12550*a^4*b^15*c^14*d^5*e^9*f^10 + 924*a^4*b^15*c^15*d^4*e^8*f^11 - 2484*a^4*b^15*c^16*d^3*e^7*f^12 + 546*a^4*b^15*c^17*d^2*e^6*f^13 - 756*a^5*b^14*c^5*d^14*e^17*f^2 + 1302*a^5*b^14*c^6*d^13*e^16*f^3 + 7728*a^5*b^14*c^7*d^12*e^15*f^4 - 26838*a^5*b^14*c^8*d^11*e^14*f^5 + 18004*a^5*b^14*c^9*d^10*e^13*f^6 + 39858*a^5*b^14*c^10*d^9*e^12*f^7 - 78624*a^5*b^14*c^11*d^8*e^11*f^8 + 39858*a^5*b^14*c^12*d^7*e^10*f^9 + 18004*a^5*b^14*c^13*d^6*e^9*f^10 - 26838*a^5*b^14*c^14*d^5*e^8*f^11 + 7728*a^5*b^14*c^15*d^4*e^7*f^12 + 1302*a^5*b^14*c^16*d^3*e^6*f^13 - 756*a^5*b^14*c^17*d^2*e^5*f^14 + 546*a^6*b^13*c^4*d^15*e^17*f^2 + 1302*a^6*b^13*c^5*d^14*e^16*f^3 - 13104*a^6*b^13*c^6*d^13*e^15*f^4 + 15288*a^6*b^13*c^7*d^12*e^14*f^5 + 39858*a^6*b^13*c^8*d^11*e^13*f^6 - 110474*a^6*b^13*c^9*d^10*e^12*f^7 + 66612*a^6*b^13*c^10*d^9*e^11*f^8 + 66612*a^6*b^13*c^11*d^8*e^10*f^9 - 110474*a^6*b^13*c^12*d^7*e^9*f^10 + 39858*a^6*b^13*c^13*d^6*e^8*f^11 + 15288*a^6*b^13*c^14*d^5*e^7*f^12 - 13104*a^6*b^13*c^15*d^4*e^6*f^13 + 1302*a^6*b^13*c^16*d^3*e^5*f^14 + 546*a^6*b^13*c^17*d^2*e^4*f^15 - 168*a^7*b^12*c^3*d^16*e^17*f^2 - 2484*a^7*b^12*c^4*d^15*e^16*f^3 + 7728*a^7*b^12*c^5*d^14*e^15*f^4 + 15288*a^7*b^12*c^6*d^13*e^14*f^5 - 78624*a^7*b^12*c^7*d^12*e^13*f^6 + 66612*a^7*b^12*c^8*d^11*e^12*f^7 + 99736*a^7*b^12*c^9*d^10*e^11*f^8 - 216216*a^7*b^12*c^10*d^9*e^10*f^9 + 99736*a^7*b^12*c^11*d^8*e^9*f^10 + 66612*a^7*b^12*c^12*d^7*e^8*f^11 - 78624*a^7*b^12*c^13*d^6*e^7*f^12 + 15288*a^7*b^12*c^14*d^5*e^6*f^13 + 7728*a^7*b^12*c^15*d^4*e^5*f^14 - 2484*a^7*b^12*c^16*d^3*e^4*f^15 - 168*a^7*b^12*c^17*d^2*e^3*f^16 - 27*a^8*b^11*c^2*d^17*e^17*f^2 + 1560*a^8*b^11*c^3*d^16*e^16*f^3 + 924*a^8*b^11*c^4*d^15*e^15*f^4 - 26838*a^8*b^11*c^5*d^14*e^14*f^5 + 39858*a^8*b^11*c^6*d^13*e^13*f^6 + 66612*a^8*b^11*c^7*d^12*e^12*f^7 - 216216*a^8*b^11*c^8*d^11*e^11*f^8 + 134134*a^8*b^11*c^9*d^10*e^10*f^9 + 134134*a^8*b^11*c^10*d^9*e^9*f^10 - 216216*a^8*b^11*c^11*d^8*e^8*f^11 + 66612*a^8*b^11*c^12*d^7*e^7*f^12 + 39858*a^8*b^11*c^13*d^6*e^6*f^13 - 26838*a^8*b^11*c^14*d^5*e^5*f^14 + 924*a^8*b^11*c^15*d^4*e^4*f^15 + 1560*a^8*b^11*c^16*d^3*e^3*f^16 - 27*a^8*b^11*c^17*d^2*e^2*f^17 - 371*a^9*b^10*c^2*d^17*e^16*f^3 - 3464*a^9*b^10*c^3*d^16*e^15*f^4 + 12550*a^9*b

```
^10*c^4*d^15*e^14*f^5 + 18004*a^9*b^10*c^5*d^14*e^13*f^6 - 110474*a^9*b^10*
c^6*d^13*e^12*f^7 + 99736*a^9*b^10*c^7*d^12*e^11*f^8 + 134134*a^9*b^10*c^8*
d^11*e^10*f^9 - 300300*a^9*b^10*c^9*d^10*e^9*f^10 + 134134*a^9*b^10*c^10*d^
9*e^8*f^11 + 99736*a^9*b^10*c^11*d^8*e^7*f^12 - 110474*a^9*b^10*c^12*d^7*e^
6*f^13 + 18004*a^9*b^10*c^13*d^6*e^5*f^14 + 12550*a^9*b^10*c^14*d^5*e^4*f^1
5 - 3464*a^9*b^10*c^15*d^4*e^3*f^16 - 371*a^9*b^10*c^16*d^3*e^2*f^17 + 1560
*a^10*b^9*c^2*d^17*e^15*f^4 + 924*a^10*b^9*c^3*d^16*e^14*f^5 - 26838*a^10*b
^9*c^4*d^15*e^13*f^6 + 39858*a^10*b^9*c^5*d^14*e^12*f^7 + 66612*a^10*b^9*c^
6*d^13*e^11*f^8 - 216216*a^10*b^9*c^7*d^12*e^10*f^9 + 134134*a^10*b^9*c^8*d
^11*e^9*f^10 + 134134*a^10*b^9*c^9*d^10*e^8*f^11 - 216216*a^10*b^9*c^10*d^9
*e^7*f^12 + 66612*a^10*b^9*c^11*d^8*e^6*f^13 + 39858*a^10*b^9*c^12*d^7*e^5*
f^14 - 26838*a^10*b^9*c^13*d^6*e^4*f^15 + 924*a^10*b^9*c^14*d^5*e^3*f^16 +
1560*a^10*b^9*c^15*d^4*e^2*f^17 - 2484*a^11*b^8*c^2*d^17*e^14*f^5 + 7728*a^
11*b^8*c^3*d^16*e^13*f^6 + 15288*a^11*b^8*c^4*d^15*e^12*f^7 - 78624*a^11*b^
8*c^5*d^14*e^11*f^8 + 66612*a^11*b^8*c^6*d^13*e^10*f^9 + 99736*a^11*b^8*c^7
*d^12*e^9*f^10 - 216216*a^11*b^8*c^8*d^11*e^8*f^11 + 99736*a^11*b^8*c^9*d^1
0*e^7*f^12 + 66612*a^11*b^8*c^10*d^9*e^6*f^13 - 78624*a^11*b^8*c^11*d^8*e^5
*f^14 + 15288*a^11*b^8*c^12*d^7*e^4*f^15 + 7728*a^11*b^8*c^13*d^6*e^3*f^16
- 2484*a^11*b^8*c^14*d^5*e^2*f^17 + 1302*a^12*b^7*c^2*d^17*e^13*f^6 - 13104
*a^12*b^7*c^3*d^16*e^12*f^7 + 15288*a^12*b^7*c^4*d^15*e^11*f^8 + 39858*a^12
*b^7*c^5*d^14*e^10*f^9 - 110474*a^12*b^7*c^6*d^13*e^9*f^10 + 66612*a^12*b^7
*c^7*d^12*e^8*f^11 + 66612*a^12*b^7*c^8*d^11*e^7*f^12 - 110474*a^12*b^7*c^9
*d^10*e^6*f^13 + 39858*a^12*b^7*c^10*d^9*e^5*f^14 + 15288*a^12*b^7*c^11*d^8
*e^4*f^15 - 13104*a^12*b^7*c^12*d^7*e^3*f^16 + 1302*a^12*b^7*c^13*d^6*e^2*f
^17 + 1302*a^13*b^6*c^2*d^17*e^12*f^7 + 7728*a^13*b^6*c^3*d^16*e^11*f^8 - 2
6838*a^13*b^6*c^4*d^15*e^10*f^9 + 18004*a^13*b^6*c^5*d^14*e^9*f^10 + 39858*
a^13*b^6*c^6*d^13*e^8*f^11 - 78624*a^13*b^6*c^7*d^12*e^7*f^12 + 39858*a^13*
b^6*c^8*d^11*e^6*f^13 + 18004*a^13*b^6*c^9*d^10*e^5*f^14 - 26838*a^13*b^6*c
^10*d^9*e^4*f^15 + 7728*a^13*b^6*c^11*d^8*e^3*f^16 + 1302*a^13*b^6*c^12*d^7
*e^2*f^17 - 2484*a^14*b^5*c^2*d^17*e^11*f^8 + 924*a^14*b^5*c^3*d^16*e^10*f^
9 + 12550*a^14*b^5*c^4*d^15*e^9*f^10 - 26838*a^14*b^5*c^5*d^14*e^8*f^11 + 1
5288*a^14*b^5*c^6*d^13*e^7*f^12 + 15288*a^14*b^5*c^7*d^12*e^6*f^13 - 26838*
a^14*b^5*c^8*d^11*e^5*f^14 + 12550*a^14*b^5*c^9*d^10*e^4*f^15 + 924*a^14*b^
5*c^10*d^9*e^3*f^16 - 2484*a^14*b^5*c^11*d^8*e^2*f^17 + 1560*a^15*b^4*c^2*d
^17*e^10*f^9 - 3464*a^15*b^4*c^3*d^16*e^9*f^10 + 924*a^15*b^4*c^4*d^15*e^8*
f^11 + 7728*a^15*b^4*c^5*d^14*e^7*f^12 - 13104*a^15*b^4*c^6*d^13*e^6*f^13 +
 7728*a^15*b^4*c^7*d^12*e^5*f^14 + 924*a^15*b^4*c^8*d^11*e^4*f^15 - 3464*a^
15*b^4*c^9*d^10*e^3*f^16 + 1560*a^15*b^4*c^10*d^9*e^2*f^17 - 371*a^16*b^3*c
^2*d^17*e^9*f^10 + 1560*a^16*b^3*c^3*d^16*e^8*f^11 - 2484*a^16*b^3*c^4*d^15
*e^7*f^12 + 1302*a^16*b^3*c^5*d^14*e^6*f^13 + 1302*a^16*b^3*c^6*d^13*e^5*f^
14 - 2484*a^16*b^3*c^7*d^12*e^4*f^15 + 1560*a^16*b^3*c^8*d^11*e^3*f^16 - 37
1*a^16*b^3*c^9*d^10*e^2*f^17 - 27*a^17*b^2*c^2*d^17*e^8*f^11 - 168*a^17*b^2
*c^3*d^16*e^7*f^12 + 546*a^17*b^2*c^4*d^15*e^6*f^13 - 756*a^17*b^2*c^5*d^14
*e^5*f^14 + 546*a^17*b^2*c^6*d^13*e^4*f^15 - 168*a^17*b^2*c^7*d^12*e^3*f^16
 - 27*a^17*b^2*c^8*d^11*e^2*f^17)/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16
*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^
16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^
4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^
10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^1
6 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^1
6*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5
*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2
 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16
*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16
*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13
- 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*
e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*
c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b
^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*
```

c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15
*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^
9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*
d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15
*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*
f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b
^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*
e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a
^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*
d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 -
224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c
^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2
+ 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6
*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f
^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^1
3*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*
e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a
^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^1
1*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 3
00*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^
14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d
^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^
4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 280
0*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12
*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8
*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^
6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*
a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*
c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9
*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^
6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480
*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^1
1*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^
11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*
f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11
396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b
^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14
*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12
*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15
400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9
*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*
e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 +
1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^
8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c
^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*
f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1
750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^
8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^1
2*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^
7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a
^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^1
1*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^1
2*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 +
7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a
^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c
^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*
e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5
+ 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264
*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5

*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7
*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^
14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750
*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4
*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8
*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14
 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a
^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3
*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*
e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11
+ 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f^14) - (x*(18*
a^9*b^10*c^17*d^2*f^19 - 74*a^10*b^9*c^16*d^3*f^19 + 184*a^11*b^8*c^15*d^4*
f^19 - 308*a^12*b^7*c^14*d^5*f^19 + 364*a^13*b^6*c^13*d^6*f^19 - 308*a^14*b
^5*c^12*d^7*f^19 + 184*a^15*b^4*c^11*d^8*f^19 - 74*a^16*b^3*c^10*d^9*f^19 +
 18*a^17*b^2*c^9*d^10*f^19 + 18*a^9*b^10*d^19*e^17*f^2 - 74*a^10*b^9*d^19*e
^16*f^3 + 184*a^11*b^8*d^19*e^15*f^4 - 308*a^12*b^7*d^19*e^14*f^5 + 364*a^1
3*b^6*d^19*e^13*f^6 - 308*a^14*b^5*d^19*e^12*f^7 + 184*a^15*b^4*d^19*e^11*f
^8 - 74*a^16*b^3*d^19*e^10*f^9 + 18*a^17*b^2*d^19*e^9*f^10 + 18*b^19*c^9*d^
10*e^17*f^2 - 74*b^19*c^10*d^9*e^16*f^3 + 184*b^19*c^11*d^8*e^15*f^4 - 308*
b^19*c^12*d^7*e^14*f^5 + 364*b^19*c^13*d^6*e^13*f^6 - 308*b^19*c^14*d^5*e^1
2*f^7 + 184*b^19*c^15*d^4*e^11*f^8 - 74*b^19*c^16*d^3*e^10*f^9 + 18*b^19*c^
17*d^2*e^9*f^10 - 2*a^8*b^11*c^18*d*f^19 - 2*a^18*b*c^8*d^11*f^19 - 2*a^8*b
^11*d^19*e^18*f - 2*a^18*b*d^19*e^8*f^11 - 2*b^19*c^8*d^11*e^18*f - 2*b^19*
c^18*d*e^8*f^11 + 16*a*b^18*c^7*d^12*e^18*f + 16*a*b^18*c^18*d*e^7*f^12 + 1
6*a^7*b^12*c*d^18*e^18*f + 16*a^7*b^12*c^18*d*e*f^18 + 16*a^18*b*c*d^18*e^7
*f^12 + 16*a^18*b*c^7*d^12*e*f^18 - 126*a*b^18*c^8*d^11*e^17*f^2 + 434*a*b^
18*c^9*d^10*e^16*f^3 - 840*a*b^18*c^10*d^9*e^15*f^4 + 936*a*b^18*c^11*d^8*e
^14*f^5 - 420*a*b^18*c^12*d^7*e^13*f^6 - 420*a*b^18*c^13*d^6*e^12*f^7 + 936
*a*b^18*c^14*d^5*e^11*f^8 - 840*a*b^18*c^15*d^4*e^10*f^9 + 434*a*b^18*c^16*
d^3*e^9*f^10 - 126*a*b^18*c^17*d^2*e^8*f^11 - 56*a^2*b^17*c^6*d^13*e^18*f -
 56*a^2*b^17*c^18*d*e^6*f^13 + 112*a^3*b^16*c^5*d^14*e^18*f + 112*a^3*b^16*
c^18*d*e^5*f^14 - 140*a^4*b^15*c^4*d^15*e^18*f - 140*a^4*b^15*c^18*d*e^4*f^
15 + 112*a^5*b^14*c^3*d^16*e^18*f + 112*a^5*b^14*c^18*d*e^3*f^16 - 56*a^6*b
^13*c^2*d^17*e^18*f - 56*a^6*b^13*c^18*d*e^2*f^17 - 126*a^8*b^11*c*d^18*e^1
7*f^2 - 126*a^8*b^11*c^17*d^2*e*f^18 + 434*a^9*b^10*c*d^18*e^16*f^3 + 434*a
^9*b^10*c^16*d^3*e*f^18 - 840*a^10*b^9*c*d^18*e^15*f^4 - 840*a^10*b^9*c^15*
d^4*e*f^18 + 936*a^11*b^8*c*d^18*e^14*f^5 + 936*a^11*b^8*c^14*d^5*e*f^18 -
420*a^12*b^7*c*d^18*e^13*f^6 - 420*a^12*b^7*c^13*d^6*e*f^18 - 420*a^13*b^6*
c*d^18*e^12*f^7 - 420*a^13*b^6*c^12*d^7*e*f^18 + 936*a^14*b^5*c*d^18*e^11*f
^8 + 936*a^14*b^5*c^11*d^8*e*f^18 - 840*a^15*b^4*c*d^18*e^10*f^9 - 840*a^15
*b^4*c^10*d^9*e*f^18 + 434*a^16*b^3*c*d^18*e^9*f^10 + 434*a^16*b^3*c^9*d^10
*e*f^18 - 126*a^17*b^2*c*d^18*e^8*f^11 - 126*a^17*b^2*c^8*d^11*e*f^18 - 56*
a^18*b*c^2*d^17*e^6*f^13 + 112*a^18*b*c^3*d^16*e^5*f^14 - 140*a^18*b*c^4*d^
15*e^4*f^15 + 112*a^18*b*c^5*d^14*e^3*f^16 - 56*a^18*b*c^6*d^13*e^2*f^17 +
360*a^2*b^17*c^7*d^12*e^17*f^2 - 882*a^2*b^17*c^8*d^11*e^16*f^3 + 728*a^2*b
^17*c^9*d^10*e^15*f^4 + 1152*a^2*b^17*c^10*d^9*e^14*f^5 - 4032*a^2*b^17*c^1
1*d^8*e^13*f^6 + 5460*a^2*b^17*c^12*d^7*e^12*f^7 - 4032*a^2*b^17*c^13*d^6*e
^11*f^8 + 1152*a^2*b^17*c^14*d^5*e^10*f^9 + 728*a^2*b^17*c^15*d^4*e^9*f^10
- 882*a^2*b^17*c^16*d^3*e^8*f^11 + 360*a^2*b^17*c^17*d^2*e^7*f^12 - 504*a^3
*b^16*c^6*d^13*e^17*f^2 + 312*a^3*b^16*c^7*d^12*e^16*f^3 + 2520*a^3*b^16*c^
8*d^11*e^15*f^4 - 7480*a^3*b^16*c^9*d^10*e^14*f^5 + 9408*a^3*b^16*c^10*d^9*
e^13*f^6 - 4368*a^3*b^16*c^11*d^8*e^12*f^7 - 4368*a^3*b^16*c^12*d^7*e^11*f^
8 + 9408*a^3*b^16*c^13*d^6*e^10*f^9 - 7480*a^3*b^16*c^14*d^5*e^9*f^10 + 252
0*a^3*b^16*c^15*d^4*e^8*f^11 + 312*a^3*b^16*c^16*d^3*e^7*f^12 - 504*a^3*b^1
6*c^17*d^2*e^6*f^13 + 252*a^4*b^15*c^5*d^14*e^17*f^2 + 1596*a^4*b^15*c^6*d^
13*e^16*f^3 - 6288*a^4*b^15*c^7*d^12*e^15*f^4 + 7380*a^4*b^15*c^8*d^11*e^14
*f^5 + 2660*a^4*b^15*c^9*d^10*e^13*f^6 - 18564*a^4*b^15*c^10*d^9*e^12*f^7 +
 26208*a^4*b^15*c^11*d^8*e^11*f^8 - 18564*a^4*b^15*c^12*d^7*e^10*f^9 + 2660
*a^4*b^15*c^13*d^6*e^9*f^10 + 7380*a^4*b^15*c^14*d^5*e^8*f^11 - 6288*a^4*b^

15*c^15*d^4*e^7*f^12 + 1596*a^4*b^15*c^16*d^3*e^6*f^13 + 252*a^4*b^15*c^17*
d^2*e^5*f^14 + 252*a^5*b^14*c^4*d^15*e^17*f^2 - 2772*a^5*b^14*c^5*d^14*e^16
*f^3 + 3696*a^5*b^14*c^6*d^13*e^15*f^4 + 7056*a^5*b^14*c^7*d^12*e^14*f^5 -
25452*a^5*b^14*c^8*d^11*e^13*f^6 + 30212*a^5*b^14*c^9*d^10*e^12*f^7 - 13104
*a^5*b^14*c^10*d^9*e^11*f^8 - 13104*a^5*b^14*c^11*d^8*e^10*f^9 + 30212*a^5*
b^14*c^12*d^7*e^9*f^10 - 25452*a^5*b^14*c^13*d^6*e^8*f^11 + 7056*a^5*b^14*c
^14*d^5*e^7*f^12 + 3696*a^5*b^14*c^15*d^4*e^6*f^13 - 2772*a^5*b^14*c^16*d^3
*e^5*f^14 + 252*a^5*b^14*c^17*d^2*e^4*f^15 - 504*a^6*b^13*c^3*d^16*e^17*f^2
 + 1596*a^6*b^13*c^4*d^15*e^16*f^3 + 3696*a^6*b^13*c^5*d^14*e^15*f^4 - 1747
2*a^6*b^13*c^6*d^13*e^14*f^5 + 17472*a^6*b^13*c^7*d^12*e^13*f^6 + 9828*a^6*
b^13*c^8*d^11*e^12*f^7 - 38584*a^6*b^13*c^9*d^10*e^11*f^8 + 48048*a^6*b^13*
c^10*d^9*e^10*f^9 - 38584*a^6*b^13*c^11*d^8*e^9*f^10 + 9828*a^6*b^13*c^12*d
^7*e^8*f^11 + 17472*a^6*b^13*c^13*d^6*e^7*f^12 - 17472*a^6*b^13*c^14*d^5*e^
6*f^13 + 3696*a^6*b^13*c^15*d^4*e^5*f^14 + 1596*a^6*b^13*c^16*d^3*e^4*f^15
 - 504*a^6*b^13*c^17*d^2*e^3*f^16 + 360*a^7*b^12*c^2*d^17*e^17*f^2 + 312*a^7
*b^12*c^3*d^16*e^16*f^3 - 6288*a^7*b^12*c^4*d^15*e^15*f^4 + 7056*a^7*b^12*c
^5*d^14*e^14*f^5 + 17472*a^7*b^12*c^6*d^13*e^13*f^6 - 43680*a^7*b^12*c^7*d^
12*e^12*f^7 + 32760*a^7*b^12*c^8*d^11*e^11*f^8 - 8008*a^7*b^12*c^9*d^10*e^1
0*f^9 - 8008*a^7*b^12*c^10*d^9*e^9*f^10 + 32760*a^7*b^12*c^11*d^8*e^8*f^11
 - 43680*a^7*b^12*c^12*d^7*e^7*f^12 + 17472*a^7*b^12*c^13*d^6*e^6*f^13 + 705
6*a^7*b^12*c^14*d^5*e^5*f^14 - 6288*a^7*b^12*c^15*d^4*e^4*f^15 + 312*a^7*b^
12*c^16*d^3*e^3*f^16 + 360*a^7*b^12*c^17*d^2*e^2*f^17 - 882*a^8*b^11*c^2*d^
17*e^16*f^3 + 2520*a^8*b^11*c^3*d^16*e^15*f^4 + 7380*a^8*b^11*c^4*d^15*e^14
*f^5 - 25452*a^8*b^11*c^5*d^14*e^13*f^6 + 9828*a^8*b^11*c^6*d^13*e^12*f^7 +
 32760*a^8*b^11*c^7*d^12*e^11*f^8 - 36036*a^8*b^11*c^8*d^11*e^10*f^9 + 2002
0*a^8*b^11*c^9*d^10*e^9*f^10 - 36036*a^8*b^11*c^10*d^9*e^8*f^11 + 32760*a^8
*b^11*c^11*d^8*e^7*f^12 + 9828*a^8*b^11*c^12*d^7*e^6*f^13 - 25452*a^8*b^11*
c^13*d^6*e^5*f^14 + 7380*a^8*b^11*c^14*d^5*e^4*f^15 + 2520*a^8*b^11*c^15*d^
4*e^3*f^16 - 882*a^8*b^11*c^16*d^3*e^2*f^17 + 728*a^9*b^10*c^2*d^17*e^15*f^
4 - 7480*a^9*b^10*c^3*d^16*e^14*f^5 + 2660*a^9*b^10*c^4*d^15*e^13*f^6 + 302
12*a^9*b^10*c^5*d^14*e^12*f^7 - 38584*a^9*b^10*c^6*d^13*e^11*f^8 - 8008*a^9
*b^10*c^7*d^12*e^10*f^9 + 20020*a^9*b^10*c^8*d^11*e^9*f^10 + 20020*a^9*b^10
*c^9*d^10*e^8*f^11 - 8008*a^9*b^10*c^10*d^9*e^7*f^12 - 38584*a^9*b^10*c^11*
d^8*e^6*f^13 + 30212*a^9*b^10*c^12*d^7*e^5*f^14 + 2660*a^9*b^10*c^13*d^6*e^
4*f^15 - 7480*a^9*b^10*c^14*d^5*e^3*f^16 + 728*a^9*b^10*c^15*d^4*e^2*f^17 +
 1152*a^10*b^9*c^2*d^17*e^14*f^5 + 9408*a^10*b^9*c^3*d^16*e^13*f^6 - 18564*
a^10*b^9*c^4*d^15*e^12*f^7 - 13104*a^10*b^9*c^5*d^14*e^11*f^8 + 48048*a^10*
b^9*c^6*d^13*e^10*f^9 - 8008*a^10*b^9*c^7*d^12*e^9*f^10 - 36036*a^10*b^9*c^
8*d^11*e^8*f^11 - 8008*a^10*b^9*c^9*d^10*e^7*f^12 + 48048*a^10*b^9*c^10*d^9
*e^6*f^13 - 13104*a^10*b^9*c^11*d^8*e^5*f^14 - 18564*a^10*b^9*c^12*d^7*e^4*
f^15 + 9408*a^10*b^9*c^13*d^6*e^3*f^16 + 1152*a^10*b^9*c^14*d^5*e^2*f^17 -
4032*a^11*b^8*c^2*d^17*e^13*f^6 - 4368*a^11*b^8*c^3*d^16*e^12*f^7 + 26208*a
^11*b^8*c^4*d^15*e^11*f^8 - 13104*a^11*b^8*c^5*d^14*e^10*f^9 - 38584*a^11*b
^8*c^6*d^13*e^9*f^10 + 32760*a^11*b^8*c^7*d^12*e^8*f^11 + 32760*a^11*b^8*c^
8*d^11*e^7*f^12 - 38584*a^11*b^8*c^9*d^10*e^6*f^13 - 13104*a^11*b^8*c^10*d^
9*e^5*f^14 + 26208*a^11*b^8*c^11*d^8*e^4*f^15 - 4368*a^11*b^8*c^12*d^7*e^3*
f^16 - 4032*a^11*b^8*c^13*d^6*e^2*f^17 + 5460*a^12*b^7*c^2*d^17*e^12*f^7 -
4368*a^12*b^7*c^3*d^16*e^11*f^8 - 18564*a^12*b^7*c^4*d^15*e^10*f^9 + 30212*
a^12*b^7*c^5*d^14*e^9*f^10 + 9828*a^12*b^7*c^6*d^13*e^8*f^11 - 43680*a^12*b
^7*c^7*d^12*e^7*f^12 + 9828*a^12*b^7*c^8*d^11*e^6*f^13 + 30212*a^12*b^7*c^9
*d^10*e^5*f^14 - 18564*a^12*b^7*c^10*d^9*e^4*f^15 - 4368*a^12*b^7*c^11*d^8*
e^3*f^16 + 5460*a^12*b^7*c^12*d^7*e^2*f^17 - 4032*a^13*b^6*c^2*d^17*e^11*f^
8 + 9408*a^13*b^6*c^3*d^16*e^10*f^9 + 2660*a^13*b^6*c^4*d^15*e^9*f^10 - 254
52*a^13*b^6*c^5*d^14*e^8*f^11 + 17472*a^13*b^6*c^6*d^13*e^7*f^12 + 17472*a^
13*b^6*c^7*d^12*e^6*f^13 - 25452*a^13*b^6*c^8*d^11*e^5*f^14 + 2660*a^13*b^6
*c^9*d^10*e^4*f^15 + 9408*a^13*b^6*c^10*d^9*e^3*f^16 - 4032*a^13*b^6*c^11*d
^8*e^2*f^17 + 1152*a^14*b^5*c^2*d^17*e^10*f^9 - 7480*a^14*b^5*c^3*d^16*e^9*
f^10 + 7380*a^14*b^5*c^4*d^15*e^8*f^11 + 7056*a^14*b^5*c^5*d^14*e^7*f^12 -
17472*a^14*b^5*c^6*d^13*e^6*f^13 + 7056*a^14*b^5*c^7*d^12*e^5*f^14 + 7380*a

^14*b^5*c^8*d^11*e^4*f^15 - 7480*a^14*b^5*c^9*d^10*e^3*f^16 + 1152*a^14*b^5
*c^10*d^9*e^2*f^17 + 728*a^15*b^4*c^2*d^17*e^9*f^10 + 2520*a^15*b^4*c^3*d^1
6*e^8*f^11 - 6288*a^15*b^4*c^4*d^15*e^7*f^12 + 3696*a^15*b^4*c^5*d^14*e^6*f
^13 + 3696*a^15*b^4*c^6*d^13*e^5*f^14 - 6288*a^15*b^4*c^7*d^12*e^4*f^15 + 2
520*a^15*b^4*c^8*d^11*e^3*f^16 + 728*a^15*b^4*c^9*d^10*e^2*f^17 - 882*a^16*
b^3*c^2*d^17*e^8*f^11 + 312*a^16*b^3*c^3*d^16*e^7*f^12 + 1596*a^16*b^3*c^4*
d^15*e^6*f^13 - 2772*a^16*b^3*c^5*d^14*e^5*f^14 + 1596*a^16*b^3*c^6*d^13*e^
4*f^15 + 312*a^16*b^3*c^7*d^12*e^3*f^16 - 882*a^16*b^3*c^8*d^11*e^2*f^17 +
360*a^17*b^2*c^2*d^17*e^7*f^12 - 504*a^17*b^2*c^3*d^16*e^6*f^13 + 252*a^17*
b^2*c^4*d^15*e^5*f^14 + 252*a^17*b^2*c^5*d^14*e^4*f^15 - 504*a^17*b^2*c^6*d
^13*e^3*f^16 + 360*a^17*b^2*c^7*d^12*e^2*f^17))/(56*a^3*b^13*c^5*d^11*e^16
- a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^16 - a^16*d^16*e^8
*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8*b^8*c^16*f^16 -
70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28*a^6*b^10*c^2*d^1
4*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*f^16 - 70*a^12*b^
4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*c^10*d^6*f^16 - 2
8*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*a^4*b^12*c^16*e^4
*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f^14 - 28*a^10*b^6
*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d^16*e^12*f^4 + 56
*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a^16*c^2*d^14*e^6*
f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^12 + 56*a^16*c^5*
d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d^6*e^14*f^2 + 56*
b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^16*c^13*d^3*e^11*
f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 + 8*a^7*b^9*c*d^15*
e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a*b^15*c^16*e^7*f^
9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15*b*d^16*e^9*f^7 +
8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^9*d^7*e^15*f + 8*
b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15*c^15*d*e^8*f^8 -
56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*a^15*b*c*d^15*e^8
*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*f^2 - 224*a*b^15*
c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b^15*c^12*d^4*e^11
*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*e^9*f^7 + 160*a^2
*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a^3*b^13*c^6*d^10*
e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d^11*e^15*f + 112*
a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f + 112*a^5*b^11*c^15
*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*c^15*d*e^3*f^13 +
160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14 + 160*a^9*b^7*c*
d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6*c*d^15*e^13*f^3
- 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*f^4 + 112*a^11*b^
5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^12*b^4*c^11*d^5*e*
f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^6*e*f^15 + 160*a^
14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*a^15*b*c^2*d^14*e
^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^12*e^5*f^11 + 112*
a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 + 160*a^15*b*c^7*d^
9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*c^10*d^6*e^12*f^4
- 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4*e^10*f^6 - 300*a
^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3 - 2800*a^3*b^13*c
^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a^3*b^13*c^11*d^5*
e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*c^13*d^3*e^8*f^8
+ 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e^13*f^3 + 1750*a^
4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 - 8624*a^4*b^12*c^
10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^4*b^12*c^12*d^4*e
^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^14*d^2*e^6*f^10 -
1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e^13*f^3 + 4480*a^
5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5 + 7392*a^5*b^11*c
^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a^5*b^11*c^11*d^5*
e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c^13*d^3*e^6*f^10
- 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*e^14*f^2 + 1568*a
^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^4 + 7392*a^6*b^10

*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 24640*a^6*b^10*c^9*d^
7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^10*c^11*d^5*e^7*f^
9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d^3*e^5*f^11 + 840
*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f^3 + 4480*a^7*b^9
*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 24640*a^7*b^9*c^7*d^9
*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*c^9*d^7*e^8*f^8 -
24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^6*f^10 + 4480*a^7
*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 - 300*a^8*b^8*c^2*d
^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b^8*c^4*d^12*e^12*
f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*d^10*e^10*f^6 + 1
5400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f^8 + 15400*a^8*b^
8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 12264*a^8*b^8*c^11*d^
5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8*c^13*d^3*e^3*f^1
3 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*e^12*f^4 + 4480*a
^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6 - 24640*a^9*b^7*c
^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^9*b^7*c^8*d^8*e^7
*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*d^6*e^5*f^11 + 44
80*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f^13 + 840*a^10*b^
6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 8624*a^10*b^6*c^4*d
^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10*b^6*c^6*d^10*e^8
*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8*d^8*e^6*f^10 + 7
392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4*f^12 + 1568*a^10
*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 - 1344*a^11*b^5*c^2
*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^11*b^5*c^4*d^12*e
^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c^6*d^10*e^7*f^9 +
7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e^5*f^11 + 4480*a^
11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13 - 1344*a^11*b^5*c
^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a^12*b^4*c^3*d^13*
e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c^5*d^11*e^7*f^9 -
8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e^5*f^11 + 1750*a^
12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 + 840*a^12*b^4*c^1
0*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^13*b^3*c^4*d^12*e^
7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^6*d^10*e^5*f^11 -
2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^3*f^13 - 300*a^14
*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1344*a^14*b^2*c^5*
d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*b^2*c^8*d^8*e^2*f
^14)) + (x*(84*a^5*b^11*c^13*d^3*f^16 - 12*a^4*b^12*c^14*d^2*f^16 - 243*a^6
*b^10*c^12*d^4*f^16 + 366*a^7*b^9*c^11*d^5*f^16 - 321*a^8*b^8*c^10*d^6*f^16
+ 252*a^9*b^7*c^9*d^7*f^16 - 321*a^10*b^6*c^8*d^8*f^16 + 366*a^11*b^5*c^7*
d^9*f^16 - 243*a^12*b^4*c^6*d^10*f^16 + 84*a^13*b^3*c^5*d^11*f^16 - 12*a^14
*b^2*c^4*d^12*f^16 - 12*a^4*b^12*d^16*e^14*f^2 + 84*a^5*b^11*d^16*e^13*f^3
- 243*a^6*b^10*d^16*e^12*f^4 + 366*a^7*b^9*d^16*e^11*f^5 - 321*a^8*b^8*d^16
*e^10*f^6 + 252*a^9*b^7*d^16*e^9*f^7 - 321*a^10*b^6*d^16*e^8*f^8 + 366*a^11
*b^5*d^16*e^7*f^9 - 243*a^12*b^4*d^16*e^6*f^10 + 84*a^13*b^3*d^16*e^5*f^11
- 12*a^14*b^2*d^16*e^4*f^12 - 12*b^16*c^4*d^12*e^14*f^2 + 84*b^16*c^5*d^11*
e^13*f^3 - 243*b^16*c^6*d^10*e^12*f^4 + 366*b^16*c^7*d^9*e^11*f^5 - 321*b^1
6*c^8*d^8*e^10*f^6 + 252*b^16*c^9*d^7*e^9*f^7 - 321*b^16*c^10*d^6*e^8*f^8 +
366*b^16*c^11*d^5*e^7*f^9 - 243*b^16*c^12*d^4*e^6*f^10 + 84*b^16*c^13*d^3*
e^5*f^11 - 12*b^16*c^14*d^2*e^4*f^12 + 48*a*b^15*c^3*d^13*e^14*f^2 - 252*a*
b^15*c^4*d^12*e^13*f^3 + 366*a*b^15*c^5*d^11*e^12*f^4 + 354*a*b^15*c^6*d^10
*e^11*f^5 - 1458*a*b^15*c^7*d^9*e^10*f^6 + 942*a*b^15*c^8*d^8*e^9*f^7 + 942
*a*b^15*c^9*d^7*e^8*f^8 - 1458*a*b^15*c^10*d^6*e^7*f^9 + 354*a*b^15*c^11*d^
5*e^6*f^10 + 366*a*b^15*c^12*d^4*e^5*f^11 - 252*a*b^15*c^13*d^3*e^4*f^12 +
48*a*b^15*c^14*d^2*e^3*f^13 + 48*a^3*b^13*c*d^15*e^14*f^2 + 48*a^3*b^13*c^1
4*d^2*e*f^15 - 252*a^4*b^12*c*d^15*e^13*f^3 - 252*a^4*b^12*c^13*d^3*e*f^15
+ 366*a^5*b^11*c*d^15*e^12*f^4 + 366*a^5*b^11*c^12*d^4*e*f^15 + 354*a^6*b^1
0*c*d^15*e^11*f^5 + 354*a^6*b^10*c^11*d^5*e*f^15 - 1458*a^7*b^9*c*d^15*e^10
*f^6 - 1458*a^7*b^9*c^10*d^6*e*f^15 + 942*a^8*b^8*c*d^15*e^9*f^7 + 942*a^8*
b^8*c^9*d^7*e*f^15 + 942*a^9*b^7*c*d^15*e^8*f^8 + 942*a^9*b^7*c^8*d^8*e*f^1

```
5 - 1458*a^10*b^6*c*d^15*e^7*f^9 - 1458*a^10*b^6*c^7*d^9*e*f^15 + 354*a^11*
b^5*c*d^15*e^6*f^10 + 354*a^11*b^5*c^6*d^10*e*f^15 + 366*a^12*b^4*c*d^15*e^
5*f^11 + 366*a^12*b^4*c^5*d^11*e*f^15 - 252*a^13*b^3*c*d^15*e^4*f^12 - 252*
a^13*b^3*c^4*d^12*e*f^15 + 48*a^14*b^2*c*d^15*e^3*f^13 + 48*a^14*b^2*c^3*d^
13*e*f^15 - 72*a^2*b^14*c^2*d^14*e^14*f^2 + 168*a^2*b^14*c^3*d^13*e^13*f^3
+ 723*a^2*b^14*c^4*d^12*e^12*f^4 - 3258*a^2*b^14*c^5*d^11*e^11*f^5 + 3156*a
^2*b^14*c^6*d^10*e^10*f^6 + 3522*a^2*b^14*c^7*d^9*e^9*f^7 - 8478*a^2*b^14*c
^8*d^8*e^8*f^8 + 3522*a^2*b^14*c^9*d^7*e^7*f^9 + 3156*a^2*b^14*c^10*d^6*e^6
*f^10 - 3258*a^2*b^14*c^11*d^5*e^5*f^11 + 723*a^2*b^14*c^12*d^4*e^4*f^12 +
168*a^2*b^14*c^13*d^3*e^3*f^13 - 72*a^2*b^14*c^14*d^2*e^2*f^14 + 168*a^3*b^
13*c^2*d^14*e^13*f^3 - 1692*a^3*b^13*c^3*d^13*e^12*f^4 + 2538*a^3*b^13*c^4*
d^12*e^11*f^5 + 5634*a^3*b^13*c^5*d^11*e^10*f^6 - 18738*a^3*b^13*c^6*d^10*e
^9*f^7 + 12042*a^3*b^13*c^7*d^9*e^8*f^8 + 12042*a^3*b^13*c^8*d^8*e^7*f^9 -
18738*a^3*b^13*c^9*d^7*e^6*f^10 + 5634*a^3*b^13*c^10*d^6*e^5*f^11 + 2538*a^
3*b^13*c^11*d^5*e^4*f^12 - 1692*a^3*b^13*c^12*d^4*e^3*f^13 + 168*a^3*b^13*c
^13*d^3*e^2*f^14 + 723*a^4*b^12*c^2*d^14*e^12*f^4 + 2538*a^4*b^12*c^3*d^13*
e^11*f^5 - 14022*a^4*b^12*c^4*d^12*e^10*f^6 + 14022*a^4*b^12*c^5*d^11*e^9*f
^7 + 21087*a^4*b^12*c^6*d^10*e^8*f^8 - 48168*a^4*b^12*c^7*d^9*e^7*f^9 + 210
87*a^4*b^12*c^8*d^8*e^6*f^10 + 14022*a^4*b^12*c^9*d^7*e^5*f^11 - 14022*a^4*
b^12*c^10*d^6*e^4*f^12 + 2538*a^4*b^12*c^11*d^5*e^3*f^13 + 723*a^4*b^12*c^1
2*d^4*e^2*f^14 - 3258*a^5*b^11*c^2*d^14*e^11*f^5 + 5634*a^5*b^11*c^3*d^13*e
^10*f^6 + 14022*a^5*b^11*c^4*d^12*e^9*f^7 - 50544*a^5*b^11*c^5*d^11*e^8*f^8
 + 33696*a^5*b^11*c^6*d^10*e^7*f^9 + 33696*a^5*b^11*c^7*d^9*e^6*f^10 - 5054
4*a^5*b^11*c^8*d^8*e^5*f^11 + 14022*a^5*b^11*c^9*d^7*e^4*f^12 + 5634*a^5*b^
11*c^10*d^6*e^3*f^13 - 3258*a^5*b^11*c^11*d^5*e^2*f^14 + 3156*a^6*b^10*c^2*
d^14*e^10*f^6 - 18738*a^6*b^10*c^3*d^13*e^9*f^7 + 21087*a^6*b^10*c^4*d^12*e
^8*f^8 + 33696*a^6*b^10*c^5*d^11*e^7*f^9 - 78624*a^6*b^10*c^6*d^10*e^6*f^10
 + 33696*a^6*b^10*c^7*d^9*e^5*f^11 + 21087*a^6*b^10*c^8*d^8*e^4*f^12 - 1873
8*a^6*b^10*c^9*d^7*e^3*f^13 + 3156*a^6*b^10*c^10*d^6*e^2*f^14 + 3522*a^7*b^
9*c^2*d^14*e^9*f^7 + 12042*a^7*b^9*c^3*d^13*e^8*f^8 - 48168*a^7*b^9*c^4*d^1
2*e^7*f^9 + 33696*a^7*b^9*c^5*d^11*e^6*f^10 + 33696*a^7*b^9*c^6*d^10*e^5*f^
11 - 48168*a^7*b^9*c^7*d^9*e^4*f^12 + 12042*a^7*b^9*c^8*d^8*e^3*f^13 + 3522
*a^7*b^9*c^9*d^7*e^2*f^14 - 8478*a^8*b^8*c^2*d^14*e^8*f^8 + 12042*a^8*b^8*c
^3*d^13*e^7*f^9 + 21087*a^8*b^8*c^4*d^12*e^6*f^10 - 50544*a^8*b^8*c^5*d^11*
e^5*f^11 + 21087*a^8*b^8*c^6*d^10*e^4*f^12 + 12042*a^8*b^8*c^7*d^9*e^3*f^13
 - 8478*a^8*b^8*c^8*d^8*e^2*f^14 + 3522*a^9*b^7*c^2*d^14*e^7*f^9 - 18738*a^
9*b^7*c^3*d^13*e^6*f^10 + 14022*a^9*b^7*c^4*d^12*e^5*f^11 + 14022*a^9*b^7*c
^5*d^11*e^4*f^12 - 18738*a^9*b^7*c^6*d^10*e^3*f^13 + 3522*a^9*b^7*c^7*d^9*e
^2*f^14 + 3156*a^10*b^6*c^2*d^14*e^6*f^10 + 5634*a^10*b^6*c^3*d^13*e^5*f^11
 - 14022*a^10*b^6*c^4*d^12*e^4*f^12 + 5634*a^10*b^6*c^5*d^11*e^3*f^13 + 315
6*a^10*b^6*c^6*d^10*e^2*f^14 - 3258*a^11*b^5*c^2*d^14*e^5*f^11 + 2538*a^11*
b^5*c^3*d^13*e^4*f^12 + 2538*a^11*b^5*c^4*d^12*e^3*f^13 - 3258*a^11*b^5*c^5
*d^11*e^2*f^14 + 723*a^12*b^4*c^2*d^14*e^4*f^12 - 1692*a^12*b^4*c^3*d^13*e^
3*f^13 + 723*a^12*b^4*c^4*d^12*e^2*f^14 + 168*a^13*b^3*c^2*d^14*e^3*f^13 +
168*a^13*b^3*c^3*d^13*e^2*f^14 - 72*a^14*b^2*c^2*d^14*e^2*f^14))/(56*a^3*b^
13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*c^8*d^8*e^1
6 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10*e^16 - a^8
*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^13*e^16 - 28
*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^5*c^13*d^3*
f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 28*a^14*b^2*
c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5*f^11 - 70*
a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10*c^16*e^2*f
^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70*a^12*b^4*d
^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10*f^6 - 28*a
^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*d^12*e^4*f^
12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*b^16*c^10*d
^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*f^4 + 56*b^
16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*d^9*e^16 +
8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7*f^16 + 8*a
```

```
*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15*f + 8*a^15
*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15 + 8*b^16*c^
9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f - 56*a*b^15
*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e*f^15 - 56*
a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^9*d^7*e^14*
f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^4 + 112*a*b
^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^15*c^14*d^2*
e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*f^9 - 224*a
^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4*b^12*c^5*d
^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^12*e^15*f +
112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 224*a^6*b^10*
c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15*d*e^2*f^14
 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 224*a^10*b^6
*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c*d^15*e^12*
f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^5 + 112*a^1
2*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*b^3*c^10*d^
6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*f^15 + 160*
a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^15*b*c^4*d^1
2*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*e^3*f^13 +
160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 840*a^2*b^14*
c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^14*c^12*d^4
*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^8*e^13*f^3
- 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f^5 + 1568*a
^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1400*a^3*b^13*
c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^12*c^7*d^9*e
^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7*e^11*f^5 -
 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^7 + 1750*a^
4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a^4*b^12*c^1
4*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11*c^6*d^10*e
^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^8*e^11*f^5
+ 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^7 - 12264*a
^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568*a^5*b^11*c
^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^10*c^4*d^12*
e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d^10*e^12*f^
4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10*f^6 - 2464
0*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7392*a^6*b^1
0*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*b^10*c^13*d
^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*d^12*e^13*f
^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11*f^5 - 2464
0*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 15400*a^7*b^9*
c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*c^11*d^5*e^
6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*e^4*f^12 -
300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 + 1750*a^8*b
^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a^8*b^8*c^6*
d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^8*d^8*e^8*f
^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*f^10 - 1226
4*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1400*a^8*b^8
*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^7*c^3*d^13*
e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^11*e^10*f^6
- 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8 + 15400*a^
9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^9*b^7*c^10*
d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^12*d^4*e^3*f
^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^11*f^5 - 86
24*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 + 11396*a^10
*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a^10*b^6*c^8
*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c^10*d^6*e^4
*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*e^2*f^14 -
1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^6 + 4480*a^
11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392*a^11*b^5*c
```

```
^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^5*c^8*d^8*e
^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^6*e^3*f^13
- 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f^6 - 2800*a
^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480*a^12*b^4*c
^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^4*c^7*d^9*e
^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7*e^3*f^13 +
 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8 - 2800*a^1
3*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*a^13*b^3*c^
6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3*c^8*d^8*e^
3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e^6*f^10 - 1
344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12 - 300*a^14*
b^2*c^8*d^8*e^2*f^14)) - (36*a^11*b^2*d^13*f^13 + 36*b^13*c^11*d^2*f^13 + 3
6*b^13*d^13*e^11*f^2 + 297*a^2*b^11*c^9*d^4*f^13 - 108*a^3*b^10*c^8*d^5*f^1
3 - 198*a^4*b^9*c^7*d^6*f^13 + 153*a^5*b^8*c^6*d^7*f^13 + 153*a^6*b^7*c^5*d
^8*f^13 - 198*a^7*b^6*c^4*d^9*f^13 - 108*a^8*b^5*c^3*d^10*f^13 + 297*a^9*b^
4*c^2*d^11*f^13 + 297*a^2*b^11*d^13*e^9*f^4 - 108*a^3*b^10*d^13*e^8*f^5 - 1
98*a^4*b^9*d^13*e^7*f^6 + 153*a^5*b^8*d^13*e^6*f^7 + 153*a^6*b^7*d^13*e^5*f
^8 - 198*a^7*b^6*d^13*e^4*f^9 - 108*a^8*b^5*d^13*e^3*f^10 + 297*a^9*b^4*d^1
3*e^2*f^11 + 297*b^13*c^2*d^11*e^9*f^4 - 108*b^13*c^3*d^10*e^8*f^5 - 198*b^
13*c^4*d^9*e^7*f^6 + 153*b^13*c^5*d^8*e^6*f^7 + 153*b^13*c^6*d^7*e^5*f^8 -
198*b^13*c^7*d^6*e^4*f^9 - 108*b^13*c^8*d^5*e^3*f^10 + 297*b^13*c^9*d^4*e^2
*f^11 - 180*a*b^12*c^10*d^3*f^13 - 180*a^10*b^3*c*d^12*f^13 - 180*a*b^12*d^
13*e^10*f^3 - 180*a^10*b^3*d^13*e*f^12 - 180*b^13*c*d^12*e^10*f^3 - 180*b^1
3*c^10*d^3*e*f^12 + 1026*a*b^12*c*d^12*e^9*f^4 + 1026*a*b^12*c^9*d^4*e*f^12
 + 1026*a^9*b^4*c*d^12*e*f^12 - 2052*a*b^12*c^2*d^11*e^8*f^5 + 1548*a*b^12*
c^3*d^10*e^7*f^6 + 297*a*b^12*c^4*d^9*e^6*f^7 - 1242*a*b^12*c^5*d^8*e^5*f^8
 + 297*a*b^12*c^6*d^7*e^4*f^9 + 1548*a*b^12*c^7*d^6*e^3*f^10 - 2052*a*b^12*
c^8*d^5*e^2*f^11 - 2052*a^2*b^11*c*d^12*e^8*f^5 - 2052*a^2*b^11*c^8*d^5*e*f
^12 + 1548*a^3*b^10*c*d^12*e^7*f^6 + 1548*a^3*b^10*c^7*d^6*e*f^12 + 297*a^4
*b^9*c*d^12*e^6*f^7 + 297*a^4*b^9*c^6*d^7*e*f^12 - 1242*a^5*b^8*c*d^12*e^5*
f^8 - 1242*a^5*b^8*c^5*d^8*e*f^12 + 297*a^6*b^7*c*d^12*e^4*f^9 + 297*a^6*b^
7*c^4*d^9*e*f^12 + 1548*a^7*b^6*c*d^12*e^3*f^10 + 1548*a^7*b^6*c^3*d^10*e*f
^12 - 2052*a^8*b^5*c*d^12*e^2*f^11 - 2052*a^8*b^5*c^2*d^11*e*f^12 + 4860*a^
2*b^11*c^2*d^11*e^7*f^6 - 4986*a^2*b^11*c^3*d^10*e^6*f^7 + 1701*a^2*b^11*c^
4*d^9*e^5*f^8 + 1701*a^2*b^11*c^5*d^8*e^4*f^9 - 4986*a^2*b^11*c^6*d^7*e^3*f
^10 + 4860*a^2*b^11*c^7*d^6*e^2*f^11 - 4986*a^3*b^10*c^2*d^11*e^6*f^7 + 633
6*a^3*b^10*c^3*d^10*e^5*f^8 - 3960*a^3*b^10*c^4*d^9*e^4*f^9 + 6336*a^3*b^10
*c^5*d^8*e^3*f^10 - 4986*a^3*b^10*c^6*d^7*e^2*f^11 + 1701*a^4*b^9*c^2*d^11*
e^5*f^8 - 3960*a^4*b^9*c^3*d^10*e^4*f^9 - 3960*a^4*b^9*c^4*d^9*e^3*f^10 + 1
701*a^4*b^9*c^5*d^8*e^2*f^11 + 1701*a^5*b^8*c^2*d^11*e^4*f^9 + 6336*a^5*b^8
*c^3*d^10*e^3*f^10 + 1701*a^5*b^8*c^4*d^9*e^2*f^11 - 4986*a^6*b^7*c^2*d^11*
e^3*f^10 - 4986*a^6*b^7*c^3*d^10*e^2*f^11 + 4860*a^7*b^6*c^2*d^11*e^2*f^11)
/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^16*
c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^10
*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d^1
3*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*b^
5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 - 2
8*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e^5
*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^10
*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 - 70
*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^10
*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^4*
d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 28*
b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^12*
f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^7*
d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d^7
*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^15
*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^15
+ 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*f
```

- 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d*e
*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*c^
9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*f^
4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b^1
5*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^7*
f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a^4
*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d^1
2*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f - 22
4*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^15
*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 - 2
24*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5*c
*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*f^
5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^13*
b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*e*
f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a^1
5*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^10*
e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 + 84
0*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*b^
14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*d^
8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11*f
^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 140
0*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b^1
2*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d^7
*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*f^
7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840*a
^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^11
*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*d^
8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*f^
7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 1568
*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b^1
0*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6*d
^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^10
*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 + 7
392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^6*
b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^4*
d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^11
*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 154
00*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^9*
c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^3*
e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3 +
 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396*a
^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*c^
8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^6*
f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 + 1
400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*b^
7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d^1
1*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f^8
 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*a^
9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c^1
2*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*e^
11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7 +
 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396*a
^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6*c
^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^4*
e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*f^
6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 7392
*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*b^
5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*d^
6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10*f

^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 4480
*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*b^
4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d^7
*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f^8
 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 1568*
a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b^3
*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12*e
^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^12
- 300*a^14*b^2*c^8*d^8*e^2*f^14) + (x*(108*a^3*b^10*c^7*d^6*f^13 - 36*b^13*
c^10*d^3*f^13 - 36*b^13*d^13*e^10*f^3 - 297*a^2*b^11*c^8*d^5*f^13 - 36*a^10
*b^3*d^13*f^13 + 324*a^4*b^9*c^6*d^7*f^13 - 594*a^5*b^8*c^5*d^8*f^13 + 324*
a^6*b^7*c^4*d^9*f^13 + 108*a^7*b^6*c^3*d^10*f^13 - 297*a^8*b^5*c^2*d^11*f^1
3 - 297*a^2*b^11*d^13*e^8*f^5 + 108*a^3*b^10*d^13*e^7*f^6 + 324*a^4*b^9*d^1
3*e^6*f^7 - 594*a^5*b^8*d^13*e^5*f^8 + 324*a^6*b^7*d^13*e^4*f^9 + 108*a^7*b
^6*d^13*e^3*f^10 - 297*a^8*b^5*d^13*e^2*f^11 - 297*b^13*c^2*d^11*e^8*f^5 +
108*b^13*c^3*d^10*e^7*f^6 + 324*b^13*c^4*d^9*e^6*f^7 - 594*b^13*c^5*d^8*e^5
*f^8 + 324*b^13*c^6*d^7*e^4*f^9 + 108*b^13*c^7*d^6*e^3*f^10 - 297*b^13*c^8*
d^5*e^2*f^11 + 180*a*b^12*c^9*d^4*f^13 + 180*a^9*b^4*c*d^12*f^13 + 180*a*b^
12*d^13*e^9*f^4 + 180*a^9*b^4*d^13*e*f^12 + 180*b^13*c*d^12*e^9*f^4 + 180*b
^13*c^9*d^4*e*f^12 - 1026*a*b^12*c*d^12*e^8*f^5 - 1026*a*b^12*c^8*d^5*e*f^1
2 - 1026*a^8*b^5*c*d^12*e*f^12 + 2052*a*b^12*c^2*d^11*e^7*f^6 - 2052*a*b^12
*c^3*d^10*e^6*f^7 + 1026*a*b^12*c^4*d^9*e^5*f^8 + 1026*a*b^12*c^5*d^8*e^4*f
^9 - 2052*a*b^12*c^6*d^7*e^3*f^10 + 2052*a*b^12*c^7*d^6*e^2*f^11 + 2052*a^2
*b^11*c*d^12*e^7*f^6 + 2052*a^2*b^11*c^7*d^6*e*f^12 - 2052*a^3*b^10*c*d^12*
e^6*f^7 - 2052*a^3*b^10*c^6*d^7*e*f^12 + 1026*a^4*b^9*c*d^12*e^5*f^8 + 1026
*a^4*b^9*c^5*d^8*e*f^12 + 1026*a^5*b^8*c*d^12*e^4*f^9 + 1026*a^5*b^8*c^4*d^
9*e*f^12 - 2052*a^6*b^7*c*d^12*e^3*f^10 - 2052*a^6*b^7*c^3*d^10*e*f^12 + 20
52*a^7*b^6*c*d^12*e^2*f^11 + 2052*a^7*b^6*c^2*d^11*e*f^12 - 4104*a^2*b^11*c
^2*d^11*e^6*f^7 + 4104*a^2*b^11*c^3*d^10*e^5*f^8 - 5130*a^2*b^11*c^4*d^9*e^
4*f^9 + 4104*a^2*b^11*c^5*d^8*e^3*f^10 - 4104*a^2*b^11*c^6*d^7*e^2*f^11 + 4
104*a^3*b^10*c^2*d^11*e^5*f^8 + 4104*a^3*b^10*c^5*d^8*e^2*f^11 - 5130*a^4*b
^9*c^2*d^11*e^4*f^9 - 5130*a^4*b^9*c^4*d^9*e^2*f^11 + 4104*a^5*b^8*c^2*d^11
*e^3*f^10 + 4104*a^5*b^8*c^3*d^10*e^2*f^11 - 4104*a^6*b^7*c^2*d^11*e^2*f^11
))/(56*a^3*b^13*c^5*d^11*e^16 - a^8*b^8*d^16*e^16 - a^16*c^8*d^8*f^16 - b^1
6*c^8*d^8*e^16 - a^16*d^16*e^8*f^8 - b^16*c^16*e^8*f^8 - 28*a^2*b^14*c^6*d^
10*e^16 - a^8*b^8*c^16*f^16 - 70*a^4*b^12*c^4*d^12*e^16 + 56*a^5*b^11*c^3*d
^13*e^16 - 28*a^6*b^10*c^2*d^14*e^16 - 28*a^10*b^6*c^14*d^2*f^16 + 56*a^11*
b^5*c^13*d^3*f^16 - 70*a^12*b^4*c^12*d^4*f^16 + 56*a^13*b^3*c^11*d^5*f^16 -
 28*a^14*b^2*c^10*d^6*f^16 - 28*a^2*b^14*c^16*e^6*f^10 + 56*a^3*b^13*c^16*e
^5*f^11 - 70*a^4*b^12*c^16*e^4*f^12 + 56*a^5*b^11*c^16*e^3*f^13 - 28*a^6*b^
10*c^16*e^2*f^14 - 28*a^10*b^6*d^16*e^14*f^2 + 56*a^11*b^5*d^16*e^13*f^3 -
70*a^12*b^4*d^16*e^12*f^4 + 56*a^13*b^3*d^16*e^11*f^5 - 28*a^14*b^2*d^16*e^
10*f^6 - 28*a^16*c^2*d^14*e^6*f^10 + 56*a^16*c^3*d^13*e^5*f^11 - 70*a^16*c^
4*d^12*e^4*f^12 + 56*a^16*c^5*d^11*e^3*f^13 - 28*a^16*c^6*d^10*e^2*f^14 - 2
8*b^16*c^10*d^6*e^14*f^2 + 56*b^16*c^11*d^5*e^13*f^3 - 70*b^16*c^12*d^4*e^1
2*f^4 + 56*b^16*c^13*d^3*e^11*f^5 - 28*b^16*c^14*d^2*e^10*f^6 + 8*a*b^15*c^
7*d^9*e^16 + 8*a^7*b^9*c*d^15*e^16 + 8*a^9*b^7*c^15*d*f^16 + 8*a^15*b*c^9*d
^7*f^16 + 8*a*b^15*c^16*e^7*f^9 + 8*a^7*b^9*c^16*e*f^15 + 8*a^9*b^7*d^16*e^
15*f + 8*a^15*b*d^16*e^9*f^7 + 8*a^16*c*d^15*e^7*f^9 + 8*a^16*c^7*d^9*e*f^1
5 + 8*b^16*c^9*d^7*e^15*f + 8*b^16*c^15*d*e^9*f^7 - 56*a*b^15*c^8*d^8*e^15*
f - 56*a*b^15*c^15*d*e^8*f^8 - 56*a^8*b^8*c*d^15*e^15*f - 56*a^8*b^8*c^15*d
*e*f^15 - 56*a^15*b*c*d^15*e^8*f^8 - 56*a^15*b*c^8*d^8*e*f^15 + 160*a*b^15*
c^9*d^7*e^14*f^2 - 224*a*b^15*c^10*d^6*e^13*f^3 + 112*a*b^15*c^11*d^5*e^12*
f^4 + 112*a*b^15*c^12*d^4*e^11*f^5 - 224*a*b^15*c^13*d^3*e^10*f^6 + 160*a*b
^15*c^14*d^2*e^9*f^7 + 160*a^2*b^14*c^7*d^9*e^15*f + 160*a^2*b^14*c^15*d*e^
7*f^9 - 224*a^3*b^13*c^6*d^10*e^15*f - 224*a^3*b^13*c^15*d*e^6*f^10 + 112*a
^4*b^12*c^5*d^11*e^15*f + 112*a^4*b^12*c^15*d*e^5*f^11 + 112*a^5*b^11*c^4*d
^12*e^15*f + 112*a^5*b^11*c^15*d*e^4*f^12 - 224*a^6*b^10*c^3*d^13*e^15*f -
224*a^6*b^10*c^15*d*e^3*f^13 + 160*a^7*b^9*c^2*d^14*e^15*f + 160*a^7*b^9*c^

```
15*d*e^2*f^14 + 160*a^9*b^7*c*d^15*e^14*f^2 + 160*a^9*b^7*c^14*d^2*e*f^15 -
 224*a^10*b^6*c*d^15*e^13*f^3 - 224*a^10*b^6*c^13*d^3*e*f^15 + 112*a^11*b^5
*c*d^15*e^12*f^4 + 112*a^11*b^5*c^12*d^4*e*f^15 + 112*a^12*b^4*c*d^15*e^11*
f^5 + 112*a^12*b^4*c^11*d^5*e*f^15 - 224*a^13*b^3*c*d^15*e^10*f^6 - 224*a^1
3*b^3*c^10*d^6*e*f^15 + 160*a^14*b^2*c*d^15*e^9*f^7 + 160*a^14*b^2*c^9*d^7*
e*f^15 + 160*a^15*b*c^2*d^14*e^7*f^9 - 224*a^15*b*c^3*d^13*e^6*f^10 + 112*a
^15*b*c^4*d^12*e^5*f^11 + 112*a^15*b*c^5*d^11*e^4*f^12 - 224*a^15*b*c^6*d^1
0*e^3*f^13 + 160*a^15*b*c^7*d^9*e^2*f^14 - 300*a^2*b^14*c^8*d^8*e^14*f^2 +
840*a^2*b^14*c^10*d^6*e^12*f^4 - 1344*a^2*b^14*c^11*d^5*e^11*f^5 + 840*a^2*
b^14*c^12*d^4*e^10*f^6 - 300*a^2*b^14*c^14*d^2*e^8*f^8 + 1400*a^3*b^13*c^8*
d^8*e^13*f^3 - 2800*a^3*b^13*c^9*d^7*e^12*f^4 + 1568*a^3*b^13*c^10*d^6*e^11
*f^5 + 1568*a^3*b^13*c^11*d^5*e^10*f^6 - 2800*a^3*b^13*c^12*d^4*e^9*f^7 + 1
400*a^3*b^13*c^13*d^3*e^8*f^8 + 840*a^4*b^12*c^6*d^10*e^14*f^2 - 2800*a^4*b
^12*c^7*d^9*e^13*f^3 + 1750*a^4*b^12*c^8*d^8*e^12*f^4 + 4480*a^4*b^12*c^9*d
^7*e^11*f^5 - 8624*a^4*b^12*c^10*d^6*e^10*f^6 + 4480*a^4*b^12*c^11*d^5*e^9*
f^7 + 1750*a^4*b^12*c^12*d^4*e^8*f^8 - 2800*a^4*b^12*c^13*d^3*e^7*f^9 + 840
*a^4*b^12*c^14*d^2*e^6*f^10 - 1344*a^5*b^11*c^5*d^11*e^14*f^2 + 1568*a^5*b^
11*c^6*d^10*e^13*f^3 + 4480*a^5*b^11*c^7*d^9*e^12*f^4 - 12264*a^5*b^11*c^8*
d^8*e^11*f^5 + 7392*a^5*b^11*c^9*d^7*e^10*f^6 + 7392*a^5*b^11*c^10*d^6*e^9*
f^7 - 12264*a^5*b^11*c^11*d^5*e^8*f^8 + 4480*a^5*b^11*c^12*d^4*e^7*f^9 + 15
68*a^5*b^11*c^13*d^3*e^6*f^10 - 1344*a^5*b^11*c^14*d^2*e^5*f^11 + 840*a^6*b
^10*c^4*d^12*e^14*f^2 + 1568*a^6*b^10*c^5*d^11*e^13*f^3 - 8624*a^6*b^10*c^6
*d^10*e^12*f^4 + 7392*a^6*b^10*c^7*d^9*e^11*f^5 + 11396*a^6*b^10*c^8*d^8*e^
10*f^6 - 24640*a^6*b^10*c^9*d^7*e^9*f^7 + 11396*a^6*b^10*c^10*d^6*e^8*f^8 +
 7392*a^6*b^10*c^11*d^5*e^7*f^9 - 8624*a^6*b^10*c^12*d^4*e^6*f^10 + 1568*a^
6*b^10*c^13*d^3*e^5*f^11 + 840*a^6*b^10*c^14*d^2*e^4*f^12 - 2800*a^7*b^9*c^
4*d^12*e^13*f^3 + 4480*a^7*b^9*c^5*d^11*e^12*f^4 + 7392*a^7*b^9*c^6*d^10*e^
11*f^5 - 24640*a^7*b^9*c^7*d^9*e^10*f^6 + 15400*a^7*b^9*c^8*d^8*e^9*f^7 + 1
5400*a^7*b^9*c^9*d^7*e^8*f^8 - 24640*a^7*b^9*c^10*d^6*e^7*f^9 + 7392*a^7*b^
9*c^11*d^5*e^6*f^10 + 4480*a^7*b^9*c^12*d^4*e^5*f^11 - 2800*a^7*b^9*c^13*d^
3*e^4*f^12 - 300*a^8*b^8*c^2*d^14*e^14*f^2 + 1400*a^8*b^8*c^3*d^13*e^13*f^3
 + 1750*a^8*b^8*c^4*d^12*e^12*f^4 - 12264*a^8*b^8*c^5*d^11*e^11*f^5 + 11396
*a^8*b^8*c^6*d^10*e^10*f^6 + 15400*a^8*b^8*c^7*d^9*e^9*f^7 - 34650*a^8*b^8*
c^8*d^8*e^8*f^8 + 15400*a^8*b^8*c^9*d^7*e^7*f^9 + 11396*a^8*b^8*c^10*d^6*e^
6*f^10 - 12264*a^8*b^8*c^11*d^5*e^5*f^11 + 1750*a^8*b^8*c^12*d^4*e^4*f^12 +
 1400*a^8*b^8*c^13*d^3*e^3*f^13 - 300*a^8*b^8*c^14*d^2*e^2*f^14 - 2800*a^9*
b^7*c^3*d^13*e^12*f^4 + 4480*a^9*b^7*c^4*d^12*e^11*f^5 + 7392*a^9*b^7*c^5*d
^11*e^10*f^6 - 24640*a^9*b^7*c^6*d^10*e^9*f^7 + 15400*a^9*b^7*c^7*d^9*e^8*f
^8 + 15400*a^9*b^7*c^8*d^8*e^7*f^9 - 24640*a^9*b^7*c^9*d^7*e^6*f^10 + 7392*
a^9*b^7*c^10*d^6*e^5*f^11 + 4480*a^9*b^7*c^11*d^5*e^4*f^12 - 2800*a^9*b^7*c
^12*d^4*e^3*f^13 + 840*a^10*b^6*c^2*d^14*e^12*f^4 + 1568*a^10*b^6*c^3*d^13*
e^11*f^5 - 8624*a^10*b^6*c^4*d^12*e^10*f^6 + 7392*a^10*b^6*c^5*d^11*e^9*f^7
 + 11396*a^10*b^6*c^6*d^10*e^8*f^8 - 24640*a^10*b^6*c^7*d^9*e^7*f^9 + 11396
*a^10*b^6*c^8*d^8*e^6*f^10 + 7392*a^10*b^6*c^9*d^7*e^5*f^11 - 8624*a^10*b^6
*c^10*d^6*e^4*f^12 + 1568*a^10*b^6*c^11*d^5*e^3*f^13 + 840*a^10*b^6*c^12*d^
4*e^2*f^14 - 1344*a^11*b^5*c^2*d^14*e^11*f^5 + 1568*a^11*b^5*c^3*d^13*e^10*
f^6 + 4480*a^11*b^5*c^4*d^12*e^9*f^7 - 12264*a^11*b^5*c^5*d^11*e^8*f^8 + 73
92*a^11*b^5*c^6*d^10*e^7*f^9 + 7392*a^11*b^5*c^7*d^9*e^6*f^10 - 12264*a^11*
b^5*c^8*d^8*e^5*f^11 + 4480*a^11*b^5*c^9*d^7*e^4*f^12 + 1568*a^11*b^5*c^10*
d^6*e^3*f^13 - 1344*a^11*b^5*c^11*d^5*e^2*f^14 + 840*a^12*b^4*c^2*d^14*e^10
*f^6 - 2800*a^12*b^4*c^3*d^13*e^9*f^7 + 1750*a^12*b^4*c^4*d^12*e^8*f^8 + 44
80*a^12*b^4*c^5*d^11*e^7*f^9 - 8624*a^12*b^4*c^6*d^10*e^6*f^10 + 4480*a^12*
b^4*c^7*d^9*e^5*f^11 + 1750*a^12*b^4*c^8*d^8*e^4*f^12 - 2800*a^12*b^4*c^9*d
^7*e^3*f^13 + 840*a^12*b^4*c^10*d^6*e^2*f^14 + 1400*a^13*b^3*c^3*d^13*e^8*f
^8 - 2800*a^13*b^3*c^4*d^12*e^7*f^9 + 1568*a^13*b^3*c^5*d^11*e^6*f^10 + 156
8*a^13*b^3*c^6*d^10*e^5*f^11 - 2800*a^13*b^3*c^7*d^9*e^4*f^12 + 1400*a^13*b
^3*c^8*d^8*e^3*f^13 - 300*a^14*b^2*c^2*d^14*e^8*f^8 + 840*a^14*b^2*c^4*d^12
*e^6*f^10 - 1344*a^14*b^2*c^5*d^11*e^5*f^11 + 840*a^14*b^2*c^6*d^10*e^4*f^1
2 - 300*a^14*b^2*c^8*d^8*e^2*f^14))*root(756756*a^10*b^10*c^10*d^10*e^10*f^
```

10*z^3 + 573300*a^12*b^8*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e
^8*f^12*z^3 + 573300*a^11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*
d^8*e^9*f^11*z^3 + 573300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*
c^11*d^9*e^11*f^9*z^3 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^1
1*b^9*c^9*d^11*e^10*f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343
980*a^10*b^10*c^9*d^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^
3 - 343980*a^9*b^11*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*
f^13*z^3 + 326340*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d
^7*e^7*f^13*z^3 + 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*
c^13*d^7*e^10*f^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^
12*b^8*c^10*d^10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 26
7540*a^10*b^10*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z
^3 - 267540*a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^1
2*f^8*z^3 + 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^
8*e^6*f^14*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^
14*d^6*e^8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^
14*c^12*d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a
^13*b^7*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191
100*a^12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3
- 191100*a^11*b^9*c^7*d^13*e^12*f^8*z^3 - 191100*a^9*b^11*c^13*d^7*e^8*f^12
*z^3 - 191100*a^9*b^11*c^8*d^12*e^13*f^7*z^3 - 191100*a^8*b^12*c^13*d^7*e^9
*f^11*z^3 - 191100*a^8*b^12*c^9*d^11*e^13*f^7*z^3 - 191100*a^7*b^13*c^12*d^
8*e^11*f^9*z^3 - 191100*a^7*b^13*c^11*d^9*e^12*f^8*z^3 - 123900*a^14*b^6*c^
9*d^11*e^7*f^13*z^3 - 123900*a^14*b^6*c^7*d^13*e^9*f^11*z^3 - 123900*a^13*b
^7*c^11*d^9*e^6*f^14*z^3 - 123900*a^13*b^7*c^6*d^14*e^11*f^9*z^3 - 123900*a
^11*b^9*c^13*d^7*e^6*f^14*z^3 - 123900*a^11*b^9*c^6*d^14*e^13*f^7*z^3 - 123
900*a^9*b^11*c^14*d^6*e^7*f^13*z^3 - 123900*a^9*b^11*c^7*d^13*e^14*f^6*z^3
- 123900*a^7*b^13*c^14*d^6*e^9*f^11*z^3 - 123900*a^7*b^13*c^9*d^11*e^14*f^6
*z^3 - 123900*a^6*b^14*c^13*d^7*e^11*f^9*z^3 - 123900*a^6*b^14*c^11*d^9*e^1
3*f^7*z^3 + 101700*a^15*b^5*c^9*d^11*e^6*f^14*z^3 + 101700*a^15*b^5*c^6*d^1
4*e^9*f^11*z^3 + 101700*a^14*b^6*c^11*d^9*e^5*f^15*z^3 + 101700*a^14*b^6*c^
5*d^15*e^11*f^9*z^3 + 101700*a^11*b^9*c^14*d^6*e^5*f^15*z^3 + 101700*a^11*b
^9*c^5*d^15*e^14*f^6*z^3 + 101700*a^9*b^11*c^15*d^5*e^6*f^14*z^3 + 101700*a
^9*b^11*c^6*d^14*e^15*f^5*z^3 + 101700*a^6*b^14*c^15*d^5*e^9*f^11*z^3 + 101
700*a^6*b^14*c^9*d^11*e^15*f^5*z^3 + 101700*a^5*b^15*c^14*d^6*e^11*f^9*z^3
+ 101700*a^5*b^15*c^11*d^9*e^14*f^6*z^3 - 65820*a^14*b^6*c^10*d^10*e^6*f^14
*z^3 - 65820*a^14*b^6*c^6*d^14*e^10*f^10*z^3 - 65820*a^10*b^10*c^14*d^6*e^6
*f^14*z^3 - 65820*a^10*b^10*c^6*d^14*e^14*f^6*z^3 - 65820*a^6*b^14*c^14*d^6
*e^10*f^10*z^3 - 65820*a^6*b^14*c^10*d^10*e^14*f^6*z^3 + 56700*a^16*b^4*c^7
*d^13*e^7*f^13*z^3 - 56700*a^15*b^5*c^8*d^12*e^7*f^13*z^3 - 56700*a^15*b^5*
c^7*d^13*e^8*f^12*z^3 + 56700*a^13*b^7*c^13*d^7*e^4*f^16*z^3 - 56700*a^13*b
^7*c^12*d^8*e^5*f^15*z^3 - 56700*a^13*b^7*c^5*d^15*e^12*f^8*z^3 + 56700*a^1
3*b^7*c^4*d^16*e^13*f^7*z^3 - 56700*a^12*b^8*c^13*d^7*e^5*f^15*z^3 - 56700*
a^12*b^8*c^5*d^15*e^13*f^7*z^3 - 56700*a^8*b^12*c^15*d^5*e^7*f^13*z^3 - 567
00*a^8*b^12*c^7*d^13*e^15*f^5*z^3 + 56700*a^7*b^13*c^16*d^4*e^7*f^13*z^3 -
56700*a^7*b^13*c^15*d^5*e^8*f^12*z^3 - 56700*a^7*b^13*c^8*d^12*e^15*f^5*z^3
+ 56700*a^7*b^13*c^7*d^13*e^16*f^4*z^3 - 56700*a^5*b^15*c^13*d^7*e^12*f^8*
z^3 - 56700*a^5*b^15*c^12*d^8*e^13*f^7*z^3 + 56700*a^4*b^16*c^13*d^7*e^13*f
^7*z^3 - 48252*a^15*b^5*c^10*d^10*e^5*f^15*z^3 - 48252*a^15*b^5*c^5*d^15*e^
10*f^10*z^3 - 48252*a^10*b^10*c^15*d^5*e^5*f^15*z^3 - 48252*a^10*b^10*c^5*d
^15*e^15*f^5*z^3 - 48252*a^5*b^15*c^15*d^5*e^10*f^10*z^3 - 48252*a^5*b^15*c
^10*d^10*e^15*f^5*z^3 - 32400*a^16*b^4*c^8*d^12*e^6*f^14*z^3 - 32400*a^16*b
^4*c^6*d^14*e^8*f^12*z^3 - 32400*a^14*b^6*c^12*d^8*e^4*f^16*z^3 - 32400*a^1
4*b^6*c^4*d^16*e^12*f^8*z^3 - 32400*a^12*b^8*c^14*d^6*e^4*f^16*z^3 - 32400*
a^12*b^8*c^4*d^16*e^14*f^6*z^3 - 32400*a^8*b^12*c^16*d^4*e^6*f^14*z^3 - 324
00*a^8*b^12*c^6*d^14*e^16*f^4*z^3 - 32400*a^6*b^14*c^16*d^4*e^8*f^12*z^3 -
32400*a^6*b^14*c^8*d^12*e^16*f^4*z^3 - 32400*a^4*b^16*c^14*d^6*e^12*f^8*z^3
- 32400*a^4*b^16*c^12*d^8*e^14*f^6*z^3 + 20565*a^16*b^4*c^10*d^10*e^4*f^16
*z^3 + 20565*a^16*b^4*c^4*d^16*e^10*f^10*z^3 + 20565*a^10*b^10*c^16*d^4*e^4

*f^16*z^3 + 20565*a^10*b^10*c^4*d^16*e^16*f^4*z^3 + 20565*a^4*b^16*c^16*d^4
*e^10*f^10*z^3 + 20565*a^4*b^16*c^10*d^10*e^16*f^4*z^3 + 15660*a^17*b^3*c^8
*d^12*e^5*f^15*z^3 + 15660*a^17*b^3*c^5*d^15*e^8*f^12*z^3 + 15660*a^15*b^5*
c^12*d^8*e^3*f^17*z^3 + 15660*a^15*b^5*c^3*d^17*e^12*f^8*z^3 + 15660*a^12*b
^8*c^15*d^5*e^3*f^17*z^3 + 15660*a^12*b^8*c^3*d^17*e^15*f^5*z^3 + 15660*a^8
*b^12*c^17*d^3*e^5*f^15*z^3 + 15660*a^8*b^12*c^5*d^15*e^17*f^3*z^3 + 15660*
a^5*b^15*c^17*d^3*e^8*f^12*z^3 + 15660*a^5*b^15*c^8*d^12*e^17*f^3*z^3 + 156
60*a^3*b^17*c^15*d^5*e^12*f^8*z^3 + 15660*a^3*b^17*c^12*d^8*e^15*f^5*z^3 -
9750*a^17*b^3*c^9*d^11*e^4*f^16*z^3 - 9750*a^17*b^3*c^4*d^16*e^9*f^11*z^3 -
9750*a^16*b^4*c^11*d^9*e^3*f^17*z^3 - 9750*a^16*b^4*c^3*d^17*e^11*f^9*z^3
- 9750*a^11*b^9*c^16*d^4*e^3*f^17*z^3 - 9750*a^11*b^9*c^3*d^17*e^16*f^4*z^3
- 9750*a^9*b^11*c^17*d^3*e^4*f^16*z^3 - 9750*a^9*b^11*c^4*d^16*e^17*f^3*z^
3 - 9750*a^4*b^16*c^17*d^3*e^9*f^11*z^3 - 9750*a^4*b^16*c^9*d^11*e^17*f^3*z
^3 - 9750*a^3*b^17*c^16*d^4*e^11*f^9*z^3 - 9750*a^3*b^17*c^11*d^9*e^16*f^4*
z^3 - 8100*a^17*b^3*c^7*d^13*e^6*f^14*z^3 - 8100*a^17*b^3*c^6*d^14*e^7*f^13
*z^3 - 8100*a^14*b^6*c^13*d^7*e^3*f^17*z^3 - 8100*a^14*b^6*c^3*d^17*e^13*f^
7*z^3 - 8100*a^13*b^7*c^14*d^6*e^3*f^17*z^3 - 8100*a^13*b^7*c^3*d^17*e^14*f
^6*z^3 - 8100*a^7*b^13*c^17*d^3*e^6*f^14*z^3 - 8100*a^7*b^13*c^6*d^14*e^17*
f^3*z^3 - 8100*a^6*b^14*c^17*d^3*e^7*f^13*z^3 - 8100*a^6*b^14*c^7*d^13*e^17
*f^3*z^3 - 8100*a^3*b^17*c^14*d^6*e^13*f^7*z^3 - 8100*a^3*b^17*c^13*d^7*e^1
4*f^6*z^3 - 7980*a^16*b^4*c^9*d^11*e^5*f^15*z^3 - 7980*a^16*b^4*c^5*d^15*e^
9*f^11*z^3 - 7980*a^15*b^5*c^11*d^9*e^4*f^16*z^3 - 7980*a^15*b^5*c^4*d^16*e
^11*f^9*z^3 - 7980*a^11*b^9*c^15*d^5*e^4*f^16*z^3 - 7980*a^11*b^9*c^4*d^16*
e^15*f^5*z^3 - 7980*a^9*b^11*c^16*d^4*e^5*f^15*z^3 - 7980*a^9*b^11*c^5*d^15
*e^16*f^4*z^3 - 7980*a^5*b^15*c^16*d^4*e^9*f^11*z^3 - 7980*a^5*b^15*c^9*d^1
1*e^16*f^4*z^3 - 7980*a^4*b^16*c^15*d^5*e^11*f^9*z^3 - 7980*a^4*b^16*c^11*d
^9*e^15*f^5*z^3 + 6300*a^18*b^2*c^6*d^14*e^6*f^14*z^3 + 6300*a^14*b^6*c^14*
d^6*e^2*f^18*z^3 + 6300*a^14*b^6*c^2*d^18*e^14*f^6*z^3 + 6300*a^6*b^14*c^18
*d^2*e^6*f^14*z^3 + 6300*a^6*b^14*c^6*d^14*e^18*f^2*z^3 + 6300*a^2*b^18*c^1
4*d^6*e^14*f^6*z^3 - 4260*a^18*b^2*c^7*d^13*e^5*f^15*z^3 - 4260*a^18*b^2*c^
5*d^15*e^7*f^13*z^3 - 4260*a^15*b^5*c^13*d^7*e^2*f^18*z^3 - 4260*a^15*b^5*c
^2*d^18*e^13*f^7*z^3 - 4260*a^13*b^7*c^15*d^5*e^2*f^18*z^3 - 4260*a^13*b^7*
c^2*d^18*e^15*f^5*z^3 - 4260*a^7*b^13*c^18*d^2*e^5*f^15*z^3 - 4260*a^7*b^13
*c^5*d^15*e^18*f^2*z^3 - 4260*a^5*b^15*c^18*d^2*e^7*f^13*z^3 - 4260*a^5*b^1
5*c^7*d^13*e^18*f^2*z^3 - 4260*a^2*b^18*c^15*d^5*e^13*f^7*z^3 - 4260*a^2*b^
18*c^13*d^7*e^15*f^5*z^3 + 1470*a^17*b^3*c^10*d^10*e^3*f^17*z^3 + 1470*a^17
*b^3*c^3*d^17*e^10*f^10*z^3 + 1470*a^10*b^10*c^17*d^3*e^3*f^17*z^3 + 1470*a
^10*b^10*c^3*d^17*e^17*f^3*z^3 + 1470*a^3*b^17*c^17*d^3*e^10*f^10*z^3 + 147
0*a^3*b^17*c^10*d^10*e^17*f^3*z^3 + 1350*a^18*b^2*c^9*d^11*e^3*f^17*z^3 + 1
350*a^18*b^2*c^3*d^17*e^9*f^11*z^3 + 1350*a^17*b^3*c^11*d^9*e^2*f^18*z^3 +
1350*a^17*b^3*c^2*d^18*e^11*f^9*z^3 + 1350*a^11*b^9*c^17*d^3*e^2*f^18*z^3 +
1350*a^11*b^9*c^2*d^18*e^17*f^3*z^3 + 1350*a^9*b^11*c^18*d^2*e^3*f^17*z^3
+ 1350*a^9*b^11*c^3*d^17*e^18*f^2*z^3 + 1350*a^3*b^17*c^18*d^2*e^9*f^11*z^3
+ 1350*a^3*b^17*c^9*d^11*e^18*f^2*z^3 + 1350*a^2*b^18*c^17*d^3*e^11*f^9*z^
3 + 1350*a^2*b^18*c^11*d^9*e^17*f^3*z^3 - 1070*a^18*b^2*c^10*d^10*e^2*f^18*
z^3 - 1070*a^18*b^2*c^2*d^18*e^10*f^10*z^3 - 1070*a^10*b^10*c^18*d^2*e^2*f^
18*z^3 - 1070*a^10*b^10*c^2*d^18*e^18*f^2*z^3 - 1070*a^2*b^18*c^18*d^2*e^10
*f^10*z^3 - 1070*a^2*b^18*c^10*d^10*e^18*f^2*z^3 + 525*a^18*b^2*c^8*d^12*e^
4*f^16*z^3 + 525*a^18*b^2*c^4*d^16*e^8*f^12*z^3 + 525*a^16*b^4*c^12*d^8*e^2
*f^18*z^3 + 525*a^16*b^4*c^2*d^18*e^12*f^8*z^3 + 525*a^12*b^8*c^16*d^4*e^2*
f^18*z^3 + 525*a^12*b^8*c^2*d^18*e^16*f^4*z^3 + 525*a^8*b^12*c^18*d^2*e^4*f
^16*z^3 + 525*a^8*b^12*c^4*d^16*e^18*f^2*z^3 + 525*a^4*b^16*c^18*d^2*e^8*f^
12*z^3 + 525*a^4*b^16*c^8*d^12*e^18*f^2*z^3 + 525*a^2*b^18*c^16*d^4*e^12*f^
8*z^3 + 525*a^2*b^18*c^12*d^8*e^16*f^4*z^3 + 900*a^19*b*c^7*d^13*e^4*f^16*z
^3 + 900*a^19*b*c^4*d^16*e^7*f^13*z^3 + 900*a^16*b^4*c^13*d^7*e*f^19*z^3 +
900*a^16*b^4*c*d^19*e^13*f^7*z^3 + 900*a^13*b^7*c^16*d^4*e*f^19*z^3 + 900*a
^13*b^7*c*d^19*e^16*f^4*z^3 + 900*a^7*b^13*c^19*d*e^4*f^16*z^3 + 900*a^7*b^
13*c^4*d^16*e^19*f*z^3 + 900*a^4*b^16*c^19*d*e^7*f^13*z^3 + 900*a^4*b^16*c^
7*d^13*e^19*f*z^3 + 900*a*b^19*c^16*d^4*e^13*f^7*z^3 + 900*a*b^19*c^13*d^7*

e^16*f^4*z^3 - 750*a^19*b*c^8*d^12*e^3*f^17*z^3 - 750*a^19*b*c^3*d^17*e^8*f
^12*z^3 - 750*a^17*b^3*c^12*d^8*e*f^19*z^3 - 750*a^17*b^3*c*d^19*e^12*f^8*z
^3 - 750*a^12*b^8*c^17*d^3*e*f^19*z^3 - 750*a^12*b^8*c*d^19*e^17*f^3*z^3 -
750*a^8*b^12*c^19*d*e^3*f^17*z^3 - 750*a^8*b^12*c^3*d^17*e^19*f*z^3 - 750*a
^3*b^17*c^19*d*e^8*f^12*z^3 - 750*a^3*b^17*c^8*d^12*e^19*f*z^3 - 750*a*b^19
*c^17*d^3*e^12*f^8*z^3 - 750*a*b^19*c^12*d^8*e^17*f^3*z^3 - 420*a^19*b*c^6*
d^14*e^5*f^15*z^3 - 420*a^19*b*c^5*d^15*e^6*f^14*z^3 - 420*a^15*b^5*c^14*d^
6*e*f^19*z^3 - 420*a^15*b^5*c*d^19*e^14*f^6*z^3 - 420*a^14*b^6*c^15*d^5*e*f
^19*z^3 - 420*a^14*b^6*c*d^19*e^15*f^5*z^3 - 420*a^6*b^14*c^19*d*e^5*f^15*z
^3 - 420*a^6*b^14*c^5*d^15*e^19*f*z^3 - 420*a^5*b^15*c^19*d*e^6*f^14*z^3 -
420*a^5*b^15*c^6*d^14*e^19*f*z^3 - 420*a*b^19*c^15*d^5*e^14*f^6*z^3 - 420*a
*b^19*c^14*d^6*e^15*f^5*z^3 + 350*a^19*b*c^9*d^11*e^2*f^18*z^3 + 350*a^19*b
*c^2*d^18*e^9*f^11*z^3 + 350*a^18*b^2*c^11*d^9*e*f^19*z^3 + 350*a^18*b^2*c*
d^19*e^11*f^9*z^3 + 350*a^11*b^9*c^18*d^2*e*f^19*z^3 + 350*a^11*b^9*c*d^19*
e^18*f^2*z^3 + 350*a^9*b^11*c^19*d*e^2*f^18*z^3 + 350*a^9*b^11*c^2*d^18*e^1
9*f*z^3 + 350*a^2*b^18*c^19*d*e^9*f^11*z^3 + 350*a^2*b^18*c^9*d^11*e^19*f*z
^3 + 350*a*b^19*c^18*d^2*e^11*f^9*z^3 + 350*a*b^19*c^11*d^9*e^18*f^2*z^3 -
90*a^19*b*c^10*d^10*e*f^19*z^3 - 90*a^19*b*c*d^19*e^10*f^10*z^3 - 90*a^10*b
^10*c^19*d*e*f^19*z^3 - 90*a^10*b^10*c*d^19*e^19*f*z^3 - 90*a*b^19*c^19*d*e
^10*f^10*z^3 - 90*a*b^19*c^10*d^10*e^19*f*z^3 + 10*b^20*c^19*d*e^11*f^9*z^3
 + 10*b^20*c^11*d^9*e^19*f*z^3 + 10*a^20*c^9*d^11*e*f^19*z^3 + 10*a^20*c*d^
19*e^9*f^11*z^3 + 10*a^19*b*d^20*e^11*f^9*z^3 + 10*a^11*b^9*d^20*e^19*f*z^3
 + 10*a^9*b^11*c^20*e*f^19*z^3 + 10*a*b^19*c^20*e^9*f^11*z^3 + 10*a^19*b*c^
11*d^9*f^20*z^3 + 10*a^11*b^9*c^19*d*f^20*z^3 + 10*a^9*b^11*c*d^19*e^20*z^3
 + 10*a*b^19*c^9*d^11*e^20*z^3 + 252*b^20*c^15*d^5*e^15*f^5*z^3 - 210*b^20*
c^16*d^4*e^14*f^6*z^3 - 210*b^20*c^14*d^6*e^16*f^4*z^3 + 120*b^20*c^17*d^3*
e^13*f^7*z^3 + 120*b^20*c^13*d^7*e^17*f^3*z^3 - 45*b^20*c^18*d^2*e^12*f^8*z
^3 - 45*b^20*c^12*d^8*e^18*f^2*z^3 + 252*a^20*c^5*d^15*e^5*f^15*z^3 - 210*a
^20*c^6*d^14*e^4*f^16*z^3 - 210*a^20*c^4*d^16*e^6*f^14*z^3 + 120*a^20*c^7*d
^13*e^3*f^17*z^3 + 120*a^20*c^3*d^17*e^7*f^13*z^3 - 45*a^20*c^8*d^12*e^2*f^
18*z^3 - 45*a^20*c^2*d^18*e^8*f^12*z^3 + 252*a^15*b^5*d^20*e^15*f^5*z^3 - 2
10*a^16*b^4*d^20*e^14*f^6*z^3 - 210*a^14*b^6*d^20*e^16*f^4*z^3 + 120*a^17*b
^3*d^20*e^13*f^7*z^3 + 120*a^13*b^7*d^20*e^17*f^3*z^3 - 45*a^18*b^2*d^20*e^
12*f^8*z^3 - 45*a^12*b^8*d^20*e^18*f^2*z^3 + 252*a^5*b^15*c^20*e^5*f^15*z^3
 - 210*a^6*b^14*c^20*e^4*f^16*z^3 - 210*a^4*b^16*c^20*e^6*f^14*z^3 + 120*a^
7*b^13*c^20*e^3*f^17*z^3 + 120*a^3*b^17*c^20*e^7*f^13*z^3 - 45*a^8*b^12*c^2
0*e^2*f^18*z^3 - 45*a^2*b^18*c^20*e^8*f^12*z^3 + 252*a^15*b^5*c^15*d^5*f^20
*z^3 - 210*a^16*b^4*c^14*d^6*f^20*z^3 - 210*a^14*b^6*c^16*d^4*f^20*z^3 + 12
0*a^17*b^3*c^13*d^7*f^20*z^3 + 120*a^13*b^7*c^17*d^3*f^20*z^3 - 45*a^18*b^2
*c^12*d^8*f^20*z^3 - 45*a^12*b^8*c^18*d^2*f^20*z^3 + 252*a^5*b^15*c^5*d^15*
e^20*z^3 - 210*a^6*b^14*c^4*d^16*e^20*z^3 - 210*a^4*b^16*c^6*d^14*e^20*z^3
+ 120*a^7*b^13*c^3*d^17*e^20*z^3 + 120*a^3*b^17*c^7*d^13*e^20*z^3 - 45*a^8*
b^12*c^2*d^18*e^20*z^3 - 45*a^2*b^18*c^8*d^12*e^20*z^3 - b^20*c^20*e^10*f^1
0*z^3 - a^20*d^20*e^10*f^10*z^3 - b^20*c^10*d^10*e^20*z^3 - a^20*c^10*d^10*
f^20*z^3 - a^10*b^10*d^20*e^20*z^3 - a^10*b^10*c^20*f^20*z^3 + 1890*a^12*b^
2*c*d^13*e*f^13*z + 1890*a*b^13*c^12*d^2*e*f^13*z + 1890*a*b^13*c*d^13*e^12
*f^2*z + 92610*a^6*b^8*c^4*d^10*e^4*f^10*z + 92610*a^4*b^10*c^6*d^8*e^4*f^1
0*z + 92610*a^4*b^10*c^4*d^10*e^6*f^8*z + 66150*a^8*b^6*c^3*d^11*e^3*f^11*z
 - 66150*a^7*b^7*c^4*d^10*e^3*f^11*z - 66150*a^7*b^7*c^3*d^11*e^4*f^10*z -
66150*a^4*b^10*c^7*d^7*e^3*f^11*z - 66150*a^4*b^10*c^3*d^11*e^7*f^7*z + 661
50*a^3*b^11*c^8*d^6*e^3*f^11*z - 66150*a^3*b^11*c^7*d^7*e^4*f^10*z - 66150*
a^3*b^11*c^4*d^10*e^7*f^7*z + 66150*a^3*b^11*c^3*d^11*e^8*f^6*z - 55566*a^5
*b^9*c^5*d^9*e^4*f^10*z - 55566*a^5*b^9*c^4*d^10*e^5*f^9*z - 55566*a^4*b^10
*c^5*d^9*e^5*f^9*z - 32130*a^9*b^5*c^3*d^11*e^2*f^12*z - 32130*a^9*b^5*c^2*
d^12*e^3*f^11*z - 32130*a^3*b^11*c^9*d^5*e^2*f^12*z - 32130*a^3*b^11*c^2*d^
12*e^9*f^5*z - 32130*a^2*b^12*c^9*d^5*e^3*f^11*z - 32130*a^2*b^12*c^3*d^11*
e^9*f^5*z + 22680*a^8*b^6*c^4*d^10*e^2*f^12*z + 22680*a^8*b^6*c^2*d^12*e^4*
f^10*z + 22680*a^4*b^10*c^8*d^6*e^2*f^12*z + 22680*a^4*b^10*c^2*d^12*e^8*f^
6*z + 22680*a^2*b^12*c^8*d^6*e^4*f^10*z + 22680*a^2*b^12*c^4*d^10*e^8*f^6*z

```
 + 19278*a^10*b^4*c^2*d^12*e^2*f^12*z + 19278*a^2*b^12*c^10*d^4*e^2*f^12*z
+ 19278*a^2*b^12*c^2*d^12*e^10*f^4*z + 18522*a^6*b^8*c^5*d^9*e^3*f^11*z + 1
8522*a^6*b^8*c^3*d^11*e^5*f^9*z + 18522*a^5*b^9*c^6*d^8*e^3*f^11*z + 18522*
a^5*b^9*c^3*d^11*e^6*f^8*z + 18522*a^3*b^11*c^6*d^8*e^5*f^9*z + 18522*a^3*b
^11*c^5*d^9*e^6*f^8*z - 13230*a^6*b^8*c^6*d^8*e^2*f^12*z - 13230*a^6*b^8*c^
2*d^12*e^6*f^8*z - 13230*a^2*b^12*c^6*d^8*e^6*f^8*z + 3402*a^7*b^7*c^5*d^9*
e^2*f^12*z + 3402*a^7*b^7*c^2*d^12*e^5*f^9*z + 3402*a^5*b^9*c^7*d^7*e^2*f^1
2*z + 3402*a^5*b^9*c^2*d^12*e^7*f^7*z + 3402*a^2*b^12*c^7*d^7*e^5*f^9*z + 3
402*a^2*b^12*c^5*d^9*e^7*f^7*z + 7938*a^10*b^4*c^3*d^11*e*f^13*z + 7938*a^1
0*b^4*c*d^13*e^3*f^11*z + 7938*a^3*b^11*c^10*d^4*e*f^13*z + 7938*a^3*b^11*c
*d^13*e^10*f^4*z + 7938*a*b^13*c^10*d^4*e^3*f^11*z + 7938*a*b^13*c^3*d^11*e
^10*f^4*z - 5670*a^11*b^3*c^2*d^12*e*f^13*z - 5670*a^11*b^3*c*d^13*e^2*f^12
*z - 5670*a^2*b^12*c^11*d^3*e*f^13*z - 5670*a^2*b^12*c*d^13*e^11*f^3*z - 56
70*a*b^13*c^11*d^3*e^2*f^12*z - 5670*a*b^13*c^2*d^12*e^11*f^3*z - 3780*a^9*
b^5*c^4*d^10*e*f^13*z - 3780*a^9*b^5*c*d^13*e^4*f^10*z - 3780*a^4*b^10*c^9*
d^5*e*f^13*z - 3780*a^4*b^10*c*d^13*e^9*f^5*z - 3780*a*b^13*c^9*d^5*e^4*f^1
0*z - 3780*a*b^13*c^4*d^10*e^9*f^5*z - 2268*a^8*b^6*c^5*d^9*e*f^13*z - 2268
*a^8*b^6*c*d^13*e^5*f^9*z - 2268*a^5*b^9*c^8*d^6*e*f^13*z - 2268*a^5*b^9*c*
d^13*e^8*f^6*z - 2268*a*b^13*c^8*d^6*e^5*f^9*z - 2268*a*b^13*c^5*d^9*e^8*f^
6*z + 1890*a^7*b^7*c^6*d^8*e*f^13*z + 1890*a^7*b^7*c*d^13*e^6*f^8*z + 1890*
a^6*b^8*c^7*d^7*e*f^13*z + 1890*a^6*b^8*c*d^13*e^7*f^7*z + 1890*a*b^13*c^7*
d^7*e^6*f^8*z + 1890*a*b^13*c^6*d^8*e^7*f^7*z - 252*b^14*c^13*d*e*f^13*z -
252*b^14*c*d^13*e^13*f*z - 252*a^13*b*d^14*e*f^13*z - 252*a*b^13*d^14*e^13*
f*z - 252*a^13*b*c*d^13*f^14*z - 252*a*b^13*c^13*d*f^14*z - 918*b^14*c^7*d^
7*e^7*f^7*z - 882*b^14*c^11*d^3*e^3*f^11*z - 882*b^14*c^3*d^11*e^11*f^3*z +
 693*b^14*c^12*d^2*e^2*f^12*z + 693*b^14*c^2*d^12*e^12*f^2*z + 567*b^14*c^8
*d^6*e^6*f^8*z + 567*b^14*c^6*d^8*e^8*f^6*z + 441*b^14*c^10*d^4*e^4*f^10*z
+ 441*b^14*c^4*d^10*e^10*f^4*z - 126*b^14*c^9*d^5*e^5*f^9*z - 126*b^14*c^5*
d^9*e^9*f^5*z - 918*a^7*b^7*d^14*e^7*f^7*z - 882*a^11*b^3*d^14*e^3*f^11*z -
 882*a^3*b^11*d^14*e^11*f^3*z + 693*a^12*b^2*d^14*e^2*f^12*z + 693*a^2*b^12
*d^14*e^12*f^2*z + 567*a^8*b^6*d^14*e^6*f^8*z + 567*a^6*b^8*d^14*e^8*f^6*z
+ 441*a^10*b^4*d^14*e^4*f^10*z + 441*a^4*b^10*d^14*e^10*f^4*z - 126*a^9*b^5
*d^14*e^5*f^9*z - 126*a^5*b^9*d^14*e^9*f^5*z - 918*a^7*b^7*c^7*d^7*f^14*z -
 882*a^11*b^3*c^3*d^11*f^14*z - 882*a^3*b^11*c^11*d^3*f^14*z + 693*a^12*b^2
*c^2*d^12*f^14*z + 693*a^2*b^12*c^12*d^2*f^14*z + 567*a^8*b^6*c^6*d^8*f^14*
z + 567*a^6*b^8*c^8*d^6*f^14*z + 441*a^10*b^4*c^4*d^10*f^14*z + 441*a^4*b^1
0*c^10*d^4*f^14*z - 126*a^9*b^5*c^5*d^9*f^14*z - 126*a^5*b^9*c^9*d^5*f^14*z
 + 36*b^14*d^14*e^14*z + 36*b^14*c^14*f^14*z + 36*a^14*d^14*f^14*z - 27054*
a^2*b^9*c^2*d^9*e^2*f^9 + 9018*a^3*b^8*c^2*d^9*e*f^10 + 9018*a^3*b^8*c*d^10
*e^2*f^9 + 9018*a^2*b^9*c^3*d^8*e*f^10 + 9018*a^2*b^9*c*d^10*e^3*f^8 + 9018
*a*b^10*c^3*d^8*e^2*f^9 + 9018*a*b^10*c^2*d^9*e^3*f^8 - 9018*a^4*b^7*c*d^10
*e*f^10 - 9018*a*b^10*c^4*d^7*e*f^10 - 9018*a*b^10*c*d^10*e^4*f^7 + 2268*b^
11*c^5*d^6*e*f^10 + 2268*b^11*c*d^10*e^5*f^6 + 2268*a^5*b^6*d^11*e*f^10 + 2
268*a*b^10*d^11*e^5*f^6 + 2268*a^5*b^6*c*d^10*f^11 + 2268*a*b^10*c^5*d^6*f^
11 - 1458*b^11*c^3*d^8*e^3*f^8 - 1161*b^11*c^4*d^7*e^2*f^9 - 1161*b^11*c^2*
d^9*e^4*f^7 - 1458*a^3*b^8*d^11*e^3*f^8 - 1161*a^4*b^7*d^11*e^2*f^9 - 1161*
a^2*b^9*d^11*e^4*f^7 - 1458*a^3*b^8*c^3*d^8*f^11 - 1161*a^4*b^7*c^2*d^9*f^1
1 - 1161*a^2*b^9*c^4*d^7*f^11 - 756*b^11*d^11*e^6*f^5 - 756*b^11*c^6*d^5*f^
11 - 756*a^6*b^5*d^11*f^11, z, k), k, 1, 3) - ((7*a*b^6*c^2*d^5*e^7 - a^3*b
^4*d^7*e^7 - a^7*c^3*d^4*f^7 - b^7*c^3*d^4*e^7 - a^7*d^7*e^3*f^4 - b^7*c^7*
e^3*f^4 - 6*a^5*b^2*c^5*d^2*f^7 - 6*a^5*b^2*d^7*e^5*f^2 - 6*b^7*c^5*d^2*e^5
*f^2 - a^3*b^4*c^7*f^7 + 7*a^2*b^5*c*d^6*e^7 + 4*a^4*b^3*c^6*d*f^7 + 4*a^6*
b*c^4*d^3*f^7 + 7*a*b^6*c^7*e^2*f^5 + 7*a^2*b^5*c^7*e*f^6 + 4*a^4*b^3*d^7*e
^6*f + 4*a^6*b*d^7*e^4*f^3 + 7*a^7*c*d^6*e^2*f^5 + 7*a^7*c^2*d^5*e*f^6 + 4*
b^7*c^4*d^3*e^6*f + 4*b^7*c^6*d*e^4*f^3 - 21*a*b^6*c^3*d^4*e^6*f - 21*a*b^6
*c^6*d*e^3*f^4 - 21*a^3*b^4*c*d^6*e^6*f - 21*a^3*b^4*c^6*d*e*f^6 - 21*a^6*b
*c*d^6*e^3*f^4 - 21*a^6*b*c^3*d^4*e*f^6 + 14*a*b^6*c^4*d^3*e^5*f^2 + 14*a*b
^6*c^5*d^2*e^4*f^3 - 26*a^2*b^5*c^2*d^5*e^6*f - 26*a^2*b^5*c^6*d*e^2*f^5 +
14*a^4*b^3*c*d^6*e^5*f^2 + 14*a^4*b^3*c^5*d^2*e*f^6 + 14*a^5*b^2*c*d^6*e^4*
```

f^3 + 14*a^5*b^2*c^4*d^3*e*f^6 - 26*a^6*b*c^2*d^5*e^2*f^5 + 52*a^2*b^5*c^3*
d^4*e^5*f^2 - 78*a^2*b^5*c^4*d^3*e^4*f^3 + 52*a^2*b^5*c^5*d^2*e^3*f^4 + 52*
a^3*b^4*c^2*d^5*e^5*f^2 + 52*a^3*b^4*c^5*d^2*e^2*f^5 - 78*a^4*b^3*c^2*d^5*e
^4*f^3 - 78*a^4*b^3*c^4*d^3*e^2*f^5 + 52*a^5*b^2*c^2*d^5*e^3*f^4 + 52*a^5*b
^2*c^3*d^4*e^2*f^5)/(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4
*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*
d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6
*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*
a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8
*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3
+ 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c
^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4
*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c
^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^
6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3
*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*
a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 +
8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*
e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*
b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f
^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c
^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 -
48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^
4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^
6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (3*x^5*(2*a^5*b^2*d^
7*f^7 + 2*b^7*c^5*d^2*f^7 + 2*b^7*d^7*e^5*f^2 + 2*a^2*b^5*c^3*d^4*f^7 + 2*a
^3*b^4*c^2*d^5*f^7 + 2*a^2*b^5*d^7*e^3*f^4 + 2*a^3*b^4*d^7*e^2*f^5 + 2*b^7*
c^2*d^5*e^3*f^4 + 2*b^7*c^3*d^4*e^2*f^5 - 5*a*b^6*c^4*d^3*f^7 - 5*a^4*b^3*c
*d^6*f^7 - 5*a*b^6*d^7*e^4*f^3 - 5*a^4*b^3*d^7*e*f^6 - 5*b^7*c*d^6*e^4*f^3
- 5*b^7*c^4*d^3*e*f^6 + 16*a*b^6*c*d^6*e^3*f^4 + 16*a*b^6*c^3*d^4*e*f^6 + 1
6*a^3*b^4*c*d^6*e*f^6 - 12*a*b^6*c^2*d^5*e^2*f^5 - 12*a^2*b^5*c*d^6*e^2*f^5
- 12*a^2*b^5*c^2*d^5*e*f^6))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*
c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b
^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*
d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f
^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*
b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e
^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f +
4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*
a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*
a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8
*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f
+ 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f
^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^
3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c
^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 +
36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^
2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^
4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2
*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3
*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4
- 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (3*x^4*(8*a^6*
b*d^7*f^7 + 8*b^7*c^6*d*f^7 + 8*b^7*d^7*e^6*f - 7*a^2*b^5*c^4*d^3*f^7 + 14*
a^3*b^4*c^3*d^4*f^7 - 7*a^4*b^3*c^2*d^5*f^7 - 7*a^2*b^5*d^7*e^4*f^3 + 14*a^
3*b^4*d^7*e^3*f^4 - 7*a^4*b^3*d^7*e^2*f^5 - 7*b^7*c^2*d^5*e^4*f^3 + 14*b^7*
c^3*d^4*e^3*f^4 - 7*b^7*c^4*d^3*e^2*f^5 - 14*a*b^6*c^5*d^2*f^7 - 14*a^5*b^2
*c*d^6*f^7 - 14*a*b^6*d^7*e^5*f^2 - 14*a^5*b^2*d^7*e*f^6 - 14*b^7*c*d^6*e^5
*f^2 - 14*b^7*c^5*d^2*e*f^6 + 34*a*b^6*c*d^6*e^4*f^3 + 34*a*b^6*c^4*d^3*e*f
^6 + 34*a^4*b^3*c*d^6*e*f^6 + 6*a*b^6*c^2*d^5*e^3*f^4 + 6*a*b^6*c^3*d^4*e^2

```
*f^5 + 6*a^2*b^5*c*d^6*e^3*f^4 + 6*a^2*b^5*c^3*d^4*e*f^6 + 6*a^3*b^4*c*d^6*
e^2*f^5 + 6*a^3*b^4*c^2*d^5*e*f^6 - 78*a^2*b^5*c^2*d^5*e^2*f^5))/(2*(4*a*b^
7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d
^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^
8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 -
 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*
c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7
 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*
c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^
4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*
d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3
*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^
7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3
*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*
b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*
a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e
^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b
^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^
2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^
5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 3
6*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3
*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6
*b^2*c^4*d^4*e^2*f^6)) + (x^2*(18*a*b^6*c^7*f^7 + 18*a*b^6*d^7*e^7 + 18*a^7
*c*d^6*f^7 + 18*b^7*c*d^6*e^7 + 18*a^7*d^7*e*f^6 + 18*b^7*c^7*e*f^6 - 3*a^3
*b^4*c^5*d^2*f^7 + 32*a^4*b^3*c^4*d^3*f^7 - 3*a^5*b^2*c^3*d^4*f^7 - 3*a^3*b
^4*d^7*e^5*f^2 + 32*a^4*b^3*d^7*e^4*f^3 - 3*a^5*b^2*d^7*e^3*f^4 - 3*b^7*c^3
*d^4*e^5*f^2 + 32*b^7*c^4*d^3*e^4*f^3 - 3*b^7*c^5*d^2*e^3*f^4 - 37*a^2*b^5*
c^6*d*f^7 - 37*a^6*b*c^2*d^5*f^7 - 37*a^2*b^5*d^7*e^6*f - 37*a^6*b*d^7*e^2*
f^5 - 37*b^7*c^2*d^5*e^6*f - 37*b^7*c^6*d*e^2*f^5 + 9*a*b^6*c^2*d^5*e^5*f^2
 + a*b^6*c^3*d^4*e^4*f^3 + a*b^6*c^4*d^3*e^3*f^4 + 9*a*b^6*c^5*d^2*e^2*f^5
+ 9*a^2*b^5*c*d^6*e^5*f^2 + 9*a^2*b^5*c^5*d^2*e*f^6 + a^3*b^4*c*d^6*e^4*f^3
 + a^3*b^4*c^4*d^3*e*f^6 + a^4*b^3*c*d^6*e^3*f^4 + a^4*b^3*c^3*d^4*e*f^6 +
9*a^5*b^2*c*d^6*e^2*f^5 + 9*a^5*b^2*c^2*d^5*e*f^6 - 34*a*b^6*c*d^6*e^6*f -
34*a*b^6*c^6*d*e*f^6 - 34*a^6*b*c*d^6*e*f^6 + 234*a^2*b^5*c^2*d^5*e^4*f^3 -
 208*a^2*b^5*c^3*d^4*e^3*f^4 + 234*a^2*b^5*c^4*d^3*e^2*f^5 - 208*a^3*b^4*c^
2*d^5*e^3*f^4 - 208*a^3*b^4*c^3*d^4*e^2*f^5 + 234*a^4*b^3*c^2*d^5*e^2*f^5))
/(2*(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d^4*f^8 - b^8*c^4*d^4*
e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2
*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^
6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 +
 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^
5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f
^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8*c^7*d*e^5*f^3 - 12*a
*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b^4*c*d^7*e^7*f - 12*a
^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b*c^4*d^4*e*f^7 + 8*a*
b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8
*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a^3*b^5*c^7*d*e^2*f^6
+ 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f
^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^
2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^
6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3
 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2
*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4*c^4*d^4*e^4*f^4 + 36
*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*
e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*
b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f
^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6)) + (x*(2*a^2*b^5*c^7*f^7 + 2*a^2*b^5*d^7*e
^7 + 2*a^7*c^2*d^5*f^7 + 2*b^7*c^2*d^5*e^7 + 2*a^7*d^7*e^2*f^5 + 2*b^7*c^7*
e^2*f^5 + 4*a^4*b^3*c^5*d^2*f^7 + 4*a^5*b^2*c^4*d^3*f^7 + 4*a^4*b^3*d^7*e^5
```

```
*f^2 + 4*a^5*b^2*d^7*e^4*f^3 + 4*b^7*c^4*d^3*e^5*f^2 + 4*b^7*c^5*d^2*e^4*f^
3 + 14*a*b^6*c*d^6*e^7 + 14*a*b^6*c^7*e*f^6 + 14*a^7*c*d^6*e*f^6 - 6*a^3*b^
4*c^6*d*f^7 - 6*a^6*b*c^3*d^4*f^7 - 6*a^3*b^4*d^7*e^6*f - 6*a^6*b*d^7*e^3*f
^4 - 6*b^7*c^3*d^4*e^6*f - 6*b^7*c^6*d*e^3*f^4 - 33*a*b^6*c^2*d^5*e^6*f - 3
3*a*b^6*c^6*d*e^2*f^5 - 33*a^2*b^5*c*d^6*e^6*f - 33*a^2*b^5*c^6*d*e*f^6 - 3
3*a^6*b*c*d^6*e^2*f^5 - 33*a^6*b*c^2*d^5*e*f^6 + 17*a*b^6*c^3*d^4*e^5*f^2 -
 8*a*b^6*c^4*d^3*e^4*f^3 + 17*a*b^6*c^5*d^2*e^3*f^4 + 17*a^3*b^4*c*d^6*e^5*
f^2 + 17*a^3*b^4*c^5*d^2*e*f^6 - 8*a^4*b^3*c*d^6*e^4*f^3 - 8*a^4*b^3*c^4*d^
3*e*f^6 + 17*a^5*b^2*c*d^6*e^3*f^4 + 17*a^5*b^2*c^3*d^4*e*f^6 + 78*a^2*b^5*
c^2*d^5*e^5*f^2 - 26*a^2*b^5*c^3*d^4*e^4*f^3 - 26*a^2*b^5*c^4*d^3*e^3*f^4 +
 78*a^2*b^5*c^5*d^2*e^2*f^5 - 26*a^3*b^4*c^2*d^5*e^4*f^3 - 26*a^3*b^4*c^4*d
^3*e^2*f^5 - 26*a^4*b^3*c^2*d^5*e^3*f^4 - 26*a^4*b^3*c^3*d^4*e^2*f^5 + 78*a
^5*b^2*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 - a^8*c^4*d
^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*a^2*b^6*c^
2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6*b^2*d^8*e
^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*c^8*f^8 +
4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 + 4*a*b^7*c
^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*d^8*e^5*f^
3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7*f + 4*b^8
*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4 - 12*a^4*b
^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4 - 12*a^7*b
*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^3 + 8*a^2*
b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e^7*f + 8*a
^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^2*e*f^7 +
8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d^6*e^3*f^5
 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*b^6*c^5*d^
3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f^2 + 36*a^
3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c^6*d^2*e^3
*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 - 90*a^4*b^4
*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^2*e^2*f^6
- 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^5*b^3*c^4*
d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4*f^4 - 48*
a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6) + (x^3*(6*a^7*d^7*f^7
 + 6*b^7*c^7*f^7 + 6*b^7*d^7*e^7 - 37*a^2*b^5*c^5*d^2*f^7 + 19*a^3*b^4*c^4*
d^3*f^7 + 19*a^4*b^3*c^3*d^4*f^7 - 37*a^5*b^2*c^2*d^5*f^7 - 37*a^2*b^5*d^7*
e^5*f^2 + 19*a^3*b^4*d^7*e^4*f^3 + 19*a^4*b^3*d^7*e^3*f^4 - 37*a^5*b^2*d^7*
e^2*f^5 - 37*b^7*c^2*d^5*e^5*f^2 + 19*b^7*c^3*d^4*e^4*f^3 + 19*b^7*c^4*d^3*
e^3*f^4 - 37*b^7*c^5*d^2*e^2*f^5 + 3*a*b^6*c^6*d*f^7 + 3*a^6*b*c*d^6*f^7 +
3*a*b^6*d^7*e^6*f + 3*a^6*b*d^7*e*f^6 + 3*b^7*c*d^6*e^6*f + 3*b^7*c^6*d*e*f
^6 - 28*a*b^6*c*d^6*e^5*f^2 - 28*a*b^6*c^5*d^2*e*f^6 - 28*a^5*b^2*c*d^6*e*f
^6 + 86*a*b^6*c^2*d^5*e^4*f^3 - 68*a*b^6*c^3*d^4*e^3*f^4 + 86*a*b^6*c^4*d^3
*e^2*f^5 + 86*a^2*b^5*c*d^6*e^4*f^3 + 86*a^2*b^5*c^4*d^3*e*f^6 - 68*a^3*b^4
*c*d^6*e^3*f^4 - 68*a^3*b^4*c^3*d^4*e*f^6 + 86*a^4*b^3*c*d^6*e^2*f^5 + 86*a
^4*b^3*c^2*d^5*e*f^6 - 52*a^2*b^5*c^2*d^5*e^3*f^4 - 52*a^2*b^5*c^3*d^4*e^2*
f^5 - 52*a^3*b^4*c^2*d^5*e^2*f^5))/(4*a*b^7*c^3*d^5*e^8 - a^4*b^4*d^8*e^8 -
 a^8*c^4*d^4*f^8 - b^8*c^4*d^4*e^8 - a^8*d^8*e^4*f^4 - b^8*c^8*e^4*f^4 - 6*
a^2*b^6*c^2*d^6*e^8 - 6*a^6*b^2*c^6*d^2*f^8 - 6*a^2*b^6*c^8*e^2*f^6 - 6*a^6
*b^2*d^8*e^6*f^2 - 6*a^8*c^2*d^6*e^2*f^6 - 6*b^8*c^6*d^2*e^6*f^2 - a^4*b^4*
c^8*f^8 + 4*a^3*b^5*c*d^7*e^8 + 4*a^5*b^3*c^7*d*f^8 + 4*a^7*b*c^5*d^3*f^8 +
 4*a*b^7*c^8*e^3*f^5 + 4*a^3*b^5*c^8*e*f^7 + 4*a^5*b^3*d^8*e^7*f + 4*a^7*b*
d^8*e^5*f^3 + 4*a^8*c*d^7*e^3*f^5 + 4*a^8*c^3*d^5*e*f^7 + 4*b^8*c^5*d^3*e^7
*f + 4*b^8*c^7*d*e^5*f^3 - 12*a*b^7*c^4*d^4*e^7*f - 12*a*b^7*c^7*d*e^4*f^4
- 12*a^4*b^4*c*d^7*e^7*f - 12*a^4*b^4*c^7*d*e*f^7 - 12*a^7*b*c*d^7*e^4*f^4
- 12*a^7*b*c^4*d^4*e*f^7 + 8*a*b^7*c^5*d^3*e^6*f^2 + 8*a*b^7*c^6*d^2*e^5*f^
3 + 8*a^2*b^6*c^3*d^5*e^7*f + 8*a^2*b^6*c^7*d*e^3*f^5 + 8*a^3*b^5*c^2*d^6*e
^7*f + 8*a^3*b^5*c^7*d*e^2*f^6 + 8*a^5*b^3*c*d^7*e^6*f^2 + 8*a^5*b^3*c^6*d^
2*e*f^7 + 8*a^6*b^2*c*d^7*e^5*f^3 + 8*a^6*b^2*c^5*d^3*e*f^7 + 8*a^7*b*c^2*d
^6*e^3*f^5 + 8*a^7*b*c^3*d^5*e^2*f^6 + 22*a^2*b^6*c^4*d^4*e^6*f^2 - 48*a^2*
b^6*c^5*d^3*e^5*f^3 + 22*a^2*b^6*c^6*d^2*e^4*f^4 - 48*a^3*b^5*c^3*d^5*e^6*f
```

```
^2 + 36*a^3*b^5*c^4*d^4*e^5*f^3 + 36*a^3*b^5*c^5*d^3*e^4*f^4 - 48*a^3*b^5*c
^6*d^2*e^3*f^5 + 22*a^4*b^4*c^2*d^6*e^6*f^2 + 36*a^4*b^4*c^3*d^5*e^5*f^3 -
90*a^4*b^4*c^4*d^4*e^4*f^4 + 36*a^4*b^4*c^5*d^3*e^3*f^5 + 22*a^4*b^4*c^6*d^
2*e^2*f^6 - 48*a^5*b^3*c^2*d^6*e^5*f^3 + 36*a^5*b^3*c^3*d^5*e^4*f^4 + 36*a^
5*b^3*c^4*d^4*e^3*f^5 - 48*a^5*b^3*c^5*d^3*e^2*f^6 + 22*a^6*b^2*c^2*d^6*e^4
*f^4 - 48*a^6*b^2*c^3*d^5*e^3*f^5 + 22*a^6*b^2*c^4*d^4*e^2*f^6))/(x*(2*a*b*
c^2*e^2 + 2*a^2*c*d*e^2 + 2*a^2*c^2*e*f) + x^3*(2*a*b*c^2*f^2 + 2*a*b*d^2*e
^2 + 2*a^2*c*d*f^2 + 2*b^2*c*d*e^2 + 2*a^2*d^2*e*f + 2*b^2*c^2*e*f + 8*a*b*
c*d*e*f) + x^2*(a^2*c^2*f^2 + a^2*d^2*e^2 + b^2*c^2*e^2 + 4*a*b*c*d*e^2 + 4
*a*b*c^2*e*f + 4*a^2*c*d*e*f) + x^5*(2*a*b*d^2*f^2 + 2*b^2*c*d*f^2 + 2*b^2*
d^2*e*f) + x^4*(a^2*d^2*f^2 + b^2*c^2*f^2 + b^2*d^2*e^2 + 4*a*b*c*d*f^2 + 4
*a*b*d^2*e*f + 4*b^2*c*d*e*f) + a^2*c^2*e^2 + b^2*d^2*f^2*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x**2+b*d*f*x**
3)**3,x)
```

```
[Out] Timed out
```

3.21 $\int \frac{1}{1+x+x^2+x^3} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x+x^2+x^3} \, dx &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right) \, dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2} \, dx \\
&= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2} \, dx-\frac{1}{2}\int \frac{x}{1+x^2} \, dx \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x+x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^2 + x^3)^(-1), x]
```

fricas [A] time = 1.12, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

giac [A] time = 0.31, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2} + \frac{\ln\left(x+1\right)}{2} - \frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

maxima [A] time = 1.45, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

mupad [B] time = 2.20, size = 25, normalized size = 1.00

$$\frac{\ln\left(x+1\right)}{2} + \ln\left(x-\mathrm{i}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^2 + x^3 + 1),x)
```

```
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy **[A]** time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)
```

```
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.22 $\int \frac{1}{-1+4x-4x^2+16x^3} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
```

```
[Out] -ArcTan[2*x]/10 + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1+4x-4x^2+16x^3} \, dx &= \int \left(\frac{4}{5(-1+4x)}+\frac{-1-4x}{5\left(1+4x^2\right)}\right) dx \\
&= \frac{1}{5}\log(1-4x)+\frac{1}{5}\int \frac{-1-4x}{1+4x^2} \, dx \\
&= \frac{1}{5}\log(1-4x)-\frac{1}{5}\int \frac{1}{1+4x^2} \, dx-\frac{4}{5}\int \frac{x}{1+4x^2} \, dx \\
&= -\frac{1}{10}\tan^{-1}(2x)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\log\left(1+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{10}\log\left(4x^2+1\right)+\frac{1}{5}\log(1-4x)-\frac{1}{10}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
[Out] -1/10*ArcTan[2*x] + Log[1 - 4*x]/5 - Log[1 + 4*x^2]/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-1+4x-4x^2+16x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1),x]
[Out] IntegrateAlgebraic[(-1 + 4*x - 4*x^2 + 16*x^3)^(-1), x]
```

fricas [A] time = 1.25, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan\left(2\,x\right) - \frac{1}{10} \log\left(4\,x^{2} + 1\right) + \frac{1}{5} \log\left(4\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

giac [A] time = 0.35, size = 26, normalized size = 0.84

$$-\frac{1}{10} \arctan\left(2\,x\right) - \frac{1}{10} \log\left(4\,x^{2} + 1\right) + \frac{1}{5} \log\left({\left| 4\,x - 1 \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(abs(4*x - 1))
```

maple [A] time = 0.01, size = 26, normalized size = 0.84

$$-\frac{\arctan\left(2x\right)}{10}+\frac{\ln\left(4x-1\right)}{5}-\frac{\ln\left(4x^{2}+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(16*x^3-4*x^2+4*x-1),x)
[Out] -1/10*ln(4*x^2+1)-1/10*arctan(2*x)+1/5*ln(-1+4*x)
```

maxima [A] time = 1.28, size = 25, normalized size = 0.81

$$-\frac{1}{10} \arctan\left(2\,x\right) - \frac{1}{10} \log\left(4\,x^{2} + 1\right) + \frac{1}{5} \log\left(4\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
[Out] -1/10*arctan(2*x) - 1/10*log(4*x^2 + 1) + 1/5*log(4*x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(x-\frac{1}{4}\right)}{5}+\ln\left(x-\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}+\frac{1}{20}\mathrm{i}\right)+\ln\left(x+\frac{1}{2}\mathrm{i}\right)\,\left(-\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x - 4*x^2 + 16*x^3 - 1),x)
```

```
[Out] log(x - 1/4)/5 - log(x - 1i/2)*(1/10 - 1i/20) - log(x + 1i/2)*(1/10 + 1i/20
)
```

sympy [A] time = 0.15, size = 24, normalized size = 0.77

$$\frac{\log\left(x-\frac{1}{4}\right)}{5}-\frac{\log\left(x^2+\frac{1}{4}\right)}{10}-\frac{\operatorname{atan}(2x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(16*x**3-4*x**2+4*x-1),x)
```

```
[Out] log(x - 1/4)/5 - log(x**2 + 1/4)/10 - atan(2*x)/10
```

3.23 $\int \frac{1}{dx^3} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{2dx^2}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 30}

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(d*x^3),x]
[Out] -1/(2*d*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{dx^3} \, dx &= \frac{\int \frac{1}{x^3} \, dx}{d} \\ &= -\frac{1}{2dx^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{2dx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(d*x^3),x]
[Out] -1/2*1/(d*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{dx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(d*x^3),x]
[Out] IntegrateAlgebraic[1/(d*x^3), x]
```

fricas [A] time = 0.81, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d*x^2)
```

giac [A] time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="giac")
```

```
[Out] -1/2/(d*x^2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{1}{2d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/d/x^3,x)
```

```
[Out] -1/2/d/x^2
```

maxima [A] time = 0.63, size = 8, normalized size = 0.80

$$-\frac{1}{2\,dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/(d*x^2)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.80

$$-\frac{1}{2\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3),x)
```

```
[Out] -1/(2*d*x^2)
```

sympy [A] time = 0.06, size = 8, normalized size = 0.80

$$-\frac{1}{2dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/d/x**3,x)
```

```
[Out] -1/(2*d*x**2)
```

3.24 $\int \frac{1}{cx^2+dx^3} \, dx$

Optimal. Leaf size=28

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2 + d*x^3)^(-1),x]
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{cx^2+dx^3} \, dx &= \int \frac{1}{x^2(c+dx)} \, dx\\
&= \int \left(\frac{1}{cx^2}-\frac{d}{c^2x}+\frac{d^2}{c^2(c+dx)}\right) dx\\
&= -\frac{1}{cx}-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{d\log(x)}{c^2}+\frac{d\log(c+dx)}{c^2}-\frac{1}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2 + d*x^3)^(-1),x]
[Out] -(1/(c*x)) - (d*Log[x])/c^2 + (d*Log[c + d*x])/c^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{cx^2+dx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2 + d*x^3)^(-1),x]
[Out] IntegrateAlgebraic[(c*x^2 + d*x^3)^(-1), x]
```

fricas [A] time = 0.57, size = 26, normalized size = 0.93

$$\frac{dx\log\left(dx+c\right)-dx\log(x)-c}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (d*x*log(d*x + c) - d*x*log(x) - c)/(c^2*x)
```

giac [A] time = 0.36, size = 30, normalized size = 1.07

$$\frac{d\log\left(|dx+c|\right)}{c^2}-\frac{d\log\left(|x|\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="giac")
[Out] d*log(abs(d*x + c))/c^2 - d*log(abs(x))/c^2 - 1/(c*x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{d\ln(x)}{c^2}+\frac{d\ln\left(dx+c\right)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2),x)
[Out] -1/c/x-d*ln(x)/c^2+d*ln(d*x+c)/c^2
```

maxima [A] time = 0.59, size = 28, normalized size = 1.00

$$\frac{d\log\left(dx+c\right)}{c^2}-\frac{d\log(x)}{c^2}-\frac{1}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2),x, algorithm="maxima")
[Out] d*log(d*x + c)/c^2 - d*log(x)/c^2 - 1/(c*x)
```

mupad [B] time = 0.06, size = 25, normalized size = 0.89

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{2\,d\,x}{c}+1\right)}{c^2}-\frac{1}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2 + d*x^3),x)
[Out] (2*d*atanh((2*d*x)/c + 1))/c^2 - 1/(c*x)
```

sympy [A] time = 0.18, size = 19, normalized size = 0.68

$$-\frac{1}{cx}+\frac{d\left(-\log\left(x\right)+\log\left(\frac{c}{d}+x\right)\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2),x)
[Out] -1/(c*x) + d*(-log(x) + log(c/d + x))/c**2
```

3.25 $\int \frac{1}{bx+dx^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{bx+dx^3}\,dx &= \int \frac{1}{x\left(b+dx^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(b+dx)}\,dx,x,x^2\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2b}-\frac{d\operatorname{Subst}\left(\int \frac{1}{b+dx}\,dx,x,x^2\right)}{2b}\\ &= \frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{\log(x)}{b}-\frac{\log\left(b+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + d*x^3)^(-1),x]
```

```
[Out] Log[x]/b - Log[b + d*x^2]/(2*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{bx+dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + d*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b*x + d*x^3)^(-1), x]
```

fricas [A] time = 0.87, size = 18, normalized size = 0.82

$$-\frac{\log\left(dx^2+b\right)-2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(d*x^2 + b) - 2*log(x))/b
```

giac [A] time = 0.30, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,b}-\frac{\log\left(\left|dx^2+b\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/b - 1/2*log(abs(d*x^2 + b))/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{b}-\frac{\ln\left(d\,x^2+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+b*x),x)
```

```
[Out] ln(x)/b-1/2*ln(d*x^2+b)/b
```

maxima [A] time = 0.79, size = 20, normalized size = 0.91

$$-\frac{\log\left(dx^2+b\right)}{2\,b}+\frac{\log(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+b*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(d*x^2 + b)/b + log(x)/b
```

mupad [B] time = 2.13, size = 18, normalized size = 0.82

$$-\frac{\ln\left(d\,x^2+b\right)-2\,\ln(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + d*x^3),x)
```

```
[Out] -(log(b + d*x^2) - 2*log(x))/(2*b)
```

sympy [A] time = 0.20, size = 15, normalized size = 0.68

$$\frac{\log{(x)}}{b}-\frac{\log{\left(\frac{b}{d}+x^{2}\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+b*x),x)
```

```
[Out] log(x)/b - log(b/d + x**2)/(2*b)
```

3.26 $\int \frac{1}{bx+cx^2+dx^3} \, dx$

Optimal. Leaf size=62

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log\left(b+cx+dx^2\right)}{2b} + \frac{\log(x)}{b}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1594, 705, 29, 634, 618, 206, 628}

$$\frac{c \tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}} - \frac{\log\left(b+cx+dx^2\right)}{2b} + \frac{\log(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] (c*ArcTanh[(c + 2*d*x)/Sqrt[c^2 - 4*b*d]])/(b*Sqrt[c^2 - 4*b*d]) + Log[x]/b
 - Log[b + c*x + d*x^2]/(2*b)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{bx+cx^2+dx^3}\,dx &= \int \frac{1}{x\left(b+cx+dx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{b}+\frac{\int \frac{-c-dx}{b+cx+dx^2}\,dx}{b}\\
&= \frac{\log(x)}{b}-\frac{\int \frac{c+2dx}{b+cx+dx^2}\,dx}{2b}-\frac{c\int \frac{1}{b+cx+dx^2}\,dx}{2b}\\
&= \frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}+\frac{c\,\text{Subst}\left(\int \frac{1}{c^2-4bd-x^2}\,dx,x,c+2dx\right)}{b}\\
&= \frac{c\tanh^{-1}\left(\frac{c+2dx}{\sqrt{c^2-4bd}}\right)}{b\sqrt{c^2-4bd}}+\frac{\log(x)}{b}-\frac{\log\left(b+cx+dx^2\right)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 61, normalized size = 0.98

$$
-\frac{\frac{2c\tan^{-1}\left(\frac{c+2dx}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}}+\log(b+x(c+dx))-2\log(x)}{2b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] -1/2*((2*c*ArcTan[(c + 2*d*x)/Sqrt[-c^2 + 4*b*d]])/Sqrt[-c^2 + 4*b*d] - 2*L
og[x] + Log[b + x*(c + d*x)])/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{bx+cx^2+dx^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2 + d*x^3)^(-1), x]
```

fricas [A] time = 1.13, size = 211, normalized size = 3.40

$$
\left[\frac{\sqrt{c^2-4\,bd}\,c\log\left(\frac{2\,d^2x^2+2\,cdx+c^2-2\,bd+\sqrt{c^2-4\,bd}\,(2\,dx+c)}{dx^2+cx+b}\right)-\left(c^2-4\,bd\right)\log\left(dx^2+cx+b\right)+2\left(c^2-4\,bd\right)\log(x)}{2\left(bc^2-4\,b^2d\right)},\ \frac{2\sqrt{-c^2+4\,bd}\,c\arctan\left(-\frac{\sqrt{-c^2+4\,bd}\,(2\,dx+c)}{c^2-4\,bd}\right)-\left(c^2-4\,bd\right)\log\left(dx^2+cx+b\right)+2\left(c^2-4\,bd\right)\log(x)}{2\left(bc^2-4\,b^2d\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c^2 - 4*b*d)*c*log((2*d^2*x^2 + 2*c*d*x + c^2 - 2*b*d + sqrt(c^2
 - 4*b*d)*(2*d*x + c))/(d*x^2 + c*x + b)) - (c^2 - 4*b*d)*log(d*x^2 + c*x +
 b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d), 1/2*(2*sqrt(-c^2 + 4*b*d)*
```

```
c*arctan(-sqrt(-c^2 + 4*b*d)*(2*d*x + c)/(c^2 - 4*b*d)) - (c^2 - 4*b*d)*log
(d*x^2 + c*x + b) + 2*(c^2 - 4*b*d)*log(x))/(b*c^2 - 4*b^2*d)]
```

giac [A]   time = 0.29, size = 62, normalized size = 1.00

$$-\frac{c\arctan\left(\frac{2\,dx+c}{\sqrt{-c^2+4\,bd}}\right)}{\sqrt{-c^2+4\,bd}\,b}-\frac{\log\left(dx^2+cx+b\right)}{2\,b}+\frac{\log\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="giac")
```

```
[Out] -c*arctan((2*d*x + c)/sqrt(-c^2 + 4*b*d))/(sqrt(-c^2 + 4*b*d)*b) - 1/2*log(
d*x^2 + c*x + b)/b + log(abs(x))/b
```

maple [A]   time = 0.01, size = 62, normalized size = 1.00

$$-\frac{c\arctan\left(\frac{2dx+c}{\sqrt{4bd-c^2}}\right)}{\sqrt{4bd-c^2}\;b}+\frac{\ln(x)}{b}-\frac{\ln\left(d\,x^2+cx+b\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+c*x^2+b*x),x)
```

```
[Out] -1/2*ln(d*x^2+c*x+b)/b-1/b*c/(4*b*d-c^2)^(1/2)*arctan((2*d*x+c)/(4*b*d-c^2)
^(1/2))+1/b*ln(x)
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b*d-c^2>0)', see `assume?` for mo
re details)Is 4*b*d-c^2 positive or negative?
```

mupad [B]   time = 0.47, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{b}-\ln\left(\left(x\,\left(6\,b\,d^2-2\,c^2\,d\right)-b\,c\,d\right)\,\left(\frac{1}{2\,b}-\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-c\,d-3\,d^2\,x\right)\,\left(\frac{1}{2\,b}-\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-\ln\left(\left(x\,\left(6\,b\,d^2-2\,c^2\,d\right)-b\,c\,d\right)\,\left(\frac{1}{2\,b}+\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)-c\,d-3\,d^2\,x\right)\,\left(\frac{1}{2\,b}+\frac{c\,\sqrt{c^2-4\,b\,d}}{2\,\left(b\,c^2-4\,b^2\,d\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + c*x^2 + d*x^3),x)
```

```
[Out] log(x)/b - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b) - (c*(c^2 - 4*b*d)^
(1/2))/(2*(b*c^2 - 4*b^2*d))) - log((x*(6*b*d^2 - 2*c^2*d) - b*c*d)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d))) - c*d - 3*d^2*x)*(1/(2*b
) + (c*(c^2 - 4*b*d)^(1/2))/(2*(b*c^2 - 4*b^2*d)))
```

sympy [B]   time = 4.19, size = 564, normalized size = 9.10

$$\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-12b^3d^2\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)+2b^2c^4\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2+3b^2c^2d\left(-\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)-12b^2d^2+11bc^2d-2c^4}{9bcd^2-2c^3d}\right)+\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)\log\left(x+\frac{24b^4d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-14b^3c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2-12b^3d^2\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)+2b^2c^4\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)^2+3b^2c^2d\left(\frac{c\sqrt{-4bd+c^2}}{2b\left(4bd-c^2\right)}-\frac{1}{2b}\right)-12b^2d^2+11bc^2d-2c^4}{9bcd^2-2c^3d}\right)+\frac{\log\left(x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+c*x**2+b*x),x)
```

```
[Out] (-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**
2*d*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d*
*2*(-c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(-
c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d - c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b
*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c**3*d)) + (c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b))*log(x + (24*b**4*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 14*b**3*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*
(4*b*d - c**2)) - 1/(2*b))**2 - 12*b**3*d**2*(c*sqrt(-4*b*d + c**2)/(2*b*(4
*b*d - c**2)) - 1/(2*b)) + 2*b**2*c**4*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
 c**2)) - 1/(2*b))**2 + 3*b**2*c**2*d*(c*sqrt(-4*b*d + c**2)/(2*b*(4*b*d -
c**2)) - 1/(2*b)) - 12*b**2*d**2 + 11*b*c**2*d - 2*c**4)/(9*b*c*d**2 - 2*c*
*3*d)) + log(x)/b
```

3.27 $\int \frac{1}{a+dx^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}$$

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + d*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3)*d^(1/3
))) + Log[a^(1/3) + d^(1/3)*x]/(3*a^(2/3)*d^(1/3)) - Log[a^(2/3) - a^(1/3)*
d^(1/3)*x + d^(2/3)*x^2]/(6*a^(2/3)*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+dx^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{d}\,x}\,dx}{3a^{2/3}} + \frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{d}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3a^{2/3}} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} + \frac{\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{2\sqrt[3]{a}} - \frac{\int \frac{-\sqrt[3]{a}\,\sqrt[3]{d}+2d^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{6a^{2/3}\sqrt[3]{d}} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{d}} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{d}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)}{3a^{2/3}\sqrt[3]{d}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{d}}
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.77

$$
-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{d}\,x\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}\sqrt[3]{d}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + d*x^3)^(-1),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + d^(1/3)*x] + Log[a^(2/3) - a^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(a^(2/3)*d^
(1/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{a+dx^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + d*x^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + d*x^3)^(-1), x]
```

fricas [A] time = 0.54, size = 299, normalized size = 2.60

$$
\left[\frac{3\sqrt{\frac{1}{3}}\,ad\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,adx^3-3\left(a^2d\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,adx^2+\left(a^2d\right)^{\frac{2}{3}}x-\left(a^2d\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+a}\right)-\left(a^2d\right)^{\frac{2}{3}}\log\left(adx^2-\left(a^2d\right)^{\frac{2}{3}}x+\left(a^2d\right)^{\frac{1}{3}}a\right)+2\left(a^2d\right)^{\frac{2}{3}}\log\left(adx+\left(a^2d\right)^{\frac{2}{3}}\right)}{6\,a^2d},\ \frac{6\sqrt{\frac{1}{3}}\,ad\sqrt{\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2d\right)^{\frac{2}{3}}x-\left(a^2d\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2d\right)^{\frac{1}{3}}}{d}}}{a^2}\right)-\left(a^2d\right)^{\frac{2}{3}}\log\left(adx^2-\left(a^2d\right)^{\frac{2}{3}}x+\left(a^2d\right)^{\frac{1}{3}}a\right)+2\left(a^2d\right)^{\frac{2}{3}}\log\left(adx+\left(a^2d\right)^{\frac{2}{3}}\right)}{6\,a^2d}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*d*sqrt(-(a^2*d)^(1/3)/d)*log((2*a*d*x^3 - 3*(a^2*d)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*d*x^2 + (a^2*d)^(2/3)*x - (a^2*d)^(1/3)*a)*
sqrt(-(a^2*d)^(1/3)/d))/(d*x^3 + a)) - (a^2*d)^(2/3)*log(a*d*x^2 - (a^2*d)^
```

```
(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x + (a^2*d)^(2/3)))/(a
^2*d), 1/6*(6*sqrt(1/3)*a*d*sqrt((a^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(a^2*
d)^(2/3)*x - (a^2*d)^(1/3)*a)*sqrt((a^2*d)^(1/3)/d)/a^2) - (a^2*d)^(2/3)*lo
g(a*d*x^2 - (a^2*d)^(2/3)*x + (a^2*d)^(1/3)*a) + 2*(a^2*d)^(2/3)*log(a*d*x
+ (a^2*d)^(2/3)))/(a^2*d)]
```

giac [A] time = 0.26, size = 112, normalized size = 0.97

$$-\frac{\left(-\frac{a}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\,\left(-ad^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,ad}+\frac{\left(-ad^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{d}\right)^{\frac{1}{3}}+\left(-\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(-a/d)^(1/3)*log(abs(x - (-a/d)^(1/3)))/a + 1/3*sqrt(3)*(-a*d^2)^(1/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/d)^(1/3))/(-a/d)^(1/3))/(a*d) + 1/6*(-a*d^2)
^(1/3)*log(x^2 + x*(-a/d)^(1/3) + (-a/d)^(2/3))/(a*d)
```

maple [A] time = 0.01, size = 91, normalized size = 0.79

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}}d}+\frac{\ln\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{2}{3}}d}-\frac{\ln\left(x^2-\left(\frac{a}{d}\right)^{\frac{1}{3}}x+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{d}\right)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+a),x)
```

```
[Out] 1/3/d/(a/d)^(2/3)*ln(x+(a/d)^(1/3))-1/6/d/(a/d)^(2/3)*ln(x^2-(a/d)^(1/3)*x+
(a/d)^(2/3))+1/3/d/(a/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/d)^(1/3)*x-
1))
```

maxima [A] time = 1.28, size = 98, normalized size = 0.85

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{d}\right)^{\frac{1}{3}}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{d}\right)^{\frac{1}{3}}+\left(\frac{a}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{a}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/d)^(1/3))/(a/d)^(1/3))/(d*(a/d)^(2
/3)) - 1/6*log(x^2 - x*(a/d)^(1/3) + (a/d)^(2/3))/(d*(a/d)^(2/3)) + 1/3*log
(x + (a/d)^(1/3))/(d*(a/d)^(2/3))
```

mupad [B] time = 0.23, size = 99, normalized size = 0.86

$$\frac{\ln\left(d^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,d^{1/3}}+\frac{\ln\left(3\,d^2\,x+\frac{3\,a^{1/3}\,d^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}-\frac{\ln\left(3\,d^2\,x-\frac{3\,a^{1/3}\,d^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,d^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + d*x^3),x)
```

```
[Out] log(d^(1/3)*x + a^(1/3))/(3*a^(2/3)*d^(1/3)) + (log(3*d^2*x + (3*a^(1/3)*d^
(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*d^(1/3)) - (log(3*d
^2*x - (3*a^(1/3)*d^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)
*d^(1/3))
```

sympy [A] time = 0.16, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3a^2d-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*d - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

3.28 $\int \left(dx^3\right)^n \, dx$

Optimal. Leaf size=16

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {15, 30}

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x^3)^n,x]
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \left(dx^3\right)^n \, dx &= \left(x^{-3n}\left(dx^3\right)^n\right)\int x^{3n}\, dx\\ &= \frac{x\left(dx^3\right)^n}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x\left(dx^3\right)^n}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x^3)^n,x]
[Out] (x*(d*x^3)^n)/(1 + 3*n)
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(dx^3\right)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x^3)^n,x]
[Out] Defer[IntegrateAlgebraic][(d*x^3)^n, x]
```

fricas [A] time = 1.17, size = 16, normalized size = 1.00

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

giac [A] time = 0.31, size = 16, normalized size = 1.00

$$\frac{\left(dx^3\right)^n x}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="giac")
```

```
[Out] (d*x^3)^n*x/(3*n + 1)
```

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x\left(d\,x^3\right)^n}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
```

```
[Out] x*(d*x^3)^n/(1+3*n)
```

maxima [A] time = 0.57, size = 17, normalized size = 1.06

$$\frac{d^n x x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3)^n,x, algorithm="maxima")
```

```
[Out] d^n*x*x^(3*n)/(3*n + 1)
```

mupad [B] time = 2.50, size = 16, normalized size = 1.00

$$\frac{x\left(d\,x^3\right)^n}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3)^n,x)
```

```
[Out] (x*(d*x^3)^n)/(3*n + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{d^n x\left(x^3\right)^n}{3n+1} & \text{for}\ n \neq -\frac{1}{3} \\ \int \frac{1}{\sqrt[3]{dx^3}}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3)**n,x)
```

```
[Out] Piecewise((d**n*x*(x**3)**n/(3*n + 1), Ne(n, -1/3)), (Integral((d*x**3)**(-
1/3), x), True))
```

3.29 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$

Optimal. Leaf size=270

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}+\frac{4c^3\left(\right.}{}$$

Rubi [A] time = 0.54, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1106, 1090}

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}-\frac{8c^4\left(4ad^2+c^3\right)\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^7}{7d^2}+\frac{4c^3\left(4ad^2+c^3\right)^2\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^5\left(4ad^2+c^3\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}+\frac{c^4x\left(4ad^2+c^3\right)^4}{d^8}-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] (c^4*(c^3 + 4*a*d^2)^4*x)/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)^3)/(3*d^
6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^4) - (8*c
^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*c^2*(35*c^6
 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c^3 + 12*a*d^
2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)/13 - (8*c^2*
d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx &= \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^4\,dx,x,\frac{c}{d}+x\right)\\ &= \text{Subst}\left(\int \left(\frac{\left(c^4+4acd^2\right)^4}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3x^2}{d^6}+\frac{24c^6\left(c^3+4ad^2\right)^2\left(\frac{7}{6}+\right.}{d^4}\right.\right.\\ &= \frac{c^4\left(c^3+4ad^2\right)^4x}{d^8}-\frac{8c^5\left(c^3+4ad^2\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}+\frac{4c^3\left(c^3+4ad^2\right)^2\left(7c^3+4\right.}{5d^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 285, normalized size = 1.06

$$256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4+512a^2c^5dx^6+\frac{256}{5}a^2c^3x^5\left(ad^2+6c^3\right)+\frac{32}{9}c^2x^9\left(3a^2d^4+120ac^3d^2+8c^6\right)+\frac{64}{11}c^3d^2x^{11}\left(15ad^2+28c^3\right)+96ac^3dx^8\left(ad^2+4c^3\right)+\frac{16}{13}cd^4x^{13}\left(ad^2+70c^3\right)+\frac{256}{5}c^4dx^{10}\left(5ad^2+2c^3\right)+\frac{256}{7}ac^4x^7\left(9ad^2+4c^3\right)+\frac{16}{3}c^2d^3x^{12}\left(3ad^2+28c^3\right)+32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] 256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6*
c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^7)
/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3*a^
2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*c^3
+ 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*d^4*(
70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + c*d^7*
x^16 + (d^8*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4, x]
```

fricas [A] time = 0.76, size = 277, normalized size = 1.03

$$\frac{1}{17}x^{17}d^8 + x^{16}d^7c + \frac{112}{15}x^{15}d^6c^2 + 32x^{14}d^5c^3 + \frac{1120}{13}x^{13}d^4c^4 + \frac{16}{13}x^{13}d^6ca + \frac{448}{3}x^{12}d^3c^5 + 16x^{12}d^5c^2a + \frac{1792}{11}x^{11}d^2c^6 + \frac{960}{11}x^{11}d^4c^3a + \frac{512}{5}x^{10}dc^7 + 256x^{10}d^3c^4a + \frac{256}{9}x^9c^8 + \frac{1280}{3}x^9d^2c^5a + \frac{32}{3}x^9d^4c^2a^2 + 384x^8dc^6a + 96x^8d^3c^3a^2 + \frac{1024}{7}x^7c^7a + \frac{2304}{7}x^7d^2c^4a^2 + 512x^6dc^5a^2 + \frac{1536}{5}x^5c^6a^2 + \frac{256}{5}x^5d^2c^3a^3 + 256x^4dc^4a^3 + \frac{1024}{3}x^3c^5a^3 + 256xc^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^8 + x^16*d^7*c + 112/15*x^15*d^6*c^2 + 32*x^14*d^5*c^3 + 1120/1
3*x^13*d^4*c^4 + 16/13*x^13*d^6*c*a + 448/3*x^12*d^3*c^5 + 16*x^12*d^5*c^2*
a + 1792/11*x^11*d^2*c^6 + 960/11*x^11*d^4*c^3*a + 512/5*x^10*d*c^7 + 256*x
^10*d^3*c^4*a + 256/9*x^9*c^8 + 1280/3*x^9*d^2*c^5*a + 32/3*x^9*d^4*c^2*a^2
 + 384*x^8*d*c^6*a + 96*x^8*d^3*c^3*a^2 + 1024/7*x^7*c^7*a + 2304/7*x^7*d^2
*c^4*a^2 + 512*x^6*d*c^5*a^2 + 1536/5*x^5*c^6*a^2 + 256/5*x^5*d^2*c^3*a^3 +
 256*x^4*d*c^4*a^3 + 1024/3*x^3*c^5*a^3 + 256*x*c^4*a^4
```

giac [A] time = 0.24, size = 277, normalized size = 1.03

$$\frac{1}{17}d^8x^{17} + cd^7x^{16} + \frac{112}{15}c^2d^6x^{15} + 32c^3d^5x^{14} + \frac{1120}{13}c^4d^4x^{13} + \frac{16}{13}acd^6x^{13} + \frac{448}{3}c^5d^3x^{12} + 16ac^2d^5x^{12} + \frac{1792}{11}c^6d^2x^{11} + \frac{960}{11}ac^3d^4x^{11} + \frac{512}{5}c^7dx^{10} + 256ac^4d^3x^{10} + \frac{256}{9}c^8x^9 + \frac{1280}{3}ac^5d^2x^9 + \frac{32}{3}a^2c^2d^4x^9 + 384ac^6dx^8 + 96a^2c^3d^3x^8 + \frac{1024}{7}ac^7x^7 + \frac{2304}{7}a^2c^4d^2x^7 + 512a^2c^5dx^6 + \frac{1536}{5}a^2c^6x^5 + \frac{256}{5}a^3c^3d^2x^5 + 256a^3c^4dx^4 + \frac{1024}{3}a^3c^5x^3 + 256a^4c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/1
3*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x^1
2 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 256*a
*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4*x^9
 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^2*c^4
*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2*x^5 +
 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

maple [A] time = 0.00, size = 392, normalized size = 1.45

$$\frac{d^8x^{17}}{17} + cd^7x^{16} + \frac{112c^2d^6x^{15}}{15} + 32c^3d^5x^{14} + 512a^2c^5dx^6 + \frac{\left(1088c^4d^4 + 2\left(8acd^2 + 16c^4\right)d^4\right)x^{13}}{13} + 256a^3c^4dx^4 + \frac{\left(64ac^2d^5 + 1536c^5d^3 + 16\left(8acd^2 + 16c^4\right)cd^3\right)x^{12}}{12} + \frac{1024a^3c^5x^3}{3} + \frac{\left(576ac^3d^4 + 1024c^6d^2 + 48\left(8acd^2 + 16c^4\right)c^2d^2\right)x^{11}}{11} + \frac{\left(2048ac^4d^3 + 64\left(8acd^2 + 16c^4\right)c^3d\right)x^{10}}{10} + 256a^4c^4x + \frac{\left(32a^2c^2d^4 + 3584ac^5d^2 + \left(8acd^2 + 16c^4\right)^2\right)x^9}{9} + \frac{\left(256a^2c^3d^3 + 2048ac^6d + 64\left(8acd^2 + 16c^4\right)ac^2d\right)x^8}{8} + \frac{\left(1792a^2c^4d^2 + 64\left(8acd^2 + 16c^4\right)ac^3\right)x^7}{7} + \frac{\left(1024a^2c^6 + 32\left(8acd^2 + 16c^4\right)a^2c^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)
```

```
[Out] 1/17*d^8*x^17+c*d^7*x^16+112/15*c^2*d^6*x^15+32*c^3*d^5*x^14+1/13*(2*(8*a*c
*d^2+16*c^4)*d^4+1088*c^4*d^4)*x^13+1/12*(64*a*c^2*d^5+16*(8*a*c*d^2+16*c^4
)*c*d^3+1536*c^5*d^3)*x^12+1/11*(576*a*c^3*d^4+48*(8*a*c*d^2+16*c^4)*c^2*d^
2+1024*c^6*d^2)*x^11+1/10*(2048*a*c^4*d^3+64*(8*a*c*d^2+16*c^4)*c^3*d)*x^10
```

```
+1/9*(32*a^2*c^2*d^4+3584*a*c^5*d^2+(8*a*c*d^2+16*c^4)^2)*x^9+1/8*(256*a^2*
c^3*d^3+2048*a*c^6*d+64*a*c^2*d*(8*a*c*d^2+16*c^4))*x^8+1/7*(1792*a^2*c^4*d
^2+64*a*c^3*(8*a*c*d^2+16*c^4))*x^7+512*a^2*c^5*d*x^6+1/5*(32*a^2*c^2*(8*a*
c*d^2+16*c^4)+1024*a^2*c^6)*x^5+256*a^3*c^4*d*x^4+1024/3*a^3*c^5*x^3+256*a^
4*c^4*x
```

maxima [A] time = 0.63, size = 372, normalized size = 1.38

$$\frac{1}{17}d^8x^{17} + cd^7x^{16} + \frac{32}{5}c^2d^6x^{15} + \frac{128}{7}c^3d^5x^{14} + \frac{256}{13}c^4d^4x^{13} + \frac{256}{9}c^8x^9 + 256a^4c^4x + \frac{256}{15}\left(3d^2x^5 + 15cdx^4 + 20c^2x^3\right)a^3c^3 + \frac{256}{55}\left(5d^2x^{11} + 22cdx^{10}\right)c^6 + \frac{32}{105}\left(35d^4x^9 + 315cd^3x^8 + 720c^2d^2x^7 + 1008c^4x^5 + 120\left(3d^2x^7 + 14cdx^6\right)c^2\right)a^2c^2 + \frac{32}{143}\left(33d^4x^{13} + 286cd^3x^{12} + 624c^2d^2x^{11}\right)c^4 + \frac{16}{15015}\left(1155d^6x^{13} + 15015cd^5x^{12} + 65520c^2d^4x^{11} + 96096c^3d^3x^{10} + 137280c^6x^7 + 40040\left(2d^2x^9 + 9cdx^8\right)c^4 + 364\left(45d^4x^{11} + 396cd^3x^{10} + 880c^2d^2x^9\right)c^2\right)ac + \frac{16}{1365}\left(91d^6x^{15} + 1170cd^5x^{14} + 5040c^2d^4x^{13} + 7280c^3d^3x^{12}\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/1
3*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c*d
*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/105
*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*
x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12 + 624
*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 65520*c^2
*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 + 9*c*d*
x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2)*a*c +
16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*c^3*d^3*x
^12)*c^2
```

mupad [B] time = 2.30, size = 261, normalized size = 0.97

$$x^{10}\left(\frac{512\,c^7\,d}{5} + 256\,a\,c^4\,d^3\right) + x^{13}\left(\frac{1120\,c^4\,d^4}{13} + \frac{16\,a\,c\,d^6}{13}\right) + x^9\left(\frac{32\,a^2\,c^2\,d^4}{3} + \frac{1280\,a\,c^5\,d^2}{3} + \frac{256\,c^8}{9}\right) + x^{12}\left(\frac{448\,c^5\,d^3}{3} + 16\,a\,c^2\,d^5\right) + x^{11}\left(\frac{1792\,c^6\,d^2}{11} + \frac{960\,a\,c^3\,d^4}{11}\right) + \frac{d^8\,x^{17}}{17} + 256\,a^4\,c^4\,x + c\,d^7\,x^{16} + \frac{1024\,a^3\,c^5\,x^3}{3} + 32\,c^3\,d^5\,x^{14} + \frac{112\,c^2\,d^6\,x^{15}}{15} + 256\,a^3\,c^4\,d\,x^4 + 512\,a^2\,c^5\,d\,x^6 + \frac{256\,a\,c^4\,x^7\,\left(4\,c^3 + 9\,a\,d^2\right)}{7} + \frac{256\,a^2\,c^3\,x^5\,\left(6\,c^3 + a\,d^2\right)}{5} + 96\,a\,c^3\,d\,x^8\,\left(4\,c^3 + a\,d^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

```
[Out] x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d^
6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^12
*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3*d^
4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3)/3
+ 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^2*c^5
*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^2 + 6*
c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

sympy [A] time = 0.13, size = 299, normalized size = 1.11

$$256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4dx^4 + 512a^2c^5dx^6 + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + cd^7x^{16} + \frac{d^8x^{17}}{17} + x^{13}\left(\frac{16acd^6}{13} + \frac{1120c^4d^4}{13}\right) + x^{12}\left(16ac^2d^5 + \frac{448c^5d^3}{3}\right) + x^{11}\left(\frac{960ac^3d^4}{11} + \frac{1792c^6d^2}{11}\right) + x^{10}\left(256ac^4d^3 + \frac{512c^7d}{5}\right) + x^9\left(\frac{32a^2c^2d^4}{3} + \frac{1280ac^5d^2}{3} + \frac{256c^8}{9}\right) + x^8\left(96a^2c^3d^3 + 384ac^6d\right) + x^7\left(\frac{2304a^2c^4d^2}{7} + \frac{1024ac^7}{7}\right) + x^5\left(\frac{256a^3c^3d^2}{5} + \frac{1536a^2c^6}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

```
[Out] 256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*c
**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 + d
**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*c**
2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/11)
+ x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 + 1280
*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d) + x*
*7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**2/5 + 1
536*a**2*c**6/5)
```

3.30 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 dx$

Optimal. Leaf size=171

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{4}{5}$$

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$48a^2c^3dx^4+64a^2c^4x^3+64a^3c^3x+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+64ac^4dx^6+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 dx = \int \left(64a^3c^3+192a^2c^4x^2+192a^2c^3dx^3+48ac^2\left(4c^3+ad^2\right)x^4+384ac^4\right.$$

$$= 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+\frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6+$$

Mathematica [A] time = 0.03, size = 171, normalized size = 1.00

$$64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4+64ac^4dx^6+\frac{4}{3}cd^2x^9\left(ad^2+20c^3\right)+\frac{32}{7}c^3x^7\left(9ad^2+2c^3\right)+12c^2dx^8\left(ad^2+2c^3\right)+\frac{48}{5}ac^2x^5\left(ad^2+4c^3\right)+16c^3d^3x^{10}+\frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] 64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^2
)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(2*
c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 + (60
*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3, x]
```

fricas [A] time = 0.74, size = 166, normalized size = 0.97

$$\frac{1}{13}x^{13}d^6 + x^{12}d^5c + \frac{60}{11}x^{11}d^4c^2 + 16x^{10}d^3c^3 + \frac{80}{3}x^9d^2c^4 + \frac{4}{3}x^9d^4ca + 24x^8dc^5 + 12x^8d^3c^2a + \frac{64}{7}x^7c^6 + \frac{288}{7}x^7d^2c^3a + 64x^6dc^4a + \frac{192}{5}x^5c^5a + \frac{48}{5}x^5d^2c^2a^2 + 48x^4dc^3a^2 + 64x^3c^4a^2 + 64xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^6 + x^12*d^5*c + 60/11*x^11*d^4*c^2 + 16*x^10*d^3*c^3 + 80/3*x^
9*d^2*c^4 + 4/3*x^9*d^4*c*a + 24*x^8*d*c^5 + 12*x^8*d^3*c^2*a + 64/7*x^7*c^
6 + 288/7*x^7*d^2*c^3*a + 64*x^6*d*c^4*a + 192/5*x^5*c^5*a + 48/5*x^5*d^2*c
^2*a^2 + 48*x^4*d*c^3*a^2 + 64*x^3*c^4*a^2 + 64*x*c^3*a^3
```

giac [A] time = 0.24, size = 166, normalized size = 0.97

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{60}{11}c^2d^4x^{11} + 16c^3d^3x^{10} + \frac{80}{3}c^4d^2x^9 + \frac{4}{3}acd^4x^9 + 24c^5dx^8 + 12ac^2d^3x^8 + \frac{64}{7}c^6x^7 + \frac{288}{7}ac^3d^2x^7 + 64ac^4dx^6 + \frac{192}{5}ac^5x^5 + \frac{48}{5}a^2c^2d^2x^5 + 48a^2c^3dx^4 + 64a^2c^4x^3 + 64a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c^
4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*x^
7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^2*d
^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

maple [A] time = 0.00, size = 231, normalized size = 1.35

$$\frac{d^6x^{13}}{13} + cd^5x^{12} + \frac{60c^2d^4x^{11}}{11} + 16c^3d^3x^{10} + 64ac^4dx^6 + 48a^2c^3dx^4 + 64a^2c^4x^3 + \frac{\left(4acd^4 + 224c^4d^2 + \left(8acd^2 + 16c^4\right)d^2\right)x^9}{9} + \frac{\left(64ac^2d^3 + 128c^5d + 4\left(8acd^2 + 16c^4\right)cd\right)x^8}{8} + 64a^3c^3x + \frac{\left(256ac^3d^2 + 4\left(8acd^2 + 16c^4\right)c^2\right)x^7}{7} + \frac{\left(16a^2c^2d^2 + 128ac^5 + 4\left(8acd^2 + 16c^4\right)ac\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)
```

```
[Out] 1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+1/9*(4*a*c*d^4+
224*c^4*d^2+d^2*(8*a*c*d^2+16*c^4))*x^9+1/8*(64*a*c^2*d^3+128*c^5*d+4*c*d*(
8*a*c*d^2+16*c^4))*x^8+1/7*(256*a*c^3*d^2+4*c^2*(8*a*c*d^2+16*c^4))*x^7+64*
a*c^4*d*x^6+1/5*(4*a*c*(8*a*c*d^2+16*c^4)+128*c^5*a+16*d^2*a^2*c^2)*x^5+48*
a^2*c^3*d*x^4+64*a^2*c^4*x^3+64*a^3*c^3*x
```

maxima [A] time = 0.80, size = 205, normalized size = 1.20

$$\frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{48}{11}c^2d^4x^{11} + \frac{32}{5}c^3d^3x^{10} + \frac{64}{7}c^6x^7 + 64a^3c^3x + \frac{16}{5}\left(3d^2x^5 + 15cdx^4 + 20c^2x^3\right)a^2c^2 + \frac{8}{3}\left(2d^2x^9 + 9cdx^8\right)c^4 + \frac{4}{105}\left(35d^4x^9 + 315cd^3x^8 + 720c^2d^2x^7 + 1008c^4x^5 + 120\left(3d^2x^7 + 14cdx^6\right)c^2\right)ac + \frac{4}{165}\left(45d^4x^{11} + 396cd^3x^{10} + 880c^2d^2x^9\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7*
c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c^2
 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 + 72
0*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c + 4/16
5*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

mupad [B] time = 2.16, size = 160, normalized size = 0.94

$$x^8\left(24c^5d + 12ac^2d^3\right) + x^9\left(\frac{80c^4d^2}{3} + \frac{4acd^4}{3}\right) + \frac{d^6x^{13}}{13} + x^7\left(\frac{64c^6}{7} + \frac{288ac^3d^2}{7}\right) + 64a^3c^3x + cd^5x^{12} + 64a^2c^4x^3 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + 48a^2c^3dx^4 + \frac{48ac^2x^5\left(4c^3 + ad^2\right)}{5} + 64ac^4dx^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

```
[Out] x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^6
*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x^1
```

```
2 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3*d*
x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

sympy [A] time = 0.10, size = 180, normalized size = 1.05

$$64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9\left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8\left(12ac^2d^3 + 24c^5d\right) + x^7\left(\frac{288ac^3d^2}{7} + \frac{64c^6}{7}\right) + x^5\left(\frac{48a^2c^2d^2}{5} + \frac{192ac^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

```
[Out] 64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**6
 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13/1
3 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**5*d
) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 + 192*
a*c**5/5)
```

3.31 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx$

Optimal. Leaf size=92

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2061}

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+8ac^2dx^4+\frac{32}{3}ac^3x^3+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3dx^6+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx=\int \left(16a^2c^2+32ac^3x^2+32ac^2dx^3+8c\left(2c^3+ad^2\right)x^4+32c^3dx^5+24c^2d^2\right.$$

$$=16a^2c^2x+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{8}{5}c\left(2c^3+ad^2\right)x^5+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7$$

Mathematica [A] time = 0.02, size = 92, normalized size = 1.00

$$16a^2c^2x+\frac{8}{5}cx^5\left(ad^2+2c^3\right)+\frac{32}{3}ac^3x^3+8ac^2dx^4+\frac{16}{3}c^3dx^6+\frac{24}{7}c^2d^2x^7+cd^3x^8+\frac{d^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] 16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5)
/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2, x]
```

fricas [A] time = 0.54, size = 83, normalized size = 0.90

$$\frac{1}{9}x^9d^4+x^8d^3c+\frac{24}{7}x^7d^2c^2+\frac{16}{3}x^6dc^3+\frac{16}{5}x^5c^4+\frac{8}{5}x^5d^2ca+8x^4dc^2a+\frac{32}{3}x^3c^3a+16xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^4 + x^8*d^3*c + 24/7*x^7*d^2*c^2 + 16/3*x^6*d*c^3 + 16/5*x^5*c^4
+ 8/5*x^5*d^2*c*a + 8*x^4*d*c^2*a + 32/3*x^3*c^3*a + 16*x*c^2*a^2
```

giac [A] time = 0.25, size = 83, normalized size = 0.90

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{24}{7}\,c^2d^2x^7 + \frac{16}{3}\,c^3dx^6 + \frac{16}{5}\,c^4x^5 + \frac{8}{5}\,acd^2x^5 + 8\,ac^2dx^4 + \frac{32}{3}\,ac^3x^3 + 16\,a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
+ 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

maple [A] time = 0.00, size = 84, normalized size = 0.91

$$\frac{d^4x^9}{9} + c\,d^3x^8 + \frac{24c^2d^2x^7}{7} + \frac{16c^3d\,x^6}{3} + 8a\,c^2d\,x^4 + \frac{32a\,c^3x^3}{3} + 16a^2c^2x + \frac{\left(8ac\,d^2 + 16c^4\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] 1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+1/5*(8*a*c*d^2+16*c^4
)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

maxima [A] time = 0.57, size = 94, normalized size = 1.02

$$\frac{1}{9}\,d^4x^9 + cd^3x^8 + \frac{16}{7}\,c^2d^2x^7 + \frac{16}{5}\,c^4x^5 + 16\,a^2c^2x + \frac{8}{15}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)ac + \frac{8}{21}\left(3\,d^2x^7 + 14\,cdx^6\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d*x
^6)*c^2
```

mupad [B] time = 0.04, size = 82, normalized size = 0.89

$$x^5\left(\frac{16\,c^4}{5} + \frac{8\,a\,c\,d^2}{5}\right) + \frac{d^4\,x^9}{9} + 16\,a^2\,c^2\,x + \frac{32\,a\,c^3\,x^3}{3} + \frac{16\,c^3\,d\,x^6}{3} + c\,d^3\,x^8 + \frac{24\,c^2\,d^2\,x^7}{7} + 8\,a\,c^2\,d\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*x
^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

sympy [A] time = 0.08, size = 95, normalized size = 1.03

$$16a^2c^2x + \frac{32ac^3x^3}{3} + 8ac^2dx^4 + \frac{16c^3dx^6}{3} + \frac{24c^2d^2x^7}{7} + cd^3x^8 + \frac{d^4x^9}{9} + x^5\left(\frac{8acd^2}{5} + \frac{16c^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] 16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 24
*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c**
4/5)
```

3.32 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx$

Optimal. Leaf size=32

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx = 4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] 4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

```
[Out] IntegrateAlgebraic[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4, x]
```

fricas [A] time = 0.46, size = 28, normalized size = 0.88

$$\frac{1}{5}x^5d^2+x^4dc+\frac{4}{3}x^3c^2+4xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2 + x^4*d*c + 4/3*x^3*c^2 + 4*x*c*a
```

giac [A] time = 0.27, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$\frac{1}{5}d^2x^5 + cd\,x^4 + \frac{4}{3}c^2x^3 + 4acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)
[Out] 4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

maxima [A] time = 0.65, size = 28, normalized size = 0.88

$$\frac{1}{5}\,d^2x^5 + cdx^4 + \frac{4}{3}\,c^2x^3 + 4\,acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
[Out] 1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

mupad [B] time = 0.04, size = 28, normalized size = 0.88

$$\frac{4\,c^2\,x^3}{3} + c\,d\,x^4 + 4\,a\,c\,x + \frac{d^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
[Out] (4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

sympy [A] time = 0.07, size = 31, normalized size = 0.97

$$4acx + \frac{4c^2x^3}{3} + cdx^4 + \frac{d^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
[Out] 4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

3.33 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=529

$$-\frac{d\log\left(\sqrt{c}\,\sqrt{4ad^2+c^3}-\sqrt{2}\,\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{c}\,\sqrt{4ad^2+c^3}+\sqrt{2}\,\sqrt[4]{c}\,d\right.}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2}}$$

Rubi [A] time = 0.90, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1106, 1094, 634, 618, 206, 628}

$$-\frac{d\log\left(-\sqrt{2}\,\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\,\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\log\left(\sqrt{2}\,\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\,\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{4\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}\,c+\sqrt{2}\,dx}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\,\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}\,(c+dx)}{\sqrt[4]{c}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{4ad^2+c^3}\,\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] -(d*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] + Sqrt
[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)
*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*ArcTanh[(c^(
1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt
[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] -
Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d
+ x)^2])/(4*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4
*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(4*Sqrt[2]*c^(3/4
)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
```

```
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{d\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}-x}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\,\text{Subst}\left(\int \cdots\right)}{2\sqrt{2}\cdots}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,x}{d}+x^2}\,dx, x, \frac{c}{d}+x\right)}{4\sqrt{c}\sqrt{c^3+4ad^2}} + \frac{\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= -\frac{d\log\left(\sqrt{c}\sqrt{c^3+4ad^2}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\,(c+dx)+(c+dx)^2\cdots\right)}{4\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}\cdots$$

$$= -\frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}\,c^{3/4}-\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\right)+\sqrt{2}\,dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{c}\left(\sqrt{2}\,c^{3/4}+\sqrt{c^{3/2}\cdots}\right)\cdots}{\sqrt[4]{c}\sqrt{c^{3/2}\cdots}}\right)}{2\sqrt{2}\,c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}\cdots}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[\#1^4d^2+4\#1^3cd+4\#1^2c^2+4ac\&,\frac{\log(x-\#1)}{\#1^3d^2+3\#1^2cd+2\#1c^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

```
[Out] RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*#
1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]

[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1), x]

fricas [B] time = 0.74, size = 905, normalized size = 1.71

$$\frac{1}{8}\sqrt{-\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}+1}{ac^3+4a^2d^2}}\log\left(d^2x+cd+\left(2acd^2+\left(ac^7+4a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}+1}{ac^3+4a^2d^2}}\right)-\frac{1}{8}\sqrt{-\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}+1}{ac^3+4a^2d^2}}\log\left(d^2x+cd-\left(2acd^2+\left(ac^7+4a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}\right)\sqrt{-\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}+1}{ac^3+4a^2d^2}}\right)+\frac{1}{8}\sqrt{\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}-1}{ac^3+4a^2d^2}}\log\left(d^2x+cd+\left(2acd^2-\left(ac^7+4a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}-1}{ac^3+4a^2d^2}}\right)-\frac{1}{8}\sqrt{\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}-1}{ac^3+4a^2d^2}}\log\left(d^2x+cd-\left(2acd^2-\left(ac^7+4a^2c^4d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}\right)\sqrt{\frac{2\left(ac^3+4a^2d^2\right)\sqrt{-\frac{d^2}{ac^9+8a^2c^6d^2+16a^3c^3d^4}}-1}{ac^3+4a^2d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")

[Out] 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d - (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))

giac [A] time = 0.34, size = 603, normalized size = 1.14

$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^3-3cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)\right)}+\frac{\log\left(x-\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^3+3cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)\right)}-\frac{\log\left(x+\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^3-3cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)\right)}+\frac{\log\left(x-\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^3+3cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}\,d^3}{d^4}}-\frac{c}{d}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")

[Out] -1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d))

maple [C] time = 0.06, size = 64, normalized size = 0.12

$$\frac{\ln\left(-\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)+x\right)}{4\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^3d^2+12\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^2cd+8\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x)

```
[Out] 1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x),_R=RootOf(_Z^4*d^2+4*_Z^
3*c*d+4*_Z^2*c^2+4*a*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

```
[Out] integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

mupad [B] time = 4.54, size = 1551, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

```
[Out] atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2
)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*
(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a
*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (-(2*
d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*
c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(-a^3*c^3
)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 2
56*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^(1/2) + a*c^
3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x
)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^
6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*
(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^
6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) +
4*c*d^5 + 4*d^6*x)*1i + ((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3
*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^
6*x)*1i)/(((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(
1/2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) -
a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - ((2*d*(
-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4
*d^5 + 256*a*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^
3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c
^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*((2*d*(-a^3*c^3)^(1/2) -
 a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i
```

sympy [A] time = 1.15, size = 88, normalized size = 0.17

$$\operatorname{RootSum}\left(t^4\left(16384a^3c^3d^2 + 4096a^2c^6\right) + 128t^2ac^3 + 1, \left(t \mapsto t\log\left(x + \frac{-1024t^3a^2c^4d^2 - 256t^3ac^7 + 16tacd^2 - 4tc^4 + cd}{d^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

```
[Out] RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 + 1
6*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

3.34 $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx$

Optimal. Leaf size=746

$$-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

Rubi [A] time = 1.33, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1106, 1092, 1169, 634, 618, 206, 628}

$$-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}-\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(-\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\left(-c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\log\left(\sqrt{2}\sqrt[4]{c}\,d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+\sqrt{c}\sqrt{4ad^2+c^3}+d^2\left(\frac{c}{d}+x\right)^2\right)}{64\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}-\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}+\sqrt{2}c+\sqrt{2}dx}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}+\frac{d\left(c^{3/2}\sqrt{4ad^2+c^3}+12ad^2+c^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{32\sqrt{2}\,ac^{7/4}\left(4ad^2+c^3\right)^{3/2}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] -((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(16*a*c*(c^3 + 4*a*d^2)*(4
*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)) - (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqr
t[c^3 + 4*a*d^2])*ArcTanh[(Sqrt[2]*c + c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]] + Sqrt[2]*d*x)/(c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/(32*
Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]
) + (d*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^2])*ArcTanh[(c^(1/4)*Sqrt
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]] - Sqrt[2]*(c + d*x))/(c^(1/4)*Sqrt[c^(3/2)
- Sqrt[c^3 + 4*a*d^2]])])/(32*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[
c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3 + 4
*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) +
 Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/(64*Sqrt[2]*a*c^(7/4)*(
c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*
d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2
]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2
])/(64*Sqrt[2]*a*c^(7/4)*(c^3 + 4*a*d^2)^(3/2)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*
a*d^2]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^2}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{\text{Subst}\left(\int \frac{4c^4-2c\left(4a+\ldots\right.}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} + \frac{d\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\ldots}}{\ldots}}{\ldots}\right.}{\ldots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{\left(d\left(c^3+12ad^2-c^{\ldots}\right.\right.}{\ldots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2-c^{3/\ldots}\right.}{\ldots}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} - \frac{d\left(c^3+12ad^2+c^{3/\ldots}\right.}{32\sqrt{\ldots}}$$

Mathematica [C] time = 0.12, size = 182, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^4d^2+4\#1^3cd+4\#1^2c^2+4ac\&,\frac{\#1^2cd^2\log(x-\#1)+12ad^2\log(x-\#1)+2c^3\log(x-\#1)+2\#1c^2d\log(x-\#1)}{\#1^3d^2+3\#1^2cd+2\#1c^2}\&\right]+\frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2}}{64ac\left(4ad^2+c^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] ((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Root
Sum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 12*
a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*c^2
*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2), x]
```

fricas [B] time = 1.09, size = 3222, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(4*c*d^2*x^3 + 12*c^2*d*x^2 + 16*a*c*d + (4*a^2*c^5 + 16*a^3*c^2*d^2 +
 (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6
 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11
 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 36
0*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4
 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^
7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*lo
g(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 32
4*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^1
9 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8
)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^2
2*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8
*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 +
2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c
^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a
^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 +
4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c
^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4
*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6
+ 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4
 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*
c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^
13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d
^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a
^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d^4 + 96
*a^3*c^5*d^6 + 432*a^4*c^2*d^8 + (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13
*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^
8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^
6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*
sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 + 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*
a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*
d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 +
3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 1
2*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + (4*a^2*c^5 + 16*a^3*c^
2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4
*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a
^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d
^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c
^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096
*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d
^6))*log(5*c^7*d^3 + 81*a*c^4*d^5 + 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d
^6 + 324*a^2*d^8)*x + (5*a^2*c^8*d^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (
a^3*c^19 + 20*a^4*c^16*d^2 + 144*a^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*
c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*
a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6
144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2
*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt
(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2
+ 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*
d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 6
4*a^6*c^2*d^6))) - (4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*
```

```
x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)*sqrt
(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^2 + 48*a^5*
c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 1296*a^2*d^10
)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16*d^6 + 3840
*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c^11 + 12*a^
4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))*log(5*c^7*d^3 + 81*a*c^4*d^5
+ 324*a^2*c*d^7 + (5*c^6*d^4 + 81*a*c^3*d^6 + 324*a^2*d^8)*x - (5*a^2*c^8*d
^4 + 96*a^3*c^5*d^6 + 432*a^4*c^2*d^8 - (a^3*c^19 + 20*a^4*c^16*d^2 + 144*a
^5*c^13*d^4 + 448*a^6*c^10*d^6 + 512*a^7*c^7*d^8)*sqrt(-(25*c^6*d^6 + 360*a
*c^3*d^8 + 1296*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 +
1280*a^6*c^16*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d
^12)))*sqrt(-(c^6 + 15*a*c^3*d^2 + 60*a^2*d^4 - 2*(a^3*c^11 + 12*a^4*c^8*d^
2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6)*sqrt(-(25*c^6*d^6 + 360*a*c^3*d^8 + 12
96*a^2*d^10)/(a^3*c^25 + 24*a^4*c^22*d^2 + 240*a^5*c^19*d^4 + 1280*a^6*c^16
*d^6 + 3840*a^7*c^13*d^8 + 6144*a^8*c^10*d^10 + 4096*a^9*c^7*d^12)))/(a^3*c
^11 + 12*a^4*c^8*d^2 + 48*a^5*c^5*d^4 + 64*a^6*c^2*d^6))) + 8*(c^3 + 2*a*d^
2)*x)/(4*a^2*c^5 + 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^
5*d + 4*a^2*c^2*d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2)
```

giac [A] time = 0.37, size = 1057, normalized size = 1.42

$$-\frac{\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2-2c^2d\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)+2c^3+12ad^2\right)\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}-\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2d\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)+2c^3+12ad^2\right)\log\left(x-\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)}+\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2-2c^2d\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)+2c^3+12ad^2\right)\log\left(x+\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}-\frac{\left(cd^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2d\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)+2c^3+12ad^2\right)\log\left(x-\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)}}{64\left(ac^4+4a^2cd^2\right)}+\frac{cd^2x^3+3c^2dx^2+2c^3x+4ad^2x+4acd}{16\left(d^2x^4+4cdx^3+4c^2x^2+4ac\right)\left(ac^4+4a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

```
[Out] -1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(s
qrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sq
rt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a
*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 + 2*sq
rt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d
)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt(
(c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(
-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4)
+ c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) +
 c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((
c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12
*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a
*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/
d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 2*c^3*x + 4*a*
d^2*x + 4*a*c*d)/((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)*(a*c^4 + 4*a^2*
c*d^2))
```

maple [C] time = 0.02, size = 232, normalized size = 0.31

$$\frac{\left(\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^2 c\,d^2+2\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)c^2d+12a\,d^2+2c^3\right)\ln\left(-\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)+x\right)}{64\left(4a\,d^2+c^3\right)ac\left(\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^3d^2+3\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)^2cd+2\,\mathrm{RootOf}\left(d^2\_Z^4+4dc\_Z^3+4c^2\_Z^2+4ac\right)c^2\right)}+\frac{\frac{d^2x^3}{16\left(4a\,d^2+c^3\right)a}+\frac{3cd\,x^2}{16\left(4a\,d^2+c^3\right)a}+\frac{d}{16a\,d^2+4c^3}+\frac{\left(2a\,d^2+c^3\right)x}{8\left(4a\,d^2+c^3\right)ac}}{d^2x^4+4cd\,x^3+4c^2x^2+4ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

```
[Out] (1/16*d^2/a/(4*a*d^2+c^3)*x^3+3/16/a*c*d/(4*a*d^2+c^3)*x^2+1/8/c*(2*a*d^2+c
^3)/(4*a*d^2+c^3)/a*x+1/4*d/(4*a*d^2+c^3))/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a
```

```
*c)+1/64/(4*a*d^2+c^3)/a/c*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_R^3
*d^2+3*_R^2*c*d+2*_R*c^2)*ln(-_R+x),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2*c^
2+4*a*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5 +
 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*d^
3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*c^2
*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/(a*c
^4 + 4*a^2*c*d^2)
```

mupad [B] time = 4.30, size = 5844, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

```
[Out] (d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 + c
^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c +
 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^
7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(40
96*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*
((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*
(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768
*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c
^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a
^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 +
 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536*a^4*
c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*
d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*
c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48
*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 +
704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36
*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*
c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2
+ 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13
*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 +
2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c
^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10
*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^
9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*
c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d
^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^
3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c
^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7
*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^
3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a
*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*d^8 +
 c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^11 +
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*
a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a
```

```
^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3
*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15
*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16
+ 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c
^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^
5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^
4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 1
2*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5
+ 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5
*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^
3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) +
 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^
16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144
*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 +
 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d
^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*
(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4
 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^
10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 +
196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(
a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 +
72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*
d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^
4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10
+ c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))
))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*
d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8
*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i - atan(((-(a^3*c^11 - 10*c^3*d^3*(-a
^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2)
)/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(
1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(
1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 +
32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*
a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 +
 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*
d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 + 65536
*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5
*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60
*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2
 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d
^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (
x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16
*a^4*c^2*d^4)))*1i + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8
*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7
*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d
^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*
d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*
a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11
 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5
*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64
*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*
c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 -
10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-
a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^
9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(102
4*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 +
 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*1i)/((9*a*
d^8 + c^3*d^6)/(512*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) - (-(a^3*c^
11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d
```

```
^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 +
64*a^9*c^7*d^6)))^(1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 419
4304*a^6*c^6*d^9)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4
096*a^3*c^11*d^6 + 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 +
8*a^3*c^5*d^2 + 16*a^4*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2)
 + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*
c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*
a^3*c^8*d^6 + 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a
^4*c^5*d^2 + 16*a^5*c^2*d^4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) +
15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^1
6 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7
*d^5 + 2304*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 1
6*a^5*c^2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 +
 8*a^3*c^5*d^2 + 16*a^4*c^2*d^4))) + (-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1
/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a
^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*((((2
62144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/(1024*(a^3*
c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6 + 32768*a^4*
c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d^
4)))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^
5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a
^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (4096*a^3*c^8*d^6 + 65536*a^4*c^5*d
^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4))
)*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d
^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*
c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 2304*a^3*c*d^9 + 704*a
^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(36*a^2*
d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 + 16*a^4*c^2*d
^4)))))*(-(a^3*c^11 - 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5
*c^5*d^4 - 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 4
8*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*2i
```

sympy [A] time = 109.97, size = 427, normalized size = 0.57

$$\frac{4acd + 3c^2dx^2 + cd^2x^3 + x\left(4ad^2 + 2c^3\right)}{256a^3c^2d^2 + 64a^2c^5 + x^4\left(64a^2cd^4 + 16ac^4d^2\right) + x^3\left(256a^2c^2d^3 + 64ac^5d\right) + x^2\left(256a^2c^3d^2 + 64ac^6\right)} + \operatorname{RootSum}\left(t^4\left(1073741824a^9c^7d^6 + 805306368a^8c^{10}d^4 + 201326592a^7c^{13}d^2 + 16777216a^6c^{16}\right) + t^2\left(491520a^5c^5d^4 + 122880a^4c^8d^2 + 8192a^3c^{11}\right) + 81a^2d^4 + 18ac^3d^2 + c^6, \left(t \mapsto t\log\left(x + \frac{-67108864t^3a^7c^7d^8 - 58720256t^3a^6c^{10}d^6 - 18874368t^3a^5c^{13}d^4 - 2621440t^3a^4c^{16}d^2 - 131072t^3a^3c^{19} + 27648ta^4c^2d^8 - 9216ta^3c^5d^6 - 5440ta^2c^8d^4 - 736tac^{11}d^2 - 32tc^{14} + 324a^2cd^7 + 81ac^4d^5 + 5c^7d^3}{324a^2d^8 + 81ac^3d^6 + 5c^6d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

```
[Out] (4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*c
**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(25
6*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6)) +
RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 + 2013
26592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5*d**4
 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**3*d**2
 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 58720256*_
t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t**3*a**4*
c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a**4*c**2*d**8 - 9216*_t*a*
*3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 - 736*_t*a*c**11*d**2 - 32*_t*c**14 +
 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3)/(324*a**2*d**8 + 81*a*c**3
*d**6 + 5*c**6*d**4))))
```

3.35 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx$

Optimal. Leaf size=295

$$\frac{1}{24}e^4\left(65536a^2e^6 + 20992ad^4e^3 + 601d^8\right)\left(\frac{d}{4e} + x\right)^9 + \frac{\left(256ae^3 + 5d^4\right)^2\left(256ae^3 + 59d^4\right)\left(\frac{d}{4e} + x\right)^5}{5120} + \frac{64}{13}e^8\left(256ae\right.$$

Rubi [A] time = 0.53, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1106, 1090}

$$\frac{1}{24}e^4\left(65536a^2e^6 + 20992ad^4e^3 + 601d^8\right)\left(\frac{d}{4e} + x\right)^9 + \frac{64}{13}e^8\left(256ae^3 + 59d^4\right)\left(\frac{d}{4e} + x\right)^{13} - \frac{72}{11}d^2e^6\left(256ae^3 + 17d^4\right)\left(\frac{d}{4e} + x\right)^{11} - \frac{9}{224}d^2e^2\left(256ae^3 + 5d^4\right)\left(256ae^3 + 17d^4\right)\left(\frac{d}{4e} + x\right)^7 + \frac{\left(256ae^3 + 5d^4\right)^2\left(256ae^3 + 59d^4\right)\left(\frac{d}{4e} + x\right)^5}{5120} - \frac{d^2\left(256ae^3 + 5d^4\right)^3\left(\frac{d}{4e} + x\right)^3}{8192e^2} + \frac{x\left(256ae^3 + 5d^4\right)^4}{1048576e^4} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e} + x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e} + x\right)^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^4*x)/(1048576*e^4) - (d^2*(5*d^4 + 256*a*e^3)^3*(d/(4*
e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*a*e^3)*(d/(4*e
) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 256*a*e^3)*(d/(4*
e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^2*e^6)*(d/(4*e)
+ x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x)^11)/11 + (64*e^
8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e^10*(d/(4*e) + x)^
15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx = \text{Subst}\left(\int \left(\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^4 dx, x, \frac{d}{4e} + x\right)$$

$$= \text{Subst}\left(\int \left(\frac{\left(5d^4 + 256ae^3\right)^4}{1048576e^4} - \frac{3d^2\left(5d^4 + 256ae^3\right)^3x^2}{8192e^2} + \frac{27}{512}d^4\left(5d^4\right.\right.\right.$$

$$= \frac{\left(5d^4 + 256ae^3\right)^4 x}{1048576e^4} - \frac{d^2\left(5d^4 + 256ae^3\right)^3\left(\frac{d}{4e} + x\right)^3}{8192e^2} + \frac{\left(5d^4 + 256ae^3\right)^2}{}$$

Mathematica [A] time = 0.05, size = 345, normalized size = 1.17

$$4096a^4e^8x - 1024a^3d^3e^6x^2 + 8ade^2x^4\left(512a^2e^6 - d^8\right) + 128a^2d^6e^4x^3 - 4de^3x^8\left(-1536a^2e^6 + 192ad^4e^3 + d^8\right) + \frac{128}{3}e^4x^9\left(64a^2e^6 - 32ad^4e^3 + d^8\right) - \frac{32}{7}d^2e^2x^7\left(-768a^2e^6 - 24ad^4e^3 + d^8\right) + \frac{1}{5}x^5\left(16384a^3e^9 - 6144a^2d^4e^6 + d^{12}\right) + \frac{2048}{13}e^8x^{13}\left(8ae^3 - d^4\right) - 512de^7x^{12}\left(d^4 - 8ae^3\right) + \frac{128}{5}d^3e^5x^{10}\left(40ae^3 + 3d^4\right) - 128ad^3e^4x^6\left(8ae^3 - d^4\right) + \frac{128}{11}d^2e^6x^{11}\left(384ae^3 - 13d^4\right) + 1024d^3e^9x^{14} + \frac{8192}{5}d^2e^{10}x^{15} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] 4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-d
^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5 -
 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 - 768
*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (128*
e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 40*a*e
^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7*(d^4
- 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9*x^14 +
 (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4, x]
```

fricas [A] time = 0.66, size = 353, normalized size = 1.20

$$\frac{4096}{17}x^{17}e^{12} + 1024x^{16}e^{11}d + \frac{8192}{5}x^{15}e^{10}d^2 + 1024x^{14}e^9d^3 - \frac{2048}{13}x^{13}e^8d^4 + \frac{16384}{13}x^{13}e^{11}a - 512x^{12}e^7d^5 + 4096x^{12}e^{10}da - \frac{1664}{11}x^{11}e^6d^6 + \frac{49152}{11}x^{11}e^9d^2a + \frac{384}{5}x^{10}e^5d^7 + 1024x^{10}e^8d^3a + \frac{128}{3}x^9e^4d^8 - \frac{4096}{3}x^9e^7d^4a + \frac{8192}{3}x^9e^{10}a^2 - 4x^8e^3d^9 - 768x^8e^6d^5a + 6144x^8e^9da^2 - \frac{32}{7}x^7e^2d^{10} + \frac{768}{7}x^7e^5d^6a + \frac{24576}{7}x^7e^8d^2a^2 + 128x^6e^4d^7a - 1024x^6e^7d^3a^2 + \frac{1}{5}x^5d^{12} - \frac{6144}{5}x^5e^6d^4a^2 + \frac{16384}{5}x^5e^9a^3 - 8x^4e^2d^9a + 4096x^4e^8da^3 + 128x^3e^4d^6a^2 - 1024x^2e^6d^3a^3 + 4096xe^8a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17*e^12 + 1024*x^16*e^11*d + 8192/5*x^15*e^10*d^2 + 1024*x^14*e^9
*d^3 - 2048/13*x^13*e^8*d^4 + 16384/13*x^13*e^11*a - 512*x^12*e^7*d^5 + 409
6*x^12*e^10*d*a - 1664/11*x^11*e^6*d^6 + 49152/11*x^11*e^9*d^2*a + 384/5*x^
10*e^5*d^7 + 1024*x^10*e^8*d^3*a + 128/3*x^9*e^4*d^8 - 4096/3*x^9*e^7*d^4*a
 + 8192/3*x^9*e^10*a^2 - 4*x^8*e^3*d^9 - 768*x^8*e^6*d^5*a + 6144*x^8*e^9*d
*a^2 - 32/7*x^7*e^2*d^10 + 768/7*x^7*e^5*d^6*a + 24576/7*x^7*e^8*d^2*a^2 +
128*x^6*e^4*d^7*a - 1024*x^6*e^7*d^3*a^2 + 1/5*x^5*d^12 - 6144/5*x^5*e^6*d^
4*a^2 + 16384/5*x^5*e^9*a^3 - 8*x^4*e^2*d^9*a + 4096*x^4*e^8*d*a^3 + 128*x^
3*e^4*d^6*a^2 - 1024*x^2*e^6*d^3*a^3 + 4096*x*e^8*a^4
```

giac [A] time = 0.31, size = 323, normalized size = 1.09

$$\frac{4096}{17}x^{17}e^{12} + 1024dx^{16}e^{11} + \frac{8192}{5}d^2x^{15}e^{10} + 1024d^3x^{14}e^9 - \frac{2048}{13}d^4x^{13}e^8 - 512d^5x^{12}e^7 - \frac{1664}{11}d^6x^{11}e^6 + \frac{384}{5}d^7x^{10}e^5 + \frac{128}{3}d^8x^9e^4 - 4d^9x^8e^3 - \frac{32}{7}d^{10}x^7e^2 + \frac{1}{5}d^{12}x^5 + \frac{16384}{13}ax^{13}e^{11} + 4096adx^{12}e^{10} + \frac{49152}{11}ad^2x^{11}e^9 + 1024ad^3x^{10}e^8 - \frac{4096}{3}ad^4x^9e^7 - 768ad^5x^8e^6 + \frac{768}{7}ad^6x^7e^5 + 128ad^7x^6e^4 - 8ad^9x^4e^2 + \frac{8192}{3}a^2x^9e^{10} + 6144a^2dx^8e^9 + \frac{24576}{7}a^2d^2x^7e^8 - 1024a^2d^3x^6e^7 - \frac{6144}{5}a^2d^4x^5e^6 + 128a^2d^6x^3e^4 + \frac{16384}{5}a^3x^5e^9 + 4096a^3dx^4e^8 - 1024a^3d^3x^2e^6 + 4096a^4xe^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17*e^12 + 1024*d*x^16*e^11 + 8192/5*d^2*x^15*e^10 + 1024*d^3*x^14
*e^9 - 2048/13*d^4*x^13*e^8 - 512*d^5*x^12*e^7 - 1664/11*d^6*x^11*e^6 + 384
/5*d^7*x^10*e^5 + 128/3*d^8*x^9*e^4 - 4*d^9*x^8*e^3 - 32/7*d^10*x^7*e^2 + 1
/5*d^12*x^5 + 16384/13*a*x^13*e^11 + 4096*a*d*x^12*e^10 + 49152/11*a*d^2*x^
11*e^9 + 1024*a*d^3*x^10*e^8 - 4096/3*a*d^4*x^9*e^7 - 768*a*d^5*x^8*e^6 + 7
68/7*a*d^6*x^7*e^5 + 128*a*d^7*x^6*e^4 - 8*a*d^9*x^4*e^2 + 8192/3*a^2*x^9*e
^10 + 6144*a^2*d*x^8*e^9 + 24576/7*a^2*d^2*x^7*e^8 - 1024*a^2*d^3*x^6*e^7 -
 6144/5*a^2*d^4*x^5*e^6 + 128*a^2*d^6*x^3*e^4 + 16384/5*a^3*x^5*e^9 + 4096*
a^3*d*x^4*e^8 - 1024*a^3*d^3*x^2*e^6 + 4096*a^4*x*e^8
```

maple [A] time = 0.00, size = 500, normalized size = 1.69

$$\frac{4096e^{12}x^{17}}{17} + 1024de^{11}x^{16} + \frac{8192d^2e^{10}x^{15}}{5} + 1024d^3e^9x^{14} + \frac{128\left(128ae^5 - 16d^4e^2\right)e^6x^{13}}{13} + 128a^2d^6e^4x^3 - 1024a^3d^3e^6x^2 + 4096a^4e^8x + \frac{\left(16384ade^{10} - 2048d^5e^7 + 256\left(128ae^5 - 16d^4e^2\right)de^5\right)x^{12}}{12} + \frac{\left(32768ad^2e^9 + 384d^6e^6 + 128\left(128ae^5 - 16d^4e^2\right)d^2e^4\right)x^{11}}{11} + \frac{\left(14336ad^3e^8 + 256d^7e^5 - 32\left(128ae^5 - 16d^4e^2\right)d^3e^3\right)x^{10}}{10} + \frac{\left(8192a^2e^{10} - 8192ad^4e^7 + 128d^8e^4 + \left(128ae^5 - 16d^4e^2\right)^2\right)x^9}{9} + \frac{\left(16384a^2de^9 - 2048ad^5e^6 - 32d^9e^3 + 256\left(128ae^5 - 16d^4e^2\right)ade^4\right)x^8}{8} + \frac{\left(24576a^2d^2e^8 + 512ad^6e^5 + 2\left(128ae^5 - 16d^4e^2\right)d^6\right)x^7}{7} + \frac{\left(-2048a^2d^3e^7 + 256ad^7e^4 - 32\left(128ae^5 - 16d^4e^2\right)ad^3e^2\right)x^6}{6} + \frac{\left(-4096a^2d^4e^6 + d^{12} + 128\left(128ae^5 - 16d^4e^2\right)a^2e^4\right)x^5}{5} + \frac{\left(16384a^3e^8d - 32ae^2d^9\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)
```

```
[Out] 4096/17*e^12*x^17+1024*d*e^11*x^16+8192/5*d^2*e^10*x^15+1024*d^3*e^9*x^14+1
28/13*(128*a*e^5-16*d^4*e^2)*e^6*x^13+1/12*(16384*a*e^10*d+256*(128*a*e^5-1
6*d^4*e^2)*d*e^5-2048*d^5*e^7)*x^12+1/11*(384*d^6*e^6+32768*a*e^9*d^2+128*(
128*a*e^5-16*d^4*e^2)*d^2*e^4)*x^11+1/10*(14336*a*e^8*d^3+256*d^7*e^5-32*(1
28*a*e^5-16*d^4*e^2)*d^3*e^3)*x^10+1/9*(8192*a^2*e^10-8192*a*e^7*d^4+128*d^
8*e^4+(128*a*e^5-16*d^4*e^2)^2)*x^9+1/8*(16384*a^2*e^9*d-2048*a*e^6*d^5-32*
d^9*e^3+256*a*e^4*d*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(24576*a^2*e^8*d^2+512*
a*e^5*d^6+2*d^6*(128*a*e^5-16*d^4*e^2))*x^7+1/6*(-2048*a^2*e^7*d^3-32*a*e^2
*d^3*(128*a*e^5-16*d^4*e^2)+256*d^7*a*e^4)*x^6+1/5*(128*a^2*e^4*(128*a*e^5-
16*d^4*e^2)-4096*a^2*e^6*d^4+d^12)*x^5+1/4*(16384*a^3*d*e^8-32*a*d^9*e^2)*x
^4+128*a^2*e^4*d^6*x^3-1024*a^3*e^6*d^3*x^2+4096*a^4*e^8*x
```

maxima [A] time = 0.75, size = 383, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e^
9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^3*
x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^3
*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128/105
*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e
^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e^8*x^1
3 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9*x^13 +
 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 - 15015*d
^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10 + 80*d*e^
5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

mupad [B] time = 0.27, size = 331, normalized size = 1.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

```
[Out] x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e^
5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11) +
(4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*(d^
8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 + 102
4*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^4) +
(32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*e^6*x^
2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^4*e^3)
- 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*e^6)
```

sympy [A] time = 0.14, size = 366, normalized size = 1.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

```
[Out] 4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 102
4*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e**
12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096*a*
d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6/11)
 + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/3 - 4
```

```
096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a*d**5*
e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5/7 - 32
*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x**5*(1638
4*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3*d*e**8 -
 8*a*d**9*e**2)
```

3.36 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx$

Optimal. Leaf size=203

$$512a^3e^6x-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{128}{3}e^5x^9\left(d^4-4ae^3\right)-24de^4x^8\left(d^4-16ae^3\right)-\frac{384}{5}ae^4x^5$$

Rubi [A] time = 0.12, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+512a^3e^6x-\frac{128}{3}e^5x^9\left(d^4-4ae^3\right)-24de^4x^8\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right)+4d^3e^2x^6\left(d^4-16ae^3\right)-\frac{384}{5}ae^4x^5\left(d^4-4ae^3\right)+8ad^6e^2x^3+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 - (384*a*e^4*(d^4 - 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3)
*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
- (128*e^5*(d^4 - 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)/1
1 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\int \left(512a^3e^6-192a^2d^3e^4x+24ad^6e^2x^2-d\left(d^8-1536a^2e^6\right)x^3-384a$$

$$=512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3-\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4-\frac{384}{5}ae^4$$

Mathematica [A] time = 0.03, size = 207, normalized size = 1.02

$$512a^3e^6x-\frac{1}{4}dx^4\left(d^8-1536a^2e^6\right)-96a^2d^3e^4x^2+8ad^6e^2x^3+\frac{128}{3}e^5x^9\left(4ae^3-d^4\right)-24de^4x^8\left(d^4-16ae^3\right)+\frac{384}{5}ae^4x^5\left(4ae^3-d^4\right)+4d^3e^2x^6\left(d^4-16ae^3\right)+\frac{24}{7}d^2e^3x^7\left(64ae^3+d^4\right)+32d^3e^6x^{10}+\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] 512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*e
^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e^3
)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*x^8
 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x^11)
/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3, x]
```

fricas [A] time = 0.70, size = 205, normalized size = 1.01

$$\frac{512}{13}x^{13}e^9+128x^{12}e^8d+\frac{1536}{11}x^{11}e^7d^2+32x^{10}e^6d^3-\frac{128}{3}x^9e^5d^4+\frac{512}{3}x^9e^8a-24x^8e^4d^5+384x^8e^7da+\frac{24}{7}x^7e^3d^6+\frac{1536}{7}x^7e^6d^2a+4x^6e^2d^7-64x^6e^5d^3a-\frac{384}{5}x^5e^4d^4a+\frac{1536}{5}x^5e^7a^2-\frac{1}{4}x^4d^9+384x^4e^6da^2+8x^3e^2d^6a-96x^2e^4d^3a^2+512xe^6a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13*e^9 + 128*x^12*e^8*d + 1536/11*x^11*e^7*d^2 + 32*x^10*e^6*d^3 -
 128/3*x^9*e^5*d^4 + 512/3*x^9*e^8*a - 24*x^8*e^4*d^5 + 384*x^8*e^7*d*a + 2
4/7*x^7*e^3*d^6 + 1536/7*x^7*e^6*d^2*a + 4*x^6*e^2*d^7 - 64*x^6*e^5*d^3*a -
 384/5*x^5*e^4*d^4*a + 1536/5*x^5*e^7*a^2 - 1/4*x^4*d^9 + 384*x^4*e^6*d*a^2
 + 8*x^3*e^2*d^6*a - 96*x^2*e^4*d^3*a^2 + 512*x*e^6*a^3
```

giac [A] time = 0.24, size = 187, normalized size = 0.92

$$\frac{512}{13}x^{13}e^9+128\,dx^{12}e^8+\frac{1536}{11}d^2x^{11}e^7+32\,d^3x^{10}e^6-\frac{128}{3}d^4x^9e^5-24\,d^5x^8e^4+\frac{24}{7}d^6x^7e^3+4\,d^7x^6e^2-\frac{1}{4}d^9x^4+\frac{512}{3}ax^9e^8+384\,adx^8e^7+\frac{1536}{7}ad^2x^7e^6-64\,ad^3x^6e^5-\frac{384}{5}ad^4x^5e^4+8\,ad^6x^3e^2+\frac{1536}{5}a^2x^5e^7+384\,a^2dx^4e^6-96\,a^2d^3x^2e^4+512\,a^3xe^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13*e^9 + 128*d*x^12*e^8 + 1536/11*d^2*x^11*e^7 + 32*d^3*x^10*e^6 -
 128/3*d^4*x^9*e^5 - 24*d^5*x^8*e^4 + 24/7*d^6*x^7*e^3 + 4*d^7*x^6*e^2 - 1/
4*d^9*x^4 + 512/3*a*x^9*e^8 + 384*a*d*x^8*e^7 + 1536/7*a*d^2*x^7*e^6 - 64*a
*d^3*x^6*e^5 - 384/5*a*d^4*x^5*e^4 + 8*a*d^6*x^3*e^2 + 1536/5*a^2*x^5*e^7 +
 384*a^2*d*x^4*e^6 - 96*a^2*d^3*x^2*e^4 + 512*a^3*x*e^6
```

maple [A] time = 0.00, size = 288, normalized size = 1.42

$$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+8a\,d^6e^2x^3-96a^2d^3e^4x^2+512a^3e^6x+\frac{\left(512a\,e^8-256d^4e^5+8\left(128a\,e^5-16d^4e^2\right)e^3\right)x^9}{9}+\frac{\left(2048ad\,e^7-64d^5e^4+8\left(128a\,e^5-16d^4e^2\right)d\,e^2\right)x^8}{8}+\frac{\left(1536a\,e^6d^2+24d^6e^3\right)x^7}{7}+\frac{\left(-256a\,d^3e^5+8d^7e^2-\left(128a\,e^5-16d^4e^2\right)d^3\right)x^6}{6}+\frac{\left(512a^2e^7-256a\,d^4e^4+8\left(128a\,e^5-16d^4e^2\right)a\,e^2\right)x^5}{5}+\frac{\left(1536a^2e^6d-d^9\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)
```

```
[Out] 512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+1/9*(51
2*a*e^8-256*d^4*e^5+8*e^3*(128*a*e^5-16*d^4*e^2))*x^9+1/8*(2048*a*e^7*d-64*
d^5*e^4+8*d*e^2*(128*a*e^5-16*d^4*e^2))*x^8+1/7*(1536*a*d^2*e^6+24*d^6*e^3)
*x^7+1/6*(-256*a*e^5*d^3-d^3*(128*a*e^5-16*d^4*e^2)+8*d^7*e^2)*x^6+1/5*(8*a
*e^2*(128*a*e^5-16*d^4*e^2)-256*d^4*a*e^4+512*e^7*a^2)*x^5+1/4*(1536*a^2*d*
e^6-d^9)*x^4+8*a*d^6*e^2*x^3-96*a^2*d^3*e^4*x^2+512*a^3*e^6*x
```

maxima [A] time = 0.60, size = 214, normalized size = 1.05

$$\frac{512}{13}e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+\frac{256}{5}d^3e^6x^{10}-\frac{1}{4}d^9x^4+512\,a^3e^6x+\frac{4}{7}\left(6\,e^3x^7+7\,de^2x^6\right)d^6+\frac{96}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)a^2e^4-\frac{8}{15}\left(36\,e^6x^{10}+80\,de^5x^9+45\,d^2e^4x^8\right)d^3+\frac{8}{105}\left(2240\,e^6x^9+5040\,de^5x^8+2880\,d^2e^4x^7+105\,d^6x^3-168\left(5\,e^3x^6+6\,de^2x^5\right)d^3\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

```
[Out] 512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^1
0 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/5*
(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80*d*
e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880
*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

mupad [B] time = 2.24, size = 201, normalized size = 0.99

$$\frac{512\,e^9\,x^{13}}{13}-x^4\,\left(\frac{d^9}{4}-384\,a^2\,d\,e^6\right)+\frac{128\,e^5\,x^9\,\left(4\,a\,e^3-d^4\right)}{3}+512\,a^3\,e^6\,x+128\,d\,e^8\,x^{12}+32\,d^3\,e^6\,x^{10}+\frac{1536\,d^2\,e^7\,x^{11}}{11}+8\,a\,d^6\,e^2\,x^3+\frac{384\,a\,e^4\,x^5\,\left(4\,a\,e^3-d^4\right)}{5}+24\,d\,e^4\,x^8\,\left(16\,a\,e^3-d^4\right)+\frac{24\,d^2\,e^3\,x^7\,\left(d^4+64\,a\,e^3\right)}{7}-96\,a^2\,d^3\,e^4\,x^2-4\,d^3\,e^2\,x^6\,\left(16\,a\,e^3-d^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

```
[Out] (512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 - d
^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4*x^
8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e^4*x
^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

sympy [**A**] time = 0.11, size = 218, normalized size = 1.07

$$512a^3e^6x - 96a^2d^3e^4x^2 + 8ad^6e^2x^3 + 32d^3e^6x^{10} + \frac{1536d^2e^7x^{11}}{11} + 128de^8x^{12} + \frac{512e^9x^{13}}{13} + x^9\left(\frac{512ae^8}{3} - \frac{128d^4e^5}{3}\right) + x^8\left(384ade^7 - 24d^5e^4\right) + x^7\left(\frac{1536ad^2e^6}{7} + \frac{24d^6e^3}{7}\right) + x^6\left(-64ad^3e^5 + 4d^7e^2\right) + x^5\left(\frac{1536a^2e^7}{5} - \frac{384ad^4e^4}{5}\right) + x^4\left(384a^2de^6 - \frac{d^9}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

```
[Out] 512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e**
6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13 +
x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**4)
+ x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 + 4*d*
*7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**2*d*e
**6 - d**9/4)
```

3.37 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx$

Optimal. Leaf size=107

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

Rubi [A] time = 0.05, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {2061}

$$64a^2e^4x-\frac{16}{5}e^2x^5\left(d^4-8ae^3\right)-8ad^3e^2x^2+32ade^4x^4+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{d^6x^3}{3}+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 - (16*e^2*(d^
4 - 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^8
 + (64*e^6*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\int \left(64a^2e^4-16ad^3e^2x+d^6x^2+128ade^4x^3-16e^2\left(d^4-8ae^3\right)x^4-16d^3e$$

$$=64a^2e^4x-8ad^3e^2x^2+\frac{d^6x^3}{3}+32ade^4x^4-\frac{16}{5}e^2\left(d^4-8ae^3\right)x^5-\frac{8}{3}d^3e^3x^6$$

Mathematica [A] time = 0.01, size = 109, normalized size = 1.02

$$64a^2e^4x+\frac{16}{5}e^2x^5\left(8ae^3-d^4\right)-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8}{3}d^3e^3x^6+\frac{64}{7}d^2e^4x^7+16de^5x^8+\frac{64e^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] 64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-d
^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*x^
8 + (64*e^6*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2, x]
```

fricas [A] time = 0.64, size = 99, normalized size = 0.93

$$\frac{64}{9}x^9e^6+16x^8e^5d+\frac{64}{7}x^7e^4d^2-\frac{8}{3}x^6e^3d^3-\frac{16}{5}x^5e^2d^4+\frac{128}{5}x^5e^5a+32x^4e^4da+\frac{1}{3}x^3d^6-8x^2e^2d^3a+64xe^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9*e^6 + 16*x^8*e^5*d + 64/7*x^7*e^4*d^2 - 8/3*x^6*e^3*d^3 - 16/5*x^5
*e^2*d^4 + 128/5*x^5*e^5*a + 32*x^4*e^4*d*a + 1/3*x^3*d^6 - 8*x^2*e^2*d^3*a
 + 64*x*e^4*a^2
```

giac [A] time = 0.29, size = 90, normalized size = 0.84

$$\frac{64}{9}x^9e^6 + 16\,dx^8e^5 + \frac{64}{7}d^2x^7e^4 - \frac{8}{3}d^3x^6e^3 - \frac{16}{5}d^4x^5e^2 + \frac{1}{3}d^6x^3 + \frac{128}{5}ax^5e^5 + 32\,adx^4e^4 - 8\,ad^3x^2e^2 + 64\,a^2xe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9*e^6 + 16*d*x^8*e^5 + 64/7*d^2*x^7*e^4 - 8/3*d^3*x^6*e^3 - 16/5*d^4
*x^5*e^2 + 1/3*d^6*x^3 + 128/5*a*x^5*e^5 + 32*a*d*x^4*e^4 - 8*a*d^3*x^2*e^2
 + 64*a^2*x*e^4
```

maple [A] time = 0.00, size = 100, normalized size = 0.93

$$\frac{64e^6x^9}{9} + 16d\,e^5x^8 + \frac{64d^2e^4x^7}{7} - \frac{8d^3e^3x^6}{3} + 32ad\,e^4x^4 + \frac{d^6x^3}{3} - 8a\,d^3e^2x^2 + 64a^2e^4x + \frac{\left(128a\,e^5 - 16d^4e^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] 64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+1/5*(128*a*e^5-1
6*d^4*e^2)*x^5+32*a*d*e^4*x^4+1/3*d^6*x^3-8*a*d^3*e^2*x^2+64*a^2*e^4*x
```

maxima [A] time = 0.61, size = 101, normalized size = 0.94

$$\frac{64}{9}e^6x^9 + 16\,de^5x^8 + \frac{64}{7}d^2e^4x^7 + \frac{1}{3}d^6x^3 + 64\,a^2e^4x - \frac{8}{15}\left(5\,e^3x^6 + 6\,de^2x^5\right)d^3 + \frac{8}{5}\left(16\,e^3x^5 + 20\,de^2x^4 - 5\,d^3x^2\right)ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] 64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*x
 - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*
d^3*x^2)*a*e^2
```

mupad [B] time = 0.04, size = 98, normalized size = 0.92

$$x^5\left(\frac{128\,a\,e^5}{5} - \frac{16\,d^4\,e^2}{5}\right) + \frac{d^6\,x^3}{3} + \frac{64\,e^6\,x^9}{9} + 64\,a^2\,e^4\,x + 16\,d\,e^5\,x^8 - \frac{8\,d^3\,e^3\,x^6}{3} + \frac{64\,d^2\,e^4\,x^7}{7} - 8\,a\,d^3\,e^2\,x^2 + 32\,a\,d\,e^4\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a^
2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3*e
^2*x^2 + 32*a*d*e^4*x^4
```

sympy [A] time = 0.09, size = 112, normalized size = 1.05

$$64a^2e^4x - 8ad^3e^2x^2 + 32ade^4x^4 + \frac{d^6x^3}{3} - \frac{8d^3e^3x^6}{3} + \frac{64d^2e^4x^7}{7} + 16de^5x^8 + \frac{64e^6x^9}{9} + x^5\left(\frac{128ae^5}{5} - \frac{16d^4e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] 64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d*
*3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 + x*
*5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

3.38 $\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx$

Optimal. Leaf size=37

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi steps

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] 8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
[Out] IntegrateAlgebraic[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4, x]
```

fricas [A] time = 0.56, size = 33, normalized size = 0.89

$$\frac{8}{5}x^5e^3 + 2x^4e^2d - \frac{1}{2}x^2d^3 + 8xe^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
[Out] 8/5*x^5*e^3 + 2*x^4*e^2*d - 1/2*x^2*d^3 + 8*x*e^2*a
```

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{8}{5}x^5e^3 + 2\,dx^4e^2 - \frac{1}{2}\,d^3x^2 + 8\,axe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
```

```
[Out] 8/5*x^5*e^3 + 2*d*x^4*e^2 - 1/2*d^3*x^2 + 8*a*x*e^2
```

maple [A] time = 0.00, size = 34, normalized size = 0.92

$$\frac{8}{5}e^3x^5 + 2d\,e^2x^4 - \frac{1}{2}d^3x^2 + 8a\,e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)
```

```
[Out] 8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

maxima [A] time = 0.67, size = 33, normalized size = 0.89

$$\frac{8}{5}\,e^3x^5 + 2\,de^2x^4 - \frac{1}{2}\,d^3x^2 + 8\,ae^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
```

```
[Out] 8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

mupad [B] time = 0.04, size = 33, normalized size = 0.89

$$-\frac{d^3\,x^2}{2} + 2\,d\,e^2\,x^4 + \frac{8\,e^3\,x^5}{5} + 8\,a\,e^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
```

```
[Out] (8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

sympy [A] time = 0.07, size = 36, normalized size = 0.97

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

```
[Out] 8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

3.39 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal. Leaf size=153

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1106, 1093, 208}

$$\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] (2*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 6
4*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(d + 4*e*x)/Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 + 2*Sq
rt[d^4 - 64*a*e^3]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Subst}\left(\int \frac{1}{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{(4e^2)\,\text{Subst}\left(\int \frac{1}{-\frac{3d^2e}{2}-e\sqrt{d^4-64ae^3}+8e^3x^2}\,dx, x, \frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}} - \frac{(4e^2)\,\text{Subst}\left(\int \frac{}{-\frac{3d^2e}{2}+e}\right.}{\sqrt{d}}$$

$$= \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.46

$$-\text{RootSum}\left[8\#1^4e^3+8\#1^3de^2-\#1d^3+8ae^2\&, \frac{\log(x-\#1)}{-32\#1^3e^3-24\#1^2de^2+d^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] -RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
- 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1), x]
```

fricas [B] time = 0.89, size = 1115, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

```
[Out] -sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*
a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 1
6384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3
- 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419
4304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(
25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3
 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 419430
4*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 -
128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 96
0*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8
- 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^
4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) -
sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a
*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16
384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 -
```

```
 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194
304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*
a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3
- 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304
*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*(2*d^4 - 1
28*a*e^3 + 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 -
 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4
*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

maple [C] time = 0.07, size = 67, normalized size = 0.44

$$\frac{\ln\left(-\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)+x\right)}{32\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)^3e^3+24\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8a\,e^2\right)^2d\,e^2-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)
```

```
[Out] sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3
*d*e^2-_Z*d^3+8*a*e^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

mupad [B] time = 3.73, size = 1264, normalized size = 8.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

```
[Out] atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*e*
x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^
(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 10485
76*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*e^9*(-
(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^
6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^
4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 +
 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^
```

```
9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^6*(-(2*(d^1
2 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192
*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))
^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(
1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1
2288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*
e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256i + a^2*d*e^6*819
2i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a
*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*1024i)/(5*d^12*((2*(d
^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 1
92*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6
))^(1/2) + 1048576*a^3*e^9*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 122
88*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 -
384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*((2*(d^12 - 26214
4*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^
3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) -
12288*a^2*d^4*e^6*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^
4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8
*e^3 - 12288*a^2*d^4*e^6))^(1/2)))*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e
^3 + 12288*a^2*d^4*e^6)^(1/2) + 3*d^6 - 192*a*d^2*e^3)/(5*d^12 + 1048576*a^
3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i
```

sympy [A] time = 1.83, size = 122, normalized size = 0.80

$$\operatorname{RootSum}\left(t^4\left(1048576a^3e^9-12288a^2d^4e^6-384ad^8e^3+5d^{12}\right)+t^2\left(384ad^2e^3-6d^6\right)+1,\left(t\mapsto t\log\left(x+\frac{-49152t^3a^2d^2e^6-192t^3ad^6e^3+15t^3d^{10}+256tae^3-13td^4+2d}{8e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

```
[Out] RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3 +
 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (-4
9152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256*_t
*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

3.40 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx$

Optimal. Leaf size=342

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\right.}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2}}$$

Rubi [A] time = 0.53, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1106, 1092, 1166, 208}

$$\frac{2e\left(\frac{d}{4e}+x\right)\left(-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2+13d^4\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}-\frac{24e\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}+\frac{24e\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\tanh^{-1}\left(\frac{d+4ex}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\left(d^4-64ae^3\right)^{3/2}\left(256ae^3+5d^4\right)\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] (2*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4)) - (24*e*(d^4 + 128*a*e^3 - d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*
x)/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 +
256*a*e^3)*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) + (24*e*(d^4 + 128*a*e^3 +
 d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(d + 4*e*x)/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]])/((d^4 - 64*a*e^3)^(3/2)*(5*d^4 + 256*a*e^3)*Sqrt[3*d^2 + 2*Sqrt[
d^4 - 64*a*e^3]])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4\right)^2}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{4\,\text{Subst}\Big(}{}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} + \frac{\left(48e^3\left(d^4\right.\right.}{}$$

$$= \frac{2e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)} - \frac{24e\left(d^4+\right.}{\left(d^4-\right.}$$

Mathematica [C] time = 0.19, size = 234, normalized size = 0.68

$$\frac{48e^2\text{RootSum}\left[8\#1^4e^3+8\#1^3de^2-\#1d^3+8ae^2\&,\frac{2\#1^2d^2e\log(x-\#1)+32ae^2\log(x-\#1)+\#1d^3\log(x-\#1)}{32\#1^3e^3+24\#1^2de^2-d^3}\&\right]}{16384a^2e^6+64ad^4e^3-5d^8}+\frac{(d+4ex)\left(-128ae^3+5d^4-12d^3ex-24d^2e^2x^2\right)}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] ((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64*
a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) + (
48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^2*L
og[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*d*e^
2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2), x]
```

fricas [B] time = 1.29, size = 4285, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

```
[Out] -(96*d^2*e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 + 12*sqrt(2)*(40*a*
d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 -
16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (
5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^
5 + 40960*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6
```

+ 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^1
5 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)
/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^
3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27
44381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 51881467707
30811392*a^8*d^4*e^24 - 7378697629483820646 4*a^9*e^27)))/(125*d^24 - 4800*a
*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d
^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*
d^5*e^6 + 226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 1382
4*sqrt(2)*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*
e^11 - 268435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22
*e^6 - 566493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a
^5*d^10*e^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
8697629483820646 4*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^
2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d
^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804651110
4*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1
800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150
994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a
^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4
*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167
360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 515396
07552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 -
512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2
*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 -
64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960
*a^2*d^2*e^8 + (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 3119513
6*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 43980
46511104*a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d
^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^
9 - 150994944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 27443810229
28896*a^6*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*
a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3
- 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 -
51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 +
226492416*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)
*(d^16*e^2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 26
8435456*a^4*e^14 - (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 56
6493184*a^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e
^15 - 30786325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^
8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 -
115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e
^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931
694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737869762948
38206464*a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 + (
125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 +
3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^1
8)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*
d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000
*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e
^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7
3786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d
^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5
*d^4*e^15 - 4398046511104*a^6*e^18))) + 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*
d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4

+ 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*
e^3 - 16384*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*
e^8 - (125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^1
2*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*
a^6*e^18)*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 180
0000*a*d^32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 15099
4944000*a^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6
*d^12*e^18 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e
^24 - 73786976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 116736
0*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607
552*a^5*d^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 22649241
6*a^2*d*e^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x + 13824*sqrt(2)*(d^16*e^
2 - 128*a*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a
^4*e^14 + (125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a
^3*d^18*e^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307
86325577728*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 5
12*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000
*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780
82505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085
312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*
a^9*e^27)))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24
- 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 38252052
48*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((
d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3
- 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20
*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709
31694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629
4838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 +
31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15
- 4398046511104*a^6*e^18))) - 12*sqrt(2)*(40*a*d^8*e^2 - 512*a^2*d^4*e^5 -
131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d
^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 - 163
84*a^2*d^3*e^6)*x)*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (12
5*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3
825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)
*sqrt((d^8*e^4 + 512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^
32*e^3 - 115200000*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a
^4*d^20*e^12 + 78082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^1
8 - 70931694131085312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 737
86976294838206464*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^1
6*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d
^4*e^15 - 4398046511104*a^6*e^18))*log(884736*a*d^5*e^6 + 226492416*a^2*d*e
^9 + 3538944*(a*d^4*e^7 + 256*a^2*e^10)*x - 13824*sqrt(2)*(d^16*e^2 - 128*a
*d^12*e^5 - 61440*a^2*d^8*e^8 + 8388608*a^3*d^4*e^11 - 268435456*a^4*e^14 +
(125*d^30 + 59200*a*d^26*e^3 - 3624960*a^2*d^22*e^6 - 566493184*a^3*d^18*e
^9 + 19797114880*a^4*d^14*e^12 + 1906965479424*a^5*d^10*e^15 - 307863255777
28*a^6*d^6*e^18 - 2251799813685248*a^7*d^2*e^21)*sqrt((d^8*e^4 + 512*a*d^4*
e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 115200000*a^2*d^28
*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 780825054412
80*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 70931694131085312*a^7*d
^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 73786976294838206464*a^9*e^27)
))*sqrt((d^10*e^2 + 160*a*d^6*e^5 + 40960*a^2*d^2*e^8 - (125*d^24 - 4800*a*
d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136*a^3*d^12*e^9 + 3825205248*a^4*d^
8*e^12 - 51539607552*a^5*d^4*e^15 - 4398046511104*a^6*e^18)*sqrt((d^8*e^4 +
512*a*d^4*e^7 + 65536*a^2*e^10)/(15625*d^36 + 1800000*a*d^32*e^3 - 1152000
00*a^2*d^28*e^6 - 21135360000*a^3*d^24*e^9 - 150994944000*a^4*d^20*e^12 + 7
8082505441280*a^5*d^16*e^15 + 2744381022928896*a^6*d^12*e^18 - 709316941310
85312*a^7*d^8*e^21 - 5188146770730811392*a^8*d^4*e^24 - 7378697629483820646
4*a^9*e^27)))/(125*d^24 - 4800*a*d^20*e^3 - 1167360*a^2*d^16*e^6 + 31195136

```
*a^3*d^12*e^9 + 3825205248*a^4*d^8*e^12 - 51539607552*a^5*d^4*e^15 - 439804
6511104*a^6*e^18))) - 8*(d^4*e - 64*a*e^4)*x)/(40*a*d^8*e^2 - 512*a^2*d^4*e
^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*e^6 - 16384*a^2*e^9)*x^4 + 8*
(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*x^3 - (5*d^11 - 64*a*d^7*e^3 -
 16384*a^2*d^3*e^6)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.02, size = 288, normalized size = 0.84

$$\frac{384e^2\left(2\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8ae^2\right)^2d^2e+\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8ae^2\right)d^3+32ae^2\right)\ln\left(-\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8ae^2\right)+x\right)}{\left(2048ae^3+40d^4\right)\left(64ae^3-d^4\right)\left(32\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8ae^2\right)^3e^3+24\,\mathrm{RootOf}\left(8e^3\_Z^4+8e^2d\_Z^3-d^3\_Z+8ae^2\right)^2de^2-d^3\right)}+\frac{\frac{12d^2e^3x^3}{\left(256ae^3+5d^4\right)\left(64ae^3-d^4\right)}+\frac{9d^3e^2x^2}{\left(256ae^3+5d^4\right)\left(64ae^3-d^4\right)}+\frac{ex}{256ae^3+5d^4}+\frac{\left(128ae^3-5d^4\right)d}{131072a^2e^6+512ad^4e^3-40d^8}}{e^3x^4+de^2x^3-\frac{1}{8}d^3x+ae^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

```
[Out] (12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^4
)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384*a
^2*e^6+64*a*d^4*e^3-5*d^8))/(e^3*x^4+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/(20
48*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*_R^3
*e^3+24*_R^2*d*e^2-d^3)*ln(-_R+x),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*d^3+
8*a*e^2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 7.11, size = 10351, normalized size = 30.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

```
[Out] ((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(256
*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4))
+ (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^3*x
 + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 -
 d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11
 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^
3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32
*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20
*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837
1998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(6871
9476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 -
5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5
*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342
17728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^
```

```
19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^
5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 -
 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a
^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^
15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*
e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
+ 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/
2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440
*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^
(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12902
40*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663
6764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*
d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^
2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2
*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(78643
2*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12
- 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + ((288*(d
^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d
^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*
d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460
6846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*
d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062
494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8
*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e
^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e
^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) -
 ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2
*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17
179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 +
 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 2
5769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798691
84*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9
+ 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 11878
4*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1717986
9184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744
*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^1
1 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e
^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^3
2*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^2
0*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483
71998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22
*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14
*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2
*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684
6976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^2
4*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494
720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^
4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))
/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097
152*a^3*d^4*e^9))*1i)/(((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2
```

```
) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*
a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(
1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 129024
0*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636
764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d
^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68719476736*a^5*e
^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 5242880*a^3*d
^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15 - 2240*
a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4
*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^14 + 199
22944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7*e^23 +
1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*
e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)
+ (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 + 12320768
*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24))/(25*d^
16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3
*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^1
8*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14
 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*
d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e
^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d
^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40
532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3
 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^
11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*
e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^
32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^
20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648
371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) + (1536*(96
*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d^5*e^16))/
(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 1
2320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a^2*e^17 +
96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*
e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - ((288*(d^22*e^2 + d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 616038
4*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a
*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27
 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250
489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*
e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2
)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2
*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*
d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(6871947
6736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 524
2880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^
15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 1342177
28*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*
e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d
^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 22
40*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*
d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15
+ 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^2
4))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2
097152*a^3*d^4*e^9))*((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
- 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^
4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/
2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*
a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9663676
4160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8
```

```
*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*((288*(d^22*e^2 + d^4*e^2*(
-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a
^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^
5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) +
 (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 2684
35456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)
) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^
3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12)))*
((288*(d^22*e^2 + d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 225
28*a^2*d^14*e^8 - 6160384*a^3*d^10*e^11 + 461373440*a^4*d^6*e^14 - 10737418
240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152
921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577
856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 +
44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 4053239664633
4464*a^8*d^4*e^24))^(1/2)*2i + atan(((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^
(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^
11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*
e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^
32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^
20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648
371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(68
719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15
- 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a
^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 13
4217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*
d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*
a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15
 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728
*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*
e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^
2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^
6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^2
2*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373
440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^
9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 12
90240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 9
6636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a
^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64
*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 61
60384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 2
56*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*
e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 1
5250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d
^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^
(1/2) + (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 19660
8*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784
*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(7
86432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e
^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*1i + (-(2
88*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*
a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240
*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921
504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856
*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 443
24062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 405323966463344 6
4*a^8*d^4*e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^
```

```
3*d*e^19 + 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d
^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^
12) - ((1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 77004
8*a^2*d^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20
 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*
a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e
^11 + 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^2
0 - 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 1717
9869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8
*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 -
118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17
179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 1
59744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4
)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^
10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(
64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800
*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a
^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7
91648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(28
8*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a
^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*
a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215
04606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*
a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432
4062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464
*a^8*d^4*e^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4
*e^14))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6
 + 2097152*a^3*d^4*e^9))*1i)/((-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) -
 d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 46
1373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d
^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3
+ 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12
 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 7916483719987
20*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2)*(((1536*(687194767
36*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12*e^15 - 52428
80*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 17179869184*a^5*e^15
 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728
*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 24576*a^2*d^19*e^
14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 25769803776*a^5*d^7
*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^5*e^15 - 2240
*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^
4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2*d^14*e^15 +
12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*a^5*d^2*e^24)
)/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 209
7152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 +
 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4
*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2
)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a
^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764
160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*
e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2*(-(64*a*e^3
- d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a
^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^
5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 -
28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489
344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^1
8 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2) +
 (1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 + 196608*a^2*d
^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^
```

```
12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(786432*a
^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(25*d^16 + 268435456*a^4*e^12 - 6
40*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152*a^3*d^4*e^9)) - (-(288*(d^4*e^
2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e
^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e
^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 11529215046068469
76*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*
e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 4432406249472
0*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*
e^24))^(1/2)*((1536*(96*d^13*e^10 + 3072*a*d^9*e^13 - 50331648*a^3*d*e^19 +
 196608*a^2*d^5*e^16))/(25*d^20 - 17179869184*a^5*e^15 - 2240*a*d^16*e^3 -
118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4*d^4*e^12) - ((15
36*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d^12
*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 1717986
9184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^
9 + 134217728*a^4*d^4*e^12) + ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11 + 2457
6*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 - 257698
03776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 17179869184*a^
5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134
217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 - 118784*a^2
*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 17179869184*
a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*
d^8*e^6 + 2097152*a^3*d^4*e^9))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2)
 - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 -
461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 -
 d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^
3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^
12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 79164837199
8720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e^24))^(1/2))*(-(288*(d^4*e^2
*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*d^18*e^5 - 22528*a^2*d^14*e^
8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e^14 + 10737418240*a^5*d^2*e^
17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 115292150460684697
6*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e
^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720
*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 - 40532396646334464*a^8*d^4*e
^24))^(1/2) + (6144*x*(786432*a^2*e^17 + 96*d^8*e^11 + 9216*a*d^4*e^14))/(2
5*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^3 - 159744*a^2*d^8*e^6 + 2097152
*a^3*d^4*e^9)) + (113246208*a*d^2*e^14)/(25*d^20 - 17179869184*a^5*e^15 - 2
240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*e^9 + 134217728*a^4
*d^4*e^12)))*(-(288*(d^4*e^2*(-(64*a*e^3 - d^4)^9)^(1/2) - d^22*e^2 + 32*a*
d^18*e^5 - 22528*a^2*d^14*e^8 + 6160384*a^3*d^10*e^11 - 461373440*a^4*d^6*e
^14 + 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(64*a*e^3 - d^4)^9)^(1/2)))/(1
25*d^36 + 1152921504606846976*a^9*e^27 - 28800*a*d^32*e^3 + 1290240*a^2*d^2
8*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*a^4*d^20*e^12 - 96636764160*a^
5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 - 791648371998720*a^7*d^8*e^21 -
 40532396646334464*a^8*d^4*e^24))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

```
[Out] Timed out
```

3.41 $\int \left(8+8x-x^3+8x^4\right)^4 \, dx$

Optimal. Leaf size=96

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+716$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \int \left(4096+16384x+24576x^2+14336x^3+14336x^4+43008x^5+47488x^6+11\right.$$

$$= 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+$$

Mathematica [A] time = 0.00, size = 96, normalized size = 1.00

$$\frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^4, x]
```

fricas [A] time = 0.68, size = 84, normalized size = 0.88

$$\frac{4096}{17}x^{17}-128x^{16}+\frac{128}{5}x^{15}+1168x^{14}+\frac{10241}{13}x^{13}-448x^{12}+\frac{25312}{11}x^{11}+\frac{21488}{5}x^{10}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336}{5}x^5+3584x^4+8192x^3+8192x^2+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

giac [A] time = 0.31, size = 84, normalized size = 0.88

$$\frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^{9} + 1376\,x^{8} + 6784\,x^{7} + 7168\,x^{6} + \frac{14336}{5}\,x^{5} + 3584\,x^{4} + 8192\,x^{3} + 8192\,x^{2} + 4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

maple [A] time = 0.00, size = 85, normalized size = 0.89

$$\frac{4096}{17}x^{17} - 128x^{16} + \frac{128}{5}x^{15} + 1168x^{14} + \frac{10241}{13}x^{13} - 448x^{12} + \frac{25312}{11}x^{11} + \frac{21488}{5}x^{10} + 1408x^{9} + 1376x^{8} + 6784x^{7} + 7168x^{6} + \frac{14336}{5}x^{5} + 3584x^{4} + 8192x^{3} + 8192x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^4,x)
```

```
[Out] 4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+14
08*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*x^
15-128*x^16+4096/17*x^17
```

maxima [A] time = 1.25, size = 84, normalized size = 0.88

$$\frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^{9} + 1376\,x^{8} + 6784\,x^{7} + 7168\,x^{6} + \frac{14336}{5}\,x^{5} + 3584\,x^{4} + 8192\,x^{3} + 8192\,x^{2} + 4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^12
 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x^6
 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

mupad [B] time = 0.19, size = 84, normalized size = 0.88

$$\frac{4096\,x^{17}}{17} - 128\,x^{16} + \frac{128\,x^{15}}{5} + 1168\,x^{14} + \frac{10241\,x^{13}}{13} - 448\,x^{12} + \frac{25312\,x^{11}}{11} + \frac{21488\,x^{10}}{5} + 1408\,x^{9} + 1376\,x^{8} + 6784\,x^{7} + 7168\,x^{6} + \frac{14336\,x^{5}}{5} + 3584\,x^{4} + 8192\,x^{3} + 8192\,x^{2} + 4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*x
^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 + (1
0241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

sympy [A] time = 0.07, size = 94, normalized size = 0.98

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^{9} + 1376x^{8} + 6784x^{7} + 7168x^{6} + \frac{14336x^{5}}{5} + 3584x^{4} + 8192x^{3} + 8192x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 448
*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x**7
 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

3.42 $\int \left(8+8x-x^3+8x^4\right)^3 \, dx$

Optimal. Leaf size=74

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+8x-x^3+8x^4\right)^3 \, dx &= \int \left(512+1536x+1536x^2+320x^3+1152x^4+2880x^5+1560x^6-360x^7+115\right.\\ &= 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{30}{}\end{aligned}$$

Mathematica [A] time = 0.00, size = 74, normalized size = 1.00

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^3, x]
```

fricas [A] time = 0.48, size = 64, normalized size = 0.86

$$\frac{512}{13}x^{13}-16x^{12}+\frac{24}{11}x^{11}+\frac{307}{2}x^{10}+128x^9-45x^8+\frac{1560}{7}x^7+480x^6+\frac{1152}{5}x^5+80x^4+512x^3+768x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

giac [A] time = 0.33, size = 64, normalized size = 0.86

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^{9} - 45\,x^{8} + \frac{1560}{7}\,x^{7} + 480\,x^{6} + \frac{1152}{5}\,x^{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.88

$$\frac{512}{13}x^{13} - 16x^{12} + \frac{24}{11}x^{11} + \frac{307}{2}x^{10} + 128x^{9} - 45x^{8} + \frac{1560}{7}x^{7} + 480x^{6} + \frac{1152}{5}x^{5} + 80x^{4} + 512x^{3} + 768x^{2} + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^3,x)
```

```
[Out] 512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+3
07/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

maxima [A] time = 0.94, size = 64, normalized size = 0.86

$$\frac{512}{13}\,x^{13} - 16\,x^{12} + \frac{24}{11}\,x^{11} + \frac{307}{2}\,x^{10} + 128\,x^{9} - 45\,x^{8} + \frac{1560}{7}\,x^{7} + 480\,x^{6} + \frac{1152}{5}\,x^{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/7
*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.86

$$\frac{512\,x^{13}}{13} - 16\,x^{12} + \frac{24\,x^{11}}{11} + \frac{307\,x^{10}}{2} + 128\,x^{9} - 45\,x^{8} + \frac{1560\,x^{7}}{7} + 480\,x^{6} + \frac{1152\,x^{5}}{5} + 80\,x^{4} + 512\,x^{3} + 768\,x^{2} + 512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
- 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/13
```

sympy [A] time = 0.07, size = 71, normalized size = 0.96

$$\frac{512x^{13}}{13} - 16x^{12} + \frac{24x^{11}}{11} + \frac{307x^{10}}{2} + 128x^{9} - 45x^{8} + \frac{1560x^{7}}{7} + 480x^{6} + \frac{1152x^{5}}{5} + 80x^{4} + 512x^{3} + 768x^{2} + 512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 512*
x
```

3.43 $\int \left(8+8x-x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(8+8x-x^3+8x^4\right)^2 \, dx &= \int \left(64+128x+64x^2-16x^3+112x^4+128x^5+x^6-16x^7+64x^8\right) dx\\ &= 64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 54, normalized size = 1.00

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+8x-x^3+8x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^2, x]
```

fricas [A] time = 0.76, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9-2x^8+\frac{1}{7}x^7+\frac{64}{3}x^6+\frac{112}{5}x^5-4x^4+\frac{64}{3}x^3+64x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

giac [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.83

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] 64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

maxima [A] time = 0.88, size = 44, normalized size = 0.81

$$\frac{64}{9}x^9 - 2x^8 + \frac{1}{7}x^7 + \frac{64}{3}x^6 + \frac{112}{5}x^5 - 4x^4 + \frac{64}{3}x^3 + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*x
^2 + 64*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.81

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*x
^8 + (64*x^9)/9
```

sympy [A] time = 0.06, size = 49, normalized size = 0.91

$$\frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3 +
 64*x**2 + 64*x
```

3.44 $\int \left(8+8x-x^3+8x^4\right)\,dx$

Optimal. Leaf size=23

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi steps

$$\int \left(8+8x-x^3+8x^4\right)\,dx=8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+8x-x^3+8x^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[8 + 8*x - x^3 + 8*x^4,x]
```

```
[Out] IntegrateAlgebraic[8 + 8*x - x^3 + 8*x^4, x]
```

fricas [A] time = 0.73, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5-\frac{1}{4}x^4+4x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

```
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

giac [A] time = 0.32, size = 19, normalized size = 0.83

$$\frac{8}{5}x^5-\frac{1}{4}x^4+4x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{8}{5}x^{5}-\frac{1}{4}x^{4}+4x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-x^3+8*x+8,x)
[Out] 8*x+4*x^2-1/4*x^4+8/5*x^5
```

maxima [A] time = 0.91, size = 19, normalized size = 0.83

$$\frac{8}{5}\,x^{5}-\frac{1}{4}\,x^{4}+4\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
[Out] 8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

mupad [B] time = 0.03, size = 19, normalized size = 0.83

$$\frac{8\,x^{5}}{5}-\frac{x^{4}}{4}+4\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x - x^3 + 8*x^4 + 8,x)
[Out] 8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

sympy [A] time = 0.06, size = 19, normalized size = 0.83

$$\frac{8x^{5}}{5}-\frac{x^{4}}{4}+4x^{2}+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-x**3+8*x+8,x)
[Out] 8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

3.45 $\int \frac{1}{8+8x-x^3+8x^4} \, dx$

Optimal. Leaf size=268

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\right.$$

Rubi [A] time = 0.40, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$-\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)+\frac{1}{24}\sqrt{\frac{67\sqrt{29}-109}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)-\frac{\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}}-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] -ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])]/(12*Sqrt[7]) - (Sqrt[(109 + 67*Sqrt[
29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])
]])/12 - (Sqrt[(109 + 67*Sqrt[29])/1218]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])]
 + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/12 - (Sqrt[(-109 + 67*Sqrt[29])/1218]*Log
[3*Sqrt[29] - Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/24 + (Sqrt[(
-109 + 67*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x)
 + (1 + 4/x)^2])/24
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^2}{8\left(1069056-393216x^2+1048576x^4\right)}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int \frac{(8-32x)^2}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int -\frac{512x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 128\,\text{Subst}\left(\int$$

$$= 65536\,\text{Subst}\left(\int \frac{x}{1069056-393216x^2+1048576x^4}\,dx, x, \frac{1}{4}+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{6\left(1+\right.}}{\frac{3\sqrt{29}}{16}-}\right.}{768}$$

$$= 32768\,\text{Subst}\left(\int \frac{1}{1069056-393216x+1048576x^2}\,dx, x, \left(\frac{1}{4}+\frac{1}{x}\right)^2\right) - \frac{\left(87+\sqrt{29}\right)\text{Su}}{}$$

$$= -\frac{1}{24}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\,\log\left(3\sqrt{29}-\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)+\left(1+\frac{4}{x}\right)^2\right)+\frac{1}{24}\sqrt{\frac{-109}{}}$$

$$= -\frac{\tan^{-1}\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} - \frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\,\tan^{-1}\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) - \frac{1}{12}\sqrt{\frac{109}{}}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 0.17

$$\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&,\frac{\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-1), x]
```

fricas [C] time = 3.62, size = 1015, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
```

```
[Out] -1/168*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696))*log(287314195
392*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^3 - 120389
06880*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696))^2 + 1687
8104*x + 4897683*I*sqrt(7) - 411405372*sqrt(65/43848*I*sqrt(7) - 109/87696)
 + 6055613) - 1/168*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696))*
log(-35914274424*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696
))^3 + 16443*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7) - 109/87696))
^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/87696) - 91520
) + 609*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
)^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/87696)) + 2109763
*x - 1911147/8*I*sqrt(7) + 40134087/2*sqrt(65/43848*I*sqrt(7) - 109/87696)
- 1461344) + 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/43
848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848
*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7
) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) - 7)
 + 261*sqrt(65/43848*I*sqrt(7) - 109/87696) + 261*sqrt(-65/43848*I*sqrt(7)
- 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I*sqrt(7)
 - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7) - 109/
87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt
(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) - 109/8
7696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) - 109/876
96))^2 + 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7)
- 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7) -
 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) -
109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sqrt(65/
43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/438
48*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt
(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87696)) -
7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt(7) - 10
9/87696) + 220336) - 1/1044*(sqrt(174)*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*sq
rt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt(-
65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/43848*
I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/8769
```

```
6)) - 7) - 261*sqrt(65/43848*I*sqrt(7) - 109/87696) - 261*sqrt(-65/43848*I*
sqrt(7) - 109/87696))*log(-16443/2*(-1/168*I*sqrt(7) - 1/2*sqrt(-65/43848*I
*sqrt(7) - 109/87696))^2*(-13001*I*sqrt(7) + 1092084*sqrt(65/43848*I*sqrt(7
) - 109/87696) - 91520) - 609/2*(351027*(1/168*I*sqrt(7) - 1/2*sqrt(65/4384
8*I*sqrt(7) - 109/87696))^2 - 613)*(I*sqrt(7) + 84*sqrt(-65/43848*I*sqrt(7)
 - 109/87696)) + 752431680*(1/168*I*sqrt(7) - 1/2*sqrt(65/43848*I*sqrt(7) -
 109/87696))^2 - 1/32*(3*(13001*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*
sqrt(7) - 109/87696)) - 91520*sqrt(174))*(I*sqrt(7) + 84*sqrt(-65/43848*I*s
qrt(7) - 109/87696)) - 274560*sqrt(174)*(-I*sqrt(7) + 84*sqrt(65/43848*I*sq
rt(7) - 109/87696)) + 1922368*sqrt(174))*sqrt(-4698*(1/168*I*sqrt(7) - 1/2*
sqrt(65/43848*I*sqrt(7) - 109/87696))^2 - 4698*(-1/168*I*sqrt(7) - 1/2*sqrt
(-65/43848*I*sqrt(7) - 109/87696))^2 - 87/784*(I*sqrt(7) + 84*sqrt(-65/4384
8*I*sqrt(7) - 109/87696))*(-I*sqrt(7) + 84*sqrt(65/43848*I*sqrt(7) - 109/87
696)) - 7) + 2109763*x - 373317/2*I*sqrt(7) + 15679314*sqrt(65/43848*I*sqrt
(7) - 109/87696) + 220336)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - x^3 + 8x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

maple [C] time = 0.02, size = 41, normalized size = 0.15

$$\frac{\ln\left(-\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right) + x\right)}{32\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right)^3 - 3\,\mathrm{RootOf}\left(8\_Z^4 - \_Z^3 + 8\_Z + 8\right)^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8),x)
```

```
[Out] sum(1/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8x^4 - x^3 + 8x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

mupad [B] time = 2.45, size = 123, normalized size = 0.46

$$\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)\left(8064\,\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)+256x+\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)x\,12285+\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)^2x\,148176+198072\,\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)^2-8\right)}{4096}\right)\mathrm{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(8
064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x + 1
2285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148176
*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 198072*r
```

```
oot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/4096)*ro
ot(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)
```

sympy [A] time = 0.96, size = 41, normalized size = 0.15

$$\operatorname{RootSum}\left(66298176t^4 + 74088t^2 + 4095t + 64, \left(t \mapsto t\log\left(\frac{35914274424t^3}{2109763} - \frac{1504863360t^2}{2109763} + \frac{102851343t}{2109763} + x + \frac{6055613}{16878104}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8),x)
```

```
[Out] RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(3591
4274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 + x
 + 6055613/16878104)))
```

3.46 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

Optimal. Leaf size=357

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^{2}\right.}{175\ldots}$$

Rubi [A] time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$-\frac{29\left(\frac{4}{x}+1\right)^2+207}{336\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}+\frac{5\left(199\left(\frac{4}{x}+1\right)^2+5157\right)\left(\frac{4}{x}+1\right)}{87696\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)}-\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2-\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}+\frac{\sqrt{\frac{45923327\sqrt{29}-180983329}{1218}}\log\left(\left(\frac{4}{x}+1\right)^2+\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{4}{x}+1\right)+3\sqrt{29}\right)}{175392}-\frac{17\tan^{-1}\left(\frac{3-\left(\frac{4}{x}+1\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}-\frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{6\left(1+\sqrt{29}\right)}+2}{\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{87696}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] -(207 + 29*(1 + 4/x)^2)/(336*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)) + (5*(515
7 + 199*(1 + 4/x)^2)*(1 + 4/x))/(87696*(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4))
 - (17*ArcTan[(3 - (1 + 4/x)^2)/(6*Sqrt[7])])/(1008*Sqrt[7]) - (Sqrt[(18098
3329 + 45923327*Sqrt[29])/1218]*ArcTan[(2 - Sqrt[6*(1 + Sqrt[29])] + 8/x)/S
qrt[6*(-1 + Sqrt[29])]])/87696 - (Sqrt[(180983329 + 45923327*Sqrt[29])/1218
]*ArcTan[(2 + Sqrt[6*(1 + Sqrt[29])] + 8/x)/Sqrt[6*(-1 + Sqrt[29])]])/87696
 - (Sqrt[(-180983329 + 45923327*Sqrt[29])/1218]*Log[3*Sqrt[29] - Sqrt[6*(1
+ Sqrt[29])]*(1 + 4/x) + (1 + 4/x)^2])/175392 + (Sqrt[(-180983329 + 4592332
7*Sqrt[29])/1218]*Log[3*Sqrt[29] + Sqrt[6*(1 + Sqrt[29])]*(1 + 4/x) + (1 +
4/x)^2])/175392
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(8-32x)^6}{64\left(1069056-393216x^2+1048576x^4\right)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(8-32x)^6}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-6291456-335544320x^2-1610612736x^4\right)}{\left(1069056-393216x^2+1048576x^4\right)^2}\,dx, x, \frac{1}{4}+\frac{1}{x}\right)\right) - 16$$

$$= \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{27892774076141524746 24+7758008804499}{1069056-393216x^2+1048576x^4}\right.}{57853663025666457}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \text{Sub}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{1392}{21}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \sqrt{-1}$$

$$= -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{17\,\text{ta}}{}$$

Mathematica [C] time = 0.02, size = 113, normalized size = 0.32

$$\frac{\text{RootSum}\left[8\#1^4-\#1^3+8\#1+8\&, \frac{392\#1^2\log(x-\#1)-1097\#1\log(x-\#1)+2243\log(x-\#1)}{32\#1^3-3\#1^2+8}\&\right]}{21924} + \frac{784x^3-1146x^2+1539x+544}{43848\left(8x^4-x^3+8x+8\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + RootS
um[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 + 3
92*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8+8x-x^3+8x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(8 + 8*x - x^3 + 8*x^4)^(-2), x]
```

fricas [C] time = 5.20, size = 1201, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

```
[Out] 1/213627456*(3819648*x^3 - 15138*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*log(621
785056787306565435997385932 8*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/3345405
96096*I*sqrt(7) - 180983329/4683568345344))^3 - 100287672431797174786327756
80*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 + 674816656554692870314 16*x + 320944207138750561964778*
I*sqrt(7) - 133210725033589645013145504*sqrt(4550065/334540596096*I*sqrt(7)
 - 180983329/4683568345344) + 333979081113202533090737) - 15138*(8*x^4 - x^
3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344))*log(-777231320984133206794996732416*(17/14112*I*sqr
t(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^
3 + 878169064752*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sq
rt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqrt(7) + 44252943
5492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) -
 1427510892508480) + 7569*(727651150781043057 3072*(17/14112*I*sqrt(7) - 1/2
*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 233594
23554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 18
0983329/4683568345344)) + 843520820693366087 8927*x - 148449195141328682772 6
33/4*I*sqrt(7) + 154037870723119880241720 36*sqrt(4550065/334540596096*I*sqr
t(7) - 180983329/4683568345344) - 4739360660669659506761 6) - 5583312*x^2 +
(56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-125452723536*(17/14112*I*sqrt(7)
 - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 -
125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7
) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-45
50065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7
056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 66301
91) + 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) + 7056*sqrt(-4550065/33454
0596096*I*sqrt(7) - 180983329/4683568345344)) + 7569*(8*x^4 - x^3 + 8*x + 8
)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468
3568345344)))*log(-439084532376*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/33
4540596096*I*sqrt(7) - 180983329/4683568345344))^2*(-1066184864424603*I*sqr
t(7) + 442529435492941104*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4
683568345344) - 1427510892508480) - 7569/2*(727651150781043057 3072*(17/1411
2*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356834
5344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*sqrt(-4550065/3345405960
96*I*sqrt(7) - 180983329/4683568345344)) + 626797952698732342414548480*(17/
14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835
68345344))^2 + 1/16*(261*(62716756730859*sqrt(87)*(-17*I*sqrt(7) + 7056*sqr
t(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 142751089250
8480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) -
180983329/4683568345344)) - 372580342944713280*sqrt(87)*(-17*I*sqrt(7) + 70
56*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 104650
21752358451264*sqrt(87))*sqrt(-125452723536*(17/14112*I*sqrt(7) - 1/2*sqrt(
4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 125452723536
*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 18098332
9/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7056*sqrt(-4550065/334540
596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I*sqrt(7) + 7056*sqrt(455
0065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 6630191) + 843520
8206933660878927*x - 3005727107011649552439/2*I*sqrt(7) + 62377677844335880
1235576*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/4683568345344) + 22
```

```
959102208397854107O4) - (56*sqrt(87)*(8*x^4 - x^3 + 8*x + 8)*sqrt(-12545272
3536*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983
329/4683568345344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-45500
65/334540596096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I
*sqrt(7) + 7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345
344))*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329
/4683568345344)) - 6630191) - 7569*(8*x^4 - x^3 + 8*x + 8)*(17*I*sqrt(7) +
7056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) - 756
9*(8*x^4 - x^3 + 8*x + 8)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I
*sqrt(7) - 180983329/4683568345344)))*log(-439084532376*(-17/14112*I*sqrt(7
) - 1/2*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))^2*
(-1066184864424603*I*sqrt(7) + 442529435492941104*sqrt(4550065/334540596096
*I*sqrt(7) - 180983329/4683568345344) - 1427510892508480) - 7569/2*(7276511
507810430573072*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(
7) - 180983329/4683568345344))^2 - 23359423554371543)*(17*I*sqrt(7) + 7056*
sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344)) + 62679795
269873234241454848O*(17/14112*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*s
qrt(7) - 180983329/4683568345344))^2 - 1/16*(261*(62716756730859*sqrt(87)*(
-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/468356
8345344)) - 1427510892508480*sqrt(87))*(17*I*sqrt(7) + 7056*sqrt(-4550065/3
34540596096*I*sqrt(7) - 180983329/4683568345344)) - 372580342944713280*sqrt
(87)*(-17*I*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/
4683568345344)) + 10465021752358451264*sqrt(87))*sqrt(-125452723536*(17/141
12*I*sqrt(7) - 1/2*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683
45344))^2 - 125452723536*(-17/14112*I*sqrt(7) - 1/2*sqrt(-4550065/334540596
096*I*sqrt(7) - 180983329/4683568345344))^2 - 658503/1568*(17*I*sqrt(7) + 7
056*sqrt(-4550065/334540596096*I*sqrt(7) - 180983329/4683568345344))*(-17*I
*sqrt(7) + 7056*sqrt(4550065/334540596096*I*sqrt(7) - 180983329/46835683453
44)) - 6630191) + 8435208206933660878927*x - 3005727107011649552439/2*I*sqr
t(7) + 623776778443358801235576*sqrt(4550065/334540596096*I*sqrt(7) - 18098
3329/4683568345344) + 229591022083978541O704) + 7498008*x + 2650368)/(8*x^4
 - x^3 + 8*x + 8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^{4} - x^{3} + 8\,x + 8\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

maple [C] time = 0.01, size = 83, normalized size = 0.23

$$\frac{\left(392\,\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)^{2}-1097\,\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)+2243\right)\ln\left(-\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)+x\right)}{701568\,\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)^{3}-65772\,\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)^{2}+175392}+\frac{\frac{7}{3132}x^{3}-\frac{191}{58464}x^{2}+\frac{57}{12992}x+\frac{17}{10962}}{x^{4}-\frac{1}{8}x^{3}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^2,x)
```

```
[Out] (7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*su
m((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(-_R+x),_R=RootOf(8*_Z^4-_Z^
3+8*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{784\,x^{3} - 1146\,x^{2} + 1539\,x + 544}{43848\,\left(8\,x^{4} - x^{3} + 8\,x + 8\right)} + \frac{1}{21924}\,\int \frac{392\,x^{2} - 1097\,x + 2243}{8\,x^{4} - x^{3} + 8\,x + 8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/219
24*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

mupad [B] time = 0.21, size = 176, normalized size = 0.49

$$\left(\sum_{k=1}^{4}\ln\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)x\,34885379}{72918171648}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2x\,191555}{475136}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3x\,9261}{256}-\frac{11205\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2}{59392}-\frac{24759\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3}{512}+\frac{10901}{107668237824}\right)\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\right)+\frac{\frac{7x^3}{3132}-\frac{191x^2}{58464}+\frac{57x}{12992}+\frac{17}{10962}}{x^4-\frac{x^3}{8}+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/67
4433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/40
375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (191555
*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 111
4096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191*z^2)/
167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709, z,
 k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/
674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (24759*root(z
^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13
723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (6630191*z^2
)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377709,
z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/3132 + 17/10962
)/(x - x^3/8 + x^4 + 1)
```

sympy [B] time = 3.22, size = 3834, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

```
[Out] (784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
+ 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/1292018853888
)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424 - 26
7658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660878927
- 2157374520970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11
3504161632499340786841712 + 3881045239007430*sqrt(29)/5326727264361229 + 43
5853770857118353330297/33740832827734643515708 + 20905585576953*sqrt(42)*sq
rt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 2942814074101429415
084030510182204250067556953*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/888496186
751485201253966401139075287452416534006272 - 142576256328563148358311429727
65102609010539559351093/27765505835983912539186450035596102732888016687696
- 75184631502818837388875900060881355871*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-
180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637799543154662112205
737970312940946635052896768 - 963314181796141259748858766106570487809406229
9*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/3063779954315466211220573
7970312940946635052896768 - 1398888334001652366855237255*sqrt(42)*sqrt(-180
983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/359456428
291497016547944746810895370264 + 91245981690030498967778233214015591679*sqr
t(42)*sqrt(-180983329 + 45923327*sqrt(29))/23005211410655809059068463795897
```

```
303696896 + 1030417535184194126067674556970150551 9*sqrt(29)*sqrt(2140954230
17213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 40699873480352667)/1934738279636153541867657805234963240908953 6 + 63911
10884897489624999840174039179843740854 85*sqrt(29)/48368456990903838546691 44
513087408102272384) + sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/129
2018853888)*log(x**2 + x*(-62716756730859*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 214095423017213*sqrt(29) + 40699873480352667)/2270083232649986815 73
683424 - 20905585576953*sqrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227
636229779664 + 3881045239007430*sqrt(29)/532672726436122 9 + 267658292345340
*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095
423017213*sqrt(29) + 40699873480352667)/843520820693366087892 7 + 2157374520
970352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/11350416163249
9340786841712 + 435853770857118353330297/337408328277346435157 08) - 1425762
563285631483583114297276510260901053955935109 3/2776550583598391253918645003
5596102732888016687696 - 103041753518419412606767455697015055 19*sqrt(29)*sq
rt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423
017213*sqrt(29) + 40699873480352667)/1934738279636153541867657805234963240 9
089536 - 912459816900304989677782332140155916 79*sqrt(42)*sqrt(-180983329 +
45923327*sqrt(29))/230052114106558090590684637958973 03696896 - 751846315028
188373888759000608813558 71*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954
23017213*sqrt(29) + 40699873480352667)/306377995431546621122057379703129409
46635052896768 - 139888833400165236685523725 5*sqrt(42)*sqrt(-180983329 + 45
923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sq
rt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/3594564282914970165
47944746810895370264 + 96331418179614125974885876610657048780940622 99*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))/30637799543154662112205737970312
940946635052896768 + 294281407410142941508403051018220425006755695 3*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721
3*sqrt(29) + 40699873480352667)/888496186751485201253966401139075287452416 5
34006272 + 6391110884897489624999840174039179843740854 85*sqrt(29)/483684569
909038385466914451308740810227238 4) - 2*sqrt(199631405/37468546762752 + sqr
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)/9367136690688 + 1583563*sqrt(29)/430672
951296)*atan(454016646529997363147366848*x/(-450967351627246742986 0*sqrt(12
18)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*s
qrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(19
9631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29
)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) +
20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*s
qrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4069987348
0352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329
 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 29
324241703266922812062382 16/(-450967351627246742986 0*sqrt(1218)*sqrt(1996314
05 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) +
214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 36016
09981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt
(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301
7213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29)) + 20905585576953*s
qrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 13776
9981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 214095423017213*sqrt(29) + 40699873480352667)) + 431474904194070573
3646*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/(-45096735162724674298
60*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1809833
29 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 1
37769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29
```

```
))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 459233
27*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sq
rt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sq
rt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) +
40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(
-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352
667)) + 7203219963597790080*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-450967
3516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)
*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873
480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45
923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1809
83329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)
+ 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-
47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230172
13*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sq
rt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) +
40699873480352667)) + 1653979129206147051605980800*sqrt(29)/(-45096735162724
67429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4069987348035266
7) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 45923327*s
qrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 +
45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) + 137769
981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822
945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(
29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqrt(1218)
*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873
480352667)) - 5568313446945998439259 8*sqrt(42)*sqrt(-180983329 + 45923327*s
qrt(29))/(-4509673516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sq
rt(29) + 40699873480352667) + 137769981*sqrt(29)) + 3601609981798895040*sqr
t(-180983329 + 45923327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40
699873480352667) + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199
631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)
) + 214095423017213*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqr
t(-47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 2140954230
17213*sqrt(29) + 40699873480352667)) - 62716756730859*sqrt(1218)*sqrt(-1809
83329 + 45923327*sqrt(29))*sqrt(-47106822945*sqrt(1218)*sqrt(-180983329 + 4
5923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667)/(-4509673
516272467429860*sqrt(1218)*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*
sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 406998734
80352667) + 137769981*sqrt(29)) + 3601609981798895040*sqrt(-180983329 + 459
23327*sqrt(29))*sqrt(199631405 + 4*sqrt(-47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 40699873480352667) +
 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(199631405 + 4*sqrt(-4
7106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 21409542301721
3*sqrt(29) + 40699873480352667) + 137769981*sqrt(29))*sqrt(-47106822945*sqr
t(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 214095423017213*sqrt(29) + 4
0699873480352667))) + 2*sqrt(-sqrt(214095423017213*sqrt(29) + 47106822945*s
qrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/9367136
690688 + 199631405/37468546762752 + 1583563*sqrt(29)/430672951296)*atan(454
0166465299973631473668 48*x/(3601609981798895040*sqrt(-180983329 + 45923327*
sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 13776
9981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sq
rt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-18098
```

```
3329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213
*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4
0699873480352667) + 199631405 + 137769981*sqrt(29))) - 62716756730859*sqrt(
1218)*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) +
47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987348035
2667)/(360160998179889504O*sqrt(-180983329 + 45923327*sqrt(29))*sqrt(-4*sqr
t(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923
327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 4509
673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106
822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)
 + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 471068229
45*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 1
99631405 + 137769981*sqrt(29))) - 7203219963597790080*sqrt(214095423017213*
sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40
699873480352667)/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*
sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983
329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(
29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(
29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 4069987
3480352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*s
qrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 459
23327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29))) - 4314749041940705733646*sqrt(12
18)*sqrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-1809833
29 + 45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945
*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199
631405 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sq
rt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592
3327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 209
05585576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(121
8)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(2
14095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327
*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 165397
9129206147051605980080*sqrt(29)/(3601609981798895040*sqrt(-180983329 + 45923
327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218
)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 1
37769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(2140954
23017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(
29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 2090558557695
3*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(21409542301
7213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))
 + 40699873480352667) + 199631405 + 137769981*sqrt(29))) + 2932424170326692
281206238216/(3601609981798895040*sqrt(-180983329 + 45923327*sqrt(29))*sqrt
(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329
+ 45923327*sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))
 + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(214095423017213*sqrt(29)
+ 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480
352667) + 199631405 + 137769981*sqrt(29)) + 20905585576953*sqrt(1218)*sqrt(
214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 4592332
7*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47
106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 406998734803526
67) + 199631405 + 137769981*sqrt(29))) + 5568313446945998439259 8*sqrt(42)*s
qrt(-180983329 + 45923327*sqrt(29))/(3601609981798895040*sqrt(-180983329 +
45923327*sqrt(29))*sqrt(-4*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt
(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667) + 19963140
```

```
5 + 137769981*sqrt(29)) + 4509673516272467429860*sqrt(1218)*sqrt(-4*sqrt(21
4095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*
sqrt(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29)) + 20905585
576953*sqrt(1218)*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sq
rt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)*sqrt(-4*sqrt(214095
423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29)) + 40699873480352667) + 199631405 + 137769981*sqrt(29))))
```

3.47 $\int \left(1+4x+4x^2+4x^4\right)^4 \, dx$

Optimal. Leaf size=97

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2061}

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx = \int \left(1+16x+112x^2+448x^3+1136x^4+1984x^5+2752x^6+3584x^7+4192x^8+3$$
$$= x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x$$

Mathematica [A] time = 0.00, size = 97, normalized size = 1.00

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+4x+4x^2+4x^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^4, x]
```

fricas [A] time = 0.72, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

giac [A] time = 0.36, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256\,x^{12} + \frac{3328}{11}x^{11} + 384\,x^{10} + \frac{4192}{9}x^{9} + 448\,x^{8} + \frac{2752}{7}x^{7} + \frac{992}{3}x^{6} + \frac{1136}{5}x^{5} + 112\,x^{4} + \frac{112}{3}x^{3} + 8\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

maple [A] time = 0.00, size = 78, normalized size = 0.80

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256x^{12} + \frac{3328}{11}x^{11} + 384x^{10} + \frac{4192}{9}x^{9} + 448x^{8} + \frac{2752}{7}x^{7} + \frac{992}{3}x^{6} + \frac{1136}{5}x^{5} + 112x^{4} + \frac{112}{3}x^{3} + 8x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^4,x)
```

```
[Out] x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x^
9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256/1
7*x^17
```

maxima [A] time = 0.57, size = 77, normalized size = 0.79

$$\frac{256}{17}x^{17} + \frac{1024}{15}x^{15} + \frac{512}{7}x^{14} + \frac{1792}{13}x^{13} + 256\,x^{12} + \frac{3328}{11}x^{11} + 384\,x^{10} + \frac{4192}{9}x^{9} + 448\,x^{8} + \frac{2752}{7}x^{7} + \frac{992}{3}x^{6} + \frac{1136}{5}x^{5} + 112\,x^{4} + \frac{112}{3}x^{3} + 8\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

```
[Out] 256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/11
*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5*x
^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

mupad [B] time = 0.15, size = 77, normalized size = 0.79

$$\frac{256\,x^{17}}{17} + \frac{1024\,x^{15}}{15} + \frac{512\,x^{14}}{7} + \frac{1792\,x^{13}}{13} + 256\,x^{12} + \frac{3328\,x^{11}}{11} + 384\,x^{10} + \frac{4192\,x^{9}}{9} + 448\,x^{8} + \frac{2752\,x^{7}}{7} + \frac{992\,x^{6}}{3} + \frac{1136\,x^{5}}{5} + 112\,x^{4} + \frac{112\,x^{3}}{3} + 8\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)
```

```
[Out] x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7)
/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (1792*
x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

sympy [A] time = 0.07, size = 94, normalized size = 0.97

$$\frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^{9}}{9} + 448x^{8} + \frac{2752x^{7}}{7} + \frac{992x^{6}}{3} + \frac{1136x^{5}}{5} + 112x^{4} + \frac{112x^{3}}{3} + 8x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

```
[Out] 256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 33
28*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6/3
 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

3.48 $\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$

Optimal. Leaf size=69

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^3 \, dx &= \int \left(1+12x+60x^2+160x^3+252x^4+288x^5+352x^6+384x^7+240x^8+192x^9+\right.\\ &= x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^1}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^3, x]
```

fricas [A] time = 0.71, size = 57, normalized size = 0.83

$$\frac{64}{13}x^{13}+\frac{192}{11}x^{11}+\frac{96}{5}x^{10}+\frac{80}{3}x^9+48x^8+\frac{352}{7}x^7+48x^6+\frac{252}{5}x^5+40x^4+20x^3+6x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

giac [A] time = 0.36, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^{9} + 48\,x^{8} + \frac{352}{7}\,x^{7} + 48\,x^{6} + \frac{252}{5}\,x^{5} + 40\,x^{4} + 20\,x^{3} + 6\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{64}{13}x^{13}+\frac{192}{11}x^{11}+\frac{96}{5}x^{10}+\frac{80}{3}x^{9}+48x^{8}+\frac{352}{7}x^{7}+48x^{6}+\frac{252}{5}x^{5}+40x^{4}+20x^{3}+6x^{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^3,x)
```

```
[Out] x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10+
192/11*x^11+64/13*x^13
```

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{64}{13}\,x^{13} + \frac{192}{11}\,x^{11} + \frac{96}{5}\,x^{10} + \frac{80}{3}\,x^{9} + 48\,x^{8} + \frac{352}{7}\,x^{7} + 48\,x^{6} + \frac{252}{5}\,x^{5} + 40\,x^{4} + 20\,x^{3} + 6\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

```
[Out] 64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*x
^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

mupad [B] time = 0.06, size = 57, normalized size = 0.83

$$\frac{64\,x^{13}}{13}+\frac{192\,x^{11}}{11}+\frac{96\,x^{10}}{5}+\frac{80\,x^9}{3}+48\,x^8+\frac{352\,x^7}{7}+48\,x^6+\frac{252\,x^5}{5}+40\,x^4+20\,x^3+6\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

```
[Out] x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8 +
 (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

sympy [A] time = 0.07, size = 66, normalized size = 0.96

$$\frac{64x^{13}}{13} + \frac{192x^{11}}{11} + \frac{96x^{10}}{5} + \frac{80x^{9}}{3} + 48x^{8} + \frac{352x^{7}}{7} + 48x^{6} + \frac{252x^{5}}{5} + 40x^{4} + 20x^{3} + 6x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

```
[Out] 64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
+ 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

3.49 $\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$

Optimal. Leaf size=45

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2061}

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1+4x+4x^2+4x^4\right)^2 \, dx &= \int \left(1+8x+24x^2+32x^3+24x^4+32x^5+32x^6+16x^8\right) dx\\ &= x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^2, x]
```

fricas [A] time = 0.74, size = 37, normalized size = 0.82

$$\frac{16}{9}x^9+\frac{32}{7}x^7+\frac{16}{3}x^6+\frac{24}{5}x^5+8x^4+8x^3+4x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

giac [A] time = 0.25, size = 37, normalized size = 0.82

$$\frac{16}{9}\,x^{9} + \frac{32}{7}\,x^{7} + \frac{16}{3}\,x^{6} + \frac{24}{5}\,x^{5} + 8\,x^{4} + 8\,x^{3} + 4\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

maple [A] time = 0.00, size = 38, normalized size = 0.84

$$\frac{16}{9}x^{9} + \frac{32}{7}x^{7} + \frac{16}{3}x^{6} + \frac{24}{5}x^{5} + 8x^{4} + 8x^{3} + 4x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

maxima [A] time = 0.57, size = 37, normalized size = 0.82

$$\frac{16}{9}\,x^{9} + \frac{32}{7}\,x^{7} + \frac{16}{3}\,x^{6} + \frac{24}{5}\,x^{5} + 8\,x^{4} + 8\,x^{3} + 4\,x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

mupad [B] time = 0.03, size = 37, normalized size = 0.82

$$\frac{16\,x^9}{9} + \frac{32\,x^7}{7} + \frac{16\,x^6}{3} + \frac{24\,x^5}{5} + 8\,x^4 + 8\,x^3 + 4\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9)
/9
```

sympy [A] time = 0.06, size = 42, normalized size = 0.93

$$\frac{16x^{9}}{9} + \frac{32x^{7}}{7} + \frac{16x^{6}}{3} + \frac{24x^{5}}{5} + 8x^{4} + 8x^{3} + 4x^{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] 16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
x
```

3.50 $\int (1 + 4x + 4x^2 + 4x^4) \, dx$

Optimal. Leaf size=21

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

```
[In] Int[1 + 4*x + 4*x^2 + 4*x^4,x]
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi steps

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = x + 2x^2 + \frac{4x^3}{3} + \frac{4x^5}{5}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1 + 4*x + 4*x^2 + 4*x^4,x]
```

```
[Out] IntegrateAlgebraic[1 + 4*x + 4*x^2 + 4*x^4, x]
```

fricas [A] time = 1.18, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

giac [A] time = 0.36, size = 17, normalized size = 0.81

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^4+4*x^2+4*x+1,x)
```

```
[Out] x+2*x^2+4/3*x^3+4/5*x^5
```

maxima [A] time = 0.57, size = 17, normalized size = 0.81

$$\frac{4}{5}\,x^5 + \frac{4}{3}\,x^3 + 2\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
```

```
[Out] 4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{4\,x^5}{5} + \frac{4\,x^3}{3} + 2\,x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 4*x^2 + 4*x^4 + 1,x)
```

```
[Out] x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

sympy [A] time = 0.06, size = 19, normalized size = 0.90

$$\frac{4x^5}{5} + \frac{4x^3}{3} + 2x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**4+4*x**2+4*x+1,x)
```

```
[Out] 4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

3.51 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal. Leaf size=234

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\right.$$

Rubi [A] time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2069, 1673, 1169, 634, 618, 204, 628, 12, 1107}

$$-\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] ArcTan[(-1 + (1 + x^(-1))^2)/2]/2 - (Sqrt[(2 + Sqrt[5])/5]*ArcTan[(2 - Sqrt
[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sqrt[(2 + Sqrt[5])/5
]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/2 - (Sq
rt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1
+ x^(-1))^2])/4 + (Sqrt[(-2 + Sqrt[5])/5]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5]
)]*(1 + x^(-1)) + (1 + x^(-1))^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = -\left(16\,\text{Subst}\left(\int \frac{(4-4x)^2}{1280-512x^2+256x^4}\,dx,x,1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int -\frac{32x}{1280-512x^2+256x^4}\,dx,x,1+\frac{1}{x}\right)\right) - 16\,\text{Subst}\left(\int \frac{16+\cdots}{1280-512\cdots}\right.$$

$$= 512\,\text{Subst}\left(\int \frac{x}{1280-512x^2+256x^4}\,dx,x,1+\frac{1}{x}\right) - \frac{\text{Subst}\left(\int \frac{16\sqrt{2\left(1+\sqrt{5}\right)}-\left(16-16\sqrt{\cdots}\right.}{\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\,x+x\cdots}\right.}{32\sqrt{10\left(1+\sqrt{\cdots}\right.}}$$

$$= 256\,\text{Subst}\left(\int \frac{1}{1280-512x+256x^2}\,dx,x,\left(1+\frac{1}{x}\right)^2\right) + \frac{\left(1-\sqrt{5}\right)\text{Subst}\left(\int \frac{-\sqrt{2(\cdots}}{\sqrt{5}-\sqrt{2\cdots}}\right.}{4\sqrt{10\left(1-\cdots\right.}}$$

$$= -\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\,\log\left(\sqrt{5}-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\left(1+\frac{1}{x}\right)^2\right)+\frac{1}{4}\sqrt{-\frac{2}{5}+\frac{1}{\sqrt{5}}}\,\text{lo}\cdots$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{\left(1+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}} - \frac{\left(1+\sqrt{5}\right)^{3/2}\cdots}{\cdots}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.20

$$\frac{1}{4}\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&,\frac{\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]/
4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-1), x]
```

fricas [C] time = 4.09, size = 499, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] -1/20*(sqrt(10)*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*
I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I
)^2 - 9) - 5*sqrt(1/10*I - 1/5) - 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/
10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I -
 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^
2 + ((6*sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1
/5) - I) - sqrt(10)*(2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I
- 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)
- 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 8*x + 3) + 1/20*(sqrt(10)*sqrt(
-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt
(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*I - 1/5) + I)^2 - 9) + 5*sqrt(1/1
0*I - 1/5) + 5*sqrt(-1/10*I - 1/5))*log(5/2*(2*sqrt(1/10*I - 1/5) - I)^2*(1
2*sqrt(-1/10*I - 1/5) + 6*I - 1) + 15*(2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1
/10*I - 1/5) + I)^2 - 5/2*(2*sqrt(-1/10*I - 1/5) + I)^2 - ((6*sqrt(10)*(2*s
qrt(-1/10*I - 1/5) + I) - sqrt(10))*(2*sqrt(1/10*I - 1/5) - I) - sqrt(10)*(
2*sqrt(-1/10*I - 1/5) + I))*sqrt(-15/8*(2*sqrt(1/10*I - 1/5) - I)^2 - 5/4*(
2*sqrt(1/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I) - 15/8*(2*sqrt(-1/10*
I - 1/5) + I)^2 - 9) + 8*x + 3) - 1/4*(2*sqrt(1/10*I - 1/5) - I)*log(-5*(2*
sqrt(1/10*I - 1/5) - I)^2*(12*sqrt(-1/10*I - 1/5) + 6*I - 1) - 30*(2*sqrt(1
/10*I - 1/5) - I)*(2*sqrt(-1/10*I - 1/5) + I)^2 - 30*(2*sqrt(-1/10*I - 1/5)
 + I)^3 + 8*x - 216*sqrt(-1/10*I - 1/5) - 108*I + 21) - 1/4*(2*sqrt(-1/10*I
 - 1/5) + I)*log(30*(2*sqrt(-1/10*I - 1/5) + I)^3 + 5*(2*sqrt(-1/10*I - 1/5
) + I)^2 + 8*x + 216*sqrt(-1/10*I - 1/5) + 108*I - 27)
```

giac [C] time = 0.52, size = 265, normalized size = 1.13

$$-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(\frac{i}{\sqrt{5}-2}+1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x+29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}+(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((i+2)\sqrt{\sqrt{5}-2}\left(-\frac{i}{\sqrt{5}-2}-1\right)+5i\right)\log\left((406i+174)\sqrt{5}x+(868i+372)x-29\sqrt{5}\sqrt{29\sqrt{5}+62}+(87i-203)\sqrt{5}-(19i+62)\sqrt{29\sqrt{5}+62}+186i-434\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(-\frac{i}{\sqrt{5}+2}-1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x+13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}+(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)-\frac{1}{20}\left((2i+1)\sqrt{\sqrt{5}+2}\left(\frac{i}{\sqrt{5}+2}+1\right)-5i\right)\log\left((26i+130)\sqrt{5}x-(44i+220)x-13\sqrt{5}\sqrt{13\sqrt{5}-22}-(65i-13)\sqrt{5}-(19i-22)\sqrt{13\sqrt{5}-22}+110i-22\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] -1/20*((I + 2)*sqrt(sqrt(5) - 2)*(I/(sqrt(5) - 2) + 1) + 5*I)*log((406*I +
174)*sqrt(5)*x + (868*I + 372)*x + 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I
 - 203)*sqrt(5) + (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*(
(I + 2)*sqrt(sqrt(5) - 2)*(-I/(sqrt(5) - 2) - 1) + 5*I)*log((406*I + 174)*s
qrt(5)*x + (868*I + 372)*x - 29*sqrt(5)*sqrt(29*sqrt(5) + 62) + (87*I - 203
)*sqrt(5) - (19*I + 62)*sqrt(29*sqrt(5) + 62) + 186*I - 434) - 1/20*((2*I +
 1)*sqrt(sqrt(5) + 2)*(-I/(sqrt(5) + 2) - 1) - 5*I)*log((26*I + 130)*sqrt(5
)*x - (44*I + 220)*x + 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(
5) + (19*I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22) - 1/20*((2*I + 1)*sqrt
```

```
(sqrt(5) + 2)*(I/(sqrt(5) + 2) + 1) - 5*I)*log((26*I + 130)*sqrt(5)*x - (44
*I + 220)*x - 13*sqrt(5)*sqrt(13*sqrt(5) - 22) - (65*I - 13)*sqrt(5) - (19*
I - 22)*sqrt(13*sqrt(5) - 22) + 110*I - 22)
```

maple [C] time = 0.01, size = 41, normalized size = 0.18

$$\frac{\ln\left(-\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+x\right)}{16\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^3+8\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1),x)
```

```
[Out] 1/4*sum(1/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

mupad [B] time = 2.36, size = 87, normalized size = 0.37

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(\frac{x}{4}+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\left(6\,x+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)(36\,x+16)\right)\right)\right)\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

```
[Out] symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
(9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1/1
280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k), k,
1, 4)
```

sympy [B] time = 2.57, size = 3432, normalized size = 14.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

```
[Out] sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/1
0 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12*s
qrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 +
sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5)*sqr
t(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 3*sq
rt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 301*s
qrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(5)*sqrt(
-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(
-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqrt(5))/10 +
 sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)*sqrt(-2 +
sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) - 3913*sqrt(2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75*sqrt(5)*sqrt
(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8 - 301*sq
rt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 +
 sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407*sqrt(5)/40 + 84
```

```
1*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/20) - 2*sqrt(3/
80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40)*
atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(
-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(
5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*
sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
 + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt
(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) - 120*
sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt
(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) +
 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 +
 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*
sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(5)/(-27*sqrt(5)*sqrt(3
 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sq
rt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
 19)) + 5*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(-27*sqrt(5)*s
qrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) +
 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt
(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5)
+ 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(
5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
 sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 80/(-
27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*sqrt(5)*sq
rt(-2 + sqrt(5)) + sqrt(5) + 19)) + 6*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-
2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19))*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19))) - 2*sqrt(-sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(
5) + 19)/40 + 3/80 + 3*sqrt(5)/80)*atan(20*x/(5*sqrt(-2 + sqrt(5))*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqr
t(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt
(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*
sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 80/(5
*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5))
+ sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(
5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 1
9) + 3 + 3*sqrt(5))) - 120*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + sqrt(5))*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sq
rt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqr
t(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2
*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))) - 5*sq
rt(5)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + s
qrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqr
t(5) + 19) + 3 + 3*sqrt(5))) + 21*sqrt(5)*sqrt(-2 + sqrt(5))/(5*sqrt(-2 + s
qrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
qrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5
) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*s
```

```
qrt(5))) + 5*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/(5*sqrt(-2 +
 sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5)
+ 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt
(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3
*sqrt(5))) + 18*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)
) + sqrt(5) + 19)/(5*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 27*sqrt(5)*sqrt(-2*sqrt(2*sqrt(5)
*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5)) + 6*sqrt(5)*sqrt(2*sqr
t(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)*sqrt(-2*sqrt(2*sqrt(5)*sqrt(-2 + sq
rt(5)) + sqrt(5) + 19) + 3 + 3*sqrt(5))))
```

3.52 $\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2}\right.$$

Rubi [A] time = 0.33, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {2069, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660, 12}

$$-\frac{17-\left(\frac{1}{x}+1\right)^2}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)-\frac{1}{40}\sqrt{\frac{1}{10}\left(2665\sqrt{5}-5959\right)}\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)+\frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(\left(\frac{1}{x}+1\right)^2-1\right)\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}-\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\tan^{-1}\left(\frac{\frac{2}{x}+\sqrt{2\left(1+\sqrt{5}\right)}+2}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] -(17 - (1 + x^(-1))^2)/(2*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) + ((59 -
 17*(1 + x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)) + (7*ArcTan[(-1 + (1 + x^(-1))^2)/2])/4 - (Sqrt[(5959 + 2665*Sqrt[5])/1
0]*ArcTan[(2 - Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqrt[2*(-1 + Sqrt[5])]])/20 - (
Sqrt[(5959 + 2665*Sqrt[5])/10]*ArcTan[(2 + Sqrt[2*(1 + Sqrt[5])] + 2/x)/Sqr
t[2*(-1 + Sqrt[5])]])/20 + (Sqrt[(-5959 + 2665*Sqrt[5])/10]*Log[Sqrt[5] - S
qrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/40 - (Sqrt[(-5959 + 26
65*Sqrt[5])/10]*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(
-1))^2])/40
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx = -\left(16\,\text{Subst}\left(\int \frac{(4-4x)^6}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-24576-81920x^2-24576x^4\right)}{\left(1280-512x^2+256x^4\right)^2}\,dx, x, 1+\frac{1}{x}\right)\right) - 16\,\text{Subst}\left(\int \frac{4}{}\right.$$

$$= \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{261993005056+115964116992x^2}{1280-512x^2+256x^4}\,dx, x, 1+\frac{1}{x}\right)}{167772160}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int -\frac{}{128}\right.}{}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + 896\,\text{Subst}\left(\int\right.$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{1}{40}\sqrt{-\frac{5959}{10}+}$$

$$= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} + \frac{7}{4}\tan^{-1}\left(\frac{1}{2}\left(-\right.\right.$$

Mathematica [C] time = 0.02, size = 108, normalized size = 0.34

$$\frac{1}{40}\left(\text{RootSum}\left[4\#1^4+4\#1^2+4\#1+1\&, \frac{18\#1^2\log(x-\#1)-16\#1\log(x-\#1)+27\log(x-\#1)}{4\#1^3+2\#1+1}\&\right]+\frac{72x^3-32x^2+84x+38}{4x^4+4x^2+4x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] ((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#1
 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #1]
*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x + 4*x^2 + 4*x^4)^(-2), x]
```

fricas [C] time = 3.89, size = 704, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(720*x^3 - 50*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19/1000*I - 5959/2000
) + 7*I)*log(33368250*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^3 - 11755375/4*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 541735337*x + 25784243612*sqrt(19
/1000*I - 5959/2000) + 45122426321*I - 71080995) - 50*(4*x^4 + 4*x^2 + 4*x
+ 1)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)*log(-33368250*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^3 - 125/4*(4271136*sqrt(19/1000*I - 5959/2000) + 74744
88*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 25*(1334730*(4*sqr
t(19/1000*I - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000)
 - 7*I) + 541735337*x - 25806203712*sqrt(19/1000*I - 5959/2000) - 451608564
96*I - 355111539) - 320*x^2 - (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-3
75/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I -
5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-
19/1000*I - 5959/2000) - 7*I)^2 - 3021) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) - 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-1
9/1000*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000
) + 7474488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/
8*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I
 - 5959/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) + 1
/2*sqrt(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19
/1000*I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32
*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*s
qrt(19/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 59
59/2000) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 8
78404*sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 3843
0175/2*I + 213096267) + (4*sqrt(10)*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(-375/32*
(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*I - 5959/2
000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sqrt(-19/100
0*I - 5959/2000) - 7*I)^2 - 3021) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 25*(4*x^4 + 4*x^2 + 4*x + 1)*(4*sqrt(-19/1000
*I - 5959/2000) - 7*I))*log(125/8*(4271136*sqrt(19/1000*I - 5959/2000) + 74
74488*I + 94043)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I)^2 + 11755375/8*(4*s
qrt(19/1000*I - 5959/2000) + 7*I)^2 + 25/2*(1334730*(4*sqrt(19/1000*I - 595
9/2000) + 7*I)^2 + 219601)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 1/2*sqr
t(-375/32*(4*sqrt(19/1000*I - 5959/2000) + 7*I)^2 - 125/16*(4*sqrt(19/1000*
I - 5959/2000) + 7*I)*(4*sqrt(-19/1000*I - 5959/2000) - 7*I) - 375/32*(4*sq
rt(-19/1000*I - 5959/2000) - 7*I)^2 - 3021)*(5*(1067784*sqrt(10)*(4*sqrt(19
/1000*I - 5959/2000) + 7*I) + 94043*sqrt(10))*(4*sqrt(-19/1000*I - 5959/200
0) - 7*I) + 470215*sqrt(10)*(4*sqrt(19/1000*I - 5959/2000) + 7*I) - 878404*
sqrt(10)) + 541735337*x + 10980050*sqrt(19/1000*I - 5959/2000) + 38430175/2
*I + 213096267) + 840*x + 380)/(4*x^4 + 4*x^2 + 4*x + 1)
```

giac [C] time = 0.72, size = 315, normalized size = 0.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

```
[Out] -1/400*(-(I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1)
- 350*I)*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*
I + 26022625108780)*x + 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118)
- (8426908086005*I - 1267318112395)*sqrt(5) + (8166407345*I - 7795873310)*s
qrt(1424281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*
((I + 3)*sqrt(2665*sqrt(5) - 4790)*(709*I/(533*sqrt(5) - 958) + 1) - 350*I)
*log((2534636224790*I + 16853816172010)*sqrt(5)*x - (3913528401620*I + 2602
2625108780)*x - 5049076145*sqrt(5)*sqrt(1424281*sqrt(5) - 2199118) - (84269
```

```
08086005*I - 1267318112395)*sqrt(5) - (8166407345*I - 7795873310)*sqrt(1424
281*sqrt(5) - 2199118) + 13011312554390*I - 1956764200810) - 1/400*((3*I +
1)*sqrt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((
16722951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 381438558
5140)*x + 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (12404110944
55*I - 8361475596225)*sqrt(5) + (8153361745*I + 7721428310)*sqrt(1416617*sq
rt(5) + 2178118) + 1907192792570*I - 12856178136150) - 1/400*(-(3*I + 1)*sq
rt(2665*sqrt(5) + 4790)*(709*I/(533*sqrt(5) + 958) + 1) + 350*I)*log((16722
951192450*I + 2480822188910)*sqrt(5)*x + (25712356272300*I + 3814385585140)
*x - 5021907265*sqrt(5)*sqrt(1416617*sqrt(5) + 2178118) + (1240411094455*I
- 8361475596225)*sqrt(5) - (8153361745*I + 7721428310)*sqrt(1416617*sqrt(5)
 + 2178118) + 1907192792570*I - 12856178136150) + 1/20*(36*x^3 - 16*x^2 + 4
2*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

maple [C] time = 0.01, size = 79, normalized size = 0.25

$$\frac{\left(18\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^2-16\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+27\right)\ln\left(-\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+x\right)}{160\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)^3+80\,\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)+40}+\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^2,x)
```

```
[Out] (9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+27
)/(4*_R^3+2*_R+1)*ln(-_R+x),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{36\,x^3-16\,x^2+42\,x+19}{20\left(4\,x^4+4\,x^2+4\,x+1\right)}+\frac{1}{10}\int\frac{18\,x^2-16\,x+27}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integra
te((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

mupad [B] time = 2.21, size = 174, normalized size = 0.55

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100}+\frac{11\,x}{1600}+\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)x\,131}{100}-\frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2x\,72}{5}-\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3x\,36+\frac{59\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2}{20}-16\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3+\frac{27}{1600}\right)\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\right)+\frac{\frac{9\,x^3}{20}-\frac{x^2}{5}+\frac{21\,x}{40}+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

```
[Out] symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 2
9/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/6
4000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/640
00, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k
)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k)^3
+ 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z, k), k,
1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1/4)
```

sympy [B] time = 3.66, size = 3834, normalized size = 12.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-595
9/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
+ 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/101638
9 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/
1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-5959 +
 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149111451*
sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36
004639)/27530691871904650 - 288583554422522791728 2997/14673858767725178450 0
```

```
[Out] (36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-595
9/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
+ 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/101638
9 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/
1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-5959 +
 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149111451*
sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36
004639)/27530691871904650 - 2885835544225227917282997/14673858767725178450 0
 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 - 5020
8805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53848575489193
3*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*
sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450 - 92532195509690
1411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/29347717535450356900 + 48430461193
8766076267*sqrt(5)/55061383743809300 + 2201303608701478540 3*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/666993580351144
4750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-94043*sqrt(665*
sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337
 - 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959
 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/13543383425 - 3131659367*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 291689395/1083470674 + 1067784
*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/1016389 + 470215*sqrt(5)/2032778) - 220
13036087014785403*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)/6669935803511444750 - 2885835544225227917282997/1467385876
77251784500 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093
 - 538485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/14673858767725178450
+ 925321955096901411*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/293477175354503569
00 + 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806926 + 48
4304611938766076267*sqrt(5)/55061383743809300 + 40634464149111451*sqrt(5)*s
qrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/27
530691871904650) + 2*sqrt(6291/16000 + 1599*sqrt(5)/3200 + sqrt(-665*sqrt(1
0)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000)*atan(54173
533700*x/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt
(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt
(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqr
t(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 2
21195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
221195*sqrt(5) + 36004639)) - 3203352*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(
-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2
665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7
995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639)) - 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(-644057
0878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqr
t(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqr
t(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
36004639)) + 6263318734*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/(-6440570878*sq
rt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*s
```

```
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) +
4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3600463
9)) + 7292234875/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351
075*sqrt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(
10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt
(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)) + 6265614875*sqrt(5)/(-6440570878*sqrt(
10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(
5)) + 221195*sqrt(5) + 36004639)) + 2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt
(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22
1195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*s
qrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))*
sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639))
 + 4702150*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) +
 36004639)/(-6440570878*sqrt(10)*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqr
t(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 2351075*sq
rt(-5959 + 2665*sqrt(5))*sqrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)) + 1067784*sqrt(10)*s
qrt(6291 + 7995*sqrt(5) + 4*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) +
 221195*sqrt(5) + 36004639))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
+ 221195*sqrt(5) + 36004639))) - 2*sqrt(-sqrt(665*sqrt(10)*sqrt(-5959 + 266
5*sqrt(5)) + 221195*sqrt(5) + 36004639)/4000 + 6291/16000 + 1599*sqrt(5)/32
00)*atan(54173533700*x/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 +
7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 +
2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784
*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5))) - 4702150*sqrt(665*sqrt(10)*sqrt(-5
959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665
*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqr
t(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(
665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqr
t(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2
665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 3203352
*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sq
rt(5)) + 221195*sqrt(5) + 36004639)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(
5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22119
5*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))
+ 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) - 6263318734*sqrt(10)*
sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt
(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 629
1 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-595
9 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 106
7784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5)
 + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*
sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 7292234875/(2351075*sqrt(-595
9 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 22
1195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(
-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 +
```

```
2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) +
 28456443600*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/(2351075*sqrt(-5959 + 2665*
sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639) + 6291 + 7995*sqrt(5)) + 6440570878*sqrt(10)*sqrt(-4*sqrt(6
65*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291
+ 7995*sqrt(5)) + 1067784*sqrt(10)*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt
(5)) + 221195*sqrt(5) + 36004639)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 26
65*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5))) + 62656148
75*sqrt(5)/(2351075*sqrt(-5959 + 2665*sqrt(5))*sqrt(-4*sqrt(665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)
) + 6440570878*sqrt(10)*sqrt(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)
) + 221195*sqrt(5) + 36004639) + 6291 + 7995*sqrt(5)) + 1067784*sqrt(10)*sq
rt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)*sqr
t(-4*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 360046
39) + 6291 + 7995*sqrt(5))))
```

3.53 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx$

Optimal. Leaf size=104

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-$$

Rubi [A] time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.045$, Rules used = {2061}

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx=\int \left(4096+49152x+237568x^2+559104x^3+538624x^4-184320x^5-5\right.$$
$$=4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{56}{}$$

Mathematica [A] time = 0.00, size = 104, normalized size = 1.00

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4, x]
```

fricas [A] time = 0.61, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-\frac{566912}{7}x^{7}-30720x^{6}+\frac{538624}{5}x^{5}+139776x^{4}+\frac{237568}{3}x^{3}+24576x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

giac [A] time = 0.36, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-\frac{566912}{7}x^{7}-30720x^{6}+\frac{538624}{5}x^{5}+139776x^{4}+\frac{237568}{3}x^{3}+24576x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

maple [A] time = 0.00, size = 85, normalized size = 0.82

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-\frac{566912}{7}x^{7}-30720x^{6}+\frac{538624}{5}x^{5}+139776x^{4}+\frac{237568}{3}x^{3}+24576x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)
```

```
[Out] 4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x^
7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13*x
^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

maxima [A] time = 0.66, size = 84, normalized size = 0.81

$$\frac{4096}{17}x^{17}-1920x^{16}+\frac{102784}{15}x^{15}-\frac{75504}{7}x^{14}-\frac{12095}{13}x^{13}+31128x^{12}-\frac{331040}{11}x^{11}-\frac{169584}{5}x^{10}+\frac{641152}{9}x^{9}+36384x^{8}-\frac{566912}{7}x^{7}-30720x^{6}+\frac{538624}{5}x^{5}+139776x^{4}+\frac{237568}{3}x^{3}+24576x^{2}+4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

```
[Out] 4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 - 56
6912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 24576*x
^2 + 4096*x
```

mupad [B] time = 2.23, size = 84, normalized size = 0.81

$$\frac{4096\,x^{17}}{17}-1920\,x^{16}+\frac{102784\,x^{15}}{15}-\frac{75504\,x^{14}}{7}-\frac{12095\,x^{13}}{13}+31128\,x^{12}-\frac{331040\,x^{11}}{11}-\frac{169584\,x^{10}}{5}+\frac{641152\,x^{9}}{9}+36384\,x^{8}-\frac{566912\,x^{7}}{7}-30720\,x^{6}+\frac{538624\,x^{5}}{5}+139776\,x^{4}+\frac{237568\,x^{3}}{3}+24576\,x^{2}+4096\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

```
[Out] 4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*x
^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (33104
0*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^15)/
15 - 1920*x^16 + (4096*x^17)/17
```

sympy [A] time = 0.08, size = 100, normalized size = 0.96

$$\frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^{9}}{9} + 36384x^{8} - \frac{566912x^{7}}{7} - 30720x^{6} + \frac{538624x^{5}}{5} + 139776x^{4} + \frac{237568x^{3}}{3} + 24576x^{2} + 4096x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

```
[Out] 4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13/
13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 36384
*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 237568*x
**3/3 + 24576*x**2 + 4096*x
```

3.54 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx$

Optimal. Leaf size=76

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+5$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx = \int \left(512+4608x+15360x^2+20160x^3-384x^4-17856x^5+5528x^6+\right.$$
$$= 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+20$$

Mathematica [A] time = 0.00, size = 76, normalized size = 1.00

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3, x]
```

fricas [A] time = 0.39, size = 64, normalized size = 0.84

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^9+2097x^8+\frac{5528}{7}x^7-2976x^6-\frac{384}{5}x^5+5040x^4+5120x^3+2304x^2+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

giac [A] time = 0.38, size = 64, normalized size = 0.84

$$\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.86

$$\frac{512}{13}x^{13}-240x^{12}+\frac{6936}{11}x^{11}-\frac{4527}{10}x^{10}-\frac{2936}{3}x^{9}+2097x^{8}+\frac{5528}{7}x^{7}-2976x^{6}-\frac{384}{5}x^{5}+5040x^{4}+5120x^{3}+2304x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)
```

```
[Out] 512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-293
6/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

maxima [A] time = 0.65, size = 64, normalized size = 0.84

$$\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

```
[Out] 512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x^
8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 + 51
2*x
```

mupad [B] time = 0.08, size = 64, normalized size = 0.84

$$\frac{512\,x^{13}}{13}-240\,x^{12}+\frac{6936\,x^{11}}{11}-\frac{4527\,x^{10}}{10}-\frac{2936\,x^{9}}{3}+2097\,x^{8}+\frac{5528\,x^{7}}{7}-2976\,x^{6}-\frac{384\,x^{5}}{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

```
[Out] 512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^7
)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^12
+ (512*x^13)/13
```

sympy [A] time = 0.08, size = 73, normalized size = 0.96

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^{9}}{3}+2097x^{8}+\frac{5528x^{7}}{7}-2976x^{6}-\frac{384x^{5}}{5}+5040x^{4}+5120x^{3}+2304x^{2}+512x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

```
[Out] 512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 20
97*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 + 23
04*x**2 + 512*x
```

3.55 $\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx$

Optimal. Leaf size=52

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx = \int \left(64+384x+704x^2+144x^3-528x^4+144x^5+353x^6-240x^7+6\right.$$
$$= 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

Mathematica [A] time = 0.00, size = 52, normalized size = 1.00

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2, x]
```

fricas [A] time = 0.86, size = 44, normalized size = 0.85

$$\frac{64}{9}x^9-30x^8+\frac{353}{7}x^7+24x^6-\frac{528}{5}x^5+36x^4+\frac{704}{3}x^3+192x^2+64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

giac [A] time = 0.36, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^{9} - 30\,x^{8} + \frac{353}{7}\,x^{7} + 24\,x^{6} - \frac{528}{5}\,x^{5} + 36\,x^{4} + \frac{704}{3}\,x^{3} + 192\,x^{2} + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

maple [A] time = 0.00, size = 45, normalized size = 0.87

$$\frac{64}{9}x^{9} - 30x^{8} + \frac{353}{7}x^{7} + 24x^{6} - \frac{528}{5}x^{5} + 36x^{4} + \frac{704}{3}x^{3} + 192x^{2} + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] 64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

maxima [A] time = 0.61, size = 44, normalized size = 0.85

$$\frac{64}{9}\,x^{9} - 30\,x^{8} + \frac{353}{7}\,x^{7} + 24\,x^{6} - \frac{528}{5}\,x^{5} + 36\,x^{4} + \frac{704}{3}\,x^{3} + 192\,x^{2} + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 + 1
92*x^2 + 64*x
```

mupad [B] time = 0.03, size = 44, normalized size = 0.85

$$\frac{64\,x^{9}}{9} - 30\,x^{8} + \frac{353\,x^{7}}{7} + 24\,x^{6} - \frac{528\,x^{5}}{5} + 36\,x^{4} + \frac{704\,x^{3}}{3} + 192\,x^{2} + 64\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] 64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
- 30*x^8 + (64*x^9)/9
```

sympy [A] time = 0.07, size = 49, normalized size = 0.94

$$\frac{64x^{9}}{9} - 30x^{8} + \frac{353x^{7}}{7} + 24x^{6} - \frac{528x^{5}}{5} + 36x^{4} + \frac{704x^{3}}{3} + 192x^{2} + 64x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] 64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x**
3/3 + 192*x**2 + 64*x
```

3.56 $\int \left(8+24x+8x^2-15x^3+8x^4\right) dx$

Optimal. Leaf size=30

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi steps

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

Antiderivative was successfully verified.

```
[In] Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
[Out] IntegrateAlgebraic[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4, x]
```

fricas [A] time = 0.73, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5-\frac{15}{4}x^4+\frac{8}{3}x^3+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

giac [A] time = 0.30, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5-\frac{15}{4}x^4+\frac{8}{3}x^3+12x^2+8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(8*x^4-15*x^3+8*x^2+24*x+8,x)
```

```
[Out] 8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

maxima [A] time = 0.46, size = 24, normalized size = 0.80

$$\frac{8}{5}x^5 - \frac{15}{4}x^4 + \frac{8}{3}x^3 + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
```

```
[Out] 8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

mupad [B] time = 0.02, size = 24, normalized size = 0.80

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
```

```
[Out] 8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

sympy [A] time = 0.06, size = 27, normalized size = 0.90

$$\frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

```
[Out] 8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

3.57 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal. Leaf size=263

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+$$

Rubi [A] time = 0.49, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2069, 12, 1673, 1169, 634, 618, 204, 628, 1107}

$$\frac{1}{4}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)+\frac{1}{8}\sqrt{\frac{235\sqrt{517}-5167}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] -(Sqrt[(5167 + 235*Sqrt[517])/40326]*ArcTan[(6 - Sqrt[2*(19 + Sqrt[517])] +
 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 - (Sqrt[(5167 + 235*Sqrt[517])/40326]*A
rcTan[(6 + Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/4 +
(Sqrt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/4 - (Sqrt[(-5167 + 2
35*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) + (
3 + 4/x)^2])/8 + (Sqrt[(-5167 + 235*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[
2*(19 + Sqrt[517])]*(3 + 4/x) + (3 + 4/x)^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(24-32x)^2}{8\left(2117632-2490368x^2+1048576x^4\right)}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int \frac{(24-32x)^2}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(128\,\text{Subst}\left(\int -\frac{1536x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right) - 128$$

$$\text{Subs}$$

$$= 196608\,\text{Subst}\left(\int \frac{x}{2117632-2490368x^2+1048576x^4}\,dx, x, \frac{3}{4}+\frac{1}{x}\right) - $$

$$(517$$

$$= 98304\,\text{Subst}\left(\int \frac{1}{2117632-2490368x+1048576x^2}\,dx, x, \left(\frac{3}{4}+\frac{1}{x}\right)^2\right) - $$

$$= -\frac{1}{8}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\,\log\left(\sqrt{517}-\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)+\left(3+\frac{4}{x}\right)\right.$$

$$= -\frac{1}{4}\sqrt{\frac{3}{13}}\,\tan^{-1}\left(\frac{19-\left(3+\frac{4}{x}\right)^2}{2\sqrt{39}}\right) - \frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\,\tan^{-1}\left(\frac{6+\sqrt{2(19}}{\sqrt{2(-19}}\right.$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.21

$$\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&,\frac{\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1 -
 45*#1^2 + 32*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

maple [C] time = 0.01, size = 49, normalized size = 0.19

$$\frac{\ln\left(-\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)+x\right)}{32\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)^3-45\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)^2+16\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x)
```

```
[Out] sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(-_R+x),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

```
[Out] integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

mupad [B] time = 0.41, size = 123, normalized size = 0.47

$$\sum_{k=1}^{4}\ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\,x+\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)x\,38259+\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)^2x\,1531920+805896\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)^2-120\right)}{4096}\right)\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

```
[Out] symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) + 2
56*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k
)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k), k, 1, 4)
```

sympy [A] time = 2.37, size = 41, normalized size = 0.16

$$\mathrm{RootSum}\left(50326848t^4+765960t^2+12753t+64,\left(t\mapsto t\log\left(\frac{100785893208t^3}{4758335}-\frac{1430512512t^2}{4758335}+\frac{72982352521t}{223641745}+x+\frac{2270349121}{1789133960}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

```
[Out] RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(10
0785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/223641
745 + x + 2270349121/1789133960)))
```

3.58 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

Optimal. Leaf size=366

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+\right.}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+5\right.}$$

Rubi [A] time = 0.51, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2069, 12, 1673, 1678, 1169, 634, 618, 204, 628, 1663, 1660}

$$\frac{73}{208}\sqrt{\frac{3}{13}}\tan^{-1}\left(\frac{-5x^2+12x+8}{\sqrt{39}\,x^2}\right)-\frac{3\left(3359-107\left(\frac{4}{x}+3\right)^2\right)}{208\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}+\frac{\left(3327931-129631\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)}{322608\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)}-\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2-\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}+\frac{\sqrt{\frac{2623170438295\sqrt{517}-59644114671451}{40326}}\log\left(\left(\frac{4}{x}+3\right)^2+\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{4}{x}+3\right)+\sqrt{517}\right)}{645216}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}-\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}-\frac{\sqrt{\frac{19+\sqrt{517}}{40326}}\left(1678181+74897\sqrt{517}\right)\tan^{-1}\left(\frac{\frac{8}{x}+\sqrt{2\left(19+\sqrt{517}\right)}+6}{\sqrt{2\left(\sqrt{517}-19\right)}}\right)}{645216}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (-3*(3359 - 107*(3 + 4/x)^2))/(208*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)) +
((3327931 - 129631*(3 + 4/x)^2)*(3 + 4/x))/(322608*(517 - 38*(3 + 4/x)^2 +
(3 + 4/x)^4)) - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*A
rcTan[(6 - Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/6452
16 - (Sqrt[(19 + Sqrt[517])/40326]*(1678181 + 74897*Sqrt[517])*ArcTan[(6 +
Sqrt[2*(19 + Sqrt[517])] + 8/x)/Sqrt[2*(-19 + Sqrt[517])]])/645216 + (73*Sq
rt[3/13]*ArcTan[(8 + 12*x - 5*x^2)/(Sqrt[39]*x^2)])/208 - (Sqrt[(-596441146
71451 + 2623170438295*Sqrt[517])/40326]*Log[Sqrt[517] - Sqrt[2*(19 + Sqrt[5
17])]*(3 + 4/x) + (3 + 4/x)^2])/645216 + (Sqrt[(-59644114671451 + 262317043
8295*Sqrt[517])/40326]*Log[Sqrt[517] + Sqrt[2*(19 + Sqrt[517])]*(3 + 4/x) +
 (3 + 4/x)^2])/645216
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2069

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1*((a*(-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 256*a^3*e - 32*a^
2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4))/(b - 4*a*x)^4)^p)/(b - 4*a*x)^2, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx = -\left(1024\,\text{Subst}\left(\int \frac{(24-32x)^6}{64\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4} -\right.\right.$$

$$= -\left(16\,\text{Subst}\left(\int \frac{(24-32x)^6}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,dx, x, \frac{3}{4}+\frac{1}{x}\right)\right)$$

$$= -\left(16\,\text{Subst}\left(\int \frac{x\left(-1528823808-9059696640x^2-4831838208x^4\right)}{\left(2117632-2490368x^2+1048576x^4\right)^2}\,d\right.\right.$$

$$= \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{12092568522016394156}{2117632-}\right.}{709}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\right.}$$

$$= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right.}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\right.}$$

Mathematica [C] time = 0.03, size = 128, normalized size = 0.35

$$\frac{\text{RootSum}\left[8\#1^4-15\#1^3+8\#1^2+24\#1+8\&, \frac{19640\#1^2\log(x-\#1)-57489\#1\log(x-\#1)+74897\log(x-\#1)}{32\#1^3-45\#1^2+16\#1+24}\&\right]}{80652} + \frac{39280x^3-94314x^2+89033x+72888}{161304\left(8x^4-15x^3+8x^2+24x+8\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] (72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Log[
x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 45*#
1^2 + 32*#1^3) & ]/80652
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

maple [C] time = 0.01, size = 96, normalized size = 0.26

$$\frac{\left(19640\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)^2-57489\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)+74897\right)\ln\left(-\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)+x\right)}{2580864\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)^3-3629340\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)^2+1290432\,\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)+1935648}+\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

```
[Out] (2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x^
2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+24)
*ln(-_R+x),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{39280\,x^3-94314\,x^2+89033\,x+72888}{161304\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)}+\frac{1}{80652}\int \frac{19640\,x^2-57489\,x+74897}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

```
[Out] 1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15*x
^3 + 8*x^2 + 24*x + 8), x)
```

mupad [B] time = 0.21, size = 181, normalized size = 0.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

```
[Out] ((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*x
 + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (1491162
5619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/
44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (92412
4364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/3
```

```
730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984 - (72
451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (95745*ro
ot(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37306361040
19968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root(z^4 + (14
911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4302
3440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1491162561931
1*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/442045
10553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (14911625619311
*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43023440/4420451
0553294663, z, k), k, 1, 4)
```

sympy [B] time = 3.95, size = 3839, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

```
[Out] (39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/1678786246808985
6 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685033
06932567484755463/603722125611976319526135612861060 - 296438698298128332309
07750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/
1936419398792394461637855141912238396080 - 181533261043120360732*sqrt(-7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314
02994079881533903215265 - 46926347979646613249222*sqrt(517)/297468603626329
12338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 + 2623170438
295*sqrt(517))/1427849297406379792240272 + 994065243322493861977*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sq
rt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715
0042935*sqrt(517) + 3557579971691991294769382675)/1290946265861596307758570
094608158930720) - 45971497067730669689218547912235602388091893135917351760
29*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623
170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912
94769382675)/18432767186998698626450604048374763890148748053806275728419558
40 - 1022132763720267175882780425063613131088601935958303878081158710949715
459967411486447/30220181238068169063463153438589206735086644165655335340962
4708723614680800 - 10638094717334280126176111526682776643728382835565338369
93*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/689619370306997
2436744723519626607862706189949382980866560 - 89036038929850064673184559559
3034670326044595870824169313*sqrt(40326)*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675)/9352473884121677079601749613489889898672849258229891014611209
554309158400 - 45113976327488809325094501633826014671791*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))*sqrt(-7120427417275887*sqrt(40326)*s
qrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqr
t(517) + 3557579971691991294769382675)/107753026610468319324136304994165747
854784217959109076040 + 426980096365154687189009427342740052122552822995528
02371283821308121207*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517))/935247388412167707960174961348988989867284925822989101461120955430915
8400 + 68548776709669674081892851407413209373218007060934353137152573209405
073*sqrt(517)/4608191796749674656612651012093690972537187013451568932104889
60 + 2741964319335541530074345707646806021327350986246447585831575286311670
33*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476938267
5)/483522899809090705015410455017427307761386306650485365455399533957783489
2800) - sqrt(-59644114671451/16787862468089856 + 5073830635*sqrt(517)/32471
```

```
687559168)*log(x**2 + x*(-112396995020468503306932567484755463/603722125611
976319526135612861060 - 994065243322493861977*sqrt(78)*sqrt(-59644114671451
 + 2623170438295*sqrt(517))/1427849297406379792240272 - 4692634797964661324
9222*sqrt(517)/29746860362632912338339 + 181533261043120360732*sqrt(7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
3621568587150042935*sqrt(517) + 3557579971691991294769382675)/1509305314029
94079881533903215265 + 29643869829812833230907750777733957*sqrt(40326)*sqrt
(-59644114671451 + 2623170438295*sqrt(517))/1936419398792394461637855141912
238396080 + 994065243322493861977*sqrt(40326)*sqrt(-59644114671451 + 262317
0438295*sqrt(517))*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169
1991294769382675)/1290946265861596307758570094608158930720) - 2741964319335
54153007434570764680602132735098624644758583157528631167033*sqrt(7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
1568587150042935*sqrt(517) + 3557579971691991294769382675)/4835228998090907
050154104550174273077613863066504853654553995339577834892800 - 102213276372
0267175882780425063613131088601935958303878081158710949715459967411486447/3
02201812380681690634631534385892067350866441656553353409624708723614680800
- 890360389298500646731845595593034670326044595870824169313*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)/9352473884121677079601749613489
889898672849258229891014611209554309158400 - 426980096365154687189009427342
74005212255282299552802371283821308121207*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517))/935247388412167707960174961348988989867284925822
9891014611209554309158400 - 45113976327488809325094501633826014671791*sqrt(
78)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 3557579971691991294769382675)/107753026610468319324136
304994165747854784217959109076040 + 106380947173342801261761115266827766437
2838283556533836993*sqrt(78)*sqrt(-59644114671451 + 2623170438295*sqrt(517)
)/6896193703069972436744723519626607862706189949382980866560 + 685487767096
69674081892851407413209373218007060934353137152573209405073*sqrt(517)/46081
9179674967465661265101209369097253718701345156893210488960 + 45971497067730
66968921854791223560238809189313591735176029*sqrt(517)*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675)/184327671869986986264
5060404837476389014874805380627572841955840) - 2*sqrt(59653665894623/167878
62468089856 + 5073830635*sqrt(517)/10823895853056 + sqrt(-7120427417275887*
sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587
150042935*sqrt(517) + 3557579971691991294769382675)/4196965617022464)*atan(
-7745677595169577846551420567648953584320*x/(-59292486929118917272637172801
533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-
7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)
) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 232
9048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*s
qrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqr
t(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150
042935*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*s
qrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274
17275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263
621568587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626
3621568587150042935*sqrt(517) + 3557579971691991294769382675)) - 2982195729
967481585931*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sq
rt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(
517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/(-
59292486929118917272637172801533436*sqrt(40326)*sqrt(59653665894623 + 78695
11314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671
```

451 + 2623170438295*sqrt(517)) + 626362156858715004293 5*sqrt(517) + 3557579
971691991294769382675)) + 232904850292582070878538 6304*sqrt(-59644114671451
 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) +
 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 7
5)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885
*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26
23170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199
1294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675)) - 269626104706077517551711257226632 8310*sqrt(78)*sqrt(-596
44114671451 + 2623170438295*sqrt(517))/(-5929248692911891727263717280153343
6*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204
27417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6
263621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290485
0292582070878538 6304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(5
9653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(4
0326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275
887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156
8587150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741727
5887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215
68587150042935*sqrt(517) + 3557579971691991294769382675)) + 610949118223023
8698537149348154570111680*sqrt(517)/(-59292486929118917272637172801533436*s
qrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274
17275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263
621568587150042935*sqrt(517) + 3557579971691991294769382675)) + 23290485029
2582070878538 6304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(5965
3665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)
*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*s
qrt(517) + 3557579971691991294769382675)) + 994065243322493861977*sqrt(4032
6)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675))*sqrt(-712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675)) + 465809700585164141
7570772608*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317
0438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294
769382675)/(-59292486929118917272637172801533436*sqrt(40326)*sqrt(596536658
94623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt
(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(5
17) + 3557579971691991294769382675)) + 2329048502925820708785386304*sqrt(-5
9644114671451 + 2623170438295*sqrt(517))*sqrt(59653665894623 + 786951131488
5*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675)) + 994065243322493861977*sqrt(40326)*sqrt(59653665894623 +
7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-596441
14671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35
57579971691991294769382675))*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644
114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3
557579971691991294769382675)) + 5928773965962566646181550155546791 4*sqrt(40
326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))/(-5929248692911891727 2
637172801533436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) +
 4*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*
sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 7
5)) + 2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sqr
t(517))*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621

568587150042935*sqrt(517) + 35575799716919912947693826 75)) + 99406524332249
3861977*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(
-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517
)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75))*sqrt
(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75)) + 7
210195296486241387297607767303872707 95368/(-592924869291189172726371728015 3
3436*sqrt(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-71
20427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))
+ 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75)) + 23290
485029258207087853 86304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqr
t(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 35575799716919912947693826 75)) + 994065243322493861977*sqr
t(40326)*sqrt(59653665894623 + 7869511314885*sqrt(517) + 4*sqrt(-7120427417
275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362
1568587150042935*sqrt(517) + 35575799716919912947693826 75))*sqrt(-712042741
7275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636
21568587150042935*sqrt(517) + 35575799716919912947693826 75))) - 2*sqrt(-sqr
t(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(51
7)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75)/4196
965617022464 + 59653665894623/16787862468089856 + 5073830635*sqrt(517)/1082
3895853056)*atan(7745677595169577846551420567648953584320*x/(23290485029258
20708785386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt
(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517
)) + 6263621568587150042935*sqrt(517) + 35575799716919912947693826 75) + 596
53665894623 + 7869511314885*sqrt(517)) + 592924869291189172726371728015 3343
6*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997
16919912947693826 75) + 59653665894623 + 7869511314885*sqrt(517)) + 99406524
3322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59
644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517)
+ 35575799716919912947693826 75) + 59653665894623 + 7869511314885*sqrt(517))
) - 7210195296486241387297607767303872707 95368/(232904850292582070878538630
4*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727
5887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215
68587150042935*sqrt(517) + 35575799716919912947693826 75) + 59653665894623 +
 7869511314885*sqrt(517)) + 5929248692911891727263717280153 3436*sqrt(40326)
*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043
8295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129 4769
382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977
*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231
70438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129
4769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451
+ 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716
91991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) - 269626104
7060775175517112572266328310*sqrt(78)*sqrt(-59644114671451 + 2623170438295*
sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 26231704382
95*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-5964411467145
1 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997
16919912947693826 75) + 59653665894623 + 7869511314885*sqrt(517)) + 59292486
929118917272637172801533436*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(
40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004
2935*sqrt(517) + 35575799716919912947693826 75) + 59653665894623 + 786951131
4885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*s
qrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685871
50042935*sqrt(517) + 35575799716919912947693826 75)*sqrt(-4*sqrt(71204274172

75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623
+ 7869511314885*sqrt(517))) - 6109491182230238698537149348154570111680*sqrt
(517)/(2329048502925820708785386304*sqrt(-59644114671451 + 2623170438295*sq
rt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5929248692911
8917272637172801533436*sqrt(40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326
)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*
sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 7869511314885*
sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(7120427417275887*sqrt(4
0326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621568587150042
935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 786
9511314885*sqrt(517))) + 4658097005851641417570772608*sqrt(7120427417275887
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858
7150042935*sqrt(517) + 3557579971691991294769382675)/(232904850292582070878
5386304*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042
7417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62
63621568587150042935*sqrt(517) + 3557579971691991294769382675) + 5965366589
4623 + 7869511314885*sqrt(517)) + 59292486929118917272637172801533436*sqrt(
40326)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262
3170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991
294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493
861977*sqrt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 +
 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169
1991294769382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441146
71451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575
79971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517))) + 592
87739659625666461815501555467914*sqrt(40326)*sqrt(-59644114671451 + 2623170
438295*sqrt(517))/(2329048502925820708785386304*sqrt(-59644114671451 + 2623
170438295*sqrt(517))*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-596441
14671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35
57579971691991294769382675) + 59653665894623 + 7869511314885*sqrt(517)) + 5
9292486929118917272637172801533436*sqrt(40326)*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 78
69511314885*sqrt(517)) + 994065243322493861977*sqrt(40326)*sqrt(71204274172
75887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 6263621
568587150042935*sqrt(517) + 3557579971691991294769382675)*sqrt(-4*sqrt(7120
427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) +
6263621568587150042935*sqrt(517) + 3557579971691991294769382675) + 59653665
894623 + 7869511314885*sqrt(517))) + 2982195729967481585931*sqrt(40326)*sqr
t(-59644114671451 + 2623170438295*sqrt(517))*sqrt(7120427417275887*sqrt(403
26)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 626362156858715004293
5*sqrt(517) + 3557579971691991294769382675)/(2329048502925820708785386304*s
qrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-4*sqrt(712042741727588
7*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62636215685
87150042935*sqrt(517) + 3557579971691991294769382675) + 59653665894623 + 78
69511314885*sqrt(517)) + 59292486929118917272637172801533436*sqrt(40326)*sq
rt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 262317043829
5*sqrt(517)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382
675) + 59653665894623 + 7869511314885*sqrt(517)) + 994065243322493861977*sq
rt(40326)*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 26231704
38295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 355757997169199129476
9382675)*sqrt(-4*sqrt(7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2
623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) + 35575799716919
91294769382675) + 59653665894623 + 7869511314885*sqrt(517))))

3.59

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx$$

Optimal. Leaf size=14

$$\frac{(a+bx)^{16}}{16b}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx &= \int (a+bx)^{15}\,dx\\ &= \frac{(a+bx)^{16}}{16b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{16}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^3,x]
```

```
[Out] (a + b*x)^16/(16*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^3,x]
```

```
[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^3, x]
```

fricas [B] time = 0.92, size = 163, normalized size = 11.64

$$\frac{1}{16}x^{16}b^{15} + x^{15}b^{14}a + \frac{15}{2}x^{14}b^{13}a^2 + 35x^{13}b^{12}a^3 + \frac{455}{4}x^{12}b^{11}a^4 + 273x^{11}b^{10}a^5 + \frac{1001}{2}x^{10}b^9a^6 + 715x^9b^8a^7 + \frac{6435}{8}x^8b^7a^8 + 715x^7b^6a^9 + \frac{1001}{2}x^6b^5a^{10} + 273x^5b^4a^{11} + \frac{455}{4}x^4b^3a^{12} + 35x^3b^2a^{13} + \frac{15}{2}x^2ba^{14} + xa^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^15 + x^15*b^14*a + 15/2*x^14*b^13*a^2 + 35*x^13*b^12*a^3 + 455/
4*x^12*b^11*a^4 + 273*x^11*b^10*a^5 + 1001/2*x^10*b^9*a^6 + 715*x^9*b^8*a^7
 + 6435/8*x^8*b^7*a^8 + 715*x^7*b^6*a^9 + 1001/2*x^6*b^5*a^10 + 273*x^5*b^4
*a^11 + 455/4*x^4*b^3*a^12 + 35*x^3*b^2*a^13 + 15/2*x^2*b*a^14 + x*a^15
```

giac [B] time = 0.28, size = 163, normalized size = 11.64

$$\frac{1}{16}b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}a^2b^{13}x^{14} + 35a^3b^{12}x^{13} + \frac{455}{4}a^4b^{11}x^{12} + 273a^5b^{10}x^{11} + \frac{1001}{2}a^6b^9x^{10} + 715a^7b^8x^9 + \frac{6435}{8}a^8b^7x^8 + 715a^9b^6x^7 + \frac{1001}{2}a^{10}b^5x^6 + 273a^{11}b^4x^5 + \frac{455}{4}a^{12}b^3x^4 + 35a^{13}b^2x^3 + \frac{15}{2}a^{14}bx^2 + a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="giac")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 15/2*a^2*b^13*x^14 + 35*a^3*b^12*x^13 + 455/
4*a^4*b^11*x^12 + 273*a^5*b^10*x^11 + 1001/2*a^6*b^9*x^10 + 715*a^7*b^8*x^9
 + 6435/8*a^8*b^7*x^8 + 715*a^9*b^6*x^7 + 1001/2*a^10*b^5*x^6 + 273*a^11*b^
4*x^5 + 455/4*a^12*b^3*x^4 + 35*a^13*b^2*x^3 + 15/2*a^14*b*x^2 + a^15*x
```

maple [B] time = 0.00, size = 164, normalized size = 11.71

$$\frac{1}{16}b^{15}x^{16} + ab^{14}x^{15} + \frac{15}{2}a^2b^{13}x^{14} + 35a^3b^{12}x^{13} + \frac{455}{4}a^4b^{11}x^{12} + 273a^5b^{10}x^{11} + \frac{1001}{2}a^6b^9x^{10} + 715a^7b^8x^9 + \frac{6435}{8}a^8b^7x^8 + 715a^9b^6x^7 + \frac{1001}{2}a^{10}b^5x^6 + 273a^{11}b^4x^5 + \frac{455}{4}a^{12}b^3x^4 + 35a^{13}b^2x^3 + \frac{15}{2}a^{14}bx^2 + a^{15}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x)
```

```
[Out] 1/16*b^15*x^16+a*b^14*x^15+15/2*a^2*b^13*x^14+35*a^3*b^12*x^13+455/4*a^4*b^
11*x^12+273*a^5*b^10*x^11+1001/2*a^6*b^9*x^10+715*a^7*b^8*x^9+6435/8*a^8*b^
7*x^8+715*a^9*b^6*x^7+1001/2*a^10*b^5*x^6+273*a^11*b^4*x^5+455/4*a^12*b^3*x
^4+35*a^13*b^2*x^3+15/2*a^14*b*x^2+a^15*x
```

maxima [B] time = 0.68, size = 592, normalized size = 42.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^15*x^16 + a*b^14*x^15 + 75/14*a^2*b^13*x^14 + 125/13*a^3*b^12*x^13 +
 100*a^6*b^9*x^10 + 1000/7*a^9*b^6*x^7 + 125/4*a^12*b^3*x^4 + a^15*x + 1/2*
(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^
10 + 25/56*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6 + 336*a^3*b^2*x^5)
*a^8*b^2 + 5/3*(18*b^5*x^10 + 100*a*b^4*x^9 + 225*a^2*b^3*x^8)*a^6*b^4 + 25
/11*(11*b^5*x^12 + 60*a*b^4*x^11)*a^4*b^6 + 1/462*(126*b^10*x^11 + 1386*a*b
^9*x^10 + 3850*a^2*b^8*x^9 + 19800*a^4*b^6*x^7 + 27720*a^6*b^4*x^5 + 11550*
a^8*b^2*x^3 + 330*(6*b^5*x^7 + 35*a*b^4*x^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*
x^4)*a^4*b + 165*(21*b^5*x^8 + 120*a*b^4*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 3
85*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3)*a^5 + 5/308*(77*b^10*x^12 + 840*a*b^
```

```
9*x^11 + 4158*a^2*b^8*x^10 + 12320*a^3*b^7*x^9 + 23100*a^4*b^6*x^8 + 26400*
a^5*b^5*x^7 + 15400*a^6*b^4*x^6)*a^4*b + 5/429*(198*b^10*x^13 + 2145*a*b^9*
x^12 + 10530*a^2*b^8*x^11 + 25740*a^3*b^7*x^10 + 28600*a^4*b^6*x^9)*a^3*b^2
 + 5/182*(78*b^10*x^14 + 840*a*b^9*x^13 + 2275*a^2*b^8*x^12)*a^2*b^3
```

mupad [B]   time = 0.17, size = 163, normalized size = 11.64

$$a^{15}x+\frac{15a^{14}bx^2}{2}+35a^{13}b^2x^3+\frac{455a^{12}b^3x^4}{4}+273a^{11}b^4x^5+\frac{1001a^{10}b^5x^6}{2}+715a^9b^6x^7+\frac{6435a^8b^7x^8}{8}+715a^7b^8x^9+\frac{1001a^6b^9x^{10}}{2}+273a^5b^{10}x^{11}+\frac{455a^4b^{11}x^{12}}{4}+35a^3b^{12}x^{13}+\frac{15a^2b^{13}x^{14}}{2}+ab^{14}x^{15}+\frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^3,x)
```

```
[Out] a^15*x + (b^15*x^16)/16 + (15*a^14*b*x^2)/2 + a*b^14*x^15 + 35*a^13*b^2*x^3
 + (455*a^12*b^3*x^4)/4 + 273*a^11*b^4*x^5 + (1001*a^10*b^5*x^6)/2 + 715*a^
9*b^6*x^7 + (6435*a^8*b^7*x^8)/8 + 715*a^7*b^8*x^9 + (1001*a^6*b^9*x^10)/2
+ 273*a^5*b^10*x^11 + (455*a^4*b^11*x^12)/4 + 35*a^3*b^12*x^13 + (15*a^2*b^
13*x^14)/2
```

sympy [B]   time = 0.11, size = 185, normalized size = 13.21

$$a^{15}x+\frac{15a^{14}bx^2}{2}+35a^{13}b^2x^3+\frac{455a^{12}b^3x^4}{4}+273a^{11}b^4x^5+\frac{1001a^{10}b^5x^6}{2}+715a^9b^6x^7+\frac{6435a^8b^7x^8}{8}+715a^7b^8x^9+\frac{1001a^6b^9x^{10}}{2}+273a^5b^{10}x^{11}+\frac{455a^4b^{11}x^{12}}{4}+35a^3b^{12}x^{13}+\frac{15a^2b^{13}x^{14}}{2}+ab^{14}x^{15}+\frac{b^{15}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**3,x)
```

```
[Out] a**15*x + 15*a**14*b*x**2/2 + 35*a**13*b**2*x**3 + 455*a**12*b**3*x**4/4 +
273*a**11*b**4*x**5 + 1001*a**10*b**5*x**6/2 + 715*a**9*b**6*x**7 + 6435*a*
*8*b**7*x**8/8 + 715*a**7*b**8*x**9 + 1001*a**6*b**9*x**10/2 + 273*a**5*b**
10*x**11 + 455*a**4*b**11*x**12/4 + 35*a**3*b**12*x**13 + 15*a**2*b**13*x**
14/2 + a*b**14*x**15 + b**15*x**16/16
```

3.60

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx$$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2059, 32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx &= \int (a+bx)^{10}\,dx\\ &= \frac{(a+bx)^{11}}{11b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^2,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^2,x]
```

```
[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^2, x]
```

fricas [B] time = 1.02, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^{9}a + 5x^{9}b^{8}a^{2} + 15x^{8}b^{7}a^{3} + 30x^{7}b^{6}a^{4} + 42x^{6}b^{5}a^{5} + 42x^{5}b^{4}a^{6} + 30x^{4}b^{3}a^{7} + 15x^{3}b^{2}a^{8} + 5x^{2}ba^{9} + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a
^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*
x^2*b*a^9 + x*a^10
```

giac [B] time = 0.24, size = 108, normalized size = 7.71

$$\frac{1}{11}\,b^{10}x^{11} + ab^{9}x^{10} + 5\,a^{2}b^{8}x^{9} + 15\,a^{3}b^{7}x^{8} + 30\,a^{4}b^{6}x^{7} + 42\,a^{5}b^{5}x^{6} + 42\,a^{6}b^{4}x^{5} + 30\,a^{7}b^{3}x^{4} + 15\,a^{8}b^{2}x^{3} + 5\,a^{9}bx^{2} + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="giac")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

maple [B] time = 0.00, size = 109, normalized size = 7.79

$$\frac{1}{11}b^{10}x^{11} + a\,b^{9}x^{10} + 5a^{2}b^{8}x^{9} + 15a^{3}b^{7}x^{8} + 30a^{4}b^{6}x^{7} + 42a^{5}b^{5}x^{6} + 42a^{6}b^{4}x^{5} + 30a^{7}b^{3}x^{4} + 15a^{8}b^{2}x^{3} + 5a^{9}b\,x^{2} + a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x)
```

```
[Out] 1/11*b^10*x^11+a*b^9*x^10+5*a^2*b^8*x^9+15*a^3*b^7*x^8+30*a^4*b^6*x^7+42*a^
5*b^5*x^6+42*a^6*b^4*x^5+30*a^7*b^3*x^4+15*a^8*b^2*x^3+5*a^9*b*x^2+a^10*x
```

maxima [B] time = 0.49, size = 228, normalized size = 16.29

$$\frac{1}{11}\,b^{10}x^{11} + ab^{9}x^{10} + \frac{25}{9}\,a^{2}b^{8}x^{9} + \frac{100}{7}\,a^{4}b^{6}x^{7} + 20\,a^{6}b^{4}x^{5} + \frac{25}{3}\,a^{8}b^{2}x^{3} + a^{10}x + \frac{1}{3}\,{\left(b^{5}x^{6} + 6\,ab^{4}x^{5} + 15\,a^{2}b^{3}x^{4} + 20\,a^{3}b^{2}x^{3} + 15\,a^{4}bx^{2}\right)}a^{5} + \frac{5}{21}\,{\left(6\,b^{5}x^{7} + 35\,ab^{4}x^{6} + 84\,a^{2}b^{3}x^{5} + 105\,a^{3}b^{2}x^{4}\right)}a^{4}b + \frac{5}{42}\,{\left(21\,b^{5}x^{8} + 120\,ab^{4}x^{7} + 280\,a^{2}b^{3}x^{6}\right)}a^{3}b^{2} + \frac{5}{18}\,{\left(8\,b^{5}x^{9} + 45\,ab^{4}x^{8}\right)}a^{2}b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)
^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 25/9*a^2*b^8*x^9 + 100/7*a^4*b^6*x^7 + 20*a^6
*b^4*x^5 + 25/3*a^8*b^2*x^3 + a^10*x + 1/3*(b^5*x^6 + 6*a*b^4*x^5 + 15*a^2*
b^3*x^4 + 20*a^3*b^2*x^3 + 15*a^4*b*x^2)*a^5 + 5/21*(6*b^5*x^7 + 35*a*b^4*x
^6 + 84*a^2*b^3*x^5 + 105*a^3*b^2*x^4)*a^4*b + 5/42*(21*b^5*x^8 + 120*a*b^4
*x^7 + 280*a^2*b^3*x^6)*a^3*b^2 + 5/18*(8*b^5*x^9 + 45*a*b^4*x^8)*a^2*b^3
```

mupad [B] time = 0.06, size = 108, normalized size = 7.71

$$a^{10}\,x + 5\,a^{9}\,b\,x^{2} + 15\,a^{8}\,b^{2}\,x^{3} + 30\,a^{7}\,b^{3}\,x^{4} + 42\,a^{6}\,b^{4}\,x^{5} + 42\,a^{5}\,b^{5}\,x^{6} + 30\,a^{4}\,b^{6}\,x^{7} + 15\,a^{3}\,b^{7}\,x^{8} + 5\,a^{2}\,b^{8}\,x^{9} + a\,b^{9}\,x^{10} + \frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*
b*x)^2,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

sympy [B] time = 0.10, size = 114, normalized size = 8.14

$$a^{10}x + 5a^9bx^2 + 15a^8b^2x^3 + 30a^7b^3x^4 + 42a^6b^4x^5 + 42a^5b^5x^6 + 30a^4b^6x^7 + 15a^3b^7x^8 + 5a^2b^8x^9 + ab^9x^{10} + \frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a*
*4*b*x+a**5)**2,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

3.61

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

Rubi [B] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 4.36, number of steps used = 1, number of rules used = 0, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4bx^2+a^5x+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*x
^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

Rubi steps

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx=a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Mathematica [B] time = 0.00, size = 61, normalized size = 4.36

$$a^5x+\frac{5}{2}a^4bx^2+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^2b^3x^4+ab^4x^5+\frac{b^5x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 +
 b^5*x^5,x]
```

```
[Out] a^5*x + (5*a^4*b*x^2)/2 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2 + a*b^4*x^
5 + (b^5*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*
b^4*x^4 + b^5*x^5,x]
```

```
[Out] IntegrateAlgebraic[a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*
b^4*x^4 + b^5*x^5, x]
```

fricas [B] time = 0.85, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5+x^5b^4a+\frac{5}{2}x^4b^3a^2+\frac{10}{3}x^3b^2a^3+\frac{5}{2}x^2ba^4+xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^
4 + x*a^5
```

giac [B] time = 0.29, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

maple [B] time = 0.00, size = 54, normalized size = 3.86

$$\frac{1}{6}b^5x^6 + a\,b^4x^5 + \frac{5}{2}a^2b^3x^4 + \frac{10}{3}a^3b^2x^3 + \frac{5}{2}a^4b\,x^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x)
```

```
[Out] a^5*x+5/2*a^4*b*x^2+10/3*a^3*b^2*x^3+5/2*a^2*b^3*x^4+a*b^4*x^5+1/6*b^5*x^6
```

maxima [B] time = 0.46, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5,x
, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

mupad [B] time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b
*x,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

sympy [B] time = 0.07, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*a**
4*b*x+a**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

3.62 $\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{4b(a+bx)^4}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-1),x]
```

```
[Out] -1/(4*b*(a + b*x)^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx &= \int \frac{1}{(a+bx)^5}\,dx\\ &= -\frac{1}{4b(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-1),x]
```

```
[Out] -1/4*1/(b*(a + b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-1), x]
```

fricas [B] time = 0.77, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^{5}x^{4}+4\,ab^{4}x^{3}+6\,a^{2}b^{3}x^{2}+4\,a^{3}b^{2}x+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="fricas")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{4\,(bx+a)^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="giac")
```

```
[Out] -1/4/((b*x + a)^4*b)
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{1}{4\,(bx+a)^{4}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5),x)
```

```
[Out] -1/4/b/(b*x+a)^4
```

maxima [B] time = 0.69, size = 46, normalized size = 3.29

$$-\frac{1}{4\left(b^{5}x^{4}+4\,ab^{4}x^{3}+6\,a^{2}b^{3}x^{2}+4\,a^{3}b^{2}x+a^{4}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5),x, algorithm="maxima")
```

```
[Out] -1/4/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
```

mupad [B] time = 0.05, size = 48, normalized size = 3.43

$$-\frac{1}{4\,a^{4}\,b+16\,a^{3}\,b^{2}\,x+24\,a^{2}\,b^{3}\,x^{2}+16\,a\,b^{4}\,x^{3}+4\,b^{5}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x),x)
```

```
[Out] -1/(4*a^4*b + 4*b^5*x^4 + 16*a^3*b^2*x + 16*a*b^4*x^3 + 24*a^2*b^3*x^2)
```

sympy [B] time = 0.29, size = 49, normalized size = 3.50

$$-\frac{1}{4a^4b + 16a^3b^2x + 24a^2b^3x^2 + 16ab^4x^3 + 4b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5),x)
```

```
[Out] -1/(4*a**4*b + 16*a**3*b**2*x + 24*a**2*b**3*x**2 + 16*a*b**4*x**3 + 4*b**5
*x**4)
```

3.63 $\displaystyle\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2058, 32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-2),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx &= \int \frac{1}{(a+bx)^{10}}\,dx\\ &= -\frac{1}{9b(a+bx)^9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-2),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-2), x]
```

fricas [B] time = 1.17, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

giac [A] time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^{9}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{9\,(bx+a)^{9}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^2,x
)
```

```
[Out] -1/9/b/(b*x+a)^9
```

maxima [B] time = 0.60, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^{9}+9\,ab^{9}x^{8}+36\,a^{2}b^{8}x^{7}+84\,a^{3}b^{7}x^{6}+126\,a^{4}b^{6}x^{5}+126\,a^{5}b^{5}x^{4}+84\,a^{6}b^{4}x^{3}+36\,a^{7}b^{3}x^{2}+9\,a^{8}b^{2}x+a^{9}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^2,x, algorithm="maxima")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

mupad [B] time = 2.10, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^{9}\,b+81\,a^{8}\,b^{2}\,x+324\,a^{7}\,b^{3}\,x^{2}+756\,a^{6}\,b^{4}\,x^{3}+1134\,a^{5}\,b^{5}\,x^{4}+1134\,a^{4}\,b^{6}\,x^{5}+756\,a^{3}\,b^{7}\,x^{6}+324\,a^{2}\,b^{8}\,x^{7}+81\,a\,b^{9}\,x^{8}+9\,b^{10}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^2,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

sympy [B] time = 0.59, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b + 81a^8b^2x + 324a^7b^3x^2 + 756a^6b^4x^3 + 1134a^5b^5x^4 + 1134a^4b^6x^5 + 756a^3b^7x^6 + 324a^2b^8x^7 + 81ab^9x^8 + 9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**2,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

3.64 $\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{14b(a+bx)^{14}}$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2058, 32}

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4 + b^5*
x^5)^(-3),x]
```

```
[Out] -1/(14*b*(a + b*x)^14)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx &= \int \frac{1}{(a+bx)^{15}}\,dx\\ &= -\frac{1}{14b(a+bx)^{14}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{14b(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a*b^4*x^4
+ b^5*x^5)^(-3),x]
```

```
[Out] -1/14*1/(b*(a + b*x)^14)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^5+5a^4bx+10a^3b^2x^2+10a^2b^3x^3+5ab^4x^4+b^5x^5\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a^5 + 5*a^4*b*x + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a
*b^4*x^4 + b^5*x^5)^(-3), x]
```

fricas [B] time = 1.13, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+3003\,a^{8}b^{7}x^{6}+2002\,a^{9}b^{6}x^{5}+1001\,a^{10}b^{5}x^{4}+364\,a^{11}b^{4}x^{3}+91\,a^{12}b^{3}x^{2}+14\,a^{13}b^{2}x+a^{14}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="fricas")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

giac [A] time = 0.35, size = 12, normalized size = 0.86

$$-\frac{1}{14\,(bx+a)^{14}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="giac")
```

```
[Out] -1/14/((b*x + a)^14*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{14\,(bx+a)^{14}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^5)^3,x
)
```

```
[Out] -1/14/b/(b*x+a)^14
```

maxima [B] time = 0.54, size = 156, normalized size = 11.14

$$-\frac{1}{14\left(b^{15}x^{14}+14\,ab^{14}x^{13}+91\,a^{2}b^{13}x^{12}+364\,a^{3}b^{12}x^{11}+1001\,a^{4}b^{11}x^{10}+2002\,a^{5}b^{10}x^{9}+3003\,a^{6}b^{9}x^{8}+3432\,a^{7}b^{8}x^{7}+3003\,a^{8}b^{7}x^{6}+2002\,a^{9}b^{6}x^{5}+1001\,a^{10}b^{5}x^{4}+364\,a^{11}b^{4}x^{3}+91\,a^{12}b^{3}x^{2}+14\,a^{13}b^{2}x+a^{14}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^5*x^5+5*a*b^4*x^4+10*a^2*b^3*x^3+10*a^3*b^2*x^2+5*a^4*b*x+a^
5)^3,x, algorithm="maxima")
```

```
[Out] -1/14/(b^15*x^14 + 14*a*b^14*x^13 + 91*a^2*b^13*x^12 + 364*a^3*b^12*x^11 +
1001*a^4*b^11*x^10 + 2002*a^5*b^10*x^9 + 3003*a^6*b^9*x^8 + 3432*a^7*b^8*x^
7 + 3003*a^8*b^7*x^6 + 2002*a^9*b^6*x^5 + 1001*a^10*b^5*x^4 + 364*a^11*b^4*
x^3 + 91*a^12*b^3*x^2 + 14*a^13*b^2*x + a^14*b)
```

mupad [B] time = 3.03, size = 158, normalized size = 11.29

$$-\frac{1}{14\,a^{14}\,b+196\,a^{13}\,b^{2}\,x+1274\,a^{12}\,b^{3}\,x^{2}+5096\,a^{11}\,b^{4}\,x^{3}+14014\,a^{10}\,b^{5}\,x^{4}+28028\,a^{9}\,b^{6}\,x^{5}+42042\,a^{8}\,b^{7}\,x^{6}+48048\,a^{7}\,b^{8}\,x^{7}+42042\,a^{6}\,b^{9}\,x^{8}+28028\,a^{5}\,b^{10}\,x^{9}+14014\,a^{4}\,b^{11}\,x^{10}+5096\,a^{3}\,b^{12}\,x^{11}+1274\,a^{2}\,b^{13}\,x^{12}+196\,a\,b^{14}\,x^{13}+14\,b^{15}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^
4*b*x)^3,x)
```

```
[Out] -1/(14*a^14*b + 14*b^15*x^14 + 196*a^13*b^2*x + 196*a*b^14*x^13 + 1274*a^12
*b^3*x^2 + 5096*a^11*b^4*x^3 + 14014*a^10*b^5*x^4 + 28028*a^9*b^6*x^5 + 420
42*a^8*b^7*x^6 + 48048*a^7*b^8*x^7 + 42042*a^6*b^9*x^8 + 28028*a^5*b^10*x^9
 + 14014*a^4*b^11*x^10 + 5096*a^3*b^12*x^11 + 1274*a^2*b^13*x^12)
```

sympy [**B**] time = 0.94, size = 168, normalized size = 12.00

$$-\frac{1}{14a^{14}b+196a^{13}b^2x+1274a^{12}b^3x^2+5096a^{11}b^4x^3+14014a^{10}b^5x^4+28028a^9b^6x^5+42042a^8b^7x^6+48048a^7b^8x^7+42042a^6b^9x^8+28028a^5b^{10}x^9+14014a^4b^{11}x^{10}+5096a^3b^{12}x^{11}+1274a^2b^{13}x^{12}+196ab^{14}x^{13}+14b^{15}x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**5*x**5+5*a*b**4*x**4+10*a**2*b**3*x**3+10*a**3*b**2*x**2+5*
a**4*b*x+a**5)**3,x)
```

```
[Out] -1/(14*a**14*b + 196*a**13*b**2*x + 1274*a**12*b**3*x**2 + 5096*a**11*b**4*
x**3 + 14014*a**10*b**5*x**4 + 28028*a**9*b**6*x**5 + 42042*a**8*b**7*x**6
+ 48048*a**7*b**8*x**7 + 42042*a**6*b**9*x**8 + 28028*a**5*b**10*x**9 + 140
14*a**4*b**11*x**10 + 5096*a**3*b**12*x**11 + 1274*a**2*b**13*x**12 + 196*a
*b**14*x**13 + 14*b**15*x**14)
```

3.65 $\int \frac{1}{1+x^2+x^3+x^5}\,dx$

Optimal. Leaf size=38

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 635, 203, 260, 628}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3 + x^5)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x^2+x^3+x^5}\,dx &= \int \left(\frac{1}{6(1+x)}+\frac{1+x}{2\left(1+x^2\right)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{1}{6}\log(1+x)+\frac{1}{3}\int \frac{1-2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\
&= \frac{1}{6}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{6}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)-\frac{1}{3}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3 + x^5)^(-1),x]
[Out] ArcTan[x]/2 + Log[1 + x]/6 + Log[1 + x^2]/4 - Log[1 - x + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+x^2+x^3+x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^3 + x^5)^(-1),x]
[Out] IntegrateAlgebraic[(1 + x^2 + x^3 + x^5)^(-1), x]
```

fricas [A] time = 1.00, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

giac [A] time = 0.29, size = 31, normalized size = 0.82

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(abs(x + 1
))
```

maple [A] time = 0.01, size = 31, normalized size = 0.82

$$\frac{\arctan(x)}{2}+\frac{\ln(x+1)}{6}+\frac{\ln\left(x^2+1\right)}{4}-\frac{\ln\left(x^2-x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5+x^3+x^2+1),x)
[Out] 1/2*arctan(x)+1/6*ln(x+1)+1/4*ln(x^2+1)-1/3*ln(x^2-x+1)
```

maxima [A] time = 1.11, size = 30, normalized size = 0.79

$$\frac{1}{2}\arctan(x)-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^5+x^3+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/3*log(x^2 - x + 1) + 1/4*log(x^2 + 1) + 1/6*log(x + 1)
```

mupad [B] time = 2.16, size = 36, normalized size = 0.95

$$\frac{\ln(x+1)}{6} - \frac{\ln\left(x^2-x+1\right)}{3} + \ln(x-\mathrm{i})\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + x^3 + x^5 + 1),x)
```

```
[Out] log(x + 1)/6 + log(x - 1i)*(1/4 - 1i/4) + log(x + 1i)*(1/4 + 1i/4) - log(x^
2 - x + 1)/3
```

sympy [A] time = 0.15, size = 29, normalized size = 0.76

$$\frac{\log(x+1)}{6} + \frac{\log\left(x^2+1\right)}{4} - \frac{\log\left(x^2-x+1\right)}{3} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**5+x**3+x**2+1),x)
```

```
[Out] log(x + 1)/6 + log(x**2 + 1)/4 - log(x**2 - x + 1)/3 + atan(x)/2
```

3.66 $\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx$

Optimal. Leaf size=84

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{6343\ldots}{9}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(3-19x^2+32x^4-16x^6\right)^4 \, dx &= \int (-1+x)^4(1+x)^4(-1+2x)^4(1+2x)^4\left(-3+4x^2\right)^4 \, dx\\
&= \int (-1+2x)^4(1+2x)^4\left(-1+x^2\right)^4\left(-3+4x^2\right)^4 \, dx\\
&= \int \left(-1+x^2\right)^4\left(-3+4x^2\right)^4\left(-1+4x^2\right)^4 \, dx\\
&= \int \left(81-2052x^2+22950x^4-149700x^6+634321x^8-1841600x^{10}+3764416\ldots\right.\\
&= 81x-684x^3+4590x^5-\frac{149700x^7}{7}+\frac{634321x^9}{9}-\frac{1841600x^{11}}{11}+\frac{3764416x^{13}}{13}\ldots
\end{aligned}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^9}{9}-\frac{149700x^7}{7}+4590x^5-684x^3+81x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4,x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^4, x]
```

fricas [A] time = 1.07, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590x^{5}-684x^{3}+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="fricas")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

giac [A] time = 0.38, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="giac")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

maple [A] time = 0.00, size = 65, normalized size = 0.77

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590x^{5}-684x^{3}+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^4,x)
```

```
[Out] 81*x-684*x^3+4590*x^5-149700/7*x^7+634321/9*x^9-1841600/11*x^11+3764416/13*
x^13-1094656/3*x^15+5633536/17*x^17-4014080/19*x^19+1884160/21*x^21-524288/
23*x^23+65536/25*x^25
```

maxima [A] time = 0.49, size = 64, normalized size = 0.76

$$\frac{65536}{25}x^{25}-\frac{524288}{23}x^{23}+\frac{1884160}{21}x^{21}-\frac{4014080}{19}x^{19}+\frac{5633536}{17}x^{17}-\frac{1094656}{3}x^{15}+\frac{3764416}{13}x^{13}-\frac{1841600}{11}x^{11}+\frac{634321}{9}x^{9}-\frac{149700}{7}x^{7}+4590\,x^{5}-684\,x^{3}+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^4,x, algorithm="maxima")
```

```
[Out] 65536/25*x^25 - 524288/23*x^23 + 1884160/21*x^21 - 4014080/19*x^19 + 563353
6/17*x^17 - 1094656/3*x^15 + 3764416/13*x^13 - 1841600/11*x^11 + 634321/9*x
^9 - 149700/7*x^7 + 4590*x^5 - 684*x^3 + 81*x
```

mupad [B] time = 2.17, size = 64, normalized size = 0.76

$$\frac{65536\,x^{25}}{25}-\frac{524288\,x^{23}}{23}+\frac{1884160\,x^{21}}{21}-\frac{4014080\,x^{19}}{19}+\frac{5633536\,x^{17}}{17}-\frac{1094656\,x^{15}}{3}+\frac{3764416\,x^{13}}{13}-\frac{1841600\,x^{11}}{11}+\frac{634321\,x^{9}}{9}-\frac{149700\,x^{7}}{7}+4590\,x^5-684\,x^3+81\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^4,x)
```

```
[Out] 81*x - 684*x^3 + 4590*x^5 - (149700*x^7)/7 + (634321*x^9)/9 - (1841600*x^11
)/11 + (3764416*x^13)/13 - (1094656*x^15)/3 + (5633536*x^17)/17 - (4014080*
x^19)/19 + (1884160*x^21)/21 - (524288*x^23)/23 + (65536*x^25)/25
```

sympy [A] time = 0.08, size = 80, normalized size = 0.95

$$\frac{65536x^{25}}{25}-\frac{524288x^{23}}{23}+\frac{1884160x^{21}}{21}-\frac{4014080x^{19}}{19}+\frac{5633536x^{17}}{17}-\frac{1094656x^{15}}{3}+\frac{3764416x^{13}}{13}-\frac{1841600x^{11}}{11}+\frac{634321x^{9}}{9}-\frac{149700x^{7}}{7}+4590x^5-684x^3+81x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**4,x)
```

```
[Out] 65536*x**25/25 - 524288*x**23/23 + 1884160*x**21/21 - 4014080*x**19/19 + 56
33536*x**17/17 - 1094656*x**15/3 + 3764416*x**13/13 - 1841600*x**11/11 + 63
4321*x**9/9 - 149700*x**7/7 + 4590*x**5 - 684*x**3 + 81*x
```

3.67 $\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx$

Optimal. Leaf size=63

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(3-19x^2+32x^4-16x^6\right)^3 \, dx &= -\int (-1+x)^3(1+x)^3(-1+2x)^3(1+2x)^3\left(-3+4x^2\right)^3 \, dx\\
&= -\int (-1+2x)^3(1+2x)^3\left(-1+x^2\right)^3\left(-3+4x^2\right)^3 \, dx\\
&= -\int \left(-1+x^2\right)^3\left(-3+4x^2\right)^3\left(-1+4x^2\right)^3 \, dx\\
&= -\int \left(-27+513x^2-4113x^4+18235x^6-49344x^8+84912x^{10}-93440x^{12}\right.\\
&= 27x-171x^3+\frac{4113x^5}{5}-2605x^7+\frac{16448x^9}{3}-\frac{84912x^{11}}{11}+\frac{93440x^{13}}{13}-
\end{aligned}$$

Mathematica [A] time = 0.00, size = 63, normalized size = 1.00

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^3\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3,x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^3, x]
```

fricas [A] time = 1.10, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

giac [A] time = 0.36, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

maple [A] time = 0.00, size = 50, normalized size = 0.79

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 27*x-171*x^3+4113/5*x^5-2605*x^7+16448/3*x^9-84912/11*x^11+93440/13*x^13-21
248/5*x^15+24576/17*x^17-4096/19*x^19
```

maxima [A] time = 0.73, size = 49, normalized size = 0.78

$$-\frac{4096}{19}x^{19}+\frac{24576}{17}x^{17}-\frac{21248}{5}x^{15}+\frac{93440}{13}x^{13}-\frac{84912}{11}x^{11}+\frac{16448}{3}x^9-2605x^7+\frac{4113}{5}x^5-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -4096/19*x^19 + 24576/17*x^17 - 21248/5*x^15 + 93440/13*x^13 - 84912/11*x^1
1 + 16448/3*x^9 - 2605*x^7 + 4113/5*x^5 - 171*x^3 + 27*x
```

mupad [B] time = 0.05, size = 49, normalized size = 0.78

$$-\frac{4096\,x^{19}}{19}+\frac{24576\,x^{17}}{17}-\frac{21248\,x^{15}}{5}+\frac{93440\,x^{13}}{13}-\frac{84912\,x^{11}}{11}+\frac{16448\,x^9}{3}-2605\,x^7+\frac{4113\,x^5}{5}-171\,x^3+27\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] 27*x - 171*x^3 + (4113*x^5)/5 - 2605*x^7 + (16448*x^9)/3 - (84912*x^11)/11
+ (93440*x^13)/13 - (21248*x^15)/5 + (24576*x^17)/17 - (4096*x^19)/19
```

sympy [A] time = 0.07, size = 60, normalized size = 0.95

$$-\frac{4096x^{19}}{19}+\frac{24576x^{17}}{17}-\frac{21248x^{15}}{5}+\frac{93440x^{13}}{13}-\frac{84912x^{11}}{11}+\frac{16448x^9}{3}-2605x^7+\frac{4113x^5}{5}-171x^3+27x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -4096*x**19/19 + 24576*x**17/17 - 21248*x**15/5 + 93440*x**13/13 - 84912*x*
*11/11 + 16448*x**9/3 - 2605*x**7 + 4113*x**5/5 - 171*x**3 + 27*x
```

3.68 $\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx$

Optimal. Leaf size=44

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2059, 517, 521}

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 521

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2059

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[u^p, x] /;  !SumQ[Non
freeFactors[u, x]]] /; PolyQ[P, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx &= \int (-1+x)^2(1+x)^2(-1+2x)^2(1+2x)^2\left(-3+4x^2\right)^2\,dx\\
&= \int (-1+2x)^2(1+2x)^2\left(-1+x^2\right)^2\left(-3+4x^2\right)^2\,dx\\
&= \int \left(-1+x^2\right)^2\left(-3+4x^2\right)^2\left(-1+4x^2\right)^2\,dx\\
&= \int \left(9-114x^2+553x^4-1312x^6+1632x^8-1024x^{10}+256x^{12}\right)dx\\
&= 9x-38x^3+\frac{553x^5}{5}-\frac{1312x^7}{7}+\frac{544x^9}{3}-\frac{1024x^{11}}{11}+\frac{256x^{13}}{13}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(3-19x^2+32x^4-16x^6\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^2, x]
```

fricas [A] time = 1.00, size = 34, normalized size = 0.77

$$\frac{256}{13}x^{13}-\frac{1024}{11}x^{11}+\frac{544}{3}x^9-\frac{1312}{7}x^7+\frac{553}{5}x^5-38x^3+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

giac [A] time = 0.36, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.80

$$\frac{256}{13}x^{13}-\frac{1024}{11}x^{11}+\frac{544}{3}x^9-\frac{1312}{7}x^7+\frac{553}{5}x^5-38x^3+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] 9*x-38*x^3+553/5*x^5-1312/7*x^7+544/3*x^9-1024/11*x^11+256/13*x^13
```

maxima [A] time = 0.75, size = 34, normalized size = 0.77

$$\frac{256}{13}\,x^{13}-\frac{1024}{11}\,x^{11}+\frac{544}{3}\,x^9-\frac{1312}{7}\,x^7+\frac{553}{5}\,x^5-38\,x^3+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 256/13*x^13 - 1024/11*x^11 + 544/3*x^9 - 1312/7*x^7 + 553/5*x^5 - 38*x^3 +
9*x
```

mupad [B] time = 0.02, size = 34, normalized size = 0.77

$$\frac{256\,x^{13}}{13}-\frac{1024\,x^{11}}{11}+\frac{544\,x^9}{3}-\frac{1312\,x^7}{7}+\frac{553\,x^5}{5}-38\,x^3+9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] 9*x - 38*x^3 + (553*x^5)/5 - (1312*x^7)/7 + (544*x^9)/3 - (1024*x^11)/11 +
(256*x^13)/13
```

sympy [A] time = 0.06, size = 41, normalized size = 0.93

$$\frac{256x^{13}}{13}-\frac{1024x^{11}}{11}+\frac{544x^9}{3}-\frac{1312x^7}{7}+\frac{553x^5}{5}-38x^3+9x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] 256*x**13/13 - 1024*x**11/11 + 544*x**9/3 - 1312*x**7/7 + 553*x**5/5 - 38*x
**3 + 9*x
```

3.69 $\int (3-19x^2+32x^4-16x^6)\, dx$

Optimal. Leaf size=25

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

Rubi steps

$$\int (3-19x^2+32x^4-16x^6)\, dx = 3x-\frac{19x^3}{3}+\frac{32x^5}{5}-\frac{16x^7}{7}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$-\frac{16x^7}{7}+\frac{32x^5}{5}-\frac{19x^3}{3}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3-19x^2+32x^4-16x^6)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3 - 19*x^2 + 32*x^4 - 16*x^6,x]
[Out] IntegrateAlgebraic[3 - 19*x^2 + 32*x^4 - 16*x^6, x]
```

fricas [A] time = 1.00, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7+\frac{32}{5}x^5-\frac{19}{3}x^3+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="fricas")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

giac [A] time = 0.28, size = 19, normalized size = 0.76

$$-\frac{16}{7}x^7+\frac{32}{5}x^5-\frac{19}{3}x^3+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="giac")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$-\frac{16}{7}x^{7}+\frac{32}{5}x^{5}-\frac{19}{3}x^{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-16*x^6+32*x^4-19*x^2+3,x)
[Out] 3*x-19/3*x^3+32/5*x^5-16/7*x^7
```

maxima [A] time = 0.64, size = 19, normalized size = 0.76

$$-\frac{16}{7}\,x^{7}+\frac{32}{5}\,x^{5}-\frac{19}{3}\,x^{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x^6+32*x^4-19*x^2+3,x, algorithm="maxima")
[Out] -16/7*x^7 + 32/5*x^5 - 19/3*x^3 + 3*x
```

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$-\frac{16\,x^{7}}{7}+\frac{32\,x^{5}}{5}-\frac{19\,x^{3}}{3}+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(32*x^4 - 19*x^2 - 16*x^6 + 3,x)
[Out] 3*x - (19*x^3)/3 + (32*x^5)/5 - (16*x^7)/7
```

sympy [A] time = 0.06, size = 22, normalized size = 0.88

$$-\frac{16x^{7}}{7}+\frac{32x^{5}}{5}-\frac{19x^{3}}{3}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-16*x**6+32*x**4-19*x**2+3,x)
[Out] -16*x**7/7 + 32*x**5/5 - 19*x**3/3 + 3*x
```

3.70 $\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx$

Optimal. Leaf size=31

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2057, 207}

$$\frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] ArcTanh[x]/3 + ArcTanh[2*x]/3 - ArcTanh[(2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{3-19x^2+32x^4-16x^6} \, dx &= \int \left(-\frac{1}{3\left(-1+x^2\right)}+\frac{2}{-3+4x^2}-\frac{2}{3\left(-1+4x^2\right)}\right) dx \\
&= -\left(\frac{1}{3}\int \frac{1}{-1+x^2} \, dx\right)-\frac{2}{3}\int \frac{1}{-1+4x^2} \, dx+2\int \frac{1}{-3+4x^2} \, dx \\
&= \frac{1}{3}\tanh^{-1}(x)+\frac{1}{3}\tanh^{-1}(2x)-\frac{\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 2.00

$$\frac{1}{6}\left(-\log\left(2x^2-3x+1\right)+\log\left(2x^2+3x+1\right)+\sqrt{3}\log\left(\sqrt{3}-2x\right)-\sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] (Sqrt[3]*Log[Sqrt[3] - 2*x] - Sqrt[3]*Log[Sqrt[3] + 2*x] - Log[1 - 3*x + 2*
x^2] + Log[1 + 3*x + 2*x^2])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3-19x^2+32x^4-16x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-1), x]
```

fricas [B] time = 1.25, size = 56, normalized size = 1.81

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{4\,x^2-4\,\sqrt{3}x+3}{4\,x^2-3}\right)+\frac{1}{6}\,\log\left(2\,x^2+3\,x+1\right)-\frac{1}{6}\,\log\left(2\,x^2-3\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 1/6*log(2*x^2 + 3*
x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

giac [B] time = 0.28, size = 62, normalized size = 2.00

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{\left|8\,x-4\,\sqrt{3}\right|}{\left|8\,x+4\,\sqrt{3}\right|}\right)+\frac{1}{6}\,\log\left(\left|2\,x+1\right|\right)-\frac{1}{6}\,\log\left(\left|2\,x-1\right|\right)+\frac{1}{6}\,\log\left(\left|x+1\right|\right)-\frac{1}{6}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) + 1/6*log(abs(2*
x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*log(abs(x - 1))
```

maple [A] time = 0.01, size = 42, normalized size = 1.35

$$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{3}-\frac{\ln\left(x-1\right)}{6}-\frac{\ln\left(2x-1\right)}{6}+\frac{\ln\left(x+1\right)}{6}+\frac{\ln\left(2x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3),x)
```

```
[Out] -1/6*ln(-1+x)-1/6*ln(-1+2*x)+1/6*ln(1+2*x)+1/6*ln(x+1)-1/3*arctanh(2/3*x*3^
(1/2))*3^(1/2)
```

maxima [B] time = 1.05, size = 54, normalized size = 1.74

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{2\,x-\sqrt{3}}{2\,x+\sqrt{3}}\right)+\frac{1}{6}\,\log\left(2\,x+1\right)-\frac{1}{6}\,\log\left(2\,x-1\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{6}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) + 1/6*log(2*x + 1) - 1/6*l
og(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

mupad [B] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{x}{4608\left(\frac{x^2}{6912}+\frac{1}{13824}\right)}\right)}{3}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3),x)
```

```
[Out] atanh(x/(4608*(x^2/6912 + 1/13824)))/3 - (3^(1/2)*atanh((2*3^(1/2)*x)/3))/3
```

sympy [B] time = 0.15, size = 63, normalized size = 2.03

$$\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{6}-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3),x)
```

```
[Out] sqrt(3)*log(x - sqrt(3)/2)/6 - sqrt(3)*log(x + sqrt(3)/2)/6 - log(x**2 - 3*
x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

3.71 $\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{2x}{3\left(3-4x^2\right)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 207, 199}

$$\frac{2x}{3\left(3-4x^2\right)}+\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(x+1)}-\frac{1}{18(2x+1)}+\frac{67}{54}\tanh^{-1}(x)-\frac{7}{27}\tanh^{-1}(2x)-\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] 1/(18*(1 - 2*x)) + 1/(36*(1 - x)) - 1/(36*(1 + x)) - 1/(18*(1 + 2*x)) + (2*
x)/(3*(3 - 4*x^2)) + (67*ArcTanh[x])/54 - (7*ArcTanh[2*x])/27 - (5*ArcTanh[
(2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx=\int \left(\frac{1}{36(-1+x)^2}+\frac{1}{36(1+x)^2}+\frac{1}{9(-1+2x)^2}+\frac{1}{9(1+2x)^2}-\frac{67}{54\left(-1+x^2\right)}+\right.$$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{14}{27}\int \frac{1}{-1+4x^2}\,dx-\frac{67}{54}$$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-1}($$

$$=\frac{1}{18(1-2x)}+\frac{1}{36(1-x)}-\frac{1}{36(1+x)}-\frac{1}{18(1+2x)}+\frac{2x}{3\left(3-4x^2\right)}+\frac{67}{54}\tanh^{-1}($$

Mathematica [A] time = 0.06, size = 103, normalized size = 1.16

$$\frac{1}{108}\left(-\frac{6x\left(80x^4-104x^2+27\right)}{16x^6-32x^4+19x^2-3}+14\log(1-2x)+30\sqrt{3}\log\left(\sqrt{3}-2x\right)-67\log(1-x)+67\log(x+1)-14\log(2x+1)-30\sqrt{3}\log\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] ((-6*x*(27 - 104*x^2 + 80*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) + 14*Log[1
- 2*x] + 30*Sqrt[3]*Log[Sqrt[3] - 2*x] - 67*Log[1 - x] + 67*Log[1 + x] - 14
*Log[1 + 2*x] - 30*Sqrt[3]*Log[Sqrt[3] + 2*x])/108
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-2), x]
```

fricas [B] time = 1.19, size = 177, normalized size = 1.99

$$-\frac{480\,x^5-624\,x^3-30\,\sqrt{3}\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(\frac{4\,x^2-4\,\sqrt{3}x+3}{4\,x^2-3}\right)+14\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(2\,x+1\right)-14\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(2\,x-1\right)-67\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(x+1\right)+67\left(16\,x^6-32\,x^4+19\,x^2-3\right)\log\left(x-1\right)+162\,x}{108\left(16\,x^6-32\,x^4+19\,x^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/108*(480*x^5 - 624*x^3 - 30*sqrt(3)*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log((
4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) + 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*l
og(2*x + 1) - 14*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(2*x - 1) - 67*(16*x^6 -
 32*x^4 + 19*x^2 - 3)*log(x + 1) + 67*(16*x^6 - 32*x^4 + 19*x^2 - 3)*log(x
- 1) + 162*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3)
```

giac [A] time = 0.28, size = 97, normalized size = 1.09

$$\frac{5}{18}\sqrt{3}\log\left(\frac{\left|8\,x-4\,\sqrt{3}\right|}{\left|8\,x+4\,\sqrt{3}\right|}\right)-\frac{80\,x^5-104\,x^3+27\,x}{18\left(16\,x^6-32\,x^4+19\,x^2-3\right)}-\frac{7}{54}\log\left(\left|2\,x+1\right|\right)+\frac{7}{54}\log\left(\left|2\,x-1\right|\right)+\frac{67}{108}\log\left(\left|x+1\right|\right)-\frac{67}{108}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/18*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/18*(80*x^5
- 104*x^3 + 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(abs(2*x + 1)) +
 7/54*log(abs(2*x - 1)) + 67/108*log(abs(x + 1)) - 67/108*log(abs(x - 1))
```

maple [A] time = 0.02, size = 84, normalized size = 0.94

$$-\frac{x}{6\left(x^2-\frac{3}{4}\right)}-\frac{5\sqrt{3}\,\operatorname{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{9}-\frac{67\ln\left(x-1\right)}{108}+\frac{7\ln\left(2x-1\right)}{54}+\frac{67\ln\left(x+1\right)}{108}-\frac{7\ln\left(2x+1\right)}{54}-\frac{1}{36\left(x-1\right)}-\frac{1}{18\left(2x-1\right)}-\frac{1}{18\left(2x+1\right)}-\frac{1}{36\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^2,x)
```

```
[Out] -1/36/(x-1)-67/108*ln(x-1)-1/18/(2*x-1)+7/54*ln(2*x-1)-1/18/(2*x+1)-7/54*ln
(2*x+1)-1/36/(x+1)+67/108*ln(x+1)-1/6*x/(x^2-3/4)-5/9*arctanh(2/3*3^(1/2)*x
)*3^(1/2)
```

maxima [A] time = 1.29, size = 89, normalized size = 1.00

$$\frac{5}{18}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{80x^5-104x^3+27x}{18\left(16x^6-32x^4+19x^2-3\right)}-\frac{7}{54}\log(2x+1)+\frac{7}{54}\log(2x-1)+\frac{67}{108}\log(x+1)-\frac{67}{108}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/18*(80*x^5 - 104*x^3
+ 27*x)/(16*x^6 - 32*x^4 + 19*x^2 - 3) - 7/54*log(2*x + 1) + 7/54*log(2*x -
 1) + 67/108*log(x + 1) - 67/108*log(x - 1)
```

mupad [B] time = 0.08, size = 64, normalized size = 0.72

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\;67\mathrm{i}}{54}+\frac{\operatorname{atan}(x\,2\mathrm{i})\;7\mathrm{i}}{27}-\frac{\frac{5x^5}{18}-\frac{13x^3}{36}+\frac{3x}{32}}{x^6-2x^4+\frac{19x^2}{16}-\frac{3}{16}}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)5\mathrm{i}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^2,x)
```

```
[Out] (atan(x*2i)*7i)/27 - (atan(x*1i)*67i)/54 - ((3*x)/32 - (13*x^3)/36 + (5*x^5
)/18)/((19*x^2)/16 - 2*x^4 + x^6 - 3/16) + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*
5i)/9
```

sympy [A] time = 1.36, size = 104, normalized size = 1.17

$$\frac{-80x^5+104x^3-27x}{288x^6-576x^4+342x^2-54}-\frac{67\log(x-1)}{108}+\frac{7\log\left(x-\frac{1}{2}\right)}{54}-\frac{7\log\left(x+\frac{1}{2}\right)}{54}+\frac{67\log(x+1)}{108}+\frac{5\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{18}-\frac{5\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**2,x)
```

```
[Out] (-80*x**5 + 104*x**3 - 27*x)/(288*x**6 - 576*x**4 + 342*x**2 - 54) - 67*log
(x - 1)/108 + 7*log(x - 1/2)/54 - 7*log(x + 1/2)/54 + 67*log(x + 1)/108 + 5
*sqrt(3)*log(x - sqrt(3)/2)/18 - 5*sqrt(3)*log(x + sqrt(3)/2)/18
```

3.72 $\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx$

Optimal. Leaf size=161

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{}{43}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 207, 199}

$$\frac{5x}{3\left(3-4x^2\right)}-\frac{2x}{3\left(3-4x^2\right)^2}-\frac{7}{108(1-2x)}+\frac{67}{432(1-x)}-\frac{67}{432(x+1)}+\frac{7}{108(2x+1)}+\frac{1}{108(1-2x)^2}+\frac{1}{432(1-x)^2}-\frac{1}{432(x+1)^2}-\frac{1}{108(2x+1)^2}+\frac{3913}{648}\tanh^{-1}(x)+\frac{67}{162}\tanh^{-1}(2x)-4\sqrt{3}\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+\frac{5\tanh^{-1}\left(\frac{2x}{\sqrt{3}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] 1/(108*(1 - 2*x)^2) - 7/(108*(1 - 2*x)) + 1/(432*(1 - x)^2) + 67/(432*(1 -
x)) - 1/(432*(1 + x)^2) - 67/(432*(1 + x)) - 1/(108*(1 + 2*x)^2) + 7/(108*(
1 + 2*x)) - (2*x)/(3*(3 - 4*x^2)^2) + (5*x)/(3*(3 - 4*x^2)) + (3913*ArcTanh
[x])/648 + (67*ArcTanh[2*x])/162 + (5*ArcTanh[(2*x)/Sqrt[3]])/(6*Sqrt[3]) -
 4*Sqrt[3]*ArcTanh[(2*x)/Sqrt[3]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx = \int \left(-\frac{1}{216(-1+x)^3}+\frac{67}{432(-1+x)^2}+\frac{1}{216(1+x)^3}+\frac{67}{432(1+x)^2}-\frac{1}{27(-1+2\cdots}\right.$$
$$= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{\cdots}{432\cdots}$$
$$= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{\cdots}{432\cdots}$$
$$= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{\cdots}{432\cdots}$$
$$= \frac{1}{108(1-2x)^2}-\frac{7}{108(1-2x)}+\frac{1}{432(1-x)^2}+\frac{67}{432(1-x)}-\frac{1}{432(1+x)^2}-\frac{\cdots}{432\cdots}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 0.85

$$\frac{\frac{36x(80x^4-104x^2+27)}{(-16x^6+32x^4-19x^2+3)^2}-\frac{6x(2288x^4-2384x^2+345)}{16x^6-32x^4+19x^2-3}-268\log(1-2x)+2412\sqrt{3}\log\left(\sqrt{3}-2x\right)-3913\log(1-x)+3913\log(x+1)+268\log(2x+1)-2412\sqrt{3}\log\left(2x+\sqrt{3}\right)}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] ((36*x*(27 - 104*x^2 + 80*x^4))/(3 - 19*x^2 + 32*x^4 - 16*x^6)^2 - (6*x*(34
5 - 2384*x^2 + 2288*x^4))/(-3 + 19*x^2 - 32*x^4 + 16*x^6) - 268*Log[1 - 2*x
] + 2412*Sqrt[3]*Log[Sqrt[3] - 2*x] - 3913*Log[1 - x] + 3913*Log[1 + x] + 2
68*Log[1 + 2*x] - 2412*Sqrt[3]*Log[Sqrt[3] + 2*x])/1296
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(3-19x^2+32x^4-16x^6\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(3 - 19*x^2 + 32*x^4 - 16*x^6)^(-3), x]
```

fricas [B] time = 1.30, size = 282, normalized size = 1.75

$$-\frac{219648x^{11}-668160x^9+751680x^7-382080x^5+85986x^3-2412\sqrt{3}\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)\log\left(\frac{4x^2-4\sqrt{3}x+3}{4x^2-3}\right)-268\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)\log\left(2x+1\right)+268\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)\log\left(2x-1\right)-3913\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)\log\left(x+1\right)+3913\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)\log\left(x-1\right)-7182x}{1296\left(256x^{12}-1024x^{10}+1632x^8-1312x^6+553x^4-114x^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/1296*(219648*x^11 - 668160*x^9 + 751680*x^7 - 382080*x^5 + 85986*x^3 - 2
412*sqrt(3)*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2
 + 9)*log((4*x^2 - 4*sqrt(3)*x + 3)/(4*x^2 - 3)) - 268*(256*x^12 - 1024*x^1
0 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x + 1) + 268*(256*x^
12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)*log(2*x - 1)
- 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9)
*log(x + 1) + 3913*(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x^6 + 553*x^4 -
114*x^2 + 9)*log(x - 1) - 7182*x)/(256*x^12 - 1024*x^10 + 1632*x^8 - 1312*x
^6 + 553*x^4 - 114*x^2 + 9)
```

giac [A] time = 0.31, size = 112, normalized size = 0.70

$$\frac{67}{36}\sqrt{3}\log\left(\frac{|8x-4\sqrt{3}|}{|8x+4\sqrt{3}|}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\left(16x^{6}-32x^{4}+19x^{2}-3\right)^{2}}+\frac{67}{324}\log\left(|2x+1|\right)-\frac{67}{324}\log\left(|2x-1|\right)+\frac{3913}{1296}\log\left(|x+1|\right)-\frac{3913}{1296}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 67/36*sqrt(3)*log(abs(8*x - 4*sqrt(3))/abs(8*x + 4*sqrt(3))) - 1/216*(36608
*x^11 - 111360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(16*x^6 -
 32*x^4 + 19*x^2 - 3)^2 + 67/324*log(abs(2*x + 1)) - 67/324*log(abs(2*x - 1
)) + 3913/1296*log(abs(x + 1)) - 3913/1296*log(abs(x - 1))
```

maple [A] time = 0.02, size = 126, normalized size = 0.78

$$-\frac{67\sqrt{3}\,\operatorname{arctanh}\left(\frac{2\sqrt{3}\,x}{3}\right)}{18}-\frac{3913\ln\left(x-1\right)}{1296}-\frac{67\ln\left(2x-1\right)}{324}+\frac{3913\ln\left(x+1\right)}{1296}+\frac{67\ln\left(2x+1\right)}{324}+\frac{1}{432\left(x-1\right)^{2}}-\frac{67}{432\left(x-1\right)}+\frac{1}{108\left(2x-1\right)^{2}}+\frac{7}{108\left(2x-1\right)}-\frac{1}{108\left(2x+1\right)^{2}}+\frac{7}{108\left(2x+1\right)}-\frac{1}{432\left(x+1\right)^{2}}-\frac{67}{432\left(x+1\right)}+\frac{-\frac{20}{3}x^{3}+\frac{13}{3}x}{\left(4x^{2}-3\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-16*x^6+32*x^4-19*x^2+3)^3,x)
```

```
[Out] 1/432/(x-1)^2-67/432/(x-1)-3913/1296*ln(x-1)+1/108/(2*x-1)^2+7/108/(2*x-1)-
67/324*ln(2*x-1)-1/108/(2*x+1)^2+7/108/(2*x+1)+67/324*ln(2*x+1)-1/432/(x+1)
^2-67/432/(x+1)+3913/1296*ln(x+1)+64*(-5/48*x^3+13/192*x)/(4*x^2-3)^2-67/18
*arctanh(2/3*3^(1/2)*x)*3^(1/2)
```

maxima [A] time = 1.32, size = 119, normalized size = 0.74

$$\frac{67}{36}\sqrt{3}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right)-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{216\left(256x^{12}-1024x^{10}+1632x^{8}-1312x^{6}+553x^{4}-114x^{2}+9\right)}+\frac{67}{324}\log\left(2x+1\right)-\frac{67}{324}\log\left(2x-1\right)+\frac{3913}{1296}\log\left(x+1\right)-\frac{3913}{1296}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x^6+32*x^4-19*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(3)*log((2*x - sqrt(3))/(2*x + sqrt(3))) - 1/216*(36608*x^11 - 11
1360*x^9 + 125280*x^7 - 63680*x^5 + 14331*x^3 - 1197*x)/(256*x^12 - 1024*x^
10 + 1632*x^8 - 1312*x^6 + 553*x^4 - 114*x^2 + 9) + 67/324*log(2*x + 1) - 6
7/324*log(2*x - 1) + 3913/1296*log(x + 1) - 3913/1296*log(x - 1)
```

mupad [B] time = 0.09, size = 93, normalized size = 0.58

$$\frac{-\frac{143x^{11}}{216}+\frac{145x^{9}}{72}-\frac{145x^{7}}{64}+\frac{995x^{5}}{864}-\frac{4777x^{3}}{18432}+\frac{133x}{6144}}{x^{12}-4x^{10}+\frac{51x^{8}}{8}-\frac{41x^{6}}{8}+\frac{553x^{4}}{256}-\frac{57x^{2}}{128}+\frac{9}{256}}-\frac{\mathrm{atan}\left(x\,2\mathrm{i}\right)\,67\mathrm{i}}{162}-\frac{\mathrm{atan}\left(x\,1\mathrm{i}\right)\,3913\mathrm{i}}{648}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x\,2\mathrm{i}}{3}\right)\,67\mathrm{i}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(19*x^2 - 32*x^4 + 16*x^6 - 3)^3,x)
```

```
[Out] ((133*x)/6144 - (4777*x^3)/18432 + (995*x^5)/864 - (145*x^7)/64 + (145*x^9)
/72 - (143*x^11)/216)/((553*x^4)/256 - (57*x^2)/128 - (41*x^6)/8 + (51*x^8)
/8 - 4*x^10 + x^12 + 9/256) - (atan(x*2i)*67i)/162 - (atan(x*1i)*3913i)/648
 + (3^(1/2)*atan((3^(1/2)*x*2i)/3)*67i)/18
```

sympy [A] time = 1.45, size = 134, normalized size = 0.83

$$-\frac{36608x^{11}-111360x^{9}+125280x^{7}-63680x^{5}+14331x^{3}-1197x}{55296x^{12}-221184x^{10}+352512x^{8}-283392x^{6}+119448x^{4}-24624x^{2}+1944}-\frac{3913\log\left(x-1\right)}{1296}-\frac{67\log\left(x-\frac{1}{2}\right)}{324}+\frac{67\log\left(x+\frac{1}{2}\right)}{324}+\frac{3913\log\left(x+1\right)}{1296}+\frac{67\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}\right)}{36}-\frac{67\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-16*x**6+32*x**4-19*x**2+3)**3,x)
```

```
[Out] -(36608*x**11 - 111360*x**9 + 125280*x**7 - 63680*x**5 + 14331*x**3 - 1197*
x)/(55296*x**12 - 221184*x**10 + 352512*x**8 - 283392*x**6 + 119448*x**4 -
24624*x**2 + 1944) - 3913*log(x - 1)/1296 - 67*log(x - 1/2)/324 + 67*log(x
+ 1/2)/324 + 3913*log(x + 1)/1296 + 67*sqrt(3)*log(x - sqrt(3)/2)/36 - 67*s
qrt(3)*log(x + sqrt(3)/2)/36
```

3.73 $\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{32\left(1-x^2\right)}+\frac{\left(99-17x^2\right)x}{128\left(x^4-6x^2+1\right)}+\frac{5}{32}\tanh^{-1}(x)+\frac{1}{512}\left(3\sqrt{2}-4\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)+\frac{1}{512}\left(4+3\sqrt{2}\right)\tanh^{-1}$$

Rubi [B] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2057, 207, 638, 618, 206, 632, 31}

$$-\frac{41-17x}{256\left(-x^2+2x+1\right)}+\frac{17x+41}{256\left(-x^2-2x+1\right)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)-\frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{256\sqrt{2}}+\frac{5}{32}\tanh^{-1}(x)+\frac{17\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] 1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41 -
 17*x)/(256*(1 + 2*x - x^2)) - (17*ArcTanh[(1 - x)/Sqrt[2]])/(256*Sqrt[2])
+ (5*ArcTanh[x])/32 + (17*ArcTanh[(1 + x)/Sqrt[2]])/(256*Sqrt[2]) + ((2 - 7
*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x]
)/512 - ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] + x])/512 - ((2 + 7*Sqrt[2])*Log[1
 + Sqrt[2] + x])/512
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
```

```
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2057

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P /. x -> Sqrt[x]]}, Int[Expan
dIntegrand[(u /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; Po
lyQ[P, x^2] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx = \int \left(\frac{1}{64(-1+x)^2}+\frac{1}{64(1+x)^2}-\frac{5}{32\left(-1+x^2\right)}+\frac{29-12x}{64\left(-1-2x+x^2\right)^2}+\frac{6+\cdots}{128\left(-1-\cdots\right.}\right.$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{1}{128}\int \frac{6+x}{-1-2x+x^2}\,dx+\frac{1}{128}\int \frac{6-x}{-1+2x+x^2}\,dx+\frac{1}{64}\cdots$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)\cdots$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}+\frac{5}{32}\tanh^{-1}(x)\cdots$$

$$= \frac{1}{64(1-x)}-\frac{1}{64(1+x)}+\frac{41+17x}{256\left(1-2x-x^2\right)}-\frac{41-17x}{256\left(1+2x-x^2\right)}-\frac{17\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\cdots\right)}{256\sqrt{2}}\cdots$$

Mathematica [A] time = 0.10, size = 132, normalized size = 1.45

$$\frac{-\frac{8x\left(21x^4-140x^2+103\right)}{x^6-7x^4+7x^2-1}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(4-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+80\log(x+1)+\left(4+3\sqrt{2}\right)\log\left(x+\sqrt{2}-1\right)+\left(3\sqrt{2}-4\right)\log\left(x+\sqrt{2}+1\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] ((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x]
 - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*Sqrt
[2])*Log[1 + Sqrt[2] + x])/1024
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2), x]
```

fricas [B] time = 1.32, size = 223, normalized size = 2.45

$$-\frac{168x^5-1120x^3-3\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2+2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)-3\sqrt{2}\left(x^6-7x^4+7x^2-1\right)\log\left(\frac{x^2+2\sqrt{2}(x-1)-2x+3}{x^2-2x-1}\right)+4\left(x^6-7x^4+7x^2-1\right)\log\left(x^2+2x-1\right)-4\left(x^6-7x^4+7x^2-1\right)\log\left(x^2-2x-1\right)-80\left(x^6-7x^4+7x^2-1\right)\log(x+1)+80\left(x^6-7x^4+7x^2-1\right)\log(x-1)+824x}{1024\left(x^6-7x^4+7x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
+ 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4 +
7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)*log
(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 - 7*x^4
 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

giac [A] time = 0.44, size = 134, normalized size = 1.47

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-2\right|}{\left|2x+2\sqrt{2}-2\right|}\right)-\frac{21x^5-140x^3+103x}{128\left(x^6-7x^4+7x^2-1\right)}-\frac{1}{256}\log\left(\left|x^2+2x-1\right|\right)+\frac{1}{256}\log\left(\left|x^2-2x-1\right|\right)+\frac{5}{64}\log\left(\left|x+1\right|\right)-\frac{5}{64}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

```
[Out] -3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3/
1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/128
*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^2 +
 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/64*lo
g(abs(x - 1))
```

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{512}-\frac{5\ln\left(x-1\right)}{64}+\frac{5\ln\left(x+1\right)}{64}+\frac{\ln\left(x^2-2x-1\right)}{256}-\frac{\ln\left(x^2+2x-1\right)}{256}-\frac{1}{64\left(x-1\right)}-\frac{\frac{17x}{2}+\frac{41}{2}}{128\left(x^2+2x-1\right)}-\frac{1}{64\left(x+1\right)}+\frac{-\frac{17x}{2}+\frac{41}{2}}{128x^2-256x-128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-7*x^4+7*x^2-1)^2,x)
```

```
[Out] -1/64/(x-1)-5/64*ln(x-1)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-1
)+3/512*2^(1/2)*arctanh(1/4*(2+2*x)*2^(1/2))-1/64/(x+1)+5/64*ln(x+1)+1/128*
(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4*(2
*x-2)*2^(1/2))
```

maxima [A] time = 1.27, size = 114, normalized size = 1.25

$$-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)-\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{21x^5-140x^3+103x}{128\left(x^6-7x^4+7x^2-1\right)}-\frac{1}{256}\log\left(x^2+2x-1\right)+\frac{1}{256}\log\left(x^2-2x-1\right)+\frac{5}{64}\log\left(x+1\right)-\frac{5}{64}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*l
og((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x)/
(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2*x
- 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

mupad [B] time = 2.18, size = 126, normalized size = 1.38

$$-\frac{\operatorname{atan}\left(x\,1\mathrm{i}\right)\,5\mathrm{i}}{32}-\frac{\frac{21x^5}{128}-\frac{35x^3}{32}+\frac{103x}{128}}{x^6-7x^4+7x^2-1}-\operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\left(\frac{275445\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}-\frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\left(\frac{275445\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}-\frac{1}{128}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\left(\frac{275445\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}+\frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\left(\frac{275445\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}+\frac{1}{128}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] - (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/134217728
- 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1342
17728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*940311i)/
```

```
(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)*x*33
2433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((2^(1/2
)*3i)/512 + 1i/128)
```

sympy [B] time = 1.43, size = 296, normalized size = 3.25

$$\frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128}-\frac{5\log(x-1)}{64}+\frac{5\log(x+1)}{64}+\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001}{202624020}-\frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001\sqrt{2}}{270165360}\right)+\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}-\frac{8071264001}{202624020}+\frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005}-\frac{471550901878784\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^3}{2979765}\right)+\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{8071264001\sqrt{2}}{270165360}+\frac{1299552375287054336\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^5}{50656005}-\frac{471550901878784\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{8071264001}{202624020}\right)+\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)\log\left(x-\frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765}+\frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005}+\frac{8071264001}{202624020}+\frac{8071264001\sqrt{2}}{270165360}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

```
[Out] (-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*lo
g(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 807126400
1/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129955
2375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2)/27
0165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165360 -
 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)**5/50
656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/256 - 3
*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287054336*(
1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*sqrt(2)/10
24)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)*log(x - 4
71550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 1299552375287054336*(1
/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 8071264001*sqrt
(2)/270165360)
```

3.74 $\int \frac{x^3}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=78

$$\frac{\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{(a+bx)^2}{2b^4}-\frac{3ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (-3*a*x)/b^3 + (a + b*x)^2/(2*b^4) - (a*(a^2 - 3*c)*ArcTan[(a + b*x)/Sqrt[c
]])/(b^4*Sqrt[c]) + ((3*a^2 - c)*Log[c + (a + b*x)^2])/(2*b^4)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\int \frac{x^3}{c+(a+bx)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-a+x)^3}{c+x^2}\,dx, x, a+bx\right)}{b^4}$$
$$= \frac{\text{Subst}\left(\int \left(-3a+x-\frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\right)dx, x, a+bx\right)}{b^4}$$
$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\text{Subst}\left(\int \frac{a^3-3ac-\left(3a^2-c\right)x}{c+x^2}\,dx, x, a+bx\right)}{b^4}$$
$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{\left(a\left(a^2-3c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{b^4}+\frac{\left(3a^2-c\right)\text{Subst}\left(\int \frac{x}{c+x^2}\,dx\right.}{b^4}$$
$$= -\frac{3ax}{b^3}+\frac{(a+bx)^2}{2b^4}-\frac{a\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{\left(3a^2-c\right)\log\left(c+(a+bx)^2\right)}{2b^4}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.94

$$\frac{-\frac{2\left(a^3-3ac\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+bx(bx-4a)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + (a + b*x)^2),x]
```

```
[Out] (b*x*(-4*a + b*x) - (2*(a^3 - 3*a*c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + (
3*a^2 - c)*Log[a^2 + c + 2*a*b*x + b^2*x^2])/(2*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{c+(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(c + (a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/(c + (a + b*x)^2), x]
```

fricas [A] time = 1.20, size = 198, normalized size = 2.54

$$\left[\frac{b^2cx^2-4\,abcx+\left(a^3-3\,ac\right)\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+\left(3\,a^2c-c^2\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4c}, \frac{b^2cx^2-4\,abcx-2\left(a^3-3\,ac\right)\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+\left(3\,a^2c-c^2\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*c*x^2 - 4*a*b*c*x + (a^3 - 3*a*c)*sqrt(-c)*log((b^2*x^2 + 2*a*b*x
 + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + (3*a^2*
c - c^2)*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c), 1/2*(b^2*c*x^2 - 4*a*b*
c*x - 2*(a^3 - 3*a*c)*sqrt(c)*arctan((b*x + a)/sqrt(c)) + (3*a^2*c - c^2)*l
og(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^4*c)]
```

giac [A] time = 0.34, size = 77, normalized size = 0.99

$$\frac{\left(3\,a^2-c\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4}-\frac{\left(a^3-3\,ac\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^4 - (a^3 - 3*a*c)*arctan
((b*x + a)/sqrt(c))/(b^4*sqrt(c)) + 1/2*(b^2*x^2 - 4*a*b*x)/b^4
```

maple [A] time = 0.01, size = 127, normalized size = 1.63

$$\frac{x^2}{2b^2}-\frac{a^3\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4\sqrt{c}}+\frac{3a^2\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^4}-\frac{2ax}{b^3}+\frac{3a\sqrt{c}\ \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^4}-\frac{c\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+(b*x+a)^2),x)
```

```
[Out] 1/2/b^2*x^2-2*a*x/b^3+3/2/b^4*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2-1/2/b^4*ln(b^2*
x^2+2*a*b*x+a^2+c)*c-1/b^4/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^
3+3/b^4*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a
```

maxima [A] time = 1.46, size = 81, normalized size = 1.04

$$\frac{bx^2-4\,ax}{2\,b^3}+\frac{\left(3\,a^2-c\right)\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^4}-\frac{\left(a^3-3\,ac\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 - 4*a*x)/b^3 + 1/2*(3*a^2 - c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/
b^4 - (a^3 - 3*a*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^4*sqrt(c))
```

mupad [B] time = 2.27, size = 87, normalized size = 1.12

$$\frac{x^2}{2\,b^2}-\frac{2\,a\,x}{b^3}-\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)\,\left(4\,b^4\,c^2-12\,a^2\,b^4\,c\right)}{8\,b^8\,c}+\frac{a\,\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)\,\left(3\,c-a^2\right)}{b^4\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + (a + b*x)^2),x)
```

```
[Out] x^2/(2*b^2) - (2*a*x)/b^3 - (log(c + a^2 + b^2*x^2 + 2*a*b*x)*(4*b^4*c^2 -
12*a^2*b^4*c))/(8*b^8*c) + (a*atan((a + b*x)/c^(1/2))*(3*c - a^2))/(b^4*c^(
1/2))
```

sympy [B] time = 0.69, size = 209, normalized size = 2.68

$$-\frac{2ax}{b^3}+\left(-\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)\log\left(x+\frac{a^4-2b^4c\left(-\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)-c^2}{a^3b-3abc}\right)+\left(\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)\log\left(x+\frac{a^4-2b^4c\left(\frac{a\sqrt{-c}\left(a^2-3c\right)}{2b^4c}+\frac{3a^2-c}{2b^4}\right)-c^2}{a^3b-3abc}\right)+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*x/b**3 + (-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4))
*log(x + (a**4 - 2*b**4*c*(-a*sqrt(-c)*(a**2 - 3*c)/(2*b**4*c) + (3*a**2 -
c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + (a*sqrt(-c)*(a**2 - 3*c)/(2*b**4
*c) + (3*a**2 - c)/(2*b**4))*log(x + (a**4 - 2*b**4*c*(a*sqrt(-c)*(a**2 - 3
*c)/(2*b**4*c) + (3*a**2 - c)/(2*b**4)) - c**2)/(a**3*b - 3*a*b*c)) + x**2/
(2*b**2)
```

3.75 $\int \frac{x^2}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=50

$$\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {371, 702, 635, 203, 260}

$$\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left((a+bx)^2+c\right)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + (a + b*x)^2),x]
```

```
[Out] x/b^2 + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(b^3*Sqrt[c]) - (a*Log[c + (a
 + b*x)^2])/b^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{c+(a+bx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{c+x^2}\,dx,x,a+bx\right)}{b^3} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a^2-c-2ax}{c+x^2}\right)dx,x,a+bx\right)}{b^3} \\
&= \frac{x}{b^2}+\frac{\text{Subst}\left(\int \frac{a^2-c-2ax}{c+x^2}\,dx,x,a+bx\right)}{b^3} \\
&= \frac{x}{b^2}-\frac{(2a)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx,x,a+bx\right)}{b^3}+\frac{\left(a^2-c\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,x,a+bx\right)}{b^3} \\
&= \frac{x}{b^2}+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\log\left(c+(a+bx)^2\right)}{b^3}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.08

$$
\frac{-a\log\left(a^2+2abx+b^2x^2+c\right)+\frac{\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+bx}{b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + (a + b*x)^2),x]
```

```
[Out] (b*x + ((a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - a*Log[a^2 + c + 2*a*
b*x + b^2*x^2])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{c+(a+bx)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(c + (a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/(c + (a + b*x)^2), x]
```

fricas [A] time = 1.23, size = 157, normalized size = 3.14

$$
\left[\frac{2\,bcx-2\,ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{-c}\,\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,b^3c},\ \frac{bcx-ac\log\left(b^2x^2+2\,abx+a^2+c\right)+\left(a^2-c\right)\sqrt{c}\,\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*x - 2*a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^2 - c)*sqrt(-c)
*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*
x + a^2 + c)))/(b^3*c), (b*c*x - a*c*log(b^2*x^2 + 2*a*b*x + a^2 + c) + (a^
2 - c)*sqrt(c)*arctan((b*x + a)/sqrt(c)))/(b^3*c)]
```

giac [A] time = 0.31, size = 54, normalized size = 1.08

$$
\frac{x}{b^2}-\frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{\left(a^2-c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^3\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b*x + a)
/sqrt(c))/(b^3*sqrt(c))
```

maple [A] time = 0.01, size = 89, normalized size = 1.78

$$\frac{a^2\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3\sqrt{c}}-\frac{a\ln\left(b^2x^2+2abx+a^2+c\right)}{b^3}+\frac{x}{b^2}-\frac{\sqrt{c}\,\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+(b*x+a)^2),x)
```

```
[Out] x/b^2-1/b^3*a*ln(b^2*x^2+2*a*b*x+a^2+c)+1/b^3/c^(1/2)*arctan(1/2*(2*b^2*x+2
*a*b)/b/c^(1/2))*a^2-1/b^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.65, size = 61, normalized size = 1.22

$$\frac{x}{b^2}-\frac{a\log\left(b^2x^2+2\,abx+a^2+c\right)}{b^3}+\frac{\left(a^2-c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] x/b^2 - a*log(b^2*x^2 + 2*a*b*x + a^2 + c)/b^3 + (a^2 - c)*arctan((b^2*x +
a*b)/(b*sqrt(c)))/(b^3*sqrt(c))
```

mupad [B] time = 0.09, size = 206, normalized size = 4.12

$$\frac{x}{b^2}-\frac{a\,\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{b^3}+\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a\,\sqrt{c}}{c-a^2}+\frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3}-\frac{a^2\,\mathrm{atan}\left(\frac{a^3}{\sqrt{c}\,\left(c-a^2\right)}-\frac{\sqrt{c}\,x}{\frac{c}{b}-\frac{a^2}{b}}-\frac{a\,\sqrt{c}}{c-a^2}+\frac{a^2\,x}{\sqrt{c}\,\left(\frac{c}{b}-\frac{a^2}{b}\right)}\right)}{b^3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + (a + b*x)^2),x)
```

```
[Out] x/b^2 - (a*log(c + a^2 + b^2*x^2 + 2*a*b*x))/b^3 + (c^(1/2)*atan(a^3/(c^(1/
2)*(c - a^2)) - (c^(1/2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)
/(c^(1/2)*(c/b - a^2/b))))/b^3 - (a^2*atan(a^3/(c^(1/2)*(c - a^2)) - (c^(1/
2)*x)/(c/b - a^2/b) - (a*c^(1/2))/(c - a^2) + (a^2*x)/(c^(1/2)*(c/b - a^2/b
))))/(b^3*c^(1/2))
```

sympy [B] time = 0.46, size = 153, normalized size = 3.06

$$\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\,\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}-\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\,\left(a^2-c\right)}{2b^3c}\right)\log\left(x+\frac{a^3+ac+2b^3c\left(-\frac{a}{b^3}+\frac{\sqrt{-c}\left(a^2-c\right)}{2b^3c}\right)}{a^2b-bc}\right)+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+(b*x+a)**2),x)
```

```
[Out] (-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(
-a/b**3 - sqrt(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + (-a/b**3 + sqr
t(-c)*(a**2 - c)/(2*b**3*c))*log(x + (a**3 + a*c + 2*b**3*c*(-a/b**3 + sqrt
(-c)*(a**2 - c)/(2*b**3*c)))/(a**2*b - b*c)) + x/b**2
```

3.76 $\int \frac{x}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=41

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {371, 635, 203, 260}

$$\frac{\log\left((a+bx)^2+c\right)}{2b^2}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + (a + b*x)^2),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(b^2*Sqrt[c])) + Log[c + (a + b*x)^2]/(2*b^
2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{c+(a+bx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{-a+x}{c+x^2} \, dx, x, a+bx\right)}{b^2}\\
&= \frac{\text{Subst}\left(\int \frac{x}{c+x^2} \, dx, x, a+bx\right)}{b^2}-\frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2} \, dx, x, a+bx\right)}{b^2}\\
&= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(c+(a+bx)^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.93

$$\frac{\log\left((a+bx)^2+c\right)-\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + (a + b*x)^2),x]
[Out] ((-2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] + Log[c + (a + b*x)^2])/(2*b^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{c+(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(c + (a + b*x)^2),x]
[Out] IntegrateAlgebraic[x/(c + (a + b*x)^2), x]
```

fricas [A] time = 1.15, size = 136, normalized size = 3.32

$$\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c},\ -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-c\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="fricas")
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) - c*log(b^2*x^2 + 2*a*b*x + a^2 + c))/(b^2*c
), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) - c*log(b^2*x^2 + 2*a*b*x +
a^2 + c))/(b^2*c)]
```

giac [A] time = 0.36, size = 43, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="giac")
[Out] -a*arctan((b*x + a)/sqrt(c))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*b*x + a^
2 + c)/b^2
```

maple [A] time = 0.00, size = 54, normalized size = 1.32

$$-\frac{a\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+(b*x+a)^2),x)
[Out] 1/2/b^2*ln(b^2*x^2+2*a*b*x+a^2+c)-a/b^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/
b/c^(1/2))
```

maxima [A] time = 1.36, size = 50, normalized size = 1.22

$$-\frac{a\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b^2\sqrt{c}}+\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/(b^2*sqrt(c)) + 1/2*log(b^2*x^2 + 2*a*
b*x + a^2 + c)/b^2
```

mupad [B] time = 2.08, size = 46, normalized size = 1.12

$$\frac{\ln\left(a^2+2\,a\,b\,x+b^2\,x^2+c\right)}{2\,b^2}-\frac{a\,\mathrm{atan}\left(\frac{a}{\sqrt{c}}+\frac{b\,x}{\sqrt{c}}\right)}{b^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + (a + b*x)^2),x)
```

```
[Out] log(c + a^2 + b^2*x^2 + 2*a*b*x)/(2*b^2) - (a*atan(a/c^(1/2) + (b*x)/c^(1/2
)))/(b^2*c^(1/2))
```

sympy [B] time = 0.25, size = 124, normalized size = 3.02

$$\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(-\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)+\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)\log\left(x+\frac{a^2-2b^2c\left(\frac{a\sqrt{-c}}{2b^2c}+\frac{1}{2b^2}\right)+c}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2))*log(x + (a**2 - 2*b**2*c*(-a*sqrt(-c)
/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b)) + (a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)
)*log(x + (a**2 - 2*b**2*c*(a*sqrt(-c)/(2*b**2*c) + 1/(2*b**2)) + c)/(a*b))
```

3.77 $\int \frac{1}{c+(a+bx)^2} \, dx$

Optimal. Leaf size=21

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = $\{247, 203\}$

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{c+(a+bx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{c+x^2} \, dx, x, a+bx\right)}{b} \\ &= \frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[c]]/(b*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+(a+bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + (a + b*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + (a + b*x)^2)^(-1), x]
```

fricas [A] time = 0.88, size = 83, normalized size = 3.95

$$\left[-\frac{\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2-2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)}{2\,bc},\ \frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^
2*x^2 + 2*a*b*x + a^2 + c))/(b*c), arctan((b*x + a)/sqrt(c))/(b*sqrt(c))]
```

giac [A] time = 0.38, size = 17, normalized size = 0.81

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(c))/(b*sqrt(c))
```

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+(b*x+a)^2),x)
```

```
[Out] 1/b/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.52, size = 24, normalized size = 1.14

$$\frac{\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] arctan((b^2*x + a*b)/(b*sqrt(c)))/(b*sqrt(c))
```

mupad [B] time = 0.04, size = 17, normalized size = 0.81

$$\frac{\mathrm{atan}\left(\frac{a+b\,x}{\sqrt{c}}\right)}{b\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + (a + b*x)^2),x)
```

[Out] atan((a + b*x)/c^(1/2))/(b*c^(1/2))

sympy [B] time = 0.19, size = 54, normalized size = 2.57

$$\frac{-\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a-c\sqrt{-\frac{1}{c}}}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{c}}\log\left(x+\frac{a+c\sqrt{-\frac{1}{c}}}{b}\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+(b*x+a)**2),x)

[Out] (-sqrt(-1/c)*log(x + (a - c*sqrt(-1/c))/b)/2 + sqrt(-1/c)*log(x + (a + c*sq
rt(-1/c))/b)/2)/b

3.78 $\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=59

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 706, 31, 635, 203, 260}

$$-\frac{\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)}-\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -((a*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c))) + Log[x]/(a^2 + c) - L
og[c + (a + b*x)^2]/(2*(a^2 + c))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-a+x)\left(c+x^2\right)}\,dx, x, a+bx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-a+x}\,dx, x, a+bx\right)}{a^2+c} + \frac{\text{Subst}\left(\int \frac{-a-x}{c+x^2}\,dx, x, a+bx\right)}{a^2+c}$$
$$= \frac{\log(x)}{a^2+c} - \frac{\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{a^2+c} - \frac{a\,\text{Subst}\left(\int \frac{1}{c+x^2}\,dx, x, a+bx\right)}{a^2+c}$$
$$= -\frac{a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)} + \frac{\log(x)}{a^2+c} - \frac{\log\left(c+(a+bx)^2\right)}{2\left(a^2+c\right)}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.81

$$-\frac{\log\left((a+bx)^2+c\right)+\frac{2a\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}-2\log(bx)}{2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*((2*a*ArcTan[(a + b*x)/Sqrt[c]])/Sqrt[c] - 2*Log[b*x] + Log[c + (a + b
*x)^2])/(a^2 + c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c+(a+bx)^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(c + (a + b*x)^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(c + (a + b*x)^2)), x]
```

fricas [A] time = 1.24, size = 154, normalized size = 2.61

$$\left[-\frac{a\sqrt{-c}\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)}, -\frac{2\,a\sqrt{c}\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+c\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,c\log(x)}{2\left(a^2c+c^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-c)*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c)/
(b^2*x^2 + 2*a*b*x + a^2 + c)) + c*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*l
og(x))/(a^2*c + c^2), -1/2*(2*a*sqrt(c)*arctan((b*x + a)/sqrt(c)) + c*log(b
^2*x^2 + 2*a*b*x + a^2 + c) - 2*c*log(x))/(a^2*c + c^2)]
```

giac [A] time = 0.38, size = 62, normalized size = 1.05

$$-\frac{a\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}} - \frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^2+c\right)} + \frac{\log\left(|x|\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan((b*x + a)/sqrt(c))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2 + 2*a*b*
x + a^2 + c)/(a^2 + c) + log(abs(x))/(a^2 + c)
```

maple [A] time = 0.01, size = 72, normalized size = 1.22

$$-\frac{a \arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}}+\frac{\ln(x)}{a^2+c}-\frac{\ln\left(b^2x^2+2abx+a^2+c\right)}{2\left(a^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c+(b*x+a)^2),x)
```

```
[Out] ln(x)/(a^2+c)-1/2/(a^2+c)*ln(b^2*x^2+2*a*b*x+a^2+c)-1/(a^2+c)*a/c^(1/2)*arc
tan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.52, size = 68, normalized size = 1.15

$$-\frac{a \arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^2+c\right)\sqrt{c}}-\frac{\log\left(b^2x^2+2\,abx+a^2+c\right)}{2\left(a^2+c\right)}+\frac{\log(x)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -a*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^2 + c)*sqrt(c)) - 1/2*log(b^2*x^2
+ 2*a*b*x + a^2 + c)/(a^2 + c) + log(x)/(a^2 + c)
```

mupad [B] time = 2.59, size = 173, normalized size = 2.93

$$\frac{\ln(x)}{a^2+c}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\left(c+a\,\sqrt{-c}\right)\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\left(a^2+c\right)}\right)\left(c+a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}-\frac{\ln\left(2\,a\,b^3+3\,b^4\,x+\frac{b^3\left(c-a\,\sqrt{-c}\right)\left(a^3+b\,x\,a^2+c\,a-3\,b\,c\,x\right)}{c\left(a^2+c\right)}\right)\left(c-a\,\sqrt{-c}\right)}{2\,\left(a^2\,c+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)/(c + a^2) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c + a*(-c)^(1/2))*(a*c +
a^3 - 3*b*c*x + a^2*b*x))/(c*(c + a^2)))*(c + a*(-c)^(1/2)))/(2*(a^2*c + c^
2)) - (log(2*a*b^3 + 3*b^4*x + (b^3*(c - a*(-c)^(1/2))*(a*c + a^3 - 3*b*c*x
 + a^2*b*x))/(c*(c + a^2)))*(c - a*(-c)^(1/2)))/(2*(a^2*c + c^2))
```

sympy [B] time = 3.40, size = 738, normalized size = 12.51

$$\left(-\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}-\frac{1}{2\left(a^2+c\right)}\right)\log\left(x+\frac{-4a^6c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+20a^2c^3\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-12a^2c^2\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+10a^2c+12c^4\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6c^3\left(-\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)-6c^2}{a^3b+9abc}\right)+\left(\frac{a\sqrt{-c}}{2c\left(a^2+c\right)}-\frac{1}{2\left(a^2+c\right)}\right)\log\left(x+\frac{-4a^6c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2+4a^4c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6a^4c\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+20a^2c^3\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-12a^2c^2\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)+10a^2c+12c^4\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)^2-6c^3\left(\frac{a\sqrt{-c}}{2c(a^2+c)}-\frac{1}{2(a^2+c)}\right)-6c^2}{a^3b+9abc}\right)+\frac{\log\left(x+\frac{-\frac{4a^6c}{(a^2+c)^2}+\frac{4a^4c^2}{(a^2+c)^2}-\frac{6a^4c}{a^2+c}+\frac{20a^2c^3}{(a^2+c)^2}-\frac{12a^2c^2}{a^2+c}+10a^2c+\frac{12c^4}{(a^2+c)^2}-\frac{6c^3}{a^2+c}-6c^2}{a^3b+9abc}\right)}{a^2+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c+(b*x+a)**2),x)
```

```
[Out] (-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*log(x + (-4*a**6*c*(-a*sq
rt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a**4*c**2*(-a*sqrt(-c)/(
2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(-a*sqrt(-c)/(2*c*(a**2 +
 c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(
2*(a**2 + c)))**2 - 12*a**2*c**2*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) + 10*a**2*c + 12*c**4*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 +
c)))**2 - 6*c**3*(-a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) - 6*c**2
)/(a**3*b + 9*a*b*c)) + (a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))*lo
```

```
g(x + (-4*a**6*c*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 + 4*a*
*4*c**2*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 6*a**4*c*(a*s
qrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2 + c))) + 20*a**2*c**3*(a*sqrt(-c)/(2*
c*(a**2 + c)) - 1/(2*(a**2 + c)))**2 - 12*a**2*c**2*(a*sqrt(-c)/(2*c*(a**2
+ c)) - 1/(2*(a**2 + c))) + 10*a**2*c + 12*c**4*(a*sqrt(-c)/(2*c*(a**2 + c)
) - 1/(2*(a**2 + c)))**2 - 6*c**3*(a*sqrt(-c)/(2*c*(a**2 + c)) - 1/(2*(a**2
 + c))) - 6*c**2)/(a**3*b + 9*a*b*c)) + log(x + (-4*a**6*c/(a**2 + c)**2 +
4*a**4*c**2/(a**2 + c)**2 - 6*a**4*c/(a**2 + c) + 20*a**2*c**3/(a**2 + c)**
2 - 12*a**2*c**2/(a**2 + c) + 10*a**2*c + 12*c**4/(a**2 + c)**2 - 6*c**3/(a
**2 + c) - 6*c**2)/(a**3*b + 9*a*b*c))/(a**2 + c)
```

3.79 $\int \frac{1}{x^2\left(c+(a+bx)^2\right)} dx$

Optimal. Leaf size=79

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 710, 801, 635, 203, 260}

$$-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left((a+bx)^2+c\right)}{\left(a^2+c\right)^2}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{1}{x\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(c + (a + b*x)^2)),x]
```

```
[Out] -(1/((a^2 + c)*x)) + (b*(a^2 - c)*ArcTan[(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2
+ c)^2) - (2*a*b*Log[x])/(a^2 + c)^2 + (a*b*Log[c + (a + b*x)^2])/(a^2 + c)
^2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx &= b\,\text{Subst}\left(\int \frac{1}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\,\text{Subst}\left(\int \left(\frac{2a}{\left(a^2+c\right)(a-x)}+\frac{a^2-c+2ax}{\left(a^2+c\right)\left(c+x^2\right)}\right)dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{\left(a^2+c\right)x}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{b\,\text{Subst}\left(\int \frac{a^2-c+2ax}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2}\\
&= -\frac{1}{\left(a^2+c\right)x}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{(2ab)\,\text{Subst}\left(\int \frac{x}{c+x^2}\,dx, x, a+bx\right)}{\left(a^2+c\right)^2}+\frac{\left(b\left(a^2-c\right)\right)\text{Subst}\left(\int \right.}{\left(a^2+\right.}\\
&= -\frac{1}{\left(a^2+c\right)x}+\frac{b\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^2}-\frac{2ab\log(x)}{\left(a^2+c\right)^2}+\frac{ab\log\left(c+(a+bx)^2\right)}{\left(a^2+c\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.03

$$
\frac{bx\left(a^2-c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)-\sqrt{c}\left(-abx\log\left(a^2+2abx+b^2x^2+c\right)+a^2+2abx\log(x)+c\right)}{\sqrt{c}\,x\left(a^2+c\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(c + (a + b*x)^2)),x]
[Out] (b*(a^2 - c)*x*ArcTan[(a + b*x)/Sqrt[c]] - Sqrt[c]*(a^2 + c + 2*a*b*x*Log[x
] - a*b*x*Log[a^2 + c + 2*a*b*x + b^2*x^2]))/(Sqrt[c]*(a^2 + c)^2*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^2\left(c+(a+bx)^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(c + (a + b*x)^2)),x]
[Out] IntegrateAlgebraic[1/(x^2*(c + (a + b*x)^2)), x]
```

fricas [A] time = 1.14, size = 229, normalized size = 2.90

$$
\left[\frac{2\,abcx\log\left(b^2x^2+2\,abx+a^2+c\right)-4\,abcx\log(x)+\left(a^2b-bc\right)\sqrt{-c}\,x\log\left(\frac{b^2x^2+2\,abx+a^2+2\,(bx+a)\sqrt{-c}-c}{b^2x^2+2\,abx+a^2+c}\right)-2\,a^2c-2\,c^2}{2\left(a^4c+2\,a^2c^2+c^3\right)x}, \frac{abcx\log\left(b^2x^2+2\,abx+a^2+c\right)-2\,abcx\log(x)+\left(a^2b-bc\right)\sqrt{c}\,x\arctan\left(\frac{bx+a}{\sqrt{c}}\right)-a^2c-c^2}{\left(a^4c+2\,a^2c^2+c^3\right)x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 4*a*b*c*x*log(x) + (a^2*
b - b*c)*sqrt(-c)*x*log((b^2*x^2 + 2*a*b*x + a^2 + 2*(b*x + a)*sqrt(-c) - c
)/(b^2*x^2 + 2*a*b*x + a^2 + c)) - 2*a^2*c - 2*c^2)/((a^4*c + 2*a^2*c^2 + c
^3)*x), (a*b*c*x*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*a*b*c*x*log(x) + (a^2
*b - b*c)*sqrt(c)*x*arctan((b*x + a)/sqrt(c)) - a^2*c - c^2)/((a^4*c + 2*a^
2*c^2 + c^3)*x)]
```

giac [A] time = 0.37, size = 117, normalized size = 1.48

$$\frac{ab\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab\log\left(|x|\right)}{a^4 + 2\,a^2c + c^2} + \frac{\left(a^2b^2 - b^2c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^4 + 2\,a^2c + c^2\right)b\sqrt{c}} - \frac{1}{\left(a^2 + c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(abs(
x))/(a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b*x + a)/sqrt(c))/((a
^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

maple [A] time = 0.01, size = 123, normalized size = 1.56

$$\frac{a^2b\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^2\sqrt{c}} - \frac{2ab\ln(x)}{\left(a^2+c\right)^2} + \frac{ab\ln\left(b^2x^2 + 2abx + a^2 + c\right)}{\left(a^2+c\right)^2} - \frac{b\sqrt{c}\,\arctan\left(\frac{2b^2x+2ba}{2b\sqrt{c}}\right)}{\left(a^2+c\right)^2} - \frac{1}{\left(a^2+c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c+(b*x+a)^2),x)
```

```
[Out] -1/(a^2+c)/x-2*a*b*ln(x)/(a^2+c)^2+b/(a^2+c)^2*a*ln(b^2*x^2+2*a*b*x+a^2+c)+
b/(a^2+c)^2/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))*a^2-b/(a^2+c)^2*c
^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2))
```

maxima [A] time = 1.59, size = 123, normalized size = 1.56

$$\frac{ab\log\left(b^2x^2 + 2\,abx + a^2 + c\right)}{a^4 + 2\,a^2c + c^2} - \frac{2\,ab\log(x)}{a^4 + 2\,a^2c + c^2} + \frac{\left(a^2b^2 - b^2c\right)\arctan\left(\frac{b^2x+ab}{b\sqrt{c}}\right)}{\left(a^4 + 2\,a^2c + c^2\right)b\sqrt{c}} - \frac{1}{\left(a^2 + c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] a*b*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^4 + 2*a^2*c + c^2) - 2*a*b*log(x)/(
a^4 + 2*a^2*c + c^2) + (a^2*b^2 - b^2*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/
((a^4 + 2*a^2*c + c^2)*b*sqrt(c)) - 1/((a^2 + c)*x)
```

mupad [B] time = 2.58, size = 425, normalized size = 5.38

$$\frac{\ln\left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} + a\,c^6 - a^{11}\,c + 35\,a^3\,c^5 + 34\,a^5\,c^4 - 34\,a^7\,c^3 - 35\,a^9\,c^2 + b\,c^6\,x - a^{10}\,b\,c\,x + 35\,a^2\,b\,c^5\,x + 34\,a^4\,b\,c^4\,x - 34\,a^6\,b\,c^3\,x - 35\,a^8\,b\,c^2\,x\right)\left(b\,(-c)^{3/2} + 2\,a\,b\,c + a^2\,b\,\sqrt{-c}\right)}{2\left(a^4\,c + 2\,a^2\,c^2 + c^3\right)} - \frac{1}{x\left(a^2+c\right)} - \frac{\ln\left((-c)^{13/2} - 35\,a^2\,(-c)^{11/2} + 34\,a^4\,(-c)^{9/2} + 34\,a^6\,(-c)^{7/2} - 35\,a^8\,(-c)^{5/2} + a^{10}\,(-c)^{3/2} - a\,c^6 + a^{11}\,c - 35\,a^3\,c^5 - 34\,a^5\,c^4 + 34\,a^7\,c^3 + 35\,a^9\,c^2 - b\,c^6\,x + a^{10}\,b\,c\,x - 35\,a^2\,b\,c^5\,x - 34\,a^4\,b\,c^4\,x + 34\,a^6\,b\,c^3\,x + 35\,a^8\,b\,c^2\,x\right)\left(b\,(-c)^{3/2} - 2\,a\,b\,c + a^2\,b\,\sqrt{-c}\right)}{2\left(a^4\,c + 2\,a^2\,c^2 + c^3\right)} - \frac{2\,a\,b\,\ln(x)}{\left(a^2+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + (a + b*x)^2)),x)
```

```
[Out] (log((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/
2) - 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) + a*c^6 - a^11*c + 35*a^3*c^5 + 34
*a^5*c^4 - 34*a^7*c^3 - 35*a^9*c^2 + b*c^6*x - a^10*b*c*x + 35*a^2*b*c^5*x
+ 34*a^4*b*c^4*x - 34*a^6*b*c^3*x - 35*a^8*b*c^2*x)*(b*(-c)^(3/2) + 2*a*b*c
 + a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - 1/(x*(c + a^2)) - (lo
```

```
g((-c)^(13/2) - 35*a^2*(-c)^(11/2) + 34*a^4*(-c)^(9/2) + 34*a^6*(-c)^(7/2)
- 35*a^8*(-c)^(5/2) + a^10*(-c)^(3/2) - a*c^6 + a^11*c - 35*a^3*c^5 - 34*a^
5*c^4 + 34*a^7*c^3 + 35*a^9*c^2 - b*c^6*x + a^10*b*c*x - 35*a^2*b*c^5*x - 3
4*a^4*b*c^4*x + 34*a^6*b*c^3*x + 35*a^8*b*c^2*x)*(b*(-c)^(3/2) - 2*a*b*c +
a^2*b*(-c)^(1/2)))/(2*(a^4*c + c^3 + 2*a^2*c^2)) - (2*a*b*log(x))/(c + a^2)
^2
```

sympy [B] time = 11.12, size = 1620, normalized size = 20.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c+(b*x+a)**2),x)
```

```
[Out] -2*a*b*log(x + (-16*a**13*b**2*c/(a**2 + c)**4 + 48*a**11*b**2*c**2/(a**2 +
 c)**4 + 352*a**9*b**2*c**3/(a**2 + c)**4 - 20*a**9*b**2*c/(a**2 + c)**2 +
608*a**7*b**2*c**4/(a**2 + c)**4 - 64*a**7*b**2*c**2/(a**2 + c)**2 + 432*a*
*5*b**2*c**5/(a**2 + c)**4 - 72*a**5*b**2*c**3/(a**2 + c)**2 + 36*a**5*b**2
*c + 112*a**3*b**2*c**6/(a**2 + c)**4 - 32*a**3*b**2*c**4/(a**2 + c)**2 - 8
8*a**3*b**2*c**2 - 4*a*b**2*c**5/(a**2 + c)**2 + 4*a*b**2*c**3)/(a**6*b**3
+ 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3))/(a**2 + c)**2 + (a*b/(a*
*2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*log(x +
(-4*a**11*c*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*
c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2
*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 - b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a**2 +
c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32*a**6*
b*c**2*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c
**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a*
*2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**2 + c)
**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3*b**2*
c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4
+ 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a
**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*(a*b/(
a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 2
*b*c**5*(a*b/(a**2 + c)**2 - b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c +
c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)) + (a
*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*lo
g(x + (-4*a**11*c*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2
*a**2*c + c**2)))**2 + 12*a**9*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 -
 c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 10*a**8*b*c*(a*b/(a**2 + c)**2 + b
*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 88*a**7*c**3*(a*b/(a
**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 32
*a**6*b*c**2*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2
*c + c**2))) + 36*a**5*b**2*c + 152*a**5*c**4*(a*b/(a**2 + c)**2 + b*sqrt(-
c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))**2 + 36*a**4*b*c**3*(a*b/(a**
2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) - 88*a**3
*b**2*c**2 + 108*a**3*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*
(a**4 + 2*a**2*c + c**2)))**2 + 16*a**2*b*c**4*(a*b/(a**2 + c)**2 + b*sqrt(
-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2))) + 4*a*b**2*c**3 + 28*a*c**6*
(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**2*c + c**2)))*
*2 + 2*b*c**5*(a*b/(a**2 + c)**2 + b*sqrt(-c)*(a**2 - c)/(2*c*(a**4 + 2*a**
2*c + c**2))))/(a**6*b**3 + 33*a**4*b**3*c - 33*a**2*b**3*c**2 - b**3*c**3)
) - 1/(x*(a**2 + c))
```

3.80 $\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx$

Optimal. Leaf size=121

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {371, 710, 801, 635, 203, 260}

$$\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\left(3a^2-c\right)\log\left((a+bx)^2+c\right)}{2\left(a^2+c\right)^3}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{2ab}{x\left(a^2+c\right)^2}-\frac{1}{2x^2\left(a^2+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/(2*(a^2 + c)*x^2) + (2*a*b)/((a^2 + c)^2*x) - (a*b^2*(a^2 - 3*c)*ArcTan[
(a + b*x)/Sqrt[c]])/(Sqrt[c]*(a^2 + c)^3) + (b^2*(3*a^2 - c)*Log[x])/(a^2 +
 c)^3 - (b^2*(3*a^2 - c)*Log[c + (a + b*x)^2])/(2*(a^2 + c)^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx &= b^2\,\text{Subst}\left(\int \frac{1}{(-a+x)^3\left(c+x^2\right)}\,dx, x, a+bx\right)\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{b^2\,\text{Subst}\left(\int \frac{-a-x}{(-a+x)^2\left(c+x^2\right)}\,dx, x, a+bx\right)}{a^2+c}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{b^2\,\text{Subst}\left(\int \left(-\frac{2a}{\left(a^2+c\right)(a-x)^2}+\frac{-3a^2+c}{\left(a^2+c\right)^2(a-x)}+\frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{\left(a^2+c\right)^2\left(c+x^2\right)}\right)dx, x, a+
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}+\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}+\frac{b^2\,\text{Subst}\left(\int \frac{-a\left(a^2-3c\right)-\left(3a^2-c\right)x}{c+x^2}\,dx,\right.}{\left(a^2+c\right)^3}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}+\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{\left(ab^2\left(a^2-3c\right)\right)\text{Subst}\left(\int \frac{1}{c+x^2}\,dx,\right.}{\left(a^2+c\right)^3}\\
&= -\frac{1}{2\left(a^2+c\right)x^2}+\frac{2ab}{\left(a^2+c\right)^2x}-\frac{ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}\left(a^2+c\right)^3}+\frac{b^2\left(3a^2-c\right)\log(x)}{\left(a^2+c\right)^3}-\frac{b^2\,(
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 106, normalized size = 0.88

$$
-\frac{b^2\left(3a^2-c\right)\log\left(a^2+2abx+b^2x^2+c\right)+2b^2\left(c-3a^2\right)\log(x)+\frac{2ab^2\left(a^2-3c\right)\tan^{-1}\left(\frac{a+bx}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(a^2+c\right)\left(a^2-4abx+c\right)}{x^2}}{2\left(a^2+c\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] -1/2*(((a^2 + c)*(a^2 + c - 4*a*b*x))/x^2 + (2*a*b^2*(a^2 - 3*c)*ArcTan[(a
+ b*x)/Sqrt[c]])/Sqrt[c] + 2*b^2*(-3*a^2 + c)*Log[x] + b^2*(3*a^2 - c)*Log[
a^2 + c + 2*a*b*x + b^2*x^2])/(a^2 + c)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x^3\left(c+(a+bx)^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(c + (a + b*x)^2)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(c + (a + b*x)^2)), x]
```

fricas [A] time = 1.44, size = 371, normalized size = 3.07

$$
\left[-\frac{a^4c-\left(a^3b^2-3ab^2c\right)\sqrt{-c}\,x^2\log\left(\frac{b^2x^2+2abx+a^2-2(bx+a)\sqrt{-c}-c}{b^2x^2+2abx+a^2+c}\right)+2a^2c^2+\left(3a^2b^2c-b^2c^2\right)x^2\log\left(b^2x^2+2abx+a^2+c\right)-2\left(3a^2b^2c-b^2c^2\right)x^2\log(x)+c^3-4\left(a^3bc+abc^2\right)x}{2\left(a^6c+3a^4c^2+3a^2c^3+c^4\right)x^2}, -\frac{a^4c+2\left(a^3b^2-3ab^2c\right)\sqrt{c}\,x^2\arctan\left(\frac{bx+a}{\sqrt{c}}\right)+2a^2c^2+\left(3a^2b^2c-b^2c^2\right)x^2\log\left(b^2x^2+2abx+a^2+c\right)-2\left(3a^2b^2c-b^2c^2\right)x^2\log(x)+c^3-4\left(a^3bc+abc^2\right)x}{2\left(a^6c+3a^4c^2+3a^2c^3+c^4\right)x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^4*c - (a^3*b^2 - 3*a*b^2*c)*sqrt(-c)*x^2*log((b^2*x^2 + 2*a*b*x +
a^2 - 2*(b*x + a)*sqrt(-c) - c)/(b^2*x^2 + 2*a*b*x + a^2 + c)) + 2*a^2*c^2
+ (3*a^2*b^2*c - b^2*c^2)*x^2*log(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b
^2*c - b^2*c^2)*x^2*log(x) + c^3 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4
*c^2 + 3*a^2*c^3 + c^4)*x^2), -1/2*(a^4*c + 2*(a^3*b^2 - 3*a*b^2*c)*sqrt(c)
*x^2*arctan((b*x + a)/sqrt(c)) + 2*a^2*c^2 + (3*a^2*b^2*c - b^2*c^2)*x^2*lo
g(b^2*x^2 + 2*a*b*x + a^2 + c) - 2*(3*a^2*b^2*c - b^2*c^2)*x^2*log(x) + c^3
 - 4*(a^3*b*c + a*b*c^2)*x)/((a^6*c + 3*a^4*c^2 + 3*a^2*c^3 + c^4)*x^2)]
```

giac [A]   time = 0.42, size = 195, normalized size = 1.61

$$-\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)}+\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log\left(|x|\right)}{a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}}-\frac{\left(a^{3}b^{3}-3\,ab^{3}c\right)\arctan\left(\frac{bx+a}{\sqrt{c}}\right)}{\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)b\sqrt{c}}-\frac{a^{4}+2\,a^{2}c+c^{2}-4\left(a^{3}b+abc\right)x}{2\left(a^{2}+c\right)^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(abs(x))/(a^6 + 3*a^4*c + 3*a^2*c
^2 + c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b*x + a)/sqrt(c))/((a^6 + 3*a^4*c
 + 3*a^2*c^2 + c^3)*b*sqrt(c)) - 1/2*(a^4 + 2*a^2*c + c^2 - 4*(a^3*b + a*b*
c)*x)/((a^2 + c)^3*x^2)
```

maple [A]   time = 0.01, size = 198, normalized size = 1.64

$$-\frac{a^{3}b^{2}\arctan\left(\frac{2b^{2}x+2ba}{2b\sqrt{c}}\right)}{\left(a^{2}+c\right)^{3}\sqrt{c}}+\frac{3a^{2}b^{2}\ln(x)}{\left(a^{2}+c\right)^{3}}-\frac{3a^{2}b^{2}\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{2\left(a^{2}+c\right)^{3}}+\frac{3a\,b^{2}\sqrt{c}\,\arctan\left(\frac{2b^{2}x+2ba}{2b\sqrt{c}}\right)}{\left(a^{2}+c\right)^{3}}-\frac{b^{2}c\ln(x)}{\left(a^{2}+c\right)^{3}}+\frac{b^{2}c\ln\left(b^{2}x^{2}+2abx+a^{2}+c\right)}{2\left(a^{2}+c\right)^{3}}+\frac{2ab}{\left(a^{2}+c\right)^{2}x}-\frac{1}{2\left(a^{2}+c\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c+(b*x+a)^2),x)
```

```
[Out] -1/2/(a^2+c)/x^2+3*b^2/(a^2+c)^3*ln(x)*a^2-b^2/(a^2+c)^3*ln(x)*c+2*a*b/(a^2
+c)^2/x-3/2*b^2/(a^2+c)^3*ln(b^2*x^2+2*a*b*x+a^2+c)*a^2+1/2*b^2/(a^2+c)^3*l
n(b^2*x^2+2*a*b*x+a^2+c)*c-b^2/(a^2+c)^3/c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)
/b/c^(1/2))*a^3+3*b^2/(a^2+c)^3*c^(1/2)*arctan(1/2*(2*b^2*x+2*a*b)/b/c^(1/2
))*a
```

maxima [A]   time = 1.61, size = 197, normalized size = 1.63

$$-\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log\left(b^{2}x^{2}+2\,abx+a^{2}+c\right)}{2\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)}+\frac{\left(3\,a^{2}b^{2}-b^{2}c\right)\log(x)}{a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}}-\frac{\left(a^{3}b^{3}-3\,ab^{3}c\right)\arctan\left(\frac{b^{2}x+ab}{b\sqrt{c}}\right)}{\left(a^{6}+3\,a^{4}c+3\,a^{2}c^{2}+c^{3}\right)b\sqrt{c}}+\frac{4\,abx-a^{2}-c}{2\left(a^{4}+2\,a^{2}c+c^{2}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c+(b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a^2*b^2 - b^2*c)*log(b^2*x^2 + 2*a*b*x + a^2 + c)/(a^6 + 3*a^4*c +
3*a^2*c^2 + c^3) + (3*a^2*b^2 - b^2*c)*log(x)/(a^6 + 3*a^4*c + 3*a^2*c^2 +
c^3) - (a^3*b^3 - 3*a*b^3*c)*arctan((b^2*x + a*b)/(b*sqrt(c)))/((a^6 + 3*a^
4*c + 3*a^2*c^2 + c^3)*b*sqrt(c)) + 1/2*(4*a*b*x - a^2 - c)/((a^4 + 2*a^2*c
 + c^2)*x^2)
```

mupad [B]   time = 2.77, size = 573, normalized size = 4.74

$$\ln(x)\left(\frac{3\,b^{2}}{\left(a^{2}+c\right)^{2}}-\frac{4\,b^{2}c}{\left(a^{2}+c\right)^{3}}\right)-\frac{\frac{1}{2\left(a^{2}+c\right)}-\frac{2\,a\,b\,x}{\left(a^{2}+c\right)^{2}}}{x^{2}}-\frac{\ln\left(27\,(-c)^{15/2}+90\,a^{2}\,(-c)^{13/2}+9\,a^{4}\,(-c)^{11/2}-324\,a^{6}\,(-c)^{9/2}+125\,a^{8}\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(-c)^{3/2}-27\,a\,c^{7}+a^{13}\,c+90\,a^{3}\,c^{6}-9\,a^{5}\,c^{5}-324\,a^{7}\,c^{4}-125\,a^{9}\,c^{3}+74\,a^{11}\,c^{2}-27\,b\,c^{7}\,x+a^{12}\,b\,c\,x+90\,a^{2}\,b\,c^{6}\,x-9\,a^{4}\,b\,c^{5}\,x-324\,a^{6}\,b\,c^{4}\,x-125\,a^{8}\,b\,c^{3}\,x+74\,a^{10}\,b\,c^{2}\,x\right)\left(a^{3}\,b^{2}\,\sqrt{-c}-b^{2}\,c^{2}+3\,a^{2}\,b^{2}\,c+3\,a\,b^{2}\,(-c)^{3/2}\right)}{2\left(a^{6}\,c+3\,a^{4}\,c^{2}+3\,a^{2}\,c^{3}+c^{4}\right)}+\frac{\ln\left(27\,(-c)^{15/2}+90\,a^{2}\,(-c)^{13/2}+9\,a^{4}\,(-c)^{11/2}-324\,a^{6}\,(-c)^{9/2}+125\,a^{8}\,(-c)^{7/2}+74\,a^{10}\,(-c)^{5/2}-a^{12}\,(-c)^{3/2}+27\,a\,c^{7}-a^{13}\,c-90\,a^{3}\,c^{6}+9\,a^{5}\,c^{5}+324\,a^{7}\,c^{4}+125\,a^{9}\,c^{3}-74\,a^{11}\,c^{2}+27\,b\,c^{7}\,x-a^{12}\,b\,c\,x-90\,a^{2}\,b\,c^{6}\,x+9\,a^{4}\,b\,c^{5}\,x+324\,a^{6}\,b\,c^{4}\,x+125\,a^{8}\,b\,c^{3}\,x-74\,a^{10}\,b\,c^{2}\,x\right)\left(b^{2}\,c^{2}+a^{3}\,b^{2}\,\sqrt{-c}-3\,a^{2}\,b^{2}\,c+3\,a\,b^{2}\,(-c)^{3/2}\right)}{2\left(a^{6}\,c+3\,a^{4}\,c^{2}+3\,a^{2}\,c^{3}+c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + (a + b*x)^2)),x)
```

```
[Out] log(x)*((3*b^2)/(c + a^2)^2 - (4*b^2*c)/(c + a^2)^3) - (1/(2*(c + a^2)) - (
2*a*b*x)/(c + a^2)^2)/x^2 - (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) + 9*a^
4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^(5/2
) - a^12*(-c)^(3/2) - 27*a*c^7 + a^13*c + 90*a^3*c^6 - 9*a^5*c^5 - 324*a^7*
c^4 - 125*a^9*c^3 + 74*a^11*c^2 - 27*b*c^7*x + a^12*b*c*x + 90*a^2*b*c^6*x
- 9*a^4*b*c^5*x - 324*a^6*b*c^4*x - 125*a^8*b*c^3*x + 74*a^10*b*c^2*x)*(a^3
*b^2*(-c)^(1/2) - b^2*c^2 + 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*c +
c^4 + 3*a^2*c^3 + 3*a^4*c^2)) + (log(27*(-c)^(15/2) + 90*a^2*(-c)^(13/2) +
9*a^4*(-c)^(11/2) - 324*a^6*(-c)^(9/2) + 125*a^8*(-c)^(7/2) + 74*a^10*(-c)^
(5/2) - a^12*(-c)^(3/2) + 27*a*c^7 - a^13*c - 90*a^3*c^6 + 9*a^5*c^5 + 324*
a^7*c^4 + 125*a^9*c^3 - 74*a^11*c^2 + 27*b*c^7*x - a^12*b*c*x - 90*a^2*b*c^
6*x + 9*a^4*b*c^5*x + 324*a^6*b*c^4*x + 125*a^8*b*c^3*x - 74*a^10*b*c^2*x)*
(b^2*c^2 + a^3*b^2*(-c)^(1/2) - 3*a^2*b^2*c + 3*a*b^2*(-c)^(3/2)))/(2*(a^6*
c + c^4 + 3*a^2*c^3 + 3*a^4*c^2))
```

sympy [B] time = 38.26, size = 3284, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c+(b*x+a)**2),x)
```

```
[Out] b**2*(3*a**2 - c)*log(x + (-4*a**16*b**4*c*(3*a**2 - c)**2/(a**2 + c)**6 +
24*a**14*b**4*c**2*(3*a**2 - c)**2/(a**2 + c)**6 + 216*a**12*b**4*c**3*(3*a
**2 - c)**2/(a**2 + c)**6 - 14*a**12*b**4*c*(3*a**2 - c)/(a**2 + c)**3 + 56
8*a**10*b**4*c**4*(3*a**2 - c)**2/(a**2 + c)**6 - 44*a**10*b**4*c**2*(3*a**
2 - c)/(a**2 + c)**3 + 720*a**8*b**4*c**5*(3*a**2 - c)**2/(a**2 + c)**6 - 4
2*a**8*b**4*c**3*(3*a**2 - c)/(a**2 + c)**3 + 78*a**8*b**4*c + 456*a**6*b**
4*c**6*(3*a**2 - c)**2/(a**2 + c)**6 - 8*a**6*b**4*c**4*(3*a**2 - c)/(a**2
+ c)**3 - 464*a**6*b**4*c**2 + 104*a**4*b**4*c**7*(3*a**2 - c)**2/(a**2 + c
)**6 - 2*a**4*b**4*c**5*(3*a**2 - c)/(a**2 + c)**3 + 380*a**4*b**4*c**3 - 2
4*a**2*b**4*c**8*(3*a**2 - c)**2/(a**2 + c)**6 - 12*a**2*b**4*c**6*(3*a**2
- c)/(a**2 + c)**3 - 96*a**2*b**4*c**4 - 12*b**4*c**9*(3*a**2 - c)**2/(a**2
 + c)**6 - 6*b**4*c**7*(3*a**2 - c)/(a**2 + c)**3 + 6*b**4*c**5)/(a**9*b**5
 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**
4))/(a**2 + c)**3 + (-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**
16*c*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c
**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 24*a**14*c**2*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 14*a**12*b**2*c*(-a*b**2*sqrt(-c)*(a**2 -
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) + 216*a**12*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 +
 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2
- 44*a**10*b**2*c**2*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 568*a**10*c*
*4*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**
3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 78*a**8*b**4*c - 42*a**8*b*
*2*c**3*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 720*a**8*c**5*(-a*b**2*sq
rt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*
a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*b**4*c**2 - 8*a**6*b**2*c**4*(-a
*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) -
b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a**6*c**6*(-a*b**2*sqrt(-c)*(a**
2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(
2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2*a**4*b**2*c**5*(-a*b**2*sqrt(
-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**
2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2
*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c
)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c**6*(-a*b**2*sqrt(-c)*(a**2 -
```

```
 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(
a**2 + c)**3)) - 24*a**2*c**8*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3
*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 +
6*b**4*c**5 - 6*b**2*c**7*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**
4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 12*c**9
*(-a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)
) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*b**5 + 72*a**7*b**5*c -
270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5*c**4)) + (a*b**2*sqrt(-
c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2
 - c)/(2*(a**2 + c)**3))*log(x + (-4*a**16*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/
(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 +
 c)**3))**2 + 24*a**14*c**2*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a*
*4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 14*
a**12*b**2*c*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c
**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 216*a**12*c**3*(a*b**
2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2
*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 44*a**10*b**2*c**2*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3)) + 568*a**10*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a
**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)
)**2 + 78*a**8*b**4*c - 42*a**8*b**2*c**3*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*
c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)
**3)) + 720*a**8*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c +
 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 464*a**6*
b**4*c**2 - 8*a**6*b**2*c**4*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a
**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 456*a
**6*c**6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2
+ c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 + 380*a**4*b**4*c**3 - 2
*a**4*b**2*c**5*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**
2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) + 104*a**4*c**7*(a*b
**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b*
*2*(3*a**2 - c)/(2*(a**2 + c)**3))**2 - 96*a**2*b**4*c**4 - 12*a**2*b**2*c*
*6*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3
)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3)) - 24*a**2*c**8*(a*b**2*sqrt(-c)*(
a**2 - 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c
)/(2*(a**2 + c)**3))**2 + 6*b**4*c**5 - 6*b**2*c**7*(a*b**2*sqrt(-c)*(a**2
- 3*c)/(2*c*(a**6 + 3*a**4*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*
(a**2 + c)**3)) - 12*c**9*(a*b**2*sqrt(-c)*(a**2 - 3*c)/(2*c*(a**6 + 3*a**4
*c + 3*a**2*c**2 + c**3)) - b**2*(3*a**2 - c)/(2*(a**2 + c)**3))**2)/(a**9*
b**5 + 72*a**7*b**5*c - 270*a**5*b**5*c**2 + 144*a**3*b**5*c**3 - 27*a*b**5
*c**4)) + (-a**2 + 4*a*b*x - c)/(x**2*(2*a**4 + 4*a**2*c + 2*c**2))
```

3.81 $\int \frac{1}{a+b(c+dx)^2}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-1), x]
```

fricas [A] time = 1.27, size = 109, normalized size = 3.52

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{2\,abd}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c
) - a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(a*b*d), sqrt(a*b)*arctan(sqrt(
a*b)*(d*x + c)/a)/(a*b*d)]
```

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*d)
```

maple [A] time = 0.01, size = 34, normalized size = 1.10

$$\frac{\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{ab}\;d}\right)}{\sqrt{ab}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2),x)
```

```
[Out] 1/d/(b*a)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/(b*a)^(1/2))
```

maxima [A] time = 1.56, size = 30, normalized size = 0.97

$$\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{\sqrt{ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*d)
```

mupad [B] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

sympy [B]   time = 0.21, size = 61, normalized size = 1.97

$$\frac{-\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{-a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(x+\frac{a\sqrt{-\frac{1}{ab}}+c}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2),x)
```

```
[Out] (-sqrt(-1/(a*b))*log(x + (-a*sqrt(-1/(a*b)) + c)/d)/2 + sqrt(-1/(a*b))*log(
x + (a*sqrt(-1/(a*b)) + c)/d)/2)/d
```

3.82 $\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*a*d*(a + b*(c + d*x)^2)) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]]
/(2*a^(3/2)*Sqrt[b]*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+b(c+dx)^2\right)^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2} dx, x, c+dx\right)}{d}\\
&= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, c+dx\right)}{2ad}\\
&= \frac{c+dx}{2ad\left(a+b(c+dx)^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}
\end{aligned}$$

Mathematica [A]   time = 0.04, size = 60, normalized size = 0.95

$$\frac{\frac{\sqrt{a}(c+dx)}{a+b(c+dx)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x))/(a + b*(c + d*x)^2) + ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[
a]]/Sqrt[b])/(2*a^(3/2)*d)
```

IntegrateAlgebraic [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-2), x]
```

fricas [A]   time = 1.12, size = 253, normalized size = 4.02

$$\left[\frac{2\,abdx+2\,abc-\left(bd^2x^2+2\,bcdx+bc^2+a\right)\sqrt{-ab}\,\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}\,(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{4\left(a^2b^2d^3x^2+2\,a^2b^2cd^2x+\left(a^2b^2c^2+a^3b\right)d\right)},\ \frac{abdx+abc+\left(bd^2x^2+2\,bcdx+bc^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,(dx+c)}{a}\right)}{2\left(a^2b^2d^3x^2+2\,a^2b^2cd^2x+\left(a^2b^2c^2+a^3b\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*d*x + 2*a*b*c - (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*sqrt(-a*b)*
log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(b*d^2*x^2
 + 2*b*c*d*x + b*c^2 + a)))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d^2*x + (a^2*b^2
*c^2 + a^3*b)*d), 1/2*(a*b*d*x + a*b*c + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a))/(a^2*b^2*d^3*x^2 + 2*a^2*b^2*c*d
^2*x + (a^2*b^2*c^2 + a^3*b)*d)]
```

giac [A]   time = 0.36, size = 65, normalized size = 1.03

$$\frac{\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ad}+\frac{dx+c}{2\left(bd^2x^2+2\,bcdx+bc^2+a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a*d) + 1/2*(d*x + c)/((b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)*a*d)
```

maple [A]   time = 0.00, size = 86, normalized size = 1.37

$$\frac{\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{ab}\,d}\right)}{2\sqrt{ab}\,ad}+\frac{2b\,d^2x+2bdc}{4\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)ab\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)+1/2/d/a/(a*b)
^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)
```

maxima [A] time = 1.56, size = 75, normalized size = 1.19

$$\frac{dx+c}{2\left(abd^3x^2+2\,abcd^2x+\left(abc^2+a^2\right)d\right)}+\frac{\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{2\,\sqrt{ab}\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*b*d^3*x^2 + 2*a*b*c*d^2*x + (a*b*c^2 + a^2)*d) + 1/2*arcta
n((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a*d)
```

mupad [B] time = 0.10, size = 76, normalized size = 1.21

$$\frac{\frac{x}{2\,a}+\frac{c}{2\,a\,d}}{b\,c^2+2\,b\,c\,d\,x+b\,d^2\,x^2+a}+\frac{\mathrm{atan}\left(2\,a\,\left(\frac{\sqrt{b}\,c}{2\,a^{3/2}}+\frac{\sqrt{b}\,d\,x}{2\,a^{3/2}}\right)\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^2,x)
```

```
[Out] (x/(2*a) + c/(2*a*d))/(a + b*c^2 + b*d^2*x^2 + 2*b*c*d*x) + atan(2*a*((b^(1
/2)*c)/(2*a^(3/2)) + (b^(1/2)*d*x)/(2*a^(3/2))))/(2*a^(3/2)*b^(1/2)*d)
```

sympy [B] time = 0.58, size = 117, normalized size = 1.86

$$\frac{c+dx}{2a^2d+2abc^2d+4abcd^2x+2abd^3x^2}+\frac{-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{-a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(x+\frac{a^2\sqrt{-\frac{1}{a^3b}}+c}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*a**2*d + 2*a*b*c**2*d + 4*a*b*c*d**2*x + 2*a*b*d**3*x**2) + (-
sqrt(-1/(a**3*b))*log(x + (-a**2*sqrt(-1/(a**3*b)) + c)/d)/4 + sqrt(-1/(a**
3*b))*log(x + (a**2*sqrt(-1/(a**3*b)) + c)/d)/4)/d
```

3.83 $\int \frac{1}{\left(a+b(c+dx)^2\right)^3} dx$

Optimal. Leaf size=91

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 205}

$$\frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*a*d*(a + b*(c + d*x)^2)^2) + (3*(c + d*x))/(8*a^2*d*(a + b*(c
+ d*x)^2)) + (3*ArcTan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^3}\,dx,x,c+dx\right)}{d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx,x,c+dx\right)}{4ad}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,c+dx\right)}{8a^2d}$$

$$= \frac{c+dx}{4ad\left(a+b(c+dx)^2\right)^2} + \frac{3(c+dx)}{8a^2d\left(a+b(c+dx)^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(c+dx)\left(5a+3b(c+dx)^2\right)}{\left(a+b(c+dx)^2\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] ((Sqrt[a]*(c + d*x)*(5*a + 3*b*(c + d*x)^2))/(a + b*(c + d*x)^2)^2 + (3*Arc
Tan[(Sqrt[b]*(c + d*x))/Sqrt[a]])/Sqrt[b])/(8*a^(5/2)*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b(c+dx)^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^2)^(-3), x]
```

fricas [B] time = 1.26, size = 595, normalized size = 6.54

$$\left[\frac{6\,ab^2d^3x^3+18\,ab^2cd^2x^2+6\,ab^2c^3+10\,a^2bc+2\left(9\,ab^2c^2+5\,a^2b\right)dx-3\left(b^2d^4x^4+4\,b^2cd^3x^3+b^2c^4+2\left(3\,b^2c^2+ab\right)d^2x^2+2\,abc^2+4\left(b^2c^3+abc\right)dx+a^2\right)\sqrt{-ab}\log\left(\frac{bd^2x^2+2\,bcdx+bc^2-2\,\sqrt{-ab}(dx+c)-a}{bd^2x^2+2\,bcdx+bc^2+a}\right)}{16\left(a^3b^3d^5x^4+4\,a^3b^3cd^4x^3+2\left(3\,a^3b^3c^2+a^4b^2\right)d^3x^2+4\left(a^3b^3c^3+a^4b^2c\right)d^2x+\left(a^3b^3c^4+2\,a^4b^2c^2+a^5b\right)d\right)}, \frac{3\,ab^2d^3x^3+9\,ab^2cd^2x^2+3\,ab^2c^3+5\,a^2bc+\left(9\,ab^2c^2+5\,a^2b\right)dx+3\left(b^2d^4x^4+4\,b^2cd^3x^3+b^2c^4+2\left(3\,b^2c^2+ab\right)d^2x^2+2\,abc^2+4\left(b^2c^3+abc\right)dx+a^2\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}(dx+c)}{a}\right)}{8\left(a^3b^3d^5x^4+4\,a^3b^3cd^4x^3+2\left(3\,a^3b^3c^2+a^4b^2\right)d^3x^2+4\left(a^3b^3c^3+a^4b^2c\right)d^2x+\left(a^3b^3c^4+2\,a^4b^2c^2+a^5b\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^2*d^3*x^3 + 18*a*b^2*c*d^2*x^2 + 6*a*b^2*c^3 + 10*a^2*b*c + 2*
(9*a*b^2*c^2 + 5*a^2*b)*d*x - 3*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 +
2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sq
rt(-a*b)*log((b*d^2*x^2 + 2*b*c*d*x + b*c^2 - 2*sqrt(-a*b)*(d*x + c) - a)/(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)))/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3
 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x
+ (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^5*b)*d), 1/8*(3*a*b^2*d^3*x^3 + 9*a*b^2*
c*d^2*x^2 + 3*a*b^2*c^3 + 5*a^2*b*c + (9*a*b^2*c^2 + 5*a^2*b)*d*x + 3*(b^2*
d^4*x^4 + 4*b^2*c*d^3*x^3 + b^2*c^4 + 2*(3*b^2*c^2 + a*b)*d^2*x^2 + 2*a*b*c
```

```
^2 + 4*(b^2*c^3 + a*b*c)*d*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)*(d*x + c)/a)
)/(a^3*b^3*d^5*x^4 + 4*a^3*b^3*c*d^4*x^3 + 2*(3*a^3*b^3*c^2 + a^4*b^2)*d^3*
x^2 + 4*(a^3*b^3*c^3 + a^4*b^2*c)*d^2*x + (a^3*b^3*c^4 + 2*a^4*b^2*c^2 + a^
5*b)*d)]
```

giac [A] time = 0.41, size = 103, normalized size = 1.13

$$\frac{3\,\arctan\left(\frac{bdx+bc}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2d}+\frac{3\,bd^3x^3+9\,bcd^2x^2+9\,bc^2dx+3\,bc^3+5\,adx+5\,ac}{8\,\left(bd^2x^2+2\,bcdx+bc^2+a\right)^2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan((b*d*x + b*c)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/8*(3*b*d^3*x^3 +
9*b*c*d^2*x^2 + 9*b*c^2*d*x + 3*b*c^3 + 5*a*d*x + 5*a*c)/((b*d^2*x^2 + 2*b*
c*d*x + b*c^2 + a)^2*a^2*d)
```

maple [A] time = 0.00, size = 147, normalized size = 1.62

$$\frac{3x}{8\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)a^2}+\frac{3c}{8\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)a^2d}+\frac{3\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{ab}\;d}\right)}{8\sqrt{ab}\;a^2d}+\frac{2b\,d^2x+2bdc}{8\left(b\,d^2x^2+2bcdx+b\,c^2+a\right)^2ab\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*b*d^2*x+2*b*c*d)/a/b/d^2/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)^2+3/8/a^2/(b*
d^2*x^2+2*b*c*d*x+b*c^2+a)*x+3/8/a^2/d/(b*d^2*x^2+2*b*c*d*x+b*c^2+a)*c+3/8/
a^2/d/(a*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/(a*b)^(1/2)/d)
```

maxima [B] time = 1.59, size = 184, normalized size = 2.02

$$\frac{3\,bd^3x^3+9\,bcd^2x^2+3\,bc^3+\left(9\,bc^2+5\,a\right)dx+5\,ac}{8\,\left(a^2b^2d^5x^4+4\,a^2b^2cd^4x^3+2\,\left(3\,a^2b^2c^2+a^3b\right)d^3x^2+4\,\left(a^2b^2c^3+a^3bc\right)d^2x+\left(a^2b^2c^4+2\,a^3bc^2+a^4\right)d\right)}+\frac{3\,\arctan\left(\frac{bd^2x+bcd}{\sqrt{ab}\,d}\right)}{8\,\sqrt{ab}\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b*d^3*x^3 + 9*b*c*d^2*x^2 + 3*b*c^3 + (9*b*c^2 + 5*a)*d*x + 5*a*c)/(
a^2*b^2*d^5*x^4 + 4*a^2*b^2*c*d^4*x^3 + 2*(3*a^2*b^2*c^2 + a^3*b)*d^3*x^2 +
 4*(a^2*b^2*c^3 + a^3*b*c)*d^2*x + (a^2*b^2*c^4 + 2*a^3*b*c^2 + a^4)*d) + 3
/8*arctan((b*d^2*x + b*c*d)/(sqrt(a*b)*d))/(sqrt(a*b)*a^2*d)
```

mupad [B] time = 2.22, size = 181, normalized size = 1.99

$$\frac{\frac{x\,\left(9\,b\,c^2+5\,a\right)}{8\,a^2}+\frac{3\,b\,c^3+5\,a\,c}{8\,a^2\,d}+\frac{3\,b\,d^2\,x^3}{8\,a^2}+\frac{9\,b\,c\,d\,x^2}{8\,a^2}}{x^2\,\left(6\,b^2\,c^2\,d^2+2\,a\,b\,d^2\right)+x\,\left(4\,d\,b^2\,c^3+4\,a\,d\,b\,c\right)+a^2+b^2\,c^4+b^2\,d^4\,x^4+2\,a\,b\,c^2+4\,b^2\,c\,d^3\,x^3}+\frac{3\,\mathrm{atan}\left(\frac{8\,a^2\,\left(\frac{3\,\sqrt{b}\,c}{8\,a^{5/2}}+\frac{3\,\sqrt{b}\,d\,x}{8\,a^{5/2}}\right)}{3}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^2)^3,x)
```

```
[Out] ((x*(5*a + 9*b*c^2))/(8*a^2) + (5*a*c + 3*b*c^3)/(8*a^2*d) + (3*b*d^2*x^3)/
(8*a^2) + (9*b*c*d*x^2)/(8*a^2))/(x^2*(6*b^2*c^2*d^2 + 2*a*b*d^2) + x*(4*b^
2*c^3*d + 4*a*b*c*d) + a^2 + b^2*c^4 + b^2*d^4*x^4 + 2*a*b*c^2 + 4*b^2*c*d^
3*x^3) + (3*atan((8*a^2*((3*b^(1/2)*c)/(8*a^(5/2)) + (3*b^(1/2)*d*x)/(8*a^(
5/2))))/3))/(8*a^(5/2)*b^(1/2)*d)
```

sympy [B] time = 1.25, size = 257, normalized size = 2.82

$$\frac{5ac+3bc^3+9bcd^2x^2+3bd^3x^3+x\left(5ad+9bc^2d\right)}{8a^4d+16a^3bc^2d+8a^2b^2c^4d+32a^2b^2cd^4x^3+8a^2b^2d^5x^4+x^2\left(16a^3bd^3+48a^2b^2c^2d^3\right)+x\left(32a^3bcd^2+32a^2b^2c^3d^2\right)}+\frac{-\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{-3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(x+\frac{3a^3\sqrt{-\frac{1}{a^5b}}+3c}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**2)**3,x)
```

```
[Out] (5*a*c + 3*b*c**3 + 9*b*c*d**2*x**2 + 3*b*d**3*x**3 + x*(5*a*d + 9*b*c**2*d
))/(8*a**4*d + 16*a**3*b*c**2*d + 8*a**2*b**2*c**4*d + 32*a**2*b**2*c*d**4*
x**3 + 8*a**2*b**2*d**5*x**4 + x**2*(16*a**3*b*d**3 + 48*a**2*b**2*c**2*d**
3) + x*(32*a**3*b*c*d**2 + 32*a**2*b**2*c**3*d**2)) + (-3*sqrt(-1/(a**5*b))
*log(x + (-3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16 + 3*sqrt(-1/(a**5*b))*
log(x + (3*a**3*sqrt(-1/(a**5*b)) + 3*c)/(3*d))/16)/d
```

3.84 $\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}\,d}$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {247, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+bx^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}\,d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)}{\sqrt[4]{-a}}\right)}{\sqrt[4]{-a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x))/(-a)^(1/4)]/((-a)^(1/4)*Sqrt[b]*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a}+b(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-a] + b*(c + d*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(Sqrt[-a] + b*(c + d*x)^2)^(-1), x]
```

fricas [A] time = 1.46, size = 279, normalized size = 7.97

$$\left[\frac{\sqrt{\frac{\sqrt{-a}}{ab}}\log\left(\frac{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4-2\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-a}+2\left(abdx+abc+\left(b^2d^3x^3+3\,b^2cd^2x^2+3\,b^2c^2dx+b^2c^3\right)\sqrt{-a}\right)\sqrt{\frac{\sqrt{-a}}{ab}}-a}{b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4+a}\right)}{2\,d},\ \frac{\sqrt{-\frac{\sqrt{-a}}{ab}}\arctan\left((bdx+bc)\sqrt{-\frac{\sqrt{-a}}{ab}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(sqrt(-a)/(a*b))*log((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^
2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-a
) + 2*(a*b*d*x + a*b*c + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*sqrt(-a))*sqrt(sqrt(-a)/(a*b)) - a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + a))/d, sqrt(-sqrt(-a)/(a*b)
)*arctan((b*d*x + b*c)*sqrt(-sqrt(-a)/(a*b)))/d]
```

giac [A] time = 0.45, size = 30, normalized size = 0.86

$$\frac{\arctan\left(\frac{bdx+bc}{(-a)^{\frac{1}{4}}\sqrt{b}}\right)}{(-a)^{\frac{1}{4}}\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan((b*d*x + b*c)/((-a)^(1/4)*sqrt(b)))/((-a)^(1/4)*sqrt(b)*d)
```

maple [A] time = 0.01, size = 42, normalized size = 1.20

$$\frac{\arctan\left(\frac{2b\,d^2x+2bdc}{2\sqrt{\sqrt{-a}\;b\;d}}\right)}{\sqrt{\sqrt{-a}\;b\;d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(d*x+c)^2+(-a)^(1/2)),x)
```

```
[Out] 1/d/((-a)^(1/2)*b)^(1/2)*arctan(1/2*(2*b*d^2*x+2*b*c*d)/d/((-a)^(1/2)*b)^(1
/2))
```

maxima [B] time = 1.40, size = 66, normalized size = 1.89

$$\frac{\log\left(\frac{bd^2x+bcd-\sqrt{-\sqrt{-a}\,b}\,d}{bd^2x+bcd+\sqrt{-\sqrt{-a}\,b}\,d}\right)}{2\sqrt{-\sqrt{-a}\,b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)^2+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*d^2*x + b*c*d - sqrt(-sqrt(-a)*b)*d)/(b*d^2*x + b*c*d + sqrt(-sq
rt(-a)*b)*d))/(sqrt(-sqrt(-a)*b)*d)
```

mupad [B] time = 0.10, size = 31, normalized size = 0.89

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,c+\sqrt{b}\,d\,x}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*(c + d*x)^2 + (-a)^(1/2)),x)
```

```
[Out] atan((b^(1/2)*c + b^(1/2)*d*x)/(-a)^(1/4))/((-a)^(1/4)*b^(1/2)*d)
```

sympy [B] time = 0.22, size = 92, normalized size = 2.63

$$\frac{-\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c-\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}+\frac{\sqrt{-\frac{1}{b\sqrt{-a}}}\log\left(x+\frac{c+\sqrt{-a}\sqrt{-\frac{1}{b\sqrt{-a}}}}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*(d*x+c)**2+(-a)**(1/2)),x)
```

```
[Out] (-sqrt(-1/(b*sqrt(-a)))*log(x + (c - sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2 +
 sqrt(-1/(b*sqrt(-a)))*log(x + (c + sqrt(-a)*sqrt(-1/(b*sqrt(-a))))/d)/2)/d
```

3.85 $\int \frac{1}{1+(c+dx)^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}(c+dx)}{d}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 203}

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+(c+dx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{d}\\ &= \frac{\tan^{-1}(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTan[c + d*x]/d
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1+(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-1), x]
```

fricas [A] time = 1.45, size = 10, normalized size = 1.00

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="fricas")
[Out] arctan(d*x + c)/d
```

giac [A] time = 0.32, size = 10, normalized size = 1.00

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="giac")
[Out] arctan(d*x + c)/d
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2),x)
[Out] arctan(d*x+c)/d
```

maxima [A] time = 1.57, size = 18, normalized size = 1.80

$$\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2),x, algorithm="maxima")
[Out] arctan((d^2*x + c*d)/d)/d
```

mupad [B] time = 0.04, size = 10, normalized size = 1.00

$$\frac{\mathrm{atan}\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1),x)
[Out] atan(c + d*x)/d
```

sympy [C] time = 0.17, size = 24, normalized size = 2.40

$$\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{2}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2),x)
[Out] (-I*log(x + (c - I)/d)/2 + I*log(x + (c + I)/d)/2)/d
```

3.86 $\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{c+dx}{2d\left((c+dx)^2+1\right)}+\frac{\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 + (c + d*x)^2)) + ArcTan[c + d*x]/(2*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx,x,c+dx\right)}{d}\\
&= \frac{c+dx}{2d\left(1+(c+dx)^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,c+dx\right)}{2d}\\
&= \frac{c+dx}{2d\left(1+(c+dx)^2\right)}+\frac{\tan^{-1}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.84

$$\frac{\frac{c+dx}{(c+dx)^2+1}+\tan^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] ((c + d*x)/(1 + (c + d*x)^2) + ArcTan[c + d*x])/(2*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+(c+dx)^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-2), x]
```

fricas [A] time = 1.15, size = 55, normalized size = 1.49

$$\frac{dx+\left(d^2x^2+2\,cdx+c^2+1\right)\arctan\left(dx+c\right)+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*x + (d^2*x^2 + 2*c*d*x + c^2 + 1)*arctan(d*x + c) + c)/(d^3*x^2 + 2*
c*d^2*x + (c^2 + 1)*d)
```

giac [A] time = 0.38, size = 41, normalized size = 1.11

$$\frac{\arctan\left(dx+c\right)}{2\,d}+\frac{dx+c}{2\left(d^2x^2+2\,cdx+c^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(d*x + c)/d + 1/2*(d*x + c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)*d)
```

maple [A] time = 0.01, size = 59, normalized size = 1.59

$$\frac{\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{2d}+\frac{2d^2x+2cd}{4\left(d^2x^2+2cdx+c^2+1\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^2,x)
```

```
[Out] 1/4*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)+1/2/d*arctan(1/2*(2*d^2*x+2
*c*d)/d)
```

maxima [A] time = 1.32, size = 51, normalized size = 1.38

$$\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2+1\right)d\right)}+\frac{\arctan\left(\frac{d^2x+cd}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 + 1)*d) + 1/2*arctan((d^2*x + c*d
)/d)/d
```

mupad [B] time = 2.07, size = 42, normalized size = 1.14

$$\frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2+1}+\frac{\mathrm{atan}\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^2,x)
```

```
[Out] (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x + 1) + atan(c + d*x)/(2*d)
```

sympy [C] time = 0.44, size = 56, normalized size = 1.51

$$\frac{c+dx}{2c^2d+4cd^2x+2d^3x^2+2d}+\frac{-\frac{i\log\left(x+\frac{c-i}{d}\right)}{4}+\frac{i\log\left(x+\frac{c+i}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**2,x)
```

```
[Out] (c + d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 + 2*d) + (-I*log(x + (c - I)
/d)/4 + I*log(x + (c + I)/d)/4)/d
```

3.87 $\int \frac{1}{\left(1+(c+dx)^2\right)^3} \, dx$

Optimal. Leaf size=60

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 203}

$$\frac{3(c+dx)}{8d\left((c+dx)^2+1\right)}+\frac{c+dx}{4d\left((c+dx)^2+1\right)^2}+\frac{3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 + (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 + (c + d*x)^2))
 + (3*ArcTan[c + d*x])/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1+(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1+(c+dx)^2\right)} + \frac{3\tan^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.87

$$\frac{\frac{3(c+dx)}{(c+dx)^2+1} + \frac{2(c+dx)}{\left((c+dx)^2+1\right)^2} + 3\tan^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] ((2*(c + d*x))/(1 + (c + d*x)^2)^2 + (3*(c + d*x))/(1 + (c + d*x)^2) + 3*Ar
cTan[c + d*x])/(8*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+(c+dx)^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(1 + (c + d*x)^2)^(-3), x]
```

fricas [B] time = 0.90, size = 153, normalized size = 2.55

$$\frac{3\,d^3x^3 + 9\,cd^2x^2 + 3\,c^3 + \left(9\,c^2+5\right)dx + 3\left(d^4x^4 + 4\,cd^3x^3 + 2\left(3\,c^2+1\right)d^2x^2 + c^4 + 4\left(c^3+c\right)dx + 2\,c^2 + 1\right)\arctan\left(dx+c\right) + 5\,c}{8\left(d^5x^4 + 4\,cd^4x^3 + 2\left(3\,c^2+1\right)d^3x^2 + 4\left(c^3+c\right)d^2x + \left(c^4+2\,c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 3*(d^4*x^4 + 4*c*d
^3*x^3 + 2*(3*c^2 + 1)*d^2*x^2 + c^4 + 4*(c^3 + c)*d*x + 2*c^2 + 1)*arctan(
d*x + c) + 5*c)/(d^5*x^4 + 4*c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c
)*d^2*x + (c^4 + 2*c^2 + 1)*d)
```

giac [A] time = 0.30, size = 73, normalized size = 1.22

$$\frac{3\,\arctan\left(dx+c\right)}{8\,d} + \frac{3\,d^3x^3 + 9\,cd^2x^2 + 9\,c^2dx + 3\,c^3 + 5\,dx + 5\,c}{8\left(d^2x^2 + 2\,cdx + c^2 + 1\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(d*x + c)/d + 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 +
5*d*x + 5*c)/((d^2*x^2 + 2*c*d*x + c^2 + 1)^2*d)
```

maple [A] time = 0.01, size = 94, normalized size = 1.57

$$\frac{3\arctan\left(\frac{2d^2x+2cd}{2d}\right)}{8d}+\frac{2d^2x+2cd}{8\left(d^2x^2+2cdx+c^2+1\right)^2d^2}+\frac{\frac{3}{8}d^2x+\frac{3}{8}cd}{\left(d^2x^2+2cdx+c^2+1\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(d*x+c)^2)^3,x)
```

```
[Out] 1/8*(2*d^2*x+2*c*d)/d^2/(d^2*x^2+2*c*d*x+c^2+1)^2+3/16*(2*d^2*x+2*c*d)/(d^2
*x^2+2*c*d*x+c^2+1)/d^2+3/8/d*arctan(1/2*(2*d^2*x+2*c*d)/d)
```

maxima [B] time = 1.51, size = 115, normalized size = 1.92

$$\frac{3\,d^3x^3+9\,cd^2x^2+3\,c^3+\left(9\,c^2+5\right)dx+5\,c}{8\left(d^5x^4+4\,cd^4x^3+2\left(3\,c^2+1\right)d^3x^2+4\left(c^3+c\right)d^2x+\left(c^4+2\,c^2+1\right)d\right)}+\frac{3\,\arctan\left(\frac{d^2x+cd}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 + 5)*d*x + 5*c)/(d^5*x^4 + 4*
c*d^4*x^3 + 2*(3*c^2 + 1)*d^3*x^2 + 4*(c^3 + c)*d^2*x + (c^4 + 2*c^2 + 1)*d
) + 3/8*arctan((d^2*x + c*d)/d)/d
```

mupad [B] time = 0.12, size = 111, normalized size = 1.85

$$\frac{3\,\mathrm{atan}\left(c+d\,x\right)}{8\,d}+\frac{x\,\left(\frac{9\,c^2}{8}+\frac{5}{8}\right)+\frac{3\,c^3+5\,c}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{x^2\,\left(6\,c^2\,d^2+2\,d^2\right)+2\,c^2+c^4+x\,\left(4\,d\,c^3+4\,d\,c\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 + 1)^3,x)
```

```
[Out] (3*atan(c + d*x))/(8*d) + (x*((9*c^2)/8 + 5/8) + (5*c + 3*c^3)/(8*d) + (3*d
^2*x^3)/8 + (9*c*d*x^2)/8)/(x^2*(2*d^2 + 6*c^2*d^2) + 2*c^2 + c^4 + x*(4*c*
d + 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

sympy [C] time = 0.93, size = 146, normalized size = 2.43

$$\frac{3c^3+9cd^2x^2+5c+3d^3x^3+x\left(9c^2d+5d\right)}{8c^4d+16c^2d+32cd^4x^3+8d^5x^4+8d+x^2\left(48c^2d^3+16d^3\right)+x\left(32c^3d^2+32cd^2\right)}+\frac{-\frac{3i\log\left(x+\frac{3c-3i}{3d}\right)}{16}+\frac{3i\log\left(x+\frac{3c+3i}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(d*x+c)**2)**3,x)
```

```
[Out] (3*c**3 + 9*c*d**2*x**2 + 5*c + 3*d**3*x**3 + x*(9*c**2*d + 5*d))/(8*c**4*d
 + 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 + 16
*d**3) + x*(32*c**3*d**2 + 32*c*d**2)) + (-3*I*log(x + (3*c - 3*I)/(3*d))/1
6 + 3*I*log(x + (3*c + 3*I)/(3*d))/16)/d
```

3.88 $\int \frac{1}{1-(c+dx)^2} dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {247, 206}

$$\frac{\tanh^{-1}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] ArcTanh[c + d*x]/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(c+dx)^2} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, c+dx\right)}{d} \\ &= \frac{\tanh^{-1}(c+dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.01, size = 32, normalized size = 3.20

$$\frac{\log(c+dx+1)}{2d} - \frac{\log(-c-dx+1)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] -1/2*Log[1 - c - d*x]/d + Log[1 + c + d*x]/(2*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-1), x]
```

fricas [B] time = 0.80, size = 22, normalized size = 2.20

$$\frac{\log(dx+c+1)-\log(dx+c-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="fricas")
[Out] 1/2*(log(d*x + c + 1) - log(d*x + c - 1))/d
```

giac [B] time = 0.38, size = 27, normalized size = 2.70

$$\frac{\log(|dx+c+1|)}{2\,d}-\frac{\log(|dx+c-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*log(abs(d*x + c + 1))/d - 1/2*log(abs(d*x + c - 1))/d
```

maple [B] time = 0.01, size = 26, normalized size = 2.60

$$-\frac{\ln(dx+c-1)}{2d}+\frac{\ln(dx+c+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2),x)
[Out] -1/2/d*ln(d*x+c-1)+1/2/d*ln(d*x+c+1)
```

maxima [B] time = 0.49, size = 25, normalized size = 2.50

$$\frac{\log(dx+c+1)}{2\,d}-\frac{\log(dx+c-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2),x, algorithm="maxima")
[Out] 1/2*log(d*x + c + 1)/d - 1/2*log(d*x + c - 1)/d
```

mupad [B] time = 2.05, size = 10, normalized size = 1.00

$$\frac{\mathrm{atanh}\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1),x)
[Out] atanh(c + d*x)/d
```

sympy [B] time = 0.18, size = 22, normalized size = 2.20

$$-\frac{\frac{\log\left(x+\frac{c-1}{d}\right)}{2}-\frac{\log\left(x+\frac{c+1}{d}\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2),x)
[Out] -(log(x + (c - 1)/d)/2 - log(x + (c + 1)/d)/2)/d
```

3.89 $\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx$

Optimal. Leaf size=39

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 199, 206}

$$\frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] (c + d*x)/(2*d*(1 - (c + d*x)^2)) + ArcTanh[c + d*x]/(2*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, c+dx\right)}{d}\\
&= \frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{2d}\\
&= \frac{c+dx}{2d\left(1-(c+dx)^2\right)}+\frac{\tanh^{-1}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.15

$$\frac{-\frac{2(c+dx)}{(c+dx)^2-1}-\log(-c-dx+1)+\log(c+dx+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] ((-2*(c + d*x))/(-1 + (c + d*x)^2) - Log[1 - c - d*x] + Log[1 + c + d*x])/(
4*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-(c+dx)^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-2), x]
```

fricas [B] time = 0.93, size = 85, normalized size = 2.18

$$-\frac{2\,dx-\left(d^2x^2+2\,cdx+c^2-1\right)\log\left(dx+c+1\right)+\left(d^2x^2+2\,cdx+c^2-1\right)\log\left(dx+c-1\right)+2\,c}{4\left(d^3x^2+2\,cd^2x+\left(c^2-1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d*x - (d^2*x^2 + 2*c*d*x + c^2 - 1)*log(d*x + c + 1) + (d^2*x^2 + 2
*c*d*x + c^2 - 1)*log(d*x + c - 1) + 2*c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*
d)
```

giac [A] time = 0.35, size = 56, normalized size = 1.44

$$\frac{\log\left(|dx+c+1|\right)}{4\,d}-\frac{\log\left(|dx+c-1|\right)}{4\,d}-\frac{dx+c}{2\left(d^2x^2+2\,cdx+c^2-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(abs(d*x + c + 1))/d - 1/4*log(abs(d*x + c - 1))/d - 1/2*(d*x + c)/(
(d^2*x^2 + 2*c*d*x + c^2 - 1)*d)
```

maple [A] time = 0.01, size = 52, normalized size = 1.33

$$-\frac{\ln\left(dx+c-1\right)}{4d}+\frac{\ln\left(dx+c+1\right)}{4d}-\frac{1}{4\left(dx+c-1\right)d}-\frac{1}{4\left(dx+c+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^2,x)
```

```
[Out] -1/4/d/(d*x+c-1)-1/4/d*ln(d*x+c-1)-1/4/d/(d*x+c+1)+1/4/d*ln(d*x+c+1)
```

maxima [A] time = 0.62, size = 56, normalized size = 1.44

$$-\frac{dx+c}{2\left(d^3x^2+2\,cd^2x+\left(c^2-1\right)d\right)}+\frac{\log\left(dx+c+1\right)}{4\,d}-\frac{\log\left(dx+c-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)/(d^3*x^2 + 2*c*d^2*x + (c^2 - 1)*d) + 1/4*log(d*x + c + 1)/d
 - 1/4*log(d*x + c - 1)/d
```

mupad [B] time = 2.06, size = 43, normalized size = 1.10

$$\frac{\operatorname{atanh}(c+d\,x)}{2\,d}-\frac{\frac{x}{2}+\frac{c}{2\,d}}{c^2+2\,c\,d\,x+d^2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x)^2 - 1)^2,x)
```

```
[Out] atanh(c + d*x)/(2*d) - (x/2 + c/(2*d))/(c^2 + d^2*x^2 + 2*c*d*x - 1)
```

sympy [A] time = 0.47, size = 54, normalized size = 1.38

$$\frac{-c-dx}{2c^2d+4cd^2x+2d^3x^2-2d}+\frac{-\frac{\log\left(x+\frac{c-1}{d}\right)}{4}+\frac{\log\left(x+\frac{c+1}{d}\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**2,x)
```

```
[Out] (-c - d*x)/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2 - 2*d) + (-log(x + (c - 1)/
d)/4 + log(x + (c + 1)/d)/4)/d
```

3.90 $\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 199, 206}

$$\frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)}+\frac{c+dx}{4d\left(1-(c+dx)^2\right)^2}+\frac{3\tanh^{-1}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] (c + d*x)/(4*d*(1 - (c + d*x)^2)^2) + (3*(c + d*x))/(8*d*(1 - (c + d*x)^2))
 + (3*ArcTanh[c + d*x])/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx, x, c+dx\right)}{d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, c+dx\right)}{4d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, c+dx\right)}{8d}$$

$$= \frac{c+dx}{4d\left(1-(c+dx)^2\right)^2} + \frac{3(c+dx)}{8d\left(1-(c+dx)^2\right)} + \frac{3\tanh^{-1}(c+dx)}{8d}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{-\frac{6(c+dx)}{(c+dx)^2-1} + \frac{4(c+dx)}{\left((c+dx)^2-1\right)^2} - 3\log(-c-dx+1) + 3\log(c+dx+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] ((4*(c + d*x))/(-1 + (c + d*x)^2)^2 - (6*(c + d*x))/(-1 + (c + d*x)^2) - 3*
Log[1 - c - d*x] + 3*Log[1 + c + d*x])/(16*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-(c+dx)^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(1 - (c + d*x)^2)^(-3), x]
```

fricas [B] time = 1.21, size = 220, normalized size = 3.44

$$-\frac{6\,d^3x^3 + 18\,cd^2x^2 + 6\,c^3 + 2\left(9\,c^2-5\right)dx - 3\left(d^4x^4 + 4\,cd^3x^3 + 2\left(3\,c^2-1\right)d^2x^2 + c^4 + 4\left(c^3-c\right)dx - 2\,c^2 + 1\right)\log\left(dx+c+1\right) + 3\left(d^4x^4 + 4\,cd^3x^3 + 2\left(3\,c^2-1\right)d^2x^2 + c^4 + 4\left(c^3-c\right)dx - 2\,c^2 + 1\right)\log\left(dx+c-1\right) - 10\,c}{16\left(d^5x^4 + 4\,cd^4x^3 + 2\left(3\,c^2-1\right)d^3x^2 + 4\left(c^3-c\right)d^2x + \left(c^4-2\,c^2+1\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*d^3*x^3 + 18*c*d^2*x^2 + 6*c^3 + 2*(9*c^2 - 5)*d*x - 3*(d^4*x^4 +
4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4*(c^3 - c)*d*x - 2*c^2 + 1)*lo
g(d*x + c + 1) + 3*(d^4*x^4 + 4*c*d^3*x^3 + 2*(3*c^2 - 1)*d^2*x^2 + c^4 + 4
*(c^3 - c)*d*x - 2*c^2 + 1)*log(d*x + c - 1) - 10*c)/(d^5*x^4 + 4*c*d^4*x^3
 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*d)
```

giac [A] time = 0.36, size = 88, normalized size = 1.38

$$\frac{3\,\log\left(|dx+c+1|\right)}{16\,d} - \frac{3\,\log\left(|dx+c-1|\right)}{16\,d} - \frac{3\,d^3x^3 + 9\,cd^2x^2 + 9\,c^2dx + 3\,c^3 - 5\,dx - 5\,c}{8\left(d^2x^2 + 2\,cdx + c^2 - 1\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/16*log(abs(d*x + c + 1))/d - 3/16*log(abs(d*x + c - 1))/d - 1/8*(3*d^3*x^
3 + 9*c*d^2*x^2 + 9*c^2*d*x + 3*c^3 - 5*d*x - 5*c)/((d^2*x^2 + 2*c*d*x + c^
2 - 1)^2*d)
```

maple [A] time = 0.01, size = 78, normalized size = 1.22

$$-\frac{3\ln\left(dx+c-1\right)}{16d}+\frac{3\ln\left(dx+c+1\right)}{16d}+\frac{1}{16\left(dx+c-1\right)^{2}d}-\frac{3}{16\left(dx+c-1\right)d}-\frac{1}{16\left(dx+c+1\right)^{2}d}-\frac{3}{16\left(dx+c+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(d*x+c)^2)^3,x)
```

```
[Out] 1/16/d/(d*x+c-1)^2-3/16/(d*x+c-1)/d-3/16/d*ln(d*x+c-1)-1/16/d/(d*x+c+1)^2-3
/16/(d*x+c+1)/d+3/16/d*ln(d*x+c+1)
```

maxima [B] time = 0.64, size = 122, normalized size = 1.91

$$-\frac{3\,d^{3}x^{3}+9\,cd^{2}x^{2}+3\,c^{3}+\left(9\,c^{2}-5\right)dx-5\,c}{8\left(d^{5}x^{4}+4\,cd^{4}x^{3}+2\left(3\,c^{2}-1\right)d^{3}x^{2}+4\left(c^{3}-c\right)d^{2}x+\left(c^{4}-2\,c^{2}+1\right)d\right)}+\frac{3\,\log\left(dx+c+1\right)}{16\,d}-\frac{3\,\log\left(dx+c-1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*d^3*x^3 + 9*c*d^2*x^2 + 3*c^3 + (9*c^2 - 5)*d*x - 5*c)/(d^5*x^4 + 4
*c*d^4*x^3 + 2*(3*c^2 - 1)*d^3*x^2 + 4*(c^3 - c)*d^2*x + (c^4 - 2*c^2 + 1)*
d) + 3/16*log(d*x + c + 1)/d - 3/16*log(d*x + c - 1)/d
```

mupad [B] time = 2.12, size = 114, normalized size = 1.78

$$\frac{3\,\mathrm{atanh}\left(c+d\,x\right)}{8\,d}-\frac{x\,\left(\frac{9\,c^2}{8}-\frac{5}{8}\right)-\frac{5\,c-3\,c^3}{8\,d}+\frac{3\,d^2\,x^3}{8}+\frac{9\,c\,d\,x^2}{8}}{c^4-2\,c^2-x^2\,\left(2\,d^2-6\,c^2\,d^2\right)-x\,\left(4\,c\,d-4\,c^3\,d\right)+d^4\,x^4+4\,c\,d^3\,x^3+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((c + d*x)^2 - 1)^3,x)
```

```
[Out] (3*atanh(c + d*x))/(8*d) - (x*((9*c^2)/8 - 5/8) - (5*c - 3*c^3)/(8*d) + (3*
d^2*x^3)/8 + (9*c*d*x^2)/8)/(c^4 - 2*c^2 - x^2*(2*d^2 - 6*c^2*d^2) - x*(4*c
*d - 4*c^3*d) + d^4*x^4 + 4*c*d^3*x^3 + 1)
```

sympy [B] time = 1.03, size = 141, normalized size = 2.20

$$-\frac{3c^{3}+9cd^{2}x^{2}-5c+3d^{3}x^{3}+x\left(9c^{2}d-5d\right)}{8c^{4}d-16c^{2}d+32cd^{4}x^{3}+8d^{5}x^{4}+8d+x^{2}\left(48c^{2}d^{3}-16d^{3}\right)+x\left(32c^{3}d^{2}-32cd^{2}\right)}-\frac{\frac{3\log\left(x+\frac{3c-3}{3d}\right)}{16}-\frac{3\log\left(x+\frac{3c+3}{3d}\right)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(d*x+c)**2)**3,x)
```

```
[Out] -(3*c**3 + 9*c*d**2*x**2 - 5*c + 3*d**3*x**3 + x*(9*c**2*d - 5*d))/(8*c**4*
d - 16*c**2*d + 32*c*d**4*x**3 + 8*d**5*x**4 + 8*d + x**2*(48*c**2*d**3 - 1
6*d**3) + x*(32*c**3*d**2 - 32*c*d**2)) - (3*log(x + (3*c - 3)/(3*d))/16 -
3*log(x + (3*c + 3)/(3*d))/16)/d
```

3.91 $\int \frac{1}{1-(1+x)^2} \, dx$

Optimal. Leaf size=4

$$\tanh^{-1}(x+1)$$

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {247, 206}

$$\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] ArcTanh[1 + x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-(1+x)^2} \, dx &= \text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, 1+x\right) \\ &= \tanh^{-1}(1+x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 15, normalized size = 3.75

$$\frac{1}{2}\log(x+2)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] -1/2*Log[x] + Log[2 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1-(1+x)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-1), x]
```

fricas [B] time = 1.28, size = 11, normalized size = 2.75

$$\frac{1}{2}\,\log\left(x+2\right)-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="fricas")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

giac [B] time = 0.35, size = 13, normalized size = 3.25

$$\frac{1}{2}\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="giac")
[Out] 1/2*log(abs(x + 2)) - 1/2*log(abs(x))
```

maple [B] time = 0.00, size = 12, normalized size = 3.00

$$-\frac{\ln(x)}{2}+\frac{\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2),x)
[Out] 1/2*ln(2+x)-1/2*ln(x)
```

maxima [B] time = 0.57, size = 11, normalized size = 2.75

$$\frac{1}{2}\,\log\left(x+2\right)-\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2),x, algorithm="maxima")
[Out] 1/2*log(x + 2) - 1/2*log(x)
```

mupad [B] time = 0.15, size = 4, normalized size = 1.00

$$\operatorname{atanh}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1),x)
[Out] atanh(x + 1)
```

sympy [B] time = 0.10, size = 10, normalized size = 2.50

$$-\frac{\log\left(x\right)}{2}+\frac{\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2),x)
[Out] -log(x)/2 + log(x + 2)/2
```

3.92 $\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-(1+x)^2\right)^2} \, dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2} \, dx,x,1+x\right)\\
&= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,1+x\right)\\
&= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-2),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-(1+x)^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-2),x]
[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-2), x]
```

fricas [A] time = 1.08, size = 39, normalized size = 1.44

$$\frac{\left(x^2+2x\right)\log\left(x+2\right)-\left(x^2+2x\right)\log(x)-2x-2}{4\left(x^2+2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

giac [A] time = 0.33, size = 27, normalized size = 1.00

$$-\frac{x+1}{2\left(x^2+2x\right)}+\frac{1}{4}\log\left(|x+2|\right)-\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4}+\frac{\ln\left(x+2\right)}{4}-\frac{1}{4x}-\frac{1}{4\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2)^2,x)
[Out] -1/4/(x+2)+1/4*ln(x+2)-1/4/x-1/4*ln(x)
```

maxima [A] time = 0.65, size = 25, normalized size = 0.93

$$-\frac{x+1}{2\left(x^2+2x\right)}+\frac{1}{4}\log\left(x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^2,x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

mupad [B] time = 0.07, size = 23, normalized size = 0.85

$$\frac{\operatorname{atanh}\left(x+1\right)}{2}-\frac{x+1}{2\left(\left(x+1\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^2 - 1)^2,x)
```

```
[Out] atanh(x + 1)/2 - (x + 1)/(2*((x + 1)^2 - 1))
```

sympy [A] time = 0.12, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**2,x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.93 $\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx$

Optimal. Leaf size=45

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 199, 206}

$$\frac{3(x+1)}{8\left(1-(x+1)^2\right)}+\frac{x+1}{4\left(1-(x+1)^2\right)^2}+\frac{3}{8}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - (1 + x)^2)^(-3),x]
```

```
[Out] (1 + x)/(4*(1 - (1 + x)^2)^2) + (3*(1 + x))/(8*(1 - (1 + x)^2)) + (3*ArcTan
h[1 + x])/8
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, 1+x\right)\\
&= \frac{1+x}{4\left(1-(1+x)^2\right)^2}+\frac{3(1+x)}{8\left(1-(1+x)^2\right)}+\frac{3}{8}\tanh^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.82

$$\frac{1}{16}\left(\frac{1}{x^2}-\frac{3}{x}-\frac{3}{x+2}-\frac{1}{(x+2)^2}-3\log(x)+3\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - (1 + x)^2)^(-3),x]
[Out] (x^(-2) - 3/x - (2 + x)^(-2) - 3/(2 + x) - 3*Log[x] + 3*Log[2 + x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-(1+x)^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - (1 + x)^2)^(-3),x]
[Out] IntegrateAlgebraic[(1 - (1 + x)^2)^(-3), x]
```

fricas [B] time = 0.82, size = 71, normalized size = 1.58

$$-\frac{6\,x^3+18\,x^2-3\,\left(x^4+4\,x^3+4\,x^2\right)\log\left(x+2\right)+3\,\left(x^4+4\,x^3+4\,x^2\right)\log(x)+8\,x-4}{16\,\left(x^4+4\,x^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="fricas")
[Out] -1/16*(6*x^3 + 18*x^2 - 3*(x^4 + 4*x^3 + 4*x^2)*log(x + 2) + 3*(x^4 + 4*x^3
 + 4*x^2)*log(x) + 8*x - 4)/(x^4 + 4*x^3 + 4*x^2)
```

giac [A] time = 0.34, size = 39, normalized size = 0.87

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,\left(x^2+2\,x\right)^2}+\frac{3}{16}\,\log\left(|x+2|\right)-\frac{3}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="giac")
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^2 + 2*x)^2 + 3/16*log(abs(x + 2)) - 3/16*
log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$-\frac{3\ln(x)}{16}+\frac{3\ln\left(x+2\right)}{16}-\frac{3}{16x}+\frac{1}{16x^2}-\frac{1}{16\left(x+2\right)^2}-\frac{3}{16\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(x+1)^2)^3,x)
[Out] -1/16/(x+2)^2-3/16/(x+2)+3/16*ln(x+2)+1/16/x^2-3/16/x-3/16*ln(x)
```

maxima [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{3\,x^3+9\,x^2+4\,x-2}{8\,\left(x^4+4\,x^3+4\,x^2\right)}+\frac{3}{16}\,\log\left(x+2\right)-\frac{3}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^3 + 9*x^2 + 4*x - 2)/(x^4 + 4*x^3 + 4*x^2) + 3/16*log(x + 2) - 3/
16*log(x)
```

mupad [B] time = 2.09, size = 36, normalized size = 0.80

$$\frac{3\,\mathrm{atanh}\left(x+1\right)}{8}+\frac{\frac{5\,x}{8}-\frac{3\,{\left(x+1\right)}^3}{8}+\frac{5}{8}}{{\left(x+1\right)}^4-2\,{\left(x+1\right)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x + 1)^2 - 1)^3,x)
```

```
[Out] (3*atanh(x + 1))/8 + ((5*x)/8 - (3*(x + 1)^3)/8 + 5/8)/((x + 1)^4 - 2*(x +
1)^2 + 1)
```

sympy [A] time = 0.14, size = 44, normalized size = 0.98

$$-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+2\right)}{16}-\frac{3x^3+9x^2+4x-2}{8x^4+32x^3+32x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**2)**3,x)
```

```
[Out] -3*log(x)/16 + 3*log(x + 2)/16 - (3*x**3 + 9*x**2 + 4*x - 2)/(8*x**4 + 32*x
**3 + 32*x**2)
```

3.94 $\int \frac{\left(1+(a+bx)^2\right)^2}{x} \, dx$

Optimal. Leaf size=59

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {371, 697}

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)bx+\left(a^2+1\right)^2\log(x)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*b*x + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a + b*x)
^4/4 + (1 + a^2)^2*Log[x]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+(a+bx)^2\right)^2}{x} \, dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{-a+x} \, dx, x, a+bx\right)\\
&= \text{Subst}\left(\int \left(a\left(2+a^2\right)-\frac{\left(1+a^2\right)^2}{a-x}+\left(2+a^2\right)x+ax^2+x^3\right) dx, x, a+bx\right)\\
&= a\left(2+a^2\right)bx+\frac{1}{2}\left(2+a^2\right)(a+bx)^2+\frac{1}{3}a(a+bx)^3+\frac{1}{4}(a+bx)^4+\left(1+a^2\right)^2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 1.08

$$\frac{1}{2}\left(a^2+2\right)(a+bx)^2+a\left(a^2+2\right)(a+bx)+\left(a^2+1\right)^2\log(bx)+\frac{1}{4}(a+bx)^4+\frac{1}{3}a(a+bx)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] a*(2 + a^2)*(a + b*x) + ((2 + a^2)*(a + b*x)^2)/2 + (a*(a + b*x)^3)/3 + (a
+ b*x)^4/4 + (1 + a^2)^2*Log[b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1+(a+bx)^2\right)^2}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + (a + b*x)^2)^2/x,x]
```

```
[Out] IntegrateAlgebraic[(1 + (a + b*x)^2)^2/x, x]
```

fricas [A] time = 1.20, size = 54, normalized size = 0.92

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+\left(3\,a^2+1\right)b^2x^2+4\left(a^3+a\right)bx+\left(a^4+2\,a^2+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

giac [A] time = 0.31, size = 62, normalized size = 1.05

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+3\,a^2b^2x^2+4\,a^3bx+b^2x^2+4\,abx+\left(a^4+2\,a^2+1\right)\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + 3*a^2*b^2*x^2 + 4*a^3*b*x + b^2*x^2 + 4*a*b*x
 + (a^4 + 2*a^2 + 1)*log(abs(x))
```

maple [A] time = 0.00, size = 64, normalized size = 1.08

$$\frac{b^4x^4}{4}+\frac{4a\,b^3x^3}{3}+3a^2b^2x^2+a^4\ln(x)+4a^3bx+b^2x^2+2a^2\ln(x)+4abx+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b*x+a)^2)^2/x,x)
```

```
[Out] 1/4*b^4*x^4+4/3*a*b^3*x^3+3*x^2*a^2*b^2+4*a^3*b*x+b^2*x^2+4*a*b*x+ln(x)*a^4
+2*ln(x)*a^2+ln(x)
```

maxima [A] time = 0.54, size = 54, normalized size = 0.92

$$\frac{1}{4}\,b^4x^4+\frac{4}{3}\,ab^3x^3+\left(3\,a^2+1\right)b^2x^2+4\left(a^3+a\right)bx+\left(a^4+2\,a^2+1\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 4/3*a*b^3*x^3 + (3*a^2 + 1)*b^2*x^2 + 4*(a^3 + a)*b*x + (a^4
+ 2*a^2 + 1)*log(x)
```

mupad [B] time = 0.05, size = 55, normalized size = 0.93

$$\ln(x)\,\left(a^4+2\,a^2+1\right)+\frac{b^4\,x^4}{4}+\frac{4\,a\,b^3\,x^3}{3}+b^2\,x^2\,\left(3\,a^2+1\right)+4\,a\,b\,x\,\left(a^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2 + 1)^2/x,x)
```

```
[Out] log(x)*(2*a^2 + a^4 + 1) + (b^4*x^4)/4 + (4*a*b^3*x^3)/3 + b^2*x^2*(3*a^2 +
 1) + 4*a*b*x*(a^2 + 1)
```

sympy [A] time = 0.17, size = 58, normalized size = 0.98

$$\frac{4ab^3x^3}{3} + \frac{b^4x^4}{4} + x^2\left(3a^2b^2 + b^2\right) + x\left(4a^3b + 4ab\right) + \left(a^2 + 1\right)^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b*x+a)**2)**2/x,x)
```

```
[Out] 4*a*b**3*x**3/3 + b**4*x**4/4 + x**2*(3*a**2*b**2 + b**2) + x*(4*a**3*b + 4
*a*b) + (a**2 + 1)**2*log(x)
```

3.95 $\int \frac{x^2}{1+(-1+x)^2} \, dx$

Optimal. Leaf size=10

$$x+\log \left((x-1)^2+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {371, 702, 260}

$$x+\log \left((x-1)^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[1 + (-1 + x)^2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 702

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Polyno
mialDivide[(d + e*x)^m, a + c*x^2, x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 + a*e^2, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+(-1+x)^2} \, dx &= \text{Subst}\left(\int \frac{(1+x)^2}{1+x^2} \, dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{2x}{1+x^2}\right) \, dx, x, -1+x\right)\\
&= x+2\,\text{Subst}\left(\int \frac{x}{1+x^2} \, dx, x, -1+x\right)\\
&= x+\log \left(1+(-1+x)^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log \left(x^2-2x+2\right)+x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x)^2),x]
```

```
[Out] x + Log[2 - 2*x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+(-1+x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + (-1 + x)^2),x]
[Out] IntegrateAlgebraic[x^2/(1 + (-1 + x)^2), x]
```

fricas [A] time = 1.22, size = 11, normalized size = 1.10

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="fricas")
[Out] x + log(x^2 - 2*x + 2)
```

giac [A] time = 0.35, size = 11, normalized size = 1.10

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="giac")
[Out] x + log(x^2 - 2*x + 2)
```

maple [A] time = 0.00, size = 12, normalized size = 1.20

$$x+\ln\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x-1)^2),x)
[Out] x+ln(x^2-2*x+2)
```

maxima [A] time = 0.68, size = 11, normalized size = 1.10

$$x+\log\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(-1+x)^2),x, algorithm="maxima")
[Out] x + log(x^2 - 2*x + 2)
```

mupad [B] time = 0.03, size = 11, normalized size = 1.10

$$x+\ln\left(x^2-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)^2 + 1),x)
[Out] x + log(x^2 - 2*x + 2)
```

sympy [A] time = 0.09, size = 10, normalized size = 1.00

$$x+\log\left(x^2-2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(-1+x)**2),x)
[Out] x + log(x**2 - 2*x + 2)
```

3.96 $\int \frac{x^2}{\sqrt{1-(1+x)^2}} \, dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{1-(x+1)^2}\,x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 671, 641, 216}

$$-\frac{1}{2}\sqrt{1-(x+1)^2}\,x+\frac{3}{2}\sqrt{1-(x+1)^2}+\frac{3}{2}\sin^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] (3*Sqrt[1 - (1 + x)^2])/2 - (x*Sqrt[1 - (1 + x)^2])/2 + (3*ArcSin[1 + x])/2
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 671

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[(2*c
*d*(m + p))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p, x], x]
/; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[m, 1] && NeQ[m
 + 2*p + 1, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{1-(1+x)^2}}\,dx &= \text{Subst}\left(\int \frac{(-1+x)^2}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= -\frac{1}{2}x\sqrt{1-(1+x)^2}-\frac{3}{2}\text{Subst}\left(\int \frac{-1+x}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1+x\right)\\
&= \frac{3}{2}\sqrt{1-(1+x)^2}-\frac{1}{2}x\sqrt{1-(1+x)^2}+\frac{3}{2}\sin^{-1}(1+x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.16

$$\frac{x\left(x^2-x-6\right)+6\sqrt{x}\sqrt{x+2}\sinh^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}}\right)}{2\sqrt{-x(x+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] (x*(-6 - x + x^2) + 6*Sqrt[x]*Sqrt[2 + x]*ArcSinh[Sqrt[x]/Sqrt[2]])/(2*Sqrt
[-(x*(2 + x))])
```

IntegrateAlgebraic [A] time = 0.13, size = 45, normalized size = 1.02

$$\frac{1}{2}(3-x)\sqrt{-x^2-2x}-3\tan^{-1}\left(\frac{\sqrt{-x^2-2x}}{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 - (1 + x)^2],x]
[Out] ((3 - x)*Sqrt[-2*x - x^2])/2 - 3*ArcTan[Sqrt[-2*x - x^2]/(2 + x)]
```

fricas [A] time = 1.26, size = 35, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-x^2-2x}(x-3)-3\arctan\left(\frac{\sqrt{-x^2-2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-x^2 - 2*x)*(x - 3) - 3*arctan(sqrt(-x^2 - 2*x)/x)
```

giac [A] time = 0.39, size = 23, normalized size = 0.52

$$-\frac{1}{2}\sqrt{-(x+1)^2+1}(x-3)+\frac{3}{2}\arcsin(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(-(x + 1)^2 + 1)*(x - 3) + 3/2*arcsin(x + 1)
```

maple [A] time = 0.01, size = 35, normalized size = 0.80

$$-\frac{\sqrt{-x^2-2x}\,x}{2}+\frac{3\arcsin(x+1)}{2}+\frac{3\sqrt{-x^2-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(x+1)^2)^(1/2),x)
[Out] -1/2*x*(-x^2-2*x)^(1/2)+3/2*(-x^2-2*x)^(1/2)+3/2*arcsin(x+1)
```

maxima [A] time = 1.29, size = 36, normalized size = 0.82

$$-\frac{1}{2}\sqrt{-x^2-2x}\,x+\frac{3}{2}\sqrt{-x^2-2x}-\frac{3}{2}\arcsin(-x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(1+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 - 2*x)*x + 3/2*sqrt(-x^2 - 2*x) - 3/2*arcsin(-x - 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{1-(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (x + 1)^2)^(1/2),x)
```

```
[Out] int(x^2/(1 - (x + 1)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-x(x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(1+x)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x + 2)), x)
```

3.97 $\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx$

Optimal. Leaf size=67

$$\frac{\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = $\{371, 743, 641, 216\}$

$$\frac{\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}+\frac{3a\sqrt{1-(a+bx)^2}}{2b^3}-\frac{x\sqrt{1-(a+bx)^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] (3*a*Sqrt[1 - (a + b*x)^2])/(2*b^3) - (x*Sqrt[1 - (a + b*x)^2])/(2*b^2) + (
(1 + 2*a^2)*ArcSin[a + b*x])/(2*b^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{1-(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{b^3} \\
&= -\frac{x\sqrt{1-(a+bx)^2}}{2b^2} - \frac{\text{Subst}\left(\int \frac{-1-2a^2+3ax}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, a+bx\right)}{2b^3} \\
&= \frac{3a\sqrt{1-(a+bx)^2}}{2b^3} - \frac{x\sqrt{1-(a+bx)^2}}{2b^2} + \frac{(1+2a^2)\sin^{-1}(a+bx)}{2b^3}
\end{aligned}
$$

Mathematica [**A**] time = 0.09, size = 55, normalized size = 0.82

$$
\frac{\sqrt{-a^2-2abx-b^2x^2+1}\,(3a-bx)+\left(2a^2+1\right)\sin^{-1}(a+bx)}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + (1 + 2*a^2)*ArcSin[a + b*x
])/(2*b^3)
```

IntegrateAlgebraic [**B**] time = 0.45, size = 171, normalized size = 2.55

$$
\frac{\left(2a^2+1\right)\sqrt{-b^2}\,\log\left(2\sqrt{-b^2}\,x\sqrt{-a^2-2abx-b^2x^2+1}+2abx+2b^2x^2-1\right)}{4b^4}+\frac{\sqrt{-a^2-2abx-b^2x^2+1}\,(3a-bx)}{2b^3}+\frac{\left(-2a^2-1\right)\tan^{-1}\left(\frac{\sqrt{-a^2-2abx-b^2x^2+1}-\sqrt{-b^2}\,x}{a}\right)}{2b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 - (a + b*x)^2],x]
```

```
[Out] ((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/(2*b^3) + ((-1 - 2*a^2)*Arc
Tan[(-(Sqrt[-b^2]*x) + Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a])/(2*b^3) + ((1
 + 2*a^2)*Sqrt[-b^2]*Log[-1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 -
 a^2 - 2*a*b*x - b^2*x^2]])/(4*b^4)
```

fricas [**A**] time = 0.93, size = 92, normalized size = 1.37

$$
-\frac{\left(2\,a^2+1\right)\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx+a)}{b^2x^2+2\,abx+a^2-1}\right)+\sqrt{-b^2x^2-2\,abx-a^2+1}\,(bx-3\,a)}{2\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 1)*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*
x^2 + 2*a*b*x + a^2 - 1)) + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x - 3*a))
/b^3
```

giac [**A**] time = 0.49, size = 55, normalized size = 0.82

$$
-\frac{1}{2}\sqrt{-(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2+1\right)\arcsin\left(-bx-a\right)\mathrm{sgn}(b)}{2\,b^2|b|}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-(b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 + 1)*arcsin(-b*x
 - a)*sgn(b)/(b^2*abs(b))
```

maple [B] time = 0.02, size = 152, normalized size = 2.27

$$\frac{a^2 \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{\sqrt{b^2}\, b^2} - \frac{\sqrt{-b^2x^2-2abx-a^2+1}\, x}{2b^2} + \frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{2\sqrt{b^2}\, b^2} + \frac{3\sqrt{-b^2x^2-2abx-a^2+1}\, a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1-(b*x+a)^2)^(1/2),x)
```

```
[Out] -1/2*x/b^2*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)+3/2*a/b^3*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2)+a^2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*x-a^
2+1)^(1/2))+1/2/b^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*(a/b+x)/(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))
```

maxima [B] time = 1.46, size = 139, normalized size = 2.07

$$-\frac{3\,a^2 \arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3} - \frac{\sqrt{-b^2x^2-2\,abx-a^2+1}\,x}{2\,b^2} + \frac{\left(a^2-1\right)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{2\,b^3} + \frac{3\,\sqrt{-b^2x^2-2\,abx-a^2+1}\,a}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1-(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 1/2*sqr
t(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 + 1/2*(a^2 - 1)*arcsin(-(b^2*x + a*b)
/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 + 3/2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{1-(a+b\,x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1 - (a + b*x)^2)^(1/2),x)
```

```
[Out] int(x^2/(1 - (a + b*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(a+bx-1)(a+bx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1-(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-(a + b*x - 1)*(a + b*x + 1)), x)
```

3.98 $\int \frac{x^2}{\sqrt{1+(a+bx)^2}} \, dx$

Optimal. Leaf size=63

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {371, 743, 641, 215}

$$-\frac{(1-2a^2)\sinh^{-1}(a+bx)}{2b^3}-\frac{3a\sqrt{(a+bx)^2+1}}{2b^3}+\frac{x\sqrt{(a+bx)^2+1}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] (-3*a*Sqrt[1 + (a + b*x)^2])/(2*b^3) + (x*Sqrt[1 + (a + b*x)^2])/(2*b^2) -
((1 - 2*a^2)*ArcSinh[a + b*x])/(2*b^3)
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+(a+bx)^2}}\,dx &= \frac{\text{Subst}\left(\int \frac{(-a+x)^2}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{b^3}\\ &= \frac{x\sqrt{1+(a+bx)^2}}{2b^2}+\frac{\text{Subst}\left(\int \frac{-1+2a^2-3ax}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3}\\ &= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{\left(1-2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, a+bx\right)}{2b^3}\\ &= -\frac{3a\sqrt{1+(a+bx)^2}}{2b^3}+\frac{x\sqrt{1+(a+bx)^2}}{2b^2}-\frac{\left(1-2a^2\right)\sinh^{-1}(a+bx)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.81

$$\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(bx-3a)+\left(2a^2-1\right)\sinh^{-1}(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2] + (-1 + 2*a^2)*ArcSinh[a +
b*x])/(2*b^3)
```

IntegrateAlgebraic [B] time = 0.55, size = 176, normalized size = 2.79

$$\frac{\left(-2a^2\sqrt{b^2}-2a^2b+\sqrt{b^2}+b\right)\log\left(\sqrt{a^2+2abx+b^2x^2+1}-a-\sqrt{b^2}\,x\right)}{4b^4}+\frac{\left(-2a^2\sqrt{b^2}+2a^2b+\sqrt{b^2}-b\right)\log\left(\sqrt{a^2+2abx+b^2x^2+1}+a-\sqrt{b^2}\,x\right)}{4b^4}+\frac{\sqrt{a^2+2abx+b^2x^2+1}\,(bx-3a)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[1 + (a + b*x)^2],x]
```

```
[Out] ((-3*a + b*x)*Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2])/(2*b^3) + ((b - 2*a^2*b +
Sqrt[b^2] - 2*a^2*Sqrt[b^2])*Log[-a - Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x
+ b^2*x^2]])/(4*b^4) + ((-b + 2*a^2*b + Sqrt[b^2] - 2*a^2*Sqrt[b^2])*Log[a
- Sqrt[b^2]*x + Sqrt[1 + a^2 + 2*a*b*x + b^2*x^2]])/(4*b^4)
```

fricas [A] time = 1.88, size = 70, normalized size = 1.11

$$-\frac{\left(2\,a^2-1\right)\log\left(-bx-a+\sqrt{b^2x^2+2\,abx+a^2+1}\right)-\sqrt{b^2x^2+2\,abx+a^2+1}\,(bx-3\,a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 - 1)*log(-b*x - a + sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)) - sqrt(
b^2*x^2 + 2*a*b*x + a^2 + 1)*(b*x - 3*a))/b^3
```

giac [A] time = 0.42, size = 70, normalized size = 1.11

$$\frac{1}{2}\sqrt{(bx+a)^2+1}\left(\frac{x}{b^2}-\frac{3\,a}{b^3}\right)-\frac{\left(2\,a^2-1\right)\log\left(-ab-\left(x|b|-\sqrt{(bx+a)^2+1}\right)|b|\right)}{2\,b^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt((b*x + a)^2 + 1)*(x/b^2 - 3*a/b^3) - 1/2*(2*a^2 - 1)*log(-a*b - (x
*abs(b) - sqrt((b*x + a)^2 + 1))*abs(b))/(b^2*abs(b))
```

maple [B] time = 0.01, size = 146, normalized size = 2.32

$$\frac{a^2 \ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{\sqrt{b^2}\, b^2} + \frac{\sqrt{b^2x^2 + 2abx + a^2 + 1}\, x}{2b^2} - \frac{\ln\left(\frac{b^2x+ab}{\sqrt{b^2}} + \sqrt{b^2x^2 + 2abx + a^2 + 1}\right)}{2\sqrt{b^2}\, b^2} - \frac{3\sqrt{b^2x^2 + 2abx + a^2 + 1}\, a}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(b*x+a)^2)^(1/2),x)
```

```
[Out] 1/2*x/b^2*(b^2*x^2+2*a*b*x+a^2+1)^(1/2)-3/2*a/b^3*(b^2*x^2+2*a*b*x+a^2+1)^(
1/2)+a^2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(b^2
)^(1/2)-1/2/b^2*ln((b^2*x+a*b)/(b^2)^(1/2)+(b^2*x^2+2*a*b*x+a^2+1)^(1/2))/(
b^2)^(1/2)
```

maxima [B] time = 0.59, size = 135, normalized size = 2.14

$$\frac{3\,a^2 \operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3} + \frac{\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,x}{2\,b^2} - \frac{\left(a^2+1\right)\operatorname{arsinh}\left(\frac{2\left(b^2x+ab\right)}{\sqrt{-4\,a^2b^2+4\left(a^2+1\right)b^2}}\right)}{2\,b^3} - \frac{3\,\sqrt{b^2x^2 + 2\,abx + a^2 + 1}\,a}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(b*x+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*a^2*arcsinh(2*(b^2*x + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 + 1
/2*sqrt(b^2*x^2 + 2*a*b*x + a^2 + 1)*x/b^2 - 1/2*(a^2 + 1)*arcsinh(2*(b^2*x
 + a*b)/sqrt(-4*a^2*b^2 + 4*(a^2 + 1)*b^2))/b^3 - 3/2*sqrt(b^2*x^2 + 2*a*b*
x + a^2 + 1)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{(a+b\,x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^2 + 1)^(1/2),x)
```

```
[Out] int(x^2/((a + b*x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2 + 2abx + b^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(b*x+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a**2 + 2*a*b*x + b**2*x**2 + 1), x)
```

3.99 $\int \frac{x^3}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+\right.}{6a^{2/3}b^{4/3}d^4}$$

Rubi [A] time = 0.37, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4}+\frac{\left(3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{4/3}d^4}+\frac{\left(-3\sqrt[3]{a}\,b^{2/3}c^2+a+bc^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4}-\frac{c\log\left(a+b(c+dx)^3\right)}{bd^4}+\frac{x}{bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] x/(b*d^3) + ((a - 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*ArcTan[(a^(1/3) - 2*b^(1/3
)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(4/3)*d^4) - ((a + 3*a^
(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(
4/3)*d^4) + ((a + 3*a^(1/3)*b^(2/3)*c^2 + b*c^3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(4/3)*d^4) - (c*Log[a +
b*(c + d*x)^3])/(b*d^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a+bc^3-3bc^2x+3bcx^2}{b\left(a+bx^3\right)}\right)dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x+3bcx^2}{a+bx^3}\,dx, x, c+dx\right)}{bd^4}$$

$$= \frac{x}{bd^3} - \frac{\text{Subst}\left(\int \frac{a+bc^3-3bc^2x}{a+bx^3}\,dx, x, c+dx\right)}{bd^4} - \frac{(3c)\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{x}{bd^3} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-3\sqrt[3]{a}\,bc^2+2\sqrt[3]{b}\left(a+bc^3\right)\right)+\sqrt[3]{b}\left(-3\sqrt[3]{a}\,bc^2-\sqrt[3]{b}\left(a+bc^3\right)\right)\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\ldots\right.}{3a^{2/3}b^{4/3}d^4}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} - \frac{c\log\left(a+b(c+dx)^3\right)}{bd^4} - \frac{(a-\ldots}{\ldots}$$

$$= \frac{x}{bd^3} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{4/3}d^4} + \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(a^{2\ldots}\right.}{6a\ldots}$$

$$= \frac{x}{bd^3} + \frac{\left(a-3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}d^4} - \frac{\left(a+3\sqrt[3]{a}\,b^{2/3}c^2+bc^3\right)\log\left(\sqrt[3]{a}+\ldots\right.}{3a^{2/3}b^{4/3}d^4}$$

Mathematica [C] time = 0.06, size = 132, normalized size = 0.56

$$-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&, \frac{3\#1^2bcd^2\log(x-\#1)+a\log(x-\#1)+bc^3\log(x-\#1)+3\#1bc^2d\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3bdx}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] -1/3*(-3*b*d*x + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*
#1^3 & , (a*Log[x - #1] + b*c^3*Log[x - #1] + 3*b*c^2*d*Log[x - #1]*#1 + 3*
b*c*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(b^2*d^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^3), x]
```

fricas [C] time = 47.20, size = 6315, normalized size = 26.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8)
)/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 +
 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2
*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(
b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4*log(-3/4*((-I*s
qrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^
12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 +
3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + b^3*c^10 - 9*a*b
^2*c^7 - 12*a^2*b*c^4 + 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(
3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12)
- 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^
2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*
c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a
^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*d^4 - 2*a^3*c + (b^3*c^9 - 24*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)*d*x) - 12*d*x - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) +
(b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(
a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^1
2) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3
) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^1
2) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54
*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(
b*d^4))*b*d^4 - 3*sqrt(1/3)*b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
 + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*
c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*
d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(
1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*
d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1
/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*
c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2
*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12
) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*
(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*s
qrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9
- 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*
b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8)) - 18*c)*log(3/4*((-I*sqrt(3) +
1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2
*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c
^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3
)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^
4*d^12))^(1/3) + 6*c/(b*d^4))^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7
 + 21*a^2*b*c^4 - 1/2*(a*b^3*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(3) + 1
)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*
(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/
(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4
*d^12))^(1/3) + 6*c/(b*d^4))*d^4 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*
```

```
a^2*b*c^3 + a^3)*d*x + 3/4*sqrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8)
+ (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c
/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d
^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1
/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d
^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/
54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c
/(b*d^4))*a^2*b^3*c^2*d^8 + 2*(a*b^3*c^6 - 7*a^2*b^2*c^3 + a^3*b)*d^4)*sqrt
(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^
3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 +
 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^
12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 +
3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(
3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12)
- 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^
2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*
c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a
^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*
d^8))) - (((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8
))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/
(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2
*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*b*d^4 + 3*sqrt(1/3)*
b*d^4*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*
d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c
^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a
*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c
^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*
b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*
((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(
b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*
a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12)
 - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3
 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^
2)/(a*b^2*d^8)) - 18*c)*log(3/4*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5
 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*
d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1
/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*
(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1
/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*
c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4)
)^2*a^2*b^3*c^2*d^8 + 2*b^3*c^10 - 63*a*b^2*c^7 + 21*a^2*b*c^4 - 1/2*(a*b^3
*c^6 + 20*a^2*b^2*c^3 + a^3*b)*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5
- 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d
^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/
54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(
I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/
54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c
^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))
*d^4 + 5*a^3*c + 2*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*d*x - 3/4*s
```

```
qrt(1/3)*(3*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d
^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^
9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*
b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^
3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 +
 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b*d^4))*a^2*b^3*c^2*d^8 +
2*(a*b^3*c^6 - 7*a^2*b^2*c^3 + a^3*b)*d^4)*sqrt(-(((-I*sqrt(3) + 1)*(3*c^2/
(b^2*d^8) + (b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 -
2*a*c^2)*c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/
(a^2*b^4*d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4
*d^12))^(1/3) + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*
c/(a*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*
d^12) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(
1/3) + 6*c/(b*d^4))^2*a*b^2*d^8 - 12*((-I*sqrt(3) + 1)*(3*c^2/(b^2*d^8) + (
b*c^5 - 2*a*c^2)/(a*b^2*d^8))/(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a
*b^3*d^12) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12
) + 1/54*(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3)
 + 3*(I*sqrt(3) + 1)*(-c^3/(b^3*d^12) - 1/2*(b*c^5 - 2*a*c^2)*c/(a*b^3*d^12
) - 1/54*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12) + 1/54*
(b^3*c^9 - 24*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)/(a^2*b^4*d^12))^(1/3) + 6*c/(b
*d^4))*a*b*c*d^4 - 48*b*c^5 - 12*a*c^2)/(a*b^2*d^8))))/(b*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.01, size = 108, normalized size = 0.46

$$\frac{x}{b\,d^3}+\frac{\left(-3\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2 bc\,d^2-3\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)b\,c^2d-b\,c^3-a\right)\ln\left(-\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+x\right)}{3b^2d^4\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2cd\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] x/b/d^3+1/3/b^2/d^4*sum((-3*_R^2*b*c*d^2-3*_R*b*c^2*d-b*c^3-a)/(_R^2*d^2+2*
_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^
3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x}{bd^3}-\frac{\int \frac{3\,bcd^2x^2+3\,bc^2dx+bc^3+a}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] x/(b*d^3) - integrate((3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a), x)/(b*d^3)
```

mupad [B] time = 2.46, size = 374, normalized size = 1.60

$$\left(\sum_{k=1}^{3}\ln\left(\frac{3\left(b\,c^5+a\,c^2\right)}{d^2}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\left(\frac{3\left(b^2\,c^4\,d^4-5\,a\,b\,c\,d^4\right)}{d^2}+\frac{3\,x\left(b^2\,c^3\,d^4+a\,b\,d^4\right)}{d}-\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)a\,b^2\,d^6\,9\right)-\frac{3\,x\left(a\,c-2\,b\,c^4\right)}{d}\right)\mathrm{root}\left(27\,a^2\,b^4\,d^{12}\,z^3+81\,a^2\,b^3\,c\,d^8\,z^2+54\,a^2\,b^2\,c^2\,d^4\,z-27\,a\,b^3\,c^5\,d^4\,z+3\,a\,b^2\,c^6+3\,a^2\,b\,c^3+b^3\,c^9+a^3,z,k\right)\right)+\frac{x}{b\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log((3*(a*c^2 + b*c^5))/d^2 - root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*
c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + b^3*c^9 + a^3, z, k)*((3*(b^2*c^4*d^4 - 5*a*b*c*d^4))/d^2 + (3*x*(
b^2*c^3*d^4 + a*b*d^4))/d - 9*root(27*a^2*b^4*d^12*z^3 + 81*a^2*b^3*c*d^8*z
^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4*z + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ b^3*c^9 + a^3, z, k)*a*b^2*d^6) - (3*x*(a*c - 2*b*c^4))/d)*root(27*a^2*b^
4*d^12*z^3 + 81*a^2*b^3*c*d^8*z^2 + 54*a^2*b^2*c^2*d^4*z - 27*a*b^3*c^5*d^4
*z + 3*a*b^2*c^6 + 3*a^2*b*c^3 + b^3*c^9 + a^3, z, k), k, 1, 3) + x/(b*d^3)
```

sympy [A] time = 2.90, size = 238, normalized size = 1.02

$$\mathrm{RootSum}\left(27t^3a^2b^4d^{12} + 81t^2a^2b^3cd^8 + t\left(54a^2b^2c^2d^4 - 27ab^3c^5d^4\right) + a^3 + 3a^2bc^3 + 3ab^2c^6 + b^3c^9, \left(t \mapsto t\log\left(x + \frac{-27t^2a^2b^3c^2d^8 - 3ta^3bd^4 - 60ta^2b^2c^3d^4 - 3tab^3c^6d^4 - 2a^3c - 12a^2bc^4 - 9ab^2c^7 + b^3c^{10}}{a^3d + 3a^2bc^3d - 24ab^2c^6d + b^3c^9d}\right)\right)\right) + \frac{x}{bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4*d**12 + 81*_t**2*a**2*b**3*c*d**8 + _t*(54*a**2*
b**2*c**2*d**4 - 27*a*b**3*c**5*d**4) + a**3 + 3*a**2*b*c**3 + 3*a*b**2*c**
6 + b**3*c**9, Lambda(_t, _t*log(x + (-27*_t**2*a**2*b**3*c**2*d**8 - 3*_t*
a**3*b*d**4 - 60*_t*a**2*b**2*c**3*d**4 - 3*_t*a*b**3*c**6*d**4 - 2*a**3*c
- 12*a**2*b*c**4 - 9*a*b**2*c**7 + b**3*c**10)/(a**3*d + 3*a**2*b*c**3*d -
24*a*b**2*c**6*d + b**3*c**9*d)))) + x/(b*d**3)
```

3.100 $\int \frac{x^2}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=210

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\right.}{}$$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {371, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^3}-\frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^3}+\frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3}+\frac{\log\left(a+b(c+dx)^3\right)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] (c*(2*a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^3) + (c*(2*a^(1/3) + b^(1/3)*c)*Log[a
^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^3) - (c*(2*a^(1/3) + b^(1
/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a
^(2/3)*b^(2/3)*d^3) + Log[a + b*(c + d*x)^3]/(3*b*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{d^3} + \frac{\text{Subst}\left(\int \frac{c^2-2cx}{a+bx^3}\,dx, x, c+dx\right)}{d^3}\\
&= \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{a}c+2\sqrt[3]{b}c^2\right)+\sqrt[3]{b}\left(-2\sqrt[3]{a}c-\sqrt[3]{b}c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}\,d^3} + \cdots\\
&= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} + \frac{\log\left(a+b(c+dx)^3\right)}{3bd^3} - \frac{\left(c\left(\frac{2}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\right)\text{Subs}\cdots}{\cdots}\\
&= \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)\cdots\right)}{6a^{2/3}b^{2/3}d^3}\cdots\\
&= \frac{c\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^3} + \frac{c\left(2\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^3} - \frac{c\left(2\sqrt[3]{a}\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 81, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&, \frac{\#1^2\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b(c + dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^3), x]
```

fricas [C] time = 4.04, size = 4759, normalized size = 22.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^
2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/
(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*
(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d
^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b
*d^3))*b*d^3*log(-1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2
*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*
b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) +
 (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) - 2/(b*d^3))^2*a^2*b^2*d^6 + b^2*c^6 - a*b*c^3 - 1/2*(a*b^2*c^3 + 4*
a^2*b)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*
d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b
^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I
*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9
) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d
^3))*d^3 + (b^2*c^5 - 8*a*b*c^2)*d*x - 2*a^2) - ((2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2
*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/
(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*
b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 - 3*sqrt(1/3)*b*d^3*sq
rt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^
6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3
*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*s
qrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9)
+ 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3
))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^
6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3
*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a
)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1
/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) + 6)*log(1/2*(2*(1/
2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3
 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^
2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)
*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))^2*a^2*b^
2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
```

3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*d^3 + 2*(b^2*c^5 -
8*a*b*c^2)*d*x + 2*a^2 + 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((
2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) +
3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^
2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2
d^9) + 3(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 +
a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b^2*c^3 - 2*a^2*b)*
d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a
*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)
/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/
3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*
c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d
^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6))) - ((2*(1/2)
^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 -
8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*(
(b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9)
+ (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*b*d^3 + 3*
sqrt(1/3)*b*d^3*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^
2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a
*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3)
+ (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9)
)^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*
c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2
*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^
3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9
) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)
/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*d^6)) +
6)*log(1/2*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/
(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) +
2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/
3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^
3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2
/(b*d^3))^2*a^2*b^2*d^6 + 2*b^2*c^6 - 23*a*b*c^3 + 1/2*(a*b^2*c^3 + 4*a^2*b
)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))
/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^
9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt
(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2
/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*
d^3 + 2*(b^2*c^5 - 8*a*b*c^2)*d*x + 2*a^2 - 3/2*sqrt(1/3)*((2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^
3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*
a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 -
8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*
c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a^2*b^2*d^6 - (a*b
^2*c^3 - 2*a^2*b)*d^3)*sqrt(-((2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*((2*b*c^3 - a
)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(2*b*c^3
- a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b^3*d^9))
^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9) + 3*(
2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/(a^2*b

```
^3*d^9))^(1/3) - 2/(b*d^3))^2*a*b^2*d^6 + 4*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)
*((2*b*c^3 - a)/(a*b^2*d^6) + 1/(b^2*d^6))/((b*c^3 - 8*a)*c^3/(a^2*b^2*d^9)
 + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3 + a^2)/
(a^2*b^3*d^9))^(1/3) + (1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 - 8*a)*c^3/(a^2*
b^2*d^9) + 3*(2*b*c^3 - a)/(a*b^3*d^9) + 2/(b^3*d^9) + (b^2*c^6 + 2*a*b*c^3
 + a^2)/(a^2*b^3*d^9))^(1/3) - 2/(b*d^3))*a*b*d^3 - 32*b*c^3 + 4*a)/(a*b^2*
d^6))))/(b*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.00, size = 74, normalized size = 0.35

$$\frac{\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2 \ln\left(-\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+x\right)}{3bd\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2cd\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R^2/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_
Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^3*b + a), x)
```

mupad [B] time = 2.30, size = 437, normalized size = 2.08

$$\sum_{k=1}^{3} \ln\left(a+b\,c^3-\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\right)\,a\,b\,d^3\,6+3\,b\,c^2\,d\,x+\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\right)^2\,a\,b^2\,d^6\,9+\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\right)\,b^2\,c^3\,d^3\,3+\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\right)\,b^2\,c^2\,d^4\,x\,3\right)\,\mathrm{root}\left(27\,a^2\,b^3\,d^9\,z^3-27\,a^2\,b^2\,d^6\,z^2-18\,a\,b^2\,c^3\,d^3\,z+9\,a^2\,b\,d^3\,z-2\,a\,b\,c^3-b^2\,c^6-a^2,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(a + b*c^3 - 6*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*
a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*a*b*d^3
+ 3*b*c^2*d*x + 9*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c
^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)^2*a*b^2*d^6 + 3
*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*
b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2, z, k)*b^2*c^3*d^3 + 3*root(27*a^2*b^3*
d^9*z^3 - 27*a^2*b^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c
^3 - b^2*c^6 - a^2, z, k)*b^2*c^2*d^4*x)*root(27*a^2*b^3*d^9*z^3 - 27*a^2*b
^2*d^6*z^2 - 18*a*b^2*c^3*d^3*z + 9*a^2*b*d^3*z - 2*a*b*c^3 - b^2*c^6 - a^2
, z, k), k, 1, 3)
```

sympy [A] time = 0.99, size = 158, normalized size = 0.75

$$\mathrm{RootSum}\left(27t^3a^2b^3d^9-27t^2a^2b^2d^6+t\left(9a^2bd^3-18ab^2c^3d^3\right)-a^2-2abc^3-b^2c^6,\left(t\mapsto t\log\left(x+\frac{18t^2a^2b^2d^6-12ta^2bd^3-3tab^2c^3d^3+2a^2+abc^3-b^2c^6}{8abc^2d-b^2c^5d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**3*d**9 - 27*_t**2*a**2*b**2*d**6 + _t*(9*a**2*b*d*
*3 - 18*a*b**2*c**3*d**3) - a**2 - 2*a*b*c**3 - b**2*c**6, Lambda(_t, _t*lo
g(x + (18*_t**2*a**2*b**2*d**6 - 12*_t*a**2*b*d**3 - 3*_t*a*b**2*c**3*d**3
+ 2*a**2 + a*b*c**3 - b**2*c**6)/(8*a*b*c**2*d - b**2*c**5*d))))
```

3.101 $\int \frac{x}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=180

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\mathrm{t}}{\sqrt{3}}$$

Rubi [A] time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {371, 1860, 31, 634, 617, 204, 628}

$$-\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}b^{2/3}d^2}+\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}b^{2/3}d^2}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^3),x]
```

```
[Out] -(((a^(1/3) - b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(2/3)*d^2)) - ((a^(1/3) + b^(1/3)*c)*Log[a^(1/3
) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*b^(2/3)*d^2) + ((a^(1/3) + b^(1/3)*c)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^
(2/3)*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^3}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^3}\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}-2\sqrt[3]{b}\,c\right)+\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}\sqrt[3]{b}\,d^2} - \frac{\left(\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+c\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x\right.}{3a^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\frac{1}{\sqrt[3]{b}}-\frac{c}{\sqrt[3]{a}}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{2d^2}$$

$$= -\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2}\right.}{6a^{2/3}b^{2/3}d^2}$$

$$= -\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{2/3}d^2} - \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}b^{2/3}d^2} + \frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Mathematica [C] time = 0.02, size = 79, normalized size = 0.44

$$\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^3),x]
```

```
[Out] RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (Log[x -
 #1]*#1)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ]/(3*b*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*(c + d*x)^3), x]
```

fricas [C] time = 3.83, size = 1950, normalized size = 10.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)
/(a^2*b^2*d^6))^(1/3) - 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3
 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3)
 + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2
*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*
(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*
log(1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3
- a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/
(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(
9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*
b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d
^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*
c^3 - a)*d*x - 4*a*c + 1/12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a
)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1
)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6
))^(1/3)))*a^2*b*d^4 + 6*a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c
^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(
3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b
^2*d^6))^(1/3)))^2*a*b*d^4 - 144*c)/(a*b*d^4))) + 1/36*(9*(I*sqrt(3) + 1)*(
-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + 3
*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/5
4*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*
c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^4
- 144*c)/(a*b*d^4)) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b
^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*log(1/36*(9*(I*sqrt(3) +
1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)
 + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 - 1/6*(9*(I*sqrt(3) + 1)*(-1/54*
(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*s
qrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + 2*b*c^4 + 2*(b*c^3 - a)*d*x - 4*a*c - 1/
12*sqrt(1/3)*((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b
*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3
+ a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a^2*b*d^4 + 6*
a*b*c^2*d^2)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1
/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(
b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a*b*d^
4 - 144*c)/(a*b*d^4))) - 1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^
4*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3))
)*log(-1/36*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c
^3 - a)/(a^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 +
a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))^2*a^2*b*d^4 + 1/
6*(9*(I*sqrt(3) + 1)*(-1/54*(b*c^3 + a)/(a^2*b^2*d^6) + 1/54*(b*c^3 - a)/(a
^2*b^2*d^6))^(1/3) + c*(-I*sqrt(3) + 1)/(a*b*d^4*(-1/54*(b*c^3 + a)/(a^2*b^
2*d^6) + 1/54*(b*c^3 - a)/(a^2*b^2*d^6))^(1/3)))*a*b*c^2*d^2 + b*c^4 + (b*c
^3 - a)*d*x + a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^3 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

maple [C] time = 0.00, size = 72, normalized size = 0.40

$$\frac{\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)\ln\left(-\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+x\right)}{3bd\left(d^2\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)^2+2cd\,\mathrm{RootOf}\left(b\,d^3\_Z^3+3b\,d^2c\,\_Z^2+3bd\,c^2\_Z+b\,c^3+a\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(_R/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^
2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^3b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^3*b + a), x)
```

mupad [B] time = 0.26, size = 145, normalized size = 0.81

$$\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\left(3\,b^2\,c^2\,d^4-\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)\,a\,b^2\,d^6\,9+3\,b^2\,c\,d^5\,x\right)+b\,d^3\,x\right)\,\mathrm{root}\left(27\,a^2\,b^2\,d^6\,z^3-9\,a\,b\,c\,d^2\,z+b\,c^3+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^3),x)
```

```
[Out] symsum(log(b*d^3*x - root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a, z
, k)*(3*b^2*c^2*d^4 - 9*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z + b*c^3 + a
, z, k)*a*b^2*d^6 + 3*b^2*c*d^5*x))*root(27*a^2*b^2*d^6*z^3 - 9*a*b*c*d^2*z
 + b*c^3 + a, z, k), k, 1, 3)
```

sympy [A] time = 0.70, size = 83, normalized size = 0.46

$$\mathrm{RootSum}\left(27t^3a^2b^2d^6-9tabcd^2+a+bc^3,\left(t\mapsto t\log\left(x+\frac{9t^2a^2bd^4+3tabc^2d^2-ac-bc^4}{ad-bc^3d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**2*d**6 - 9*_t*a*b*c*d**2 + a + b*c**3, Lambda(_t,
_t*log(x + (9*_t**2*a**2*b*d**4 + 3*_t*a*b*c**2*d**2 - a*c - b*c**4)/(a*d -
 b*c**3*d))))
```

3.102 $\int \frac{1}{a+b(c+dx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

Rubi [A] time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*(c + d*x)]/(3*a^(2/3)*b^(1/3)*d) - Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2]/(6*a^(2/3)*b^(
1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{a+b(c+dx)^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, c+dx\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, c+dx\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{3a^{2/3}d} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c+dx\right)}{2\sqrt[3]{a}\,d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}}{a^{2/3}-\sqrt[3]{a}}\right.}{6} \\
&= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{-3-}\right.}{} \\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+\right.}{6a^{2/3}\sqrt[3]{b}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.83

$$
\frac{-\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)+2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{b}\,(c+dx)-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{6a^{2/3}\sqrt[3]{b}\,d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a^(1/3) + 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))] + 2*L
og[a^(1/3) + b^(1/3)*(c + d*x)] - Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) +
 b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*b^(1/3)*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{a+b(c+dx)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^3)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^3)^(-1), x]
```

fricas [A] time = 1.58, size = 442, normalized size = 3.16

$$
\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2abd^3x^3+6abcd^2x^2+6abc^2dx+2abc^3-a^2+3\sqrt{\frac{1}{3}}\left(2abd^2x^2+4abcdx+2abc^2+(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}(adx+ac)}{bd^3x^3+3bcd^2x^2+3bc^2dx+bc^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abd^2x^2+2abcdx+abc^2-(a^2b)^{\frac{2}{3}}(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2(a^2b)^{\frac{2}{3}}\log\left(abdx+abc+(a^2b)^{\frac{2}{3}}\right)}{6a^2bd}, \frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(a^2b)^{\frac{2}{3}}(dx+c)-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(a^2b)^{\frac{2}{3}}\log\left(abd^2x^2+2abcdx+abc^2-(a^2b)^{\frac{2}{3}}(dx+c)+(a^2b)^{\frac{1}{3}}a\right)+2(a^2b)^{\frac{2}{3}}\log\left(abdx+abc+(a^2b)^{\frac{2}{3}}\right)}{6a^2bd}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*d^3*x^3 + 6*a*b*c*d
^2*x^2 + 6*a*b*c^2*d*x + 2*a*b*c^3 - a^2 + 3*sqrt(1/3)*(2*a*b*d^2*x^2 + 4*a
*b*c*d*x + 2*a*b*c^2 + (a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)*sqrt(-(a^
2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*(a*d*x + a*c))/(b*d^3*x^3 + 3*b*c*d^2*x^2 +
 3*b*c^2*d*x + b*c^3 + a)) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(
a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2
*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*(d*x + c) - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*log(a*b*d^2*x^2 + 2*a*b*c*d*x +
 a*b*c^2 - (a^2*b)^(2/3)*(d*x + c) + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*d*x + a*b*c + (a^2*b)^(2/3)))/(a^2*b*d)]
```

giac [A] time = 0.45, size = 160, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\arctan\left(-\frac{bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}}{\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^{2}\right)^{\frac{1}{3}}}\right)-\frac{1}{6}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\log\left(4\left(\sqrt{3}\,bdx+\sqrt{3}\,bc-\sqrt{3}\left(ab^{2}\right)^{\frac{1}{3}}\right)^{2}+4\left(bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}\right)^{2}\right)+\frac{1}{3}\left(\frac{1}{a^{2}bd^{3}}\right)^{\frac{1}{3}}\log\left(\left|bdx+bc+\left(ab^{2}\right)^{\frac{1}{3}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(1/(a^2*b*d^3))^(1/3)*arctan(-(b*d*x + b*c + (a*b^2)^(1/3))/(sq
rt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))) - 1/6*(1/(a^2*b*d^3))^(
1/3)*log(4*(sqrt(3)*b*d*x + sqrt(3)*b*c - sqrt(3)*(a*b^2)^(1/3))^2 + 4*(b*d
*x + b*c + (a*b^2)^(1/3))^2) + 1/3*(1/(a^2*b*d^3))^(1/3)*log(abs(b*d*x + b*
c + (a*b^2)^(1/3)))
```

maple [C] time = 0.00, size = 71, normalized size = 0.51

$$\frac{\ln\left(-\mathrm{RootOf}\left(b\,d^{3}\_Z^{3}+3b\,d^{2}c\,\_Z^{2}+3bd\,c^{2}\_Z+b\,c^{3}+a\right)+x\right)}{3bd\left(d^{2}\,\mathrm{RootOf}\left(b\,d^{3}\_Z^{3}+3b\,d^{2}c\,\_Z^{2}+3bd\,c^{2}\_Z+b\,c^{3}+a\right)^{2}+2cd\,\mathrm{RootOf}\left(b\,d^{3}\_Z^{3}+3b\,d^{2}c\,\_Z^{2}+3bd\,c^{2}\_Z+b\,c^{3}+a\right)+c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3/b/d*sum(1/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2
*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^{3}b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^3*b + a), x)
```

mupad [B] time = 2.31, size = 144, normalized size = 1.03

$$\frac{\ln\left(b^{1/3}\,c+a^{1/3}+b^{1/3}\,d\,x\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^{2}\,c\,d^{5}+3\,b^{2}\,d^{6}\,x+\frac{3\,a^{1/3}\,b^{5/3}\,d^{5}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^{2}\,c\,d^{5}+3\,b^{2}\,d^{6}\,x-\frac{3\,a^{1/3}\,b^{5/3}\,d^{5}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*c + a^(1/3) + b^(1/3)*d*x)/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*c
*d^5 + 3*b^2*d^6*x + (3*a^(1/3)*b^(5/3)*d^5*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1
i - 1))/(6*a^(2/3)*b^(1/3)*d) - (log(3*b^2*c*d^5 + 3*b^2*d^6*x - (3*a^(1/3)
*b^(5/3)*d^5*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

sympy [A] time = 0.26, size = 26, normalized size = 0.19

$$\frac{\text{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(x+\frac{3ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**3),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(x + (3*_t*a + c)/d)))/d
```

3.103 $\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=224

$$-\frac{\left(2\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}+\frac{\sqrt[3]{b}c\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}c+b^{2/3}c^2\right)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c\cdots\right.}{3a^{2/3}\left(\sqrt[3]{a}+\cdots\right.}$$

Rubi [A] time = 0.48, antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)}+\frac{\sqrt[3]{b}c\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\left(a+bc^3\right)}-\frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)}+\frac{\log(x)}{a+bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] (b^(1/3)*c*(a^(1/3) + b^(1/3)*c)*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sq
rt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)) + Log[x]/(a + b*c^3) + (b^(1
/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(1/3) + b^(1/3)*(c + d*x)])/(3*a^(2/3)*(a
 + b*c^3)) - (b^(1/3)*c*(a^(1/3) - b^(1/3)*c)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)) - Log[a + b*(c +
 d*x)^3]/(3*(a + b*c^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx &= \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^3\right)}\,dx, x, c+dx\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^3\right)(c-x)} - \frac{b\left(c^2+cx+x^2\right)}{\left(a+bc^3\right)\left(a+bx^3\right)}\right)dx, x, c+dx\right)\\
&= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx+x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}\\
&= \frac{\log(x)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3} - \frac{b\,\text{Subst}\left(\int \frac{c^2+cx}{a+bx^3}\,dx, x, c+dx\right)}{a+bc^3}\\
&= \frac{\log(x)}{a+bc^3} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{b^{2/3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(\sqrt[3]{a}\,c+2\sqrt[3]{b}\,c^2\right)+\sqrt[3]{b}\left(\sqrt[3]{a}\,c-\sqrt[3]{b}\,c^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, c \cdots}{3a^{2/3}\left(a+bc^3\right)}\\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\log\left(a+b(c+dx)^3\right)}{3\left(a+bc^3\right)} - \frac{\left(\sqrt[3]{b}\,c\left(\cdots\right.\right.}{\cdots}\\
&= \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)} - \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\cdots\right.}{6a^{2/3}\left(\cdots\right.}\\
&= \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)} + \frac{\log(x)}{a+bc^3} + \frac{\sqrt[3]{b}\,c\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c\cdots\right.}{3a^{2/3}\left(a+bc^3\right)}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 119, normalized size = 0.53

$$
-\frac{\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{\#1^2d^2\log(x-\#1)+3c^2\log(x-\#1)+3\#1cd\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3\log(x)}{3\left(a+bc^3\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/3*(-3*Log[x] + RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^3
*#1^3 & , (3*c^2*Log[x - #1] + 3*c*d*Log[x - #1]*#1 + d^2*Log[x - #1]*#1^2)
/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(a + b*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{x\left(a+b(c+dx)^3\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^3)), x]
```

fricas [C] time = 3.97, size = 4370, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(b*c^3 + a)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))*log(b*c^2*d*x + b*c^3
+ 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2)
- 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a
*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3
) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2
)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 - 1/2*(a*b*c^3 -
 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^
2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + a) - ((b*c^3 + a)*(2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)
^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3
+ a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - 2/(b*c^3 + a)) + 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 +
 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/
(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c
^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) +
1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b
*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*
(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^
3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a
)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 +
a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^
3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) +
6)*log(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqr
t(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2)
- 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^
(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c
^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b
*c^3 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*
b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1
/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((
a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 3/4
*sqrt(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(
1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*
c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/
2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) +
 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))
+ 2*a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(
-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a
^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3)
- 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(
a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^
(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/
((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4
*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) - ((b*c^3 + a)*(2*(1/2)^(2/3)*(-I
*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a
^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)
```

```
^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2
*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) -
2/(b*c^3 + a)) - 3*sqrt(1/3)*(b*c^3 + a)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a
^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c
^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 +
 a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(
b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3
 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(
1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((
b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) -
 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2
*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 +
a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) + 6)*l
og(2*b*c^2*d*x + 2*b*c^3 - 1/4*(a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3)
 + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/
(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3
) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 +
 a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3
 + a))^2 + 1/2*(a*b*c^3 - 2*a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*c^
3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3
) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3)*
(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*
c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) - 3/4*sqr
t(1/3)*(2*a*b*c^3 + (a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a
*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3
+ a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(
1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/(
(a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 2*
a^2)*sqrt(-(16*b*c^3 + (a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(2*(1/2)^(2/3)*(-I*s
qrt(3) + 1)*(1/(a*b*c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2
) - 1/(a^2*b*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3
)^(1/3) - (1/2)^(1/3)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b
*c^3 + a^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/
(b*c^3 + a))^2 + 4*(a*b*c^3 + a^2)*(2*(1/2)^(2/3)*(-I*sqrt(3) + 1)*(1/(a*b*
c^3 + a^2) - 1/(b*c^3 + a)^2)/(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a
^3) + 3/((a*b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - (1/2)^(1/3
)*(I*sqrt(3) + 1)*(b*c^3/((b*c^3 + a)^2*a^2) - 1/(a^2*b*c^3 + a^3) + 3/((a*
b*c^3 + a^2)*(b*c^3 + a)) - 2/(b*c^3 + a)^3)^(1/3) - 2/(b*c^3 + a)) + 4*a)/
(a*b^2*c^6 + 2*a^2*b*c^3 + a^3)) - a) + 12*log(x))/(b*c^3 + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x), x)
```

maple [C] time = 0.01, size = 105, normalized size = 0.47

$$\frac{\ln(x)}{b c^3+a}-\frac{\left(d^2 \operatorname{RootOf}\left(b d^3\_Z^3+3b d^2 c\_Z^2+3bd c^2\_Z+b c^3+a\right)^2+3cd \operatorname{RootOf}\left(b d^3\_Z^3+3b d^2 c\_Z^2+3bd c^2\_Z+b c^3+a\right)+3c^2\right)\ln\left(-\operatorname{RootOf}\left(b d^3\_Z^3+3b d^2 c\_Z^2+3bd c^2\_Z+b c^3+a\right)+x\right)}{3\left(b c^3+a\right)\left(d^2 \operatorname{RootOf}\left(b d^3\_Z^3+3b d^2 c\_Z^2+3bd c^2\_Z+b c^3+a\right)^2+2cd \operatorname{RootOf}\left(b d^3\_Z^3+3b d^2 c\_Z^2+3bd c^2\_Z+b c^3+a\right)+c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^3),x)
```

```
[Out] -1/3/(b*c^3+a)*sum((_R^2*d^2+3*_R*c*d+3*c^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R
+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))+ln(x)/(b*c^3
+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\int \frac{d^{2}x^{2}+3\,cdx+3\,c^{2}}{bd^{3}x^{3}+3\,bcd^{2}x^{2}+3\,bc^{2}dx+bc^{3}+a}\,dx}{bc^{3}+a}+\frac{\log\left(x\right)}{bc^{3}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^2*x^2 + 3*c*d*x + 3*c^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3 + a), x)/(b*c^3 + a) + log(x)/(b*c^3 + a)
```

mupad [B] time = 0.12, size = 553, normalized size = 2.47

$$\frac{\ln(x)}{b\,c^3+a}+\left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,b^4\,c^4\,d^8\,3-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,c\,d^8\,3-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\,b^3\,d^9\,x\,4-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,a\,b^3\,c\,d^8\,6-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,a\,b^3\,d^9\,x\,24+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a^2\,b^3\,c\,d^8\,9+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a\,b^4\,c^4\,d^8\,9-\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a^2\,b^3\,d^9\,x\,36+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^2\,b^4\,c^3\,d^9\,x\,3+\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)^3\,a\,b^4\,c^3\,d^9\,x\,18\right)\,\mathrm{root}\left(27\,a^2\,b\,c^3\,z^3+27\,a^3\,z^3+27\,a^2\,z^2+9\,a\,z+1,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^3)),x)
```

```
[Out] log(x)/(a + b*c^3) + symsum(log(3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a
^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^4*d^8 - 3*root(27*a^2*b*c^3*z^3 + 27*a^3*
z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*c*d^8 - 4*root(27*a^2*b*c^3*z^3 + 2
7*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)*b^3*d^9*x - 6*root(27*a^2*b*c^3*z
^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*c*d^8 - 24*root(27*
a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^2*a*b^3*d^9*x +
9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a^2*
b^3*c*d^8 + 9*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1,
z, k)^3*a*b^4*c^4*d^8 - 36*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2
+ 9*a*z + 1, z, k)^3*a^2*b^3*d^9*x + 3*root(27*a^2*b*c^3*z^3 + 27*a^3*z^3 +
 27*a^2*z^2 + 9*a*z + 1, z, k)^2*b^4*c^3*d^9*x + 18*root(27*a^2*b*c^3*z^3 +
 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k)^3*a*b^4*c^3*d^9*x)*root(27*a^2*
b*c^3*z^3 + 27*a^3*z^3 + 27*a^2*z^2 + 9*a*z + 1, z, k), k, 1, 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.104 $\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=314

$$-\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}\,c\left(2a-bc^3\right)\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}\right.}{3a^{2}}$$

Rubi [A] time = 0.54, antiderivative size = 312, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}d\left(-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}+2ac-bc^4\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}\left(a-2bc^3\right)-\sqrt[3]{b}\,c\left(2a-bc^3\right)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}+\frac{\sqrt[3]{b}\,d\left(\sqrt[3]{a}-\sqrt[3]{b}\,c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^2}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}-\frac{1}{x\left(a+bc^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] -(1/((a + b*c^3)*x)) + (b^(1/3)*(a^(1/3) - b^(1/3)*c)*(a^(1/3) + b^(1/3)*c)
^3*d*ArcTan[(a^(1/3) - 2*b^(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(2/3)*(a + b*c^3)^2) - (3*b*c^2*d*Log[x])/(a + b*c^3)^2 + (b^(1/3)*(a^(1/3)
*(a - 2*b*c^3) - b^(1/3)*c*(2*a - b*c^3))*d*Log[a^(1/3) + b^(1/3)*(c + d*x)
])/(3*a^(2/3)*(a + b*c^3)^2) + (b^(2/3)*(2*a*c - b*c^4 - (a^(1/3)*(a - 2*b*
c^3))/b^(1/3))*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x
)^2])/(6*a^(2/3)*(a + b*c^3)^2) + (b*c^2*d*Log[a + b*(c + d*x)^3])/(a + b*c
^3)^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d\,\text{Subst}\left(\int\left(\frac{1}{\left(a+bc^3\right)(c-x)^2}+\frac{3bc^2}{\left(a+bc^3\right)^2(c-x)}+\frac{b\left(-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x\right.}{\left(a+bc^3\right)^2\left(a+bx^3\right)}\right.\right.$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)-\left(a-2bc^3\right)x+3bc^2x^2}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{(bd)\,\text{Subst}\left(\int \frac{-c\left(2a-bc^3\right)+\left(-a+2bc^3\right)x}{a+bx^3}\,dx, x, c+dx\right)}{\left(a+bc^3\right)^2}+\frac{(3b}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}+\frac{bc^2d\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^2}+\frac{\left(b^{2/3}d\right)\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2\sqrt[3]{b}\right.}{}\right.}{}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-\frac{b^{2/3}\left(2ac-bc^4-\frac{\sqrt[3]{a}\left(a-2bc^3\right)}{\sqrt[3]{b}}\right)d\log\left(\sqrt[3]{a}+\sqrt[3]{b}(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^2}$$

$$= -\frac{1}{\left(a+bc^3\right)x}+\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{b}c\right)\left(\sqrt[3]{a}+\sqrt[3]{b}c\right)^3 d\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^2}-\frac{3bc^2d\log(x)}{\left(a+bc^3\right)^2}-$$

Mathematica [C] time = 0.10, size = 173, normalized size = 0.55

$$\frac{dx\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{3\#1^2bc^2d^2\log(x-\#1)-3ac\log(x-\#1)-\#1ad\log(x-\#1)+6bc^4\log(x-\#1)+8\#1bc^3d\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]-3\left(a+bc^3+3bc^2dx\log(x)\right)}{3x\left(a+bc^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] (-3*(a + b*c^3 + 3*b*c^2*d*x*Log[x]) + d*x*RootSum[a + b*c^3 + 3*b*c^2*d*#1
 + 3*b*c*d^2*#1^2 + b*d^3*#1^3 & , (-3*a*c*Log[x - #1] + 6*b*c^4*Log[x - #1
] - a*d*Log[x - #1]*#1 + 8*b*c^3*d*Log[x - #1]*#1 + 3*b*c^2*d^2*Log[x - #1]
*#1^2)/(c^2 + 2*c*d*#1 + d^2*#1^2) & ])/(3*(a + b*c^3)^2*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b(c+dx)^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^3)), x]
```

fricas [C] time = 3.93, size = 8919, normalized size = 28.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(36*b*c^2*d*x*log(x) + 12*b*c^3 - 2*(b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))*x*log((b^3*c^6 - a^2*b)*d^3*x + 1/4*(2*a^2*b^3*c^9 + 3*a^3*
b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^
2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^
3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)
) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a
)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a
^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
+ a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 + 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 1
0*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*
c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3
+ a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 -
 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2
)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (b^3*c^7 + 5*a*b^2*c^4 - 5*a^2*b*c)*d
^2) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 + a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*x -
3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^
2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2
/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6
 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^
2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^
3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*
c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*
(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b
*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c
^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*
(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2
*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*
b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 +
 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*
b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2
```

```
*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^
6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 -
a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*
a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4
*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*
a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(
9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*
b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^
3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3
 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^
2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3
 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*
c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5
 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*
b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*
c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3
- 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^
2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 +
 a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)
^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
 a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^
3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*
b*c)*d^2 + 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/
(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b
*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)
/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2
*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1
/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*
c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6
 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt
(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*d)*sqrt(-((a*b^4*c^12
 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*c^2*d/(b^2*c^6 +
 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2
)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c
^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b
*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^
3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^
2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*
c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^
2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*
c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2
*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/
(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 +
a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4)
 + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + 4*(
8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*
a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) - (18*b*c^2*d*x - (b^2*c^6 + 2*a*b*c^3 +
 a^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2
/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))
*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^
3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2
```

```
*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*
d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^
2))^(1/3)*(I*sqrt(3) + 1))*x + 3*sqrt(1/3)*(b^2*c^6 + 2*a*b*c^3 + a^2)*x*sq
rt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*(6*b*
c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 +
 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3
) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b
^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c
^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1
/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((
a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^
2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(
I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^2)*(6*b*c^2*d/(
b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*
c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/
(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6
+ 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*
a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/
3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c
^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 +
 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(
3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(a*b^4*c^12 + 4*
a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)))*log(2*(b^3*c^6 - a^2*b)*
d^3*x - 1/4*(2*a^2*b^3*c^9 + 3*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a
*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2
- 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d
^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3
 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a
^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^
6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*
c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3
+ a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 -
1/2*(a*b^3*c^8 - 16*a^2*b^2*c^5 + 10*a^3*b*c^2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b
*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 -
2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3
/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 +
 a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4
) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*
d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^
3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 +
a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1))*d + (2
*b^3*c^7 - 5*a*b^2*c^4 + 2*a^2*b*c)*d^2 - 3/4*sqrt(1/3)*((2*a^2*b^3*c^9 + 3
*a^3*b^2*c^6 - a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*
(9*b^2*c^4*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2
*b*c^3 + a^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)
^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 +
 a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^
3 + a)^3*a^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a
^2)^3 - 18*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^
3 + a^2)) + b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b
*c^3 + a)^3*a^2))^(1/3)*(I*sqrt(3) + 1)) - 2*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a
^3*b*c^2)*d)*sqrt(-((a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c
^3 + a^5)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4
*d^2/(b^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a
^3))*(-I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^
2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b
*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a
^2))^(1/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18
*b^2*c^3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2))
```

```
+ b*d^3/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^
3*a^2))^(1/3)*(I*sqrt(3) + 1))^2 - 12*(a*b^3*c^8 + 2*a^2*b^2*c^5 + a^3*b*c^
2)*(6*b*c^2*d/(b^2*c^6 + 2*a*b*c^3 + a^2) - 2*(1/2)^(2/3)*(9*b^2*c^4*d^2/(b
^2*c^6 + 2*a*b*c^3 + a^2)^2 - 2*b*c*d^2/(a*b^2*c^6 + 2*a^2*b*c^3 + a^3))*(-
I*sqrt(3) + 1)/(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^3*d
^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3/(a
^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))^(1
/3) - (1/2)^(1/3)*(54*b^3*c^6*d^3/(b^2*c^6 + 2*a*b*c^3 + a^2)^3 - 18*b^2*c^
3*d^3/((a*b^2*c^6 + 2*a^2*b*c^3 + a^3)*(b^2*c^6 + 2*a*b*c^3 + a^2)) + b*d^3
/(a^2*b^2*c^6 + 2*a^3*b*c^3 + a^4) + (b*c^3 - a)*b*d^3/((b*c^3 + a)^3*a^2))
^(1/3)*(I*sqrt(3) + 1))*d + 4*(8*b^3*c^7 - 11*a*b^2*c^4 + 8*a^2*b*c)*d^2)/(
a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5))) + 12*a)/(
(b^2*c^6 + 2*a*b*c^3 + a^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^2), x)
```

maple [C] time = 0.01, size = 144, normalized size = 0.46

$$-\frac{3bc^2d\ln(x)}{\left(bc^3+a\right)^2}+\frac{d\left(3\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)^2bc^2d^2+8\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)bc^3d+6bc^4-\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)ad-3ac\right)\ln\left(-\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)+x\right)}{3\left(bc^3+a\right)^2\left(d^2\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)^2+2cd\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)+c^2\right)}-\frac{1}{\left(bc^3+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d/(b*c^3+a)^2*sum((3*_R^2*b*c^2*d^2+8*_R*b*c^3*d+6*b*c^4-_R*a*d-3*a*c)/
(_R^2*d^2+2*_R*c*d+c^2)*ln(-_R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*
b*c^2*d+b*c^3+a))-1/(b*c^3+a)/x-3*b*c^2*d*ln(x)/(b*c^3+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,bc^2d\log(x)}{b^2c^6+2\,abc^3+a^2}+\frac{bd^2\int\frac{3\,bc^2d^2x^2+6\,bc^4+\left(8\,bc^3-a\right)dx-3\,ac}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^2c^6+2\,abc^3+a^2}-\frac{1}{\left(bc^3+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -3*b*c^2*d*log(x)/(b^2*c^6 + 2*a*b*c^3 + a^2) + b*d^2*integrate((3*b*c^2*d^
2*x^2 + 6*b*c^4 + (8*b*c^3 - a)*d*x - 3*a*c)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3
*b*c^2*d*x + b*c^3 + a), x)/(b^2*c^6 + 2*a*b*c^3 + a^2) - 1/((b*c^3 + a)*x)
```

mupad [B] time = 2.33, size = 1588, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((b^4*d^12*x - 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*
a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^3*b^3*d^9
- 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*
d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^9*d^9 - 9*root(27*a^2*b^2*c^6
```

```
*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z
- b*d^3, z, k)*b^5*c^5*d^10 + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3
 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^
4*c^3*d^9 + 27*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81
*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^4*d^8 + 27*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^2*b^5*c^7*d^8 - 9*root(27*a^2*b^2*c^6*z^
3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b
*d^3, z, k)*a*b^4*c^2*d^10 - 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 +
 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*b^5*c^4*d^
11*x + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b
*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*c*d^8 + 18*root(27*a^2
*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*
c*d^2*z - b*d^3, z, k)^2*a*b^5*c^6*d^9 + 9*root(27*a^2*b^2*c^6*z^3 + 54*a^3
*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k
)^3*a*b^6*c^10*d^8 - 36*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^3*a^4*b^3*d^9*x -
 3*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*b^6*c^8*d^10*x + 48*root(27*a^2*b^2*
c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2
*z - b*d^3, z, k)^2*a*b^5*c^5*d^10*x + 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*
b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)
^3*a*b^6*c^9*d^9*x - 18*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4
*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)*a*b^4*c*d^11*x +
51*root(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d
*z^2 + 18*a*b*c*d^2*z - b*d^3, z, k)^2*a^2*b^4*c^2*d^10*x - 54*root(27*a^2*
b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 + 18*a*b*c
*d^2*z - b*d^3, z, k)^3*a^3*b^4*c^3*d^9*x)/(a^2 + b^2*c^6 + 2*a*b*c^3))*roo
t(27*a^2*b^2*c^6*z^3 + 54*a^3*b*c^3*z^3 + 27*a^4*z^3 - 81*a^2*b*c^2*d*z^2 +
 18*a*b*c*d^2*z - b*d^3, z, k), k, 1, 3) - 1/(a*x + b*c^3*x) - (3*b*c^2*d*l
og(x))/(a^2 + b^2*c^6 + 2*a*b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.105 $\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx$

Optimal. Leaf size=393

$$\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}\,c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+\right.}{3a^{2/3}\left(a+bc^3\right)^3}$$

Rubi [A] time = 0.60, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,(c+dx)\right)}{3a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(6a^{4/3}b^{2/3}c^2+a^2-3\sqrt[3]{a}\,b^{5/3}c^5-7abc^3+b^2c^6\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,(c+dx)+b^{2/3}(c+dx)^2\right)}{6a^{2/3}\left(a+bc^3\right)^3}+\frac{b^{2/3}d^2\left(-3a^{2/3}\sqrt[3]{b}\,c+a+bc^3\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,(c+dx)}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}-\frac{3bcd^2\log(x)\left(a-2bc^3\right)}{\left(a+bc^3\right)^3}+\frac{bcd^2\left(a-2bc^3\right)\log\left(a+b(c+dx)^3\right)}{\left(a+bc^3\right)^3}+\frac{3bc^2d}{x\left(a+bc^3\right)^2}-\frac{1}{2x^2\left(a+bc^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/(2*(a + b*c^3)*x^2) + (3*b*c^2*d)/((a + b*c^3)^2*x) + (b^(2/3)*(a^(1/3)
+ b^(1/3)*c)^3*(a - 3*a^(2/3)*b^(1/3)*c + b*c^3)*d^2*ArcTan[(a^(1/3) - 2*b^
(1/3)*(c + d*x))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a + b*c^3)^3) - (3*b
*c*(a - 2*b*c^3)*d^2*Log[x])/(a + b*c^3)^3 - (b^(2/3)*(a^2 + 6*a^(4/3)*b^(2
/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(1/3) + b^
(1/3)*(c + d*x)])/(3*a^(2/3)*(a + b*c^3)^3) + (b^(2/3)*(a^2 + 6*a^(4/3)*b^(
2/3)*c^2 - 7*a*b*c^3 - 3*a^(1/3)*b^(5/3)*c^5 + b^2*c^6)*d^2*Log[a^(2/3) - a
^(1/3)*b^(1/3)*(c + d*x) + b^(2/3)*(c + d*x)^2])/(6*a^(2/3)*(a + b*c^3)^3)
+ (b*c*(a - 2*b*c^3)*d^2*Log[a + b*(c + d*x)^3])/(a + b*c^3)^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx = d^2\,\text{Subst}\left(\int \frac{1}{(-c+x)^3\left(a+bx^3\right)}\,dx, x, c+dx\right)$$

$$= d^2\,\text{Subst}\left(\int\left(-\frac{1}{\left(a+bc^3\right)(c-x)^3} - \frac{3bc^2}{\left(a+bc^3\right)^2(c-x)^2} - \frac{3bc\left(-a+2bc^3\right)}{\left(a+bc^3\right)^3(c-x)} + \frac{b\left(-a^2\right.}{}\right.\right.$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{-a^2+7abc^3-}{}\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{\left(bd^2\right)\text{Subst}\left(\int \frac{-a^2+7abc^3-}{}\right.}{(a}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} + \frac{bc\left(a-2bc^3\right)d^2\log\left(a+\right.}{\left(a+bc^3\right)^3}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} - \frac{3bc\left(a-2bc^3\right)d^2\log(x)}{\left(a+bc^3\right)^3} - \frac{b^{2/3}\left(a^2+6a^{4/3}b^{2/3}c^2-7\right.}{}$$

$$= -\frac{1}{2\left(a+bc^3\right)x^2} + \frac{3bc^2d}{\left(a+bc^3\right)^2x} + \frac{b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,c\right)^3\left(a-3a^{2/3}\sqrt[3]{b}\,c+bc^3\right)d^2\tan^{-1}\left(\frac{1}{}\right.}{\sqrt{3}\,a^{2/3}\left(a+bc^3\right)^3}$$

Mathematica [C] time = 0.17, size = 244, normalized size = 0.62

$$-\frac{2d^2x^2\text{RootSum}\left[\#1^3bd^3+3\#1^2bcd^2+3\#1bc^2d+a+bc^3\&,\frac{-3\#1^2abcd^2\log(x-\#1)+6\#1^2b^2c^4d^2\log(x-\#1)+a^2\log(x-\#1)-16abc^3\log(x-\#1)-12\#1abc^2d\log(x-\#1)+10b^2c^6\log(x-\#1)+15\#1b^2c^5d\log(x-\#1)}{\#1^2d^2+2\#1cd+c^2}\&\right]+18bcd^2x^2\log(x)\left(a-2bc^3\right)+3\left(a+bc^3\right)\left(a+bc^2(c-6dx)\right)}{6x^2\left(a+bc^3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] -1/6*(3*(a + b*c^3)*(a + b*c^2*(c - 6*d*x)) + 18*b*c*(a - 2*b*c^3)*d^2*x^2*
Log[x] + 2*d^2*x^2*RootSum[a + b*c^3 + 3*b*c^2*d*#1 + 3*b*c*d^2*#1^2 + b*d^
3*#1^3 & , (a^2*Log[x - #1] - 16*a*b*c^3*Log[x - #1] + 10*b^2*c^6*Log[x - #
1] - 12*a*b*c^2*d*Log[x - #1]*#1 + 15*b^2*c^5*d*Log[x - #1]*#1 - 3*a*b*c*d^
2*Log[x - #1]*#1^2 + 6*b^2*c^4*d^2*Log[x - #1]*#1^2)/(c^2 + 2*c*d*#1 + d^2*
#1^2) & ])/((a + b*c^3)^3*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b(c+dx)^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*(c + d*x)^3)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*(c + d*x)^3)), x]
```

fricas [C] time = 12.24, size = 14765, normalized size = 37.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*c^6 - 36*(2*b^2*c^4 - a*b*c)*d^2*x^2*log(x) + 12*a*b*c^3 - 2*(
b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b
^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)
^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b
^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c
^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c
^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b
*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2
*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2*
log((b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (b^4*c^10 + 15
*a*b^3*c^7 - 63*a^2*b^2*c^4 + 4*a^3*b*c)*d^4 - 1/2*(a*b^4*c^12 - 50*a^2*b^3
*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a
*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^
2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2
*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b
*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3
*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2
*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^
3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*
(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^
2 + 3/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*
c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 +
 a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^
3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((
a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a
^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5
) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2
) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 +
 a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3
*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2) - 36*(b^2*c^5 + a*b*c^2)*d*x + 6*a^2 +
(18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^
3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a
^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3
+ a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
```

```
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5)
+ (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2)
- 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a
*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2 + 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)*x^2*sqrt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5
 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 -
 a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*
c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*
d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6
+ 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6
/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*
c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*
c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a
^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c
^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d
^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3
+ a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^
6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^
9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*
a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2
*(b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a
*b^3*c^7 - 9*a^2*b^2*c^4 - a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9
+ 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*
c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*
c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3
+ a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9
+ 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^
3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^
3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^
2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3
/4*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*
```

(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))^2 + 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^
9 + 6*a^3*b^2*c^6 + 4*a^4*b*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11
 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*
b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2
)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3
/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*
b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*
c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*
c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*
b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3
/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(
2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sq
rt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 +
12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*
(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
 - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a
^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3
*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*
c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (
b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 5
4*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)
^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^
2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4
*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)
*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^
4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqr
t(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1
/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b
^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) -
b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*
b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d
^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*s
qrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*

b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7))) + (18*(2*b^2*c^4 - a*b*c)*d^2*x^2 + (b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*x^2 - 3*sqrt(1/3)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2*sqrt(-(12*(4*b^5*c^11 - 24*a*b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)))*log(2*(b^4*c^9 + 3*a*b^3*c^6 - 24*a^2*b^2*c^3 + a^3*b)*d^5*x + (2*b^4*c^10 - 6*a*b^3*c^7 - 9*a^2*b^2*c^4 - a^3*b*c)*d^4 + 1/2*(a*b^4*c^12 - 50*a^2*b^3*c^9 + 141*a^3*b^2*c^6 - 50*a^4*b*c^3 + a^5)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^

2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c
^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c
^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 +
a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2
*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3
*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))*d^2 - 3/4*(a^2*b^4*c^14 + a^3*b^3*
c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
^2 - 3/4*sqrt(1/3)*(2*(a*b^4*c^12 + 4*a^2*b^3*c^9 + 6*a^3*b^2*c^6 + 4*a^4*b
*c^3 + a^5)*d^2 + 3*(a^2*b^4*c^14 + a^3*b^3*c^11 - 3*a^4*b^2*c^8 - 5*a^5*b*
c^5 - 2*a^6*c^2)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3
*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6
+ 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^
2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b
^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4
*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^3)*b^2*d^6/((b*c^
3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3
*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*d^2 - a*b*c*d^2)/
(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)))*sqrt(-(12*(4*b^5*c^11 - 24*a*
b^4*c^8 + 48*a^2*b^3*c^5 - 5*a^3*b^2*c^2)*d^4 + 12*(2*a*b^5*c^13 + 5*a^2*b^
4*c^10 + 3*a^3*b^3*c^7 - a^4*b^2*c^4 - a^5*b*c)*(6*(1/2)^(2/3)*(b^2*c^2*d^4
/(a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c
*d^2)^2/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27
*(2*b^2*c^4*d^2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^
3*b*c^3 + a^4)*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*
b^3*c^9 + 3*a^3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*
a^2*b*c^3 + a^3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^
2)^3/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) -
6*(2*b^2*c^4*d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))
*d^2 + (a*b^6*c^18 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15
*a^5*b^2*c^6 + 6*a^6*b*c^3 + a^7)*(6*(1/2)^(2/3)*(b^2*c^2*d^4/(a*b^3*c^9 +
3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4) - 3*(2*b^2*c^4*d^2 - a*b*c*d^2)^2/(b^3*c
^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^2)*(-I*sqrt(3) + 1)/(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9

```
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3) - (1/2)^(1/3)*(27*(2*b^2*c^4*d^
2 - a*b*c*d^2)*b^2*c^2*d^4/((a*b^3*c^9 + 3*a^2*b^2*c^6 + 3*a^3*b*c^3 + a^4)
*(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)) - b^2*d^6/(a^2*b^3*c^9 + 3*a^
3*b^2*c^6 + 3*a^4*b*c^3 + a^5) + (b^3*c^9 + 3*a*b^2*c^6 - 24*a^2*b*c^3 + a^
3)*b^2*d^6/((b*c^3 + a)^6*a^2) - 54*(2*b^2*c^4*d^2 - a*b*c*d^2)^3/(b^3*c^9
+ 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)^3)^(1/3)*(I*sqrt(3) + 1) - 6*(2*b^2*c^4*
d^2 - a*b*c*d^2)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3))^2)/(a*b^6*c^1
8 + 6*a^2*b^5*c^15 + 15*a^3*b^4*c^12 + 20*a^4*b^3*c^9 + 15*a^5*b^2*c^6 + 6*
a^6*b*c^3 + a^7))))/((b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^3 b + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^3*b + a)*x^3), x)
```

maple [C] time = 0.01, size = 217, normalized size = 0.55

$$\frac{6b^2c^4d^2\ln(x)}{\left(bc^3+a\right)^3}-\frac{3abcd^2\ln(x)}{\left(bc^3+a\right)^3}+\frac{3bc^2d}{\left(bc^3+a\right)^2x}+\frac{d^2\left(-6\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)^2b^2c^4d^2-15\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)b^2c^5d-10b^2c^6+3\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)^2abcd^2+12\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)abc^2d+16abc^3-a^2\right)\ln\left(-\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)+x\right)}{3\left(bc^3+a\right)^3\left(d^2\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)^2+2cd\,\mathrm{RootOf}\left(bd^3\_Z^3+3bd^2c\_Z^2+3bdc^2\_Z+bc^3+a\right)+c^2\right)}-\frac{1}{2\left(bc^3+a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^3),x)
```

```
[Out] 1/3*d^2/(b*c^3+a)^3*sum((-6*_R^2*b^2*c^4*d^2-15*_R*b^2*c^5*d-10*b^2*c^6+3*_
R^2*a*b*c*d^2+12*_R*a*b*c^2*d+16*a*b*c^3-a^2)/(_R^2*d^2+2*_R*c*d+c^2)*ln(-_
R+x),_R=RootOf(_Z^3*b*d^3+3*_Z^2*b*c*d^2+3*_Z*b*c^2*d+b*c^3+a))-1/2/(b*c^3+
a)/x^2+3*b*c^2*d/(b*c^3+a)^2/x+6*b^2*d^2*c^4/(b*c^3+a)^3*ln(x)-3*b*d^2*c/(b
*c^3+a)^3*ln(x)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd^3\int\frac{10\,b^2c^6-16\,abc^3+3\left(2\,b^2c^4-abc\right)d^2x^2+3\left(5\,b^2c^5-4\,abc^2\right)dx+a^2}{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\,dx}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{3\left(2\,b^2c^4-abc\right)d^2\log(x)}{b^3c^9+3\,ab^2c^6+3\,a^2bc^3+a^3}+\frac{6\,bc^2dx-bc^3-a}{2\left(b^2c^6+2\,abc^3+a^2\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*d^3*integrate((10*b^2*c^6 - 16*a*b*c^3 + 3*(2*b^2*c^4 - a*b*c)*d^2*x^2 +
 3*(5*b^2*c^5 - 4*a*b*c^2)*d*x + a^2)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a), x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 3*(2*b^2
*c^4 - a*b*c)*d^2*log(x)/(b^3*c^9 + 3*a*b^2*c^6 + 3*a^2*b*c^3 + a^3) + 1/2*
(6*b*c^2*d*x - b*c^3 - a)/((b^2*c^6 + 2*a*b*c^3 + a^2)*x^2)
```

mupad [B] time = 2.49, size = 1328, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^3)),x)
```

```
[Out] symsum(log((6*b^6*c^4*d^14 - 3*a*b^5*c*d^14)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((a^3*b^4*d^12 + 19*b^7*c^9*d^
```

```
12 + 12*a*b^6*c^6*d^12 - 6*a^2*b^5*c^3*d^12)/(a^4 + b^4*c^12 + 4*a^3*b*c^3
+ 4*a*b^3*c^9 + 6*a^2*b^2*c^6) - root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z
^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d
^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*(root(81*a^3*b^2*c^6*z^3 + 27*
a^2*b^3*c^9*z^3 + 81*a^4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*
a^2*b^2*c^4*d^2*z^2 + 27*a*b^2*c^2*d^4*z + b^2*d^6, z, k)*((9*a^6*b^3*c*d^8
 + 9*a*b^8*c^16*d^8 + 45*a^5*b^4*c^4*d^8 + 90*a^4*b^5*c^7*d^8 + 90*a^3*b^6*
c^10*d^8 + 45*a^2*b^7*c^13*d^8)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9
 + 6*a^2*b^2*c^6) - (x*(36*a^6*b^3*d^9 - 18*a*b^8*c^15*d^9 + 126*a^5*b^4*c^
3*d^9 + 144*a^4*b^5*c^6*d^9 + 36*a^3*b^6*c^9*d^9 - 36*a^2*b^7*c^12*d^9))/(a
^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)) + (3*b^8*c^14*d
^10 - 42*a*b^7*c^11*d^10 + 30*a^4*b^4*c^2*d^10 + 12*a^3*b^5*c^5*d^10 - 63*a
^2*b^6*c^8*d^10)/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^
6) + (x*(3*b^8*c^13*d^11 + 66*a^4*b^4*c*d^11 - 87*a*b^7*c^10*d^11 + 39*a^3*
b^5*c^4*d^11 - 117*a^2*b^6*c^7*d^11))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b
^3*c^9 + 6*a^2*b^2*c^6)) + (x*(18*b^7*c^8*d^13 + 90*a*b^6*c^5*d^13 - 9*a^2*
b^5*c^2*d^13))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3*c^9 + 6*a^2*b^2*c^6)
) - (x*(a*b^5*d^15 + b^6*c^3*d^15))/(a^4 + b^4*c^12 + 4*a^3*b*c^3 + 4*a*b^3
*c^9 + 6*a^2*b^2*c^6))*root(81*a^3*b^2*c^6*z^3 + 27*a^2*b^3*c^9*z^3 + 81*a^
4*b*c^3*z^3 + 27*a^5*z^3 - 81*a^3*b*c*d^2*z^2 + 162*a^2*b^2*c^4*d^2*z^2 + 2
7*a*b^2*c^2*d^4*z + b^2*d^6, z, k), k, 1, 3) - 1/(2*(a*x^2 + b*c^3*x^2)) +
(3*b*c^2*d)/(a^2*x + b^2*c^6*x + 2*a*b*c^3*x) + (6*b^2*c^4*d^2*log(x))/(a^3
 + b^3*c^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6) - (3*a*b*c*d^2*log(x))/(a^3 + b^3*c
^9 + 3*a^2*b*c^3 + 3*a*b^2*c^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.106 $\int \frac{x^3}{a+b(c+dx)^4} \, dx$

Optimal. Leaf size=356

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}$$

Rubi [A] time = 0.43, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {371, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}-\frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4}+\frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4}+\frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] (3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*d^4) + (c*
(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) - (c*(
3*Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) +
Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + (c*(3*Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^4) + Log[a + b*(c + d*x)^4]/(4*b*d^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{x(3c^2+x^2)}{a+bx^4} + \frac{-c^3-3cx^2}{a+bx^4}\right) dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \frac{x(3c^2+x^2)}{a+bx^4}\,dx, x, c+dx\right)}{d^4} + \frac{\text{Subst}\left(\int \frac{-c^3-3cx^2}{a+bx^4}\,dx, x, c+dx\right)}{d^4}$$
$$= \frac{\text{Subst}\left(\int \frac{3c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{\left(c\left(3-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^4} - \frac{\left(c\left(3\right.\right.}{}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} + \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^4} - \frac{\left(c\left(3\sqrt{a}-\sqrt{b}\,c\right.\right.}{}$$
$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4} - \frac{c\left(3\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^4} +$$
$$= \frac{3c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}}{2\sqrt{2}\,a^{3/4}b}$$

Mathematica [C] time = 0.06, size = 106, normalized size = 0.30

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1^3\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^4),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*(c + d*x)^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.02, size = 97, normalized size = 0.27

$$\frac{\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^3 \ln\left(-\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)+x\right)}{4 b d\left(d^3\,\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^3+3\,\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^2 c d^2+3\,\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right) c^2 d+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.69, size = 1003, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256*
a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^3*
b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z +
 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*root(25
6*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*z
 + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 + 24*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 9
6*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d
^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*d^9*x - 2*
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^4*d^5*x +
 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*
z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*
a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*c*d^4 +
 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z
^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a
^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*a*b*d^5*x))*
```

```
root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2
+ 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*
b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k), k, 1, 4)
```

sympy [A] time = 3.74, size = 374, normalized size = 1.05

$$\operatorname{RootSum}\left(256t^4a^3b^4d^{16} - 256t^3a^3b^3d^{12} + t^2\left(96a^3b^2d^8 + 480a^2b^3c^4d^8\right) + t\left(-16a^3bd^4 + 192a^2b^2c^4d^4 - 48ab^3c^8d^4\right) + a^3 + 3a^2bc^4 + 3ab^2c^8 + b^3c^{12}, \left(t \mapsto t\log\left(x + \frac{-1728t^3a^4b^3d^{12} - 960t^3a^3b^4c^4d^{12} + 1296t^2a^4b^2d^8 + 2016t^2a^3b^3c^4d^8 - 48t^2a^2b^4c^8d^8 - 324ta^4bd^4 - 4716ta^3b^2c^4d^4 - 1452ta^2b^3c^8d^4 - 4tab^4c^{12}d^4 + 27a^4 - 390a^3bc^4 - 444a^2b^2c^8 - 26ab^3c^{12} + b^4c^{16}}{729a^3bc^3d - 1053a^2b^2c^7d - 117ab^3c^{11}d + b^4c^{15}d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*a
**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2*b
**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c**8
 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960*_t*
*3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*b**3*
c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*_t*a**
3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d**4 + 2
7*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b**4*c**1
6)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d + b**4*c
**15*d))))
```

3.107 $\int \frac{x^2}{a+b(c+dx)^4} \, dx$

Optimal. Leaf size=318

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\right.}{4\sqrt{2}a^{3/4}b^{3/4}d^3}$$

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {371, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}+\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}-\frac{c\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] -((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*d^3)) - ((Sqrt
[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(
1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) + ((Sqrt[a] - Sqr
t[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*
x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)*d^3) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a
^(3/4)*b^(3/4)*d^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{2cx}{a+bx^4} + \frac{c^2+x^2}{a+bx^4}\right)dx, x, c+dx\right)}{d^3}$$
$$= \frac{\text{Subst}\left(\int \frac{c^2+x^2}{a+bx^4}\,dx, x, c+dx\right)}{d^3} - \frac{(2c)\,\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^3}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{d^3} - \frac{\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x, c+dx\right)}{2bd^3} + \frac{(1}{}$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, c+dx\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}-\right.}{}$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} + \frac{\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}d^3} -$$
$$= -\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d^3} - \frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1\right.}{2\sqrt{2}\,a^{3/4}b^{3/}}$$

Mathematica [C] time = 0.04, size = 106, normalized size = 0.33

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1^2\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*(c + d*x)^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.00, size = 97, normalized size = 0.31

$$\frac{\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^2 \ln\left(-\mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)+x\right)}{4 b d\left(d^3 \mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^3+3 \mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^2 c d^2+3 \mathrm{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right) c^2 d+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(dx+c)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.59, size = 625, normalized size = 1.97

$$\sum_{k=1}^{4} \ln\left(-b d^4 \left(a+b c^4+4 b c^3 d x+\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right) b^2 c^5 d^3\, 4+\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right) b^2 c^4 d^4 x\, 4-\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right) a b c d^3\, 20-\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right) a b d^4 x\, 4+\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right)^2 a b^2 c^2 d^6\, 48+\mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right)^2 a b^2 c d^7 x\, 32\right)\right) \mathrm{root}\left(256 a^3 b^3 d^{12} z^4+192 a^2 b^2 c^2 d^6 z^2+32 a b^2 c^5 d^3 z-32 a^2 b c d^3 z+2 a b c^4+b^2 c^8+a^2, z, k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4
 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2
*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*
c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2
*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8
+ a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*
z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
 k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32
*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*
b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a
*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k)^2*a*b^
2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*
c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z, k), k, 1, 4)
```

sympy [A] time = 2.67, size = 274, normalized size = 0.86

$$\mathrm{RootSum}\left(256 t^4 a^3 b^3 d^{12}+192 t^2 a^2 b^2 c^2 d^6+t\left(-32 a^2 b c d^3+32 a b^2 c^5 d^3\right)+a^2+2 a b c^4+b^2 c^8, \left(t \mapsto t \log\left(x+\frac{64 t^3 a^4 b^2 d^9+448 t^3 a^3 b^3 c^4 d^9+160 t^2 a^3 b^2 c^3 d^6-32 t^2 a^2 b^3 c^7 d^6+60 t a^3 b c^2 d^3+256 t a^2 b^2 c^6 d^3+4 t a b^3 c^{10} d^3-5 a^3 c-9 a^2 b c^5-3 a b^2 c^9+b^3 c^{13}}{a^3 d-33 a^2 b c^4 d-33 a b^2 c^8 d+b^3 c^{12} d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-32
*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d**9
+ 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*a**3
*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5*a**3
*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*c**4*d
 - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

3.108 $\int \frac{x}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=261

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3}}$$

Rubi [A] time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {371, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*(c + d*x)^4),x]
```

```
[Out] ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]*d^2) + (c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) -
(c*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1
/4)*d^2) + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c
+ d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d^2
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{c}{a+bx^4}+\frac{x}{a+bx^4}\right)dx, x, c+dx\right)}{d^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+bx^4}\,dx, x, c+dx\right)}{d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x}{a+bx^4}\right.}{2\sqrt{a}\,\ldots}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\,\sqrt{b}\,d^2} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx\ldots\right.}{4\sqrt{a}\,\sqrt{b}\,d^2}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} + \frac{c\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\,\sqrt[4]{b}\,d^2} - \frac{c\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{\ldots}\right.}{4\ldots}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{b}\,d^2} + \frac{c\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,\sqrt[4]{b}\,d^2} - \frac{c\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,\sqrt[4]{b}\,d^2} + \frac{c\log\left(\sqrt{a}-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.03, size = 104, normalized size = 0.40

$$\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&, \frac{\#1\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*(c + d*x)^4),x]
```

```
[Out] RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^
4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) &
]/(4*b*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b(c+dx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*(c + d*x)^4),x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*(c + d*x)^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

maple [C] time = 0.01, size = 95, normalized size = 0.36

$$\frac{\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)\ln\left(-\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)+x\right)}{4bd\left(d^3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^3+3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)^2c\,d^2+3\,\mathrm{RootOf}\left(b\,d^4\_Z^4+4b\,d^3c\_Z^3+6b\,d^2c^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)c^2d+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_
Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx+c)^4 b + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x + c)^4*b + a), x)
```

mupad [B] time = 2.36, size = 205, normalized size = 0.79

$$\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(-\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)\left(16\,a\,x\,b^3\,d^{12}+32\,a\,c\,b^3\,d^{11}\right)+4\,b^3\,c^3\,d^9+4\,b^3\,c^2\,d^{10}\,x\right)+b^2\,d^8\,x\right)\mathrm{root}\left(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^4),x)
```

```
[Out] symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*b
*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 + 3
2*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 + 16
*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4
*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

sympy [A] time = 0.88, size = 131, normalized size = 0.50

$$\mathrm{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+8ta^2d^2+4tabc^4d^2-ac^2-bc^6}{4acd-bc^5d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d*
*2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*a*
*2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/(4*
a*c*d - b*c**5*d))))
```

3.109 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal. Leaf size=221

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}a^{3/4}}$$

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {247, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^(1/4)
*d) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*b^
(1/4)*d) - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^4}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx, x, c+dx\right)}{2\sqrt{a}\,d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, c+dx\right)}{4\sqrt{a}\sqrt{b}\,d} - \frac{\text{Subst}}{\ldots}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}\ldots\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\ldots\right.}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

Mathematica [A] time = 0.10, size = 161, normalized size = 0.73

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d
*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b(c+dx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*(c + d*x)^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*(c + d*x)^4)^(-1), x]
```

fricas [A] time = 1.00, size = 189, normalized size = 0.86

$$\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(a^2bd^4\sqrt{\frac{a^2d^2\sqrt{-\frac{1}{a^3bd^4}}+d^2x^2+2\,cdx+c^2}{d^2}}\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}-\left(a^2bd^4x+a^2bcd^3\right)\left(-\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*b*d^4))^(1/4)*arctan(a^2*b*d^4*sqrt((a^2*d^2*sqrt(-1/(a^3*b*d^4))
+ d^2*x^2 + 2*c*d*x + c^2)/d^2)*(-1/(a^3*b*d^4))^(3/4) - (a^2*b*d^4*x + a^2
*b*c*d^3)*(-1/(a^3*b*d^4))^(3/4)) + 1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/
(a^3*b*d^4))^(1/4) + d*x + c) - 1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^
3*b*d^4))^(1/4) + d*x + c)
```

giac [A] time = 0.50, size = 103, normalized size = 0.47

$$-\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{\left(-ab^3\right)^{\frac{1}{4}}}\right)+\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)-\frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-1
/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3*b
*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

maple [C] time = 0.00, size = 94, normalized size = 0.43

$$\frac{\ln\left(-\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)+x\right)}{4bd\left(d^3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^3+3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^2cd^2+3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)c^2d+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^4b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^4*b + a), x)
```

mupad [B] time = 0.12, size = 60, normalized size = 0.27

$$-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)+\mathrm{atanh}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^4),x)
```

```
[Out] -(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)*
c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

<u>**sympy**</u> **[A]** time = 0.30, size = 26, normalized size = 0.12

$$\frac{\text{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**4),x)
```

```
[Out] RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

3.110 $\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx$

Optimal. Leaf size=393

$$-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)\right.}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}$$

Rubi [A] time = 0.47, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx)+\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}-\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}+\frac{\log(x)}{a+bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*(a + b*c^4)
) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x
))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + Sqrt[b
]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(a + b*c^4)) + Log[x]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt
[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/
4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{(-c+x)\left(a+bx^4\right)}\,dx, x, c+dx\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\left(a+bc^4\right)(c-x)} - \frac{b\left(c^3+c^2x+cx^2+x^3\right)}{\left(a+bc^4\right)\left(a+bx^4\right)}\right)dx, x, c+dx\right)$$
$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+c^2x+cx^2+x^3}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$
$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \left(\frac{x\left(c^2+x^2\right)}{a+bx^4} + \frac{c^3+cx^2}{a+bx^4}\right)dx, x, c+dx\right)}{a+bc^4}$$
$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x\left(c^2+x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^3+cx^2}{a+bx^4}\,dx, x, c+dx\right)}{a+bc^4}$$
$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{c^2+x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} + \frac{\left(c\left(1-\frac{\sqrt{b}\,c^2}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx, x,\right.}{2\left(a+bc^4\right)}$$

$$= \frac{\log(x)}{a+bc^4} - \frac{b\,\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} - \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)} -$$
$$= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\sqrt[4]{b}\,c\left(\sqrt{a}-\sqrt{b}\,c^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,(c+dx\right.}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)}$$
$$= -\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\sqrt{b}\,c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)} - \frac{\sqrt[4]{b}\,c\left(\sqrt{a}+\right.}{2}$$

Mathematica [C] time = 0.07, size = 163, normalized size = 0.41

$$-\frac{\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{\#1^3d^3\log(x-\#1)+4\#1^2cd^2\log(x-\#1)+4c^3\log(x-\#1)+6\#1c^2d\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]-4\log(x)}{4\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] -1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b
*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1 +
4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^
2*#1^2 + d^3*#1^3) & ])/(a + b*c^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*(c + d*x)^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^4 b + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x), x)
```

maple [C] time = 0.01, size = 139, normalized size = 0.35

$$\frac{\ln(x)}{bc^4+a}-\frac{\left(d^3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^3+4\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^2cd^2+6\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)c^2d+4c^3\right)\ln\left(-\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)+x\right)}{4\left(bc^4+a\right)\left(d^3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^3+3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)^2cd^2+3\,\mathrm{RootOf}\left(bd^4\_Z^4+4bd^3c\_Z^3+6bd^2c^2\_Z^2+4bc^3d\_Z+bc^4+a\right)c^2d+c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^4),x)
```

```
[Out] -1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/(_R^3*d^3+3*_R^
2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^
2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))+ln(x)/(b*c^4+a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{bd\int \frac{d^3x^3+4\,cd^2x^2+6\,c^2dx+4\,c^3}{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}\,dx}{bc^4+a}+\frac{\log(x)}{bc^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*b
*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) + l
og(x)/(b*c^4 + a)
```

mupad [B] time = 2.18, size = 882, normalized size = 2.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^4)),x)
```

```
[Out] log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 25
6*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2
+ 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 +
256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*root(2
56*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z,
k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 +
```

```
 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b*c^4*z^4
+ 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^2*b^4*c*d^
15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 1
6*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4
 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^15 + 240*root
(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z
, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*
z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*root(256*a^3*b*c^
4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5*c^
4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z
^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256*a^3*b*c^4*z^4 + 25
6*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^4*d^16
*x)*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*
z + 1, z, k), k, 1, 4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.111 $\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx$

Optimal. Leaf size=496

$$-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}\,c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\left(a-3bc\right.\right.}{}$$

Rubi [A] time = 0.89, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {371, 6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}\,c^2\left(3a-bc^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{a}\left(a-3bc^4\right)-\sqrt{b}\,c^2\left(3a-bc^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}+\frac{\sqrt[4]{b}\,d\left(\sqrt{b}\,c^2\left(3a-bc^4\right)+\sqrt{a}\left(a-3bc^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}-\frac{\sqrt[4]{b}\,d\left(\sqrt{b}\,c^2\left(3a-bc^4\right)+\sqrt{a}\left(a-3bc^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(a+bc^4\right)^2}-\frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2}+\frac{bc^3d\log\left(a+b(c+dx)^4\right)}{\left(a+bc^4\right)^2}-\frac{\sqrt{b}\,cd\left(a-bc^4\right)\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2}-\frac{1}{x\left(a+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] -(1/((a + b*c^4)*x)) - (Sqrt[b]*c*(a - b*c^4)*d*ArcTan[(Sqrt[b]*(c + d*x)^2
)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqr
t[b]*c^2*(3*a - b*c^4))*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) + Sqrt[
b]*c^2*(3*a - b*c^4))*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*d*Log[x])/(a + b*c^4)^2 - (b^(1/
4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*(a
 + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))
*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/
(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*d*Log[a + b*(c + d*x)^4])/(a + b
*c^4)^2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = d\,\text{Subst}\left(\int \frac{1}{(-c+x)^2\left(a+bx^4\right)}\,dx, x, c+dx\right)$$

$$= d\,\text{Subst}\left(\int\left(\frac{1}{\left(a+bc^4\right)(c-x)^2} + \frac{4bc^3}{\left(a+bc^4\right)^2(c-x)} + \frac{b\left(-c^2\left(3a-bc^4\right)-2c\left(a-b\right.\right.}{\left(a+bc^4\right.}\right.\right.$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} + \frac{(bd)\,\text{Subst}\left(\int \frac{-c^2\left(3a-bc^4\right)-2c\left(a-bc^4\right)x-\left(a-3bc^4\right)x^2+4bc^3x^3}{a+bx^4}\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} + \frac{(bd)\,\text{Subst}\left(\int\left(\frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4} + \frac{-c^2\left(3a-bc^4\right)+\left(-a+\right.}{a+bx^4}\right.\right.}{\left(a+bc^4\right)^2}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} + \frac{(bd)\,\text{Subst}\left(\int \frac{x\left(-2c\left(a-bc^4\right)+4bc^3x^2\right)}{a+bx^4}\,dx, x, c+dx\right)}{\left(a+bc^4\right)^2} + \frac{(b}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} + \frac{(bd)\,\text{Subst}\left(\int \frac{-2c\left(a-bc^4\right)+4bc^3x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{2\left(a+bc^4\right)^2} + \frac{((}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} + \frac{\left(2b^2c^3d\right)\text{Subst}\left(\int \frac{x}{a+bx^2}\,dx, x, (c+dx)^2\right)}{\left(a+bc^4\right)^2} - \frac{\left(bc\left(a-\right.\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{\sqrt{b}\,c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2} - \frac{4bc^3d\log(x)}{\left(a+bc^4\right)^2} - \frac{\sqrt[4]{b}\left(a-3bc^4\right.}{}$$

$$= -\frac{1}{\left(a+bc^4\right)x} - \frac{\sqrt{b}\,c\left(a-bc^4\right)d\tan^{-1}\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\left(a+bc^4\right)^2} + \frac{\sqrt[4]{b}\left(a-3bc^4+\frac{\sqrt{b}c^2\left(3a-bc^4\right)}{\sqrt{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(a-\right.}$$

Mathematica [C] time = 0.13, size = 238, normalized size = 0.48

$$\frac{dx\text{RootSum}\left[\#1^4bd^4+4\#1^3bcd^3+6\#1^2bc^2d^2+4\#1bc^3d+a+bc^4\&,\frac{4\#1^3bc^3d^3\log(x-\#1)-\#1^2ad^2\log(x-\#1)+15\#1^2bc^4d^2\log(x-\#1)-6ac^2\log(x-\#1)-4\#1acd\log(x-\#1)+10bc^6\log(x-\#1)+20\#1bc^5d\log(x-\#1)}{\#1^3d^3+3\#1^2cd^2+3\#1c^2d+c^3}\&\right]-4\left(a+bc^4+4bc^3dx\log(x)\right)}{4x\left(a+bc^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] (-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#1
 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #1]
 + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1]*#
1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^3*Lo
g[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(a + b
*c^4)^2*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^4)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*(c + d*x)^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((dx+c)^4 b + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

maple [C] time = 0.01, size = 188, normalized size = 0.38

$$-\frac{4 b c^3 d \ln(x)}{\left(b c^4+a\right)^2}+\frac{d\left(4 \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^3 b c^3 d^3+10 b c^6+\left(15 b c^4-a\right) \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^2 d^2+4\left(5 b c^4-a\right) \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right) c d-6 a c^2\right) \ln\left(-\operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)+x\right)}{4\left(b c^4+a\right)^2\left(d^3 \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^3+3 \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right)^2 c d^2+3 \operatorname{RootOf}\left(b d^4\_Z^4+4 b d^3 c\_Z^3+6 b d^2 c^2\_Z^2+4 b c^3 d\_Z+b c^4+a\right) c^2 d+c^3\right)}-\frac{1}{\left(b c^4+a\right) x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^4),x)
```

```
[Out] 1/4*d/(b*c^4+a)^2*sum((4*b*d^3*c^3*_R^3+d^2*(15*b*c^4-a)*_R^2+4*c*d*(5*b*c^
4-a)*_R+10*b*c^6-6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(-_R+x),
_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\,bc^3 d\log(x)}{b^2c^8+2\,abc^4+a^2}+\frac{bd^2\int \frac{4\,bc^3d^3x^3+10\,bc^6+\left(15\,bc^4-a\right)d^2x^2-6\,ac^2+4\left(5\,bc^5-ac\right)dx}{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}\,dx}{b^2c^8+2\,abc^4+a^2}-\frac{1}{\left(bc^4+a\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d^
3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d*x
)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a),
x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

mupad [B] time = 2.48, size = 2440, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

```
[Out] symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
```

```
k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*
a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b
*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4 + 512
*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*c^8*z^4
 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d
^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*root(256*a^
3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 32
0*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^4*d^16 + 19
2*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c
^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^4*b^5*
c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*c^8*z^4 + 512*a^4*
b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32
*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16*root(256*a^3*b^2*c^8*z
^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2
*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^5*d^19*x + 64*root(256*a^3*b
^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a
^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^5*b^4*c*d^15 - 184*roo
t(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*
z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^7*d^1
7 - 48*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^
3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b
^7*c^12*d^16 - 320*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z
^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4,
 z, k)^4*a^5*b^4*d^16*x + 4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^2*b^7*c^10*d^18*x - 248*root(256*a^3*b^2*c^8*z^4 + 512*a^4*
b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32
*a*b*c*d^3*z + b*d^4, z, k)^2*a*b^6*c^6*d^18*x - 64*root(256*a^3*b^2*c^8*z^
4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*
d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a*b^7*c^11*d^17*x + 32*root(256*a
^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 3
20*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*a*b^5*c*d^19*x - 316*r
oot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*
d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^2
*d^18*x + 704*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k
)^3*a^3*b^5*c^3*d^17*x - 448*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*
z + b*d^4, z, k)^4*a^4*b^5*c^4*d^16*x + 640*root(256*a^3*b^2*c^8*z^4 + 512*
a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
- 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^2*b^6*c^7*d^17*x + 64*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*
b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^8*d^16*x + 192*ro
ot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d
*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^12
*d^16*x)/(a^2 + b^2*c^8 + 2*a*b*c^4))*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*
c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a
*b*c*d^3*z + b*d^4, z, k), k, 1, 4) - 1/(a*x + b*c^4*x) - (4*b*c^3*d*log(x)
)/(a^2 + b^2*c^8 + 2*a*b*c^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.112 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=123

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5$$

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1106, 1090}

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3+(a+3)^4x+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (-8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1 + x)^5)/5 + (8*(3 +
a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-1 + x)^9)/9 - (8*(5 +
3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 + (8*(-1 + x)^15)/15 + (-
1 + x)^17/17 + (3 + a)^4*x
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}
\int \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx &= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^4 \, dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \left(81\left(1+\frac{1}{81}a\left(108+54a+12a^2+a^3\right)\right)-216\left(1+a\left(1+\frac{1}{27}a(9\right.\right.\right.\right.\\
&= -\frac{8}{3}(3+a)^3(-1+x)^3+\frac{4}{5}(3-a)(3+a)^2(-1+x)^5+\frac{8}{7}(3+a)(5+3a)(-1+
\end{aligned}$$

Mathematica [A] time = 0.04, size = 195, normalized size = 1.59

$$a^4x+16a^3x^2+\frac{2}{9}\left(3a^2-1536a+20480\right)x^9-6\left(a^2-128a+896\right)x^8+\frac{64}{7}\left(3a^2-140a+512\right)x^7-\frac{16}{3}\left(15a^2-288a+512\right)x^6+4a\left(a^2-48a+128\right)x^4-\frac{32}{3}(a-12)a^2x^3-\frac{4}{5}\left(a^3-192a^2+1536a-1024\right)x^5-\frac{4}{13}(a-640)x^{13}+\frac{4}{3}(3a-464)x^{12}-\frac{32}{11}(9a-524)x^{11}+\frac{16}{5}(35a-928)x^{10}+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
- (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)*x
^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (2*(
20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524 + 9*
```

```
a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^14 +
(128*x^15)/15 - x^16 + x^17/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]
```

fricas [B] time = 1.03, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} - \frac{4}{13}x^{13}a + \frac{2560}{13}x^{13} + 4x^{12}a - \frac{1856}{3}x^{12} - \frac{288}{11}x^{11}a + \frac{16768}{11}x^{11} + 112x^{10}a + \frac{2}{3}x^9a^2 - \frac{14848}{5}x^{10} - \frac{1024}{3}x^9a - 6x^8a^2 + \frac{40960}{9}x^9 + 768x^8a + \frac{192}{7}x^7a^2 - 5376x^8 - 1280x^7a - 80x^6a^2 - \frac{4}{5}x^5a^3 + \frac{32768}{7}x^7 + 1536x^6a + \frac{768}{5}x^5a^2 + 4x^4a^3 - \frac{8192}{3}x^6 - \frac{6144}{5}x^5a - 192x^4a^2 - \frac{32}{3}x^3a^3 + \frac{4096}{5}x^5 + 512x^4a + 128x^3a^2 + 16x^2a^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 - 4/13*x^13*a + 2560/13*x^13 + 4*x
^12*a - 1856/3*x^12 - 288/11*x^11*a + 16768/11*x^11 + 112*x^10*a + 2/3*x^9*
a^2 - 14848/5*x^10 - 1024/3*x^9*a - 6*x^8*a^2 + 40960/9*x^9 + 768*x^8*a + 1
92/7*x^7*a^2 - 5376*x^8 - 1280*x^7*a - 80*x^6*a^2 - 4/5*x^5*a^3 + 32768/7*x
^7 + 1536*x^6*a + 768/5*x^5*a^2 + 4*x^4*a^3 - 8192/3*x^6 - 6144/5*x^5*a - 1
92*x^4*a^2 - 32/3*x^3*a^3 + 4096/5*x^5 + 512*x^4*a + 128*x^3*a^2 + 16*x^2*a
^3 + x*a^4
```

giac [B] time = 0.33, size = 219, normalized size = 1.78

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - \frac{4}{13}ax^{13} - 48x^{14} + 4ax^{12} + \frac{2560}{13}x^{13} - \frac{288}{11}ax^{11} - \frac{1856}{3}x^{12} + \frac{2}{3}a^2x^9 + 112ax^{10} + \frac{16768}{11}x^{11} - 6a^2x^8 - \frac{1024}{3}ax^9 - \frac{14848}{5}x^{10} + \frac{192}{7}a^2x^7 + 768ax^8 + \frac{40960}{9}x^9 - \frac{4}{5}a^3x^5 - 80a^2x^6 - 1280ax^7 - 5376x^8 + 4a^3x^4 + \frac{768}{5}a^2x^5 + 1536ax^6 + \frac{32768}{7}x^7 - \frac{32}{3}a^3x^3 - 192a^2x^4 - \frac{6144}{5}ax^5 - \frac{8192}{3}x^6 + a^4x + 16a^3x^2 + 128a^2x^3 + 512ax^4 + \frac{4096}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/13
*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11*x
^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^8 +
 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3*x^4
 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x^4 -
6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*x^4 +
4096/5*x^5
```

maple [B] time = 0.00, size = 264, normalized size = 2.15

$$\frac{x^{17}}{17} - x^{16} + \frac{128x^{15}}{15} - 48x^{14} + \frac{(-4a+2560)\,x^{13}}{13} + \frac{(48a-7424)\,x^{12}}{12} + \frac{(-288a+16768)\,x^{11}}{11} + \frac{(1120a-29696)\,x^{10}}{10} + \frac{\left(2a^2-2560a+24576+(-2a+128)^2\right)x^9}{9} + \frac{\left(-16a^2+3584a-10240+2\,(8a-128)\,(-2a+128)\right)x^8}{8} + \frac{\left(64a^2-2560a+2\,(-16a+64)\,(-2a+128)+(8a-128)^2\right)x^7}{7} + \frac{\left(-160a^2+32\,(-2a+128)\,a+2\,(-16a+64)\,(8a-128)\right)x^6}{6} + a^4x + 16a^3x^2 + \frac{\left(2\,(-2a+128)\,a^2+32\,(8a-128)\,a+(-16a+64)^2\right)x^5}{5} + \frac{\left(2\,(8a-128)\,a^2+32\,(-16a+64)\,a\right)x^4}{4} + \frac{\left(2\,(-16a+64)\,a^2+256a^2\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/17*x^17-x^16+128/15*x^15-48*x^14+1/13*(-4*a+2560)*x^13+1/12*(48*a-7424)*x
^12+1/11*(-288*a+16768)*x^11+1/10*(1120*a-29696)*x^10+1/9*(2*a^2-2560*a+245
76+(-2*a+128)^2)*x^9+1/8*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128))*x^8+
1/7*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^7+1/6*(-160*a^2+3
2*a*(-2*a+128)+2*(-16*a+64)*(8*a-128))*x^6+1/5*(2*a^2*(-2*a+128)+32*a*(8*a-
128)+(-16*a+64)^2)*x^5+1/4*(2*a^2*(8*a-128)+32*a*(-16*a+64))*x^4+1/3*(2*a^2
*(-16*a+64)+256*a^2)*x^3+16*a^3*x^2+a^4*x
```

maxima [A] time = 0.69, size = 192, normalized size = 1.56

$$\frac{1}{17}x^{17} - x^{16} + \frac{128}{15}x^{15} - 48x^{14} + \frac{2560}{13}x^{13} - \frac{1856}{3}x^{12} + \frac{16768}{11}x^{11} - \frac{14848}{5}x^{10} + \frac{40960}{9}x^9 - 5376x^8 + \frac{32768}{7}x^7 - \frac{8192}{3}x^6 + a^4x + \frac{4096}{5}x^5 - \frac{4}{15}\left(3x^5 - 15x^4 + 40x^3 - 60x^2\right)a^3 + \frac{2}{105}\left(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3\right)a^2 - \frac{4}{2145}\left(165x^{13} - 2145x^{12} + 14040x^{11} - 60060x^{10} + 183040x^9 - 411840x^8 + 686400x^7 - 823680x^6 + 658944x^5 - 274560x^4\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 167
68/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3*x
^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 + 2/1
05*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 6720*x^
3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 183040*x^
9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

mupad [B] time = 0.21, size = 175, normalized size = 1.42

$$x^{12}\left(4a-\frac{1856}{3}\right)-x^{13}\left(\frac{4a}{13}-\frac{2560}{13}\right)+x^{10}\left(112a-\frac{14848}{5}\right)-x^{11}\left(\frac{288a}{11}-\frac{16768}{11}\right)-x^8\left(6a^2-768a+5376\right)-x^6\left(80a^2-1536a+\frac{8192}{3}\right)+x^7\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right)+x^9\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)-x^5\left(\frac{4a^3}{5}-\frac{768a^2}{5}+\frac{6144a}{5}-\frac{4096}{5}\right)+a^4x-48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}+16a^3x^2+4ax^4\left(a^2-48a+128\right)-\frac{32a^2x^3\left(a-12\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 - 1
536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3 - (
1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096/5)
+ a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*x^4*(
a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

sympy [A] time = 0.12, size = 199, normalized size = 1.62

$$a^4x+16a^3x^2+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+x^{13}\left(\frac{2560}{13}-\frac{4a}{13}\right)+x^{12}\left(4a-\frac{1856}{3}\right)+x^{11}\left(\frac{16768}{11}-\frac{288a}{11}\right)+x^{10}\left(112a-\frac{14848}{5}\right)+x^9\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)+x^8\left(-6a^2+768a-5376\right)+x^7\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right)+x^6\left(-80a^2+1536a-\frac{8192}{3}\right)+x^5\left(-\frac{4a^3}{5}+\frac{768a^2}{5}-\frac{6144a}{5}+\frac{4096}{5}\right)+x^4\left(4a^3-192a^2+512a\right)+x^3\left(-\frac{32a^3}{3}+128a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13*
(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) + x
**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6*a*
*2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a**2
+ 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5) + x*
*4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

3.113 $\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=120

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6$$

Rubi [A] time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+a^3x-\frac{1}{3}(256-a)x^9+3(64-a)x^8-\frac{32}{7}(70-3a)x^7+8(48-5a)x^6+8(8-a)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 + 8*(8 - a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512 -
 128*a + a^2)*x^5)/5 + 8*(48 - 5*a)*x^6 - (32*(70 - 3*a)*x^7)/7 + 3*(64 - a
)*x^8 - ((256 - a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = \int \left(a^3+24a^2x+24(8-a)ax^2+4\left(128-96a+3a^2\right)x^3-3\left(512-128a+a^2\right.\right.$$

$$= a^3x+12a^2x^2+8(8-a)ax^3+\left(128-96a+3a^2\right)x^4-\frac{3}{5}\left(512-128a+a^2\right)x^5$$

Mathematica [A] time = 0.02, size = 114, normalized size = 0.95

$$a^3x-\frac{3}{5}\left(a^2-128a+512\right)x^5+\left(3a^2-96a+128\right)x^4+12a^2x^2+\frac{1}{3}(a-256)x^9-3(a-64)x^8+\frac{32}{7}(3a-70)x^7-8(5a-48)x^6-8(a-8)ax^3-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
- 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-64
 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]
```

fricas [A] time = 1.08, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^9a-\frac{256}{3}x^9-3x^8a+192x^8+\frac{96}{7}x^7a-320x^7-40x^6a-\frac{3}{5}x^5a^2+384x^6+\frac{384}{5}x^5a+3x^4a^2-\frac{1536}{5}x^5-96x^4a-8x^3a^2+128x^4+64x^3a+12x^2a^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 + 1/3*x^9*a - 256/3*x^9 - 3*x^8*a
+ 192*x^8 + 96/7*x^7*a - 320*x^7 - 40*x^6*a - 3/5*x^5*a^2 + 384*x^6 + 384/5
*x^5*a + 3*x^4*a^2 - 1536/5*x^5 - 96*x^4*a - 8*x^3*a^2 + 128*x^4 + 64*x^3*a
 + 12*x^2*a^2 + x*a^3
```

giac [A] time = 0.30, size = 128, normalized size = 1.07

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + \frac{1}{3}ax^9 + 28x^{10} - 3ax^8 - \frac{256}{3}x^9 + \frac{96}{7}ax^7 + 192x^8 - \frac{3}{5}a^2x^5 - 40ax^6 - 320x^7 + 3a^2x^4 + \frac{384}{5}ax^5 + 384x^6 - 8a^2x^3 - 96ax^4 - \frac{1536}{5}x^5 + a^3x + 12a^2x^2 + 64ax^3 + 128x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
+ 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 384
/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*x^2
 + 64*a*x^3 + 128*x^4
```

maple [A] time = 0.00, size = 138, normalized size = 1.15

$$-\frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + \frac{(3a-768)\,x^9}{9} + \frac{(-24a+1536)\,x^8}{8} + \frac{(96a-2240)\,x^7}{7} + \frac{(-240a+2304)\,x^6}{6} + \frac{\left(-a^2+(-2a+128)\,a+256a-1536\right)x^5}{5} + a^3x + 12a^2x^2 + \frac{\left(4a^2+(8a-128)\,a-256a+512\right)x^4}{4} + \frac{\left(-8a^2+(-16a+64)\,a+128a\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/13*x^13+x^12-72/11*x^11+28*x^10+1/9*(3*a-768)*x^9+1/8*(-24*a+1536)*x^8+1
/7*(96*a-2240)*x^7+1/6*(-240*a+2304)*x^6+1/5*((-2*a+128)*a+256*a-1536-a^2)*
x^5+1/4*((8*a-128)*a-256*a+512+4*a^2)*x^4+1/3*((-16*a+64)*a+128*a-8*a^2)*x^
3+12*a^2*x^2+a^3*x
```

maxima [A] time = 0.67, size = 119, normalized size = 0.99

$$-\frac{1}{13}x^{13} + x^{12} - \frac{72}{11}x^{11} + 28x^{10} - \frac{256}{3}x^9 + 192x^8 - 320x^7 + 384x^6 - \frac{1536}{5}x^5 + a^3x + 128x^4 - \frac{1}{5}\left(3x^5 - 15x^4 + 40x^3 - 60x^2\right)a^2 + \frac{1}{105}\left(35x^9 - 315x^8 + 1440x^7 - 4200x^6 + 8064x^5 - 10080x^4 + 6720x^3\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 60*
x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080
*x^4 + 6720*x^3)*a
```

mupad [B] time = 0.10, size = 108, normalized size = 0.90

$$x^9\left(\frac{a}{3}-\frac{256}{3}\right) - x^8\,(3a-192) - x^6\,(40a-384) + x^7\left(\frac{96a}{7}-320\right) + x^4\left(3a^2-96a+128\right) - x^5\left(\frac{3a^2}{5}-\frac{384a}{5}+\frac{1536}{5}\right) + a^3x + 28x^{10} - \frac{72x^{11}}{11} + x^{12} - \frac{x^{13}}{13} + 12a^2x^2 - 8ax^3\,(a-8)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 32
0) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^3*
x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a - 8)
```

sympy [A] time = 0.09, size = 114, normalized size = 0.95

$$a^3x + 12a^2x^2 - \frac{x^{13}}{13} + x^{12} - \frac{72x^{11}}{11} + 28x^{10} + x^9\left(\frac{a}{3}-\frac{256}{3}\right) + x^8\,(192-3a) + x^7\left(\frac{96a}{7}-320\right) + x^6\,(384-40a) + x^5\left(-\frac{3a^2}{5}+\frac{384a}{5}-\frac{1536}{5}\right) + x^4\left(3a^2-96a+128\right) + x^3\left(-8a^2+64a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(a
/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a) +
x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*(-8
*a**2 + 64*a)
```

3.114 $\int \left(a+8x-8x^2+4x^3-x^4\right)^2 dx$

Optimal. Leaf size=72

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2061}

$$a^2x+\frac{2}{5}(64-a)x^5-2(16-a)x^4+\frac{16}{3}(4-a)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 + (16*(4 - a)*x^3)/3 - 2*(16 - a)*x^4 + (2*(64 - a)*x^5)/5
- (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rule 2061

```
Int[(P_)^(p_), x_Symbol] :> Int[ExpandToSum[P^p, x], x] /; PolyQ[P, x] && I
GtQ[p, 0]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 dx = \int \left(a^2+16ax+16(4-a)x^2-8(16-a)x^3+2(64-a)x^4-80x^5+32x^6-\right.$$

$$=a^2x+8ax^2+\frac{16}{3}(4-a)x^3-2(16-a)x^4+\frac{2}{5}(64-a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.92

$$a^2x-\frac{2}{5}(a-64)x^5+2(a-16)x^4-\frac{16}{3}(a-4)x^3+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5)
/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]
```

fricas [A] time = 1.10, size = 65, normalized size = 0.90

$$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}x^5a+\frac{128}{5}x^5+2x^4a-32x^4-\frac{16}{3}x^3a+\frac{64}{3}x^3+8x^2a+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 - 2/5*x^5*a + 128/5*x^5 + 2*x^4*a - 32*
x^4 - 16/3*x^3*a + 64/3*x^3 + 8*x^2*a + x*a^2
```

giac [A] time = 0.31, size = 65, normalized size = 0.90

$$\frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{2}{5}\,ax^5 - \frac{40}{3}\,x^6 + 2\,ax^4 + \frac{128}{5}\,x^5 - \frac{16}{3}\,ax^3 - 32\,x^4 + a^2x + 8\,ax^2 + \frac{64}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16/
3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

maple [A] time = 0.00, size = 63, normalized size = 0.88

$$\frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + \frac{(-2a+128)\,x^5}{5} + \frac{(8a-128)\,x^4}{4} + a^2x + 8a\,x^2 + \frac{(-16a+64)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/9*x^9-x^8+32/7*x^7-40/3*x^6+1/5*(-2*a+128)*x^5+1/4*(8*a-128)*x^4+1/3*(-16
*a+64)*x^3+8*a*x^2+a^2*x
```

maxima [A] time = 0.71, size = 65, normalized size = 0.90

$$\frac{1}{9}\,x^9 - x^8 + \frac{32}{7}\,x^7 - \frac{40}{3}\,x^6 + \frac{128}{5}\,x^5 - 32\,x^4 + a^2x + \frac{64}{3}\,x^3 - \frac{2}{15}\left(3\,x^5 - 15\,x^4 + 40\,x^3 - 60\,x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^3
 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

mupad [B] time = 0.04, size = 61, normalized size = 0.85

$$x^4\,\left(2\,a-32\right) - x^3\,\left(\frac{16\,a}{3} - \frac{64}{3}\right) - x^5\,\left(\frac{2\,a}{5} - \frac{128}{5}\right) + 8\,a\,x^2 + a^2\,x - \frac{40\,x^6}{3} + \frac{32\,x^7}{7} - x^8 + \frac{x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

sympy [A] time = 0.08, size = 65, normalized size = 0.90

$$a^2x + 8ax^2 + \frac{x^9}{9} - x^8 + \frac{32x^7}{7} - \frac{40x^6}{3} + x^5\left(\frac{128}{5} - \frac{2a}{5}\right) + x^4\,(2a-32) + x^3\left(\frac{64}{3} - \frac{16a}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 - 2
*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

3.115 $\int \left(a+8x-8x^2+4x^3-x^4\right) \, dx$

Optimal. Leaf size=26

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right) \, dx=ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+8x-8x^2+4x^3-x^4\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
[Out] IntegrateAlgebraic[a + 8*x - 8*x^2 + 4*x^3 - x^4, x]
```

fricas [A] time = 0.68, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5+x^4-\frac{8}{3}x^3+4x^2+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + 4*x^2 + x*a
```

giac [A] time = 0.30, size = 22, normalized size = 0.85

$$-\frac{1}{5}x^5+x^4-\frac{8}{3}x^3+ax+4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.88

$$-\frac{1}{5}x^5+x^4-\frac{8}{3}x^3+ax+4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^4+4*x^3-8*x^2+a+8*x,x)
```

```
[Out] a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

maxima [A] time = 0.63, size = 22, normalized size = 0.85

$$-\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
```

```
[Out] -1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

mupad [B] time = 0.02, size = 22, normalized size = 0.85

$$-\frac{x^5}{5}+x^4-\frac{8\,x^3}{3}+4\,x^2+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
```

```
[Out] a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

sympy [A] time = 0.06, size = 22, normalized size = 0.85

$$ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

```
[Out] a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

3.116 $\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal. Leaf size=89

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1106, 1093, 204}

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}} + \frac{\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x^2}\,dx, x, -1+x\right)}{2\sqrt{4+a}}$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.64

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1 -
 3*#1^2 + #1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1), x]
```

fricas [B] time = 0.88, size = 457, normalized size = 5.13

$$\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)-\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(-\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(-\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a +
 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(((
a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) + x -
 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 +
 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*
sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)
) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1
)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36
) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)/(a^2 + 7
*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 +
33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34]-sqrt(1/16)*sqrt(1/256*(256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3
-11*a^2-40*a-48))*ln(abs(-a^5*sqrt(a+4)+a^5+a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*x-a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-17*a^4*sqrt(a+
4)+17*a^4-a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x+a^3*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+14*a^3*sqrt(sqrt(a+4)*(-a^2-
7*a-12)+a^2+7*a+12)*x-14*a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-111*a
^3*sqrt(a+4)+111*a^3-10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a
+4)*x+10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)+69*a^2*sqrt
(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-69*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+
a^2+7*a+12)-351*a^2*sqrt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2
+7*a+12)*sqrt(a+4)*x+29*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4
)+144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-144*a*sqrt(sqrt(a+4)*(-a
^2-7*a-12)+a^2+7*a+12)-544*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(-a^2-7*a-12
)+a^2+7*a+12)*sqrt(a+4)*x+28*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(
a+4)+112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x-112*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)-336*sqrt(a+4)+336))+sqrt(1/16)*sqrt(1/256*(256*sqrt(a
+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*ln(abs(-a^5*sqrt(a+4)+a^5-a^4*
sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+a^4*sqrt(sqrt(a+4)*(-a^2-7*a-12)
+a^2+7*a+12)-17*a^4*sqrt(a+4)+17*a^4+a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7
*a+12)*sqrt(a+4)*x-a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)-1
4*a^3*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+14*a^3*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)-111*a^3*sqrt(a+4)+111*a^3+10*a^2*sqrt(sqrt(a+4)*(-a^2
-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-10*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*
a+12)*sqrt(a+4)-69*a^2*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*x+69*a^2*sq
rt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-351*a^2*sqrt(a+4)+351*a^2+29*a*sqrt(
sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-29*a*sqrt(sqrt(a+4)*(-a^2-7
*a-12)+a^2+7*a+12)*sqrt(a+4)-144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)
*x+144*a*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-544*a*sqrt(a+4)+544*a+28*
sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)*sqrt(a+4)*x-28*sqrt(sqrt(a+4)*(-a^
2-7*a-12)+a^2+7*a+12)*sqrt(a+4)-112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12
)*x+112*sqrt(sqrt(a+4)*(-a^2-7*a-12)+a^2+7*a+12)-336*sqrt(a+4)+336))-sqrt(1
/16)*sqrt(1/256*(-256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*a-48))*ln
(abs(a^5*sqrt(a+4)+a^5+a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x-a^4*sq
rt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+17*a^4*sqrt(a+4)+17*a^4+a^3*sqrt(sqrt
(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+
a^2+7*a+12)*sqrt(a+4)+14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*x-14*a
^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+111*a^3*sqrt(a+4)+111*a^3+10*a^2
*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-10*a^2*sqrt(sqrt(a+4)*
(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7
*a+12)*x-69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+351*a^2*sqrt(a+4)+3
51*a^2+29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-29*a*sqrt(s
qrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+144*a*sqrt(sqrt(a+4)*(a^2+7*a+1
2)+a^2+7*a+12)*x-144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+544*a*sqrt(a
+4)+544*a+28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x-28*sqrt(sq
rt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)+112*sqrt(sqrt(a+4)*(a^2+7*a+12)+
a^2+7*a+12)*x-112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+336*sqrt(a+4)+336
))+sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(a+4)-256*a-1024)/(-a^3-11*a^2-40*
a-48))*ln(abs(a^5*sqrt(a+4)+a^5-a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)
```

```
*x+a^4*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+17*a^4*sqrt(a+4)+17*a^4-a^3*
sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+a^3*sqrt(sqrt(a+4)*(a^2
+7*a+12)+a^2+7*a+12)*sqrt(a+4)-14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+1
2)*x+14*a^3*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+111*a^3*sqrt(a+4)+111*a
^3-10*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+10*a^2*sqrt(s
qrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+
12)+a^2+7*a+12)*x+69*a^2*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+351*a^2*sq
rt(a+4)+351*a^2-29*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+29
*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-144*a*sqrt(sqrt(a+4)*(
a^2+7*a+12)+a^2+7*a+12)*x+144*a*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+544
*a*sqrt(a+4)+544*a-28*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)*x+2
8*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)*sqrt(a+4)-112*sqrt(sqrt(a+4)*(a^2
+7*a+12)+a^2+7*a+12)*x+112*sqrt(sqrt(a+4)*(a^2+7*a+12)+a^2+7*a+12)+336*sqrt
(a+4)+336))
```

maple [C] time = 0.02, size = 49, normalized size = 0.55

$$-\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{4\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(1/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.58, size = 571, normalized size = 6.42

$$-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x\,1\mathrm{i}+a^2\,1\mathrm{i}+16\mathrm{i}}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}}\right)\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}\,2\mathrm{i}-\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}-x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}+\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a^2\,x\,1\mathrm{i}+a^2\,1\mathrm{i}+16\mathrm{i}}{160\,a\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+44\,a^2\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}}\right)\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] - atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a -
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/
2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(64
0*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3 + 64)
^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a + 12*a^2
+ a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i - atan(-(
a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (48*a + 12*
a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a + (48*a + 12
*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*(
(a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768)
)^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a
^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) +
4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 12*a^2 + a^3 + 6
4)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

sympy [A] time = 0.93, size = 66, normalized size = 0.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a-128\right)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3a+768t^3-4ta-20t+x-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 12
8) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*a
- 20*_t + x - 1)))
```

3.117 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$

Optimal. Leaf size=169

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-}}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a}}}$$

Rubi [A] time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1106, 1092, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 +
 a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[
4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sq
rt[1 + Sqrt[4 + a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-2(4+}{3+a-2x}\right.}{8\left(12+\right.}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} - \frac{\left(10+3a-\sqrt{4+a}\right)\text{S}}{8(3}$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(10+3a+\sqrt{4+a}\right)\text{t}}{8(3+a)(4+a)^{3/2}}$$

Mathematica [C] time = 0.07, size = 150, normalized size = 0.89

$$\frac{(x-1)\left(a+x^2-2x+6\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2\log(x-\#1)+3a\log(x-\#1)-2\#1\log(x-\#1)+12\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] ((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1] +
3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2
+ #1^3) & ]/(16*(12 + 7*a + a^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2), x]
```

fricas [B] time = 0.86, size = 1948, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^
2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^
6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 1
83*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a
 + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 29
75*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 96
1)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3
+ 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21
```

```
*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a +
 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a
^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4
 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7*a + 12)*
x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^
2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 +
2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 302
4*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 408*a^3 + 2
309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 +
24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088
*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656))
 + 5800*a + 5456)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a
^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3
088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 4665
6)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a +
1728)) - 567*a - 992) - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^
3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*
a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((8
1*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598
*a^4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 2
1*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^
2 + 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513
*a^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 5
58*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 -
(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 +
 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5
+ 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 8
47*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^
8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 +
128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*
a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x - (27*a^4 + 40
8*a^3 + 2309*a^2 + 2*(2*a^7 + 49*a^6 + 513*a^5 + 2975*a^4 + 10321*a^3 + 214
20*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a
^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304*a
+ 46656)) + 5800*a + 5456)*sqrt((15*a^2 - (a^6 + 21*a^5 + 183*a^4 + 847*a^3
 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9 + 30*a^8 + 39
9*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 156492*a^2 + 128304
*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 +
3024*a + 1728)) - 567*a - 992) + 4*(a + 8)*x - 12*x^2 - 4*a - 24)/((a^2 + 7
*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^
2 - 8*(a^2 + 7*a + 12)*x - 12*a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
```

for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34](x^3-3*x^2+x*a+8*x-a-6)/(-4*a^2-28*a-48)/(x^4-4*x^3+8*x^2-8*x
-a)+(sqrt(1/16)*sqrt(1/256*(256*sqrt(a+4)*(9*a^3+103*a^2+392*a+496)+3840*a^
3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(243*a^10*sqrt(a+4
)+324*a^10+8640*a^9*sqrt(a+4)+11466*a^9+81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^
3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-81*a^8*sqrt(s
qrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a
+2208)+138027*a^8*sqrt(a+4)+182314*a^8+81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3
+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-81*a
^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a
^2+2548*a+2208)*sqrt(a+4)+2340*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+16
72*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^7*sqrt(sqrt(a+4)*(
9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+130
4648*a^7*sqrt(a+4)+1715172*a^7+2016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a
^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-2016*a^6*s
qrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2
548*a+2208)*sqrt(a+4)+29518*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*
a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-29518*a^6*sqrt(sqrt(a+4)*(9*
a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+80797
49*a^6*sqrt(a+4)+10572392*a^6+21454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a
^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-21454*a^5*
sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)+212356*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+167
2*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-212356*a^5*sqrt(sqrt(a+4)*
(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34
255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+
701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-12654
0*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+109
9*a^2+2548*a+2208)*sqrt(a+4)+952845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a
^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-952845*a^4*sqrt(sqrt
(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+22
08)+100679657*a^4*sqrt(a+4)+130513730*a^4+446685*a^3*sqrt(sqrt(a+4)*(9*a^4+
130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)
*x-446685*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*
a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2730184*a^3
sqrt(sqrt(a+4)(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2
+2548*a+2208)+202540404*a^3*sqrt(a+4)+261341928*a^3+943444*a^2*sqrt(sqrt(a+
4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)
*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+4877364*a^2*sqrt(sqrt(a+4)*(
9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4
877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^
3+1099*a^2+2548*a+2208)+266882676*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqr
t(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+254
8*a+2208)*sqrt(a+4)*x-1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*
a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+4965684*a*sqrt(sqrt(
a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*x-4965684*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+21
0*a^3+1099*a^2+2548*a+2208)+207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(s
qrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a
+2208)*sqrt(a+4)*x-551332*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2205328*sqrt(sqrt(a+4)*(9*
a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-220
5328*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099
*a^2+2548*a+2208)+72775824*sqrt(a+4)+92623776))-sqrt(1/16)*sqrt(1/256*(256*
sqrt(a+4)*(9*a^3+103*a^2+392*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^

```
3+11*a^2+40*a+48))*ln(abs(243*a^10*sqrt(a+4)+324*a^10+8640*a^9*sqrt(a+4)+11
466*a^9-81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+21
0*a^3+1099*a^2+2548*a+2208)*x+81*a^8*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+
1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+138027*a^8*sqrt(a+4)+1823
14*a^8-81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+81*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3
+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2340*a
^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a
^2+2548*a+2208)*x+2340*a^7*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+148
8)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+1304648*a^7*sqrt(a+4)+1715172*a^7-2
016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*sqrt(a+4)*x+2016*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701
*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-29518*a^6*
sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*x+29518*a^6*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)
+15*a^4+210*a^3+1099*a^2+2548*a+2208)+8079749*a^6*sqrt(a+4)+10572392*a^6-21
454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*sqrt(a+4)*x+21454*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+70
1*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^
5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*x+212356*a^5*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+34255200*a^5*sqrt(a+4)+44613658*a^
5-126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*
a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+126540*a^4*sqrt(sqrt(a+4)*(9*a^4+130*
a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952
845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+167
2*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+100679657*a^4*sqrt(a+4)+1305
13730*a^4-446685*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*
a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+446685*a^3*sqrt(sqrt(a+4)*(9*
a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(
a+4)-2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+
210*a^3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(9*a^4+130*a^3+7
01*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+202540404*a^3*sqrt
(a+4)+261341928*a^3-943444*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a
+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+943444*a^2*sqrt(sqr
t(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2
208)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+148
8)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(9*a^4
+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+26688267
6*a^2*sqrt(a+4)+342778384*a^2-1103588*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a
^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+1103588*a*
sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672
*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)*(9
*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+2079
74132*a*sqrt(a+4)+265897256*a-551332*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+
1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+551332*sqrt(s
qrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a
+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(9*a^4+130*a^3+701*a^2+1672*a+1488)
+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(9*a^4+130*a
^3+701*a^2+1672*a+1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)+72775824*sqrt(
a+4)+92623776))+sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(9*a^3+103*a^2+392*a+
496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(-243*
a^10*sqrt(a+4)+324*a^10-8640*a^9*sqrt(a+4)+11466*a^9+81*a^8*sqrt(sqrt(a+4)*
(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x
-81*a^8*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+
1099*a^2+2548*a+2208)-138027*a^8*sqrt(a+4)+182314*a^8-81*a^7*sqrt(sqrt(a+4)
*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*
```

```
sqrt(a+4)*x+81*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a
^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+2340*a^7*sqrt(sqrt(a+4)*(-9*a^4-
130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2340*a^
7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a
^2+2548*a+2208)-1304648*a^7*sqrt(a+4)+1715172*a^7-2016*a^6*sqrt(sqrt(a+4)*(
-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sq
rt(a+4)*x+2016*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a
^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+29518*a^6*sqrt(sqrt(a+4)*(-9*a^4
-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-29518*
a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099
*a^2+2548*a+2208)-8079749*a^6*sqrt(a+4)+10572392*a^6-21454*a^5*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*sqrt(a+4)*x+21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)
+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+212356*a^5*sqrt(sqrt(a+4)*(
-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-
212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a
^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5-126540*a^4*sqr
t(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+25
48*a+2208)*sqrt(a+4)*x+126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-16
72*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+952845*a^4*sqrt(s
qrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*
a+2208)*x-952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15
*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^4-44
6685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3
+1099*a^2+2548*a+2208)*sqrt(a+4)*x+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^
3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)+27301
84*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1
099*a^2+2548*a+2208)*x-2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4)+26
1341928*a^3-943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+943444*a^2*sqrt(sqrt(a+4)*
(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*s
qrt(a+4)+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15
*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130
*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-266882676*a^
2*sqrt(a+4)+342778384*a^2-1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-
1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+1103588*a*sqr
t(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+25
48*a+2208)*sqrt(a+4)+4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-4965684*a*sqrt(sqrt(a+4)*(-9
*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-2079
74132*a*sqrt(a+4)+265897256*a-551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2
-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x+551332*sqrt(
sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548
*a+2208)*sqrt(a+4)+2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x-2205328*sqrt(sqrt(a+4)*(-9*a^4-1
30*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-72775824*s
qrt(a+4)+92623776))-sqrt(1/16)*sqrt(1/256*(-256*sqrt(a+4)*(9*a^3+103*a^2+39
2*a+496)+3840*a^3+42240*a^2+154624*a+188416)/(a^3+11*a^2+40*a+48))*ln(abs(-
243*a^10*sqrt(a+4)+324*a^10-8640*a^9*sqrt(a+4)+11466*a^9-81*a^8*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*x+81*a^8*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*
a^3+1099*a^2+2548*a+2208)-138027*a^8*sqrt(a+4)+182314*a^8+81*a^7*sqrt(sqrt(
a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+22
08)*sqrt(a+4)*x-81*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2340*a^7*sqrt(sqrt(a+4)*(-9*
a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+234
0*a^7*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+10
99*a^2+2548*a+2208)-1304648*a^7*sqrt(a+4)+1715172*a^7+2016*a^6*sqrt(sqrt(a+
```

```
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*sqrt(a+4)*x-2016*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+
15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-29518*a^6*sqrt(sqrt(a+4)*(-9
*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+29
518*a^6*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+
1099*a^2+2548*a+2208)-8079749*a^6*sqrt(a+4)+10572392*a^6+21454*a^5*sqrt(sqr
t(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+
2208)*sqrt(a+4)*x-21454*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1
488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-212356*a^5*sqrt(sqrt(a+
4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208
)*x+212356*a^5*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+2
10*a^3+1099*a^2+2548*a+2208)-34255200*a^5*sqrt(a+4)+44613658*a^5+126540*a^4
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)*x-126540*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^
2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-952845*a^4*sq
rt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2
548*a+2208)*x+952845*a^4*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-100679657*a^4*sqrt(a+4)+130513730*a^
4+446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210
*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-446685*a^3*sqrt(sqrt(a+4)*(-9*a^4-13
0*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)-2
730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a
^3+1099*a^2+2548*a+2208)*x+2730184*a^3*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a
^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-202540404*a^3*sqrt(a+4
)+261341928*a^3+943444*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-14
88)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-943444*a^2*sqrt(sqrt(a
+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+220
8)*sqrt(a+4)-4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488
)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4877364*a^2*sqrt(sqrt(a+4)*(-9*a^4
-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-26688267
6*a^2*sqrt(a+4)+342778384*a^2+1103588*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*
a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-1103588*a
*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^
2+2548*a+2208)*sqrt(a+4)-4965684*a*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1
672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+4965684*a*sqrt(sqrt(a+4)
*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-
207974132*a*sqrt(a+4)+265897256*a+551332*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701
*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*sqrt(a+4)*x-551332*s
qrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+
2548*a+2208)*sqrt(a+4)-2205328*sqrt(sqrt(a+4)*(-9*a^4-130*a^3-701*a^2-1672*
a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)*x+2205328*sqrt(sqrt(a+4)*(-9*a
^4-130*a^3-701*a^2-1672*a-1488)+15*a^4+210*a^3+1099*a^2+2548*a+2208)-727758
24*sqrt(a+4)+92623776)))/(4*a^2+28*a+48)
```

maple [C] time = 0.01, size = 158, normalized size = 0.93

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-3a-12\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{16\,(a+3)\,(a+4)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}+\frac{-\frac{x^3}{4\left(a^2+7a+12\right)}+\frac{3x^2}{4\left(a^2+7a+12\right)}-\frac{(a+8)x}{4\left(a^2+7a+12\right)}+\frac{a+6}{4a^2+28a+48}}{x^4-4x^3+8x^2-a-8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3+3/4/(a^2+7*a+12)*x^2-1/4*(8+a)/(a^2+7*a+12)*x+1/4*(6
+a)/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(3+a)/(4+a)*sum((-_R^2+2*_R-
3*a-12)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{x^3+(a+8)x-3x^2-a-6}{4\left(\left(a^2+7a+12\right)x^4-4\left(a^2+7a+12\right)x^3-a^3+8\left(a^2+7a+12\right)x^2-7a^2-8\left(a^2+7a+12\right)x-12a\right)}-\frac{\int\frac{x^2+3a-2x+12}{x^4-4x^3+8x^2-a-8x}\,dx}{4\left(a^2+7a+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a
 + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x -
12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x
), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 5.35, size = 4591, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
+ 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632
*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592))
)^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^
5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576
)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/
(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2
) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/
(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 1067
52*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2)
 + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*
a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^
9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a
+ 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a + 4)^9)^(1/2) - 3
1*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*
(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 48
3*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997
696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2
 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 +
 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*(
(15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(
816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 1
44*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(
61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a +
 32)/(32*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a - 9*
a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
+ 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22
488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*
a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 943718
4)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 1
17760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1
/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 3064
32*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 +
 a^9 + 110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((
```

15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^
3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81
744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) +
(5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a
^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a
^4 + 144))) - ((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 82
08*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 +
197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1
10592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9
0112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14
7456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4
115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2
+ 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^
3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9
)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a +
306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a
^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*
(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))))*((15552*a - 9*a*((a + 4)^9)^(1/2)
- 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(
256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6
+ 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*2i + atan(-(((15552*a + 9*a*((a
+ 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a
^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^
5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10
878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64
*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*
a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3
+ a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) +
8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2
+ 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +
110592)))^(1/2) - (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^
5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*
a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 28
5*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^
4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568
*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a
^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 1
44)))*1i + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*
a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 19
7632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 1105
92)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9011
2*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 14745
6))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^
(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 117
76)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115
*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 +
106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 +
18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a +
460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(16
8*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i)/((9*a + 32)/(32*(816*a + 460*a^2 +
129*a^3 + 18*a^4 + a^5 + 576)) + ((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((

```
a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276
480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^
7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*
a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 460
8*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((155
52*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 +
 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744
*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (7
33184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*
a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*((15552*a + 9*a*((a + 4)^9)^(
1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 1177
6)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*
a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a
^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a
 + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))) - ((15552*a + 9
*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4
 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 2
2488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*((((15728640
*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 90112*a^5 + 4096*a^6 + 94371
84)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(208896*a +
117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147456))/(4*(168*a + 73*a^2 +
 14*a^3 + a^4 + 144)))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(
1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306
432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8
+ a^9 + 110592)))^(1/2) + (733184*a + 396288*a^2 + 106752*a^3 + 14336*a^4 +
 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*(
(15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a
^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 8
1744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)
+ (5568*a + 1552*a^2 + 144*a^3 + 6656)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 + 104))/(4*(168*a + 73*a^2 + 14*a^3 +
a^4 + 144)))))*((15552*a + 9*a*((a + 4)^9)^(1/2) + 31*((a + 4)^9)^(1/2) + 8
208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2
+ 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 +
110592)))^(1/2)*2i + (x^3/(4*(7*a + a^2 + 12)) - (a + 6)/(4*(a + 3)*(a + 4)
) - (3*x^2)/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(4*(a + 3)*(a + 4)))/(a + 8*x
 - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 6.32, size = 294, normalized size = 1.74

$$\frac{a - x^3 + 3x^2 + x(-a-8) + 6}{-4a^3 - 28a^2 - 48a + x^4(4a^2 + 28a + 48) + x^3(-16a^2 - 112a - 192) + x^2(32a^2 + 224a + 384) + x(-32a^2 - 224a - 384)} + \operatorname{RootSum}\left(t^4(65536a^9 + 2162688a^8 + 31653888a^7 + 269680640a^6 + 1473773568a^5 + 5357174784a^4 + 12952010752a^3 + 20082327552a^2 + 18119393280a + 7247757312) + t^2(-7680a^5 - 145920a^4 - 1107968a^3 - 4202496a^2 - 7962624a - 6029312) - 81a^2 - 576a - 1024, \left(t \mapsto t\log\left(x + \frac{-16384t^3a^7 - 401408t^3a^6 - 4202496t^3a^5 - 24371200t^3a^4 - 84549632t^3a^3 - 175472640t^3a^2 - 201719808t^3a - 99090432t^3 + 432ta^4 + 7488ta^3 + 47024ta^2 + 128096ta + 128512t - 81a^2 - 567a - 992}{81a^2 + 567a + 992}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a*
*2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 3
84) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**
8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-7
680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 6029312)
 - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7 - 40140
8*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*_t**3*a*
*3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 432*_t*a**
4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a**2 - 567*
a - 992)/(81*a**2 + 567*a + 992))))
```

3.118 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$

Optimal. Leaf size=252

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\cdots$$

Rubi [A] time = 0.53, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1106, 1092, 1178, 1166, 204}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(32*(3 + a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a
^2 + 14*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt
[4 + a]]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*
a - (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]
)/(64*(3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^3}\,dx, x, -1+x\right)$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{4+2(3+a)-4(4 \cdots}{(3+a-2 \cdots}\right.}{16\left(12 \cdots\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(\cdots\right.}{8\left(12+7a+a^2\right)\left(3+ \cdots\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(\cdots\right.}{8\left(12+7a+a^2\right)\left(3+ \cdots\right.}$$

$$= -\frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32\left(12+7a+a^2\right)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(\cdots\right.}{8\left(12+7a+a^2\right)\left(3+ \cdots\right.}$$

Mathematica [C] time = 0.15, size = 254, normalized size = 1.01

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{4\#1^2 a\log(x-\#1)+14\#1^2\log(x-\#1)+7a^2\log(x-\#1)+55a\log(x-\#1)-8\#1 a\log(x-\#1)+108\log(x-\#1)-28\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{\left(a^2+7a+12\right)^2}+\frac{4(x-1)\left(7a^2+a\left(12x^2-24x+79\right)+6\left(7x^2-14x+32\right)\right)}{(a+3)^2(a+4)^2\left(a-x\left(x^3-4x^2+8x-8\right)\right)}+\frac{16(x-1)\left(a+x^2-2x+6\right)}{(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] ((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^
2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x +
12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*RootS
um[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x - #1
] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Log[x -
 #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*
a + a^2)^2)/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3), x]
```

fricas [B] time = 1.67, size = 3971, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a + 7)*x^7 - 168*(2*a + 7)*x^6 + 4*(7*a^2 + 343*a + 1116)*x^5
 - 20*(7*a^2 + 175*a + 528)*x^4 + 8*(34*a^2 + 679*a + 1968)*x^3 + 44*a^3 -
8*(32*a^2 + 623*a + 1800)*x^2 - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2
 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 +
14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^
4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16
*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 +
14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 +
 (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4
+ 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 +
 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 2
41870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^
2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 +
5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 +
725760*a + 248832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^
4 + 33614*a^3 + 177061*a^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6
+ 100811*a^5 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11
 + 8881*a^10 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410
692*a^5 + 117844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 338
41152)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 +
 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^
9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940
*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a
 + 3543424)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8
+ 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2
+ 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 3
46921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^1
0 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a
^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176
)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129
367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 1122
8868*a - 9923472) + 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 +
 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 1
44)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(
a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a
^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*
a^3 + 168*a^2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*
a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3
 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2
+ 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11
 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6
 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 27713664
0*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 3
1085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 2
48832))*log(-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^
3 + 177061*a^2 + 415884*a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5
 + 628887*a^4 + 2354874*a^3 + 5293208*a^2 - (11*a^12 + 462*a^11 + 8881*a^10
 + 103320*a^9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 11
7844800*a^4 + 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt(
(2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1
```

```
165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320
*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 78207
1200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*
sqrt((105*a^4 + 1470*a^3 + 7749*a^2 + (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 +
 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a +
 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15
 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*
a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 7143179
40*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a
 + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 37
3020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 992
3472) - 3*((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^
6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4
 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151
*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
- 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^
2 + 144*a)*x)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^
8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^
2 + 725760*a + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a +
 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a
^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313
*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 604661
76)) + 18228*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 1
29367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(
-64827*a^4 - 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a
^2 + 415884*a + 367536)*x + 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^
4 + 2354874*a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^
9 + 810205*a^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4
+ 181238400*a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 +
33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 1
6780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320
045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 5
92064640*a^2 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^
4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 +
 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqr
t((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 +
 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 285703
20*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782
071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a
^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 7
35840*a^3 + 950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 3*((
a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4
+ 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 -
 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*
a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 -
528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x
)*sqrt((105*a^4 + 1470*a^3 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7
 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a
 + 248832)*sqrt((2401*a^4 + 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^
15 + 50*a^14 + 1165*a^13 + 16780*a^12 + 167090*a^11 + 1218460*a^10 + 672213
0*a^9 + 28570320*a^8 + 94320045*a^7 + 241870050*a^6 + 477857313*a^5 + 71431
7940*a^4 + 782071200*a^3 + 592064640*a^2 + 277136640*a + 60466176)) + 18228
*a + 16144)/(a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 +
373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832))*log(-64827*a^4 -
 907578*a^3 - 4780647*a^2 + 27*(2401*a^4 + 33614*a^3 + 177061*a^2 + 415884*
a + 367536)*x - 27*(343*a^7 + 8981*a^6 + 100811*a^5 + 628887*a^4 + 2354874*
a^3 + 5293208*a^2 + (11*a^12 + 462*a^11 + 8881*a^10 + 103320*a^9 + 810205*a
```

```
^8 + 4511542*a^7 + 18292039*a^6 + 54410692*a^5 + 117844800*a^4 + 181238400*
a^3 + 187875072*a^2 + 117863424*a + 33841152)*sqrt((2401*a^4 + 33124*a^3 +
171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 + 16780*a^12 +
167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 94320045*a^7 + 24
1870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 + 592064640*a^2
 + 277136640*a + 60466176)) + 6613472*a + 3543424)*sqrt((105*a^4 + 1470*a^3
 + 7749*a^2 - (a^10 + 35*a^9 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5
+ 373020*a^4 + 735840*a^3 + 950400*a^2 + 725760*a + 248832)*sqrt((2401*a^4
+ 33124*a^3 + 171966*a^2 + 398164*a + 346921)/(a^15 + 50*a^14 + 1165*a^13 +
 16780*a^12 + 167090*a^11 + 1218460*a^10 + 6722130*a^9 + 28570320*a^8 + 943
20045*a^7 + 241870050*a^6 + 477857313*a^5 + 714317940*a^4 + 782071200*a^3 +
 592064640*a^2 + 277136640*a + 60466176)) + 18228*a + 16144)/(a^10 + 35*a^9
 + 550*a^8 + 5110*a^7 + 31085*a^6 + 129367*a^5 + 373020*a^4 + 735840*a^3 +
950400*a^2 + 725760*a + 248832)) - 11228868*a - 9923472) + 524*a^2 - 4*(11*
a^3 + 107*a^2 - 84*a - 1152)*x + 1632*a + 1152)/((a^4 + 14*a^3 + 73*a^2 + 1
68*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 1
4*a^3 + 73*a^2 + 168*a + 144)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a
 + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 921
6)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3
+ 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^
2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a^2 + 144*a)*x)
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a]=[86]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a]=[12]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a]=[48]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a]=[34]-(-12*x^7*a-42*x^7+84*x^6*a+294*x^6-7*x^5*a^2-343*x^5*a-1116*
x^5+35*x^4*a^2+875*x^4*a+2640*x^4-68*x^3*a^2-1358*x^3*a-3936*x^3+64*x^2*a^2
+1246*x^2*a+3600*x^2+11*x*a^3+107*x*a^2-84*x*a-1152*x-11*a^3-131*a^2-408*a-
288)/(-32*a^4-448*a^3-2336*a^2-5376*a-4608)/(x^4-4*x^3+8*x^2-8*x-a)^2-(sqrt
(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a
+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-22798336*a-16531456)/
(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+26411*a^15-908950*a^14*
sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2401*a^
12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-35
7887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401*a^11*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+10515
4*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*x-105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*
a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)-3865394166*a^11*sqrt(a+4)+5945365998*a^11-95550*a^10*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
```

6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+9555
0*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*sqrt(a+4)+2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^
3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)*x-2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^
4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)-30600511272*a^10*sqrt(a+4)+46810709868*a^10-1
727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-1
87408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)*sqrt(a+4)*x+1727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)+25624212*a^9*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-25624212*a^9*sqrt(sqrt(a+4)*(-
49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-183431725500*a^9*sqrt(
a+4)+279067335420*a^9-18715896*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^
4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+18715896*a^8*sqrt(sqrt(a+4)*(-49*
a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^
5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+209972223*a^8*s
qrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-1130
88)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2099
72223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)-847810669320*a^8*sqrt(a+4)+1282741275000*a^8-135108639*a^7*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1351
08639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x-1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-
9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+9
0111*a^3+236728*a^2+331744*a+193728)-3046208716923*a^7*sqrt(a+4)+4583471076
759*a^7-682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1
29188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+
331744*a+193728)*sqrt(a+4)*x+682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+
90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+5187446733*a^6*sqrt(sqrt(a+
4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-5187446733*a^6*
sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113
088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-85085
27261290*a^6*sqrt(a+4)+12731345334296*a^6-2458605429*a^5*sqrt(sqrt(a+4)*(-4
9*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*
a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2458605429*
a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a
-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*s
qrt(a+4)+16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^
3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)*x-16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775
*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)-18324012543204*a^5*sqrt(a+4)+2726574704738
0*a^5-6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-12
9188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3
31744*a+193728)*sqrt(a+4)*x+6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+

90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+36673732452*a^4*sqrt(sqrt(a
+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-36673732452*a^
4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-1
13088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-298
80339194272*a^4*sqrt(a+4)+44213263379848*a^4-11377675428*a^3*sqrt(sqrt(a+4)
*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1137767
5428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)+59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x-59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5
-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+
90111*a^3+236728*a^2+331744*a+193728)-35712575419864*a^3*sqrt(a+4)+52547642
661032*a^3-13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*
a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)*sqrt(a+4)*x+13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1
073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+64340036640*a^2*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-6434003
6640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)-29533943648028*a^2*sqrt(a+4)+43213040637212*a^2-9797208656*a*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+979
7208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18
7408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+42385864836*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4741
9*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)*x-42385864836*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-97
75*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+901
11*a^3+236728*a^2+331744*a+193728)-15111479733208*a*sqrt(a+4)+2198667320430
4*a-3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a
^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)*sqrt(a+4)*x+3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)*sqrt(a+4)+12788120848*sqrt(sqrt(a+4)*(-49*a^6-1
073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*sqrt(sqrt(a+4)*(
-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+220
5*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3606250079136*sqrt(a+
4)+5217553305984))-sqrt(9/16)*sqrt(1/256*(256*sqrt(a+4)*(49*a^5+926*a^4+699
7*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^
2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(-16807*a^15*sqrt(a+4)+
26411*a^15-908950*a^14*sqrt(a+4)+1420804*a^14-22929088*a^13*sqrt(a+4)+35650
176*a^13-2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1291
88*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*x+2401*a^12*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a
^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*
a^2+331744*a+193728)-357887692*a^12*sqrt(a+4)+553458148*a^12+2401*a^11*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)
*x-2401*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2
-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)*sqrt(a+4)-105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x+105154*a^11*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-977

```
5*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)-3865394166*a^11*sqrt(a+4)+5945365998*a^11
+95550*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-
187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*sqrt(a+4)*x-95550*a^10*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-474
19*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)-2109279*a^10*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2109279*a^10*sqrt(sqrt(a+4)*(-
49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-30600511272*a^10*sqrt(
a+4)+46810709868*a^10+1727079*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4
-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3
+236728*a^2+331744*a+193728)*sqrt(a+4)*x-1727079*a^9*sqrt(sqrt(a+4)*(-49*a^
6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+
19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-25624212*a^9*sqrt
(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+2562421
2*a^9*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408
*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)
-183431725500*a^9*sqrt(a+4)+279067335420*a^9+18715896*a^8*sqrt(sqrt(a+4)*(-
49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-18715896*a
^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sq
rt(a+4)-209972223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1
29188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+
331744*a+193728)*x+209972223*a^8*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)-847810669320*a^8*sqrt(a+4)+1282741275000*a^8+13
5108639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-
187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*sqrt(a+4)*x-135108639*a^7*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)*sqrt(a+4)-1222644882*a^7*sqrt(sqrt(a+4)*(-49*a^
6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+
19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+1222644882*a^7*sqrt(sqrt(
a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-3046208716923*a
^7*sqrt(a+4)+4583471076759*a^7+682210326*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a
^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-682210326*a^6*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-5187
446733*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-1
87408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)*x+5187446733*a^6*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-
129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2
+331744*a+193728)-8508527261290*a^6*sqrt(a+4)+12731345334296*a^6+2458605429
*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*
a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*
sqrt(a+4)*x-2458605429*a^5*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*
a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)*sqrt(a+4)-16158435972*a^5*sqrt(sqrt(a+4)*(-49*a^6-107
3*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+16158435972*a^5*sqrt(sqrt(a+4)
*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-18324012543204*a^5*
sqrt(a+4)+27265747047380*a^5+6324014256*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^
```

```
5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4
+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-6324014256*a^4*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-3667
3732452*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-
187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*x+36673732452*a^4*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^
3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a
^2+331744*a+193728)-29880339194272*a^4*sqrt(a+4)+44213263379848*a^4+1137767
5428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187
408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)*x-11377675428*a^3*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4
7419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
36728*a^2+331744*a+193728)*sqrt(a+4)-59146408708*a^3*sqrt(sqrt(a+4)*(-49*a^
6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+
19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+59146408708*a^3*sqrt(sqrt
(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-35712575419864
*a^3*sqrt(a+4)+52547642661032*a^3+13635706996*a^2*sqrt(sqrt(a+4)*(-49*a^6-1
073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-13635706996*a^2*sq
rt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-11308
8)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+
4)-64340036640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-1291
88*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*x+64340036640*a^2*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-4
7419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
36728*a^2+331744*a+193728)-29533943648028*a^2*sqrt(a+4)+43213040637212*a^2+
9797208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2
-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)*sqrt(a+4)*x-9797208656*a*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-
47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+
236728*a^2+331744*a+193728)*sqrt(a+4)-42385864836*a*sqrt(sqrt(a+4)*(-49*a^6
-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+1
9299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+42385864836*a*sqrt(sqrt(a+
4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)-15111479733208*a*
sqrt(a+4)+21986673204304*a+3197030212*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775
*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-3197030212*sqrt(sqrt(a+4)*(-49
*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-113088)+105*a^6+2205*a
^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-12788120848*sq
rt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-187408*a-11308
8)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+12788
120848*sqrt(sqrt(a+4)*(-49*a^6-1073*a^5-9775*a^4-47419*a^3-129188*a^2-18740
8*a-113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)-3606250079136*sqrt(a+4)+5217553305984))+sqrt(9/16)*sqrt(1/256*(-256*sqrt(
a+4)*(49*a^5+926*a^4+6997*a^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4
-3489024*a^3-12601344*a^2-22798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(ab
s(16807*a^15*sqrt(a+4)+26411*a^15+908950*a^14*sqrt(a+4)+1420804*a^14+229290
88*a^13*sqrt(a+4)+35650176*a^13+2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9
775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90
111*a^3+236728*a^2+331744*a+193728)*x-2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073
*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*
a^4+90111*a^3+236728*a^2+331744*a+193728)+357887692*a^12*sqrt(a+4)+55345814
8*a^12+2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*
a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)*sqrt(a+4)*x-2401*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4
7419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2
```

36728*a^2+331744*a+193728)*sqrt(a+4)+105154*a^11*sqrt(sqrt(a+4)*(49*a^6+107
3*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299
*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-105154*a^11*sqrt(sqrt(a+4)*(49
*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a
^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3865394166*a^11*sqrt(a+4
)+5945365998*a^11+95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*sqrt(a+4)*x-95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+2109279*a^10*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-2109279*a^10*sqr
t(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)
+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+306005112
72*a^10*sqrt(a+4)+46810709868*a^10+1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*
a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a
^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-1727079*a^9*sqrt(sqrt(
a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+256242
12*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408
*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)
*x-25624212*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a
^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)+183431725500*a^9*sqrt(a+4)+279067335420*a^9+18715896*a^8*sqrt(sqr
t(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-18
715896*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+18
7408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193
728)*sqrt(a+4)+209972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*x-209972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775
*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)+847810669320*a^8*sqrt(a+4)+1282741275000*a
^8+135108639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*
a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744
*a+193728)*sqrt(a+4)*x-135108639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a
^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a
^3+236728*a^2+331744*a+193728)*sqrt(a+4)+1222644882*a^7*sqrt(sqrt(a+4)*(49*
a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^
5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-1222644882*a^7*sqrt(sqr
t(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*
a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3046208716923*
a^7*sqrt(a+4)+4583471076759*a^7+682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-682210326*a^6*sqrt(sqrt
(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a
^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+51874
46733*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*x-5187446733*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129
188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+33
1744*a+193728)+8508527261290*a^6*sqrt(a+4)+12731345334296*a^6+2458605429*a^
5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11
3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt
(a+4)*x-2458605429*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+1
29188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+
331744*a+193728)*sqrt(a+4)+16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+
9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9
0111*a^3+236728*a^2+331744*a+193728)*x-16158435972*a^5*sqrt(sqrt(a+4)*(49*a

^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5
+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+18324012543204*a^5*sqrt(a+
4)+27265747047380*a^5+6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a
^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a
^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-6324014256*a^4*sqrt(sqrt(a+4)*(4
9*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*
a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+36673732452*a
^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+1
13088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-3
6673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^
2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a
+193728)+29880339194272*a^4*sqrt(a+4)+44213263379848*a^4+11377675428*a^3*sq
rt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088
)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4
)*x-11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+1291
88*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*sqrt(a+4)+59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+977
5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)*x-59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+
1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19
299*a^4+90111*a^3+236728*a^2+331744*a+193728)+35712575419864*a^3*sqrt(a+4)+
52547642661032*a^3+13635706996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4
+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3
+236728*a^2+331744*a+193728)*sqrt(a+4)*x-13635706996*a^2*sqrt(sqrt(a+4)*(49
*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a
^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)+64340036640*a^
2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11
3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x-64
340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2
+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)+29533943648028*a^2*sqrt(a+4)+43213040637212*a^2+9797208656*a*sqrt(s
qrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+10
5*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x-
9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+
187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*sqrt(a+4)+42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+474
19*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*x-42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+97
75*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+901
11*a^3+236728*a^2+331744*a+193728)+15111479733208*a*sqrt(a+4)+2198667320430
4*a+3197030212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^
2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a
+193728)*sqrt(a+4)*x-3197030212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47
419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23
6728*a^2+331744*a+193728)*sqrt(a+4)+12788120848*sqrt(sqrt(a+4)*(49*a^6+1073
*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*
a^4+90111*a^3+236728*a^2+331744*a+193728)*x-12788120848*sqrt(sqrt(a+4)*(49*
a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^
5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+3606250079136*sqrt(a+4)+5
217553305984))-sqrt(9/16)*sqrt(1/256*(-256*sqrt(a+4)*(49*a^5+926*a^4+6997*a
^3+26428*a^2+49904*a+37696)-26880*a^5-483840*a^4-3489024*a^3-12601344*a^2-2
2798336*a-16531456)/(-a^3-11*a^2-40*a-48))*ln(abs(16807*a^15*sqrt(a+4)+2641
1*a^15+908950*a^14*sqrt(a+4)+1420804*a^14+22929088*a^13*sqrt(a+4)+35650176*
a^13-2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^
2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a
+193728)*x+2401*a^12*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129
188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+33
1744*a+193728)+357887692*a^12*sqrt(a+4)+553458148*a^12-2401*a^11*sqrt(sqrt(
a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^

```
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2401
*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*
a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*
sqrt(a+4)-105154*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12
9188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3
31744*a+193728)*x+105154*a^11*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4741
9*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)+3865394166*a^11*sqrt(a+4)+5945365998*a^11-95550*a^1
0*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11
3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt
(a+4)*x+95550*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918
8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317
44*a+193728)*sqrt(a+4)-2109279*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^
4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^
3+236728*a^2+331744*a+193728)*x+2109279*a^10*sqrt(sqrt(a+4)*(49*a^6+1073*a^
5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4
+90111*a^3+236728*a^2+331744*a+193728)+30600511272*a^10*sqrt(a+4)+468107098
68*a^10-1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+1291
88*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331
744*a+193728)*sqrt(a+4)*x+1727079*a^9*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*
a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*
a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-25624212*a^9*sqrt(sqrt(a+4)*(49*a
^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5
+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+25624212*a^9*sqrt(sqrt(a
+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6
+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+183431725500*a^9*
sqrt(a+4)+279067335420*a^9-18715896*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+977
5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+18715896*a^8*sqrt(sqrt(a+4)*(
49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205
*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-209972223*a^
8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+11
3088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+20
9972223*a^8*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+1
87408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+19
3728)+847810669320*a^8*sqrt(a+4)+1282741275000*a^8-135108639*a^7*sqrt(sqrt(
a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^
6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+1351
08639*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)-1222644882*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)*x+1222644882*a^7*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+977
5*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+9011
1*a^3+236728*a^2+331744*a+193728)+3046208716923*a^7*sqrt(a+4)+4583471076759
*a^7-682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+12918
8*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+3317
44*a+193728)*sqrt(a+4)*x+682210326*a^6*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775
*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111
*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-5187446733*a^6*sqrt(sqrt(a+4)*(4
9*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*
a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+5187446733*a^6*sqrt(s
qrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+10
5*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+850852726129
0*a^6*sqrt(a+4)+12731345334296*a^6-2458605429*a^5*sqrt(sqrt(a+4)*(49*a^6+10
73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+1929
9*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+2458605429*a^5*sqrt
(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+
105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-
```

```
16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a
^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*
a+193728)*x+16158435972*a^5*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*
a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728
*a^2+331744*a+193728)+18324012543204*a^5*sqrt(a+4)+27265747047380*a^5-63240
14256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187
408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1937
28)*sqrt(a+4)*x+6324014256*a^4*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+474
19*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236
728*a^2+331744*a+193728)*sqrt(a+4)-36673732452*a^4*sqrt(sqrt(a+4)*(49*a^6+1
073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+192
99*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+36673732452*a^4*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29880339194272*a^4
*sqrt(a+4)+44213263379848*a^4-11377675428*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a
^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^
4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+11377675428*a^3*sqrt(sq
rt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105
*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-591
46408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+
187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+1
93728)*x+59146408708*a^3*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3
+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^
2+331744*a+193728)+35712575419864*a^3*sqrt(a+4)+52547642661032*a^3-13635706
996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+18740
8*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728
)*sqrt(a+4)*x+13635706996*a^2*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+4741
9*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+2367
28*a^2+331744*a+193728)*sqrt(a+4)-64340036640*a^2*sqrt(sqrt(a+4)*(49*a^6+10
73*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+1929
9*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+64340036640*a^2*sqrt(sqrt(a+4
)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2
205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)+29533943648028*a^2*
sqrt(a+4)+43213040637212*a^2-9797208656*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9
775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90
111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+9797208656*a*sqrt(sqrt(a+4)
*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+22
05*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)-4238586483
6*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+
113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*x+
42385864836*a*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2
+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+
193728)+15111479733208*a*sqrt(a+4)+21986673204304*a-3197030212*sqrt(sqrt(a+
4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a+113088)+105*a^6+
2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*sqrt(a+4)*x+319703
0212*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129188*a^2+187408*a
+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+331744*a+193728)*s
qrt(a+4)-12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419*a^3+129
188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+236728*a^2+33
1744*a+193728)*x+12788120848*sqrt(sqrt(a+4)*(49*a^6+1073*a^5+9775*a^4+47419
*a^3+129188*a^2+187408*a+113088)+105*a^6+2205*a^5+19299*a^4+90111*a^3+23672
8*a^2+331744*a+193728)+3606250079136*sqrt(a+4)+5217553305984)))/(32*a^4+448
*a^3+2336*a^2+5376*a+4608)
```

maple [C] time = 0.02, size = 398, normalized size = 1.58

$$-\frac{3\left(2\left(2a+7\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+7a^2+4\left(-2a-7\right)\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+55a+108\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{128\left(a^3+10a^2+33a+36\right)\left(a+4\right)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}-\frac{\frac{3(2a+7)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{21(2a+7)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}+\frac{\left(7a^2+343a+1116\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(34a^2+679a+1968\right)x^3}{16a^4+224a^3+1168a^2+2688a+2304}-\frac{\left(32a^2+623a+1800\right)x^2}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(11a^3+107a^2-84a-1152\right)x}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{11a^3+131a^2+408a+288}{32a^4+448a^3+2336a^2+5376a+4608}}{\left(x^4-4x^3+8x^2-a-8x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16)
/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^5-
5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+679*a
+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+14*a
^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+73*a^2
+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a^4+14*a^3+73*a^2+168*a+144)
)/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(a+4)*sum((108+2*(7+
2*a)*_R^2+4*(-2*a-7)*_R+7*a^2+55*a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootO
f(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{6\,(2\,a+7)x^7-42\,(2\,a+7)x^6+\left(7\,a^2+343\,a+1116\right)x^5-5\left(7\,a^2+175\,a+528\right)x^4+2\left(34\,a^2+679\,a+1968\right)x^3+11\,a^3-2\left(32\,a^2+623\,a+1800\right)x^2+131\,a^2-\left(11\,a^3+107\,a^2-84\,a-1152\right)x+408\,a+288}{32\left(\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^8-8\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^7+32\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^6+a^6-80\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^5+14\,a^5-2\left(a^5-50\,a^4-823\,a^3-4504\,a^2-10608\,a-9216\right)x^4+73\,a^4+8\left(a^5-2\,a^4-151\,a^3-1000\,a^2-2544\,a-2304\right)x^3+168\,a^3-16\left(a^5+10\,a^4+17\,a^3-124\,a^2-528\,a-576\right)x^2+144\,a^2+16\left(a^5+14\,a^4+73\,a^3+168\,a^2+144\,a\right)x\right)}-\frac{3\int\frac{2\,(2\,a+7)x^2+7\,a^2-4\,(2\,a+7)x+55\,a+108}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx}{32\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5*
(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(32*
a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x + 40
8*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3 + 7
3*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6 + a
^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 50*a^
4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4 - 15
1*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17*a^3
 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 168*a
^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a + 7)*x +
55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73*a^2 + 16
8*a + 144)
```

mupad [B] time = 6.41, size = 8242, normalized size = 32.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5
+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) -
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
```

68)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^
15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i - (((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a +
4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +
a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 58
9*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a
^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 5713
9003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 72850636
8*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(9400
32*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 556
4*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001143054
336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255
6119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a
^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328
*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 +
a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144
324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 1638
4*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 23
80*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a - 338*a*((a + 4)

```
^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*
a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 +
3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247
4311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a
^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12
+ 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*
((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 4
1598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 5394
9*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a
^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203
166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 223
50*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a +
74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 662077
44)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 +
 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a +
31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979
2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +
2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((
52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 40271
70816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290
287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a
^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((42906723
28704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 10902
00272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266
176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2
678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7
 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*
a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3932979
2*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^1
5)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 47560
8*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a +
2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5736217
60*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 24496
5*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*
(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((
a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5
 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683
200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 +
 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10
 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/
2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5
+ 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 44286
4*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 33177
6)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a
+ 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 2
0736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/2)
 - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 +
 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16
384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247
703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 +
1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548
03968)))^(1/2) - (99468*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a
```

+ 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
 + 582*a^8 + 36*a^9 + a^10 + 331776))))*((9*(39329792*a - 338*a*((a + 4)^15
)^(1/2) - 589*((a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)*2i + atan((((52357496832*a
+ 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 72
8506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + ((4290672328704*a + 6001
143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 +
 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 4194
3040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 +
899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*
a^9 + a^10 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3
+ 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8
+ 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^
4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((
a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 415
98976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*
a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2
 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 20316
6720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350
*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a +
338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/
2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6
+ 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 206143
4880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6
 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11
 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (10834329
6*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 +
66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208
*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(7347
6*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 +
28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(3
9329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a
+ 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 +
 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168320
0*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 5
73621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 +
 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2)
*1i - (((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a^
4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^
9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 +
 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (
(4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 2800520003584*a^
4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 8275361792*a^8
 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16384*(94
0032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5
564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a + 402024038
4*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 61407232*a^6 + 74
38336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a + 49248*a^2
 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9
*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*(
(a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^
5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(106168
3200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5

+ 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^1
0 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1
/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) +
49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 26
95744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384
*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703
040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 196
6491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 2548039
68)))^(1/2) - (108343296*a + 74059776*a^2 + 27065088*a^3 + 5576256*a^4 + 61
4016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) + (x*(73476*a + 31545*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*
(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7
+ a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^
15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*1i)/((((52357496832*a + 57139003392*a^2 + 363221483
52*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 +
5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 89
9328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^
9 + a^10 + 331776)) + ((4290672328704*a + 6001143054336*a^2 + 5025917042688
*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 598621552
64*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 13
91569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149
208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3
510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*
a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(2
56*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a
^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a +
4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10
187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16
531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 205320192
0*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119
44200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 +
a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 58
9*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*
a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*
a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2
053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a
^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 5
5*a^14 + a^15 + 254803968)))^(1/2) + (108343296*a + 74059776*a^2 + 27065088
*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)/(16384*(940032*a +
1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 +
582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(73476*a + 31545*a^2 + 6066*a^3 +
441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15
)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
+ 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) + (((52357496832*a + 5713900
3392*a^2 + 36322148352*a^3 + 14822473728*a^4 + 4027170816*a^5 + 728506368*a
^6 + 84615168*a^7 + 5726208*a^8 + 172032*a^9 + 21290287104)/(16384*(940032*
a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a
^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - ((4290672328704*a + 6001143054336

```
*a^2 + 5025917042688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 30255611
9040*a^6 + 59862155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10
 + 1048576*a^11 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^
3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^1
0 + 331776)) - (x*(3510632448*a + 4020240384*a^2 + 2678587392*a^3 + 1144324
096*a^4 + 325074944*a^5 + 61407232*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a
^9 + 1358954496))/(256*(48384*a + 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*
a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a + 338*a*((a + 4)^15
)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2
 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 357
0*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2 + 247431
1680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7
+ 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1
410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2))*((9*(39329792*a + 338*a*((a
 + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 4159
8976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a
^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 2061434880*a^2
+ 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*a^6 + 203166
720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a^11 + 22350*
a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (108343296*a + 740
59776*a^2 + 27065088*a^3 + 5576256*a^4 + 614016*a^5 + 28224*a^6 + 66207744)
/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34
833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) + (x*(73476*a + 315
45*a^2 + 6066*a^3 + 441*a^4 + 64656))/(256*(48384*a + 49248*a^2 + 28560*a^3
 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 + 20736)))*((9*(39329792*a
 + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^15)^(1/2) + 49*a^2*((a + 4)^15)^
(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^4 + 2695744*a^5 + 475608*a
^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))/(16384*(1061683200*a + 206
1434880*a^2 + 2474311680*a^3 + 2053201920*a^4 + 1247703040*a^5 + 573621760*
a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*a^9 + 1966491*a^10 + 244965*a
^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15 + 254803968)))^(1/2) - (99468
*a + 28053*a^2 + 2646*a^3 + 117936)/(8192*(940032*a + 1195776*a^2 + 899328*
a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a
^10 + 331776))))*((9*(39329792*a + 338*a*((a + 4)^15)^(1/2) + 589*((a + 4)^
15)^(1/2) + 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187
840*a^4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531
456))/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a
^4 + 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 119442
00*a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^
15 + 254803968)))^(1/2)*2i - ((408*a + 131*a^2 + 11*a^3 + 288)/(32*(a + 4)*
(33*a + 10*a^2 + a^3 + 36)) - (21*x^6*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a
^3 + a^4 + 144)) + (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 1
44)) + (x*(84*a - 107*a^2 - 11*a^3 + 1152))/(32*(a + 4)*(33*a + 10*a^2 + a^
3 + 36)) - (5*x^4*(175*a + 7*a^2 + 528))/(32*(a + 4)*(33*a + 10*a^2 + a^3 +
 36)) + (x^5*(343*a + 7*a^2 + 1116))/(32*(a + 4)*(33*a + 10*a^2 + a^3 + 36)
) - (x^2*(623*a + 32*a^2 + 1800))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)) +
 (x^3*(679*a + 34*a^2 + 1968))/(16*(a + 4)*(33*a + 10*a^2 + a^3 + 36)))/(16
*a*x - x^2*(16*a - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 +
 32*x^6 - 8*x^7 + x^8)
```

sympy [B] time = 15.62, size = 697, normalized size = 2.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**5
*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2 +
 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a**2
```

```
+ 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 4608*a*
*2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-256*a**
4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14336*a**
3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3 - 18688
0*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**3 + 288
256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a**3 - 25
6000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*a**3 +
63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a**3 + 86
016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*a**14 +
 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 52787590
8304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 5453715112
7224320*a**7 + 153980418717122560*a**6 + 334927734494986240*a**5 + 55115219
3655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2 + 28499
3413919539200*a + 68398419340689408) + _t**2*(-30965760*a**9 - 1052835840*a
**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528*a**5 - 3004516270
080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11598827618304*a - 48
75324751872) - 194481*a**4 - 2762424*a**3 - 14762736*a**2 - 35178624*a - 31
539456, Lambda(_t, _t*log(x + (23068672*_t**3*a**12 + 968884224*_t**3*a**11
 + 18624806912*_t**3*a**10 + 216677744640*_t**3*a**9 + 1699123036160*_t**3*
a**8 + 9461389328384*_t**3*a**7 + 38361186172928*_t**3*a**6 + 1141074915491
84*_t**3*a**5 + 247138458009600*_t**3*a**4 + 380084473036800*_t**3*a**3 + 3
94002582994944*_t**3*a**2 + 247177515368448*_t**3*a + 70970039599104*_t**3
- 395136*_t*a**7 - 11676672*_t*a**6 - 144076032*_t*a**5 - 969518592*_t*a**4
 - 3861475200*_t*a**3 - 9133300224*_t*a**2 - 11906574336*_t*a - 6611337216*
_t - 64827*a**4 - 907578*a**3 - 4780647*a**2 - 11228868*a - 9923472)/(64827
*a**4 + 907578*a**3 + 4780647*a**2 + 11228868*a + 9923472))))
```

3.119 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$

Optimal. Leaf size=210

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48\right.$$

Rubi [A] time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{32a^3x^3}{3}+\frac{a^4x^2}{2}+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^2
)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a + 1
5*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)/3
+ ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(524 -
 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224*x^15
)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\int\left(a^4x+32a^3x^2-32(-12+a)a^2x^3+16a\left(128-48a+a^2\right)x^4-4\left(-1024\right.\right.$$
$$=\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5+\frac{2}{3}\left(1024-153\right.$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9+8\left(3a^2-140a+512\right)x^8-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5-8(a-12)a^2x^4-\frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6-\frac{2}{7}(a-640)x^{14}+\frac{16}{13}(3a-464)x^{13}-\frac{8}{3}(9a-524)x^{12}+\frac{32}{11}(35a-928)x^{11}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*x^9
)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 - (8*(
-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/7 - (
224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]
```

fricas [A] time = 0.78, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{224}{5}x^{15} - \frac{2}{7}x^{14}a + \frac{1280}{7}x^{14} + \frac{48}{13}x^{13}a - \frac{7424}{13}x^{13} - 24x^{12}a + \frac{4192}{3}x^{12} + \frac{1120}{11}x^{11}a + \frac{3}{5}x^{10}a^2 - \frac{29696}{11}x^{11} - \frac{1536}{5}x^{10}a - \frac{16}{3}x^9a^2 + 4096x^{10} + \frac{2048}{3}x^9a + 24x^8a^2 - \frac{14336}{3}x^9 - 1120x^8a - \frac{480}{7}x^7a^2 - \frac{2}{3}x^6a^3 + 4096x^8 + \frac{9216}{7}x^7a + 128x^6a^2 + \frac{16}{5}x^5a^3 - \frac{16384}{7}x^7 - 1024x^6a - \frac{768}{5}x^5a^2 - 8x^4a^3 + \frac{2048}{3}x^6 + \frac{2048}{5}x^5a + 96x^4a^2 + \frac{32}{3}x^3a^3 + \frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 224/5*x^15 - 2/7*x^14*a + 1280/7*x^14 + 4
8/13*x^13*a - 7424/13*x^13 - 24*x^12*a + 4192/3*x^12 + 1120/11*x^11*a + 3/5
*x^10*a^2 - 29696/11*x^11 - 1536/5*x^10*a - 16/3*x^9*a^2 + 4096*x^10 + 2048
/3*x^9*a + 24*x^8*a^2 - 14336/3*x^9 - 1120*x^8*a - 480/7*x^7*a^2 - 2/3*x^6*
a^3 + 4096*x^8 + 9216/7*x^7*a + 128*x^6*a^2 + 16/5*x^5*a^3 - 16384/7*x^7 -
1024*x^6*a - 768/5*x^5*a^2 - 8*x^4*a^3 + 2048/3*x^6 + 2048/5*x^5*a + 96*x^4
*a^2 + 32/3*x^3*a^3 + 1/2*x^2*a^4
```

giac [A] time = 0.39, size = 222, normalized size = 1.06

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}ax^{14} - \frac{224}{5}x^{15} + \frac{48}{13}ax^{13} + \frac{1280}{7}x^{14} - 24ax^{12} - \frac{7424}{13}x^{13} + \frac{3}{5}a^2x^{10} + \frac{1120}{11}ax^{11} + \frac{4192}{3}x^{12} - \frac{16}{3}a^2x^9 - \frac{1536}{5}ax^{10} - \frac{29696}{11}x^{11} + 24a^2x^8 + \frac{2048}{3}ax^9 + 4096x^{10} - \frac{2}{3}a^3x^6 - \frac{480}{7}a^2x^7 - 1120ax^8 - \frac{14336}{3}x^9 + \frac{16}{5}a^3x^5 + 128a^2x^6 + \frac{9216}{7}ax^7 + 4096x^8 - 8a^3x^4 - \frac{768}{5}a^2x^5 - 1024ax^6 - \frac{16384}{7}x^7 + \frac{1}{2}a^4x^2 + \frac{32}{3}a^3x^3 + 96a^2x^4 + \frac{2048}{5}ax^5 + \frac{2048}{3}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 + 41
92/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 + 204
8/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 14336/3*
x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4 - 76
8/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 96*a^
2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{18}}{18} - \frac{16x^{17}}{17} + 8x^{16} - \frac{224x^{15}}{5} + \frac{(-4a+2560)x^{14}}{14} + \frac{(48a-7424)x^{13}}{13} + \frac{(-288a+16768)x^{12}}{12} + \frac{(1120a-29696)x^{11}}{11} + \frac{\left(2a^2-2560a+24576+(-2a+128)^2\right)x^{10}}{10} + \frac{\left(-16a^2+3584a-10240+2(8a-128)(-2a+128)\right)x^9}{9} + \frac{\left(64a^2-2560a+2(-16a+64)(-2a+128)+(8a-128)^2\right)x^8}{8} + \frac{\left(-160a^2+32(-2a+128)a+2(-16a+64)(8a-128)\right)x^7}{7} + \frac{a^4x^2}{2} + \frac{32a^3x^3}{3} + \frac{\left(2(-2a+128)a^2+32(8a-128)a+(-16a+64)^2\right)x^6}{6} + \frac{\left(2(8a-128)a^2+32(-16a+64)a\right)x^5}{5} + \frac{\left(2(-16a+64)a^2+256a^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/18*x^18-16/17*x^17+8*x^16-224/5*x^15+1/14*(-4*a+2560)*x^14+1/13*(48*a-742
4)*x^13+1/12*(-288*a+16768)*x^12+1/11*(1120*a-29696)*x^11+1/10*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^10+1/9*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128)
)*x^9+1/8*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^8+1/7*(-160
*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^7+1/6*(2*(-2*a+128)*a^2+32*(
8*a-128)*a+(-16*a+64)^2)*x^6+1/5*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^5+1/4*
(2*(-16*a+64)*a^2+256*a^2)*x^4+32/3*a^3*x^3+1/2*a^4*x^2
```

maxima [A] time = 0.62, size = 182, normalized size = 0.87

$$\frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}(a-640)x^{14} - \frac{224}{5}x^{15} + \frac{16}{13}(3a-464)x^{13} - \frac{8}{3}(9a-524)x^{12} + \frac{32}{11}(35a-928)x^{11} + \frac{1}{5}\left(3a^2-1536a+20480\right)x^{10} - \frac{16}{3}\left(a^2-128a+896\right)x^9 + 8\left(3a^2-140a+512\right)x^8 - \frac{32}{7}\left(15a^2-288a+512\right)x^7 - \frac{2}{3}\left(a^3-192a^2+1536a-1024\right)x^6 + \frac{1}{2}a^4x^2 + \frac{32}{3}a^3x^3 + \frac{16}{5}\left(a^3-48a^2+128a\right)x^5 - 8\left(a^3-12a^2\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*(
3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3*a
^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 140*a
 + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 1536*a
```

```
- 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a)*x^5
- 8*(a^3 - 12*a^2)*x^4
```

mupad [B] time = 0.22, size = 178, normalized size = 0.85

$$x^{13}\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\left(24\,a-\frac{4192}{3}\right)-x^{14}\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\left(\frac{1120\,a}{11}-\frac{29696}{11}\right)+x^8\left(24\,a^2-1120\,a+4096\right)+x^{10}\left(\frac{3\,a^2}{5}-\frac{1536\,a}{5}+4096\right)-x^9\left(\frac{16\,a^2}{3}-\frac{2048\,a}{3}+\frac{14336}{3}\right)-x^7\left(\frac{480\,a^2}{7}-\frac{9216\,a}{7}+\frac{16384}{7}\right)-x^6\left(\frac{2\,a^3}{3}-128\,a^2+1024\,a-\frac{2048}{3}\right)-\frac{224\,x^{15}}{5}+8\,x^{16}-\frac{16\,x^{17}}{17}+\frac{x^{18}}{18}+\frac{32\,a^3\,x^3}{3}+\frac{a^4\,x^2}{2}+\frac{16\,a\,x^5\,\left(a^2-48\,a+128\right)}{5}-8\,a^2\,x^4\,\left(a-12\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7)
 + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((3*
a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) - x^7
*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)/3 -
 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^3)/3
+ (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

sympy [A] time = 0.12, size = 212, normalized size = 1.01

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13}\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\left(\frac{4192}{3}-24a\right)+x^{11}\left(\frac{1120a}{11}-\frac{29696}{11}\right)+x^{10}\left(\frac{3a^2}{5}-\frac{1536a}{5}+4096\right)+x^9\left(-\frac{16a^2}{3}+\frac{2048a}{3}-\frac{14336}{3}\right)+x^8\left(24a^2-1120a+4096\right)+x^7\left(-\frac{480a^2}{7}+\frac{9216a}{7}-\frac{16384}{7}\right)+x^6\left(-\frac{2a^3}{3}+128a^2-1024a+\frac{2048}{3}\right)+x^5\left(\frac{16a^3}{5}-\frac{768a^2}{5}+\frac{2048a}{5}\right)+x^4\left(-8a^3+96a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**15
/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 - 2
4*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096) +
x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096) + x
**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 - 1024*
a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**3 + 96
*a**2)
```

3.120 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$

Optimal. Leaf size=134

$$\frac{a^3x^2}{2}-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48$$

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$-\frac{1}{2}\left(a^2-128a+512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{a^3x^2}{2}-\frac{3}{10}(256-a)x^{10}+\frac{8}{3}(64-a)x^9-4(70-3a)x^8+\frac{48}{7}(48-5a)x^7+6(8-a)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
- ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8 +
(8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (12*x
^13)/13 - x^14/14
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\int\left(a^3x+24a^2x^2-24(-8+a)ax^3+4\left(128-96a+3a^2\right)x^4-3\left(512-1\right.\right.$$
$$=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5-\frac{1}{2}\left(512-128a+\right.$$

Mathematica [A] time = 0.02, size = 130, normalized size = 0.97

$$\frac{a^3x^2}{2}+\frac{1}{2}\left(-a^2+128a-512\right)x^6+\frac{4}{5}\left(3a^2-96a+128\right)x^5+8a^2x^3+\frac{3}{10}(a-256)x^{10}-\frac{8}{3}(a-64)x^9+4(3a-70)x^8-\frac{48}{7}(5a-48)x^7-6(a-8)ax^4-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*x^
8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^12 +
 (12*x^13)/13 - x^14/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]
```

fricas [A] time = 0.98, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{280}{11}x^{11}+\frac{3}{10}x^{10}a-\frac{384}{5}x^{10}-\frac{8}{3}x^{9}a+\frac{512}{3}x^{9}+12x^{8}a-280x^{8}-\frac{240}{7}x^{7}a-\frac{1}{2}x^{6}a^{2}+\frac{2304}{7}x^{7}+64x^{6}a+\frac{12}{5}x^{5}a^{2}-256x^{6}-\frac{384}{5}x^{5}a-6x^{4}a^{2}+\frac{512}{5}x^{5}+48x^{4}a+8x^{3}a^{2}+\frac{1}{2}x^{2}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 280/11*x^11 + 3/10*x^10*a - 384/5*x^10 -
 8/3*x^9*a + 512/3*x^9 + 12*x^8*a - 280*x^8 - 240/7*x^7*a - 1/2*x^6*a^2 + 2
304/7*x^7 + 64*x^6*a + 12/5*x^5*a^2 - 256*x^6 - 384/5*x^5*a - 6*x^4*a^2 + 5
12/5*x^5 + 48*x^4*a + 8*x^3*a^2 + 1/2*x^2*a^3
```

giac [A] time = 0.35, size = 133, normalized size = 0.99

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}ax^{10}+\frac{280}{11}x^{11}-\frac{8}{3}ax^{9}-\frac{384}{5}x^{10}+12ax^{8}+\frac{512}{3}x^{9}-\frac{1}{2}a^{2}x^{6}-\frac{240}{7}ax^{7}-280x^{8}+\frac{12}{5}a^{2}x^{5}+64ax^{6}+\frac{2304}{7}x^{7}-6a^{2}x^{4}-\frac{384}{5}ax^{5}-256x^{6}+\frac{1}{2}a^{3}x^{2}+8a^{2}x^{3}+48ax^{4}+\frac{512}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 + 1
2/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6 + 1
/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

maple [A] time = 0.00, size = 143, normalized size = 1.07

$$-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+\frac{(3a-768)\,x^{10}}{10}+\frac{(-24a+1536)\,x^{9}}{9}+\frac{(96a-2240)\,x^{8}}{8}+\frac{(-240a+2304)\,x^{7}}{7}+\frac{\left(-a^{2}+(-2a+128)\,a+256a-1536\right)x^{6}}{6}+\frac{a^{3}x^{2}}{2}+8a^{2}x^{3}+\frac{\left(4a^{2}+(8a-128)\,a-256a+512\right)x^{5}}{5}+\frac{\left(-8a^{2}+(-16a+64)\,a+128a\right)x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/14*x^14+12/13*x^13-6*x^12+280/11*x^11+1/10*(3*a-768)*x^10+1/9*(-24*a+153
6)*x^9+1/8*(96*a-2240)*x^8+1/7*(-240*a+2304)*x^7+1/6*(-a^2+(-2*a+128)*a+256
*a-1536)*x^6+1/5*(4*a^2+(8*a-128)*a-256*a+512)*x^5+1/4*(-8*a^2+(-16*a+64)*a
+128*a)*x^4+8*a^2*x^3+1/2*a^3*x^2
```

maxima [A] time = 0.62, size = 113, normalized size = 0.84

$$-\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}(a-256)x^{10}+\frac{280}{11}x^{11}-\frac{8}{3}(a-64)x^{9}+4(3a-70)x^{8}-\frac{48}{7}(5a-48)x^{7}-\frac{1}{2}\left(a^{2}-128a+512\right)x^{6}+\frac{4}{5}\left(3a^{2}-96a+128\right)x^{5}+\frac{1}{2}a^{3}x^{2}+8a^{2}x^{3}-6\left(a^{2}-8a\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3*
(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a +
512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a^2
- 8*a)*x^4
```

mupad [B] time = 2.12, size = 113, normalized size = 0.84

$$x^{8}\,(12a-280)+x^{10}\left(\frac{3a}{10}-\frac{384}{5}\right)-x^{9}\left(\frac{8a}{3}-\frac{512}{3}\right)-x^{7}\left(\frac{240a}{7}-\frac{2304}{7}\right)-x^{6}\left(\frac{a^{2}}{2}-64a+256\right)+x^{5}\left(\frac{12a^{2}}{5}-\frac{384a}{5}+\frac{512}{5}\right)+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}+8a^{2}x^{3}+\frac{a^{3}x^{2}}{2}-6ax^{4}\,(a-8)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*((
240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)/5
 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3 + (
a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

sympy [A] time = 0.10, size = 128, normalized size = 0.96

$$\frac{a^3x^2}{2}+8a^2x^3-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+x^{10}\left(\frac{3a}{10}-\frac{384}{5}\right)+x^9\left(\frac{512}{3}-\frac{8a}{3}\right)+x^8\left(12a-280\right)+x^7\left(\frac{2304}{7}-\frac{240a}{7}\right)+x^6\left(-\frac{a^2}{2}+64a-256\right)+x^5\left(\frac{12a^2}{5}-\frac{384a}{5}+\frac{512}{5}\right)+x^4\left(-6a^2+48a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/11
 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x**7
*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 384*a
/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

3.121 $\int x\left(a+8x-8x^2+4x^3-x^4\right)^2 dx$

Optimal. Leaf size=79

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {6742}

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a)
*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2 dx=\int\left(a^2x+16ax^2-16(-4+a)x^3+8(-16+a)x^4-2(-64+a)x^5-80x^6+3\right.$$
$$=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8}{9}$$

Mathematica [A] time = 0.01, size = 75, normalized size = 0.95

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6+\frac{8}{5}(a-16)x^5-4(a-4)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]
```

fricas [A] time = 0.86, size = 68, normalized size = 0.86

$$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7-\frac{1}{3}x^6a+\frac{64}{3}x^6+\frac{8}{5}x^5a-\frac{128}{5}x^5-4x^4a+16x^4+\frac{16}{3}x^3a+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 80/7*x^7 - 1/3*x^6*a + 64/3*x^6 + 8/5*x^5*a -
 128/5*x^5 - 4*x^4*a + 16*x^4 + 16/3*x^3*a + 1/2*x^2*a^2
```

giac [A] time = 0.35, size = 68, normalized size = 0.86

$$\frac{1}{10}\,x^{10} - \frac{8}{9}\,x^{9} + 4\,x^{8} - \frac{1}{3}\,ax^{6} - \frac{80}{7}\,x^{7} + \frac{8}{5}\,ax^{5} + \frac{64}{3}\,x^{6} - 4\,ax^{4} - \frac{128}{5}\,x^{5} + \frac{1}{2}\,a^{2}x^{2} + \frac{16}{3}\,ax^{3} + 16\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6 -
 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{10}}{10} - \frac{8x^{9}}{9} + 4x^{8} - \frac{80x^{7}}{7} + \frac{(-2a+128)\,x^{6}}{6} + \frac{(8a-128)\,x^{5}}{5} + \frac{a^{2}x^{2}}{2} + \frac{16a\,x^{3}}{3} + \frac{(-16a+64)\,x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+1/6*(-2*a+128)*x^6+1/5*(8*a-128)*x^5+1/4*(
-16*a+64)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

maxima [A] time = 0.62, size = 59, normalized size = 0.75

$$\frac{1}{10}\,x^{10} - \frac{8}{9}\,x^{9} + 4\,x^{8} - \frac{1}{3}\,(a-64)x^{6} - \frac{80}{7}\,x^{7} + \frac{8}{5}\,(a-16)x^{5} - 4\,(a-4)x^{4} + \frac{1}{2}\,a^{2}x^{2} + \frac{16}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x^
5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^{5}\left(\frac{8\,a}{5} - \frac{128}{5}\right) - x^{6}\left(\frac{a}{3} - \frac{64}{3}\right) - x^{4}\,(4\,a-16) + \frac{16\,a\,x^{3}}{3} - \frac{80\,x^{7}}{7} + 4\,x^{8} - \frac{8\,x^{9}}{9} + \frac{x^{10}}{10} + \frac{a^{2}\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
(80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

sympy [A] time = 0.08, size = 70, normalized size = 0.89

$$\frac{a^{2}x^{2}}{2} + \frac{16ax^{3}}{3} + \frac{x^{10}}{10} - \frac{8x^{9}}{9} + 4x^{8} - \frac{80x^{7}}{7} + x^{6}\left(\frac{64}{3} - \frac{a}{3}\right) + x^{5}\left(\frac{8a}{5} - \frac{128}{5}\right) + x^{4}\,(16-4a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**6
*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

3.122 $\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx$

Optimal. Leaf size=35

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {14}

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx &= \int\left(ax+8x^2-8x^3+4x^4-x^5\right)\,dx\\ &= \frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] IntegrateAlgebraic[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

fricas [A] time = 1.02, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6+\frac{4}{5}x^5-2x^4+\frac{8}{3}x^3+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 8/3*x^3 + 1/2*x^2*a
```

giac [A] time = 0.36, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}a\,x^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] 1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^6}{6} + \frac{4\,x^5}{5} - 2\,x^4 + \frac{8\,x^3}{3} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

3.123 $\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=116

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Rubi [A] time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1680, 1673, 1093, 204, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 - Sqrt[4 + a]
]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqrt[4
+ a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*Sqrt[4 + a])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1+x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{1}{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{3+a-2x^2-x^4}\,dx, x,\right.$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-1+x)^2\right) - \frac{\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -\right.}{2\sqrt{4+a}}$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} - \text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

Mathematica [C] time = 0.02, size = 59, normalized size = 0.51

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 + 4
*#1 - 3*#1^2 + #1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

maple [C] time = 0.00, size = 50, normalized size = 0.43

$$-\frac{\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right) + x\right)}{4\left(\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^3 - 3\,\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^2 + 4\,\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

```
[Out] -1/4*sum(_R/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_
Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^4 - 4x^3 + 8x^2 - a - 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.58, size = 275, normalized size = 2.37

$$\sum_{k=1}^{4}\ln\left(-x-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\left(\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\left(32\,a-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\left(64\,a-x\,\left(64\,a+256\right)+256\right)-x\,\left(16\,a+64\right)+128\right)-8\right)\right)\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

```
[Out] symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*a^
2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 51
2*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^4 +
10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z
+ a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))*root(
2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z
^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

sympy [A] time = 4.43, size = 155, normalized size = 1.34

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t\left(-16a-64\right)+a,\left(t\mapsto t\log\left(x+\frac{-128t^3a^4-1728t^3a^3-8640t^3a^2-18944t^3a-15360t^3+48t^2a^3+464t^2a^2+1472t^2a+1536t^2+8ta^3+88ta^2+312ta+352t-a^2-2a}{4a^2+21a+28}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2 -
 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a**4
 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*_t*
*2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_t*a*
*2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

3.124 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal. Leaf size=231

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}}{8(a+3)(a+4)^{3/2}\sqrt{1-}}$$

Rubi [A] time = 0.24, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1680, 1673, 1092, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(3a+\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{1-\sqrt{a+4}}}+\frac{\left(3a-\sqrt{a+4}+10\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)^{3/2}\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{4(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(4*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
+ (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x
)^4)) - ((10 + 3*a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/(
8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 - Sqrt[4 + a]]) + ((10 + 3*a - Sqrt[4 + a])*
ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)^(3/2)*Sqrt[1 + S
qrt[4 + a]]) + ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(4*(4 + a)^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x\right.}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{}{\left(3+a-\right.}\right.$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} + \frac{\left(5+a+(-1+x)^2\right)(-}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

Mathematica [C] time = 0.07, size = 166, normalized size = 0.72

$$\frac{ax^2-ax+a+x^3+2x}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)}-\frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)+2\#1a\log(x-\#1)+a\log(x-\#1)+4\#1\log(x-\#1)+6\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] + a*
Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2
 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^4-4\,x^3+8\,x^2-a-8\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

maple [C] time = 0.01, size = 162, normalized size = 0.70

$$\frac{\left(-\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2(-a-2)\,\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-a-6\right)\ln\left(-\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{16\left(a^2+7a+12\right)\left(\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}+\frac{-\frac{a\,x^2}{4\left(a^2+7a+12\right)}-\frac{x^3}{4\left(a^2+7a+12\right)}-\frac{a}{4\left(a^2+7a+12\right)}+\frac{(a-2)x}{4a^2+28a+48}}{x^4-4x^3+8x^2-a-8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a^2+7*a+12)*x^3-1/4*a/(a^2+7*a+12)*x^2+1/4*(a-2)/(a^2+7*a+12)*x-1/4*
a/(a^2+7*a+12))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a^2+7*a+12)*sum((-6-_R^2+2*(-
a-2)*_R-a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax^2+x^3-(a-2)x+a}{4\left(\left(a^2+7\,a+12\right)x^4-4\left(a^2+7\,a+12\right)x^3-a^3+8\left(a^2+7\,a+12\right)x^2-7\,a^2-8\left(a^2+7\,a+12\right)x-12\,a\right)}-\frac{\int \frac{2\,(a+2)x+x^2+a+6}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx}{4\left(a^2+7\,a+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 1
2)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a
) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a - 8*
x), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 2.82, size = 1167, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^
5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z
^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*
z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 141
55776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 26419
2*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a + 3600*a
^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))
 + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^
9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357
174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 284
2624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z
^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 2
53952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(12952010752*a^3*z^4 +
 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2
0082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^
6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2
 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 102912*a^2
*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57*a^2 + 16
*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(3932160*a + 2719744*a^2 + 999424*a^3 + 205824*a^4 + 22528
*a^5 + 1024*a^6 + 2359296))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 +
 576))) - (1150976*a + 631808*a^2 + 172800*a^3 + 23552*a^4 + 1280*a^5 + 835
584)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a +
 58880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(16*(816*a + 460*a^2
+ 129*a^3 + 18*a^4 + a^5 + 576))) - (x*(864*a + 228*a^2 + 20*a^3 + 1088))/(
16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))) + (x*(9*a + 2*a^2 + 8
))/(16*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)))*root(12952010752*
a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*
a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269
680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2 - 52070
4*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 9568256*z^2 + 1
02912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z - 984*a - 57
*a^2 + 16*a^3 - 2064, z, k), k, 1, 4) + (x^3/(4*(7*a + a^2 + 12)) + a/(4*(a
 + 3)*(a + 4)) - (x*(a - 2))/(4*(a + 3)*(a + 4)) + (a*x^2)/(4*(a + 3)*(a +
4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 31.42, size = 539, normalized size = 2.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2 +
 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384)
+ x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8 +
31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 129520
10752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-2048*
a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155776*a
- 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 253952)
+ 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t**3*a**1
2 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a**9 + 583
9372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*a**6 + 303
864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t**3*a**3 +
683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t**3 + 30208*
_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156928*_t**2*a**
7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312*_t**2*a**4 +
12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600*_t**2*a + 2944
401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**7 - 6200656*_t*a
**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*a**3 - 228144640*
_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a**6 + 71132*a**5 +
 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 956416)/(576*a**7 +
10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3 + 1826304*a**2 + 18677
76*a + 917504))))
```

3.125 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal. Leaf size=349

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right.}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}$$

Rubi [A] time = 0.37, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1680, 1673, 1092, 1178, 1166, 204, 1107, 614, 618, 206}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{3\left(7a^2+\left(4\sqrt{a+4}+47\right)a+14\sqrt{a+4}+80\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{64(a+3)^2(a+4)^{5/2}\sqrt{1-\sqrt{a+4}}}-\frac{3\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{64(a+3)^2(a+4)^2\sqrt{\sqrt{a+4}+1}}+\frac{3\left((x-1)^2+1\right)}{16(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{8(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{32(a+3)^2(a+4)^2\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{3\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{16(a+4)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (1 + (-1 + x)^2)/(8*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^2) + (3*(1
+ (-1 + x)^2))/(16*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((5 + a
 + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 +
x)^4)^2) + (((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(32*(3
+ a)^2*(4 + a)^2*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - (3*(80 + 7*a^2 + 14
*Sqrt[4 + a] + a*(47 + 4*Sqrt[4 + a]))*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]
]])/(64*(3 + a)^2*(4 + a)^(5/2)*Sqrt[1 - Sqrt[4 + a]]) - (3*(14 + 4*a - (80
 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(64*(
3 + a)^2*(4 + a)^2*Sqrt[1 + Sqrt[4 + a]]) + (3*ArcTanh[(1 + (-1 + x)^2)/Sqr
t[4 + a]])/(16*(4 + a)^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx = \text{Subst}\left(\int \frac{1+x}{\left(3+a-2x^2-x^4\right)^3}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\left(3+a-2x^2-x^4\right)^3}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\left(3+a-2x^2-x\right.}\right.$$

$$= \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{8\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{}{\left(3+a-\right.}\right.$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} - \frac{\left((6+a)(25+7a)+6(7+2a)(\right.}{32\left(12+7a+a^2\right)^2\left(3+a-2(1\right.}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} + \frac{3\left(1+(-1+x)^2\right)}{16(4+a)^2\left(3+a-2(1-x)^2-(\right.}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} + \frac{3\left(1+(-1+x)^2\right)}{16(4+a)^2\left(3+a-2(1-x)^2-(\right.}$$

$$= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} + \frac{3\left(1+(-1+x)^2\right)}{16(4+a)^2\left(3+a-2(1-x)^2-(\right.}$$

Mathematica [C] time = 0.16, size = 284, normalized size = 0.81

$$\frac{1}{128}\left(-\frac{3\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{4\#1^2a\log(x-\#1)+14\#1^2\log(x-\#1)+3a^2\log(x-\#1)+4\#1a^2\log(x-\#1)+31a\log(x-\#1)+16\#1a\log(x-\#1)+72\log(x-\#1)+8\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{\left(a^2+7a+12\right)^2}+\frac{4\left(a^2\left(6x^2-5x+5\right)+a\left(12x^3+31x-7\right)+6\left(7x^3-12x^2+28x-14\right)\right)}{(a+3)^2(a+4)^2\left(a-x\left(x^3-4x^2+8x-8\right)\right)}+\frac{16\left(ax^2-ax+a+x^3+2x\right)}{(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] ((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^3)
 + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 +
 x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x - #1]
+ 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x - #1
]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(
-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

maple [C] time = 0.02, size = 405, normalized size = 1.16

$$-\frac{3\left(2\left(2a+7\right)\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+3a^2+4\left(a^2+4a+2\right)\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+31a+72\right)\ln\left(-\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{128\left(a^4+14a^3+73a^2+168a+144\right)\left(\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}-\frac{\frac{3(2a+7)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{3\left(a^2-8a-40\right)x^6}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(29a^2-127a-792\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(73a^2-227a-1668\right)x^4}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{\left(62a^2-103a-1104\right)x^3}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(5a^3-26a^2+140a+1008\right)x^2}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{3\left(3a^2+7a-12\right)a}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{3\left(3a^3-17a^2-40a+192\right)x}{32\left(a^4+14a^3+73a^2+168a+144\right)}}{\left(x^4-4x^3+8x^2-a-8x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -(3/16*(2*a+7)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14*
a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*a+
144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16*(6
2*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+140*
a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)/(a^4
+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+168*a+
144))/(x^4-4*x^3+8*x^2-a-8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)*sum((72
+2*(2*a+7)*_R^2+4*(a^2+4*a+2)*_R+3*a^2+31*a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x)
,_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{6\,(2\,a+7)x^7+6\left(a^2-8\,a-40\right)x^6-\left(29\,a^2-127\,a-792\right)x^5+\left(73\,a^2-227\,a-1668\right)x^4-2\left(62\,a^2-103\,a-1104\right)x^3-9\,a^3-2\left(5\,a^3-26\,a^2+140\,a+1008\right)x^2-21\,a^2+3\left(3\,a^3-17\,a^2-40\,a+192\right)x+36\,a}{32\left(\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^8-8\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^7+32\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^6+a^6-80\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)x^5+14\,a^5-2\left(a^5-50\,a^4-823\,a^3-4504\,a^2-10608\,a-9216\right)x^4+73\,a^4+8\left(a^5-2\,a^4-151\,a^3-1000\,a^2-2544\,a-2304\right)x^3+168\,a^3-16\left(a^5+10\,a^4+17\,a^3-124\,a^2-528\,a-576\right)x^2+144\,a^2+16\left(a^5+14\,a^4+73\,a^3+168\,a^2+144\,a\right)x\right)}-\frac{3\int\frac{2\,(2\,a+7)x^2+3\,a^2+4\left(a^2+4\,a+2\right)x+31\,a+72}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx}{32\left(a^4+14\,a^3+73\,a^2+168\,a+144\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x^
5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 - 2
*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a +
192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^6
 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 - 5
0*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a^4
- 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4 + 17
*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^3 + 1
68*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*(a^2 + 4*a
 + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 + 73
*a^2 + 168*a + 144)
```

mupad [B] time = 3.39, size = 2200, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 153
980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a
^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995324
41600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 3206
246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 378
493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a
^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224*a*z^2
- 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 11
043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 60494614
36416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 728801
28*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3335100825
6*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 68
345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 13340736*a^4 +
 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195776*a^2 + 8993
28*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9
+ a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*
z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 55115219365
5275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 +
 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z
^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*
z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 -
4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 150233929482
24*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7
*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 -
 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z
 + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 3
3351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736
*a^5 - 68345856, z, k)*(root(15003759578972160*a^8*z^4 + 54537151127224320*
a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 5511521
93655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z
^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200
*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a
^11*z^4 + 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^
4 - 4718592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392
948224*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608
*a^7*z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z
^2 - 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a
^2*z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z
 + 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 2
0736*a^5 - 68345856, z, k)*((4290672328704*a + 6001143054336*a^2 + 50259170
42688*a^3 + 2800520003584*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 5986
2155264*a^7 + 8275361792*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11
 + 1391569403904)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4
+ 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) -
(x*(536334041088*a + 750142881792*a^2 + 628239630336*a^3 + 350065000448*a^4
 + 136275034112*a^5 + 37819514880*a^6 + 7482769408*a^7 + 1034420224*a^8 + 9
5158272*a^9 + 5242880*a^10 + 131072*a^11 + 173946175488))/(2048*(940032*a +
 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7
+ 582*a^8 + 36*a^9 + a^10 + 331776))) - (73421291520*a + 81260445696*a^2 +
52393672704*a^3 + 21688418304*a^4 + 5977620480*a^5 + 1096949760*a^6 + 12924
5184*a^7 + 8871936*a^8 + 270336*a^9 + 29444014080)/(16384*(940032*a + 11957
76*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*
a^8 + 36*a^9 + a^10 + 331776)) + (x*(2632974336*a + 3015180288*a^2 + 200894
0544*a^3 + 858243072*a^4 + 243806208*a^5 + 46055424*a^6 + 5578752*a^7 + 393
216*a^8 + 12288*a^9 + 1019215872))/(2048*(940032*a + 1195776*a^2 + 899328*a
^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^
10 + 331776))) - (x*(10805760*a + 7173504*a^2 + 2539872*a^3 + 505800*a^4 +
53712*a^5 + 2376*a^6 + 6782976))/(2048*(940032*a + 1195776*a^2 + 899328*a^3
 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10
```

```
 + 331776))) - (133812*a + 56187*a^2 + 10098*a^3 + 648*a^4 + 115776)/(16384
*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6
 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(1971*a^2 - 1539*a +
918*a^3 + 108*a^4 - 6372))/(2048*(940032*a + 1195776*a^2 + 899328*a^3 + 442
864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331
776)))*root(15003759578972160*a^8*z^4 + 5453715127224320*a^7*z^4 + 1539804
1871712256*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520*a^4*z
^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 599953244160
0*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 + 32062467
73555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4 + 3784939
92960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 4718592*a^10*
z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 1502339294822*a*z^2 - 16
75258704691*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*z^2 - 110433
92716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 - 604946143641
6*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*z + 72880128*a
^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z + 33351008256*z
 - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20736*a^5 - 683458
56, z, k), k, 1, 4) + ((3*(7*a^2 - 12*a + 3*a^3))/(32*(6*a + a^2 + 9)*(8*a
 + a^2 + 16)) - (3*x^7*(2*a + 7))/(16*(168*a + 73*a^2 + 14*a^3 + a^4 + 144))
 + (x^2*(140*a - 26*a^2 + 5*a^3 + 1008))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 1
6)) + (3*x*(40*a + 17*a^2 - 3*a^3 - 192))/(32*(6*a + a^2 + 9)*(8*a + a^2 +
16)) + (3*x^6*(8*a - a^2 + 40))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^
5*(127*a - 29*a^2 + 792))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)) - (x^3*(103
*a - 62*a^2 + 1104))/(16*(6*a + a^2 + 9)*(8*a + a^2 + 16)) + (x^4*(227*a -
73*a^2 + 1668))/(32*(6*a + a^2 + 9)*(8*a + a^2 + 16)))/(16*a*x - x^2*(16*a
 - 64) - x^4*(2*a - 128) + x^3*(8*a - 128) + a^2 - 80*x^5 + 32*x^6 - 8*x^7 +
 x^8)
```

sympy [B] time = 88.09, size = 1102, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] -(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
+ x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-12
4*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9*a*
*3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 +
4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-
256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 14
336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a**
3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*a*
*3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 8704*
a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 37376*a*
*3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 14763950080*
a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**11 + 5
27875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**8 + 545
3715127224320*a**7 + 15398041871712256*a**6 + 334927734494986240*a**5 + 5
51152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105280*a**2
+ 284993413919539200*a + 68398419340689408) + _t**2*(-4718592*a**10 - 19611
6480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 286939938816*a**6 - 140543
7345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3 - 16752587046912*a*
*2 - 15023392948224*a - 6049461436416) + _t*(-2709504*a**7 - 72880128*a**6
- 839890944*a**5 - 5375877120*a**4 - 20640890880*a**3 - 47542173696*a**2 -
60827369472*a - 33351008256) + 20736*a**5 - 155601*a**4 - 4706424*a**3 - 29
249424*a**2 - 74027520*a - 68345856, Lambda(_t, _t*log(x + (-469762048*_t**
3*a**20 - 31417434112*_t**3*a**19 - 992305217536*_t**3*a**18 - 196635766292
48*_t**3*a**17 - 273880031690752*_t**3*a**16 - 2846116194287616*_t**3*a**15
```

```
 - 22853982892326912*_t**3*a**14 - 144840417605582848*_t**3*a**13 - 7331931
54773123072*_t**3*a**12 - 2977941469704224768*_t**3*a**11 - 967719737311730
0736*_t**3*a**10 - 24850421452415959040*_t**3*a**9 - 48984708931769073664*_
t**3*a**8 - 69124682329943441408*_t**3*a**7 - 54921507243737219072*_t**3*a*
*6 + 18833423088924753920*_t**3*a**5 + 128767022044444360704*_t**3*a**4 + 1
97893824476545548288*_t**3*a**3 + 170576989286005997568*_t**3*a**2 + 837098
68624400351232*_t**3*a + 18392762450832261120*_t**3 + 136642560*_t**2*a**17
 + 7616593920*_t**2*a**16 + 198980665344*_t**2*a**15 + 3234300690432*_t**2*
a**14 + 36614363283456*_t**2*a**13 + 306155605721088*_t**2*a**12 + 19563396
56687616*_t**2*a**11 + 9747894775578624*_t**2*a**10 + 38291841445330944*_t*
*2*a**9 + 119050488573591552*_t**2*a**8 + 292236772188880896*_t**2*a**7 + 5
61261720373297152*_t**2*a**6 + 828898581078343680*_t**2*a**5 + 914439454498
750464*_t**2*a**4 + 718255692208668672*_t**2*a**3 + 369227414724673536*_t**
2*a**2 + 104815442748506112*_t**2*a + 10263520138493952*_t**2 + 4128768*_t*
a**15 + 235608192*_t*a**14 + 6050117376*_t*a**13 + 92875570560*_t*a**12 + 9
50838962688*_t*a**11 + 6825858397056*_t*a**10 + 34932826734336*_t*a**9 + 12
5262778564224*_t*a**8 + 287989861404672*_t*a**7 + 257684685023232*_t*a**6 -
 836263788945408*_t*a**5 - 4002432415137792*_t*a**4 - 8409454278082560*_t*a
**3 - 10371340262965248*_t*a**2 - 7285247072796672*_t*a - 2270140431335424*
_t + 1000512*a**12 + 42546357*a**11 + 777344580*a**10 + 7998006582*a**9 + 5
0045408388*a**8 + 182866499613*a**7 + 247394170512*a**6 - 1063305068832*a**
5 - 6960658344192*a**4 - 19132655580288*a**3 - 30001872614400*a**2 - 261928
92672000*a - 9953981595648)/(1354752*a**12 + 44550027*a**11 + 663517980*a**
10 + 5951170602*a**9 + 36270700668*a**8 + 162289912419*a**7 + 567868212432*
a**6 + 1626099007104*a**5 + 3825839091456*a**4 + 7035734732544*a**3 + 92167
60449024*a**2 + 7467334520832*a + 2773884911616))))
```

3.126 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^4 \, dx$

Optimal. Leaf size=210

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+\right.$$

Rubi [A] time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}-4\left(15a^2-288a+512\right)x^8+\frac{4}{7}\left(-a^3+192a^2-1536a+1024\right)x^7+\frac{8}{3}a\left(a^2-48a+128\right)x^6+\frac{32}{5}(12-a)a^2x^5+8a^3x^4+\frac{a^4x^3}{3}+\frac{4}{15}(640-a)x^{15}-\frac{8}{7}(464-3a)x^{14}+\frac{32}{13}(524-9a)x^{13}-\frac{8}{3}(928-35a)x^{12}+\frac{64}{9}(128-3a)(4-a)x^9+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2)
*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15*a
^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)/5
+ (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*(524
 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42*x^16
 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4 dx = \int \left(a^4x^2+32a^3x^3-32(-12+a)a^2x^4+16a\left(128-48a+a^2\right)x^5-4\left(-102\right.\right.$$
$$= \frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6+\frac{4}{7}\left(1024-153\right.$$

Mathematica [A] time = 0.04, size = 204, normalized size = 0.97

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{2}{11}\left(3a^2-1536a+20480\right)x^{11}-\frac{24}{5}\left(a^2-128a+896\right)x^{10}+\frac{64}{9}\left(3a^2-140a+512\right)x^9-4\left(15a^2-288a+512\right)x^8+\frac{8}{3}a\left(a^2-48a+128\right)x^6-\frac{32}{5}(a-12)a^2x^5-\frac{4}{7}\left(a^3-192a^2+1536a-1024\right)x^7-\frac{4}{15}(a-640)x^{15}+\frac{8}{7}(3a-464)x^{14}-\frac{32}{13}(9a-524)x^{13}+\frac{8}{3}(35a-928)x^{12}+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] (a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10
)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 - (32
*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)/15 -
 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4, x]
```

fricas [A] time = 1.25, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-42x^{16}-\frac{4}{15}x^{15}a+\frac{512}{3}x^{15}+\frac{24}{7}x^{14}a-\frac{3712}{7}x^{14}-\frac{288}{13}x^{13}a+\frac{16768}{13}x^{13}+\frac{280}{3}x^{12}a+\frac{6}{11}x^{11}a^2-\frac{7424}{3}x^{12}-\frac{3072}{11}x^{11}a-\frac{24}{5}x^{10}a^2+\frac{40960}{11}x^{11}+\frac{3072}{5}x^{10}a+\frac{64}{3}x^9a^2-\frac{21504}{5}x^{10}-\frac{8960}{9}x^9a-60x^8a^2-\frac{4}{7}x^7a^3+\frac{32768}{9}x^9+1152x^8a+\frac{768}{7}x^7a^2+\frac{8}{3}x^6a^3-2048x^8-\frac{6144}{7}x^7a-128x^6a^2-\frac{32}{5}x^5a^3+\frac{4096}{7}x^7+\frac{1024}{3}x^6a+\frac{384}{5}x^5a^2+8x^4a^3+\frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 42*x^16 - 4/15*x^15*a + 512/3*x^15 + 2
4/7*x^14*a - 3712/7*x^14 - 288/13*x^13*a + 16768/13*x^13 + 280/3*x^12*a + 6
/11*x^11*a^2 - 7424/3*x^12 - 3072/11*x^11*a - 24/5*x^10*a^2 + 40960/11*x^11
 + 3072/5*x^10*a + 64/3*x^9*a^2 - 21504/5*x^10 - 8960/9*x^9*a - 60*x^8*a^2
- 4/7*x^7*a^3 + 32768/9*x^9 + 1152*x^8*a + 768/7*x^7*a^2 + 8/3*x^6*a^3 - 20
48*x^8 - 6144/7*x^7*a - 128*x^6*a^2 - 32/5*x^5*a^3 + 4096/7*x^7 + 1024/3*x^
6*a + 384/5*x^5*a^2 + 8*x^4*a^3 + 1/3*x^3*a^4
```

giac [A] time = 0.24, size = 222, normalized size = 1.06

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}ax^{15}-42x^{16}+\frac{24}{7}ax^{14}+\frac{512}{3}x^{15}-\frac{288}{13}ax^{13}-\frac{3712}{7}x^{14}+\frac{6}{11}a^2x^{11}+\frac{280}{3}ax^{12}+\frac{16768}{13}x^{13}-\frac{24}{5}a^2x^{10}-\frac{3072}{11}ax^{11}-\frac{7424}{3}x^{12}+\frac{64}{3}a^2x^9+\frac{3072}{5}ax^{10}+\frac{40960}{11}x^{11}-\frac{4}{7}a^3x^7-60a^2x^8-\frac{8960}{9}ax^9-\frac{21504}{5}x^{10}+\frac{8}{3}a^3x^6+\frac{768}{7}a^2x^7+1152ax^8+\frac{32768}{9}x^9-\frac{32}{5}a^3x^5-128a^2x^6-\frac{6144}{7}ax^7-2048x^8+\frac{1}{3}a^4x^3+8a^3x^4+\frac{384}{5}a^2x^5+\frac{1024}{3}ax^6+\frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 + 1
6768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x^9
+ 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x^9 -
 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9 - 32
/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*a^3*x^
4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

maple [A] time = 0.00, size = 267, normalized size = 1.27

$$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)x^{15}}{15}+\frac{(48a-7424)x^{14}}{14}+\frac{(-288a+16768)x^{13}}{13}+\frac{(1120a-29696)x^{12}}{12}+\frac{(2a^2-2560a+24576+(-2a+128)^2)x^{11}}{11}+\frac{(-16a^2+3584a-10240+2(8a-128)(-2a+128))x^{10}}{10}+\frac{(64a^2-2560a+2(-16a+64)(-2a+128)+(8a-128)^2)x^9}{9}+\frac{(-160a^2+32(-2a+128)a+2(-16a+64)(8a-128))x^8}{8}+\frac{a^4x^3}{3}+8a^3x^4+\frac{(2(-2a+128)a^2+32(8a-128)a+(-16a+64)^2)x^7}{7}+\frac{(2(8a-128)a^2+32(-16a+64)a)x^6}{6}+\frac{(2(-16a+64)a^2+256a^2)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

```
[Out] 1/19*x^19-8/9*x^18+128/17*x^17-42*x^16+1/15*(-4*a+2560)*x^15+1/14*(48*a-742
4)*x^14+1/13*(-288*a+16768)*x^13+1/12*(1120*a-29696)*x^12+1/11*(2*a^2-2560*
a+24576+(-2*a+128)^2)*x^11+1/10*(-16*a^2+3584*a-10240+2*(8*a-128)*(-2*a+128
))*x^10+1/9*(64*a^2-2560*a+2*(-16*a+64)*(-2*a+128)+(8*a-128)^2)*x^9+1/8*(-1
60*a^2+32*(-2*a+128)*a+2*(-16*a+64)*(8*a-128))*x^8+1/7*(2*(-2*a+128)*a^2+32
*(8*a-128)*a+(-16*a+64)^2)*x^7+1/6*(2*(8*a-128)*a^2+32*(-16*a+64)*a)*x^6+1/
5*(2*(-16*a+64)*a^2+256*a^2)*x^5+8*a^3*x^4+1/3*a^4*x^3
```

maxima [A] time = 0.64, size = 182, normalized size = 0.87

$$\frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}(a-640)x^{15}-42x^{16}+\frac{8}{7}(3a-464)x^{14}-\frac{32}{13}(9a-524)x^{13}+\frac{8}{3}(35a-928)x^{12}+\frac{2}{11}(3a^2-1536a+20480)x^{11}-\frac{24}{5}(a^2-128a+896)x^{10}+\frac{64}{9}(3a^2-140a+512)x^9-4(15a^2-288a+512)x^8-\frac{4}{7}(a^3-192a^2+1536a-1024)x^7+\frac{1}{3}a^4x^3+8a^3x^4+\frac{8}{3}(a^3-48a^2+128a)x^6-\frac{32}{5}(a^3-12a^2)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

```
[Out] 1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(3
*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3*a
^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2 - 1
40*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1536*
```

```
a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x^6 -
32/5*(a^3 - 12*a^2)*x^5
```

mupad [B] time = 2.29, size = 178, normalized size = 0.85

$$x^{14}\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^8\,\left(60\,a^2-1152\,a+2048\right)-x^{10}\left(\frac{24\,a^2}{5}-\frac{3072\,a}{5}+\frac{21504}{5}\right)+x^9\left(\frac{64\,a^2}{3}-\frac{8960\,a}{9}+\frac{32768}{9}\right)+x^{11}\left(\frac{6\,a^2}{11}-\frac{3072\,a}{11}+\frac{40960}{11}\right)-x^7\left(\frac{4\,a^3}{7}-\frac{768\,a^2}{7}+\frac{6144\,a}{7}-\frac{4096}{7}\right)-42\,x^{16}+\frac{128\,x^{17}}{17}-\frac{8\,x^{18}}{9}+\frac{x^{19}}{19}+8\,a^3\,x^4+\frac{a^4\,x^3}{3}+\frac{8\,a\,x^6\,\left(a^2-48\,a+128\right)}{3}-\frac{32\,a^2\,x^5\,\left(a-12\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

```
[Out] x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 7424
/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*((
24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768/9)
 + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*a^2)
/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19 +
 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x^5*(a
- 12))/5
```

sympy [A] time = 0.15, size = 219, normalized size = 1.04

$$\frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+x^{15}\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\left(\frac{24a}{7}-\frac{3712}{7}\right)+x^{13}\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\left(\frac{280a}{3}-\frac{7424}{3}\right)+x^{11}\left(\frac{6a^2}{11}-\frac{3072a}{11}+\frac{40960}{11}\right)+x^{10}\left(-\frac{24a^2}{5}+\frac{3072a}{5}-\frac{21504}{5}\right)+x^9\left(\frac{64a^2}{3}-\frac{8960a}{9}+\frac{32768}{9}\right)+x^8\left(-60a^2+1152a-2048\right)+x^7\left(-\frac{4a^3}{7}+\frac{768a^2}{7}-\frac{6144a}{7}+\frac{4096}{7}\right)+x^6\left(\frac{8a^3}{3}-128a^2+\frac{1024a}{3}\right)+x^5\left(-\frac{32a^3}{5}+\frac{384a^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

```
[Out] a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
+ x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 288*
a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/11)
 + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9 + 3
2768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2/7 -
6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-32*a**3
/5 + 384*a**2/5)
```

3.127 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx$

Optimal. Leaf size=138

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6($$

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {6742}

$$-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{a^3x^3}{3}-\frac{3}{11}(256-a)x^{11}+\frac{12}{5}(64-a)x^{10}-\frac{32}{9}(70-3a)x^9+6(48-5a)x^8+\frac{24}{5}(8-a)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x^
6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*x^
9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*x^1
3)/13 + (6*x^14)/7 - x^15/15
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3 dx = \int \left(a^3x^2+24a^2x^3-24(-8+a)ax^4+4\left(128-96a+3a^2\right)x^5-3\left(512-\right.\right.$$

$$=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128\right.$$

Mathematica [A] time = 0.02, size = 132, normalized size = 0.96

$$\frac{a^3x^3}{3}-\frac{3}{7}\left(a^2-128a+512\right)x^7+\frac{2}{3}\left(3a^2-96a+128\right)x^6+6a^2x^4+\frac{3}{11}(a-256)x^{11}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{24}{5}(a-8)ax^5-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*a)
*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 - (7
2*x^13)/13 + (6*x^14)/7 - x^15/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3, x]
```

fricas [A] time = 0.91, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{70}{3}x^{12}+\frac{3}{11}x^{11}a-\frac{768}{11}x^{11}-\frac{12}{5}x^{10}a+\frac{768}{5}x^{10}+\frac{32}{3}x^{9}a-\frac{2240}{9}x^{9}-30x^{8}a-\frac{3}{7}x^{7}a^{2}+288x^{8}+\frac{384}{7}x^{7}a+2x^{6}a^{2}-\frac{1536}{7}x^{7}-64x^{6}a-\frac{24}{5}x^{5}a^{2}+\frac{256}{3}x^{6}+\frac{192}{5}x^{5}a+6x^{4}a^{2}+\frac{1}{3}x^{3}a^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 70/3*x^12 + 3/11*x^11*a - 768/11*x^11
- 12/5*x^10*a + 768/5*x^10 + 32/3*x^9*a - 2240/9*x^9 - 30*x^8*a - 3/7*x^7*a
^2 + 288*x^8 + 384/7*x^7*a + 2*x^6*a^2 - 1536/7*x^7 - 64*x^6*a - 24/5*x^5*a
^2 + 256/3*x^6 + 192/5*x^5*a + 6*x^4*a^2 + 1/3*x^3*a^3
```

giac [A] time = 0.29, size = 133, normalized size = 0.96

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}ax^{11}+\frac{70}{3}x^{12}-\frac{12}{5}ax^{10}-\frac{768}{11}x^{11}+\frac{32}{3}ax^{9}+\frac{768}{5}x^{10}-\frac{3}{7}a^{2}x^{7}-30ax^{8}-\frac{2240}{9}x^{9}+2a^{2}x^{6}+\frac{384}{7}ax^{7}+288x^{8}-\frac{24}{5}a^{2}x^{5}-64ax^{6}-\frac{1536}{7}x^{7}+\frac{1}{3}a^{3}x^{3}+6a^{2}x^{4}+\frac{192}{5}ax^{5}+\frac{256}{3}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
- 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9*x
^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/7*x
^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

maple [A] time = 0.00, size = 143, normalized size = 1.04

$$-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+\frac{(3a-768)\,x^{11}}{11}+\frac{(-24a+1536)\,x^{10}}{10}+\frac{(96a-2240)\,x^{9}}{9}+\frac{(-240a+2304)\,x^{8}}{8}+\frac{\left(-a^{2}+(-2a+128)\,a+256a-1536\right)x^{7}}{7}+\frac{a^{3}x^{3}}{3}+6a^{2}x^{4}+\frac{\left(4a^{2}+(8a-128)\,a-256a+512\right)x^{6}}{6}+\frac{\left(-8a^{2}+(-16a+64)\,a+128a\right)x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

```
[Out] -1/15*x^15+6/7*x^14-72/13*x^13+70/3*x^12+1/11*(3*a-768)*x^11+1/10*(-24*a+15
36)*x^10+1/9*(96*a-2240)*x^9+1/8*(-240*a+2304)*x^8+1/7*(-a^2+(-2*a+128)*a+2
56*a-1536)*x^7+1/6*(4*a^2+(8*a-128)*a-256*a+512)*x^6+1/5*(-8*a^2+(-16*a+64)
*a+128*a)*x^5+6*a^2*x^4+1/3*a^3*x^3
```

maxima [A] time = 0.56, size = 113, normalized size = 0.82

$$-\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^{9}-6(5a-48)x^{8}-\frac{3}{7}\left(a^{2}-128a+512\right)x^{7}+\frac{2}{3}\left(3a^{2}-96a+128\right)x^{6}+\frac{1}{3}a^{3}x^{3}+6a^{2}x^{4}-\frac{24}{5}\left(a^{2}-8a\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/5
*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*a
+ 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24/5*
(a^2 - 8*a)*x^5
```

mupad [B] time = 0.09, size = 113, normalized size = 0.82

$$x^{11}\left(\frac{3a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12a}{5}-\frac{768}{5}\right)-x^{8}(30a-288)+x^{9}\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^{6}\left(2a^{2}-64a+\frac{256}{3}\right)-x^{7}\left(\frac{3a^{2}}{7}-\frac{384a}{7}+\frac{1536}{7}\right)+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}+6a^{2}x^{4}+\frac{a^{3}x^{3}}{3}-\frac{24ax^{5}(a-8)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

```
[Out] x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^9
*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384*a
)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^2*x
^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

sympy [A] time = 0.10, size = 134, normalized size = 0.97

$$\frac{a^3x^3}{3}+6a^2x^4-\frac{x^{15}}{15}+\frac{6x^{14}}{7}-\frac{72x^{13}}{13}+\frac{70x^{12}}{3}+x^{11}\left(\frac{3a}{11}-\frac{768}{11}\right)+x^{10}\left(\frac{768}{5}-\frac{12a}{5}\right)+x^9\left(\frac{32a}{3}-\frac{2240}{9}\right)+x^8(288-30a)+x^7\left(-\frac{3a^2}{7}+\frac{384a}{7}-\frac{1536}{7}\right)+x^6\left(2a^2-64a+\frac{256}{3}\right)+x^5\left(-\frac{24a^2}{5}+\frac{192a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

```
[Out] a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/3
 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240/9
) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a**2
- 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

3.128 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$

Optimal. Leaf size=79

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6742}

$$\frac{a^2x^3}{3}+\frac{2}{7}(64-a)x^7-\frac{4}{3}(16-a)x^6+\frac{16}{5}(4-a)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx = \int \left(a^2x^2+16ax^3-16(-4+a)x^4+8(-16+a)x^5-2(-64+a)x^6-80x^7+\right.$$
$$=\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-$$

Mathematica [A] time = 0.01, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3}-\frac{2}{7}(a-64)x^7+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-64
 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]
```

fricas [A] time = 0.93, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^9-10x^8-\frac{2}{7}x^7a+\frac{128}{7}x^7+\frac{4}{3}x^6a-\frac{64}{3}x^6-\frac{16}{5}x^5a+\frac{64}{5}x^5+4x^4a+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 10*x^8 - 2/7*x^7*a + 128/7*x^7 + 4/3*x^6*
a - 64/3*x^6 - 16/5*x^5*a + 64/5*x^5 + 4*x^4*a + 1/3*x^3*a^2
```

giac [A] time = 0.29, size = 68, normalized size = 0.86

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}ax^7 - 10x^8 + \frac{4}{3}ax^6 + \frac{128}{7}x^7 - \frac{16}{5}ax^5 - \frac{64}{3}x^6 + \frac{1}{3}a^2x^3 + 4ax^4 + \frac{64}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x^
7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

maple [A] time = 0.00, size = 66, normalized size = 0.84

$$\frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + \frac{(-2a+128)x^7}{7} + \frac{(8a-128)x^6}{6} + \frac{a^2x^3}{3} + 4ax^4 + \frac{(-16a+64)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] 1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+1/7*(-2*a+128)*x^7+1/6*(8*a-128)*x^6+1/5
*(-16*a+64)*x^5+4*a*x^4+1/3*a^2*x^3
```

maxima [A] time = 0.55, size = 59, normalized size = 0.75

$$\frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}(a-64)x^7 - 10x^8 + \frac{4}{3}(a-16)x^6 - \frac{16}{5}(a-4)x^5 + \frac{1}{3}a^2x^3 + 4ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)*
x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

mupad [B] time = 0.04, size = 64, normalized size = 0.81

$$x^6\left(\frac{4a}{3} - \frac{64}{3}\right) - x^5\left(\frac{16a}{5} - \frac{64}{5}\right) - x^7\left(\frac{2a}{7} - \frac{128}{7}\right) + 4ax^4 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11} + \frac{a^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a*
x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

sympy [A] time = 0.08, size = 73, normalized size = 0.92

$$\frac{a^2x^3}{3} + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8 + x^7\left(\frac{128}{7} - \frac{2a}{7}\right) + x^6\left(\frac{4a}{3} - \frac{64}{3}\right) + x^5\left(\frac{64}{5} - \frac{16a}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7*
(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

3.129 $\int x^2 \left(a+8x-8x^2+4x^3-x^4\right) dx$

Optimal. Leaf size=35

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {14}

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)dx &= \int \left(ax^2+8x^3-8x^4+4x^5-x^6\right)dx\\ &= \frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

fricas [A] time = 1.06, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7+\frac{2}{3}x^6-\frac{8}{5}x^5+2x^4+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 2*x^4 + 1/3*x^3*a
```

giac [A] time = 0.31, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

maple [A] time = 0.00, size = 28, normalized size = 0.80

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}a x^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] 1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

maxima [A] time = 0.61, size = 27, normalized size = 0.77

$$-\frac{1}{7}x^7 + \frac{2}{3}x^6 - \frac{8}{5}x^5 + \frac{1}{3}ax^3 + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

mupad [B] time = 0.02, size = 27, normalized size = 0.77

$$-\frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4 + \frac{a x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] (a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

sympy [A] time = 0.06, size = 29, normalized size = 0.83

$$\frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
[Out] a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

3.130 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1680, 1673, 1166, 204, 12, 1107, 618, 206}

$$-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(2*Sqrt[1 - Sqrt[4 + a]]) - ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) + ArcTanh[(1 + (-
1 + x)^2)/Sqrt[4 + a]]/Sqrt[4 + a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \frac{(1+x)^2}{3+a-2x^2-x^4}\,dx, x, -1+x\right)\\
&= \text{Subst}\left(\int \frac{2x}{3+a-2x^2-x^4}\,dx, x, -1+x\right)+\text{Subst}\left(\int \frac{1+x^2}{3+a-2x^2-x^4}\,dx, x,\right.\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-\sqrt{4+a}-x^2}\,dx, x, -1+x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+\sqrt{4+a}-x}\right.\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\text{Subst}\left(\int \frac{1}{3+a-2x-x^2}\,dx, x, (-\right.\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}-2\,\text{Subst}\left(\int \frac{1}{4(4+a)-x^2}\,dx, x, -2\right.\\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\frac{\tanh^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.62

$$-\frac{1}{4}\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&,\frac{\#1^2\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

```
[Out] -1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2 +
 4*#1 - 3*#1^2 + #1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + 8x - 8x^2 + 4x^3 - x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
[Out] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
[Out] integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

maple [C] time = 0.00, size = 52, normalized size = 0.53

$$-\frac{\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right) + x\right)}{4\left(\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^3 - 3\,\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right)^2 + 4\,\mathrm{RootOf}\left(\_Z^4 - 4\_Z^3 + 8\_Z^2 - 8\_Z - a\right) - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)
[Out] -1/4*sum(_R^2/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8
*_Z-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
[Out] -integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

mupad [B] time = 2.78, size = 878, normalized size = 8.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
[Out] symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k
```

```
) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160
*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^
2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a
 + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*
z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*x -
 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^
2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*x - 1
92*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2
- 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2 + 256*r
oot(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 11
52*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3 - 4*root(28
16*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z
^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a*x + 32*root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2
- 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^2*a*x - 64*root(2816*a
^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 -
 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)^3*a*x)*root(2816*a^2*z^
4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048
*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k), k, 1, 4)
```

sympy [B] time = 7.61, size = 172, normalized size = 1.74

$$-\operatorname{RootSum}\left(t^4\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256a-512\right)-a^2,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4-448t^3a^3-256t^3a^2+3584t^3a+6144t^3-224t^2a^3-2208t^2a^2-7168t^2a-7680t^2+56ta^3+400ta^2+864ta+512t+5a^3+34a^2+56a}{a^3+60a^2+320a+448}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

```
[Out] -RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
- 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(x
+ (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 6144*_
t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 56*_t
*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a**3 +
60*a**2 + 320*a + 448))))
```

3.131 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2} \, dx$

Optimal. Leaf size=225

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\right.}{8(a+3)(a+4)\sqrt{1-}}$$

Rubi [A] time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1680, 1673, 1178, 1166, 204, 12, 1107, 614, 618, 206}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{(x-1)^2+1}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\left(a+\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{8(a+3)(a+4)\sqrt{1-\sqrt{a+4}}}-\frac{\left(a-\sqrt{a+4}+4\right)\tan^{-1}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{8(a+3)(a+4)\sqrt{\sqrt{a+4}+1}}+\frac{\tanh^{-1}\left(\frac{(x-1)^2+1}{\sqrt{a+4}}\right)}{2(a+4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + ((4 + a)
*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)) - ((4 + a + Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])/
(8*(3 + a)*(4 + a)*Sqrt[1 - Sqrt[4 + a]]) - ((4 + a - Sqrt[4 + a])*ArcTan[(
-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(8*(3 + a)*(4 + a)*Sqrt[1 + Sqrt[4 + a]]) +
 ArcTanh[(1 + (-1 + x)^2)/Sqrt[4 + a]]/(2*(4 + a)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\left(3+a-2x^2-x^4\right)^2}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+x^2}{\left(3+a-2x^2-x^4\right)^2}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + 2\,\text{Subst}\left(\int \frac{x}{\left(3+a-2x^2\right.}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{\left(4+a-\sqrt{4+a}\right)\text{Subst}}{8\left(12\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)} + \frac{(4+a)\left(2+(-1+x)^2\right)}{4\left(12+7a+a^2\right)\left(3+a-2(-1+\right.}$$

Mathematica [C] time = 0.08, size = 182, normalized size = 0.81

$$\frac{a\left(x^3-x^2+x+1\right)+2x\left(2x^2-3x+4\right)}{4(a+3)(a+4)\left(a-x\left(x^3-4x^2+8x-8\right)\right)} - \frac{\text{RootSum}\left[-\#1^4+4\#1^3-8\#1^2+8\#1+a\&, \frac{\#1^2 a\log(x-\#1)+4\#1^2\log(x-\#1)+2\#1 a\log(x-\#1)-a\log(x-\#1)+4\#1\log(x-\#1)}{\#1^3-3\#1^2+4\#1-2}\&\right]}{16\left(a^2+7a+12\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] (2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*(
-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (
-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]*#1
^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^
2))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^4 - 4\,x^3 + 8\,x^2 - a - 8\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

maple [C] time = 0.01, size = 160, normalized size = 0.71

$$\frac{\left((-a-4)\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+2\,(-a-2)\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+a\right)\ln\left(-\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)+x\right)}{16\,(a+3)\,(a+4)\left(\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^3-3\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)^2+4\,\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)-2\right)}+\frac{-\frac{x^3}{4(a+3)}+\frac{(a+6)x^2}{4(a+3)(a+4)}-\frac{a}{4(a+3)(a+4)}-\frac{(a+8)x}{4(a+3)(a+4)}}{x^4-4x^3+8x^2-a-8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

```
[Out] (-1/4/(a+3)*x^3+1/4*(a+6)/(a+3)/(a+4)*x^2-1/4*(a+8)/(a+3)/(a+4)*x-1/4*a/(a+
3)/(a+4))/(x^4-4*x^3+8*x^2-a-8*x)+1/16/(a+3)/(a+4)*sum(((-a-4)*_R^2+2*(-a-2
)*_R+a)/(_R^3-3*_R^2+4*_R-2)*ln(-_R+x),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(a+4)x^3-(a+6)x^2+(a+8)x+a}{4\left(\left(a^2+7\,a+12\right)x^4-4\left(a^2+7\,a+12\right)x^3-a^3+8\left(a^2+7\,a+12\right)x^2-7\,a^2-8\left(a^2+7\,a+12\right)x-12\,a\right)}-\frac{\int \frac{(a+4)x^2+2\,(a+2)x-a}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx}{4\left(a^2+7\,a+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*
(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a
+ 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4*x^
3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

mupad [B] time = 2.85, size = 1218, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

```
[Out] symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
 a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^
4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8
*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568
*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 242155
52*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*
z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5888*a^4
*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^3 - a^4,
 z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(816*a + 460
*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4 + 31653888
*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 2008232755
```

```
2*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7
247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 2094
08*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z +
53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 102
4*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31653888*a^7*z^4 +
 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4
+ 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*
z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2
 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*
z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 14
4*a^3 - a^4, z, k)*((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 +
 90112*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 +
a^5 + 576)) - (x*(1966080*a + 1359872*a^2 + 499712*a^3 + 102912*a^4 + 11264
*a^5 + 512*a^6 + 1179648))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 5
76))) - (1359872*a + 749568*a^2 + 205824*a^3 + 28160*a^4 + 1536*a^5 + 98304
0)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + (x*(104448*a + 5
8880*a^2 + 16512*a^3 + 2304*a^4 + 128*a^5 + 73728))/(8*(816*a + 460*a^2 + 1
29*a^3 + 18*a^4 + a^5 + 576))) + (x*(448*a + 104*a^2 - 2*a^3 - 2*a^4 + 512)
)/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576))))*root(12952010752*a
^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a
*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 2696
80640*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 18780
16*a^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2
+ 237568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752
*z + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k), k, 1, 4) + (x^3/(4*(a + 3))
+ a/(4*(a + 3)*(a + 4)) - (x^2*(a + 6))/(4*(a + 3)*(a + 4)) + (x*(a + 8))/(
4*(a + 3)*(a + 4)))/(a + 8*x - 8*x^2 + 4*x^3 - x^4)
```

sympy [B] time = 43.73, size = 561, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

```
[Out] (-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
+ x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2
 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 +
 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 53571747
84*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312)
 + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24215552
*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a**3 + 23
7568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792*a, Lamb
da(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*_t**3*a**
10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t**3*a**7 -
 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832*_t**3*a**4
 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 439848271872*_t**3*a
 - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9 - 15400960*_t
**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414427648*_t**2*a
**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938255360*_t**2*a*
*2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 - 6096*_t*a**8 +
174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 119009280*_t*a**4 + 3
32017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 225837056*_t + 11*a
**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4 + 1567552*a**3 + 20
49792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**6 + 165176*a**5 + 82456
0*a**4 + 2452288*a**3 + 4340224*a**2 + 4229120*a + 1748992))))
```

3.132 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=545

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(-1)^{2/}\cdots}{3\sqrt{3}\,(\cdots}$$

Rubi [A] time = 1.48, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{5/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(-1)^{2/3}\left(3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+2b\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{5/6}b^2c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2 a^{2/3}b^2\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1
/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*a
^(5/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((-1)^(2/3)*(2*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(3*Sqrt[3]*(1 - (
-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(5/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c
^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(18*a^(2/3)*b^2*c
^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(6*(1 + (-1)^(1
/3))^2*a^(2/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^
(1/3)*x + b*x^2])/(18*a^(2/3)*b^2*c^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{c}\,x}{59049\left(1+\sqrt[3]{-1}\right)^2 a^{20/3}bc^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$=\frac{\int\frac{-\sqrt[3]{a}-\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{9a^{2/3}bc^{2/3}}-\frac{(-1)^{2/3}\int\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{c}\,x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{9a^{2/3}bc^{2/3}}+\frac{\int}{3}$$

$$=\frac{\left(3-\frac{2b}{\sqrt[3]{a}\,c^{2/3}}\right)\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{18b^2}+\frac{\left(3-\frac{2(-1)^{2/3}b}{\sqrt[3]{a}\,c^{2/3}}\right)\int\frac{}{3a+3(-1)^{2/}}}{18b^2}$$

$$=-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+b\right.}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}}$$

$$=-\frac{\left(3ib+\sqrt{3}\left(b+3\sqrt[3]{a}\,c^{2/3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}-$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.18

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^3\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.02, size = 93, normalized size = 0.17

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^4\ln\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{6\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5b^3+36\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3a\,b^2+81\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2a^2c+54\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.18, size = 1563, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z
^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*
a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 15
94323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 3280
5*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*
a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z
```

```
+ 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^
6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*
c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^3*c^2 - 19
683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^
4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^
2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*root(918330048*a^
5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 10235
16*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c
*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b^9*c^4*z^6 - 3874
20489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 -
 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*
b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^
6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3
+ 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^5*b^6*c^5 + 108*root(9
18330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*
z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
+ 324*a*b*c*z + 1, z, k)^2*a*b^5*c*x + 108*root(918330048*a^5*b^9*c^4*z^6 -
 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*
z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*a
*b*c^2*x + 1458*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b^2*c^3*x - 2916*root(
918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4
*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2
 + 324*a*b*c*z + 1, z, k)^3*a^2*b^6*c^2*x + 78732*root(918330048*a^5*b^9*c^
4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b
^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1,
z, k)^4*a^3*b^7*c^3*x + 1062882*root(918330048*a^5*b^9*c^4*z^6 - 387420489*
a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 53144
1*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^5*a^4*b^8*c^
4*x))*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*
a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*
b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=487

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx\right.}{54a^{4/3}bc^{2/3}}$$

Rubi [A] time = 0.76, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}+\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\,a^{7/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(3*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqr
t[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*a^(7/6
)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
)*c^(2/3)])])/(3*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(7/6)*b*Sqrt
[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/3)
*x + b*x^2]/(54*a^(4/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(
2/3)*c^(1/3)*x + b*x^2])/(18*(1 + (-1)^(1/3))^2*a^(4/3)*b*c^(2/3)) + ((-1)^
(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(54*a^(4/3)*b*c^(2
/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int \left(\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$= \frac{\int \frac{x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-}{}$$

$$= \frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} + \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{54a^{4/3}bc^{2/3}} - \frac{(-}{}$$

$$= \frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{54a^{4/3}bc^{2/3}} - \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\right.}{18\left(1+\sqrt[3]{-1}\right)^2 a^{4/3}bc^{2/}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,\sqrt[3]{c}} - \frac{\tan^{-1}}{9\sqrt{3}\,a^{7/}}$$

Mathematica [C] time = 0.05, size = 99, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^2\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.00, size = 93, normalized size = 0.19

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3\ln\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{6\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5b^3+36\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3a\,b^2+81\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2a^2c+54\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.10, size = 1354, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c
^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4
*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12914
0163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348
907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a
^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*a^9*b^
3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928
*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^
```

```
11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a
^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441
*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 - 94143178827
*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*
a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b
*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^7*c + 4
374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143489
07*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^
3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353203*a^9*b^3*c^6*z^6
 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*
c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b^9*c -
 59049*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 143
48907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683
*a^3*b*c^2*z^2 - 1, z, k)*a^7*b^6*c^2*x - 531441*root(10460353203*a^9*b^3*c
^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^
4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^8*b
^8*c^2*x - 688747536*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6
*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c
^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^4*a^10*b^12*c^2*x + 1162261467*root
(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b
^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*
z^2 - 1, z, k)^4*a^11*b^9*c^4*x - 20920706406*root(10460353203*a^9*b^3*c^6*
z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b
^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^1
1*c^4*x)*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1
4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 196
83*a^3*b*c^2*z^2 - 1, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=334

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\ldots\right.}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)\ldots}$$

Rubi [A] time = 0.47, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2097, 618, 204}

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{2\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{11/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{11/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a
]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 + (-1)^(1/3))^2
*a^(11/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*ArcTan[(3*
a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/
(27*Sqrt[3]*a^(11/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + (2*(-1)^(2/3)
*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b +
3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3
))^2*a^(11/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int \left(\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2 a^{22/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$= \frac{\int \frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-1)^{2/3}\int \frac{1}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{27a^{4/3}c^{2/3}} + \frac{(-}{}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-3a\left(4b-3\sqrt[3]{a}\,c^{2/3}\right)-x^2}\,dx, x, 3a^{2/3}\sqrt[3]{c}+2bx\right)}{27a^{4/3}c^{2/3}} - \frac{\left(2(-1\right.}{}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}} + \frac{2\tan^{-1}}{27\sqrt{3}\,a^{11}}$$

Mathematica [C] time = 0.06, size = 97, normalized size = 0.29

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&, \frac{\#1\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6
),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) &
]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6),x]
```

```
[Out] IntegrateAlgebraic[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
+ b^3*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

maple [C] time = 0.00, size = 93, normalized size = 0.28

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2\ln\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{6\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+36\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3 a\,b^2+81\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2 a^2c+54\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right) a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
), x)
```

mupad [B] time = 3.34, size = 825, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 28242
9536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12
*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c^3 -
 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784401*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(669462604992*
a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 1968
3*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^4*z^6 - 28
2429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^3
*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^
6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^3*c^3
+ 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129
140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^2*c*x + 708588*ro
ot(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^
8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x + 38263752*root(66
9462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4
*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^4*a^7*b^2*c^3*x + 774840978*root(669462
604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4
 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^9*b^2*c^4*x + 1))*root(669462604992*a^1
```

```
1*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a
^4*c^2*z^2 + 1, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

```
[Out] Timed out
```

3.135 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=469

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+\right.}{162a^{7/3}c^{2/3}}$$

Rubi [A] time = 0.68, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}-\frac{(-1)^{2/3}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{7/3}c^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

```
[Out] -ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b -
3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(9*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt
[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan[(3*a^(2/3)*c^(1/3) +
 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])]/(27*Sqrt[3]*a^(13/
6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(1/3)*ArcTan[(3*(-1)^(2/3
)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)
*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(13/6)*Sqrt[4
*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
 + b*x^2]/(162*a^(7/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a^(2/3
)*c^(1/3)*x + b*x^2])/(54*(1 + (-1)^(1/3))^2*a^(7/3)*c^(2/3)) - ((-1)^(2/3)
*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*a^(7/3)*c^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
```

```
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-3a^{2/3}\sqrt[3]{c}-(-1)^{2/3}bx}{531441\left(1+\sqrt[3]{-1}\right)^2 a^{25/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$= \frac{\int \frac{-3a^{2/3}\sqrt[3]{c}-bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{81a^{7/3}c^{2/3}} + \frac{\int}{2}$$

$$= -\frac{\int \frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}} - \frac{(-1)^{2/3}\int \frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2}\,dx}{162a^{7/3}c^{2/3}} + $$

$$= -\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{7/3}c^{2/3}} + \frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\right.}{54\left(1+\sqrt[3]{-1}\right)^2 a^{7/3}c}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\sqrt[3]{c}} - \frac{\tan^{-1}}{27\sqrt{3}\,a}$$

Mathematica [C] time = 0.05, size = 95, normalized size = 0.20

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),
x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 +
b^3*x^6),x]
```

```
[Out] IntegrateAlgebraic[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 +
b^3*x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

maple [C] time = 0.00, size = 91, normalized size = 0.19

$$\frac{\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)\ln\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{6\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+36\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3 a\,b^2+81\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2 a^2c+54\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_
R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

mupad [B] time = 2.90, size = 1057, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^11*c
^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15
*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94143178
827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 116
2261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 1
77147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(18075490334784*
a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 +
 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 +
b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
```

```
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^2*a^5*b^11*c^
2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^
6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*
c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c^2*x - 14348907*r
oot(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 11622614
67*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147
*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 229582512*root(180754903347
84*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^
4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2
 + b^3, z, k)^4*a^9*b^13*c^2*x + 387420489*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7
*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4
*a^10*b^10*c^4*x - 20920706406*root(18075490334784*a^14*b^3*c^4*z^6 - 76255
97484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^11*c
^4*x)*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1
162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 +
 177147*a^5*b^2*c^2*z^2 + b^3, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)
```

```
[Out] Timed out
```

3.136 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{(2\cdots}{27\sqrt{3}\cdots}$$

Rubi [A] time = 0.86, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2070, 634, 618, 204, 628}

$$-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}\,c^{2/3}+2\sqrt[3]{-1}\,b\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\,a^{17/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\,\sqrt{a}\,\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}+\frac{\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}-\frac{\sqrt[3]{-1}\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

```
[Out] -((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2
/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)])])/(27*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1
/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*a^
(17/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*(-1)^(2/3)*b - 3*a^(1/3
)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*S
qrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*(1 - (-1)^(1/3))*(1
+ (-1)^(1/3))^2*a^(17/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(162*a^(8/3)*c^(1/3)) - Log[3*a -
3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(54*(1 + (-1)^(1/3))^2*a^(8/3)*c^(1
/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*
a^(8/3)*c^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \left(19683a^6\right)\int\left(\frac{-(-1)^{2/3}\sqrt[3]{a}\,b-3\sqrt[3]{-1}\,a^{2/3}c^{2/3}+b\sqrt[3]{c}}{531441\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(-3a+3\sqrt[3]{-1}\,a^{2/3}\right.}\right.$$

$$=\frac{\int\frac{-\sqrt[3]{a}\,b+3a^{2/3}c^{2/3}+b\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-\frac{\int\frac{(-1)^{2/3}\sqrt[3]{a}\,b-3a^{2/3}c^{2/3}+\sqrt[3]{-1}\,b\sqrt[3]{c}\,x}{3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{81a^{8/3}c^{2/3}}-$$

$$=-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{1}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{162a^{7/3}c^{2/3}}-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}\,c^{2/\cdots}\right.}{16\cdots}$$

$$=\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+bx^2\right.}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}}$$

$$=-\frac{\left(2(-1)^{2/3}b+3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\,c^{2/3}}$$

Mathematica [C] time = 0.07, size = 99, normalized size = 0.19

$$\frac{1}{3}\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\log(x-\#1)}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-
1),x]
```

```
[Out] RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
, Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5) &
]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^
3*x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^
3*x^6)^(-1), x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="giac")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

maple [**C**] time = 0.00, size = 90, normalized size = 0.17

$$\frac{\ln\left(-\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{6\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+36\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3 a\,b^2+81\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2 a^2c+54\operatorname{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] 1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3),
 x)
```

mupad [**B**] time = 0.71, size = 1394, normalized size = 2.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

```
[Out] symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^1
8*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 3874
20489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k
)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209464
9*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 +
 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6,
 z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 20589113209
4649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^
3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b
```

```
^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^3*c^4*z^6
- 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a
^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^
3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(488038239039168*a^17
*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4
- 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*
z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 167365651248*root(488
038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*
a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 265
7205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^12*c^3 - 9414
3178827*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^13
*b^9*c^5 + 14580*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^
18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387
420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z,
k)^2*a^3*b^14*c*x - 10628820*root(488038239039168*a^17*b^3*c^4*z^6 - 205891
132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c
^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*
z + b^6, z, k)^3*a^6*b^13*c^2*x + 2238429492*root(488038239039168*a^17*b^3*
c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746
143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 -
 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^12*c^3*x - 1162261467*root(488038239
039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b
^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a
^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^9*c^5*x - 209207064
06*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 1
0460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^
5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a^12*b^11
*c^4*x)*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^
6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^
10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k), k, 1,
 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3)
,x)

[Out] Timed out
```

3.137 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=563

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}\,c^{2/3}+i\sqrt{3}\,b+b\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\right.}{}$$

Rubi [A] time = 1.16, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.109, Rules used = {2097, 634, 618, 204, 628}

$$-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}-\frac{\left(6\sqrt[3]{a}\,c^{2/3}+i\sqrt{3}\,b+b\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{972a^{10/3}c^{2/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{10/3}}+\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{27\sqrt{3}\,a^{19/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt[3]{c}\sqrt{3\sqrt[3]{-1}\sqrt[3]{a}\,c^{2/3}+4b}}+\frac{\log(x)}{27a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x]
```

```
[Out] ((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3
]*(1 + (-1)^(1/3))^2*a^(19/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1
/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*S
qrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(27*Sqrt[3]*a^(19/6)*Sqrt[4*b - 3*a
^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3)*b - a^(1/3)*c^(2/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-
1)^(1/3)*a^(1/3)*c^(2/3)])])/(9*Sqrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2
*a^(19/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(27*a^
3) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(486*
a^(10/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(972*a^(10/3)*c^(2/3)) - ((3*a^(1/3) - ((-1)^(2/
3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(1
0/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x}+\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}+b}{4782969a^{28/3}c^{2/3}\left(3a+3a\right.}\right.$$

$$=\frac{\log(x)}{27a^3}+\frac{\int\frac{3a^{2/3}\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}+b\left(b-3\sqrt[3]{a}\,c^{2/3}\right)x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{243a^{10/3}c^{2/3}}+\frac{(-1)^{2/3}}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{486a^{10/3}}-\frac{\left(3\sqrt[3]{a}-\frac{(-}{c}\right.}{}$$

$$=\frac{\log(x)}{27a^3}-\frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{486a^{10/3}}-\frac{\left(b+\right.}{}$$

$$=\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}\sqrt[3]{c}}+\frac{\left(b\right.}{}$$

Mathematica [C] time = 0.12, size = 157, normalized size = 0.28

$$-\frac{\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^4b^3\log(x-\#1)+9\#1^2ab^2\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c\log(x-\#1)}{2\#1^4b^3+12\#1^2ab^2+27\#1a^2c+18a^2b}\&\right]-3\log(x)}{81a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6)),x]
```

```
[Out] -1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2
*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*
b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 12*a
*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
 + b^3*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4
 + b^3*x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^{3}x^{6} + 9\,ab^{2}x^{4} + 27\,a^{2}cx^{3} + 27\,a^{2}bx^{2} + 27\,a^{3}\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x), x)
```

maple [C] time = 0.01, size = 134, normalized size = 0.24

$$\frac{\ln(x)}{27a^{3}} - \frac{\left(\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{5} b^{3} + 9\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{3} a\,b^{2} + 27\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{2} a^{2}c + 27\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right) a^{2}b\right) \ln\left(-\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right) + x\right)}{81a^{3}\left(2\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{5} b^{3} + 12\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{3} a\,b^{2} + 27\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right)^{2} a^{2}c + 18\,\mathrm{RootOf}\left(b^{3}\_Z^{6} + 9b^{2}a\,\_Z^{4} + 27a^{2}c\,\_Z^{3} + 27b\,a^{2}\_Z^{2} + 27a^{3}\right) a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2*b)/(2*_R^5*b^3
+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b^3+9*_Z
^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))+1/27*ln(x)/a^3
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.55, size = 4002, normalized size = 7.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x)
```

```
[Out] log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^20*b^3
*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5
```

 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658
28329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*
z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^
6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093442*root(13
177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 48803823
9039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^1
4*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c
^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^8*b
^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*z^6 - 55590605665
55523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^1
8*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 2
7506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12
*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b
^6*c^2*z + b^9, z, k)^4*a^11*b^13*c^3 - 10460353203*root(13177032454057536*
a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3
*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*
b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009737
9*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^12*b^10*c^5 + 15062
90861232*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^5*a^14*b^13*c^3 - 564859072962*root(13177032454057536*a^20*b^3*c^4
*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254186582832
9*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3
 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^
2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^10*c^5 - 67783088755440*ro
ot(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488
038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930662375
5*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*
z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*
b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*
a^17*b^13*c^3 + 22876792454961*root(13177032454057536*a^20*b^3*c^4*z^6 - 55
59060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911320
94649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^
6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046035
3203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6
561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^10*c^5 + 17496*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^4*b^16*c - 47239
2*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 +
 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 61193066
23755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*
c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*
a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k
)^3*a^7*b^16*c - 39366*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566
555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^
18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^1
2*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*
b^6*c^2*z + b^9, z, k)^2*a^4*b^15*c^2*x + 51372630*root(13177032454057536*a
^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*

```
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2
541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b
^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379
*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^3*a^7*b^15*c^2*x + 71744
535*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6
 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 611930
6623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^
3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 1434890
7*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z,
 k)^3*a^8*b^12*c^4*x - 2008846980*root(13177032454057536*a^20*b^3*c^4*z^6 -
 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2058911
32094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15
*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 1046
0353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2
- 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^10*b^15*c^2*x + 108477736920*root(131
77032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239
039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14
*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 -
229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^
4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^4*a^11*b
^12*c^4*x - 41841412812*root(13177032454057536*a^20*b^3*c^4*z^6 - 555906056
6555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a
^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 +
 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^
12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4
*b^6*c^2*z + b^9, z, k)^4*a^12*b^9*c^6*x + 18596183472*root(131770324540575
36*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*
b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4
 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^
10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 1009
7379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^13*b^15*c^2*x +
16129864639026*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a
^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z
^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854
719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^
3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*
z + b^9, z, k)^5*a^14*b^12*c^4*x - 6778308875544*root(13177032454057536*a^2
0*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^
4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 254
1865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6
*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a
^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^5*a^15*b^9*c^6*x + 6456339
20395566*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^
6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6
119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^
11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14
348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^6*a^17*b^12*c^4*x - 274521509459532*root(13177032454057536*a^20*b^
3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^
5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865
828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2
*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b
^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^6*a^18*b^9*c^6*x)*root(1317703
2454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 4880382390391
68*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3
*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 2295
82512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^
2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k), k, 1, 6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**
3),x)
```

```
[Out] Timed out
```

3.138 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal. Leaf size=645

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2b^2\right)\tan^{-}}{243\sqrt{3}\,a^{23/6}c^{2/3}\sqrt{4b-}}$$

Rubi [A] time = 1.38, antiderivative size = 640, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 5, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.109$, Rules used = {2097, 634, 618, 204, 628}

$$\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\,\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2b^2\right)\tan^{-1}\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}\right)}{243\sqrt{3}\,a^{23/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}\,c^{2/3}}}+\frac{\left(-9\sqrt[3]{-1}\,a^{2/3}c^{4/3}-12\sqrt[3]{a}\,bc^{2/3}+2(-1)^{2/3}b^2\right)\tan^{-1}\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}c^{2/3}\sqrt{3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+4b}}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\log\left(3a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}\,c^{2/3}\right)\log\left(-3\sqrt[3]{-1}\,a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}}+\frac{\sqrt[3]{-1}\left(3\sqrt[3]{-1}\,\sqrt[3]{a}\,c^{2/3}+2b\right)\log\left(3(-1)^{2/3}a^{2/3}\sqrt[3]{c}\,x+3a+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}-\frac{1}{27a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),
x]
```

```
[Out] -1/(27*a^3*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3)*b*c^(2/3) + 9*a^
(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[
a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(81*Sqrt[3]*(1 + (-1)^(1/3))
^2*a^(23/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*b^2 - 1
2*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)
/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(243*Sqrt[3]*a^(23/6)*Sq
rt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*c^
(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3)
+ 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(81*S
qrt[3]*(1 - (-1)^(1/3))*(1 + (-1)^(1/3))^2*a^(23/6)*Sqrt[4*b + 3*(-1)^(1/3)
*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a^(2/3)
*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/3)*a^(1/3)*
c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(162*(1 + (-1)^
(1/3))^2*a^(11/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3
))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(486*a^(11/3)*c^(1/3)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \left(19683a^6\right)\int\left(\frac{1}{531441a^9x^2}+\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc^{2/3}+\right.}{1594323\left(1-\sqrt[3]{-1}\right)\left(1+\right.}\right.$$

$$= -\frac{1}{27a^3x}+\frac{\int\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\sqrt[3]{c}\,x}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{243a^{11/3}c^{2/3}}$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\int\frac{3a^{2/3}\sqrt[3]{c}+2bx}{3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2}\,dx}{486a^{11/3}\sqrt[3]{c}}+\frac{\left(\sqrt[3]{-1}\right.}{}$$

$$= -\frac{1}{27a^3x}-\frac{\left(2b-3\sqrt[3]{a}\,c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}\,x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}+$$

$$= -\frac{1}{27a^3x}+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}\,bc^{2/3}+9a^{2/3}c^{4/3}\right)\tan}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(}}$$

Mathematica [C] time = 0.14, size = 163, normalized size = 0.25

$$-\frac{x\text{RootSum}\left[\#1^6b^3+9\#1^4ab^2+27\#1^3a^2c+27\#1^2a^2b+27a^3\&,\frac{\#1^4b^3\log(x-\#1)+9\#1^2ab^2\log(x-\#1)+27a^2b\log(x-\#1)+27\#1a^2c\log(x-\#1)}{2\#1^5b^3+12\#1^3ab^2+27\#1^2a^2c+18\#1a^2b}\&\right]+3}{81a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*
x^6)),x]
```

```
[Out] -1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
+ b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*Lo
g[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*
b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x
^4 + b^3*x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x
^4 + b^3*x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
*x^2), x)
```

maple [C] time = 0.01, size = 133, normalized size = 0.21

$$\frac{\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^4 b^3-9\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2 a\,b^2-27\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2c-27b\,a^2\right)\ln\left(-\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)+x\right)}{81a^3\left(2\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^5 b^3+12\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^3 a\,b^2+27\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)^2 a^2c+18\,\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)a^2b\right)}-\frac{1}{27a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

```
[Out] -1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*_
R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(-_R+x),_R=RootOf(_Z^6*b
^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, a
lgorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 2.72, size = 2663, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),
x)
```

```
[Out] symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946
35296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*
a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^
```

3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x + 2541865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 387420489*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^12*b^5*c^2 + 6973568802*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^13*b^2*c^4 - 4518872583696*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^3*c^3 - 328050*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^6*c^2 - 177147*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^6*b^3*c^4 + 387420489*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^10*b*c^5 + 23328*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^4*b^8*c*x + 196830*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^5*b^5*c^3*x - 20920706406*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^3*a^13*b*c^5*x + 74401740*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352

```
96999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^1
6*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z
^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*
b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^2*a^8*b^6*c^2*x - 746143164*
root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6
- 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145
430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14
*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^
4*b^10*c*z + b^12, z, k)^2*a^9*b^3*c^4*x + 55788550416*root(355779876259553
472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^1
7*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*
z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326
*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z
, k)^3*a^12*b^4*c^3*x + 564859072962*root(355779876259553472*a^23*b^3*c^4*z
^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 1093
00230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104
*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 +
 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^16*b^2*c^
4*x))*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^
6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 -
753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 28242953648
1*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17
496*a^4*b^10*c*z + b^12, z, k), k, 1, 6) - 1/(27*a^3*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*
a**3),x)
```

```
[Out] Timed out
```

3.139 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=395

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)+\frac{1}{10}$$

Rubi [A] time = 1.44, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)+\frac{1}{108}\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)-\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}\,x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(1-\sqrt[3]{2}\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -(((-2)^(1/3)*(1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/
Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6
) + 3*2^(1/3)*3^(2/3)])) + ((3/2)^(1/6)*(1 - (-3)^(2/3)*2^(1/3))*ArcTan[(2^
(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/((1
+ (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1/3)*3^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/
(2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((36 + 2^(2/3)*3^(1/3)*(1
+ I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/216 + ((18 - (-2)^(2/3
)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/108 + ((18 - 2^(2/3)*3^(1
/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/108
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}+\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{3779136\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\right.$$

$$=\frac{1}{54}\int\frac{6\sqrt[3]{2}\,3^{2/3}+\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx+\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}\right.}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+}}{9\sqrt[3]{2}\,3^{2/3}}$$

$$=\frac{\left((-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)\int\frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{6\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left((-1)^{2/3}\sqrt[3]{\frac{3}{2}}\left(6+\right.\right.}{}$$

$$=\frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}+i2^{2/3}3^{5/6}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)+\frac{1}{108}\left(18\right.$$

$$=\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+\right.}{\sqrt[6]{6}\left(1+\right.}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.15

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

[Out] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.14

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5 \ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 0.65, size = 427, normalized size = 1.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 26467
86132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 19131876
*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416355
9388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4374*
z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266
2865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^
3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4 + 264678
6132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 1704742
2*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72

```
662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5 + 6626610*z^
4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4
 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 +
 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 + 4374*z^5 + 662
6610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048,
z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24
163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*root(z^6 - z^5 + (421
*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 - z/16612452 - 1/7266
2865048, z, k), k, 1, 6)
```

sympy [**A**] time = 0.26, size = 70, normalized size = 0.18

$$\operatorname{RootSum}\left(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1, \left(t \mapsto t\log\left(-\frac{89236417131047376t^5}{833243797} + \frac{89301949532998128t^4}{833243797} - \frac{29740560281805852t^3}{833243797} + \frac{192466080408420t^2}{49014341} + \frac{5867255361684t}{833243797} + x + \frac{5365044886}{2499731391}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 2646786
132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-892364171310473
76*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 29740560281805852*
_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/8332437
97 + x + 5365044886/2499731391)))
```

3.140 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2}\right.}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^{2}}$$

Rubi [A] time = 0.91, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{18\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{27\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{27\sqrt{6\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4
- 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*
3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(27*Sqrt[3*(8 + (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(2^(
1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(27*Sqrt[6
*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(6*2^(2
/3)*3^(1/3)*(1 + (-1)^(1/3))^2) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/(18*2^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(18*2^(2/3)*3
^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int \left(\frac{(-1)^{2/3}\left(-2+\sqrt[3]{-3}\,2^{2/3}x\right)}{7558272\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)} \cdots$$

$$= -\frac{(-1)^{2/3}\int \frac{2+(-2)^{2/3}\sqrt[3]{3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\sqrt[3]{2}\,3^{2/3}} - \frac{\int \frac{\sqrt[3]{2}+\sqrt[3]{3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{9\,6^{2/3}} + \frac{(-1)^{2/3}\int \cdots}{6\sqrt[3]{2}\,3^{2/3}} \cdots$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\,2^{2/3}} - \frac{\int \frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\,2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{3\sqrt[3]{-3}\,2^{2/3}-2\cdots}{-6+3\sqrt[3]{-3}\,2^{2/3}x\cdots}}{6\,2^{2/3}\sqrt[3]{3}\left(1+\cdots\right.} \cdots$$

$$= \frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{6\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\,2^{2/3}} - \cdots$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\,3^{2/3}\right)\cdots}{27\sqrt[6]{3}\sqrt{2\cdots}} \cdots$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^3\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

[Out] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

maple [C] time = 0.01, size = 56, normalized size = 0.15

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4 \ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)

[Out] 1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")

[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)

mupad [B] time = 2.70, size = 390, normalized size = 1.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)

[Out] symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066

```
242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 1180980*z^4 -
1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 7558272*root(z^
6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156951788503
68, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^
3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^4/7596 + (217*z^
3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/15695178850368, z, k), k,
1, 6)
```

sympy [A] time = 0.28, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t\log\left(\frac{6147145261785517462 08t^5}{57121295165} - \frac{1270857362386176t^4}{57121295165} - \frac{80483053187684376t^3}{57121295165} + \frac{72431318325103884t^2}{57121295165} - \frac{45358602689088t}{57121295165} + x - \frac{44532180783}{57121295165}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 118098
0*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/571212
95165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571212
95165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/57121295165
 + x - 44532180783/57121295165)))
```

3.141 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}-\frac{\tan}{6\sqrt[6]{2}\,3^{5/6}\left(1\right.}$$

Rubi [A] time = 0.51, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{108\sqrt[3]{2}\,3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{9\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{18\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(6
*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1)
^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3)
)]])/(9*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]
) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3
))]]/(18*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(36*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) +
((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(108*2^(1/3)*3^(2/3)) +
Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(108*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}\right.$$

$$=\frac{\int\frac{x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{}{-6+3}}{18\sqrt[3]{2}\,3^{2/3}(}$$

$$=\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{18\,2^{2/3}}+\frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{3(-2)}{6+3(-2}}{108\sqrt[3]{2}}$$

$$=-\frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{36\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x\right.}{108\sqrt[3]{2}\,3^{2/3}}$$

$$=-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\right.}}\right.}{18\sqrt[6]{2}\,3^{5/6}\sqrt{4+3\sqrt[3]{-2}}}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.17

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^2\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/(
36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.16

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3 \ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.52, size = 276, normalized size = 0.76

$$\sum_{k=1}^{6} \ln\left(\frac{23328\,\left(297538935552\,x - 7992872640\,x\,\mathrm{root}(\ldots,z,k) + 52488\,x\,\mathrm{root}(\ldots,z,k)^3 + 2904\,x\,\mathrm{root}(\ldots,z,k)^4 + x\,\mathrm{root}(\ldots,z,k)^5 - 153055008\,\mathrm{root}(\ldots,z,k)^2 - 2764368\,\mathrm{root}(\ldots,z,k)^3 - 1620\,\mathrm{root}(\ldots,z,k)^4 - 3673320192\,\mathrm{root}(\ldots,z,k) + 7240114098432\right)}{\mathrm{root}\left(z^6+1417176\,z^4+1332145440\,z^3+74384733888\,z^2-3390158631679488,z,k\right)}\right) \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root(z
^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z,
k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3
390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 7438
4733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 1417176*z^4
+ 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 2764368*ro
ot(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
 z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 -
 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 + 1332145440
*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432))/root(z^6
 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)
```

```
^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/2392193088 - 1/3390158
631679488, z, k), k, 1, 6)
```

sympy [A] time = 0.25, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{8482372214243328t^5}{415817} + \frac{2216055910930560t^4}{415817} - \frac{2062546612992t^3}{415817} - \frac{57027208896t^2}{415817} - \frac{416583756t}{415817} + x - \frac{89938}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 141
7176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 22160559
10930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4158
17 - 416583756*_t/415817 + x - 89938/415817)))
```

3.142 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=248

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Rubi [A] time = 0.32, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2097, 618, 204, 206}

$$\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{81\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] ((-1)^(2/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^
(1/3))]])/(27*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1
/3)]) + ((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(81*2^(1/3)*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^
(1/3)*3^(2/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]]/(81*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}}{22674816\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right)}\cdots\right.$$

$$=\frac{\int\frac{1}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{54\sqrt[3]{2}\,3^{2/3}}+\frac{(-1)^{2/3}\int\frac{\cdots}{-6+3\cdots}}{18\sqrt[3]{2}\,3^{2/3}\left(\cdots\right.}$$

$$=-\frac{\text{Subst}\left(\int\frac{1}{-6\left(4-3\sqrt[3]{2}\,3^{2/3}\right)-x^2}\,dx,x,3\,2^{2/3}\sqrt[3]{3}+2x\right)}{27\sqrt[3]{2}\,3^{2/3}}-\frac{(-1)^{2/3}\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$=\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{(-1)^{2/3}\tan^{-1}\left(\frac{3(-2)^{2/3}\cdots}{\sqrt{6\left(4+3\sqrt[3]{\cdots}\right.}}\right.}{81\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{\cdots}}}$$

Mathematica [C] time = 0.01, size = 59, normalized size = 0.24

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(36
 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
```

fricas [B] time = 4.13, size = 1277, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] 1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)*
(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867
)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt(1/6
33)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(6*18^(
2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2*x + 72
9/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/136728*sqrt(1266)*sqrt(-2
/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 4
8*18^(1/3) + 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 18*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(
```

```
1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3
) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36
*18^(2/3) + 48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)
 - 211*sqrt(1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273
*sqrt(1266))*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)
^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*
18^(2/3) - 3888*18^(1/3) - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3)
+ sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
 371) - 8/9*18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 18*sqrt
(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*
(6*18^(2/3) + 8*18^(1/3) + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/
3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) +
48*18^(1/3) + 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(
1266)) - 1247*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))
*sqrt(-2/3*18^(2/3) + sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(
2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3
888*18^(1/3) - 16578) - 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27
*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*
18^(1/3) + 18)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3)
+ 8*18^(1/3) + 81) + 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 -
 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3)
+ 371)*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 124
7*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*1
8^(2/3) - sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18
^(1/3) + 371) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3)
 - 16578) + 1/136728*sqrt(1266)*sqrt(-2/3*18^(2/3) - sqrt(-1/27*(6*18^(2/3)
 + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 8/9*18^(1/3) + 1
8)*log(2*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 18*sqrt(-1/27*(6*18^(2/3) + 8*1
8^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3)
 + 81) - 1/211*(6*sqrt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 9*sqrt(-1/2
7*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(6*sq
rt(1266)*(6*18^(2/3) + 8*18^(1/3) + 81) - 211*sqrt(1266)) - 1247*sqrt(1266)
*(6*18^(2/3) + 8*18^(1/3) + 81) + 51273*sqrt(1266))*sqrt(-2/3*18^(2/3) - sq
rt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371
) - 8/9*18^(1/3) + 18) + 3376*x - 2916*18^(2/3) - 3888*18^(1/3) - 16578)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.23

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.68, size = 247, normalized size = 1.00

$$\sum_{k=1}^{6}\ln\left(-\frac{216\,\left(32134205039616\,x-1836660096\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^2-1889568\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^3+972\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^4+\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^5+132239526912\,x\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)+204073344\,x\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^2+139968\,x\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^3+36\,x\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^4+863230245120\,\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)+781932322630656\right)}{\mathrm{root}\left(z^6-2834352\,z^4+2677850419968\,z^2-732274264442769408,z,k\right)^5}\right)\,\mathrm{root}\left(z^6-\frac{z^4}{273456}+\frac{z^2}{258356853504}-\frac{1}{732274264442769408},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 26
77850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 2834352*
z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 - 2834
352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 132239526912*x*
root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k) + 20
4073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408,
z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 73227426444
2769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274
264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 2677850419968
*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 - 2834352*z^4
 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 - z^4/273456 +
 z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

sympy [A] time = 0.21, size = 48, normalized size = 0.19

$$\operatorname{RootSum}\left(732274264442769408t^6-2677850419968t^4+2834352t^2-1,\left(t\mapsto t\log\left(10170475895038464t^5-5231726283456t^4-31809932496t^3+19131876t^2+19683t+x-\frac{27}{2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1,
 Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 31809932
496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

3.143 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=361

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\tan^{-}}{36\sqrt[6]{2}\,3^{5/6}\left(1\right.}$$

Rubi [A] time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2097, 634, 618, 204, 628, 206}

$$\frac{(-1)^{2/3}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{(-1)^{2/3}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\sqrt[3]{2}\,3^{2/3}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{54\,2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{108\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] -ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(3
6*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((-1
)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(54*2^(2/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3
)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]]/(108*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(216*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2
) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(648*2^(1/3)*3^(2/3)
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}\,2^{2/3}-x\right)}{136048896\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right.}\right.$$

$$= -\frac{(-1)^{2/3}\int\frac{3(-2)^{2/3}\sqrt[3]{3}+x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\sqrt[3]{2}\,3^{2/3}} - \frac{\int\frac{6\sqrt[3]{3}+\sqrt[3]{2}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{324\,6^{2/3}} + \frac{(-1)^{2/3}\int\frac{3}{-6+}}{108\sqrt[3]{2}\,3^{2/3}}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int\frac{1}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{108\,2^{2/3}} - \frac{\int\frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}\,3^{2/3}} - \frac{(-1)^{2/3}\int\frac{3(-2)}{6+3(-2}}{648\sqrt[3]{2}}$$

$$\vdots$$

$$= \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+\right.}{648\sqrt[3]{2}\,3^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{}}{\sqrt{6\left(4+3\sqrt[3]{}\right.}}\right.}{108\sqrt[6]{2}\,3^{5/6}\sqrt{4+3\sqrt[3]{}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 16
2*#1 + 12*#1^2 + #1^4) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

```
[Out] IntegrateAlgebraic[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.15

$$\frac{\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 176, normalized size = 0.49

$$\sum_{k=1}^{6}\ln\left(x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(216\,x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(51018336\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(277947894528\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(33192121254912\,x-\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(6940988288557056\,x+1688973816882216696\right)+28563737812992\right)\right)\right)\right)\right)\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(216*x + root(z^6 - z^4/1640736 +
(235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)
*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/310028
2242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 - z^4/1
640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411196381921
28, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/1291784267
52 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288557056*x
 + 1688973816882216696) + 28563737812992)))))*root(z^6 - z^4/1640736 + (235*
z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k), k,
1, 6)
```

sympy [A] time = 0.26, size = 61, normalized size = 0.17

$$\operatorname{RootSum}\left(1581712411196381921\,28t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto t \log\left(\frac{6541839944572114096\,1280t^5}{415817} + \frac{2480926457425102848t^4}{415817} - \frac{39451802929737984t^3}{415817} + \frac{118071997444800t^2}{415817} - \frac{16745884920t}{415817} + x - \frac{268790}{415817}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*_
t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(654183994457211409612 80*_t**5/
415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/415817
+ 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415817)
))
```

3.144 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal. Leaf size=377

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2\right.}{324\sqrt[6]{3}\left(1+\sqrt[3]{-\right.}}$$

Rubi [A] time = 0.72, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2070, 634, 618, 204, 628, 206}

$$-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\sqrt[3]{-\frac{1}{3}}\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{648\,2^{2/3}\sqrt[3]{3}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}+\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{972\sqrt{3\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)}}-\frac{\left(9-2^{2/3}\sqrt[3]{3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{972\sqrt{6\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sq
rt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(324*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(
4 - 3*(-3)^(2/3)*2^(1/3))]) + ((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(972*Sqrt[3*(8 + (9*I
)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))]) - ((9 - 2^(2/3)*3^(1/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(972*Sq
rt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(21
6*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2])/(648*2^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(648*2
^(2/3)*3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2070

```
Int[(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6, x, 2]
, c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist[1/(3
^(3*p)*a^(2*p)), Int[ExpandIntegrand[(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2
)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-1)^(2/3
)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0] && Eq
Q[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[Q6, x,
1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = 1259712\int\left(\frac{(-1)^{2/3}\left(-2+6(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}\,2^{2/3}x\right)}{272097792\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(-6+3\sqrt[3]{-3}\,2^{2/3}x-x^2\right.}\right.$$

$$= \frac{\int \frac{18-2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{1944} - \frac{\int \frac{2(-1)^{2/3}-6\sqrt[3]{2}\,3^{2/3}+\sqrt[3]{-3}\,2^{2/3}x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\sqrt[3]{2}\,3^{2/3}} + \frac{(-1)^{2/3}\int}{216}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int \frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{6+3(-2)^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\,2^{2/3}} + \frac{\int \frac{3\,2^{2/3}\sqrt[3]{3}+2x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{648\,2^{2/3}\sqrt[3]{3}} - \frac{\int \frac{3\sqrt[3]{-3}\,2^{2/3}}{-6+3\sqrt[3]{-3}\,2^{2/3}}}{216\,2^{2/3}\sqrt[3]{3}\,(1}$$

$$= -\frac{\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\,2^{2/3}} +$$

$$= -\frac{\sqrt[3]{-1}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\left(1+\sqrt[3]{-1}\right)^2\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\,3^{2/3}\right)}{972\sqrt[6]{3}\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 62, normalized size = 0.16

$$\frac{1}{6}\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1 +
 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

maple [C] time = 0.01, size = 53, normalized size = 0.14

$$\frac{\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{6\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+972\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+216\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.67, size = 306, normalized size = 0.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

```
[Out] symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/
111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2
*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071
3728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 61222003
20*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728
 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582637960
59136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713
728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x - 6940988
288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5*x + 944
784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016071372
```

```
8 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 - 16529940864
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^3 - 33192121254912
*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 +
 z/488182842961846272 - 1/34164988081841849499648, z, k)^4 - 16889738168822
1696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607137
28 + z/488182842961846272 - 1/34164988081841849499648, z, k)^5)*root(z^6 -
z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111610160713728 + z/488182842
961846272 - 1/34164988081841849499648, z, k), k, 1, 6)
```

sympy [A] time = 0.27, size = 65, normalized size = 0.17

$$\operatorname{RootSum}\left(34164988081841849499648t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 + 69984t - 1, \left(t \mapsto t\log\left(\frac{185904446699109611410573787136t^5}{57121295165} + \frac{637730125326791738276659 2t^4}{57121295165} - \frac{189046360023885643115 52t^3}{57121295165} - \frac{469080552915181723968t^2}{57121295165} - \frac{24358640509989936t}{57121295165} + x + \frac{152427895956}{57121295165}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 8608793201
9712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(18590444669
9109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/571
21295165 - 1890463600238856431155 2*_t**3/57121295165 - 46908055291518172396
8*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895956/5
7121295165)))
```

3.145 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal. Leaf size=415

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{23328}-\frac{\left(18-2^{2/3}\right.}{}$$

Rubi [A] time = 0.90, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{46656}-\frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{23328}-\frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{23328}+\frac{\log(x)}{216}+\frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\sqrt[3]{2}\,3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(1-\sqrt[3]{2}\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{216\sqrt[6]{2}\,3^{5/6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] ((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sq
rt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(216*2^(1/3)*3^(5/6)*Sqrt[8 + (9*I)*2^(1
/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))
]])/(216*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 -
 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*
2^(1/3)*3^(2/3))]])/(216*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + Lo
g[x]/216 - ((36 + 2^(2/3)*3^(1/3)*(1 + I*Sqrt[3]))*Log[6 - 3*(-3)^(1/3)*2^(
2/3)*x + x^2])/46656 - ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2])/23328 - ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/23328
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int \left(\frac{1}{272097792x} + \frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}\,2^{2/3}\right)-\left(1-\cdots\right.\right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(6-\cdots\right.}\cdots$$

$$= \frac{\log(x)}{216} + \frac{\int \frac{-6\sqrt[3]{6}\left(9\sqrt[3]{2}-2\sqrt[3]{3}\right)-\left(18-2^{2/3}\sqrt[3]{3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664} + \frac{(-1)^{2/3}\int \frac{-6\left(9-(-2)\cdots\right.}{6+\cdots}}{194\cdots}$$

$$= \frac{\log(x)}{216} + \frac{\left(\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\right)\int \frac{1}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{72\left(1+\sqrt[3]{-1}\right)^2} + \frac{\left((-1)\cdots\right.}{\cdots}$$

$$= \frac{\log(x)}{216} - \frac{(-1)^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \cdots$$

$$= \frac{(-1)^{2/3}\left((-2)^{2/3}-2\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{216\,6^{5/6}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}} - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}\cdots\right.}{216\sqrt[6]{6}\cdots}$$

Mathematica [C] time = 0.02, size = 103, normalized size = 0.25

$$\frac{\log(x)}{216} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^4+12\#1^2+162\#1+36}\&\right]}{1296}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

maple [C] time = 0.01, size = 75, normalized size = 0.18

$$\frac{\ln(x)}{216}-\frac{\left(\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+18\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+324\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+108\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{1296\left(\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+12\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+162\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+36\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] 1/216*ln(x)-1/1296*sum((_R^5+18*_R^3+324*_R^2+108*_R)/(_R^5+12*_R^3+162*_R^
2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216}\int \frac{x^5+18\,x^3+324\,x^2+108\,x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx+\frac{1}{216}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] -1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3 +
 108*x^2 + 216), x) + 1/216*log(x)
```

mupad [B] time = 2.33, size = 432, normalized size = 1.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853
*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352
- 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 +
 z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 154687594
7520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^3*x - 10696114790560972 8*root(z^6 + z^5/216 + (421*z^4)/590
```

```
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/78109
25487389540352 - 1/7379637425677839491923968, z, k)^4*x - 14051199585413401
8048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432
- (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796374256778394
91923968, z, k)^5*x - 45607290567387619000320*root(z^6 + z^5/216 + (421*z^4
)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/
7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x + 839808*root(
z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2
)/12053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968,
z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^
3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1
/7379637425677839491923968, z, k)^3 - 8483430130458624*root(z^6 + z^5/216 +
 (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1205389735708
2624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4 - 38314
25535283494912*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902
540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637
425677839491923968, z, k)^5 + 1217393817906599165952*root(z^6 + z^5/216 + (
421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6)*root(z^6
 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/1
2053897357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z,
k), k, 1, 6)
```

sympy [**A**] time = 0.42, size = 82, normalized size = 0.20

$$\frac{\log(x)}{216} + \operatorname{RootSum}\left(7379637425677839491923968t^6 + 34164988081841849499648t^5 + 52598809250685370368t^4 + 26673506015311872t^3 - 309171116160t^2 + 944784t - 1, \left(t \mapsto t\log\left(\frac{8145570099668817936783362115119297360560128t^6}{143425799309052440063} + \frac{977068766770806381087358257564745728t^5}{143425799309052440063} - \frac{116529526608851264288400971539061538816t^4}{143425799309052440063} - \frac{239359794985242202542501440710766592t^3}{143425799309052440063} - \frac{136678312638137094439887341418240t^2}{143425799309052440063} + \frac{1563115569067663795735413696t}{143425799309052440063} + x - \frac{3164446315075236190044}{143425799309052440063}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 34164988081841849499
648*_t**5 + 52598809250685370368*_t**4 + 26673506015311872*_t**3 - 30917111
6160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(81455700996688179367833621151
19297360560128*_t**6/143425799309052440063 + 977068766770806381087358257564
745728*_t**5/143425799309052440063 - 11652952660885126428840097153906153881
6*_t**4/143425799309052440063 - 239359794985242202542501440710766592*_t**3/
143425799309052440063 - 136678312638137094439887341418240*_t**2/14342579930
9052440063 + 1563115569067663795735413696*_t/143425799309052440063 + x - 31
64446315075236190044/143425799309052440063))
```

3.146 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)} \, dx$

Optimal. Leaf size=448

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right.}{}$$

Rubi [A] time = 1.10, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 634, 618, 204, 628, 206}

$$-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{7776\sqrt[3]{3}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{3888\sqrt[3]{6}}-\frac{1}{216x}-\frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\,6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{5832\sqrt[6]{6}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/(216*x) - ((27*(-6)^(1/3) - (-2)^(2/3) + 12*3^(2/3))*ArcTan[(3*(-2)^(2/3
)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(5832*3^(1/6)*Sqrt[8
+ (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(6*(-6)^(2/3) +
 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1944*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3
*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6
)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(5832*6^(1/6)
*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6
 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(1296*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2)
 + ((3*(-6)^(2/3) + 2*(-2)^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(7
776*3^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(
3888*6^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = 1259712\int\left(\frac{1}{272097792x^2}+\frac{(-1)^{2/3}\left(-1+9(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{\cdots}\right.}{816293376\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)\cdots}\cdots\right.$$

$$= -\frac{1}{216x}+\frac{\int\frac{-54+2^{2/3}\sqrt[3]{3}+54\sqrt[3]{2}\,3^{2/3}-6^{2/3}\left(2^{2/3}-3\,3^{2/3}\right)x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{11664}+\frac{(-1)^{2/3}\int\cdots}{\cdots}$$

$$= -\frac{1}{216x}-\frac{\left((-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\right)\int\frac{-3\sqrt[3]{-3}\,2^{2/3}+2x}{6-3\sqrt[3]{-3}\,2^{2/3}x+x^2}\,dx}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left((-1)^{2\cdots}\right.}{\cdots}$$

$$= -\frac{1}{216x}-\frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\cdots$$

$$= -\frac{1}{216x}+\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3(-2\cdots}{\sqrt{6(4\cdots}}\right.}{5832\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}\cdots}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{\#1^4\log(x-\#1)+18\#1^2\log(x-\#1)+324\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{1296}-\frac{1}{216x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] -1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*Lo
g[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/(3
6*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

```
[Out] integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 0.17

$$\frac{\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-18\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2-324\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)-108\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{1296\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+15552\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+209952\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+46656\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}-\frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

```
[Out] -1/216/x+1/1296*sum((-_R^4-18*_R^2-324*_R-108)/(_R^5+12*_R^3+162*_R^2+36*_R
)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{216\,x} - \frac{1}{216}\int \frac{x^4 + 18\,x^2 + 324\,x + 108}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

```
[Out] -1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 324
*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.29, size = 340, normalized size = 0.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

```
[Out] symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594001683946413
330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300
846726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/1594
001683946413330255577088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/118132
992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189805489
```

```
3435658305536 - 1/15940016839464133302555770888, z, k)^2*x - 598229670528*ro
ot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155
89428330496 - z/1898054893435658305536 - 1/15940016839464133302555770888, z,
 k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008
46726144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/15940
016839464133302555770888, z, k)^4*x - 6940988288557056*root(z^6 + (281*z^4)/
118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
8054893435658305536 - 1/15940016839464133302555770888, z, k)^5*x + 2344464*r
oot(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215
589428330496 - z/1898054893435658305536 - 1/15940016839464133302555770888, z
, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/93008467
26144 - (331*z^2)/48215589428330496 - z/1898054893435658305536 - 1/15940016
839464133302555770888, z, k)^3 - 10535082310656*root(z^6 + (281*z^4)/1181329
92 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/1898054893
435658305536 - 1/15940016839464133302555770888, z, k)^4 - 168897381688221696
*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482
15589428330496 - z/1898054893435658305536 - 1/15940016839464133302555770888,
 z, k)^5)*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331
*z^2)/48215589428330496 - z/1898054893435658305536 - 1/159400168394641333302
55577088, z, k), k, 1, 6) - 1/(216*x)
```

sympy [A] time = 0.32, size = 70, normalized size = 0.16

$$\operatorname{RootSum}\left(15940016839464133302555770888t^6 + 3791612026460331638784t^4 - 8643672699589509120t^3 - 10942820851968t^2 - 839808t - 1, \left(t \mapsto t\log\left(-\frac{49875532761902496003293561236914468028416t^5}{12350449784703991795} + \frac{126254898724316203880059752004976 64t^4}{12350449784703991795} - \frac{1186376926075737712385507988526448 64t^3}{12350449784703991795} + \frac{270486324927832147818193778754816t^2}{12350449784703991795} + \frac{273914194897479402961199352t}{12350449784703991795} + x - \frac{12798926329353908292}{12350449784703991795}\right)\right)\right) - \frac{1}{216x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

```
[Out] RootSum(15940016839464133302555770888*_t**6 + 3791612026460331638784*_t**4 -
 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda(_
t, _t*log(-49875532761902496003293561236914468028416*_t**5/1235044978470399
1795 + 126254898724316203880059752004976664*_t**4/12350449784703991795 - 118
63769260757377123855079885264486 4*_t**3/12350449784703991795 + 270486324927
832147818193778754816*_t**2/12350449784703991795 + 273914194897479402961199
352*_t/12350449784703991795 + x - 12798926329353908292/12350449784703991795
))) - 1/(216*x)
```

3.147 $\displaystyle\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1064

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\right.}{162\sqrt[6]{2}\,3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\right.}$$

Rubi [A] time = 2.50, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

$$-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{729\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{162\sqrt[6]{2}\,3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)^{3/2}}-\frac{i\left((-2)^{2/3}+6\,3^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{162\,2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}+\frac{\left(i2^{2/3}-9\sqrt[6]{3}-3i3^{2/3}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\,2^{5/6}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\sqrt[3]{-1}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\tan^{-1}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\sqrt{2}\,3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\,6^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}\,3^{2/3}\right)}}\right)}{81\sqrt{2}\,3^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}\,3^{2/3}\right)^{3/2}}-\frac{\left(1+3\sqrt[3]{2}\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}\,3^{2/3}\right)}}\right)}{1458\sqrt[6]{2}\,3^{5/6}\sqrt{-4+3\sqrt[3]{2}\,3^{2/3}}}-\frac{\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{972\sqrt[3]{2}\,3^{2/3}\left(1+\sqrt[3]{-1}\right)^4}+\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}{972\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}-\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}{8748\sqrt[3]{2}\,3^{2/3}}-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2-3\,2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x+9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)\right)}{162\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\left(2+2^{2/3}\left(27\sqrt[3]{3}-6\,6^{2/3}\right)\right)x+9\left(6-2^{2/3}\sqrt[3]{3}\right)}{1458\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)) + (2 - 3*2^(2/3)*(2*(-6)^(2/3) +
 9*(-3)^(1/3)))*x))/(162*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/
3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*
3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1/3)*3^(2/3))*x))/(729*2^(
2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1
/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 + 2^(2/3)*(27*3^(1/3) - 6*6^(
2/3)))*x)/(1458*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1
/3)*x + x^2)) - ((I/162)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^
(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3
^(1/3) + 12*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*
(4 + 3*(-2)^(1/3)*3^(2/3))]])/(162*2^(1/6)*3^(5/6)*(1 - (-1)^(1/3))^2*(1 +
(-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3)
 + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqr
t[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(81*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4
 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*
ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3
))]])/(162*2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)
]) - ((1 + 3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqr
t[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1458*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3
^(2/3)]) + ((2^(1/3) + 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(81*Sqrt[2]*3^(5/6)*(1 - (
-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - Log[6 - 3
*(-3)^(1/3)*2^(2/3)*x + x^2]/(972*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I
/972)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/
3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(8748*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{\sqrt[3]{-\frac{1}{3}}\left(-1+3(-3)^{2/3}\sqrt[3]{2}+\left(9+\sqrt[3]{-3}\,2^{2/}\right.\right.}{42845606719488\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(6-3\sqrt[3]{-3}\,2\right.} \right.$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\int \frac{-1-3\sqrt[3]{-2}\,3^{2/3}+\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{243\,2^{2/3}} - \frac{\int \frac{-27+x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{4374\sqrt[3]{2}\,3^{2/3}} + \frac{\int \frac{1-3}{(}}{}$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\,2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \dots$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\,2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \dots$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\left(9\left(6+\sqrt[3]{-3}\,2^{2/3}\right)+\left(2-3\,2^{2/3}\left(2(-6)^{2/3}+9\sqrt[3]{-3}\right)\right)x\right)}{162\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \dots$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.16

$$\frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182\left(x^6+18x^4+324x^3+108x^2+216\right)} - \frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&, \frac{9\#1^4\log(x-\#1)+406\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)-96\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{205092}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108*
x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 +
406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/205092
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.11

$$\frac{\left(-9\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-406\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3-324\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+96\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)-324\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{205092\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+2461104\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+33224904\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+7383312\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+1
8*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5+203\,x^4+11610\,x^3+3990\,x^2-324\,x+7884}{34182\,\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{34182}\int \frac{9\,x^4+406\,x^3+324\,x^2-96\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18*
x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324*x
^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.34, size = 388, normalized size = 0.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141
49992416343982992 + (5171*z^2)/5093997269883833877l2 + (505*z)/133686864350
83133627113728 + 4513/852560170529641874151233606645 76, z, k)*x)/876306843
- (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619 + (8
113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*
z)/13368686435083133627113728 + 4513/852560170529641874151233606645 76, z, k
))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868643
5083133627113728 + 4513/852560170529641874151233606645 76, z, k)^2*x)/44521
- (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/141499924
16343982992 + (5171*z^2)/509399726988383387712 + (505*z)/133686864350831336
27113728 + 4513/852560170529641874151233606645 76, z, k)^3*x)/44521 + (13078
9789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343
982992 + (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113
728 + 4513/852560170529641874151233606645 76, z, k)^4*x)/211 - 6940988288557
056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149992416343982992
+ (5171*z^2)/509399726988383387712 + (505*z)/13368686435083133627113728 + 4
513/852560170529641874151233606645 76, z, k)^5*x - (4264220928*root(z^6 + (3
```

```
26*z^4)/554702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093
99726988383387712 + (505*z)/1336868643508313362711 3728 + 4513/8525601705296
4187415123360664576, z, k)^2)/44521 - (5086414725120*root(z^6 + (326*z^4)/5
54702231619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883
83387712 + (505*z)/1336868643508313362711 3728 + 4513/852560170529641874151 2
3360664576, z, k)^3)/44521 + (243585208571904*root(z^6 + (326*z^4)/55470223
1619 + (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883833877 1
2 + (505*z)/1336868643508313362711 3728 + 4513/852560170529641874151233606 64
576, z, k)^4)/211 - 168897381688221696*root(z^6 + (326*z^4)/554702231619 +
(8113597*z^3)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (50
5*z)/1336868643508313362711 3728 + 4513/8525601705296418741512336066 4576, z,
 k)^5 - 48160/23660284761)*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3
)/14149992416343982992 + (5171*z^2)/509399726988383387712 + (505*z)/1336868
6435083133627113728 + 4513/8525601705296418741512336066 4576, z, k), k, 1, 6
) - ((665*x^2)/5697 - (2*x)/211 + (215*x^3)/633 + (203*x^4)/34182 + x^5/379
8 + 146/633)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

$$\mathrm{RootSum}\left(8525601705296418741512336066 4576t^6 + 501051915333854345687 04t^4 + 48885748051277486016t^3 + 865447782603408t^2 + 3220532460t + 4513, \left(t \mapsto t\log\left(\frac{35492036204084174404119193135483487466590764032t^5}{356900697070792948475845} - \frac{19474160067218837086826809631017022308224t^4}{71380139414158589695169} + \frac{207799630765451322338945827649033965 44t^3}{356900697070792948475845} + \frac{20265219154367004972162198012037344t^2}{356900697070792948475845} + \frac{27519246894921053204907514 5372t}{356900697070792948475845} + x + \frac{129028519129217728962 2012}{107070209121237884542 7535}\right)\right)\right) + \frac{-9x^5 - 203x^4 - 11610x^3 - 3990x^2 + 324x - 7884}{34182x^6 + 615276x^4 + 11074968x^3 + 3691656x^2 + 7383312}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(8525601705296418741512336066 4576*_t**6 + 501051915333854345687 04*_t
**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t +
4513, Lambda(_t, _t*log(35492036204084174404119193135483487466590764032*_t*
*5/356900697070792948475845 - 19474160067218837086826809631017022308224*_t*
*4/71380139414158589695169 + 207799630765451322338945827649033965 44*_t**3/3
56900697070792948475845 + 20265219154367004972162198012037344*_t**2/3569006
97070792948475845 + 27519246894921053204907514 5372*_t/356900697070792948475
845 + x + 129028519129217728962 2012/107070209121237884542 7535))) + (-9*x**5
 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 + 615276*x
**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

3.148 $\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=1005

$$\frac{2\left(2-3\sqrt[3]{2}\,3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{2916\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(2i2^{2/3}-9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2\ldots}{\sqrt{6\left(4-3(-3)^{2/3}\ldots\right.}}\right.}{5832\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}$$

Rubi [A] time = 2.40, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 634, 618, 204, 628, 638, 206}

Antiderivative was successfully verified.

```
[In] Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) - 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/
3))*x)/(972*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1
+ (-2)^(1/3)*3^(2/3))*x)/(4374*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2 - 3*2^(1/3)*3^(2/3)) - 3*(6
 - 2^(2/3)*3^(1/3))*x)/(2916*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3
*2^(2/3)*3^(1/3)*x + x^2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1/6) + 2
*2^(2/3)*Sqrt[3]))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(
2/3)*2^(1/3))]])/(5832*(1 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))
]) + ((1 + (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(54*Sqrt[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3)
)^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*3^(1/6) + I*(4*2^(2/3) - 3*3^(2
/3)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]])/(1944*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) -
((-1)^(1/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(54*Sqrt[2]*3^(5/6)*(1 + (-1)^(1
/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(54*Sqr
t[6]*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
+ ((2*2^(2/3) + 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(26244*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))
]) + ((I/648)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 +
(-1)^(1/3))^5) - ((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(129
6*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/
(17496*2^(2/3)*3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(\frac{-27+3\,2^{2/3}\sqrt[3]{3}+9i\sqrt{3}+i2^{2/3}3^{5/6}-3i\cdots}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(-6+3\sqrt[3]{-}\cdots\right.}\right.$$

$$= \frac{\int \frac{-18-2\,2^{2/3}\sqrt[3]{3}-\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}\,dx}{52488} + \frac{\int \frac{9\,2^{2/3}+\sqrt[3]{-1}\,3^{2/3}\left(1+3\sqrt[3]{-2}\,3^{2/3}\right)x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}\,dx}{4374\,2^{2/3}} + \cdots$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \cdots$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \cdots$$

$$= -\frac{2\left(2\sqrt[3]{-1}\,3^{2/3}+9\sqrt[3]{6}\right)-9\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)x}{972\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \cdots$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{73\#1^4\log(x-\#1)-36\#1^3\log(x-\#1)+96\#1^2\log(x-\#1)-216\#1\log(x-\#1)+96\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{410184}+\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*Log[
x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]
/410184
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.12

$$\frac{\left(73 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-36 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+96 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2-216 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+96\right) \ln\left(-\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{410184 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+4922208 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+66449808 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+14766624 \operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(_R
^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z
^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{73\,x^5 - 18\,x^4 + 908\,x^3 + 432\,x^2 - 96\,x + 648}{68364\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)} + \frac{1}{68364}\int \frac{73\,x^4 - 36\,x^3 + 96\,x^2 - 216\,x + 96}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 - 21
6*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.30, size = 387, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3
)/75466626220501242624 - (2479189*z^2)/264072818470777948189900 8 - (1989787
*z)/3118647171576193412533090467 84 - 7197829/58928958987008846341333266891 3
549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6 - (
292589*z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (247918
9*z^2)/264072818470777948189900 8 - (1989787*z)/311864717157619341253309046 7
84 - 7197829/58928958987008846341333266891 3549312, z, k)*x)/7886761587 - (2
07565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262
20501242624 - (2479189*z^2)/264072818470777948189900 8 - (1989787*z)/3118647
171576193412533090467 84 - 7197829/58928958987008846341333266891 3549312, z,
k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485412544 + (1
1805253*z^3)/75466626220501242624 - (2479189*z^2)/264072818470777948189900 8
 - (1989787*z)/3118647171576193412533090467 84 - 7197829/589289589870088463 4
13332668913549312, z, k)^3*x)/44521 - (147138513610752*root(z^6 - (292589*z
^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2
640728184707779481899008 - (1989787*z)/3118647171576193412533090467 84 - 719
7829/5892895898700884634133326689135 49312, z, k)^4*x)/211 - 694098828855705
```

```
6*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124
2624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/31186471715761
9341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5*x
- (1156135728*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546
6626220501242624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/31
1864717157619341253309046784 - 7197829/589289589870088463413332668913549312
, z, k)^2)/44521 + (6458021903232*root(z^6 - (292589*z^4)/319508485412544 +
 (11805253*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899
008 - (1989787*z)/311864717157619341253309046784 - 7197829/5892895898700884
63413332668913549312, z, k)^3)/44521 - (102226052063232*root(z^6 - (292589*
z^4)/319508485412544 + (11805253*z^3)/75466626220501242624 - (2479189*z^2)/
2640728184707779481899008 - (1989787*z)/311864717157619341253309046784 - 71
97829/589289589870088463413332668913549312, z, k)^4)/211 - 1688973816882216
96*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/754666262205012
42624 - (2479189*z^2)/2640728184707779481899008 - (1989787*z)/3118647171576
19341253309046784 - 7197829/589289589870088463413332668913549312, z, k)^5 +
 2207561/7665932262564)*root(z^6 - (292589*z^4)/319508485412544 + (11805253
*z^3)/75466626220501242624 - (2479189*z^2)/2640728184707779481899008 - (198
9787*z)/311864717157619341253309046784 - 7197829/58928958987008846341333266
8913549312, z, k), k, 1, 6) + ((4*x^2)/633 - (8*x)/5697 + (227*x^3)/17091 -
 x^4/3798 + (73*x^5)/68364 + 2/211)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216
)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(589289589870088463413332668913549312t^6 - 539640290266075248405737472t^4 + 92182638168509682392064t^3 - 553241442069170496t^2 - 3759837842016t - 7197829, \left(t \mapsto t\log\left(\frac{42996027639727447714003743305160746111018438501025999323136t^5}{154206009791052044490694380303237521} - \frac{42584766259508194684689715474422251405157209835847680t^4}{154206009791052044490694380303237521} - \frac{37512446128849588150108369449323754078317341082112t^3}{154206009791052044490694380303237521} + \frac{7152037594021675267638890715531672481920222144t^2}{154206009791052044490694380303237521} - \frac{44227546998835297723830291794974310524032t}{154206009791052044490694380303237521} + x - \frac{174573349036676047734132569583024855}{154206009791052044490694380303237521}\right)\right)\right) + \frac{73x^5 - 18x^4 + 908x^3 + 432x^2 - 96x + 648}{68364x^6 + 1230552x^4 + 22149936x^3 + 7383312x^2 + 14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(589289589870088463413332668913549312*_t**6 - 5396402902660752484057
37472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 - 37
59837842016*_t - 7197829, Lambda(_t, _t*log(4299602763972744771400374330516
0746111018438501025999323136*_t**5/154206009791052044490694380303237521 - 4
2584766259508194684689715474422251405157209835847680*_t**4/1542060097910520
44490694380303237521 - 37512446128849588150108369449323754078317341082112*_
t**3/154206009791052044490694380303237521 + 7152037594021675267638890715531
672481920222144*_t**2/154206009791052044490694380303237521 - 44227546998835
297723830291794974310524032*_t/154206009791052044490694380303237521 + x - 1
74573349036676047734132569583024855/154206009791052044490694380303237521)))
 + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364*x**6 + 123
0552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.149 $\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal. Leaf size=677

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\,3^{2/3}\left(2+3\sqrt[3]{-2}\,3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2\right.}$$

Rubi [A] time = 1.55, antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2097, 638, 618, 204, 628, 206}

$$\frac{\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x+9(-2)^{2/3}}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\,3^{2/3}\left(2+3\sqrt[3]{-2}\,3^{2/3}\right)x+9\,2^{2/3}}{13122\,2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{3\,2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}\,3^{2/3}\right)x}{8748\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}-\frac{i\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(x^2+3\,2^{2/3}\sqrt[3]{3}x+6\right)}{52488\sqrt[3]{2}\,3^{2/3}}+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\,6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}\,2^{2/3}\right)\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)^{3/2}}-\frac{\left(2^{2/3}-3\,3^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{486\,6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(3\sqrt[3]{2}\,3^{2/3}-4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (9*(-2)^(2/3) + 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))*x)/(2916*2^(2/3)*(1 + (-1)
^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) +
(9*2^(2/3) + (-1)^(1/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13122*2^(2/3
)*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)
*x + x^2)) + (3*2^(2/3)*3^(1/3) - (2 - 3*2^(1/3)*3^(2/3))*x)/(8748*2^(2/3)*
3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1
/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4
- 3*(-3)^(2/3)*2^(1/3))]])/(486*6^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3
)*2^(1/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))*ArcTan[(3*(-2)^(2/
3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-
1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((2^(2/3
) - 3*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]])/(486*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3
*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x
^2])/(5832*2^(1/3)*(1 + (-1)^(1/3))^5) - ((I/5832)*Log[6 + 3*(-2)^(2/3)*3^(
1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(
1/3)*x + x^2]/(52488*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{-2\sqrt[3]{-1}\,3^{2/3}+3(-2)^{2/3}x}{1542441841901568\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\cdots\right.}\cdots\right.$$

$$= -\frac{\int\frac{-2\sqrt[3]{-1}\,3^{2/3}+3\,2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{8748\,2^{2/3}} - \frac{\int\frac{2+2^{2/3}\sqrt[3]{3}x}{\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{2916\,2^{2/3}\sqrt[3]{3}} + \frac{\int\frac{3\sqrt[3]{3}+\sqrt[3]{2}\cdots}{6+3\,2^{2/3}\sqrt[3]{3}\cdots}}{26244\,6\cdots}$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\cdots$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\cdots$$

$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{2916\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}+\cdots$$

Mathematica [C] time = 0.05, size = 167, normalized size = 0.25

$$\frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{3\#1^4\log(x-\#1)-146\#1^3\log(x-\#1)+108\#1^2\log(x-\#1)-32\#1\log(x-\#1)+108\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{410184}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 32
4*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6
 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*Log[
x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/
410184
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.02, size = 122, normalized size = 0.18

$$\frac{\left(-3\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4+146\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3-108\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+32\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)-108\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{410184\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+4922208\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+66449808\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+14766624\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-108)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+10
8*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{3\,x^5-73\,x^4+72\,x^3+64\,x^2-108\,x+96}{68364\,\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{68364}\int \frac{3\,x^4-146\,x^3+108\,x^2-32\,x+108}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 32*
x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.33, size = 388, normalized size = 0.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)
/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)
/311864717157619341253309046784 - 880007/3977704731623097128039995515166457
856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)*x)/70980854283 - (
6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619
30300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715
7619341253309046784 - 880007/3977704731623097128039995515166457856, z, k)^2
*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408 - (1549690
9*z^3)/3056398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3
971*z)/311864717157619341253309046784 - 880007/3977704731623097128039995515
166457856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60865*z^4)/23963
1364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/594163841
5592503834272768 - (3971*z)/311864717157619341253309046784 - 880007/3977704
731623097128039995515166457856, z, k)^4*x)/211 - 6940988288557056*root(z^6
- (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (16
8169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090467
84 - 880007/3977704731623097128039995515166457856, z, k)^5*x + (100886752*r
oot(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30563983619303003262
72 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31186471715761934125
3309046784 - 880007/3977704731623097128039995515166457856, z, k)^2)/133563
+ (1715052538368*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/30
56398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31
1864717157619341253309046784 - 880007/3977704731623097128039995515166457856
, z, k)^3)/44521 + (115004308571136*root(z^6 - (60865*z^4)/239631364059408
- (15496909*z^3)/3056398361930300326272 - (168169*z^2)/59416384155925038342
72768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309712
8039995515166457856, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^
4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/5
941638415592503834272768 - (3971*z)/311864717157619341253309046784 - 880007
/3977704731623097128039995515166457856, z, k)^5 - 265/5749449196923)*root(z
^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 -
(168169*z^2)/5941638415592503834272768 - (3971*z)/3118647171576193412533090
46784 - 880007/3977704731623097128039995515166457856, z, k), k, 1, 6) - ((1
6*x^2)/17091 - x/633 + (2*x^3)/1899 - (73*x^4)/68364 + x^5/22788 + 8/5697)/
(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.38, size = 112, normalized size = 0.17

$$\operatorname{RootSum}\left(3977704731623097128039995515166457856t^6 - 1010314319415295961050951680t^4 - 20168224477093957151232t^3 - 112582856818899648t^2 - 50648453064t - 880007, \left(t \mapsto t\log\left(-\frac{273655567090018991570649941414395560986199688040644608t^5}{49797855396139900267573395695} + \frac{11837008470196046085308646230764354292805044570112t^4}{49797855396139900267573395695} - \frac{10570581900446717266374077482873315047787008t^3}{49797855396139900267573395695} - \frac{1552547411569469872387563218792789323392t^2}{49797855396139900267573395695} - \frac{12542923791159140826909003250295928t}{49797855396139900267573395695} + x - \frac{23066533870320322410834348296}{49797855396139900267573395695}\right)\right)\right) + \frac{-3x^5 + 73x^4 - 72x^3 - 64x^2 + 108x - 96}{68364x^6 + 1230552x^4 + 22149936x^3 + 7383312x^2 + 14766624}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(3977704731623097128039995515166457856*_t**6 - 101031431941529596105
0951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2 -
50648453064*_t - 880007, Lambda(_t, _t*log(-2736555670900189915706499414143
95560986199688040644608*_t**5/49797855396139900267573395695 + 1183700847019
6046085308646230764354292805044570112*_t**4/49797855396139900267573395695 -
 10570581900446717266374077482873315047787008*_t**3/49797855396139900267573
395695 - 1552547411569469872387563218792789323392*_t**2/4979785539613990026
7573395695 - 12542923791159140826909003250295928*_t/49797855396139900267573
395695 + x - 23066533870320322410834348296/49797855396139900267573395695)))
 + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x**6 + 12305
52*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

3.150 $\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=682

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}\,x+4\right)}{8748\,2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}\,+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)\right.}$$

Rubi [A] time = 1.24, antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2097, 638, 618, 204, 206}

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-\frac{1}{3}}\left((-2)^{2/3}\sqrt[3]{3}\,x+4\right)}{8748\,2^{2/3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}\,+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}\,x+6\right)}-\frac{2^{2/3}\sqrt[3]{3}\,x+4}{17496\,2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2+3\,2^{2/3}\sqrt[3]{3}\,x+6\right)}+\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\,\sqrt[6]{3}\,+3\sqrt[3]{2}\,3^{2/3}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\,2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\,\sqrt[6]{3}\,+3\sqrt[3]{2}\,3^{2/3}\right)^{3/2}}-\frac{i\tan^{-1}\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}\,3^{2/3}\right)}}\right)}{1458\,2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}\,3^{2/3}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{39366\,2^{5/6}\sqrt[6]{3}\sqrt{3\sqrt[3]{2}\,3^{2/3}-4}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}\,x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}\,3^{2/3}-4\right)}}\right)}{8748\sqrt{6}\left(3\sqrt[3]{2}\,3^{2/3}-4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(1944*2^(2/3)*(1 + (-1)^(1/3))^4*
(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + ((-1/3)^(1
/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(8748*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/6) +
3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (4 + 2^(2/3)*3^(1/
3)*x)/(17496*2^(2/3)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)
*x + x^2)) - ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2
^(1/3))]]/(4374*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^
(1/3)]) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1
/3))]]/(4374*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
 - ((I/1458)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3
^(2/3))]])/(2^(5/6)*3^(2/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))
]]/(4374*Sqrt[3]*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/
(8748*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3)
 + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(39366*2^(5/6)*3^(1/6)*Sqr
t[-4 + 3*2^(1/3)*3^(2/3)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
```

```
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{\sqrt[3]{-\frac{1}{3}}\,x}{1542441841901568\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int\frac{x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)^2}dx}{8748\,2^{2/3}} + \frac{\int\frac{1}{6+3\,2^{2/3}\sqrt[3]{3}x+x^2}dx}{26244\sqrt[3]{2}\,3^{2/3}} + \frac{\int\frac{}{\left(6+3\,2\right.}}{874}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} +$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} +$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}\,2^{2/3}x\right)}{1944\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} +$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.24

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{4\#1^4\log(x-\#1)-54\#1^3\log(x-\#1)+2043\#1^2\log(x-\#1)-324\#1\log(x-\#1)+144\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{3691656}+\frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #
1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 - 54*
Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/3691656
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
[Out] IntegrateAlgebraic[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [B] time = 3.89, size = 1445, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
[Out] 1/28041818976*(182304*x^5 - 1230552*x^4 + 33224904*x^3 + 422*sqrt(1/633)*(x
^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 - 447902374840040617697 9673*18^(2/3) - 83
3430652250766125864 5112*18^(1/3) - 268625598114228853471 20477)*sqrt(5034474
*18^(2/3) + 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 513225545496
0803463351330/9393931*18^(2/3) + 954980203637704604075 3520/9393931*18^(1/3)
 + 272789282330339400324 25830/9393931) - 422*sqrt(1/633)*(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
*log(-2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(
1/3) + 44687457)^2 - 447902374840040617697 9673*18^(2/3) - 83343065225076612
58645112*18^(1/3) - 268625598114228853471 20477)*sqrt(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*
18^(1/3) + 44687457)^2 + 247458158879850620*x + 5132255454960803463351330/9
393931*18^(2/3) + 954980203637704604075 3520/9393931*18^(1/3) + 272789282330
33940032425830/9393931) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 2
16)*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397
4962699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 3064230/211*sqrt(-1/19683*(5034474*
18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787
01088/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*
18^(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(
2/3) + 9367856*18^(1/3) + 44687457)^2 + 243*sqrt(-1/19683*(5034474*18^(2/3)
 + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81
*18^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 936785
6*18^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 177934129698570542
9*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805
69051992480475*sqrt(422))*sqrt(-20718*18^(2/3) + sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387
65959317812080*x - 102645109099216069267026 60/211*18^(2/3) - 19099604072754
092081507040/211*18^(1/3) - 545578564660678800648 51660/211) + 9*sqrt(422)*(
x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) + sqrt(-1/1968
3*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3)
 + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 36779
8)*log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
 3064230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)
^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895
278433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*
(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 +
```

```
 243*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228
60116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*
sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061
5865*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856
*18^(1/3) + 44687457) + 265058558805690519924804 75*sqrt(422))*sqrt(-20718*1
8^(2/3) + sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 936785
6/243*18^(1/3) + 367798) + 44068338765959317812080*x - 1026451090992160692 6
702660/211*18^(2/3) - 190996040727540920815070 40/211*18^(1/3) - 54557856466
067880064851660/211) - 9*sqrt(422)*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)
*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
+ 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 27397496
2699) - 9367856/243*18^(1/3) + 367798)*log(14766083020/211*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 3064230/211*sqrt(-1/19683*(5034474*18^
(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 34454787010
88/81*18^(1/3) + 273974962699)*(5895278433468*18^(2/3) + 10969590754592*18^
(1/3) + 57028339027521) + 9/9393931*(14476041114*sqrt(422)*(5034474*18^(2/3
) + 9367856*18^(1/3) + 44687457)^2 - 243*sqrt(-1/19683*(5034474*18^(2/3) +
9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18
^(1/3) + 273974962699)*(14476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*1
8^(1/3) + 44687457) - 161351097450615865*sqrt(422)) - 1779341296985705429*s
qrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) + 265058558805690
51992480475*sqrt(422))*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(5034474*18^(2/
3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/
81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798) + 440683387659
59317812080*x - 1026451090992160692670 2660/211*18^(2/3) - 19099604072754092
081507040/211*18^(1/3) - 54557856466067880064851660/211) + 9*sqrt(422)*(x^6
 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-20718*18^(2/3) - sqrt(-1/19683*(
5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 22860116892*18^(2/3) +
3445478701088/81*18^(1/3) + 273974962699) - 9367856/243*18^(1/3) + 367798)*
log(14766083020/211*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 30
64230/211*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2
+ 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(5895278
433468*18^(2/3) + 10969590754592*18^(1/3) + 57028339027521) - 9/9393931*(14
476041114*sqrt(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 - 24
3*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 228601
16892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699)*(14476041114*sqr
t(422)*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457) - 16135109745061586
5*sqrt(422)) - 1779341296985705429*sqrt(422)*(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457) + 26505855880569051992480475*sqrt(422))*sqrt(-20718*18^(
2/3) - sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)^2 + 2
2860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974962699) - 9367856/2
43*18^(1/3) + 367798) + 44068338765959317812080*x - 102645109099216069267 02
660/211*18^(2/3) - 19099604072754092081507040/211*18^(1/3) - 54557856466067
880064851660/211) + 29533248*x^2 - 6562944*x + 44299872)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
[Out] integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.18

$$\frac{\left(4\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-54\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+2043\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2-324\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+144\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{3691656\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+44299872\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+598048272\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+132899616\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^4
+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*
_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,x^5 - 27\,x^4 + 729\,x^3 + 648\,x^2 - 144\,x + 972}{615276\,\left(x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216\right)} + \frac{1}{615276}\int \frac{4\,x^4 - 54\,x^3 + 2043\,x^2 - 324\,x + 144}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2 -
 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 0.31, size = 299, normalized size = 0.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/383
4101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510
4978847349012449000830556700672, z, k))/189282278088 - (16340881*root(z^6 -
 (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39
753025/27493895104978847349012449000830556700672, z, k)*x)/5110621508376 -
(43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651
774823903461376 - 39753025/27493895104978847349012449000830556700672, z, k)
^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950528 + (6
209*z^2)/14083883651774823903461376 - 39753025/274938951049788473490124490000
0830556700672, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183899*z^4)/3
834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
104978847349012449000830556700672, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/27493895104978847349012449000830556700672, z, k)^5*x + (5943884*
root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838836517748239034
61376 - 39753025/27493895104978847349012449000830556700672, z, k)^2)/400689
 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/1408
3883651774823903461376 - 39753025/27493895104978847349012449000830556700672
, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z^4)/383410182495052
8 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734901
2449000830556700672, z, k)^4)/211 - 168897381688221696*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/274
93895104978847349012449000830556700672, z, k)^5 - 13082875/1788308678210929
92)*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823
903461376 - 39753025/27493895104978847349012449000830556700672, z, k), k, 1
, 6) + ((2*x^2)/1899 - (4*x)/17091 + x^3/844 - x^4/22788 + x^5/153819 + 1/6
33)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.31, size = 104, normalized size = 0.15

$$\operatorname{RootSum}\left(27493895104978847349012449000830556700672t^6 - 1318718189226950088862983192576t^4 + 12120917704776776448t^2 - 39753025, \left(t \mapsto t\log\left(\frac{947842259001288723909832054550209950242045952t^5}{61864539719962655} - \frac{243458646817775607639654889480814592t^4}{9811980923071} - \frac{416825564750675004317873107796674 56t^3}{61864539719962655} + \frac{12026877442664328616462272t^2}{9811980923071} + \frac{216142618488859793668428t}{61864539719962655} + x - \frac{308574300024117}{39247923692284}\right)\right)\right) + \frac{4x^5 - 27x^4 + 729x^3 + 648x^2 - 144x + 972}{615276x^6 + 11074968x^4 + 199349424x^3 + 66449808x^2 + 132899616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(27493895104978847349012449000830556700672*_t**6 - 13187181892269500
88862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t, _t
*log(947842259001288723909832054550209950242045952*_t**5/61864539719962655
- 243458646817775607639654889480814592*_t**4/9811980923071 - 41682556475067
500431787310779667456*_t**3/61864539719962655 + 12026877442664328616462272*
_t**2/9811980923071 + 216142618488859793668428*_t/61864539719962655 + x - 3
08574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 648*x**2
- 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 66449808*x**
2 + 132899616)
```

3.151 $\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2} dx$

Optimal. Leaf size=850

$$\frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}+\frac{\sqrt[3]{-1}\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{729\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2}\cdots\right.}$$

Rubi [A] time = 1.47, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 614, 618, 204, 634, 628, 206}

Antiderivative was successfully verified.

```
[In] Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-1/3)^(1/3)*(3*(-3)^(1/3)*2^(2/3) - 2*x))/(5832*2^(2/3)*(1 + (-1)^(1/3))^
4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-1/3)^
(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(26244*2^(2/3)*(8 + (9*I)*2^(1/3)*3^(1/
6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (3*3^(1/3) +
2^(1/3)*x)/(52488*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2))
+ ((-1)^(1/3)*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*
2^(1/3))]])/(729*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1
/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3)
+ 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2916*2^(1/6)*3^(5/6)*(1 - (-1)
^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I + Sqrt
[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]
])/(11664*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]
) - ((I/5832)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3
)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/
3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(
1/3)*3^(2/3))]]/(26244*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(
52488*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2]/(34992*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^4) + ((I/34992)*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)*(1 + (-1)^(1/3))^5) -
 Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(314928*2^(1/3)*3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}}{1542441841901568\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\ldots\right.}\right.$$

$$= -\frac{\sqrt[3]{-\frac{1}{3}}\int \frac{1}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{8748\, 2^{2/3}} - \frac{\int \frac{3\, 2^{2/3}\sqrt[3]{3}+x}{6+3\, 2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{157464\sqrt[3]{2}\, 3^{2/3}} + \frac{\int \frac{\ldots}{\left(6+3\, 2^{\ldots}\right.}}{874\ldots}$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \ldots$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \ldots$$

$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(3\sqrt[3]{-3}\,2^{2/3}-2x\right)}{5832\, 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \ldots$$

Mathematica [C] time = 0.04, size = 167, normalized size = 0.20

$$\frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)-16\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)-2628\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{7383312}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 - 1
6*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5
) & ]/7383312
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(-9\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4+16\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3-324\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+2628\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)-324\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{7383312\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+88599744\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+1196096544\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+265799232\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18*
x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-32
4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5-8\,x^4+216\,x^3+1458\,x^2-324\,x+288}{1230552\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{1230552}\int \frac{9\,x^4-16\,x^3+324\,x^2-2628\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x^2
 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 388, normalized size = 0.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/
660182046176944870474752 - (168169*z^2)/770036338660788496921750 7328 + (397
1*z)/242506004061764799758573114779 2384 - 880007/18558379195860721960583403
0755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 - (10
90723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
0406176479975857311477923 84 - 880007/185583791958607219605834030755606257 72
9536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/86267291061
38688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660
7884969217507328 + (3971*z)/24250600406176479975857311477923 84 - 880007/185
583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (64554687936*
root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870
474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617
647997585731147792384 - 880007/185583791958607219605834030755606257729536,
z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/8626729106138688
+ (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770036338660788496
9217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/185583791
958607219605834030755606257729536, z, k)^4*x)/211 - 6940988288557056*root(z
^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176944870474752
 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060040617647997
585731147792384 - 880007/185583791958607219605834030755606257729536, z, k)^
5*x - (1697552*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660
182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z
)/2425060040617647997585731147792384 - 880007/185583791958607219605834030 75
5606257729536, z, k)^2)/10818603 + (12229983936*root(z^6 - (60865*z^4)/8626
729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77003
63386607884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880
007/185583791958607219605834030755606257729536, z, k)^3)/44521 + (253679492
45952*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182046176
944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2425060
040617647997585731147792384 - 880007/185583791958607219605834030755606257 72
9536, z, k)^4)/211 - 168897381688221696*root(z^6 - (60865*z^4)/862672910613
8688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/7700363386607
884969217507328 + (3971*z)/2425060040617647997585731147792384 - 880007/1855
83791958607219605834030755606257729536, z, k)^5 - 971/22353858477636624)*ro
ot(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018204617694487047
4752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/242506004061764
7997585731147792384 - 880007/185583791958607219605834030755606257729536, z,
 k), k, 1, 6) - (x^2/844 - x/3798 + x^3/5697 - x^4/153819 + x^5/136728 + 4/
17091)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.39, size = 112, normalized size = 0.13

$$\operatorname{RootSum}\left(185583791958607219605834030755606257729536 t^{6} - 130936735796222356552203337728 0 t^{4} + 435633648705229474466611 2 t^{3} - 405298284548038 7328 t^{2} + 303890718384 t - 880007, \left(t \mapsto t \log{\left(\frac{39083462657955593476841044707333565976412952759280634691584 t^{5}}{497978553961399002675733956 95} + \frac{8836979346223785538912817601414711102396804462575616 t^{4}}{49797855396139900267573395695} - \frac{2649305813483085325888442495971346957068050677 76 t^{3}}{49797855396139900267573395695} + \frac{8861353335473631852015151098261583762506 24 t^{2}}{49797855396139900267573395695} - \frac{682321479574909906511394635855601936 t}{49797855396139900267573395695} + x - \frac{2137556077084648622 4291519568}{49797855396139900267573395695} \right)} \right)\right) + \frac{- 9 x^{5} + 8 x^{4} - 216 x^{3} - 1458 x^{2} + 324 x - 288}{1230552 x^{6} + 22149936 x^{4} + 398698848 x^{3} + 132899616 x^{2} + 265799232}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1855837919586072196058340307556062577295336*_t**6 - 1309367357962223
565522033377280*_t**4 + 4356336487052294744666112*_t**3 - 40529828454803873
28*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(39083462657955593476
84104470733356597641295275928063469158 4*_t**5/4979785539613990026757339569 5
 + 883697934622378553891281760141471110239680446257561 6*_t**4/4979785539613
9900267573395695 - 26493058134830853258884424959713469570680506777 6*_t**3/4
979785539613990026757339569 5 + 88613533354736318520151510982615837625062 4*_
t**2/497978553961399002675733956 95 - 68232147957490990651139463585560193 6*_
t/497978553961399002675733956 95 + x - 213755607708464862242915195 68/4979785
5396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x**2 + 324*
x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899616*x**2 +
265799232)
```

3.152 $\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=873

$$\frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(2i2^{2/3}+9\sqrt[6]{3}+2\,2^{2/3}\sqrt{3}\right)\right)\tan^{-1}\left(\frac{3\sqrt[3]{\cdots}}{\sqrt{6\left(4-\cdots\right.}}\right.}{209952\left(1+\sqrt[3]{-1}\right)^5\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\cdots\right.}}$$

Rubi [A] time = 1.92, antiderivative size = 873, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] ((-6)^(1/3)*(2*(-3)^(1/3) + 9*2^(1/3)) - 3*x)/(157464*(8 - (9*I)*2^(1/3)*3^
(1/6) + 3*2^(1/3)*3^(2/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3
)*(9*(-2)^(1/3) + 2*3^(1/3)) + 3*x)/(157464*(8 + (9*I)*2^(1/3)*3^(1/6) + 3*
2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) - (2*2^(1/3) - 3*6^(2/
3) - 3^(1/3)*x)/(104976*(9*2^(1/3) - 4*3^(1/3))*(6 + 3*2^(2/3)*3^(1/3)*x +
x^2)) + ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3
))]]/(26244*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
- ((9*I - 3^(1/3)*((2*I)*2^(2/3) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(
3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(209952*(1
 + (-1)^(1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(26244*Sqrt[3]*(8 + (9*
I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((4*I)*2^(
2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(209952*(1 + (-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3
))]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]]/(52488*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) - 3*3^
(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(
2/3))]])/(944784*3^(1/6)*Sqrt[2*(-4 + 3*2^(1/3)*3^(2/3))]) - ((I/23328)*Log
[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^5) +
((I + Sqrt[3])*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(46656*2^(2/3)*3^(5/6
)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(629856*2^(2/3)*
3^(1/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = 1586874322944\int\left(-\frac{9(-2)^{2/3}-\sqrt[3]{-1}\,3^{2/3}x}{27763953154228224\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+\right.}\right.$$

$$= \frac{\int \frac{18-2\,2^{2/3}\sqrt[3]{3}+\sqrt[3]{2}\,3^{2/3}x}{6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2}\,dx}{1889568} - \frac{\int \frac{9\,2^{2/3}-\sqrt[3]{-1}\,3^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}\,dx}{157464\,2^{2/3}} - \frac{\int \frac{3\,2^{2/3}\sqrt[3]{3}+}{\left(6+3\,2^{2/3}\sqrt[3]{3}\,x\right.}}{52488\,2^{2/3}}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

$$= \frac{\sqrt[3]{-6}\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)-3x}{157464\left(8-9i\sqrt[3]{2}\,\sqrt[6]{3}+3\sqrt[3]{2}\,3^{2/3}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)} - \frac{}{314928}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.19

$$\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{2\#1^4\log(x-\#1)-27\#1^3\log(x-\#1)+72\#1^2\log(x-\#1)-162\#1\log(x-\#1)+1971\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{11074968}+\frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656\left(x^6+18x^4+324x^3+108x^2+216\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 - 27*
Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/11074968
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.14

$$\frac{\left(2\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4-27\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+72\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2-162\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+1971\right)\ln\left(-\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{11074968\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+132899616\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+1794144816\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+398698848\,\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R+1
971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,x^5-27\,x^4+96\,x^3+648\,x^2-3942\,x+972}{3691656\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}+\frac{1}{1845828}\int \frac{2\,x^4-27\,x^3+72\,x^2-162\,x+1971}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] 1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2 -
 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.42, size = 387, normalized size = 0.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/41408
2997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/44
35409310686141742269284220928 + (1989787*z)/1885726687584283082922664540523
3577984 - 7197829/12827551700178931019155524820582750453426552832, z, k))/30
663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (11805
253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068614174226928
4220928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/1282
7551700178931019155524820582750453426552832, z, k)*x)/6623365474855296 + (66
03523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/352097091
2943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787
*z)/18857266875842830829226645405233577984 - 7197829/1282755170017893101915
524820582750453426552832, z, k)^2*x)/584204562 - (1762321104*root(z^6 - (29
2589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (
2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/1885726687584283
0829226645405233577984 - 7197829/1282755170017893101915552482058275045342655
2832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589*z^4)/4140829970
94657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/4435409
310686141742269284220928 + (1989787*z)/188572668758428308292266454052335779
84 - 7197829/12827551700178931019155524820582750453426552832, z, k)^4*x)/211
 - 6940988288557056*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*
z^3)/3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220
928 + (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551
700178931019155524820582750453426552832, z, k)^5*x + (166697*root(z^6 - (292
589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2
479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18857266875842830
829226645405233577984 - 7197829/12827551700178931019155524820582750453426552
832, z, k)^2)/43274412 + (639193032*root(z^6 - (292589*z^4)/414082997094657
024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068
6141742269284220928 + (1989787*z)/18857266875842830829226645405233577984 -
7197829/12827551700178931019155524820582750453426552832, z, k)^3)/44521 - (9
815247601920*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35
20970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928 + (
1989787*z)/18857266875842830829226645405233577984 - 7197829/128275517001789
31019155524820582750453426552832, z, k)^4)/211 - 168897381688221696*root(z^6
 - (292589*z^4)/414082997094657024 - (11805253*z^3)/35209709129437059758653
44 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/1885726687
5842830829226645405233577984 - 7197829/1282755170017893101915552482058275045
3426552832, z, k)^5 + 661/28970600587017064704)*root(z^6 - (292589*z^4)/414
082997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/
4435409310686141742269284220928 + (1989787*z)/18857266875842830829226645405
233577984 - 7197829/12827551700178931019155524820582750453426552832, z, k),
k, 1, 6) + (x^2/5697 - (73*x)/68364 + (4*x^3)/153819 - x^4/136728 + x^5/922
914 + 1/3798)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [A] time = 0.40, size = 112, normalized size = 0.13

$$\operatorname{RootSum}\left(1282755170017893101915524820582750453426552832t^6 - 906388465775544244426251149770752t^4 - 430087316638998774168413798 4t^3 - 717000908921644962816t^2 + 135354162312576t - 7197829, \left(t \mapsto t \log\left(\frac{172579355928104499014095565978918829956040010833393680413614806138 88t^5}{154206009791052044490694380303237521} + \frac{2389607400620985524376358853572652207181956324560587684052992t^4}{154206009791052044490694380303237521} - \frac{122860721608832839307117159488782600789969921934883880 96t^3}{154206009791052044490694380303237521} - \frac{59490553573959173161125496013527909754156558410752t^2}{154206009791052044490694380303237521} - \frac{17520149679836691112367064197713753004827200t}{154206009791052044490694380303237521} + x + \frac{7664229887072296150558552870408873 32}{154206009791052044490694380303237521}\right)\right)\right) + \frac{4x^5 - 27x^4 + 96x^3 + 648x^2 - 3942x + 972}{3691656x^6 + 66449808x^4 + 1196096544x^3 + 398698848x^2 + 797397696}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(1282755170017893101915524820582750453426552832*_t**6 - 906388465775
544244426251149770752*_t**4 - 430087316638998774168413798 4*_t**3 - 71700090
8921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17257
935592810449901409556597891882995604001083339368041361480613888*_t**5/15420
6009791052044490694380303237521 + 238960740062098552437635885357265220718 19
56324560587684052992*_t**4/154206009791052044490694380303237521 - 122860721
608832839307117159488782600789969921934883880 96*_t**3/154206009791052044490
694380303237521 - 59490553573959173161125496013527909754156558410752*_t**2/
154206009791052044490694380303237521 - 175201496798366911123670641977137530
04827200*_t/154206009791052044490694380303237521 + x + 766422988707229615 05
5855287040887332/154206009791052044490694380303237521))) + (4*x**5 - 27*x**
4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808*x**4 + 1196
096544*x**3 + 398698848*x**2 + 797397696)
```

3.153 $\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal. Leaf size=986

$$-\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}\,2^{2/3}x+6\right)}-\frac{\left(1+i\sqrt{3}+3\sqrt[3]{2}\,3^{2/3}\right)\tan^{-1}\left(\frac{3\sqrt[3]{-3}\,2^{2/3}-\ldots}{\sqrt{6\left(4-3(-3)^{2/3}\ldots\right.}}\right.}{8748\,2^{2/3}3^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\ldots\right.}$$

Rubi [A] time = 1.93, antiderivative size = 986, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2097, 638, 618, 204, 634, 628, 206}

Antiderivative was successfully verified.

```
[In] Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] -(27*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3)) - 6^(1/3)*(9 + (-3)^(1/3)*2^(2/3))
*x)/(104976*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-
3)^(1/3)*2^(2/3)*x + x^2)) - (27*2^(2/3)*(1 + (-2)^(1/3)*3^(2/3)) - (-1)^(1
/3)*3^(2/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(472392*2^(2/3)*(8 + (9*I)*2^(1/3
)*3^(1/6) + 3*2^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6
- 2^(2/3)*3^(1/3)) - (2 - 3*2^(1/3)*3^(2/3))*x)/(314928*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((1 + I*Sqrt[3] + 3
*2^(1/3)*3^(2/3))*ArcTan[(3*(-3)^(1/3)*2^(2/3) - 2*x)/Sqrt[6*(4 - 3*(-3)^(2
/3)*2^(1/3))]])/(8748*2^(2/3)*3^(5/6)*(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)
*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^(2/3))
*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(
17496*6^(5/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(4 + 3*(-2)^(1/3)*3^(2/
3))^(3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(34992*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4
+ 3*(-2)^(1/3)*3^(2/3)]) + ((I/17496)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/
3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3
))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1
/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(17496*6^(5
/6)*(1 - (-1)^(1/3))^2*(1 + (-1)^(1/3))^4*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) -
 ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]
]/(157464*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((I + Sqrt[3])*Lo
g[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(419904*2^(1/3)*3^(1/6)*(1 + (-1)^(1/3
))^5) - ((I/209952)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3)*3^(1/6)
*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1889568*2^(1/3)*
3^(2/3))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2097

```
Int[(Q6_)^(p_)*(u_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Dist
[1/(3^(3*p)*a^(2*p)), Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*x
+ b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*(-
1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d, 0
] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coeff[
Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Rubi steps

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = 1586874322944\int\left(\frac{2\sqrt[3]{-1}\,3^{2/3}+18\sqrt[3]{6}+3(-2)^{2/3}x}{55527906308456448\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(-6+3\right.}\right.$$

$$= \frac{\int \frac{2\sqrt[3]{-1}\,3^{2/3}+18(-1)^{2/3}\sqrt[3]{6}+3\,2^{2/3}x}{\left(6+3(-2)^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}dx}{314928\,2^{2/3}} + \frac{\int \frac{-2+6\sqrt[3]{2}\,3^{2/3}+2^{2/3}\sqrt[3]{3}\,x}{\left(6+3\,2^{2/3}\sqrt[3]{3}\,x+x^2\right)^2}dx}{104976\,2^{2/3}\sqrt[3]{3}} + \frac{\int \frac{9}{6+3}}{94}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

$$= -\frac{27\left((-2)^{2/3}+2\sqrt[3]{-1}\,3^{2/3}\right)-\sqrt[3]{6}\left(9+\sqrt[3]{-3}\,2^{2/3}\right)x}{104976\,2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}\,2^{2/3}x+x^2\right)}$$

Mathematica [C] time = 0.03, size = 167, normalized size = 0.17

$$\frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{\text{RootSum}\left[\#1^6+18\#1^4+324\#1^3+108\#1^2+216\&,\frac{9\#1^4\log(x-\#1)-16\#1^3\log(x-\#1)+324\#1^2\log(x-\#1)+2436\#1\log(x-\#1)+324\log(x-\#1)}{\#1^5+12\#1^3+162\#1^2+36\#1}\&\right]}{44299872}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] (-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2 -
16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^
5) & ]/44299872
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

maple [C] time = 0.01, size = 122, normalized size = 0.12

$$\frac{\left(-9\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^4+16\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3-324\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2-2436\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)-324\right)\ln\left(-\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)+x\right)}{44299872\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^5+531598464\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^3+7176579264\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)^2+1594795392\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}+\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

```
[Out] (-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)/
(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2-2
436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(-_R+x),_R=RootOf(_Z^6+18*_Z^4+
324*_Z^3+108*_Z^2+216))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{9\,x^5-8\,x^4+216\,x^3+2724\,x^2-324\,x+7884}{7383312\left(x^6+18\,x^4+324\,x^3+108\,x^2+216\right)}-\frac{1}{7383312}\int \frac{9\,x^4-16\,x^3+324\,x^2+2436\,x+324}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

```
[Out] -1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*x^
2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

mupad [B] time = 2.48, size = 388, normalized size = 0.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

```
[Out] symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/12
940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11
088523276715354355673210552 32 - (505*z)/628575562528094360974221513507785 93
28 + 4513/86585973976207784379297925389335655606292 31616, z, k))/1103894245
809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/14
2599321974220092022546432 + (5171*z^2)/110885232767153543556732105 5232 - (5
05*z)/62857556252809436097422151350778 59328 + 4513/865859739762077843792979
253893356556062 9231616, z, k)*x)/29805144636848832 + (452809*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/110885232767153543556732105 5232 - (505*z)/628575562528094360974221 51
35077859328 + 4513/865859739762077843792979253893356556062 9231616, z, k)^2*
x)/194734854 - (1241776944*root(z^6 + (163*z^4)/12940093659208032 - (811359
7*z^3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105
5232 - (505*z)/62857556252809436097422151350778 59328 + 4513/865859739762077
843792979253893356556062 9231616, z, k)^3*x)/44521 + (452407928832*root(z^6
+ (163*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 +
 (5171*z^2)/110885232767153543556732105 5232 - (505*z)/628575562528094360974
2215135077859328 + 4513/865859739762077843792979253893356556062 9231616, z,
```

```
k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/12940093659208032 - (81
13597*z^3)/1425993219742200920225464 32 + (5171*z^2)/11088523276715354355673
21055232 - (505*z)/628575562528094360974221513507 7859328 + 4513/86585973976
207784379297925389335655606292316 16, z, k)^5*x + (114155*root(z^6 + (163*z^
4)/12940093659208032 - (8113597*z^3)/1425993219742200920225464 32 + (5171*z^
2)/110885232767153543556732105 5232 - (505*z)/628575562528094360974221513507
7859328 + 4513/865859739762077843792979253893356556062923161 6, z, k)^2)/292
102281 - (163984176*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/
142599321974220092022546432 + (5171*z^2)/110885232767153543556732105 5232 -
(505*z)/628575562528094360974221513507 7859328 + 4513/8658597397620778437929
7925389335655606292316 16, z, k)^3)/44521 + (94281884928*root(z^6 + (163*z^4
)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2
)/110885232767153543556732105 5232 - (505*z)/628575562528094360974221513507 7
859328 + 4513/865859739762077843792979253893356556062923161 6, z, k)^4)/211
- 168897381688221696*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)
/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105 5232 -
 (505*z)/628575562528094360974221513507 7859328 + 4513/865859739762077843792
979253893356556062923161 6, z, k)^5 + 1/19313733724678043136)*root(z^6 + (16
3*z^4)/12940093659208032 - (8113597*z^3)/142599321974220092022546432 + (517
1*z^2)/110885232767153543556732105 5232 - (505*z)/628575562528094360974221 51
35077859328 + 4513/865859739762077843792979253893356556062923161 6, z, k), k
, 1, 6) - ((227*x^2)/615276 - x/22788 + x^3/34182 - x^4/922914 + x^5/820368
 + 73/68364)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)
```

sympy [**A**] time = 0.39, size = 112, normalized size = 0.11

$$\operatorname{RootSum}\left(865859739762077843792979253893356556062923161 6t^6 + 1090680958717701682488386456125 44t^4 - 49265570759336691571349913 6t^3 + 40378331745144603648t^2 - 695635011360t + 4513, \left(t \mapsto t\log\left(\frac{1014425315618041811131612870398593498518816196536317121658 88t^5}{35690069707079294847 5845} - \frac{14979655008235933511270943497197508896705021005004 8t^4}{35690069707079294847 5845} + \frac{12224097544582728185058987776867078361723699 2t^3}{35690069707079294847 5845} - \frac{5775055524251595723022901938558261453824t^2}{35690069707079294847 5845} + \frac{96165242200260265765603930470432t}{71380139414158589695169} + x - \frac{17059152341129698120545584}{10707020912123788845427535}\right)\right)\right) + \frac{-9x^5 + 8x^4 - 216x^3 - 2724x^2 + 324x - 7884}{7383312x^6 + 132899616x^4 + 2392193088x^3 + 797397696x^2 + 1594795392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

```
[Out] RootSum(865859739762077843792979253893356556062923161 6*_t**6 + 109068095871
7701682488386456125 44*_t**4 - 49265570759336691571349913 6*_t**3 + 403783317
45144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315618
041811131612870398593498518816196536317121658 88*_t**5/35690069707079294847 5
845 - 14979655008235933511270943497197508896705021005004 8*_t**4/35690069707
079294847 5845 + 12224097544582728185058987776867078361723699 2*_t**3/356900
69707079294847 5845 - 5775055524251595723022901938558261453824*_t**2/3569006
97070792948475845 + 96165242200260265765603930470432*_t/7138013941415858969
5169 + x - 17059152341129698120545584/10707020912123788845427535))) + (-9*x*
*5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6 + 13289961
6*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

3.154 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx} dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} dx &= \int \left(a^2 + 2abx^2 + b^2x^4\right) dx\\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x),x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

IntegrateAlgebraic [A] time = 0.03, size = 26, normalized size = 1.04

$$\frac{1}{15}\left(15a^2x + 10abx^3 + 3b^2x^5\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(c + d*x),x]
```

```
[Out] (15*a^2*x + 10*a*b*x^3 + 3*b^2*x^5)/15
```

fricas [A] time = 0.79, size = 21, normalized size = 0.84

$$\frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

giac [A] time = 0.37, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{1}{5}b^2x^5 + \frac{2}{3}ab\,x^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 0.44, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c),x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.04, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x),x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.09, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c),x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.155 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Rubi [A] time = 0.13, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {1586, 28, 697}

$$\frac{bx^2\left(2ad^2+bc^2\right)}{2d^3}-\frac{bcx\left(2ad^2+bc^2\right)}{d^4}+\frac{\left(ad^2+bc^2\right)^2\log(c+dx)}{d^5}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
c + d*x)^2,x]
```

```
[Out] -((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^2
*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx &= \int \frac{a^2+2abx^2+b^2x^4}{c+dx}\,dx\\ &= \frac{\int \frac{\left(ab+b^2x^2\right)^2}{c+dx}\,dx}{b^2}\\ &= \frac{\int \left(-\frac{b^3c\left(bc^2+2ad^2\right)}{d^4}+\frac{b^3\left(bc^2+2ad^2\right)x}{d^3}-\frac{b^4cx^2}{d^2}+\frac{b^4x^3}{d}+\frac{b^2\left(bc^2+\right.}{d^4(c+}\right.}{b^2}\\ &= -\frac{bc\left(bc^2+2ad^2\right)x}{d^4}+\frac{b\left(bc^2+2ad^2\right)x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x}{4c}\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.84

$$\frac{12\left(ad^2+bc^2\right)^2\log(c+dx)+bdx\left(12ad^2(dx-2c)+b\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(c + d*x)^2,x]
```

```
[Out] (b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^
3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c + dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(c + d*x)^2, x]
```

fricas [A] time = 0.94, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^4x^4 - 4\,b^2cd^3x^3 + 6\left(b^2c^2d^2 + 2\,abd^4\right)x^2 - 12\left(b^2c^3d + 2\,abcd^3\right)x + 12\left(b^2c^4 + 2\,abc^2d^2 + a^2d^4\right)\log\left(dx + c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 - 1
2*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c))/d^5
```

giac [B] time = 0.26, size = 365, normalized size = 3.88

$$-\frac{1}{12}\,b^2d\left(\frac{(dx+c)^4\left(\frac{20c}{dx+c} - \frac{60c^2}{(dx+c)^2} + \frac{120c^3}{(dx+c)^3} - 3\right)}{d^6} + \frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6} - \frac{12\,c^5}{(dx+c)d^6}\right) - \frac{1}{3}\,b^2c\left(\frac{(dx+c)^3\left(\frac{6c}{dx+c} - \frac{18c^2}{(dx+c)^2} - 1\right)}{d^5} - \frac{12\,c^3\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5} + \frac{3\,c^4}{(dx+c)d^5}\right) - abd\left(\frac{(dx+c)^2\left(\frac{6c}{dx+c} - 1\right)}{d^4} + \frac{6\,c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4} - \frac{2\,c^3}{(dx+c)d^4}\right) + 2\,abc\left(\frac{2\,c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} + \frac{dx+c}{d^3} - \frac{c^2}{(dx+c)d^3}\right) - a^2\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d} - \frac{c}{(dx+c)d}\right) - \frac{a^2c}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="giac")
```

```
[Out] -1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d*
x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 12*
c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d*x
+ c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3*c^4
/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(
abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a*b*c*(
2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x
+ c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d
)) - a^2*c/((d*x + c)*d)
```

maple [A] time = 0.00, size = 114, normalized size = 1.21

$$\frac{b^2x^4}{4d} - \frac{b^2c\,x^3}{3d^2} + \frac{ab\,x^2}{d} + \frac{b^2c^2x^2}{2d^3} + \frac{a^2\ln\left(dx + c\right)}{d} + \frac{2ab\,c^2\ln\left(dx + c\right)}{d^3} - \frac{2abcx}{d^2} + \frac{b^2c^4\ln\left(dx + c\right)}{d^5} - \frac{b^2c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,x
)
```

```
[Out] 1/4*b^2*x^4/d-1/3*b^2*c*x^3/d^2+b/d*x^2*a+1/2*b^2/d^3*x^2*c^2-2*b/d^2*a*c*x
-b^2/d^4*c^3*x+1/d*ln(d*x+c)*a^2+2/d^3*ln(d*x+c)*a*b*c^2+1/d^5*ln(d*x+c)*b^
2*c^4
```

maxima [A] time = 0.44, size = 105, normalized size = 1.12

$$\frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\left(b^2c^2d+2\,abd^3\right)x^2-12\left(b^2c^3+2\,abcd^2\right)x}{12\,d^4}+\frac{\left(b^2c^4+2\,abc^2d^2+a^2d^4\right)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+
c)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12*
(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(d*
x + c)/d^5
```

mupad [B] time = 0.06, size = 106, normalized size = 1.13

$$x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5}+\frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
c + d*x)^2,x)
```

```
[Out] x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a*
b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)/d
^3 + (2*a*b)/d))/d
```

sympy [A] time = 0.31, size = 88, normalized size = 0.94

$$-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}+\frac{b^2c^2}{2d^3}\right)+x\left(-\frac{2abc}{d^2}-\frac{b^2c^3}{d^4}\right)+\frac{\left(ad^2+bc^2\right)^2\log\left(c+dx\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3))
 + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)/d
**5
```

3.156 $\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx$

Optimal. Leaf size=15

$$\frac{1}{14} \left(bx+cx^2\right)^{14}$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{1}{14} \left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx = \frac{1}{14} \left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b+2cx) \left(bx+cx^2\right)^{13} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^13, x]
```

fricas [B] time = 0.71, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8+143x^{19}c^5b^9+\frac{143}{2}x^{18}c^4b^{10}+26x^{17}c^3b^{11}+\frac{13}{2}x^{16}c^2b^{12}+x^{15}cb^{13}+\frac{1}{14}x^{14}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2 + bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28} + b\,c^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20} + 143b^9c^5x^{19} + \frac{143}{2}b^{10}c^4x^{18} + 26b^{11}c^3x^{17} + \frac{13}{2}b^{12}c^2x^{16} + b^{13}c\,x^{15} + \frac{1}{14}b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2 + bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

mupad [B] time = 2.22, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14} + b^{13}\,c\,x^{15} + \frac{13\,b^{12}\,c^2\,x^{16}}{2} + 26\,b^{11}\,c^3\,x^{17} + \frac{143\,b^{10}\,c^4\,x^{18}}{2} + 143\,b^9\,c^5\,x^{19} + \frac{429\,b^8\,c^6\,x^{20}}{2} + \frac{1716\,b^7\,c^7\,x^{21}}{7} + \frac{429\,b^6\,c^8\,x^{22}}{2} + 143\,b^5\,c^9\,x^{23} + \frac{143\,b^4\,c^{10}\,x^{24}}{2} + 26\,b^3\,c^{11}\,x^{25} + \frac{13\,b^2\,c^{12}\,x^{26}}{2} + b\,c^{13}\,x^{27} + \frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

3.157 $\int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{28} x^{28} \left(b+c x^2\right)^{14}$$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$\frac{1}{28} x^{28} \left(b+c x^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^{14} \left(b+2 c x^2\right) \left(b x+c x^3\right)^{13} \, dx &= \int x^{27} \left(b+c x^2\right)^{13} \left(b+2 c x^2\right) \, dx\\ &= \frac{1}{2} \text{Subst}\left(\int x^{13} (b+c x)^{13} (b+2 c x) \, dx,x,x^2\right)\\ &= \frac{1}{28} x^{28} \left(b+c x^2\right)^{14} \end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14} x^{28}}{28}+\frac{1}{2} b^{13} c x^{30}+\frac{13}{4} b^{12} c^2 x^{32}+13 b^{11} c^3 x^{34}+\frac{143}{4} b^{10} c^4 x^{36}+\frac{143}{2} b^9 c^5 x^{38}+\frac{429}{4} b^8 c^6 x^{40}+\frac{858}{7} b^7 c^7 x^{42}+\frac{429}{4} b^6 c^8 x^{44}+\frac{143}{2} b^5 c^9 x^{46}+\frac{143}{4} b^4 c^{10} x^{48}+13 b^3 c^{11} x^{50}+\frac{13}{4} b^2 c^{12} x^{52}+\frac{1}{2} b c^{13} x^{54}+\frac{c^{14} x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
```

```
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\left(b+2cx^2\right)\left(bx+cx^3\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13,x]
```

```
[Out] IntegrateAlgebraic[x^14*(b + 2*c*x^2)*(b*x + c*x^3)^13, x]
```

fricas [B] time = 1.24, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^{2}+13x^{50}c^{11}b^{3}+\frac{143}{4}x^{48}c^{10}b^{4}+\frac{143}{2}x^{46}c^{9}b^{5}+\frac{429}{4}x^{44}c^{8}b^{6}+\frac{858}{7}x^{42}c^{7}b^{7}+\frac{429}{4}x^{40}c^{6}b^{8}+\frac{143}{2}x^{38}c^{5}b^{9}+\frac{143}{4}x^{36}c^{4}b^{10}+13x^{34}c^{3}b^{11}+\frac{13}{4}x^{32}c^{2}b^{12}+\frac{1}{2}x^{30}cb^{13}+\frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

giac [B] time = 0.31, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{143}{4}b^{10}c^{4}x^{36}+13b^{11}c^{3}x^{34}+\frac{13}{4}b^{12}c^{2}x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{143}{4}b^{10}c^{4}x^{36}+13b^{11}c^{3}x^{34}+\frac{13}{4}b^{12}c^{2}x^{32}+\frac{1}{2}b^{13}c\,x^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

maxima [B] time = 0.61, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^{2}c^{12}x^{52}+13b^{3}c^{11}x^{50}+\frac{143}{4}b^{4}c^{10}x^{48}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{858}{7}b^{7}c^{7}x^{42}+\frac{429}{4}b^{8}c^{6}x^{40}+\frac{143}{2}b^{9}c^{5}x^{38}+\frac{143}{4}b^{10}c^{4}x^{36}+13b^{11}c^{3}x^{34}+\frac{13}{4}b^{12}c^{2}x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(2*c*x^2+b)*(c*x^3+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
```

```
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

mupad [B] time = 0.14, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(b*x + c*x^3)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

sympy [B] time = 0.14, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{143b^5c^9x^{46}}{2}+\frac{143b^4c^{10}x^{48}}{4}+13b^3c^{11}x^{50}+\frac{13b^2c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(2*c*x**2+b)*(c*x**3+b*x)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

3.158 $\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{28}\left(b+2cx^3\right)\left(bx+cx^4\right)^{13}\,dx &= \int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx\\ &= \frac{1}{3}\,\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
```

```
(572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{28}\left(b+2cx^{3}\right)\left(bx+cx^{4}\right)^{13}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13,x]
```

```
[Out] IntegrateAlgebraic[x^28*(b + 2*c*x^3)*(b*x + c*x^4)^13, x]
```

fricas [B] time = 1.17, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^{2}+\frac{26}{3}x^{75}c^{11}b^{3}+\frac{143}{6}x^{72}c^{10}b^{4}+\frac{143}{3}x^{69}c^{9}b^{5}+\frac{143}{2}x^{66}c^{8}b^{6}+\frac{572}{7}x^{63}c^{7}b^{7}+\frac{143}{2}x^{60}c^{6}b^{8}+\frac{143}{3}x^{57}c^{5}b^{9}+\frac{143}{6}x^{54}c^{4}b^{10}+\frac{26}{3}x^{51}c^{3}b^{11}+\frac{13}{6}x^{48}c^{2}b^{12}+\frac{1}{3}x^{45}cb^{13}+\frac{1}{42}x^{42}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

giac [B] time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}b\,c^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}c\,x^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

maxima [B] time = 0.55, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^{2}c^{12}x^{78}+\frac{26}{3}b^{3}c^{11}x^{75}+\frac{143}{6}b^{4}c^{10}x^{72}+\frac{143}{3}b^{5}c^{9}x^{69}+\frac{143}{2}b^{6}c^{8}x^{66}+\frac{572}{7}b^{7}c^{7}x^{63}+\frac{143}{2}b^{8}c^{6}x^{60}+\frac{143}{3}b^{9}c^{5}x^{57}+\frac{143}{6}b^{10}c^{4}x^{54}+\frac{26}{3}b^{11}c^{3}x^{51}+\frac{13}{6}b^{12}c^{2}x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^28*(2*c*x^3+b)*(c*x^4+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
```

```
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

mupad [B] time = 2.17, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^28*(b*x + c*x^4)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

sympy [B] time = 0.14, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**28*(2*c*x**3+b)*(c*x**4+b*x)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

3.159 $\int x^{14(-1+n)} (b+2cx^n) \left(bx+cx^{1+n}\right)^{13} dx$

Optimal. Leaf size=21

$$\frac{x^{14n} (b+cx^n)^{14}}{14n}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n} (b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x^{14(-1+n)} (b+2cx^n) \left(bx+cx^{1+n}\right)^{13} dx &= \int x^{13+14(-1+n)} (b+cx^n)^{13} (b+2cx^n)\, dx \\
&= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\, dx, x, x^n\right)}{n} \\
&= \frac{x^{14n} (b+cx^n)^{14}}{14n}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

IntegrateAlgebraic [A] time = 0.05, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(14*(-1 + n))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

fricas [B] time = 0.98, size = 262, normalized size = 12.48

$$\frac{b^{14}x^{14}x^{14n+14}+14\,b^{13}cx^{13}x^{15n+15}+91\,b^{12}c^{2}x^{12}x^{16n+16}+364\,b^{11}c^{3}x^{11}x^{17n+17}+1001\,b^{10}c^{4}x^{10}x^{18n+18}+2002\,b^{9}c^{5}x^{9}x^{19n+19}+3003\,b^{8}c^{6}x^{8}x^{20n+20}+3432\,b^{7}c^{7}x^{7}x^{21n+21}+3003\,b^{6}c^{8}x^{6}x^{22n+22}+2002\,b^{5}c^{9}x^{5}x^{23n+23}+1001\,b^{4}c^{10}x^{4}x^{24n+24}+364\,b^{3}c^{11}x^{3}x^{25n+25}+91\,b^{2}c^{12}x^{2}x^{26n+26}+14\,bc^{13}xx^{27n+27}+c^{14}x^{28n+28}}{14\,nx^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(b^14*x^14*x^(14*n + 14) + 14*b^13*c*x^13*x^(15*n + 15) + 91*b^12*c^2*
x^12*x^(16*n + 16) + 364*b^11*c^3*x^11*x^(17*n + 17) + 1001*b^10*c^4*x^10*x
^(18*n + 18) + 2002*b^9*c^5*x^9*x^(19*n + 19) + 3003*b^8*c^6*x^8*x^(20*n +
20) + 3432*b^7*c^7*x^7*x^(21*n + 21) + 3003*b^6*c^8*x^6*x^(22*n + 22) + 200
2*b^5*c^9*x^5*x^(23*n + 23) + 1001*b^4*c^10*x^4*x^(24*n + 24) + 364*b^3*c^1
1*x^3*x^(25*n + 25) + 91*b^2*c^12*x^2*x^(26*n + 26) + 14*b*c^13*x*x^(27*n +
 27) + c^14*x^(28*n + 28))/(n*x^28)
```

giac [B] time = 1.86, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n}+14\,bc^{13}x^{27n}+91\,b^{2}c^{12}x^{26n}+364\,b^{3}c^{11}x^{25n}+1001\,b^{4}c^{10}x^{24n}+2002\,b^{5}c^{9}x^{23n}+3003\,b^{6}c^{8}x^{22n}+3432\,b^{7}c^{7}x^{21n}+3003\,b^{8}c^{6}x^{20n}+2002\,b^{9}c^{5}x^{19n}+1001\,b^{10}c^{4}x^{18n}+364\,b^{11}c^{3}x^{17n}+91\,b^{12}c^{2}x^{16n}+14\,b^{13}cx^{15n}+b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

maple [B] time = 0.05, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n}+\frac{b^{13}c\,x^{15n}}{n}+\frac{13b^{12}c^{2}x^{16n}}{2n}+\frac{26b^{11}c^{3}x^{17n}}{n}+\frac{143b^{10}c^{4}x^{18n}}{2n}+\frac{143b^{9}c^{5}x^{19n}}{n}+\frac{429b^{8}c^{6}x^{20n}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+\frac{429b^{6}c^{8}x^{22n}}{2n}+\frac{143b^{5}c^{9}x^{23n}}{n}+\frac{143b^{4}c^{10}x^{24n}}{2n}+\frac{26b^{3}c^{11}x^{25n}}{n}+\frac{13b^{2}c^{12}x^{26n}}{2n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{c^{14}x^{28n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

maxima [B] time = 0.65, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14\,n}+\frac{bc^{13}x^{27n}}{n}+\frac{13\,b^{2}c^{12}x^{26n}}{2\,n}+\frac{26\,b^{3}c^{11}x^{25n}}{n}+\frac{143\,b^{4}c^{10}x^{24n}}{2\,n}+\frac{143\,b^{5}c^{9}x^{23n}}{n}+\frac{429\,b^{6}c^{8}x^{22n}}{2\,n}+\frac{1716\,b^{7}c^{7}x^{21n}}{7\,n}+\frac{429\,b^{8}c^{6}x^{20n}}{2\,n}+\frac{143\,b^{9}c^{5}x^{19n}}{n}+\frac{143\,b^{10}c^{4}x^{18n}}{2\,n}+\frac{26\,b^{11}c^{3}x^{17n}}{n}+\frac{13\,b^{12}c^{2}x^{16n}}{2\,n}+\frac{b^{13}cx^{15n}}{n}+\frac{b^{14}x^{14n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-14+14*n)*(b+2*c*x^n)*(b*x+c*x^(1+n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

mupad [B] time = 4.02, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7\,x^{21\,n}}{7\,n}+\frac{429\,b^6\,c^8\,x^{22\,n}}{2\,n}+\frac{143\,b^5\,c^9\,x^{23\,n}}{n}+\frac{143\,b^4\,c^{10}\,x^{24\,n}}{2\,n}+\frac{26\,b^3\,c^{11}\,x^{25\,n}}{n}+\frac{13\,b^2\,c^{12}\,x^{26\,n}}{2\,n}+\frac{b^{13}\,c\,x^{15\,n}}{n}+\frac{b\,c^{13}\,x^{27\,n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 14)*(b*x + c*x^(n + 1))^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-14+14*n)*(b+2*c*x**n)*(b*x+c*x**(1+n))**13,x)
```

```
[Out] Timed out
```

3.160 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{bx+cx^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2), x]
```

fricas [A] time = 0.73, size = 10, normalized size = 1.00

$$\log \left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

giac [A] time = 0.23, size = 11, normalized size = 1.10

$$\log\left(\left|cx^2 + bx\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln\left(\left(cx + b\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
```

```
[Out] ln(x*(c*x+b))
```

maxima [A] time = 0.55, size = 10, normalized size = 1.00

$$\log\left(cx^2 + bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x)
```

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\ \left(b + c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
```

```
[Out] log(x*(b + c*x))
```

sympy [A] time = 0.13, size = 8, normalized size = 0.80

$$\log\left(bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
```

```
[Out] log(b*x + c*x**2)
```

3.161 $\int \frac{b+2cx^2}{bx+cx^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1593, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+2cx^2}{bx+cx^3}\,dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^2\right)\\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(b*x + c*x^3),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^2}{bx+cx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^2)/(b*x + c*x^3),x]
[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(b*x + c*x^3), x]
```

fricas [A] time = 0.77, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="fricas")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

giac [A] time = 0.33, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="giac")
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/(c*x^3+b*x),x)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

maxima [A] time = 0.64, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/(c*x^3+b*x),x, algorithm="maxima")
[Out] 1/2*log(c*x^2 + b) + log(x)
```

mupad [B] time = 2.08, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(b*x + c*x^3),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/(c*x**3+b*x),x)
[Out] log(x) + log(b/c + x**2)/2
```

3.162 $\int \frac{b+2cx^3}{bx+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1593, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+2cx^3}{bx+cx^4}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(b*x + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^3}{bx + cx^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^3)/(b*x + c*x^4),x]
[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(b*x + c*x^4), x]
```

fricas [A] time = 0.96, size = 13, normalized size = 0.87

$$\frac{1}{3} \log\left(cx^3 + b\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="fricas")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

giac [A] time = 0.31, size = 15, normalized size = 1.00

$$\frac{1}{3} \log\left(\left|cx^3 + b\right|\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="giac")
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\ln(x) + \frac{\ln\left(c\,x^3 + b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/(c*x^4+b*x),x)
[Out] ln(x)+1/3*ln(c*x^3+b)
```

maxima [A] time = 0.52, size = 13, normalized size = 0.87

$$\frac{1}{3} \log\left(cx^3 + b\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/(c*x^4+b*x),x, algorithm="maxima")
[Out] 1/3*log(c*x^3 + b) + log(x)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3 + b\right)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(b*x + c*x^4),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/(c*x**4+b*x),x)
[Out] log(x) + log(b/c + x**3)/3
```

3.163 $\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1593, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{b+2cx^n}{bx+cx^{1+n}}\,dx &= \int \frac{b+2cx^n}{x(b+cx^n)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^n\right)}{n}\\
&= \log(x)+\frac{\log(b+cx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
[Out] Log[x] + Log[b + c*x^n]/n
```

IntegrateAlgebraic [A] time = 0.04, size = 24, normalized size = 1.60

$$\frac{\log\left(bn+cnx^{n}\right)}{n}+\frac{\log\left(x^{n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*c*x^n)/(b*x + c*x^(1 + n)),x]
[Out] Log[x^n]/n + Log[b*n + c*n*x^n]/n
```

fricas [A] time = 0.89, size = 23, normalized size = 1.53

$$\frac{(n-1)\log(x)+\log\left(bx+cx^{n+1}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="fricas")
[Out] ((n - 1)*log(x) + log(b*x + c*x^(n + 1)))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\,cx^{n}+b}{bx+cx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/(b*x + c*x^(n + 1)), x)
```

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x)+\frac{\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(b*x+c*x^(1+n)),x)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

maxima [B] time = 0.58, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^{n}+b}{c}\right)}{bn}\right)+\frac{2\,\log\left(\frac{cx^{n}+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(b*x+c*x^(1+n)),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int\frac{b+2\,c\,x^{n}}{b\,x+c\,x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)),x)
```

```
[Out] int((b + 2*c*x^n)/(b*x + c*x^(n + 1)), x)
```

sympy [A] time = 1.48, size = 29, normalized size = 1.93

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \log(x) & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(b*x+c*x**(1+n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

3.164 $\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8,x]
[Out] IntegrateAlgebraic[(b + 2*c*x)/(b*x + c*x^2)^8, x]
```

fricas [B] time = 0.94, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

giac [A] time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7\left(cx+b\right)^7b^7}+\frac{c^7}{\left(cx+b\right)^6b^8}+\frac{4c^7}{\left(cx+b\right)^5b^9}+\frac{12c^7}{\left(cx+b\right)^4b^{10}}+\frac{30c^7}{\left(cx+b\right)^3b^{11}}+\frac{66c^7}{\left(cx+b\right)^2b^{12}}+\frac{132c^7}{\left(cx+b\right)b^{13}}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}-\frac{30c^4}{b^{11}x^3}+\frac{12c^3}{b^{10}x^4}-\frac{4c^2}{b^9x^5}+\frac{c}{b^8x^6}-\frac{1}{7b^7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+1/b^8*c^7/(c*x+b)^6+1/7/b^7*c^7/(c*x+b)^7-1
/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4
/b^9*c^2/x^5+1/b^8*c/x^6
```

maxima [A] time = 0.53, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

mupad [B] time = 4.38, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,{\left(b+c\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

sympy [B] time = 0.93, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

3.165 $\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x^7(bx+cx^3)^8}\,dx &= \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}(b+cx^2)^7}\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^2}{x^7\left(bx+cx^3\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8), x]
```

fricas [B] time = 0.91, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

giac [A] time = 0.41, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^6}{7\left(cx^2+b\right)^7c}-\frac{b^5}{\left(cx^2+b\right)^6c}-\frac{4b^4}{\left(cx^2+b\right)^5c}-\frac{12b^3}{\left(cx^2+b\right)^4c}-\frac{30b^2}{\left(cx^2+b\right)^3c}-\frac{66b}{\left(cx^2+b\right)^2c}-\frac{132}{\left(cx^2+b\right)c}\right)c^8}{2b^{13}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{1}{14b^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x)
```

```
[Out] -1/2/b^13*c^8*(-1/7*b^6/c/(c*x^2+b)^7-b^5/c/(c*x^2+b)^6-132/c/(c*x^2+b)-66*
b/c/(c*x^2+b)^2-4*b^4/c/(c*x^2+b)^5-30*b^2/c/(c*x^2+b)^3-12*b^3/c/(c*x^2+b)
^4)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

maxima [B] time = 0.67, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^7/(c*x^3+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

mupad [B] time = 2.22, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,{\left(c\,x^2+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^7*(b*x + c*x^3)^8),x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

sympy [B] time = 1.43, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**7/(c*x**3+b*x)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

3.166 $\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8} dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8} dx\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b+2cx^3}{x^{14}\left(bx+cx^4\right)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8), x]
```

fricas [B] time = 1.12, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

giac [A] time = 0.30, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^6+bx^3\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^6}{7\left(cx^3+b\right)^7c}-\frac{b^5}{\left(cx^3+b\right)^6c}-\frac{4b^4}{\left(cx^3+b\right)^5c}-\frac{12b^3}{\left(cx^3+b\right)^4c}-\frac{30b^2}{\left(cx^3+b\right)^3c}-\frac{66b}{\left(cx^3+b\right)^2c}-\frac{132}{\left(cx^3+b\right)c}\right)c^8}{3b^{13}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}-\frac{10c^4}{b^{11}x^9}+\frac{4c^3}{b^{10}x^{12}}-\frac{4c^2}{3b^9x^{15}}+\frac{c}{3b^8x^{18}}-\frac{1}{21b^7x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-1/7*b^6/c/(c*x^3+b)^7-b^5/c/(c*x^3+b)^6-132/c/(c*x^3+b)-66*
b/c/(c*x^3+b)^2-4*b^4/c/(c*x^3+b)^5-30*b^2/c/(c*x^3+b)^3-12*b^3/c/(c*x^3+b)
^4)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

maxima [B] time = 0.80, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^14/(c*x^4+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

mupad [B] time = 7.22, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,{\left(c\,x^3+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^14*(b*x + c*x^4)^8),x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

sympy [B] time = 2.06, size = 87, normalized size = 5.44

$$-\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**14/(c*x**4+b*x)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

3.167 $\int \frac{x^{-7(-1+n)}(b+2cx^n)}{\left(bx+cx^{1+n}\right)^8} dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-7(-1+n)}\left(b+2cx^n\right)}{\left(bx+cx^{1+n}\right)^8} dx &= \int \frac{x^{-8-7(-1+n)}\left(b+2cx^n\right)}{\left(b+cx^n\right)^8} dx\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}\end{aligned}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\left(b+cx^n\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

IntegrateAlgebraic [A] time = 0.11, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(b + 2*c*x^n)/(x^(7*(-1 + n))*(b*x + c*x^(1 + n))^8),x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

fricas [B] time = 1.73, size = 143, normalized size = 6.81

$$-\frac{x^{14}}{7\left(b^7nx^7x^{7n+7}+7\,b^6cnx^6x^{8n+8}+21\,b^5c^2nx^5x^{9n+9}+35\,b^4c^3nx^4x^{10n+10}+35\,b^3c^4nx^3x^{11n+11}+21\,b^2c^5nx^2x^{12n+12}+7\,bc^6nxx^{13n+13}+c^7nx^{14n+14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="fricas")
```

```
[Out] -1/7*x^14/(b^7*n*x^7*x^(7*n + 7) + 7*b^6*c*n*x^6*x^(8*n + 8) + 21*b^5*c^2*n
*x^5*x^(9*n + 9) + 35*b^4*c^3*n*x^4*x^(10*n + 10) + 35*b^3*c^4*n*x^3*x^(11*
n + 11) + 21*b^2*c^5*n*x^2*x^(12*n + 12) + 7*b*c^6*n*x*x^(13*n + 13) + c^7*
n*x^(14*n + 14))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^n+b}{\left(bx+cx^{n+1}\right)^8x^{7\,n-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)/((b*x + c*x^(n + 1))^8*x^(7*n - 7)), x)
```

maple [B] time = 0.06, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n}+\frac{c\,x^{-6n}}{b^8n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{66c^5x^{-2n}}{b^{12}n}+\frac{\left(9009b^5c\,x^n+20020b^4c^2x^{2n}+24024b^3c^3x^{3n}+16380b^2c^4x^{4n}+6006b\,c^5x^{5n}+924c^6x^{6n}+1716b^6\right)c^7}{7\,(c\,x^n+b)^7\,b^{13}n}-\frac{132c^6x^{-n}}{b^{13}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

maxima [B] time = 1.03, size = 612, normalized size = 29.14

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13n}+2342340\,bc^{12}x^{12n}+6426420\,b^2c^{11}x^{11n}+9579570\,b^3c^{10}x^{10n}+8270262\,b^4c^9x^{9n}+4018014\,b^5c^8x^{8n}+934362\,b^6c^7x^{7n}+45045\,b^7c^6x^{6n}-5005\,b^8c^5x^{5n}+1001\,b^9c^4x^{4n}-273\,b^{10}c^3x^{3n}+91\,b^{11}c^2x^{2n}-35\,b^{12}cx^n+15\,b^{13}}{b^{14}c^7nx^{14n}+7\,b^{15}c^6nx^{13n}+21\,b^{16}c^5nx^{12n}+35\,b^{17}c^4nx^{11n}+35\,b^{18}c^3nx^{10n}+21\,b^{19}c^2nx^{9n}+7\,b^{20}cnx^{8n}+b^{21}nx^{7n}}+\frac{360360\,c^7\log(x)}{b^{15}}-\frac{360360\,c^7\log\left(\frac{cx^n+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}\,c\left(\frac{360360\,c^{12}x^{12n}+2342340\,bc^{11}x^{11n}+6426420\,b^2c^{10}x^{10n}+9579570\,b^3c^9x^{9n}+8270262\,b^4c^8x^{8n}+4018014\,b^5c^7x^{7n}+934362\,b^6c^6x^{6n}+45045\,b^7c^5x^{5n}-5005\,b^8c^4x^{4n}+1001\,b^9c^3x^{3n}-273\,b^{10}c^2x^{2n}+91\,b^{11}cx^n-35\,b^{12}}{b^{13}c^7nx^{13n}+7\,b^{14}c^6nx^{12n}+21\,b^{15}c^5nx^{11n}+35\,b^{16}c^4nx^{10n}+35\,b^{17}c^3nx^{9n}+21\,b^{18}c^2nx^{8n}+7\,b^{19}cnx^{7n}+b^{20}nx^{6n}}+\frac{360360\,c^6\log(x)}{b^{14}}-\frac{360360\,c^6\log\left(\frac{cx^n+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/(x^(-7+7*n))/(b*x+c*x^(1+n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
```

```
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x^{7-7n}\,(b+2\,c\,x^n)}{\left(b\,x+c\,x^{n+1}\right)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8,x)
[Out] int((x^(7 - 7*n)*(b + 2*c*x^n))/(b*x + c*x^(n + 1))^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/(x**(-7+7*n))/(b*x+c*x**(1+n))**8,x)
[Out] Timed out
```

3.168 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x)*(b*x + c*x^2)^p, x]
```

fricas [A] time = 1.07, size = 26, normalized size = 1.37

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

giac [A] time = 0.37, size = 19, normalized size = 1.00

$$\frac{\left(cx^2 + bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx+b)\,x\left(c\,x^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] (c*x+b)*x/(1+p)*(c*x^2+b*x)^p
```

maxima [A] time = 0.56, size = 19, normalized size = 1.00

$$\frac{\left(cx^2 + bx\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

mupad [B] time = 2.10, size = 23, normalized size = 1.21

$$\frac{x\left(c\,x^2+b\,x\right)^p\,\left(b+c\,x\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

sympy [A] time = 0.71, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx\left(bx+cx^2\right)^p}{p+1} + \frac{cx^2\left(bx+cx^2\right)^p}{p+1} & \text{for}\ p \neq -1 \\ \log{(x)} + \log{\left(\frac{b}{c} + x\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.169 $\int x^{1+p} \left(b+2cx^2\right) \left(bx+cx^3\right)^p \, dx$

Optimal. Leaf size=27

$$\frac{x^{p+1} \left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1590}

$$\frac{x^{p+1} \left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(1 + p)*(b*x + c*x^3)^(1 + p))/(2*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{1+p} \left(b+2cx^2\right) \left(bx+cx^3\right)^p \, dx = \frac{x^{1+p} \left(bx+cx^3\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 3.59

$$\frac{x^{p+2} \left(x \left(b+cx^2\right)\right)^p \left(\frac{cx^2}{b}+1\right)^{-p} \left(2c(p+1)x^2 \, {}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2) \, {}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x^{1+p} \left(b+2cx^2\right) \left(bx+cx^3\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(1 + p)*(b + 2*c*x^2)*(b*x + c*x^3)^p, x]
```

fricas [A] time = 1.23, size = 32, normalized size = 1.19

$$\frac{\left(cx^3+bx\right)\left(cx^3+bx\right)^p x^{p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^3 + b*x)*(c*x^3 + b*x)^p*x^(p + 1)/(p + 1)
```

giac [B] time = 0.33, size = 54, normalized size = 2.00

$$\frac{cx^3e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="giac")
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(c\,x^2+b\right)x^{p+2}\left(c\,x^3+bx\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(p+1)*(2*c*x^2+b)*(c*x^3+b*x)^p,x)
[Out] 1/2*x^(2+p)*(c*x^2+b)/(p+1)*(c*x^3+b*x)^p
```

maxima [A] time = 0.99, size = 35, normalized size = 1.30

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+p)*(2*c*x^2+b)*(c*x^3+b*x)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [B] time = 2.21, size = 45, normalized size = 1.67

$$\left(c\,x^3+b\,x\right)^p\left(\frac{b\,x\,x^{p+1}}{2\,p+2}+\frac{c\,x^{p+1}\,x^3}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(p + 1)*(b*x + c*x^3)^p*(b + 2*c*x^2),x)
[Out] (b*x + c*x^3)^p*((b*x*x^(p + 1))/(2*p + 2) + (c*x^(p + 1)*x^3)/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+p)*(2*c*x**2+b)*(c*x**3+b*x)**p,x)
[Out] Timed out
```

3.170 $\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx$

Optimal. Leaf size=27

$$\frac{x^{p+1}\left(bx+cx^3\right)^{p+1}}{2(p+1)}$$

Rubi [C] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 4.30, number of steps used = 7, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2032, 365, 364}

$$\frac{bx^{p+2}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)}{2(p+1)}+\frac{cx^{p+4}\left(bx+cx^3\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)}{p+2}$$

Antiderivative was successfully verified.

```
[In] Int[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (b*x^(2 + p)*(b*x + c*x^3)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x^2)/
b)])/(2*(1 + p)*(1 + (c*x^2)/b)^p) + (c*x^(4 + p)*(b*x + c*x^3)^p*Hypergeom
etric2F1[-p, 2 + p, 3 + p, -((c*x^2)/b)])/((2 + p)*(1 + (c*x^2)/b)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx &= b\int x^{1+p}\left(bx+cx^3\right)^p\,dx+(2c)\int x^{3+p}\left(bx+cx^3\right)^p\,dx\\ &= \left(bx^{-p}\left(b+cx^2\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(b+cx^2\right)^p\,dx+\left(2cx\right.\\ &= \left(bx^{-p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p\right)\int x^{1+2p}\left(1+\frac{cx^2}{b}\right)^p\,dx+\left(2c\right.\\ &= \frac{bx^{2+p}\left(1+\frac{cx^2}{b}\right)^{-p}\left(bx+cx^3\right)^p{}_2F_1\left(-p,1+p;2+p;-\frac{cx^2}{b}\right)}{2(1+p)}+\end{aligned}$$

Mathematica [C] time = 0.03, size = 97, normalized size = 3.59

$$\frac{x^{p+2}\left(x\left(b+cx^2\right)\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3)^p,x]
```

```
[Out] (x^(2 + p)*(x*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int\left(bx^{1+p}\left(bx+cx^3\right)^p+2cx^{3+p}\left(bx+cx^3\right)^p\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x + c*x^3
)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][b*x^(1 + p)*(b*x + c*x^3)^p + 2*c*x^(3 + p)*(b*x
+ c*x^3)^p, x]
```

fricas [A] time = 0.83, size = 33, normalized size = 1.22

$$\frac{\left(cx^2+b\right)\left(cx^3+bx\right)^p x^{p+3}}{2\left(p+1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="f
ricas")
```

```
[Out] 1/2*(c*x^2 + b)*(c*x^3 + b*x)^p*x^(p + 3)/((p + 1)*x)
```

giac [B] time = 0.49, size = 54, normalized size = 2.00

$$\frac{cx^3e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}+bxe^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="g
iac")
```

```
[Out] 1/2*(c*x^3*e^(p*log(c*x^2 + b) + 2*p*log(x) + log(x)) + b*x*e^(p*log(c*x^2
+ b) + 2*p*log(x) + log(x)))/(p + 1)
```

maple [C] time = 0.26, size = 142, normalized size = 5.26

$$\frac{\left(c\,x^2+b\right)x\,x^{p+1}\mathrm{e}^{\frac{\left(-i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)+i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^2+i\pi\,\mathrm{csgn}\left(i\left(c\,x^2+b\right)\right)\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^2-i\pi\mathrm{csgn}\left(i\left(c\,x^2+b\right)x\right)^3+2\ln(x)+2\ln\left(c\,x^2+b\right)\right)p}{2}}}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(p+1)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x)
```

```
[Out] 1/2*(c*x^2+b)*x*x^(p+1)/(p+1)*exp(1/2*p*(-I*csgn(I*x*(c*x^2+b))^3*Pi+I*csgn
(I*x*(c*x^2+b))^2*csgn(I*x)*Pi+I*csgn(I*x*(c*x^2+b))^2*csgn(I*(c*x^2+b))*Pi
-I*csgn(I*x*(c*x^2+b))*csgn(I*x)*csgn(I*(c*x^2+b))*Pi+2*ln(x)+2*ln(c*x^2+b)
))
```

maxima [A] time = 1.07, size = 35, normalized size = 1.30

$$\frac{\left(cx^4 + bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^(1+p)*(c*x^3+b*x)^p+2*c*x^(3+p)*(c*x^3+b*x)^p,x, algorithm="m
axima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int b\,x^{p+1}\,\left(c\,x^3+b\,x\right)^p+2\,c\,x^{p+3}\,\left(c\,x^3+b\,x\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p,x)
```

```
[Out] int(b*x^(p + 1)*(b*x + c*x^3)^p + 2*c*x^(p + 3)*(b*x + c*x^3)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**(1+p)*(c*x**3+b*x)**p+2*c*x**(3+p)*(c*x**3+b*x)**p,x)
```

```
[Out] Timed out
```

3.171 $\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1590}

$$\frac{x^{2(p+1)}\left(bx+cx^4\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(2*(1 + p))*(b*x + c*x^4)^(1 + p))/(3*(1 + p))
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx=\frac{x^{2(1+p)}\left(bx+cx^4\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.08, size = 99, normalized size = 3.41

$$\frac{x^{2p+3}\left(x\left(b+cx^3\right)\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] (x^(3 + 2*p)*(x*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 +
p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c
*x^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^{2(1+p)}\left(b+2cx^3\right)\left(bx+cx^4\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(2*(1 + p))*(b + 2*c*x^3)*(b*x + c*x^4)^p, x]
```

fricas [A] time = 1.08, size = 34, normalized size = 1.17

$$\frac{\left(cx^4+bx\right)\left(cx^4+bx\right)^p x^{2p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^4 + b*x)^p*x^(2*p + 2)/(p + 1)
```

giac [B] time = 0.35, size = 58, normalized size = 2.00

$$\frac{cx^4e^{\left(p\log\left(cx^3+b\right)+3p\log(x)+2\log(x)\right)}+bxe^{\left(p\log\left(cx^3+b\right)+3p\log(x)+2\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^4*e^(p*log(c*x^3 + b) + 3*p*log(x) + 2*log(x)) + b*x*e^(p*log(c*x^
3 + b) + 3*p*log(x) + 2*log(x)))/(p + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 1.14

$$\frac{\left(c\,x^3+b\right)x^{2p+3}\left(c\,x^4+bx\right)^p}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x)
```

```
[Out] 1/3*x^(3+2*p)*(c*x^3+b)/(p+1)*(c*x^4+b*x)^p
```

maxima [A] time = 1.01, size = 35, normalized size = 1.21

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3p\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+2*p)*(2*c*x^3+b)*(c*x^4+b*x)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

mupad [B] time = 2.21, size = 49, normalized size = 1.69

$$\left(c\,x^4+b\,x\right)^p\,\left(\frac{c\,x^{2\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{2\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 2)*(b*x + c*x^4)^p*(b + 2*c*x^3),x)
```

```
[Out] (b*x + c*x^4)^p*((c*x^(2*p + 2)*x^4)/(3*p + 3) + (b*x*x^(2*p + 2))/(3*p + 3
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+2*p)*(2*c*x**3+b)*(c*x**4+b*x)**p,x)
```

```
[Out] Timed out
```

3.172 $\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$

Optimal. Leaf size=36

$$\frac{x^{-((1-n)(p+1))}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2036}

$$\frac{x^{-(1-n)(p+1)}\left(bx+cx^{n+1}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] (b*x + c*x^(1 + n))^(1 + p)/(n*(1 + p)*x^((1 - n)*(1 + p)))
```

Rule 2036

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*e^(j - 1)*(e*x)^(m - j + 1)*(a*x^j
+ b*x^(j + n))^(p + 1))/(a*(m + j*p + 1)), x] /; FreeQ[{a, b, c, d, e, j, m
, n, p}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && EqQ[
a*d*(m + j*p + 1) - b*c*(m + n + p*(j + n) + 1), 0] && (GtQ[e, 0] || Intege
rsQ[j]) && NeQ[m + j*p + 1, 0]
```

Rubi steps

$$\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx=\frac{x^{-((1-n)(1+p))}\left(bx+cx^{1+n}\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.17, size = 108, normalized size = 3.00

$$\frac{x^{-p}\left(x\left(b+cx^n\right)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p,x]
```

```
[Out] ((x*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2
+ p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p,
 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^p*(1 + (c*x^n)/b)^p)
```

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^{(-1+n)(1+p)}\left(b+2cx^n\right)\left(bx+cx^{1+n}\right)^p\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1 + n))^p
,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^((-1 + n)*(1 + p))*(b + 2*c*x^n)*(b*x + c*x^(1
+ n))^p, x]
```

fricas [A] time = 0.85, size = 42, normalized size = 1.17

$$\frac{\left(bx + cx^{n+1}\right)\left(bx + cx^{n+1}\right)^{p} x^{(n-1)p+n-1}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="fric
as")
```

```
[Out] (b*x + c*x^(n + 1))*(b*x + c*x^(n + 1))^p*x^((n - 1)*p + n - 1)/(n*p + n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2\,cx^{n} + b)\left(bx + cx^{n+1}\right)^{p} x^{(n-1)(p+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="giac
")
```

```
[Out] integrate((2*c*x^n + b)*(b*x + c*x^(n + 1))^p*x^((n - 1)*(p + 1)), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (2c\,x^{n} + b)\,x^{(n-1)(p+1)}\left(bx + c\,x^{n+1}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

```
[Out] int(x^((-1+n)*(p+1))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x)
```

maxima [A] time = 1.07, size = 39, normalized size = 1.08

$$\frac{\left(cx^{2n} + bx^{n}\right)e^{\left(np\log(x) + p\log(cx^{n}+b)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^((-1+n)*(1+p))*(b+2*c*x^n)*(b*x+c*x^(1+n))^p,x, algorithm="maxi
ma")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{(n-1)\,(p+1)}\left(b\,x + c\,x^{n+1}\right)^{p}\,\left(b + 2\,c\,x^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n),x)
```

```
[Out] int(x^((n - 1)*(p + 1))*(b*x + c*x^(n + 1))^p*(b + 2*c*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**((-1+n)*(1+p))*(b+2*c*x**n)*(b*x+c*x**(1+n))**p,x)
```

```
[Out] Timed out
```

3.173 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2} dx$

Optimal. Leaf size=32

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} dx &= \int \left(ac + adx + bcx^2 + bdx^3\right) dx\\ &= acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 1.00

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

IntegrateAlgebraic [A] time = 0.04, size = 29, normalized size = 0.91

$$\frac{1}{12}x\left(12ac + 6adx + 4bcx^2 + 3bdx^3\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(a + b*x^2),x]
```

```
[Out] (x*(12*a*c + 6*a*d*x + 4*b*c*x^2 + 3*b*d*x^3))/12
```

fricas [A] time = 0.93, size = 26, normalized size = 0.81

$$\frac{1}{4}bdx^4 + \frac{1}{3}bcx^3 + \frac{1}{2}adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="fricas")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

giac [A] time = 0.31, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="giac")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{1}{4}bd\,x^4 + \frac{1}{3}bc\,x^3 + \frac{1}{2}ad\,x^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),x
)
```

```
[Out] a*c*x+1/2*a*d*x^2+1/3*b*c*x^3+1/4*b*d*x^4
```

maxima [A] time = 0.78, size = 26, normalized size = 0.81

$$\frac{1}{4}\,bdx^4 + \frac{1}{3}\,bcx^3 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a),x, algorithm="maxima")
```

```
[Out] 1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

mupad [B] time = 2.14, size = 26, normalized size = 0.81

$$\frac{b\,d\,x^4}{4} + \frac{b\,c\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2),x)
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$acx + \frac{adx^2}{2} + \frac{bcx^3}{3} + \frac{bdx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a),x)
```

```
[Out] a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

3.174 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

Rubi [A] time = 0.05, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1586}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^2}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}\,dx\\ &= \int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^2,x]
```

```
[Out] c*x + (d*x^2)/2
```

IntegrateAlgebraic [A] time = 0.04, size = 14, normalized size = 1.17

$$\frac{(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(a + b*x^2)^2,x]
```

[Out] (c + d*x)^2/(2*d)

fricas [A] time = 0.93, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="fricas")
```

[Out] 1/2*d*x^2 + c*x

giac [A] time = 0.40, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="giac")
```

[Out] 1/2*d*x^2 + c*x

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}d\,x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^2
,x)
```

[Out] c*x+1/2*d*x^2

maxima [A] time = 0.47, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^2,x, algorithm="maxima")
```

[Out] 1/2*d*x^2 + c*x

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^2,x)
```

[Out] c*x + (d*x^2)/2

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**2,x)
```

```
[Out] c*x + d*x**2/2
```

3.175 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=42

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1586, 635, 205, 260}

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/(
a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}\,dx &= \int \frac{ac+adx+bcx^2+bdx^3}{\left(a+bx^2\right)^2}\,dx\\
&= \int \frac{c+dx}{a+bx^2}\,dx\\
&= c\int \frac{1}{a+bx^2}\,dx+d\int \frac{x}{a+bx^2}\,dx\\
&= \frac{c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{c\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*
x^5)/(a + b*x^2)^3,x]
```

```
[Out] (c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*b
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(a + b*x^2)^3,x]
```

```
[Out] IntegrateAlgebraic[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4
 + b^2*d*x^5)/(a + b*x^2)^3, x]
```

fricas [A] time = 0.81, size = 98, normalized size = 2.33

$$\left[\frac{ad\log\left(bx^2+a\right)-\sqrt{-ab}\,c\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{ad\log\left(bx^2+a\right)+2\sqrt{ab}\,c\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*b)
*x/a))/(a*b)]
```

giac [A] time = 0.40, size = 31, normalized size = 0.74

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="giac")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.76

$$\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{d\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^3
,x)
```

```
[Out] 1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

maxima [A] time = 1.49, size = 31, normalized size = 0.74

$$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d \log\left(bx^2 + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^
2+a)^3,x, algorithm="maxima")
```

```
[Out] c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

mupad [B] time = 2.13, size = 32, normalized size = 0.76

$$\frac{d\,\ln\left(b\,x^2 + a\right)}{2\,b} + \frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/(
a + b*x^2)^3,x)
```

```
[Out] (d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.30, size = 124, normalized size = 2.95

$$\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right) + \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right)\log\left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2}\right) - ad}{bc}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2*
c)/(b*x**2+a)**3,x)
```

```
[Out] (d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a*
b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*lo
g(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

3.176 $\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{(a+x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n
, x]
```

fricas [A] time = 1.02, size = 38, normalized size = 1.52

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^n/(n + 1)
```

giac [A] time = 0.29, size = 25, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$\frac{\left(d\,x^3 + c\,x^2 + bx + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x)
[Out] (d*x^3+c*x^2+b*x+a)^(n+1)/(n+1)
```

maxima [A] time = 0.67, size = 25, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + b*x + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.19, size = 54, normalized size = 2.16

$$\left(d\,x^3 + c\,x^2 + b\,x + a\right)^n \left(\frac{a}{n+1} + \frac{b\,x}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^n,x)
[Out] (a + b*x + c*x^2 + d*x^3)^n*(a/(n + 1) + (b*x)/(n + 1) + (c*x^2)/(n + 1) +
(d*x^3)/(n + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**n,x)
[Out] Timed out
```

3.177 $\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{\left(bx+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx=\frac{\left(bx+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.05, size = 21, normalized size = 0.88

$$\frac{(x(b+x(c+dx)))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] (x*(b + x*(c + d*x)))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(b+2cx+3dx^2\right)\left(bx+cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n, x]
```

fricas [A] time = 0.86, size = 36, normalized size = 1.50

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^3+cx^2+bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^3 + c*x^2 + b*x)^n/(n + 1)
```

giac [A] time = 0.31, size = 24, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 1.42

$$\frac{\left(d\,x^2 + cx + b\right)x\left(d\,x^3 + c\,x^2 + bx\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x)
```

```
[Out] x*(d*x^2+c*x+b)/(n+1)*(d*x^3+c*x^2+b*x)^n
```

maxima [A] time = 0.63, size = 24, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)^(n + 1)/(n + 1)
```

mupad [B] time = 2.12, size = 46, normalized size = 1.92

$$\left(\frac{b\,x}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\left(d\,x^3 + c\,x^2 + b\,x\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^n,x)
```

```
[Out] ((b*x)/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(b*x + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**n,x)
```

```
[Out] Timed out
```

3.178 $\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx$

Optimal. Leaf size=25

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1590}

$$\frac{x^{n+1}\left(b+cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(b + c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx = \frac{x^{1+n}\left(b+cx+dx^2\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.96

$$\frac{x^{n+1}(b+x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
[Out] (x^(1 + n)*(b + x*(c + d*x))^(1 + n))/(1 + n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n \left(b+cx+dx^2\right)^n \left(b+2cx+3dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x]
[Out] Defer[IntegrateAlgebraic][x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2), x]
```

fricas [A] time = 1.02, size = 35, normalized size = 1.40

$$\frac{\left(dx^3+cx^2+bx\right)\left(dx^2+cx+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*(d*x^2 + c*x + b)^n*x^n/(n + 1)
```

giac [**B**] time = 0.40, size = 65, normalized size = 2.60

$$\frac{\left(dx^2+cx+b\right)^n dx^3x^n+\left(dx^2+cx+b\right)^n cx^2x^n+\left(dx^2+cx+b\right)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + c*x + b)^n*d*x^3*x^n + (d*x^2 + c*x + b)^n*c*x^2*x^n + (d*x^2 + c
*x + b)^n*b*x*x^n)/(n + 1)
```

maple [**A**] time = 0.01, size = 26, normalized size = 1.04

$$\frac{x^{n+1}\left(d\,x^2+cx+b\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x)
```

```
[Out] x^(n+1)*(d*x^2+c*x+b)^(n+1)/(n+1)
```

maxima [**A**] time = 0.86, size = 39, normalized size = 1.56

$$\frac{\left(dx^3+cx^2+bx\right)e^{\left(n\log\left(dx^2+cx+b\right)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x+b)^n*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x)*e^(n*log(d*x^2 + c*x + b) + n*log(x))/(n + 1)
```

mupad [**B**] time = 2.15, size = 51, normalized size = 2.04

$$\left(\frac{c\,x^n\,x^2}{n+1}+\frac{d\,x^n\,x^3}{n+1}+\frac{b\,x\,x^n}{n+1}\right)\left(d\,x^2+c\,x+b\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + c*x + d*x^2)^n*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] ((c*x^n*x^2)/(n + 1) + (d*x^n*x^3)/(n + 1) + (b*x*x^n)/(n + 1))*(b + c*x +
d*x^2)^n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x+b)**n*(3*d*x**2+2*c*x+b),x)
```

```
[Out] Timed out
```

3.179 $\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx=\frac{\left(a+bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\left(a+bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] (a + b*x + d*x^3)^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(b+3dx^2\right)\left(a+bx+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 3*d*x^2)*(a + b*x + d*x^3)^n, x]
```

fricas [A] time = 0.95, size = 28, normalized size = 1.40

$$\frac{\left(dx^3+bx+a\right)\left(dx^3+bx+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x + a)*(d*x^3 + b*x + a)^n/(n + 1)
```

giac [A] time = 0.28, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{(d\,x^3 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^n,x)
```

```
[Out] (d*x^3+b*x+a)^(n+1)/(n+1)
```

maxima [A] time = 0.65, size = 20, normalized size = 1.00

$$\frac{(dx^3 + bx + a)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.14, size = 39, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{b\,x}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + b\,x + a\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^n,x)
```

```
[Out] (a/(n + 1) + (b*x)/(n + 1) + (d*x^3)/(n + 1))*(a + b*x + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**n,x)
```

```
[Out] Timed out
```

3.180 $\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1588}

$$\frac{\left(bx+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (b*x + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx=\frac{\left(bx+dx^3\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 5.58

$$\frac{x\left(x\left(b+dx^2\right)\right)^n\left(\frac{dx^2}{b}+1\right)^{-n}\left(3d(n+1)x^2\,{}_2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3)\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] (x*(x*(b + d*x^2))^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n)/2,
 -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 + n)/2
, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b + 3*d*x^2)*(b*x + d*x^3)^n, x]
```

fricas [**A**] time = 1.38, size = 26, normalized size = 1.37

$$\frac{\left(dx^{3}+bx\right)\left(dx^{3}+bx\right)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="fricas")
[Out] (d*x^3 + b*x)*(d*x^3 + b*x)^n/(n + 1)
```

giac [**A**] time = 0.34, size = 19, normalized size = 1.00

$$\frac{\left(dx^{3}+bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="giac")
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

maple [**A**] time = 0.00, size = 26, normalized size = 1.37

$$\frac{\left(d\,x^{2}+b\right)x\left(d\,x^{3}+bx\right)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^n,x)
[Out] x*(d*x^2+b)/(n+1)*(d*x^3+b*x)^n
```

maxima [**A**] time = 0.53, size = 19, normalized size = 1.00

$$\frac{\left(dx^{3}+bx\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^n,x, algorithm="maxima")
[Out] (d*x^3 + b*x)^(n + 1)/(n + 1)
```

mupad [**B**] time = 2.13, size = 25, normalized size = 1.32

$$\frac{x\left(d\,x^{3}+b\,x\right)^{n}\left(d\,x^{2}+b\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^n*(b + 3*d*x^2),x)
[Out] (x*(b*x + d*x^3)^n*(b + d*x^2))/(n + 1)
```

sympy [**B**] time = 11.34, size = 73, normalized size = 3.84

$$\begin{cases} \frac{bx\left(bx+dx^{3}\right)^{n}}{n+1}+\frac{dx^{3}\left(bx+dx^{3}\right)^{n}}{n+1} & \text{for } n\neq -1 \\ \log\left(x\right)+\log\left(-i\sqrt{b}\sqrt{\frac{1}{d}}+x\right)+\log\left(i\sqrt{b}\sqrt{\frac{1}{d}}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**n,x)
```

```
[Out] Piecewise((b*x*(b*x + d*x**3)**n/(n + 1) + d*x**3*(b*x + d*x**3)**n/(n + 1)
, Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)*sqrt(
1/d) + x), True))
```

3.181 $\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^{n+1}\left(b+dx^2\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {449}

$$\frac{x^{n+1}\left(b+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^(1 + n))/(1 + n)
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) dx = \frac{x^{1+n}\left(b+dx^2\right)^{1+n}}{1+n}$$

Mathematica [C] time = 0.04, size = 108, normalized size = 4.91

$$\frac{x^{n+1}\left(b+dx^2\right)^n \left(\frac{dx^2}{b}+1\right)^{-n} \left(3d(n+1)x^2\,{}_2F_1\left(-n,\frac{n+3}{2};\frac{n+5}{2};-\frac{dx^2}{b}\right)+b(n+3)\,{}_2F_1\left(-n,\frac{n+1}{2};\frac{n+3}{2};-\frac{dx^2}{b}\right)\right)}{(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(b + d*x^2)^n*(b*(3 + n)*Hypergeometric2F1[-n, (1 + n)/2, (3 + n
)/2, -((d*x^2)/b)] + 3*d*(1 + n)*x^2*Hypergeometric2F1[-n, (3 + n)/2, (5 +
n)/2, -((d*x^2)/b)]))/((1 + n)*(3 + n)*(1 + (d*x^2)/b)^n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n \left(b+dx^2\right)^n \left(b+3dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n*(b + d*x^2)^n*(b + 3*d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^n*(b + d*x^2)^n*(b + 3*d*x^2), x]
```

fricas [A] time = 1.03, size = 27, normalized size = 1.23

$$\frac{\left(dx^3+bx\right)\left(dx^2+b\right)^n x^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] (d*x^3 + b*x)*(d*x^2 + b)^n*x^n/(n + 1)
```

giac [A] time = 0.34, size = 39, normalized size = 1.77

$$\frac{\left(dx^2+b\right)^n dx^3x^n+\left(dx^2+b\right)^n bxx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] ((d*x^2 + b)^n*d*x^3*x^n + (d*x^2 + b)^n*b*x*x^n)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{n+1}\left(d\,x^2+b\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+b)^n*(3*d*x^2+b),x)
```

```
[Out] x^(n+1)*(d*x^2+b)^(n+1)/(n+1)
```

maxima [A] time = 1.21, size = 31, normalized size = 1.41

$$\frac{\left(dx^3+bx\right)e^{\left(n\log\left(dx^2+b\right)+n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+b)^n*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] (d*x^3 + b*x)*e^(n*log(d*x^2 + b) + n*log(x))/(n + 1)
```

mupad [B] time = 2.16, size = 26, normalized size = 1.18

$$\frac{x\,x^n\,\left(d\,x^2+b\right)^n\,\left(d\,x^2+b\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(b + d*x^2)^n*(b + 3*d*x^2),x)
```

```
[Out] (x*x^n*(b + d*x^2)^n*(b + d*x^2))/(n + 1)
```

sympy [B] time = 52.84, size = 76, normalized size = 3.45

$$\begin{cases} \frac{bxx^n\left(b+dx^2\right)^n}{n+1}+\frac{dx^3x^n\left(b+dx^2\right)^n}{n+1} & \text{for } n\neq -1 \\ \log\left(x\right)+\log\left(-i\sqrt{b}\sqrt{\frac{1}{d}}+x\right)+\log\left(i\sqrt{b}\sqrt{\frac{1}{d}}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+b)**n*(3*d*x**2+b),x)
```

```
[Out] Piecewise((b*x*x**n*(b + d*x**2)**n/(n + 1) + d*x**3*x**n*(b + d*x**2)**n/(
n + 1), Ne(n, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/d) + x) + log(I*sqrt(b)
*sqrt(1/d) + x), True))
```

3.182 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n, x]
```

fricas [A] time = 0.99, size = 32, normalized size = 1.45

$$\frac{\left(dx^3+cx^2+a\right)\left(dx^3+cx^2+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

giac [A] time = 0.34, size = 22, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(d\,x^3 + c\,x^2 + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x)
[Out] (d*x^3+c*x^2+a)^(n+1)/(n+1)
```

maxima [A] time = 0.66, size = 22, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + c\,x^2 + a\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^n,x)
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**n,x)
[Out] Timed out
```

3.183 $\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \left(2cx+3dx^2\right)\left(cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n, x]
```

fricas [A] time = 0.91, size = 30, normalized size = 1.43

$$\frac{\left(dx^3+cx^2\right)\left(dx^3+cx^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

giac [A] time = 0.31, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx+c)\, x^2 \left(d\, x^3 + c\, x^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x^3+c*x^2)^n*x^2*(d*x+c)/(n+1)
```

maxima [A] time = 0.59, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

mupad [B] time = 2.16, size = 27, normalized size = 1.29

$$\frac{x^2 \left(d\, x^3 + c\, x^2\right)^n\, (c + d\, x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 1.16, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2\left(cx^2+dx^3\right)^n}{n+1} + \frac{dx^3\left(cx^2+dx^3\right)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.184 $\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right) \, dx$

Optimal. Leaf size=24

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 763}

$$\frac{x^{n+1}\left(cx+dx^2\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(c*x + d*x^2)^(1 + n))/(1 + n)
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right) \, dx &= \int x^{1+n}(2c+3dx)\left(cx+dx^2\right)^n \, dx \\ &= \frac{x^{1+n}\left(cx+dx^2\right)^{1+n}}{1+n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$\frac{x^{n+1}(x(c+dx))^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(1 + n)*(x*(c + d*x))^(1 + n))/(1 + n)
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^n \left(cx+dx^2\right)^n \left(2cx+3dx^2\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2), x]
```

fricas [A] time = 0.79, size = 31, normalized size = 1.29

$$\frac{\left(dx^3 + cx^2\right)\left(dx^2 + cx\right)^n x^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x^2 + c*x)^n*x^n/(n + 1)
```

giac [B] time = 0.46, size = 51, normalized size = 2.12

$$\frac{dx^3 x^n e^{\left(n\log(dx+c)+n\log(x)\right)} + cx^2 x^n e^{\left(n\log(dx+c)+n\log(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
[Out] (d*x^3*x^n*e^(n*log(d*x + c) + n*log(x)) + c*x^2*x^n*e^(n*log(d*x + c) + n*
log(x)))/(n + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 1.17

$$\frac{(dx + c)\, x^{n+2} \left(d\, x^2 + cx\right)^n}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x)
[Out] (d*x^2+c*x)^n*x^(2+n)*(d*x+c)/(n+1)
```

maxima [A] time = 0.83, size = 32, normalized size = 1.33

$$\frac{\left(dx^3 + cx^2\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(d*x^2+c*x)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

mupad [B] time = 2.20, size = 28, normalized size = 1.17

$$\frac{x^n\, x^2 \left(d\, x^2 + c\, x\right)^n\, (c + d\, x)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(c*x + d*x^2)^n*(2*c*x + 3*d*x^2),x)
[Out] (x^n*x^2*(c*x + d*x^2)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 6.17, size = 56, normalized size = 2.33

$$\begin{cases} \frac{cx^2x^n\left(cx+dx^2\right)^n}{n+1} + \frac{dx^3x^n\left(cx+dx^2\right)^n}{n+1} & \text{for } n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(d*x**2+c*x)**n*(3*d*x**2+2*c*x),x)
```

```
[Out] Piecewise((c*x**2*x**n*(c*x + d*x**2)**n/(n + 1) + d*x**3*x**n*(c*x + d*x**
2)**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.185 $\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {845}

$$\frac{x^{2(n+1)}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2*(1 + n))*(c + d*x)^(1 + n))/(1 + n)
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx = \frac{x^{2(1+n)}(c+dx)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x^{2n+2}(c+dx)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^(2 + 2*n)*(c + d*x)^(1 + n))/(1 + n)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^{2n}(c+dx)^n \left(2cx+3dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(2*n)*(c + d*x)^n*(2*c*x + 3*d*x^2), x]
```

fricas [A] time = 0.97, size = 29, normalized size = 1.32

$$\frac{\left(dx^3+cx^2\right)(dx+c)^n x^{2n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] (d*x^3 + c*x^2)*(d*x + c)^n*x^(2*n)/(n + 1)
```

giac [A] time = 0.31, size = 41, normalized size = 1.86

$$\frac{(dx+c)^n dx^3 x^{2n} + (dx+c)^n cx^2 x^{2n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="giac")
[Out] ((d*x + c)^n*d*x^3*x^(2*n) + (d*x + c)^n*c*x^2*x^(2*n))/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^{2n+2}(dx+c)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x)
[Out] x^(2+2*n)*(d*x+c)^(n+1)/(n+1)
```

maxima [A] time = 0.90, size = 32, normalized size = 1.45

$$\frac{\left(dx^3+cx^2\right)e^{\left(n\log(dx+c)+2n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2*n)*(d*x+c)^n*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

mupad [B] time = 2.19, size = 26, normalized size = 1.18

$$\frac{x^{2n}\,x^2\,(c+d\,x)^n\,(c+d\,x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n)*(2*c*x + 3*d*x^2)*(c + d*x)^n,x)
[Out] (x^(2*n)*x^2*(c + d*x)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 6.15, size = 53, normalized size = 2.41

$$\begin{cases} \frac{cx^2x^{2n}(c+dx)^n}{n+1} + \frac{dx^3x^{2n}(c+dx)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2*n)*(d*x+c)**n*(3*d*x**2+2*c*x),x)
[Out] Piecewise((c*x**2*x**(2*n)*(c + d*x)**n/(n + 1) + d*x**3*x**(2*n)*(c + d*x)
**n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.186 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=22

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{\left(a+cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx=\frac{\left(a+cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{\left(a+x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] (a + x^2*(c + d*x))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n, x]
```

fricas [A] time = 1.07, size = 32, normalized size = 1.45

$$\frac{\left(dx^3+cx^2+a\right)\left(dx^3+cx^2+a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

giac [A] time = 0.32, size = 22, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2 + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + a)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{\left(d\,x^3 + c\,x^2 + a\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x)
```

```
[Out] 1/(n+1)*(d*x^3+c*x^2+a)^(n+1)
```

maxima [A] time = 0.75, size = 32, normalized size = 1.45

$$\frac{\left(dx^3 + cx^2 + a\right)\left(dx^3 + cx^2 + a\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + a)*(d*x^3 + c*x^2 + a)^n/(n + 1)
```

mupad [B] time = 2.14, size = 43, normalized size = 1.95

$$\left(\frac{a}{n+1} + \frac{c\,x^2}{n+1} + \frac{d\,x^3}{n+1}\right)\,\left(d\,x^3 + c\,x^2 + a\right)^n$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^n,x)
```

```
[Out] (a/(n + 1) + (c*x^2)/(n + 1) + (d*x^3)/(n + 1))*(a + c*x^2 + d*x^3)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**n,x)
```

```
[Out] Timed out
```

3.187 $\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$

Optimal. Leaf size=21

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{\left(cx^2+dx^3\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (c*x^2 + d*x^3)^(1 + n)/(1 + n)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx=\frac{\left(cx^2+dx^3\right)^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{\left(x^2(c+dx)\right)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] (x^2*(c + d*x))^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n, x]
```

fricas [A] time = 0.66, size = 30, normalized size = 1.43

$$\frac{\left(dx^3+cx^2\right)\left(dx^3+cx^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2)*(d*x^3 + c*x^2)^n/(n + 1)
```

giac [A] time = 0.40, size = 21, normalized size = 1.00

$$\frac{\left(dx^3 + cx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 1.33

$$\frac{(dx + c)\,x^2\left(d\,x^3 + c\,x^2\right)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x)
```

```
[Out] (d*x+c)/(n+1)*x^2*(d*x^3+c*x^2)^n
```

maxima [A] time = 0.93, size = 32, normalized size = 1.52

$$\frac{\left(dx^3 + cx^2\right)e^{\left(n\log(dx+c)+2\,n\log(x)\right)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^n,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2)*e^(n*log(d*x + c) + 2*n*log(x))/(n + 1)
```

mupad [B] time = 2.15, size = 27, normalized size = 1.29

$$\frac{x^2\left(d\,x^3 + c\,x^2\right)^n\,(c + d\,x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^n,x)
```

```
[Out] (x^2*(c*x^2 + d*x^3)^n*(c + d*x))/(n + 1)
```

sympy [A] time = 1.14, size = 53, normalized size = 2.52

$$\begin{cases} \frac{cx^2\left(cx^2+dx^3\right)^n}{n+1} + \frac{dx^3\left(cx^2+dx^3\right)^n}{n+1} & \text{for}\ n \neq -1 \\ 2\log(x) + \log\left(\frac{c}{d} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**n,x)
```

```
[Out] Piecewise((c*x**2*(c*x**2 + d*x**3)**n/(n + 1) + d*x**3*(c*x**2 + d*x**3)**
n/(n + 1), Ne(n, -1)), (2*log(x) + log(c/d + x), True))
```

3.188 $\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=21

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Rubi [A] time = 0.13, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1588}

$$\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+bx+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.15, size = 143, normalized size = 6.81

$$\frac{1}{8}x(b+x(c+dx))\left(8a^7+28a^6x(b+x(c+dx))+56a^5x^2(b+x(c+dx))^2+70a^4x^3(b+x(c+dx))^3+56a^3x^4(b+x(c+dx))^4+28a^2x^5(b+x(c+dx))^5+8ax^6(b+x(c+dx))^6+x^7(b+x(c+dx))^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x*(b + x*(c + d*x))*(8*a^7 + 28*a^6*x*(b + x*(c + d*x)) + 56*a^5*x^2*(b +
x*(c + d*x))^2 + 70*a^4*x^3*(b + x*(c + d*x))^3 + 56*a^3*x^4*(b + x*(c + d*
x))^4 + 28*a^2*x^5*(b + x*(c + d*x))^5 + 8*a*x^6*(b + x*(c + d*x))^6 + x^7*
(b + x*(c + d*x))^7))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b+2cx+3dx^2\right)\left(a+bx+cx^2+dx^3\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 0.57, size = 1956, normalized size = 93.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + x^21*d^7*a + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2
*x^20*d^6*b^2 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^
19*d^5*c*b^2 + 21*x^19*d^5*c^2*a + 7*x^19*d^6*b*a + 7/2*x^18*d^2*c^6 + 35*x
^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + 35*x^18*d^4*c^3*a
 + 42*x^18*d^5*c*b*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 21*x^17*d^2*c^5*b +
70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 35*x^17*d^3*c^4*a + 105*x^17*d^4*
c^2*b*a + 21*x^17*d^5*b^2*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 7*x^16*d*c
^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*x^16*d^4*b^4 + 2
1*x^16*d^2*c^5*a + 140*x^16*d^3*c^3*b*a + 105*x^16*d^4*c*b^2*a + 105/2*x^16
*d^4*c^2*a^2 + 21*x^16*d^5*b*a^2 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15
*d^2*c^3*b^3 + 35*x^15*d^3*c*b^4 + 7*x^15*d*c^6*a + 105*x^15*d^2*c^4*b*a +
210*x^15*d^3*c^2*b^2*a + 35*x^15*d^4*b^3*a + 70*x^15*d^3*c^3*a^2 + 105*x^15
*d^4*c*b*a^2 + 7*x^15*d^5*a^3 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/
2*x^14*d^2*c^2*b^4 + 7*x^14*d^3*b^5 + x^14*c^7*a + 42*x^14*d*c^5*b*a + 210*
x^14*d^2*c^3*b^2*a + 140*x^14*d^3*c*b^3*a + 105/2*x^14*d^2*c^4*a^2 + 210*x^
14*d^3*c^2*b*a^2 + 105/2*x^14*d^4*b^2*a^2 + 35*x^14*d^4*c*a^3 + 7*x^13*c^5*
b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5 + 7*x^13*c^6*b*a + 105*x^13*d*c
^4*b^2*a + 210*x^13*d^2*c^2*b^3*a + 35*x^13*d^3*b^4*a + 21*x^13*d*c^5*a^2 +
 210*x^13*d^2*c^3*b*a^2 + 210*x^13*d^3*c*b^2*a^2 + 70*x^13*d^3*c^2*a^3 + 35
*x^13*d^4*b*a^3 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6
+ 21*x^12*c^5*b^2*a + 140*x^12*d*c^3*b^3*a + 105*x^12*d^2*c*b^4*a + 7/2*x^1
2*c^6*a^2 + 105*x^12*d*c^4*b*a^2 + 315*x^12*d^2*c^2*b^2*a^2 + 70*x^12*d^3*b
^3*a^2 + 70*x^12*d^2*c^3*a^3 + 140*x^12*d^3*c*b*a^3 + 35/4*x^12*d^4*a^4 + 7
*x^11*c^3*b^5 + 7*x^11*d*c*b^6 + 35*x^11*c^4*b^3*a + 105*x^11*d*c^2*b^4*a +
 21*x^11*d^2*b^5*a + 21*x^11*c^5*b*a^2 + 210*x^11*d*c^3*b^2*a^2 + 210*x^11*
d^2*c*b^3*a^2 + 35*x^11*d*c^4*a^3 + 210*x^11*d^2*c^2*b*a^3 + 70*x^11*d^3*b^
2*a^3 + 35*x^11*d^3*c*a^4 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + 35*x^10*c^3*b^4
*a + 42*x^10*d*c*b^5*a + 105/2*x^10*c^4*b^2*a^2 + 210*x^10*d*c^2*b^3*a^2 +
105/2*x^10*d^2*b^4*a^2 + 7*x^10*c^5*a^3 + 140*x^10*d*c^3*b*a^3 + 210*x^10*d
^2*c*b^2*a^3 + 105/2*x^10*d^2*c^2*a^4 + 35*x^10*d^3*b*a^4 + x^9*c*b^7 + 21*
x^9*c^2*b^5*a + 7*x^9*d*b^6*a + 70*x^9*c^3*b^3*a^2 + 105*x^9*d*c*b^4*a^2 +
35*x^9*c^4*b*a^3 + 210*x^9*d*c^2*b^2*a^3 + 70*x^9*d^2*b^3*a^3 + 35*x^9*d*c^
3*a^4 + 105*x^9*d^2*c*b*a^4 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 7*x^8*c*b^6*a +
 105/2*x^8*c^2*b^4*a^2 + 21*x^8*d*b^5*a^2 + 70*x^8*c^3*b^2*a^3 + 140*x^8*d*
c*b^3*a^3 + 35/4*x^8*c^4*a^4 + 105*x^8*d*c^2*b*a^4 + 105/2*x^8*d^2*b^2*a^4
+ 21*x^8*d^2*c*a^5 + x^7*b^7*a + 21*x^7*c*b^5*a^2 + 70*x^7*c^2*b^3*a^3 + 35
*x^7*d*b^4*a^3 + 35*x^7*c^3*b*a^4 + 105*x^7*d*c*b^2*a^4 + 21*x^7*d*c^2*a^5
+ 21*x^7*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*c*b^4*a^3 + 105/2*x^6*c^2*b^2
*a^4 + 35*x^6*d*b^3*a^4 + 7*x^6*c^3*a^5 + 42*x^6*d*c*b*a^5 + 7/2*x^6*d^2*a^
6 + 7*x^5*b^5*a^3 + 35*x^5*c*b^3*a^4 + 21*x^5*c^2*b*a^5 + 21*x^5*d*b^2*a^5
+ 7*x^5*d*c*a^6 + 35/4*x^4*b^4*a^4 + 21*x^4*c*b^2*a^5 + 7/2*x^4*c^2*a^6 + 7
*x^4*d*b*a^6 + 7*x^3*b^3*a^5 + 7*x^3*c*b*a^6 + x^3*d*a^7 + 7/2*x^2*b^2*a^6
+ x^2*c*a^7 + x*b*a^7
```

giac [B] time = 0.30, size = 160, normalized size = 7.62

$$\frac{1}{8}\left(dx^3+cx^2+bx\right)^8+\left(dx^3+cx^2+bx\right)^7a+\frac{7}{2}\left(dx^3+cx^2+bx\right)^6a^2+7\left(dx^3+cx^2+bx\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2+bx\right)^4a^4+7\left(dx^3+cx^2+bx\right)^3a^5+\frac{7}{2}\left(dx^3+cx^2+bx\right)^2a^6+\left(dx^3+cx^2+bx\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8 + (d*x^3 + c*x^2 + b*x)^7*a + 7/2*(d*x^3 + c*x^
2 + b*x)^6*a^2 + 7*(d*x^3 + c*x^2 + b*x)^5*a^3 + 35/4*(d*x^3 + c*x^2 + b*x)
^4*a^4 + 7*(d*x^3 + c*x^2 + b*x)^3*a^5 + 7/2*(d*x^3 + c*x^2 + b*x)^2*a^6 +
(d*x^3 + c*x^2 + b*x)*a^7
```

maple [B] time = 0.00, size = 25686, normalized size = 1223.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x)
[Out] result too large to display
```

maxima [A] time = 0.74, size = 19, normalized size = 0.90

$$\frac{1}{8}\left(dx^3 + cx^2 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x+a)^7,x, algorithm="maxima")
[Out] 1/8*(d*x^3 + c*x^2 + b*x + a)^8
```

mupad [B] time = 2.98, size = 1576, normalized size = 75.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(a + b*x + c*x^2 + d*x^3)^7,x)
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + (35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*
a*b^2*c^5 + 21*b^5*c^2*d + 70*a^2*b^3*d^3 + 70*a^3*c^3*d^2 + 315*a^2*b^2*c^
2*d^2 + 140*a*b^3*c^3*d + 105*a*b^4*c*d^2 + 105*a^2*b*c^4*d + 140*a^3*b*c*d
^3) + x^11*(7*b^5*c^3 + 35*a*b^3*c^4 + 21*a^2*b*c^5 + 21*a*b^5*d^2 + 35*a^3
*c^4*d + 35*a^4*c*d^3 + 70*a^3*b^2*d^3 + 7*b^6*c*d + 210*a^2*b^2*c^3*d + 21
0*a^2*b^3*c*d^2 + 210*a^3*b*c^2*d^2 + 105*a*b^4*c^2*d) + x^13*(7*b^3*c^5 +
35*a*b^4*d^3 + 35*a^3*b*d^4 + 21*a^2*c^5*d + 35*b^4*c^3*d + 21*b^5*c*d^2 +
70*a^3*c^2*d^3 + 7*a*b*c^6 + 210*a*b^3*c^2*d^2 + 210*a^2*b*c^3*d^2 + 210*a^
2*b^2*c*d^3 + 105*a*b^2*c^4*d) + x^5*(7*a^3*b^5 + 35*a^4*b^3*c + 21*a^5*b*c
^2 + 21*a^5*b^2*d + 7*a^6*c*d) + x^19*(7*c^5*d^3 + 21*a*c^2*d^5 + 35*b*c^3*
d^4 + 21*b^2*c*d^5 + 7*a*b*d^6) + x^8*(b^8/8 + (35*a^4*c^4)/4 + 21*a^2*b^5*
d + 21*a^5*c*d^2 + (105*a^2*b^4*c^2)/2 + 70*a^3*b^2*c^3 + (105*a^4*b^2*d^2)
/2 + 7*a*b^6*c + 140*a^3*b^3*c*d + 105*a^4*b*c^2*d) + x^9*(b^7*c + 7*a^5*d^
3 + 21*a*b^5*c^2 + 35*a^3*b*c^4 + 35*a^4*c^3*d + 70*a^2*b^3*c^3 + 70*a^3*b^
3*d^2 + 7*a*b^6*d + 210*a^3*b^2*c^2*d + 105*a^2*b^4*c*d + 105*a^4*b*c*d^2)
+ x^16*(c^8/8 + (35*b^4*d^4)/4 + 21*a^2*b*d^5 + 21*a*c^5*d^2 + (105*a^2*c^2
*d^4)/2 + (105*b^2*c^4*d^2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d + 140*a*b*c^3*d^
3 + 105*a*b^2*c*d^4) + x^10*(b^7*d + 7*a^3*c^5 + (7*b^6*c^2)/2 + 35*a*b^4*c
^3 + 35*a^4*b*d^3 + (105*a^2*b^2*c^4)/2 + (105*a^2*b^4*d^2)/2 + (105*a^4*c^
2*d^2)/2 + 210*a^2*b^3*c^2*d + 210*a^3*b^2*c*d^2 + 42*a*b^5*c*d + 140*a^3*b
*c^3*d) + x^15*(b*c^7 + 7*a^3*d^5 + 35*a*b^3*d^4 + 21*b^2*c^5*d + 35*b^4*c*
d^3 + 70*a^2*c^3*d^3 + 70*b^3*c^3*d^2 + 7*a*c^6*d + 210*a*b^2*c^2*d^3 + 105
*a*b*c^4*d^2 + 105*a^2*b*c*d^4) + x^14*(a*c^7 + (7*b^2*c^6)/2 + 7*b^5*d^3 +
 35*a^3*c*d^4 + 35*b^3*c^4*d + (105*a^2*b^2*d^4)/2 + (105*a^2*c^4*d^2)/2 +
(105*b^4*c^2*d^2)/2 + 210*a*b^2*c^3*d^2 + 210*a^2*b*c^2*d^3 + 42*a*b*c^5*d
+ 140*a*b^3*c*d^3) + x^4*((35*a^4*b^4)/4 + (7*a^6*c^2)/2 + 21*a^5*b^2*c + 7
*a^6*b*d) + x^20*((7*b^2*d^6)/2 + (35*c^4*d^4)/4 + 21*b*c^2*d^5 + 7*a*c*d^6
) + x^6*((7*a^2*b^6)/2 + 7*a^5*c^3 + (7*a^6*d^2)/2 + 35*a^3*b^4*c + 35*a^4*
b^3*d + (105*a^4*b^2*c^2)/2 + 42*a^5*b*c*d) + x^7*(a*b^7 + 21*a^2*b^5*c + 3
5*a^4*b*c^3 + 35*a^3*b^4*d + 21*a^5*b*d^2 + 21*a^5*c^2*d + 70*a^3*b^3*c^2 +
 105*a^4*b^2*c*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5 + (7*c^6*d^2)/2 + 35*a*
c^3*d^4 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2 + 42*a*b*c*d^5) + x^17*(c^7*d
+ 21*a*b^2*d^5 + 35*a*c^4*d^3 + 21*a^2*c*d^5 + 21*b*c^5*d^2 + 35*b^3*c*d^4
+ 70*b^2*c^3*d^3 + 105*a*b*c^2*d^4) + x^3*(a^7*d + 7*a^5*b^3 + 7*a^6*b*c) +
```

```
(d^8*x^24)/8 + x^2*(a^7*c + (7*a^6*b^2)/2) + c*d^7*x^23 + d^5*x^21*(a*d^2
+ 7*c^3 + 7*b*c*d) + (d^6*x^22*(2*b*d + 7*c^2))/2 + a^7*b*x
```

sympy [B] time = 0.42, size = 1771, normalized size = 84.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x+a)**7,x)
```

```
[Out] a**7*b*x + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7*c**2*d**6/2) + x
**21*(a*d**7 + 7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 7*b**2*d**6/
2 + 21*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(7*a*b*d**6 + 21*a*c**2*d**5 +
 21*b**2*c*d**5 + 35*b*c**3*d**4 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 42
*a*b*c*d**5 + 35*a*c**3*d**4 + 7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c*
*4*d**3 + 7*c**6*d**2/2) + x**17*(21*a**2*c*d**5 + 21*a*b**2*d**5 + 105*a*b
*c**2*d**4 + 35*a*c**4*d**3 + 35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**
5*d**2 + c**7*d) + x**16*(21*a**2*b*d**5 + 105*a**2*c**2*d**4/2 + 105*a*b**
2*c*d**4 + 140*a*b*c**3*d**3 + 21*a*c**5*d**2 + 35*b**4*d**4/4 + 70*b**3*c*
*2*d**3 + 105*b**2*c**4*d**2/2 + 7*b*c**6*d + c**8/8) + x**15*(7*a**3*d**5
+ 105*a**2*b*c*d**4 + 70*a**2*c**3*d**3 + 35*a*b**3*d**4 + 210*a*b**2*c**2*
d**3 + 105*a*b*c**4*d**2 + 7*a*c**6*d + 35*b**4*c*d**3 + 70*b**3*c**3*d**2
+ 21*b**2*c**5*d + b*c**7) + x**14*(35*a**3*c*d**4 + 105*a**2*b**2*d**4/2 +
 210*a**2*b*c**2*d**3 + 105*a**2*c**4*d**2/2 + 140*a*b**3*c*d**3 + 210*a*b*
*2*c**3*d**2 + 42*a*b*c**5*d + a*c**7 + 7*b**5*d**3 + 105*b**4*c**2*d**2/2
+ 35*b**3*c**4*d + 7*b**2*c**6/2) + x**13*(35*a**3*b*d**4 + 70*a**3*c**2*d*
*3 + 210*a**2*b**2*c*d**3 + 210*a**2*b*c**3*d**2 + 21*a**2*c**5*d + 35*a*b*
*4*d**3 + 210*a*b**3*c**2*d**2 + 105*a*b**2*c**4*d + 7*a*b*c**6 + 21*b**5*c
*d**2 + 35*b**4*c**3*d + 7*b**3*c**5) + x**12*(35*a**4*d**4/4 + 140*a**3*b*
c*d**3 + 70*a**3*c**3*d**2 + 70*a**2*b**3*d**3 + 315*a**2*b**2*c**2*d**2 +
105*a**2*b*c**4*d + 7*a**2*c**6/2 + 105*a*b**4*c*d**2 + 140*a*b**3*c**3*d +
 21*a*b**2*c**5 + 7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*
(35*a**4*c*d**3 + 70*a**3*b**2*d**3 + 210*a**3*b*c**2*d**2 + 35*a**3*c**4*d
 + 210*a**2*b**3*c*d**2 + 210*a**2*b**2*c**3*d + 21*a**2*b*c**5 + 21*a*b**5
*d**2 + 105*a*b**4*c**2*d + 35*a*b**3*c**4 + 7*b**6*c*d + 7*b**5*c**3) + x*
*10*(35*a**4*b*d**3 + 105*a**4*c**2*d**2/2 + 210*a**3*b**2*c*d**2 + 140*a**
3*b*c**3*d + 7*a**3*c**5 + 105*a**2*b**4*d**2/2 + 210*a**2*b**3*c**2*d + 10
5*a**2*b**2*c**4/2 + 42*a*b**5*c*d + 35*a*b**4*c**3 + b**7*d + 7*b**6*c**2/
2) + x**9*(7*a**5*d**3 + 105*a**4*b*c*d**2 + 35*a**4*c**3*d + 70*a**3*b**3*
d**2 + 210*a**3*b**2*c**2*d + 35*a**3*b*c**4 + 105*a**2*b**4*c*d + 70*a**2*
b**3*c**3 + 7*a*b**6*d + 21*a*b**5*c**2 + b**7*c) + x**8*(21*a**5*c*d**2 +
105*a**4*b**2*d**2/2 + 105*a**4*b*c**2*d + 35*a**4*c**4/4 + 140*a**3*b**3*c
*d + 70*a**3*b**2*c**3 + 21*a**2*b**5*d + 105*a**2*b**4*c**2/2 + 7*a*b**6*c
 + b**8/8) + x**7*(21*a**5*b*d**2 + 21*a**5*c**2*d + 105*a**4*b**2*c*d + 35
*a**4*b*c**3 + 35*a**3*b**4*d + 70*a**3*b**3*c**2 + 21*a**2*b**5*c + a*b**7
) + x**6*(7*a**6*d**2/2 + 42*a**5*b*c*d + 7*a**5*c**3 + 35*a**4*b**3*d + 10
5*a**4*b**2*c**2/2 + 35*a**3*b**4*c + 7*a**2*b**6/2) + x**5*(7*a**6*c*d + 2
1*a**5*b**2*d + 21*a**5*b*c**2 + 35*a**4*b**3*c + 7*a**3*b**5) + x**4*(7*a*
*6*b*d + 7*a**6*c**2/2 + 21*a**5*b**2*c + 35*a**4*b**4/4) + x**3*(a**7*d +
7*a**6*b*c + 7*a**5*b**3) + x**2*(a**7*c + 7*a**6*b**2/2)
```

3.189 $\int \left(b+2 c x+3 d x^2\right) \left(b x+c x^2+d x^3\right)^7 \, dx$

Optimal. Leaf size=20

$$\frac{1}{8} \left(b x+c x^2+d x^3\right)^8$$

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1588}

$$\frac{1}{8} \left(b x+c x^2+d x^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (b*x + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+2 c x+3 d x^2\right) \left(b x+c x^2+d x^3\right)^7 \, dx=\frac{1}{8} \left(b x+c x^2+d x^3\right)^8$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\frac{1}{8} x^8 (b+x (c+d x))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b+2 c x+3 d x^2\right) \left(b x+c x^2+d x^3\right)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 0.62, size = 496, normalized size = 24.80

$$\frac{1}{8} x^{24} d^{8} + x^{23} d^{7} c + \frac{7}{2} x^{22} d^{6} c^{2} + x^{22} d^{7} b + 7 x^{21} d^{5} c^{3} + 7 x^{21} d^{6} c b + \frac{35}{4} x^{20} d^{4} c^{4} + 21 x^{20} d^{5} c^{2} b + \frac{7}{2} x^{20} d^{6} b^{2} + 7 x^{19} d^{3} c^{5} + 35 x^{19} d^{4} c^{3} b + 21 x^{19} d^{5} c b^{2} + \frac{7}{2} x^{18} d^{2} c^{6} + 35 x^{18} d^{3} c^{4} b + \frac{105}{2} x^{18} d^{4} c^{2} b^{2} + 7 x^{18} d^{5} b^{3} + x^{17} d c^{7} + 21 x^{17} d^{2} c^{5} b + 70 x^{17} d^{3} c^{3} b^{2} + 35 x^{17} d^{4} c b^{3} + \frac{1}{8} x^{16} c^{8} + 7 x^{16} d c^{6} b + \frac{105}{2} x^{16} d^{2} c^{4} b^{2} + 70 x^{16} d^{3} c^{2} b^{3} + \frac{35}{4} x^{16} d^{4} b^{4} + x^{15} c^{7} b + 21 x^{15} d c^{5} b^{2} + 70 x^{15} d^{2} c^{3} b^{3} + 35 x^{15} d^{3} c b^{4} + \frac{7}{2} x^{14} c^{6} b^{2} + 35 x^{14} d c^{4} b^{3} + \frac{105}{2} x^{14} d^{2} c^{2} b^{4} + 7 x^{14} d^{3} b^{5} + 7 x^{13} c^{5} b^{3} + 35 x^{13} d c^{3} b^{4} + 21 x^{13} d^{2} c b^{5} + \frac{35}{4} x^{12} c^{4} b^{4} + 21 x^{12} d c^{2} b^{5} + \frac{7}{2} x^{12} d^{2} b^{6} + 7 x^{11} c^{3} b^{5} + 7 x^{11} d c b^{6} + \frac{7}{2} x^{10} c^{2} b^{6} + x^{10} d b^{7} + x^{9} c b^{7} + \frac{1}{8} x^{8} b^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

giac [A] time = 0.31, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^3 + cx^2 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

maple [B] time = 0.00, size = 5596, normalized size = 279.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.63, size = 18, normalized size = 0.90

$$\frac{1}{8}\left(dx^3 + cx^2 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x+b)*(d*x^3+c*x^2+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

mupad [B] time = 2.32, size = 418, normalized size = 20.90

$$x^{14}\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{18}\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{12}\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{16}\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+\frac{b^8x^8}{8}+\frac{d^8x^{24}}{8}+x^{10}\left(db^7+\frac{7b^6c^2}{2}\right)+b^7cx^9+cd^7x^{23}+\frac{d^6x^{22}\left(7c^2+2bd\right)}{2}+7b^3cx^{13}\left(3b^2d^2+5bc^2d+c^4\right)+7cd^3x^{19}\left(3b^2d^2+5bc^2d+c^4\right)+bcx^{15}\left(35b^3d^3+70b^2c^2d^2+21bc^4d+c^6\right)+cdx^{17}\left(35b^3d^3+70b^2c^2d^2+21bc^4d+c^6\right)+7b^5cx^{11}\left(c^2+bd\right)+7cd^5x^{21}\left(c^2+bd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x + 3*d*x^2)*(b*x + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
```

```
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

sympy [B] time = 0.18, size = 469, normalized size = 23.45

$$\frac{b^8x^8}{8} + b^7cx^9 + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{22}\left(bd^7 + \frac{7c^2d^6}{2}\right) + x^{21}\left(7bcd^6 + 7c^3d^5\right) + x^{20}\left(\frac{7b^2d^6}{2} + 21bc^2d^5 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21b^2cd^5 + 35bc^3d^4 + 7c^5d^3\right) + x^{18}\left(7b^3d^5 + \frac{105b^2c^2d^4}{2} + 35bc^4d^3 + \frac{7c^6d^2}{2}\right) + x^{17}\left(35b^3cd^4 + 70b^2c^3d^3 + 21bc^5d^2 + c^7d\right) + x^{16}\left(\frac{35b^4d^4}{4} + 70b^3c^2d^3 + \frac{105b^2c^4d^2}{2} + 7bc^6d + \frac{c^8}{8}\right) + x^{15}\left(35b^4cd^3 + 70b^3c^3d^2 + 21b^2c^5d + bc^7\right) + x^{14}\left(7b^5d^3 + \frac{105b^4c^2d^2}{2} + 35b^3c^4d + \frac{7b^2c^6}{2}\right) + x^{13}\left(21b^5cd^2 + 35b^4c^3d + 7b^3c^5\right) + x^{12}\left(\frac{7b^6d^2}{2} + 21b^5c^2d + \frac{35b^4c^4}{4}\right) + x^{11}\left(7b^6cd + 7b^5c^3\right) + x^{10}\left(b^7d + \frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x+b)*(d*x**3+c*x**2+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.190 $\int x^7 \left(b+cx+dx^2\right)^7 \left(b+2cx+3dx^2\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1588}

$$\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + c*x + d*x^2)^8)/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx=\frac{1}{8}x^8\left(b+cx+dx^2\right)^8$$

Mathematica [A] time = 0.02, size = 18, normalized size = 0.95

$$\frac{1}{8}x^8(b+x(c+dx))^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] (x^8*(b + x*(c + d*x))^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(b+cx+dx^2\right)^7\left(b+2cx+3dx^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2), x]
```

fricas [B] time = 0.80, size = 496, normalized size = 26.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + x^22*d^7*b + 7*x^21*d^5*c^3
+ 7*x^21*d^6*c*b + 35/4*x^20*d^4*c^4 + 21*x^20*d^5*c^2*b + 7/2*x^20*d^6*b^2
 + 7*x^19*d^3*c^5 + 35*x^19*d^4*c^3*b + 21*x^19*d^5*c*b^2 + 7/2*x^18*d^2*c^
6 + 35*x^18*d^3*c^4*b + 105/2*x^18*d^4*c^2*b^2 + 7*x^18*d^5*b^3 + x^17*d*c^
7 + 21*x^17*d^2*c^5*b + 70*x^17*d^3*c^3*b^2 + 35*x^17*d^4*c*b^3 + 1/8*x^16*
c^8 + 7*x^16*d*c^6*b + 105/2*x^16*d^2*c^4*b^2 + 70*x^16*d^3*c^2*b^3 + 35/4*
x^16*d^4*b^4 + x^15*c^7*b + 21*x^15*d*c^5*b^2 + 70*x^15*d^2*c^3*b^3 + 35*x^
15*d^3*c*b^4 + 7/2*x^14*c^6*b^2 + 35*x^14*d*c^4*b^3 + 105/2*x^14*d^2*c^2*b^
4 + 7*x^14*d^3*b^5 + 7*x^13*c^5*b^3 + 35*x^13*d*c^3*b^4 + 21*x^13*d^2*c*b^5
 + 35/4*x^12*c^4*b^4 + 21*x^12*d*c^2*b^5 + 7/2*x^12*d^2*b^6 + 7*x^11*c^3*b^
5 + 7*x^11*d*c*b^6 + 7/2*x^10*c^2*b^6 + x^10*d*b^7 + x^9*c*b^7 + 1/8*x^8*b^
8
```

giac [A] time = 0.30, size = 18, normalized size = 0.95

$$\frac{1}{8}\left(dx^3 + cx^2 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + b*x)^8
```

maple [B] time = 0.00, size = 5596, normalized size = 294.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x)
```

```
[Out] result too large to display
```

maxima [B] time = 0.60, size = 441, normalized size = 23.21

$$\frac{1}{8} d^8 x^{24} + c d^7 x^{23} + \frac{1}{2} \left(7 c^2 d^6 + 2 b d^7\right) x^{22} + 7 \left(c^3 d^5 + b c d^6\right) x^{21} + \frac{7}{4} \left(5 c^4 d^4 + 12 b c^2 d^5 + 2 b^2 d^6\right) x^{20} + 7 \left(c^5 d^3 + 5 b c^3 d^4 + 3 b^2 c d^5\right) x^{19} + \frac{7}{2} \left(c^6 d^2 + 10 b c^4 d^3 + 15 b^2 c^2 d^4 + 2 b^3 d^5\right) x^{18} + \left(c^7 d + 21 b c^5 d^2 + 70 b^2 c^3 d^3 + 35 b^3 c d^4\right) x^{17} + b^7 c x^9 + \frac{1}{8} \left(c^8 + 56 b c^6 d + 420 b^2 c^4 d^2 + 560 b^3 c^2 d^3 + 70 b^4 d^4\right) x^{16} + \frac{1}{8} b^8 x^8 + \left(b c^7 + 21 b^2 c^5 d + 70 b^3 c^3 d^2 + 35 b^4 c d^3\right) x^{15} + \frac{7}{2} \left(b^2 c^6 + 10 b^3 c^4 d + 15 b^4 c^2 d^2 + 2 b^5 d^3\right) x^{14} + 7 \left(b^3 c^5 + 5 b^4 c^3 d + 3 b^5 c d^2\right) x^{13} + \frac{7}{4} \left(5 b^4 c^4 + 12 b^5 c^2 d + 2 b^6 d^2\right) x^{12} + 7 \left(b^5 c^3 + b^6 c d\right) x^{11} + \frac{1}{2} \left(7 b^6 c^2 + 2 b^7 d\right) x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x+b)^7*(3*d*x^2+2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 1/2*(7*c^2*d^6 + 2*b*d^7)*x^22 + 7*(c^3*d^5 + b
*c*d^6)*x^21 + 7/4*(5*c^4*d^4 + 12*b*c^2*d^5 + 2*b^2*d^6)*x^20 + 7*(c^5*d^3
 + 5*b*c^3*d^4 + 3*b^2*c*d^5)*x^19 + 7/2*(c^6*d^2 + 10*b*c^4*d^3 + 15*b^2*c
^2*d^4 + 2*b^3*d^5)*x^18 + (c^7*d + 21*b*c^5*d^2 + 70*b^2*c^3*d^3 + 35*b^3*
c*d^4)*x^17 + b^7*c*x^9 + 1/8*(c^8 + 56*b*c^6*d + 420*b^2*c^4*d^2 + 560*b^3
*c^2*d^3 + 70*b^4*d^4)*x^16 + 1/8*b^8*x^8 + (b*c^7 + 21*b^2*c^5*d + 70*b^3*
c^3*d^2 + 35*b^4*c*d^3)*x^15 + 7/2*(b^2*c^6 + 10*b^3*c^4*d + 15*b^4*c^2*d^2
 + 2*b^5*d^3)*x^14 + 7*(b^3*c^5 + 5*b^4*c^3*d + 3*b^5*c*d^2)*x^13 + 7/4*(5*
b^4*c^4 + 12*b^5*c^2*d + 2*b^6*d^2)*x^12 + 7*(b^5*c^3 + b^6*c*d)*x^11 + 1/2
*(7*b^6*c^2 + 2*b^7*d)*x^10
```

mupad [B] time = 2.27, size = 418, normalized size = 22.00

$$x^{14} \left(7 b^5 d^3 + \frac{105 b^4 c^2 d^2}{2} + 35 b^3 c^4 d + \frac{7 b^2 c^6}{2}\right) + x^{18} \left(7 b^3 d^5 + \frac{105 b^2 c^2 d^4}{2} + 35 b c^4 d^3 + \frac{7 c^6 d^2}{2}\right) + x^{12} \left(\frac{7 b^6 d^2}{2} + 21 b^5 c^2 d + \frac{35 b^4 c^4}{4}\right) + x^{20} \left(\frac{7 b^2 d^6}{2} + 21 b c^2 d^5 + \frac{35 c^4 d^4}{4}\right) + x^{16} \left(\frac{35 b^4 d^4}{4} + 70 b^3 c^2 d^3 + \frac{105 b^2 c^4 d^2}{2} + 7 b c^6 d + \frac{c^8}{8}\right) + \frac{b^8 x^8}{8} + \frac{d^8 x^{24}}{8} + x^{10} \left(d b^7 + \frac{7 b^6 c^2}{2}\right) + b^7 c x^9 + c d^7 x^{23} + \frac{d^6 x^{22} \left(7 c^2 + 2 b d\right)}{2} + 7 b^3 c x^{13} \left(3 b^2 d^2 + 5 b c^2 d + c^4\right) + 7 c d^3 x^{19} \left(3 b^2 d^2 + 5 b c^2 d + c^4\right) + b c x^{15} \left(35 b^3 d^3 + 70 b^2 c^2 d^2 + 21 b c^4 d + c^6\right) + c d x^{17} \left(35 b^3 d^3 + 70 b^2 c^2 d^2 + 21 b c^4 d + c^6\right) + 7 b^5 c x^{11} \left(c^2 + b d\right) + 7 c d^5 x^{21} \left(c^2 + b d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + c*x + d*x^2)^7*(b + 2*c*x + 3*d*x^2),x)
```

```
[Out] x^14*((7*b^2*c^6)/2 + 7*b^5*d^3 + 35*b^3*c^4*d + (105*b^4*c^2*d^2)/2) + x^1
8*(7*b^3*d^5 + (7*c^6*d^2)/2 + 35*b*c^4*d^3 + (105*b^2*c^2*d^4)/2) + x^12*(
(35*b^4*c^4)/4 + (7*b^6*d^2)/2 + 21*b^5*c^2*d) + x^20*((7*b^2*d^6)/2 + (35*
c^4*d^4)/4 + 21*b*c^2*d^5) + x^16*(c^8/8 + (35*b^4*d^4)/4 + (105*b^2*c^4*d^
2)/2 + 70*b^3*c^2*d^3 + 7*b*c^6*d) + (b^8*x^8)/8 + (d^8*x^24)/8 + x^10*(b^7
*d + (7*b^6*c^2)/2) + b^7*c*x^9 + c*d^7*x^23 + (d^6*x^22*(2*b*d + 7*c^2))/2
 + 7*b^3*c*x^13*(c^4 + 3*b^2*d^2 + 5*b*c^2*d) + 7*c*d^3*x^19*(c^4 + 3*b^2*d
^2 + 5*b*c^2*d) + b*c*x^15*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d)
 + c*d*x^17*(c^6 + 35*b^3*d^3 + 70*b^2*c^2*d^2 + 21*b*c^4*d) + 7*b^5*c*x^11
*(b*d + c^2) + 7*c*d^5*x^21*(b*d + c^2)
```

sympy [B] time = 0.17, size = 469, normalized size = 24.68

$$\frac{b^8x^8}{8}+b^7cx^9+cd^7x^{23}+\frac{d^8x^{24}}{8}+x^{22}\left(bd^7+\frac{7c^2d^6}{2}\right)+x^{21}\left(7bcd^6+7c^3d^5\right)+x^{20}\left(\frac{7b^2d^6}{2}+21bc^2d^5+\frac{35c^4d^4}{4}\right)+x^{19}\left(21b^2cd^5+35bc^3d^4+7c^5d^3\right)+x^{18}\left(7b^3d^5+\frac{105b^2c^2d^4}{2}+35bc^4d^3+\frac{7c^6d^2}{2}\right)+x^{17}\left(35b^3cd^4+70b^2c^3d^3+21bc^5d^2+c^7d\right)+x^{16}\left(\frac{35b^4d^4}{4}+70b^3c^2d^3+\frac{105b^2c^4d^2}{2}+7bc^6d+\frac{c^8}{8}\right)+x^{15}\left(35b^4cd^3+70b^3c^3d^2+21b^2c^5d+bc^7\right)+x^{14}\left(7b^5d^3+\frac{105b^4c^2d^2}{2}+35b^3c^4d+\frac{7b^2c^6}{2}\right)+x^{13}\left(21b^5cd^2+35b^4c^3d+7b^3c^5\right)+x^{12}\left(\frac{7b^6d^2}{2}+21b^5c^2d+\frac{35b^4c^4}{4}\right)+x^{11}\left(7b^6cd+7b^5c^3\right)+x^{10}\left(b^7d+\frac{7b^6c^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x+b)**7*(3*d*x**2+2*c*x+b),x)
```

```
[Out] b**8*x**8/8 + b**7*c*x**9 + c*d**7*x**23 + d**8*x**24/8 + x**22*(b*d**7 + 7
*c**2*d**6/2) + x**21*(7*b*c*d**6 + 7*c**3*d**5) + x**20*(7*b**2*d**6/2 + 2
1*b*c**2*d**5 + 35*c**4*d**4/4) + x**19*(21*b**2*c*d**5 + 35*b*c**3*d**4 +
7*c**5*d**3) + x**18*(7*b**3*d**5 + 105*b**2*c**2*d**4/2 + 35*b*c**4*d**3 +
 7*c**6*d**2/2) + x**17*(35*b**3*c*d**4 + 70*b**2*c**3*d**3 + 21*b*c**5*d**
2 + c**7*d) + x**16*(35*b**4*d**4/4 + 70*b**3*c**2*d**3 + 105*b**2*c**4*d**
2/2 + 7*b*c**6*d + c**8/8) + x**15*(35*b**4*c*d**3 + 70*b**3*c**3*d**2 + 21
*b**2*c**5*d + b*c**7) + x**14*(7*b**5*d**3 + 105*b**4*c**2*d**2/2 + 35*b**
3*c**4*d + 7*b**2*c**6/2) + x**13*(21*b**5*c*d**2 + 35*b**4*c**3*d + 7*b**3
*c**5) + x**12*(7*b**6*d**2/2 + 21*b**5*c**2*d + 35*b**4*c**4/4) + x**11*(7
*b**6*c*d + 7*b**5*c**3) + x**10*(b**7*d + 7*b**6*c**2/2)
```

3.191 $\int \left(b+3 d x^2\right) \left(a+b x+d x^3\right)^7 \, dx$

Optimal. Leaf size=16

$$\frac{1}{8} \left(a+b x+d x^3\right)^8$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1588}

$$\frac{1}{8} \left(a+b x+d x^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (a + b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3 d x^2\right) \left(a+b x+d x^3\right)^7 \, dx=\frac{1}{8} \left(a+b x+d x^3\right)^8$$

Mathematica [B] time = 0.06, size = 127, normalized size = 7.94

$$\frac{1}{8} x \left(b+d x^2\right) \left(8 a^7+28 a^6 x \left(b+d x^2\right)+56 a^5 x^2 \left(b+d x^2\right)^2+70 a^4 x^3 \left(b+d x^2\right)^3+56 a^3 x^4 \left(b+d x^2\right)^4+28 a^2 x^5 \left(b+d x^2\right)^5+8 a x^6 \left(b+d x^2\right)^6+x^7 \left(b+d x^2\right)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] (x*(b + d*x^2)*(8*a^7 + 28*a^6*x*(b + d*x^2) + 56*a^5*x^2*(b + d*x^2)^2 + 7
0*a^4*x^3*(b + d*x^2)^3 + 56*a^3*x^4*(b + d*x^2)^4 + 28*a^2*x^5*(b + d*x^2)
^5 + 8*a*x^6*(b + d*x^2)^6 + x^7*(b + d*x^2)^7))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b+3 d x^2\right) \left(a+b x+d x^3\right)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(b + 3*d*x^2)*(a + b*x + d*x^3)^7, x]
```

fricas [B] time = 0.94, size = 486, normalized size = 30.38

$$\frac{1}{8} x^{24} d^{8} + x^{22} d^{7} b + x^{21} d^{7} a + \frac{7}{2} x^{20} d^{6} b^{2} + 7 x^{19} d^{6} b a + 7 x^{18} d^{5} b^{3} + \frac{7}{2} x^{18} d^{6} a^{2} + 21 x^{17} d^{5} b^{2} a + \frac{35}{4} x^{16} d^{4} b^{4} + 21 x^{16} d^{5} b a^{2} + 35 x^{15} d^{4} b^{3} a + 7 x^{15} d^{5} a^{3} + 7 x^{14} d^{3} b^{5} + \frac{105}{2} x^{14} d^{4} b^{2} a^{2} + 35 x^{13} d^{3} b^{4} a + 35 x^{13} d^{4} b a^{3} + \frac{7}{2} x^{12} d^{2} b^{6} + 70 x^{12} d^{3} b^{3} a^{2} + \frac{35}{4} x^{12} d^{4} a^{4} + 21 x^{11} d^{2} b^{5} a + 70 x^{11} d^{3} b^{2} a^{3} + x^{10} d b^{7} + \frac{105}{2} x^{10} d^{2} b^{4} a^{2} + 35 x^{10} d^{3} b a^{4} + 7 x^{9} d b^{6} a + 70 x^{9} d^{2} b^{3} a^{3} + 7 x^{9} d^{3} a^{5} + \frac{1}{8} x^{8} b^{8} + 21 x^{8} d b^{5} a^{2} + \frac{105}{2} x^{8} d^{2} b^{2} a^{4} + x^{7} b^{7} a + 35 x^{7} d b^{4} a^{3} + 21 x^{7} d^{2} b a^{5} + \frac{7}{2} x^{6} b^{6} a^{2} + 35 x^{6} d b^{3} a^{4} + \frac{7}{2} x^{6} d^{2} a^{6} + 7 x^{5} b^{5} a^{3} + 21 x^{5} d b^{2} a^{5} + \frac{35}{4} x^{4} b^{4} a^{4} + 7 x^{4} d b a^{6} + 7 x^{3} b^{3} a^{5} + x^{3} d a^{7} + \frac{7}{2} x^{2} b^{2} a^{6} + x b a^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + x^21*d^7*a + 7/2*x^20*d^6*b^2 + 7*x^19*d^6*b*a
+ 7*x^18*d^5*b^3 + 7/2*x^18*d^6*a^2 + 21*x^17*d^5*b^2*a + 35/4*x^16*d^4*b^4
 + 21*x^16*d^5*b*a^2 + 35*x^15*d^4*b^3*a + 7*x^15*d^5*a^3 + 7*x^14*d^3*b^5
+ 105/2*x^14*d^4*b^2*a^2 + 35*x^13*d^3*b^4*a + 35*x^13*d^4*b*a^3 + 7/2*x^12
*d^2*b^6 + 70*x^12*d^3*b^3*a^2 + 35/4*x^12*d^4*a^4 + 21*x^11*d^2*b^5*a + 70
*x^11*d^3*b^2*a^3 + x^10*d*b^7 + 105/2*x^10*d^2*b^4*a^2 + 35*x^10*d^3*b*a^4
 + 7*x^9*d*b^6*a + 70*x^9*d^2*b^3*a^3 + 7*x^9*d^3*a^5 + 1/8*x^8*b^8 + 21*x^
8*d*b^5*a^2 + 105/2*x^8*d^2*b^2*a^4 + x^7*b^7*a + 35*x^7*d*b^4*a^3 + 21*x^7
*d^2*b*a^5 + 7/2*x^6*b^6*a^2 + 35*x^6*d*b^3*a^4 + 7/2*x^6*d^2*a^6 + 7*x^5*b
^5*a^3 + 21*x^5*d*b^2*a^5 + 35/4*x^4*b^4*a^4 + 7*x^4*d*b*a^6 + 7*x^3*b^3*a^
5 + x^3*d*a^7 + 7/2*x^2*b^2*a^6 + x*b*a^7
```

giac [B] time = 0.41, size = 120, normalized size = 7.50

$$\frac{1}{8}\left(dx^3+bx\right)^8+\left(dx^3+bx\right)^7a+\frac{7}{2}\left(dx^3+bx\right)^6a^2+7\left(dx^3+bx\right)^5a^3+\frac{35}{4}\left(dx^3+bx\right)^4a^4+7\left(dx^3+bx\right)^3a^5+\frac{7}{2}\left(dx^3+bx\right)^2a^6+\left(dx^3+bx\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8 + (d*x^3 + b*x)^7*a + 7/2*(d*x^3 + b*x)^6*a^2 + 7*(d*x^
3 + b*x)^5*a^3 + 35/4*(d*x^3 + b*x)^4*a^4 + 7*(d*x^3 + b*x)^3*a^5 + 7/2*(d*
x^3 + b*x)^2*a^6 + (d*x^3 + b*x)*a^7
```

maple [B] time = 0.00, size = 2185, normalized size = 136.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+d^7*a*x^21+7/2*b^2*d^6*x^20+7*b*a*d^6*x^19+1/18*(21
*b^3*d^5+3*d*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*
d^4)))*x^18+21*b^2*a*d^5*x^17+1/16*(b*(6*a^2*d^5+15*b^3*d^4+d*(2*(3*a^2*d+b
^3)*d^3+18*b^3*d^3+9*a^2*d^4))+3*d*(30*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*
b^3*d^3+9*a^2*d^4)+d*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)))*x^16+
1/15*(105*b^3*a*d^4+3*d*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^
3*a*d^3+d*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)))*x^15+1/14*(b*(30
*a^2*d^4*b+b*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+d*(42*b*a^2*d^3+6*(
3*a^2*d+b^3)*b*d^2+9*d^2*b^4))+3*d*(60*a^2*b^2*d^3+b*(42*b*a^2*d^3+6*(3*a^2
*d+b^3)*b*d^2+9*d^2*b^4)+d*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)))*x^14+1/
13*(b*(a*(2*(3*a^2*d+b^3)*d^3+18*b^3*d^3+9*a^2*d^4)+60*b^3*a*d^3+d*(2*a^3*d
^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2))+3*d*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3
)*b*d^2+9*d^2*b^4)+b*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a
^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)))*x^13+1/12*(b*(60*a^2*b^2*d^3+
b*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+d*(72*b^2*a^2*d^2+6*(3*a^2
*d+b^3)*d*b^2))+3*d*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(72
*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)
^2)))*x^12+1/11*(b*(a*(42*b*a^2*d^3+6*(3*a^2*d+b^3)*b*d^2+9*d^2*b^4)+b*(2*a
^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+d*(24*a^3*b*d^2+18*b^4*a*d+12*(3
*a^2*d+b^3)*d*a*b))+3*d*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+b*(24*a^3
*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+
b^3))))*x^11+1/10*(b*(a*(2*a^3*d^3+54*b^3*a*d^2+6*(3*a^2*d+b^3)*a*d^2)+b*(7
2*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+d*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3
)^2))+3*d*(a*(24*a^3*b*d^2+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+
54*b^3*a^2*d+(3*a^2*d+b^3)^2)+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2
)))*x^10+1/9*(b*(a*(72*b^2*a^2*d^2+6*(3*a^2*d+b^3)*d*b^2)+b*(24*a^3*b*d^2+1
8*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+d*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3)))+3
*d*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*b^2+6*b^2*a*(3*a
^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)))*x^9+1/8*(b*(a*(24*a^3*b*d^2
```

```
+18*b^4*a*d+12*(3*a^2*d+b^3)*d*a*b)+b*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)
^2)+d*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2))+3*d*(a*(42*a^3*d*b^2+6*
b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^
2*a^4))*x^8+1/7*(b*(a*(6*a^4*d^2+54*b^3*a^2*d+(3*a^2*d+b^3)^2)+b*(42*a^3*d*
b^2+6*b^2*a*(3*a^2*d+b^3))+d*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3))+3*d*(a*(12*a
^4*d*b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+
6*d*a^5*b))*x^7+1/6*(b*(a*(42*a^3*d*b^2+6*b^2*a*(3*a^2*d+b^3))+b*(12*a^4*d*
b+6*b*a^2*(3*a^2*d+b^3)+9*b^4*a^2)+15*d*b^2*a^4)+3*d*(a*(2*a^3*(3*a^2*d+b^3
)+18*b^3*a^3)+15*b^3*a^4+d*a^6))*x^6+1/5*(b*(a*(12*a^4*d*b+6*b*a^2*(3*a^2*d
+b^3)+9*b^4*a^2)+b*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+6*d*a^5*b)+63*d*b^2*a^5
)*x^5+1/4*(b*(a*(2*a^3*(3*a^2*d+b^3)+18*b^3*a^3)+15*b^3*a^4+d*a^6)+21*d*a^6
*b)*x^4+1/3*(3*a^7*d+21*a^5*b^3)*x^3+7/2*b^2*a^6*x^2+b*a^7*x
```

maxima [A] time = 0.55, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(dx^3 + bx + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x + a)^8
```

mupad [B] time = 2.63, size = 438, normalized size = 27.38

$$x^{12}\left(\frac{35a^4d^4}{4}+70a^2b^3d^3+\frac{7b^6d^2}{2}\right)+x^4\left(7da^6b+\frac{35a^4b^4}{4}\right)+x^{18}\left(\frac{7a^2d^6}{2}+7b^3d^5\right)+x^6\left(\frac{7a^6d^2}{2}+35a^4b^3d+\frac{7a^2b^6}{2}\right)+x^8\left(\frac{105a^4b^2d^2}{2}+21a^2b^5d+\frac{b^8}{8}\right)+\frac{d^8x^{24}}{8}+x^3\left(da^7+7a^5b^3\right)+ad^7x^{21}+bd^7x^{22}+\frac{7a^6b^2x^2}{2}+\frac{7b^2d^6x^{20}}{2}+a^7bx+21ab^2d^5x^{17}+abx^7\left(21a^4d^2+35a^2b^3d+b^6\right)+7adx^9\left(b^6+a^4d^2+10a^2b^3d\right)+7a^3b^2x^5\left(3a^2d+b^3\right)+7ad^4x^{15}\left(a^2d+5b^3\right)+\frac{7bd^4x^{16}\left(12a^2d+5b^3\right)}{4}+\frac{bdx^{10}\left(2b^6+70a^4d^2+105a^2b^3d\right)}{2}+7abd^6x^{19}+\frac{7b^2d^3x^{14}\left(15a^2d+2b^3\right)}{2}+7ab^2d^2x^{11}\left(10a^2d+3b^3\right)+35abd^3x^{13}\left(a^2d+b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 3*d*x^2)*(a + b*x + d*x^3)^7,x)
```

```
[Out] x^12*((35*a^4*d^4)/4 + (7*b^6*d^2)/2 + 70*a^2*b^3*d^3) + x^4*((35*a^4*b^4)/
4 + 7*a^6*b*d) + x^18*((7*a^2*d^6)/2 + 7*b^3*d^5) + x^6*((7*a^2*b^6)/2 + (7
*a^6*d^2)/2 + 35*a^4*b^3*d) + x^8*(b^8/8 + 21*a^2*b^5*d + (105*a^4*b^2*d^2)
/2) + (d^8*x^24)/8 + x^3*(a^7*d + 7*a^5*b^3) + a*d^7*x^21 + b*d^7*x^22 + (7
*a^6*b^2*x^2)/2 + (7*b^2*d^6*x^20)/2 + a^7*b*x + 21*a*b^2*d^5*x^17 + a*b*x^
7*(b^6 + 21*a^4*d^2 + 35*a^2*b^3*d) + 7*a*d*x^9*(b^6 + a^4*d^2 + 10*a^2*b^3
*d) + 7*a^3*b^2*x^5*(3*a^2*d + b^3) + 7*a*d^4*x^15*(a^2*d + 5*b^3) + (7*b*d
^4*x^16*(12*a^2*d + 5*b^3))/4 + (b*d*x^10*(2*b^6 + 70*a^4*d^2 + 105*a^2*b^3
*d))/2 + 7*a*b*d^6*x^19 + (7*b^2*d^3*x^14*(15*a^2*d + 2*b^3))/2 + 7*a*b^2*d
^2*x^11*(10*a^2*d + 3*b^3) + 35*a*b*d^3*x^13*(a^2*d + b^3)
```

sympy [B] time = 0.17, size = 483, normalized size = 30.19

$$a^7bx+\frac{7a^6b^2x^2}{2}+21ab^2d^5x^{17}+7abd^6x^{19}+ad^7x^{21}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}+x^{18}\left(\frac{7a^2d^6}{2}+7b^3d^5\right)+x^{16}\left(21a^2bd^5+\frac{35b^4d^4}{4}\right)+x^{15}\left(7a^3d^5+35ab^3d^4\right)+x^{14}\left(\frac{105a^2b^2d^4}{2}+7b^5d^3\right)+x^{13}\left(35a^3bd^4+35ab^4d^3\right)+x^{12}\left(\frac{35a^4d^4}{4}+70a^2b^3d^3+\frac{7b^6d^2}{2}\right)+x^{11}\left(70a^3b^2d^3+21ab^5d^2\right)+x^{10}\left(35a^4bd^3+\frac{105a^2b^4d^2}{2}+b^7d\right)+x^9\left(7a^5d^3+70a^3b^3d^2+7ab^6d\right)+x^8\left(\frac{105a^4b^2d^2}{2}+21a^2b^5d+\frac{b^8}{8}\right)+x^7\left(21a^5bd^2+35a^3b^4d+ab^7\right)+x^6\left(\frac{7a^6d^2}{2}+35a^4b^3d+\frac{7a^2b^6}{2}\right)+x^5\left(21a^5b^2d+7a^3b^5\right)+x^4\left(7a^6bd+\frac{35a^4b^4}{4}\right)+x^3\left(a^7d+7a^5b^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x+a)**7,x)
```

```
[Out] a**7*b*x + 7*a**6*b**2*x**2/2 + 21*a*b**2*d**5*x**17 + 7*a*b*d**6*x**19 + a
*d**7*x**21 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 + d**8*x**24/8 + x**18*(7*
a**2*d**6/2 + 7*b**3*d**5) + x**16*(21*a**2*b*d**5 + 35*b**4*d**4/4) + x**1
5*(7*a**3*d**5 + 35*a*b**3*d**4) + x**14*(105*a**2*b**2*d**4/2 + 7*b**5*d**
3) + x**13*(35*a**3*b*d**4 + 35*a*b**4*d**3) + x**12*(35*a**4*d**4/4 + 70*a
**2*b**3*d**3 + 7*b**6*d**2/2) + x**11*(70*a**3*b**2*d**3 + 21*a*b**5*d**2)
 + x**10*(35*a**4*b*d**3 + 105*a**2*b**4*d**2/2 + b**7*d) + x**9*(7*a**5*d*
*3 + 70*a**3*b**3*d**2 + 7*a*b**6*d) + x**8*(105*a**4*b**2*d**2/2 + 21*a**2
*b**5*d + b**8/8) + x**7*(21*a**5*b*d**2 + 35*a**3*b**4*d + a*b**7) + x**6*
(7*a**6*d**2/2 + 35*a**4*b**3*d + 7*a**2*b**6/2) + x**5*(21*a**5*b**2*d + 7
*a**3*b**5) + x**4*(7*a**6*b*d + 35*a**4*b**4/4) + x**3*(a**7*d + 7*a**5*b*
*3)
```

3.192 $\int (b+3dx^2)(bx+dx^3)^7\,dx$

Optimal. Leaf size=15

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1588}

$$\frac{1}{8}\left(bx+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b*x + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx=\frac{1}{8}\left(bx+dx^3\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.53

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b+3dx^2\right)\left(bx+dx^3\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(b + 3*d*x^2)*(b*x + d*x^3)^7, x]
```

fricas [B] time = 0.71, size = 88, normalized size = 5.87

$$\frac{1}{8}x^{24}d^8+x^{22}d^7b+\frac{7}{2}x^{20}d^6b^2+7x^{18}d^5b^3+\frac{35}{4}x^{16}d^4b^4+7x^{14}d^3b^5+\frac{7}{2}x^{12}d^2b^6+x^{10}db^7+\frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.93

$$\frac{1}{8}d^8x^{24}+b\,d^7x^{22}+\frac{7}{2}b^2d^6x^{20}+7b^3d^5x^{18}+\frac{35}{4}b^4d^4x^{16}+7b^5d^3x^{14}+\frac{7}{2}b^6d^2x^{12}+b^7d\,x^{10}+\frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+b)*(d*x^3+b*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+d*b^7*x^10+1/8*b^8*x^8
```

maxima [A] time = 0.51, size = 13, normalized size = 0.87

$$\frac{1}{8}\left(dx^3+bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+b)*(d*x^3+b*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

mupad [B] time = 0.05, size = 88, normalized size = 5.87

$$\frac{b^8\,x^8}{8}+b^7\,d\,x^{10}+\frac{7\,b^6\,d^2\,x^{12}}{2}+7\,b^5\,d^3\,x^{14}+\frac{35\,b^4\,d^4\,x^{16}}{4}+7\,b^3\,d^5\,x^{18}+\frac{7\,b^2\,d^6\,x^{20}}{2}+b\,d^7\,x^{22}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + d*x^3)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

sympy [B] time = 0.10, size = 97, normalized size = 6.47

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7b^6d^2x^{12}}{2}+7b^5d^3x^{14}+\frac{35b^4d^4x^{16}}{4}+7b^3d^5x^{18}+\frac{7b^2d^6x^{20}}{2}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+b)*(d*x**3+b*x)**7,x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

3.193 $\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx$

Optimal. Leaf size=16

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{8}x^8 \left(b+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (x^8*(b + d*x^2)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx &= \frac{1}{2}\text{Subst}\left(\int x^3(b+dx)^7(b+3dx) \, dx, x, x^2\right)\\ &= \frac{1}{8}x^8 \left(b+dx^2\right)^8\end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 6.12

$$\frac{b^8x^8}{8}+b^7dx^{10}+\frac{7}{2}b^6d^2x^{12}+7b^5d^3x^{14}+\frac{35}{4}b^4d^4x^{16}+7b^3d^5x^{18}+\frac{7}{2}b^2d^6x^{20}+bd^7x^{22}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] (b^8*x^8)/8 + b^7*d*x^10 + (7*b^6*d^2*x^12)/2 + 7*b^5*d^3*x^14 + (35*b^4*d^
4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2 + b*d^7*x^22 + (d^8*x^24)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(b+dx^2\right)^7 \left(b+3dx^2\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(b + d*x^2)^7*(b + 3*d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^7*(b + d*x^2)^7*(b + 3*d*x^2), x]
```

fricas [B] time = 0.80, size = 88, normalized size = 5.50

$$\frac{1}{8}x^{24}d^8 + x^{22}d^7b + \frac{7}{2}x^{20}d^6b^2 + 7x^{18}d^5b^3 + \frac{35}{4}x^{16}d^4b^4 + 7x^{14}d^3b^5 + \frac{7}{2}x^{12}d^2b^6 + x^{10}db^7 + \frac{1}{8}x^8b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^22*d^7*b + 7/2*x^20*d^6*b^2 + 7*x^18*d^5*b^3 + 35/4*x^16*d
^4*b^4 + 7*x^14*d^3*b^5 + 7/2*x^12*d^2*b^6 + x^10*d*b^7 + 1/8*x^8*b^8
```

giac [A] time = 0.27, size = 13, normalized size = 0.81

$$\frac{1}{8}\left(dx^3 + bx\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + b*x)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.56

$$\frac{1}{8}d^8x^{24} + b\,d^7x^{22} + \frac{7}{2}b^2d^6x^{20} + 7b^3d^5x^{18} + \frac{35}{4}b^4d^4x^{16} + 7b^5d^3x^{14} + \frac{7}{2}b^6d^2x^{12} + b^7d\,x^{10} + \frac{1}{8}b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+b)^7*(3*d*x^2+b),x)
```

```
[Out] 1/8*d^8*x^24+b*d^7*x^22+7/2*b^2*d^6*x^20+7*b^3*d^5*x^18+35/4*b^4*d^4*x^16+7
*b^5*d^3*x^14+7/2*b^6*d^2*x^12+b^7*d*x^10+1/8*b^8*x^8
```

maxima [B] time = 0.64, size = 88, normalized size = 5.50

$$\frac{1}{8}\,d^8x^{24} + bd^7x^{22} + \frac{7}{2}\,b^2d^6x^{20} + 7\,b^3d^5x^{18} + \frac{35}{4}\,b^4d^4x^{16} + 7\,b^5d^3x^{14} + \frac{7}{2}\,b^6d^2x^{12} + b^7dx^{10} + \frac{1}{8}\,b^8x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+b)^7*(3*d*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + b*d^7*x^22 + 7/2*b^2*d^6*x^20 + 7*b^3*d^5*x^18 + 35/4*b^4*d^
4*x^16 + 7*b^5*d^3*x^14 + 7/2*b^6*d^2*x^12 + b^7*d*x^10 + 1/8*b^8*x^8
```

mupad [B] time = 0.04, size = 88, normalized size = 5.50

$$\frac{b^8\,x^8}{8} + b^7\,d\,x^{10} + \frac{7\,b^6\,d^2\,x^{12}}{2} + 7\,b^5\,d^3\,x^{14} + \frac{35\,b^4\,d^4\,x^{16}}{4} + 7\,b^3\,d^5\,x^{18} + \frac{7\,b^2\,d^6\,x^{20}}{2} + b\,d^7\,x^{22} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b + d*x^2)^7*(b + 3*d*x^2),x)
```

```
[Out] (b^8*x^8)/8 + (d^8*x^24)/8 + b^7*d*x^10 + b*d^7*x^22 + (7*b^6*d^2*x^12)/2 +
 7*b^5*d^3*x^14 + (35*b^4*d^4*x^16)/4 + 7*b^3*d^5*x^18 + (7*b^2*d^6*x^20)/2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.06

$$\frac{b^8x^8}{8} + b^7dx^{10} + \frac{7b^6d^2x^{12}}{2} + 7b^5d^3x^{14} + \frac{35b^4d^4x^{16}}{4} + 7b^3d^5x^{18} + \frac{7b^2d^6x^{20}}{2} + bd^7x^{22} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+b)**7*(3*d*x**2+b),x)
```

```
[Out] b**8*x**8/8 + b**7*d*x**10 + 7*b**6*d**2*x**12/2 + 7*b**5*d**3*x**14 + 35*b
**4*d**4*x**16/4 + 7*b**3*d**5*x**18 + 7*b**2*d**6*x**20/2 + b*d**7*x**22 +
 d**8*x**24/8
```

3.194 $\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.05, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2cx+3dx^2\right)\left(a+cx^2+dx^3\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 0.64, size = 488, normalized size = 27.11

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+x^{21}d^7a+\frac{35}{4}x^{20}d^4c^4+7x^{20}d^6ca+7x^{19}d^3c^5+21x^{19}d^5c^2a+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^4c^3a+\frac{7}{2}x^{18}d^6a^2+x^{17}dc^7+35x^{17}d^3c^4a+21x^{17}d^5ca^2+\frac{1}{8}x^{16}c^8+21x^{16}d^2c^5a+\frac{105}{2}x^{16}d^4c^2a^2+7x^{15}dc^6a+70x^{15}d^3c^3a^2+7x^{15}d^5a^3+x^{14}c^7a+\frac{105}{2}x^{14}d^2c^4a^2+35x^{14}d^4ca^3+21x^{13}dc^5a^2+70x^{13}d^3c^2a^3+\frac{7}{2}x^{12}c^6a^2+70x^{12}d^2c^3a^3+\frac{35}{4}x^{12}d^4a^4+35x^{11}dc^4a^3+35x^{11}d^3ca^4+7x^{10}c^5a^3+\frac{105}{2}x^{10}d^2c^2a^4+35x^9dc^3a^4+7x^9d^3a^5+\frac{35}{4}x^8c^4a^4+21x^8d^2ca^5+21x^7dc^2a^5+7x^6c^3a^5+\frac{7}{2}x^6d^2a^6+7x^5dca^6+\frac{7}{2}x^4c^2a^6+x^3da^7+x^2ca^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

giac [B] time = 0.43, size = 136, normalized size = 7.56

$$\frac{1}{8}\left(dx^3+cx^2\right)^8+\left(dx^3+cx^2\right)^7a+\frac{7}{2}\left(dx^3+cx^2\right)^6a^2+7\left(dx^3+cx^2\right)^5a^3+\frac{35}{4}\left(dx^3+cx^2\right)^4a^4+7\left(dx^3+cx^2\right)^3a^5+\frac{7}{2}\left(dx^3+cx^2\right)^2a^6+\left(dx^3+cx^2\right)a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8 + (d*x^3 + c*x^2)^7*a + 7/2*(d*x^3 + c*x^2)^6*a^2 + 7
*(d*x^3 + c*x^2)^5*a^3 + 35/4*(d*x^3 + c*x^2)^4*a^4 + 7*(d*x^3 + c*x^2)^3*a
^5 + 7/2*(d*x^3 + c*x^2)^2*a^6 + (d*x^3 + c*x^2)*a^7
```

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*d*(a*d^6+15*c^3
*d^4+d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)))*x^21+1/20*(2*c*(a*d^6+15*c^3*d^4+
d*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3))+3*d*(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+1
8*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)))*x^20+1/19*(2*c*
(6*a*c*d^5+c*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+d*(12*a*c*d^4+6*(3*a*d^2+c^3)
*c*d^2+9*c^4*d^2))+3*d*(15*a*c^2*d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*
c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)))*x^19+1/18*(2*c*(15*a*c^2*
d^4+c*(12*a*c*d^4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+d*(42*a*c^2*d^3+6*(3*a*d
^2+c^3)*c^2*d))+3*d*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)))*x^18+1/17*
(2*c*(a*(2*(3*a*d^2+c^3)*d^3+18*c^3*d^3)+c*(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^
2*d)+d*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2))+3*d*(a*(12*a*c*d^4+6*(3*a*
d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)+d*(24*
c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))))*x^17+1/16*(2*c*(a*(12*a*c*d^
4+6*(3*a*d^2+c^3)*c*d^2+9*c^4*d^2)+c*(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^
2)+d*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3)))+3*d*(a*(42*a*c^2*d^3
+6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+
d*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))))*x^16+1/15*(2*c*(a*(42*a*c^2*d^3+
6*(3*a*d^2+c^3)*c^2*d)+c*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*a*d^2+c^3))+d
*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3)))+3*d*(a*(6*a^2*d^4+54*a*c^3*d^2+(3*
a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*c^3*
a^2*d+6*d*a^2*(3*a*d^2+c^3))))*x^15+1/14*(2*c*(a*(6*a^2*d^4+54*a*c^3*d^2+(3
*a*d^2+c^3)^2)+c*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+d*(2*a^3*d^3+54*c^3
*a^2*d+6*d*a^2*(3*a*d^2+c^3)))+3*d*(a*(24*c*a^2*d^3+18*a*c^4*d+12*a*c*d*(3*
a*d^2+c^3))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3))+d*(42*a^3*c*d^
2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)))*x^14+1/13*(2*c*(a*(24*c*a^2*d^3+18*a*c
^4*d+12*a*c*d*(3*a*d^2+c^3))+c*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3
))+d*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4))+3*d*(a*(72*a^2*c^2*d^2
+6*a*c^2*(3*a*d^2+c^3))+c*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+60
```

```
*d^2*a^3*c^2))*x^13+1/12*(2*c*(a*(72*a^2*c^2*d^2+6*a*c^2*(3*a*d^2+c^3))+c*(
42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+60*d^2*a^3*c^2)+3*d*(a*(2*a^3
*d^3+54*c^3*a^2*d+6*d*a^2*(3*a*d^2+c^3))+60*c^3*a^3*d+d*(2*a^3*(3*a*d^2+c^3
)+18*c^3*a^3+9*a^4*d^2)))*x^12+1/11*(2*c*(a*(2*a^3*d^3+54*c^3*a^2*d+6*d*a^2
*(3*a*d^2+c^3))+60*c^3*a^3*d+d*(2*a^3*(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2))+
3*d*(a*(42*a^3*c*d^2+6*c*a^2*(3*a*d^2+c^3)+9*a^2*c^4)+c*(2*a^3*(3*a*d^2+c^3
)+18*c^3*a^3+9*a^4*d^2)+30*d^2*a^4*c))*x^11+1/10*(2*c*(a*(42*a^3*c*d^2+6*c*
a^2*(3*a*d^2+c^3)+9*a^2*c^4)+c*(2*a^3*(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2)+3
0*d^2*a^4*c)+315*d^2*a^4*c^2)*x^10+1/9*(210*c^3*a^4*d+3*d*(a*(2*a^3*(3*a*d^
2+c^3)+18*c^3*a^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5))*x^9+1/8*(2*c*(a*(2*a^3*
(3*a*d^2+c^3)+18*c^3*a^3+9*a^4*d^2)+15*c^3*a^4+6*d^2*a^5)+126*d^2*a^5*c)*x^
8+21*c^2*a^5*d*x^7+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6+7*c*a^6*d*x^5+7/2*c^2*a^
6*x^4+d*a^7*x^3+c*a^7*x^2
```

maxima [A] time = 0.62, size = 16, normalized size = 0.89

$$\frac{1}{8}\left(dx^3 + cx^2 + a\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2 + a)^8
```

mupad [B] time = 2.63, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^6\left(\frac{7a^6d^2}{2} + 7a^5c^3\right) + x^{20}\left(\frac{35c^4d^4}{4} + 7acd^6\right) + x^{16}\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{18}\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + \frac{d^8x^{24}}{8} + x^{21}\left(7c^3d^5 + ad^7\right) + a^7cx^2 + a^7dx^3 + cd^7x^{23} + \frac{7a^6c^2x^4}{2} + \frac{7c^2d^6x^{22}}{2} + 21a^5c^2dx^7 + 7adx^{15}\left(a^2d^4 + 10ac^3d^2 + c^6\right) + cdx^{17}\left(21a^2d^4 + 35ac^3d^2 + c^6\right) + \frac{7a^4cx^8\left(5c^3 + 12ad^2\right)}{4} + 7a^4dx^9\left(5c^3 + ad^2\right) + 7c^2d^3x^{19}\left(c^3 + 3ad^2\right) + \frac{acx^{14}\left(70a^2d^4 + 105ac^3d^2 + 2c^6\right)}{2} + 7a^6cdx^5 + \frac{7a^3c^2x^{10}\left(2c^3 + 15ad^2\right)}{2} + 7a^2c^2dx^{13}\left(3c^3 + 10ad^2\right) + 35a^3cdx^{11}\left(c^3 + ad^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(a + c*x^2 + d*x^3)^7,x)
```

```
[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (
7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*
d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3
*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c
*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a
*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*
c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3)
 + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c
^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*
c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)
```

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\left(ad^7 + 7c^3d^5\right) + x^{20}\left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21ac^2d^5 + 7c^5d^3\right) + x^{18}\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17}\left(21a^2cd^5 + 35ac^4d^3 + c^7d\right) + x^{16}\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15}\left(7a^3d^5 + 70a^2c^3d^3 + 7ac^6d\right) + x^{14}\left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13}\left(70a^3c^2d^3 + 21a^2c^5d\right) + x^{12}\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11}\left(35a^4cd^3 + 35a^3c^4d\right) + x^{10}\left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9\left(7a^5d^3 + 35a^4c^3d\right) + x^8\left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6\left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

3.195 $\int (2cx+3dx^2)(cx^2+dx^3)^7\,dx$

Optimal. Leaf size=17

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1588}

$$\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2cx+3dx^2)(cx^2+dx^3)^7\,dx=\frac{1}{8}\left(cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.76

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2cx+3dx^2)(cx^2+dx^3)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[(2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 0.95, size = 88, normalized size = 5.18

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.37, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 5.24

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x^2+2*c*x)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

mupad [B] time = 2.07, size = 88, normalized size = 5.18

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x + 3*d*x^2)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 5.71

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*d*x**2+2*c*x)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.196 $\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) \, dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

Rubi [A] time = 0.23, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1584, 845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) \, dx &= \int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) \, dx \\ &= \frac{1}{8}x^{16}(c+dx)^8 \end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7 \left(cx+dx^2\right)^7 \left(2cx+3dx^2\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x]
[Out] IntegrateAlgebraic[x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2), x]
```

fricas [B] time = 0.55, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.23, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x)
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.52, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^2+c*x)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.05, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*x + d*x^2)^7*(2*c*x + 3*d*x^2),x)
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**2+c*x)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.197 $\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {845}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 845

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((b_.)*(x_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(c*x^(m + 2)*(f + g*x)^(n + 1))/(g*(m + n + 3)), x] /; FreeQ[{
b, c, f, g, m, n}, x] && EqQ[c*f*(m + 2) - b*g*(m + n + 3), 0] && NeQ[m + n
 + 3, 0]
```

Rubi steps

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx = \frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}(c+dx)^7 \left(2cx+3dx^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2),x]
```

```
[Out] IntegrateAlgebraic[x^14*(c + d*x)^7*(2*c*x + 3*d*x^2), x]
```

fricas [B] time = 0.73, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [A] time = 0.26, size = 15, normalized size = 1.07

$$\frac{1}{8}\left(dx^3 + cx^2\right)^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="giac")
```

```
[Out] 1/8*(d*x^3 + c*x^2)^8
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.66, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(d*x+c)^7*(3*d*x^2+2*c*x),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(2*c*x + 3*d*x^2)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(d*x+c)**7*(3*d*x**2+2*c*x),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.198 $\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1588}

$$\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (a + c*x^2 + d*x^3)^8/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx=\frac{1}{8}\left(a+cx^2+dx^3\right)^8$$

Mathematica [B] time = 0.01, size = 115, normalized size = 6.39

$$\frac{1}{8}x^2(c+dx)\left(8a^7+28a^6x^2(c+dx)+56a^5x^4(c+dx)^2+70a^4x^6(c+dx)^3+56a^3x^8(c+dx)^4+28a^2x^{10}(c+dx)^5+8ax^{12}(c+dx)^6+x^{14}(c+dx)^7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^2*(c + d*x)*(8*a^7 + 28*a^6*x^2*(c + d*x) + 56*a^5*x^4*(c + d*x)^2 + 70*
a^4*x^6*(c + d*x)^3 + 56*a^3*x^8*(c + d*x)^4 + 28*a^2*x^10*(c + d*x)^5 + 8*
a*x^12*(c + d*x)^6 + x^14*(c + d*x)^7))/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(2c+3dx)\left(a+cx^2+dx^3\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 0.63, size = 488, normalized size = 27.11

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+x^{21}d^7a+\frac{35}{4}x^{20}d^4c^4+7x^{20}d^6ca+7x^{19}d^3c^5+21x^{19}d^5c^2a+\frac{7}{2}x^{18}d^2c^6+35x^{18}d^4c^3a+\frac{7}{2}x^{18}d^6a^2+x^{17}dc^7+35x^{17}d^3c^4a+21x^{17}d^5ca^2+\frac{1}{8}x^{16}c^8+21x^{16}d^2c^5a+\frac{105}{2}x^{16}d^4c^2a^2+7x^{15}dc^6a+70x^{15}d^3c^3a^2+7x^{15}d^5a^3+x^{14}c^7a+\frac{105}{2}x^{14}d^2c^4a^2+35x^{14}d^4ca^3+21x^{13}dc^5a^2+70x^{13}d^3c^2a^3+\frac{7}{2}x^{12}c^6a^2+70x^{12}d^2c^3a^3+\frac{35}{4}x^{12}d^4a^4+35x^{11}dc^4a^3+35x^{11}d^3ca^4+7x^{10}c^5a^3+\frac{105}{2}x^{10}d^2c^2a^4+35x^9dc^3a^4+7x^9d^3a^5+\frac{35}{4}x^8c^4a^4+21x^8d^2ca^5+21x^7dc^2a^5+7x^6c^3a^5+\frac{7}{2}x^6d^2a^6+7x^5dca^6+\frac{7}{2}x^4c^2a^6+x^3da^7+x^2ca^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + x^21*d^7*a
+ 35/4*x^20*d^4*c^4 + 7*x^20*d^6*c*a + 7*x^19*d^3*c^5 + 21*x^19*d^5*c^2*a +
 7/2*x^18*d^2*c^6 + 35*x^18*d^4*c^3*a + 7/2*x^18*d^6*a^2 + x^17*d*c^7 + 35*
x^17*d^3*c^4*a + 21*x^17*d^5*c*a^2 + 1/8*x^16*c^8 + 21*x^16*d^2*c^5*a + 105
/2*x^16*d^4*c^2*a^2 + 7*x^15*d*c^6*a + 70*x^15*d^3*c^3*a^2 + 7*x^15*d^5*a^3
 + x^14*c^7*a + 105/2*x^14*d^2*c^4*a^2 + 35*x^14*d^4*c*a^3 + 21*x^13*d*c^5*
a^2 + 70*x^13*d^3*c^2*a^3 + 7/2*x^12*c^6*a^2 + 70*x^12*d^2*c^3*a^3 + 35/4*x
^12*d^4*a^4 + 35*x^11*d*c^4*a^3 + 35*x^11*d^3*c*a^4 + 7*x^10*c^5*a^3 + 105/
2*x^10*d^2*c^2*a^4 + 35*x^9*d*c^3*a^4 + 7*x^9*d^3*a^5 + 35/4*x^8*c^4*a^4 +
21*x^8*d^2*c*a^5 + 21*x^7*d*c^2*a^5 + 7*x^6*c^3*a^5 + 7/2*x^6*d^2*a^6 + 7*x
^5*d*c*a^6 + 7/2*x^4*c^2*a^6 + x^3*d*a^7 + x^2*c*a^7
```

giac [B] time = 0.32, size = 488, normalized size = 27.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + a*d^7*x^21
+ 35/4*c^4*d^4*x^20 + 7*a*c*d^6*x^20 + 7*c^5*d^3*x^19 + 21*a*c^2*d^5*x^19 +
 7/2*c^6*d^2*x^18 + 35*a*c^3*d^4*x^18 + 7/2*a^2*d^6*x^18 + c^7*d*x^17 + 35*
a*c^4*d^3*x^17 + 21*a^2*c*d^5*x^17 + 1/8*c^8*x^16 + 21*a*c^5*d^2*x^16 + 105
/2*a^2*c^2*d^4*x^16 + 7*a*c^6*d*x^15 + 70*a^2*c^3*d^3*x^15 + 7*a^3*d^5*x^15
 + a*c^7*x^14 + 105/2*a^2*c^4*d^2*x^14 + 35*a^3*c*d^4*x^14 + 21*a^2*c^5*d*x
^13 + 70*a^3*c^2*d^3*x^13 + 7/2*a^2*c^6*x^12 + 70*a^3*c^3*d^2*x^12 + 35/4*a
^4*d^4*x^12 + 35*a^3*c^4*d*x^11 + 35*a^4*c*d^3*x^11 + 7*a^3*c^5*x^10 + 105/
2*a^4*c^2*d^2*x^10 + 35*a^4*c^3*d*x^9 + 7*a^5*d^3*x^9 + 35/4*a^4*c^4*x^8 +
21*a^5*c*d^2*x^8 + 21*a^5*c^2*d*x^7 + 7*a^5*c^3*x^6 + 7/2*a^6*d^2*x^6 + 7*a
^6*c*d*x^5 + 7/2*a^6*c^2*x^4 + a^7*d*x^3 + a^7*c*x^2
```

maple [B] time = 0.00, size = 2205, normalized size = 122.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+1/21*(42*c^3*d^5+3*(a*d^6+15*c^3*d
^4+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*d)*x^21+1/20*(2*(a*d^6+15*c^3*d^4+(1
8*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*d)*c+3*(6*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3
)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*d)*d)*x^20+1/19*(2*(6
*a*c*d^5+(18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*c+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^
2+c^3)*c*d^2)*d)*c+3*(15*a*c^2*d^4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*
d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*d)*d)*x^19+1/18*(2*(15*a*c^2*d^
4+(12*a*c*d^4+9*c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*c+(42*a*c^2*d^3+6*(3*a*d^2+c
^3)*c^2*d)*d)*c+3*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*
d^2+c^3)*c^2*d)*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*d)*x^18+1/17*
(2*((18*c^3*d^3+2*(3*a*d^2+c^3)*d^3)*a+(42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)
*c+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*d)*c+3*((12*a*c*d^4+9*c^4*d^2+6
*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c+(24*a^2*
c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*d)*x^17+1/16*(2*((12*a*c*d^4+9*
c^4*d^2+6*(3*a*d^2+c^3)*c*d^2)*a+(6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*c
+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*d)*c+3*((42*a*c^2*d^3+6*(
3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*c+(7
2*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*d)*x^16+21*a^5*c^2*d*x^7+1/15*(2*((
42*a*c^2*d^3+6*(3*a*d^2+c^3)*c^2*d)*a+(24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+
c^3)*a*c*d)*c+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*d)*c+3*((6*a^2*d^4+54*
a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a^
```

3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*d)*x^15+1/14*(2*((6*a^2*d^4+54*a*c^3*d^2+(3*a*d^2+c^3)^2)*a+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*c+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*d)*c+3*((24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*d)*d)*x^14+7*a^6*c*d*x^5+1/13*(2*((24*a^2*c*d^3+18*a*c^4*d+12*(3*a*d^2+c^3)*a*c*d)*a+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*c+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*d)*c+3*(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*d)*x^13+7/2*a^6*c^2*x^4+1/12*(2*(60*a^3*c^2*d^2+(72*a^2*c^2*d^2+6*(3*a*d^2+c^3)*a*c^2)*a+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*c)*c+3*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*d)*x^12+a^7*d*x^3+1/11*(2*(60*a^3*c^3*d+(2*a^3*d^3+54*a^2*c^3*d+6*(3*a*d^2+c^3)*a^2*d)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*d)*c+3*(30*a^4*c*d^2+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*c)*d)*x^11+a^7*c*x^2+1/10*(315*a^4*c^2*d^2+2*(30*a^4*c*d^2+(42*a^3*c*d^2+9*a^2*c^4+6*(3*a*d^2+c^3)*a^2*c)*a+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*c)*c)*x^10+1/9*(210*a^4*c^3*d+3*(6*a^5*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*d)*x^9+1/8*(126*a^5*c*d^2+2*(6*a^5*d^2+15*a^4*c^3+(9*a^4*d^2+18*a^3*c^3+2*(3*a*d^2+c^3)*a^3)*a)*c)*x^8+1/6*(21*a^6*d^2+42*a^5*c^3)*x^6

maxima [B] time = 0.59, size = 458, normalized size = 25.44

$$\frac{1}{8}d^8x^{24} + cd^7x^{23} + \frac{7}{2}c^2d^6x^{22} + \left(7c^3d^5 + ad^7\right)x^{21} + \frac{7}{4}\left(5c^4d^4 + 4acd^6\right)x^{20} + 7\left(c^5d^3 + 3ac^2d^5\right)x^{19} + \frac{7}{2}\left(c^6d^2 + 10ac^3d^4 + a^2d^6\right)x^{18} + \left(c^7d + 35ac^4d^3 + 21a^2cd^5\right)x^{17} + \frac{1}{8}\left(c^8 + 168ac^5d^2 + 420a^2c^2d^4\right)x^{16} + 7\left(ac^6d + 10a^2c^3d^3 + a^3d^5\right)x^{15} + 21a^5c^2dx^7 + \frac{1}{2}\left(2ac^7 + 105a^2c^4d^2 + 70a^3cd^4\right)x^{14} + 7\left(3a^2c^5d + 10a^3c^2d^3\right)x^{13} + 7a^6cdx^5 + \frac{7}{4}\left(2a^2c^6 + 40a^3c^3d^2 + 5a^4d^4\right)x^{12} + \frac{7}{2}a^6c^2x^4 + 35\left(a^3c^4d + a^4cd^3\right)x^{11} + a^7dx^3 + \frac{7}{2}\left(2a^3c^5 + 15a^4c^2d^2\right)x^{10} + a^7cx^2 + 7\left(5a^4c^3d + a^5d^3\right)x^9 + \frac{7}{4}\left(5a^4c^4 + 12a^5cd^2\right)x^8 + \frac{7}{2}\left(2a^5c^3 + a^6d^2\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2+a)^7,x, algorithm="maxima")

[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + (7*c^3*d^5 + a*d^7)*x^21 + 7/4*(5*c^4*d^4 + 4*a*c*d^6)*x^20 + 7*(c^5*d^3 + 3*a*c^2*d^5)*x^19 + 7/2*(c^6*d^2 + 10*a*c^3*d^4 + a^2*d^6)*x^18 + (c^7*d + 35*a*c^4*d^3 + 21*a^2*c*d^5)*x^17 + 1/8*(c^8 + 168*a*c^5*d^2 + 420*a^2*c^2*d^4)*x^16 + 7*(a*c^6*d + 10*a^2*c^3*d^3 + a^3*d^5)*x^15 + 21*a^5*c^2*d*x^7 + 1/2*(2*a*c^7 + 105*a^2*c^4*d^2 + 70*a^3*c*d^4)*x^14 + 7*(3*a^2*c^5*d + 10*a^3*c^2*d^3)*x^13 + 7*a^6*c*d*x^5 + 7/4*(2*a^2*c^6 + 40*a^3*c^3*d^2 + 5*a^4*d^4)*x^12 + 7/2*a^6*c^2*x^4 + 35*(a^3*c^4*d + a^4*c*d^3)*x^11 + a^7*d*x^3 + 7/2*(2*a^3*c^5 + 15*a^4*c^2*d^2)*x^10 + a^7*c*x^2 + 7*(5*a^4*c^3*d + a^5*d^3)*x^9 + 7/4*(5*a^4*c^4 + 12*a^5*c*d^2)*x^8 + 7/2*(2*a^5*c^3 + a^6*d^2)*x^6

mupad [B] time = 0.57, size = 440, normalized size = 24.44

$$x^{12}\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^6\left(\frac{7a^6d^2}{2} + 7a^5c^3\right) + x^{20}\left(\frac{35c^4d^4}{4} + 7acd^6\right) + x^{16}\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{18}\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + \frac{d^8x^{24}}{8} + x^{21}\left(7c^3d^5 + ad^7\right) + a^7cx^2 + a^7dx^3 + cd^7x^{23} + \frac{7a^6c^2x^4}{2} + \frac{7c^2d^6x^{22}}{2} + 21a^5c^2dx^7 + 7adx^{15}\left(a^2d^4 + 10ac^3d^2 + c^6\right) + cdx^{17}\left(21a^2d^4 + 35ac^3d^2 + c^6\right) + \frac{7a^4cx^8\left(5c^3 + 12ad^2\right)}{4} + 7a^4dx^9\left(5c^3 + ad^2\right) + 7c^2d^3x^{19}\left(c^3 + 3ad^2\right) + \frac{acx^{14}\left(70a^2d^4 + 105ac^3d^2 + 2c^6\right)}{2} + 7a^6cdx^5 + \frac{7a^3c^2x^{10}\left(2c^3 + 15ad^2\right)}{2} + 7a^2c^2dx^{13}\left(3c^3 + 10ad^2\right) + 35a^3cdx^{11}\left(c^3 + ad^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c + 3*d*x)*(a + c*x^2 + d*x^3)^7,x)

[Out] x^12*((7*a^2*c^6)/2 + (35*a^4*d^4)/4 + 70*a^3*c^3*d^2) + x^6*(7*a^5*c^3 + (7*a^6*d^2)/2) + x^20*((35*c^4*d^4)/4 + 7*a*c*d^6) + x^16*(c^8/8 + 21*a*c^5*d^2 + (105*a^2*c^2*d^4)/2) + x^18*((7*a^2*d^6)/2 + (7*c^6*d^2)/2 + 35*a*c^3*d^4) + (d^8*x^24)/8 + x^21*(a*d^7 + 7*c^3*d^5) + a^7*c*x^2 + a^7*d*x^3 + c*d^7*x^23 + (7*a^6*c^2*x^4)/2 + (7*c^2*d^6*x^22)/2 + 21*a^5*c^2*d*x^7 + 7*a*d*x^15*(c^6 + a^2*d^4 + 10*a*c^3*d^2) + c*d*x^17*(c^6 + 21*a^2*d^4 + 35*a*c^3*d^2) + (7*a^4*c*x^8*(12*a*d^2 + 5*c^3))/4 + 7*a^4*d*x^9*(a*d^2 + 5*c^3) + 7*c^2*d^3*x^19*(3*a*d^2 + c^3) + (a*c*x^14*(2*c^6 + 70*a^2*d^4 + 105*a*c^3*d^2))/2 + 7*a^6*c*d*x^5 + (7*a^3*c^2*x^10*(15*a*d^2 + 2*c^3))/2 + 7*a^2*c^2*d*x^13*(10*a*d^2 + 3*c^3) + 35*a^3*c*d*x^11*(a*d^2 + c^3)

sympy [B] time = 0.17, size = 484, normalized size = 26.89

$$a^7cx^2 + a^7dx^3 + \frac{7a^6c^2x^4}{2} + 7a^6cdx^5 + 21a^5c^2dx^7 + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8} + x^{21}\left(ad^7 + 7c^3d^5\right) + x^{20}\left(7acd^6 + \frac{35c^4d^4}{4}\right) + x^{19}\left(21ac^2d^5 + 7c^5d^3\right) + x^{18}\left(\frac{7a^2d^6}{2} + 35ac^3d^4 + \frac{7c^6d^2}{2}\right) + x^{17}\left(21a^2cd^5 + 35ac^4d^3 + c^7d\right) + x^{16}\left(\frac{105a^2c^2d^4}{2} + 21ac^5d^2 + \frac{c^8}{8}\right) + x^{15}\left(7a^3d^5 + 70a^2c^3d^3 + 7ac^6d\right) + x^{14}\left(35a^3cd^4 + \frac{105a^2c^4d^2}{2} + ac^7\right) + x^{13}\left(70a^3c^2d^3 + 21a^2c^5d\right) + x^{12}\left(\frac{35a^4d^4}{4} + 70a^3c^3d^2 + \frac{7a^2c^6}{2}\right) + x^{11}\left(35a^4cd^3 + 35a^3c^4d\right) + x^{10}\left(\frac{105a^4c^2d^2}{2} + 7a^3c^5\right) + x^9\left(7a^5d^3 + 35a^4c^3d\right) + x^8\left(21a^5cd^2 + \frac{35a^4c^4}{4}\right) + x^6\left(\frac{7a^6d^2}{2} + 7a^5c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2+a)**7,x)
```

```
[Out] a**7*c*x**2 + a**7*d*x**3 + 7*a**6*c**2*x**4/2 + 7*a**6*c*d*x**5 + 21*a**5*
c**2*d*x**7 + 7*c**2*d**6*x**22/2 + c*d**7*x**23 + d**8*x**24/8 + x**21*(a*
d**7 + 7*c**3*d**5) + x**20*(7*a*c*d**6 + 35*c**4*d**4/4) + x**19*(21*a*c**
2*d**5 + 7*c**5*d**3) + x**18*(7*a**2*d**6/2 + 35*a*c**3*d**4 + 7*c**6*d**2
/2) + x**17*(21*a**2*c*d**5 + 35*a*c**4*d**3 + c**7*d) + x**16*(105*a**2*c*
*2*d**4/2 + 21*a*c**5*d**2 + c**8/8) + x**15*(7*a**3*d**5 + 70*a**2*c**3*d*
*3 + 7*a*c**6*d) + x**14*(35*a**3*c*d**4 + 105*a**2*c**4*d**2/2 + a*c**7) +
 x**13*(70*a**3*c**2*d**3 + 21*a**2*c**5*d) + x**12*(35*a**4*d**4/4 + 70*a*
*3*c**3*d**2 + 7*a**2*c**6/2) + x**11*(35*a**4*c*d**3 + 35*a**3*c**4*d) + x
**10*(105*a**4*c**2*d**2/2 + 7*a**3*c**5) + x**9*(7*a**5*d**3 + 35*a**4*c**
3*d) + x**8*(21*a**5*c*d**2 + 35*a**4*c**4/4) + x**6*(7*a**6*d**2/2 + 7*a**
5*c**3)
```

3.199 $\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx &= \int x^{15}(c+dx)^7(2c+3dx)\,dx\\ &= \frac{1}{8}x^{16}(c+dx)^8\end{aligned}$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(2c+3dx)\left(cx^2+dx^3\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x]
```

```
[Out] IntegrateAlgebraic[x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7, x]
```

fricas [B] time = 1.04, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8 + x^{23}d^7c + \frac{7}{2}x^{22}d^6c^2 + 7x^{21}d^5c^3 + \frac{35}{4}x^{20}d^4c^4 + 7x^{19}d^3c^5 + \frac{7}{2}x^{18}d^2c^6 + x^{17}dc^7 + \frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.31, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.60, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x^3+c*x^2)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c + 3*d*x)*(c*x^2 + d*x^3)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*d*x+2*c)*(d*x**3+c*x**2)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.200 $\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$

Optimal. Leaf size=18

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {763}

$$\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (x^8*(c*x + d*x^2)^8)/8
```

Rule 763

```
Int[((e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.
), x_Symbol] :> Simp[(g*(e*x)^m*(b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 2)), x
] /; FreeQ[{b, c, e, f, g, m, p}, x] && EqQ[b*g*(m + p + 1) - c*f*(m + 2*p
+ 2), 0] && NeQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx=\frac{1}{8}x^8\left(cx+dx^2\right)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 5.44

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(2c+3dx)\left(cx+dx^2\right)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7,x]
```

```
[Out] IntegrateAlgebraic[x^8*(2*c + 3*d*x)*(c*x + d*x^2)^7, x]
```

fricas [B] time = 1.01, size = 88, normalized size = 4.89

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.29, size = 88, normalized size = 4.89

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 4.94

$$\frac{1}{8}d^8x^{24}+c\,d^7x^{23}+\frac{7}{2}c^2d^6x^{22}+7c^3d^5x^{21}+\frac{35}{4}c^4d^4x^{20}+7c^5d^3x^{19}+\frac{7}{2}c^6d^2x^{18}+c^7d\,x^{17}+\frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.57, size = 88, normalized size = 4.89

$$\frac{1}{8}\,d^8x^{24}+cd^7x^{23}+\frac{7}{2}\,c^2d^6x^{22}+7\,c^3d^5x^{21}+\frac{35}{4}\,c^4d^4x^{20}+7\,c^5d^3x^{19}+\frac{7}{2}\,c^6d^2x^{18}+c^7dx^{17}+\frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(3*d*x+2*c)*(d*x^2+c*x)^7,x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 4.89

$$\frac{c^8\,x^{16}}{8}+c^7\,d\,x^{17}+\frac{7\,c^6\,d^2\,x^{18}}{2}+7\,c^5\,d^3\,x^{19}+\frac{35\,c^4\,d^4\,x^{20}}{4}+7\,c^3\,d^5\,x^{21}+\frac{7\,c^2\,d^6\,x^{22}}{2}+c\,d^7\,x^{23}+\frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x + d*x^2)^7*(2*c + 3*d*x),x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.10, size = 97, normalized size = 5.39

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7c^6d^2x^{18}}{2}+7c^5d^3x^{19}+\frac{35c^4d^4x^{20}}{4}+7c^3d^5x^{21}+\frac{7c^2d^6x^{22}}{2}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(3*d*x+2*c)*(d*x**2+c*x)**7,x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.201 $\int x^{15}(c+dx)^7(2c+3dx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{8}x^{16}(c+dx)^8$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {74}

$$\frac{1}{8}x^{16}(c+dx)^8$$

Antiderivative was successfully verified.

```
[In] Int[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (x^16*(c + d*x)^8)/8
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx=\frac{1}{8}x^{16}(c+dx)^8$$

Mathematica [B] time = 0.00, size = 98, normalized size = 7.00

$$\frac{c^8x^{16}}{8}+c^7dx^{17}+\frac{7}{2}c^6d^2x^{18}+7c^5d^3x^{19}+\frac{35}{4}c^4d^4x^{20}+7c^3d^5x^{21}+\frac{7}{2}c^2d^6x^{22}+cd^7x^{23}+\frac{d^8x^{24}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] (c^8*x^16)/8 + c^7*d*x^17 + (7*c^6*d^2*x^18)/2 + 7*c^5*d^3*x^19 + (35*c^4*d
^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/2 + c*d^7*x^23 + (d^8*x^24)/
8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{15}(c+dx)^7(2c+3dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^15*(c + d*x)^7*(2*c + 3*d*x),x]
```

```
[Out] IntegrateAlgebraic[x^15*(c + d*x)^7*(2*c + 3*d*x), x]
```

fricas [B] time = 1.07, size = 88, normalized size = 6.29

$$\frac{1}{8}x^{24}d^8+x^{23}d^7c+\frac{7}{2}x^{22}d^6c^2+7x^{21}d^5c^3+\frac{35}{4}x^{20}d^4c^4+7x^{19}d^3c^5+\frac{7}{2}x^{18}d^2c^6+x^{17}dc^7+\frac{1}{8}x^{16}c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="fricas")
```

```
[Out] 1/8*x^24*d^8 + x^23*d^7*c + 7/2*x^22*d^6*c^2 + 7*x^21*d^5*c^3 + 35/4*x^20*d
^4*c^4 + 7*x^19*d^3*c^5 + 7/2*x^18*d^2*c^6 + x^17*d*c^7 + 1/8*x^16*c^8
```

giac [B] time = 0.24, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="giac")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

maple [B] time = 0.00, size = 89, normalized size = 6.36

$$\frac{1}{8}d^8x^{24} + c\,d^7x^{23} + \frac{7}{2}c^2d^6x^{22} + 7c^3d^5x^{21} + \frac{35}{4}c^4d^4x^{20} + 7c^5d^3x^{19} + \frac{7}{2}c^6d^2x^{18} + c^7d\,x^{17} + \frac{1}{8}c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(d*x+c)^7*(3*d*x+2*c),x)
```

```
[Out] 1/8*d^8*x^24+c*d^7*x^23+7/2*c^2*d^6*x^22+7*c^3*d^5*x^21+35/4*c^4*d^4*x^20+7
*c^5*d^3*x^19+7/2*c^6*d^2*x^18+c^7*d*x^17+1/8*c^8*x^16
```

maxima [B] time = 0.67, size = 88, normalized size = 6.29

$$\frac{1}{8}\,d^8x^{24} + cd^7x^{23} + \frac{7}{2}\,c^2d^6x^{22} + 7\,c^3d^5x^{21} + \frac{35}{4}\,c^4d^4x^{20} + 7\,c^5d^3x^{19} + \frac{7}{2}\,c^6d^2x^{18} + c^7dx^{17} + \frac{1}{8}\,c^8x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15*(d*x+c)^7*(3*d*x+2*c),x, algorithm="maxima")
```

```
[Out] 1/8*d^8*x^24 + c*d^7*x^23 + 7/2*c^2*d^6*x^22 + 7*c^3*d^5*x^21 + 35/4*c^4*d^
4*x^20 + 7*c^5*d^3*x^19 + 7/2*c^6*d^2*x^18 + c^7*d*x^17 + 1/8*c^8*x^16
```

mupad [B] time = 0.04, size = 88, normalized size = 6.29

$$\frac{c^8\,x^{16}}{8} + c^7\,d\,x^{17} + \frac{7\,c^6\,d^2\,x^{18}}{2} + 7\,c^5\,d^3\,x^{19} + \frac{35\,c^4\,d^4\,x^{20}}{4} + 7\,c^3\,d^5\,x^{21} + \frac{7\,c^2\,d^6\,x^{22}}{2} + c\,d^7\,x^{23} + \frac{d^8\,x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15*(2*c + 3*d*x)*(c + d*x)^7,x)
```

```
[Out] (c^8*x^16)/8 + (d^8*x^24)/8 + c^7*d*x^17 + c*d^7*x^23 + (7*c^6*d^2*x^18)/2
+ 7*c^5*d^3*x^19 + (35*c^4*d^4*x^20)/4 + 7*c^3*d^5*x^21 + (7*c^2*d^6*x^22)/
2
```

sympy [B] time = 0.09, size = 97, normalized size = 6.93

$$\frac{c^8x^{16}}{8} + c^7dx^{17} + \frac{7c^6d^2x^{18}}{2} + 7c^5d^3x^{19} + \frac{35c^4d^4x^{20}}{4} + 7c^3d^5x^{21} + \frac{7c^2d^6x^{22}}{2} + cd^7x^{23} + \frac{d^8x^{24}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15*(d*x+c)**7*(3*d*x+2*c),x)
```

```
[Out] c**8*x**16/8 + c**7*d*x**17 + 7*c**6*d**2*x**18/2 + 7*c**5*d**3*x**19 + 35*
c**4*d**4*x**20/4 + 7*c**3*d**5*x**21 + 7*c**2*d**6*x**22/2 + c*d**7*x**23
+ d**8*x**24/8
```

3.202 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx$

Optimal. Leaf size=28

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{1}{160}x^5(2a+bx)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (x^5*(2*a + b*x)^5)/160
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx &= \text{Subst}\left(\int \left(1+x^4\right)dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{160}x^5(2a+bx)^5\end{aligned}$$

Mathematica [B] time = 0.01, size = 80, normalized size = 2.86

$$\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7+\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3*
x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4), x]
```

fricas [B] time = 1.52, size = 66, normalized size = 2.36

$$\frac{1}{160}x^{10}b^5 + \frac{1}{16}x^9b^4a + \frac{1}{4}x^8b^3a^2 + \frac{1}{2}x^7b^2a^3 + \frac{1}{2}x^6ba^4 + \frac{1}{5}x^5a^5 + \frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/4*x^8*b^3*a^2 + 1/2*x^7*b^2*a^3 + 1/2*x
^6*b*a^4 + 1/5*x^5*a^5 + 1/2*x^2*b + x*a
```

giac [A] time = 0.37, size = 24, normalized size = 0.86

$$\frac{1}{160}\left(bx^2 + 2\,ax\right)^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 67, normalized size = 2.39

$$\frac{1}{160}b^5x^{10} + \frac{1}{16}a\,b^4x^9 + \frac{1}{4}a^2b^3x^8 + \frac{1}{2}a^3b^2x^7 + \frac{1}{2}a^4b\,x^6 + \frac{1}{5}a^5x^5 + \frac{1}{2}b\,x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/4*a^2*b^3*x^8+1/2*a^3*b^2*x^7+1/2*a^4*b*x^6
+1/5*a^5*x^5+1/2*b*x^2+a*x
```

maxima [B] time = 0.61, size = 66, normalized size = 2.36

$$\frac{1}{160}\,b^5x^{10} + \frac{1}{16}\,ab^4x^9 + \frac{1}{4}\,a^2b^3x^8 + \frac{1}{2}\,a^3b^2x^7 + \frac{1}{2}\,a^4bx^6 + \frac{1}{5}\,a^5x^5 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*a
^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

mupad [B] time = 0.05, size = 66, normalized size = 2.36

$$\frac{a^5\,x^5}{5} + \frac{a^4\,b\,x^6}{2} + \frac{a^3\,b^2\,x^7}{2} + \frac{a^2\,b^3\,x^8}{4} + \frac{a\,b^4\,x^9}{16} + a\,x + \frac{b^5\,x^{10}}{160} + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^9
)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

sympy [B] time = 0.09, size = 70, normalized size = 2.50

$$\frac{a^5x^5}{5} + \frac{a^4bx^6}{2} + \frac{a^3b^2x^7}{2} + \frac{a^2b^3x^8}{4} + \frac{ab^4x^9}{16} + ax + \frac{b^5x^{10}}{160} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

```
[Out] a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4*
x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

3.203 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx &= \text{Subst}\left(\int \left(1+x^4\right)\,dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{1}{5}\left(c+ax+\frac{bx^2}{2}\right)^5\end{aligned}$$

Mathematica [B] time = 0.04, size = 108, normalized size = 3.48

$$\frac{1}{160}x(2a+bx)\left(16a^4x^4+32a^3bx^5+24a^2b^2x^6+8ab^3x^7+80c^3x(2a+bx)+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3+b^4x^8+80c^4+80\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] (x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 + 1
0*c*x^3*(2*a + b*x)^3))/160
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4), x]
```

fricas [B] time = 0.95, size = 208, normalized size = 6.71

$$\frac{1}{160}x^{10}b^5+\frac{1}{16}x^9b^4a+\frac{1}{16}x^8cb^4+\frac{1}{4}x^8b^3a^2+\frac{1}{2}x^7cb^3a+\frac{1}{2}x^7b^2a^3+\frac{1}{4}x^6c^2b^3+\frac{3}{2}x^6cb^2a^2+\frac{1}{2}x^6ba^4+\frac{3}{2}x^5c^2b^2a+2x^5cba^3+\frac{1}{5}x^5a^5+\frac{1}{2}x^4c^3b^2+3x^4c^2ba^2+x^4ca^4+2x^3c^3ba+2x^3c^2a^3+\frac{1}{2}x^2c^4b+2x^2c^3a^2+xc^4a+\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

```
[Out] 1/160*x^10*b^5 + 1/16*x^9*b^4*a + 1/16*x^8*c*b^4 + 1/4*x^8*b^3*a^2 + 1/2*x^
7*c*b^3*a + 1/2*x^7*b^2*a^3 + 1/4*x^6*c^2*b^3 + 3/2*x^6*c*b^2*a^2 + 1/2*x^6
*b*a^4 + 3/2*x^5*c^2*b^2*a + 2*x^5*c*b*a^3 + 1/5*x^5*a^5 + 1/2*x^4*c^3*b^2
+ 3*x^4*c^2*b*a^2 + x^4*c*a^4 + 2*x^3*c^3*b*a + 2*x^3*c^2*a^3 + 1/2*x^2*c^4
*b + 2*x^2*c^3*a^2 + x*c^4*a + 1/2*x^2*b + x*a
```

giac [B] time = 0.37, size = 88, normalized size = 2.84

$$\frac{1}{160}\left(bx^2+2\,ax\right)^5+\frac{1}{16}\left(bx^2+2\,ax\right)^4c+\frac{1}{4}\left(bx^2+2\,ax\right)^3c^2+\frac{1}{2}\left(bx^2+2\,ax\right)^2c^3+\frac{1}{2}\left(bx^2+2\,ax\right)c^4+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

```
[Out] 1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3*
c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 325, normalized size = 10.48

$$\frac{b^5x^{10}}{160}+\frac{a\,b^4x^9}{16}+\frac{\left(\frac{a^2b^3}{2}+\left(a^2b^2+\frac{(a^2+bc)b^2}{2}\right)b\right)x^8}{8}+\frac{\left(\left(a^2b^2+\frac{(a^2+bc)b^2}{2}\right)a+\left(a\,b^2c+2\left(a^2+bc\right)ab\right)b\right)x^7}{7}+\frac{\left(\left(a\,b^2c+2\left(a^2+bc\right)ab\right)a+\left(4a^2bc+\frac{b^2c^2}{2}+\left(a^2+bc\right)^2\right)b\right)x^6}{6}+\frac{\left(\left(4a^2bc+\frac{b^2c^2}{2}+\left(a^2+bc\right)^2\right)a+\left(2ab\,c^2+4\left(a^2+bc\right)ac\right)b\right)x^5}{5}+\frac{\left(\left(2ab\,c^2+4\left(a^2+bc\right)ac\right)a+\left(4a^2c^2+2\left(a^2+bc\right)c^2\right)b\right)x^4}{4}+\frac{\left(4ab\,c^3+\left(4a^2c^2+2\left(a^2+bc\right)c^2\right)a\right)x^3}{3}+\left(c^4+1\right)ax+\frac{\left(4a^2c^3+\left(c^4+1\right)b\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)
```

```
[Out] 1/160*b^5*x^10+1/16*a*b^4*x^9+1/8*(1/2*a^2*b^3+b*(1/2*(a^2+b*c)*b^2+a^2*b^2
))*x^8+1/7*(a*(1/2*(a^2+b*c)*b^2+a^2*b^2)+b*(a*c*b^2+2*(a^2+b*c)*a*b))*x^7+
1/6*(a*(a*c*b^2+2*(a^2+b*c)*a*b)+b*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2))*x^6
+1/5*(a*(1/2*c^2*b^2+4*a^2*c*b+(a^2+b*c)^2)+b*(2*c^2*a*b+4*a*c*(a^2+b*c)))*
x^5+1/4*(a*(2*c^2*a*b+4*a*c*(a^2+b*c))+b*(2*c^2*(a^2+b*c)+4*a^2*c^2))*x^4+1
/3*(a*(2*c^2*(a^2+b*c)+4*a^2*c^2)+4*a*b*c^3)*x^3+1/2*(4*a^2*c^3+b*(c^4+1))*
x^2+a*(c^4+1)*x
```

maxima [B] time = 0.44, size = 187, normalized size = 6.03

$$\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{16}\left(4\,a^2b^3+b^4c\right)x^8+\frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6+\frac{1}{10}\left(2\,a^5+20\,a^3bc+15\,ab^2c^2\right)x^5+\frac{1}{2}\left(2\,a^4c+6\,a^2bc^2+b^2c^3\right)x^4+2\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\left(4\,a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

```
[Out] 1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*b
^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a^5
 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3)*x
^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c^4 +
 a)*x
```

mupad [B] time = 0.10, size = 180, normalized size = 5.81

$$x^6\left(\frac{a^4\,b}{2}+\frac{3\,a^2\,b^2\,c}{2}+\frac{b^3\,c^2}{4}\right)+x^4\left(a^4\,c+3\,a^2\,b\,c^2+\frac{b^2\,c^3}{2}\right)+x^2\left(2\,a^2\,c^3+\frac{b\,c^4}{2}+\frac{b}{2}\right)+x^5\left(\frac{a^5}{5}+2\,a^3\,b\,c+\frac{3\,a\,b^2\,c^2}{2}\right)+\frac{b^5\,x^{10}}{160}+x^8\left(\frac{a^2\,b^3}{4}+\frac{c\,b^4}{16}\right)+\frac{a\,b^4\,x^9}{16}+a\,x\left(c^4+1\right)+\frac{a\,b^2\,x^7\left(a^2+b\,c\right)}{2}+2\,a\,c^2\,x^3\left(a^2+b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

```
[Out] x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
+ 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^2*
c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) + (a*
b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(b*c
+ a^2)
```

sympy [B] time = 0.11, size = 194, normalized size = 6.26

$$\frac{ab^4x^9}{16} + \frac{b^5x^{10}}{160} + x^8\left(\frac{a^2b^3}{4} + \frac{b^4c}{16}\right) + x^7\left(\frac{a^3b^2}{2} + \frac{ab^3c}{2}\right) + x^6\left(\frac{a^4b}{2} + \frac{3a^2b^2c}{2} + \frac{b^3c^2}{4}\right) + x^5\left(\frac{a^5}{5} + 2a^3bc + \frac{3ab^2c^2}{2}\right) + x^4\left(a^4c + 3a^2bc^2 + \frac{b^2c^3}{2}\right) + x^3\left(2a^3c^2 + 2abc^3\right) + x^2\left(2a^2c^3 + \frac{bc^4}{2} + \frac{b}{2}\right) + x\left(ac^4 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

```
[Out] a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a*
*3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4) +
 x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*c**
2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3 + b*
c**4/2 + b/2) + x*(a*c**4 + a)
```

3.204 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 1.00

$$\frac{x(2a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+n+1\right)}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] (x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

IntegrateAlgebraic [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

```
[Out] (2*a*x + b*x^2)/2 + Defer[IntegrateAlgebraic][(a + b*x)*(a*x + (b*x^2)/2)^n
, x]
```

fricas [A] time = 0.63, size = 48, normalized size = 1.41

$$\frac{(bn+b)x^2 + (bx^2+2\,ax)\left(\frac{1}{2}\,bx^2+ax\right)^n + 2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/(
n + 1)
```

giac [A] time = 0.40, size = 30, normalized size = 0.88

$$\frac{1}{2}\,bx^2 + ax + \frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{b\,x^2}{2} + ax + \frac{\left(\frac{1}{2}b\,x^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)
```

```
[Out] a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(n+1)/(n+1)
```

maxima [A] time = 1.13, size = 52, normalized size = 1.53

$$\frac{1}{2}\,bx^2 + ax + \frac{(bx^2+2\,ax)e^{(n\log(bx+2\,a)+n\log(x))}}{2^{n+1}n+2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n + 1
)*n + 2^(n + 1))
```

mupad [B] time = 2.12, size = 31, normalized size = 0.91

$$\frac{x\,(2\,a+b\,x)\,\left(n+\left(\frac{b\,x^2}{2}+a\,x\right)^n+1\right)}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] (x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

sympy [A] time = 50.75, size = 230, normalized size = 6.76

$$\begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for } b=0 \wedge n=-1 \\ a\left(\frac{a^n x x^n}{n+1}+\frac{nx}{n+1}+\frac{x}{n+1}\right) & \text{for } b=0 \\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for } n=-1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2 x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx\left(2ax+bx^2\right)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2\left(2ax+bx^2\right)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

```
[Out] Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(a**n*x*x**n/(n + 1)
 + n*x/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a/
b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*x/
(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**n*b
**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*b*n
+ 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), True))
```

3.205 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal. Leaf size=35

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)dx &= \text{Subst}\left(\int (1+x^n)\ dx, x, c+ax+\frac{bx^2}{2}\right)\\ &= ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 1.00

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

```
[Out] (2*a*x + b*x^2)/2 + Defer[IntegrateAlgebraic][(a + b*x)*(c + a*x + (b*x^2)/
2)^n, x]
```

fricas [A] time = 1.05, size = 52, normalized size = 1.49

$$\frac{(bn+b)x^2 + \left(bx^2 + 2\,ax + 2\,c\right)\left(\frac{1}{2}\,bx^2 + ax + c\right)^n + 2\,(an+a)x}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

```
[Out] 1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*n
 + a)*x)/(n + 1)
```

giac [A] time = 0.25, size = 32, normalized size = 0.91

$$\frac{1}{2}\,bx^2 + ax + c + \frac{\left(\frac{1}{2}\,bx^2 + ax + c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 33, normalized size = 0.94

$$\frac{b\,x^2}{2} + ax + c + \frac{\left(\frac{1}{2}b\,x^2 + ax + c\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)
```

```
[Out] c+a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(n+1)/(n+1)
```

maxima [A] time = 1.16, size = 54, normalized size = 1.54

$$\frac{1}{2}\,bx^2 + ax + \frac{\left(bx^2 + 2\,ax + 2\,c\right)\left(bx^2 + 2\,ax + 2\,c\right)^n}{2^{n+1}n + 2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)*
n + 2^(n + 1))
```

mupad [B] time = 2.11, size = 58, normalized size = 1.66

$$a\,x + \left(\frac{b\,x^2}{2} + a\,x + c\right)^n\,\left(\frac{2\,c}{2\,n+2} + \frac{b\,x^2}{2\,n+2} + \frac{2\,a\,x}{2\,n+2}\right) + \frac{b\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

```
[Out] a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*x
)/(2*n + 2)) + (b*x^2)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

```
[Out] Timed out
```

3.206 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=30

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx,x,ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.01, size = 93, normalized size = 3.10

$$\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*a
^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/4374
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5), x]
```

fricas [B] time = 0.97, size = 77, normalized size = 2.57

$$\frac{1}{4374}x^{18}c^6 + \frac{1}{243}x^{16}c^5a + \frac{5}{162}x^{14}c^4a^2 + \frac{10}{81}x^{12}c^3a^3 + \frac{5}{18}x^{10}c^2a^4 + \frac{1}{3}x^8ca^5 + \frac{1}{6}x^6a^6 + \frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 5/162*x^14*c^4*a^2 + 10/81*x^12*c^3*a^
3 + 5/18*x^10*c^2*a^4 + 1/3*x^8*c*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + x*a
```

giac [A] time = 0.22, size = 24, normalized size = 0.80

$$\frac{1}{4374}\left(cx^3 + 3\,ax\right)^6 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

maple [B] time = 0.00, size = 78, normalized size = 2.60

$$\frac{1}{4374}c^6x^{18} + \frac{1}{243}a\,c^5x^{16} + \frac{5}{162}a^2c^4x^{14} + \frac{10}{81}a^3c^3x^{12} + \frac{5}{18}a^4c^2x^{10} + \frac{1}{3}a^5c\,x^8 + \frac{1}{6}a^6x^6 + \frac{1}{3}c\,x^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+5/162*a^2*c^4*x^14+10/81*a^3*c^3*x^12+5/18
*a^4*c^2*x^10+1/3*a^5*c*x^8+1/6*a^6*x^6+1/3*c*x^3+a*x
```

maxima [B] time = 0.44, size = 77, normalized size = 2.57

$$\frac{1}{4374}\,c^6x^{18} + \frac{1}{243}\,ac^5x^{16} + \frac{5}{162}\,a^2c^4x^{14} + \frac{10}{81}\,a^3c^3x^{12} + \frac{5}{18}\,a^4c^2x^{10} + \frac{1}{3}\,a^5cx^8 + \frac{1}{6}\,a^6x^6 + \frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^1
2 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

mupad [B] time = 0.05, size = 77, normalized size = 2.57

$$\frac{a^6\,x^6}{6} + \frac{a^5\,c\,x^8}{3} + \frac{5\,a^4\,c^2\,x^{10}}{18} + \frac{10\,a^3\,c^3\,x^{12}}{81} + \frac{5\,a^2\,c^4\,x^{14}}{162} + \frac{a\,c^5\,x^{16}}{243} + a\,x + \frac{c^6\,x^{18}}{4374} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
```

```
[Out] a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x^
16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162
```

sympy [B] time = 0.09, size = 87, normalized size = 2.90

$$\frac{a^6x^6}{6} + \frac{a^5cx^8}{3} + \frac{5a^4c^2x^{10}}{18} + \frac{10a^3c^3x^{12}}{81} + \frac{5a^2c^4x^{14}}{162} + \frac{ac^5x^{16}}{243} + ax + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
+ 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x**3
/3
```

3.207 $\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx, x, d+ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.05, size = 140, normalized size = 4.52

$$\frac{x\left(3a+cx^2\right)\left(243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+1215d^4\left(3ax+cx^3\right)+540d^3\left(3ax+cx^3\right)^2+135d^2\left(3ax+cx^3\right)^3+18d\left(3ax+cx^3\right)^4+c^5x^{15}+1458d^5+1458\right)}{4374}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] (x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*c
^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c*x
^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*x +
 c*x^3)^4))/4374
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5), x]
```

fricas [B] time = 0.82, size = 291, normalized size = 9.39

$$\frac{1}{4374}x^{18}c^{6}+\frac{1}{243}x^{16}c^{5}a+\frac{1}{243}x^{15}dc^{5}+\frac{5}{162}x^{14}c^{4}a^{2}+\frac{5}{81}x^{13}dc^{4}a+\frac{5}{162}x^{12}d^{2}c^{4}+\frac{10}{81}x^{12}c^{3}a^{3}+\frac{10}{27}x^{11}dc^{3}a^{2}+\frac{10}{27}x^{10}d^{2}c^{3}a+\frac{5}{18}x^{10}c^{2}a^{4}+\frac{10}{81}x^{9}d^{3}c^{3}+\frac{10}{9}x^{9}dc^{2}a^{3}+\frac{5}{3}x^{8}d^{2}c^{2}a^{2}+\frac{1}{3}x^{8}ca^{5}+\frac{10}{9}x^{7}d^{3}c^{2}a+\frac{5}{3}x^{7}dca^{4}+\frac{5}{18}x^{6}d^{4}c^{2}+\frac{10}{3}x^{6}d^{2}ca^{3}+\frac{1}{6}x^{6}a^{6}+\frac{10}{3}x^{5}d^{3}ca^{2}+x^{5}da^{5}+\frac{5}{3}x^{4}d^{4}ca+\frac{5}{2}x^{4}d^{2}a^{4}+\frac{1}{3}x^{3}d^{5}c+\frac{10}{3}x^{3}d^{3}a^{3}+\frac{5}{2}x^{2}d^{4}a^{2}+xd^{5}a+\frac{1}{3}x^{3}c+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/243*x^16*c^5*a + 1/243*x^15*d*c^5 + 5/162*x^14*c^4*a^2
+ 5/81*x^13*d*c^4*a + 5/162*x^12*d^2*c^4 + 10/81*x^12*c^3*a^3 + 10/27*x^11*
d*c^3*a^2 + 10/27*x^10*d^2*c^3*a + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 +
10/9*x^9*d*c^2*a^3 + 5/3*x^8*d^2*c^2*a^2 + 1/3*x^8*c*a^5 + 10/9*x^7*d^3*c^2
*a + 5/3*x^7*d*c*a^4 + 5/18*x^6*d^4*c^2 + 10/3*x^6*d^2*c*a^3 + 1/6*x^6*a^6
+ 10/3*x^5*d^3*c*a^2 + x^5*d*a^5 + 5/3*x^4*d^4*c*a + 5/2*x^4*d^2*a^4 + 1/3*
x^3*d^5*c + 10/3*x^3*d^3*a^3 + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*c + x*a
```

giac [B] time = 0.34, size = 105, normalized size = 3.39

$$\frac{1}{4374}\left(cx^{3}+3\,ax\right)^{6}+\frac{1}{243}\left(cx^{3}+3\,ax\right)^{5}d+\frac{5}{162}\left(cx^{3}+3\,ax\right)^{4}d^{2}+\frac{10}{81}\left(cx^{3}+3\,ax\right)^{3}d^{3}+\frac{5}{18}\left(cx^{3}+3\,ax\right)^{2}d^{4}+\frac{1}{3}\left(cx^{3}+3\,ax\right)d^{5}+\frac{1}{3}\,cx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*x
)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*(c
*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x
```

maple [B] time = 0.00, size = 618, normalized size = 19.94

$$\frac{c^{6}x^{18}}{4374}+\frac{a\,c^{5}x^{16}}{243}+\frac{c^{5}d\,x^{15}}{243}+\frac{5a^{2}c^{4}x^{14}}{162}+\frac{5a\,c^{4}d\,x^{13}}{81}+\frac{10a^{2}c^{3}d\,x^{11}}{27}+\frac{\left(\frac{10a^{3}c^{3}}{27}+\left(\frac{2a^{3}c^{2}}{3}+\frac{4c^{3}d^{2}}{27}+\frac{\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)c}{3}\right)c\right)x^{12}}{12}+\frac{\left(\left(\frac{2a^{3}c^{2}}{3}+\frac{4c^{3}d^{2}}{27}+\frac{\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)c}{3}\right)a+\left(\frac{4ac^{2}d^{2}}{3}+\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)a+\frac{\left(a^{4}+4acd^{2}\right)c}{3}\right)c\right)x^{10}}{10}+\frac{\left(\frac{10a^{3}c^{2}d}{3}+\left(4a^{3}cd+\frac{\left(4a^{3}d+\frac{4}{3}cd^{3}\right)c}{3}+\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)d\right)c\right)x^{9}}{9}+\frac{5a^{2}d^{4}x^{2}}{2}+\frac{\left(\left(\frac{4ac^{2}d^{2}}{3}+\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)a+\frac{\left(a^{4}+4acd^{2}\right)c}{3}\right)a+\left(6a^{2}cd^{2}+\left(a^{4}+4acd^{2}\right)a\right)c\right)x^{8}}{8}+\frac{\left(\left(4a^{3}cd+\frac{\left(4a^{3}d+\frac{4}{3}cd^{3}\right)c}{3}+\left(\frac{4}{3}a^{3}c+\frac{2}{3}c^{2}d^{2}\right)d\right)a+\left(\left(a^{4}+4acd^{2}\right)d+\left(4a^{3}d+\frac{4}{3}cd^{3}\right)a+\frac{4}{3}acd^{3}\right)c\right)x^{7}}{7}+\frac{\left(\left(6a^{2}cd^{2}+\left(a^{4}+4acd^{2}\right)a\right)a+\left(\left(4a^{3}d+\frac{4}{3}cd^{3}\right)d+6a^{3}d^{2}+\frac{1}{3}cd^{4}\right)c\right)x^{6}}{6}+\frac{\left(\left(\left(a^{4}+4acd^{2}\right)d+\left(4a^{3}d+\frac{4}{3}cd^{3}\right)a+\frac{4}{3}acd^{3}\right)a+10a^{2}cd^{3}\right)x^{5}}{5}+\frac{\left(\left(\left(4a^{3}d+\frac{4}{3}cd^{3}\right)d+6a^{3}d^{2}+\frac{1}{3}cd^{4}\right)a+5acd^{4}\right)x^{4}}{4}+\frac{\left(10a^{3}d^{3}+\left(d^{5}+1\right)c\right)x^{3}}{3}+\left(d^{5}+1\right)ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/243*a*c^5*x^16+1/243*c^5*d*x^15+5/162*a^2*c^4*x^14+5/81*a
*c^4*d*x^13+1/12*(10/27*a^3*c^3+c*(4/27*d^2*c^3+2/3*a^3*c^2+1/3*c*(2/3*d^2*
c^2+4/3*a^3*c)))*x^12+10/27*a^2*c^3*d*x^11+1/10*(a*(4/27*d^2*c^3+2/3*a^3*c^
2+1/3*c*(2/3*d^2*c^2+4/3*a^3*c))+c*(4/3*d^2*a*c^2+a*(2/3*d^2*c^2+4/3*a^3*c)
+1/3*c*(a^4+4*a*c*d^2)))*x^10+1/9*(10/3*a^3*d*c^2+c*(d*(2/3*d^2*c^2+4/3*a^3
*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d)))*x^9+1/8*(a*(4/3*d^2*a*c^2+a*(2/3*
d^2*c^2+4/3*a^3*c)+1/3*c*(a^4+4*a*c*d^2))+c*(6*d^2*a^2*c+a*(a^4+4*a*c*d^2))
)*x^8+1/7*(a*(d*(2/3*d^2*c^2+4/3*a^3*c)+4*a^3*d*c+1/3*c*(4/3*d^3*c+4*a^3*d)
)+c*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a))*x^7+1/6*(a*(6*d^
2*a^2*c+a*(a^4+4*a*c*d^2))+c*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4))*x
^6+1/5*(a*(d*(a^4+4*a*c*d^2)+a*(4/3*d^3*c+4*a^3*d)+4/3*c*d^3*a)+10*a^2*c*d^
3)*x^5+1/4*(a*(d*(4/3*d^3*c+4*a^3*d)+6*a^3*d^2+1/3*c*d^4)+5*a*c*d^4)*x^4+1/
3*(10*a^3*d^3+c*(d^5+1))*x^3+5/2*a^2*d^4*x^2+a*(d^5+1)*x
```

maxima [B] time = 0.46, size = 280, normalized size = 9.03

$$\frac{1}{4374}\,c^{6}x^{18}+\frac{1}{243}\,ac^{5}x^{16}+\frac{1}{243}\,c^{5}dx^{15}+\frac{5}{162}\,a^{2}c^{4}x^{14}+\frac{5}{81}\,ac^{4}dx^{13}+\frac{10}{27}\,a^{2}c^{3}dx^{11}+\frac{5}{162}\left(4\,a^{3}c^{3}+c^{4}d^{2}\right)x^{12}+\frac{5}{54}\left(3\,a^{4}c^{2}+4\,ac^{3}d^{2}\right)x^{10}+\frac{10}{81}\left(9\,a^{3}c^{2}d+c^{3}d^{3}\right)x^{9}+\frac{1}{3}\left(a^{5}c+5\,a^{2}c^{2}d^{2}\right)x^{8}+\frac{5}{2}\,a^{2}d^{4}x^{2}+\frac{5}{9}\left(3\,a^{4}cd+2\,ac^{2}d^{3}\right)x^{7}+\frac{1}{18}\left(3\,a^{6}+60\,a^{3}cd^{2}+5\,c^{2}d^{4}\right)x^{6}+\frac{1}{3}\left(3\,a^{5}d+10\,a^{2}cd^{3}\right)x^{5}+\frac{5}{6}\left(3\,a^{4}d^{2}+2\,acd^{4}\right)x^{4}+\frac{1}{3}\left(10\,a^{3}d^{3}+cd^{5}+c\right)x^{3}+\left(ad^{5}+a\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
+ 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*x^
```

```
12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)*x^
9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d + 2*
a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*a^5*d
 + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*d^3 +
c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

mupad [B] time = 2.27, size = 266, normalized size = 8.58

$$x^5\left(a^5 d+\frac{10 c a^2 d^3}{3}\right)+x^4\left(\frac{5 a^4 d^2}{2}+\frac{5 c a d^4}{3}\right)+x^3\left(\frac{10 a^3 d^3}{3}+\frac{c d^5}{3}+\frac{c}{3}\right)+x^6\left(\frac{a^6}{6}+\frac{10 a^3 c d^2}{3}+\frac{5 c^2 d^4}{18}\right)+\frac{c^6 x^{18}}{4374}+\frac{a c^5 x^{16}}{243}+a x\left(d^5+1\right)+\frac{c^5 d x^{15}}{243}+\frac{5 a^2 c^4 x^{14}}{162}+\frac{5 a^2 d^4 x^2}{2}+\frac{5 c^3 x^{12}\left(4 a^3+c d^2\right)}{162}+\frac{a^2 c x^8\left(a^3+5 c d^2\right)}{3}+\frac{10 a^2 c^3 d x^{11}}{27}+\frac{5 a c^2 x^{10}\left(3 a^3+4 c d^2\right)}{54}+\frac{10 c^2 d x^9\left(9 a^3+c d^2\right)}{81}+\frac{5 a c^4 d x^{13}}{81}+\frac{5 a c d x^7\left(3 a^3+2 c d^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)
```

```
[Out] x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3*
(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^3*
c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*x^1
5)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2 + 4*
a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (5*a*c
^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (5*a*c^
4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9
```

sympy [B] time = 0.14, size = 314, normalized size = 10.13

$$\frac{5a^2c^4x^{14}}{162}+\frac{10a^2c^3dx^{11}}{27}+\frac{5a^2d^4x^2}{2}+\frac{ac^5x^{16}}{243}+\frac{5ac^4dx^{13}}{81}+\frac{c^6x^{18}}{4374}+\frac{c^5dx^{15}}{243}+x^{12}\left(\frac{10a^3c^3}{81}+\frac{5c^4d^2}{162}\right)+x^{10}\left(\frac{5a^4c^2}{18}+\frac{10ac^3d^2}{27}\right)+x^9\left(\frac{10a^3c^2d}{9}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^2c^2d^2}{3}\right)+x^7\left(\frac{5a^4cd}{3}+\frac{10ac^2d^3}{9}\right)+x^6\left(\frac{a^6}{6}+\frac{10a^3cd^2}{3}+\frac{5c^2d^4}{18}\right)+x^5\left(a^5d+\frac{10a^2cd^3}{3}\right)+x^4\left(\frac{5a^4d^2}{2}+\frac{5acd^4}{3}\right)+x^3\left(\frac{10a^3d^3}{3}+\frac{cd^5}{3}+\frac{c}{3}\right)+x\left(ad^5+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

```
[Out] 5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c*
*5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 + x
**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*c**
3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a**6/6
+ 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/3) + x
**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3 + c/3)
 + x*(a*d**5 + a)
```

3.208 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=34

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{x^{12}(3b+2cx)^6}{279936}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (x^12*(3*b + 2*c*x)^6)/279936
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int (1+x^5)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}\end{aligned}$$

Mathematica [B] time = 0.01, size = 98, normalized size = 2.88

$$\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14)
/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (c^
6*x^18)/4374
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5), x]
```

fricas [B] time = 0.64, size = 80, normalized size = 2.35

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{5}{324}x^{15}c^3b^3+\frac{5}{288}x^{14}c^2b^4+\frac{1}{96}x^{13}cb^5+\frac{1}{384}x^{12}b^6+\frac{1}{3}x^3c+\frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 5/324*x^15*c^3*b^
3 + 5/288*x^14*c^2*b^4 + 1/96*x^13*c*b^5 + 1/384*x^12*b^6 + 1/3*x^3*c + 1/2
*x^2*b
```

giac [A] time = 0.29, size = 30, normalized size = 0.88

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

maple [B] time = 0.00, size = 81, normalized size = 2.38

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}b\,c^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}b^5c\,x^{13}+\frac{1}{384}b^6x^{12}+\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+5/324*b^3*c^3*x^15+5/28
8*b^4*c^2*x^14+1/96*b^5*c*x^13+1/384*b^6*x^12+1/3*c*x^3+1/2*b*x^2
```

maxima [B] time = 0.44, size = 80, normalized size = 2.35

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{5}{324}\,b^3c^3x^{15}+\frac{5}{288}\,b^4c^2x^{14}+\frac{1}{96}\,b^5cx^{13}+\frac{1}{384}\,b^6x^{12}+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^1
5 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1/2
*b*x^2
```

mupad [B] time = 0.07, size = 80, normalized size = 2.35

$$\frac{b^6\,x^{12}}{384}+\frac{b^5\,c\,x^{13}}{96}+\frac{5\,b^4\,c^2\,x^{14}}{288}+\frac{5\,b^3\,c^3\,x^{15}}{324}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,c^5\,x^{17}}{486}+\frac{b\,x^2}{2}+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
+ (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c
^4*x^16)/648
```

sympy [B] time = 0.10, size = 90, normalized size = 2.65

$$\frac{b^6x^{12}}{384}+\frac{b^5cx^{13}}{96}+\frac{5b^4c^2x^{14}}{288}+\frac{5b^3c^3x^{15}}{324}+\frac{5b^2c^4x^{16}}{648}+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**1
5/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/43
74 + c*x**3/3
```

3.209 $\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1591}

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx &= \text{Subst}\left(\int \left(1+x^5\right)\,dx, x, d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.05, size = 146, normalized size = 3.56

$$\frac{x^2(3b+2cx)\left(243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+19440d^4x^2(3b+2cx)+4320d^3x^4(3b+2cx)^2+540d^2x^6(3b+2cx)^3+36dx^8(3b+2cx)^4+32c^5x^{15}+46656d^5+46656\right)}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 108
0*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*d^
4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b + 2*c
*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5), x]
```

fricas [B] time = 0.92, size = 298, normalized size = 7.27

$$\frac{1}{4374}x^{18}c^6+\frac{1}{486}x^{17}c^5b+\frac{5}{648}x^{16}c^4b^2+\frac{1}{243}x^{15}dc^5+\frac{5}{324}x^{15}c^3b^3+\frac{5}{162}x^{14}dc^4b+\frac{5}{288}x^{14}c^2b^4+\frac{5}{54}x^{13}dc^3b^2+\frac{1}{96}x^{13}cb^5+\frac{5}{162}x^{12}d^2c^4+\frac{5}{36}x^{12}dc^2b^3+\frac{1}{384}x^{12}b^6+\frac{5}{27}x^{11}d^2c^3b+\frac{5}{48}x^{11}dcb^4+\frac{5}{12}x^{10}d^2c^2b^2+\frac{1}{32}x^{10}db^5+\frac{10}{81}x^9d^3c^3+\frac{5}{12}x^9d^2cb^3+\frac{5}{9}x^8d^3c^2b+\frac{5}{32}x^8d^2b^4+\frac{5}{6}x^7d^3cb^2+\frac{5}{18}x^6d^4c^2+\frac{5}{12}x^6d^3b^3+\frac{5}{6}x^5d^4cb+\frac{5}{8}x^4d^4b^2+\frac{1}{3}x^3d^5c+\frac{1}{2}x^2d^5b+\frac{1}{3}x^3c+\frac{1}{2}x^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^15*d*c^5
+ 5/324*x^15*c^3*b^3 + 5/162*x^14*d*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^13*
d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/
384*x^12*b^6 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/12*x^10*d^2*c^2*
b^2 + 1/32*x^10*d*b^5 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^2*c*b^3 + 5/9*x^8*d^
3*c^2*b + 5/32*x^8*d^2*b^4 + 5/6*x^7*d^3*c*b^2 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 5/6*x^5*d^4*c*b + 5/8*x^4*d^4*b^2 + 1/3*x^3*d^5*c + 1/2*x^2*d^5
*b + 1/3*x^3*c + 1/2*x^2*b
```

giac [B] time = 0.31, size = 126, normalized size = 3.07

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(2
*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3 +
 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

maple [B] time = 0.00, size = 646, normalized size = 15.76

$$\frac{c^6x^{18}}{4374}+\frac{bc^5x^{17}}{486}+\frac{5b^2c^4x^{16}}{648}+$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+5/648*b^2*c^4*x^16+1/15*(5/54*b^3*c^3+c*(1
/81*d*c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c)))*x^15+1/14*(b*(1/81*d*
c^4+1/12*b^3*c^2+1/3*c*(4/27*c^3*d+1/6*b^3*c))+c*(2/27*d*b*c^3+1/2*b*(4/27*
c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4)))*x^14+1/13*(b*(2/27*d*b*c^3+
1/2*b*(4/27*c^3*d+1/6*b^3*c)+1/3*c*(2/3*d*b*c^2+1/16*b^4))+c*(1/2*d*b^2*c^2
+1/2*b*(2/3*d*b*c^2+1/16*b^4)))*x^13+1/12*(b*(1/2*d*b^2*c^2+1/2*b*(2/3*d*b*
c^2+1/16*b^4))+c*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c^2*d^2+1
/2*d*b^3)))*x^12+1/11*(b*(d*(4/27*c^3*d+1/6*b^3*c)+1/2*b^3*d*c+1/3*c*(2/3*c
^2*d^2+1/2*d*b^3))+c*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2+1/2*d*b^3
)+2/3*c^2*d^2*b))*x^11+1/10*(b*(d*(2/3*d*b*c^2+1/16*b^4)+1/2*b*(2/3*c^2*d^2
+1/2*d*b^3)+2/3*c^2*d^2*b)+5/2*b^2*c^2*d^2)*x^10+1/9*(5/2*b^3*c*d^2+c*(d*(2
/3*c^2*d^2+1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3))*x^9+1/8*(b*(d*(2/3*c^2*d^2+
1/2*d*b^3)+3/4*b^3*d^2+4/9*c^2*d^3)+10/3*c^2*d^3*b)*x^8+5/6*b^2*c*d^3*x^7+1
/6*(5/2*b^3*d^3+5/3*c^2*d^4)*x^6+5/6*b*c*d^4*x^5+5/8*b^2*d^4*x^4+1/3*(d^5+1
)*c*x^3+1/2*b*(d^5+1)*x^2
```

maxima [B] time = 0.45, size = 289, normalized size = 7.05

$$\frac{1}{4374}\,c^6x^{18}+\frac{1}{486}\,bc^5x^{17}+\frac{5}{648}\,b^2c^4x^{16}+\frac{1}{972}\left(15\,b^3c^3+4\,c^5d\right)x^{15}+\frac{5}{2592}\left(9\,b^4c^2+16\,bc^4d\right)x^{14}+\frac{1}{864}\left(9\,b^5c+80\,b^2c^3d\right)x^{13}+\frac{5}{6}\,b^2cd^3x^7+\frac{1}{10368}\left(27\,b^6+1440\,b^3c^2d+320\,c^4d^2\right)x^{12}+\frac{5}{432}\left(9\,b^4cd+16\,bc^3d^2\right)x^{11}+\frac{5}{6}\,bcd^4x^5+\frac{1}{96}\left(3\,b^5d+40\,b^2c^2d^2\right)x^{10}+\frac{5}{8}\,b^2d^4x^4+\frac{5}{324}\left(27\,b^3cd^2+8\,c^3d^3\right)x^9+\frac{5}{288}\left(9\,b^4d^2+32\,bc^2d^3\right)x^8+\frac{5}{36}\left(3\,b^3d^3+2\,c^2d^4\right)x^6+\frac{1}{3}\left(cd^5+c\right)x^3+\frac{1}{2}\left(bd^5+b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^3
 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c +
80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*d +
 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^4*x^
5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27*b^3*
c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*(3*b^3
*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

mupad [B] time = 2.28, size = 273, normalized size = 6.66

$$x^{13}\left(\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)+x^{14}\left(\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right)+\frac{c^6\,x^{18}}{4374}+x^{15}\left(\frac{5\,b^3\,c^3}{324}+\frac{d\,c^5}{243}\right)+\frac{5\,d^3\,x^6\,\left(3\,b^3+2\,d\,c^2\right)}{36}+\frac{b\,c^5\,x^{17}}{486}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,x^2\,\left(d^5+1\right)}{2}+\frac{5\,b^2\,d^4\,x^4}{8}+\frac{c\,x^3\,\left(d^5+1\right)}{3}+\frac{5\,b^2\,c\,d^3\,x^7}{6}+\frac{5\,b\,d^2\,x^8\,\left(9\,b^3+32\,d\,c^2\right)}{288}+\frac{b^2\,d\,x^{10}\,\left(3\,b^3+40\,d\,c^2\right)}{96}+\frac{5\,c\,d^2\,x^9\,\left(27\,b^3+8\,d\,c^2\right)}{324}+\frac{5\,b\,c\,d^4\,x^5}{6}+\frac{5\,b\,c\,d\,x^{11}\,\left(9\,b^3+16\,d\,c^2\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

```
[Out] x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)/
162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/437
4 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))/36
 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b^2*d
^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(32*c^
2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9*(8*c^
2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9*b^3))/
432
```

sympy [B] time = 0.15, size = 321, normalized size = 7.83

$$\frac{5b^2c^4x^{16}}{648}+\frac{5b^2cd^3x^7}{6}+\frac{5b^2d^4x^4}{8}+\frac{bc^5x^{17}}{486}+\frac{5bcd^4x^5}{6}+\frac{c^6x^{18}}{4374}+x^{15}\left(\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\left(\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\left(\frac{b^5c}{96}+\frac{5b^2c^3d}{54}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)+x^{11}\left(\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)+x^{10}\left(\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)+x^9\left(\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)+x^6\left(\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)+x^3\left(\frac{cd^5}{3}+\frac{c}{3}\right)+x^2\left(\frac{bd^5}{2}+\frac{b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] 5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5*
x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324 +
c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/96 +
 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162)
+ x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2*c**2
*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*d**2/3
2 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3*(c*d**
5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

3.210 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal. Leaf size=46

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.06, size = 244, normalized size = 5.30

$$\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}+a\left(\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}+x\right)+\frac{x^2\left(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b\left(c^5x^{15}+243\right)+64cx\left(c^5x^{15}+1458\right)\right)}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
(5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(x
+ (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x^14
)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916*b^5
*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 + 576*b
*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),
x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),
 x]
```

fricas [B] time = 0.93, size = 309, normalized size = 6.72

$$\frac{1}{4374}x^{18}c^{6}+\frac{1}{486}x^{17}c^{5}b+\frac{5}{648}x^{16}c^{4}b^{2}+\frac{1}{243}x^{16}c^{5}a+\frac{5}{324}x^{15}c^{3}b^{3}+\frac{5}{162}x^{15}c^{4}ba+\frac{5}{288}x^{14}c^{2}b^{4}+\frac{5}{54}x^{14}c^{3}b^{2}a+\frac{5}{162}x^{14}c^{4}a^{2}+\frac{1}{96}x^{13}cb^{5}+\frac{5}{36}x^{13}c^{2}b^{3}a+\frac{5}{27}x^{13}c^{3}ba^{2}+\frac{1}{384}x^{12}b^{6}+\frac{5}{48}x^{12}cb^{4}a+\frac{5}{12}x^{12}c^{2}b^{2}a^{2}+\frac{10}{81}x^{12}c^{3}a^{3}+\frac{1}{32}x^{11}b^{5}a+\frac{5}{12}x^{11}cb^{3}a^{2}+\frac{5}{9}x^{11}c^{2}ba^{3}+\frac{5}{32}x^{10}b^{4}a^{2}+\frac{5}{6}x^{10}cb^{2}a^{3}+\frac{5}{18}x^{10}c^{2}a^{4}+\frac{5}{12}x^{9}b^{3}a^{3}+\frac{5}{6}x^{9}cba^{4}+\frac{5}{8}x^{8}b^{2}a^{4}+\frac{1}{3}x^{8}ca^{5}+\frac{1}{2}x^{7}ba^{5}+\frac{1}{6}x^{6}a^{6}+\frac{1}{3}x^{3}c+\frac{1}{2}x^{2}b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="frica
s")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/288*x^14*c^2*b^4 + 5/54*x^14*
c^3*b^2*a + 5/162*x^14*c^4*a^2 + 1/96*x^13*c*b^5 + 5/36*x^13*c^2*b^3*a + 5/
27*x^13*c^3*b*a^2 + 1/384*x^12*b^6 + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*
a^2 + 10/81*x^12*c^3*a^3 + 1/32*x^11*b^5*a + 5/12*x^11*c*b^3*a^2 + 5/9*x^11
*c^2*b*a^3 + 5/32*x^10*b^4*a^2 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 5
/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 1/2*x
^7*b*a^5 + 1/6*x^6*a^6 + 1/3*x^3*c + 1/2*x^2*b + x*a
```

giac [A] time = 0.31, size = 37, normalized size = 0.80

$$\frac{1}{279936}\left(2\,cx^{3}+3\,bx^{2}+6\,ax\right)^{6}+\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac"
)
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 1523, normalized size = 33.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+c*(1/81*a*
c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)))*x^16+1/15*
(5/162*a*b*c^4+b*(1/81*a*c^4+1/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+
1/9*b^2*c^2))+c*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)
+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+1/14*(a*(1/81*a*c^4+1
/27*b^2*c^3+1/3*c*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2))+b*(2/27*a*b*c^3+
1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a
*c+1/4*b^2)*b*c))+c*(a*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a
*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1
/4*b^2)^2)))*x^14+1/13*(a*(2/27*a*b*c^3+1/2*b*(2/9*(2/3*a*c+1/4*b^2)*c^2+1/
9*b^2*c^2)+1/3*c*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+b*(a*(2/9*(2/3*a*
c+1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1
/3*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+c*(a*(2/9*a*b*c^2+2/3*(
2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1
/3*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(a*(2/9*(2/3*a*c+
1/4*b^2)*c^2+1/9*b^2*c^2)+1/2*b*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3
*c*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2))+b*(a*(2/9*a*b*c^2+2/3*(2/
3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3
```

```
*c*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+c*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/
3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*
(2/3*a*c+1/4*b^2)+a^2*b^2)))*x^12+1/11*(a*(a*(2/9*a*b*c^2+2/3*(2/3*a*c+1/4*
b^2)*b*c)+1/2*b*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(2/3*a^
2*b*c+2*a*b*(2/3*a*c+1/4*b^2)))+b*(a*(2/9*a^2*c^2+2/3*a*b^2*c+(2/3*a*c+1/4*
b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*c*(2*a^2*(2/3*a*c+1
/4*b^2)+a^2*b^2))+c*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(
2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b))*x^11+1/10*(a*(a*(2/9*a^2*c^2+2/3*a*
b^2*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b^2))+1/3*
c*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2))+b*(a*(2/3*a^2*b*c+2*a*b*(2/3*a*c+1/4*b
^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+c*(a*(2*a^2*(2/3*
a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4))*x^10+1/9*(a*(a*(2/3*a^2*b*c+2*a*b
*(2/3*a*c+1/4*b^2))+1/2*b*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+2/3*c*a^3*b)+b*
(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*c*a^4*b)*x^9+1/
8*(a*(a*(2*a^2*(2/3*a*c+1/4*b^2)+a^2*b^2)+b^2*a^3+1/3*c*a^4)+5/2*b^2*a^4+c*
a^5)*x^8+1/2*a^5*b*x^7+1/6*a^6*x^6+1/3*c*x^3+1/2*b*x^2+a*x
```

maxima [B] time = 0.45, size = 289, normalized size = 6.28

$$\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15b^2c^4+8ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2abc^4\right)x^{15}+\frac{5}{2592}\left(9b^4c^2+48ab^2c^3+16a^2c^4\right)x^{14}+\frac{1}{864}\left(9b^5c+120ab^3c^2+160a^2bc^3\right)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}\left(27b^6+1080ab^4c+4320a^2b^2c^2+1280a^3c^3\right)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}\left(9ab^5+120a^2b^3c+160a^3bc^2\right)x^{11}+\frac{5}{288}\left(9a^2b^4+48a^3b^2c+16a^4c^2\right)x^{10}+\frac{5}{12}\left(a^3b^3+2a^4bc\right)x^9+\frac{1}{24}\left(15a^4b^2+8a^5c\right)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxim
a")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 5
/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a^2
*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*a^5
*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3)*
x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x^11 +
 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 + 2*a^4
*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

mupad [B] time = 2.24, size = 270, normalized size = 5.87

$$x^{12}\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{b^6}{384}\right)+ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{a^6x^6}{6}+\frac{c^6x^{18}}{4374}+\frac{5a^2x^{10}\left(16a^2c^2+48ab^2c+9b^4\right)}{288}+\frac{5c^2x^{14}\left(16a^2c^2+48ab^2c+9b^4\right)}{2592}+\frac{a^5bx^7}{2}+\frac{bc^5x^{17}}{486}+\frac{a^4x^8\left(15b^2+8ac\right)}{24}+\frac{c^4x^{16}\left(15b^2+8ac\right)}{1944}+\frac{abx^{11}\left(160a^2c^2+120ab^2c+9b^4\right)}{288}+\frac{bcx^{13}\left(160a^2c^2+120ab^2c+9b^4\right)}{864}+\frac{5a^3bx^9\left(b^2+2ac\right)}{12}+\frac{5bc^3x^{15}\left(b^2+2ac\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a*
x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(9*
b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 + 48*
a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 15*b^
2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2*c^2
+ 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/864 +
(5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

sympy [B] time = 0.16, size = 323, normalized size = 7.02

$$\frac{a^6x^6}{6}+\frac{a^5bx^7}{2}+ax+\frac{bc^5x^{17}}{486}+\frac{bx^2}{2}+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}+x^{16}\left(\frac{ac^5}{243}+\frac{5b^2c^4}{648}\right)+x^{15}\left(\frac{5abc^4}{162}+\frac{5b^3c^3}{324}\right)+x^{14}\left(\frac{5a^2c^4}{162}+\frac{5ab^2c^3}{54}+\frac{5b^4c^2}{288}\right)+x^{13}\left(\frac{5a^2bc^3}{27}+\frac{5ab^3c^2}{36}+\frac{b^5c}{96}\right)+x^{12}\left(\frac{10a^3c^3}{81}+\frac{5a^2b^2c^2}{12}+\frac{5ab^4c}{48}+\frac{b^6}{384}\right)+x^{11}\left(\frac{5a^3bc^2}{9}+\frac{5a^2b^3c}{12}+\frac{ab^5}{32}\right)+x^{10}\left(\frac{5a^4c^2}{18}+\frac{5a^3b^2c}{6}+\frac{5a^2b^4}{32}\right)+x^9\left(\frac{5a^4bc}{6}+\frac{5a^3b^3}{12}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^4b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**1
8/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c**
4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5*b*
*4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96) + x*
```

```
*12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384) + x*
*11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*c**2/1
8 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*b**3/12
) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

3.211 $\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$

Optimal. Leaf size=47

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1591}

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx &= \text{Subst}\left(\int \left(1+x^5\right)dx, x, d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{1}{6}\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6\end{aligned}$$

Mathematica [B] time = 0.12, size = 248, normalized size = 5.28

$$\frac{x(6a+x(3b+2cx))\left(7776a^5x^5+6480a^4x^6(3b+2cx)+2160a^3x^7(3b+2cx)^2+360a^2x^8(3b+2cx)^3+19440d^4x(6a+x(3b+2cx))+4320d^3x^2(6a+x(3b+2cx))^2+540d^2x^3(6a+x(3b+2cx))^3+36dx^4(6a+x(3b+2cx))^4+30ax^9(3b+2cx)^4+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x^{15}+46656d^5+46656\right)}{279936}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^10
 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 +
 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2 +
360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6*a +
 x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x^3*(6
*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)
^5),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)
^5), x]
```

fricas [B] time = 0.96, size = 928, normalized size = 19.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fri
cas")
```

```
[Out] 1/4374*x^18*c^6 + 1/486*x^17*c^5*b + 5/648*x^16*c^4*b^2 + 1/243*x^16*c^5*a
+ 1/243*x^15*d*c^5 + 5/324*x^15*c^3*b^3 + 5/162*x^15*c^4*b*a + 5/162*x^14*d
*c^4*b + 5/288*x^14*c^2*b^4 + 5/54*x^14*c^3*b^2*a + 5/162*x^14*c^4*a^2 + 5/
54*x^13*d*c^3*b^2 + 1/96*x^13*c*b^5 + 5/81*x^13*d*c^4*a + 5/36*x^13*c^2*b^3
*a + 5/27*x^13*c^3*b*a^2 + 5/162*x^12*d^2*c^4 + 5/36*x^12*d*c^2*b^3 + 1/384
*x^12*b^6 + 10/27*x^12*d*c^3*b*a + 5/48*x^12*c*b^4*a + 5/12*x^12*c^2*b^2*a^
2 + 10/81*x^12*c^3*a^3 + 5/27*x^11*d^2*c^3*b + 5/48*x^11*d*c*b^4 + 5/6*x^11
*d*c^2*b^2*a + 1/32*x^11*b^5*a + 10/27*x^11*d*c^3*a^2 + 5/12*x^11*c*b^3*a^2
 + 5/9*x^11*c^2*b*a^3 + 5/12*x^10*d^2*c^2*b^2 + 1/32*x^10*d*b^5 + 10/27*x^1
0*d^2*c^3*a + 5/6*x^10*d*c*b^3*a + 5/3*x^10*d*c^2*b*a^2 + 5/32*x^10*b^4*a^2
 + 5/6*x^10*c*b^2*a^3 + 5/18*x^10*c^2*a^4 + 10/81*x^9*d^3*c^3 + 5/12*x^9*d^
2*c*b^3 + 5/3*x^9*d^2*c^2*b*a + 5/16*x^9*d*b^4*a + 5/2*x^9*d*c*b^2*a^2 + 10
/9*x^9*d*c^2*a^3 + 5/12*x^9*b^3*a^3 + 5/6*x^9*c*b*a^4 + 5/9*x^8*d^3*c^2*b +
 5/32*x^8*d^2*b^4 + 5/2*x^8*d^2*c*b^2*a + 5/3*x^8*d^2*c^2*a^2 + 5/4*x^8*d*b
^3*a^2 + 10/3*x^8*d*c*b*a^3 + 5/8*x^8*b^2*a^4 + 1/3*x^8*c*a^5 + 5/6*x^7*d^3
*c*b^2 + 10/9*x^7*d^3*c^2*a + 5/4*x^7*d^2*b^3*a + 5*x^7*d^2*c*b*a^2 + 5/2*x
^7*d*b^2*a^3 + 5/3*x^7*d*c*a^4 + 1/2*x^7*b*a^5 + 5/18*x^6*d^4*c^2 + 5/12*x^
6*d^3*b^3 + 10/3*x^6*d^3*c*b*a + 15/4*x^6*d^2*b^2*a^2 + 10/3*x^6*d^2*c*a^3
+ 5/2*x^6*d*b*a^4 + 1/6*x^6*a^6 + 5/6*x^5*d^4*c*b + 5/2*x^5*d^3*b^2*a + 10/
3*x^5*d^3*c*a^2 + 5*x^5*d^2*b*a^3 + x^5*d*a^5 + 5/8*x^4*d^4*b^2 + 5/3*x^4*d
^4*c*a + 5*x^4*d^3*b*a^2 + 5/2*x^4*d^2*a^4 + 1/3*x^3*d^5*c + 5/2*x^3*d^4*b*
a + 10/3*x^3*d^3*a^3 + 1/2*x^2*d^5*b + 5/2*x^2*d^4*a^2 + x*d^5*a + 1/3*x^3*
c + 1/2*x^2*b + x*a
```

giac [B] time = 0.44, size = 153, normalized size = 3.26

$$\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2+6\,ax\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2+6\,ax\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2+6\,ax\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2+6\,ax\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2+6\,ax\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gia
c")
```

```
[Out] 1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x)
^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2
+ 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 + 3*
b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [B] time = 0.00, size = 4284, normalized size = 91.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

```
[Out] 1/4374*c^6*x^18+1/486*b*c^5*x^17+1/16*(1/243*a*c^5+5/162*b^2*c^4+(1/81*a*c^
      4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*c)*c)*x^16+1/15*
      (5/162*a*b*c^4+(1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b
      ^2)*c^2)*c)*b+c*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*
      b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)))*x^15+
      1/14*((1/81*a*c^4+1/27*b^2*c^3+1/3*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*
      c)*a+b*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)
      *b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c))+c*(2/27*b*c^3*d
      +(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b)*c^2+2/3
      *(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3
      *a*c+1/4*b^2)^2)))*x^14+1/13*(a*(1/81*c^4*d+2/27*a*b*c^3+1/2*(1/9*b^2*c^2+2
      /9*(2/3*a*c+1/4*b^2)*c^2)*b+1/3*c*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b
      ^2)*b*c))+b*(2/27*b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(
      2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2
      /3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2))+c*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1
      /4*b^2)*c^2)+a*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9
      *(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^
      2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))))*x^13+1/12*(a*(2/27
      *b*c^3*d+(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)*a+1/2*b*(2/9*(2/3*c*d+a*b)
      *c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/3*c*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*
      b*c+(2/3*a*c+1/4*b^2)^2))+b*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2
      /9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/
      3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b
      *c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+c*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3
      *a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*
      b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
      ^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
      a*b)^2)))*x^12+1/11*(a*(d*(1/9*b^2*c^2+2/9*(2/3*a*c+1/4*b^2)*c^2)+a*(2/9*(2
      /3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+1/2*b*(2/9*(a^2+b*d)*c^2+2/3*(2/
      3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/3*c*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*
      (2/3*c*d+a*b)*(2/3*a*c+1/4*b^2)))+b*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+
      1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^
      2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
      1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
      2))+c*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4
      /9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*
      c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2
      /3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))))*x^11+1/10*(
      a*(d*(2/9*(2/3*c*d+a*b)*c^2+2/3*(2/3*a*c+1/4*b^2)*b*c)+a*(2/9*(a^2+b*d)*c^2
      +2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+1/2*b*(4/9*a*d*c^2+2/3*(a^2+b*d
      )*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/3*c*(2/9*c^2*d^2+4/3*a*d*b*c+2*(
      a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2))+b*(d*(2/9*(a^2+b*d)*c^2+2/3*(2
      /3*c*d+a*b)*b*c+(2/3*a*c+1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/
      3*c*d+a*b)*(2/3*a*c+1/4*b^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2
      /3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)
      +2*(a^2+b*d)*(2/3*c*d+a*b)))+c*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d
      +a*b)*(2/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/
      4*b^2)+(2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b
      *d)*(2/3*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+
      b*d)^2)))*x^10+1/9*(a*(d*(2/9*(a^2+b*d)*c^2+2/3*(2/3*c*d+a*b)*b*c+(2/3*a*c+
      1/4*b^2)^2)+a*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b
      ^2))+1/2*b*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+
      a*b)^2)+1/3*c*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b
      )))+b*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2/3*a*c+1/4*b^2))+
      a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+1
      /2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/3*c*
      (2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))+c*(d*(2/9*c^2*d^
      2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c
      +4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1
```

```
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(
a^2+b*d))))*x^9+1/8*(a*(d*(4/9*a*d*c^2+2/3*(a^2+b*d)*b*c+2*(2/3*c*d+a*b)*(2
/3*a*c+1/4*b^2))+a*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(
2/3*c*d+a*b)^2)+1/2*b*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3
*c*d+a*b))+1/3*c*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2))
+b*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4*b^2)+(2/3*c*d+a*b)^
2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+1/2*b*
(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+1/3*c*(2*d^2*(2/3
*c*d+a*b)+4*a*d*(a^2+b*d)))+c*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^
2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b
*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^2*(a^2+b*d)+4
*a^2*d^2)))*x^8+1/7*(a*(d*(2/9*c^2*d^2+4/3*a*d*b*c+2*(a^2+b*d)*(2/3*a*c+1/4
*b^2)+(2/3*c*d+a*b)^2)+a*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1/4*b^2)+2*(a^2+b*d)*(
2/3*c*d+a*b))+1/2*b*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^
2)+1/3*c*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d)))+b*(d*(2/3*d^2*b*c+4*a*d*(2/
3*a*c+1/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*
(2/3*c*d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*
c*(2*d^2*(a^2+b*d)+4*a^2*d^2))+c*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d
+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^
2+b*d)+4*a^2*d^2)+4/3*a*c*d^3))*x^7+1/6*(a*(d*(2/3*d^2*b*c+4*a*d*(2/3*a*c+1
/4*b^2)+2*(a^2+b*d)*(2/3*c*d+a*b))+a*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*
d+a*b)+(a^2+b*d)^2)+1/2*b*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/3*c*(2*d^
2*(a^2+b*d)+4*a^2*d^2))+b*(d*(2*d^2*(2/3*a*c+1/4*b^2)+4*a*d*(2/3*c*d+a*b)+(
a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+1/2*b*(2*d^2*(a^2+b*d)+
4*a^2*d^2)+4/3*a*c*d^3)+c*(d*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2
*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4))*x^6+1/5*(a*(d*(2*d^2*(2/3*a*c+1
/4*b^2)+4*a*d*(2/3*c*d+a*b)+(a^2+b*d)^2)+a*(2*d^2*(2/3*c*d+a*b)+4*a*d*(a^2+
b*d))+1/2*b*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4/3*a*c*d^3)+b*(d*(2*d^2*(2/3*c*d+a
*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*d^4)+c*(
d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4))*x^5+1/4*(a*(d*(2*d^2*(2
/3*c*d+a*b)+4*a*d*(a^2+b*d))+a*(2*d^2*(a^2+b*d)+4*a^2*d^2)+2*b*d^3*a+1/3*c*
d^4)+b*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*a*c*d^4)*x^4+1
/3*(a*(d*(2*d^2*(a^2+b*d)+4*a^2*d^2)+4*a^2*d^3+1/2*b*d^4)+5*b*d^4*a+(d^5+1)
*c)*x^3+1/2*(5*a^2*d^4+b*(d^5+1))*x^2+(d^5+1)*a*x
```

maxima [B] time = 0.48, size = 773, normalized size = 16.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="max
ima")
```

```
[Out] 1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 + 1
/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^
2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2 +
480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 1080*a
*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^2 + 8
*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 + 160*b
*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*(135*a^
2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2 + 9*(3*b
^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 216*a^4*b*
c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^2*b^2*c +
32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d^3 + 15*(3*
b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c)*d)*x^8 + 1
/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2*b*c)*d^2 + 3
0*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 10*c^2*d^4 + 15
*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1/6*(6*a^5*d + 3
0*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 + 5/24*(12*a^4*d^2
```

```
 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d^3 + 15*a*b*d^4 +
 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (a*d^5 + a)*x
```

mupad [B] time = 2.45, size = 753, normalized size = 16.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

```
[Out] x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10*
a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3) +
 x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*b*c
^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) + x^9
*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/12 +
(5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/2) + x
^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4*d)/162
) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36 +
(5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^6/6 + (5*
b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2)/4 + (5*a
^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3 + (5*
a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^2)/9 + (10*a
^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^2*d)/6) + x^7
*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d^3)/9 + (5*b^2
*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d^5)/2 + (5*a^2*
d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^3)/27 + (5*b^2*c
^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2 + 5*a^3*b*d^2 + (
10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d^2*x^4*(12*a^4 + 3
*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) + (b*c^5*x^17)/486 +
 (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4*x^16*(8*a*c + 15*b^2))
/1944
```

sympy [B] time = 0.27, size = 930, normalized size = 19.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

```
[Out] b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648) +
 x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c**4
/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a**2
*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3*d/5
4) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10*a*b*
c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5*a**3*
b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b**2*c**2
*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 + 5*a**3*b
**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10*a*c**3*d
**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 + 5*a**3*b**
3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*a*b*c**2*d*
*2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5*a**4*b**2/8
 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*a*b**2*c*d**2
/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a**4*c*d/3 + 5*
a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**2*d**3/9 + 5*b*
*2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2/3 + 15*a**2*b**
2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/18) + x**5*(a**5*
d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 + 5*b*c*d**4/6) + x*
*4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**2*d**4/8) + x**3*(1
0*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**2*(5*a**2*d**4/2 + b*d*
*5/2 + b/2) + x*(a*d**5 + a)
```

3.212 $\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=34

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1591}

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
[Out] a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{cx^3}{3}\right)\\ &= ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.07, size = 36, normalized size = 1.06

$$\frac{x\left(3a+cx^2\right)\left(\left(ax+\frac{cx^3}{3}\right)^n+n+1\right)}{3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
[Out] (x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

IntegrateAlgebraic [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+cx^2\right)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

```
[Out] (3*a*x + c*x^3)/3 + Defer[IntegrateAlgebraic][(a + c*x^2)*(a*x + (c*x^3)/3)
^n, x]
```

fricas [A] time = 1.03, size = 48, normalized size = 1.41

$$\frac{(cn+c)x^3 + \left(cx^3 + 3\,ax\right)\left(\frac{1}{3}\,cx^3 + ax\right)^n + 3\,(an+a)x}{3\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

```
[Out] 1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/(
n + 1)
```

giac [A] time = 0.42, size = 30, normalized size = 0.88

$$\frac{1}{3}\,cx^3 + ax + \frac{\left(\frac{1}{3}\,cx^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 31, normalized size = 0.91

$$\frac{c\,x^3}{3} + ax + \frac{\left(\frac{1}{3}c\,x^3 + ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(n+1)/(n+1)
```

maxima [A] time = 1.14, size = 54, normalized size = 1.59

$$\frac{1}{3}\,cx^3 + ax + \frac{\left(cx^3 + 3\,ax\right)e^{\left(n\log\left(cx^2+3\,a\right)+n\log\left(x\right)\right)}}{3^{n+1}n + 3^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n +
 1)*n + 3^(n + 1))
```

mupad [B] time = 2.11, size = 33, normalized size = 0.97

$$\frac{x\,\left(c\,x^2+3\,a\right)\,\left(n+{\left(\frac{c\,x^3}{3}+a\,x\right)}^n+1\right)}{3\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
```

```
[Out] (x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

sympy [B] time = 112.00, size = 201, normalized size = 5.91

$$\begin{cases} \frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n} + \frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n} + \frac{3ax\left(3ax+cx^3\right)^n}{3\cdot 3^n n+3\cdot 3^n} + \frac{cx^3\left(3ax+cx^3\right)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n \neq -1 \\ ax + \frac{cx^3}{3} + \log(x) + \log\left(-\sqrt{3}\,i\sqrt{a}\sqrt{\frac{1}{c}} + x\right) + \log\left(\sqrt{3}\,i\sqrt{a}\sqrt{\frac{1}{c}} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

```
[Out] Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**n
) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) + 3
*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**n/(
3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(-sqrt(3)*I*s
qrt(a)*sqrt(1/c) + x) + log(sqrt(3)*I*sqrt(a)*sqrt(1/c) + x), True))
```

3.213 $\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal. Leaf size=44

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1591}

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx &= \text{Subst}\left(\int \left(1+x^n\right)\,dx, x, \frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= \frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.09, size = 42, normalized size = 0.95

$$\frac{x^2(3b+2cx)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

IntegrateAlgebraic [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \left(bx+cx^2\right)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

[Out] (x^2*(3*b + 2*c*x))/6 + Defer[IntegrateAlgebraic][x*(b + c*x)*((b*x^2)/2 + (c*x^3)/3)^n, x]

fricas [A] time = 1.05, size = 57, normalized size = 1.30

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")

[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 + 1/2*b*x^2)^n)/(n + 1)

giac [A] time = 0.39, size = 36, normalized size = 0.82

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)

maple [A] time = 0.00, size = 37, normalized size = 0.84

$$\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+\frac{\left(\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)

[Out] 1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)

maxima [A] time = 1.17, size = 71, normalized size = 1.61

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(2\,cx^3+3\,bx^2\right)e^{\left(n\log(2\,cx+3\,b)+2\,n\log(x)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))

mupad [B] time = 2.21, size = 37, normalized size = 0.84

$$\frac{x^2\,\left(3\,b+2\,c\,x\right)\,\left(n+{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)}^n+1\right)}{6\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)

[Out] (x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

3.214 $\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$

Optimal. Leaf size=50

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1591}

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx &= \text{Subst}\left(\int (1+x^n)\,dx, x, ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)\\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}\end{aligned}$$

Mathematica [A] time = 0.19, size = 49, normalized size = 0.98

$$\frac{x(6a+x(3b+2cx))\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+n+1\right)}{6(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

```
[Out] (x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(1
 + n))
```

IntegrateAlgebraic [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),
x]
```

```
[Out] (x*(6*a + 3*b*x + 2*c*x^2))/6 + Defer[IntegrateAlgebraic][(a + b*x + c*x^2)
*(a*x + (b*x^2)/2 + (c*x^3)/3)^n, x]
```

fricas [A] time = 1.16, size = 72, normalized size = 1.44

$$\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2+6\,ax\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="frica
s")
```

```
[Out] 1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*c
*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

giac [A] time = 0.39, size = 42, normalized size = 0.84

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac"
)
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 43, normalized size = 0.86

$$\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+ax+\frac{\left(\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+ax\right)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(n+1)/(n+1)
```

maxima [A] time = 1.22, size = 83, normalized size = 1.66

$$\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(2\,cx^3+3\,bx^2+6\,ax\right)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log(x)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxim
a")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
+ 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

mupad [B] time = 2.17, size = 73, normalized size = 1.46

$$a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\,{\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)}^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
```

```
[Out] a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
+ (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

```
[Out] Timed out
```

3.215 $\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1588}

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] (5 - 12*x + 6*x^2 + x^3)^2/6
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.74

$$\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] -20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3), x]
```

fricas [A] time = 0.74, size = 29, normalized size = 1.53

$$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
[Out] 1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

giac [A] time = 0.35, size = 30, normalized size = 1.58

$$\frac{5}{3}\,x^3 + \frac{1}{6}\left(x^3 + 6\,x^2 - 12\,x\right)^2 + 10\,x^2 - 20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
[Out] 5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

maple [A] time = 0.00, size = 30, normalized size = 1.58

$$\frac{1}{6}x^6 + 2x^5 + 2x^4 - \frac{67}{3}x^3 + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)
[Out] 1/6*x^6+2*x^5+2*x^4-67/3*x^3+34*x^2-20*x
```

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{6}\left(x^3 + 6\,x^2 - 12\,x + 5\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
[Out] 1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

mupad [B] time = 0.03, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2\,x^5 + 2\,x^4 - \frac{67\,x^3}{3} + 34\,x^2 - 20\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
[Out] 34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

sympy [A] time = 0.06, size = 29, normalized size = 1.53

$$\frac{x^6}{6} + 2x^5 + 2x^4 - \frac{67x^3}{3} + 34x^2 - 20x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
[Out] x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

3.216 $\int (2x + x^3) (1 + 4x^2 + x^4) \, dx$

Optimal. Leaf size=16

$$\frac{1}{8} \left(x^4 + 4x^2 + 1\right)^2$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{8} \left(x^4 + 4x^2 + 1\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] (1 + 4*x^2 + x^4)^2/8
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{1}{8} \left(1 + 4x^2 + x^4\right)^2$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.31

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(2*x + x^3)*(1 + 4*x^2 + x^4), x]
```

fricas [A] time = 0.94, size = 17, normalized size = 1.06

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

giac [A] time = 0.23, size = 22, normalized size = 1.38

$$\frac{1}{4}x^4 + \frac{1}{8}\left(x^4 + 4x^2\right)^2 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

maple [A] time = 0.00, size = 18, normalized size = 1.12

$$\frac{1}{8}x^8 + x^6 + \frac{9}{4}x^4 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)*(x^4+4*x^2+1),x)
[Out] 1/8*x^8+x^6+9/4*x^4+x^2
```

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$\frac{1}{8}\left(x^4 + 4x^2 + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
[Out] 1/8*(x^4 + 4*x^2 + 1)^2
```

mupad [B] time = 0.03, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
[Out] x^2 + (9*x^4)/4 + x^6 + x^8/8
```

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$\frac{x^8}{8} + x^6 + \frac{9x^4}{4} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)*(x**4+4*x**2+1),x)
[Out] x**8/8 + x**6 + 9*x**4/4 + x**2
```

3.217 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

Rubi [B] time = 0.20, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1593, 1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx &= \int x^3(1+x)^3(1+2x)\left(-18+7\left(x+x^2\right)^3\right)^2\,dx\\ &= \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-\right.\\ &= 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-\end{aligned}$$

Mathematica [B] time = 0.01, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2, x]
```

fricas [B] time = 0.73, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^{9} - 171x^{8} + 288x^{7} + 486x^{6} + 324x^{5} + 81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

giac [A] time = 0.29, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^{2} + x\right)^{10} - 36\left(x^{2} + x\right)^{7} + 81\left(x^{2} + x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^{9} - 171x^{8} + 288x^{7} + 486x^{6} + 324x^{5} + 81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

maxima [B] time = 0.44, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20} + 49x^{19} + \frac{441}{2}x^{18} + 588x^{17} + 1029x^{16} + \frac{6174}{5}x^{15} + 993x^{14} + 336x^{13} - \frac{1071}{2}x^{12} - 1211x^{11} - \frac{12551}{10}x^{10} - 756x^{9} - 171x^{8} + 288x^{7} + 486x^{6} + 324x^{5} + 81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

mupad [B] time = 0.22, size = 86, normalized size = 2.61

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^{9} - 171x^{8} + 288x^{7} + 486x^{6} + 324x^{5} + 81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

sympy [B] time = 0.08, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10} + 49x^{19} + \frac{441x^{18}}{2} + 588x^{17} + 1029x^{16} + \frac{6174x^{15}}{5} + 993x^{14} + 336x^{13} - \frac{1071x^{12}}{2} - 1211x^{11} - \frac{12551x^{10}}{10} - 756x^9 - 171x^8 + 288x^7 + 486x^6 + 324x^5 + 81x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.218 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal. Leaf size=33

$$\frac{49}{10}x^{10}(x+1)^{10}-36x^7(x+1)^7+81x^4(x+1)^4$$

Rubi [B] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1612}

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \int \left(324x^3+1620x^4+2916x^5+2016x^6-1368x^7-6804x^8-12\right.$$

$$=81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1$$

Mathematica [B] time = 0.00, size = 96, normalized size = 2.91

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2, x]
```

fricas [B] time = 0.66, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

giac [A] time = 0.36, size = 28, normalized size = 0.85

$$\frac{49}{10}\left(x^{2}+x\right)^{10}-36\left(x^{2}+x\right)^{7}+81\left(x^{2}+x\right)^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

```
[Out] 49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

maple [B] time = 0.00, size = 87, normalized size = 2.64

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x+1)^3*(2*x+1)*(-18+7*x^3*(x+1)^3)^2,x)
```

```
[Out] 49/10*x^20+49*x^19+441/2*x^18+588*x^17+1029*x^16+6174/5*x^15+993*x^14+336*x
^13-1071/2*x^12-1211*x^11-12551/10*x^10-756*x^9-171*x^8+288*x^7+486*x^6+324
*x^5+81*x^4
```

maxima [B] time = 0.45, size = 86, normalized size = 2.61

$$\frac{49}{10}x^{20}+49x^{19}+\frac{441}{2}x^{18}+588x^{17}+1029x^{16}+\frac{6174}{5}x^{15}+993x^{14}+336x^{13}-\frac{1071}{2}x^{12}-1211x^{11}-\frac{12551}{10}x^{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

```
[Out] 49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 99
3*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 171
*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

mupad [B] time = 0.19, size = 86, normalized size = 2.61

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

```
[Out] 81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
- 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*x^
16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

sympy [B] time = 0.09, size = 94, normalized size = 2.85

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

```
[Out] 49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15/
5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 - 75
6*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

3.219 $\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$

Optimal. Leaf size=14

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1588}

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5}\,dx=\frac{1}{12\left(1-6x+x^3\right)^4}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{12\left(x^3-6x+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] 1/(12*(1 - 6*x + x^3)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

```
[Out] IntegrateAlgebraic[(2 - x^2)/(1 - 6*x + x^3)^5, x]
```

fricas [B] time = 0.84, size = 57, normalized size = 4.07

$$\frac{1}{12\left(x^{12}-24\,x^{10}+4\,x^{9}+216\,x^{8}-72\,x^{7}-858\,x^{6}+432\,x^{5}+1224\,x^{4}-860\,x^{3}+216\,x^{2}-24\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
```

```
[Out] 1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224*
x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

giac [A] time = 0.32, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^{3}-6\,x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{12\left(x^{3}-6x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)/(x^3-6*x+1)^5,x)
```

```
[Out] 1/12/(x^3-6*x+1)^4
```

maxima [A] time = 0.44, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^{3}-6\,x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
```

```
[Out] 1/12/(x^3 - 6*x + 1)^4
```

mupad [B] time = 2.10, size = 12, normalized size = 0.86

$$\frac{1}{12\left(x^{3}-6\,x+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

```
[Out] 1/(12*(x^3 - 6*x + 1)^4)
```

sympy [B] time = 0.19, size = 56, normalized size = 4.00

$$\frac{1}{12x^{12}-288x^{10}+48x^{9}+2592x^{8}-864x^{7}-10296x^{6}+5184x^{5}+14688x^{4}-10320x^{3}+2592x^{2}-288x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

```
[Out] 1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 518
4*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

3.220 $\int \frac{2x+x^2}{4+3x^2+x^3} dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(2*x + x^2)/(4 + 3*x^2 + x^3), x]
```

fricas [A] time = 1.02, size = 13, normalized size = 0.87

$$\frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

giac [A] time = 0.37, size = 14, normalized size = 0.93

$$\frac{1}{3}\log\left(\left|x^3+3x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x)
[Out] 1/3*ln(x^3+3*x^2+4)
```

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
[Out] log(3*x^2 + x^3 + 4)/3
```

sympy [A] time = 0.09, size = 12, normalized size = 0.80

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
[Out] log(x**3 + 3*x**2 + 4)/3
```

3.221 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\, dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1587}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\, dx = \frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^3)/(4*x + 2*x^2 + x^4), x]
```

fricas [A] time = 0.77, size = 15, normalized size = 0.88

$$\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

giac [A] time = 0.27, size = 18, normalized size = 1.06

$$\frac{1}{4}\log\left(4\left|\frac{1}{4}x^4+\frac{1}{2}x^2+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
[Out] 1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln\left(\left(x^3+2x+4\right)x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)
[Out] 1/4*ln(x*(x^3+2*x+4))
```

maxima [A] time = 0.45, size = 15, normalized size = 0.88

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

mupad [B] time = 0.07, size = 13, normalized size = 0.76

$$\frac{\ln\left(x\left(x^3+2x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
[Out] log(x*(2*x + x^3 + 4))/4
```

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

3.222 $\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx$

Optimal. Leaf size=40

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Rubi [A] time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {6, 2102, 1588}

$$\frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x + e*x^
2 + f*x^3)^2,x]
```

```
[Out] a/(c + d*x + e*x^2 + f*x^3) + (b*x)/(c + d*x + e*x^2 + f*x^3)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{bc-ad-2aex-bex^2-3afx^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx &= \int \frac{bc-ad-2aex+(-be-3af)x^2-2bfx^3}{(c+dx+ex^2+fx^3)^2}\,dx\\ &= \frac{bx}{c+dx+ex^2+fx^3}-\frac{\int \frac{2adf+4aefx+6af^2x^2}{(c+dx+ex^2+fx^3)^2}\,dx}{2f}\\ &= \frac{a}{c+dx+ex^2+fx^3}+\frac{bx}{c+dx+ex^2+fx^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.58

$$\frac{a+bx}{c+dx+ex^2+fx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/(c + d*x
+ e*x^2 + f*x^3)^2,x]
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bc - ad - 2aex - bex^2 - 3afx^2 - 2bfx^3}{\left(c + dx + ex^2 + fx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/
(c + d*x + e*x^2 + f*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(b*c - a*d - 2*a*e*x - b*e*x^2 - 3*a*f*x^2 - 2*b*f*x^3)/
(c + d*x + e*x^2 + f*x^3)^2, x]
```

fricas [A] time = 1.00, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="fricas")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

giac [A] time = 0.54, size = 24, normalized size = 0.60

$$\frac{bx + a}{fx^3 + x^2e + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)/(f*x^3 + x^2*e + d*x + c)
```

maple [A] time = 0.01, size = 28, normalized size = 0.70

$$-\frac{-bx - a}{f\,x^3 + e\,x^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c)^2,x)
```

```
[Out] -(-b*x-a)/(f*x^3+e*x^2+d*x+c)
```

maxima [A] time = 0.50, size = 23, normalized size = 0.58

$$\frac{bx + a}{fx^3 + ex^2 + dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x^3-3*a*f*x^2-b*e*x^2-2*a*e*x-a*d+b*c)/(f*x^3+e*x^2+d*x+c
)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)/(f*x^3 + e*x^2 + d*x + c)
```

mupad [B] time = 0.11, size = 23, normalized size = 0.58

$$\frac{a+b\,x}{f\,x^3+e\,x^2+d\,x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*d - b*c + 2*a*e*x + 3*a*f*x^2 + b*e*x^2 + 2*b*f*x^3)/(c + d*x + e*x
^2 + f*x^3)^2,x)
```

```
[Out] (a + b*x)/(c + d*x + e*x^2 + f*x^3)
```

sympy [A] time = 32.56, size = 22, normalized size = 0.55

$$-\frac{-a-bx}{c+dx+ex^2+fx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*b*f*x**3-3*a*f*x**2-b*e*x**2-2*a*e*x-a*d+b*c)/(f*x**3+e*x**2+
d*x+c)**2,x)
```

```
[Out] -(-a - b*x)/(c + d*x + e*x**2 + f*x**3)
```

3.223 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal. Leaf size=605

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\right.\right.}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}$$

Rubi [A] time = 4.54, antiderivative size = 605, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2086, 634, 618, 204, 628}

$$\frac{\tan^{-1}\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2B\right)}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}-\frac{\tan^{-1}\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)\left(-a\left(A\left(\sqrt{8a^2-4ac+b^2}+b\right)+C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2B\right)}{\sqrt{2}\,a\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}-\frac{\log\left(x\left(b-\sqrt{8a^2-4ac+b^2}\right)+2ax^2+2a\right)\left(D\left(b-\sqrt{8a^2-4ac+b^2}\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}+\frac{\log\left(x\left(\sqrt{8a^2-4ac+b^2}+b\right)+2ax^2+2a\right)\left(D\left(\sqrt{8a^2-4ac+b^2}+b\right)+2a(A-C)\right)}{4a\sqrt{8a^2-4ac+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] ((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 + b^
2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sqrt[
8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])]])/(Sqrt[2]*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*
a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8*a^2
+ b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(
Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]])/(Sqrt[2]
*a*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 -
 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (
b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c]
) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b + Sqrt[
8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*a*Sqrt[8*a^2 + b^2 - 4*a*c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = -\frac{\int \frac{Ab-2aB-A\sqrt{8a^2+b^2-4ac}+2aD+\left(2aA-2aC+bD-\sqrt{8a^2+b^2-4ac}\,D\right)x}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{\sqrt{8a^2+b^2-4ac}} + \frac{\int \frac{Ab-2aB+A\sqrt{\cdots}}{\cdots}}{\cdots}$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\int \frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2}\,dx}{4a\sqrt{8a^2+b^2-4ac}} + \cdots$$

$$= -\frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)\cdots\right.}{4a\sqrt{8a^2+b^2-4ac}}\cdots$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{\cdots}\right.\right.}{\sqrt{2}\,a\sqrt{8a^2+b^2-4ac}\,\sqrt{4a^2+2ac-\cdots}}\cdots$$

Mathematica [C] time = 0.07, size = 98, normalized size = 0.16

$$\text{RootSum}\left[\#1^4a+\#1^3b+\#1^2c+\#1b+a\&,\frac{\#1^3D\log(x-\#1)+\#1^2C\log(x-\#1)+A\log(x-\#1)+\#1B\log(x-\#1)}{4\#1^3a+3\#1^2b+2\#1c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

```
[Out] RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
#1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2 +
4*a*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x
^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.16Not invertible Error: B
ad Argument Value
```

maple [B] time = 0.09, size = 2105, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

```
[Out] -1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)
*A+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*
a)*C+1/4/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D-1/4/a/(8*a^2-4*
a*c+b^2)^(1/2)*ln(-2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x-b*x-2*a)*D*b-1/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+
b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*A+1/(8
*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)
*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*
a*c+b^2)^(1/2))^(1/2))*b*A-1/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2)
)^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8
*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^
2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/(8*a^2-4*a
*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan(
(-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)
^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^
2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+
4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2+1/a/(8*a^2+4*a*c-2*b^
2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)
-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(8*a^2-4
*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arcta
n((-4*a*x+(8*a^2-4*a*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^
2)^(1/2))^(1/2))*B+1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)
^(1/2)*x+b*x+2*a)*A-1/2/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2
)^(1/2)*x+b*x+2*a)*C+1/4/a*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)*D+
1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*ln(2*a*x^2+(8*a^2-4*a*c+b^2)^(1/2)*x+b*x+2*a)
*D*b+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*
x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^
(1/2))*A+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)
^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2
*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*A+1/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*
c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2
*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*C+1/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2
+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*
c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*b*C+2/
```

```
(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/
2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4
*a*c+b^2)^(1/2))^(1/2))*D*c-1/a/(8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-
2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))
/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b^2-1/a/(8*a^2+4*
a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((b+4*a*x+(8*a^2-4*a*c+b
^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))*D*b-4*a/(
8*a^2-4*a*c+b^2)^(1/2)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2
)*arctan((b+4*a*x+(8*a^2-4*a*c+b^2)^(1/2))/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*
a*c+b^2)^(1/2))^(1/2))*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Dx^3 + Cx^2 + Bx + A}{ax^4 + bx^3 + cx^2 + bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxim
a")
```

```
[Out] integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x + C\,x^2 + x^3\,D}{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

3.224 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2086, 628}

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(1 + Sqrt[5])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx &= -\frac{\int \frac{-2\sqrt{5}+\left(10-2\sqrt{5}\right)x}{2+\left(-1-\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}+\frac{\int \frac{2\sqrt{5}+\left(10+2\sqrt{5}\right)x}{2+\left(-1+\sqrt{5}\right)x+2x^2}\,dx}{\sqrt{5}}\\ &= -\frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}}-\frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.87

$$\frac{1}{2}\left(\left(1+\sqrt{5}\right)\log\left(2x^2+\left(\sqrt{5}-1\right)x+2\right)-\left(\sqrt{5}-1\right)\log\left(-2x^2+\sqrt{5}x+x-2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] (-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
(-1 + Sqrt[5])*x + 2*x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4), x]
```

fricas [A] time = 1.87, size = 83, normalized size = 1.32

$$\frac{1}{2}\sqrt{5}\log\left(\frac{2x^4-2x^3+7x^2+\sqrt{5}\left(2x^3-x^2+2x\right)-2x+2}{x^4-x^3+x^2-x+1}\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
+ 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

giac [A] time = 0.27, size = 58, normalized size = 0.92

$$-\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/2
*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

maple [A] time = 0.05, size = 82, normalized size = 1.30

$$\frac{\ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{2}-\frac{\sqrt{5}\,\ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{2}+\frac{\ln\left(2x^2+\sqrt{5}\,x-x+2\right)}{2}+\frac{\sqrt{5}\,\ln\left(2x^2+\sqrt{5}\,x-x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)
```

```
[Out] 1/2*ln(x*5^(1/2)+2*x^2-x+2)+1/2*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)+1/2*ln(-x*5
^(1/2)+2*x^2-x+2)-1/2*ln(-x*5^(1/2)+2*x^2-x+2)*5^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^3-4x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

mupad [B] time = 0.18, size = 75, normalized size = 1.19

$$\frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}-\frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
```

```
[Out] log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/2
 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)*x
)/2 - x/2 + x^2 + 1))/2
```

sympy [A] time = 0.13, size = 58, normalized size = 0.92

$$\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

```
[Out] (1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)*
log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

3.225 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Rubi [A] time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1680, 14}

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx &= \int \frac{x\left(3+3x+x^2\right)}{1+4x+6x^2+4x^3+x^4}\,dx\\ &= \text{Subst}\left(\int \frac{-1+x^3}{x^4}\,dx,x,1+x\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x}\right)dx,x,1+x\right)\\ &= \frac{1}{3(1+x)^3}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] 1/(3*(1 + x)^3) + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
[Out] IntegrateAlgebraic[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4), x]
```

fricas [B] time = 1.17, size = 38, normalized size = 2.71

$$\frac{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)\log\left(x + 1\right) + 1}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
[Out] 1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

giac [A] time = 0.29, size = 13, normalized size = 0.93

$$\frac{1}{3\,(x + 1)^3} + \log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
[Out] 1/3/(x + 1)^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$\ln\left(x + 1\right) + \frac{1}{3\,(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)
[Out] 1/3/(x+1)^3+ln(x+1)
```

maxima [A] time = 0.44, size = 22, normalized size = 1.57

$$\frac{1}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)} + \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
[Out] 1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$\ln\left(x + 1\right) + \frac{1}{3\,(x + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + 1/(3*(x + 1)^3)
```

sympy [A] time = 0.10, size = 20, normalized size = 1.43

$$\log(x+1) + \frac{1}{3x^3 + 9x^2 + 9x + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

3.226 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal. Leaf size=28

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1680, 43}

$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] 8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1680

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (
b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;
 EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq,
x] && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx &= \text{Subst}\left(\int \frac{(-2+x)^3}{x^4} \, dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \left(-\frac{8}{x^4}+\frac{12}{x^3}-\frac{6}{x^2}+\frac{1}{x}\right) \, dx, x, 1+x\right)\\
&= \frac{8}{3(1+x)^3}-\frac{6}{(1+x)^2}+\frac{6}{1+x}+\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.86

$$\frac{2\left(9x^2+9x+4\right)}{3(x+1)^3}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

```
[Out] (2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4)
,x]
```

```
[Out] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4)
, x]
```

fricas [A] time = 1.10, size = 46, normalized size = 1.64

$$\frac{18\,x^2+3\left(x^3+3\,x^2+3\,x+1\right)\log\left(x+1\right)+18\,x+8}{3\left(x^3+3\,x^2+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

```
[Out] 1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^2
 + 3*x + 1)
```

giac [A] time = 0.39, size = 23, normalized size = 0.82

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\,(x+1)^3}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.96

$$\ln\left(x+1\right)+\frac{8}{3\,(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

```
[Out] 8/3/(x+1)^3-6/(x+1)^2+6/(x+1)+ln(x+1)
```

maxima [A] time = 0.45, size = 32, normalized size = 1.14

$$\frac{2\left(9\,x^2+9\,x+4\right)}{3\left(x^3+3\,x^2+3\,x+1\right)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

mupad [B] time = 0.04, size = 21, normalized size = 0.75

$$\ln\left(x+1\right)+\frac{6\,x^2+6\,x+\frac{8}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
[Out] log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

sympy [A] time = 0.11, size = 29, normalized size = 1.04

$$\frac{18x^2 + 18x + 8}{3x^3 + 9x^2 + 9x + 3} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
[Out] (18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

3.227 $$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$$

Optimal. Leaf size=59

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {1673, 1678, 1588, 1663, 1660, 8}

$$\frac{13x}{x^4+2x^2+3}-\frac{2\left(13x^2+18\right)x}{\left(x^4+2x^2+3\right)^2}+\frac{2\left(1-2x^2\right)}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 + x
^4)^3,x]
```

```
[Out] (2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4)
^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
```

```
&& !PolyQ[Pq, x^2]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx = \int \frac{x\left(-40+24x^4\right)}{\left(3+2x^2+x^4\right)^3}\,dx + \int \frac{9-18x^2+174x^4+26x^6-39}{\left(3+2x^2+x^4\right)^3}$$

$$= -\frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{3744-2496x^2-3744x^4}{\left(3+2x^2+x^4\right)^2}\,dx +$$

$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4} + \frac{1}{32}$$

$$= \frac{2\left(1-2x^2\right)}{\left(3+2x^2+x^4\right)^2} - \frac{2x\left(18+13x^2\right)}{\left(3+2x^2+x^4\right)^2} + \frac{13x}{3+2x^2+x^4}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{13x^5-4x^2+3x+2}{\left(x^4+2x^2+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x
^2 + x^4)^3,x]
```

```
[Out] (2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{\left(3+2x^2+x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)
/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)
/(3 + 2*x^2 + x^4)^3, x]
```

fricas [A] time = 1.87, size = 38, normalized size = 0.64

$$\frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="fricas")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [A] time = 1.44, size = 28, normalized size = 0.47

$$\frac{13\,x^5 - 4\,x^2 + 3\,x + 2}{\left(x^4 + 2\,x^2 + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="giac")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

maple [A] time = 0.01, size = 30, normalized size = 0.51

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{\left(x^4 + 2x^2 + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)
```

```
[Out] -(-13*x^5+4*x^2-3*x-2)/(x^4+2*x^2+3)^2
```

maxima [A] time = 0.46, size = 38, normalized size = 0.64

$$\frac{13\,x^5 - 4\,x^2 + 3\,x + 2}{x^8 + 4\,x^6 + 10\,x^4 + 12\,x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
algorithm="maxima")
```

```
[Out] (13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

mupad [B] time = 0.05, size = 28, normalized size = 0.47

$$\frac{13\,x^5 - 4\,x^2 + 3\,x + 2}{\left(x^4 + 2\,x^2 + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4 +
 3)^3,x)
```

```
[Out] (3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

sympy [A] time = 0.18, size = 36, normalized size = 0.61

$$-\frac{-13x^5 + 4x^2 - 3x - 2}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+3
)**3,x)
```

```
[Out] -(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

3.228 $\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} dx = -\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] -(x/(1 + x + x^5))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

```
[Out] IntegrateAlgebraic[(-1 + 4*x^5)/(1 + x + x^5)^2, x]
```

fricas [A] time = 1.22, size = 11, normalized size = 1.00

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^5 + x + 1)
```

giac [A] time = 0.33, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

maple [B] time = 0.01, size = 41, normalized size = 3.73

$$-\frac{-3x^2 + 5x - 1}{7\left(x^3 - x^2 + 1\right)} + \frac{-3x - 1}{7x^2 + 7x + 7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
```

```
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

maxima [A] time = 0.44, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

mupad [B] time = 2.30, size = 11, normalized size = 1.00

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

```
[Out] -x/(x + x^5 + 1)
```

sympy [A] time = 0.12, size = 8, normalized size = 0.73

$$-\frac{x}{x^5 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

3.229 $\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$

Optimal. Leaf size=91

$$\frac{x}{16\left(1-x^2\right)}+\frac{\left(29-5x^2\right)x}{32\left(x^4-6x^2+1\right)}+\frac{1}{4}\tanh^{-1}(x)+\frac{1}{64}\left(\left(3-2\sqrt{2}\right)\tanh^{-1}\left(\left(\sqrt{2}-1\right)x\right)-\left(3+2\sqrt{2}\right)\tanh^{-1}\left(\left(1+\sqrt{2}\right.\right.\right.$$

Rubi [B] time = 0.15, antiderivative size = 205, normalized size of antiderivative = 2.25, number of steps used = 15, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2073, 207, 638, 618, 206, 632, 31}

$$-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)-\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)+\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)-\frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}}+\frac{1}{4}\tanh^{-1}(x)+\frac{5\tanh^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] 1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 - 5
*x)/(64*(1 + 2*x - x^2)) - (5*ArcTanh[(1 - x)/Sqrt[2]])/(64*Sqrt[2]) + ArcT
anh[x]/4 + (5*ArcTanh[(1 + x)/Sqrt[2]])/(64*Sqrt[2]) - (3*(2 + 3*Sqrt[2])*L
og[1 - Sqrt[2] - x])/256 - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 + (
3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] + x])/256 + (3*(2 - 3*Sqrt[2])*Log[1 + Sq
rt[2] + x])/256
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
```

```
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx = \int \left(\frac{1}{32(-1+x)^2}+\frac{1}{32(1+x)^2}-\frac{1}{4\left(-1+x^2\right)}+\frac{17-7x}{32\left(-1-2x+x^2\right)^2}-\frac{3(-4 \ldots}{64\left(-1-2\ldots\right.}\right.$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{1}{32}\int \frac{17-7x}{\left(-1-2x+x^2\right)^2}\,dx+\frac{1}{32}\int \frac{17+7x}{\left(-1+2x+x^2\right)^2}\,dx \ldots$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)-\ldots$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}+\frac{1}{4}\tanh^{-1}(x)-\ldots$$

$$= \frac{1}{32(1-x)}-\frac{1}{32(1+x)}+\frac{12+5x}{64\left(1-2x-x^2\right)}-\frac{12-5x}{64\left(1+2x-x^2\right)}-\frac{5\tanh^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{64\sqrt{2}} \ldots$$

Mathematica [A] time = 0.09, size = 132, normalized size = 1.45

$$\frac{1}{128}\left(-\frac{4x\left(7x^4-46x^2+31\right)}{x^6-7x^4+7x^2-1}-16\log(1-x)+\left(3+2\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)+\left(2\sqrt{2}-3\right)\log\left(-x+\sqrt{2}+1\right)+16\log(x+1)-\left(3+2\sqrt{2}\right)\log\left(x+\sqrt{2}-1\right)+\left(3-2\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] ((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
(3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2] -
x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2, x]
```

fricas [B] time = 2.23, size = 223, normalized size = 2.45

$$-\frac{28\,x^5-184\,x^3-2\,\sqrt{2}\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(\frac{x^2-2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)-2\,\sqrt{2}\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(\frac{x^2-2\,\sqrt{2}(x-1)-2\,x+3}{x^2-2\,x-1}\right)-3\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(x^2+2\,x-1\right)+3\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(x^2-2\,x-1\right)-16\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(x+1\right)+16\left(x^6-7\,x^4+7\,x^2-1\right)\log\left(x-1\right)+124\,x}{128\left(x^6-7\,x^4+7\,x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 - 2
*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x
^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^6 -
 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^
2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x^4 +
7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

giac [A] time = 0.37, size = 134, normalized size = 1.47

$$\frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}-2|}{|2x+2\sqrt{2}-2|}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(|x^2+2x-1|\right)-\frac{3}{128}\log\left(|x^2-2x-1|\right)+\frac{1}{8}\log\left(|x+1|\right)-\frac{1}{8}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64*
sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*x^
5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x - 1)
) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x - 1
))
```

maple [A] time = 0.02, size = 116, normalized size = 1.27

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x-2)\sqrt{2}}{4}\right)}{32}-\frac{\ln(x-1)}{8}+\frac{\ln(x+1)}{8}-\frac{3\ln\left(x^2-2x-1\right)}{128}+\frac{3\ln\left(x^2+2x-1\right)}{128}-\frac{1}{32(x-1)}+\frac{-5x-12}{64x^2+128x-64}-\frac{1}{32(x+1)}-\frac{5x-12}{64\left(x^2-2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

```
[Out] -1/32/(x-1)-1/8*ln(x-1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/32
*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x+1)+1/8*ln(x+1)-1/64*(-12+5*x)
/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2))
```

maxima [A] time = 0.99, size = 114, normalized size = 1.25

$$\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right)-\frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)}+\frac{3}{128}\log\left(x^2+2x-1\right)-\frac{3}{128}\log\left(x^2-2x-1\right)+\frac{1}{8}\log\left(x+1\right)-\frac{1}{8}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((x
 - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 - 7*
x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) + 1/
8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 2.37, size = 124, normalized size = 1.36

$$-\frac{\operatorname{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4}-\frac{\frac{7x^5}{32}-\frac{23x^3}{16}+\frac{31x}{32}}{x^6-7x^4+7x^2-1}+\operatorname{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)}-\frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right)+\operatorname{atan}\left(\frac{x\,23313\mathrm{i}}{8192\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)}+\frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\left(\frac{27309\sqrt{2}}{32768}+\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}+\frac{3}{64}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

```
[Out] atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*6
5943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 - 3i
/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1) - (
atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317/163
```

```
84)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)))*((
2^(1/2)*1i)/32 + 3i/64)
```

sympy [B] time = 1.36, size = 272, normalized size = 2.99

$$\frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}+\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}-\frac{38423555\sqrt{2}}{1363992}+\frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}\right)+\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555}{909328}+\frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499}-\frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833}+\frac{38423555\sqrt{2}}{1363992}\right)+\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)\log\left(x-\frac{38423555\sqrt{2}}{1363992}-\frac{56267374592\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833}+\frac{9549859782656\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499}+\frac{38423555}{909328}\right)+\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)\log\left(x-\frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833}+\frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499}+\frac{38423555\sqrt{2}}{1363992}+\frac{38423555}{909328}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

```
[Out] (-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x - 1
)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 384235
55*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562673
74592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 38423
555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 56267374592*(-
3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - sqrt(2)
/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)/64)**3
/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/909328) +
(sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/56833 + 95
49859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/1363992 + 384
23555/909328)
```

3.230 $\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$

Optimal. Leaf size=25

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {1590}

$$x^{m+1}\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x*(c*(3
 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx = x^{1+m}\left(a+b\right.$$

Mathematica [A] time = 0.37, size = 23, normalized size = 0.92

$$x^{m+1}(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m + p) + x
*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] x^(1 + m)*(a + x*(b + x*(c + d*x)))^(1 + p)
```

IntegrateAlgebraic [F] time = 1.13, size = 0, normalized size = 0.00

$$\int x^m \left(a+bx+cx^2+dx^3\right)^p (a(1+m)+x(b(2+m+p)+x(c(3+m+2p)+d(4+m+3p)x)))\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b*(2 + m
 + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x + c*x^2 + d*x^3)^p*(a*(1 + m) + x*(b
*(2 + m + p) + x*(c*(3 + m + 2*p) + d*(4 + m + 3*p)*x))), x]
```

fricas [**A**] time = 2.02, size = 40, normalized size = 1.60

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p*x^m
```

giac [**B**] time = 2.09, size = 99, normalized size = 3.96

$$\left(dx^3+cx^2+bx+a\right)^p dx^4x^m+\left(dx^3+cx^2+bx+a\right)^p cx^3x^m+\left(dx^3+cx^2+bx+a\right)^p bx^2x^m+\left(dx^3+cx^2+bx+a\right)^p axx^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4*x^m + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3*x
^m + (d*x^3 + c*x^2 + b*x + a)^p*b*x^2*x^m + (d*x^3 + c*x^2 + b*x + a)^p*a*
x*x^m
```

maple [**A**] time = 0.01, size = 26, normalized size = 1.04

$$x^{m+1}\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*(4+m+3
*p)*x))),x)
```

```
[Out] x^(1+m)*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [**A**] time = 0.70, size = 44, normalized size = 1.76

$$\left(dx^4+cx^3+bx^2+ax\right)e^{\left(p\log\left(dx^3+cx^2+bx+a\right)+m\log\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(d*x^3+c*x^2+b*x+a)^p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p)+d*
(4+m+3*p)*x))),x, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*e^(p*log(d*x^3 + c*x^2 + b*x + a) + m*log(x))
```

mupad [**B**] time = 2.66, size = 49, normalized size = 1.96

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(a\,x\,x^m+b\,x^m\,x^2+c\,x^m\,x^3+d\,x^m\,x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*(m + 1) + x*(x*(c*(m + 2*p + 3) + d*x*(m + 3*p + 4)) + b*(m + p
+ 2)))*(a + b*x + c*x^2 + d*x^3)^p,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x*x^m + b*x^m*x^2 + c*x^m*x^3 + d*x^m*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(d*x**3+c*x**2+b*x+a)**p*(a*(1+m)+x*(b*(2+m+p)+x*(c*(3+m+2*p
)+d*(4+m+3*p)*x))),x)
```

```
[Out] Timed out
```

3.231

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right)$$

Optimal. Leaf size=23

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

Rubi [A] time = 0.08, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1588}

$$x^3 \left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*x^2 +
d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right) dx = x^3 \left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.23, size = 21, normalized size = 0.91

$$x^3(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(5 + 2*p)*
x^2 + d*(6 + 3*p)*x^3),x]
```

```
[Out] x^3*(a + x*(b + x*(c + d*x)))^(1 + p)
```

IntegrateAlgebraic [F] time = 0.96, size = 0, normalized size = 0.00

$$\int x^2 \left(a+bx+cx^2+dx^3\right)^p \left(3a+b(4+p)x+c(5+2p)x^2+d(6+3p)x^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*x + c*(
5 + 2*p)*x^2 + d*(6 + 3*p)*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*(4 + p)*
x + c*(5 + 2*p)*x^2 + d*(6 + 3*p)*x^3), x]
```

fricas [A] time = 1.81, size = 39, normalized size = 1.70

$$\left(dx^6+cx^5+bx^4+ax^3\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="fricas")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 1.89, size = 89, normalized size = 3.87

$$(dx^3 + cx^2 + bx + a)^p dx^6 + (dx^3 + cx^2 + bx + a)^p cx^5 + (dx^3 + cx^2 + bx + a)^p bx^4 + (dx^3 + cx^2 + bx + a)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^6 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^5 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^3
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^3 \left(d x^3 + c x^2 + bx + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*x^3),x
)
```

```
[Out] x^3*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.66, size = 39, normalized size = 1.70

$$(dx^6 + cx^5 + bx^4 + ax^3)(dx^3 + cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c*x^2+b*x+a)^p*(3*a+b*(4+p)*x+c*(5+2*p)*x^2+d*(6+3*p)*
x^3),x, algorithm="maxima")
```

```
[Out] (d*x^6 + c*x^5 + b*x^4 + a*x^3)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.55, size = 39, normalized size = 1.70

$$\left(d x^3 + c x^2 + b x + a\right)^p \left(d x^6 + c x^5 + b x^4 + a x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x + c*x^2 + d*x^3)^p*(3*a + b*x*(p + 4) + c*x^2*(2*p + 5) +
d*x^3*(3*p + 6)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^3 + b*x^4 + c*x^5 + d*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c*x**2+b*x+a)**p*(3*a+b*(4+p)*x+c*(5+2*p)*x**2+d*(6+
3*p)*x**3),x)
```

```
[Out] Timed out
```

3.232

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)$$

Optimal. Leaf size=23

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1588}

$$x^2\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^2 + d*
(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx=x^2\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$x^2(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4 + 2*p)*x^
2 + d*(5 + 3*p)*x^3),x]
```

```
[Out] x^2*(a + x*(b + x*(c + d*x)))^(1 + p)
```

IntegrateAlgebraic [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x\left(a+bx+cx^2+dx^3\right)^p\left(2a+b(3+p)x+c(4+2p)x^2+d(5+3p)x^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x + c*(4
+ 2*p)*x^2 + d*(5 + 3*p)*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*(3 + p)*x
+ c*(4 + 2*p)*x^2 + d*(5 + 3*p)*x^3), x]
```

fricas [A] time = 0.80, size = 39, normalized size = 1.70

$$\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="fricas")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 0.73, size = 89, normalized size = 3.87

$$\left(dx^3+cx^2+bx+a\right)^p dx^5+\left(dx^3+cx^2+bx+a\right)^p cx^4+\left(dx^3+cx^2+bx+a\right)^p bx^3+\left(dx^3+cx^2+bx+a\right)^p ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^5 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^4 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^3 + (d*x^3 + c*x^2 + b*x + a)^p*a*x^2
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$x^2\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^3),x)
```

```
[Out] x^2*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.63, size = 39, normalized size = 1.70

$$\left(dx^5+cx^4+bx^3+ax^2\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c*x^2+b*x+a)^p*(2*a+b*(3+p)*x+c*(4+2*p)*x^2+d*(5+3*p)*x^
3),x, algorithm="maxima")
```

```
[Out] (d*x^5 + c*x^4 + b*x^3 + a*x^2)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.32, size = 39, normalized size = 1.70

$$\left(d\,x^3+c\,x^2+b\,x+a\right)^p\,\left(d\,x^5+c\,x^4+b\,x^3+a\,x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2 + d*x^3)^p*(2*a + b*x*(p + 3) + c*x^2*(2*p + 4) + d*
x^3*(3*p + 5)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x^2 + b*x^3 + c*x^4 + d*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c*x**2+b*x+a)**p*(2*a+b*(3+p)*x+c*(4+2*p)*x**2+d*(5+3*p
)*x**3),x)
```

```
[Out] Timed out
```

3.233

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) d$$

Optimal. Leaf size=21

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Rubi [A] time = 0.04, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {1588}

$$x\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 + d*(4 +
 3*p)*x^3),x]
```

```
[Out] x*(a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx = x\left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.12, size = 19, normalized size = 0.90

$$x(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*p)*x^2 +
d*(4 + 3*p)*x^3),x]
```

```
[Out] x*(a + x*(b + x*(c + d*x)))^(1 + p)
```

IntegrateAlgebraic [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2+dx^3\right)^p \left(a+b(2+p)x+c(3+2p)x^2+d(4+3p)x^3\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*(3 + 2*
p)*x^2 + d*(4 + 3*p)*x^3),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x + c*x^2 + d*x^3)^p*(a + b*(2 + p)*x + c*
(3 + 2*p)*x^2 + d*(4 + 3*p)*x^3), x]
```

fricas [A] time = 1.26, size = 37, normalized size = 1.76

$$\left(dx^4+cx^3+bx^2+ax\right)\left(dx^3+cx^2+bx+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="fricas")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 1.08, size = 87, normalized size = 4.14

$$\left(dx^3 + cx^2 + bx + a\right)^p dx^4 + \left(dx^3 + cx^2 + bx + a\right)^p cx^3 + \left(dx^3 + cx^2 + bx + a\right)^p bx^2 + \left(dx^3 + cx^2 + bx + a\right)^p ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^p*d*x^4 + (d*x^3 + c*x^2 + b*x + a)^p*c*x^3 + (d*
x^3 + c*x^2 + b*x + a)^p*b*x^2 + (d*x^3 + c*x^2 + b*x + a)^p*a*x
```

maple [A] time = 0.00, size = 22, normalized size = 1.05

$$x\left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(a+b*(p+2)*x+c*(2*p+3)*x^2+d*(4+3*p)*x^3),x)
```

```
[Out] x*(d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.61, size = 37, normalized size = 1.76

$$\left(dx^4 + cx^3 + bx^2 + ax\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(a+b*(2+p)*x+c*(3+2*p)*x^2+d*(4+3*p)*x^3),x
, algorithm="maxima")
```

```
[Out] (d*x^4 + c*x^3 + b*x^2 + a*x)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.27, size = 37, normalized size = 1.76

$$\left(d\,x^3 + c\,x^2 + b\,x + a\right)^p\,\left(d\,x^4 + c\,x^3 + b\,x^2 + a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2 + d*x^3)^p*(a + b*x*(p + 2) + c*x^2*(2*p + 3) + d*x^3*
(3*p + 4)),x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^p*(a*x + b*x^2 + c*x^3 + d*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(a+b*(2+p)*x+c*(3+2*p)*x**2+d*(4+3*p)*x*
*3),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx$

Optimal. Leaf size=19

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Rubi [A] time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1585, 1588}

$$\left(a+bx+cx^2+dx^3\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(3 + 3*
p)*x^3))/x,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx = \int \left(b(1+p)+c(2+2p)x+d(3+3p)x^2\right)(\ldots$$
$$= \left(a+bx+cx^2+dx^3\right)^{1+p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$(a+x(b+x(c+dx)))^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*x^2 + d*(
3 + 3*p)*x^3))/x,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(b(1+p)x+c(2+2p)x^2+d(3+3p)x^3\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2 + 2*p)*
x^2 + d*(3 + 3*p)*x^3))/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(b*(1 + p)*x + c*(2
+ 2*p)*x^2 + d*(3 + 3*p)*x^3))/x, x]
```

fricas [A] time = 0.82, size = 33, normalized size = 1.74

$$\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

giac [B] time = 0.30, size = 52, normalized size = 2.74

$$\frac{\left(dx^3 + cx^2 + bx + a\right)^{p+1} p}{p+1} + \frac{\left(dx^3 + cx^2 + bx + a\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="giac")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)^(p + 1)*p/(p + 1) + (d*x^3 + c*x^2 + b*x + a)^(p
+ 1)/(p + 1)
```

maple [A] time = 0.01, size = 20, normalized size = 1.05

$$\left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(b*(p+1)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)
```

maxima [A] time = 0.62, size = 33, normalized size = 1.74

$$\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(b*(1+p)*x+c*(2+2*p)*x^2+d*(3+3*p)*x^3)/x,x
, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p
```

mupad [B] time = 2.19, size = 19, normalized size = 1.00

$$\left(d\,x^3 + c\,x^2 + b\,x + a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x*(p + 1) + c*x^2*(2*p + 2) + d*x^3*(3*p + 3))*(a + b*x + c*x^2 + d
*x^3)^p)/x,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(b*(1+p)*x+c*(2+2*p)*x**2+d*(3+3*p)*x**3
)/x,x)
```

```
[Out] Timed out
```

3.235 $\int \frac{(a+bx+cx^2+dx^3)^p(-a+bpx+c(1+2p)x^2+d(2+3p)x^3)}{x^2}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {1590}

$$\frac{\left(a+bx+cx^2+dx^3\right)^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2 + 3*p
)*x^3))/x^2,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2}\,dx=\frac{\left(a+bx+cx^2+dx^3\right)^{1+p}}{x}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x^2 + d*(2
 + 3*p)*x^3))/x^2,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x
```

IntegrateAlgebraic [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2+dx^3\right)^p\left(-a+bpx+c(1+2p)x^2+d(2+3p)x^3\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 + 2*p)*x
^2 + d*(2 + 3*p)*x^3))/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(-a + b*p*x + c*(1 +
 2*p)*x^2 + d*(2 + 3*p)*x^3))/x^2, x]
```

fricas [A] time = 1.28, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(3p+2\right)x^3+c\left(2p+1\right)x^2+bpx-a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 2)*x^3 + c*(2*p + 1)*x^2 + b*p*x - a)*(d*x^3 + c*x^2 +
b*x + a)^p/x^2, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x
```

maxima [A] time = 0.62, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-a+b*p*x+c*(1+2*p)*x^2+d*(2+3*p)*x^3)/x^2,
x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x
```

mupad [B] time = 3.20, size = 23, normalized size = 1.00

$$\frac{{\left(d\,x^3+c\,x^2+b\,x+a\right)}^{p+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*p*x - a + c*x^2*(2*p + 1) + d*x^3*(3*p
+ 2)))/x^2,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-a+b*p*x+c*(1+2*p)*x**2+d*(2+3*p)*x**3)
/x**2,x)
```

```
[Out] Timed out
```

3.236 $\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^2
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^2}$$

Mathematica [A] time = 0.20, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^2
```

IntegrateAlgebraic [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2+dx^3)^p(-2a+b(-1+p)x+2cpx^2+d(1+3p)x^3)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(-2*a + b*(-1 + p)*x
 + 2*c*p*x^2 + d*(1 + 3*p)*x^3))/x^3, x]
```

fricas [A] time = 1.08, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\left(3\,p+1\right)x^3+2\,cpx^2+b\left(p-1\right)x-2\,a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="giac")
```

```
[Out] integrate((d*(3*p + 1)*x^3 + 2*c*p*x^2 + b*(p - 1)*x - 2*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^3, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3+c\,x^2+bx+a\right)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(3*p+1)*x^3)/x^3,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^2
```

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{\left(dx^3+cx^2+bx+a\right)\left(dx^3+cx^2+bx+a\right)^p}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-2*a+b*(-1+p)*x+2*c*p*x^2+d*(1+3*p)*x^3)/x
^3,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^2
```

mupad [B] time = 3.32, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3+c\,x^2+b\,x+a\right)^{p+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 1) - 2*a + 2*c*p*x^2 + d*x^3*(3*
p + 1)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^2
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-2*a+b*(-1+p)*x+2*c*p*x**2+d*(1+3*p)*x*
*3)/x**3,x)
```

```
[Out] Timed out
```

3.237 $\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {1590}

$$\frac{(a+bx+cx^2+dx^3)^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*x^2 + 
3*d*p*x^3))/x^4,x]
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(1 + p)/x^3
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx = \frac{(a+bx+cx^2+dx^3)^{1+p}}{x^3}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 0.91

$$\frac{(a+x(b+x(c+dx)))^{p+1}}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-1 + 2*p)*
x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] (a + x*(b + x*(c + d*x)))^(1 + p)/x^3
```

IntegrateAlgebraic [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2+dx^3)^p(-3a+b(-2+p)x+c(-1+2p)x^2+3dpx^3)}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x + c*(-
1 + 2*p)*x^2 + 3*d*p*x^3))/x^4,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2 + d*x^3)^p*(-3*a + b*(-2 + p)*x
 + c*(-1 + 2*p)*x^2 + 3*d*p*x^3))/x^4, x]
```

fricas [A] time = 1.50, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="fricas")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3\,dpx^3 + c\left(2\,p - 1\right)x^2 + b\left(p - 2\right)x - 3\,a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="giac")
```

```
[Out] integrate((3*d*p*x^3 + c*(2*p - 1)*x^2 + b*(p - 2)*x - 3*a)*(d*x^3 + c*x^2
+ b*x + a)^p/x^4, x)
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{\left(d\,x^3 + c\,x^2 + bx + a\right)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/x^4,x)
```

```
[Out] (d*x^3+c*x^2+b*x+a)^(p+1)/x^3
```

maxima [A] time = 0.64, size = 36, normalized size = 1.57

$$\frac{\left(dx^3 + cx^2 + bx + a\right)\left(dx^3 + cx^2 + bx + a\right)^p}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c*x^2+b*x+a)^p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x^2+3*d*p*x^3)/
x^4,x, algorithm="maxima")
```

```
[Out] (d*x^3 + c*x^2 + b*x + a)*(d*x^3 + c*x^2 + b*x + a)^p/x^3
```

mupad [B] time = 3.35, size = 23, normalized size = 1.00

$$\frac{\left(d\,x^3 + c\,x^2 + b\,x + a\right)^{p+1}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2 + d*x^3)^p*(b*x*(p - 2) - 3*a + 3*d*p*x^3 + c*x^2*(2*
p - 1)))/x^4,x)
```

```
[Out] (a + b*x + c*x^2 + d*x^3)^(p + 1)/x^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c*x**2+b*x+a)**p*(-3*a+b*(-2+p)*x+c*(-1+2*p)*x**2+3*d*p*x
**3)/x**4,x)
```

```
[Out] Timed out
```

3.238 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$

Optimal. Leaf size=97

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.14, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]
)/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 - (1
3*Log[2 - x + 2*x^2])/48
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^4\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(\frac{5}{4}-\frac{3x}{2}+x^2+x^3+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2-13x}{12\left(2-x+2x^2\right)}\right)dx$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{12}\int \frac{2-13x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx-\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{}{1}$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{3}\log\left(1+x+x^2\right)-\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}\,\text{Subst}$$
$$= \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+\right.$$

Mathematica [A] time = 0.04, size = 83, normalized size = 0.86

$$\frac{1}{144}\left(36x^4+48x^3-108x^2+48\log\left(x^2+x+1\right)-39\log\left(2x^2-x+2\right)+180x-160\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
- 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2 -
 x + 2*x^2])/144
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4), x]
```

fricas [A] time = 1.34, size = 79, normalized size = 0.81

$$\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{3}{4}x^2-\frac{1}{72}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{4}x-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt
(3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48
*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

giac [A] time = 0.31, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{3}{4}x^2-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{4}x-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^2+x+1\right)}{3}-\frac{13\ln\left(2x^2-x+2\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/4*x^4+1/3*x^3-3/4*x^2+5/4*x-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15
*(4*x-1)*15^(1/2))+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
```

maxima [A] time = 1.52, size = 73, normalized size = 0.75

$$\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{3}{4}x^2-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{4}x-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1))
 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 - x
 + 2) + 1/3*log(x^2 + x + 1)
```

mupad [B] time = 0.19, size = 97, normalized size = 1.00

$$\frac{5x}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1i}{144}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{13}{48}+\frac{\sqrt{15}\,1i}{144}\right)-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] (5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3
^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i
)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

sympy [A] time = 0.25, size = 97, normalized size = 1.00

$$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\ \mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}-\frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2 +
 x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*at
an(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.239 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=90

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{3x}{2}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (8
*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 -
x + 2*x^2]/24
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3 \left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(-\frac{3}{2}+x+x^2+\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{-6-x}{6\left(2-x+2x^2\right)}\right)dx$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{1}{6}\int \frac{-6-x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{1}{2-x+2x^2}$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\,\text{Subst}\left(\int \frac{}{-15}\right.$$
$$= -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)$$

Mathematica [A] time = 0.02, size = 78, normalized size = 0.87

$$\frac{1}{72}\left(24x^3+36x^2+48\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)-108x+64\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-10\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt[
15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2
])/72
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4), x]
```

fricas [A] time = 1.22, size = 74, normalized size = 0.82

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x -
 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 -
 x + 2) + 2/3*log(x^2 + x + 1)
```

giac [A] time = 0.38, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 69, normalized size = 0.77

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{5\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{2\ln\left(x^2+x+1\right)}{3}-\frac{\ln\left(2x^2-x+2\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-3/2*x-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*
15^(1/2))+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.88, size = 68, normalized size = 0.76

$$\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{3}{2}x-\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqr
t(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/3*
log(x^2 + x + 1)
```

mupad [B] time = 0.18, size = 92, normalized size = 1.02

$$\frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{3x}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/4)
*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5i)/
72 + 1/24) + x^2/2 + x^3/3
```

sympy [A] time = 0.25, size = 92, normalized size = 1.02

$$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\ \operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt(3
)*x/3 + sqrt(3)/3)/9
```

3.240 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=77

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2075, 634, 618, 204, 628}

$$\frac{x^2}{2}-\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(2x^2-x+2\right)+x+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(1+x-\frac{2(1+3x)}{3\left(1+x+x^2\right)}+\frac{-2+3x}{3\left(2-x+2x^2\right)}\right)dx$$
$$= x+\frac{x^2}{2}+\frac{1}{3}\int \frac{-2+3x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx$$
$$= x+\frac{x^2}{2}+\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx-$$
$$= x+\frac{x^2}{2}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,$$
$$= x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(1+x+x^2\right)+\frac{1}{4}\log\left(\right.$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.94

$$\frac{1}{36}\left(9\left(-4\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+2x(x+2)\right)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x
^4), x]
```

fricas [A] time = 0.83, size = 67, normalized size = 0.87

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9
*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x
^2 + x + 1)
```

giac [A] time = 0.39, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 62, normalized size = 0.81

$$\frac{x^2}{2}+x-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}-\ln\left(x^2+x+1\right)+\frac{\ln\left(2x^2-x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-ln(
x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.44, size = 61, normalized size = 0.79

$$\frac{1}{2}x^2-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

mupad [B] time = 2.29, size = 85, normalized size = 1.10

$$x-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*1
i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4) +
 x^2/2
```

sympy [A] time = 0.23, size = 78, normalized size = 1.01

$$\frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.241 $\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2075, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3]
])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(1+\frac{2(-2+x)}{3\left(1+x+x^2\right)}+\frac{2(1+x)}{3\left(2-x+2x^2\right)}\right)dx$$
$$= x+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx+\frac{2}{3}\int \frac{1+x}{2-x+2x^2}\,dx$$
$$= x+\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx-\frac{5}{3}\int \frac{}{1+x}$$
$$= x+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\,\mathrm{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+\right.$$
$$= x-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x\right.$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.96

$$\frac{1}{18}\left(3\left(2\log\left(x^2+x+1\right)+\log\left(2x^2-x+2\right)+6x\right)-20\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+3x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4
),x]
```

```
[Out] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4
), x]
```

fricas [A] time = 1.29, size = 62, normalized size = 0.86

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 10/9*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x
 + 1)
```

giac [A] time = 0.31, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 57, normalized size = 0.79

$$x+\frac{\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^{2}+x+1\right)}{3}+\frac{\ln\left(2x^{2}-x+2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] x+1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+1/3*ln(x^2+x
+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.59, size = 56, normalized size = 0.78

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\left(4x-1\right)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+x+\frac{1}{6}\log\left(2x^{2}-x+2\right)+\frac{1}{3}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

mupad [B] time = 2.29, size = 80, normalized size = 1.11

$$x+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5{}\mathrm{i}}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5{}\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1{}\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1{}\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1{}\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1{}\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2)
*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^
(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 + 1/
6)
```

sympy [A] time = 0.23, size = 75, normalized size = 1.04

$$x+\frac{\log\left(x^{2}-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^{2}+x+1\right)}{3}+\frac{\sqrt{15}\ \mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.242 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=71

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2074, 634, 618, 204, 628}

$$\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)-\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 + (8*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \int \left(\frac{2(3+2x)}{3\left(1+x+x^2\right)}+\frac{3-2x}{3\left(2-x+2x^2\right)}\right)dx$$
$$= \frac{1}{3}\int \frac{3-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$
$$= -\left(\frac{1}{6}\int \frac{-1+4x}{2-x+2x^2}\,dx\right)+\frac{2}{3}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{5}{6}\int \frac{1}{2-x+2x^2}\,dx+\frac{4}{3}\int \frac{}{1}$$
$$= \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)-\frac{5}{3}\,\mathrm{Subst}\left(\int \frac{1}{-15-x^2}\,dx,x,-1+4\right.$$
$$= -\frac{1}{3}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x\right.$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.92

$$\frac{1}{18}\left(12\log\left(x^2+x+1\right)-3\log\left(2x^2-x+2\right)+16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4), x
]
```

fricas [A] time = 1.33, size = 61, normalized size = 0.86

$$\frac{1}{9}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 8/9*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

giac [A] time = 0.29, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

maple [A] time = 0.01, size = 56, normalized size = 0.79

$$\frac{\sqrt{15}\ \arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{2\ln\left(x^{2}+x+1\right)}{3}-\frac{\ln\left(2x^{2}-x+2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -1/6*ln(2*x^2-x+2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+2/3*ln(x^2+x+
1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.32, size = 55, normalized size = 0.77

$$\frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}\,(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{6}\log\left(2x^{2}-x+2\right)+\frac{2}{3}\log\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

mupad [B] time = 0.15, size = 79, normalized size = 1.11

$$-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i)
/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2
)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6)
```

sympy [A] time = 0.23, size = 75, normalized size = 1.06

$$-\frac{\log\left(x^{2}-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^{2}+x+1\right)}{3}+\frac{\sqrt{15}\ \mathrm{atan}\left(\frac{4\sqrt{15}\,x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*x
/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.243 $\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)} \, dx$

Optimal. Leaf size=75

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
```

```
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x\left(4+4x+4x^2\right)}\,dx\\
&= \frac{1}{3}\int \left(\frac{6}{x}-\frac{2(1+3x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x}+\frac{1+6x}{2\left(2-x+2x^2\right)}\right)dx\\
&= \frac{5\log(x)}{2}-\frac{1}{6}\int \frac{1+6x}{2-x+2x^2}\,dx-\frac{2}{3}\int \frac{1+3x}{1+x+x^2}\,dx\\
&= \frac{5\log(x)}{2}-\frac{1}{4}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx-\frac{5}{12}\int \frac{1}{2-x+2x^2}\,dx\\
&= \frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\
&= \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.92

$$\frac{1}{36}\left(-36\log\left(x^2+x+1\right)-9\log\left(2x^2-x+2\right)+90\log(x)+8\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15
]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)
),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)
), x]
```

fricas [A] time = 1.16, size = 65, normalized size = 0.87

$$-\frac{1}{18}\sqrt{5}\sqrt{3}\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(5)*sqrt(3)*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 2/9*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) +
 5/2*log(x)
```

giac [A] time = 0.31, size = 60, normalized size = 0.80

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs(x
))
```

maple [A] time = 0.01, size = 60, normalized size = 0.80

$$-\frac{\sqrt{15}\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}+\frac{5\ln(x)}{2}-\ln\left(x^2+x+1\right)-\frac{\ln\left(2x^2-x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] -1/4*ln(2*x^2-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))+5/2*ln(x)-ln
(x^2+x+1)+2/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.57, size = 59, normalized size = 0.79

$$-\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{4}\log\left(2x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

mupad [B] time = 0.15, size = 83, normalized size = 1.11

$$\frac{5\ln(x)}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(1+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-1+\frac{\sqrt{3}\,1i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1i}{36}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1i}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x +
 (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4
)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/
36 + 1/4)
```

sympy [A] time = 0.29, size = 78, normalized size = 1.04

$$\frac{5\log(x)}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right)-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*sq
rt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

3.244 $\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2*
x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x +
2*x^2]/24
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
```

```
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx &= -\left(\frac{1}{3}\int \frac{-6+4x}{x^2\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^2\left(4+4x+4x^2\right)}\,dx\\ &= \frac{1}{3}\int \left(\frac{6}{x^2}-\frac{2}{x}+\frac{2(-2+x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^2}+\frac{1}{4x}+\frac{13-2x}{4\left(2-x+2x^2\right)}\right)d\\ &= -\frac{5}{2x}-\frac{3\log(x)}{4}-\frac{1}{12}\int \frac{13-2x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{-2+x}{1+x+x^2}\,dx\\ &= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{24}\int \frac{-1+4x}{2-x+2x^2}\,dx+\frac{1}{3}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{24}\int \frac{}{2-}\\ &= -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{24}\log\left(2-x+2x^2\right)+\frac{25}{12}\text{Subst}\\ &= -\frac{5}{2x}+\frac{5}{12}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{3\log(x)}{4}+\frac{1}{3}\log(1+\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.93

$$-\frac{-24x\log\left(x^2+x+1\right)-3x\log\left(2x^2-x+2\right)+54x\log(x)+80\sqrt{3}\,x\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+10\sqrt{15}\,x\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)+180}{72x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan[
(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 - x
+ 2*x^2])/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^
4)),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^
4)), x]
```

fricas [A] time = 1.16, size = 76, normalized size = 0.90

$$-\frac{10\sqrt{5}\sqrt{3}\,x\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}\,(4x-1)\right)+80\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-3x\log\left(2x^2-x+2\right)-24x\log\left(x^2+x+1\right)+54x\log(x)+180}{72x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/72*(10*sqrt(5)*sqrt(3)*x*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) + 80*sqr
t(3)*x*arctan(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^
2 + x + 1) + 54*x*log(x) + 180)/x
```

giac [A] time = 0.26, size = 65, normalized size = 0.77

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{3}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(abs(x))
```

maple [A] time = 0.01, size = 65, normalized size = 0.77

$$-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}-\frac{3\ln(x)}{4}+\frac{\ln\left(x^2+x+1\right)}{3}+\frac{\ln\left(2x^2-x+2\right)}{24}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/2/x-3/4*ln
(x)+1/3*ln(x^2+x+1)-10/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.44, size = 64, normalized size = 0.76

$$-\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{5}{2x}+\frac{1}{24}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{3}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1) -
 3/4*log(x)
```

mupad [B] time = 2.28, size = 88, normalized size = 1.05

$$-\frac{3\ln(x)}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5i}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5i}{72}\right)-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 -
 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*5i)/72 - 1/24) - 5/(2*x)
```

sympy [A] time = 0.31, size = 87, normalized size = 1.04

$$-\frac{3\log(x)}{4}+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{\log\left(x^2+x+1\right)}{3}-\frac{5\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}-\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*ata
n(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(
3)/3)/9 - 5/(2*x)
```

3.245 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=91

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcTa
n[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/3
+ (13*Log[2 - x + 2*x^2])/48
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
```

```
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\left(\frac{1}{3}\int \frac{-6+4x}{x^3\left(4-2x+4x^2\right)}\,dx\right)+\frac{1}{3}\int \frac{24+16x}{x^3\left(4+4x+4x^2\right)}\,dx$$

$$= \frac{1}{3}\int \left(\frac{6}{x^3}-\frac{2}{x^2}-\frac{4}{x}+\frac{2(3+2x)}{1+x+x^2}\right)dx-\frac{1}{3}\int \left(-\frac{3}{2x^3}+\frac{1}{4x^2}+\frac{13}{8x}+\frac{9-}{8\left(2-\right.}\right.$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{1}{24}\int \frac{9-26x}{2-x+2x^2}\,dx+\frac{2}{3}\int \frac{3+2x}{1+x+x^2}\,dx$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}-\frac{5}{48}\int \frac{1}{2-x+2x^2}\,dx+\frac{13}{48}\int \frac{-1+4x}{2-x+2x^2}\,dx+$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)+\frac{5}{24}$$

$$= -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\,\mathrm{l}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.90

$$\frac{1}{144}\left(3\left(-\frac{60}{x^2}+32\log\left(x^2+x+1\right)+13\log\left(2x^2-x+2\right)+\frac{36}{x}-90\log(x)\right)+128\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)-2\sqrt{15}\tan^{-1}\left(\frac{4x-1}{\sqrt{15}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x +
 2*x^2]))/144
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^
4)),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^
4)), x]
```

fricas [A] time = 1.13, size = 89, normalized size = 0.98

$$-\frac{2\sqrt{5}\sqrt{3}x^2\arctan\left(\frac{1}{15}\sqrt{5}\sqrt{3}(4x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-39x^2\log\left(2x^2-x+2\right)-96x^2\log\left(x^2+x+1\right)+270x^2\log(x)-108x+180}{144x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/144*(2*sqrt(5)*sqrt(3)*x^2*arctan(1/15*sqrt(5)*sqrt(3)*(4*x - 1)) - 128*
sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*
x^2*log(x^2 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

giac [A] time = 0.29, size = 70, normalized size = 0.77

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{15}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(abs(x))
```

maple [A] time = 0.01, size = 70, normalized size = 0.77

$$-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{9}-\frac{15\ln(x)}{8}+\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{13\ln\left(2x^2-x+2\right)}{48}+\frac{3}{4x}-\frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)
```

```
[Out] 13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/4/x^2+3/4
/x-15/8*ln(x)+2/3*ln(x^2+x+1)+8/9*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.26, size = 69, normalized size = 0.76

$$-\frac{1}{72}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{3x-5}{4x^2}+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{15}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(x^
2 + x + 1) - 15/8*log(x)
```

mupad [B] time = 0.15, size = 92, normalized size = 1.01

$$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{15\ln(x)}{8}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{13}{48}+\frac{\sqrt{15}\,1i}{144}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1i}{144}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] ((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/
2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) + lo
g(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(1/2
)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

sympy [A] time = 0.32, size = 94, normalized size = 1.03

$$-\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48}+\frac{2\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\,\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72}+\frac{8\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

```
[Out] -15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15)
*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sq
rt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

3.246 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} dx$

Optimal. Leaf size=307

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{}\right.\right.$$

Rubi [A] time = 0.58, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{42}\left(7+5i\sqrt{7}\right)x^3+\frac{1}{42}\left(7-5i\sqrt{7}\right)x^3+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)-\frac{1}{28}\left(35+9i\sqrt{7}\right)x-\frac{1}{28}\left(35-9i\sqrt{7}\right)x+\frac{11\left(5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{11\left(-5\sqrt{7}+9i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] -((35 - (9*I)*Sqrt[7])*x)/28 - ((35 + (9*I)*Sqrt[7])*x)/28 + ((7 - (5*I)*Sq
rt[7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 + ((7 - (5*I)*Sqrt[7])*x^3)/4
2 + ((7 + (5*I)*Sqrt[7])*x^3)/42 + (11*(9*I + 5*Sqrt[7])*ArcTan[(1 - I*Sqrt
[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (11*(
9*I - 5*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4
*Sqrt[14*(35 - I*Sqrt[7])]) + (3*(7 - (11*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/112 + (3*(7 + (11*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*
x^2])/112
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^3\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^3\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^3\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{4}\left(-9+5i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{1}{2}\left(5-i\sqrt{7}\right)x^2+\frac{2\left(9-5i\sqrt{7}\right)-3\left(11+i\sqrt{7}\right)x}{2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\right)}{\sqrt{7}}$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)$$

$$= -\frac{1}{28}\left(35-9i\sqrt{7}\right)x-\frac{1}{28}\left(35+9i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.36

$$\frac{1}{6}\left(3\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{3\#1^3\log(x-\#1)+19\#1^2\log(x-\#1)+\#1\log(x-\#1)+10\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+x\left(2x^2+3x-15\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] (x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10*
Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/(1
 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x
^4), x]
```

fricas [B] time = 3.80, size = 1202, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 1/112*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55
/32) - 21)*log(23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/
32) + 3/16)^3 - 23765*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^2 + 7744*x + 19470*I*sqrt(7) - 33040*sqrt(2101/1568*I*sqrt(7)
- 55/32) + 38950) - 1/112*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*log(-23324*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55
/32) + 3/16)^3 + 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) -
55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) -
869) + 1/256*(53312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) -
36681)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 17493*
(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 7744*
x - 15708*I*sqrt(7) + 26656*sqrt(2101/1568*I*sqrt(7) - 55/32) - 29132) + 1/
112*(2*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) -
 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7)
- 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55
/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99
/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 28*sqrt(210
1/1568*I*sqrt(7) - 55/32) + 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 21)*log
(-49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-561*I*sqrt(7) + 952*sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(5
3312*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 -
11781*I*sqrt(7) + 19992*sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sq
rt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7
) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((17*sqrt(7)*(-
33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(3
3*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-3
3*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*s
qrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)
^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 8
4*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) +
6384*sqrt(2101/1568*I*sqrt(7) - 55/32) - 5946) - 1/112*(2*sqrt(7)*sqrt(-336
*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*
(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/5
6*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7
) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 1859/2) - 28*sqrt(2101/1568*I*sqrt(7) - 55/32
) - 28*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log(-49/4*(-33/112*I*sqrt(7
) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-561*I*sqrt(7) + 952*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 869) - 1/256*(53312*(33/112*I*sqrt(7) -
 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 11781*I*sqrt(7) + 19992*
sqrt(2101/1568*I*sqrt(7) - 55/32) - 36681)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) + 6272*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I
*sqrt(7) - 55/32) + 3/16)^2 - 1/256*((17*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2
101/1568*I*sqrt(7) - 55/32) - 21) - 512*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(21
01/1568*I*sqrt(7) - 55/32) - 21) + 73728*sqrt(7))*sqrt(-336*(33/112*I*sqrt(
```

```
7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(
7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7)
+ 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101
/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(
7) - 55/32) - 1859/2) + 15488*x - 3762*I*sqrt(7) + 6384*sqrt(2101/1568*I*sq
rt(7) - 55/32) - 5946) - 5/2*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3 + 3\,x^2 + x + 5\right)x^3}{2\,x^4 + x^3 + 5\,x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 74, normalized size = 0.24

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \frac{\left(3\,\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right)^3 + 19\,\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right)^2 + \mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right) + 10\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right) + x\right)}{16\,\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right)^3 + 6\,\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right)^2 + 20\,\mathrm{RootOf}\left(2\_Z^4 + \_Z^3 + 5\_Z^2 + \_Z + 2\right) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+1
)*ln(-_R+x),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{5}{2}\,x + \frac{1}{2}\,\int \frac{3\,x^3 + 19\,x^2 + x + 10}{2\,x^4 + x^3 + 5\,x^2 + x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
+ x^3 + 5*x^2 + x + 2), x)
```

mupad [B] time = 2.17, size = 128, normalized size = 0.42

$$\left(\sum_{k=1}^{4}\ln\left(-29\,x + \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(-\frac{289\,x}{4} + \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(\frac{581\,x}{16} - \mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\left(\frac{147\,x}{4} - \frac{49}{16}\right) + \frac{1141}{64}\right) + \frac{47}{4}\right) + 7\right)\mathrm{root}\left(z^4 - \frac{3\,z^3}{4} + \frac{16\,z^2}{7} + \frac{96\,z}{49} + \frac{128}{343}, z, k\right)\right) - \frac{5\,x}{2} + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*(
root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/16
- root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x)/4
 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)/4 +
 (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 + x^3/
3
```

sympy [A] time = 0.99, size = 61, normalized size = 0.20

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2} + \mathrm{RootSum}\left(1372t^4 - 1029t^3 + 3136t^2 + 2688t + 512, \left(t \mapsto t\log\left(\frac{5831t^3}{1936} - \frac{23765t^2}{7744} + \frac{2065t}{242} + x + \frac{415}{121}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 26
88*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*_t
/242 + x + 415/121)))
```

3.247 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=269

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2\right.$$

Rubi [A] time = 0.39, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-\sqrt{7}+53i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 + ((7 - (5*I)*Sqrt[
7])*x^2)/28 + ((7 + (5*I)*Sqrt[7])*x^2)/28 - ((53*I + Sqrt[7])*ArcTan[(1 -
I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) +
 ((53*I - Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I*Sqrt[7])]])/
(2*Sqrt[14*(35 - I*Sqrt[7])]) - ((35 + (9*I)*Sqrt[7])*Log[4 + (1 - I*Sqrt[7
])*x + 4*x^2])/56 - ((35 - (9*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x + 4*x^2
])/56
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x^2\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x^2\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{i\int \left(\frac{1}{2}\left(5-i\sqrt{7}\right)+\frac{1}{2}\left(5-i\sqrt{7}\right)x+\frac{i\left(2\left(5i+\sqrt{7}\right)+\left(9i+5\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\right)dx}{\sqrt{7}} - \frac{i\int \left(\frac{1}{2}\left(5+i\sqrt{7}\right)\right.}{}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \cdots$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \cdots$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \cdots$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{28}\left(7-5i\sqrt{7}\right)x^2+\frac{1}{28}\left(7+5i\sqrt{7}\right)x^2 - \cdots$$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.38

$$-\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{5\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+3\#1\log(x-\#1)+2\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2
 + 8*#1^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x
^4),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x
^4), x]
```

fricas [B] time = 4.35, size = 1145, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] 1/2*x^2 - 1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(
49/4*(135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*s
qrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 10290*(-9/56*I*sqrt(
7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 25725*(-9/56*I*sqrt(7)
- 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 3/64*(3920*(-9/56*I*sqrt(7
) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sq
rt(-37/392*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt
(7) + 79/56) + 35) + 8384*x + 6615/2*I*sqrt(7) + 10290*sqrt(-37/392*I*sqrt(
7) + 79/56) + 13373/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*
I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt
(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79
/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14
*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*s
qrt(7) + 79/56) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(135*I*s
qrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*
sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt
(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392
*I*sqrt(7) + 79/56) + 5587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 7/64*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/5
6) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5
/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*
I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*
sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*
I*sqrt(7) + 79/56) - 1459)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56
) + 35) - 17856*I*sqrt(7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776)
+ 16768*x - 4941*I*sqrt(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391)
- 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/
8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 -
 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(
7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-3
7/392*I*sqrt(7) + 79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqr
t(-37/392*I*sqrt(7) + 79/56) + 5)*log(-49/4*(135*I*sqrt(7) + 420*sqrt(-37/3
92*I*sqrt(7) + 79/56) - 1459)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) +
 79/56) - 5/8)^2 + 24304*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 3/64*(3920*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^2 - 1575*I*sqrt(7) - 4900*sqrt(-37/392*I*sqrt(7) + 79/56) + 5
587)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-1
2*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56
*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt
(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/
392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7)
+ 79/56) + 11/2)*((135*I*sqrt(7) + 420*sqrt(-37/392*I*sqrt(7) + 79/56) - 14
59)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 17856*I*sqrt(
7) - 55552*sqrt(-37/392*I*sqrt(7) + 79/56) + 67776) + 16768*x - 4941*I*sqrt
(7) - 15372*sqrt(-37/392*I*sqrt(7) + 79/56) - 9391) - 1/56*(9*I*sqrt(7) + 2
8*sqrt(-37/392*I*sqrt(7) + 79/56) + 35)*log(10290*(-9/56*I*sqrt(7) - 1/2*sq
rt(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 + 1421*(-9/56*I*sqrt(7) - 1/2*sqrt(-
37/392*I*sqrt(7) + 79/56) - 5/8)^2 + 8384*x + 3267/2*I*sqrt(7) + 5082*sqrt(
```

```
-37/392*I*sqrt(7) + 79/56) + 13793/2) + x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+3\,x^2+x+5\right)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 67, normalized size = 0.25

$$\frac{x^2}{2}+x+\frac{\left(-5\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2-3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)-2\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{8\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+10\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x^2+x-\int \frac{5\,x^3+x^2+3\,x+2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
2), x)
```

mupad [B] time = 0.13, size = 188, normalized size = 0.70

$$x+\frac{x^2}{2}+\left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8}-7\,x-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,459}{8}-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,665}{8}-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}-\frac{35\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32}+\frac{49\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{16}-15\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
 k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k)^
2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k))/8 -
 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1, 4)
```

sympy [B] time = 2.73, size = 3662, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(1
4)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15*s
qrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268288
```

```
 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))
*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) - 51089
5297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71
978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/2249326
592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt(22)*sqrt
(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 290832444193/359892
25472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448 + sqrt(77)/49
))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*sqrt(553 + 64*sqr
t(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sqrt(77)*sqrt(553 +
 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*sqrt(333*sqrt(553 +
64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8313499*sqrt(14)*sqrt(55
3 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/
71978450944 - 153195*sqrt(22)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77))/2249326592 + 4417610843*sqrt(553 + 64*
sqrt(77))/35989225472 + 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))/224932659
2 + 290832444193/35989225472 + 2303470247*sqrt(77)/2249326592) + 2*sqrt(-sq
rt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1568 + 5/1
4 + 3*sqrt(77)/49)*atan(1073152*x/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqr
t(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqr
t(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77
)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqr
t(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 45*sqrt
(14)*sqrt(553 + 64*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) +
 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sq
rt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 2
1975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) - 1459*sqrt(14)*sqrt(333*sqrt(5
53 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
 + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(
333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)))
+ 20782*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(
7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))
 + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(
77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 93376*sqr
t(77)/(4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333
*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt
(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))) + 1045866/(4313*sqrt(2)*sqrt(-sqrt(14
)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt
(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*
sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
```

```
7))) + 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77
)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77)) + 1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr
t(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
)) + 2*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77))/1568 + 5/14 + 3*sqrt(77)/49)*atan(1073152*x/(-1459*sqrt(2)*sqrt(553 +
 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 76
48*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*s
qrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*s
qrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt
(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648
*sqrt(77))) + 45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*s
qrt(77)) + 21975 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sq
rt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 5
60 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqr
t(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqr
t(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96
*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 10
45866/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(
553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqr
t(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 6
4*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))) + 93376*sqrt(77)/(-1459*sqrt(2)*
sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) +
21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*s
qrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(7
7)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21
975 + 7648*sqrt(77))) - 20782*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt(5
53 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-3
33*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) +
30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*
sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
7648*sqrt(77))) - 3840*sqrt(77)*sqrt(553 + 64*sqrt(77))/(-1459*sqrt(2)*sqrt
(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 2197
5 + 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))
+ 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 764
8*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
+ 7648*sqrt(77))) - 1459*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975
 + 7648*sqrt(77))/(-1459*sqrt(2)*sqrt(553 + 64*sqrt(77))*sqrt(-sqrt(14)*sqr
t(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77)
) + 4313*sqrt(2)*sqrt(-sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 +
 7648*sqrt(77)) + 560 + 96*sqrt(77)) + 30*sqrt(7)*sqrt(-sqrt(14)*sqrt(-333*
sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77)) + 560 + 96*sqrt(77))*sqrt(
-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))))
```

3.248 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4} \, dx$

Optimal. Leaf size=230

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+$$

Rubi [A] time = 0.36, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2087, 773, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{1}{14}\left(7+5i\sqrt{7}\right)x+\frac{1}{14}\left(7-5i\sqrt{7}\right)x-\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((7 - (5*I)*Sqrt[7])*x)/14 + ((7 + (5*I)*Sqrt[7])*x)/14 - ((19*I + 7*Sqrt[7
])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*
Sqrt[7])] + ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*x)/Sqrt[2*(35 - I
*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[7])*Log[4 + (1 -
I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (1 + I*Sqrt[7])*x
+ 4*x^2])/28
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{x\left(9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x\right)}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{x\left(9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x\right)}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{i\int \frac{-4\left(10-2i\sqrt{7}\right)+\left(-\left(\left(1-i\sqrt{7}\right)\left(10-2i\sqrt{7}\right)\right)+4\left(9-5i\sqrt{}\right.\right.}{4+\left(1-i\sqrt{7}\right)x+4x^2}}{4\sqrt{7}}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right.$$

$$= \frac{1}{14}\left(7-5i\sqrt{7}\right)x + \frac{1}{14}\left(7+5i\sqrt{7}\right)x - \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{\left(19i-\right.}{}$$

Mathematica [C] time = 0.01, size = 94, normalized size = 0.41

$$2\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{\#1^3\log(x-\#1)-2\#1^2\log(x-\#1)+2\#1\log(x-\#1)-\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x -
 #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1
^3) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(5+x+3x^2+2x^3\right)}{2+x+5x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4
),x]
```

```
[Out] IntegrateAlgebraic[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4
), x]
```

fricas [B] time = 3.81, size = 1190, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/28*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(49/4*(55*I*
sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqr
t(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/9
8*I*sqrt(7) - 1/14) + 1/4)^3 + 3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqr
t(7) - 1/14) + 1/4)^2 + 11/16*(196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt
(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)
*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x + 1155/2*I*s
qrt(7) + 1617*sqrt(53/98*I*sqrt(7) - 1/14) + 1903/2) + 1/28*(2*sqrt(7)*sqrt
(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/
28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/
2) + 7*sqrt(53/98*I*sqrt(7) - 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*
log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1/14) + 147)*(5/28*I*s
qrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 2744*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(196*(-5/28*I*sqrt(7)
- 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqrt(7) + 98*sqrt(53/98*
I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) -
7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/
4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/
56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7)
 - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14
) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7)
 + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 3456*s
qrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqrt(7) - 1/14) + 636) -
 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(-49/4*(55*I*sqrt(7) + 154*sqrt(53/98*I*sqrt(7) - 1
/14) + 147)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
2744*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 11/16*(
196*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 35*I*sqr
t(7) + 98*sqrt(53/98*I*sqrt(7) - 1/14) + 15)*(-5*I*sqrt(7) + 14*sqrt(-53/98
*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I
*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7)
- 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21
)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7
*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((11*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(5
3/98*I*sqrt(7) - 1/14) - 7) + 224*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/98*I
*sqrt(7) - 1/14) - 7) + 224*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7)
- 1/14) - 7) + 3456*sqrt(7)) + 608*x - 220*I*sqrt(7) - 616*sqrt(53/98*I*sqr
t(7) - 1/14) + 636) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) -
 7)*log(3773*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^3 -
 1029*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 304*x
- 715/2*I*sqrt(7) - 1001*sqrt(53/98*I*sqrt(7) - 1/14) - 2871/2) + x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2\,x^3+3\,x^2+x+5\right)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 62, normalized size = 0.27

$$x+\frac{2\left(\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-2\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+2\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)-1\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{8\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+10\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*
_Z^4+_Z^3+5*_Z^2+_Z+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x+2\int\frac{x^3-2\,x^2+2\,x-1}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

mupad [B] time = 0.19, size = 183, normalized size = 0.80

$$x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)}{8}+15x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^2x\,133}{8}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^3x\,147}{4}-\frac{189\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)^3}{16}-4\right)\mathrm{root}\left(z^4-z^3+\frac{6z^2}{7}-\frac{48z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

```
[Out] x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
)/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x
)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/8 -
 (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 - (189
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z
^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4 - z^3
 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

sympy [A] time = 0.95, size = 48, normalized size = 0.21

$$x+\mathrm{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(\frac{3773t^3}{304}-\frac{1029t^2}{304}+\frac{1001t}{152}+x-\frac{121}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t
*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

3.249 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal. Leaf size=198

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}}{\sqrt{14\left(35+\right.}}$$

Rubi [A] time = 0.19, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2086, 634, 618, 204, 628}

$$\frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4x^2+\left(1-i\sqrt{7}\right)x+4\right)+\frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4x^2+\left(1+i\sqrt{7}\right)x+4\right)+\frac{\left(7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x-i\sqrt{7}+1}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{\left(-7\sqrt{7}+19i\right)\tan^{-1}\left(\frac{8x+i\sqrt{7}+1}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

```
[Out] ((19*I + 7*Sqrt[7])*ArcTan[(1 - I*Sqrt[7] + 8*x)/Sqrt[2*(35 + I*Sqrt[7])]])
/Sqrt[14*(35 + I*Sqrt[7])] - ((19*I - 7*Sqrt[7])*ArcTan[(1 + I*Sqrt[7] + 8*
x)/Sqrt[2*(35 - I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] + ((7 + (5*I)*Sqrt[
7])*Log[4 + (1 - I*Sqrt[7])*x + 4*x^2])/28 + ((7 - (5*I)*Sqrt[7])*Log[4 + (
1 + I*Sqrt[7])*x + 4*x^2])/28
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2086

```
Int[(P3_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4),
 x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P3, x, 0], B =
 Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, Dist[1/q, Int[
(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*x)/(2*a + (b + q)*
x + 2*a*x^2), x], x] - Dist[1/q, Int[(b*A - 2*a*B + 2*a*D - A*q + (2*a*A -
2*a*C + b*D - D*q)*x)/(2*a + (b - q)*x + 2*a*x^2), x], x]] /; FreeQ[{a, b,
```

```
c}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{4+\left(1-i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx}{\sqrt{7}}$$
$$= -\left(\frac{1}{28}\left(-7+5i\sqrt{7}\right)\int \frac{1+i\sqrt{7}+8x}{4+\left(1+i\sqrt{7}\right)x+4x^2}\,dx\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\int \frac{1-i\sqrt{}}{4+\left(1-i\right.}$$
$$= \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right) + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4+\left(1+i\sqrt{7}\right)\right.$$
$$= \frac{\left(19i+7\sqrt{7}\right)\tan^{-1}\left(\frac{1-i\sqrt{7}+8x}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{\left(19i-7\sqrt{7}\right)\tan^{-1}\left(\frac{1+i\sqrt{7}+8x}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{1}{28}\left(7+5\right.$$

Mathematica [C] time = 0.01, size = 90, normalized size = 0.45

$$\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{2\#1^3\log(x-\#1)+3\#1^2\log(x-\#1)+\#1\log(x-\#1)+5\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
[Out] RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#1
 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) &
 ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4), x
]
```

fricas [B] time = 5.34, size = 1189, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
[Out] -1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/1
4) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)
^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7
) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*
sqrt(7) - 1/14) - 27/2) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 7*sqrt(-53/98*I*
sqrt(7) - 1/14) - 7)*log(49/4*(105*I*sqrt(7) + 294*sqrt(53/98*I*sqrt(7) - 1
/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 1/16*(4
```

```
116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 + 735*I*sq
rt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*sqrt(-53
/98*I*sqrt(7) - 1/14) - 7) + 1/16*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sqrt(-53/9
8*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(
7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) +
 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*sqrt(7)
- 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt(7)*(5*I*sqrt(7) + 14*sqr
t(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*I*sqrt(7) + 14*sqrt(-53/9
8*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(
7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sqrt(7) + 910*sqrt(53/98*I*
sqrt(7) - 1/14) - 1247) + 1/28*(2*sqrt(7)*sqrt(-21*(5/28*I*sqrt(7) - 1/2*sq
rt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98
*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) -
 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 5/2*I*
sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2) + 7*sqrt(53/98*I*sqrt(7) -
 1/14) + 7*sqrt(-53/98*I*sqrt(7) - 1/14) + 7)*log(49/4*(105*I*sqrt(7) + 294
*sqrt(53/98*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sq
rt(7) - 1/14) + 1/4)^2 + 4900*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
 1/14) + 1/4)^2 + 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) -
1/14) + 1/4)^2 + 735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-
5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) - 1/16*sqrt(-21*(5/28*I
*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 21*(-5/28*I*sqrt(7)
 - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 - 1/56*(5*I*sqrt(7) + 14*sqrt(
53/98*I*sqrt(7) - 1/14) + 21)*(-5*I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/
14) - 7) - 5/2*I*sqrt(7) - 7*sqrt(53/98*I*sqrt(7) - 1/14) - 27/2)*((21*sqrt
(7)*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7))*(-5*
I*sqrt(7) + 14*sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 400*sqrt(7)*(5*I*sqrt(7
) + 14*sqrt(53/98*I*sqrt(7) - 1/14) - 7) + 7040*sqrt(7)) + 608*x + 325*I*sq
rt(7) + 910*sqrt(53/98*I*sqrt(7) - 1/14) - 1247) - 1/28*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7)*log(-49/4*(105*I*sqrt(7) + 294*sqrt(53/9
8*I*sqrt(7) - 1/14) + 253)*(5/28*I*sqrt(7) - 1/2*sqrt(-53/98*I*sqrt(7) - 1/
14) + 1/4)^2 + 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^3 - 7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 -
 1/16*(4116*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1/4)^2 +
735*I*sqrt(7) + 2058*sqrt(53/98*I*sqrt(7) - 1/14) + 11)*(-5*I*sqrt(7) + 14*
sqrt(-53/98*I*sqrt(7) - 1/14) - 7) + 304*x - 2205/2*I*sqrt(7) - 3087*sqrt(5
3/98*I*sqrt(7) - 1/14) - 3025/2) - 1/28*(5*I*sqrt(7) + 14*sqrt(53/98*I*sqrt
(7) - 1/14) - 7)*log(-7203*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/
14) + 1/4)^3 + 2303*(-5/28*I*sqrt(7) - 1/2*sqrt(53/98*I*sqrt(7) - 1/14) + 1
/4)^2 + 304*x + 1555/2*I*sqrt(7) + 2177*sqrt(53/98*I*sqrt(7) - 1/14) + 5823
/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

maple [C] time = 0.01, size = 58, normalized size = 0.29

$$\frac{\left(2\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+5\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{8\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+3\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+10\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

[Out] sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf(2*_Z^4
+_Z^3+5*_Z^2+_Z+2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")

[Out] integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)

mupad [B] time = 2.34, size = 181, normalized size = 0.91

$$\sum_{k=1}^{4}\ln\left(-\frac{193\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8}+4\,x-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8}+\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,651}{16}-\frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}+\frac{273\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16}+\frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16}+7\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)

[Out] symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8 +
(651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 - (14
7*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273*roo
t(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(z^4 -
z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z^3 + (
6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)

sympy [A] time = 0.91, size = 46, normalized size = 0.23

$$\mathrm{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(-\frac{7203t^3}{304}+\frac{2303t^2}{304}-\frac{2177t}{152}+x+\frac{250}{19}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)

[Out] RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*log
(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))

3.250 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=245

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{\right.$$

Rubi [A] time = 0.47, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)-\frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x)+\frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x)-\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]
])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7] +
 (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 + I*Sqrt[7])]) + ((35 -
 (9*I)*Sqrt[7])*Log[x])/28 + ((35 + (9*I)*Sqrt[7])*Log[x])/28 - ((35 - (9*I
)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/56 - ((35 + (9*I)*Sqrt[7
])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/56
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x}+\frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{2\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x}+\frac{3\left(11i-\sqrt{7}\right)-2\left(9i+5\sqrt{7}\right)x}{2\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)}\right.}{\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{i\int \frac{3\left(11i+\sqrt{7}\right)-2\left(9i-5\sqrt{7}\right)x}{4i+\left(i-\sqrt{7}\right)x+4ix^2}\,dx}{2\sqrt{7}}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\int \frac{i}{4i+(}$$

$$= \frac{1}{28}\left(35-9i\sqrt{7}\right)\log(x) + \frac{1}{28}\left(35+9i\sqrt{7}\right)\log(x) - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\right.$$

$$= -\frac{\left(53+i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\tanh^{-1}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{1}{28}($$

Mathematica [C] time = 0.02, size = 101, normalized size = 0.41

$$\frac{5\log(x)}{2} - \frac{1}{2}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&, \frac{10\#1^3\log(x-\#1)+\#1^2\log(x-\#1)+19\#1\log(x-\#1)+3\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] (5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1] +
 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 + 3
*#1^2 + 8*#1^3) & ]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)
),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)
), x]
```

fricas [B] time = 6.12, size = 1143, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

```
[Out] -1/56*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35)*log(49/4*(27*
I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/
2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 2058*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^3 - 5145*(-9/56*I*sqrt(7) - 1/2*sqrt(-3
7/392*I*sqrt(7) + 79/56) - 5/8)^2 + 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(
-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*s
qrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56)
+ 35) + 8384*x + 1323/2*I*sqrt(7) + 2058*sqrt(-37/392*I*sqrt(7) + 79/56) +
16089/2) + 1/8*(2*sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79
/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) -
 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-
9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 1
0*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2) + 2*sqrt(37/392*I*sqrt(7) + 79/56
) + 2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5)*log(-49/4*(27*I*sqrt(7) + 84*sqr
t(-37/392*I*sqrt(7) + 79/56) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sq
rt(7) + 79/56) - 5/8)^2 - 15680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(
7) + 79/56) - 5/8)^2 - 1/64*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqr
t(7) + 79/56) - 5/8)^2 - 945*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/5
6) - 28507)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 7/64*
sqrt(-12*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12
*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9
*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*s
qrt(37/392*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*s
qrt(7) + 79/56) + 11/2)*((27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56)
 + 1385)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*
sqrt(7) + 35840*sqrt(-37/392*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I
*sqrt(7) + 10864*sqrt(-37/392*I*sqrt(7) + 79/56) + 5484) - 1/8*(2*sqrt(-12*
(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I
*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7
) + 28*sqrt(-37/392*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/39
2*I*sqrt(7) + 79/56) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) +
79/56) + 11/2) - 2*sqrt(37/392*I*sqrt(7) + 79/56) - 2*sqrt(-37/392*I*sqrt(7
) + 79/56) + 5)*log(-49/4*(27*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56
) + 1385)*(9/56*I*sqrt(7) - 1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1
5680*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/64
*(2352*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 94
5*I*sqrt(7) - 2940*sqrt(-37/392*I*sqrt(7) + 79/56) - 28507)*(-9*I*sqrt(7) +
 28*sqrt(37/392*I*sqrt(7) + 79/56) + 35) - 7/64*sqrt(-12*(9/56*I*sqrt(7) -
1/2*sqrt(37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 12*(-9/56*I*sqrt(7) - 1/2*sqr
t(-37/392*I*sqrt(7) + 79/56) - 5/8)^2 - 1/392*(9*I*sqrt(7) + 28*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 105)*(-9*I*sqrt(7) + 28*sqrt(37/392*I*sqrt(7) + 79/5
6) + 35) + 45/14*I*sqrt(7) + 10*sqrt(-37/392*I*sqrt(7) + 79/56) + 11/2)*((2
7*I*sqrt(7) + 84*sqrt(-37/392*I*sqrt(7) + 79/56) + 1385)*(-9*I*sqrt(7) + 28
*sqrt(37/392*I*sqrt(7) + 79/56) + 35) + 11520*I*sqrt(7) + 35840*sqrt(-37/39
2*I*sqrt(7) + 79/56) - 35072) + 16768*x + 3492*I*sqrt(7) + 10864*sqrt(-37/3
92*I*sqrt(7) + 79/56) + 5484) - 1/56*(9*I*sqrt(7) + 28*sqrt(-37/392*I*sqrt(
7) + 79/56) + 35)*log(2058*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) +
79/56) - 5/8)^3 + 20825*(-9/56*I*sqrt(7) - 1/2*sqrt(-37/392*I*sqrt(7) + 79/
56) - 5/8)^2 + 8384*x - 8307/2*I*sqrt(7) - 12922*sqrt(-37/392*I*sqrt(7) + 7
9/56) - 18673/2) + 5/2*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{\left(2\,x^4+x^3+5\,x^2+x+2\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

maple [C] time = 0.01, size = 67, normalized size = 0.27

$$\frac{5\ln(x)}{2}+\frac{\left(-10\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2-19\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)-3\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{16\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+6\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+20\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/2*sum((-10*_R^3-_R^2-19*_R-3)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=RootOf
(2*_Z^4+_Z^3+5*_Z^2+_Z+2))+5/2*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\int \frac{10\,x^3+x^2+19\,x+3}{2\,x^4+x^3+5\,x^2+x+2}\,dx+\frac{5}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

```
[Out] -1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
+ 5/2*log(x)
```

mupad [B] time = 2.34, size = 237, normalized size = 0.97

$$\frac{5\,\ln(x)}{2}+\left(\sum_{k=1}^{4}\ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)}{8}-\frac{31\,x}{2}+\frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)x\,71}{16}-\frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^2x\,4463}{64}+\frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^3x\,1449}{16}+\frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^4x\,3675}{32}+\frac{257\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^2}{32}+\frac{1673\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^3}{64}-\frac{441\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^4}{32}+10\right)\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] (5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 12
8/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128
/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z,
k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k
, 1, 4)
```

sympy [A] time = 12.66, size = 60, normalized size = 0.24

$$\frac{5\log(x)}{2}+\operatorname{RootSum}\left(686t^4+1715t^3+1372t^2+448t+256,\left(t\mapsto t\log\left(-\frac{160344611t^4}{532759184}-\frac{16880402t^3}{33297449}+\frac{4010520787t^2}{2131036736}+\frac{1537535671t}{532759184}+x+\frac{46660495}{66594898}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] 5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, La
mbda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 4010
520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/66594898))
)
```

3.251 $\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)} \, dx$

Optimal. Leaf size=281

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-$$

Rubi [A] time = 0.47, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)-\frac{35+9i\sqrt{7}}{28x}-\frac{35-9i\sqrt{7}}{28x}-\frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x)-\frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x)+\frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11\left(9-5i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(35 - (9*I)*Sqrt[7])/(28*x) - (35 + (9*I)*Sqrt[7])/(28*x) + (11*(9 + (5*I)
*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqr
t[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8
*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(4*Sqrt[14*(35 + I*Sqrt[7])]) - (3*(7 - (
11*I)*Sqrt[7])*Log[x])/56 - (3*(7 + (11*I)*Sqrt[7])*Log[x])/56 + (3*(7 + (1
1*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/112 + (3*(7 - (11*I)*
Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/112
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^2\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^2\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^2}+\frac{3\left(11-i\sqrt{7}\right)}{8x}+\frac{-7\left(9i-5\sqrt{7}\right)-6\left(11i+\sqrt{7}\right)x}{4\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int \left(\frac{9-5i\sqrt{7}}{4x^2}+\frac{3(11+i\sqrt{}}{8x}\right.}{}$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} - \frac{3}{56}\left(7-11i\sqrt{7}\right)\log(x) - \frac{3}{56}\left(7+11i\sqrt{7}\right)\log(x$$

$$= -\frac{35-9i\sqrt{7}}{28x} - \frac{35+9i\sqrt{7}}{28x} + \frac{11\left(9+5i\sqrt{7}\right)\tanh^{-1}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{11\left(9-5\right.}{}$$

Mathematica [C] time = 0.02, size = 109, normalized size = 0.39

$$\frac{1}{4}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{6\#1^3\log(x-\#1)-17\#1^2\log(x-\#1)+13\#1\log(x-\#1)-35\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]-\frac{5}{2x}-\frac{3\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35*
Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^3)
/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^
4)),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^
4)), x]
```

fricas [B] time = 6.70, size = 1245, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)

[Out] -1/224*(2*x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*log
(91924*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3
- 49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839) - 1/256
*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^
2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33
*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 68943*(33/112*I*
sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x + 61908
*I*sqrt(7) - 105056*sqrt(2101/1568*I*sqrt(7) - 55/32) + 123428) + 2*x*(-33*
I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21)*log(-91924*(33/112*I
*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^3 + 98735*(33/112*
I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 + 15488*x - 146
487/2*I*sqrt(7) + 124292*sqrt(2101/1568*I*sqrt(7) - 55/32) - 285347/2) + (4
*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32
) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/3
2) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) -
 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*s
qrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2)*x - x*(33*I*sqrt(7)
 + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - x*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) - 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1
/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqr
t(2101/1568*I*sqrt(7) - 55/32) - 3839) + 1/256*(210112*(33/112*I*sqrt(7) -
1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 46431*I*sqrt(7) + 78792*s
qrt(2101/1568*I*sqrt(7) - 55/32) - 117483)*(33*I*sqrt(7) + 56*sqrt(-2101/15
68*I*sqrt(7) - 55/32) - 21) - 29792*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*
I*sqrt(7) - 55/32) + 3/16)^2 + 1/256*((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(
-2101/1568*I*sqrt(7) - 55/32) - 21) - 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt
(2101/1568*I*sqrt(7) - 55/32) - 21) + 147456*sqrt(7))*sqrt(-336*(33/112*I*s
qrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*s
qrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt
(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(
2101/1568*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*s
qrt(7) - 55/32) - 1859/2) + 30976*x + 22671/2*I*sqrt(7) - 19236*sqrt(2101/1
568*I*sqrt(7) - 55/32) + 53979/2) - (4*sqrt(7)*sqrt(-336*(33/112*I*sqrt(7)
- 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7)
- 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 5
6*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/15
68*I*sqrt(7) - 55/32) + 63) + 99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7)
- 55/32) - 1859/2)*x + x*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/
32) - 21) + x*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 84*x)*log(49/4*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2*(-2211*I*sqrt(7) + 3752*sqrt(2101/1568*I*sqrt(7) - 55/32) - 3839
) + 1/256*(210112*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32)
 + 3/16)^2 - 46431*I*sqrt(7) + 78792*sqrt(2101/1568*I*sqrt(7) - 55/32) - 11
7483)*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) - 29792*(
33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7) - 55/32) + 3/16)^2 - 1/256*
((67*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) -
2432*sqrt(7))*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) - 55/32) - 21) -
```

```
 2432*sqrt(7)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) - 21) +
 147456*sqrt(7))*sqrt(-336*(33/112*I*sqrt(7) - 1/2*sqrt(2101/1568*I*sqrt(7)
 - 55/32) + 3/16)^2 - 336*(-33/112*I*sqrt(7) - 1/2*sqrt(-2101/1568*I*sqrt(7
) - 55/32) + 3/16)^2 - 1/56*(33*I*sqrt(7) + 56*sqrt(-2101/1568*I*sqrt(7) -
55/32) - 21)*(-33*I*sqrt(7) + 56*sqrt(2101/1568*I*sqrt(7) - 55/32) + 63) +
99/2*I*sqrt(7) - 84*sqrt(2101/1568*I*sqrt(7) - 55/32) - 1859/2) + 30976*x +
 22671/2*I*sqrt(7) - 19236*sqrt(2101/1568*I*sqrt(7) - 55/32) + 53979/2) + 1
68*x*log(x) + 560)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{\left(2\,x^4+x^3+5\,x^2+x+2\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

maple [C] time = 0.01, size = 72, normalized size = 0.26

$$-\frac{3\ln(x)}{4}+\frac{\left(6\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3-17\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+13\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)-35\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{32\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+12\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+40\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+4}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/4*sum((6*_R^3-17*_R^2+13*_R-35)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Root
Of(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/2/x-3/4*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{5}{2\,x}+\frac{1}{4}\int\frac{6\,x^3-17\,x^2+13\,x-35}{2\,x^4+x^3+5\,x^2+x+2}\,dx-\frac{3}{4}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] -5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
x + 2), x) - 3/4*log(x)
```

mupad [B] time = 2.30, size = 242, normalized size = 0.86

$$\left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)}{32}+25\,x+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)x\,4169}{32}+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2x\,43993}{256}+\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3x\,28+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4x\,3675}{32}+\frac{11647\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2}{128}+\frac{7273\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3}{128}-\frac{441\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4}{32}+\frac{21}{4}\right)\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)\right)-\frac{3\ln(x)}{4}-\frac{5}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z
, k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128
/343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 +
128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
+ 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z
)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (9
6*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z^2)/7 +
(96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (16*z^2)/7
+ (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
[Out] Timed out
```

3.252 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal. Leaf size=317

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right.\right.$$

Rubi [A] time = 0.54, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2087, 800, 634, 618, 206, 628}

$$-\frac{35+9i\sqrt{7}}{56x^2}-\frac{35-9i\sqrt{7}}{56x^2}+\frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\left(-\sqrt{7}+i\right)x+4i\right)+\frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4ix^2+\left(\sqrt{7}+i\right)x+4i\right)+\frac{3\left(7+11i\sqrt{7}\right)}{56x}+\frac{3\left(7-11i\sqrt{7}\right)}{56x}-\frac{1}{16}\left(35+9i\sqrt{7}\right)\log(x)-\frac{1}{16}\left(35-9i\sqrt{7}\right)\log(x)+\frac{\left(355-73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{\left(355+73i\sqrt{7}\right)\tanh^{-1}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -(35 - (9*I)*Sqrt[7])/(56*x^2) - (35 + (9*I)*Sqrt[7])/(56*x^2) + (3*(7 - (1
1*I)*Sqrt[7]))/(56*x) + (3*(7 + (11*I)*Sqrt[7]))/(56*x) + ((355 - (73*I)*Sq
rt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(8*Sqrt[1
4*(35 - I*Sqrt[7])]) - ((355 + (73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)
*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(8*Sqrt[14*(35 + I*Sqrt[7])]) - ((35 - (9*I)
*Sqrt[7])*Log[x])/16 - ((35 + (9*I)*Sqrt[7])*Log[x])/16 + ((35 - (9*I)*Sqrt
[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/32 + ((35 + (9*I)*Sqrt[7])*Log
[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 2087

```
Int[((P3_)*(x_)^(m_.))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (
e_.)*(x_)^4), x_Symbol] :> With[{q = Sqrt[8*a^2 + b^2 - 4*a*c], A = Coeff[P
3, x, 0], B = Coeff[P3, x, 1], C = Coeff[P3, x, 2], D = Coeff[P3, x, 3]}, D
ist[1/q, Int[(x^m*(b*A - 2*a*B + 2*a*D + A*q + (2*a*A - 2*a*C + b*D + D*q)*
x))/(2*a + (b + q)*x + 2*a*x^2), x], x] - Dist[1/q, Int[(x^m*(b*A - 2*a*B +
 2*a*D - A*q + (2*a*A - 2*a*C + b*D - D*q)*x))/(2*a + (b - q)*x + 2*a*x^2),
 x], x]] /; FreeQ[{a, b, c, m}, x] && PolyQ[P3, x, 3] && EqQ[a, e] && EqQ[b
, d]
```

Rubi steps

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{i\int \frac{9-5i\sqrt{7}+\left(10-2i\sqrt{7}\right)x}{x^3\left(4+\left(1-i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}} - \frac{i\int \frac{9+5i\sqrt{7}+\left(10+2i\sqrt{7}\right)x}{x^3\left(4+\left(1+i\sqrt{7}\right)x+4x^2\right)}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \left(\frac{9+5i\sqrt{7}}{4x^3}+\frac{3\left(11-i\sqrt{7}\right)}{8x^2}-\frac{7i\left(-9i+5\sqrt{7}\right)}{16x}+\frac{-223i-61\sqrt{7}+14\left(9i-5\sqrt{7}\right)x}{8\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)}\right)dx}{\sqrt{7}} + \frac{i\int}{}$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-\right.$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-\right.$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} - \frac{1}{16}\left(35-\right.$$

$$= -\frac{35-9i\sqrt{7}}{56x^2} - \frac{35+9i\sqrt{7}}{56x^2} + \frac{3\left(7-11i\sqrt{7}\right)}{56x} + \frac{3\left(7+11i\sqrt{7}\right)}{56x} + \frac{\left(355-7\right.}{}$$

Mathematica [C] time = 0.02, size = 116, normalized size = 0.37

$$\frac{1}{8}\text{RootSum}\left[2\#1^4+\#1^3+5\#1^2+\#1+2\&,\frac{70\#1^3\log(x-\#1)+47\#1^2\log(x-\#1)+141\#1\log(x-\#1)+61\log(x-\#1)}{8\#1^3+3\#1^2+10\#1+1}\&\right]-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

```
[Out] -5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#
1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*Log
[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^
4)),x]
```

```
[Out] IntegrateAlgebraic[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^
4)), x]
```

fricas [B] time = 5.63, size = 1274, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas"
)
```

```
[Out] -1/448*(14*x^2*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
)*log(-49/4*(207711*I*sqrt(7) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896
) - 957269)*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35
/32)^2 + 9046968*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 35/32)^3 - 39580485*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) +
 2815/896) + 35/32)^2 - 21/1024*(13785856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803
/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 16963065*I*sqrt(7) + 30156560*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/
6272*I*sqrt(7) + 2815/896) - 35) + 9662336*x - 68336919/4*I*sqrt(7) - 30371
964*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 257023549/4) + 14*x^2*(9*I*sqrt
(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)*log(-9046968*(-9/32*I*
sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^3 + 41411909*(
-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 96
62336*x + 70198191/4*I*sqrt(7) + 31199196*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 240366533/4) + 1960*x^2*log(x) + (4*sqrt(7)*sqrt(-1344*(9/32*I*sqrt(
7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*I*sq
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I*sqr
t(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) + 16*s
qrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqrt(-9
803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 - 7*x^2*(9*I*sqrt(7) + 16*sqrt
(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 7*x^2*(-9*I*sqrt(7) + 16*sqrt(980
3/6272*I*sqrt(7) + 2815/896) - 35) - 980*x^2)*log(49/4*(207711*I*sqrt(7) +
369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7) - 1/
2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sqrt(7
) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(1378585
6*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 +
 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 6848
8563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) + 1/102
4*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 3
5/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) +
 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/
2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(2307
9*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
49504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35
) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896)
 - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sqrt(-
9803/6272*I*sqrt(7) + 2815/896) + 666914) - (4*sqrt(7)*sqrt(-1344*(9/32*I*s
qrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1344*(-9/32*
I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 7/8*(9*I
*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 105)*(-9*I*sqrt(7) +
16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2205/2*I*sqrt(7) - 1960*sqr
t(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*x^2 + 7*x^2*(9*I*sqrt(7) + 16*
sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35) + 7*x^2*(-9*I*sqrt(7) + 16*sqrt
(9803/6272*I*sqrt(7) + 2815/896) - 35) + 980*x^2)*log(49/4*(207711*I*sqrt(7
) + 369264*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 957269)*(9/32*I*sqrt(7)
- 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 - 1831424*(-9/32*I*sq
```

```
rt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)^2 + 21/1024*(137
85856*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 35/32)
^2 + 16963065*I*sqrt(7) + 30156560*sqrt(-9803/6272*I*sqrt(7) + 2815/896) -
68488563)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 1
/1024*sqrt(-1344*(9/32*I*sqrt(7) - 1/2*sqrt(9803/6272*I*sqrt(7) + 2815/896)
 + 35/32)^2 - 1344*(-9/32*I*sqrt(7) - 1/2*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) + 35/32)^2 - 7/8*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/89
6) + 105)*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896) - 35) - 2
205/2*I*sqrt(7) - 1960*sqrt(-9803/6272*I*sqrt(7) + 2815/896) + 1661/2)*(7*(
23079*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/896) - 35)
 - 149504*sqrt(7))*(-9*I*sqrt(7) + 16*sqrt(9803/6272*I*sqrt(7) + 2815/896)
- 35) - 1046528*sqrt(7)*(9*I*sqrt(7) + 16*sqrt(-9803/6272*I*sqrt(7) + 2815/
896) - 35) - 116260864*sqrt(7)) + 19324672*x - 465318*I*sqrt(7) - 827232*sq
rt(-9803/6272*I*sqrt(7) + 2815/896) + 666914) - 336*x + 560)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,x^3+3\,x^2+x+5}{\left(2\,x^4+x^3+5\,x^2+x+2\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

```
[Out] integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

maple [C] time = 0.01, size = 77, normalized size = 0.24

$$-\frac{35\ln(x)}{8}+\frac{\left(70\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+47\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+141\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+61\right)\ln\left(-\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+x\right)}{64\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^3+24\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)^2+80\,\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)+8}+\frac{3}{4x}-\frac{5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)
```

```
[Out] 1/8*sum((70*_R^3+47*_R^2+141*_R+61)/(8*_R^3+3*_R^2+10*_R+1)*ln(-_R+x),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))-5/4/x^2+3/4/x-35/8*ln(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,x-5}{4\,x^2}+\frac{1}{8}\int \frac{70\,x^3+47\,x^2+141\,x+61}{2\,x^4+x^3+5\,x^2+x+2}\,dx-\frac{35}{8}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima"
)
```

```
[Out] 1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 + x
^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

mupad [B] time = 2.25, size = 246, normalized size = 0.78

$$\left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69\,x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)x\,14945}{128}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2x\,269991}{1024}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3x\,1393}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4x\,3675}{32}-\frac{35697\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2}{512}-\frac{18487\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3}{256}-\frac{441\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4}{32}+\frac{245}{8}\right)\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\right)-\frac{35\,\ln(x)}{8}+\frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

```
[Out] symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z
, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7
 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/7 -
```

```
(8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^2)/7
- (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (47*z^2
)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8 + (47*
z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)/8 + (47
*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 + ((3*x)/4 -
 5/4)/x^2
```

sympy [**A**] time = 2.70, size = 70, normalized size = 0.22

$$-\frac{35\log(x)}{8} + \text{RootSum}\left(2744t^4 - 12005t^3 + 18424t^2 - 3136t + 1024, \left(t \mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586} + \frac{944515214496t^3}{45953952180793} + \frac{16572327093911939t^2}{5882105879141504} - \frac{4564471749800865t}{735263234892688} + x + \frac{70084064010625}{91907904361586}\right)\right)\right) + \frac{3x-5}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

```
[Out] -35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t + 1
024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 944515214496*
_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447174
9800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*x -
5)/(4*x**2)
```

3.253 $\int \frac{x^2(3a+bx^2)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$

Optimal. Leaf size=19

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Rubi [A] time = 0.10, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] ArcTan[(c*x^3)/(a + b*x^2)]/c
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2094

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(3a+bx^2\right)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx &= \left(3a^2\right)\text{Subst}\left(\int \frac{1}{a^2+9a^2c^2x^2}\,dx, x, \frac{x^3}{3a+3bx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{cx^3}{a+bx^2}\right)}{c}\end{aligned}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 4.58

$$\frac{1}{2}\text{RootSum}\left[\#1^6c^2+\#1^4b^2+2\#1^2ab+a^2\&,\frac{\#1^3b\log(x-\#1)+3\#1a\log(x-\#1)}{3\#1^4c^2+2\#1^2b^2+2ab}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6),x]
```

```
[Out] RootSum[a^2 + 2*a*b*#1^2 + b^2*#1^4 + c^2*#1^6 & , (3*a*Log[x - #1]*#1 + b*
Log[x - #1]*#1^3)/(2*a*b + 2*b^2*#1^2 + 3*c^2*#1^4) & ]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(3a+bx^2\right)}{a^2+2abx^2+b^2x^4+c^2x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6
),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(3*a + b*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4 + c^2*x^6
), x]
```

fricas [B] time = 2.11, size = 83, normalized size = 4.37

$$\frac{\arctan\left(\frac{cx}{b}\right) - \arctan\left(\frac{bc^2x^5 + ab^2x + (b^3 - ac^2)x^3}{a^2c}\right) + \arctan\left(\frac{bc^2x^3 + (b^3 - ac^2)x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="fri
cas")
```

```
[Out] (arctan(c*x/b) - arctan((b*c^2*x^5 + a*b^2*x + (b^3 - a*c^2)*x^3)/(a^2*c))
+ arctan((b*c^2*x^3 + (b^3 - a*c^2)*x)/(a*b*c)))/c
```

giac [B] time = 4.08, size = 87, normalized size = 4.58

$$\frac{\arctan\left(\frac{cx}{b}\right) + \arctan\left(-\frac{bc^2x^5 + b^3x^3 - ac^2x^3 + ab^2x}{a^2c}\right) - \arctan\left(-\frac{bc^2x^3 + b^3x - ac^2x}{abc}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="gia
c")
```

```
[Out] (arctan(c*x/b) + arctan(-(b*c^2*x^5 + b^3*x^3 - a*c^2*x^3 + a*b^2*x)/(a^2*c
)) - arctan(-(b*c^2*x^3 + b^3*x - a*c^2*x)/(a*b*c)))/c
```

maple [C] time = 0.10, size = 75, normalized size = 3.95

$$\frac{\left(\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right)^4 b + 3\,\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right)^2 a\right)\ln\left(-\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right) + x\right)}{6\,\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right)^5 c^2 + 4\,\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right)^3 b^2 + 4\,\mathrm{RootOf}\left(c^2\_Z^6 + b^2\_Z^4 + 2ab\_Z^2 + a^2\right) ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*sum((_R^4*b+3*_R^2*a)/(3*_R^5*c^2+2*_R^3*b^2+2*_R*a*b)*ln(-_R+x),_R=Roo
tOf(_Z^6*c^2+_Z^4*b^2+2*_Z^2*a*b+a^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^2 + 3\,a)x^2}{c^2x^6 + b^2x^4 + 2\,abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+3*a)/(c^2*x^6+b^2*x^4+2*a*b*x^2+a^2),x, algorithm="max
ima")
```

```
[Out] integrate((b*x^2 + 3*a)*x^2/(c^2*x^6 + b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

mupad [B] time = 2.27, size = 252, normalized size = 13.26

$$\frac{\mathrm{atan}\left(\frac{27\,a\,c^5\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2} - \frac{27\,b\,c^5\,x^5}{27\,a^2\,c^4-4\,a\,b^3\,c^2} - \frac{31\,b^3\,c^3\,x^3}{27\,a^2\,c^4-4\,a\,b^3\,c^2} + \frac{4\,b^6\,c\,x^3}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2} + \frac{4\,b^5\,c\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2} + \frac{4\,b^4\,c^3\,x^5}{27\,a^3\,c^4-4\,a^2\,b^3\,c^2} - \frac{27\,a\,b^2\,c^3\,x}{27\,a^2\,c^4-4\,a\,b^3\,c^2}\right) + \mathrm{atan}\left(\frac{c\,x^3}{a} - \frac{c\,x}{b} + \frac{b^2\,x}{a\,c}\right) + \mathrm{atan}\left(\frac{c\,x}{b}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*a + b*x^2))/(a^2 + b^2*x^4 + c^2*x^6 + 2*a*b*x^2),x)
```

```
[Out] (atan((27*a*c^5*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) - (27*b*c^5*x^5)/(27*a^2*c^
4 - 4*a*b^3*c^2) - (31*b^3*c^3*x^3)/(27*a^2*c^4 - 4*a*b^3*c^2) + (4*b^6*c*x
^3)/(27*a^3*c^4 - 4*a^2*b^3*c^2) + (4*b^5*c*x)/(27*a^2*c^4 - 4*a*b^3*c^2) +
 (4*b^4*c^3*x^5)/(27*a^3*c^4 - 4*a^2*b^3*c^2) - (27*a*b^2*c^3*x)/(27*a^2*c^
4 - 4*a*b^3*c^2)) + atan((c*x^3)/a - (c*x)/b + (b^2*x)/(a*c)) + atan((c*x)/
b))/c
```

sympy [C] time = 1.05, size = 44, normalized size = 2.32

$$\frac{-\frac{i \log\left(-\frac{ia}{c}-\frac{ibx^2}{c}+x^3\right)}{2}+\frac{i \log\left(\frac{ia}{c}+\frac{ibx^2}{c}+x^3\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+3*a)/(c**2*x**6+b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/c - I*b*x**2/c + x**3)/2 + I*log(I*a/c + I*b*x**2/c + x**3)/2)
/c
```

3.254 $\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{1-2x}{5\left(x^2+1\right)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1647, 1629, 635, 203, 260}

$$-\frac{1-2x}{5\left(x^2+1\right)}-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(2-x)-\frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
```

```
[Out] -(1 - 2*x)/(5*(1 + x^2)) - (46*ArcTan[x])/25 - (47*Log[2 - x])/25 - (14*Log
[1 + x^2])/25
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx = -\frac{1-2x}{5\left(1+x^2\right)} - \frac{1}{2}\int \frac{-\frac{18}{5}-\frac{4x}{5}+6x^2}{(-2+x)\left(1+x^2\right)}\,dx$$
$$= -\frac{1-2x}{5\left(1+x^2\right)} - \frac{1}{2}\int \left(\frac{94}{25(-2+x)} + \frac{4(23+14x)}{25\left(1+x^2\right)}\right)dx$$
$$= -\frac{1-2x}{5\left(1+x^2\right)} - \frac{47}{25}\log(2-x) - \frac{2}{25}\int \frac{23+14x}{1+x^2}\,dx$$
$$= -\frac{1-2x}{5\left(1+x^2\right)} - \frac{47}{25}\log(2-x) - \frac{28}{25}\int \frac{x}{1+x^2}\,dx - \frac{46}{25}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1-2x}{5\left(1+x^2\right)} - \frac{46}{25}\tan^{-1}(x) - \frac{47}{25}\log(2-x) - \frac{14}{25}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.33

$$\frac{2(x-2)+3}{5\left((x-2)^2+4(x-2)+5\right)} - \frac{14}{25}\log\left((x-2)^2+4(x-2)+5\right) - \frac{47}{25}\log(x-2) - \frac{46}{25}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
[Out] (3 + 2*(-2 + x))/(5*(5 + 4*(-2 + x) + (-2 + x)^2)) - (46*ArcTan[x])/25 - (1
4*Log[5 + 4*(-2 + x) + (-2 + x)^2])/25 - (47*Log[-2 + x])/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x^4}{(-2+x)\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2),x]
[Out] IntegrateAlgebraic[(1 - 3*x^4)/((-2 + x)*(1 + x^2)^2), x]
```

fricas [A] time = 0.88, size = 47, normalized size = 1.09

$$-\frac{46\left(x^2+1\right)\arctan(x) + 14\left(x^2+1\right)\log\left(x^2+1\right) + 47\left(x^2+1\right)\log(x-2) - 10\,x + 5}{25\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/25*(46*(x^2 + 1)*arctan(x) + 14*(x^2 + 1)*log(x^2 + 1) + 47*(x^2 + 1)*lo
g(x - 2) - 10*x + 5)/(x^2 + 1)
```

giac [A] time = 0.25, size = 34, normalized size = 0.79

$$\frac{2\,x-1}{5\left(x^2+1\right)} - \frac{46}{25}\arctan(x) - \frac{14}{25}\log\left(x^2+1\right) - \frac{47}{25}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
abs(x - 2))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{46\arctan(x)}{25}-\frac{47\ln(x-2)}{25}-\frac{14\ln\left(x^2+1\right)}{25}-\frac{2\left(-5x+\frac{5}{2}\right)}{25\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^4+1)/(-2+x)/(x^2+1)^2,x)
```

```
[Out] -2/25*(-5*x+5/2)/(x^2+1)-14/25*ln(x^2+1)-46/25*arctan(x)-47/25*ln(-2+x)
```

maxima [A] time = 2.10, size = 33, normalized size = 0.77

$$\frac{2x-1}{5\left(x^2+1\right)}-\frac{46}{25}\arctan(x)-\frac{14}{25}\log\left(x^2+1\right)-\frac{47}{25}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^4+1)/(-2+x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/5*(2*x - 1)/(x^2 + 1) - 46/25*arctan(x) - 14/25*log(x^2 + 1) - 47/25*log(
x - 2)
```

mupad [B] time = 0.05, size = 38, normalized size = 0.88

$$\frac{\frac{2x}{5}-\frac{1}{5}}{x^2+1}-\frac{47\ln(x-2)}{25}+\ln(x-\mathrm{i})\left(-\frac{14}{25}+\frac{23}{25}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{14}{25}-\frac{23}{25}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^4 - 1)/((x^2 + 1)^2*(x - 2)),x)
```

```
[Out] ((2*x)/5 - 1/5)/(x^2 + 1) - log(x - 1i)*(14/25 - 23i/25) - log(x + 1i)*(14/
25 + 23i/25) - (47*log(x - 2))/25
```

sympy [A] time = 0.17, size = 37, normalized size = 0.86

$$-\frac{1-2x}{5x^2+5}-\frac{47\log(x-2)}{25}-\frac{14\log\left(x^2+1\right)}{25}-\frac{46\operatorname{atan}(x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**4+1)/(-2+x)/(x**2+1)**2,x)
```

```
[Out] -(1 - 2*x)/(5*x**2 + 5) - 47*log(x - 2)/25 - 14*log(x**2 + 1)/25 - 46*atan(
x)/25
```

3.255 $\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx$

Optimal. Leaf size=17

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1593, 1802}

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx &= \int \frac{-9-9x+2x^2}{x\left(-9+x^2\right)}\,dx\\ &= \int \left(\frac{1}{3-x}+\frac{1}{x}+\frac{2}{3+x}\right)dx\\ &= -\log(3-x)+\log(x)+2\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\log(3-x)+\log(x)+2\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] -Log[3 - x] + Log[x] + 2*Log[3 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9-9x+2x^2}{-9x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-9 - 9*x + 2*x^2)/(-9*x + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-9 - 9*x + 2*x^2)/(-9*x + x^3), x]
```

fricas [A] time = 1.15, size = 15, normalized size = 0.88

$$2 \log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="fricas")
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

giac [A] time = 0.31, size = 18, normalized size = 1.06

$$2 \log(|x+3|) - \log(|x-3|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="giac")
[Out] 2*log(abs(x + 3)) - log(abs(x - 3)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) - \ln(x-3) + 2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-9*x-9)/(x^3-9*x),x)
[Out] 2*ln(3+x)-ln(-3+x)+ln(x)
```

maxima [A] time = 1.12, size = 15, normalized size = 0.88

$$2 \log(x+3) - \log(x-3) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-9*x-9)/(x^3-9*x),x, algorithm="maxima")
[Out] 2*log(x + 3) - log(x - 3) + log(x)
```

mupad [B] time = 2.20, size = 21, normalized size = 1.24

$$2 \ln(x+3) - 2 \operatorname{atanh}\left(\frac{1296}{18x+162} - 7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x - 2*x^2 + 9)/(9*x - x^3),x)
[Out] 2*log(x + 3) - 2*atanh(1296/(18*x + 162) - 7)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\log(x) - \log(x-3) + 2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-9*x-9)/(x**3-9*x),x)
[Out] log(x) - log(x - 3) + 2*log(x + 3)
```

3.256 $\int \frac{1+2x^2+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1593, 1802}

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2+x^5}{-x+x^3}\,dx &= \int \frac{1+2x^2+x^5}{x\left(-1+x^2\right)}\,dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}+x^2+\frac{1}{1+x}\right)dx\\ &= x+\frac{x^3}{3}+2\log(1-x)-\log(x)+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^3}{3}+x+2\log(1-x)-\log(x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + 2*Log[1 - x] - Log[x] + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x^2+x^5}{-x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x^2 + x^5)/(-x + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x^2 + x^5)/(-x + x^3), x]
```

fricas [A] time = 1.38, size = 21, normalized size = 0.84

$$\frac{1}{3}\,x^3 + x + \log\left(x + 1\right) + 2\,\log\left(x - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

giac [A] time = 0.25, size = 24, normalized size = 0.96

$$\frac{1}{3}\,x^3 + x + \log\left(|x + 1|\right) + 2\,\log\left(|x - 1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x + log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^3}{3} + x - \ln(x) + 2\ln\left(x - 1\right) + \ln\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+2*x^2+1)/(x^3-x),x)
```

```
[Out] 1/3*x^3+x+2*ln(x-1)+ln(x+1)-ln(x)
```

maxima [A] time = 1.15, size = 21, normalized size = 0.84

$$\frac{1}{3}\,x^3 + x + \log\left(x + 1\right) + 2\,\log\left(x - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+2*x^2+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x + log(x + 1) + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.05, size = 30, normalized size = 1.20

$$x + 2\,\ln\left(x - 1\right) + \frac{x^3}{3} + \mathrm{atan}\left(\frac{48\mathrm{i}}{11\,\left(22\,x - 2\right)} + \frac{13}{11}\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + x^5 + 1)/(x - x^3),x)
```

```
[Out] x + 2*log(x - 1) + atan(48i/(11*(22*x - 2)) + 13i/11)*2i + x^3/3
```

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$\frac{x^3}{3} + x - \log\left(x\right) + 2\log\left(x - 1\right) + \log\left(x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+2*x**2+1)/(x**3-x),x)
```

```
[Out] x**3/3 + x - log(x) + 2*log(x - 1) + log(x + 1)
```

3.257 $\int \frac{3+2x^2}{(-1+x)^2x} \, dx$

Optimal. Leaf size=22

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {894}

$$\frac{5}{1-x} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)/((-1 + x)^2*x),x]
[Out] 5/(1 - x) - Log[1 - x] + 3*Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x^2}{(-1+x)^2x} \, dx &= \int \left(\frac{1}{1-x} + \frac{5}{(-1+x)^2} + \frac{3}{x}\right) dx \\ &= \frac{5}{1-x} - \log(1-x) + 3\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.91

$$-\frac{5}{x-1} - \log(1-x) + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/((-1 + x)^2*x),x]
[Out] -5/(-1 + x) - Log[1 - x] + 3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+2x^2}{(-1+x)^2x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 2*x^2)/((-1 + x)^2*x),x]
[Out] IntegrateAlgebraic[(3 + 2*x^2)/((-1 + x)^2*x), x]
```

fricas [A] time = 1.43, size = 24, normalized size = 1.09

$$-\frac{(x-1)\log(x-1) - 3(x-1)\log(x) + 5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="fricas")
```

```
[Out] -((x - 1)*log(x - 1) - 3*(x - 1)*log(x) + 5)/(x - 1)
```

giac [A] time = 0.37, size = 28, normalized size = 1.27

$$-\frac{5}{x-1}+2\log\left(|x-1|\right)+3\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="giac")
```

```
[Out] -5/(x - 1) + 2*log(abs(x - 1)) + 3*log(abs(-1/(x - 1) - 1))
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$3\ln(x)-\ln\left(x-1\right)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x-1)^2/x,x)
```

```
[Out] -5/(x-1)-ln(x-1)+3*ln(x)
```

maxima [A] time = 1.09, size = 18, normalized size = 0.82

$$-\frac{5}{x-1}-\log\left(x-1\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(-1+x)^2/x,x, algorithm="maxima")
```

```
[Out] -5/(x - 1) - log(x - 1) + 3*log(x)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$3\ln(x)-\ln\left(x-1\right)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x*(x - 1)^2),x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

sympy [A] time = 0.11, size = 14, normalized size = 0.64

$$3\log\left(x\right)-\log\left(x-1\right)-\frac{5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(-1+x)**2/x,x)
```

```
[Out] 3*log(x) - log(x - 1) - 5/(x - 1)
```

3.258 $\int \frac{-1+2x^2}{(-1+4x)\left(1+x^2\right)}\,dx$

Optimal. Leaf size=27

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1629, 635, 203, 260}

$$\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(1-4x)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
```

```
[Out] (3*ArcTan[x])/17 - (7*Log[1 - 4*x])/34 + (6*Log[1 + x^2])/17
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x^2}{(-1+4x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{14}{17(-1+4x)}+\frac{3(1+4x)}{17\left(1+x^2\right)}\right)dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1+4x}{1+x^2}\,dx\\ &= -\frac{7}{34}\log(1-4x)+\frac{3}{17}\int \frac{1}{1+x^2}\,dx+\frac{12}{17}\int \frac{x}{1+x^2}\,dx\\ &= \frac{3}{17}\tan^{-1}(x)-\frac{7}{34}\log(1-4x)+\frac{6}{17}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.41

$$-\frac{7}{34}\log(4x-1)+\frac{6}{17}\log\left((4x-1)^2+2(4x-1)+17\right)+\frac{3}{17}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
[Out] (3*ArcTan[x])/17 - (7*Log[-1 + 4*x])/34 + (6*Log[17 + 2*(-1 + 4*x) + (-1 +
4*x)^2])/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+2x^2}{(-1+4x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(-1 + 2*x^2)/((-1 + 4*x)*(1 + x^2)), x]
```

fricas [A] time = 1.52, size = 21, normalized size = 0.78

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

giac [A] time = 0.29, size = 22, normalized size = 0.81

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(|4x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="giac")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(abs(4*x - 1))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3\arctan(x)}{17}-\frac{7\ln(4x-1)}{34}+\frac{6\ln\left(x^2+1\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-1)/(4*x-1)/(x^2+1),x)
[Out] 6/17*ln(x^2+1)+3/17*arctan(x)-7/34*ln(4*x-1)
```

maxima [A] time = 2.31, size = 21, normalized size = 0.78

$$\frac{3}{17}\arctan(x)+\frac{6}{17}\log\left(x^2+1\right)-\frac{7}{34}\log(4x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-1)/(-1+4*x)/(x^2+1),x, algorithm="maxima")
[Out] 3/17*arctan(x) + 6/17*log(x^2 + 1) - 7/34*log(4*x - 1)
```

mupad [B] time = 2.26, size = 25, normalized size = 0.93

$$-\frac{7\ln\left(x-\frac{1}{4}\right)}{34}+\ln(x-\mathrm{i})\left(\frac{6}{17}-\frac{3}{34}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{6}{17}+\frac{3}{34}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 1)/((4*x - 1)*(x^2 + 1)),x)
```

```
[Out] log(x - 1i)*(6/17 - 3i/34) - (7*log(x - 1/4))/34 + log(x + 1i)*(6/17 + 3i/3
4)
```

sympy [A] time = 0.14, size = 26, normalized size = 0.96

$$-\frac{7\log\left(x-\frac{1}{4}\right)}{34}+\frac{6\log\left(x^2+1\right)}{17}+\frac{3\,\mathrm{atan}\,(x)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-1)/(-1+4*x)/(x**2+1),x)
```

```
[Out] -7*log(x - 1/4)/34 + 6*log(x**2 + 1)/17 + 3*atan(x)/17
```

3.259 $\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1810, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx &= \int \left(-3+x+\frac{x}{1+x^2}\right)dx\\ &= -3x+\frac{x^2}{2}+\int \frac{x}{1+x^2}\,dx\\ &= -3x+\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{x^2}{2}+\frac{1}{2}\log\left(x^2+1\right)-3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] -3*x + x^2/2 + Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+2x-3x^2+x^3}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2),x]
```

```
[Out] IntegrateAlgebraic[(-3 + 2*x - 3*x^2 + x^3)/(1 + x^2), x]
```

fricas [A] time = 1.49, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

giac [A] time = 0.27, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="giac")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{x^2}{2} - 3x + \frac{\ln\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+2*x-3)/(x^2+1),x)
[Out] -3*x+1/2*x^2+1/2*ln(x^2+1)
```

maxima [A] time = 2.28, size = 17, normalized size = 0.81

$$\frac{1}{2}x^2 - 3x + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+2*x-3)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 3*x + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^2 + 1\right)}{2} - 3x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3*x^2 + x^3 - 3)/(x^2 + 1),x)
[Out] log(x^2 + 1)/2 - 3*x + x^2/2
```

sympy [A] time = 0.08, size = 15, normalized size = 0.71

$$\frac{x^2}{2} - 3x + \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+2*x-3)/(x**2+1),x)
[Out] x**2/2 - 3*x + log(x**2 + 1)/2
```

3.260 $\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2} \, dx$

Optimal. Leaf size=27

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
```

```
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx &= \int \left(x^2+\frac{x}{10+6x+x^2}\right)dx \\
&= \frac{x^3}{3}+\int \frac{x}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\int \frac{6+2x}{10+6x+x^2}\,dx-3\int \frac{1}{10+6x+x^2}\,dx \\
&= \frac{x^3}{3}+\frac{1}{2}\log\left(10+6x+x^2\right)+6\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,6+2x\right) \\
&= \frac{x^3}{3}-3\tan^{-1}(3+x)+\frac{1}{2}\log\left(10+6x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{x^3}{3}+\frac{1}{2}\log\left(x^2+6x+10\right)-3\tan^{-1}(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
[Out] x^3/3 - 3*ArcTan[3 + x] + Log[10 + 6*x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+10x^2+6x^3+x^4}{10+6x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2),x]
[Out] IntegrateAlgebraic[(x + 10*x^2 + 6*x^3 + x^4)/(10 + 6*x + x^2), x]
```

fricas [A] time = 1.45, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3-3\arctan\left(x+3\right)+\frac{1}{2}\log\left(x^2+6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="fricas")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

giac [A] time = 0.28, size = 23, normalized size = 0.85

$$\frac{1}{3}x^3-3\arctan\left(x+3\right)+\frac{1}{2}\log\left(x^2+6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="giac")
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

maple [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{x^3}{3}-3\arctan\left(x+3\right)+\frac{\ln\left(x^2+6x+10\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x)
```

```
[Out] 1/3*x^3-3*arctan(x+3)+1/2*ln(x^2+6*x+10)
```

maxima [A] time = 2.08, size = 23, normalized size = 0.85

$$\frac{1}{3}\,x^{3}-3\,\arctan\left(x+3\right)+\frac{1}{2}\,\log\left(x^{2}+6\,x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+6*x^3+10*x^2+x)/(x^2+6*x+10),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3*arctan(x + 3) + 1/2*log(x^2 + 6*x + 10)
```

mupad [B] time = 2.14, size = 23, normalized size = 0.85

$$\frac{\ln\left(x^{2}+6\,x+10\right)}{2}-3\,\mathrm{atan}\left(x+3\right)+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 10*x^2 + 6*x^3 + x^4)/(6*x + x^2 + 10),x)
```

```
[Out] log(6*x + x^2 + 10)/2 - 3*atan(x + 3) + x^3/3
```

sympy [A] time = 0.11, size = 22, normalized size = 0.81

$$\frac{x^{3}}{3}+\frac{\log\left(x^{2}+6x+10\right)}{2}-3\,\mathrm{atan}\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+6*x**3+10*x**2+x)/(x**2+6*x+10),x)
```

```
[Out] x**3/3 + log(x**2 + 6*x + 10)/2 - 3*atan(x + 3)
```

3.261 $\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2058}

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx &= \int \left(\frac{1}{12(-3+x)}-\frac{1}{5(-2+x)}+\frac{1}{8(-1+x)}-\frac{1}{120(3+x)}\right)dx\\ &= \frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{1}{8}\log(1-x)-\frac{1}{5}\log(2-x)+\frac{1}{12}\log(3-x)-\frac{1}{120}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] Log[1 - x]/8 - Log[2 - x]/5 + Log[3 - x]/12 - Log[3 + x]/120
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-18+27x-7x^2-3x^3+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(-18 + 27*x - 7*x^2 - 3*x^3 + x^4)^(-1), x]
```

fricas [A] time = 1.13, size = 25, normalized size = 0.64

$$-\frac{1}{120}\log(x+3)+\frac{1}{8}\log(x-1)-\frac{1}{5}\log(x-2)+\frac{1}{12}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="fricas")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

giac [A] time = 0.32, size = 29, normalized size = 0.74

$$-\frac{1}{120}\,\log\left(|x+3|\right)+\frac{1}{8}\,\log\left(|x-1|\right)-\frac{1}{5}\,\log\left(|x-2|\right)+\frac{1}{12}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="giac")
```

```
[Out] -1/120*log(abs(x + 3)) + 1/8*log(abs(x - 1)) - 1/5*log(abs(x - 2)) + 1/12*l
og(abs(x - 3))
```

maple [A] time = 0.01, size = 26, normalized size = 0.67

$$\frac{\ln\left(x-3\right)}{12}-\frac{\ln\left(x-2\right)}{5}+\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^3-7*x^2+27*x-18),x)
```

```
[Out] 1/8*ln(x-1)-1/5*ln(x-2)-1/120*ln(x+3)+1/12*ln(x-3)
```

maxima [A] time = 1.27, size = 25, normalized size = 0.64

$$-\frac{1}{120}\,\log\left(x+3\right)+\frac{1}{8}\,\log\left(x-1\right)-\frac{1}{5}\,\log\left(x-2\right)+\frac{1}{12}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^3-7*x^2+27*x-18),x, algorithm="maxima")
```

```
[Out] -1/120*log(x + 3) + 1/8*log(x - 1) - 1/5*log(x - 2) + 1/12*log(x - 3)
```

mupad [B] time = 2.14, size = 25, normalized size = 0.64

$$\frac{\ln\left(x-1\right)}{8}-\frac{\ln\left(x-2\right)}{5}+\frac{\ln\left(x-3\right)}{12}-\frac{\ln\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(7*x^2 - 27*x + 3*x^3 - x^4 + 18),x)
```

```
[Out] log(x - 1)/8 - log(x - 2)/5 + log(x - 3)/12 - log(x + 3)/120
```

sympy [A] time = 0.24, size = 26, normalized size = 0.67

$$\frac{\log\left(x-3\right)}{12}-\frac{\log\left(x-2\right)}{5}+\frac{\log\left(x-1\right)}{8}-\frac{\log\left(x+3\right)}{120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**3-7*x**2+27*x-18),x)
```

```
[Out] log(x - 3)/12 - log(x - 2)/5 + log(x - 1)/8 - log(x + 3)/120
```

3.262 $\int \frac{1+x^3}{-2+x}\,dx$

Optimal. Leaf size=22

$$\frac{x^3}{3}+x^2+4x+9\log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1850}

$$\frac{x^3}{3}+x^2+4x+9\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-2 + x),x]
```

```
[Out] 4*x + x^2 + x^3/3 + 9*Log[2 - x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{-2+x}\,dx &= \int \left(4+\frac{9}{-2+x}+2x+x^2\right)dx\\ &= 4x+x^2+\frac{x^3}{3}+9\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{x^3}{3}+x^2+4x+9\log(x-2)-\frac{44}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-2 + x),x]
```

```
[Out] -44/3 + 4*x + x^2 + x^3/3 + 9*Log[-2 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{-2+x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(-2 + x),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3)/(-2 + x), x]
```

fricas [A] time = 1.01, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^3+x^2+4\,x+9\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

giac [A] time = 0.28, size = 19, normalized size = 0.86

$$\frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(abs(x - 2))
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^3}{3} + x^2 + 4x + 9\ln\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x-2),x)
```

```
[Out] 1/3*x^3+x^2+4*x+9*ln(x-2)
```

maxima [A] time = 1.13, size = 18, normalized size = 0.82

$$\frac{1}{3}\,x^3 + x^2 + 4\,x + 9\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(-2+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + 4*x + 9*log(x - 2)
```

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$4\,x + 9\,\ln\left(x-2\right) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - 2),x)
```

```
[Out] 4*x + 9*log(x - 2) + x^2 + x^3/3
```

sympy [A] time = 0.07, size = 17, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + 4x + 9\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(-2+x),x)
```

```
[Out] x**3/3 + x**2 + 4*x + 9*log(x - 2)
```

3.263 $\int \frac{3x-4x^2+3x^3}{1+x^2} dx$

Optimal. Leaf size=15

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1594, 1802, 203}

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{3x-4x^2+3x^3}{1+x^2} dx &= \int \frac{x\left(3-4x+3x^2\right)}{1+x^2} dx \\
&= \int \left(-4+3x+\frac{4}{1+x^2}\right) dx \\
&= -4x+\frac{3x^2}{2}+4\int \frac{1}{1+x^2} dx \\
&= -4x+\frac{3x^2}{2}+4\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{3x^2}{2} - 4x + 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
```

```
[Out] -4*x + (3*x^2)/2 + 4*ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x-4x^2+3x^3}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*x - 4*x^2 + 3*x^3)/(1 + x^2),x]
[Out] IntegrateAlgebraic[(3*x - 4*x^2 + 3*x^3)/(1 + x^2), x]
```

fricas [A] time = 1.00, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="fricas")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

giac [A] time = 0.34, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="giac")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{3x^2}{2}-4x+4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x^2+3*x)/(x^2+1),x)
[Out] -4*x+3/2*x^2+4*arctan(x)
```

maxima [A] time = 2.30, size = 13, normalized size = 0.87

$$\frac{3}{2}\,x^2-4\,x+4\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x^2+3*x)/(x^2+1),x, algorithm="maxima")
[Out] 3/2*x^2 - 4*x + 4*arctan(x)
```

mupad [B] time = 2.13, size = 13, normalized size = 0.87

$$4\,\mathrm{atan}(x)-4\,x+\frac{3\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4*x^2 + 3*x^3)/(x^2 + 1),x)
```

```
[Out] 4*atan(x) - 4*x + (3*x^2)/2
```

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\frac{3x^2}{2} - 4x + 4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x**2+3*x)/(x**2+1),x)
```

```
[Out] 3*x**2/2 - 4*x + 4*atan(x)
```

3.264 $\int \frac{5+3x}{1-x-x^2+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2074, 206}

$$\frac{4}{1-x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] 4/(1 - x) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+3x}{1-x-x^2+x^3} \, dx &= \int \left(\frac{4}{(-1+x)^2}+\frac{1}{1-x^2}\right) dx\\
&= \frac{4}{1-x}+\int \frac{1}{1-x^2} \, dx\\
&= \frac{4}{1-x}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 2.00

$$-\frac{4}{x-1}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] -4/(-1 + x) - Log[-1 + x]/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+3x}{1-x-x^2+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + 3*x)/(1 - x - x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(5 + 3*x)/(1 - x - x^2 + x^3), x]
```

fricas [B] time = 1.08, size = 26, normalized size = 2.17

$$\frac{(x-1)\log(x+1)-(x-1)\log(x-1)-8}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="fricas")
[Out] 1/2*((x - 1)*log(x + 1) - (x - 1)*log(x - 1) - 8)/(x - 1)
```

giac [B] time = 0.30, size = 22, normalized size = 1.83

$$-\frac{4}{x-1}+\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="giac")
[Out] -4/(x - 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 1.75

$$-\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+3*x)/(x^3-x^2-x+1),x)
[Out] -4/(x-1)-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 1.08, size = 20, normalized size = 1.67

$$-\frac{4}{x-1}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x^3-x^2-x+1),x, algorithm="maxima")
[Out] -4/(x - 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.07, size = 10, normalized size = 0.83

$$\mathrm{atanh}(x)-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 5)/(x + x^2 - x^3 - 1),x)
[Out] atanh(x) - 4/(x - 1)
```

sympy [B] time = 0.10, size = 17, normalized size = 1.42

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+3*x)/(x**3-x**2-x+1),x)
[Out] -log(x - 1)/2 + log(x + 1)/2 - 4/(x - 1)
```

3.265 $\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1593, 1620}

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx &= \int \frac{-1-x-x^3+x^4}{(-1+x)x^2}\,dx\\ &= \int \left(-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}+x\right)dx\\ &= -\frac{1}{x}+\frac{x^2}{2}-2\log(1-x)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
```

```
[Out] -x^(-1) + x^2/2 - 2*Log[1 - x] + 2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1-x-x^3+x^4}{-x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 - x - x^3 + x^4)/(-x^2 + x^3),x]
[Out] IntegrateAlgebraic[(-1 - x - x^3 + x^4)/(-x^2 + x^3), x]
```

fricas [A] time = 1.19, size = 22, normalized size = 0.88

$$\frac{x^3 - 4\,x\log(x-1) + 4\,x\log(x) - 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="fricas")
[Out] 1/2*(x^3 - 4*x*log(x - 1) + 4*x*log(x) - 2)/x
```

giac [A] time = 0.27, size = 23, normalized size = 0.92

$$\frac{1}{2}\,x^2 - \frac{1}{x} - 2\,\log(|x-1|) + 2\,\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/2*x^2 - 1/x - 2*log(abs(x - 1)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.88

$$\frac{x^2}{2} + 2\ln(x) - 2\ln(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3-x-1)/(x^3-x^2),x)
[Out] 1/2*x^2-2*ln(x-1)-1/x+2*ln(x)
```

maxima [A] time = 1.13, size = 21, normalized size = 0.84

$$\frac{1}{2}\,x^2 - \frac{1}{x} - 2\,\log(x-1) + 2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3-x-1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/x - 2*log(x - 1) + 2*log(x)
```

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4\,\mathrm{atanh}(2\,x-1) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - x^4 + 1)/(x^2 - x^3),x)
[Out] 4*atanh(2*x - 1) - 1/x + x^2/2
```

sympy [A] time = 0.10, size = 19, normalized size = 0.76

$$\frac{x^2}{2} + 2\log(x) - 2\log(x-1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3-x-1)/(x**3-x**2),x)
[Out] x**2/2 + 2*log(x) - 2*log(x - 1) - 1/x
```

3.266 $\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1673, 1149, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+2\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x] + Log[2 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1149

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c*x^2)/e)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx &= \int \frac{x(1+x^2)}{2+3x^2+x^4}\,dx + \int \frac{2+x^2}{2+3x^2+x^4}\,dx\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x}{2+3x+x^2}\,dx, x, x^2\right) + \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)\\ &= \tan^{-1}(x) + \frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+2\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
[Out] ArcTan[x] + Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x+x^2+x^3}{2+3x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(2 + x + x^2 + x^3)/(2 + 3*x^2 + x^4), x]
```

fricas [A] time = 1.22, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="fricas")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

giac [A] time = 0.31, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="giac")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+2)/(x^4+3*x^2+2),x)
[Out] arctan(x)+1/2*ln(x^2+2)
```

maxima [A] time = 2.16, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2}\log\left(x^2 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+2)/(x^4+3*x^2+2),x, algorithm="maxima")
[Out] arctan(x) + 1/2*log(x^2 + 2)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^2 + 2\right)}{2} + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 2)/(3*x^2 + x^4 + 2),x)
[Out] log(x^2 + 2)/2 + atan(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2 + 2\right)}{2} + \mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+2)/(x**4+3*x**2+2),x)
[Out] log(x**2 + 2)/2 + atan(x)
```

3.267 $\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log(x^2+2)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1814, 1586, 635, 203, 260}

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log(x^2+2)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
```

```
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx = -\frac{1}{(2+x^2)^2}-\frac{1}{8}\int \frac{16-16x+8x^2-8x^3}{(2+x^2)^2}\,dx$$
$$= -\frac{1}{(2+x^2)^2}-\frac{1}{8}\int \frac{8-8x}{2+x^2}\,dx$$
$$= -\frac{1}{(2+x^2)^2}-\int \frac{1}{2+x^2}\,dx+\int \frac{x}{2+x^2}\,dx$$
$$= -\frac{1}{(2+x^2)^2}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
[Out] -(2 + x^2)^(-2) - ArcTan[x/Sqrt[2]]/Sqrt[2] + Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+8x-4x^2+4x^3-x^4+x^5}{(2+x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3,x]
[Out] IntegrateAlgebraic[(-4 + 8*x - 4*x^2 + 4*x^3 - x^4 + x^5)/(2 + x^2)^3, x]
```

fricas [A] time = 1.61, size = 55, normalized size = 1.57

$$-\frac{\sqrt{2}\left(x^4+4x^2+4\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\left(x^4+4x^2+4\right)\log\left(x^2+2\right)+2}{2\left(x^4+4x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="fricas")
[Out] -1/2*(sqrt(2)*(x^4 + 4*x^2 + 4)*arctan(1/2*sqrt(2)*x) - (x^4 + 4*x^2 + 4)*l
og(x^2 + 2) + 2)/(x^4 + 4*x^2 + 4)
```

giac [A] time = 0.33, size = 30, normalized size = 0.86

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{(x^2+2)^2}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^2 + 2)^2 + 1/2*log(x^2 + 2)
```

maple [A] time = 0.01, size = 31, normalized size = 0.89

$$-\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\ln\left(x^2+2\right)}{2}-\frac{1}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x)
[Out] -1/(x^2+2)^2+1/2*ln(x^2+2)-1/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

maxima [A] time = 2.22, size = 35, normalized size = 1.00

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{1}{x^4+4\,x^2+4}+\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-x^4+4*x^3-4*x^2+8*x-4)/(x^2+2)^3,x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/(x^4 + 4*x^2 + 4) + 1/2*log(x^2 + 2)
```

mupad [B] time = 2.12, size = 35, normalized size = 1.00

$$\frac{\ln\left(x^2+2\right)}{2}-\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}-\frac{1}{x^4+4\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 4*x^2 + 4*x^3 - x^4 + x^5 - 4)/(x^2 + 2)^3,x)
[Out] log(x^2 + 2)/2 - (2^(1/2)*atan((2^(1/2)*x)/2))/2 - 1/(4*x^2 + x^4 + 4)
```

sympy [A] time = 0.15, size = 36, normalized size = 1.03

$$\frac{\log\left(x^2+2\right)}{2}-\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}-\frac{1}{x^4+4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-x**4+4*x**3-4*x**2+8*x-4)/(x**2+2)**3,x)
[Out] log(x**2 + 2)/2 - sqrt(2)*atan(sqrt(2)*x/2)/2 - 1/(x**4 + 4*x**2 + 4)
```

3.268 $\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=23

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1594, 1628}

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx &= \int \frac{-1-3x+x^2}{x\left(-2+x+x^2\right)} \, dx \\
&= \int \left(\frac{1}{1-x}+\frac{1}{2x}+\frac{3}{2(2+x)}\right) dx \\
&= -\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\log(1-x)+\frac{\log(x)}{2}+\frac{3}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] -Log[1 - x] + Log[x]/2 + (3*Log[2 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1-3x+x^2}{-2x+x^2+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3),x]
[Out] IntegrateAlgebraic[(-1 - 3*x + x^2)/(-2*x + x^2 + x^3), x]
```

fricas [A] time = 2.09, size = 17, normalized size = 0.74

$$\frac{3}{2}\,\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

giac [A] time = 0.24, size = 20, normalized size = 0.87

$$\frac{3}{2}\,\log\left(|x+2|\right)-\log\left(|x-1|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
[Out] 3/2*log(abs(x + 2)) - log(abs(x - 1)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\frac{\ln(x)}{2}-\ln\left(x-1\right)+\frac{3\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-1)/(x^3+x^2-2*x),x)
[Out] -ln(x-1)+3/2*ln(x+2)+1/2*ln(x)
```

maxima [A] time = 1.40, size = 17, normalized size = 0.74

$$\frac{3}{2}\,\log\left(x+2\right)-\log\left(x-1\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
[Out] 3/2*log(x + 2) - log(x - 1) + 1/2*log(x)
```

mupad [B] time = 2.19, size = 17, normalized size = 0.74

$$\frac{3\,\ln\left(x+2\right)}{2}-\ln\left(x-1\right)+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - x^2 + 1)/(x^2 - 2*x + x^3),x)
[Out] (3*log(x + 2))/2 - log(x - 1) + log(x)/2
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\frac{\log\left(x\right)}{2}-\log\left(x-1\right)+\frac{3\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-1)/(x**3+x**2-2*x),x)
[Out] log(x)/2 - log(x - 1) + 3*log(x + 2)/2
```

3.269 $\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1594, 1628, 628}

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx &= \int \frac{3-x+3x^2-2x^3+x^4}{x\left(3-2x+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+x+\frac{1-x}{3-2x+x^2}\right)dx\\
&= \frac{x^2}{2}+\log(x)+\int \frac{1-x}{3-2x+x^2}\,dx\\
&= \frac{x^2}{2}+\log(x)-\frac{1}{2}\log\left(3-2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-2x+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
```

```
[Out] x^2/2 + Log[x] - Log[3 - 2*x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-x+3x^2-2x^3+x^4}{3x-2x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3),x]
[Out] IntegrateAlgebraic[(3 - x + 3*x^2 - 2*x^3 + x^4)/(3*x - 2*x^2 + x^3), x]
```

fricas [A] time = 1.49, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

giac [A] time = 0.29, size = 20, normalized size = 0.87

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{x^2}{2}+\ln(x)-\frac{\ln\left(x^2-2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x)
[Out] 1/2*x^2+ln(x)-1/2*ln(x^2-2*x+3)
```

maxima [A] time = 1.12, size = 19, normalized size = 0.83

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\log\left(x^2-2\,x+3\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+3*x^2-x+3)/(x^3-2*x^2+3*x),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*log(x^2 - 2*x + 3) + log(x)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.83

$$\ln(x)-\frac{\ln\left(x^2-2\,x+3\right)}{2}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - x - 2*x^3 + x^4 + 3)/(3*x - 2*x^2 + x^3),x)
```

```
[Out] log(x) - log(x^2 - 2*x + 3)/2 + x^2/2
```

sympy [A] time = 0.11, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + \log(x) - \frac{\log(x^2 - 2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+3*x**2-x+3)/(x**3-2*x**2+3*x),x)
[Out] x**2/2 + log(x) - log(x**2 - 2*x + 3)/2
```

3.270 $\int \frac{-1+x+x^3}{\left(1+x^2\right)^2} dx$

Optimal. Leaf size=29

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1814, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^3)/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) - ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^3}{\left(1+x^2\right)^2} dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1-2x}{1+x^2} dx \\
&= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{1}{1+x^2} dx+\int \frac{x}{1+x^2} dx \\
&= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^3)/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) - ArcTan[x] + Log[1 + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x+x^3}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x + x^3)/(1 + x^2)^2,x]
[Out] IntegrateAlgebraic[(-1 + x + x^3)/(1 + x^2)^2, x]
```

fricas [A] time = 1.16, size = 32, normalized size = 1.10

$$-\frac{\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + x)/(x^2 + 1)
```

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 24, normalized size = 0.83

$$-\frac{x}{2\left(x^2+1\right)}-\frac{\arctan(x)}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x-1)/(x^2+1)^2,x)
[Out] -1/2*x/(x^2+1)-1/2*arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.33, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x-1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) - 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 2.13, size = 25, normalized size = 0.86

$$\frac{\ln\left(x^2+1\right)}{2}-\frac{\mathrm{atan}(x)}{2}-\frac{x}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 - 1)/(x^2 + 1)^2,x)
[Out] log(x^2 + 1)/2 - atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 20, normalized size = 0.69

$$-\frac{x}{2x^2+2}+\frac{\log\left(x^2+1\right)}{2}-\frac{\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x-1)/(x**2+1)**2,x)
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 - atan(x)/2
```

3.271 $\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx$

Optimal. Leaf size=44

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.24, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 6725, 634, 618, 204, 628}

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) - (2*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
+ Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx = \int \frac{1+2x-x^2+8x^3+x^4}{x(1+x)(1+x^3)}\,dx$$
$$= \int \left(\frac{1}{x}+\frac{3}{(1+x)^2}-\frac{2}{1+x}+\frac{2x}{1-x+x^2}\right)dx$$
$$= -\frac{3}{1+x}+\log(x)-2\log(1+x)+2\int \frac{x}{1-x+x^2}\,dx$$
$$= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\int \frac{1}{1-x+x^2}\,dx+\int \frac{-1+2x}{1-x+x^2}\,dx$$
$$= -\frac{3}{1+x}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)-2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,$$
$$= -\frac{3}{1+x}-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-2\log(1+x)+\log\left(1-x+x^2\right)$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$\log\left(x^2-x+1\right)-\frac{3}{x+1}+\log(x)-2\log(x+1)+\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] -3/(1 + x) + (2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[x] - 2*Log[1 + x]
 + Log[1 - x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x-x^2+8x^3+x^4}{(x+x^2)(1+x^3)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)),x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x - x^2 + 8*x^3 + x^4)/((x + x^2)*(1 + x^3)), x]
```

fricas [A] time = 1.41, size = 58, normalized size = 1.32

$$\frac{2\sqrt{3}(x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3(x+1)\log\left(x^2-x+1\right)-6(x+1)\log(x+1)+3(x+1)\log(x)-9}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*(x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x + 1)*log(x^2 -
x + 1) - 6*(x + 1)*log(x + 1) + 3*(x + 1)*log(x) - 9)/(x + 1)
```

giac [A] time = 0.36, size = 43, normalized size = 0.98

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1}+\log\left(x^2-x+1\right)-2\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.95

$$\frac{2\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\ln(x)-2\ln(x+1)+\ln\left(x^{2}-x+1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x)
```

```
[Out] ln(x^2-x+1)+2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-3/(x+1)-2*ln(x+1)+ln(x)
```

maxima [A] time = 2.25, size = 41, normalized size = 0.93

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{x+1}+\log\left(x^{2}-x+1\right)-2\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+8*x^3-x^2+2*x+1)/(x^2+x)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/(x + 1) + log(x^2 - x + 1) -
2*log(x + 1) + log(x)
```

mupad [B] time = 0.13, size = 55, normalized size = 1.25

$$\ln(x)-2\ln(x+1)-\frac{3}{x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 8*x^3 + x^4 + 1)/((x^3 + 1)*(x + x^2)),x)
```

```
[Out] log(x) - 2*log(x + 1) - 3/(x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/3 - 1) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/3 + 1)
```

sympy [A] time = 0.21, size = 49, normalized size = 1.11

$$\log(x)-2\log(x+1)+\log\left(x^{2}-x+1\right)+\frac{2\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+8*x**3-x**2+2*x+1)/(x**2+x)/(x**3+1),x)
```

```
[Out] log(x) - 2*log(x + 1) + log(x**2 - x + 1) + 2*sqrt(3)*atan(2*sqrt(3)*x/3 -
sqrt(3)/3)/3 - 3/(x + 1)
```

3.272 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{15-5x+x^2+x^3}{\left(5+x^2\right)\left(3+2x+x^2\right)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right)dx\\ &= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx\\ &= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx\\ &= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\ &= -\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15-5x+x^2+x^3}{\left(5+x^2\right)\left(3+2x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
[Out] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]
```

fricas [A] time = 1.70, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

giac [A] time = 0.30, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5}\,\arctan\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] -arctan(1/5*5^(1/2)*x)*5^(1/2)+1/2*ln(x^2+2*x+3)+5/2*2^(1/2)*arctan(1/4*(2*
x+2)*2^(1/2))
```

maxima [A] time = 2.29, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

mupad [B] time = 0.16, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\,\mathrm{atan}\left(\frac{2000\sqrt{5}}{2000x+1120}-\frac{224\sqrt{5}\,x}{2000x+1120}\right)-\frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}+\frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,x}{5}\right)+\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

3.273 $\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{(1+x^2)^2(2+x+x^2)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Rubi [A] time = 0.17, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {6742, 261, 260, 629, 628}

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2*(2 +
x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{\left(1+x^2\right)^2\left(2+x+x^2\right)^2}\,dx &= \int \left(\frac{6x}{\left(1+x^2\right)^2}+\frac{2x}{1+x^2}+\frac{-1-2x}{\left(2+x+x^2\right)^2}+\frac{-1-2x}{2+x+x^2}\right)dx\\ &= 2\int \frac{x}{1+x^2}\,dx+6\int \frac{x}{\left(1+x^2\right)^2}\,dx+\int \frac{-1-2x}{\left(2+x+x^2\right)^2}\,dx+\int\\ &= -\frac{3}{1+x^2}+\frac{1}{2+x+x^2}+\log\left(1+x^2\right)-\log\left(2+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1 + x^2)^2
*(2 + x + x^2)^2),x]
```

```
[Out] -3/(1 + x^2) + (2 + x + x^2)^(-1) + Log[1 + x^2] - Log[2 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+25x+23x^2+32x^3+15x^4+7x^5+x^6}{\left(1+x^2\right)^2\left(2+x+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1
 + x^2)^2*(2 + x + x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(-3 + 25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6)/((1
 + x^2)^2*(2 + x + x^2)^2), x]
```

fricas [B] time = 2.04, size = 72, normalized size = 2.18

$$-\frac{2\,x^2+\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+x+2\right)-\left(x^4+x^3+3\,x^2+x+2\right)\log\left(x^2+1\right)+3\,x+5}{x^4+x^3+3\,x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="fricas")
```

```
[Out] -(2*x^2 + (x^4 + x^3 + 3*x^2 + x + 2)*log(x^2 + x + 2) - (x^4 + x^3 + 3*x^2
 + x + 2)*log(x^2 + 1) + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2)
```

giac [A] time = 0.30, size = 44, normalized size = 1.33

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="giac")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$\ln\left(x^2+1\right)-\ln\left(x^2+x+2\right)-\frac{3}{x^2+1}+\frac{1}{x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x)
```

```
[Out] -3/(x^2+1)+1/(x^2+x+2)+ln(x^2+1)-ln(x^2+x+2)
```

maxima [A] time = 1.16, size = 44, normalized size = 1.33

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}-\log\left(x^2+x+2\right)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+7*x^5+15*x^4+32*x^3+23*x^2+25*x-3)/(x^2+1)^2/(x^2+x+2)^2,x,
algorithm="maxima")
```

```
[Out] -(2*x^2 + 3*x + 5)/(x^4 + x^3 + 3*x^2 + x + 2) - log(x^2 + x + 2) + log(x^2
 + 1)
```

mupad [B] time = 2.16, size = 56, normalized size = 1.70

$$-\frac{2\,x^2+3\,x+5}{x^4+x^3+3\,x^2+x+2}+\mathrm{atan}\left(\frac{\frac{x\,224\mathrm{i}}{11}+\frac{224}{11}\mathrm{i}}{44\,x^2+16\,x+60}-\frac{3}{11}\mathrm{i}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((25*x + 23*x^2 + 32*x^3 + 15*x^4 + 7*x^5 + x^6 - 3)/((x^2 + 1)^2*(x + x
^2 + 2)^2),x)
```

```
[Out] atan(((x*224i)/11 + 224i/11)/(16*x + 44*x^2 + 60) - 3i/11)*2i - (3*x + 2*x^
2 + 5)/(x + 3*x^2 + x^3 + x^4 + 2)
```

sympy [A] time = 0.19, size = 41, normalized size = 1.24

$$\frac{-2x^2-3x-5}{x^4+x^3+3x^2+x+2}+\log\left(x^2+1\right)-\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+7*x**5+15*x**4+32*x**3+23*x**2+25*x-3)/(x**2+1)**2/(x**2+x+
2)**2,x)
```

```
[Out] (-2*x**2 - 3*x - 5)/(x**4 + x**3 + 3*x**2 + x + 2) + log(x**2 + 1) - log(x*
*2 + x + 2)
```

3.274 $\int \frac{1}{(1+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{391, 203\}$

$$\frac{1}{3}\tan^{-1}(x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/6 + ArcTan[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^2)(4+x^2)} \, dx &= \frac{1}{3}\int \frac{1}{1+x^2} \, dx - \frac{1}{3}\int \frac{1}{4+x^2} \, dx\\ &= -\frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{6}\tan^{-1}\left(\frac{2}{x}\right) + \frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] ArcTan[2/x]/6 + ArcTan[x]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x^2)(4+x^2)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[1/((1 + x^2)*(4 + x^2)), x]
```

fricas [A] time = 1.77, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

giac [A] time = 0.29, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{\arctan(x)}{3}-\frac{\arctan\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+4),x)
[Out] -1/6*arctan(1/2*x)+1/3*arctan(x)
```

maxima [A] time = 2.20, size = 11, normalized size = 0.65

$$-\frac{1}{6}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+4),x, algorithm="maxima")
[Out] -1/6*arctan(1/2*x) + 1/3*arctan(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{\mathrm{atan}(x)}{3}-\frac{\mathrm{atan}\left(\frac{x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + 4)),x)
[Out] atan(x)/3 - atan(x/2)/6
```

sympy [A] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\mathrm{atan}\left(\frac{x}{2}\right)}{6}+\frac{\mathrm{atan}\,(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+4),x)
[Out] -atan(x/2)/6 + atan(x)/3
```

3.275 $\int \frac{a+bx^3}{1+x^2} dx$

Optimal. Leaf size=24

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1810, 635, 203, 260}

$$a\tan^{-1}(x)+\frac{bx^2}{2}-\frac{1}{2}b\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] (b*x^2)/2 + a*ArcTan[x] - (b*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3}{1+x^2}\,dx &= \int \left(bx+\frac{a-bx}{1+x^2}\right)dx\\
&= \frac{bx^2}{2}+\int \frac{a-bx}{1+x^2}\,dx\\
&= \frac{bx^2}{2}+a\int \frac{1}{1+x^2}\,dx-b\int \frac{x}{1+x^2}\,dx\\
&= \frac{bx^2}{2}+a\tan^{-1}(x)-\frac{1}{2}b\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$a\tan^{-1}(x)+\frac{1}{2}b\left(x^2-\log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] a*ArcTan[x] + (b*(x^2 - Log[1 + x^2]))/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^3}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)/(1 + x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)/(1 + x^2), x]
```

fricas [A] time = 1.30, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + a\arctan(x) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

giac [A] time = 0.29, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + a\arctan(x) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{b\,x^2}{2} + a\arctan(x) - \frac{b\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(x^2+1),x)
```

```
[Out] 1/2*b*x^2+a*arctan(x)-1/2*b*ln(x^2+1)
```

maxima [A] time = 2.25, size = 20, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + a\arctan(x) - \frac{1}{2}\,b\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*arctan(x) - 1/2*b*log(x^2 + 1)
```

mupad [B] time = 2.13, size = 20, normalized size = 0.83

$$\frac{b\,x^2}{2} - \frac{b\,\ln\left(x^2+1\right)}{2} + a\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(x^2 + 1),x)
```

```
[Out] (b*x^2)/2 - (b*log(x^2 + 1))/2 + a*atan(x)
```

sympy [C] time = 0.17, size = 34, normalized size = 1.42

$$\frac{bx^2}{2} + \left(-\frac{ia}{2} - \frac{b}{2}\right)\log\left(x - i\right) + \left(\frac{ia}{2} - \frac{b}{2}\right)\log\left(x + i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(x**2+1),x)
```

```
[Out] b*x**2/2 + (-I*a/2 - b/2)*log(x - I) + (I*a/2 - b/2)*log(x + I)
```

3.276 $\int \frac{x+x^2}{(4+x)(-4+x^2)} \, dx$

Optimal. Leaf size=15

$$\log(x+4) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.06, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1629, 207}

$$\log(x+4) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] -ArcTanh[x/2]/2 + Log[4 + x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)} \, dx &= \int \frac{x(1+x)}{(4+x)\left(-4+x^2\right)} \, dx \\
&= \int \left(\frac{1}{4+x} + \frac{1}{-4+x^2}\right) dx \\
&= \log(4+x) + \int \frac{1}{-4+x^2} \, dx \\
&= -\frac{1}{2}\tanh^{-1}\left(\frac{x}{2}\right) + \log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.53

$$\frac{1}{4}\log(2-x) - \frac{1}{4}\log(x+2) + \log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/((4 + x)*(-4 + x^2)),x]
```

```
[Out] Log[2 - x]/4 - Log[2 + x]/4 + Log[4 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+x^2}{(4+x)\left(-4+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^2)/((4 + x)*(-4 + x^2)),x]
[Out] IntegrateAlgebraic[(x + x^2)/((4 + x)*(-4 + x^2)), x]
```

fricas [A] time = 1.42, size = 17, normalized size = 1.13

$$\log\left(x+4\right)-\frac{1}{4}\,\log\left(x+2\right)+\frac{1}{4}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="fricas")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

giac [A] time = 0.32, size = 20, normalized size = 1.33

$$\log\left(|x+4|\right)-\frac{1}{4}\,\log\left(|x+2|\right)+\frac{1}{4}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 4)) - 1/4*log(abs(x + 2)) + 1/4*log(abs(x - 2))
```

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{\ln\left(x-2\right)}{4}-\frac{\ln\left(x+2\right)}{4}+\ln\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(4+x)/(x^2-4),x)
[Out] ln(4+x)+1/4*ln(x-2)-1/4*ln(x+2)
```

maxima [A] time = 2.31, size = 17, normalized size = 1.13

$$\log\left(x+4\right)-\frac{1}{4}\,\log\left(x+2\right)+\frac{1}{4}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(4+x)/(x^2-4),x, algorithm="maxima")
[Out] log(x + 4) - 1/4*log(x + 2) + 1/4*log(x - 2)
```

mupad [B] time = 0.06, size = 19, normalized size = 1.27

$$\ln\left(x+4\right)+\frac{\mathrm{atanh}\left(\frac{90}{7\,\left(21\,x+48\right)}-\frac{8}{7}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2)/((x^2 - 4)*(x + 4)),x)
```

```
[Out] log(x + 4) + atanh(90/(7*(21*x + 48)) - 8/7)/2
```

sympy [A] time = 0.14, size = 17, normalized size = 1.13

$$\frac{\log\left(x-2\right)}{4}-\frac{\log\left(x+2\right)}{4}+\log\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(4+x)/(x**2-4),x)
```

```
[Out] log(x - 2)/4 - log(x + 2)/4 + log(x + 4)
```

3.277 $\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=20

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {522, 203}

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx &= -\left(2\int \frac{1}{2+x^2}\,dx\right)+3\int \frac{1}{1+x^2}\,dx\\ &= 3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$3\tan^{-1}(x)-\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - Sqrt[2]*ArcTan[x/Sqrt[2]]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+x^2}{(1+x^2)(2+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x^2)/((1 + x^2)*(2 + x^2)),x]
[Out] IntegrateAlgebraic[(4 + x^2)/((1 + x^2)*(2 + x^2)), x]
```

fricas [A] time = 1.82, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

giac [A] time = 0.38, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

maple [A] time = 0.01, size = 18, normalized size = 0.90

$$3\arctan(x)-\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4)/(x^2+1)/(x^2+2),x)
[Out] 3*arctan(x)-2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.40, size = 17, normalized size = 0.85

$$-\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+3\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.85

$$3\,\mathrm{atan}(x)-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 4)/((x^2 + 1)*(x^2 + 2)),x)
[Out] 3*atan(x) - 2^(1/2)*atan((2^(1/2)*x)/2)
```

sympy [A] time = 0.15, size = 19, normalized size = 0.95

$$3\,\mathrm{atan}\,(x)-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*atan(x) - sqrt(2)*atan(sqrt(2)*x/2)
```

3.278 $\int \frac{5-4x+3x^2+x^4}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2(1-x)}+\frac{1}{2}\log(1-x)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] 5/(2*(1 - x)) + x + 2*ArcTan[x] + Log[1 - x]/2 + (3*Log[1 + x^2])/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(1+\frac{5}{2(-1+x)^2}+\frac{1}{2(-1+x)}+\frac{4+3x}{2\left(1+x^2\right)}\right)dx\\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{4+3x}{1+x^2}\,dx\\
&= \frac{5}{2(1-x)}+x+\frac{1}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx+2\int \frac{1}{1+x^2}\,dx\\
&= \frac{5}{2(1-x)}+x+2\tan^{-1}(x)+\frac{1}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.89

$$\frac{3}{4}\log\left(x^2+1\right)+x+\frac{5}{2-2x}+\frac{1}{2}\log(x-1)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
[Out] 5/(2 - 2*x) + x + 2*ArcTan[x] + Log[-1 + x]/2 + (3*Log[1 + x^2])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5-4x+3x^2+x^4}{(-1+x)^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(5 - 4*x + 3*x^2 + x^4)/((-1 + x)^2*(1 + x^2)), x]
```

fricas [A] time = 1.39, size = 44, normalized size = 1.19

$$\frac{4\,x^2+8\,(x-1)\arctan(x)+3\,(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)-4\,x-10}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
[Out] 1/4*(4*x^2 + 8*(x - 1)*arctan(x) + 3*(x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x
 - 1) - 4*x - 10)/(x - 1)
```

giac [B] time = 0.29, size = 60, normalized size = 1.62

$$\frac{1}{2}\,\pi-2\,\pi\left\lfloor\frac{\pi+4\,\arctan(x)}{4\,\pi}+\frac{1}{2}\right\rfloor+x-\frac{5}{2\,(x-1)}+2\,\arctan(x)+\frac{3}{4}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)+2\,\log\left(|x-1|\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="giac")
[Out] 1/2*pi - 2*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + x - 5/2/(x - 1) + 2*
arctan(x) + 3/4*log(2/(x - 1) + 2/(x - 1)^2 + 1) + 2*log(abs(x - 1)) - 1
```

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$x+2\arctan(x)+\frac{\ln\left(x-1\right)}{2}+\frac{3\ln\left(x^2+1\right)}{4}-\frac{5}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2-4*x+5)/(x-1)^2/(x^2+1),x)
[Out] x-5/2/(x-1)+1/2*ln(x-1)+3/4*ln(x^2+1)+2*arctan(x)
```

maxima [A] time = 2.22, size = 27, normalized size = 0.73

$$x-\frac{5}{2\,(x-1)}+2\,\arctan(x)+\frac{3}{4}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2-4*x+5)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 5/2/(x - 1) + 2*arctan(x) + 3/4*log(x^2 + 1) + 1/2*log(x - 1)
```

mupad [B] time = 2.14, size = 35, normalized size = 0.95

$$x+\frac{\ln(x-1)}{2}-\frac{5}{2\,(x-1)}+\ln(x-\mathrm{i})\,\left(\frac{3}{4}-\mathrm{i}\right)+\ln(x+1\mathrm{i})\,\left(\frac{3}{4}+1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + x^4 + 5)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] x + log(x - 1)/2 + log(x - 1i)*(3/4 - 1i) + log(x + 1i)*(3/4 + 1i) - 5/(2*(
x - 1))
```

sympy [A] time = 0.16, size = 29, normalized size = 0.78

$$x+\frac{\log(x-1)}{2}+\frac{3\log\left(x^2+1\right)}{4}+2\operatorname{atan}(x)-\frac{5}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2-4*x+5)/(-1+x)**2/(x**2+1),x)
```

```
[Out] x + log(x - 1)/2 + 3*log(x**2 + 1)/4 + 2*atan(x) - 5/(2*x - 2)
```

3.279 $\int \frac{1+x^4}{2+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}-2x+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1154, 203}

$$\frac{x^3}{3}-2x+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(2 + x^2),x]
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^4}{2+x^2} \, dx &= \int \left(-2+x^2+\frac{5}{2+x^2}\right) dx \\ &= -2x+\frac{x^3}{3}+5\int \frac{1}{2+x^2} \, dx \\ &= -2x+\frac{x^3}{3}+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x^3}{3}-2x+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(2 + x^2),x]
[Out] -2*x + x^3/3 + (5*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{2+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(2 + x^2),x]
[Out] IntegrateAlgebraic[(1 + x^4)/(2 + x^2), x]
```

fricas [A] time = 1.49, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="fricas")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

giac [A] time = 0.31, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="giac")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

maple [A] time = 0.00, size = 22, normalized size = 0.85

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+2),x)
[Out] -2*x+1/3*x^3+5/2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.22, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+2),x, algorithm="maxima")
[Out] 1/3*x^3 + 5/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.81

$$\frac{5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - 2x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^2 + 2),x)
[Out] (5*2^(1/2)*atan((2^(1/2)*x)/2))/2 - 2*x + x^3/3
```

sympy [A] time = 0.09, size = 26, normalized size = 1.00

$$\frac{x^3}{3} - 2x + \frac{5\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+2),x)
```

```
[Out] x**3/3 - 2*x + 5*sqrt(2)*atan(sqrt(2)*x/2)/2
```

3.280 $\int \frac{2+2x+x^4}{x^4+x^5}\,dx$

Optimal. Leaf size=12

$$\log(x+1)-\frac{2}{3x^3}$$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1593, 1620}

$$\log(x+1)-\frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x+x^4}{x^4+x^5}\,dx &= \int \frac{2+2x+x^4}{x^4(1+x)}\,dx\\ &= \int \left(\frac{2}{x^4}+\frac{1}{1+x}\right)dx\\ &= -\frac{2}{3x^3}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(x+1)-\frac{2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^4)/(x^4 + x^5),x]
```

```
[Out] -2/(3*x^3) + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+2x+x^4}{x^4+x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 2*x + x^4)/(x^4 + x^5),x]
[Out] IntegrateAlgebraic[(2 + 2*x + x^4)/(x^4 + x^5), x]
```

fricas [A] time = 1.64, size = 16, normalized size = 1.33

$$\frac{3\,x^3\log\left(x+1\right)-2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x + 1) - 2)/x^3
```

giac [A] time = 0.29, size = 11, normalized size = 0.92

$$-\frac{2}{3\,x^3}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="giac")
[Out] -2/3/x^3 + log(abs(x + 1))
```

maple [A] time = 0.01, size = 11, normalized size = 0.92

$$\ln\left(x+1\right)-\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x+2)/(x^5+x^4),x)
[Out] -2/3/x^3+ln(x+1)
```

maxima [A] time = 1.11, size = 10, normalized size = 0.83

$$-\frac{2}{3\,x^3}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x+2)/(x^5+x^4),x, algorithm="maxima")
[Out] -2/3/x^3 + log(x + 1)
```

mupad [B] time = 2.12, size = 10, normalized size = 0.83

$$\ln\left(x+1\right)-\frac{2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4 + 2)/(x^4 + x^5),x)
[Out] log(x + 1) - 2/(3*x^3)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.83

$$\log\left(x+1\right)-\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x+2)/(x**5+x**4),x)
[Out] log(x + 1) - 2/(3*x**3)
```

3.281 $\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx$

Optimal. Leaf size=21

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2074}

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx &= \int \left(\frac{1}{2-x}+\frac{2}{-1+x}+\frac{1}{1+x}\right)dx\\ &= 2\log(1-x)-\log(2-x)+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$2\log(1-x)-\log(2-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] - Log[2 - x] + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1-5x+2x^2}{2-x-2x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-1 - 5*x + 2*x^2)/(2 - x - 2*x^2 + x^3), x]
```

fricas [A] time = 1.42, size = 17, normalized size = 0.81

$$\log(x+1)+2\log(x-1)-\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="fricas")
```

```
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

giac [A] time = 0.28, size = 20, normalized size = 0.95

$$\log\left(|x+1|\right) + 2\,\log\left(|x-1|\right) - \log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="giac")
[Out] log(abs(x + 1)) + 2*log(abs(x - 1)) - log(abs(x - 2))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\ln\left(x-2\right) + 2\ln\left(x-1\right) + \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x)
[Out] 2*ln(x-1)-ln(x-2)+ln(x+1)
```

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$\log\left(x+1\right) + 2\,\log\left(x-1\right) - \log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5*x-1)/(x^3-2*x^2-x+2),x, algorithm="maxima")
[Out] log(x + 1) + 2*log(x - 1) - log(x - 2)
```

mupad [B] time = 2.14, size = 21, normalized size = 1.00

$$2\,\ln\left(x-1\right) - 2\,\mathrm{atanh}\left(\frac{144}{11\,\left(22\,x-50\right)} + \frac{13}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x - 2*x^2 + 1)/(x + 2*x^2 - x^3 - 2),x)
[Out] 2*log(x - 1) - 2*atanh(144/(11*(22*x - 50)) + 13/11)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.71

$$-\log\left(x-2\right) + 2\log\left(x-1\right) + \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5*x-1)/(x**3-2*x**2-x+2),x)
[Out] -log(x - 2) + 2*log(x - 1) + log(x + 1)
```

3.282 $\int \frac{2+x+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {28, 1814, 635, 203, 260}

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx &= \int \frac{2+x+x^3}{\left(1+x^2\right)^2}\,dx \\ &= \frac{x}{1+x^2}-\frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx \\ &= \frac{x}{1+x^2}+\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx \\ &= \frac{x}{1+x^2}+\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{x}{x^2+1}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
[Out] x/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x+x^3}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x + x^3)/(1 + 2*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(2 + x + x^3)/(1 + 2*x^2 + x^4), x]
```

fricas [A] time = 1.67, size = 34, normalized size = 1.55

$$\frac{2\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)+2\,x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x)/(x^2 + 1)
```

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{x}{x^2+1}+\arctan(x)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+2)/(x^4+2*x^2+1),x)
```

```
[Out] 1/(x^2+1)*x+arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.23, size = 20, normalized size = 0.91

$$\frac{x}{x^2+1}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+2)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 2.12, size = 20, normalized size = 0.91

$$\frac{\ln\left(x^2+1\right)}{2}+\mathrm{atan}(x)+\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 2)/(2*x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + 1)/2 + atan(x) + x/(x^2 + 1)
```

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$\frac{x}{x^2+1}+\frac{\log\left(x^2+1\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+2)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1) + log(x**2 + 1)/2 + atan(x)
```

3.283 $\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=24

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {28, 1814, 635, 203, 260}

$$-\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x] + Log[1 + x^2]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx = \int \frac{1+2x+x^2+x^3}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int \frac{-2-2x}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \tan^{-1}(x) + \frac{1}{2}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{1}{2\left(x^2+1\right)} + \frac{1}{2}\log\left(x^2+1\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x] + Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x+x^2+x^3}{1+2x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(1 + 2*x + x^2 + x^3)/(1 + 2*x^2 + x^4), x]
```

fricas [A] time = 1.44, size = 32, normalized size = 1.33

$$\frac{2\left(x^2+1\right)\arctan(x) + \left(x^2+1\right)\log\left(x^2+1\right) - 1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 1)/(x^2 + 1)
```

giac [A] time = 0.40, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^2+1\right)} + \arctan(x) + \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x) + \frac{\ln\left(x^2+1\right)}{2} - \frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x)
[Out] -1/2/(x^2+1)+arctan(x)+1/2*ln(x^2+1)
```

maxima [A] time = 2.18, size = 20, normalized size = 0.83

$$-\frac{1}{2\left(x^2+1\right)}+\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2*x+1)/(x^4+2*x^2+1),x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x) + 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.92

$$\frac{\ln\left(x^2+1\right)}{2}+\mathrm{atan}(x)-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + x^3 + 1)/(2*x^2 + x^4 + 1),x)
[Out] log(x^2 + 1)/2 + atan(x) - 1/(2*(x^2 + 1))
```

sympy [A] time = 0.12, size = 19, normalized size = 0.79

$$\frac{\log\left(x^2+1\right)}{2}+\mathrm{atan}\,(x)-\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2*x+1)/(x**4+2*x**2+1),x)
[Out] log(x**2 + 1)/2 + atan(x) - 1/(2*x**2 + 2)
```

3.284 $\int \frac{3+4x}{(1+x^2)(2+x^2)} dx$

Optimal. Leaf size=36

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1010, 391, 203, 444, 36, 31}

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\tan^{-1}(x)-\frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx = 3\int \frac{1}{(1+x^2)(2+x^2)}\,dx + 4\int \frac{x}{(1+x^2)(2+x^2)}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{(1+x)(2+x)}\,dx, x, x^2\right) + 3\int \frac{1}{1+x^2}\,dx - 3\int \frac{1}{2+x^2}\,dx$$
$$= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - 2\,\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right)$$
$$= 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}} + 2\log(1+x^2) - 2\log(2+x^2)$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$2\log(x^2+1) - 2\log(x^2+2) + 3\tan^{-1}(x) - \frac{3\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
[Out] 3*ArcTan[x] - (3*ArcTan[x/Sqrt[2]])/Sqrt[2] + 2*Log[1 + x^2] - 2*Log[2 + x^
2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+4x}{(1+x^2)(2+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 4*x)/((1 + x^2)*(2 + x^2)),x]
[Out] IntegrateAlgebraic[(3 + 4*x)/((1 + x^2)*(2 + x^2)), x]
```

fricas [A] time = 1.43, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log(x^2+2) + 2\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

giac [A] time = 0.25, size = 33, normalized size = 0.92

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) + 3\arctan(x) - 2\log(x^2+2) + 2\log(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

maple [A] time = 0.00, size = 34, normalized size = 0.94

$$3\arctan(x)-\frac{3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+2\ln\left(x^2+1\right)-2\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*x)/(x^2+1)/(x^2+2),x)
```

```
[Out] 3*arctan(x)+2*ln(x^2+1)-2*ln(x^2+2)-3/2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.06, size = 33, normalized size = 0.92

$$-\frac{3}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+3\,\arctan(x)-2\,\log\left(x^2+2\right)+2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 3*arctan(x) - 2*log(x^2 + 2) + 2*log(x
^2 + 1)
```

mupad [B] time = 0.10, size = 56, normalized size = 1.56

$$\ln\left(x-\mathrm{i}\right)\,\left(2-\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(2+\frac{3}{2}\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\,\left(-2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\,\left(2+\frac{\sqrt{2}\,3\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(2 - 3i/2) + log(x + 1i)*(2 + 3i/2) + log(x - 2^(1/2)*1i)*((2^(
1/2)*3i)/4 - 2) - log(x + 2^(1/2)*1i)*((2^(1/2)*3i)/4 + 2)
```

sympy [A] time = 0.19, size = 39, normalized size = 1.08

$$2\log\left(x^2+1\right)-2\log\left(x^2+2\right)+3\,\mathrm{atan}\left(x\right)-\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(x**2+1)/(x**2+2),x)
```

```
[Out] 2*log(x**2 + 1) - 2*log(x**2 + 2) + 3*atan(x) - 3*sqrt(2)*atan(sqrt(2)*x/2)
/2
```

3.285 $\int \frac{2+x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1010, 391, 203, 444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)-\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/3 + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= 2\int \frac{1}{\left(1+x^2\right)\left(4+x^2\right)}\,dx + \int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx, x, x^2\right) + \frac{2}{3}\int \frac{1}{1+x^2}\,dx - \frac{2}{3}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3}\tan^{-1}(x) + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right)\\ &= -\frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3}\tan^{-1}(x) + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6}\log\left(x^2+1\right) - \frac{1}{6}\log\left(x^2+4\right) - \frac{1}{3}\tan^{-1}\left(\frac{x}{2}\right) + \frac{2}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((1 + x^2)*(4 + x^2)),x]
[Out] -1/3*ArcTan[x/2] + (2*ArcTan[x])/3 + Log[1 + x^2]/6 - Log[4 + x^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x)/((1 + x^2)*(4 + x^2)),x]
[Out] IntegrateAlgebraic[(2 + x)/((1 + x^2)*(4 + x^2)), x]
```

fricas [A] time = 1.03, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="fricas")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

giac [A] time = 0.29, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right) + \frac{2}{3}\arctan(x) - \frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="giac")
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2\arctan(x)}{3} - \frac{\arctan\left(\frac{x}{2}\right)}{3} + \frac{\ln\left(x^2+1\right)}{6} - \frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+1)/(x^2+4),x)
```

```
[Out] -1/3*arctan(1/2*x)+2/3*arctan(x)+1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

maxima [A] time = 2.00, size = 27, normalized size = 0.73

$$-\frac{1}{3}\arctan\left(\frac{1}{2}x\right)+\frac{2}{3}\arctan(x)-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/3*arctan(1/2*x) + 2/3*arctan(x) - 1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

mupad [B] time = 2.14, size = 37, normalized size = 1.00

$$\ln(x-\mathrm{i})\left(\frac{1}{6}-\frac{1}{3}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{6}+\frac{1}{3}\mathrm{i}\right)+\ln(x-2\mathrm{i})\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+\ln(x+2\mathrm{i})\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/((x^2 + 1)*(x^2 + 4)),x)
```

```
[Out] log(x - 1i)*(1/6 - 1i/3) + log(x + 1i)*(1/6 + 1i/3) - log(x - 2i)*(1/6 - 1i
/6) - log(x + 2i)*(1/6 + 1i/6)
```

sympy [A] time = 0.17, size = 29, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{2\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6 - atan(x/2)/3 + 2*atan(x)/3
```

3.286 $\int \frac{2-x+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{2(22+21x)}{-7-6x+x^2}\right)dx\\ &= 6x+\frac{x^2}{2}+2\int \frac{22+21x}{-7-6x+x^2}\,dx\\ &= 6x+\frac{x^2}{2}-\frac{1}{4}\int \frac{1}{1+x}\,dx+\frac{169}{4}\int \frac{1}{-7+x}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2}+6x+\frac{169}{4}\log(7-x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (169*Log[7 - x])/4 - Log[1 + x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-x+x^3}{-7-6x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - x + x^3)/(-7 - 6*x + x^2),x]
[Out] IntegrateAlgebraic[(2 - x + x^3)/(-7 - 6*x + x^2), x]
```

fricas [A] time = 1.19, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

giac [A] time = 0.37, size = 23, normalized size = 0.79

$$\frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(|x+1|\right)+\frac{169}{4}\,\log\left(|x-7|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x - 1/4*log(abs(x + 1)) + 169/4*log(abs(x - 7))
```

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+6x-\frac{\ln\left(x+1\right)}{4}+\frac{169\ln\left(x-7\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x+2)/(x^2-6*x-7),x)
[Out] 1/2*x^2+6*x-1/4*ln(x+1)+169/4*ln(x-7)
```

maxima [A] time = 1.08, size = 21, normalized size = 0.72

$$\frac{1}{2}\,x^2+6\,x-\frac{1}{4}\,\log\left(x+1\right)+\frac{169}{4}\,\log\left(x-7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x+2)/(x^2-6*x-7),x, algorithm="maxima")
[Out] 1/2*x^2 + 6*x - 1/4*log(x + 1) + 169/4*log(x - 7)
```

mupad [B] time = 2.21, size = 21, normalized size = 0.72

$$6\,x-\frac{\ln\left(x+1\right)}{4}+\frac{169\,\ln\left(x-7\right)}{4}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x + 2)/(6*x - x^2 + 7),x)
```

```
[Out] 6*x - log(x + 1)/4 + (169*log(x - 7))/4 + x^2/2
```

sympy [**A**] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{169\log(x-7)}{4} - \frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x+2)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 169*log(x - 7)/4 - log(x + 1)/4
```

3.287 $\int \frac{-1+x^5}{-1+x^2} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1810, 627, 31}

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^5}{-1+x^2} dx &= \int \left(x + x^3 - \frac{1-x}{-1+x^2}\right) dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1-x}{-1+x^2} dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} - \int \frac{1}{-1-x} dx \\
&= \frac{x^2}{2} + \frac{x^4}{4} + \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-1 + x^2),x]
```

```
[Out] x^2/2 + x^4/4 + Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^5}{-1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^5)/(-1 + x^2),x]
[Out] IntegrateAlgebraic[(-1 + x^5)/(-1 + x^2), x]
```

fricas [A] time = 1.23, size = 15, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="fricas")
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

giac [A] time = 0.27, size = 16, normalized size = 0.84

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="giac")
[Out] 1/4*x^4 + 1/2*x^2 + log(abs(x + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{x^4}{4}+\frac{x^2}{2}+\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^2-1),x)
[Out] 1/2*x^2+1/4*x^4+ln(x+1)
```

maxima [A] time = 1.07, size = 15, normalized size = 0.79

$$\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^2-1),x, algorithm="maxima")
[Out] 1/4*x^4 + 1/2*x^2 + log(x + 1)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.79

$$\ln\left(x+1\right)+\frac{x^2}{2}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5 - 1)/(x^2 - 1),x)
[Out] log(x + 1) + x^2/2 + x^4/4
```

sympy [A] time = 0.08, size = 14, normalized size = 0.74

$$\frac{x^4}{4} + \frac{x^2}{2} + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**2-1),x)
```

```
[Out] x**4/4 + x**2/2 + log(x + 1)
```

3.288 $\int \frac{5+2x-x^2+x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx &= \int \left(-2+x+\frac{7+3x}{1+x+x^2}\right)dx\\ &= -2x+\frac{x^2}{2}+\int \frac{7+3x}{1+x+x^2}\,dx\\ &= -2x+\frac{x^2}{2}+\frac{3}{2}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{11}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= -2x+\frac{x^2}{2}+\frac{3}{2}\log\left(1+x+x^2\right)-11\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -2x+\frac{x^2}{2}+\frac{11\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{x^2}{2}+\frac{3}{2}\log\left(x^2+x+1\right)-2x+\frac{11\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] -2*x + x^2/2 + (11*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + (3*Log[1 + x + x^2]
)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+2x-x^2+x^3}{1+x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + 2*x - x^2 + x^3)/(1 + x + x^2),x]
```

```
[Out] IntegrateAlgebraic[(5 + 2*x - x^2 + x^3)/(1 + x + x^2), x]
```

fricas [A] time = 1.53, size = 34, normalized size = 0.83

$$\frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

giac [A] time = 0.29, size = 34, normalized size = 0.83

$$\frac{1}{2}\,x^2+\frac{11}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-2\,x+\frac{3}{2}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$\frac{x^2}{2} - 2x + \frac{11\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{3\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2+2*x+5)/(x^2+x+1),x)
```

```
[Out] -2*x+1/2*x^2+3/2*ln(x^2+x+1)+11/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.18, size = 34, normalized size = 0.83

$$\frac{1}{2}\,x^2 + \frac{11}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right) - 2\,x + \frac{3}{2}\,\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2+2*x+5)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 11/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*x + 3/2*log(x^2 +
x + 1)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.88

$$\frac{3\,\ln\left(x^2+x+1\right)}{2} - 2\,x + \frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + x^3 + 5)/(x + x^2 + 1),x)
```

```
[Out] (3*log(x + x^2 + 1))/2 - 2*x + (11*3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3
))/3 + x^2/2
```

sympy [A] time = 0.12, size = 46, normalized size = 1.12

$$\frac{x^2}{2} - 2x + \frac{3\log\left(x^2+x+1\right)}{2} + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2+2*x+5)/(x**2+x+1),x)
```

```
[Out] x**2/2 - 2*x + 3*log(x**2 + x + 1)/2 + 11*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt
(3)/3)/3
```

3.289 $\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2} \, dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1657, 634, 618, 204, 628}

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3}{4}\log\left(x^2-4x+5\right)+\frac{3x}{2}+6\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
```

```
[Out] (3*x)/2 + x^2/2 + x^3/6 + 6*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx = \int \left(\frac{3}{2}+x+\frac{x^2}{2}-\frac{3(6-x)}{10-8x+2x^2}\right)dx$$
$$= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-3\int \frac{6-x}{10-8x+2x^2}\,dx$$
$$= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\int \frac{-8+4x}{10-8x+2x^2}\,dx-12\int \frac{1}{10-8x+2x^2}\,dx$$
$$= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+\frac{3}{4}\log\left(5-4x+x^2\right)+24\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,-8+4x\right)$$
$$= \frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{6}+6\tan^{-1}(2-x)+\frac{3}{4}\log\left(5-4x+x^2\right)$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.95

$$\frac{1}{2}\left(\frac{x^3}{3}+x^2+\frac{3}{2}\log\left(x^2-4x+5\right)+3x+12\tan^{-1}(2-x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
[Out] (3*x + x^2 + x^3/3 + 12*ArcTan[2 - x] + (3*Log[5 - 4*x + x^2])/2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+x-2x^3+x^4}{10-8x+2x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2),x]
[Out] IntegrateAlgebraic[(-3 + x - 2*x^3 + x^4)/(10 - 8*x + 2*x^2), x]
```

fricas [A] time = 1.76, size = 31, normalized size = 0.76

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="fricas")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

giac [A] time = 0.28, size = 31, normalized size = 0.76

$$\frac{1}{6}\,x^3+\frac{1}{2}\,x^2+\frac{3}{2}\,x-6\,\arctan\left(x-2\right)+\frac{3}{4}\,\log\left(x^2-4\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="giac")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

maple [A] time = 0.00, size = 32, normalized size = 0.78

$$\frac{x^3}{6}+\frac{x^2}{2}+\frac{3x}{2}-6\arctan\left(x-2\right)+\frac{3\ln\left(x^2-4x+5\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x)
[Out] 3/2*x+1/2*x^2+1/6*x^3-6*arctan(x-2)+3/4*ln(x^2-4*x+5)
```

maxima [A] time = 2.07, size = 31, normalized size = 0.76

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{3}{2}x - 6\arctan(x-2) + \frac{3}{4}\log\left(x^2 - 4x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^3+x-3)/(2*x^2-8*x+10),x, algorithm="maxima")
[Out] 1/6*x^3 + 1/2*x^2 + 3/2*x - 6*arctan(x - 2) + 3/4*log(x^2 - 4*x + 5)
```

mupad [B] time = 2.12, size = 31, normalized size = 0.76

$$\frac{3x}{2} - 6\,\mathrm{atan}(x-2) + \frac{3\ln\left(x^2 - 4x + 5\right)}{4} + \frac{x^2}{2} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3 + x^4 - 3)/(2*x^2 - 8*x + 10),x)
[Out] (3*x)/2 - 6*atan(x - 2) + (3*log(x^2 - 4*x + 5))/4 + x^2/2 + x^3/6
```

sympy [A] time = 0.12, size = 34, normalized size = 0.83

$$\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{2} + \frac{3\log\left(x^2 - 4x + 5\right)}{4} - 6\,\mathrm{atan}(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**3+x-3)/(2*x**2-8*x+10),x)
[Out] x**3/6 + x**2/2 + 3*x/2 + 3*log(x**2 - 4*x + 5)/4 - 6*atan(x - 2)
```

3.290 $\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=30

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1612}

$$x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] x + (7*Log[1 - x])/2 - 25*Log[2 - x] + (61*Log[3 - x])/2
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(1+\frac{61}{2(-3+x)}-\frac{25}{-2+x}+\frac{7}{2(-1+x)}\right)dx\\ &= x+\frac{7}{2}\log(1-x)-25\log(2-x)+\frac{61}{2}\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.80

$$x+\frac{61}{2}\log(x-3)-25\log(x-2)+\frac{7}{2}\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] x + (61*Log[-3 + x])/2 - 25*Log[-2 + x] + (7*Log[-1 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x+3x^2+x^3}{(-3+x)(-2+x)(-1+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)),x]
[Out] IntegrateAlgebraic[(1 + 2*x + 3*x^2 + x^3)/((-3 + x)*(-2 + x)*(-1 + x)), x]
```

fricas [A] time = 0.69, size = 20, normalized size = 0.67

$$x+\frac{7}{2}\log\left(x-1\right)-25\log\left(x-2\right)+\frac{61}{2}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

giac [A] time = 0.25, size = 23, normalized size = 0.77

$$x + \frac{7}{2}\log\left(|x-1|\right) - 25\log\left(|x-2|\right) + \frac{61}{2}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] x + 7/2*log(abs(x - 1)) - 25*log(abs(x - 2)) + 61/2*log(abs(x - 3))
```

maple [A] time = 0.01, size = 21, normalized size = 0.70

$$x + \frac{61\ln\left(x-3\right)}{2} - 25\ln\left(x-2\right) + \frac{7\ln\left(x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+2*x+1)/(x-3)/(x-2)/(x-1),x)
[Out] x+7/2*ln(x-1)-25*ln(x-2)+61/2*ln(x-3)
```

maxima [A] time = 1.04, size = 20, normalized size = 0.67

$$x + \frac{7}{2}\log\left(x-1\right) - 25\log\left(x-2\right) + \frac{61}{2}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] x + 7/2*log(x - 1) - 25*log(x - 2) + 61/2*log(x - 3)
```

mupad [B] time = 2.13, size = 20, normalized size = 0.67

$$x + \frac{7\ln\left(x-1\right)}{2} - 25\ln\left(x-2\right) + \frac{61\ln\left(x-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 + x^3 + 1)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] x + (7*log(x - 1))/2 - 25*log(x - 2) + (61*log(x - 3))/2
```

sympy [A] time = 0.15, size = 24, normalized size = 0.80

$$x + \frac{61\log\left(x-3\right)}{2} - 25\log\left(x-2\right) + \frac{7\log\left(x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+2*x+1)/(-3+x)/(-2+x)/(-1+x),x)
[Out] x + 61*log(x - 3)/2 - 25*log(x - 2) + 7*log(x - 1)/2
```

3.291 $\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$

Optimal. Leaf size=35

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2074}

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx &= \int \left(-2+\frac{13}{3(-4+x)}+x-\frac{22}{3(2+x)}+\frac{20}{3+x}\right)dx\\ &= -2x+\frac{x^2}{2}+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(2+x)+20\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{x^2}{2}-2x+\frac{13}{3}\log(4-x)-\frac{22}{3}\log(x+2)+20\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] -2*x + x^2/2 + (13*Log[4 - x])/3 - (22*Log[2 + x])/3 + 20*Log[3 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-7x+x^2-x^3+x^4}{-24-14x+x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3),x]
[Out] IntegrateAlgebraic[(2 - 7*x + x^2 - x^3 + x^4)/(-24 - 14*x + x^2 + x^3), x]
```

fricas [A] time = 1.24, size = 27, normalized size = 0.77

$$\frac{1}{2}x^2-2x+20\log(x+3)-\frac{22}{3}\log(x+2)+\frac{13}{3}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

giac [A] time = 0.32, size = 30, normalized size = 0.86

$$\frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(|x+3|\right) - \frac{22}{3}\,\log\left(|x+2|\right) + \frac{13}{3}\,\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x + 20*log(abs(x + 3)) - 22/3*log(abs(x + 2)) + 13/3*log(abs(x
- 4))
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{x^2}{2} - 2x - \frac{22\ln\left(x+2\right)}{3} + 20\ln\left(x+3\right) + \frac{13\ln\left(x-4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x)
[Out] -2*x+1/2*x^2-22/3*ln(x+2)+13/3*ln(x-4)+20*ln(x+3)
```

maxima [A] time = 1.12, size = 27, normalized size = 0.77

$$\frac{1}{2}\,x^2 - 2\,x + 20\,\log\left(x+3\right) - \frac{22}{3}\,\log\left(x+2\right) + \frac{13}{3}\,\log\left(x-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+x^2-7*x+2)/(x^3+x^2-14*x-24),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x + 20*log(x + 3) - 22/3*log(x + 2) + 13/3*log(x - 4)
```

mupad [B] time = 0.04, size = 27, normalized size = 0.77

$$20\,\ln\left(x+3\right) - \frac{22\,\ln\left(x+2\right)}{3} - 2\,x + \frac{13\,\ln\left(x-4\right)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 7*x - x^3 + x^4 + 2)/(14*x - x^2 - x^3 + 24),x)
[Out] 20*log(x + 3) - (22*log(x + 2))/3 - 2*x + (13*log(x - 4))/3 + x^2/2
```

sympy [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{x^2}{2} - 2x + \frac{13\log\left(x-4\right)}{3} - \frac{22\log\left(x+2\right)}{3} + 20\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+x**2-7*x+2)/(x**3+x**2-14*x-24),x)
[Out] x**2/2 - 2*x + 13*log(x - 4)/3 - 22*log(x + 2)/3 + 20*log(x + 3)
```

3.292 $\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$

Optimal. Leaf size=34

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Rubi [A] time = 0.06, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1612}

$$\frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
[Out] 3/(2*(1 - x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx &= \int \left(\frac{3}{2(-1+x)^2}-\frac{5}{4(-1+x)}+\frac{2}{x}-\frac{3}{4(1+x)}\right)dx\\ &= \frac{3}{2(1-x)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$-\frac{3}{2(x-1)}-\frac{5}{4}\log(1-x)+2\log(x)-\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
[Out] -3/(2*(-1 + x)) - (5*Log[1 - x])/4 + 2*Log[x] - (3*Log[1 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x^2}{(-1+x)^2x(1+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-1 + x)^2*x*(1 + x)),x]
[Out] IntegrateAlgebraic[(2 + x^2)/((-1 + x)^2*x*(1 + x)), x]
```

fricas [A] time = 1.57, size = 34, normalized size = 1.00

$$-\frac{3\,(x-1)\log\,(x+1)+5\,(x-1)\log\,(x-1)-8\,(x-1)\log(x)+6}{4\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="fricas")
```

```
[Out] -1/4*(3*(x - 1)*log(x + 1) + 5*(x - 1)*log(x - 1) - 8*(x - 1)*log(x) + 6)/(
x - 1)
```

giac [A] time = 0.39, size = 34, normalized size = 1.00

$$-\frac{3}{2(x-1)} + 2\log\left(\left|-\frac{1}{x-1} - 1\right|\right) - \frac{3}{4}\log\left(\left|-\frac{2}{x-1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="giac")
```

```
[Out] -3/2/(x - 1) + 2*log(abs(-1/(x - 1) - 1)) - 3/4*log(abs(-2/(x - 1) - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$2\ln(x) - \frac{5\ln(x-1)}{4} - \frac{3\ln(x+1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x-1)^2/x/(x+1),x)
```

```
[Out] -3/2/(x-1)-5/4*ln(x-1)-3/4*ln(x+1)+2*ln(x)
```

maxima [A] time = 1.13, size = 24, normalized size = 0.71

$$-\frac{3}{2(x-1)} - \frac{3}{4}\log(x+1) - \frac{5}{4}\log(x-1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-1+x)^2/x/(1+x),x, algorithm="maxima")
```

```
[Out] -3/2/(x - 1) - 3/4*log(x + 1) - 5/4*log(x - 1) + 2*log(x)
```

mupad [B] time = 2.11, size = 26, normalized size = 0.76

$$2\ln(x) - \frac{3\ln(x+1)}{4} - \frac{5\ln(x-1)}{4} - \frac{3}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x*(x - 1)^2*(x + 1)),x)
```

```
[Out] 2*log(x) - (3*log(x + 1))/4 - (5*log(x - 1))/4 - 3/(2*(x - 1))
```

sympy [A] time = 0.14, size = 27, normalized size = 0.79

$$2\log(x) - \frac{5\log(x-1)}{4} - \frac{3\log(x+1)}{4} - \frac{3}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-1+x)**2/x/(1+x),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/4 - 3*log(x + 1)/4 - 3/(2*x - 2)
```

3.293 $\int \frac{3+x^2+x^3}{(2+x^2)^2} dx$

Optimal. Leaf size=42

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log(x^2+2)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1814, 635, 203, 260}

$$\frac{x+4}{4(x^2+2)}+\frac{1}{2}\log(x^2+2)+\frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2}\,dx = \frac{4+x}{4\left(2+x^2\right)} - \frac{1}{4}\int \frac{-5-4x}{2+x^2}\,dx$$
$$= \frac{4+x}{4\left(2+x^2\right)} + \frac{5}{4}\int \frac{1}{2+x^2}\,dx + \int \frac{x}{2+x^2}\,dx$$
$$= \frac{4+x}{4\left(2+x^2\right)} + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}} + \frac{1}{2}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{x+4}{4\left(x^2+2\right)} + \frac{1}{2}\log\left(x^2+2\right) + \frac{5\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] (4 + x)/(4*(2 + x^2)) + (5*ArcTan[x/Sqrt[2]])/(4*Sqrt[2]) + Log[2 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+x^2+x^3}{\left(2+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + x^2 + x^3)/(2 + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(3 + x^2 + x^3)/(2 + x^2)^2, x]
```

fricas [A] time = 0.91, size = 44, normalized size = 1.05

$$\frac{5\sqrt{2}\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right) + 4\left(x^2+2\right)\log\left(x^2+2\right) + 2\,x + 8}{8\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) + 4*(x^2 + 2)*log(x^2 + 2) +
 2*x + 8)/(x^2 + 2)
```

giac [A] time = 0.30, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right) + \frac{x+4}{4\left(x^2+2\right)} + \frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$\frac{5\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{\ln\left(x^{2}+2\right)}{2}+\frac{\frac{x}{4}+1}{x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+3)/(x^2+2)^2,x)
```

```
[Out] (1/4*x+1)/(x^2+2)+1/2*ln(x^2+2)+5/8*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.36, size = 33, normalized size = 0.79

$$\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{x+4}{4\left(x^{2}+2\right)}+\frac{1}{2}\log\left(x^{2}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+3)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(x + 4)/(x^2 + 2) + 1/2*log(x^2 + 2
)
```

mupad [B] time = 2.19, size = 39, normalized size = 0.93

$$\frac{\ln\left(x^{2}+2\right)}{2}+\frac{5\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}+\frac{x}{4\left(x^{2}+2\right)}+\frac{1}{x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 3)/(x^2 + 2)^2,x)
```

```
[Out] log(x^2 + 2)/2 + (5*2^(1/2)*atan((2^(1/2)*x)/2))/8 + x/(4*(x^2 + 2)) + 1/(x
^2 + 2)
```

sympy [A] time = 0.13, size = 36, normalized size = 0.86

$$\frac{x+4}{4x^{2}+8}+\frac{\log\left(x^{2}+2\right)}{2}+\frac{5\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+3)/(x**2+2)**2,x)
```

```
[Out] (x + 4)/(4*x**2 + 8) + log(x**2 + 2)/2 + 5*sqrt(2)*atan(sqrt(2)*x/2)/8
```

3.294 $\int \frac{-35+70x-4x^2+2x^3}{\left(26-10x+x^2\right)\left(17-2x+x^2\right)}\,dx$

Optimal. Leaf size=49

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Rubi [A] time = 0.16, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {6728, 634, 618, 204, 628}

$$\frac{1003\log\left(x^2-10x+26\right)}{1025}+\frac{22\log\left(x^2-2x+17\right)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Int[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2)),x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx = \int \left(\frac{5003+2006x}{1025(26-10x+x^2)}+\frac{-4651+44x}{1025(17-2x+x^2)}\right)dx$$
$$= \frac{\int \frac{5003+2006x}{26-10x+x^2}\,dx}{1025}+\frac{\int \frac{-4651+44x}{17-2x+x^2}\,dx}{1025}$$
$$= \frac{22\int \frac{-2+2x}{17-2x+x^2}\,dx}{1025}+\frac{1003\int \frac{-10+2x}{26-10x+x^2}\,dx}{1025}-\frac{4607\int \frac{1}{17-2x+x^2}\,dx}{1025}+\frac{15033\int}{1}$$
$$= \frac{1003\log(26-10x+x^2)}{1025}+\frac{22\log(17-2x+x^2)}{1025}+\frac{9214\,\mathrm{Subst}\left(\int \frac{1}{-64-x}\right.}{1025}$$
$$= -\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{1}{4}(-1+x)\right)}{4100}+\frac{1003\log(26-10x+x}{1025}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{1003\log(x^2-10x+26)}{1025}+\frac{22\log(x^2-2x+17)}{1025}-\frac{15033\tan^{-1}(5-x)}{1025}-\frac{4607\tan^{-1}\left(\frac{x-1}{4}\right)}{4100}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*x + x^2))
,x]
```

```
[Out] (-15033*ArcTan[5 - x])/1025 - (4607*ArcTan[(-1 + x)/4])/4100 + (1003*Log[26
 - 10*x + x^2])/1025 + (22*Log[17 - 2*x + x^2])/1025
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-35+70x-4x^2+2x^3}{(26-10x+x^2)(17-2x+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*
x + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(-35 + 70*x - 4*x^2 + 2*x^3)/((26 - 10*x + x^2)*(17 - 2*
x + x^2)), x]
```

fricas [A] time = 1.42, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log(x^2-2x+17)+\frac{1003}{1025}\log(x^2-10x+26)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="fr
icas")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

giac [A] time = 0.32, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan(x-5)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log(x^2-2x+17)+\frac{1003}{1025}\log(x^2-10x+26)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="gi
ac")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{15033\arctan\left(x-5\right)}{1025}-\frac{4607\arctan\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{1003\ln\left(x^{2}-10x+26\right)}{1025}+\frac{22\ln\left(x^{2}-2x+17\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x)
```

```
[Out] 15033/1025*arctan(-5+x)-4607/4100*arctan(-1/4+1/4*x)+1003/1025*ln(x^2-10*x+
26)+22/1025*ln(x^2-2*x+17)
```

maxima [A] time = 2.80, size = 37, normalized size = 0.76

$$\frac{15033}{1025}\arctan\left(x-5\right)-\frac{4607}{4100}\arctan\left(\frac{1}{4}x-\frac{1}{4}\right)+\frac{22}{1025}\log\left(x^{2}-2x+17\right)+\frac{1003}{1025}\log\left(x^{2}-10x+26\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-4*x^2+70*x-35)/(x^2-10*x+26)/(x^2-2*x+17),x, algorithm="ma
xima")
```

```
[Out] 15033/1025*arctan(x - 5) - 4607/4100*arctan(1/4*x - 1/4) + 22/1025*log(x^2
- 2*x + 17) + 1003/1025*log(x^2 - 10*x + 26)
```

mupad [B] time = 0.07, size = 41, normalized size = 0.84

$$\ln\left(x-1-4\mathrm{i}\right)\left(\frac{22}{1025}+\frac{4607}{8200}\mathrm{i}\right)+\ln\left(x-1+4\mathrm{i}\right)\left(\frac{22}{1025}-\frac{4607}{8200}\mathrm{i}\right)+\ln\left(x-5-\mathrm{i}\right)\left(\frac{1003}{1025}-\frac{15033}{2050}\mathrm{i}\right)+\ln\left(x-5+1\mathrm{i}\right)\left(\frac{1003}{1025}+\frac{15033}{2050}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((70*x - 4*x^2 + 2*x^3 - 35)/((x^2 - 2*x + 17)*(x^2 - 10*x + 26)),x)
```

```
[Out] log(x - (1 + 4i))*(22/1025 + 4607i/8200) + log(x - (1 - 4i))*(22/1025 - 460
7i/8200) + log(x - (5 + 1i))*(1003/1025 - 15033i/2050) + log(x - (5 - 1i))*
(1003/1025 + 15033i/2050)
```

sympy [A] time = 0.23, size = 46, normalized size = 0.94

$$\frac{1003\log\left(x^{2}-10x+26\right)}{1025}+\frac{22\log\left(x^{2}-2x+17\right)}{1025}-\frac{4607\,\mathrm{atan}\left(\frac{x}{4}-\frac{1}{4}\right)}{4100}+\frac{15033\,\mathrm{atan}\left(x-5\right)}{1025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-4*x**2+70*x-35)/(x**2-10*x+26)/(x**2-2*x+17),x)
```

```
[Out] 1003*log(x**2 - 10*x + 26)/1025 + 22*log(x**2 - 2*x + 17)/1025 - 4607*atan(
x/4 - 1/4)/4100 + 15033*atan(x - 5)/1025
```

3.295 $\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)} dx$

Optimal. Leaf size=29

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1612}

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx &= \int \left(\frac{3}{2(-5+x)}-\frac{11}{14(-3+x)}+\frac{2}{7(4+x)}\right) dx\\ &= -\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{11}{14}\log(3-x)+\frac{3}{2}\log(5-x)+\frac{2}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] (-11*Log[3 - x])/14 + (3*Log[5 - x])/2 + (2*Log[4 + x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x^2}{(-5+x)(-3+x)(4+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)),x]
[Out] IntegrateAlgebraic[(2 + x^2)/((-5 + x)*(-3 + x)*(4 + x)), x]
```

fricas [A] time = 1.61, size = 19, normalized size = 0.66

$$\frac{2}{7}\log(x+4)-\frac{11}{14}\log(x-3)+\frac{3}{2}\log(x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="fricas")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

giac [A] time = 0.27, size = 22, normalized size = 0.76

$$\frac{2}{7}\log\left(|x+4|\right)-\frac{11}{14}\log\left(|x-3|\right)+\frac{3}{2}\log\left(|x-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="giac")
[Out] 2/7*log(abs(x + 4)) - 11/14*log(abs(x - 3)) + 3/2*log(abs(x - 5))
```

maple [A] time = 0.01, size = 20, normalized size = 0.69

$$\frac{3\ln\left(x-5\right)}{2}-\frac{11\ln\left(x-3\right)}{14}+\frac{2\ln\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x-5)/(x-3)/(x+4),x)
[Out] 2/7*ln(x+4)-11/14*ln(x-3)+3/2*ln(x-5)
```

maxima [A] time = 1.17, size = 19, normalized size = 0.66

$$\frac{2}{7}\log\left(x+4\right)-\frac{11}{14}\log\left(x-3\right)+\frac{3}{2}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(-5+x)/(-3+x)/(4+x),x, algorithm="maxima")
[Out] 2/7*log(x + 4) - 11/14*log(x - 3) + 3/2*log(x - 5)
```

mupad [B] time = 2.18, size = 19, normalized size = 0.66

$$\frac{2\,\ln\left(x+4\right)}{7}-\frac{11\,\ln\left(x-3\right)}{14}+\frac{3\,\ln\left(x-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/((x - 3)*(x + 4)*(x - 5)),x)
[Out] (2*log(x + 4))/7 - (11*log(x - 3))/14 + (3*log(x - 5))/2
```

sympy [A] time = 0.14, size = 24, normalized size = 0.83

$$\frac{3\log\left(x-5\right)}{2}-\frac{11\log\left(x-3\right)}{14}+\frac{2\log\left(x+4\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(-5+x)/(-3+x)/(4+x),x)
[Out] 3*log(x - 5)/2 - 11*log(x - 3)/14 + 2*log(x + 4)/7
```

3.296 $\int \frac{x^4}{(-1+x)\left(2+x^2\right)} \, dx$

Optimal. Leaf size=46

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1629, 635, 203, 260}

$$\frac{x^2}{2}-\frac{2}{3}\log\left(x^2+2\right)+x+\frac{1}{3}\log(1-x)-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] x + x^2/2 - (2*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 + Log[1 - x]/3 - (2*Log[2 + x^2
])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(-1+x)\left(2+x^2\right)} \, dx &= \int \left(1+\frac{1}{3(-1+x)}+x-\frac{4(1+x)}{3\left(2+x^2\right)}\right) dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1+x}{2+x^2} \, dx \\
&= x+\frac{x^2}{2}+\frac{1}{3}\log(1-x)-\frac{4}{3}\int \frac{1}{2+x^2} \, dx-\frac{4}{3}\int \frac{x}{2+x^2} \, dx \\
&= x+\frac{x^2}{2}-\frac{2}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{1}{3}\log(1-x)-\frac{2}{3}\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{1}{6}\left(3x^2-4\log\left(x^2+2\right)+6x+2\log(x-1)-4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] (-9 + 6*x + 3*x^2 - 4*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Log[-1 + x] - 4*Log[2 +
 x^2])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-1+x)\left(2+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/((-1 + x)*(2 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^4/((-1 + x)*(2 + x^2)), x]
```

fricas [A] time = 1.56, size = 33, normalized size = 0.72

$$\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

giac [A] time = 0.32, size = 34, normalized size = 0.74

$$\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(abs(x - 1))
```

maple [A] time = 0.01, size = 34, normalized size = 0.74

$$\frac{x^2}{2}+x-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{3}+\frac{\ln\left(x-1\right)}{3}-\frac{2\ln\left(x^2+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x-1)/(x^2+2),x)
```

```
[Out] 1/2*x^2+x+1/3*ln(x-1)-2/3*ln(x^2+2)-2/3*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.16, size = 33, normalized size = 0.72

$$\frac{1}{2}x^2-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+x-\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-1+x)/(x^2+2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + x - 2/3*log(x^2 + 2) + 1/3*lo
g(x - 1)
```

mupad [B] time = 0.09, size = 50, normalized size = 1.09

$$x+\frac{\ln\left(x-1\right)}{3}+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\,\left(\frac{2}{3}+\frac{\sqrt{2}\,1\mathrm{i}}{3}\right)+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((x^2 + 2)*(x - 1)),x)
```

```
[Out] x + log(x - 1)/3 + log(x - 2^(1/2)*1i)*((2^(1/2)*1i)/3 - 2/3) - log(x + 2^(
1/2)*1i)*((2^(1/2)*1i)/3 + 2/3) + x^2/2
```

sympy [A] time = 0.14, size = 41, normalized size = 0.89

$$\frac{x^2}{2}+x+\frac{\log\left(x-1\right)}{3}-\frac{2\log\left(x^2+2\right)}{3}-\frac{2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-1+x)/(x**2+2),x)
```

```
[Out] x**2/2 + x + log(x - 1)/3 - 2*log(x**2 + 2)/3 - 2*sqrt(2)*atan(sqrt(2)*x/2)
/3
```

3.297 $\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx$

Optimal. Leaf size=16

$$2\log(1-x)-\frac{3}{x+1}$$

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$2\log(1-x)-\frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[1 - x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx &= \int \left(\frac{2}{-1+x}+\frac{3}{(1+x)^2}\right)dx\\ &= -\frac{3}{1+x}+2\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.88

$$2\log(x-1)-\frac{3}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] -3/(1 + x) + 2*Log[-1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+7x+2x^2}{-1-x+x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-1 + 7*x + 2*x^2)/(-1 - x + x^2 + x^3), x]
```

fricas [A] time = 0.80, size = 17, normalized size = 1.06

$$\frac{2\,(x+1)\log\,(x-1)-3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="fricas")
[Out] (2*(x + 1)*log(x - 1) - 3)/(x + 1)
```

giac [A] time = 0.40, size = 15, normalized size = 0.94

$$-\frac{3}{x+1}+2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="giac")
[Out] -3/(x + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$2\ln\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+7*x-1)/(x^3+x^2-x-1),x)
[Out] 2*ln(x-1)-3/(x+1)
```

maxima [A] time = 1.09, size = 14, normalized size = 0.88

$$-\frac{3}{x+1}+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+7*x-1)/(x^3+x^2-x-1),x, algorithm="maxima")
[Out] -3/(x + 1) + 2*log(x - 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.88

$$2\,\ln\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x + 2*x^2 - 1)/(x - x^2 - x^3 + 1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

sympy [A] time = 0.09, size = 10, normalized size = 0.62

$$2\log\left(x-1\right)-\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+7*x-1)/(x**3+x**2-x-1),x)
[Out] 2*log(x - 1) - 3/(x + 1)
```

3.298 $\int \frac{1+2x}{-1+3x-3x^2+x^3} dx$

Optimal. Leaf size=21

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2074}

$$\frac{2}{1-x} - \frac{3}{2(1-x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x)
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{-1+3x-3x^2+x^3} dx &= \int \left(\frac{3}{(-1+x)^3} + \frac{2}{(-1+x)^2}\right) dx \\ &= -\frac{3}{2(1-x)^2} + \frac{2}{1-x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{1-4x}{2(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] (1 - 4*x)/(2*(-1 + x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+2x}{-1+3x-3x^2+x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + 2*x)/(-1 + 3*x - 3*x^2 + x^3), x]
```

fricas [A] time = 1.29, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="fricas")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.30, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="giac")
```

```
[Out] -1/2*(4*x - 1)/(x - 1)^2
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{3}{2(x-1)^2}-\frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^3-3*x^2+3*x-1),x)
```

```
[Out] -3/2/(x-1)^2-2/(x-1)
```

maxima [A] time = 1.09, size = 17, normalized size = 0.81

$$-\frac{4x-1}{2\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^3-3*x^2+3*x-1),x, algorithm="maxima")
```

```
[Out] -1/2*(4*x - 1)/(x^2 - 2*x + 1)
```

mupad [B] time = 2.09, size = 12, normalized size = 0.57

$$-\frac{4x-1}{2(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(3*x - 3*x^2 + x^3 - 1),x)
```

```
[Out] -(4*x - 1)/(2*(x - 1)^2)
```

sympy [A] time = 0.09, size = 14, normalized size = 0.67

$$\frac{1-4x}{2x^2-4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**3-3*x**2+3*x-1),x)
```

```
[Out] (1 - 4*x)/(2*x**2 - 4*x + 2)
```

3.299 $\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1620}

$$\frac{1}{1-x}-\frac{2}{(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] (1 - x)^(-1) - 2/(1 + x)^2
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx &= \int \left(\frac{1}{(-1+x)^2}+\frac{4}{(1+x)^3}\right)dx\\ &= \frac{1}{1-x}-\frac{2}{(1+x)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2}{(x+1)^2}-\frac{1}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] -(-1 + x)^(-1) - 2/(1 + x)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5-5x+7x^2+x^3}{(-1+x)^2(1+x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] IntegrateAlgebraic[(5 - 5*x + 7*x^2 + x^3)/((-1 + x)^2*(1 + x)^3), x]
```

fricas [A] time = 1.30, size = 23, normalized size = 1.53

$$-\frac{x^2+4\,x-1}{x^3+x^2-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

giac [A] time = 0.26, size = 30, normalized size = 2.00

$$-\frac{1}{x-1}+\frac{\frac{4}{x-1}+1}{2\left(\frac{2}{x-1}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="giac")
[Out] -1/(x - 1) + 1/2*(4/(x - 1) + 1)/(2/(x - 1) + 1)^2
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{2}{(x+1)^{2}}-\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+7*x^2-5*x+5)/(x-1)^2/(x+1)^3,x)
[Out] -2/(x+1)^2-1/(x-1)
```

maxima [A] time = 1.27, size = 23, normalized size = 1.53

$$-\frac{x^{2}+4\,x-1}{x^{3}+x^{2}-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+7*x^2-5*x+5)/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
[Out] -(x^2 + 4*x - 1)/(x^3 + x^2 - x - 1)
```

mupad [B] time = 2.09, size = 15, normalized size = 1.00

$$-\frac{1}{x-1}-\frac{2}{(x+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^2 - 5*x + x^3 + 5)/((x - 1)^2*(x + 1)^3),x)
[Out] - 1/(x - 1) - 2/(x + 1)^2
```

sympy [A] time = 0.11, size = 17, normalized size = 1.13

$$\frac{-x^{2}-4x+1}{x^{3}+x^{2}-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+7*x**2-5*x+5)/(-1+x)**2/(1+x)**3,x)
[Out] (-x**2 - 4*x + 1)/(x**3 + x**2 - x - 1)
```

3.300 $\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx$

Optimal. Leaf size=31

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2074, 634, 618, 204, 628}

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
```

```
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{2x}{1+x+x^2}\right)dx \\
&= \log(1+x)+2\int \frac{x}{1+x+x^2}\,dx \\
&= \log(1+x)-\int \frac{1}{1+x+x^2}\,dx+\int \frac{1+2x}{1+x+x^2}\,dx \\
&= \log(1+x)+\log\left(1+x+x^2\right)+2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right) \\
&= -\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\log\left(x^2+x+1\right)+\log(x+1)-\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
[Out] (-2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 + x] + Log[1 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+3x+3x^2}{1+2x+2x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3),x]
[Out] IntegrateAlgebraic[(1 + 3*x + 3*x^2)/(1 + 2*x + 2*x^2 + x^3), x]
```

fricas [A] time = 1.26, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

giac [A] time = 0.28, size = 29, normalized size = 0.94

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x +
 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$-\frac{2\sqrt{3}\ \arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\ln\left(x+1\right)+\ln\left(x^{2}+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x)
```

```
[Out] ln(x+1)+ln(x^2+x+1)-2/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.36, size = 28, normalized size = 0.90

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\log\left(x^{2}+x+1\right)+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+3*x+1)/(x^3+2*x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x + 1)
```

mupad [B] time = 0.11, size = 57, normalized size = 1.84

$$\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+\ln\left(x+1\right)+\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}-\frac{\sqrt{3}\ \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 3*x^2 + 1)/(2*x + 2*x^2 + x^3 + 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2) + log(x + (3^(1/2)*1i)/2 + 1/2) + log(x + 1)
+ (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - (3^(1/2)*log(x + (3^(1/2)*
1i)/2 + 1/2)*1i)/3
```

sympy [A] time = 0.13, size = 3, normalized size = 0.10

$$\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+3*x+1)/(x**3+2*x**2+2*x+1),x)
```

```
[Out] log(x + 1)
```

3.301 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)} dx \\
&= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right) dx \\
&= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
[Out] IntegrateAlgebraic[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3), x]
```

fricas [A] time = 1.07, size = 19, normalized size = 0.76

$$\frac{1}{10}\log\left(2x-1\right)-\frac{1}{10}\log\left(x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

giac [A] time = 0.29, size = 22, normalized size = 0.88

$$\frac{1}{10}\log\left(|2x-1|\right)-\frac{1}{10}\log\left(|x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$\frac{\ln(x)}{2}+\frac{\ln\left(2x-1\right)}{10}-\frac{\ln\left(x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
[Out] 1/10*ln(2*x-1)-1/10*ln(x+2)+1/2*ln(x)
```

maxima [A] time = 1.09, size = 19, normalized size = 0.76

$$\frac{1}{10}\log\left(2x-1\right)-\frac{1}{10}\log\left(x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

mupad [B] time = 0.06, size = 19, normalized size = 0.76

$$\frac{\mathrm{atanh}\left(\frac{24}{145\left(\frac{29x}{100}-\frac{11}{50}\right)}+\frac{35}{29}\right)}{5}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 - 1)/(3*x^2 - 2*x + 2*x^3),x)
[Out] atanh(24/(145*((29*x)/100 - 11/50)) + 35/29)/5 + log(x)/2
```

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log\left(x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

3.302 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2074}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right)dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3), x]
```

fricas [A] time = 1.14, size = 36, normalized size = 1.20

$$\frac{x^3+x^2-2\,(x-1)\log\left(x+1\right)+2\,(x-1)\log\left(x-1\right)-2\,x-4}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

giac [A] time = 0.28, size = 26, normalized size = 0.87

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(|x+1|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$\frac{x^2}{2} + x + \ln(x-1) - \ln(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)
```

```
[Out] 1/2*x^2+x+ln(x-1)-2/(x-1)-ln(x+1)
```

maxima [A] time = 1.21, size = 24, normalized size = 0.80

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(x+1) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

mupad [B] time = 2.11, size = 22, normalized size = 0.73

$$x - \frac{2}{x-1} + \frac{x^2}{2} + \operatorname{atan}(x\,1\mathrm{i})\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2*x^2 + x^4 + 1)/(x + x^2 - x^3 - 1),x)
```

```
[Out] x + atan(x*1i)*2i - 2/(x - 1) + x^2/2
```

sympy [A] time = 0.10, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

3.303 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx\\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx\\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] -1/2*ArcTan[x/2] + Log[x] + Log[4 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4-x+2x^2}{4x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - x + 2*x^2)/(4*x + x^3),x]
[Out] IntegrateAlgebraic[(4 - x + 2*x^2)/(4*x + x^3), x]
```

fricas [A] time = 1.04, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x)
```

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

maxima [A] time = 2.08, size = 17, normalized size = 0.74

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

mupad [B] time = 2.12, size = 21, normalized size = 0.91

$$\ln(x)+\ln\left(x-2\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 4)/(4*x + x^3),x)
```

```
[Out] log(x - 2i)*(1/2 + 1i/4) + log(x + 2i)*(1/2 - 1i/4) + log(x)
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

3.304 $\int \frac{1+x^2+x^3}{(-1+x)x(1+x^2)^3(1+x+x^2)}\,dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8(x^2+1)}+\frac{3x}{16(x^2+1)}+\frac{x+1}{8(x^2+1)^2}+\frac{15}{16}\log(x^2+1)-\frac{1}{2}\log(x^2+x+1)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

Rubi [A] time = 0.48, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8(x^2+1)}+\frac{3x}{16(x^2+1)}+\frac{x+1}{8(x^2+1)^2}+\frac{15}{16}\log(x^2+1)-\frac{1}{2}\log(x^2+x+1)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int \left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2\left(1+x^2\right)^3} + \frac{3(1+x)}{4\left(1+x^2\right)^2} + \frac{-1+15x}{8\left(1+x^2\right)} + \frac{-1-x}{1+x+x\ldots}\right.$$

$$= \frac{1}{8}\log(1-x) - \log(x) + \frac{1}{8}\int \frac{-1+15x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)^3}\,dx + \frac{3}{4}\int \ldots$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{1}{8}\log(1-x) - \log(x) - \frac{1}{8}\int \frac{1}{1+x^2}\,dx + \frac{3}{8}\int \ldots$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{8}\log(1-x) - \mathrm{lo}\ldots$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \ldots$$

Mathematica [A] time = 0.04, size = 93, normalized size = 0.90

$$\frac{1}{48}\left(-14\log\left(1-x^3\right) + \frac{6(x+1)}{\left(x^2+1\right)^2} + \frac{9(3x-2)}{x^2+1} + 45\log\left(x^2+1\right) - 10\log\left(x^2+x+1\right) + 20\log(1-x) - 48\log(x) + 21\tan^{-1}(x) - 16\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x
]
```

```
[Out] IntegrateAlgebraic[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),
x]
```

fricas [A] time = 0.98, size = 136, normalized size = 1.32

$$\frac{27\,x^3-16\,\sqrt{3}\left(x^4+2\,x^2+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)-18\,x^2+21\left(x^4+2\,x^2+1\right)\arctan(x)-24\left(x^4+2\,x^2+1\right)\log\left(x^2+x+1\right)+45\left(x^4+2\,x^2+1\right)\log\left(x^2+1\right)+6\left(x^4+2\,x^2+1\right)\log(x-1)-48\left(x^4+2\,x^2+1\right)\log(x)+33\,x-12}{48\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.34, size = 74, normalized size = 0.72

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{9\,x^3-6\,x^2+11\,x-4}{16\left(x^2+1\right)^2}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log\left(|x-1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 73, normalized size = 0.71

$$\frac{7\arctan(x)}{16}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\ln(x)+\frac{\ln(x-1)}{8}+\frac{15\ln\left(x^2+1\right)}{16}-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\frac{9}{2}x^3-3x^2+\frac{11}{2}x-2}{8\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x-1)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*ln(x-1)+1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arc
tan(x)-ln(x)-1/2*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.25, size = 77, normalized size = 0.75

$$-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{9\,x^3-6\,x^2+11\,x-4}{16\left(x^4+2\,x^2+1\right)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{15}{16}\log\left(x^2+1\right)+\frac{1}{8}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

mupad [B] time = 2.20, size = 96, normalized size = 0.93

$$\frac{\ln(x-1)}{8}-\ln(x)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{\frac{9x^3}{16}-\frac{3x^2}{8}+\frac{11x}{16}-\frac{1}{4}}{x^4+2x^2+1}+\ln(x-\mathrm{i})\left(\frac{15}{16}-\frac{7}{32}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{15}{16}+\frac{7}{32}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(x*(x^2 + 1)^3*(x - 1)*(x + x^2 + 1)),x)
```

```
[Out] log(x - 1)/8 + log(x - 1i)*(15/16 - 7i/32) + log(x + 1i)*(15/16 + 7i/32) -
log(x) + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) + ((11*x)/16 - (3*x^2)/8 + (9*x^3
)/16 - 1/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.51, size = 88, normalized size = 0.85

$$-\log(x)+\frac{\log(x-1)}{8}+\frac{15\log\left(x^2+1\right)}{16}-\frac{\log\left(x^2+x+1\right)}{2}+\frac{7\operatorname{atan}(x)}{16}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{9x^3-6x^2+11x-4}{16x^4+32x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

3.305 $\int \frac{1-3x+2x^2-x^3}{(1+x^2)^2}\,dx$

Optimal. Leaf size=33

$$\frac{2-x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1814, 635, 203, 260}

$$\frac{2-x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
```

```
[Out] (2 - x)/(2*(1 + x^2)) + (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx &= \frac{2-x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-3+2x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{2-x}{2\left(1+x^2\right)}+\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.91

$$\frac{1}{2}\left(\frac{2-x}{x^2+1}-\log\left(x^2+1\right)+3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
[Out] ((2 - x)/(1 + x^2) + 3*ArcTan[x] - Log[1 + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x+2x^2-x^3}{\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2,x]
[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(1 + x^2)^2, x]
```

fricas [A] time = 1.68, size = 36, normalized size = 1.09

$$\frac{3\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)-x+2}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) - x + 2)/(x^2 + 1)
```

giac [A] time = 0.32, size = 25, normalized size = 0.76

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 28, normalized size = 0.85

$$\frac{3\arctan(x)}{2}-\frac{\ln\left(x^2+1\right)}{2}-\frac{\frac{x}{2}-1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x)
[Out] -(1/2*x-1)/(x^2+1)-1/2*ln(x^2+1)+3/2*arctan(x)
```

maxima [A] time = 2.15, size = 25, normalized size = 0.76

$$-\frac{x-2}{2\left(x^2+1\right)}+\frac{3}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*(x - 2)/(x^2 + 1) + 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

mupad [B] time = 0.03, size = 32, normalized size = 0.97

$$\frac{3\,\mathrm{atan}(x)}{2} - \frac{\ln\left(x^2+1\right)}{2} - \frac{x}{2\,\left(x^2+1\right)} + \frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x^2 + 1)^2,x)
[Out] (3*atan(x))/2 - log(x^2 + 1)/2 - x/(2*(x^2 + 1)) + 1/(x^2 + 1)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.73

$$-\frac{x-2}{2x^2+2} - \frac{\log\left(x^2+1\right)}{2} + \frac{3\,\mathrm{atan}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/(x**2+1)**2,x)
[Out] -(x - 2)/(2*x**2 + 2) - log(x**2 + 1)/2 + 3*atan(x)/2
```

3.306 $\int \frac{1-3x+2x^2-x^3}{x(1+x^2)^2} \, dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2), x]
```

fricas [A] time = 1.54, size = 44, normalized size = 1.33

$$-\frac{4\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log(x)+2\,x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

giac [A] time = 0.38, size = 30, normalized size = 0.91

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

maple [A]   time = 0.01, size = 28, normalized size = 0.85

$$-2\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}-\frac{x+\frac{1}{2}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

maxima [A]   time = 2.25, size = 29, normalized size = 0.88

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

mupad [B]   time = 2.11, size = 33, normalized size = 1.00

$$\ln(x)-\frac{x+\frac{1}{2}}{x^2+1}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+1\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x - 2*x^2 + x^3 - 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i) - log(x - 1i)*(1/2 - 1i) - (x + 1/2)/(x^2 +
1)
```

sympy [A]   time = 0.15, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}-2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

3.307 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx$

Optimal. Leaf size=25

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} dx &= \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)} dx \\
&= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right) dx \\
&= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2} dx \\
&= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{x^2}{2} + \frac{1}{2}\log\left(1-x^2\right) + x - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
[Out] IntegrateAlgebraic[(1 - x - x^2 + x^3 + x^4)/(-x + x^3), x]
```

fricas [A] time = 1.28, size = 19, normalized size = 0.76

$$\frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(x^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

giac [A] time = 0.29, size = 26, normalized size = 1.04

$$\frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(|x+1|\right) + \frac{1}{2}\,\log\left(|x-1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{x^2}{2} + x - \ln(x) + \frac{\ln\left(x-1\right)}{2} + \frac{\ln\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)
[Out] 1/2*x^2+x+1/2*ln(x-1)+1/2*ln(x+1)-ln(x)
```

maxima [A] time = 1.05, size = 23, normalized size = 0.92

$$\frac{1}{2}\,x^2 + x + \frac{1}{2}\,\log\left(x+1\right) + \frac{1}{2}\,\log\left(x-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.76

$$x + \frac{\ln\left(x^2-1\right)}{2} - \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - x^2 - x + x^4 + 1)/(x - x^3),x)
[Out] x + log(x^2 - 1)/2 - log(x) + x^2/2
```

sympy [**A**] time = 0.10, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

3.308 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-4x^2+x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx\\
&= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\
&= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= 6\tan^{-1}(x)-5\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2-4x^2+x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] IntegrateAlgebraic[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)), x]
```

fricas [A] time = 1.59, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

giac [A] time = 0.34, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

maple [A] time = 0.00, size = 32, normalized size = 0.89

$$6\arctan(x)-5\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 2.10, size = 31, normalized size = 0.86

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

mupad [B] time = 0.11, size = 56, normalized size = 1.56

$$\ln(x-\mathrm{i})\left(-\frac{1}{2}-3\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(-\frac{1}{2}+3\mathrm{i}\right)+\ln\left(x-\sqrt{2}\,1\mathrm{i}\right)\left(1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)-\ln\left(x+\sqrt{2}\,1\mathrm{i}\right)\left(-1+\frac{\sqrt{2}\,5\mathrm{i}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 4*x^2 + 2)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 2^(1/2)*1i)*((2^(1/2)*5i)/2 + 1) - log(x + 1i)*(1/2 - 3i) - log(x -
 1i)*(1/2 + 3i) - log(x + 2^(1/2)*1i)*((2^(1/2)*5i)/2 - 1)
```

sympy [A] time = 0.20, size = 36, normalized size = 1.00

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}\,(x)-5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

3.309 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2}\,dx &= \int \left(\frac{1}{9(1+x^2)}-\frac{13}{3(4+x^2)^2}+\frac{8}{9(4+x^2)}\right)dx\\
&= \frac{1}{9}\int \frac{1}{1+x^2}\,dx+\frac{8}{9}\int \frac{1}{4+x^2}\,dx-\frac{13}{3}\int \frac{1}{(4+x^2)^2}\,dx\\
&= -\frac{13x}{24(4+x^2)}+\frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)-\frac{13}{24}\int \frac{1}{4+x^2}\,dx\\
&= -\frac{13x}{24(4+x^2)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{13x}{24(x^2+4)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^4}{(1+x^2)\left(4+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2), x]
```

fricas [A] time = 1.06, size = 33, normalized size = 1.14

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)+16\left(x^2+4\right)\arctan\left(x\right)-78\,x}{144\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

giac [A] time = 0.23, size = 21, normalized size = 0.72

$$-\frac{13\,x}{24\left(x^2+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

maple [A] time = 0.01, size = 22, normalized size = 0.76

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{\arctan\left(x\right)}{9}+\frac{25\arctan\left(\frac{x}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

maxima [A] time = 2.09, size = 21, normalized size = 0.72

$$-\frac{13\,x}{24\left(x^2+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

mupad [B] time = 0.04, size = 23, normalized size = 0.79

$$\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}(x)}{9}-\frac{13\,x}{24\,\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)/((x^2 + 1)*(x^2 + 4)^2),x)
[Out] (25*atan(x/2))/144 + atan(x)/9 - (13*x)/(24*(x^2 + 4))
```

sympy [A] time = 0.17, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\,\mathrm{atan}\left(\frac{x}{2}\right)}{144}+\frac{\mathrm{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

3.310 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx \\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx \\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right) \\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/2*1/x + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4), x]
```

fricas [A] time = 1.52, size = 39, normalized size = 0.85

$$\frac{2\,\sqrt{7}\,x\arctan\left(\frac{1}{7}\,\sqrt{7}\,(2\,x+1)\right)+35\,x\log\left(x^2+x+2\right)-14\,x\log(x)-28}{56\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

giac [A] time = 0.30, size = 36, normalized size = 0.78

$$\frac{1}{28}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,(2\,x+1)\right)-\frac{1}{2\,x}+\frac{5}{8}\,\log\left(x^2+x+2\right)-\frac{1}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 36, normalized size = 0.78

$$\frac{\sqrt{7}\ \arctan\left(\frac{(2x+1)\sqrt{7}}{7}\right)}{28}-\frac{\ln(x)}{4}+\frac{5\ln\left(x^2+x+2\right)}{8}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(2*x+1)*7^(1/2))*7^(1/2)
```

maxima [A] time = 2.09, size = 35, normalized size = 0.76

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

mupad [B] time = 2.17, size = 49, normalized size = 1.07

$$-\frac{\ln(x)}{4}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(-\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{7}\,1\mathrm{i}}{2}\right)\left(\frac{5}{8}+\frac{\sqrt{7}\,1\mathrm{i}}{56}\right)-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 + 1)/(2*x^2 + x^3 + x^4),x)
```

```
[Out] log(x + (7^(1/2)*1i)/2 + 1/2)*((7^(1/2)*1i)/56 + 5/8) - log(x - (7^(1/2)*1i
)/2 + 1/2)*((7^(1/2)*1i)/56 - 5/8) - log(x)/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 46, normalized size = 1.00

$$-\frac{\log(x)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

3.311 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal. Leaf size=22

$$\frac{x^2}{2}-\frac{2}{7}\tanh^{-1}\left(\frac{1}{7}(2x+1)\right)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1657, 616, 31}

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx &= \int \left(x+\frac{1}{-12+x+x^2}\right) \, dx \\
&= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2} \, dx \\
&= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x} \, dx-\frac{1}{7}\int \frac{1}{4+x} \, dx \\
&= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 - 12x + x^2 + x^3}{-12 + x + x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
[Out] IntegrateAlgebraic[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2), x]
```

fricas [A] time = 1.61, size = 18, normalized size = 0.82

$$\frac{1}{2} x^2 - \frac{1}{7} \log(x + 4) + \frac{1}{7} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

giac [A] time = 0.28, size = 20, normalized size = 0.91

$$\frac{1}{2} x^2 - \frac{1}{7} \log(|x + 4|) + \frac{1}{7} \log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{x^2}{2} + \frac{\ln(x - 3)}{7} - \frac{\ln(x + 4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
[Out] 1/2*x^2-1/7*ln(x+4)+1/7*ln(x-3)
```

maxima [A] time = 0.97, size = 18, normalized size = 0.82

$$\frac{1}{2} x^2 - \frac{1}{7} \log(x + 4) + \frac{1}{7} \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.64

$$\frac{x^2}{2} - \frac{2 \operatorname{atanh}\left(\frac{2x}{7} + \frac{1}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 12*x + x^3 + 1)/(x + x^2 - 12),x)
```

```
[Out] x^2/2 - (2*atanh((2*x)/7 + 1/7))/7
```

sympy [A] time = 0.10, size = 17, normalized size = 0.77

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

3.312 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1594, 1628}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\
&= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\
&= 2\log(1-x)+\log(x)+3\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3), x]
```

fricas [A] time = 1.27, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

giac [A] time = 0.38, size = 18, normalized size = 1.06

$$3 \log(|x+3|) + 2 \log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\ln(x) + 2\ln(x-1) + 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
[Out] 2*ln(x-1)+3*ln(x+3)+ln(x)
```

maxima [A] time = 0.85, size = 15, normalized size = 0.88

$$3 \log(x+3) + 2 \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

mupad [B] time = 0.07, size = 15, normalized size = 0.88

$$2 \ln(x-1) + 3 \ln(x+3) + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 6*x^2 - 3)/(2*x^2 - 3*x + x^3),x)
[Out] 2*log(x - 1) + 3*log(x + 3) + log(x)
```

sympy [A] time = 0.14, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x-1) + 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

3.313 $\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$

Optimal. Leaf size=14

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+3x+5x^2}{2x^2+x^3} dx &= \int \frac{-2+3x+5x^2}{x^2(2+x)} dx \\
&= \int \left(-\frac{1}{x^2} + \frac{2}{x} + \frac{3}{2+x}\right) dx \\
&= \frac{1}{x} + 2\log(x) + 3\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+3x+5x^2}{2x^2+x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
[Out] IntegrateAlgebraic[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3), x]
```

fricas [A] time = 0.96, size = 18, normalized size = 1.29

$$\frac{3 x \log (x+2)+2 x \log (x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

giac [A] time = 0.37, size = 16, normalized size = 1.14

$$\frac{1}{x}+3 \log (|x+2|)+2 \log (|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

maple [A] time = 0.01, size = 15, normalized size = 1.07

$$2 \ln (x)+3 \ln (x+2)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)
[Out] 1/x+2*ln(x)+3*ln(x+2)
```

maxima [A] time = 1.00, size = 14, normalized size = 1.00

$$\frac{1}{x}+3 \log (x+2)+2 \log (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

mupad [B] time = 2.12, size = 14, normalized size = 1.00

$$3 \ln (x+2)+2 \ln (x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 5*x^2 - 2)/(2*x^2 + x^3),x)
[Out] 3*log(x + 2) + 2*log(x) + 1/x
```

sympy [A] time = 0.12, size = 14, normalized size = 1.00

$$2 \log (x)+3 \log (x+2)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

3.314 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right)dx\\ &= \log(1-x)-2\log(2+x)-3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3), x]
```

fricas [A] time = 1.34, size = 17, normalized size = 0.89

$$-3\log(x+3)-2\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

giac [**A**] time = 0.29, size = 20, normalized size = 1.05

$$-3 \log(|x+3|) - 2 \log(|x+2|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

maple [**A**] time = 0.01, size = 18, normalized size = 0.95

$$\ln(x-1) - 2\ln(x+2) - 3\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
[Out] ln(x-1)-2*ln(x+2)-3*ln(x+3)
```

maxima [**A**] time = 1.03, size = 17, normalized size = 0.89

$$-3 \log(x+3) - 2 \log(x+2) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

mupad [**B**] time = 2.12, size = 17, normalized size = 0.89

$$\ln(x-1) - 2 \ln(x+2) - 3 \ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

sympy [**A**] time = 0.13, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

3.315 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx + \int \frac{x\left(1+x^2\right)}{4+5x^2+x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx, x, x^2\right) - 3\int \frac{1}{4+x^2}\,dx + \int \frac{1}{1+x^2}\,dx\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{4+x}\,dx, x, x^2\right)\\ &= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x) + \frac{1}{2}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+4\right) - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4), x]
```

fricas [A] time = 1.09, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right) + \arctan(x) + \frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$\arctan(x) - \frac{3\arctan\left(\frac{x}{2}\right)}{2} + \frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
```

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

maxima [A] time = 2.12, size = 17, normalized size = 0.74

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan(x)+\frac{1}{2}\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

mupad [B] time = 0.05, size = 33, normalized size = 1.43

$$-\mathrm{atan}\left(\frac{1305}{4\,(144\,x-162)}+\frac{9}{8}\right)+\ln\left(x-2\mathrm{i}\right)\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^2 + x^3 + 1)/(5*x^2 + x^4 + 4),x)
```

```
[Out] log(x - 2i)*(1/2 + 3i/4) + log(x + 2i)*(1/2 - 3i/4) - atan(1305/(4*(144*x -
 162)) + 9/8)
```

sympy [A] time = 0.18, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

3.316 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5} \, dx$

Optimal. Leaf size=63

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

Rubi [A] time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049 \log \left(x^2+x+5\right)}{260015}-\frac{3146 \log (7-3 x)}{80155}-\frac{334}{323} \log (2 x+1)+\frac{4822 \log (5 x+2)}{4879}+\frac{3988 \tan ^{-1}\left(\frac{2 x+1}{\sqrt{19}}\right)}{13685 \sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int \left(-\frac{668}{323(1+2x)} - \frac{9438}{80155(-7+3x)} + \frac{24110}{4879(2+5x)} + \frac{48\cdots}{2600\cdots}\right.$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \cdots$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \cdots$$

$$= -\frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \frac{4822\log(2+5x)}{4879} + \cdots$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}} - \frac{3146\log(7-3x)}{80155} - \frac{334}{323}\log(1+2x) + \cdots$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.90

$$\frac{453009\log\left(x^2+x+5\right) - 418418\log(7-3x) - 11023670\log(2x+1) + 10536070\log(5x+2) + 163508\sqrt{19}\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50
*x^3 - 13*x^4 + 30*x^5),x]
```

```
[Out] IntegrateAlgebraic[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50
*x^3 - 13*x^4 + 30*x^5), x]
```

fricas [A] time = 0.90, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right) + \frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log(5x+2) - \frac{3146}{80155}\log(3x-7) - \frac{334}{323}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

giac [A] time = 0.31, size = 53, normalized size = 0.84

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right) + \frac{11049}{260015}\log\left(x^2+x+5\right) + \frac{4822}{4879}\log(|5x+2|) - \frac{3146}{80155}\log(|3x-7|) - \frac{334}{323}\log(|2x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

maple [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{3988\sqrt{19}\,\arctan\left(\frac{(2x+1)\sqrt{19}}{19}\right)}{260015}-\frac{334\ln(2x+1)}{323}+\frac{4822\ln(5x+2)}{4879}-\frac{3146\ln(3x-7)}{80155}+\frac{11049\ln\left(x^2+x+5\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)-3146/80155*ln(3*x-7)-334/323*ln(2*x+1)+11049/260015*ln(
x^2+x+5)+3988/260015*arctan(1/19*(2*x+1)*19^(1/2))*19^(1/2)
```

maxima [A] time = 2.11, size = 50, normalized size = 0.79

$$\frac{3988}{260015}\sqrt{19}\,\arctan\left(\frac{1}{19}\sqrt{19}\,(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log(5x+2)-\frac{3146}{80155}\log(3x-7)-\frac{334}{323}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

mupad [B] time = 2.21, size = 58, normalized size = 0.92

$$\frac{4822\ln\left(x+\frac{2}{5}\right)}{4879}-\frac{334\ln\left(x+\frac{1}{2}\right)}{323}-\frac{3146\ln\left(x-\frac{7}{3}\right)}{80155}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(-\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{19}\,1\mathrm{i}}{2}\right)\left(\frac{11049}{260015}+\frac{\sqrt{19}\,1994\mathrm{i}}{260015}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x - 27*x^2 + 4*x^3 - 32)/(299*x + 286*x^2 - 50*x^3 + 13*x^4 - 30*x^
5 + 70),x)
```

```
[Out] (4822*log(x + 2/5))/4879 - (334*log(x + 1/2))/323 - (3146*log(x - 7/3))/801
55 - log(x - (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 - 11049/260015
) + log(x + (19^(1/2)*1i)/2 + 1/2)*((19^(1/2)*1994i)/260015 + 11049/260015)
```

sympy [A] time = 0.35, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\,\operatorname{atan}\left(\frac{2\sqrt{19}\,x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

3.317 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6} \, dx$

Optimal. Leaf size=69

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{503\tan^{-1}\left(\sqrt{2}x\right)}{7986\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \int \left(\frac{5828}{1815(-2+5x)^2} - \frac{59096}{19965(-2+5x)} + \frac{-251+31\ldots}{363\left(1+2x\ldots\right.}\right.$$

$$= \frac{5828}{9075(2-5x)} - \frac{59096\log(2-5x)}{99825} + \frac{2\int \frac{816+2843x}{1+2x^2}\,dx}{3993}$$

$$= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\left(1+2x^2\right)} - \frac{59096\log(2-5x)}{99825}$$

$$= \frac{5828}{9075(2-5x)} - \frac{313+502x}{1452\left(1+2x^2\right)} - \frac{251\tan^{-1}\left(\sqrt{2}\,x\right)}{726\sqrt{2}} + $$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.97

$$\frac{142150\log\left(2x^2+1\right) - \frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2} - 236384\log(2-5x) + 12575\sqrt{2}\tan^{-1}\left(\sqrt{2}\,x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^
3 + 116*x^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] IntegrateAlgebraic[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^
3 + 116*x^4 - 80*x^5 + 100*x^6), x]
```

fricas [A] time = 1.24, size = 103, normalized size = 1.49

$$\frac{12575\sqrt{2}\left(10x^3-4x^2+5x-2\right)\arctan\left(\sqrt{2}\,x\right) - 1203114x^2 + 142150\left(10x^3-4x^2+5x-2\right)\log\left(2x^2+1\right) - 236384\left(10x^3-4x^2+5x-2\right)\log(5x-2) - 154275x - 84282}{399300\left(10x^3-4x^2+5x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

giac [A] time = 0.24, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}\,x\right) - \frac{36458x^2+4675x+2554}{12100\left(2x^2+1\right)(5x-2)} + \frac{2843}{7986}\log\left(2x^2+1\right) - \frac{59096}{99825}\log\left(|5x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

maple [A] time = 0.01, size = 54, normalized size = 0.78

$$\frac{503\sqrt{2}\ \arctan\left(\sqrt{2}\ x\right)}{15972}-\frac{59096\ln\left(5x-2\right)}{99825}+\frac{2843\ln\left(2x^{2}+1\right)}{7986}-\frac{5828}{9075\left(5x-2\right)}+\frac{-\frac{2761x}{4}-\frac{3443}{8}}{3993x^{2}+\frac{3993}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
)
```

```
[Out] -5828/9075/(5*x-2)-59096/99825*ln(5*x-2)+1/3993*(-2761/4*x-3443/8)/(x^2+1/2
)+2843/7986*ln(2*x^2+1)+503/15972*arctan(2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 2.07, size = 59, normalized size = 0.86

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^{2}+4675\,x+2554}{12100\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)}+\frac{2843}{7986}\log\left(2\,x^{2}+1\right)-\frac{59096}{99825}\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

mupad [B] time = 2.18, size = 71, normalized size = 1.03

$$-\frac{59096\ln\left(x-\frac{2}{5}\right)}{99825}-\frac{\frac{18229\,x^{2}}{60500}+\frac{17\,x}{440}+\frac{1277}{60500}}{x^{3}-\frac{2\,x^{2}}{5}+\frac{x}{2}-\frac{1}{5}}-\ln\left(x-\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(-\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)+\ln\left(x+\frac{\sqrt{2}\,1\mathrm{i}}{2}\right)\left(\frac{2843}{7986}+\frac{\sqrt{2}\,503\mathrm{i}}{31944}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(13*x^2 + 7*x^3 - 12*x^5 - 8)/(41*x^2 - 20*x - 80*x^3 + 116*x^4 - 80*x
^5 + 100*x^6 + 4),x)
```

```
[Out] log(x + (2^(1/2)*1i)/2)*((2^(1/2)*503i)/31944 + 2843/7986) - ((17*x)/440 +
(18229*x^2)/60500 + 1277/60500)/(x/2 - (2*x^2)/5 + x^3 - 1/5) - log(x - (2^
(1/2)*1i)/2)*((2^(1/2)*503i)/31944 - 2843/7986) - (59096*log(x - 2/5))/9982
5
```

sympy [A] time = 0.21, size = 65, normalized size = 0.94

$$\frac{-36458x^{2}-4675x-2554}{121000x^{3}-48400x^{2}+60500x-24200}-\frac{59096\log\left(x-\frac{2}{5}\right)}{99825}+\frac{2843\log\left(x^{2}+\frac{1}{2}\right)}{7986}+\frac{503\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\,x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] (-36458*x**2 - 4675*x - 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

3.318 $\int \frac{9+x^4}{x^2(9+x^2)}\,dx$

Optimal. Leaf size=17

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1262, 203}

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx &= \int \left(1+\frac{1}{x^2}-\frac{10}{9+x^2}\right)dx\\ &= -\frac{1}{x}+x-10\int \frac{1}{9+x^2}\,dx\\ &= -\frac{1}{x}+x-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x-\frac{1}{x}-\frac{10}{3}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x^4)/(x^2*(9 + x^2)),x]
```

```
[Out] -x^(-1) + x - (10*ArcTan[x/3])/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9+x^4}{x^2\left(9+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 + x^4)/(x^2*(9 + x^2)),x]
[Out] IntegrateAlgebraic[(9 + x^4)/(x^2*(9 + x^2)), x]
```

fricas [A] time = 1.62, size = 19, normalized size = 1.12

$$\frac{3\,x^2 - 10\,x\arctan\left(\frac{1}{3}\,x\right) - 3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="fricas")
[Out] 1/3*(3*x^2 - 10*x*arctan(1/3*x) - 3)/x
```

giac [A] time = 0.30, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="giac")
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$x - \frac{10\arctan\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+9)/x^2/(x^2+9),x)
[Out] -1/x+x-10/3*arctan(1/3*x)
```

maxima [A] time = 2.04, size = 13, normalized size = 0.76

$$x - \frac{1}{x} - \frac{10}{3}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+9)/x^2/(x^2+9),x, algorithm="maxima")
[Out] x - 1/x - 10/3*arctan(1/3*x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 9)/(x^2*(x^2 + 9)),x)
[Out] x - (10*atan(x/3))/3 - 1/x
```

sympy [A] time = 0.11, size = 12, normalized size = 0.71

$$x - \frac{10\,\mathrm{atan}\left(\frac{x}{3}\right)}{3} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+9)/x**2/(x**2+9),x)
```

```
[Out] x - 10*atan(x/3)/3 - 1/x
```

3.319 $\int \frac{2x+x^4}{1+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^3}{3}+\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{x^3}{3}+\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^4}{1+x^2}\,dx &= \int \frac{x\left(2+x^3\right)}{1+x^2}\,dx\\ &= \int \left(-1+x^2+\frac{1+2x}{1+x^2}\right)dx\\ &= -x+\frac{x^3}{3}+\int \frac{1+2x}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\ &= -x+\frac{x^3}{3}+\tan^{-1}(x)+\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^3}{3}+\log\left(x^2+1\right)-x+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^4)/(1 + x^2),x]
[Out] -x + x^3/3 + ArcTan[x] + Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+x^4}{1+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + x^4)/(1 + x^2),x]
[Out] IntegrateAlgebraic[(2*x + x^4)/(1 + x^2), x]
```

fricas [A] time = 0.81, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3-x+\arctan(x)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

giac [A] time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3-x+\arctan(x)+\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{x^3}{3}-x+\arctan(x)+\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x)/(x^2+1),x)
```

```
[Out] -x+1/3*x^3+arctan(x)+ln(x^2+1)
```

maxima [A] time = 2.03, size = 17, normalized size = 0.89

$$\frac{1}{3}x^3 - x + \arctan(x) + \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - x + arctan(x) + log(x^2 + 1)
```

mupad [B] time = 2.10, size = 17, normalized size = 0.89

$$\ln\left(x^2 + 1\right) - x + \operatorname{atan}(x) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^4)/(x^2 + 1),x)
```

```
[Out] log(x^2 + 1) - x + atan(x) + x^3/3
```

sympy [A] time = 0.10, size = 15, normalized size = 0.79

$$\frac{x^3}{3} - x + \log\left(x^2 + 1\right) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x)/(x**2+1),x)
```

```
[Out] x**3/3 - x + log(x**2 + 1) + atan(x)
```

3.320 $\int \frac{-x+x^3}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\log(1-x)+\tan^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1586, 1593, 1629, 203}

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+x^3}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \frac{x+x^2}{(-1+x)\left(1+x^2\right)}\,dx\\
&= \int \frac{x(1+x)}{(-1+x)\left(1+x^2\right)}\,dx\\
&= \int \left(\frac{1}{-1+x}+\frac{1}{1+x^2}\right)dx\\
&= \log(1-x)+\int \frac{1}{1+x^2}\,dx\\
&= \tan^{-1}(x)+\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 1.00

$$\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x+x^3}{(-1+x)^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + x^3)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(-x + x^3)/((-1 + x)^2*(1 + x^2)), x]
```

fricas [A] time = 0.97, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x) + log(x - 1)
```

giac [B] time = 0.29, size = 28, normalized size = 3.11

$$\frac{1}{4}\pi - \pi\left\lfloor\frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \arctan(x) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + arctan(x) + log(abs(x
- 1))
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$\arctan(x) + \ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x-1)^2/(x^2+1),x)
```

```
[Out] ln(x-1)+arctan(x)
```

maxima [A] time = 2.17, size = 7, normalized size = 0.78

$$\arctan(x) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(x) + log(x - 1)
```

mupad [B] time = 2.10, size = 19, normalized size = 2.11

$$\ln(x-1) - \mathrm{atan}\left(\frac{5}{4\,x+2} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/((x^2 + 1)*(x - 1)^2),x)
[Out] log(x - 1) - atan(5/(4*x + 2) - 1/2)
```

sympy [A] time = 0.14, size = 7, normalized size = 0.78

$$\log(x-1) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(-1+x)**2/(x**2+1),x)
[Out] log(x - 1) + atan(x)
```

3.321 $\int \frac{2+5x+3x^2+2x^3}{1+x+x^2} \, dx$

Optimal. Leaf size=12

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1657, 628}

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2 + 5x + 3x^2 + 2x^3}{1 + x + x^2} \, dx &= \int \left(1 + 2x + \frac{1 + 2x}{1 + x + x^2}\right) dx \\
&= x + x^2 + \int \frac{1 + 2x}{1 + x + x^2} \, dx \\
&= x + x^2 + \log\left(1 + x + x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x^2 + \log\left(x^2 + x + 1\right) + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] x + x^2 + Log[1 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2 + 5x + 3x^2 + 2x^3}{1 + x + x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2),x]
```

```
[Out] IntegrateAlgebraic[(2 + 5*x + 3*x^2 + 2*x^3)/(1 + x + x^2), x]
```

fricas [A] time = 1.06, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="fricas")
[Out] x^2 + x + log(x^2 + x + 1)
```

giac [A] time = 0.29, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="giac")
[Out] x^2 + x + log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x^2 + x + \ln\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x)
[Out] x+x^2+ln(x^2+x+1)
```

maxima [A] time = 1.37, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+5*x+2)/(x^2+x+1),x, algorithm="maxima")
[Out] x^2 + x + log(x^2 + x + 1)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$x + \ln\left(x^2 + x + 1\right) + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 3*x^2 + 2*x^3 + 2)/(x + x^2 + 1),x)
[Out] x + log(x + x^2 + 1) + x^2
```

sympy [A] time = 0.09, size = 12, normalized size = 1.00

$$x^2 + x + \log\left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+5*x+2)/(x**2+x+1),x)
[Out] x**2 + x + log(x**2 + x + 1)
```

3.322 $\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)} dx$

Optimal. Leaf size=65

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1628, 632, 31}

$$\frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] 3/(2*x^2) - x^(-1) + 3*Log[x] - ((15 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10
- ((15 + Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)} dx &= \int \left(-\frac{3}{x^3}+\frac{1}{x^2}+\frac{3}{x}+\frac{-1-3x}{-1+x+x^2}\right) dx\\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\int \frac{-1-3x}{-1+x+x^2} dx\\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)+\frac{1}{10}\left(-15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x} dx-\frac{1}{10}\left(15+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+
\\
&= \frac{3}{2x^2}-\frac{1}{x}+3\log(x)-\frac{1}{10}\left(15-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right)-\frac{1}{10}\left(15+\sqrt{5}\right)\log\left(1+\sqrt{5}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.89

$$\frac{1}{10}\left(\frac{15}{x^2}-\frac{10}{x}+\left(\sqrt{5}-15\right)\log\left(-2x+\sqrt{5}-1\right)+30\log(x)-\left(15+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] (15/x^2 - 10/x + (-15 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x] + 30*Log[x] - (15
+ Sqrt[5])*Log[1 + Sqrt[5] + 2*x])/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3-4x-5x^2+3x^3}{x^3\left(-1+x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(3 - 4*x - 5*x^2 + 3*x^3)/(x^3*(-1 + x + x^2)), x]
```

fricas [A] time = 1.51, size = 66, normalized size = 1.02

$$\frac{\sqrt{5}\,x^2\log\left(\frac{2\,x^2-\sqrt{5}\,(2\,x+1)+2\,x+3}{x^2+x-1}\right)-15\,x^2\log\left(x^2+x-1\right)+30\,x^2\log(x)-10\,x+15}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^2*log((2*x^2 - sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1))
- 15*x^2*log(x^2 + x - 1) + 30*x^2*log(x) - 10*x + 15)/x^2
```

giac [A] time = 0.30, size = 55, normalized size = 0.85

$$\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{\left|2\,x-\sqrt{5}+1\right|}{\left|2\,x+\sqrt{5}+1\right|}\right)-\frac{2\,x-3}{2\,x^2}-\frac{3}{2}\,\log\left(\left|x^2+x-1\right|\right)+3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/2*(2*x
- 3)/x^2 - 3/2*log(abs(x^2 + x - 1)) + 3*log(abs(x))
```

maple [A] time = 0.01, size = 41, normalized size = 0.63

$$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5}+3\ln(x)-\frac{3\ln\left(x^2+x-1\right)}{2}-\frac{1}{x}+\frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x)
```

```
[Out] -3/2*ln(x^2+x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))-1/x+3/2/x^2+3*ln(
x)
```

maxima [A] time = 1.77, size = 51, normalized size = 0.78

$$\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{2\,x-3}{2\,x^2}-\frac{3}{2}\,\log\left(x^2+x-1\right)+3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-5*x^2-4*x+3)/x^3/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/2*(2*x - 3)/x
^2 - 3/2*log(x^2 + x - 1) + 3*log(x)
```

mupad [B] time = 0.10, size = 48, normalized size = 0.74

$$3\ln(x)-\frac{x-\frac{3}{2}}{x^2}+\ln\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}-\frac{3}{2}\right)-\ln\left(x+\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x + 5*x^2 - 3*x^3 - 3)/(x^3*(x + x^2 - 1)),x)
```

```
[Out] 3*log(x) - (x - 3/2)/x^2 + log(x - 5^(1/2)/2 + 1/2)*(5^(1/2)/10 - 3/2) - lo
g(x + 5^(1/2)/2 + 1/2)*(5^(1/2)/10 + 3/2)
```

sympy [A] time = 0.45, size = 99, normalized size = 1.52

$$3\log(x)+\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}-\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}+\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{405}{202}+\frac{35\sqrt{5}}{202}+\frac{110\left(-\frac{3}{2}-\frac{\sqrt{5}}{10}\right)^2}{101}\right)+\frac{3-2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-5*x**2-4*x+3)/x**3/(x**2+x-1),x)
```

```
[Out] 3*log(x) + (-3/2 + sqrt(5)/10)*log(x - 405/202 - 35*sqrt(5)/202 + 110*(-3/2
 + sqrt(5)/10)**2/101) + (-3/2 - sqrt(5)/10)*log(x - 405/202 + 35*sqrt(5)/2
02 + 110*(-3/2 - sqrt(5)/10)**2/101) + (3 - 2*x)/(2*x**2)
```

3.323 $\int \frac{4+8x+5x^2+2x^3}{\left(2+2x+x^2\right)^2} dx$

Optimal. Leaf size=28

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1660, 634, 617, 204, 628}

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
```

```
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx = -\frac{1}{2+2x+x^2}+\frac{1}{4}\int \frac{4+8x}{2+2x+x^2}\,dx$$
$$= -\frac{1}{2+2x+x^2}-\int \frac{1}{2+2x+x^2}\,dx+\int \frac{2+2x}{2+2x+x^2}\,dx$$
$$= -\frac{1}{2+2x+x^2}+\log\left(2+2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right)$$
$$= -\frac{1}{2+2x+x^2}-\tan^{-1}(1+x)+\log\left(2+2x+x^2\right)$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2x+2}+\log\left(x^2+2x+2\right)-\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
[Out] -(2 + 2*x + x^2)^(-1) - ArcTan[1 + x] + Log[2 + 2*x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+8x+5x^2+2x^3}{(2+2x+x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2,x]
[Out] IntegrateAlgebraic[(4 + 8*x + 5*x^2 + 2*x^3)/(2 + 2*x + x^2)^2, x]
```

fricas [A] time = 0.99, size = 46, normalized size = 1.64

$$-\frac{(x^2+2x+2)\arctan(x+1)-(x^2+2x+2)\log\left(x^2+2x+2\right)+1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="fricas")
[Out] -((x^2 + 2*x + 2)*arctan(x + 1) - (x^2 + 2*x + 2)*log(x^2 + 2*x + 2) + 1)/(
x^2 + 2*x + 2)
```

giac [A] time = 0.28, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2x+2}-\arctan(x+1)+\log\left(x^2+2x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="giac")
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\arctan(x+1)+\ln\left(x^2+2x+2\right)-\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x)
```

```
[Out] -1/(x^2+2*x+2)-arctan(x+1)+ln(x^2+2*x+2)
```

maxima [A] time = 1.66, size = 28, normalized size = 1.00

$$-\frac{1}{x^2+2\,x+2}-\arctan\left(x+1\right)+\log\left(x^2+2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+5*x^2+8*x+4)/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 2*x + 2) - arctan(x + 1) + log(x^2 + 2*x + 2)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.00

$$\ln\left(x^2+2\,x+2\right)-\mathrm{atan}\left(x+1\right)-\frac{1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 5*x^2 + 2*x^3 + 4)/(2*x + x^2 + 2)^2,x)
```

```
[Out] log(2*x + x^2 + 2) - atan(x + 1) - 1/(2*x + x^2 + 2)
```

sympy [A] time = 0.13, size = 24, normalized size = 0.86

$$\log\left(x^2+2x+2\right)-\mathrm{atan}\left(x+1\right)-\frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+5*x**2+8*x+4)/(x**2+2*x+2)**2,x)
```

```
[Out] log(x**2 + 2*x + 2) - atan(x + 1) - 1/(x**2 + 2*x + 2)
```

3.324 $\int \frac{(-1+x)^4 x^4}{1+x^2} \, dx$

Optimal. Leaf size=32

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1629, 203}

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-1+x)^4 x^4}{1+x^2} \, dx &= \int \left(4 - 4x^2 + 5x^4 - 4x^5 + x^6 - \frac{4}{1+x^2}\right) dx \\
&= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4\int \frac{1}{1+x^2} \, dx \\
&= 4x - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7} - 4\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*x^4)/(1 + x^2),x]
```

```
[Out] 4*x - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7 - 4*ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-1+x)^4 x^4}{1+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((-1 + x)^4*x^4)/(1 + x^2),x]
[Out] IntegrateAlgebraic[((-1 + x)^4*x^4)/(1 + x^2), x]
```

fricas [A] time = 0.61, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="fricas")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

giac [A] time = 0.37, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="giac")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^4*x^4/(x^2+1),x)
[Out] 4*x-4/3*x^3+x^5-2/3*x^6+1/7*x^7-4*arctan(x)
```

maxima [A] time = 1.51, size = 26, normalized size = 0.81

$$\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^4*x^4/(x^2+1),x, algorithm="maxima")
[Out] 1/7*x^7 - 2/3*x^6 + x^5 - 4/3*x^3 + 4*x - 4*arctan(x)
```

mupad [B] time = 0.02, size = 26, normalized size = 0.81

$$4x - 4\,\mathrm{atan}(x) - \frac{4x^3}{3} + x^5 - \frac{2x^6}{3} + \frac{x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x - 1)^4)/(x^2 + 1),x)
[Out] 4*x - 4*atan(x) - (4*x^3)/3 + x^5 - (2*x^6)/3 + x^7/7
```

sympy [A] time = 0.10, size = 29, normalized size = 0.91

$$\frac{x^7}{7} - \frac{2x^6}{3} + x^5 - \frac{4x^3}{3} + 4x - 4\,\mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**4*x**4/(x**2+1),x)
[Out] x**7/7 - 2*x**6/3 + x**5 - 4*x**3/3 + 4*x - 4*atan(x)
```

3.325 $\int \frac{-20x+4x^2}{9-10x^2+x^4} dx$

Optimal. Leaf size=31

$$\log(1-x)-\frac{1}{2}\log(3-x)+\frac{3}{2}\log(x+1)-2\log(x+3)$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 1662, 12, 1107, 616, 31, 1130, 207}

$$\frac{5}{4}\log\left(1-x^2\right)-\frac{5}{4}\log\left(9-x^2\right)-\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right)+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
```

```
[Out] (-3*ArcTanh[x/3])/2 + ArcTanh[x]/2 + (5*Log[1 - x^2])/4 - (5*Log[9 - x^2])/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx &= \int \frac{x(-20+4x)}{9-10x^2+x^4}\,dx\\
&= \int -\frac{20x}{9-10x^2+x^4}\,dx + \int \frac{4x^2}{9-10x^2+x^4}\,dx\\
&= 4\int \frac{x^2}{9-10x^2+x^4}\,dx - 20\int \frac{x}{9-10x^2+x^4}\,dx\\
&= -\left(\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\right) + \frac{9}{2}\int \frac{1}{-9+x^2}\,dx - 10\,\text{Subst}\left(\int \frac{1}{9-10x+x^2}\,dx, x, x^2\right)\\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) - \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-9+x}\,dx, x, x^2\right) + \frac{5}{4}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx\right.\\
&= -\frac{3}{2}\tanh^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}\tanh^{-1}(x) + \frac{5}{4}\log\left(1-x^2\right) - \frac{5}{4}\log\left(9-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.26

$$4\left(\frac{1}{4}\log(1-x) - \frac{1}{8}\log(3-x) + \frac{3}{8}\log(x+1) - \frac{1}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
[Out] 4*(Log[1 - x]/4 - Log[3 - x]/8 + (3*Log[1 + x])/8 - Log[3 + x]/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-20x+4x^2}{9-10x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4),x]
[Out] IntegrateAlgebraic[(-20*x + 4*x^2)/(9 - 10*x^2 + x^4), x]
```

fricas [A] time = 1.40, size = 23, normalized size = 0.74

$$-2\,\log\left(x+3\right) + \frac{3}{2}\,\log\left(x+1\right) + \log\left(x-1\right) - \frac{1}{2}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="fricas")
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

giac [A] time = 0.31, size = 27, normalized size = 0.87

$$-2 \log(|x+3|)+\frac{3}{2} \log(|x+1|)+\log(|x-1|)-\frac{1}{2} \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="giac")
```

```
[Out] -2*log(abs(x + 3)) + 3/2*log(abs(x + 1)) + log(abs(x - 1)) - 1/2*log(abs(x
- 3))
```

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$-\frac{\ln(x-3)}{2}+\ln(x-1)+\frac{3\ln(x+1)}{2}-2\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-20*x)/(x^4-10*x^2+9),x)
```

```
[Out] ln(x-1)+3/2*ln(x+1)-2*ln(x+3)-1/2*ln(x-3)
```

maxima [A] time = 0.80, size = 23, normalized size = 0.74

$$-2 \log(x+3)+\frac{3}{2} \log(x+1)+\log(x-1)-\frac{1}{2} \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-20*x)/(x^4-10*x^2+9),x, algorithm="maxima")
```

```
[Out] -2*log(x + 3) + 3/2*log(x + 1) + log(x - 1) - 1/2*log(x - 3)
```

mupad [B] time = 0.05, size = 23, normalized size = 0.74

$$\ln(x-1)+\frac{3 \ln(x+1)}{2}-\frac{\ln(x-3)}{2}-2 \ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(20*x - 4*x^2)/(x^4 - 10*x^2 + 9),x)
```

```
[Out] log(x - 1) + (3*log(x + 1))/2 - log(x - 3)/2 - 2*log(x + 3)
```

sympy [A] time = 0.19, size = 26, normalized size = 0.84

$$-\frac{\log(x-3)}{2}+\log(x-1)+\frac{3\log(x+1)}{2}-2\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-20*x)/(x**4-10*x**2+9),x)
```

```
[Out] -log(x - 3)/2 + log(x - 1) + 3*log(x + 1)/2 - 2*log(x + 3)
```

3.326 $\int \frac{-1+x+4x^3}{(-1+x)x^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Rubi [A] time = 0.17, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6725, 635, 203, 260}

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
```

```
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{1}{x^2}+\frac{1-2x}{1+x^2}\right)dx\\
&= -\frac{1}{x}+2\log(1-x)+\int \frac{1-2x}{1+x^2}\,dx\\
&= -\frac{1}{x}+2\log(1-x)-2\int \frac{x}{1+x^2}\,dx+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{x}+\tan^{-1}(x)+2\log(1-x)-\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\log\left(x^2+1\right)-\frac{1}{x}+2\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] -x^(-1) + ArcTan[x] + 2*Log[1 - x] - Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x+4x^3}{(-1+x)x^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(-1 + x + 4*x^3)/((-1 + x)*x^2*(1 + x^2)), x]
```

fricas [A] time = 1.27, size = 26, normalized size = 1.08

$$\frac{x\arctan(x) - x\log\left(x^2+1\right) + 2\,x\log\left(x-1\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="fricas")
[Out] (x*arctan(x) - x*log(x^2 + 1) + 2*x*log(x - 1) - 1)/x
```

giac [A] time = 0.29, size = 23, normalized size = 0.96

$$-\frac{1}{x} + \arctan(x) - \log\left(x^2+1\right) + 2\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="giac")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$\arctan(x) + 2\ln\left(x-1\right) - \ln\left(x^2+1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3+x-1)/(x-1)/x^2/(x^2+1),x)
[Out] 2*ln(x-1)-1/x-ln(x^2+1)+arctan(x)
```

maxima [A] time = 1.22, size = 22, normalized size = 0.92

$$-\frac{1}{x} + \arctan(x) - \log\left(x^2+1\right) + 2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3+x-1)/(-1+x)/x^2/(x^2+1),x, algorithm="maxima")
[Out] -1/x + arctan(x) - log(x^2 + 1) + 2*log(x - 1)
```

mupad [B] time = 2.13, size = 30, normalized size = 1.25

$$2\,\ln\left(x-1\right) - \frac{1}{x} + \ln\left(x-\mathrm{i}\right)\left(-1-\frac{1}{2}\mathrm{i}\right) + \ln\left(x+1\mathrm{i}\right)\left(-1+\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^3 - 1)/(x^2*(x^2 + 1)*(x - 1)),x)
[Out] 2*log(x - 1) - log(x - 1i)*(1 + 1i/2) - log(x + 1i)*(1 - 1i/2) - 1/x
```

sympy [A] time = 0.15, size = 19, normalized size = 0.79

$$2\log(x-1) - \log\left(x^2+1\right) + \operatorname{atan}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3+x-1)/(-1+x)/x**2/(x**2+1),x)
[Out] 2*log(x - 1) - log(x**2 + 1) + atan(x) - 1/x
```

3.327 $\int \frac{1-3x+2x^2-4x^3+x^4}{(1+x^2)^3}\,dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1814, 12, 203}

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-4x^3+x^4}{\left(1+x^2\right)^3}\,dx &= -\frac{1}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-4+16x-4x^2}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\frac{1}{8}\int \frac{8}{1+x^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)^2}+\frac{2}{1+x^2}+\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2+1}-\frac{1}{4\left(x^2+1\right)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{\left(1+x^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3,x]
[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + x^2)^3, x]
```

fricas [A] time = 1.10, size = 35, normalized size = 1.52

$$\frac{8\,x^2+4\left(x^4+2\,x^2+1\right)\arctan(x)+7}{4\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="fricas")
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.39, size = 19, normalized size = 0.83

$$\frac{8\,x^2+7}{4\left(x^2+1\right)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="giac")
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

maple [A] time = 0.01, size = 19, normalized size = 0.83

$$\arctan(x)+\frac{2x^2+\frac{7}{4}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x)
[Out] (2*x^2+7/4)/(x^2+1)^2+arctan(x)
```

maxima [A] time = 1.43, size = 24, normalized size = 1.04

$$\frac{8\,x^2+7}{4\left(x^4+2\,x^2+1\right)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^2+1)^3,x, algorithm="maxima")
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

mupad [**B**] time = 0.03, size = 23, normalized size = 1.00

$$\mathrm{atan}(x) + \frac{2\,x^2 + \frac{7}{4}}{x^4 + 2\,x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(x^2 + 1)^3,x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

sympy [**A**] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2 + 7}{4x^4 + 8x^2 + 4} + \mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**2+1)**3,x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.328 $\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} \, dx$

Optimal. Leaf size=23

$$\frac{2}{x^2+1} - \frac{1}{4\left(x^2+1\right)^2} + \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2073, 261, 203}

$$\frac{2}{x^2+1} - \frac{1}{4\left(x^2+1\right)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/(4*(1 + x^2)^2) + 2/(1 + x^2) + ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6} \, dx &= \int \left(\frac{x}{\left(1+x^2\right)^3} - \frac{4x}{\left(1+x^2\right)^2} + \frac{1}{1+x^2}\right) dx \\
&= -\left(4 \int \frac{x}{\left(1+x^2\right)^2} \, dx\right) + \int \frac{x}{\left(1+x^2\right)^3} \, dx + \int \frac{1}{1+x^2} \, dx \\
&= -\frac{1}{4\left(1+x^2\right)^2} + \frac{2}{1+x^2} + \tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2}{x^2+1} - \frac{1}{4\left(x^2+1\right)^2} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6),x]
```

```
[Out] -1/4*1/(1 + x^2)^2 + 2/(1 + x^2) + ArcTan[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-3x+2x^2-4x^3+x^4}{1+3x^2+3x^4+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6
),x]
```

```
[Out] IntegrateAlgebraic[(1 - 3*x + 2*x^2 - 4*x^3 + x^4)/(1 + 3*x^2 + 3*x^4 + x^6
), x]
```

fricas [A] time = 1.88, size = 35, normalized size = 1.52

$$\frac{8\,x^2+4\,\left(x^4+2\,x^2+1\right)\arctan(x)+7}{4\,\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/4*(8*x^2 + 4*(x^4 + 2*x^2 + 1)*arctan(x) + 7)/(x^4 + 2*x^2 + 1)
```

giac [A] time = 0.27, size = 19, normalized size = 0.83

$$\frac{8\,x^2+7}{4\,\left(x^2+1\right)^2}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(8*x^2 + 7)/(x^2 + 1)^2 + arctan(x)
```

maple [A] time = 0.00, size = 19, normalized size = 0.83

$$\arctan(x)+\frac{2x^2+\frac{7}{4}}{\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x)
```

```
[Out] arctan(x)+(2*x^2+7/4)/(x^2+1)^2
```

maxima [A] time = 1.42, size = 24, normalized size = 1.04

$$\frac{8\,x^2+7}{4\,\left(x^4+2\,x^2+1\right)}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-4*x^3+2*x^2-3*x+1)/(x^6+3*x^4+3*x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/4*(8*x^2 + 7)/(x^4 + 2*x^2 + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\operatorname{atan}(x)+\frac{2\,x^2+\frac{7}{4}}{x^4+2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - 3*x - 4*x^3 + x^4 + 1)/(3*x^2 + 3*x^4 + x^6 + 1),x)
[Out] atan(x) + (2*x^2 + 7/4)/(2*x^2 + x^4 + 1)
```

sympy [A] time = 0.13, size = 20, normalized size = 0.87

$$\frac{8x^2+7}{4x^4+8x^2+4}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-4*x**3+2*x**2-3*x+1)/(x**6+3*x**4+3*x**2+1),x)
[Out] (8*x**2 + 7)/(4*x**4 + 8*x**2 + 4) + atan(x)
```

3.329 $\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx$

Optimal. Leaf size=13

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Rubi [A] time = 0.05, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1594, 1628, 628}

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+2x^2+2x^3}{x^2+x^3+x^4}\,dx &= \int \frac{1+x+2x^2+2x^3}{x^2\left(1+x+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x^2}+\frac{1+2x}{1+x+x^2}\right)dx\\
&= -\frac{1}{x}+\int \frac{1+2x}{1+x+x^2}\,dx\\
&= -\frac{1}{x}+\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\log\left(x^2+x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
```

```
[Out] -x^(-1) + Log[1 + x + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1 + x + 2x^2 + 2x^3}{x^2 + x^3 + x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4),x]
[Out] IntegrateAlgebraic[(1 + x + 2*x^2 + 2*x^3)/(x^2 + x^3 + x^4), x]
```

fricas [A] time = 2.06, size = 15, normalized size = 1.15

$$\frac{x \log \left(x^2 + x + 1\right) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="fricas")
[Out] (x*log(x^2 + x + 1) - 1)/x
```

giac [A] time = 0.28, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log \left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="giac")
[Out] -1/x + log(x^2 + x + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\ln \left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x)
[Out] -1/x+ln(x^2+x+1)
```

maxima [A] time = 0.71, size = 13, normalized size = 1.00

$$-\frac{1}{x} + \log \left(x^2 + x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+2*x^2+x+1)/(x^4+x^3+x^2),x, algorithm="maxima")
[Out] -1/x + log(x^2 + x + 1)
```

mupad [B] time = 2.14, size = 13, normalized size = 1.00

$$\ln \left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^2 + 2*x^3 + 1)/(x^2 + x^3 + x^4),x)
```

```
[Out] log(x + x^2 + 1) - 1/x
```

sympy [A] time = 0.10, size = 10, normalized size = 0.77

$$\log\left(x^2 + x + 1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+2*x**2+x+1)/(x**4+x**3+x**2),x)
[Out] log(x**2 + x + 1) - 1/x
```

3.330 $\int \frac{x^2(c+dx)^2}{a+bx^3} \, dx$

Optimal. Leaf size=206

$$\frac{\sqrt[3]{a}\, d \left(2 \sqrt[3]{b}\, c-\sqrt[3]{a}\, d\right) \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b}\, x+b^{2/3} x^2\right)}{6 b^{5/3}}-\frac{\sqrt[3]{a}\, d \left(2 \sqrt[3]{b}\, c-\sqrt[3]{a}\, d\right) \log \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{3 b^{5/3}}+\frac{\sqrt[3]{a}\, d \left(\sqrt[3]{a}\, d+2 \sqrt[3]{b} \cdots\right.}{\sqrt{\cdots}}$$

Rubi [A] time = 0.27, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{a}\, d \left(2 \sqrt[3]{b}\, c-\sqrt[3]{a}\, d\right) \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b}\, x+b^{2/3} x^2\right)}{6 b^{5/3}}-\frac{\sqrt[3]{a}\, d \left(2 \sqrt[3]{b}\, c-\sqrt[3]{a}\, d\right) \log \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{3 b^{5/3}}+\frac{\sqrt[3]{a}\, d \left(\sqrt[3]{a}\, d+2 \sqrt[3]{b}\, c\right) \tan ^{-1}\left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b}\, x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3}\, b^{5/3}}+\frac{c^2 \log \left(a+b x^3\right)}{3 b}+\frac{2 c d x}{b}+\frac{d^2 x^2}{2 b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (2*c*d*x)/b + (d^2*x^2)/(2*b) + (a^(1/3)*d*(2*b^(1/3)*c + a^(1/3)*d)*ArcTan
[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)) - (a^(1/3)*d
*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(5/3)) + (a^(1/3)
*d*(2*b^(1/3)*c - a^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)) + (c^2*Log[a + b*x^3])/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx = \int \left(\frac{2cd}{b}+\frac{d^2x}{b}-\frac{2acd+ad^2x-bc^2x^2}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x-bc^2x^2}{a+bx^3}\,dx}{b}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\int \frac{2acd+ad^2x}{a+bx^3}\,dx}{b}+c^2\int \frac{x^2}{a+bx^3}\,dx$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{c^2\log\left(a+bx^3\right)}{3b}-\frac{\int \frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}\,cd+a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b^{4/3}}-\frac{\left(\sqrt[3]{a}\,d\left(2c-\right.\right.}{}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{c^2\log\left(a+bx^3\right)}{3b}+\frac{\left(\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\right.\right.}{}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}}+\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(a^{2/3}-\sqrt[3]{a}\right.}{6b^{5/3}}$$

$$= \frac{2cdx}{b}+\frac{d^2x^2}{2b}+\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c+\sqrt[3]{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}}-\frac{\sqrt[3]{a}\,d\left(2\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{3b^{5/3}}$$

Mathematica [A] time = 0.11, size = 193, normalized size = 0.94

$$\frac{-\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d-2\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d-2\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2\sqrt{3}\,\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+2\sqrt[3]{b}\,c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+12b^{2/3}cdx+3b^{2/3}d^2x^2}{6b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] (12*b^(2/3)*c*d*x + 3*b^(2/3)*d^2*x^2 + 2*Sqrt[3]*a^(1/3)*d*(2*b^(1/3)*c +
a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(-2*b^
```

```
(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - a^(1/3)*d*(-2*b^(1/3)*c + a
^(1/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Lo
g[a + b*x^3])/(6*b^(5/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(c+dx)^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(c + d*x)^2)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(x^2*(c + d*x)^2)/(a + b*x^3), x]
```

fricas [C] time = 5.14, size = 4545, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*d^2*x^2 + 24*c*d*x - 2*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b*log(1/4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*s
qrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2
*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b
^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b
^3 + 3*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (
8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a
*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 + 5*b*c
^4 + 4*a*c*d^3 + (8*b*c^3*d + a*d^4)*x) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^
5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5
)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqr
t(3) + 1) - 2*c^2/b)*b + 6*c^2 + 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b
^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^
3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a
^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4
+ 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)
^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt
(3) + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)
^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8
*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*
b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3
)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^
2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/
b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d
^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 +
a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5
- 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^
```

```
(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3
*d + a*d^4)*x + 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3
)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1
/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*
c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) -
2*c^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d
^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c
^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) +
(1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3
)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1)
- 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I
*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d
^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*
(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 +
(b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b
^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)) + ((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 +
2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(
1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*
a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3
) + 1) - 2*c^2/b)*b + 6*c^2 - 3*sqrt(1/3)*b*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2
- (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*
a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*
d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*
(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3
)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2
*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 -
3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1
/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a
*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/b)*b^2*c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3))*log(-1/4*(2*(1/2)^(2
/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*
c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c
^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a
*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d
^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)^2*b^3 - 3*(2*(1/2)^(2/3)*(c^4/b^2
 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)
*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2
*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3
*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/
3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*c^2 - 5*b*c^4 - 4*a*c*d^3 + 2*(8*b*c^3*d
+ a*d^4)*x - 3/4*sqrt(1/3)*((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b
^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 +
2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)
^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2
/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c
^2/b)*b^3 - 6*b^2*c^2)*sqrt(-((2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)
/b^3)*(-I*sqrt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4
+ 2*a*c*d^3)*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/
2)^(1/3)*(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c
^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2
*c^2/b)^2*b^3 + 4*(2*(1/2)^(2/3)*(c^4/b^2 - (b*c^4 + 2*a*c*d^3)/b^3)*(-I*sq
rt(3) + 1)/(2*c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)
*c^2/b^4 + (b^2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3) + (1/2)^(1/3)*(2*
c^6/b^3 + (8*b*c^3 + a*d^3)*a*d^3/b^5 - 3*(b*c^4 + 2*a*c*d^3)*c^2/b^4 + (b^
2*c^6 - 2*a*b*c^3*d^3 + a^2*d^6)/b^5)^(1/3)*(I*sqrt(3) + 1) - 2*c^2/b)*b^2*
c^2 + 4*b*c^4 + 32*a*c*d^3)/b^3)))/b
```

giac [A] time = 0.44, size = 208, normalized size = 1.01

$$\frac{c^2\log\left(\left|bx^3+a\right|\right)}{3\,b}-\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd-\left(-ab^2\right)^{\frac{2}{3}}d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}+\frac{bd^2x^2+4\,bcdx}{2\,b^2}-\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bcd+\left(-ab^2\right)^{\frac{2}{3}}d^2\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}+\frac{\left(ab^4d^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,ab^4cd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*c^2*log(abs(b*x^3 + a))/b - 1/3*sqrt(3)*(2*(-a*b^2)^(1/3)*b*c*d - (-a*b
^2)^(2/3)*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 +
1/2*(b*d^2*x^2 + 4*b*c*d*x)/b^2 - 1/6*(2*(-a*b^2)^(1/3)*b*c*d + (-a*b^2)^(2
/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 1/3*(a*b^4*d^2*(-a/
b)^(1/3) + 2*a*b^4*c*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5)
```

maple [A] time = 0.01, size = 236, normalized size = 1.15

$$\frac{d^2x^2}{2b}-\frac{2\sqrt{3}\,acd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{2acd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{acd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\sqrt{3}\,ad^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{ad^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{ad^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}+\frac{c^2\ln\left(bx^3+a\right)}{3b}+\frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^2/(b*x^3+a),x)
```

```
[Out] 1/2*d^2*x^2/b+2*c*d*x/b-2/3/b^2*a*c*d/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/3/b^2
*a*c*d/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-2/3/b^2*a*c*d/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/b^2*a*d^2/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))-1/6/b^2*a*d^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))-1/3/b^2*a*d^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
-1))+1/3*c^2*ln(b*x^3+a)/b
```

maxima [A] time = 1.65, size = 199, normalized size = 0.97

$$-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\,acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{d^2x^2+4\,cdx}{2\,b}+\frac{\left(2\,bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) + 2*a*c*d*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/2*(d^2*x^2 + 4*c*d*x)/b + 1/6*(2*
b*c^2*(a/b)^(2/3) - a*d^2*(a/b)^(1/3) + 2*a*c*d)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)^(1/3)
 - 2*a*c*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 0.23, size = 357, normalized size = 1.73

$$\left(\sum_{k=1}^{3}\ln\left(\frac{a\left(b\,c^4+\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)^2\,b^3\,9-\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)\,b^2\,c^2\,6+2\,a\,c\,d^3+a\,d^4\,x+2\,b\,c^3\,d\,x-\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)\,b^2\,c\,d\,x\,6\right)}{b}\right)\,\mathrm{root}\left(27\,b^5\,z^3-27\,b^4\,c^2\,z^2+18\,a\,b^2\,c\,d^3\,z+9\,b^3\,c^4\,z+2\,a\,b\,c^3\,d^3-b^2\,c^6-a^2\,d^6,z,k\right)\right)+\frac{d^2\,x^2}{2\,b}+\frac{2\,c\,d\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x)^2)/(a + b*x^3),x)
```

```
[Out] symsum(log((a*(b*c^4 + 9*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*
z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)^2*b^3 - 6*root(2
7*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3
 - b^2*c^6 - a^2*d^6, z, k)*b^2*c^2 + 2*a*c*d^3 + a*d^4*x + 2*b*c^3*d*x - 6
*root(27*b^5*z^3 - 27*b^4*c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*
```

```
c^3*d^3 - b^2*c^6 - a^2*d^6, z, k)*b^2*c*d*x))/b)*root(27*b^5*z^3 - 27*b^4*
c^2*z^2 + 18*a*b^2*c*d^3*z + 9*b^3*c^4*z + 2*a*b*c^3*d^3 - b^2*c^6 - a^2*d^
6, z, k), k, 1, 3) + (d^2*x^2)/(2*b) + (2*c*d*x)/b
```

sympy [A] time = 1.24, size = 138, normalized size = 0.67

$$\operatorname{RootSum}\left(27t^3b^5 - 27t^2b^4c^2 + t\left(18ab^2cd^3 + 9b^3c^4\right) - a^2d^6 + 2abc^3d^3 - b^2c^6, \left(t \mapsto t\log\left(x + \frac{9t^2b^3 - 18tb^2c^2 + 4acd^3 + 5bc^4}{ad^4 + 8bc^3d}\right)\right)\right) + \frac{2cdx}{b} + \frac{d^2x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*b**5 - 27*_t**2*b**4*c**2 + _t*(18*a*b**2*c*d**3 + 9*b**3*
c**4) - a**2*d**6 + 2*a*b*c**3*d**3 - b**2*c**6, Lambda(_t, _t*log(x + (9*_
t**2*b**3 - 18*_t*b**2*c**2 + 4*a*c*d**3 + 5*b*c**4)/(a*d**4 + 8*b*c**3*d))
)) + 2*c*d*x/b + d**2*x**2/(2*b)
```

3.331 $\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=45

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}$$

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1594, 1663, 1660, 12, 618, 204}

$$\frac{5-7x^2}{8\left(x^4+2x^2+3\right)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(-1+2x^2+4x^4\right)}{\left(3+2x^2+x^4\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-1+2x+4x^2}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{18}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{9}{8}\,\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} - \frac{9}{4}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right) \\
&= \frac{5-7x^2}{8\left(3+2x^2+x^4\right)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} + \frac{5-7x^2}{8\left(x^4+2x^2+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
[Out] (5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x+2x^3+4x^5}{\left(3+2x^2+x^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
[Out] IntegrateAlgebraic[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2, x]
```

fricas [A] time = 1.12, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-14x^2+10}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.11, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{7x^2-5}{8\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2 +
 3)
```

maple [A] time = 0.01, size = 41, normalized size = 0.91

$$\frac{9\sqrt{2}\ \arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}+\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(-7/4*x^2+5/4)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{7x^2-5}{8\left(x^4+2x^2+3\right)}+\frac{9}{4}\int\frac{x}{x^4+2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 2.19, size = 42, normalized size = 0.93

$$\frac{9\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{\frac{7x^2}{8}-\frac{5}{8}}{x^4+2x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^2
 + x^4 + 3)
```

sympy [A] time = 0.15, size = 44, normalized size = 0.98

$$\frac{5-7x^2}{8x^4+16x^2+24}+\frac{9\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

3.332 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3} dx$

Optimal. Leaf size=59

$$\tan^{-1}(2x^2+1)+\frac{2x^2+1}{2(2x^4+2x^2+1)}+\frac{4x^2+3}{16(2x^4+2x^2+1)^2}$$

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1593, 1663, 1660, 12, 614, 617, 204}

$$\frac{2x^2+1}{2(2x^4+2x^2+1)}+\frac{4x^2+3}{16(2x^4+2x^2+1)^2}+\tan^{-1}(2x^2+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] (3 + 4*x^2)/(16*(1 + 2*x^2 + 2*x^4)^2) + (1 + 2*x^2)/(2*(1 + 2*x^2 + 2*x^4)
) + ArcTan[1 + 2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx &= \int \frac{x\left(1+x^4\right)}{\left(1+2x^2+2x^4\right)^3}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{\left(1+2x+2x^2\right)^3}\,dx,x,x^2\right)\\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{16}{\left(1+2x+2x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2}+\operatorname{Subst}\left(\int \frac{1}{\left(1+2x+2x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2}+\frac{1+2x^2}{2\left(1+2x^2+2x^4\right)}+\operatorname{Subst}\left(\int \frac{1}{1+2x+2x^2}\,dx,x,x^2\right)\\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2}+\frac{1+2x^2}{2\left(1+2x^2+2x^4\right)}-\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+2x^2\right)\\
&= \frac{3+4x^2}{16\left(1+2x^2+2x^4\right)^2}+\frac{1+2x^2}{2\left(1+2x^2+2x^4\right)}+\tan^{-1}\left(1+2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.75

$$\tan^{-1}\left(2x^2+1\right)+\frac{32x^6+48x^4+36x^2+11}{16\left(2x^4+2x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
[Out] (11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*x
^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+x^5}{\left(1+2x^2+2x^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3, x]
```

fricas [A] time = 0.89, size = 75, normalized size = 1.27

$$\frac{32\,x^{6}+48\,x^{4}+36\,x^{2}+16\left(4\,x^{8}+8\,x^{6}+8\,x^{4}+4\,x^{2}+1\right)\arctan\left(2\,x^{2}+1\right)+11}{16\left(4\,x^{8}+8\,x^{6}+8\,x^{4}+4\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*arc
tan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

giac [A] time = 1.80, size = 42, normalized size = 0.71

$$\frac{32\,x^{6}+48\,x^{4}+36\,x^{2}+11}{16\left(2\,x^{4}+2\,x^{2}+1\right)^{2}}+\arctan\left(2\,x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2 +
 1)
```

maple [A] time = 0.01, size = 41, normalized size = 0.69

$$\arctan\left(2x^{2}+1\right)+\frac{2x^{6}+3x^{4}+\frac{9}{4}x^{2}+\frac{11}{16}}{\left(2x^{4}+2x^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+x)/(2*x^4+2*x^2+1)^3,x)
```

```
[Out] 2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{32\,x^{6}+48\,x^{4}+36\,x^{2}+11}{16\left(4\,x^{8}+8\,x^{6}+8\,x^{4}+4\,x^{2}+1\right)}+2\int\frac{x}{2\,x^{4}+2\,x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

mupad [B] time = 0.05, size = 47, normalized size = 0.80

$$\mathrm{atan}\left(2\,x^{2}+1\right)+\frac{\frac{x^{6}}{2}+\frac{3\,x^{4}}{4}+\frac{9\,x^{2}}{16}+\frac{11}{64}}{x^{8}+2\,x^{6}+2\,x^{4}+x^{2}+\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

```
[Out] atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 + 2
*x^6 + x^8 + 1/4)
```

sympy [A] time = 0.18, size = 46, normalized size = 0.78

$$\frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16}+\operatorname{atan}\left(2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

```
[Out] (32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**2
 + 16) + atan(2*x**2 + 1)
```

3.333 $\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$

Optimal. Leaf size=209

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Rubi [A] time = 0.37, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\left(\frac{ce-2af}{\sqrt{e^2-4df}}+c\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{2}\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}-\frac{b\tanh^{-1}\left(\frac{e+2fx^2}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((c - (c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e -
Sqrt[e^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + ((c +
(c*e - 2*a*f)/Sqrt[e^2 - 4*d*f])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e + Sqrt[e
^2 - 4*d*f]]])/(Sqrt[2]*Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) - (b*ArcTanh[(
e + 2*f*x^2)/Sqrt[e^2 - 4*d*f]])/Sqrt[e^2 - 4*d*f]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx = \int \frac{bx}{d+ex^2+fx^4}\,dx + \int \frac{a+cx^2}{d+ex^2+fx^4}\,dx$$

$$= b\int \frac{x}{d+ex^2+fx^4}\,dx + \frac{1}{2}\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\int \frac{1}{\frac{e}{2}-\frac{1}{2}\sqrt{e^2-4df}+fx^2}\,dx + \frac{1}{2}\left(c+\frac{ce}{\sqrt{e^2}}\right.$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} + \frac{1}{2}b\,\mathrm{Subst}\left(\int\right.$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - b\,\mathrm{Subst}\left(\int \right.$$

$$= \frac{\left(c-\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}} + \frac{\left(c+\frac{ce-2af}{\sqrt{e^2-4df}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e+\sqrt{e^2-4df}}}\right)}{\sqrt{2}\sqrt{f}\sqrt{e+\sqrt{e^2-4df}}} - \frac{b\tanh^{-1}\left(\frac{e}{\sqrt{e}}\right.}{\sqrt{e^2-4d}}$$

Mathematica [A] time = 0.25, size = 234, normalized size = 1.12

$$\frac{\frac{\sqrt{2}\left(2af+c\left(\sqrt{e^2-4df}-e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{e-\sqrt{e^2-4df}}}\right)}{\sqrt{f}\sqrt{e-\sqrt{e^2-4df}}}+\frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}+e\right)-2af\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}\,x}{\sqrt{\sqrt{e^2-4df}+e}}\right)}{\sqrt{f}\sqrt{\sqrt{e^2-4df}+e}}+b\log\left(\sqrt{e^2-4df}-e-2fx^2\right)-b\log\left(\sqrt{e^2-4df}+e+2fx^2\right)}{2\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/S
qrt[e - Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e - Sqrt[e^2 - 4*d*f]]) + (Sqrt[
2]*(-2*a*f + c*(e + Sqrt[e^2 - 4*d*f]))*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[e +
 Sqrt[e^2 - 4*d*f]]])/(Sqrt[f]*Sqrt[e + Sqrt[e^2 - 4*d*f]]) + b*Log[-e + Sq
rt[e^2 - 4*d*f] - 2*f*x^2] - b*Log[e + Sqrt[e^2 - 4*d*f] + 2*f*x^2])/(2*Sqr
t[e^2 - 4*d*f])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{d+ex^2+fx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x]
[Out] IntegrateAlgebraic[(a + b*x + c*x^2)/(d + e*x^2 + f*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 4.70, size = 1587, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="giac")
[Out] 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*sqrt(-4*d*f + e^2)*b*log(x^2 + 1/2*
(sqrt(-4*d*f + e^2) + e)/f)/((16*d^2*f^2 - 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2
+ f^2*e^2 - 2*f*e^3 + e^4)*f^2) - 1/2*(4*d*f^3 - f^4 + 2*f^3*e - f^2*e^2)*s
qrt(-4*d*f + e^2)*b*log(x^2 - 1/2*(sqrt(-4*d*f + e^2) - e)/f)/((16*d^2*f^2
- 4*d*f^3 + 8*d*f^2*e - 8*d*f*e^2 + f^2*e^2 - 2*f*e^3 + e^4)*f^2) + 1/4*((1
6*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e + s
qrt(-4*d*f + e^2)*f)*d*f^3 - 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f^2*e - 8*d*f^3*e + 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqr
t(-4*d*f + e^2)*f)*d*f*e - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*f^2*e + 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e + sqrt(-4*d*
f + e^2)*f)*d*f*e^2 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*f^2*e^2 + 16
*d*f^2*e^2 + 2*(4*d*f - e^2)*f^2*e + 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
+ sqrt(-4*d*f + e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*
f*e^3 + 2*f^2*e^3 - 2*(4*d*f - e^2)*f*e^2 - sqrt(2)*sqrt(-4*d*f + e^2)*sqrt
(f*e + sqrt(-4*d*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)
*e^4 - 2*f*e^4)*a + 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e +
sqrt(-4*d*f + e^2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4
*d*f + e^2)*f)*d*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e + sqrt(-4*d*f
+ e^2)*f)*d*f*e - 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f
 + e^2)*sqrt(f*e + sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqr
t((sqrt(-4*d*f + e^2) + e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d
^2*f*e^2 + d*f^2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f)) + 1/4*((16*sqrt(2)*sqrt(f
*e - sqrt(-4*d*f + e^2)*f)*d^2*f^2 - 4*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2
)*f)*d*f^3 + 32*d^2*f^3 + 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f^2*
e + 8*d*f^3*e - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*
f)*d*f*e + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*
e - 8*(4*d*f - e^2)*d*f^2 - 8*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*
e^2 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f^2*e^2 - 16*d*f^2*e^2 - 2*(
4*d*f - e^2)*f^2*e - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f +
e^2)*f)*f*e^2 - 2*sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*f*e^3 - 2*f^2*e^
3 + 2*(4*d*f - e^2)*f*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d
*f + e^2)*f)*e^3 + sqrt(2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*e^4 + 2*f*e^4)*
a - 2*(8*d^2*f^3 - 4*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^
2)*f)*d^2*f + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d
*f^2 - 2*sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e - sqrt(-4*d*f + e^2)*f)*d*f*e
- 2*(4*d*f - e^2)*d*f^2 - 2*d*f^2*e^2 + sqrt(2)*sqrt(-4*d*f + e^2)*sqrt(f*e
 - sqrt(-4*d*f + e^2)*f)*d*e^2)*c)*arctan(2*sqrt(1/2)*x/sqrt(-(sqrt(-4*d*f
+ e^2) - e)/f))/((16*d^3*f^2 - 4*d^2*f^3 + 8*d^2*f^2*e - 8*d^2*f*e^2 + d*f^
2*e^2 - 2*d*f*e^3 + d*e^4)*abs(f))
```

maple [B] time = 0.06, size = 616, normalized size = 2.95

$$-\frac{2\sqrt{2}\,cdf\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}+\frac{2\sqrt{2}\,cdf\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}+\frac{\sqrt{2}\,ce^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{2\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{2}\,ce^2\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{2\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}+\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,af\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}+\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,af\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,ce\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}\right)}{2\left(4df-e^2\right)\sqrt{\left(-e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{-4df+e^2}\,\sqrt{2}\,ce\arctan\left(\frac{\sqrt{2}\,fx}{\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}\right)}{2\left(4df-e^2\right)\sqrt{\left(e+\sqrt{-4df+e^2}\right)f}}-\frac{\sqrt{-4df+e^2}\,b\ln\left(-2fx^2-e+\sqrt{-4df+e^2}\right)}{2\left(4df-e^2\right)}+\frac{\sqrt{-4df+e^2}\,b\ln\left(2fx^2+e+\sqrt{-4df+e^2}\right)}{8df-2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x)
```

```
[Out] -1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*b*ln(-2*f*x^2+(-4*d*f+e^2)^(1/2)-e)-2*f
/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(
((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*d+1/2/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)
^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*
e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/
2)*arctanh(f*x*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*a-1/2*(-4*d*f+e^2)
^(1/2)/(4*d*f-e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2)*arctanh(f*x*2^(
1/2)/(((-4*d*f+e^2)^(1/2)-e)*f)^(1/2))*c*e+1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^
2)*b*ln(2*f*x^2+(-4*d*f+e^2)^(1/2)+e)+2*f/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e
^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*
d-1/2/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/
2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))*c*e^2+f*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)
*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^
2)^(1/2))*f)^(1/2))*a-1/2*(-4*d*f+e^2)^(1/2)/(4*d*f-e^2)*2^(1/2)/((e+(-4*d*
f+e^2)^(1/2))*f)^(1/2)*arctan(f*x*2^(1/2)/((e+(-4*d*f+e^2)^(1/2))*f)^(1/2))
*c*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^2 + bx + a}{fx^4 + ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(f*x^4+e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)/(f*x^4 + e*x^2 + d), x)
```

mupad [B] time = 3.44, size = 3942, normalized size = 18.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(d + e*x^2 + f*x^4),x)
```

```
[Out] symsum(log(a*b^2*f^2 - a^2*c*f^2 + b^3*f^2*x - c^3*d*f - 8*root(16*d*e^4*f*
z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d
^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 +
32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z +
 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a
^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*
f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*e^3*f^2*x + a*c^2*e*f - 16*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*d*f^3 - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a^2*f^3*x + 4*root(16
```

```
*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2 -
 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*f
^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2*
d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c*
d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*f
 + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*a*e^2*f^2 + 16*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*d*f^3*x + 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b^2*e*f^2*x + 4*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*d*f^2*x - 2*root(1
6*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
- 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2*
f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^2
*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*c
*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e*
f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*c^2*e^2*f*x + 32*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^3*d*e*f^3*x - 4*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)^2*b*e^2*f^2*x + 4*roo
t(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z
^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d
^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*
c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^
2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2
*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*a*b*e*f^2 - 8*root(
16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3*z^4 - 16*a*c*d*e^2*f*z^2
 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2*d*e*f^2*z^2 + 64*a*c*d^2
*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 + 4*a^2*e^3*f*z^2 + 16*b*c^
2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16*a^2*b*d*f^2*z - 4*a*b^2*
c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e + b^2*c^2*d*e + a^2*b^2*e
*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z, k)*b*c*d*f^2 - 2*a*b*c*f
^2*x + b*c^2*e*f*x + 4*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*
f^3*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*
a^2*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2
 + 4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z -
 16*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d
```

```
*e + b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2,
 z, k)*a*c*e*f^2*x)*root(16*d*e^4*f*z^4 - 128*d^2*e^2*f^2*z^4 + 256*d^3*f^3
*z^4 - 16*a*c*d*e^2*f*z^2 - 16*c^2*d^2*e*f*z^2 - 8*b^2*d*e^2*f*z^2 - 16*a^2
*d*e*f^2*z^2 + 64*a*c*d^2*f^2*z^2 + 32*b^2*d^2*f^2*z^2 + 4*c^2*d*e^3*z^2 +
4*a^2*e^3*f*z^2 + 16*b*c^2*d^2*f*z + 4*a^2*b*e^2*f*z - 4*b*c^2*d*e^2*z - 16
*a^2*b*d*f^2*z - 4*a*b^2*c*d*f + 2*a^2*c^2*d*f - 2*a^3*c*e*f - 2*a*c^3*d*e
+ b^2*c^2*d*e + a^2*b^2*e*f + b^4*d*f + a^2*c^2*e^2 + c^4*d^2 + a^4*f^2, z,
 k), k, 1, 4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(f*x**4+e*x**2+d),x)
[Out] Timed out
```

3.334 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=224

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Rubi [A] time = 0.39, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
(2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx &= \int \frac{2dex}{a+bx^2+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+bx^2+cx^4}\,dx \\
&= (2de)\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\Bigg(e^2 + \\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + (de)\,\mathrm{Subst}\Bigg(\\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - (2de)\,\mathrm{Subst} \\
&= \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\tanh^{-1}}{\sqrt{b^2-}}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 245, normalized size = 1.09

$$
\frac{\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}-b\right)+2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)-2cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+2de\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-2de\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{2\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2*d*e
*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*c] +
2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + b*x^2 + c*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.56, size = 1625, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c
^2 + b^2*c^2 - 4*a*c^3)*c^2) + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2
 - 4*a*c)*d*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c -
2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c +
16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*
(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 +
2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(
b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/
c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c
)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d^2 + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e^2)*arctan(2*sq
rt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.06, size = 633, normalized size = 2.83

$$-\frac{2\sqrt{2}\,a c e^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,a c e^2 \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,b^2 e^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2 e^2 \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{-4ac+b^2}\,\sqrt{2}\,b e^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{-4ac+b^2}\,\sqrt{2}\,b e^2 \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{-4ac+b^2}\,\sqrt{2}\,c d^2 \operatorname{arctanh}\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{-4ac+b^2}\,\sqrt{2}\,c d^2 \arctan\left(\frac{\sqrt{2}\,c x}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{-4ac+b^2}\,d e \ln\left(-2c x^2-b+\sqrt{-4ac+b^2}\right)}{4ac-b^2}+\frac{\sqrt{-4ac+b^2}\,d e \ln\left(2c x^2+b+\sqrt{-4ac+b^2}\right)}{4ac-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2/(c*x^4+b*x^2+a),x)

[Out] -(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d*e*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-2*c/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*a+1/2/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2)*arctanh(c*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)-b)*c)^(1/2))*d^2+(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*d*e*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*a-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e^2*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e^2+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)

mupad [B] time = 3.22, size = 3046, normalized size = 13.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^2/(a + b*x^2 + c*x^4),x)

[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3

```
*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*
d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)^3*a*b*c^3*x + 4*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2
*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a
*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^
2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z
 + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 +
a^2*e^8, z, k)*a*c^2*e^4*x - 2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 +
256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*
d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b
^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*
c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8
, z, k)*b^2*c*e^4*x + 2*b*c*d*e^5*x - 16*root(16*a*b^4*c*z^4 - 128*a^2*b^2*
c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4
*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^
5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8
 + a^2*e^8, z, k)*a*c^2*d*e^3 + 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4
 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c
^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 +
8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2
*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*
e^8, z, k)*b*c^2*d^3*e + 32*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256
*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4
*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*
c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d
^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k)^2*a*c^3*d*e*x + 12*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3
*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2
 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^
5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e
^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)
*b*c^2*d^2*e^2*x - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^
3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 +
192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e
*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2
+ 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*
b^2*c^2*d*e*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4
- 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^
2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 3
2*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*
c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k), k, 1, 4
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.335 $\int \frac{x^2}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {72}

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)(c+dx)}\,dx &= \int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx\\ &= \frac{x}{bd}+\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\frac{a^2\log(a+bx)}{b^2(bc-ad)}-\frac{c^2\log(c+dx)}{d^2(bc-ad)}+\frac{x}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] x/(b*d) + (a^2*Log[a + b*x])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x])/(d^2*(b
*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((a + b*x)*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((a + b*x)*(c + d*x)), x]
```

fricas [A] time = 1.20, size = 65, normalized size = 1.16

$$\frac{a^2d^2\log(bx+a)-b^2c^2\log(dx+c)+\left(b^2cd-abd^2\right)x}{b^3cd^2-ab^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x + a) - b^2*c^2*log(d*x + c) + (b^2*c*d - a*b*d^2)*x)/(b^3*
c*d^2 - a*b^2*d^3)
```

giac [A] time = 0.31, size = 62, normalized size = 1.11

$$\frac{a^2\log(|bx+a|)}{b^3c-ab^2d}-\frac{c^2\log(|dx+c|)}{bcd^2-ad^3}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/(b^3*c - a*b^2*d) - c^2*log(abs(d*x + c))/(b*c*d^2 -
a*d^3) + x/(b*d)
```

maple [A] time = 0.01, size = 57, normalized size = 1.02

$$-\frac{a^2\ln(bx+a)}{(ad-bc)\,b^2}+\frac{c^2\ln(dx+c)}{(ad-bc)\,d^2}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] x/b/d+1/d^2*c^2/(a*d-b*c)*ln(d*x+c)-1/b^2*a^2/(a*d-b*c)*ln(b*x+a)
```

maxima [A] time = 0.68, size = 60, normalized size = 1.07

$$\frac{a^2\log(bx+a)}{b^3c-ab^2d}-\frac{c^2\log(dx+c)}{bcd^2-ad^3}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/(b^3*c - a*b^2*d) - c^2*log(d*x + c)/(b*c*d^2 - a*d^3) + x
/(b*d)
```

mupad [B] time = 0.21, size = 61, normalized size = 1.09

$$-\frac{a^2\,d^2\,\ln(a+b\,x)-b^2\,c^2\,\ln(c+d\,x)-a\,b\,d^2\,x+b^2\,c\,d\,x}{b^2\,d^2\,(a\,d-b\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)*(c + d*x)),x)
```

```
[Out] -(a^2*d^2*log(a + b*x) - b^2*c^2*log(c + d*x) - a*b*d^2*x + b^2*c*d*x)/(b^2
*d^2*(a*d - b*c))
```

sympy [B] time = 1.06, size = 190, normalized size = 3.39

$$-\frac{a^2\log\left(x+\frac{\frac{a^4d^3}{b(ad-bc)}-\frac{2a^3cd^2}{ad-bc}+\frac{a^2bc^2d}{ad-bc}+a^2cd+abc^2}{a^2d^2+b^2c^2}\right)}{b^2(ad-bc)}+\frac{c^2\log\left(x+\frac{-\frac{a^2bc^2d}{ad-bc}+a^2cd+\frac{2ab^2c^3}{ad-bc}+abc^2-\frac{b^3c^4}{d(ad-bc)}}{a^2d^2+b^2c^2}\right)}{d^2(ad-bc)}+\frac{x}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(d*x+c),x)
```

```
[Out] -a**2*log(x + (a**4*d**3/(b*(a*d - b*c)) - 2*a**3*c*d**2/(a*d - b*c) + a**2
*b*c**2*d/(a*d - b*c) + a**2*c*d + a*b*c**2)/(a**2*d**2 + b**2*c**2))/(b**2
*(a*d - b*c)) + c**2*log(x + (-a**2*b*c**2*d/(a*d - b*c) + a**2*c*d + 2*a*b
**2*c**3/(a*d - b*c) + a*b*c**2 - b**3*c**4/(d*(a*d - b*c)))/(a**2*d**2 + b
**2*c**2))/(d**2*(a*d - b*c)) + x/(b*d)
```

3.336 $\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=96

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

Rubi [A] time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1629, 635, 205, 260}

$$\frac{ad\log\left(a+bx^2\right)}{2b\left(ad^2+bc^2\right)}+\frac{c^2\log(c+dx)}{d\left(ad^2+bc^2\right)}-\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(ad^2+bc^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] -((Sqrt[a]*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c^2 + a*d^2))) + (c^2
*Log[c + d*x])/(d*(b*c^2 + a*d^2)) + (a*d*Log[a + b*x^2])/(2*b*(b*c^2 + a*d
^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx &= \int \left(\frac{c^2}{\left(bc^2+ad^2\right)(c+dx)} - \frac{a(c-dx)}{\left(bc^2+ad^2\right)\left(a+bx^2\right)}\right)dx \\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)} - \frac{a\int \frac{c-dx}{a+bx^2}\,dx}{bc^2+ad^2} \\
&= \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)} - \frac{(ac)\int \frac{1}{a+bx^2}\,dx}{bc^2+ad^2} + \frac{(ad)\int \frac{x}{a+bx^2}\,dx}{bc^2+ad^2} \\
&= -\frac{\sqrt{a}\,c\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}\left(bc^2+ad^2\right)} + \frac{c^2\log(c+dx)}{d\left(bc^2+ad^2\right)} + \frac{ad\log\left(a+bx^2\right)}{2b\left(bc^2+ad^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.76

$$
\frac{-2\sqrt{a}\,\sqrt{b}\,cd\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right) + ad^2\log\left(a+bx^2\right) + 2bc^2\log(c+dx)}{2abd^3+2b^2c^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*c*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*b*c^2*Log[c + d*x]
+ a*d^2*Log[a + b*x^2])/(2*b^2*c^2*d + 2*a*b*d^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^2}{(c+dx)\left(a+bx^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^2)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^2)), x]
```

fricas [A] time = 1.23, size = 162, normalized size = 1.69

$$
\left[\frac{bcd\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + ad^2\log\left(bx^2+a\right) + 2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)},\ -\frac{2\,bcd\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - ad^2\log\left(bx^2+a\right) - 2\,bc^2\log\left(dx+c\right)}{2\left(b^2c^2d+abd^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(b*c*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + a*
d^2*log(b*x^2 + a) + 2*b*c^2*log(d*x + c))/(b^2*c^2*d + a*b*d^3), -1/2*(2*b
*c*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - a*d^2*log(b*x^2 + a) - 2*b*c^2*log
(d*x + c))/(b^2*c^2*d + a*b*d^3)]
```

giac [A] time = 0.26, size = 85, normalized size = 0.89

$$
\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)} + \frac{c^2\log\left(|dx+c|\right)}{bc^2d+ad^3} - \frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(abs(d*x + c))/(b*c^2*d
 + a*d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

maple [A] time = 0.01, size = 87, normalized size = 0.91

$$-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(a\,d^2+b\,c^2\right)\sqrt{ab}}+\frac{ad\ln\left(b\,x^2+a\right)}{2\left(a\,d^2+b\,c^2\right)b}+\frac{c^2\ln\left(dx+c\right)}{\left(a\,d^2+b\,c^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/2*a*d*ln(b*x^2+a)/b/(a*d^2+b*c^2)-a/(a*d^2+b*c^2)*c/(a*b)^(1/2)*arctan(1/
(a*b)^(1/2)*b*x)+c^2*ln(d*x+c)/d/(a*d^2+b*c^2)
```

maxima [A] time = 1.22, size = 84, normalized size = 0.88

$$\frac{ad\log\left(bx^2+a\right)}{2\left(b^2c^2+abd^2\right)}+\frac{c^2\log\left(dx+c\right)}{bc^2d+ad^3}-\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\left(bc^2+ad^2\right)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*log(b*x^2 + a)/(b^2*c^2 + a*b*d^2) + c^2*log(d*x + c)/(b*c^2*d + a*
d^3) - a*c*arctan(b*x/sqrt(a*b))/((b*c^2 + a*d^2)*sqrt(a*b))
```

mupad [B] time = 1.13, size = 347, normalized size = 3.61

$$\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\left(x\left(2\,b^2\,c^2-5\,a\,b\,d^2\right)-5\,a\,b\,c\,d+\frac{2\,b^2\,d\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}\right)\left(c\,\sqrt{-a\,b^3}+a\,b\,d\right)}{2\,b^3\,c^2+2\,a\,b^2\,d^2}-\frac{\ln\left(a\,c+a\,d\,x+\frac{\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\left(b\,x\left(5\,a\,d^2-2\,b\,c^2\right)+5\,a\,b\,c\,d+\frac{d\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)\left(-b\,x\,c^2+4\,a\,c\,d+3\,a\,x\,d^2\right)}{b\,c^2+a\,d^2}\right)}{2\,b^2\left(b\,c^2+a\,d^2\right)}\right)\left(c\,\sqrt{-a\,b^3}-a\,b\,d\right)}{2\left(b^3\,c^2+a\,b^2\,d^2\right)}+\frac{c^2\,\ln\left(c+d\,x\right)}{b\,c^2\,d+a\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^2)*(c + d*x)),x)
```

```
[Out] (log(a*c + a*d*x + ((c*(-a*b^3)^(1/2) + a*b*d)*(x*(2*b^2*c^2 - 5*a*b*d^2) -
 5*a*b*c*d + (2*b^2*d*(c*(-a*b^3)^(1/2) + a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c
^2*x))/(2*b^3*c^2 + 2*a*b^2*d^2)))/(2*b^3*c^2 + 2*a*b^2*d^2))*(c*(-a*b^3)^(
1/2) + a*b*d))/(2*b^3*c^2 + 2*a*b^2*d^2) - (log(a*c + a*d*x + ((c*(-a*b^3)^
(1/2) - a*b*d)*(b*x*(5*a*d^2 - 2*b*c^2) + 5*a*b*c*d + (d*(c*(-a*b^3)^(1/2)
- a*b*d)*(4*a*c*d + 3*a*d^2*x - b*c^2*x))/(a*d^2 + b*c^2)))/(2*b^2*(a*d^2 +
 b*c^2)))*(c*(-a*b^3)^(1/2) - a*b*d))/(2*(b^3*c^2 + a*b^2*d^2)) + (c^2*log(
c + d*x))/(a*d^3 + b*c^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**2+a),x)
```

```
[Out] Timed out
```

3.337 $\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=264

$$-\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}{3b^{2/3}\left(bc^3\right.}$$

Rubi [A] time = 0.47, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{2/3}\left(bc^3-ad^3\right)}-\frac{\sqrt[3]{a}\,d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{2/3}\left(a^{2/3}d^2+\sqrt[3]{a}\,\sqrt[3]{b}\,cd+b^{2/3}c^2\right)}+\frac{\sqrt[3]{a}\,d\left(\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^3)),x]
```

```
[Out] -((a^(1/3)*d*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^
(2/3)*(b^(2/3)*c^2 + a^(1/3)*b^(1/3)*c*d + a^(2/3)*d^2))) + (a^(1/3)*d*(b^(
1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(2/3)*(b*c^3 - a*d^3)) -
 (c^2*Log[c + d*x])/(b*c^3 - a*d^3) - (a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(b*c^3 - a*d^3)) +
 (c^2*Log[a + b*x^3])/(3*(b*c^3 - a*d^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx = \int \left(-\frac{c^2 d}{\left(bc^3-ad^3\right)(c+dx)}+\frac{acd-ad^2x+bc^2x^2}{\left(bc^3-ad^3\right)\left(a+bx^3\right)}\right)dx$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{\int \frac{acd-ad^2x+bc^2x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{\int \frac{acd-ad^2x}{a+bx^3}\,dx}{bc^3-ad^3}+\frac{\left(bc^2\right)\int \frac{x^2}{a+bx^3}\,dx}{bc^3-ad^3}$$

$$= -\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}+\frac{\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}cd-a^{4/3}d^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}cd-a^{4/3}d^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}\left(bc^3-ad^3\right)}+\frac{\left(\sqrt[3]{a}\right.}{}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}+\frac{c^2\log\left(a+bx^3\right)}{3\left(bc^3-ad^3\right)}+\frac{\left(a^{2/3}d\right)}{2\sqrt[3]{b}\left(b^{2/3}\right.}$$

$$= \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log(c+dx)}{bc^3-ad^3}-\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(a^{2/3}\right.}{6b^{2/3}\left(bc^3-\right.}$$

$$= -\frac{\sqrt[3]{a}d\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}\left(b^{2/3}c^2+\sqrt[3]{a}\sqrt[3]{b}cd+a^{2/3}d^2\right)}+\frac{\sqrt[3]{a}d\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}\left(bc^3-ad^3\right)}-\frac{c^2\log}{bc^3}$$

Mathematica [A] time = 0.09, size = 228, normalized size = 0.86

$$\frac{-\sqrt[3]{a}\sqrt[3]{b}cd\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-a^{2/3}d^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+2b^{2/3}c^2\log\left(a+bx^3\right)+2\sqrt[3]{a}d\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{a}d-\sqrt[3]{b}c\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-6b^{2/3}c^2\log(c+dx)}{6b^{2/3}\left(bc^3-ad^3\right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((c + d*x)*(a + b*x^3)),x]

[Out] (2*Sqrt[3]*a^(1/3)*d*(-(b^(1/3)*c) + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*d*(b^(1/3)*c + a^(1/3)*d)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(2/3)*c^2*Log[c + d*x] - a^(1/3)*b^(1/3)*c*d*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - a^(2/3)*d^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(2/3)*c^2*Log[a + b*x^3])/(6*b^(2/3)*(b*c^3 - a*d^3))

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+dx)\left(a+bx^3\right)}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^3)),x]

[Out] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^3)), x]

fricas [C] time = 4.87, size = 5975, normalized size = 22.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] -1/12*(2*(b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*log(-3/2*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*b*c^2 - 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + d*x - 2*c) + 12*c^2*log(d*x + c) - ((b*c^3 - a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 - 3*sqrt(1/3)*(b*c^3 - a*d^3)*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)

```
))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
- a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^
3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3
*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2)
+ 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2
 + 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2
- c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/
((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(
b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))*sqrt(-
(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(2*(1/2)^(2
/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6
/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((
b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*
c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c
^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d
^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d^3
)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3 -
a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x + 2*c) - ((b*c^3
- a*d^3)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I
*sqrt(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3
- a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)
+ (1/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3
- a*d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*
(I*sqrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3)) + 6*c^2 + 3*sqrt(1/3)*(b*c^3 - a*d
^3)*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)*(
2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) +
 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3))
+ a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1
/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3))
+ a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)^(2/3)
*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b
*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c
^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2
/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)))*log(3/2*(2*(1/
2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(
2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d
^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1)
 - 2*c^2/(b*c^3 - a*d^3))*b*c^2 + 1/4*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c
^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
- a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^
```

```
3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3
 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b
*c^3 - a*d^3))^2 - 3/4*sqrt(1/3)*(b^2*c^3 - a*b*d^3)*(2*(1/2)^(2/3)*(c^4/(b
*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*c^6/(b*c^3 - a
*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3/((b*c^3 - a*d
^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) - 2*c^2/(b*c^3
 - a*d^3))*sqrt(-(4*b*c^4 - 16*a*c*d^3 + (b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*
d^6)*(2*(1/2)^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqr
t(3) + 1)/(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1
/2)^(1/3)*(2*c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*
d^3)) + a*d^3/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*s
qrt(3) + 1) - 2*c^2/(b*c^3 - a*d^3))^2 + 4*(b^2*c^5 - a*b*c^2*d^3)*(2*(1/2)
^(2/3)*(c^4/(b*c^3 - a*d^3)^2 - c/(b^2*c^3 - a*b*d^3))*(-I*sqrt(3) + 1)/(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3) + (1/2)^(1/3)*(2*
c^6/(b*c^3 - a*d^3)^3 - 3*c^3/((b^2*c^3 - a*b*d^3)*(b*c^3 - a*d^3)) + a*d^3
/((b*c^3 - a*d^3)^2*b^2) + 1/(b^3*c^3 - a*b^2*d^3))^(1/3)*(I*sqrt(3) + 1) -
 2*c^2/(b*c^3 - a*d^3)))/(b^3*c^6 - 2*a*b^2*c^3*d^3 + a^2*b*d^6)) + 2*d*x +
 2*c))/(b*c^3 - a*d^3)
```

giac [A] time = 0.30, size = 320, normalized size = 1.21

$$-\frac{c^{2} d \log\left(\left|dx+c\right|\right)}{bc^{3} d-ad^{4}}+\frac{c^{2} \log\left(\left|bx^{3}+a\right|\right)}{3\left(bc^{3}-ad^{3}\right)}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}} d \arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3} b^{2} c^{2}-\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}} bcd+\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}} d^{2}}+\frac{\left(ab^{2} c^{3} d^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^{2} b d^{5}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2} c^{4} d+a^{2} bcd^{4}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^{3} c^{6}-2 a^{2} b^{2} c^{3} d^{3}+a^{3} b d^{6}\right)}+\frac{\left(\left(-ab^{2}\right)^{\frac{1}{3}} bcd-\left(-ab^{2}\right)^{\frac{2}{3}} d^{2}\right) \log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3} c^{3}-ab^{2} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -c^2*d*log(abs(d*x + c))/(b*c^3*d - a*d^4) + 1/3*c^2*log(abs(b*x^3 + a))/(b
*c^3 - a*d^3) + (-a*b^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(sqrt(3)*b^2*c^2 - sqrt(3)*(-a*b^2)^(1/3)*b*c*d + sqrt(3)*(-a*b
^2)^(2/3)*d^2) + 1/3*(a*b^2*c^3*d^2*(-a/b)^(1/3) - a^2*b*d^5*(-a/b)^(1/3) -
 a*b^2*c^4*d + a^2*b*c*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*
c^6 - 2*a^2*b^2*c^3*d^3 + a^3*b*d^6) + 1/6*((-a*b^2)^(1/3)*b*c*d - (-a*b^2)
^(2/3)*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c^3 - a*b^2*d^3)
```

maple [A] time = 0.01, size = 336, normalized size = 1.27

$$-\frac{\sqrt{3}\, acd \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}-\frac{acd \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{acd \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b}+\frac{\sqrt{3}\, a d^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{a d^{2} \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}+\frac{a d^{2} \ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a d^{3}-b c^{3}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} b}-\frac{c^{2} \ln\left(b x^{3}+a\right)}{3\left(a d^{3}-b c^{3}\right)}+\frac{c^{2} \ln\left(dx+c\right)}{a d^{3}-b c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/3/(a*d^3-b*c^3)*a*c*d/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*
a*c*d/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d^3-b*c^3)*a*c
*d/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3
-b*c^3)*a*d^2/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/(a*d^3-b*c^3)*a*d^2/b/(a/
b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/(a*d^3-b*c^3)*a*d^2*3^(1/2)/
b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d^3-b*c^3)*c^2
*ln(b*x^3+a)+c^2/(a*d^3-b*c^3)*ln(d*x+c)
```

maxima [A] time = 1.71, size = 279, normalized size = 1.06

$$-\frac{c^2\log(dx+c)}{bc^3-ad^3}-\frac{\sqrt{3}\left(ad^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-acd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(2bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-acd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\left(bc^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ad^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+acd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -c^2*log(d*x + c)/(b*c^3 - a*d^3) - 1/3*sqrt(3)*(a*d^2*(a/b)^(2/3) - a*c*d*
(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^3*
(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3))*(a/b)^(1/3)) + 1/6*(2*b*c^2*(a/b)^(2/3)
- a*d^2*(a/b)^(1/3) - a*c*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^
3*(a/b)^(2/3) - a*b*d^3*(a/b)^(2/3)) + 1/3*(b*c^2*(a/b)^(2/3) + a*d^2*(a/b)
^(1/3) + a*c*d)*log(x + (a/b)^(1/3))/(b^2*c^3*(a/b)^(2/3) - a*b*d^3*(a/b)^(
2/3))
```

mupad [B] time = 2.50, size = 570, normalized size = 2.16

$$\left(\sum_{k=1}^{3}\ln\left(-a\,b\,d\left(c+d\,x+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2\,b^2\,c^3\,3+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^3\,b^3\,c^4\,9-\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\,b\,c^2\,5-\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2\,a\,b\,d^3\,3-\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\,b\,c\,d\,x\,8+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^3\,a\,b^2\,c\,d^3\,45+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^3\,a\,b^2\,d^4\,x\,36+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^2\,b^2\,c^2\,d\,x\,9+\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)^3\,b^3\,c^3\,d\,x\,18\right)\right)\,\mathrm{root}\left(27\,a\,b^2\,d^3\,z^3-27\,b^3\,c^3\,z^3+27\,b^2\,c^2\,z^2-9\,b\,c\,z+1,z,k\right)\right)+\frac{c^2\,\ln\left(c+d\,x\right)}{a\,d^3-b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x)),x)
```

```
[Out] symsum(log(-a*b*d*(c + d*x + 3*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2*b^2*c^3 + 9*root(27*a*b^2*d^3*z^3 - 27*b
^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*b^3*c^4 - 5*root(27*a*b^
2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)*b*c^2 - 3*
root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k
)^2*a*b*d^3 - 8*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9
*b*c*z + 1, z, k)*b*c*d*x + 45*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*
b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*c*d^3 + 36*root(27*a*b^2*d^3*z^3 -
 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^3*a*b^2*d^4*x + 9*roo
t(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k)^2
*b^2*c^2*d*x + 18*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3 + 27*b^2*c^2*z^2 -
 9*b*c*z + 1, z, k)^3*b^3*c^3*d*x))*root(27*a*b^2*d^3*z^3 - 27*b^3*c^3*z^3
+ 27*b^2*c^2*z^2 - 9*b*c*z + 1, z, k), k, 1, 3) + (c^2*log(c + d*x))/(a*d^3
 - b*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.338 $\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=417

$$\frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^4+bc^4\right)}-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}+\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

Rubi [A] time = 0.55, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6725, 1461, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^2d\log\left(a+bx^4\right)}{4\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}-\frac{c\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}+\frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\left(ad^4+bc^4\right)}+\frac{c^2d\log(c+dx)}{ad^4+bc^4}-\frac{c\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}+\frac{c\left(\sqrt{b}\,c^2-\sqrt{a}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\left(ad^4+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (Sqrt[a]*d^3*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c^4 + a*d^4)) - (
c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(1/4)*b^(1/4)*(b*c^4 + a*d^4)) + (c*(Sqrt[b]*c^2 - Sqrt[a]*d^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c^4 +
a*d^4)) + (c^2*d*Log[c + d*x])/(b*c^4 + a*d^4) + (c*(Sqrt[b]*c^2 + Sqrt[a]*
d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*b^(1/4)*(b*c^4 + a*d^4)) - (c*(Sqrt[b]*c^2 + Sqrt[a]*d^2)*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(1/4)*(b*c
^4 + a*d^4)) - (c^2*d*Log[a + b*x^4])/(4*(b*c^4 + a*d^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1461

```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Dist[A, Int[(d + e*x^n)^q*(a + c*x^(2*n))^p
, x], x] + Dist[B, Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ
[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx = \int \left(\frac{c^2d^2}{\left(bc^4+ad^4\right)(c+dx)} + \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{\left(bc^4+ad^4\right)\left(a+bx^4\right)}\right) dx$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\int \frac{(c-dx)\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{c\int \frac{-ad^2+bc^2x^2}{a+bx^4}\,dx}{bc^4+ad^4} - \frac{d\int \frac{x\left(-ad^2+bc^2x^2\right)}{a+bx^4}\,dx}{bc^4+ad^4}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{d\,\mathrm{Subst}\left(\int \frac{-ad^2+bc^2x}{a+bx^2}\,dx,x,x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\right)\int \frac{\sqrt{a}\,\sqrt{b}+bx^2}{a+bx^4}\,dx}{2\left(bc^4+ad^4\right)}$$

$$= \frac{c^2d\log(c+dx)}{bc^4+ad^4} - \frac{\left(bc^2d\right)\mathrm{Subst}\left(\int \frac{x}{a+bx^2}\,dx,x,x^2\right)}{2\left(bc^4+ad^4\right)} + \frac{\left(ad^3\right)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2\left(bc^4+ad^4\right)}$$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,\left(bc^4+ad^4\right)} + \frac{c^2d\log(c+dx)}{bc^4+ad^4} + \frac{\sqrt[4]{b}\,c\left(c^2+\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\right.}{4\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4+ad^4\right)}$$

$$= \frac{\sqrt{a}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}\,\left(bc^4+ad^4\right)} - \frac{\sqrt[4]{b}\,c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4+ad^4\right)} + \frac{\sqrt[4]{b}\,c\left(c^2-\frac{\sqrt{a}\,d^2}{\sqrt{b}}\right)\tan^{-1}}{2\sqrt{2}\,\sqrt[4]{a}\,\left(bc^4\right.}$$

Mathematica [A] time = 0.23, size = 370, normalized size = 0.89

$$\frac{-2\left(2a^{3/4}d^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2+\sqrt{2}\,b^{3/4}c^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(-2a^{3/4}d^3-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{b}\,cd^2+\sqrt{2}\,b^{3/4}c^3\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)+\sqrt[4]{b}\,c\left(\sqrt{2}\,\left(\sqrt{a}\,d^2+\sqrt{b}\,c^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-\sqrt{2}\,\sqrt{b}\,c^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-2\sqrt[4]{a}\,\sqrt[4]{b}\,cd\log\left(a+bx^4\right)+8\sqrt[4]{a}\,\sqrt[4]{b}\,cd\log(c+dx)-\sqrt{2}\,\sqrt{a}\,d^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)\right)}{8\sqrt[4]{a}\,\sqrt{b}\,\left(ad^4+bc^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] (-2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*Sqrt[a]*b^(1/4)*c*d^2 + 2*a^(3/4)*d^3)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*b^(3/4)*c^3 - Sqrt[2]*S
qrt[a]*b^(1/4)*c*d^2 - 2*a^(3/4)*d^3)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + b^(1/4)*c*(8*a^(1/4)*b^(1/4)*c*d*Log[c + d*x] + Sqrt[2]*(Sqrt[b]*c^2 +
 Sqrt[a]*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt
[2]*Sqrt[b]*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sq
rt[2]*Sqrt[a]*d^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] -
2*a^(1/4)*b^(1/4)*c*d*Log[a + b*x^4]))/(8*a^(1/4)*Sqrt[b]*(b*c^4 + a*d^4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(c+dx)\left(a+bx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^4)),x]
```

```
[Out] IntegrateAlgebraic[x^2/((c + d*x)*(a + b*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A]   time = 0.57, size = 401, normalized size = 0.96

$$\frac{c^2d^2\log\left(|dx+c|\right)}{bc^4d+ad^5}-\frac{c^2d\log\left(\left|bx^4+a\right|\right)}{4\left(bc^4+ad^4\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2d^2+\sqrt{2}\sqrt{ab}\,b^2c^2-2\left(ab^3\right)^{\frac{3}{4}}cd\right)}+\frac{\left(\sqrt{2}\sqrt{ab}\,bd+\left(ab^3\right)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^2d^2+\sqrt{2}\sqrt{ab}\,b^2c^2+2\left(ab^3\right)^{\frac{3}{4}}cd\right)}-\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}+\frac{\left(\left(ab^3\right)^{\frac{1}{4}}abcd^2+\left(ab^3\right)^{\frac{3}{4}}c^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c^4+\sqrt{2}a^2b^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] c^2*d^2*log(abs(d*x + c))/(b*c^4*d + a*d^5) - 1/4*c^2*d*log(abs(b*x^4 + a))
/(b*c^4 + a*d^4) + 1/2*(sqrt(2)*sqrt(a*b)*b*d + (a*b^3)^(1/4)*b*c)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*d^2 + sq
rt(2)*sqrt(a*b)*b^2*c^2 - 2*(a*b^3)^(3/4)*c*d) + 1/2*(sqrt(2)*sqrt(a*b)*b*d
 + (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(sqrt(2)*a*b^2*d^2 + sqrt(2)*sqrt(a*b)*b^2*c^2 + 2*(a*b^3)^(3/4)*c*d
) - 1/4*((a*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4) + 1/4*((a
*b^3)^(1/4)*a*b*c*d^2 + (a*b^3)^(3/4)*c^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
+ sqrt(a/b))/(sqrt(2)*a*b^3*c^4 + sqrt(2)*a^2*b^2*d^4)
```

maple [A]   time = 0.01, size = 422, normalized size = 1.01

$$\frac{a\,d^3\arctan\left(\sqrt{\frac{b}{a}}\,x^2\right)}{2\left(a\,d^4+b\,c^4\right)\sqrt{ab}}+\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,c^3\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(a\,d^4+b\,c^4\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c^2d\ln\left(b\,x^4+a\right)}{4\left(a\,d^4+b\,c^4\right)}+\frac{c^2d\ln\left(dx+c\right)}{a\,d^4+b\,c^4}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,d^4+b\,c^4\right)}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,d^4+b\,c^4\right)}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(a\,d^4+b\,c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x+c)/(b*x^4+a),x)
```

```
[Out] c^2*d*ln(d*x+c)/(a*d^4+b*c^4)-1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/8/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*
ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2)))-1/4/(a*d^4+b*c^4)*c*d^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+1/2/(a*d^4+b*c^4)*a*d^3/(a*b)^(1/2)*arctan(x^2*(b/a)^(1/2))+1/8/(
a*d^4+b*c^4)*c^3/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a*d^4+b*c^4)*c^3/(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-1/4*c^2*d*ln(b*x^4+a)/(a*d^4+b*
c^4)
```

maxima [A]   time = 1.48, size = 349, normalized size = 0.84

$$\frac{c^2d\log\left(dx+c\right)}{bc^4+ad^4}-\frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d+\sqrt{a}\,b^{\frac{3}{2}}c^3+abcd^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c^2d-\sqrt{a}\,b^{\frac{3}{2}}c^3-abcd^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}c^3-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}cd^2-2a^{\frac{3}{2}}bd^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{7}{4}}c^3-\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}cd^2+2a^{\frac{3}{2}}bd^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}\,b^{\frac{5}{4}}}}{8\left(bc^4+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] c^2*d*log(d*x + c)/(b*c^4 + a*d^4) - 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c^2*d + sqrt(a)*b^(3/2)*c^3 + a*b*c*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*
b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c
^2*d - sqrt(a)*b^(3/2)*c^3 - a*b*c*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sq
rt(2)*a^(5/4)*b^(5/4)*c*d^2 - 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)
```

```
*sqrt(b))*b^(5/4)) - 2*(sqrt(2)*a^(3/4)*b^(7/4)*c^3 - sqrt(2)*a^(5/4)*b^(5/
4)*c*d^2 + 2*a^(3/2)*b*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))
/(b*c^4 + a*d^4)
```

mupad [B] time = 2.45, size = 823, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x)),x)
```

```
[Out] symsum(log(a*b^2*d*(c*d + d^2*x - root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*
z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^3 + 4*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^2*b^2*c^4*x + 36*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 25
6*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*d^4*x - 128*root(256*a^
2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 +
1, z, k)^4*a*b^3*c^5*d - 5*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 2
56*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)*b*c^2*d*x + 96*root(256*a^2*
b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1,
 z, k)^3*a*b^2*c^3*d^2 + 384*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 +
 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a^2*b^2*c*d^5 + 320*root
(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2
*z^2 + 1, z, k)^4*a^2*b^2*d^6*x + 32*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c
^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^2*a*b*c*d^3 + 160*
root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b
*d^2*z^2 + 1, z, k)^3*a*b^2*c^2*d^3*x - 192*root(256*a^2*b^2*d^4*z^4 + 256*
a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3 + 32*a*b*d^2*z^2 + 1, z, k)^4*a*b^3*c^4
*d^2*x))*root(256*a^2*b^2*d^4*z^4 + 256*a*b^3*c^4*z^4 + 256*a*b^2*c^2*d*z^3
 + 32*a*b*d^2*z^2 + 1, z, k), k, 1, 4) + (c^2*d*log(c + d*x))/(a*d^4 + b*c^
4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x+c)/(b*x**4+a),x)
```

```
[Out] Timed out
```

3.339 $\int \frac{x}{(1-x)(1+x)^2}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {77, 207}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTanh[x]/2
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1-x)(1+x)^2}\,dx &= \int \left(-\frac{1}{2(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right)dx\\
&= \frac{1}{2(1+x)}-\frac{1}{2}\int \frac{1}{-1+x^2}\,dx\\
&= \frac{1}{2(1+x)}+\frac{1}{2}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.50

$$\frac{1}{4}\left(\frac{2}{x+1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] (2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-x)(1+x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((1 - x)*(1 + x)^2),x]
```

```
[Out] IntegrateAlgebraic[x/((1 - x)*(1 + x)^2), x]
```

fricas [B] time = 1.30, size = 26, normalized size = 1.62

$$\frac{(x+1)\log(x+1)-(x+1)\log(x-1)+2}{4\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

giac [A] time = 0.24, size = 21, normalized size = 1.31

$$\frac{1}{2\,(x+1)}-\frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

maple [A] time = 0.01, size = 21, normalized size = 1.31

$$-\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x)/(x+1)^2,x)
```

```
[Out] -1/4*ln(x-1)+1/2/(x+1)+1/4*ln(x+1)
```

maxima [A] time = 0.75, size = 20, normalized size = 1.25

$$\frac{1}{2\,(x+1)}+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.75

$$\frac{\operatorname{atanh}(x)}{2}+\frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x - 1)*(x + 1)^2),x)
```

```
[Out] atanh(x)/2 + 1/(2*(x + 1))
```

sympy [A] time = 0.10, size = 19, normalized size = 1.19

$$-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x)/(1+x)**2,x)
```

```
[Out] -log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

3.340 $\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {471, 206}

$$\frac{1}{4}\tanh^{-1}(x) - \frac{x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
[Out] -x/(4*(1 + x^2)) + ArcTanh[x]/4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx &= -\frac{x}{4(1+x^2)} + \frac{1}{4}\int \frac{1}{1-x^2} \, dx \\
&= -\frac{x}{4(1+x^2)} + \frac{1}{4}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.42

$$\frac{1}{8}\left(-\frac{2x}{x^2+1} - \log(1-x) + \log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
[Out] ((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((1 - x^2)*(1 + x^2)^2),x]
[Out] IntegrateAlgebraic[x^2/((1 - x^2)*(1 + x^2)^2), x]
```

fricas [B] time = 0.65, size = 34, normalized size = 1.79

$$\frac{(x^2+1)\log(x+1) - (x^2+1)\log(x-1) - 2x}{8(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

giac [A] time = 0.30, size = 30, normalized size = 1.58

$$-\frac{1}{4\left(x+\frac{1}{x}\right)} + \frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right) - \frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

maple [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{x}{4(x^2+1)} - \frac{\ln(x-1)}{8} + \frac{\ln(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^2+1)^2,x)
[Out] -1/8*ln(x-1)-1/4/(x^2+1)*x+1/8*ln(x+1)
```

maxima [A] time = 0.65, size = 23, normalized size = 1.21

$$-\frac{x}{4(x^2+1)} + \frac{1}{8}\log(x+1) - \frac{1}{8}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

mupad [B] time = 2.14, size = 17, normalized size = 0.89

$$\frac{\mathrm{atanh}(x)}{4} - \frac{x}{4\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
```

```
[Out] atanh(x)/4 - x/(4*(x^2 + 1))
```

sympy **[A]** time = 0.12, size = 20, normalized size = 1.05

$$-\frac{x}{4x^2+4}-\frac{\log(x-1)}{8}+\frac{\log(x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

```
[Out] -x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

3.341 $\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {471, 522, 200, 31, 634, 618, 204, 628}

$$-\frac{x}{6(x^3+1)}+\frac{1}{72}\log\left(x^2-x+1\right)+\frac{1}{24}\log\left(x^2+x+1\right)-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] -x/(6*(1 + x^3)) + ArcTan[(1 - 2*x)/Sqrt[3]]/(12*Sqrt[3]) + ArcTan[(1 + 2*x
)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x]/12 - Log[1 + x]/36 + Log[1 - x + x^2]/7
2 + Log[1 + x + x^2]/24
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx &= -\frac{x}{6(1+x^3)}+\frac{1}{6}\int \frac{1+2x^3}{(1-x^3)(1+x^3)}\,dx\\
&= -\frac{x}{6(1+x^3)}-\frac{1}{12}\int \frac{1}{1+x^3}\,dx+\frac{1}{4}\int \frac{1}{1-x^3}\,dx\\
&= -\frac{x}{6(1+x^3)}-\frac{1}{36}\int \frac{1}{1+x}\,dx-\frac{1}{36}\int \frac{2-x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{1-x}\,dx+\frac{1}{12}\int \frac{2\cdot}{1+x}\\
&= -\frac{x}{6(1+x^3)}-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(1+x)+\frac{1}{72}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{24}\int \frac{1}{1-x+}\\
&= -\frac{x}{6(1+x^3)}-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(1+x)+\frac{1}{72}\log(1-x+x^2)+\frac{1}{24}\log(1+x\\
&= -\frac{x}{6(1+x^3)}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log(1-x)-\frac{1}{36}\log(1+x)+\frac{1}{72}\mathrm{l}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 0.88

$$\frac{1}{72}\left(-\frac{12x}{x^3+1}+\log(x^2-x+1)+3\log(x^2+x+1)-6\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] ((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcTa
n[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3*L
og[1 + x + x^2])/72
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[x^3/((1 - x^3)*(1 + x^3)^2), x]
```

fricas [A] time = 1.19, size = 106, normalized size = 1.09

$$\frac{6\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-2\sqrt{3}\left(x^3+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+3\left(x^3+1\right)\log\left(x^2+x+1\right)+\left(x^3+1\right)\log\left(x^2-x+1\right)-2\left(x^3+1\right)\log\left(x+1\right)-6\left(x^3+1\right)\log\left(x-1\right)-12x}{72\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 + 1)
*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12*x)
/(x^3 + 1)
```

giac [A] time = 0.31, size = 77, normalized size = 0.79

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-x+1\right)-\frac{1}{36}\log\left(|x+1|\right)-\frac{1}{12}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

maple [A] time = 0.01, size = 90, normalized size = 0.93

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(x-1\right)}{12}-\frac{\ln\left(x+1\right)}{36}+\frac{\ln\left(x^2-x+1\right)}{72}+\frac{\ln\left(x^2+x+1\right)}{24}+\frac{-2x-2}{36x^2-36x+36}+\frac{1}{18x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)/(x^3+1)^2,x)
```

```
[Out] -1/12*ln(x-1)+1/36*(-2*x-2)/(x^2-x+1)+1/72*ln(x^2-x+1)-1/36*3^(1/2)*arctan(
1/3*(2*x-1)*3^(1/2))+1/18/(x+1)-1/36*ln(x+1)+1/24*ln(x^2+x+1)+1/12*3^(1/2)*
arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 1.93, size = 75, normalized size = 0.77

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)-\frac{x}{6\left(x^3+1\right)}+\frac{1}{24}\log\left(x^2+x+1\right)+\frac{1}{72}\log\left(x^2-x+1\right)-\frac{1}{36}\log\left(x+1\right)-\frac{1}{12}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x +
 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

mupad [B] time = 0.18, size = 103, normalized size = 1.06

$$-\frac{\ln\left(x-1\right)}{12}-\frac{\ln\left(x+1\right)}{36}-\frac{x}{6\left(x^3+1\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1i}{24}\right)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{72}+\frac{\sqrt{3}\,1i}{72}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{72}+\frac{\sqrt{3}\,1i}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

[Out] log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x/(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) - log(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)

sympy [A] time = 0.36, size = 92, normalized size = 0.95

$$-\frac{x}{6x^3+6}-\frac{\log(x-1)}{12}-\frac{\log(x+1)}{36}+\frac{\log(x^2-x+1)}{72}+\frac{\log(x^2+x+1)}{24}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{36}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(-x**3+1)/(x**3+1)**2,x)

[Out] -x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + log(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/12

3.342 $\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{9+x+3x^2+x^3}{\left(1+x^2\right)\left(3+x^2\right)}\,dx &= \int \left(\frac{3}{1+x^2}+\frac{x}{3+x^2}\right)dx\\ &= 3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= 3\tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+3\right)+3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] 3*ArcTan[x] + Log[3 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{9+x+3x^2+x^3}{(1+x^2)(3+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] IntegrateAlgebraic[(9 + x + 3*x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]
```

fricas [A] time = 1.06, size = 13, normalized size = 0.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

giac [A] time = 0.29, size = 13, normalized size = 0.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$3\arctan(x)+\frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x)
[Out] 3*arctan(x)+1/2*ln(x^2+3)
```

maxima [A] time = 1.40, size = 13, normalized size = 0.87

$$3\arctan(x)+\frac{1}{2}\log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x^2+x+9)/(x^2+1)/(x^2+3),x, algorithm="maxima")
[Out] 3*arctan(x) + 1/2*log(x^2 + 3)
```

mupad [B] time = 2.12, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^2+3\right)}{2}+3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + x^3 + 9)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + 3*atan(x)
```

sympy [A] time = 0.12, size = 12, normalized size = 0.80

$$\frac{\log\left(x^2+3\right)}{2}+3\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x**2+x+9)/(x**2+1)/(x**2+3),x)
```

```
[Out] log(x**2 + 3)/2 + 3*atan(x)
```

3.343 $\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} \, dx$

Optimal. Leaf size=13

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6725, 203, 260}

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x+x^2+x^3}{\left(1+x^2\right)\left(3+x^2\right)}\,dx &= \int \left(\frac{1}{1+x^2}+\frac{x}{3+x^2}\right)dx\\ &= \int \frac{1}{1+x^2}\,dx+\int \frac{x}{3+x^2}\,dx\\ &= \tan^{-1}(x)+\frac{1}{2}\log\left(3+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+3\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
```

```
[Out] ArcTan[x] + Log[3 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+x+x^2+x^3}{(1+x^2)(3+x^2)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)),x]
[Out] IntegrateAlgebraic[(3 + x + x^2 + x^3)/((1 + x^2)*(3 + x^2)), x]
```

fricas [A] time = 1.05, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="fricas")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

giac [A] time = 0.30, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="giac")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\arctan(x) + \frac{\ln\left(x^2+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x)
[Out] arctan(x)+1/2*ln(x^2+3)
```

maxima [A] time = 1.33, size = 11, normalized size = 0.85

$$\arctan(x) + \frac{1}{2} \log\left(x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+x+3)/(x^2+1)/(x^2+3),x, algorithm="maxima")
[Out] arctan(x) + 1/2*log(x^2 + 3)
```

mupad [B] time = 0.04, size = 11, normalized size = 0.85

$$\frac{\ln\left(x^2+3\right)}{2} + \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + x^3 + 3)/((x^2 + 1)*(x^2 + 3)),x)
```

```
[Out] log(x^2 + 3)/2 + atan(x)
```

sympy [**A**] time = 0.12, size = 10, normalized size = 0.77

$$\frac{\log\left(x^2+3\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+x+3)/(x**2+1)/(x**2+3),x)
[Out] log(x**2 + 3)/2 + atan(x)
```

3.344 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Rubi [A] time = 0.12, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+6x-x^2+3x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\
&= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\
&= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\
&= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{-4+6x-x^2+3x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
[Out] IntegrateAlgebraic[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)), x]
```

fricas [A] time = 1.23, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

giac [A] time = 0.26, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 25, normalized size = 0.86

$$-3\arctan(x)+\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{3\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
[Out] -3*arctan(x)+3/2*ln(x^2+1)+2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.48, size = 24, normalized size = 0.83

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-3\arctan(x)+\frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

mupad [B] time = 2.15, size = 51, normalized size = 1.76

$$-\sqrt{2}\,\mathrm{atan}\left(\frac{24\sqrt{2}}{24\,x-64}+\frac{32\sqrt{2}\,x}{24\,x-64}\right)+\ln\left(x-\mathrm{i}\right)\,\left(\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x - x^2 + 3*x^3 - 4)/((x^2 + 1)*(x^2 + 2)),x)
```

```
[Out] log(x - 1i)*(3/2 + 3i/2) + log(x + 1i)*(3/2 - 3i/2) - 2^(1/2)*atan((24*2^(1
/2))/(24*x - 64) + (32*2^(1/2)*x)/(24*x - 64))
```

sympy [A] time = 0.19, size = 29, normalized size = 1.00

$$\frac{3\log\left(x^2+1\right)}{2}-3\,\mathrm{atan}\left(x\right)+\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

3.345 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)}\,dx$

Optimal. Leaf size=14

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x}+\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(4-4x+x^2\right)\left(5-4x+x^2\right)}\,dx &= \int \frac{1}{(-2+x)^2\left(5-4x+x^2\right)}\,dx\\
&= \frac{1}{2-x}-\int \frac{1}{5-4x+x^2}\,dx\\
&= \frac{1}{2-x}+2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-4+2x\right)\\
&= \frac{1}{2-x}+\tan^{-1}(2-x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(2-x)-\frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(4-4x+x^2\right)\left(5-4x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
[Out] IntegrateAlgebraic[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)), x]
```

fricas [A] time = 1.15, size = 17, normalized size = 1.21

$$-\frac{(x-2)\arctan(x-2)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
[Out] -1/(x - 2) - arctan(x - 2)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$-\arctan(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
[Out] -arctan(x-2)-1/(x-2)
```

maxima [A] time = 2.07, size = 14, normalized size = 1.00

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
[Out] -1/(x - 2) - arctan(x - 2)
```

mupad [B] time = 0.04, size = 14, normalized size = 1.00

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 4*x + 4)*(x^2 - 4*x + 5)),x)
[Out] - atan(x - 2) - 1/(x - 2)
```

sympy [A] time = 0.14, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
[Out] -atan(x - 2) - 1/(x - 2)
```

3.346 $\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx$

Optimal. Leaf size=12

$$\log(3-x)-\frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {893}

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx &= \int \left(\frac{1}{-3+x}+\frac{1}{x^2}\right)dx\\ &= -\frac{1}{x}+\log(3-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(3-x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] -x^(-1) + Log[3 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+x+x^2}{(-3+x)x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x + x^2)/((-3 + x)*x^2),x]
```

```
[Out] IntegrateAlgebraic[(-3 + x + x^2)/((-3 + x)*x^2), x]
```

fricas [A] time = 1.23, size = 12, normalized size = 1.00

$$\frac{x\log(x-3)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="fricas")
[Out] (x*log(x - 3) - 1)/x
```

giac [A] time = 0.42, size = 11, normalized size = 0.92

$$-\frac{1}{x} + \log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="giac")
[Out] -1/x + log(abs(x - 3))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\ln\left(x-3\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-3)/(x-3)/x^2,x)
[Out] -1/x+ln(x-3)
```

maxima [A] time = 0.75, size = 10, normalized size = 0.83

$$-\frac{1}{x} + \log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-3)/(-3+x)/x^2,x, algorithm="maxima")
[Out] -1/x + log(x - 3)
```

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$\ln\left(x-3\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 3)/(x^2*(x - 3)),x)
[Out] log(x - 3) - 1/x
```

sympy [A] time = 0.09, size = 7, normalized size = 0.58

$$\log\left(x-3\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-3)/(-3+x)/x**2,x)
[Out] log(x - 3) - 1/x
```

3.347 $\int \frac{1+x+4x^2}{x+4x^3}\,dx$

Optimal. Leaf size=11

$$\log(x)+\frac{1}{2}\tan^{-1}(2x)$$

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1593, 1802, 203}

$$\log(x)+\frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+4x^2}{x+4x^3}\,dx&=\int \frac{1+x+4x^2}{x\left(1+4x^2\right)}\,dx\\&=\int \left(\frac{1}{x}+\frac{1}{1+4x^2}\right)dx\\&=\log(x)+\int \frac{1}{1+4x^2}\,dx\\&=\frac{1}{2}\tan^{-1}(2x)+\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x)+\frac{1}{2}\tan^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

```
[Out] ArcTan[2*x]/2 + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + 4*x^2)/(x + 4*x^3),x]
[Out] IntegrateAlgebraic[(1 + x + 4*x^2)/(x + 4*x^3), x]
```

fricas [A] time = 1.13, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan(2x)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
[Out] 1/2*arctan(2*x) + log(x)
```

giac [A] time = 0.29, size = 10, normalized size = 0.91

$$\frac{1}{2}\arctan(2x)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
[Out] 1/2*arctan(2*x) + log(abs(x))
```

maple [A] time = 0.01, size = 10, normalized size = 0.91

$$\frac{\arctan(2x)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(4*x^3+x),x)
[Out] 1/2*arctan(2*x)+ln(x)
```

maxima [A] time = 1.61, size = 9, normalized size = 0.82

$$\frac{1}{2}\arctan(2x)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
[Out] 1/2*arctan(2*x) + log(x)
```

mupad [B] time = 2.17, size = 17, normalized size = 1.55

$$\ln(x)-\frac{\mathrm{atan}\left(\frac{17}{32\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4*x^2 + 1)/(x + 4*x^3),x)
```

```
[Out] log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

sympy [A] time = 0.13, size = 8, normalized size = 0.73

$$\log(x) + \frac{\operatorname{atan}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(4*x**3+x),x)
```

```
[Out] log(x) + atan(2*x)/2
```

3.348 $\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$

Optimal. Leaf size=12

$$\frac{1}{x}+3\log(1-x)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+3x^2}{-x^2+x^3}\,dx &= \int \frac{1-x+3x^2}{(-1+x)x^2}\,dx\\ &= \int \left(\frac{3}{-1+x}-\frac{1}{x^2}\right)dx\\ &= \frac{1}{x}+3\log(1-x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{1}{x}+3\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + 3*Log[1 - x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x + 3*x^2)/(-x^2 + x^3),x]
[Out] IntegrateAlgebraic[(1 - x + 3*x^2)/(-x^2 + x^3), x]
```

fricas [A] time = 1.14, size = 13, normalized size = 1.08

$$\frac{3\,x\log\left(x-1\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (3*x*log(x - 1) + 1)/x
```

giac [A] time = 0.38, size = 11, normalized size = 0.92

$$\frac{1}{x}+3\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
[Out] 1/x + 3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$3\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+1)/(x^3-x^2),x)
[Out] 3*ln(x-1)+1/x
```

maxima [A] time = 0.71, size = 10, normalized size = 0.83

$$\frac{1}{x}+3\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
[Out] 1/x + 3*log(x - 1)
```

mupad [B] time = 0.04, size = 10, normalized size = 0.83

$$3\,\ln\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
[Out] 3*log(x - 1) + 1/x
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$3\log\left(x-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+1)/(x**3-x**2),x)
[Out] 3*log(x - 1) + 1/x
```

3.349 $\int \frac{4+3x+x^2}{x+x^2} \, dx$

Optimal. Leaf size=12

$$x + 4\log(x) - 2\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1593, 893}

$$x + 4\log(x) - 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+3x+x^2}{x+x^2} \, dx &= \int \frac{4+3x+x^2}{x(1+x)} \, dx \\
&= \int \left(1 + \frac{4}{x} - \frac{2}{1+x}\right) dx \\
&= x + 4\log(x) - 2\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + 4\log(x) - 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] x + 4*Log[x] - 2*Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+3x+x^2}{x+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 3*x + x^2)/(x + x^2),x]
```

```
[Out] IntegrateAlgebraic[(4 + 3*x + x^2)/(x + x^2), x]
```

fricas [A] time = 0.93, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
[Out] x - 2*log(x + 1) + 4*log(x)
```

giac [A] time = 0.23, size = 14, normalized size = 1.17

$$x - 2 \log(|x + 1|) + 4 \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
[Out] x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$x + 4 \ln(x) - 2 \ln(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x^2+x),x)
[Out] x+4*ln(x)-2*ln(x+1)
```

maxima [A] time = 0.60, size = 12, normalized size = 1.00

$$x - 2 \log(x + 1) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
[Out] x - 2*log(x + 1) + 4*log(x)
```

mupad [B] time = 0.05, size = 12, normalized size = 1.00

$$x - 2 \ln(x + 1) + 4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/(x + x^2),x)
[Out] x - 2*log(x + 1) + 4*log(x)
```

sympy [A] time = 0.10, size = 12, normalized size = 1.00

$$x + 4 \log(x) - 2 \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(x**2+x),x)
[Out] x + 4*log(x) - 2*log(x + 1)
```

3.350 $\int \frac{4+x+3x^2}{x+x^3} dx$

Optimal. Leaf size=17

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1593, 1802, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + 3*x^2)/(x + x^3),x]
```

```
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+x+3x^2}{x+x^3}\,dx &= \int \frac{4+x+3x^2}{x(1+x^2)}\,dx\\ &= \int \left(\frac{4}{x}+\frac{1-x}{1+x^2}\right)dx\\ &= 4\log(x)+\int \frac{1-x}{1+x^2}\,dx\\ &= 4\log(x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \tan^{-1}(x)+4\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + 3*x^2)/(x + x^3),x]
[Out] ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+x+3x^2}{x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x + 3*x^2)/(x + x^3),x]
[Out] IntegrateAlgebraic[(4 + x + 3*x^2)/(x + x^3), x]
```

fricas [A] time = 1.26, size = 15, normalized size = 0.88

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

giac [A] time = 0.24, size = 16, normalized size = 0.94

$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\arctan(x)+4\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x+4)/(x^3+x),x)
```

```
[Out] arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

maxima [A] time = 1.53, size = 15, normalized size = 0.88

$$\arctan(x) - \frac{1}{2} \log\left(x^2 + 1\right) + 4 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

mupad [B] time = 2.28, size = 23, normalized size = 1.35

$$4 \ln(x) + \ln(x - \mathrm{i}) \left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln(x + 1\mathrm{i}) \left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 + 4)/(x + x^3),x)
```

```
[Out] 4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$4\log(x) - \frac{\log\left(x^2 + 1\right)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x+4)/(x**3+x),x)
```

```
[Out] 4*log(x) - log(x**2 + 1)/2 + atan(x)
```

3.351 $\int \frac{7-4x+8x^2}{(1+4x)(1+x^2)}\,dx$

Optimal. Leaf size=13

$$2\log(4x+1)-\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1629, 204}

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx &= \int \left(\frac{8}{1+4x}+\frac{1}{-1-x^2}\right)dx\\ &= 2\log(1+4x)+\int \frac{1}{-1-x^2}\,dx\\ &= -\tan^{-1}(x)+2\log(1+4x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$2\log(4x+1)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x] + 2*Log[1 + 4*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7-4x+8x^2}{(1+4x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)),x]
```

[Out] IntegrateAlgebraic[(7 - 4*x + 8*x^2)/((1 + 4*x)*(1 + x^2)), x]

fricas [A] time = 1.18, size = 13, normalized size = 1.00

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="fricas")

[Out] -arctan(x) + 2*log(4*x + 1)

giac [A] time = 0.34, size = 14, normalized size = 1.08

$$-\arctan(x) + 2\log(|4x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="giac")

[Out] -arctan(x) + 2*log(abs(4*x + 1))

maple [A] time = 0.01, size = 14, normalized size = 1.08

$$-\arctan(x) + 2\ln(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x)

[Out] -arctan(x)+2*ln(1+4*x)

maxima [A] time = 1.41, size = 13, normalized size = 1.00

$$-\arctan(x) + 2\log(4x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x^2-4*x+7)/(1+4*x)/(x^2+1),x, algorithm="maxima")

[Out] -arctan(x) + 2*log(4*x + 1)

mupad [B] time = 0.06, size = 19, normalized size = 1.46

$$\mathrm{atan}\left(\frac{4x + 1}{x - 4}\right) + 2\ln\left(x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^2 - 4*x + 7)/((4*x + 1)*(x^2 + 1)),x)

[Out] atan((4*x + 1)/(x - 4)) + 2*log(x + 1/4)

sympy [A] time = 0.13, size = 10, normalized size = 0.77

$$2\log\left(x + \frac{1}{4}\right) - \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8*x**2-4*x+7)/(1+4*x)/(x**2+1),x)

[Out] 2*log(x + 1/4) - atan(x)

3.352 $\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {27, 88}

$$\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + Log[1 - x]/4 + (3*Log[1 + x])/4
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx &= \int \frac{x^2}{(-1+x)(1+x)^2}\,dx\\ &= \int \left(\frac{1}{4(-1+x)}-\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right)dx\\ &= \frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)+\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.79

$$\frac{1}{4}\left(\frac{2}{x+1}+\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
```

```
[Out] (2/(1 + x) + Log[-1 + x] + 3*Log[1 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(-1+x)\left(1+2x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/((-1 + x)*(1 + 2*x + x^2)),x]
[Out] IntegrateAlgebraic[x^2/((-1 + x)*(1 + 2*x + x^2)), x]
```

fricas [A] time = 1.10, size = 26, normalized size = 0.93

$$\frac{3(x+1)\log(x+1)+(x+1)\log(x-1)+2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="fricas")
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) + 2)/(x + 1)
```

giac [A] time = 0.30, size = 22, normalized size = 0.79

$$\frac{1}{2(x+1)}+\frac{3}{4}\log(|x+1|)+\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="giac")
[Out] 1/2/(x + 1) + 3/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 21, normalized size = 0.75

$$\frac{\ln(x-1)}{4}+\frac{3\ln(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x-1)/(x^2+2*x+1),x)
[Out] 1/4*ln(x-1)+1/2/(x+1)+3/4*ln(x+1)
```

maxima [A] time = 0.66, size = 20, normalized size = 0.71

$$\frac{1}{2(x+1)}+\frac{3}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)/(x^2+2*x+1),x, algorithm="maxima")
[Out] 1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 0.07, size = 20, normalized size = 0.71

$$\frac{\ln(x-1)}{4}+\frac{3\,\ln(x+1)}{4}+\frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x - 1)*(2*x + x^2 + 1)),x)
[Out] log(x - 1)/4 + (3*log(x + 1))/4 + 1/(2*(x + 1))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.71

$$\frac{\log(x-1)}{4}+\frac{3\log(x+1)}{4}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)/(x**2+2*x+1),x)
[Out] log(x - 1)/4 + 3*log(x + 1)/4 + 1/(2*x + 2)
```

3.353 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {893}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
[Out] IntegrateAlgebraic[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)), x]
```

fricas [A] time = 1.16, size = 37, normalized size = 1.16

$$-\frac{25\,(2\,x-1)\log(2\,x+3)-41\,(2\,x-1)\log(2\,x-1)-36}{128\,(2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

giac [A] time = 0.30, size = 43, normalized size = 1.34

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right)-\frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{41\ln(2x-1)}{128}-\frac{25\ln(2x+3)}{128}+\frac{9}{32\,(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] 9/32/(2*x-1)+41/128*ln(2*x-1)-25/128*ln(3+2*x)
```

maxima [A] time = 0.75, size = 26, normalized size = 0.81

$$\frac{9}{32\,(2\,x-1)}-\frac{25}{128}\,\log(2\,x+3)+\frac{41}{128}\,\log(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

mupad [B] time = 2.23, size = 22, normalized size = 0.69

$$\frac{41\,\ln\left(x-\frac{1}{2}\right)}{128}-\frac{25\,\ln\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64\,\left(x-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 - 4)/((2*x - 1)^2*(2*x + 3)),x)
```

```
[Out] (41*log(x - 1/2))/128 - (25*log(x + 3/2))/128 + 9/(64*(x - 1/2))
```

sympy [A] time = 0.13, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

3.354 $\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right)dx\\
&= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx\\
&= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\
&= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)), x]
```

fricas [A] time = 1.19, size = 19, normalized size = 0.83

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

giac [A] time = 0.31, size = 20, normalized size = 0.87

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-3\arctan(x)+2\ln(x-1)+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(x-1)/(x^2+1),x)
[Out] 2*ln(x-1)+1/2*ln(x^2+1)-3*arctan(x)
```

maxima [A] time = 1.56, size = 19, normalized size = 0.83

$$-3\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

mupad [B] time = 0.05, size = 25, normalized size = 1.09

$$2\ln(x-1)+\ln(x-\mathrm{i})\left(\frac{1}{2}+\frac{3}{2}\mathrm{i}\right)+\ln(x+1\mathrm{i})\left(\frac{1}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 4*x + 5)/((x^2 + 1)*(x - 1)),x)
```

```
[Out] 2*log(x - 1) + log(x - 1i)*(1/2 + 3i/2) + log(x + 1i)*(1/2 - 3i/2)
```

sympy [**A**] time = 0.14, size = 19, normalized size = 0.83

$$2\log(x-1)+\frac{\log(x^2+1)}{2}-3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

3.355 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)), x]
```

fricas [A] time = 1.35, size = 36, normalized size = 1.50

$$\frac{2\,(x-1)\arctan(x)-(x-1)\log\left(x^2+1\right)+2\,(x-1)\log\left(x-1\right)+2}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

giac [B] time = 0.30, size = 47, normalized size = 1.96

$$\frac{1}{4}\,\pi-\pi\left\lfloor\frac{\pi+4\,\arctan(x)}{4\,\pi}+\frac{1}{2}\right\rfloor+\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\,\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

maple [A] time = 0.01, size = 21, normalized size = 0.88

$$\arctan(x)+\ln\left(x-1\right)-\frac{\ln\left(x^2+1\right)}{2}+\frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(x-1)^2/(x^2+1),x)
```

```
[Out] ln(x-1)+1/(x-1)-1/2*ln(x^2+1)+arctan(x)
```

maxima [A] time = 1.60, size = 20, normalized size = 0.83

$$\frac{1}{x-1}+\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

mupad [B] time = 2.11, size = 28, normalized size = 1.17

$$\ln(x-1) + \frac{1}{x-1} + \ln(x-\mathrm{i})\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - x^2 + 1)/((x^2 + 1)*(x - 1)^2),x)
```

```
[Out] log(x - 1) - log(x - 1i)*(1/2 + 1i/2) - log(x + 1i)*(1/2 - 1i/2) + 1/(x - 1
)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log(x^2+1)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

3.356 $$\int \frac{5+x^3}{\left(10-6x+x^2\right)\left(\frac{1}{2}-x+x^2\right)}\,dx$$

Optimal. Leaf size=49

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Rubi [A] time = 0.14, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6728, 634, 618, 204, 628, 617}

$$\frac{56}{221}\log\left(x^2-6x+10\right)+\frac{109}{442}\log\left(2x^2-2x+1\right)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx &= \int \left(\frac{2(345+56x)}{221(10-6x+x^2)}+\frac{2(76+109x)}{221(1-2x+2x^2)}\right)dx\\
&= \frac{2}{221}\int \frac{345+56x}{10-6x+x^2}\,dx+\frac{2}{221}\int \frac{76+109x}{1-2x+2x^2}\,dx\\
&= \frac{109}{442}\int \frac{-2+4x}{1-2x+2x^2}\,dx+\frac{56}{221}\int \frac{-6+2x}{10-6x+x^2}\,dx+\frac{261}{221}\int \frac{1}{1-2x+2x^2}\\
&= \frac{56}{221}\log(10-6x+x^2)+\frac{109}{442}\log(1-2x+2x^2)+\frac{261}{221}\text{Subst}\left(\int \frac{1}{-1-x}\right.\\
&= -\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)+\frac{56}{221}\log(10-6x+x^2)+\frac{109}{442}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$\frac{56}{221}\log(x^2-6x+10)+\frac{109}{442}\log(2x^2-2x+1)-\frac{261}{221}\tan^{-1}(1-2x)-\frac{1026}{221}\tan^{-1}(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] (-261*ArcTan[1 - 2*x])/221 - (1026*ArcTan[3 - x])/221 + (56*Log[10 - 6*x +
x^2])/221 + (109*Log[1 - 2*x + 2*x^2])/442
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5+x^3}{(10-6x+x^2)\left(\frac{1}{2}-x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(5 + x^3)/((10 - 6*x + x^2)*(1/2 - x + x^2)), x]
```

fricas [A] time = 1.30, size = 37, normalized size = 0.76

$$\frac{261}{221}\arctan(2x-1)+\frac{1026}{221}\arctan(x-3)+\frac{109}{442}\log\left(x^2-x+\frac{1}{2}\right)+\frac{56}{221}\log(x^2-6x+10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="fricas")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(x^2 - x + 1/
2) + 56/221*log(x^2 - 6*x + 10)
```

giac [A] time = 0.28, size = 39, normalized size = 0.80

$$\frac{261}{221}\arctan(2x-1)+\frac{1026}{221}\arctan(x-3)+\frac{109}{442}\log(2x^2-2x+1)+\frac{56}{221}\log(x^2-6x+10)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="giac")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

maple [A] time = 0.01, size = 40, normalized size = 0.82

$$\frac{1026\arctan(x-3)}{221}+\frac{261\arctan(2x-1)}{221}+\frac{56\ln\left(x^2-6x+10\right)}{221}+\frac{109\ln\left(2x^2-2x+1\right)}{442}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x)
```

```
[Out] 261/221*arctan(2*x-1)+1026/221*arctan(x-3)+56/221*ln(x^2-6*x+10)+109/442*ln
(2*x^2-2*x+1)
```

maxima [A] time = 1.71, size = 39, normalized size = 0.80

$$\frac{261}{221}\arctan(2x-1)+\frac{1026}{221}\arctan(x-3)+\frac{109}{442}\log\left(2x^2-2x+1\right)+\frac{56}{221}\log\left(x^2-6x+10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5)/(x^2-6*x+10)/(1/2-x+x^2),x, algorithm="maxima")
```

```
[Out] 261/221*arctan(2*x - 1) + 1026/221*arctan(x - 3) + 109/442*log(2*x^2 - 2*x
+ 1) + 56/221*log(x^2 - 6*x + 10)
```

mupad [B] time = 2.16, size = 41, normalized size = 0.84

$$\ln(x-3-\mathrm{i})\left(\frac{56}{221}-\frac{513}{221}\mathrm{i}\right)+\ln(x-3+1\mathrm{i})\left(\frac{56}{221}+\frac{513}{221}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}-\frac{261}{442}\mathrm{i}\right)+\ln\left(x-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{109}{442}+\frac{261}{442}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 5)/((x^2 - x + 1/2)*(x^2 - 6*x + 10)),x)
```

```
[Out] log(x - (3 + 1i))*(56/221 - 513i/221) + log(x - (3 - 1i))*(56/221 + 513i/22
1) + log(x - (1/2 + 1i/2))*(109/442 - 261i/442) + log(x - (1/2 - 1i/2))*(10
9/442 + 261i/442)
```

sympy [A] time = 0.22, size = 44, normalized size = 0.90

$$\frac{56\log\left(x^2-6x+10\right)}{221}+\frac{109\log\left(x^2-x+\frac{1}{2}\right)}{442}+\frac{1026\operatorname{atan}(x-3)}{221}+\frac{261\operatorname{atan}(2x-1)}{221}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5)/(x**2-6*x+10)/(1/2-x+x**2),x)
```

```
[Out] 56*log(x**2 - 6*x + 10)/221 + 109*log(x**2 - x + 1/2)/442 + 1026*atan(x - 3
)/221 + 261*atan(2*x - 1)/221
```

3.357 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=25

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1612}

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned} \int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(\frac{11}{-3+x}-\frac{14}{-2+x}+\frac{4}{-1+x}\right)dx\\ &= 4\log(1-x)-14\log(2-x)+11\log(3-x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.76

$$11\log(x-3)-14\log(x-2)+4\log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] 11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

```
[Out] IntegrateAlgebraic[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)), x]
```

fricas [A] time = 1.13, size = 19, normalized size = 0.76

$$4\,\log\left(x-1\right)-14\,\log\left(x-2\right)+11\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

giac [A] time = 0.39, size = 22, normalized size = 0.88

$$4 \log(|x-1|) - 14 \log(|x-2|) + 11 \log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
[Out] 4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$11 \ln(x-3) - 14 \ln(x-2) + 4 \ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x+4)/(x-3)/(x-2)/(x-1),x)
[Out] 4*ln(x-1)-14*ln(x-2)+11*ln(x-3)
```

maxima [A] time = 1.07, size = 19, normalized size = 0.76

$$4 \log(x-1) - 14 \log(x-2) + 11 \log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

mupad [B] time = 2.14, size = 19, normalized size = 0.76

$$4 \ln(x-1) - 14 \ln(x-2) + 11 \ln(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
[Out] 4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

sympy [A] time = 0.14, size = 19, normalized size = 0.76

$$11 \log(x-3) - 14 \log(x-2) + 4 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
[Out] 11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

3.358 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Rubi [A] time = 0.25, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right)$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,dx}{5586} - \ldots$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}$$

$$= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.90

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)), x]
```

fricas [A] time = 1.60, size = 60, normalized size = 1.00

$$\frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-243867(x+5)\log\left(x^2+x+1\right)+176400(x+5)\log(2x-3)+311334(x+5)\log(x+5)-819546}{2832102(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

giac [A] time = 0.34, size = 60, normalized size = 1.00

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273(x+5)}-\frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

maple [A] time = 0.01, size = 48, normalized size = 0.80

$$\frac{451\sqrt{3}\,\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{8379}+\frac{200\ln\left(2x-3\right)}{3211}+\frac{2731\ln\left(x+5\right)}{24843}-\frac{481\ln\left(x^{2}+x+1\right)}{5586}-\frac{79}{273\left(x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)-481/5586*ln(x^2+x+1)+4
51/8379*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))
```

maxima [A] time = 2.80, size = 47, normalized size = 0.78

$$\frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)-\frac{79}{273\left(x+5\right)}-\frac{481}{5586}\log\left(x^{2}+x+1\right)+\frac{200}{3211}\log\left(2x-3\right)+\frac{2731}{24843}\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

mupad [B] time = 0.13, size = 61, normalized size = 1.02

$$\frac{200\ln\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\ln\left(x+5\right)}{24843}-\frac{79}{273\left(x+5\right)}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

```
[Out] (200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log(
x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^(1
/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

sympy [A] time = 0.25, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^{2}+x+1\right)}{5586}+\frac{451\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

3.359 $\int \frac{-1+x^3}{1+x+x^2} dx$

Optimal. Leaf size=11

$$\frac{x^2}{2} - x$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1586}

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(1 + x + x^2),x]
[Out] -x + x^2/2
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{1+x+x^2} \, dx &= \int (-1+x) \, dx \\ &= -x+\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2} - x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(1 + x + x^2),x]
[Out] -x + x^2/2
```

IntegrateAlgebraic [A] time = 0.02, size = 9, normalized size = 0.82

$$\frac{1}{2}(x-1)^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(1 + x + x^2),x]
[Out] (-1 + x)^2/2
```

fricas [A] time = 1.17, size = 9, normalized size = 0.82

$$\frac{1}{2} x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
[Out] 1/2*x^2 - x
```

giac [A] time = 0.23, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
[Out] 1/2*x^2 - x
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+x+1),x)
[Out] -x+1/2*x^2
```

maxima [A] time = 0.84, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
[Out] 1/2*x^2 - x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x\,\left(x-2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x + x^2 + 1),x)
[Out] (x*(x - 2))/2
```

sympy [A] time = 0.06, size = 5, normalized size = 0.45

$$\frac{x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+x+1),x)
[Out] x**2/2 - x
```

3.360 $\int \frac{-3+x^3}{-7-6x+x^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x^3)/(-7 - 6*x + x^2),x]
```

```
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x^3}{-7-6x+x^2}\,dx &= \int \left(6+x+\frac{39+43x}{-7-6x+x^2}\right)dx\\ &= 6x+\frac{x^2}{2}+\int \frac{39+43x}{-7-6x+x^2}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{85}{2}\int \frac{1}{-7+x}\,dx\\ &= 6x+\frac{x^2}{2}+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{x^2}{2}+6x+\frac{85}{2}\log(7-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x^3)/(-7 - 6*x + x^2),x]
[Out] 6*x + x^2/2 + (85*Log[7 - x])/2 + Log[1 + x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3 + x^3}{-7 - 6x + x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x^3)/(-7 - 6*x + x^2),x]
[Out] IntegrateAlgebraic[(-3 + x^3)/(-7 - 6*x + x^2), x]
```

fricas [A] time = 1.14, size = 21, normalized size = 0.72

$$\frac{1}{2} x^2 + 6 x + \frac{1}{2} \log(x + 1) + \frac{85}{2} \log(x - 7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="fricas")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

giac [A] time = 0.35, size = 23, normalized size = 0.79

$$\frac{1}{2} x^2 + 6 x + \frac{1}{2} \log(|x + 1|) + \frac{85}{2} \log(|x - 7|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="giac")
[Out] 1/2*x^2 + 6*x + 1/2*log(abs(x + 1)) + 85/2*log(abs(x - 7))
```

maple [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{\ln(x + 1)}{2} + \frac{85 \ln(x - 7)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3)/(x^2-6*x-7),x)
[Out] 1/2*x^2+6*x+1/2*ln(x+1)+85/2*ln(x-7)
```

maxima [A] time = 1.10, size = 21, normalized size = 0.72

$$\frac{1}{2} x^2 + 6 x + \frac{1}{2} \log(x + 1) + \frac{85}{2} \log(x - 7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3)/(x^2-6*x-7),x, algorithm="maxima")
[Out] 1/2*x^2 + 6*x + 1/2*log(x + 1) + 85/2*log(x - 7)
```

mupad [B] time = 2.12, size = 21, normalized size = 0.72

$$6 x + \frac{\ln(x + 1)}{2} + \frac{85 \ln(x - 7)}{2} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 3)/(6*x - x^2 + 7),x)
```

```
[Out] 6*x + log(x + 1)/2 + (85*log(x - 7))/2 + x^2/2
```

sympy [A] time = 0.11, size = 22, normalized size = 0.76

$$\frac{x^2}{2} + 6x + \frac{85\log(x-7)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3)/(x**2-6*x-7),x)
```

```
[Out] x**2/2 + 6*x + 85*log(x - 7)/2 + log(x + 1)/2
```

3.361 $\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1660, 634, 618, 204, 628}

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx = \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{36}\int \frac{-50+36x}{13+4x+x^2}\,dx$$
$$= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\int \frac{4+2x}{13+4x+x^2}\,dx-\frac{61}{18}\int \frac{1}{13+4x+x^2}\,dx$$
$$= \frac{67+47x}{18\left(13+4x+x^2\right)}+\frac{1}{2}\log\left(13+4x+x^2\right)+\frac{61}{9}\,\mathrm{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,4+2x\right)$$
$$= \frac{67+47x}{18\left(13+4x+x^2\right)}-\frac{61}{54}\tan^{-1}\left(\frac{2+x}{3}\right)+\frac{1}{2}\log\left(13+4x+x^2\right)$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.00

$$\frac{47x+67}{18\left(x^2+4x+13\right)}+\frac{1}{2}\log\left(x^2+4x+13\right)-\frac{61}{54}\tan^{-1}\left(\frac{x+2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] (67 + 47*x)/(18*(13 + 4*x + x^2)) - (61*ArcTan[(2 + x)/3])/54 + Log[13 + 4*
x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{\left(13+4x+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(13 + 4*x + x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3)/(13 + 4*x + x^2)^2, x]
```

fricas [A] time = 0.74, size = 52, normalized size = 1.16

$$-\frac{61\left(x^2+4x+13\right)\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)-27\left(x^2+4x+13\right)\log\left(x^2+4x+13\right)-141x-201}{54\left(x^2+4x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="fricas")
```

```
[Out] -1/54*(61*(x^2 + 4*x + 13)*arctan(1/3*x + 2/3) - 27*(x^2 + 4*x + 13)*log(x^
2 + 4*x + 13) - 141*x - 201)/(x^2 + 4*x + 13)
```

giac [A] time = 0.30, size = 37, normalized size = 0.82

$$\frac{47x+67}{18\left(x^2+4x+13\right)}-\frac{61}{54}\arctan\left(\frac{1}{3}x+\frac{2}{3}\right)+\frac{1}{2}\log\left(x^2+4x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="giac")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

maple [A] time = 0.01, size = 37, normalized size = 0.82

$$-\frac{61\arctan\left(\frac{x}{3}+\frac{2}{3}\right)}{54}+\frac{\ln\left(x^2+4x+13\right)}{2}+\frac{\frac{47x}{18}+\frac{67}{18}}{x^2+4x+13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^2+4*x+13)^2,x)
```

```
[Out] (47/18*x+67/18)/(x^2+4*x+13)+1/2*ln(x^2+4*x+13)-61/54*arctan(2/3+1/3*x)
```

maxima [A] time = 1.84, size = 37, normalized size = 0.82

$$\frac{47\,x+67}{18\,\left(x^2+4\,x+13\right)}-\frac{61}{54}\,\arctan\left(\frac{1}{3}\,x+\frac{2}{3}\right)+\frac{1}{2}\,\log\left(x^2+4\,x+13\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^2+4*x+13)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(47*x + 67)/(x^2 + 4*x + 13) - 61/54*arctan(1/3*x + 2/3) + 1/2*log(x^2
 + 4*x + 13)
```

mupad [B] time = 0.04, size = 49, normalized size = 1.09

$$\frac{\ln\left(x^2+4\,x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}+\frac{47\,x}{18\,\left(x^2+4\,x+13\right)}+\frac{67}{18\,\left(x^2+4\,x+13\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(4*x + x^2 + 13)^2,x)
```

```
[Out] log(4*x + x^2 + 13)/2 - (61*atan(x/3 + 2/3))/54 + (47*x)/(18*(4*x + x^2 + 1
3)) + 67/(18*(4*x + x^2 + 13))
```

sympy [A] time = 0.14, size = 37, normalized size = 0.82

$$\frac{47x+67}{18x^2+72x+234}+\frac{\log\left(x^2+4x+13\right)}{2}-\frac{61\,\mathrm{atan}\left(\frac{x}{3}+\frac{2}{3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**2+4*x+13)**2,x)
```

```
[Out] (47*x + 67)/(18*x**2 + 72*x + 234) + log(x**2 + 4*x + 13)/2 - 61*atan(x/3 +
 2/3)/54
```

3.362 $\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x(1+x^2)(4+x^2)^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Rubi [A] time = 0.25, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {6725, 203, 261, 635, 260}

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 + x^2)^
2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx = \int \left(-\frac{2}{x}+\frac{2}{1+x^2}-\frac{2x}{\left(4+x^2\right)^2}+\frac{1+2x}{4+x^2}\right)dx$$
$$= -2\log(x)+2\int \frac{1}{1+x^2}\,dx-2\int \frac{x}{\left(4+x^2\right)^2}\,dx+\int \frac{1+2x}{4+x^2}\,dx$$
$$= \frac{1}{4+x^2}+2\tan^{-1}(x)-2\log(x)+2\int \frac{x}{4+x^2}\,dx+\int \frac{1}{4+x^2}\,d$$
$$= \frac{1}{4+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-2\log(x)+\log\left(4+x^2\right)$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{1}{x^2+4}+\log\left(x^2+4\right)-2\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 + x^2)*(4 +
 x^2)^2),x]
```

```
[Out] (4 + x^2)^(-1) + ArcTan[x/2]/2 + 2*ArcTan[x] - 2*Log[x] + Log[4 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-32+36x-42x^2+21x^3-10x^4+3x^5}{x\left(1+x^2\right)\left(4+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 +
x^2)*(4 + x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[(-32 + 36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5)/(x*(1 +
x^2)*(4 + x^2)^2), x]
```

fricas [A] time = 1.22, size = 52, normalized size = 1.62

$$\frac{\left(x^2+4\right)\arctan\left(\frac{1}{2}x\right)+4\left(x^2+4\right)\arctan(x)+2\left(x^2+4\right)\log\left(x^2+4\right)-4\left(x^2+4\right)\log(x)+2}{2\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="fricas")
```

```
[Out] 1/2*((x^2 + 4)*arctan(1/2*x) + 4*(x^2 + 4)*arctan(x) + 2*(x^2 + 4)*log(x^2
+ 4) - 4*(x^2 + 4)*log(x) + 2)/(x^2 + 4)
```

giac [A] time = 0.32, size = 29, normalized size = 0.91

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="giac")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(abs(x)
)
```

maple [A] time = 0.01, size = 29, normalized size = 0.91

$$2\arctan(x)+\frac{\arctan\left(\frac{x}{2}\right)}{2}-2\ln(x)+\ln\left(x^2+4\right)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] 1/(x^2+4)+1/2*arctan(1/2*x)+2*arctan(x)-2*ln(x)+ln(x^2+4)
```

maxima [A] time = 2.02, size = 28, normalized size = 0.88

$$\frac{1}{x^2+4}+\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+2\arctan(x)+\log\left(x^2+4\right)-2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-10*x^4+21*x^3-42*x^2+36*x-32)/x/(x^2+1)/(x^2+4)^2,x, algor
ithm="maxima")
```

```
[Out] 1/(x^2 + 4) + 1/2*arctan(1/2*x) + 2*arctan(x) + log(x^2 + 4) - 2*log(x)
```

mupad [B] time = 0.07, size = 44, normalized size = 1.38

$$\frac{1}{x^2+4}-2\ln(x)-2\,\mathrm{atan}\left(\frac{328000}{7\,(36288\,x-19584)}+\frac{34}{63}\right)+\ln\left(x-2\mathrm{i}\right)\left(1-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+2\mathrm{i}\right)\left(1+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((36*x - 42*x^2 + 21*x^3 - 10*x^4 + 3*x^5 - 32)/(x*(x^2 + 1)*(x^2 + 4)^2
),x)
```

```
[Out] log(x - 2i)*(1 - 1i/4) + log(x + 2i)*(1 + 1i/4) - 2*atan(328000/(7*(36288*x
 - 19584)) + 34/63) - 2*log(x) + 1/(x^2 + 4)
```

sympy [A] time = 0.26, size = 29, normalized size = 0.91

$$-2\log(x)+\log\left(x^2+4\right)+\frac{\mathrm{atan}\left(\frac{x}{2}\right)}{2}+2\,\mathrm{atan}(x)+\frac{1}{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-10*x**4+21*x**3-42*x**2+36*x-32)/x/(x**2+1)/(x**2+4)**2,x
)
```

```
[Out] -2*log(x) + log(x**2 + 4) + atan(x/2)/2 + 2*atan(x) + 1/(x**2 + 4)
```

3.363 $\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$

Optimal. Leaf size=148

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}\,7^{3/4}}$$

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1790, 1403, 211, 1165, 628, 1162, 617, 204, 1584, 1478, 275, 321, 207}

$$\frac{x^2}{2}-\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{7}x+\sqrt{7}\right)}{4\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)}{2\sqrt{2}\,7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] x^2/2 - ArcTan[1 - (Sqrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) + ArcTan[1 + (S
qrt[2]*x)/7^(1/4)]/(2*Sqrt[2]*7^(3/4)) - ArcTanh[x^2]/2 - Log[Sqrt[7] - Sqr
t[2]*7^(1/4)*x + x^2]/(4*Sqrt[2]*7^(3/4)) + Log[Sqrt[7] + Sqrt[2]*7^(1/4)*x
 + x^2]/(4*Sqrt[2]*7^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1403

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_))^(p_.), x_Symbol] :> Int[(d + e*x^n)^(p + q)*(a/d + (c*x^n)/e)^p, x] /;
FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 1478

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n
_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Int[(f*x)^m*(d + e*x^n)^(q + p)*
(a/d + (c*x^n)/e)^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2,
 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p
]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1790

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx &= \int \left(\frac{-1+x^4}{-7+6x^4+x^8}+\frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\right)dx\\
&= \int \frac{-1+x^4}{-7+6x^4+x^8}\,dx+\int \frac{x\left(7x^4+x^8\right)}{-7+6x^4+x^8}\,dx\\
&= \int \frac{1}{7+x^4}\,dx+\int \frac{x^5\left(7+x^4\right)}{-7+6x^4+x^8}\,dx\\
&= \frac{\int \frac{\sqrt{7}-x^2}{7+x^4}\,dx}{2\sqrt{7}}+\frac{\int \frac{\sqrt{7}+x^2}{7+x^4}\,dx}{2\sqrt{7}}+\int \frac{x^5}{-1+x^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,x^2\right)-\frac{\int \frac{\sqrt{2}\sqrt[4]{7}+2x}{-\sqrt{7}-\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}\,7^{3/4}}-\frac{\int \frac{\sqrt{2}\sqrt[4]{7}-2x}{-\sqrt{7}+\sqrt{2}\sqrt[4]{7}x-x^2}\,dx}{4\sqrt{2}\,7^{3/4}}+\frac{\int}{}\\
&= \frac{x^2}{2}-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}\,7^{3/4}}+\frac{\log\left(\sqrt{7}+\sqrt{2}\sqrt[4]{7}x+x^2\right)}{4\sqrt{2}\,7^{3/4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+}\right.\\
&= \frac{x^2}{2}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)}{2\sqrt{2}\,7^{3/4}}-\frac{1}{2}\tanh^{-1}\left(x^2\right)-\frac{\log\left(\sqrt{7}-\sqrt{2}\sqrt[4]{7}x\right.}{4\sqrt{2}\,7^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 159, normalized size = 1.07

$$\frac{1}{56}\left(28x^2-14\log\left(x^2+1\right)-\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2-\sqrt{2}\,7^{3/4}x+7\right)+\sqrt{2}\sqrt[4]{7}\log\left(\sqrt{7}x^2+\sqrt{2}\,7^{3/4}x+7\right)+14\log(1-x)+14\log(x+1)-2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{7}}\right)+2\sqrt{2}\sqrt[4]{7}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{7}}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] (28*x^2 - 2*Sqrt[2]*7^(1/4)*ArcTan[1 - (Sqrt[2]*x)/7^(1/4)] + 2*Sqrt[2]*7^(
1/4)*ArcTan[1 + (Sqrt[2]*x)/7^(1/4)] + 14*Log[1 - x] + 14*Log[1 + x] - 14*L
og[1 + x^2] - Sqrt[2]*7^(1/4)*Log[7 - Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2] + Sq
rt[2]*7^(1/4)*Log[7 + Sqrt[2]*7^(3/4)*x + Sqrt[7]*x^2])/56
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^4+7x^5+x^9}{-7+6x^4+x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8),x]
```

```
[Out] IntegrateAlgebraic[(-1 + x^4 + 7*x^5 + x^9)/(-7 + 6*x^4 + x^8), x]
```

fricas [A] time = 1.29, size = 178, normalized size = 1.20

$$-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{49}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}}-1\right)-\frac{1}{686}\cdot 343^{\frac{3}{4}}\sqrt{2}\arctan\left(-\frac{1}{7}\cdot 343^{\frac{1}{4}}\sqrt{2}x+\frac{1}{49}\cdot 343^{\frac{1}{4}}\sqrt{2}\sqrt{-343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}}+1\right)+\frac{1}{2744}\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}\right)-\frac{1}{2744}\cdot 343^{\frac{3}{4}}\sqrt{2}\log\left(-343^{\frac{3}{4}}\sqrt{2}x+49x^2+49\sqrt{7}\right)+\frac{1}{2}x^2-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="fricas")
```

```
[Out] -1/686*343^(3/4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*s
qrt(2)*sqrt(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1) - 1/686*343^(3/
4)*sqrt(2)*arctan(-1/7*343^(1/4)*sqrt(2)*x + 1/49*343^(1/4)*sqrt(2)*sqrt(-3
43^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1) + 1/2744*343^(3/4)*sqrt(2)*l
og(343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) - 1/2744*343^(3/4)*sqrt(2)*lo
```

```
g(-343^(3/4)*sqrt(2)*x + 49*x^2 + 49*sqrt(7)) + 1/2*x^2 - 1/4*log(x^2 + 1)
+ 1/4*log(x^2 - 1)
```

giac [A] time = 0.39, size = 122, normalized size = 0.82

$$\frac{1}{2}x^2 + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x + 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 28^{\frac{1}{4}}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x - 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2 + 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{56}\cdot 28^{\frac{1}{4}}\log\left(x^2 - 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{4}\log\left(|x + 1|\right) + \frac{1}{4}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*sqrt(2))
) + 1/28*28^(1/4)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*sqrt(2))) + 1/
56*28^(1/4)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/56*28^(1/4)*log(x^2
- 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 + 1) + 1/4*log(abs(x + 1)) + 1
/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 110, normalized size = 0.74

$$\frac{x^2}{2} + \frac{7^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,7^{\frac{3}{4}}x}{7} - 1\right)}{28} + \frac{7^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,7^{\frac{3}{4}}x}{7} + 1\right)}{28} + \frac{7^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2 + 7^{\frac{1}{4}}\sqrt{2}\,x + \sqrt{7}}{x^2 - 7^{\frac{1}{4}}\sqrt{2}\,x + \sqrt{7}}\right)}{56} + \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x)
```

```
[Out] 1/2*x^2+1/4*ln(x-1)-1/4*ln(x^2+1)+1/56*7^(1/4)*2^(1/2)*ln((x^2+7^(1/4)*x*2^
(1/2)+7^(1/2))/(x^2-7^(1/4)*x*2^(1/2)+7^(1/2)))+1/28*arctan(1+1/7*x*2^(1/2)
*7^(3/4))*7^(1/4)*2^(1/2)+1/28*arctan(-1+1/7*x*2^(1/2)*7^(3/4))*7^(1/4)*2^(
1/2)+1/4*ln(x+1)
```

maxima [A] time = 2.01, size = 132, normalized size = 0.89

$$\frac{1}{2}x^2 + \frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x + 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{28}\cdot 7^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{1}{14}\cdot 7^{\frac{3}{4}}\sqrt{2}\left(2x - 7^{\frac{1}{4}}\sqrt{2}\right)\right) + \frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2 + 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{56}\cdot 7^{\frac{1}{4}}\sqrt{2}\log\left(x^2 - 7^{\frac{1}{4}}\sqrt{2}x + \sqrt{7}\right) - \frac{1}{4}\log\left(x^2 + 1\right) + \frac{1}{4}\log\left(x + 1\right) + \frac{1}{4}\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^9+7*x^5+x^4-1)/(x^8+6*x^4-7),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x + 7^(1/4)*s
qrt(2))) + 1/28*7^(1/4)*sqrt(2)*arctan(1/14*7^(3/4)*sqrt(2)*(2*x - 7^(1/4)*
sqrt(2))) + 1/56*7^(1/4)*sqrt(2)*log(x^2 + 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1
/56*7^(1/4)*sqrt(2)*log(x^2 - 7^(1/4)*sqrt(2)*x + sqrt(7)) - 1/4*log(x^2 +
1) + 1/4*log(x + 1) + 1/4*log(x - 1)
```

mupad [B] time = 2.19, size = 124, normalized size = 0.84

$$\frac{\mathrm{atan}\left(x^2\,1i\right)\,1i}{2} + \frac{x^2}{2} + \sqrt{2}\,7^{1/4}\,\mathrm{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\,\left(\frac{89653248}{2401} + \frac{89653248}{2401}i\right)}{-\frac{1048576}{49} + \frac{\sqrt{7}\,179306496i}{2401}} + \frac{\sqrt{2}\,7^{3/4}\,x\,\left(-\frac{524288}{343} + \frac{524288}{343}i\right)}{-\frac{1048576}{49} + \frac{\sqrt{7}\,179306496i}{2401}}\right)\left(\frac{1}{28} + \frac{1}{28}i\right) + \sqrt{2}\,7^{1/4}\,\mathrm{atan}\left(\frac{\sqrt{2}\,7^{1/4}\,x\,\left(\frac{89653248}{2401} - \frac{89653248}{2401}i\right)}{\frac{1048576}{49} + \frac{\sqrt{7}\,179306496i}{2401}} + \frac{\sqrt{2}\,7^{3/4}\,x\,\left(-\frac{524288}{343} - \frac{524288}{343}i\right)}{\frac{1048576}{49} + \frac{\sqrt{7}\,179306496i}{2401}}\right)\left(-\frac{1}{28} + \frac{1}{28}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 7*x^5 + x^9 - 1)/(6*x^4 + x^8 - 7),x)
```

```
[Out] (atan(x^2*1i)*1i)/2 + x^2/2 + 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(8965
3248/2401 + 89653248i/2401))/((7^(1/2)*179306496i)/2401 - 1048576/49) - (2^
(1/2)*7^(3/4)*x*(524288/343 - 524288i/343))/((7^(1/2)*179306496i)/2401 - 10
48576/49))*(1/28 + 1i/28) - 2^(1/2)*7^(1/4)*atan((2^(1/2)*7^(1/4)*x*(896532
48/2401 - 89653248i/2401))/((7^(1/2)*179306496i)/2401 + 1048576/49) - (2^(1
/2)*7^(3/4)*x*(524288/343 + 524288i/343))/((7^(1/2)*179306496i)/2401 + 1048
576/49))*(1/28 - 1i/28)
```

sympy [A] time = 0.43, size = 146, normalized size = 0.99

$$\frac{x^2}{2}+\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}-\frac{\sqrt{2}\,\sqrt[4]{7}\log\left(x^2-\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\,\sqrt[4]{7}\log\left(x^2+\sqrt{2}\,\sqrt[4]{7}\,x+\sqrt{7}\right)}{56}+\frac{\sqrt{2}\,\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot7^{\frac{3}{4}}x}{7}-1\right)}{28}+\frac{\sqrt{2}\,\sqrt[4]{7}\operatorname{atan}\left(\frac{\sqrt{2}\cdot7^{\frac{3}{4}}x}{7}+1\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**9+7*x**5+x**4-1)/(x**8+6*x**4-7),x)
```

```
[Out] x**2/2 + log(x**2 - 1)/4 - log(x**2 + 1)/4 - sqrt(2)*7**(1/4)*log(x**2 - sq
rt(2)*7**(1/4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*log(x**2 + sqrt(2)*7**(1/
4)*x + sqrt(7))/56 + sqrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 - 1)/28 + s
qrt(2)*7**(1/4)*atan(sqrt(2)*7**(3/4)*x/7 + 1)/28
```

3.364 $\int \frac{1+x^3+x^6}{x+x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\right.}{2}$$

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.812, Rules used = {1593, 1833, 297, 1162, 617, 204, 1165, 628, 1834, 1248, 635, 203, 260}

$$\frac{x^2}{2}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{4\sqrt{2}}-\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{2}\tan^{-1}\left(x^2\right)+\log(x)-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt
[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1
+ Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1833

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j +
(k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1}]*(a + b*x^n)^p)/c^j, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \int \frac{1+x^3+x^6}{x\left(1+x^4\right)}\,dx$$
$$= \int \left(\frac{x^2}{1+x^4}+\frac{1+x^6}{x\left(1+x^4\right)}\right)dx$$
$$= \int \frac{x^2}{1+x^4}\,dx + \int \frac{1+x^6}{x\left(1+x^4\right)}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx + \int \left(\frac{1}{x}+x+\frac{x\left(-1-x^2\right)}{1+x^4}\right)dx$$
$$= \frac{x^2}{2} + \log(x) + \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{\sqrt{2}-2}{-1+\sqrt{2}x}}{4\sqrt{2}}$$
$$= \frac{x^2}{2} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx, x, x^2\right) +$$
$$= \frac{x^2}{2} - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+\right.}{4\sqrt{2}}$$
$$= \frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right) - \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \log(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}} -$$

Mathematica [A] time = 0.05, size = 101, normalized size = 0.90

$$\frac{1}{8}\left(-2\log\left(x^4+1\right)+4x^2+\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+8\log(x)-2\left(\sqrt{2}-2\right)\tan^{-1}\left(1-\sqrt{2}x\right)+2\left(2+\sqrt{2}\right)\tan^{-1}\left(\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] (4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
+ Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1
+ Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3 + x^6)/(x + x^5),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3 + x^6)/(x + x^5), x]
```

fricas [C] time = 4.32, size = 515, normalized size = 4.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/4*(2*sqrt(1/4*I) + I + 1)*log((2*sqrt(1/4*I) + I + 1)^3 - 5*(2*
sqrt(1/4*I) + I + 1)^2 + 3*x + 20*sqrt(1/4*I) + 10*I + 5) - 1/4*(2*sqrt(-1/
4*I) - I + 1)*log(-(2*sqrt(1/4*I) + I + 1)^3 - (2*sqrt(1/4*I) + I + 2)*(2*s
qrt(-1/4*I) - I + 1)^2 + 4*(2*sqrt(1/4*I) + I + 1)^2 - ((2*sqrt(1/4*I) + I
+ 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) + 3*x - 16*sqrt(
```

```
1/4*I) - 8*I - 9) + 1/4*(sqrt(1/4*I) + sqrt(-1/4*I) - 2*sqrt(-3/16*(2*sqrt(
1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) -
3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(
2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I
+ 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(
-1/4*I) - I + 1) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4
*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + s
qrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1)
 + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*sqrt(1/4*I) - I + 2) + 1/4*(sqrt(1/4*I)
 + sqrt(-1/4*I) + 2*sqrt(-3/16*(2*sqrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*
I) + I - 3)*(2*sqrt(-1/4*I) - I + 1) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sq
rt(1/4*I) + 1/2*I - 1/2) - 1)*log(1/2*(2*sqrt(1/4*I) + I + 2)*(2*sqrt(-1/4*
I) - I + 1)^2 + 1/2*(2*sqrt(1/4*I) + I + 1)^2 + 1/2*((2*sqrt(1/4*I) + I + 1
)^2 - 8*sqrt(1/4*I) - 4*I - 6)*(2*sqrt(-1/4*I) - I + 1) - 2*sqrt(-3/16*(2*s
qrt(1/4*I) + I + 1)^2 - 1/8*(2*sqrt(1/4*I) + I - 3)*(2*sqrt(-1/4*I) - I + 1
) - 3/16*(2*sqrt(-1/4*I) - I + 1)^2 + sqrt(1/4*I) + 1/2*I - 1/2)*((2*sqrt(1
/4*I) + I + 2)*(2*sqrt(-1/4*I) - I + 1) + 2*sqrt(1/4*I) + I - 1) + 3*x - 2*
sqrt(1/4*I) - I + 2) + log(x)
```

giac [A] time = 0.39, size = 92, normalized size = 0.82

$$\frac{1}{2}\,x^2 + \frac{1}{4}\left(\sqrt{2} + 2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2} - 2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{1}{8}\,\sqrt{2}\,\log\left(x^2 + \sqrt{2}\,x + 1\right) + \frac{1}{8}\,\sqrt{2}\,\log\left(x^2 - \sqrt{2}\,x + 1\right) - \frac{1}{4}\,\log\left(x^4 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqrt
(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2
)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log(ab
s(x))
```

maple [A] time = 0.01, size = 79, normalized size = 0.71

$$\frac{x^2}{2} - \frac{\arctan\left(x^2\right)}{2} + \frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x - 1\right)}{4} + \frac{\sqrt{2}\ \arctan\left(\sqrt{2}\ x + 1\right)}{4} + \ln(x) + \frac{\sqrt{2}\ \ln\left(\frac{x^2 - \sqrt{2}\ x + 1}{x^2 + \sqrt{2}\ x + 1}\right)}{8} - \frac{\ln\left(x^4 + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+x^3+1)/(x^5+x),x)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)+1/4*arctan(-1+2^(1/2)*x)*2^(1/2)+1/8*2^(1/2)*ln((1+
x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+1/4*arctan(1+2^(1/2)*x)*2^(1/2)-1/4*ln(x^
4+1)+ln(x)
```

maxima [A] time = 2.06, size = 99, normalized size = 0.88

$$\frac{1}{4}\,\sqrt{2}\left(\sqrt{2} + 1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x + \sqrt{2}\right)\right) - \frac{1}{4}\,\sqrt{2}\left(\sqrt{2} - 1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x - \sqrt{2}\right)\right) - \frac{1}{8}\,\sqrt{2}\left(\sqrt{2} + 1\right)\log\left(x^2 + \sqrt{2}\,x + 1\right) - \frac{1}{8}\,\sqrt{2}\left(\sqrt{2} - 1\right)\log\left(x^2 - \sqrt{2}\,x + 1\right) + \frac{1}{2}\,x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2)
*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2) +
 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(2)*
x + 1) + 1/2*x^2 + log(x)
```

mupad [B] time = 2.23, size = 170, normalized size = 1.52

$$\ln(x) + \left(\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\left(8\,\mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right) + x + \mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\,x\,96 + \mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^2\,x\,240 + \mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^3\,x\,320 - 16\,\mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)^2 + 8\right)\right)\,\mathrm{root}\left(z^4 + z^3 + \frac{z^2}{2} + \frac{z}{16} + \frac{1}{256}, z, k\right)\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^6 + 1)/(x + x^5),x)
```

```
[Out] log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z^
4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z/1
6 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*x +
 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z^3 +
 z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1/256,
 z, k), k, 1, 4) + x^2/2
```

sympy [A] time = 0.96, size = 61, normalized size = 0.54

$$\frac{x^2}{2} + \log(x) + \text{RootSum}\left(256t^4 + 256t^3 + 128t^2 + 16t + 1, \left(t \mapsto t\log\left(\frac{1792t^4}{73} + \frac{704t^3}{219} - \frac{3152t^2}{219} - \frac{2584t}{219} + x - \frac{344}{219}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+x**3+1)/(x**5+x),x)
```

```
[Out] x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, La
mbda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/21
9 + x - 344/219)))
```

3.365 $\int \frac{1+x^2}{-x+x^2} dx$

Optimal. Leaf size=14

$$x + 2\log(1-x) - \log(x)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 894}

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{-x+x^2} dx &= \int \frac{1+x^2}{(-1+x)x} dx \\
&= \int \left(1 + \frac{2}{-1+x} - \frac{1}{x}\right) dx \\
&= x + 2\log(1-x) - \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$x + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{-x+x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(-x + x^2),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(-x + x^2), x]
```

fricas [A] time = 1.35, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
[Out] x + 2*log(x - 1) - log(x)
```

giac [A] time = 0.26, size = 14, normalized size = 1.00

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$x - \ln(x) + 2\ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x)
[Out] x+2*ln(x-1)-ln(x)
```

maxima [A] time = 0.91, size = 12, normalized size = 0.86

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
[Out] x + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.04, size = 12, normalized size = 0.86

$$x + 2 \ln(x - 1) - \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 + 1)/(x - x^2),x)
[Out] x + 2*log(x - 1) - log(x)
```

sympy [A] time = 0.10, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
[Out] x - log(x) + 2*log(x - 1)
```

3.366 $\int \frac{1+x^3}{-x+x^3} dx$

Optimal. Leaf size=12

$$x + \log(1 - x) - \log(x)$$

Rubi [A] time = 0.03, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1802}

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^3}{-x+x^3} dx &= \int \frac{1+x^3}{x\left(-1+x^2\right)} dx \\ &= \int \left(1+\frac{1}{-1+x}-\frac{1}{x}\right) dx \\ &= x+\log(1-x)-\log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$x + \log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x + x^3),x]
```

```
[Out] x + Log[1 - x] - Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{-x+x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(-x + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^3)/(-x + x^3), x]
```

fricas [A] time = 1.56, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
[Out] x + log(x - 1) - log(x)
```

giac [A] time = 0.37, size = 12, normalized size = 1.00

$$x + \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="giac")
[Out] x + log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$x - \ln(x) + \ln(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x),x)
[Out] x+ln(x-1)-ln(x)
```

maxima [A] time = 1.01, size = 10, normalized size = 0.83

$$x + \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
[Out] x + log(x - 1) - log(x)
```

mupad [B] time = 2.14, size = 10, normalized size = 0.83

$$x - 2\,\mathrm{atanh}(2\,x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x - 1)
```

sympy [A] time = 0.10, size = 8, normalized size = 0.67

$$x - \log(x) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x),x)
[Out] x - log(x) + log(x - 1)
```

3.367 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1620}

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{-x^2+x^3} \, dx &= \int \frac{1+x^3}{(-1+x)x^2} \, dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right) dx\\ &= \frac{1}{x}+x+2\log(1-x)-\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^3}{-x^2+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^3)/(-x^2 + x^3),x]
[Out] IntegrateAlgebraic[(1 + x^3)/(-x^2 + x^3), x]
```

fricas [A] time = 1.18, size = 21, normalized size = 1.24

$$\frac{x^2 + 2\,x\log\left(x - 1\right) - x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

giac [A] time = 0.32, size = 17, normalized size = 1.00

$$x + \frac{1}{x} + 2\,\log\left(|x - 1|\right) - \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$x - \ln(x) + 2\ln\left(x - 1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x)
[Out] x+2*ln(x-1)+1/x-ln(x)
```

maxima [A] time = 0.89, size = 15, normalized size = 0.88

$$x + \frac{1}{x} + 2\,\log\left(x - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x + 2\,\ln\left(x - 1\right) - \ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 + 1)/(x^2 - x^3),x)
[Out] x + 2*log(x - 1) - log(x) + 1/x
```

sympy [A] time = 0.11, size = 14, normalized size = 0.82

$$x - \log\left(x\right) + 2\log\left(x - 1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

3.368 $\int \frac{-1+x^5}{-x+x^3} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1802}

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^5}{-x+x^3} \, dx &= \int \frac{-1+x^5}{x\left(-1+x^2\right)} \, dx\\
&= \int \left(1+\frac{1}{-1-x}+\frac{1}{x}+x^2\right) \, dx\\
&= x+\frac{x^3}{3}+\log(x)-\log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^5)/(-x + x^3),x]
```

```
[Out] x + x^3/3 + Log[x] - Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^5}{-x+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^5)/(-x + x^3),x]
[Out] IntegrateAlgebraic[(-1 + x^5)/(-x + x^3), x]
```

fricas [A] time = 1.01, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

giac [A] time = 0.26, size = 17, normalized size = 1.00

$$\frac{1}{3}x^3 + x - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="giac")
[Out] 1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^3}{3} + x + \ln(x) - \ln(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-1)/(x^3-x),x)
[Out] x+1/3*x^3+ln(x)-ln(x+1)
```

maxima [A] time = 0.90, size = 15, normalized size = 0.88

$$\frac{1}{3}x^3 + x - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
[Out] 1/3*x^3 + x - log(x + 1) + log(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$x - 2\,\mathrm{atanh}(2\,x+1) + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 - 1)/(x - x^3),x)
[Out] x - 2*atanh(2*x + 1) + x^3/3
```

sympy [A] time = 0.10, size = 14, normalized size = 0.82

$$\frac{x^3}{3} + x + \log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-1)/(x**3-x),x)
[Out] x**3/3 + x + log(x) - log(x + 1)
```

3.369 $\int \frac{1+x^4}{x^3+x^5}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 1252, 894}

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x^3 + x^5),x]
```

```
[Out] -1/(2*x^2) - Log[x] + Log[1 + x^2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x^3+x^5}\,dx &= \int \frac{1+x^4}{x^3\left(1+x^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^2(1+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{2}{1+x}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\log(x)+\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{1}{2x^2}+\log\left(x^2+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x^3 + x^5),x]
[Out] -1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^4}{x^3+x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^4)/(x^3 + x^5),x]
[Out] IntegrateAlgebraic[(1 + x^4)/(x^3 + x^5), x]
```

fricas [A] time = 1.02, size = 25, normalized size = 1.39

$$\frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log(x)-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.27, size = 23, normalized size = 1.28

$$\frac{x^2-1}{2\,x^2}+\log\left(x^2+1\right)-\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
[Out] 1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

maple [A] time = 0.01, size = 17, normalized size = 0.94

$$-\ln(x)+\ln\left(x^2+1\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^5+x^3),x)
[Out] -1/2/x^2-ln(x)+ln(x^2+1)
```

maxima [A] time = 1.92, size = 16, normalized size = 0.89

$$-\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
[Out] -1/2/x^2 + log(x^2 + 1) - log(x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.89

$$\ln\left(x^2+1\right)-\ln(x)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 1)/(x^3 + x^5),x)
```

```
[Out] log(x^2 + 1) - log(x) - 1/(2*x^2)
```

sympy [**A**] time = 0.11, size = 15, normalized size = 0.83

$$-\log(x) + \log\left(x^2 + 1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**5+x**3),x)
```

```
[Out] -log(x) + log(x**2 + 1) - 1/(2*x**2)
```

3.370 $\int \frac{1+x^2}{x+2x^2+x^3} \, dx$

Optimal. Leaf size=10

$$\frac{2}{x+1}+\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1594, 27, 894}

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{x+2x^2+x^3} \, dx &= \int \frac{1+x^2}{x\left(1+2x+x^2\right)} \, dx \\
&= \int \frac{1+x^2}{x(1+x)^2} \, dx \\
&= \int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right) dx \\
&= \frac{2}{1+x}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] 2/(1 + x) + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(x + 2*x^2 + x^3), x]
```

fricas [A] time = 1.03, size = 14, normalized size = 1.40

$$\frac{(x+1)\log(x)+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x) + 2)/(x + 1)
```

giac [A] time = 0.36, size = 11, normalized size = 1.10

$$\frac{2}{x+1}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
```

```
[Out] 2/(x + 1) + log(abs(x))
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\ln(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+2*x^2+x),x)
```

```
[Out] 2/(x+1)+ln(x)
```

maxima [A] time = 0.89, size = 10, normalized size = 1.00

$$\frac{2}{x+1}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

```
[Out] 2/(x + 1) + log(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$\ln(x)+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 2*x^2 + x^3),x)
```

```
[Out] log(x) + 2/(x + 1)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.70

$$\log(x) + \frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+2*x**2+x),x)
```

```
[Out] log(x) + 2/(x + 1)
```

3.371 $\int \frac{1+x^5}{-10x-3x^2+x^3} dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1594, 1628}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx &= \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx\\ &= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx\\ &= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^5)/(-10*x - 3*x^2 + x^3), x]
```

fricas [A] time = 0.93, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

giac [A] time = 0.29, size = 33, normalized size = 0.79

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}+19\,x-\frac{31}{14}\,\log\left(|x+2|\right)+\frac{3126}{35}\,\log\left(|x-5|\right)-\frac{1}{10}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

maple [A] time = 0.01, size = 31, normalized size = 0.74

$$\frac{x^{3}}{3}+\frac{3x^{2}}{2}+19x-\frac{\ln(x)}{10}+\frac{3126\ln\left(x-5\right)}{35}-\frac{31\ln\left(x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(x+2)-1/10*ln(x)+3126/35*ln(x-5)
```

maxima [A] time = 0.88, size = 30, normalized size = 0.71

$$\frac{1}{3}\,x^{3}+\frac{3}{2}\,x^{2}+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

mupad [B] time = 0.05, size = 30, normalized size = 0.71

$$19\,x-\frac{31\,\ln\left(x+2\right)}{14}+\frac{3126\,\ln\left(x-5\right)}{35}-\frac{\ln(x)}{10}+\frac{3\,x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

```
[Out] 19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
x^3/3
```

sympy [**A**] time = 0.15, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

3.372 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {6725, 203, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx &= \int \left(-\frac{5}{5+x^2}+\frac{6+x}{3+2x+x^2}\right) dx \\
&= -\left(5\int \frac{1}{5+x^2}\,dx\right)+\int \frac{6+x}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\int \frac{2+2x}{3+2x+x^2}\,dx+5\int \frac{1}{3+2x+x^2}\,dx \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{2}\log\left(3+2x+x^2\right)-10\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right) \\
&= -\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{1}{2}\log\left(x^2+2x+3\right)-\sqrt{5}\,\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{5\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] -(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
+ 2*x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)), x]
```

fricas [A] time = 1.15, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)+\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

giac [A] time = 0.36, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-\sqrt{5}\,\arctan\left(\frac{1}{5}\sqrt{5}\,x\right)+\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

maple [A] time = 0.00, size = 41, normalized size = 0.89

$$-\sqrt{5}\ \arctan\left(\frac{\sqrt{5}\ x}{5}\right)+\frac{5\sqrt{2}\ \arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

```
[Out] -5^(1/2)*arctan(1/5*5^(1/2)*x)+5/2*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))+1/2*
ln(x^2+2*x+3)
```

maxima [A] time = 1.97, size = 38, normalized size = 0.83

$$\frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) + 1
/2*log(x^2 + 2*x + 3)
```

mupad [B] time = 0.00, size = 88, normalized size = 1.91

$$\frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2}+\frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2}+\sqrt{5}\ \mathrm{atan}\left(\frac{2000\sqrt{5}}{2000x+1120}-\frac{224\sqrt{5}\ x}{2000x+1120}\right)-\frac{\sqrt{2}\ \ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}+\frac{\sqrt{2}\ \ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)5\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

```
[Out] log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000*
5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*log(
x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

sympy [A] time = 0.21, size = 51, normalized size = 1.11

$$\frac{\log\left(x^2+2x+3\right)}{2}-\sqrt{5}\ \mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+\frac{5\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

```
[Out] log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)*
x/2 + sqrt(2)/2)/2
```

3.373 $\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)} dx$

Optimal. Leaf size=19

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6688, 616, 31}

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

```
[Out] -Log[3 + x]/8 + Log[1 + 3*x]/8
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^2\right)\left(3+\frac{10x}{1+x^2}\right)} dx &= \int \frac{1}{3+10x+3x^2} dx\\ &= \frac{3}{8}\int \frac{1}{1+3x} dx-\frac{3}{8}\int \frac{1}{9+3x} dx\\ &= -\frac{1}{8}\log(3+x)+\frac{1}{8}\log(1+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{8}\log(3x+1)-\frac{1}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
```

```
[Out] -1/8*Log[3 + x] + Log[1 + 3*x]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x^2)\left(3+\frac{10x}{1+x^2}\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))),x]
[Out] IntegrateAlgebraic[1/((1 + x^2)*(3 + (10*x)/(1 + x^2))), x]
```

fricas [A] time = 1.08, size = 15, normalized size = 0.79

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="fricas")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

giac [A] time = 0.28, size = 17, normalized size = 0.89

$$\frac{1}{8}\log(|3x+1|) - \frac{1}{8}\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="giac")
[Out] 1/8*log(abs(3*x + 1)) - 1/8*log(abs(x + 3))
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{\ln(3x+1)}{8} - \frac{\ln(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(3+10/(x^2+1)*x),x)
[Out] -1/8*ln(x+3)+1/8*ln(1+3*x)
```

maxima [A] time = 1.03, size = 15, normalized size = 0.79

$$\frac{1}{8}\log(3x+1) - \frac{1}{8}\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(3+10*x/(x^2+1)),x, algorithm="maxima")
[Out] 1/8*log(3*x + 1) - 1/8*log(x + 3)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.42

$$-\frac{\operatorname{atanh}\left(\frac{3x}{4}+\frac{5}{4}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*((10*x)/(x^2 + 1) + 3)),x)
```

```
[Out] -atanh((3*x)/4 + 5/4)/4
```

sympy [A] time = 0.11, size = 14, normalized size = 0.74

$$\frac{\log\left(x+\frac{1}{3}\right)}{8}-\frac{\log\left(x+3\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(3+10*x/(x**2+1)),x)
```

```
[Out] log(x + 1/3)/8 - log(x + 3)/8
```

3.374 $\int \frac{x^3}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=40

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1386, 701, 632, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx = \int \frac{x^4}{2+13x+15x^2}\,dx$$
$$= \int \left(\frac{139}{3375} - \frac{13x}{225} + \frac{x^2}{15} - \frac{278+1417x}{3375\left(2+13x+15x^2\right)}\right)dx$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} + \frac{3}{875}\int \frac{1}{3+15x}\,dx - \frac{80}{189}\int \frac{1}{10+15x}\,dx$$
$$= \frac{139x}{3375} - \frac{13x^2}{450} + \frac{x^3}{45} - \frac{16}{567}\log(2+3x) + \frac{\log(1+5x)}{4375}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} - \frac{16}{567}\log(3x+2) + \frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{13+\frac{2}{x}+15x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(13 + 2/x + 15*x),x]
```

```
[Out] IntegrateAlgebraic[x^3/(13 + 2/x + 15*x), x]
```

fricas [A] time = 1.09, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x+1) - \frac{16}{567}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

giac [A] time = 0.28, size = 32, normalized size = 0.80

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(|5x+1|) - \frac{16}{567}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

maple [A] time = 0.01, size = 31, normalized size = 0.78

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\ln(5x+1)}{4375} - \frac{16\ln(3x+2)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(13+2/x+15*x),x)
```

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

maxima [A] time = 0.99, size = 30, normalized size = 0.75

$$\frac{1}{45}x^3 - \frac{13}{450}x^2 + \frac{139}{3375}x + \frac{1}{4375}\log(5x+1) - \frac{16}{567}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

mupad [B] time = 2.09, size = 26, normalized size = 0.65

$$\frac{139\,x}{3375} - \frac{16\,\ln\left(x+\frac{2}{3}\right)}{567} + \frac{\ln\left(x+\frac{1}{5}\right)}{4375} - \frac{13\,x^2}{450} + \frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(15*x + 2/x + 13),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

sympy [A] time = 0.12, size = 34, normalized size = 0.85

$$\frac{x^3}{45} - \frac{13x^2}{450} + \frac{139x}{3375} + \frac{\log\left(x+\frac{1}{5}\right)}{4375} - \frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(13+2/x+15*x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

3.375 $\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1386, 701, 632, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 + 2/x + 15*x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx = \int \frac{x^3}{2+13x+15x^2}\,dx$$
$$= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\left(2+13x+15x^2\right)}\right)dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx$$
$$= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

Mathematica [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 + 2/x + 15*x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{13+\frac{2}{x}+15x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(13 + 2/x + 15*x),x]
[Out] IntegrateAlgebraic[x^2/(13 + 2/x + 15*x), x]
```

fricas [A] time = 0.93, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

giac [A] time = 0.34, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 26, normalized size = 0.79

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\ln\left(5x+1\right)}{875}+\frac{8\ln\left(3x+2\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13+2/x+15*x),x)
[Out] -13/225*x+1/30*x^2+8/189*ln(3*x+2)-1/875*ln(5*x+1)
```

maxima [A] time = 1.02, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2 - \frac{13}{225}\,x - \frac{1}{875}\,\log\left(5\,x+1\right) + \frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

mupad [B] time = 0.03, size = 21, normalized size = 0.64

$$\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189} - \frac{13\,x}{225} - \frac{\ln\left(x+\frac{1}{5}\right)}{875} + \frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15*x + 2/x + 13),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

sympy [A] time = 0.12, size = 27, normalized size = 0.82

$$\frac{x^2}{30} - \frac{13x}{225} - \frac{\log\left(x+\frac{1}{5}\right)}{875} + \frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(13+2/x+15*x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

3.376 $\int \frac{x}{13+\frac{2}{x}+15x}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1386, 703, 632, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(13 + 2/x + 15*x),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{13+\frac{2}{x}+15x}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\
&= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\
&= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(13 + 2/x + 15*x),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{13+\frac{2}{x}+15x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(13 + 2/x + 15*x),x]
[Out] IntegrateAlgebraic[x/(13 + 2/x + 15*x), x]
```

fricas [A] time = 1.47, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

giac [A] time = 0.27, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{15}+\frac{\ln\left(5x+1\right)}{175}-\frac{4\ln\left(3x+2\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13+2/x+15*x),x)
[Out] 1/15*x-4/63*ln(3*x+2)+1/175*ln(5*x+1)
```

maxima [A] time = 1.03, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

mupad [B] time = 2.12, size = 16, normalized size = 0.62

$$\frac{x}{15} - \frac{4\,\ln\left(x+\frac{2}{3}\right)}{63} + \frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15*x + 2/x + 13),x)
```

```
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

sympy [A] time = 0.12, size = 20, normalized size = 0.77

$$\frac{x}{15} + \frac{\log\left(x+\frac{1}{5}\right)}{175} - \frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(13+2/x+15*x),x)
```

```
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

3.377 $\int \frac{1}{13+\frac{2}{x}+15x} \, dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1350, 632, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(13 + 2/x + 15*x)^(-1),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1350

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol] :> Int[(b
 + a*x^n + c*x^(2*n))^p/x^(n*p), x] /; FreeQ[{a, b, c, n}, x] && EqQ[mn, -n
] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{13+\frac{2}{x}+15x} \, dx &= \int \frac{x}{2+13x+15x^2} \, dx\\ &= -\left(\frac{3}{7}\int \frac{1}{3+15x} \, dx\right)+\frac{10}{7}\int \frac{1}{10+15x} \, dx\\ &= \frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 + 2/x + 15*x)^(-1),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{13+\frac{2}{x}+15x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(13 + 2/x + 15*x)^(-1),x]
[Out] IntegrateAlgebraic[(13 + 2/x + 15*x)^(-1), x]
```

fricas [A] time = 0.93, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5\,x+1\right)+\frac{2}{21}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

giac [A] time = 0.29, size = 19, normalized size = 0.90

$$-\frac{1}{35}\log\left(|5\,x+1|\right)+\frac{2}{21}\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\frac{\ln\left(5x+1\right)}{35}+\frac{2\ln\left(3x+2\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13+2/x+15*x),x)
[Out] 2/21*ln(3*x+2)-1/35*ln(5*x+1)
```

maxima [A] time = 1.17, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5\,x+1\right)+\frac{2}{21}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

mupad [B] time = 2.11, size = 13, normalized size = 0.62

$$\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x + 2/x + 13),x)
```

```
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

sympy [A] time = 0.11, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(13+2/x+15*x),x)
```

```
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

3.378 $\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 616, 31}

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(13 + 2/x + 15*x)),x]
[Out] -Log[2 + 3*x]/7 + Log[1 + 5*x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{2+13x+15x^2}\,dx\\ &= \frac{15}{7}\int \frac{1}{3+15x}\,dx-\frac{15}{7}\int \frac{1}{10+15x}\,dx\\ &= -\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{7}\log(5x+1)-\frac{1}{7}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(13 + 2/x + 15*x)),x]
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(13+\frac{2}{x}+15x\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(13 + 2/x + 15*x)),x]
[Out] IntegrateAlgebraic[1/(x*(13 + 2/x + 15*x)), x]
```

fricas [A] time = 1.10, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

giac [A] time = 0.27, size = 19, normalized size = 0.90

$$\frac{1}{7}\,\log\left(|5\,x+1|\right)-\frac{1}{7}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln\left(5x+1\right)}{7}-\frac{\ln\left(3x+2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(13+2/x+15*x),x)
[Out] -1/7*ln(3*x+2)+1/7*ln(5*x+1)
```

maxima [A] time = 0.93, size = 17, normalized size = 0.81

$$\frac{1}{7}\,\log\left(5\,x+1\right)-\frac{1}{7}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x, algorithm="maxima")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

mupad [B] time = 0.08, size = 8, normalized size = 0.38

$$-\frac{2\,\mathrm{atanh}\left(\frac{30\,x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(15*x + 2/x + 13)),x)
```

```
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

sympy [**A**] time = 0.11, size = 15, normalized size = 0.71

$$\frac{\log\left(x+\frac{1}{5}\right)}{7}-\frac{\log\left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(13+2/x+15*x),x)
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

3.379 $\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1386, 705, 29, 632, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(13 + 2/x + 15*x)),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\ &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(13+\frac{2}{x}+15x\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(13 + 2/x + 15*x)),x]
[Out] IntegrateAlgebraic[1/(x^2*(13 + 2/x + 15*x)), x]
```

fricas [A] time = 0.83, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log\left(5x+1\right)+\frac{3}{14}\log\left(3x+2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

giac [A] time = 0.37, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log\left(|5x+1|\right)+\frac{3}{14}\log\left(|3x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{\ln(x)}{2}-\frac{5\ln\left(5x+1\right)}{7}+\frac{3\ln\left(3x+2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(13+2/x+15*x),x)
[Out] 1/2*ln(x)+3/14*ln(3*x+2)-5/7*ln(5*x+1)
```

maxima [A] time = 0.92, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(13+2/x+15*x),x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

mupad [B] time = 0.09, size = 17, normalized size = 0.63

$$\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(15*x + 2/x + 13)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

sympy [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(13+2/x+15*x),x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

3.380 $\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(2*x) - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx\\
&= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(13 + 2/x + 15*x)),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(13 + \frac{2}{x} + 15x\right)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(13 + 2/x + 15*x)),x]
[Out] IntegrateAlgebraic[1/(x^3*(13 + 2/x + 15*x)), x]
```

fricas [A] time = 0.97, size = 30, normalized size = 0.88

$$\frac{100 \, x \log\left(5 \, x + 1\right) - 9 \, x \log\left(3 \, x + 2\right) - 91 \, x \log\left(x\right) - 14}{28 \, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

giac [A] time = 0.27, size = 29, normalized size = 0.85

$$-\frac{1}{2 \, x} + \frac{25}{7} \, \log\left({\left| 5 \, x + 1 \right|}\right) - \frac{9}{28} \, \log\left({\left| 3 \, x + 2 \right|}\right) - \frac{13}{4} \, \log\left({\left| x \right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

maple [A] time = 0.01, size = 27, normalized size = 0.79

$$-\frac{13 \ln\left(x\right)}{4}+\frac{25 \ln\left(5 x +1\right)}{7}-\frac{9 \ln\left(3 x +2\right)}{28}-\frac{1}{2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(13+2/x+15*x),x)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(3*x+2)+25/7*ln(5*x+1)
```

maxima [A] time = 1.06, size = 26, normalized size = 0.76

$$-\frac{1}{2 \, x} + \frac{25}{7} \, \log\left(5 \, x + 1\right) - \frac{9}{28} \, \log\left(3 \, x + 2\right) - \frac{13}{4} \, \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(13+2/x+15*x),x, algorithm="maxima")
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

mupad [B] time = 0.03, size = 22, normalized size = 0.65

$$\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(15*x + 2/x + 13)),x)
```

```
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

sympy [A] time = 0.16, size = 31, normalized size = 0.91

$$-\frac{13\log(x)}{4} + \frac{25\log\left(x+\frac{1}{5}\right)}{7} - \frac{9\log\left(x+\frac{2}{3}\right)}{28} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(13+2/x+15*x),x)
```

```
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

3.381 $\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(4*x^2) + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx\\
&= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(13+\frac{2}{x}+15x\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(13 + 2/x + 15*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(13 + 2/x + 15*x)), x]
```

fricas [A] time = 1.12, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log(5\,x+1)-27\,x^2\log(3\,x+2)-973\,x^2\log(x)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

giac [A] time = 0.28, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(|5\,x+1|)+\frac{27}{56}\log(|3\,x+2|)+\frac{139}{8}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.78

$$\frac{139\ln(x)}{8}-\frac{125\ln(5x+1)}{7}+\frac{27\ln(3x+2)}{56}+\frac{13}{4x}-\frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(13+2/x+15*x),x)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(3*x+2)-125/7*ln(5*x+1)
```

maxima [A] time = 1.02, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\log(5\,x+1)+\frac{27}{56}\log(3\,x+2)+\frac{139}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

mupad [B] time = 0.04, size = 26, normalized size = 0.63

$$\frac{27\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\ln(x)}{8}+\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(15*x + 2/x + 13)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

sympy [A] time = 0.17, size = 36, normalized size = 0.88

$$\frac{139\log(x)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(13+2/x+15*x),x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

3.382 $\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1386, 709, 800}

$$\frac{13}{8x^2}-\frac{1}{6x^3}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/(6*x^3) + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1386

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx\\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx\\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.00

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(13+\frac{2}{x}+15x\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(13 + 2/x + 15*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(13 + 2/x + 15*x)), x]
```

fricas [A] time = 1.02, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log(5\,x+1)-243\,x^3\log(3\,x+2)-29757\,x^3\log(x)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

giac [A] time = 0.24, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(|5\,x+1|)-\frac{81}{112}\log(|3\,x+2|)-\frac{1417}{16}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{1417\ln(x)}{16}+\frac{625\ln(5x+1)}{7}-\frac{81\ln(3x+2)}{112}-\frac{139}{8x}+\frac{13}{8x^2}-\frac{1}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(13+2/x+15*x),x)
```

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(3*x+2)+625/7*ln(5*x+1)
```

maxima [A] time = 1.07, size = 36, normalized size = 0.75

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\log(5\,x+1)-\frac{81}{112}\log(3\,x+2)-\frac{1417}{16}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(13+2/x+15*x),x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

mupad [B] time = 0.04, size = 32, normalized size = 0.67

$$\frac{625\ \ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\ \ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\ \ln(x)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(15*x + 2/x + 13)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

sympy [A] time = 0.18, size = 41, normalized size = 0.85

$$-\frac{1417\log{(x)}}{16}+\frac{625\log{\left(x+\frac{1}{5}\right)}}{7}-\frac{81\log{\left(x+\frac{2}{3}\right)}}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(13+2/x+15*x),x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

3.383 $\int \frac{x^2}{2-\left(1+x^2\right)^4} \, dx$

Optimal. Leaf size=157

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\,\tanh^{-1}\left(\ldots\right.}{4\ 2^{3/4}}$$

Rubi [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 203, 1972, 205}

$$\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}-\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{\sqrt[4]{2}-1}\,\tanh^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 + x^2)^4),x]
```

```
[Out] ((I/4)*Sqrt[1 - I*2^(1/4)]*ArcTan[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) - ((I/4)*
Sqrt[1 + I*2^(1/4)]*ArcTan[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) - (Sqrt[1 + 2^(1
/4)]*ArcTan[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4)) + (Sqrt[-1 + 2^(1/4)]*ArcTanh
[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx = \int \left(\frac{-\sqrt[4]{2}+\sqrt{2}}{8\left(-1+\sqrt[4]{2}-x^2\right)} + \frac{-\sqrt[4]{2}-\sqrt{2}}{8\left(1+\sqrt[4]{2}+x^2\right)} + \frac{-\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1+x^2\right)\right)} + \frac{-\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1+x^2\right)\right)}\right) dx$$

$$= -\frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}-x^2}\,dx}{4\,2^{3/4}} - \frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i(1+x^2)}\,dx}{4\,2^{3/4}} - \frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i(1+x^2)}\,dx}{4\,2^{3/4}} - \frac{(1 \ldots}{}$$

$$= -\frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}} + \frac{\sqrt{-1+\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}} - \frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}+ix^2}\,dx}{4\,2^{3/4}} - \ldots$$

$$= \frac{i\sqrt{1-i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}} - \frac{i\sqrt{1+i\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\,2^{3/4}} - \frac{\sqrt{1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}} + \ldots$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 + x^2)^4),x]
[Out] -1/8*RootSum[-1 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 +
 3*#1^2 + 3*#1^4 + #1^6) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2-(1+x^2)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 - (1 + x^2)^4),x]
[Out] IntegrateAlgebraic[x^2/(2 - (1 + x^2)^4), x]
```

fricas [B] time = 3.45, size = 1506, normalized size = 9.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="fricas")
[Out] -1/16*sqrt(2)*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^
(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log
(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(
2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(
3/4) - sqrt(2)) + 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(
2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4
) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) -
 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) +
 sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(1/2*sqrt
(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
 sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 -
 (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) + 4*sqrt(-
```

```
3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) -
3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2
^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2
^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4)
+ sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4)
- sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4
) - sqrt(2))^2 + sqrt(2)*(2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2
))^2 - 4*sqrt(2))*(2^(3/4) - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2
+ 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2
+ 1)*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)
*(2^(3/4) + sqrt(2)) - 4*sqrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(
2))*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sq
rt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16
*sqrt(2)*sqrt(1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4)
 + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4
*((sqrt(2)*(2^(3/4) + sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2))^2 + sqrt(2)*(
2^(3/4) + sqrt(2))^2 - (sqrt(2)*(2^(3/4) + sqrt(2))^2 - 4*sqrt(2))*(2^(3/4)
 - sqrt(2)) - 4*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*
(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)*((sqrt(2)*(2^(3/4) +
sqrt(2)) + sqrt(2))*(2^(3/4) - sqrt(2)) - sqrt(2)*(2^(3/4) + sqrt(2)) - 4*s
qrt(2)) - 4*sqrt(2)*(2^(3/4) + sqrt(2)) + 4*sqrt(2))*sqrt(1/2*sqrt(2) - sqr
t(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2))
 - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) - sqrt(2))*l
og(1/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))
^2 - ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2
) - 6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) - sqrt(2))*log(-1
/4*((2^(3/4) + sqrt(2))^3 + (2^(3/4) + sqrt(2) + 1)*(2^(3/4) - sqrt(2))^2 -
 ((2^(3/4) + sqrt(2))^2 - 4)*(2^(3/4) - sqrt(2)) - 4*2^(3/4) - 4*sqrt(2) -
6)*sqrt(2^(3/4) - sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*lo
g(4*((2^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4
) - 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) - 1/256*sqrt(2))*log(-4*((2
^(3/4) + sqrt(2))^3 - (2^(3/4) + sqrt(2))^2 - 10)*sqrt(-1/256*2^(3/4) - 1/2
56*sqrt(2)) + 3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(x^2+1\right)^4 - 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 + 1)^4 - 2), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.34

$$-\frac{\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^2\ln\left(-\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)+x\right)}{8\left(\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^7+3\,\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^5+3\,\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)^3+\mathrm{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2-1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7+3*_R^5+3*_R^3+_R)*ln(-_R+x),_R=RootOf(_Z^8+4*_Z^6+6*_Z^
4+4*_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(x^2+1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 + 1)^4 - 2), x)
```

mupad [B] time = 2.75, size = 144, normalized size = 0.92

$$\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(4096\,x-\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2\left(262144\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2 x\,67108864\right)\right)+256\right)\right)-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}+\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 + 1)^4 - 2),x)
```

```
[Out] symsum(log(- root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(56*x
 - root(z^8 - z^4/16384 + z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 + z^2/1048576 - 1/1073741824, z, k)*(4096*x - root(z^8 - z^4/16384 +
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x + 67108864*root(z^8 - z^4/16
384 + z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 + z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.22, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4+1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}-\frac{262144t^5}{3}-\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 + 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 - 262144*_t**5/3 - 40*_t/3 + x)))
```

3.384 $\int \frac{x^2}{2-\left(1-x^2\right)^4} \, dx$

Optimal. Leaf size=157

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}}{4\ 2^{3/4}}$$

Rubi [A] time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6740, 206, 203, 1972, 208}

$$-\frac{\sqrt{\sqrt[4]{2}-1}\tan^{-1}\left(\frac{x}{\sqrt{\sqrt[4]{2}-1}}\right)}{4\ 2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -(Sqrt[-1 + 2^(1/4)]*ArcTan[x/Sqrt[-1 + 2^(1/4)]])/(4*2^(3/4)) - ((I/4)*Sqr
t[1 - I*2^(1/4)]*ArcTanh[x/Sqrt[1 - I*2^(1/4)]])/2^(3/4) + ((I/4)*Sqrt[1 +
I*2^(1/4)]*ArcTanh[x/Sqrt[1 + I*2^(1/4)]])/2^(3/4) + (Sqrt[1 + 2^(1/4)]*Arc
Tanh[x/Sqrt[1 + 2^(1/4)]])/(4*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2-\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{2}+\sqrt{2}}{8\left(1+\sqrt[4]{2}-x^2\right)}+\frac{\sqrt[4]{2}-\sqrt{2}}{8\left(-1+\sqrt[4]{2}+x^2\right)}+\frac{\sqrt[4]{2}+i\sqrt{2}}{8\left(\sqrt[4]{2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{2}-i\sqrt{2}}{8\left(\sqrt[4]{2}+i\left(1-x^2\right)\right)}\right) dx$$

$$= \frac{\left(1-\sqrt[4]{2}\right)\int \frac{1}{-1+\sqrt[4]{2}+x^2}\,dx}{4\,2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}+i\left(1-x^2\right)}\,dx}{4\,2^{3/4}}+\frac{\left(1+i\sqrt[4]{2}\right)\int \frac{1}{\sqrt[4]{2}-i\left(1-x^2\right)}\,dx}{4\,2^{3/4}}+\frac{(1+}{}$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt{1+\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{\left(1-i\sqrt[4]{2}\right)\int \frac{1}{i+\sqrt[4]{2}-ix^2}\,dx}{4\,2^{3/4}}+$$

$$= -\frac{\sqrt{-1+\sqrt[4]{2}}\,\tan^{-1}\left(\frac{x}{\sqrt{-1+\sqrt[4]{2}}}\right)}{4\,2^{3/4}}-\frac{i\sqrt{1-i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-i\sqrt[4]{2}}}\right)}{4\,2^{3/4}}+\frac{i\sqrt{1+i\sqrt[4]{2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+}}\right.}{4\,2^{3/4}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.39

$$-\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2-1\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[-1 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1
+ 3*#1^2 - 3*#1^4 + #1^6) & ]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2-\left(1-x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 - (1 - x^2)^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(2 - (1 - x^2)^4), x]
```

fricas [B] time = 4.05, size = 1546, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2
^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*lo
g(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt
(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^
(3/4) + sqrt(2)) + 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sq
rt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqr
t(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqr
t(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2)
 + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sq
rt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2)*sqrt(-1/2*s
qrt(2) + sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4
) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*log(-1/4*((sqrt(2)*(2^(3/4)
 - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^
2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) + 4*((s
```

```
qrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4
) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) +
 sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)
*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(2) + sqrt(-3/16*(2^(3/4) +
 sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) -
 sqrt(2))^2 + 1)) + 6*x) - 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(
3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(
3/4) - sqrt(2))^2 + 1))*log(1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2
^(3/4) + sqrt(2))^2 - sqrt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - s
qrt(2))^2 - 4*sqrt(2))*(2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)
) - sqrt(2))*(2^(3/4) + sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))
*sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt
(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*
sqrt(2))*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4
) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x)
+ 1/16*sqrt(2)*sqrt(-1/2*sqrt(2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(
2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1))*l
og(-1/4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) + sqrt(2))^2 - sq
rt(2)*(2^(3/4) - sqrt(2))^2 - (sqrt(2)*(2^(3/4) - sqrt(2))^2 - 4*sqrt(2))*(
2^(3/4) + sqrt(2)) - 4*((sqrt(2)*(2^(3/4) - sqrt(2)) - sqrt(2))*(2^(3/4) +
sqrt(2)) + sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-3/16*(2^(3/4) + s
qrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) - sqrt(2)) - 3/16*(2^(3/4) - s
qrt(2))^2 + 1) - 4*sqrt(2)*(2^(3/4) - sqrt(2)) - 4*sqrt(2))*sqrt(-1/2*sqrt(
2) - sqrt(-3/16*(2^(3/4) + sqrt(2))^2 + 1/8*(2^(3/4) + sqrt(2))*(2^(3/4) -
sqrt(2)) - 3/16*(2^(3/4) - sqrt(2))^2 + 1)) + 6*x) + 1/16*sqrt(2^(3/4) + sq
rt(2))*log(1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sq
rt(2) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) +
4*sqrt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - 1/16*sqrt(2^(3/4) + sqrt(2)
)*log(-1/4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) + sqrt(2))^2*(2^(3/4) - sqrt(2
) - 1) - ((2^(3/4) - sqrt(2))^2 - 4)*(2^(3/4) + sqrt(2)) - 4*2^(3/4) + 4*sq
rt(2) + 6)*sqrt(2^(3/4) + sqrt(2)) + 3*x) - sqrt(-1/256*2^(3/4) + 1/256*sqr
t(2))*log(4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/25
6*2^(3/4) + 1/256*sqrt(2)) + 3*x) + sqrt(-1/256*2^(3/4) + 1/256*sqrt(2))*lo
g(-4*((2^(3/4) - sqrt(2))^3 + (2^(3/4) - sqrt(2))^2 + 10)*sqrt(-1/256*2^(3/
4) + 1/256*sqrt(2)) + 3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((x^2 - 1)^4 - 2), x)
```

maple [C] time = 0.02, size = 56, normalized size = 0.36

$$-\frac{\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)+x\right)}{8\left(\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^7-3\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^5+3\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)^3-\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2-1\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2-(-x^2+1)^4),x)
```

```
[Out] -1/8*sum(_R^2/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^6+6*_Z^
4-4*_Z^2-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\left(x^2-1\right)^4-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2-(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/((x^2 - 1)^4 - 2), x)
```

mupad [B] time = 2.80, size = 142, normalized size = 0.90

$$\sum_{k=1}^{8}\ln\left(-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(56\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)\left(4096\,x+\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2\left(262144\,x-\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)^2 x\,67108864\right)\right)+256\right)\right)-1\right)\mathrm{root}\left(z^8-\frac{z^4}{16384}-\frac{z^2}{1048576}-\frac{1}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)^4 - 2),x)
```

```
[Out] symsum(log(- root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(56*x
 + root(z^8 - z^4/16384 - z^2/1048576 - 1/1073741824, z, k)*(root(z^8 - z^4
/16384 - z^2/1048576 - 1/1073741824, z, k)*(4096*x + root(z^8 - z^4/16384 -
 z^2/1048576 - 1/1073741824, z, k)^2*(262144*x - 67108864*root(z^8 - z^4/16
384 - z^2/1048576 - 1/1073741824, z, k)^2*x)) + 256)) - 1)*root(z^8 - z^4/1
6384 - z^2/1048576 - 1/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.23, size = 41, normalized size = 0.26

$$-\operatorname{RootSum}\left(1073741824t^8-65536t^4-1024t^2-1,\left(t\mapsto t\log\left(-\frac{67108864t^7}{3}+\frac{262144t^5}{3}+\frac{40t}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2-(-x**2+1)**4),x)
```

```
[Out] -RootSum(1073741824*_t**8 - 65536*_t**4 - 1024*_t**2 - 1, Lambda(_t, _t*log
(-67108864*_t**7/3 + 262144*_t**5/3 + 40*_t/3 + x)))
```

3.385 $\int \frac{x^2}{2+\left(1+x^2\right)^4} \, dx$

Optimal. Leaf size=188

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+$$

Rubi [A] time = 0.28, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6740, 204, 203, 1972, 205}

$$\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}-\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\tan^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTan[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/4)
) - ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTan[x/Sqrt[1 + I*(-2)^(1/4)]])/(4
*2^(3/4)) - ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTan[x/Sqrt[1 + (-2)^(1/4)]]
)/(4*2^(3/4)) + (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^(3/4
))]*ArcTan[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx = \int \left(\frac{-\sqrt[4]{-2}+i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}-x^2\right)} + \frac{-\sqrt[4]{-2}-i\sqrt{2}}{8\left(1+\sqrt[4]{-2}+x^2\right)} + \frac{-\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1+x^2\right)\right)} + \frac{-\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1+x^2\right.\right.}\right.$$

$$= \frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}+i\left(1+x^2\right)}\,dx + \frac{1}{8}\left(-\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{1+\sqrt[4]{-2}+x^2}\,dx + \frac{1}{8}\left(-\right.$$

$$= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}} - \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}} + \frac{1}{8}\left(-\sqrt[4]{-2}-\sqrt{2}\right)$$

$$= \frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}} - \frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}} - \frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-}}}{}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8+4\#1^6+6\#1^4+4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6+3\#1^4+3\#1^2+1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] RootSum[3 + 4*#1^2 + 6*#1^4 + 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(1 + 3*#1^
2 + 3*#1^4 + #1^6) & ]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2+\left(1+x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 + (1 + x^2)^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(2 + (1 + x^2)^4), x]
```

fricas [B] time = 4.13, size = 2271, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
 128*sqrt(1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2))) - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 -
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12
288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(
2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) - sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))
```

```
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)
*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*
(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32
*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(
-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/
8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 - sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2
)*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I
*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-
I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqr
t(-1/8192*I*sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))
- sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(sqrt(-
12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*
I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192
*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(
-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192
*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/819
2*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqr
t(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(
2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*
(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*
sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1
/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) +
128*sqrt(1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*
sqrt(2))) - sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I
*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2))
 + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/2
56*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/
2*sqrt(-1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))
)*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)
) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2
*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + 163
84*(sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - sqrt(2))*(1/256*I*s
qrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 16384*sqrt(2)*(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(1/
8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))
^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
- sqrt(2))*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) - sqrt(2)*(I*sqrt(2) +
 128*sqrt(-1/8192*I*sqrt(2)))) + sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2
) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8
192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))*(-I*sqrt(
2) + 128*sqrt(1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt
(-1/8192*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 3
2768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(1/8192*I*sqrt(2))) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)
))^2*(-I*sqrt(2) - 128*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqr
```

```
t(-1/8192*I*sqrt(2)) + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))
) + x) + sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(-8*(8388608
*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(
2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)
)) + 32768*(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 1
28*sqrt(-1/8192*I*sqrt(2)) + 1) - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2))
 + 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2))) + x) + sqrt(-1/256
*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/81
92*I*sqrt(2)))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/2
56*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) -
1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(-
1/8192*I*sqrt(2)))^3 - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)
))^2 - 2*I*sqrt(2) - 256*sqrt(-1/8192*I*sqrt(2)) + 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(-1/8192*I*sqrt(2))) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

maple [C] time = 0.01, size = 54, normalized size = 0.29

$$\frac{\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^2 \ln\left(-\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)+x\right)}{8\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^7+24\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^5+24\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)^3+8\operatorname{RootOf}\left(\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(x^2+1)^4),x)
```

```
[Out] 1/8*sum(1/(_R^7+3*_R^5+3*_R^3+_R)*_R^2*ln(-_R+x),_R=RootOf(_Z^8+4*_Z^6+6*_Z
^4+4*_Z^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2+1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 + 1)^4 + 2), x)
```

mupad [B] time = 2.78, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8} \ln\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x+\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(4096\,x-\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2\left(786432\,x-\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2 x\,67108864\right)\right)-768\right)\right)-3\right)\operatorname{root}\left(z^8+\frac{z^4}{16384}+\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(40*x +
 root(z^8 + z^4/16384 + z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 + z^2/1048576 + 3/1073741824, z, k)*(4096*x - root(z^8 + z^4/16384 + z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x - 67108864*root(z^8 + z^4/1638
```

```
4 + z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 + z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [A] time = 0.21, size = 39, normalized size = 0.21

$$\mathrm{RootSum}\left(1073741824t^8+65536t^4+1024t^2+3,\left(t\mapsto t\log\left(67108864t^7-262144t^5+4096t^3+40t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 + 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 - 262144*_t**5 + 4096*_t**3 + 40*_t + x)))
```

3.386 $\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx$

Optimal. Leaf size=188

$$-\frac{\sqrt[4]{-1}\,\sqrt{1-\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\,\sqrt{1+\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right.}{4\ 2^{3/4}}$$

Rubi [A] time = 0.16, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6740, 206, 207, 1972, 208}

$$-\frac{\sqrt[4]{-1}\,\sqrt{1-\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}+\frac{\sqrt[4]{-1}\,\sqrt{1+\sqrt[4]{-2}}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\ 2^{3/4}}-\frac{1}{8}i\left(\sqrt[4]{-2}+\sqrt{2}\right)\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,\tanh^{-1}\left(\sqrt{\frac{1+i}{2^{3/4}+(1+i)}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] -((-1)^(1/4)*Sqrt[1 - (-2)^(1/4)]*ArcTanh[x/Sqrt[1 - (-2)^(1/4)]])/(4*2^(3/
4)) + ((-1)^(3/4)*Sqrt[1 + I*(-2)^(1/4)]*ArcTanh[x/Sqrt[1 + I*(-2)^(1/4)]])
/(4*2^(3/4)) + ((-1)^(1/4)*Sqrt[1 + (-2)^(1/4)]*ArcTanh[x/Sqrt[1 + (-2)^(1/
4)]])/(4*2^(3/4)) - (I/8)*((-2)^(1/4) + Sqrt[2])*Sqrt[(1 + I)/((1 + I) + 2^
(3/4))]*ArcTanh[Sqrt[(1 + I)/((1 + I) + 2^(3/4))]*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx = \int \left(\frac{\sqrt[4]{-2}+i\sqrt{2}}{8\left(1+\sqrt[4]{-2}-x^2\right)}+\frac{\sqrt[4]{-2}-i\sqrt{2}}{8\left(-1+\sqrt[4]{-2}+x^2\right)}+\frac{\sqrt[4]{-2}-\sqrt{2}}{8\left(\sqrt[4]{-2}-i\left(1-x^2\right)\right)}+\frac{\sqrt[4]{-2}+\sqrt{2}}{8\left(\sqrt[4]{-2}+i\left(1-\right.\right.}\right.$$

$$=\frac{1}{8}\left(\sqrt[4]{-2}-\sqrt{2}\right)\int \frac{1}{\sqrt[4]{-2}-i\left(1-x^2\right)}\,dx+\frac{1}{8}\left(\sqrt[4]{-2}-i\sqrt{2}\right)\int \frac{1}{-1+\sqrt[4]{-2}+x^2}\,dx+\frac{1}{8}\left(\right.$$

$$=-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}\sqrt{1+\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{1}{8}\left(\sqrt[4]{-2}\right.$$

$$=-\frac{\sqrt[4]{-1}\sqrt{1-\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1-\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{(-1)^{3/4}\sqrt{1+i\sqrt[4]{-2}}\tanh^{-1}\left(\frac{x}{\sqrt{1+i\sqrt[4]{-2}}}\right)}{4\,2^{3/4}}+\frac{\sqrt[4]{-1}}{}$$

Mathematica [C] time = 0.01, size = 61, normalized size = 0.32

$$\frac{1}{8}\text{RootSum}\left[\#1^8-4\#1^6+6\#1^4-4\#1^2+3\&,\frac{\#1\log(x-\#1)}{\#1^6-3\#1^4+3\#1^2-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] RootSum[3 - 4*#1^2 + 6*#1^4 - 4*#1^6 + #1^8 & , (Log[x - #1]*#1)/(-1 + 3*#1
^2 - 3*#1^4 + #1^6) & ]/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{2+\left(1-x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(2 + (1 - x^2)^4),x]
```

```
[Out] IntegrateAlgebraic[x^2/(2 + (1 - x^2)^4), x]
```

fricas [B] time = 4.00, size = 2259, normalized size = 12.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*sqrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2
)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sq
rt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2
))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((163
84*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) +
128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*
I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 +
16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2
) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*s
```

qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*s
qrt(sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(
-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqr
t(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*s
qrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1
/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/81
92*I*sqrt(2))) + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) +
sqrt(2))*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*
(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1
/8192*I*sqrt(2))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + s
qrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(sqrt(-1228
8*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqr
t(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*s
qrt(2)) + 32*sqrt(-1/8192*I*sqrt(2))) + 2*x) - 1/16*sqrt(2)*sqrt(-sqrt(-122
88*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sq
rt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*
sqrt(2)) + 32*sqrt(-1/8192*I*sqrt(2)))*log((16384*sqrt(2)*(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2)))
 + 16384*(sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/25
6*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt
(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sq
rt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt
(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sq
rt(-1/8192*I*sqrt(2))) - 1)*((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2
))) + sqrt(2))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt
(2) + 128*sqrt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sq
rt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt
(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sq
rt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*
sqrt(-1/8192*I*sqrt(2))) + 2*x) + 1/16*sqrt(2)*sqrt(-sqrt(-12288*(1/256*I*s
qrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*s
qrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32
*sqrt(-1/8192*I*sqrt(2)))*log(-(16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sqrt(
1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + 16384*(sq
rt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2))*(1/256*I*sqrt(2)
- 1/2*sqrt(-1/8192*I*sqrt(2)))^2 + 16384*sqrt(2)*(-1/256*I*sqrt(2) - 1/2*sq
rt(1/8192*I*sqrt(2)))^2 + sqrt(-12288*(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I
sqrt(2)))^2 - 12288(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 1/
8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*sqrt(-1/8192*I
sqrt(2))) - 1)((sqrt(2)*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2))) + sqrt(2
))*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) + sqrt(2)*(I*sqrt(2) + 128*sq
rt(1/8192*I*sqrt(2)))) - sqrt(2))*sqrt(-sqrt(-12288*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2 - 12288*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sq
rt(2)))^2 - 1/8*(I*sqrt(2) + 128*sqrt(1/8192*I*sqrt(2)))*(-I*sqrt(2) + 128*
sqrt(-1/8192*I*sqrt(2))) - 1) + 32*sqrt(1/8192*I*sqrt(2)) + 32*sqrt(-1/8192
*I*sqrt(2))) + 2*x) - sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*l
og(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/
256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/819
2*I*sqrt(2)) - 1) - 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2
*(-I*sqrt(2) + 128*sqrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192

```
*I*sqrt(2)) - 3)*sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) +
 sqrt(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2)))*log(-8*(8388608*(-1/25
6*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^3 + 32768*(1/256*I*sqrt(2) - 1/2*
sqrt(-1/8192*I*sqrt(2)))^2*(-I*sqrt(2) - 128*sqrt(1/8192*I*sqrt(2)) - 1) -
32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2*(-I*sqrt(2) + 128*s
qrt(-1/8192*I*sqrt(2))) - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 3)*sqr
t(1/256*I*sqrt(2) - 1/2*sqrt(-1/8192*I*sqrt(2))) + x) + sqrt(-1/256*I*sqrt(
2) - 1/2*sqrt(1/8192*I*sqrt(2)))*log(8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqr
t(1/8192*I*sqrt(2)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^2 - 2*I*sqrt(2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2)
 - 1/2*sqrt(1/8192*I*sqrt(2))) + x) - sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/81
92*I*sqrt(2)))*log(-8*(8388608*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2
)))^3 + 32768*(-1/256*I*sqrt(2) - 1/2*sqrt(1/8192*I*sqrt(2)))^2 - 2*I*sqrt(
2) - 256*sqrt(1/8192*I*sqrt(2)) - 5)*sqrt(-1/256*I*sqrt(2) - 1/2*sqrt(1/819
2*I*sqrt(2))) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

maple [C] time = 0.01, size = 56, normalized size = 0.30

$$\frac{\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^2 \ln\left(-\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)+x\right)}{8\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^7-24\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^5+24\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)^3-8\,\mathrm{RootOf}\left(\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2+(-x^2+1)^4),x)
```

```
[Out] 1/8*sum(1/(_R^7-3*_R^5+3*_R^3-_R)*_R^2*ln(-_R+x),_R=RootOf(_Z^8-4*_Z^6+6*_Z
^4-4*_Z^2+3))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2+(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^4 + 2), x)
```

mupad [B] time = 2.74, size = 142, normalized size = 0.76

$$\sum_{k=1}^{8} \ln\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(40\,x-\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)\left(4096\,x+\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2\left(786432\,x+\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)^2 x\,67108864\right)\right)-768\right)\right)-3\right)\mathrm{root}\left(z^8+\frac{z^4}{16384}-\frac{z^2}{1048576}+\frac{3}{1073741824},z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^4 + 2),x)
```

```
[Out] symsum(log(root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(40*x -
 root(z^8 + z^4/16384 - z^2/1048576 + 3/1073741824, z, k)*(root(z^8 + z^4/1
6384 - z^2/1048576 + 3/1073741824, z, k)*(4096*x + root(z^8 + z^4/16384 - z
^2/1048576 + 3/1073741824, z, k)^2*(786432*x + 67108864*root(z^8 + z^4/1638
```

```
4 - z^2/1048576 + 3/1073741824, z, k)^2*x)) - 768)) - 3)*root(z^8 + z^4/163
84 - z^2/1048576 + 3/1073741824, z, k), k, 1, 8)
```

sympy [**A**] time = 0.21, size = 39, normalized size = 0.21

$$\operatorname{RootSum}\left(1073741824t^8 + 65536t^4 - 1024t^2 + 3, \left(t \mapsto t\log\left(67108864t^7 + 262144t^5 + 4096t^3 - 40t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2+(-x**2+1)**4),x)
```

```
[Out] RootSum(1073741824*_t**8 + 65536*_t**4 - 1024*_t**2 + 3, Lambda(_t, _t*log(
67108864*_t**7 + 262144*_t**5 + 4096*_t**3 - 40*_t + x)))
```

3.387 $\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(-\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{}}}{}$$

Rubi [A] time = 1.11, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(-\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\sqrt{2}\,\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\,\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\,\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int \left(-\frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2\right)} - \frac{\sqrt{b}\left(1-x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2\right)}\right)dx$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}-b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\sqrt{-a}\sqrt{b}+b\left(1-x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} - \frac{\sqrt{b}\int \frac{1-x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(1+\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(1+\cdots\right.}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\cdots}}}{\sqrt[8]{b}}\cdots}}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{\cdots}}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{\cdots}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\cdots}}{8\sqrt{-a}\sqrt{b}\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\sqrt{\cdots}\right.}{8\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} + \cdots$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^2)/(a + b*(1 - x^2)^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\left(x^2-1\right)^4 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

maple [C] time = 0.07, size = 69, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^2+1\right)\ln\left(-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)+x\right)}{8b\left(\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^7-3\,\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^5+3\,\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^3-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(-x^2+1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\left(x^2-1\right)^4 b+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

mupad [B] time = 2.70, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8}\ln\left(a\,b^5\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,64+1\right)\,\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^4\,a^2\,b^2\,4096+\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2\,a\,b\,128-\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^5\,a^3\,b^2\,x\,32768+1\right)\right)\,\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
```

```
z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

sympy [A] time = 3.92, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7a^4b^3-6291456t^7a^3b^4+65536t^5a^3b^2-327680t^5a^2b^3-512t^3a^2b-5632t^3ab^2-32tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

3.388 $\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4} \, dx$

Optimal. Leaf size=663

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(-\sqrt{2}\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{}$$

Rubi [A] time = 0.65, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {6740, 1990, 1166, 205, 208, 1169, 634, 618, 204, 628}

$$\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(-\sqrt{2}\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}} \log\left(\sqrt{2}\sqrt[8]{b}\,x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}\,x^2\right)}{8\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\,b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\,b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4)
 - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[
-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^
(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[-
a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]]
 - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) + ArcTa
nh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^(1/4) + b^
(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-
a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x
+ b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)) - (Sq
rt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sqrt[b]] + Sqrt[
2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sq
rt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1990

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx = -\int \frac{-1+x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

$$= -\int \left(-\frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2\right)} - \frac{\sqrt{b}\left(-1+x^2\right)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2\right)}\right)dx$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}-b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\sqrt{-a}\sqrt{b}+b\left(-1+x^2\right)^2}\,dx}{2\sqrt{-a}}$$

$$= \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}-\sqrt{b}\right)\sqrt{b}+2bx^2-bx^4}\,dx}{2\sqrt{-a}} + \frac{\sqrt{b}\int \frac{-1+x^2}{\left(\sqrt{-a}+\sqrt{b}\right)\sqrt{b}-2bx^2+bx^4}\,dx}{2\sqrt{-a}}$$

$$= \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}-\left(-1-\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}\right)x}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,x}{\sqrt[8]{b}}+x^2}\,dx}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt{\sqrt{-a}+\sqrt{b}}\sqrt[8]{b}} + \frac{\int \frac{-\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}}{\sqrt[8]{b}}+\left(-1-\ldots\right.}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}+\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\ldots}}{\sqrt[8]{b}}}\ldots}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\left(1-\frac{\sqrt[4]{b}}{\sqrt{\sqrt{-a}+\sqrt{b}}}\right)\int \frac{\ldots}{\frac{\sqrt{\sqrt{-a}+\sqrt{b}}}{\sqrt[4]{b}}-\frac{\sqrt{2}\ldots}{\ldots}}\ldots}{8\sqrt{-a}\sqrt{b}\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} + \frac{\tanh^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\,\log\left(\sqrt{\ldots}\right.}{8\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[8]{b}\,x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}\,x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}\sqrt{\sqrt{-a}+\sqrt{b}}\,b^{3/8}} + \ldots$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.10

$$-\frac{\text{RootSum}\left[\#1^8 b-4\#1^6 b+6\#1^4 b-4\#1^2 b+a+b\&,\frac{\log(x-\#1)}{\#1^5-2\#1^3+\#1}\&\right]}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] -1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #1
]/(#1 - 2*#1^3 + #1^5) & ]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{a+b\left(-1+x^2\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - x^2)/(a + b*(-1 + x^2)^4),x]
```

```
[Out] IntegrateAlgebraic[(1 - x^2)/(a + b*(-1 + x^2)^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2-1}{\left(x^2-1\right)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

maple [C] time = 0.00, size = 69, normalized size = 0.10

$$\frac{\left(-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^2+1\right)\ln\left(-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)+x\right)}{8b\left(\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^7-3\,\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^5+3\,\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)^3-\mathrm{RootOf}\left(b\_Z^8-4b\_Z^6+6b\_Z^4-4b\_Z^2+a+b\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(a+b*(x^2-1)^4),x)
```

```
[Out] 1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(-_R+x),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2-1}{\left(x^2-1\right)^4 b + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(a+b*(x^2-1)^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

mupad [B] time = 0.00, size = 328, normalized size = 0.49

$$\sum_{k=1}^{8}\ln\left(a\,b^5\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2 a\,b\,64+1\right)\left(\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^4 a^2\,b^2\,4096+\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^2 a\,b\,128-\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)^5 a^3\,b^2\,x\,32768+1\right)\right)\mathrm{root}\left(16777216\,a^5\,b^3\,z^8+16777216\,a^4\,b^4\,z^8+1048576\,a^3\,b^3\,z^6+24576\,a^2\,b^2\,z^4+256\,a\,b\,z^2+1,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

```
[Out] symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 104
8576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*(4
096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6
+ 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777216*
a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^
4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 + 167772
```

```
16*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1,
 z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 +
 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k), k, 1, 8)
```

sympy [**A**] time = 3.92, size = 133, normalized size = 0.20

$$-\operatorname{RootSum}\left(t^8\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\log\left(-6291456t^7a^4b^3-6291456t^7a^3b^4+65536t^5a^3b^2-327680t^5a^2b^3-512t^3a^2b-5632t^3ab^2-32tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(a+b*(x**2-1)**4),x)
```

```
[Out] -RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-629
1456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 - 32
7680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b + x))
)
```

3.389 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}{\sqrt[6]{b}}\right)}{3b^{5/6}\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}}$$

Rubi [F] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] Defer[Int][(a*x^6 + b*(1 + x^2)^3)^(-1), x] + 2*Defer[Int][x^2/(a*x^6 + b*(
1 + x^2)^3), x] + Defer[Int][x^4/(a*x^6 + b*(1 + x^2)^3), x]
```

Rubi steps

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx=\int \left(\frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{2x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}+\frac{\cdots}{b+3bx^2+\cdots}\right)$$

$$=2\int \frac{x^2}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \frac{1}{b+3bx^2+3bx^4+a\left(1+\frac{b}{a}\right)x^6}\,dx+\int \cdots$$

$$=2\int \frac{x^2}{ax^6+b(1+x^2)^3}\,dx+\int \frac{1}{ax^6+b(1+x^2)^3}\,dx+\int \frac{x^4}{ax^6+b(1+x^2)^3}\,dx$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.57

$$\frac{1}{6}\text{RootSum}\left[\#1^6a+\#1^6b+3\#1^4b+3\#1^2b+b\&,\frac{\#1^4\log(x-\#1)+2\#1^2\log(x-\#1)+\log(x-\#1)}{\#1^5a+\#1^5b+2\#1^3b+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log[
x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3), x]
```

fricas [C] time = 4.55, size = 5653, normalized size = 33.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-
1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b
+ b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9331
2/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)
^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*log(1/6*sqrt(1/2)*s
qrt((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 +
 b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/933
12*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))*b + x) - 1/36*sqrt(1/2)*sqrt((-I*sq
rt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/
31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a +
 b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/
((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*
b^5))^(1/3) - 72/(a*b + b^2))*log(-1/6*sqrt(1/2)*sqrt((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
 b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2))*b + x) + 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2)) + 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*s
qrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1
/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
+ b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104
/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2
*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(
a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
 b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/
3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*
(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 7
2/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b +
 b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1
/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2
)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqr
t(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1
/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) +
 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)
*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*
b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5)
)^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 +
b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3
) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4
) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(
I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)
) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^
```

```
2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x)
 - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^
2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/466
56/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*
(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*
b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/
3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3
 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)
*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) -
1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b
+ b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*
b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b
 + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3)
+ 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/4665
6/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 2073
6*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(-1/12*b*sq
rt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 3*sqrt(1/3)*(a*b +
b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) -
1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^
2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sq
rt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) -
1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^
2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)
/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/9
3312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b
^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184
*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2)) + x) + 1/72*sqrt(-((a*b
+ b^2)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(
-((a^2*b^3 + 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b
^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46
656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)
*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a
*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a
*b^2 + b^3)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312
/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^
3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^
5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/
93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b
^3 + 2*a*b^4 + b^5)) + 216)/(a*b + b^2))*log(1/12*b*sqrt(-((a*b + b^2)*((-I
*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) +
 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((
a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/311
04/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)
^2*b^5))^(1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3
+ 2*a*b^4 + b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/
93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b +
b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/
(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3
 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)
```

```
*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) +
1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a
 + b)^2*b^5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^
4 + b^5)) + 216)/(a*b + b^2)) + x) - 1/72*sqrt(-((a*b + b^2)*((-I*sqrt(3) +
 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/(
(a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b
^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3
 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(
1/3) - 72/(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4
+ b^5)*((-I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b
^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1
/93312*a/((a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b
^6) + 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312
*a/((a + b)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt
(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((
a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^
5))^(1/3) - 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5))
+ 216)/(a*b + b^2))*log(-1/12*b*sqrt(-((a*b + b^2)*((-I*sqrt(3) + 1)*(1/(a*
b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b
^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
 - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^4)*(a
*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3) - 72/
(a*b + b^2)) - 3*sqrt(1/3)*(a*b + b^2)*sqrt(-((a^2*b^3 + 2*a*b^4 + b^5)*((-
I*sqrt(3) + 1)*(1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6)
+ 1/31104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/(
(a + b)^2*b^5))^(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31
104/((a*b^3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b
)^2*b^5))^(1/3) - 72/(a*b + b^2))^2 + 144*(a*b^2 + b^3)*((-I*sqrt(3) + 1)*(
1/(a*b^3 + b^4) - 1/(a*b + b^2)^2)/(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^
3 + b^4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^
(1/3) - 1296*(I*sqrt(3) + 1)*(-1/93312/(a*b^5 + b^6) + 1/31104/((a*b^3 + b^
4)*(a*b + b^2)) - 1/46656/(a*b + b^2)^3 + 1/93312*a/((a + b)^2*b^5))^(1/3)
- 72/(a*b + b^2)) + 20736*a + 5184*b)/(a^2*b^3 + 2*a*b^4 + b^5)) + 216)/(a*
b + b^2)) + x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{ax^6+\left(x^2+1\right)^3 b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

maple [C] time = 0.28, size = 67, normalized size = 0.40

$$\frac{\left(\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^4+2\,\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^2+1\right)\ln\left(-\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)+x\right)}{6\,\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^5 a+6\,\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^5 b+12\,\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right)^3 b+6\,\mathrm{RootOf}\left((a+b)\_Z^6+3\_Z^4b+3\_Z^2b+b\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(-_R+x),_R=RootOf((
a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+1\right)^2}{a x^6+\left(x^2+1\right)^3 b}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

mupad [B] time = 3.08, size = 504, normalized size = 3.00

$$\sum_{k=1}^{6} \ln\left(-a^3\,(a+b)\,\left(-\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^2\,b^2\,60-\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^4\,b^4\,864-\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^4\,a\,b^3\,864+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^3\,b^3\,x\,504+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^5\,b^5\,x\,7776+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)\,a\,x\,2+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)\,b\,x\,8+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^2\,a\,b\,12-\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^3\,a\,b^2\,x\,144+\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)^5\,a\,b^4\,x\,7776-1\right)\,3\right)\,\mathrm{root}\left(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

```
[Out] symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*
b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 46656
*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*a*b
^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3 - 60
*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b
^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b
^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^5*z^6 + 4
6656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*root(46656
*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*a*b^2*
x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2
+ 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*
z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

sympy [A] time = 1.87, size = 42, normalized size = 0.25

$$\mathrm{RootSum}\left(t^6\left(46656 a b^5+46656 b^6\right)+3888 t^4 b^4+108 t^2 b^2+1,\left(t\mapsto t\log\left(6 t b+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

```
[Out] RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b**
2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

3.390 $\int \frac{(d+ex)^3}{a+cx^4}\,dx$

Optimal. Leaf size=320

$$-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{d\,(\cdots}{\cdots}$$

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{d\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{d\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}+\frac{e^3\log\left(a+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d
^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/
4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a + c*x^
4])/(4*c)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\right)dx$$
$$= \int \frac{d^3+3de^2x^2}{a+cx^4}\,dx + \int \frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{3d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-3e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+3e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{}}{a+}}{2c}$$
$$= \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right) + \frac{1}{2}e^3\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right) + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+3e^2\right)\right)\int}{4c}$$
$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}} - \frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)}{}$$
$$= \frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}} - \frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}\,d^2+3\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{2\sqrt{2}\,a^{3/4}c^{3/4}}$$

Mathematica [A] time = 0.25, size = 322, normalized size = 1.01

$$\frac{-\sqrt{2}\,\sqrt[4]{c}\left(\sqrt[4]{a}\,\sqrt{c}\,d^3-3a^{3/4}de^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\sqrt{2}\,\sqrt[4]{c}\left(\sqrt[4]{a}\,\sqrt{c}\,d^3-3a^{3/4}de^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-2\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(6\sqrt[4]{a}\,\sqrt[4]{c}\,de+3\sqrt{2}\,\sqrt{a}\,e^2+\sqrt{2}\,\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\,\sqrt[4]{c}\,d\left(-6\sqrt[4]{a}\,\sqrt[4]{c}\,de+3\sqrt{2}\,\sqrt{a}\,e^2+\sqrt{2}\,\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)+2ae^3\log\left(a+cx^4\right)}{8ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] (-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt
[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1/4
)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d
^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8*a*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 311, normalized size = 0.97

$$\frac{e^{3}\log\left(|cx^{4}+a|\right)}{4c}+\frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}\,c^{2}d^{2}e+\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}+3\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^{3}}+\frac{\sqrt{2}\left(3\sqrt{2}\sqrt{ac}\,c^{2}d^{2}e+\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}+3\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^{3}}+\frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}-3\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^{3}}-\frac{\sqrt{2}\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{2}d^{3}-3\left(ac^{3}\right)^{\frac{3}{4}}de^{2}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
+ (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c
)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d
*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

maple [A] time = 0.01, size = 314, normalized size = 0.98

$$\frac{3d^{2}e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{2\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^{3}\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}+\frac{3\sqrt{2}\,d\,e^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{3\sqrt{2}\,d\,e^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{3\sqrt{2}\,d\,e^{2}\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{e^{3}\ln\left(c\,x^{4}+a\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a),x)
```

```
[Out] 1/8*d^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+3/2*e*d^2/(a*c)^(1/2)*arctan(x^2*(c/a)^(1/2))+3/8*d*e^2/c/(a/c
)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+3/4*d*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-
1)+1/4*e^3*ln(c*x^4+a)/c
```

maxima [A] time = 1.96, size = 310, normalized size = 0.97

$$\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^{3}+cd^{3}-3\sqrt{a}\sqrt{c}de^{2}\right)\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^{3}-cd^{3}+3\sqrt{a}\sqrt{c}de^{2}\right)\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^{3}+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^{2}-6\sqrt{a}cd^{2}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^{3}+3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^{2}+6\sqrt{a}cd^{2}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 + c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2)
*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) +
 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 - c*d^3 + 3*sqrt(a)*sqrt(c)*d*e^2
)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
+ 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 - 6*sq
rt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*(sqrt(2
)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 + 6*sqrt(a)*c*d^2*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

mupad [B] time = 2.84, size = 894, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4),x)
```

```
[Out] symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*c
^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2
 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*
e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(256*a
^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6
*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*
d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5*c*d^4*
e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2
*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a
^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)
^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*
d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z
 - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12
, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*
c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8
*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*
e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^
2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d
^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^
3*e^12, z, k), k, 1, 4)
```

sympy [A] time = 5.29, size = 384, normalized size = 1.20

$$\mathrm{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2\left(96a^3c^2e^6 + 480a^2c^3d^4e^2\right) + t\left(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48ac^3d^8e\right) + a^3e^{12} + 3a^2cd^4e^8 + 3ac^2d^8e^4 + c^3d^{12}, \left(t \mapsto t\log\left(x + \frac{1728t^3a^4c^3e^6 + 960t^3a^3c^4d^4e^2 - 1296t^2a^4c^2e^9 - 2016t^2a^3c^3d^4e^5 + 48t^2a^2c^4d^8e + 324ta^4ce^{12} + 4716ta^3c^2d^4e^8 + 1452ta^2c^3d^8e^4 + 4tac^4d^{12} - 27a^4e^{15} + 1119a^3cd^4e^{11} - 609a^2c^2d^8e^7 - 91ac^3d^{12}e^3}{729a^3cd^3e^{12} - 1053a^2c^2d^7e^8 - 117ac^3d^{11}e^4 + c^4d^{15}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c**
2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d**
4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*d**
8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 + 960
*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a**3*c*
*3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716*_t*a*
*3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12 - 27*a*
*4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*c**3*d**
12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a*c**3*d**
11*e**4 + c**4*d**15))))
```

3.391 $\int \frac{(d+ex)^2}{a+cx^4} dx$

Optimal. Leaf size=291

$$-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e\right.}{}$$

Rubi [A] time = 0.20, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sqr
t[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{a+cx^4}\,dx &= \int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right) dx \\
&= (2de)\int \frac{x}{a+cx^4}\,dx + \int \frac{d^2+e^2x^2}{a+cx^4}\,dx \\
&= (de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right) + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c} \\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} - \frac{\left(\sqrt{c}\,d^2 \cdots\right.}{\cdots} \\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}\cdots\right.}{4\sqrt{2}\,a\cdots} \\
&= \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} - \cdots
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 243, normalized size = 0.84

$$
\frac{-\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)\right)-2\left(4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\left(-4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8a^{3/4}c^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)*c
```

```
^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
)/(8*a^(3/4)*c^(3/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4),x]
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.34, size = 285, normalized size = 0.98

$$\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}\,c^2de+\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^
(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c
^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a
*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*
c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^3)
```

maple [A] time = 0.00, size = 292, normalized size = 1.00

$$\frac{de\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}+\frac{\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}c}+\frac{\sqrt{2}\,e^2\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(\frac{a}{c}\right)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a),x)
[Out] 1/8*d^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x+1)+1/4*d^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)+d*e/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/8*e^2/c/(a/c)^(1/4
)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2)))+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
*x+1)+1/4*e^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.05, size = 275, normalized size = 0.95

$$\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\sqrt{c}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+4\sqrt{a}\sqrt{c}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqr
t(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4)
)/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/4*(sqr
t(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*
d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

mupad [B] time = 2.66, size = 556, normalized size = 1.91

$$\sum_{k=1}^{4}\ln\left(3c^2d^4e^2-a\,c\,e^6+4c^2d^3e^3x-\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)c^3d^4x\,4-\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)^2a\,c^3d^2\,16+\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)a\,c^2e^4x\,4-\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)a\,c^2d\,e^3\,16+\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)^2a\,c^3d\,e\,x\,32\right)\mathrm{root}\left(256a^3c^3z^4+192a^2c^2d^2e^2z^2+32a^2c\,d\,e^5z-32a\,c^2d^5e\,z+2a\,c\,d^4e^4+c^2d^8+a^2e^8,z,k\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4),x)
```

```
[Out] symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*z
^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*
d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 + 19
2*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192*a^2*
c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d
^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^
2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2
*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^2*e^2*z^2 +
 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8 + a^2*e^8, z
, k), k, 1, 4)
```

sympy [A] time = 2.68, size = 277, normalized size = 0.95

$$\mathrm{RootSum}\left(256t^4a^3c^3+192t^2a^2c^2d^2e^2+t\left(32a^2cde^5-32ac^2d^5e\right)+a^2e^8+2acd^4e^4+c^2d^8,\left(t\mapsto t\log\left(x+\frac{64t^3a^4c^2e^6+448t^3a^3c^3d^4e^2-160t^2a^3c^2d^3e^5+32t^2a^2c^3d^7e+60ta^3cd^2e^8+256ta^2c^2d^6e^4+4tac^3d^{10}+6a^3de^{11}-24a^2cd^5e^7-30ac^2d^9e^3}{a^3e^{12}-33a^2cd^4e^8-33ac^2d^8e^4+c^3d^{12}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*c
*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, Lamb
da(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e**2
- 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**3*c*
d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*e**11
 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c*d**4*
e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

3.392 $\int \frac{d+ex}{a+cx^4}\,dx$

Optimal. Leaf size=219

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}}{\ldots}\right.}{2\sqrt{2}\,a^{3/\ldots}}$$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1
/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{a+cx^4}\,dx = \int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx$$

$$= d\int \frac{1}{a+cx^4}\,dx + e\int \frac{x}{a+cx^4}\,dx$$

$$= \frac{d\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{d\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{1}{2}e\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{d\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{d\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \ldots$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\,\text{Su}\ldots}{\ldots}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x\ldots\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.06, size = 184, normalized size = 0.84

$$\frac{-2\left(2\sqrt[4]{a}e+\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\sqrt[4]{c}d\left(\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)}{8a^{3/4}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
```

```
1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a^(3/
4)*Sqrt[c])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{a+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4),x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 215, normalized size = 0.98

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}\,ce-\left(ac^{3}\right)^{\frac{1}{4}} cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{2}}-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}\,ce-\left(ac^{3}\right)^{\frac{1}{4}} cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt(2)*
(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

maple [A] time = 0.00, size = 151, normalized size = 0.69

$$\frac{e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{2\sqrt{ac}}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a),x)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/4*d*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)+1/2*e/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)
```

maxima [A] time = 2.67, size = 207, normalized size = 0.95

$$\frac{\sqrt{2}\,d\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\,d\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/
4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt
(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4
)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

mupad [B] time = 2.32, size = 160, normalized size = 0.73

$$\begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if } a=0\\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\,\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}}-\frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\,\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}}+\frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if } a\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4),x)
```

```
[Out] piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)*
x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) - (a
tan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d))/(
8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^(1/4
)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*a^(3/
4)*c^(1/4)))
```

sympy [A] time = 0.82, size = 124, normalized size = 0.57

$$\operatorname{RootSum}\left(256t^4a^3c^2+32t^2a^2ce^2-16tacd^2e+ae^4+cd^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3ce^2-16t^2a^2cd^2e-8ta^2e^4-4tacd^4+5ad^2e^3}{4ade^4-cd^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*e
**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a**2
*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4 - c
*d**5))))
```

3.393 $\int \frac{1}{a+cx^4} dx$

Optimal. Leaf size=185

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Rubi [A] time = 0.10, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1 -\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.72

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^4)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^4)^(-1), x]
```

fricas [A] time = 1.07, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\arctan\left(-a^2cx\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}+\sqrt{a^2\sqrt{-\frac{1}{a^3c}}+x^2}\,a^2c\left(-\frac{1}{a^3c}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

giac [A] time = 0.33, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}-\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

maple [A] time = 0.00, size = 128, normalized size = 0.69

$$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+1/4*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.37, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

mupad [B] time = 0.08, size = 33, normalized size = 0.18

$$-\frac{\operatorname{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2(-a)^{3/4}c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

sympy [A] time = 0.16, size = 20, normalized size = 0.11

$$\mathrm{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

3.394 $\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=416

$$-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x\right.}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}$$

Rubi [A] time = 0.43, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}-\frac{\sqrt[4]{c}\,d\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^4+cd^4\right)}-\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^4+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4))
- (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4
 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*
d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx = \int \left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \left(\frac{d^3+de^2x^2}{a+cx^4} + \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\right)dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\int \frac{d^3+de^2x^2}{a+cx^4}\,dx}{cd^4+ae^4} + \frac{c\int \frac{x\left(-d^2e-e^3x^2\right)}{a+cx^4}\,dx}{cd^4+ae^4}$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} + \frac{c\,\text{Subst}\left(\int \frac{-d^2e-e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \frac{\left(d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^4+ae^4\right)} + \cdots$$

$$= \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\left(cd^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)} + \cdots$$

$$= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)} + \frac{e^3\log(d+ex)}{cd^4+ae^4} - \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{\cdots}\right.}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)} \cdots$$

$$= -\frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)} + \frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\cdots\right.}{2\sqrt{2}\cdots}$$

Mathematica [**A**] time = 0.15, size = 404, normalized size = 0.97

$$\frac{-2a^{3/4}e^3\log\left(a+cx^4\right)+8a^{3/4}e^3\log(d+ex)-\sqrt{2}\,c^{3/4}d^3\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\sqrt{2}\,c^{3/4}d^3\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-2\sqrt[4]{c}\,d\left(-2\sqrt[4]{a}\,\sqrt[4]{c}\,de+\sqrt{2}\,\sqrt{a}\,e^2+\sqrt{2}\,\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt[4]{c}\,d\left(2\sqrt[4]{a}\,\sqrt[4]{c}\,de+\sqrt{2}\,\sqrt{a}\,e^2+\sqrt{2}\,\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)+\sqrt{2}\,\sqrt{a}\,\sqrt[4]{c}\,de^2\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\sqrt{2}\,\sqrt{a}\,\sqrt[4]{c}\,de^2\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{8a^{3/4}\left(ae^4+cd^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] (-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a
]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqrt[
c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^(1/4)
*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c^(
3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*S
qrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
 - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

IntegrateAlgebraic [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.42, size = 371, normalized size = 0.89

$$\frac{\left(ac^{3}\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{2}d^{2} - 2\left(ac^{3}\right)^{\frac{1}{4}} acde + \sqrt{2}\sqrt{ac}\,ace^{2}\right)} + \frac{\left(ac^{3}\right)^{\frac{1}{4}} cd \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^{2}d^{2} + 2\left(ac^{3}\right)^{\frac{1}{4}} acde + \sqrt{2}\sqrt{ac}\,ace^{2}\right)} + \frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2}d^{3} - \left(ac^{3}\right)^{\frac{3}{4}} de^{2}\right)\log\left(x^{2} + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^{3}d^{4} + \sqrt{2}\,a^{2}c^{2}e^{4}\right)} - \frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2}d^{3} - \left(ac^{3}\right)^{\frac{3}{4}} de^{2}\right)\log\left(x^{2} - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^{3}d^{4} + \sqrt{2}\,a^{2}c^{2}e^{4}\right)} - \frac{e^{3}\log\left(\left|cx^{4} + a\right|\right)}{4\left(cd^{4} + ae^{4}\right)} + \frac{e^{4}\log\left(\left|xe + d\right|\right)}{cd^{4}e + ae^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(sqrt(2)*a*c^2*d^2 - 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(a*c)*a*c
*e^2) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + 2*(a*c^3)^(1/4)*a*c*d*e + sqrt(2)*sqrt(
a*c)*a*c*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 +
 sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^
4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*
(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4) - 1/4*e^
3*log(abs(c*x^4 + a))/(c*d^4 + a*e^4) + e^4*log(abs(x*e + d))/(c*d^4*e + a*
e^5)
```

maple [A] time = 0.01, size = 433, normalized size = 1.04

$$-\frac{c\,d^{2}e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{2\left(a\,e^{4}+c\,d^{4}\right)\sqrt{ac}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)a} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,c\,d^{3}\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{4}+c\,d^{4}\right)a} + \frac{\sqrt{2}\,d\,e^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}} + \frac{\sqrt{2}\,d\,e^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{4\left(a\,e^{4}+c\,d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}} + \frac{\sqrt{2}\,d\,e^{2}\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{8\left(a\,e^{4}+c\,d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}} - \frac{e^{3}\ln\left(c\,x^{4}+a\right)}{4\left(a\,e^{4}+c\,d^{4}\right)} + \frac{e^{3}\ln\left(ex+d\right)}{a\,e^{4}+c\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a),x)
```

```
[Out] e^3*ln(e*x+d)/(a*e^4+c*d^4)+1/8*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x+1)+1/4*c/(a*e^4+c*d^4)*d^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)-1/2*c/(a*e^4+c*d^4)*e*d^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^
2)+1/8/(a*e^4+c*d^4)*d*e^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/4/(a*e^4+c*d^4)*d
*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+1/4/(a*e^4+c*d^4)*
d*e^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)-1/4*e^3*ln(c*x^4+
a)/(a*e^4+c*d^4)
```

maxima [A] time = 2.73, size = 345, normalized size = 0.83

$$\frac{e^{3}\log\left(ex+d\right)}{cd^{4}+ae^{4}} - \frac{c\left(\frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^{3}-cd^{3}+\sqrt{a}\sqrt{c}\,de^{2}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^{3}+cd^{3}-\sqrt{a}\sqrt{c}\,de^{2}\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} - \frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^{3}+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^{2}+2\sqrt{a}\,cd^{2}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}} - \frac{2\left(\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{5}{4}}d^{3}+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{3}{4}}de^{2}-2\sqrt{a}\,cd^{2}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}\right)}{8\left(cd^{4}+ae^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*
e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1
```

```
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 +
 c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x
 + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^
(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(
c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(3/4)*d*e
^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c*
d^4 + a*e^4)
```

mupad [B] time = 0.42, size = 874, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)),x)
```

```
[Out] symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a^3
*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z
+ 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*
a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 25
6*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^
3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*
z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 2
56*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d*e^5 - 128*roo
t(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +
16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*z^4 + 256*a^4*e^
4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^4*
e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*
a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(256*a^3*c*d^4*z^4 +
256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k), k
, 1, 4) + (e^3*log(d + e*x))/(a*e^4 + c*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.395 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e\right.}{4}$$

Rubi [A] time = 0.81, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1248, 635, 205, 260, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)-\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)+\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d^2\left(cd^4-3ae^4\right)+\sqrt{a}\,e^2\left(3cd^4-ae^4\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^2}-\frac{cd^3e^3\log\left(a+cx^4\right)}{\left(ae^4+cd^4\right)^2}-\frac{\sqrt{c}\,de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(ae^4+cd^4\right)^2}-\frac{e^3}{(d+ex)\left(ae^4+cd^4\right)}+\frac{4cd^3e^3\log(d+ex)}{\left(ae^4+cd^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] -(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2*(
c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d
^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*e^3*Lo
g[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - S
qrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[c]*d^2*(c
*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c*d^
3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)}\,dx = \int\left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^2}+\frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae\right.\right.}{\left(cd^4+ae\right.}\right.$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(3cd^4-ae^4\right)x^2}{a+cx^4}}{\left(cd^4+ae^4\right)^2}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\left(\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}+\frac{d^2\left(cd^4-3ae^4\right.}{}\right.}{\left(cd^4+ae^4\right)^2}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{x\left(-2de\left(cd^4-ae^4\right)-4cd^3e^3x^2\right)}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2}+\frac{c\int\frac{d^2(c}{}}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{c\,\text{Subst}\left(\int\frac{-2de\left(cd^4-ae^4\right)-4cd^3e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}-\frac{\left(2c^2d^3e^3\right)\text{Subst}\left(\int\frac{x}{a+cx^2}\,dx,x,x^2\right)}{\left(cd^4+ae^4\right)^2}-\frac{(c}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}\,de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}+\frac{4cd^3e^3\log(d+ex)}{\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,(}{}$$

$$=-\frac{e^3}{\left(cd^4+ae^4\right)(d+ex)}-\frac{\sqrt{c}\,de\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\left(3cd^4e^2-ae^6+\frac{\sqrt{c}\,d^2(cd}{\sqrt{}}\right.}{2\sqrt{2}\sqrt[4]{a}\,(c}$$

Mathematica [A] time = 0.61, size = 524, normalized size = 0.95

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(a^{3/2}e^6-3\sqrt{a}cd^4e^2-3a\sqrt{c}d^2e^4+c^{3/2}d^6\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}}+\frac{2\sqrt[4]{c}\left(\sqrt{a}e^2-\sqrt{c}d^2\right)\left(-4a^{3/4}\sqrt[4]{c}de^3-4\sqrt[4]{a}c^{3/4}d^3e+4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2+\sqrt{2}ae^4+\sqrt{2}cd^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(4a^{3/4}\sqrt[4]{c}de^3+4\sqrt[4]{a}c^{3/4}d^3e+4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2+\sqrt{2}ae^4+\sqrt{2}cd^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8e^3\left(ae^4+cd^4\right)}{d+ex}-8cd^3e^3\log\left(a+cx^4\right)+32cd^3e^3\log(d+ex)}{8\left(ae^4+cd^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] ((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]*
e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d
^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[2]*c
*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4*a^(3
/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 - 3*Sq
rt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6 -
 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log[a + c*x^4]
)/(8*(c*d^4 + a*e^4)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.78, size = 646, normalized size = 1.17

$$-\frac{cd^3e^3\log\left(\left|cx^4+a\right|\right)}{c^2d^8+2\,acd^4e^4+a^2e^8}+\frac{4\,cd^3e^4\log\left(\left|xe+d\right|\right)}{c^2d^8e+2\,acd^4e^5+a^2e^9}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4-4\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e+4\sqrt{2}\sqrt{ac}\,ac^2d^2e^2+\sqrt{2}\,a^2c^2e^4-4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4+4\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e+4\sqrt{2}\sqrt{ac}\,ac^2d^2e^2+\sqrt{2}\,a^2c^2e^4+4\left(ac^3\right)^{\frac{3}{4}}ade^3\right)}+\frac{\left(\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^6-3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^4e^2-3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^4+\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ae^6\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}-\frac{\left(\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^6-3\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^4e^2-3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e^4+\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ae^6\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\left(ac^4d^8+2\,a^2c^3d^4e^4+a^3c^2e^8\right)}-\frac{cd^4e^3+ae^7}{\left(cd^4+ae^4\right)^2\left(xe+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -c*d^3*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 4*c*d^
3*e^4*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/2*((a*c^3
)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e + 4*
sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 - 4*(a*c^3)^(3/4)*a*d
*e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 4*(a*c^3)^(1/
4)*a*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + sqrt(2)*a^2*c^2*e^4 +
4*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*sqrt(2)*(
a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + sqrt(2)*(a
*c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4*d^8
+ 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3
*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 +
sqrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a*e^7)/((c*d^4
+ a*e^4)^2*(x*e + d))
```

maple [A] time = 0.01, size = 866, normalized size = 1.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a),x)
```

```
[Out] -e^3/(a*e^4+c*d^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-3/4*c/(a*e
^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^2*e^4+1/4
*c^2/(a*e^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^6-3/8*c/(a*e^4+c*d^4)^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^2*e^4+1/8*c^2/(a*e
^4+c*d^4)^2*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
)/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6-3/4*c/(a*e^4+c*d^4)^2*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^4+1/4*c^2/(a*e^4+c*d^4)
^2*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6+c/(a*e^4+c*d^4
)^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d*a-c^2/(a*e^4+c*d^4)^2/(a*c)
^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^5-1/8/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1
/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(
a/c)^(1/2)))*a*e^6+3/8*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^
(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e
^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
```

a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4*e^2-1/4/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*e^6+3/4*c/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4*e^2-c*d^3*e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2

maxima [A] time = 2.11, size = 561, normalized size = 1.02

$$\frac{4\,cd^3e^3\log\left(ex+d\right)}{c^2d^8+2\,acd^4e^4+a^2e^8}-\frac{e^3}{cd^5+ade^4+\left(cd^4e+ae^5\right)x}-\frac{c\left(\frac{\sqrt{2}\left(4\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-c^2d^6+3\,\sqrt{a}c^{\frac{3}{2}}d^4e^2+3\,acd^2e^4-a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3+c^2d^6-3\,\sqrt{a}c^{\frac{3}{2}}d^4e^2-3\,acd^2e^4+a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^6+3\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^4e^2-3\,\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^2e^4-\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}e^6+4\,\sqrt{a}c^2d^5e-4\,a^{\frac{3}{2}}cde^5\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{9}{4}}d^6+3\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{7}{4}}d^4e^2-3\,\sqrt{2}a^{\frac{5}{4}}c^{\frac{5}{4}}d^2e^4-\sqrt{2}a^{\frac{7}{4}}c^{\frac{3}{4}}e^6-4\,\sqrt{a}c^2d^5e+4\,a^{\frac{3}{2}}cde^5\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}\right)}{8\left(c^2d^8+2\,acd^4e^4+a^2e^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5 +
 a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)
*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)*sq
rt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*
c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sqrt(a)
*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*
c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)*c^(5/4)
*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e + 4*a^(3/2)*c*
d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^2*d^8 + 2*a*c*d^4
*e^4 + a^2*e^8)
```

mupad [B] time = 2.78, size = 2436, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^2),x)
```

```
[Out] symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3
*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2
*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e^4*
z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*
a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*root(51
2*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^
3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6*d^8*e
^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4
 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)
^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 +
 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*
d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^4*z^4 + 256*
a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*
e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d^5*e^10 + 32*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d^2
*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^
4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k
)^3*a*c^7*d^12*e + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 25
6*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e
*z + c, z, k)*c^6*d^5*e^4*x - 4*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^
```

```
8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +
32*a*c*d*e*z + c, z, k)^2*c^7*d^10*e*x + 64*root(512*a^4*c*d^4*e^4*z^4 + 25
6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^
2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a*c^6*d^7*e^4 + 384*root(512*a^4*c*d^
4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3
+ 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*d*e^14 + 320*ro
ot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3
*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^5*c^4*
e^15*x + 248*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8
*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z
, k)^2*a*c^6*d^6*e^5*x - 64*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^
4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a
*c*d*e*z + c, z, k)^3*a*c^7*d^11*e^2*x + 32*root(512*a^4*c*d^4*e^4*z^4 + 25
6*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^
2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)*a*c^5*d*e^8*x + 316*root(512*a^4*c*d^4*
e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 +
320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^2*e^9*x + 640*r
oot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^6
*d^7*e^6*x + 704*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5
*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
c, z, k)^3*a^3*c^5*d^3*e^10*x - 192*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^
2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^
2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^12*e^3*x - 64*root(512*a^4*c*d^4*e^
4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 32
0*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^8*e^7*x + 448*roo
t(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*
c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5*d
^4*e^11*x)/(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4))*root(512*a^4*c*d^4*e^4*z^4
+ 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*
c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k), k, 1, 4) - e^3/(c*d^5 + a*d*e^4 +
a*e^5*x + c*d^4*e*x) + (4*c*d^3*e^3*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*a*
c*d^4*e^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.396 $\int \frac{1}{(d+ex)^3(a+cx^4)}\,dx$

Optimal. Leaf size=680

$$-\frac{\sqrt{c}\,e\left(a^2e^8-12acd^4e^4+3c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\right.}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}$$

Rubi [A] time = 0.95, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 205, 260}

$$-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4-2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{\sqrt{c}\,e\left(a^2e^8-12acd^4e^4+3c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(ae^4+cd^4\right)^3}-\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}+\frac{c^{3/4}d\left(3a^2e^8-12acd^4e^4+2\sqrt{a}\,\sqrt{c}\,d^2e^2\left(3cd^4-5ae^4\right)+c^2d^8\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\left(ae^4+cd^4\right)^3}-\frac{cd^2e^3\left(5cd^4-3ae^4\right)\log\left(a+cx^4\right)}{2\left(ae^4+cd^4\right)^3}-\frac{4cd^3e^3}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{e^3}{2(d+ex)^2\left(ae^4+cd^4\right)}+\frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(ae^4+cd^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] -e^3/(2*(c*d^4 + a*e^4)*(d + e*x)^2) - (4*c*d^3*e^3)/((c*d^4 + a*e^4)^2*(d
+ e*x)) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*(c^2*d^8 - 12*a*
c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3)
+ (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*
e^2*(3*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2
]*a^(3/4)*(c*d^4 + a*e^4)^3) + (2*c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[d + e*x
])/(c*d^4 + a*e^4)^3 - (c^(3/4)*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2
*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(3/4)
*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c*d^2*e^3*(5*c*d^4 - 3*a*e^4)*Log[a +
c*x^4])/(2*(c*d^4 + a*e^4)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)}\,dx = \int\left(\frac{e^4}{\left(cd^4+ae^4\right)(d+ex)^3} + \frac{4cd^3e^4}{\left(cd^4+ae^4\right)^2(d+ex)^2} + \frac{2cd^2e^4\left(5cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d\left(c^2d\right.\right.}{}\right.$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} +$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{2cd^2e^3\left(5cd^4-3ae^4\right)\log(d+ex)}{\left(cd^4+ae^4\right)^3} -$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\sqrt{c}\,e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^3}{2\left(cd^4+ae^4\right)(d+ex)^2} - \frac{4cd^3e^3}{\left(cd^4+ae^4\right)^2(d+ex)} - \frac{\sqrt{c}\,e\left(3c^2d^8-12acd^4e^4+a^2e^8\right)\tan}{2\sqrt{a}\left(cd^4+ae^4\right)^3}$$

Mathematica [**A**] time = 0.93, size = 738, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] (-4*a^(3/4)*e^3*(c*d^4 + a*e^4)^2 - 32*a^(3/4)*c*d^3*e^3*(c*d^4 + a*e^4)*(d
 + e*x) - 2*Sqrt[c]*(Sqrt[2]*c^(9/4)*d^9 - 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*
Sqrt[a]*c^(7/4)*d^7*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 + 24*a^(5/4)*c*d^4*e
^5 - 10*Sqrt[2]*a^(3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 - 2*a
^(9/4)*e^9)*(d + e*x)^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[c]
*(Sqrt[2]*c^(9/4)*d^9 + 6*a^(1/4)*c^2*d^8*e + 6*Sqrt[2]*Sqrt[a]*c^(7/4)*d^7
*e^2 - 12*Sqrt[2]*a*c^(5/4)*d^5*e^4 - 24*a^(5/4)*c*d^4*e^5 - 10*Sqrt[2]*a^(
3/2)*c^(3/4)*d^3*e^6 + 3*Sqrt[2]*a^2*c^(1/4)*d*e^8 + 2*a^(9/4)*e^9)*(d + e*
x)^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 16*a^(3/4)*c*d^2*e^3*(5*c*d^
4 - 3*a*e^4)*(d + e*x)^2*Log[d + e*x] - Sqrt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt
[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e
^8)*(d + e*x)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sq
rt[2]*c^(3/4)*d*(c^2*d^8 - 6*Sqrt[a]*c^(3/2)*d^6*e^2 - 12*a*c*d^4*e^4 + 10*
a^(3/2)*Sqrt[c]*d^2*e^6 + 3*a^2*e^8)*(d + e*x)^2*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2] + 4*a^(3/4)*c*d^2*e^3*(-5*c*d^4 + 3*a*e^4)*(d
+ e*x)^2*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4)^3*(d + e*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3 \left(a+cx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.89, size = 901, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 + 2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*
d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d
^6 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e + 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^
2*c^2*d^2*e^4 - 8*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^3 - 3*sqrt(2)*(a*c^3)^(1/4)
*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) + 1/4*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^3 -
2*a*c^2*e^3 - 3*sqrt(2)*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3*d^6 + 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^
5*e - 9*sqrt(a*c)*a*c^2*d^4*e^2 + 9*a^2*c^2*d^2*e^4 + 8*sqrt(2)*(a*c^3)^(3/
4)*a*d^3*e^3 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^5 + sqrt(a*c)*a^2*c*e^6) +
 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 - 1
2*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6
+ 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sq
rt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12)
- 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^9 - 6*sqrt(2)*(a*c^3)^(3/4)*c*d^7*e^2 -
12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^5*e^4 + 10*sqrt(2)*(a*c^3)^(3/4)*a*d^3*e^6
 + 3*sqrt(2)*(a*c^3)^(1/4)*a^2*c*d*e^8)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a*c^4*d^12 + 3*a^2*c^3*d^8*e^4 + 3*a^3*c^2*d^4*e^8 + a^4*c*e^12)
 - 1/2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*
c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^4 - 3*a*c*d^2*e^
8)*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3*
e^13) - 1/2*(9*c^2*d^8*e^3 + 10*a*c*d^4*e^7 + a^2*e^11 + 8*(c^2*d^7*e^4 + a
*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(x*e + d)^2)
```

maple [B] time = 0.01, size = 1201, normalized size = 1.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a),x)
```

```
[Out] -1/2*e^3/(a*e^4+c*d^4)/(e*x+d)^2-4*c*d^3*e^3/(a*e^4+c*d^4)^2/(e*x+d)-6*e^7*
d^2*c/(a*e^4+c*d^4)^3*ln(e*x+d)*a+10*e^3*d^6*c^2/(a*e^4+c*d^4)^3*ln(e*x+d)+
3/4*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d*e^8-3*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)
```

```
*x-1)*d^5*e^4+1/4*c^3/(a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^9+3/8*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*a*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d
*e^8-3/2*c^2/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^4+1/8*c^3/(
a*e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9+3/4*c/(a*e^4+c*d^4)^3*(a/
c)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^8-3*c^2/(a*e^4+c*d^4
)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^4+1/4*c^3/(a*
e^4+c*d^4)^3*(a/c)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9-1/2*
c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*a^2+6*c^2/(a*e^
4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*d^4*e^5*a-3/2*c^3/(a*e^4+c
*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^8-5/4*c/(a*e^4+c*d^4)^3/(
a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2)))*a*d^3*e^6+3/4*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2
^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^7*e^2-5/2*c/(a*e^4+c*d^4)^3/(a/c)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*a*d^3*e^6+3/2*c^2/(a*e^4+c*d^4)^3/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^2+3/2*c/(a*e^4+c*
d^4)^3*ln(c*x^4+a)*e^7*d^2*a-5/2*c^2/(a*e^4+c*d^4)^3*ln(c*x^4+a)*e^3*d^6
```

maxima [A] time = 2.34, size = 817, normalized size = 1.20

$$-\frac{c\left(\frac{\sqrt{2}\left(10\sqrt{2}a^{\frac{3}{4}}c^{\frac{9}{4}}d^6e^3-6\sqrt{2}a^{\frac{7}{4}}c^{\frac{5}{4}}d^2e^7-c^3d^9+6\sqrt{a}c^{\frac{5}{2}}d^7e^2+12ac^2d^5e^4-10a^{\frac{3}{2}}c^{\frac{3}{2}}d^3e^6-3a^2cde^8\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(10\sqrt{2}a^{\frac{3}{4}}c^{\frac{9}{4}}d^6e^3-6\sqrt{2}a^{\frac{7}{4}}c^{\frac{5}{4}}d^2e^7+c^3d^9-6\sqrt{a}c^{\frac{5}{2}}d^7e^2-12ac^2d^5e^4+10a^{\frac{3}{2}}c^{\frac{3}{2}}d^3e^6+3a^2cde^8\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{13}{4}}d^9+6\sqrt{2}a^{\frac{3}{4}}c^{\frac{11}{4}}d^7e^2-12\sqrt{2}a^{\frac{5}{4}}c^{\frac{9}{4}}d^5e^4-10\sqrt{2}a^{\frac{7}{4}}c^{\frac{7}{4}}d^3e^6+3\sqrt{2}a^{\frac{9}{4}}c^{\frac{5}{4}}de^8+6\sqrt{a}c^3d^8e-24a^{\frac{3}{2}}c^2d^4e^5+2a^{\frac{5}{2}}ce^9\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}-\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{13}{4}}d^9+6\sqrt{2}a^{\frac{3}{4}}c^{\frac{11}{4}}d^7e^2-12\sqrt{2}a^{\frac{5}{4}}c^{\frac{9}{4}}d^5e^4-10\sqrt{2}a^{\frac{7}{4}}c^{\frac{7}{4}}d^3e^6+3\sqrt{2}a^{\frac{9}{4}}c^{\frac{5}{4}}de^8-6\sqrt{a}c^3d^8e+24a^{\frac{3}{2}}c^2d^4e^5-2a^{\frac{5}{2}}ce^9\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}\right)}{8\left(c^3d^{12}+3ac^2d^8e^4+3a^2cd^4e^8+a^3e^{12}\right)}+\frac{2\left(5c^2d^6e^3-3acd^2e^7\right)\log(ex+d)}{c^3d^{12}+3ac^2d^8e^4+3a^2cd^4e^8+a^3e^{12}}-\frac{8cd^3e^4x+9cd^4e^3+ae^7}{2\left(c^2d^{10}+2acd^6e^4+a^2d^2e^8+\left(c^2d^8e^2+2acd^4e^6+a^2e^{10}\right)x^2+2\left(c^2d^9e+2acd^5e^5+a^2de^9\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(
5/4)*d^2*e^7 - c^3*d^9 + 6*sqrt(a)*c^(5/2)*d^7*e^2 + 12*a*c^2*d^5*e^4 - 10*
a^(3/2)*c^(3/2)*d^3*e^6 - 3*a^2*c*d*e^8)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(10*sqrt(2)*a^(3/4)*c^(9/4
)*d^6*e^3 - 6*sqrt(2)*a^(7/4)*c^(5/4)*d^2*e^7 + c^3*d^9 - 6*sqrt(a)*c^(5/2)
*d^7*e^2 - 12*a*c^2*d^5*e^4 + 10*a^(3/2)*c^(3/2)*d^3*e^6 + 3*a^2*c*d*e^8)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(sqrt(2)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sq
rt(2)*a^(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt
(2)*a^(9/4)*c^(5/4)*d*e^8 + 6*sqrt(a)*c^3*d^8*e - 24*a^(3/2)*c^2*d^4*e^5 +
2*a^(5/2)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2
)*a^(1/4)*c^(13/4)*d^9 + 6*sqrt(2)*a^(3/4)*c^(11/4)*d^7*e^2 - 12*sqrt(2)*a^
(5/4)*c^(9/4)*d^5*e^4 - 10*sqrt(2)*a^(7/4)*c^(7/4)*d^3*e^6 + 3*sqrt(2)*a^(9
/4)*c^(5/4)*d*e^8 - 6*sqrt(a)*c^3*d^8*e + 24*a^(3/2)*c^2*d^4*e^5 - 2*a^(5/2
)*c*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^3*d^12 + 3*a*c^
2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 2*(5*c^2*d^6*e^3 - 3*a*c*d^2*e^7)
*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) - 1
/2*(8*c*d^3*e^4*x + 9*c*d^4*e^3 + a*e^7)/(c^2*d^10 + 2*a*c*d^6*e^4 + a^2*d^
2*e^8 + (c^2*d^8*e^2 + 2*a*c*d^4*e^6 + a^2*e^10)*x^2 + 2*(c^2*d^9*e + 2*a*c
*d^5*e^5 + a^2*d*e^9)*x)
```

mupad [B] time = 3.67, size = 1955, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x)^3),x)
```

```
[Out] symsum(log((c^7*d^5*e^6 + a*c^6*d*e^10)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12
*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 +
 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a
^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*
a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((208*a*c^7*d^7*e^7 - 48*a^2*
c^6*d^3*e^11)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 +
6*a^2*c^2*d^8*e^8) + root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 +
 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^
3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d
^2*e*z + c^2, z, k)*((144*a*c^8*d^13*e^4 + 16*a^4*c^5*d*e^16 + 2608*a^2*c^7
*d^9*e^8 - 592*a^3*c^6*d^5*e^12)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 +
4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^
4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^
2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e
^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k)*((896*a^4*c^6*d^7*e^13 - 1120*a^3*c^
7*d^11*e^9 - 1024*a^2*c^8*d^15*e^5 + 976*a^5*c^5*d^3*e^17 + 16*a*c^9*d^19*e
)/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^
8*e^8) - root(768*a^5*c*d^4*e^8*z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3
*d^12*z^4 + 256*a^6*e^12*z^4 - 1536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^
3*z^3 + 672*a^2*c^2*d^4*e^2*z^2 + 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2
, z, k)*((384*a^7*c^4*d*e^22 - 128*a^2*c^9*d^21*e^2 - 128*a^3*c^8*d^17*e^6
+ 768*a^4*c^7*d^13*e^10 + 1792*a^5*c^6*d^9*e^14 + 1408*a^6*c^5*d^5*e^18)/(a
^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^
8) + (x*(320*a^7*c^4*e^23 - 192*a^2*c^9*d^20*e^3 - 448*a^3*c^8*d^16*e^7 + 1
28*a^4*c^7*d^12*e^11 + 1152*a^5*c^6*d^8*e^15 + 1088*a^6*c^5*d^4*e^19))/(a^4
*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
) + (x*(80*a*c^9*d^18*e^2 - 1536*a^2*c^8*d^14*e^6 - 2016*a^3*c^7*d^10*e^10
+ 896*a^4*c^6*d^6*e^14 + 1296*a^5*c^5*d^2*e^18))/(a^4*e^16 + c^4*d^16 + 4*a
*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x*(36*a^4*c^5*e^1
7 - 4*c^9*d^16*e + 792*a*c^8*d^12*e^5 + 1632*a^2*c^7*d^8*e^9 - 152*a^3*c^6*
d^4*e^13))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a
^2*c^2*d^8*e^8)) + (x*(40*c^8*d^10*e^4 - 16*a*c^7*d^6*e^8 + 72*a^2*c^6*d^2*
e^12))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c
^2*d^8*e^8)) + (x*(a*c^6*e^11 + c^7*d^4*e^7))/(a^4*e^16 + c^4*d^16 + 4*a*c^
3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8))*root(768*a^5*c*d^4*e^8*
z^4 + 768*a^4*c^2*d^8*e^4*z^4 + 256*a^3*c^3*d^12*z^4 + 256*a^6*e^12*z^4 - 1
536*a^4*c*d^2*e^7*z^3 + 2560*a^3*c^2*d^6*e^3*z^3 + 672*a^2*c^2*d^4*e^2*z^2
+ 32*a^3*c*e^6*z^2 + 48*a*c^2*d^2*e*z + c^2, z, k), k, 1, 4) - ((a*e^7 + 9*
c*d^4*e^3)/(2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (4*c*d^3*e^4*x)/(a^2*e
^8 + c^2*d^8 + 2*a*c*d^4*e^4))/(d^2 + e^2*x^2 + 2*d*e*x) + (log(d + e*x)*(1
0*c^2*d^6*e^3 - 6*a*c*d^2*e^7))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*
a^2*c*d^4*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a),x)
```

```
[Out] Timed out
```

3.397 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{3d}{}$$

Rubi [A] time = 0.30, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {1854, 27, 12, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{3d\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{ae^3-cx\left(3d^2ex+d^3+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] -(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(4*a*c*(a + c*x^4)) + (3*d^2
*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*Sqrt[c]) - (3*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c
^(3/4)) + (3*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4)) + (3*d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx &= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}-\frac{\int \frac{-3d^3-6d^2ex-3de^2x^2}{a+cx^4}\,dx}{4a} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}-\frac{\int -\frac{3d(d+ex)^2}{a+cx^4}\,dx}{4a} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{(3d)\int \frac{(d+ex)^2}{a+cx^4}\,dx}{4a} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{(3d)\int \left(\frac{2dex}{a+cx^4}+\frac{d^2+e^2x^2}{a+cx^4}\right)dx}{4a} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{(3d)\int \frac{d^2+e^2x^2}{a+cx^4}\,dx}{4a}+\frac{\left(3d^2e\right)\int \frac{x}{a+cx^4}\,dx}{2a} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{\left(3d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{4a}+\frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}-e^2\right)\right)\int \frac{\sqrt{a}\ \cdots}{a+\cdots}}{8ac} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{\left(3d\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16ac} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\cdots\right.}{16\sqrt{2}\,a^{7/4}c^{3/4}} \\
&= -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}+\frac{3d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}-\frac{3d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\cdots\right.}{8\sqrt{2}\,a^{7/4}c^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.37, size = 347, normalized size = 0.99

$$
\frac{3\sqrt{2}\sqrt[4]{c}\left(a^{3/4}de^2-\sqrt[4]{a}\sqrt{c}\,d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+3\sqrt{2}\sqrt[4]{c}\left(\sqrt[4]{a}\sqrt{c}\,d^3-a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-6\sqrt[4]{a}\sqrt[4]{c}\,d\left(4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt[4]{a}\sqrt[4]{c}\,d\left(-4\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)-\frac{8a\left(ae^3-cdx\left(d^2+3dex+3e^2x^2\right)\right)}{a+cx^4}}{32a^2c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4)
*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e
^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[2]*
Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) + a^(3/
4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2
]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(d+ex)^3}{\left(a+cx^4\right)^2}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^2, x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.34, size = 342, normalized size = 0.98

$$\frac{3cdx^3e^2+3cd^2x^2e+cd^3x-ae^3}{4(cx^4+a)ac}+\frac{3\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}c^2d^3+(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3}+\frac{3\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}c^2d^3+(ac^3)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3}+\frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^3-(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3}-\frac{3\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^3-(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d*x^3*e^2 + 3*c*d^2*x^2*e + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) + 3
/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c^3
)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/
(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2
*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4
)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/32*sqrt
(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)
^(1/4) + sqrt(a/c))/(a^2*c^3)
```

maple [**A**] time = 0.00, size = 390, normalized size = 1.12

$$\frac{e^3x^4}{4(cx^4+a)a}+\frac{3de^2x^3}{4(cx^4+a)a}+\frac{3d^2ex^2}{4(cx^4+a)a}+\frac{d^3x}{4(cx^4+a)a}+\frac{3d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{4\sqrt{ac}\,a}+\frac{3\sqrt{2}\,de^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\sqrt{2}\,de^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\sqrt{2}\,de^2\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^3*x/a/(c*x^4+a)+3/32*d^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^3/a^
2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^3/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+3/4*e*d^2*x^2/a/(c*x^4+a)+3/4*e
*d^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/4*d*e^2*x^3/a/(c*x^4+a)+3/32
*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)+1/4*e^3*x^4/a/(c*x^4+a)
```

maxima [**A**] time = 2.20, size = 332, normalized size = 0.95

$$\frac{3d\left(\frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}\right)}{32a}+\frac{3cde^2x^3+3cd^2ex^2+cd^3x-ae^3}{4\left(ac^2x^4+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*
```

```
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4
)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*sqrt(
a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(sqrt(2)
*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(c)*d*e)
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*e^2*x^3 + 3
*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

mupad [B] time = 0.43, size = 670, normalized size = 1.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(6
5536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*
a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a
^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81
*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^
4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 +
 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a^7*c^3*z
^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*
z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c^2*e*x - 1
92*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*
z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12,
z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^
2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*
a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a) - e^3/(4*c) + (
3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)
```

sympy [A] time = 8.33, size = 350, normalized size = 1.00

$$\operatorname{RootSum}\left(65536t^4a^7c^3 + 27648t^2a^4c^2d^4e^2 + t\left(3456a^3cd^4e^5 - 3456a^2c^2d^8e\right) + 81a^2d^4e^8 + 162acd^8e^4 + 81c^2d^{12}, \left(t \mapsto t\log\left(x + \frac{4096t^3a^7c^2e^6 + 28672t^3a^6c^3d^4e^2 - 7680t^2a^5c^2d^4e^5 + 1536t^2a^4c^3d^8e + 2160ta^4cd^4e^8 + 9216ta^3c^2d^8e^4 + 144ta^2c^3d^{12} + 162a^3d^4e^{11} - 648a^2cd^8e^7 - 810ac^2d^{12}e^3}{27a^3d^3e^{12} - 891a^2cd^7e^8 - 891ac^2d^{11}e^4 + 27c^3d^{15}}\right)\right)\right) + \frac{-ae^3 + cd^3x + 3cd^2ex^2 + 3cde^2x^3}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456*
a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d**
8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 +
28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536*_t*
*2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**8*e**
4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e**7 - 8
10*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 891*a*c*
*2*d**11*e**4 + 27*c**3*d**15)))) + (-a*e**3 + c*d**3*x + 3*c*d**2*e*x**2 +
 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

3.398 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=322

$$-\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{(\sqrt{}}{}$$

Rubi [A] time = 0.27, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{\left(\sqrt{a}\,e^2+3\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\left(\sqrt{a}\,e^2+3\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x)^2)/(4*a*(a + c*x^4)) + (d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
a^(3/2)*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) - ((
3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^
(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-4dex-e^2x^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{4dex}{a+cx^4} + \frac{-3d^2-e^2x^2}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d^2-e^2x^2}{a+cx^4}\,dx}{4a} + \frac{(de)\int \frac{x}{a+cx^4}\,dx}{a}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}} - e^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}} + e^2\right)}{8}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}} + e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}\,d^2}{\sqrt{a}} + e^2\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}} + }}{16ac}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2 - \sqrt{a}\,e^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{(3}{}$$

$$= \frac{x(d+ex)^2}{4a\left(a+cx^4\right)} + \frac{de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{\left(3\sqrt{c}\,d^2 + \sqrt{a}\,e^2\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{\left(3\sqrt{c}\,d^2 + \sqrt{a}\,e^2\right)}{8\sqrt{2}}$$

Mathematica [A] time = 0.32, size = 321, normalized size = 1.00

$$\frac{\frac{\sqrt{2}\left(a^{3/4}e^2-3\sqrt[4]{a}\sqrt{c}\,d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\sqrt{c}\,d^2-a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}} - \frac{2\sqrt[4]{a}\left(8\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+3\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(-8\sqrt[4]{a}\sqrt[4]{c}\,de+\sqrt{2}\sqrt{a}\,e^2+3\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{c^{3/4}} + \frac{8ax(d+ex)^2}{a+cx^4}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a^
(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d*e
+ Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (
Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2 - a^(
3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/
(32*a^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 323, normalized size = 1.00

$$\frac{x^3e^2+2dx^2e+d^2x}{4\left(cx^4+a\right)a}+\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{ac}\,c^2de+3\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3}+\frac{\sqrt{2}\left(4\sqrt{2}\sqrt{ac}\,c^2de+3\left(ac^3\right)^{\frac{1}{4}}c^2d^2+\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3}+\frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3}-\frac{\sqrt{2}\left(3\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e^2 + 2*d*x^2*e + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2)
*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)
*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1
/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d
^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^3)
```

maple [A] time = 0.01, size = 362, normalized size = 1.12

$$\frac{e^2x^3}{4\left(cx^4+a\right)a}+\frac{dex^2}{2\left(cx^4+a\right)a}+\frac{d^2x}{4\left(cx^4+a\right)a}+\frac{de\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{2\sqrt{ac}\,a}+\frac{\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{\sqrt{2}\,e^2\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32\left(\frac{a}{c}\right)^{\frac{1}{4}}ac}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d^2*x/a/(c*x^4+a)+3/32*d^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d^2/a^
2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d^2/a^2*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/2*d*e*x^2/a/(c*x^4+a)+1/2*d*e
/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+1/4*e^2*x^3/a/(c*x^4+a)+1/32*e^2/a
/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+1/16*e^2/a/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)
^(1/4)*x-1)
```

maxima [A] time = 2.63, size = 318, normalized size = 0.99

$$\frac{e^2x^3+2dex^2+d^2x}{4\left(acx^4+a^2\right)}+\frac{\frac{\sqrt{2}\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-8\sqrt{a}\sqrt{c}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+8\sqrt{a}\sqrt{c}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(c
)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2
- sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)*a^(
1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 8*sqrt(a)*sqrt(c)*d*e)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(3/4)*
d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 8*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(
2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*
sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

mupad [B] time = 2.48, size = 391, normalized size = 1.21

$$\frac{\frac{d^2 x}{4 a}+\frac{e^2 x^3}{4 a}+\frac{d e x^2}{2 a}}{c x^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39 c^2 d^4 e^2-a c e^6}{64 a^3}-\mathrm{root}\left(65536 a^7 c^3 z^4+11264 a^4 c^2 d^2 e^2 z^2-2304 a^2 c^2 d^5 e z+256 a^3 c d e^5 z+82 a c d^4 e^4+81 c^2 d^8+a^2 e^8,z,k\right)\left(\mathrm{root}\left(65536 a^7 c^3 z^4+11264 a^4 c^2 d^2 e^2 z^2-2304 a^2 c^2 d^5 e z+256 a^3 c d e^5 z+82 a c d^4 e^4+81 c^2 d^8+a^2 e^8,z,k\right)\left(12 c^3 d^2-16 c^3 d e x\right)+\frac{x\left(18 a c^3 d^4-2 a^2 c^2 e^4\right)}{8 a^3}+\frac{2 c^2 d e^3}{a}\right)+\frac{5 c^2 d^3 e^3 x}{8 a^3}\right)\mathrm{root}\left(65536 a^7 c^3 z^4+11264 a^4 c^2 d^2 e^2 z^2-2304 a^2 c^2 d^5 e z+256 a^3 c d e^5 z+82 a c d^4 e^4+81 c^2 d^8+a^2 e^8,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^2,x)
```

```
[Out] ((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(lo
g((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^4*
c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4
 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*
e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^
2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4 - 2*a
^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))*root(6
5536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a
^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1, 4)
```

sympy [A] time = 3.52, size = 318, normalized size = 0.99

$$\mathrm{RootSum}\left(65536t^4a^7c^3+11264t^2a^4c^2d^2e^2+t\left(256a^3cde^5-2304a^2c^2d^5e\right)+a^2e^8+82acd^4e^4+81c^2d^8,\left(t\mapsto t\log\left(x+\frac{4096t^3a^7c^2e^6+356352t^3a^6c^3d^4e^2-23552t^2a^5c^2d^3e^5+27648t^2a^4c^3d^7e+912ta^4cd^2e^8+43584ta^3c^2d^6e^4+3888ta^2c^3d^{10}+12a^3de^{11}-1088a^2cd^5e^7-7020ac^2d^9e^3}{a^3e^{12}-649a^2cd^4e^8-5841ac^2d^8e^4+729c^3d^{12}}\right)\right)\right)+\frac{d^2x+2dex^2+e^2x^3}{4a^2+4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*a
**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81*c
**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**3*a
**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4*c**
3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 3888*_t
*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c**2*d*
*9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4 + 729*c
**3*d**12)))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x**4)
```

3.399 $\int \frac{d+ex}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=241

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\cdots\right.}{8\sqrt{2}a\cdots}$$

Rubi [A] time = 0.19, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x))/(4*a*(a + c*x^4)) + (e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3
/2)*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(
7/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx = \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \frac{-3d-2ex}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int \left(-\frac{3d}{a+cx^4} - \frac{2ex}{a+cx^4}\right)dx}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{1}{a+cx^4}\,dx}{4a} + \frac{e\int \frac{x}{a+cx^4}\,dx}{2a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{(3d)\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{(3d)\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{4a}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{(3d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{(3d)\int \cdots}{16\cdots}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\cdots\right)}{16\sqrt{2}\,a^{7/4}\cdots}$$

$$= \frac{x(d+ex)}{4a\left(a+cx^4\right)} + \frac{e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3d\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3d\log\left(\sqrt{\cdots}\right.}{\cdots}$$

Mathematica [A] time = 0.19, size = 224, normalized size = 0.93

$$\frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2\left(4\sqrt[4]{a}\,e+3\sqrt{2}\sqrt[4]{c}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2\left(3\sqrt{2}\sqrt[4]{c}\,d-4\sqrt[4]{a}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{3\sqrt{2}\,d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\,d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)*
e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/4)
*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*Sqr
t[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (3
*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))
/(32*a^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.34, size = 241, normalized size = 1.00

$$\frac{3\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^{2}c}-\frac{3\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}} d\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^{2}c}+\frac{x^{2}e+dx}{4\left(cx^{4}+a\right)a}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\left(ac^{3}\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^{2}c^{2}}+\frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\left(ac^{3}\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqr
t(a/c))/(a^2*c) + 1/4*(x^2*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(
2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c*e
+ 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(a^2*c^2)
```

maple [A] time = 0.00, size = 188, normalized size = 0.78

$$\frac{e\,x^{2}}{4\left(c\,x^{4}+a\right)a}+\frac{dx}{4\left(c\,x^{4}+a\right)a}+\frac{e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{4\sqrt{ac}\,a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*d*x/a/(c*x^4+a)+3/32*d/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16*d/a^2*(a/c
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16*d/a^2*(a/c)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/4*e*x^2/a/(c*x^4+a)+1/4*e/a/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)
```

maxima [A] time = 2.43, size = 238, normalized size = 0.99

$$\frac{ex^{2}+dx}{4\left(acx^{4}+a^{2}\right)}+\frac{\frac{3\sqrt{2}d\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}d\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sqr
t(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*sqrt
(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*
c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

mupad [B] time = 0.27, size = 282, normalized size = 1.17

$$\left(\sum_{k=1}^{4}\ln\left(\frac{c^{2}\left(3\,d\,e^{2}+2\,e^{3}\,x-\mathrm{root}\left(65536\,a^{7}\,c^{2}\,z^{4}+2048\,a^{4}\,c\,e^{2}\,z^{2}-1152\,a^{2}\,c\,d^{2}\,e\,z+81\,c\,d^{4}+16\,a\,e^{4},z,k\right)^{2}\,a^{3}\,c\,d\,192+\mathrm{root}\left(65536\,a^{7}\,c^{2}\,z^{4}+2048\,a^{4}\,c\,e^{2}\,z^{2}-1152\,a^{2}\,c\,d^{2}\,e\,z+81\,c\,d^{4}+16\,a\,e^{4},z,k\right)^{2}\,a^{3}\,c\,e\,x\,128-\mathrm{root}\left(65536\,a^{7}\,c^{2}\,z^{4}+2048\,a^{4}\,c\,e^{2}\,z^{2}-1152\,a^{2}\,c\,d^{2}\,e\,z+81\,c\,d^{4}+16\,a\,e^{4},z,k\right)\,a\,c\,d^{2}\,x\,36\right)}{a^{3}\,16}\right)\mathrm{root}\left(65536\,a^{7}\,c^{2}\,z^{4}+2048\,a^{4}\,c\,e^{2}\,z^{2}-1152\,a^{2}\,c\,d^{2}\,e\,z+81\,c\,d^{4}+16\,a\,e^{4},z,k\right)\right)+\frac{\frac{e\,x^{2}}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^2,x)
```

```
[Out] symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 128
*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^
4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^
2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(16*a^3
))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*
d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a + c*x^4)
```

sympy [A] time = 1.09, size = 155, normalized size = 0.64

$$\operatorname{RootSum}\left(65536t^4a^7c^2 + 2048t^2a^4ce^2 - 1152ta^2cd^2e + 16ae^4 + 81cd^4, \left(t \mapsto t\log\left(x + \frac{-32768t^3a^6ce^2 - 4608t^2a^4cd^2e - 512ta^3e^4 - 1296ta^2cd^4 + 360ad^2e^3}{192ade^4 - 243cd^5}\right)\right)\right) + \frac{dx + ex^2}{4a^2 + 4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d**
2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e**
2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 + 360
*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 + 4*a
*c*x**4)
```

3.400 $\int \frac{1}{(a+cx^4)^2} \, dx$

Optimal. Leaf size=202

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\ldots}\right.}{8\sqrt{2}\,a^{7/\ldots}}$$

Rubi [A] time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}}+\frac{x}{4a\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}+\frac{3\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a\left(a+cx^4\right)}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}+\frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}}-\frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\int \cdots}{16\cdots}$$

$$= \frac{x}{4a\left(a+cx^4\right)}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\,\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{x}{4a\left(a+cx^4\right)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}\cdots\right.}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

Mathematica [A] time = 0.11, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^4)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^4)^(-2), x]
```

fricas [A] time = 1.16, size = 173, normalized size = 0.86

$$\frac{12\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)+4x}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

giac [A] time = 0.25, size = 194, normalized size = 0.96

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c}+\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c}-\frac{3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

maple [A] time = 0.00, size = 143, normalized size = 0.71

$$\frac{x}{4\left(cx^4+a\right)a}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{32a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+3/16/a^2*(a/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+3/16/a^2*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.11, size = 189, normalized size = 0.94

$$\frac{x}{4\left(acx^4+a^2\right)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

mupad [**B**] time = 0.09, size = 58, normalized size = 0.29

$$\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{8\,{\left(-a\right)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

sympy [**A**] time = 0.30, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

3.401 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^2} \, dx$

Optimal. Leaf size=855

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\,\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\cdots$$

Rubi [A] time = 0.85, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

$$\frac{\log(d+ex)e^7}{\left(cd^4+ae^4\right)^2}-\frac{\log\left(cx^4+a\right)e^7}{4\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\,\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)e^4}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)e^4}{4\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e}{4a^{3/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{4a\left(cd^4+ae^4\right)\left(cx^4+a\right)}-\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)}{16\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}\,d\left(3\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}\right)}{16\sqrt{2}\,a^{7/4}\left(cd^4+ae^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4))
 - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^
4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a
*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(3*Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2)
+ (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 +
a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4
*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2
 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16
*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)
^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2 \cdots\right.}{\left(cd^4+ae^4\right)^2\left(a+\cdots\right.}\right.$$

$$= \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} + \frac{c\int \frac{d^3-d^2ex+de^2x^2-e^3x^3}{\left(a+cx^4\right)^2}\,dx}{cd^4+ae^4}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \left(\frac{-d^3-de^2x^2}{a+cx^4}+\frac{x\left(d^2e+e^3x^2\right)}{a+cx^4}\right) d\cdots}{\left(cd^4+ae^4\right)^2}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{-d^3-de^2x^2}{a+cx^4}\,dx}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\int \frac{x\left(d^2\cdots\right.}{a\cdots}}{\left(cd^4+\cdots\right.}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(ce^4\right)\text{Subst}\left(\int \frac{d^2e+e^3x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} + \cdots$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{e^7\log(d+ex)}{\left(cd^4+ae^4\right)^2} - \frac{\left(cd^2e^5\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^4+ae^4\right)^2} - \cdots$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} + \frac{e^7\,\mathrm{l}\cdots}{\left(c\cdots\right.}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\cdots}{\cdots}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{4a\left(cd^4+ae^4\right)\left(a+cx^4\right)} - \frac{\sqrt{c}\,d^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)^2} - \frac{\sqrt{c}\,d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^4+ae^4\right)} - \frac{\sqrt[4]{c}\cdots}{\cdots}$$

Mathematica [A] time = 0.39, size = 558, normalized size = 0.65

$$\frac{\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}de^6+\sqrt{a}cd^5e^2-7a\sqrt{c}d^3e^4-3c^{3/2}d^7\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(-5a^{3/2}de^6-\sqrt{a}cd^5e^2+7a\sqrt{c}d^3e^4+3c^{3/2}d^7\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}}-\frac{2\sqrt[4]{c}d\left(-12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6-4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4+3\sqrt{2}c^{3/2}d^6\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt[4]{c}d\left(12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6+4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4+3\sqrt{2}c^{3/2}d^6\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8\left(ae^4+cd^4\right)\left(ae^3+cdx\left(d^2-dex+e^2x^2\right)\right)}{a\left(a+cx^4\right)}-8e^7\log\left(a+cx^4\right)+32e^7\log(d+ex)}{32\left(ae^4+cd^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4)
) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]
*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*e^5
 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4)
+ (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[2]*S
qrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d*e^5 +
 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) +
32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*d^5*e^2
- 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 - Sqrt[a]*c
*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/(32*(c*d^4 +
 a*e^4)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.50, size = 771, normalized size = 0.90

$$\frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e+3\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^3d^4-4\left(ac^3\right)^{\frac{1}{4}}a^2c^2d^3e+4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2+\sqrt{2}a^3c^2e^4-4\left(ac^3\right)^{\frac{3}{4}}a^2de^3\right)}+\frac{\left(4\sqrt{2}\sqrt{ac}c^2d^2e+3\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^3d^4+4\left(ac^3\right)^{\frac{1}{4}}a^2c^2d^3e+4\sqrt{2}\sqrt{ac}a^2c^2d^2e^2+\sqrt{2}a^3c^2e^4+4\left(ac^3\right)^{\frac{3}{4}}a^2de^3\right)}+\frac{\left(3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^7-\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^5e^2+7\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e^4-5\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ade^6\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\left(a^2c^4d^8+2a^3c^3d^4e^4+a^4c^2e^8\right)}-\frac{\left(3\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}c^3d^7-\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}cd^5e^2+7\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}ac^2d^3e^4-5\sqrt{2}\left(ac^3\right)^{\frac{3}{4}}ade^6\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\left(a^2c^4d^8+2a^3c^3d^4e^4+a^4c^2e^8\right)}-\frac{e^7\log\left(|cx^4+a|\right)}{4\left(c^2d^8+2acd^4e^4+a^2e^8\right)}+\frac{e^8\log\left(|xe+d|\right)}{c^2d^8e+2acd^4e^5+a^2e^9}+\frac{acd^4e^3+\left(c^2d^5e^2+acde^6\right)x^3-\left(c^2d^6e+acd^2e^5\right)x^2+a^2e^7+\left(c^2d^7+acd^3e^4\right)x}{4\left(cd^4+ae^4\right)^2\left(cx^4+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3
/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqr
t(2)*a^2*c^3*d^4 - 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*
c^2*d^2*e^2 + sqrt(2)*a^3*c^2*e^4 - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqr
t(2)*sqrt(a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c
^3*d^4 + 4*(a*c^3)^(1/4)*a^2*c^2*d^3*e + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^
2 + sqrt(2)*a^3*c^2*e^4 + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c
^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sqrt(2)*(a*c^3)^(1/
4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/3
2*(3*sqrt(2)*(a*c^3)^(1/4)*c^3*d^7 - sqrt(2)*(a*c^3)^(3/4)*c*d^5*e^2 + 7*sq
rt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^
2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a
^4*c^2*e^8) - 1/4*e^7*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^
8) + e^8*log(abs(x*e + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) + 1/4*(a*c
*d^4*e^3 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 +
a^2*e^7 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(c*x^4 + a)*a)
```

maple [A] time = 0.02, size = 1122, normalized size = 1.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^2,x)
```

```
[Out] e^7*ln(e*x+d)/(a*e^4+c*d^4)^2+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*d*e^6*x^3+1/4
*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^5*e^2/a*x^3-1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a
)*e^5*d^2*x^2-1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*e*d^6/a*x^2+1/4*c/(a*e^4+c*
d^4)^2/(c*x^4+a)*d^3*x*e^4+1/4*c^2/(a*e^4+c*d^4)^2/(c*x^4+a)*d^7/a*x+1/4/(a
*e^4+c*d^4)^2/(c*x^4+a)*e^7*a+1/4*c/(a*e^4+c*d^4)^2/(c*x^4+a)*e^3*d^4+7/16*
c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3
*e^4+3/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)*d^7+7/32*c/(a*e^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^
4+3/32*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7+7/16*c/(a*e
^4+c*d^4)^2/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^4+3
/16*c^2/(a*e^4+c*d^4)^2/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^7-3/4*c/(a*e^4+c*d^4)^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^
2-1/4*c^2/(a*e^4+c*d^4)^2/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^6+5/3
2/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(
1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^6+1/32*c/(a*e^4+c*d^4)^2
/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^2+5/16/(a*e^4+c*d^4)^2/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^6+1/16*c/(a*e^4+c*d^4)^2/a/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^2+5/16/(a*e^4+c*d^4)^2/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^6+1/16*c/(a*e^4+c*d
^4)^2/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^2-1/4*e^7
*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

maxima [A] time = 2.29, size = 601, normalized size = 0.70

$$\frac{e^7\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8}-\frac{c\left(\frac{\sqrt{2}\left(4\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{1}{4}}e^7-3c^2d^7+\sqrt{a}\,c^{\frac{3}{2}}d^5e^2-7\,acd^3e^4+5\,a^{\frac{3}{2}}\sqrt{c}\,de^6\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\left(4\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{1}{4}}e^7+3c^2d^7-\sqrt{a}\,c^{\frac{3}{2}}d^5e^2+7\,acd^3e^4-5\,a^{\frac{3}{2}}\sqrt{c}\,de^6\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{9}{4}}d^7+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{7}{4}}d^5e^2+7\sqrt{2}\,a^{\frac{5}{4}}c^{\frac{5}{4}}d^3e^4+5\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{3}{4}}de^6+4\sqrt{a}\,c^2d^6e+12\,a^{\frac{3}{2}}cd^2e^5\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}-\frac{2\left(3\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{9}{4}}d^7+\sqrt{2}\,a^{\frac{3}{4}}c^{\frac{7}{4}}d^5e^2+7\sqrt{2}\,a^{\frac{5}{4}}c^{\frac{5}{4}}d^3e^4+5\sqrt{2}\,a^{\frac{7}{4}}c^{\frac{3}{4}}de^6-4\sqrt{a}\,c^2d^6e-12\,a^{\frac{3}{2}}cd^2e^5\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{5}{4}}}\right)}{32\left(ac^2d^8+2\,a^2cd^4e^4+a^3e^8\right)}+\frac{cde^2x^3-cd^2ex^2+cd^3x+ae^3}{4\left(a^2cd^4+a^3e^4+\left(ac^2d^4+a^2ce^4\right)x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] e^7*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(sqrt(2)*(4*s
qrt(2)*a^(7/4)*c^(1/4)*e^7 - 3*c^2*d^7 + sqrt(a)*c^(3/2)*d^5*e^2 - 7*a*c*d^
3*e^4 + 5*a^(3/2)*sqrt(c)*d*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(7/4)*c^(1/4)*e^7 + 3
*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)*d*e^
6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
 - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^2 + 7*s
qrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 + 4*sqrt(a
)*c^2*d^6*e + 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^
2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 - 4
*sqrt(a)*c^2*d^6*e - 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/4*(c*d*e^2*x^3
- c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*c^2*d^4 + a^2*c*
e^4)*x^4)
```

mupad [B] time = 2.99, size = 1591, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)),x)
```

```
[Out] e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c^
5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)
) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z
^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*
a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a
^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^
7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4
096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4
+ 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*
c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 8
1*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d
^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 2
4576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e
^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 32768*a^7*c^6*d
^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 - 16384*a^7*c^6*d^
8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4
*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 13312*a^5*c^6*d^9*e
^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - 4096*a^5*c^6*d^8*e^6 +
65536*a^6*c^5*d^4*e^10))/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) +
 (8704*a^5*c^4*d*e^12 + 3584*a^3*c^6*d^9*e^4 + 15360*a^4*c^5*d^5*e^8)/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)) + (x*(15360*a^5*c^4*e^13 - 576*a
^2*c^7*d^12*e + 1920*a^3*c^6*d^8*e^5 + 18880*a^4*c^5*d^4*e^9))/(256*(a^6*e^
8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (192*a*c^6*d^9*e^3 + 704*a^3*c^4*d*e
^11 + 1536*a^2*c^5*d^5*e^7)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))
 + (x*(1280*a^3*c^4*e^12 + 256*a*c^6*d^8*e^4 + 2240*a^2*c^5*d^4*e^8))/(256*
(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4))) + (81*c^5*d^4*e^7*x)/(256*(a^6*
e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^4)))*root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*
e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^
4 + 256*a*e^4, z, k), k, 1, 4) + (e^7*log(d + e*x))/(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4) + (c*d^3*x)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^
4*x^4)) - (c*d^2*e*x^2)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4
*x^4)) + (c*d*e^2*x^3)/(4*(a^3*e^4 + a^2*c*d^4 + a*c^2*d^4*x^4 + a^2*c*e^4*
x^4))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.402 $\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=1141

$$\frac{8cd^3\log(d+ex)e^7}{\left(cd^4+ae^4\right)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^3}-\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)}-\frac{\sqrt{c}\,d\left(3cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\right.\cdots}{\cdots}$$

Rubi [A] time = 1.66, antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] -(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*
a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4
 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]
*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(3*
Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3
) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(
c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (c^(1/4)
*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
+ a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(
7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a
*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(
Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a
*e^4)^3) - (2*c*d^3*e^7*Log[a + c*x^4])/(c*d^4 + a*e^4)^3
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
```

```
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^8}{\left(cd^4+ae^4\right)^2(d+ex)^2} + \frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4 \cdots\right.\right.}{\left(cd^4 \cdots\right.}\right.$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{8cd^3e^7\log(d+ex)}{\left(cd^4+ae^4\right)^3} + \frac{\left(ce^4\right)\int \frac{d^2\left(5cd^4-3ae^4\right)-2de\left(3cd^4-ae^4\right)x+e \cdots}{a+cx^4}}{\left(cd^4+ae^4\right)^3}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^7}{\left(cd^4+ae^4\right)^2(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\left(\cdots\right.\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 0.86, size = 807, normalized size = 0.71

$$\frac{256cd^3\log(d+ex)e^7-64cd^3\log\left(cx^4+a\right)e^7-\frac{32\left(cd^4+ae^4\right)e^7}{d+ex}+\frac{8c\left(cd^4+ae^4\right)\left(cx\left(d^2-2exd+3e^2x^2\right)d^4+ae^3\left(4d^3-3exd^2+2e^2x^2d-e^3x^3\right)\right)}{a\left(cx^4+a\right)}+\frac{2\sqrt[4]{c}\left(-3\sqrt{2}c^{5/2}d^{10}+8\sqrt[4]{a}c^{9/4}ed^9-3\sqrt{2}\sqrt{a}c^2e^2d^8-14\sqrt{2}ac^{3/2}e^4d^6+48a^{5/4}c^{5/4}e^5d^5-30\sqrt{2}a^{3/2}ce^6d^4+21\sqrt{2}a^2\sqrt{c}e^8d^2-24a^{9/4}\sqrt[4]{c}e^9d+5\sqrt{2}a^{5/2}e^{10}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt[4]{c}\left(3\sqrt{2}c^{5/2}d^{10}+8\sqrt[4]{a}c^{9/4}ed^9+3\sqrt{2}\sqrt{a}c^2e^2d^8+14\sqrt{2}ac^{3/2}e^4d^6+48a^{5/4}c^{5/4}e^5d^5+30\sqrt{2}a^{3/2}ce^6d^4-21\sqrt{2}a^2\sqrt{c}e^8d^2-24a^{9/4}\sqrt[4]{c}e^9d-5\sqrt{2}a^{5/2}e^{10}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}-\frac{\sqrt{2}\sqrt[4]{c}\left(3c^{5/2}d^{10}-3\sqrt{a}c^2e^2d^8+14ac^{3/2}e^4d^6-30a^{3/2}ce^6d^4-21a^2\sqrt{c}e^8d^2+5a^{5/2}e^{10}\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{a^{7/4}}+\frac{\sqrt{2}\sqrt[4]{c}\left(3c^{5/2}d^{10}-3\sqrt{a}c^2e^2d^8+14ac^{3/2}e^4d^6-30a^{3/2}ce^6d^4-21a^2\sqrt{c}e^8d^2+5a^{5/2}e^{10}\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{a^{7/4}}}{32\left(cd^4+ae^4\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] ((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2 -
 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))
/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/4)*
d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 + 48*a
^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2*Sqrt
[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 -
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/2)*d^10
 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c*d^4*e^6
- 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*Sqrt[2]*a^(
5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 256*c*d^3*e^7
*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 +
14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^
(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4)
 + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2 + 14*a*c^(3/2)*
d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5*a^(5/2)*e^10)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 64*c*d^3*e
^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 103.64, size = 1104, normalized size = 0.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*
e^8 + a^3*e^12) + 8*c*d^3*e^8*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e
^5 + 3*a^2*c*d^4*e^9 + a^3*e^13) + 1/8*(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(
a*c^3)^(1/4)*c^2*d^4 + 3*sqrt(2)*a*c^2*d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*
(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)
^(1/4))/(sqrt(2)*a^2*c^3*d^6 - 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sq
rt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^2*
```

```
d^3*e^3 - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/8*
(5*sqrt(2)*sqrt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 3*sqrt(2)*a*c^2*
d*e^3 + 6*(a*c^3)^(3/4)*d^2*e^2 - 5*(a*c^3)^(1/4)*a*c*e^4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 6*(a*c^3
)^(1/4)*a^2*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + 9*sqrt(2)*a^3
*c^2*d^2*e^4 + 16*(a*c^3)^(3/4)*a^2*d^3*e^3 + 6*(a*c^3)^(1/4)*a^3*c*d*e^5 +
 sqrt(2)*sqrt(a*c)*a^3*c*e^6) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*
sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4
- 30*sqrt(2)*(a*c^3)^(3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d
^2*e^8 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4)
+ sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^
2*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 - 3*sqrt(2)*(a*c^3)^(3/4)*
c^2*d^8*e^2 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 30*sqrt(2)*(a*c^3)^(
3/4)*a*c*d^4*e^6 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2*e^8 + 5*sqrt(2)*(a*
c^3)^(3/4)*a^2*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*
d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3*d^4*e^8 + a^5*c^2*e^12) + 1/4*(3*c^2*d
^4*x^4*e^3 + c^2*d^5*x^3*e^2 - c^2*d^6*x^2*e + c^2*d^7*x - 5*a*c*x^4*e^7 +
a*c*d*x^3*e^6 - a*c*d^2*x^2*e^5 + a*c*d^3*x*e^4 + 4*a*c*d^4*e^3 - 4*a^2*e^7
)/((a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8)*(c*x^5*e + c*d*x^4 + a*x*e + a*d
))
```

maple [A] time = 0.02, size = 1636, normalized size = 1.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^2,x)
```

```
[Out] 8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)
^3-e^7/(a*e^4+c*d^4)^2/(e*x+d)-1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^6*x*e^4-
3*c^2/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^5-5/32/(a
*e^4+c*d^4)^3*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^10-5/16/(a*e^4+c*d^4)^3*a/(a
/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^10-5/16/(a*e^4+c*d^4)^3
*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^10+1/4*c^3/(a*e^4+
c*d^4)^3/(c*x^4+a)*d^10/a*x-1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*e^10*a*x^3+c/(a
*e^4+c*d^4)^3/(c*x^4+a)*e^7*d^3*a+1/2*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*e^6*x^3
*d^4+c^2/(a*e^4+c*d^4)^3/(c*x^4+a)*d^7*e^3-21/16*c/(a*e^4+c*d^4)^3*(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^
3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^10-1/2*c^3/(a*e
^4+c*d^4)^3/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^9+15/16*c/(a*e^4+c*
d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+
(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/(a/c)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4*e^6+15/8*c/(a*e^4+c*d^4)^3/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^4*e^6+3/2*c/(a*e^4+c*
d^4)^3*a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d+3/4*c^3/(a*e^4+c*d^4)^
3/(c*x^4+a)*e^2/a*x^3*d^8-1/2*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)*d^9*e/a*x^2+1/2
*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d*e^9*a*x^2-3/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)*d^
2*a*x*e^8-21/16*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^
(1/4)*x-1)*d^2*e^8+3/16*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^10-21/32*c/(a*e^4+c*d^4)^3*(a/c)^(1/4)*2^(1/2)*l
n((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^
(1/2)))*d^2*e^8+3/32*c^3/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a
/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d
^10+7/16*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6*e^4+7/8*c^2
/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6*
e^4+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^8*e^2+3/16*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)*d^8*e^2+7/8*c^2/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*a
```

```
rctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^6*e^4+3/32*c^2/(a*e^4+c*d^4)^3/a/(a/c)^(1/
4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1
/2)*x+(a/c)^(1/2)))*d^8*e^2
```

maxima [A] time = 2.54, size = 961, normalized size = 0.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 8*c*d^3*e^7*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^
3*e^12) - 1/32*c*(sqrt(2)*(32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 3*c^3*d^10
+ 3*sqrt(a)*c^(5/2)*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^(3/2)*c^(3/2)*d^4*e^6
 + 21*a^2*c*d^2*e^8 - 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(7/4)*c
^(5/4)*d^3*e^7 + 3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^6*e^4
- 30*a^(3/2)*c^(3/2)*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*e^10)*l
og(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2
*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14
*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*
sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 + 8*sqrt(
a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt
(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)
*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sq
rt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sq
rt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt
(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/2)*c^2*d^5*e^5 +
24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^3*
d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^12) + 1/4*(4*a*c*d^4*e^3
 - 4*a^2*e^7 + (3*c^2*d^4*e^3 - 5*a*c*e^7)*x^4 + (c^2*d^5*e^2 + a*c*d*e^6)*
x^3 - (c^2*d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/(a^2*c^2*d
^9 + 2*a^3*c*d^5*e^4 + a^4*d*e^8 + (a*c^3*d^8*e + 2*a^2*c^2*d^4*e^5 + a^3*c
*e^9)*x^5 + (a*c^3*d^9 + 2*a^2*c^2*d^5*e^4 + a^3*c*d*e^8)*x^4 + (a^2*c^2*d^
8*e + 2*a^3*c*d^4*e^5 + a^4*e^9)*x)
```

mupad [B] time = 4.10, size = 2246, normalized size = 1.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 655
36*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 1812
48*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 1
9200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e^3 +
 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^15)/(
256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^
6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^
4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3
 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*
e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((4096*a^3*c^8*
d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 144384*a^6*c^5*
d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12
*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2
*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*
d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a
^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*(roo
t(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^1
```

```
2*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e
^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e
^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*c^4*d*e^22 - 32768*a^6*
c^9*d^21*e^2 - 32768*a^7*c^8*d^17*e^6 + 196608*a^8*c^7*d^13*e^10 + 458752*a
^9*c^6*d^9*e^14 + 360448*a^10*c^5*d^5*e^18)/(256*(a^8*e^16 + a^4*c^4*d^16 +
 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(81920*a^
11*c^4*e^23 - 49152*a^6*c^9*d^20*e^3 - 114688*a^7*c^8*d^16*e^7 + 32768*a^8*
c^7*d^12*e^11 + 294912*a^9*c^6*d^8*e^15 + 278528*a^10*c^5*d^4*e^19))/(256*(
a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2
*d^8*e^8))) + (5120*a^9*c^4*e^21 - 3072*a^4*c^9*d^20*e + 17408*a^5*c^8*d^16
*e^5 + 337920*a^6*c^7*d^12*e^9 + 616448*a^7*c^6*d^8*e^13 + 304128*a^8*c^5*d
^4*e^17)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*
e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(356352*a^6*c^7*d^11*e^10 - 32768*a^5*c^8*d^
15*e^6 - 10240*a^4*c^9*d^19*e^2 + 770048*a^7*c^6*d^7*e^14 + 391168*a^8*c^5*
d^3*e^18))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^1
2*e^4 + 6*a^6*c^2*d^8*e^8))) + (x*(768*a^3*c^8*d^14*e^5 - 576*a^2*c^9*d^18*
e + 105088*a^4*c^7*d^10*e^9 + 221952*a^5*c^6*d^6*e^13 + 183744*a^6*c^5*d^2*
e^17))/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^
4 + 6*a^6*c^2*d^8*e^8))) + (x*(200*a*c^8*d^13*e^4 + 19400*a^4*c^5*d*e^16 +
7512*a^2*c^7*d^9*e^8 + 2136*a^3*c^6*d^5*e^12))/(256*(a^8*e^16 + a^4*c^4*d^1
6 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8))) + (81*c^7*
d^9*e^6 - 254*a*c^6*d^5*e^10 + 625*a^2*c^5*d*e^14)/(256*(a^8*e^16 + a^4*c^4
*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + (x*(6
25*a^2*c^5*e^15 + 81*c^7*d^8*e^7 - 894*a*c^6*d^4*e^11))/(256*(a^8*e^16 + a^
4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)))*r
oot(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d
^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2
*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d
*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k), k, 1, 4) + ((x^4*(3*c^2*d^4*e^3 -
 5*a*c*e^7))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (a*e^7 - c*d^4*e^3
)/(a*e^4 + c*d^4)^2 + (c*d^3*x)/(4*a*(a*e^4 + c*d^4)) - (c*d^2*e*x^2)/(4*a*
(a*e^4 + c*d^4)) + (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)))/(a*d + a*e*x + c*d*
x^4 + c*e*x^5) + (8*c*d^3*e^7*log(d + e*x))/(a^3*e^12 + c^3*d^12 + 3*a*c^2*
d^8*e^4 + 3*a^2*c*d^4*e^8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.403 $\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=1384

$$\frac{12cd^2\left(3cd^4-ae^4\right)\log(d+ex)e^7}{\left(cd^4+ae^4\right)^4}-\frac{3cd^2\left(3cd^4-ae^4\right)\log\left(cx^4+a\right)e^7}{\left(cd^4+ae^4\right)^4}-\frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3(d+ex)}-\frac{e^7}{2\left(cd^4+ae^4\right)^2(d+ex}$$

Rubi [A] time = 1.96, antiderivative size = 1384, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.824, Rules used = {6742, 1854, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] -e^7/(2*(c*d^4 + a*e^4)^2*(d + e*x)^2) - (8*c*d^3*e^7)/((c*d^4 + a*e^4)^3*(
d + e*x)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e^4) + x*(d*(c^2*d^8 - 12*a*c*d^
4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 2*c*d^3*e
^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^4)) - (Sqrt[c
]*e^5*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 +
a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4 + a*e^4)^3) - (c^
(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2
*(3*c*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*
d^4 - 5*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(3/4)*d
*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d
^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) + 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^2*e^7*(3*
c*d^4 - a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(3/4)*d*(3*c^2*d^8 - 36
*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^3) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5
*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c
^(3/4)*d*(3*c^2*d^8 - 36*a*c*d^4*e^4 + 9*a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^
2*(3*c*d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt
[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) - 3*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*(c*d^4 + a*e^4)^4) - (3*c*d^2*e^7*(3*c*d^4 - a*e^4)*Log[a + c*x^4])/(c*d^4
 + a*e^4)^4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^8}{\left(cd^4+ae^4\right)^2 (d+ex)^3} + \frac{8cd^3e^8}{\left(cd^4+ae^4\right)^3 (d+ex)^2} + \frac{12cd^2e^8\left(3cd^4-ae^4\right)}{\left(cd^4+ae^4\right)^4 (d+ex)} + \frac{c\,(a}{}\right.$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{12cd^2e^7\left(3cd^4-ae^4\right)\log(d+}{\left(cd^4+ae^4\right)^4}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

$$= -\frac{e^7}{2\left(cd^4+ae^4\right)^2 (d+ex)^2} - \frac{8cd^3e^7}{\left(cd^4+ae^4\right)^3 (d+ex)} + \frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\,(a\right.}{}$$

Mathematica [A] time = 1.36, size = 996, normalized size = 0.72

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] ((-16*e^7*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (256*c*d^3*e^7*(c*d^4 + a*e^4))/
(d + e*x) + (8*c*(c*d^4 + a*e^4)*(-(a^2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) +
c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2) + 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x +
6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x^4)) - (6*Sqrt[c]*(Sqrt[2]*c^(13/4)*d
^13 - 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9*Sqrt[2
]*a*c^(9/4)*d^9*e^4 - 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^(7/4)*d
^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 + 84*a^(9/4)*c*d^4*e^9 - 30*Sqrt[2]
*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 - 4*a^(13/4)*e^13)
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[c]*(Sqrt[2]*c^(
13/4)*d^13 + 4*a^(1/4)*c^3*d^12*e + 2*Sqrt[2]*Sqrt[a]*c^(11/4)*d^11*e^2 + 9
```

```
*Sqrt[2]*a*c^(9/4)*d^9*e^4 + 44*a^(5/4)*c^2*d^8*e^5 + 36*Sqrt[2]*a^(3/2)*c^
(7/4)*d^7*e^6 - 49*Sqrt[2]*a^2*c^(5/4)*d^5*e^8 - 84*a^(9/4)*c*d^4*e^9 - 30*
Sqrt[2]*a^(5/2)*c^(3/4)*d^3*e^10 + 7*Sqrt[2]*a^3*c^(1/4)*d*e^12 + 4*a^(13/4
)*e^13)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 384*c*d^2*e^7*(3
*c*d^4 - a*e^4)*Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*Sqrt[a]*c^(
5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 49*a^2*c*d^5
*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*c^(3/4)*(c^3*d^13 - 2*
Sqrt[a]*c^(5/2)*d^11*e^2 + 9*a*c^2*d^9*e^4 - 36*a^(3/2)*c^(3/2)*d^7*e^6 - 4
9*a^2*c*d^5*e^8 + 30*a^(5/2)*Sqrt[c]*d^3*e^10 + 7*a^3*d*e^12)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 96*c*d^2*e^7*(3*c*d^4
- a*e^4)*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.15, size = 1488, normalized size = 1.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/8*(2*sqrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 + 4*sqrt(2)*a*c^
3*d^2*e^3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqr
t(2)*sqrt(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(sqrt(2)*a^2*c^4*d^8 - 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2
)*sqrt(a*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 - 40*(a*c^3)^(3/4)
*a^2*c*d^5*e^3 - 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^
3*c^2*d^2*e^6 + sqrt(2)*a^4*c^2*e^8 - 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/8*(2*s
qrt(2)*sqrt(a*c)*c^3*d^4*e + (a*c^3)^(1/4)*c^3*d^5 - 4*sqrt(2)*a*c^3*d^2*e^
3 + 2*(a*c^3)^(3/4)*c*d^3*e^2 - 9*(a*c^3)^(1/4)*a*c^2*d*e^4 + 2*sqrt(2)*sqr
t(a*c)*a*c^2*e^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(sqrt(2)*a^2*c^4*d^8 + 8*(a*c^3)^(1/4)*a^2*c^3*d^7*e + 16*sqrt(2)*sqrt(a
*c)*a^2*c^3*d^6*e^2 + 34*sqrt(2)*a^3*c^3*d^4*e^4 + 40*(a*c^3)^(3/4)*a^2*c*d
^5*e^3 + 40*(a*c^3)^(1/4)*a^3*c^2*d^3*e^5 + 16*sqrt(2)*sqrt(a*c)*a^3*c^2*d^
2*e^6 + sqrt(2)*a^4*c^2*e^8 + 8*(a*c^3)^(3/4)*a^3*d*e^7) + 3/32*(sqrt(2)*(a
*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*
c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2
)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 30*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7
*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt
(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d
^4*e^12 + a^6*c*e^16) - 3/32*(sqrt(2)*(a*c^3)^(1/4)*c^4*d^13 - 2*sqrt(2)*(a
*c^3)^(3/4)*c^2*d^11*e^2 + 9*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^9*e^4 - 36*sqrt(
```

```
2)*(a*c^3)^(3/4)*a*c*d^7*e^6 - 49*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^5*e^8 + 3
0*sqrt(2)*(a*c^3)^(3/4)*a^2*d^3*e^10 + 7*sqrt(2)*(a*c^3)^(1/4)*a^3*c*d*e^12
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^16 + 4*a^3*c^4*d^
12*e^4 + 6*a^4*c^3*d^8*e^8 + 4*a^5*c^2*d^4*e^12 + a^6*c*e^16) - 3*(3*c^2*d^
6*e^7 - a*c*d^2*e^11)*log(abs(c*x^4 + a))/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*
a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) + 12*(3*c^2*d^6*e^8 - a*c*d^
2*e^12)*log(abs(x*e + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2*d^8*e^
9 + 4*a^3*c*d^4*e^13 + a^4*e^17) + 1/4*(10*a*c^3*d^12*e^3 - 30*a^2*c^2*d^8*
e^7 - 42*a^3*c*d^4*e^11 + 6*(c^4*d^11*e^4 - 6*a*c^3*d^7*e^8 - 7*a^2*c^2*d^3
*e^12)*x^5 + 3*(3*c^4*d^12*e^3 - 11*a*c^3*d^8*e^7 - 15*a^2*c^2*d^4*e^11 - a
^3*c*e^15)*x^4 - 2*a^4*e^15 + (c^4*d^13*e^2 + 3*a*c^3*d^9*e^6 + 3*a^2*c^2*d
^5*e^10 + a^3*c*d*e^14)*x^3 - (c^4*d^14*e + 3*a*c^3*d^10*e^5 + 3*a^2*c^2*d^
6*e^9 + a^3*c*d^2*e^13)*x^2 + (c^4*d^15 + 9*a*c^3*d^11*e^4 - 33*a^2*c^2*d^7
*e^8 - 41*a^3*c*d^3*e^12)*x)/((c*d^4 + a*e^4)^4*(c*x^4 + a)*(x*e + d)^2*a)
```

maple [A] time = 0.03, size = 2121, normalized size = 1.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^2,x)
```

```
[Out] -1/2*e^7/(a*e^4+c*d^4)^2/(e*x+d)^2-8*c*d^3*e^7/(a*e^4+c*d^4)^3/(e*x+d)-12*e
^11*d^2*c/(a*e^4+c*d^4)^4*ln(e*x+d)*a-1/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^13*
a^2*x^2-3/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)*e^11*d^2*a^2-c^3/(a*e^4+c*d^4)^4/(c
*x^4+a)*d^7*e^6*x^3+9/4*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)*e^5*x^2*d^8-11/4*c^3/
(a*e^4+c*d^4)^4/(c*x^4+a)*d^9*x*e^4+1/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^13/
a*x+c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*a*d^6*e^7-33/4*c^3/(a*e^4+c*d^4)^4/(a*c)^
(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^8-3/4*c/(a*e^4+c*d^4)^4*a^2/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)*e^13+3*c/(a*e^4+c*d^4)^4*a*ln(c*x^4+a)*e^11*d^
2+36*e^7*d^6*c^2/(a*e^4+c*d^4)^4*ln(e*x+d)+5/2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a
)*d^10*e^3-9*c^2/(a*e^4+c*d^4)^4*ln(c*x^4+a)*e^7*d^6+27/16*c^3/(a*e^4+c*d^4
)^4/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^4+21/32*c/(
a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d*e^12+21/16*c/(a*e^4+c*d^4)^
4*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d*e^12-45/16*c/(a*e
^4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
)/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^10-45/8*c/(a*e^4+c*d^4)^4*
a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^10-45/8*c/(a*e^
4+c*d^4)^4*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^10-1
47/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)*d^5*e^8+27/8*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)
*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^6+27
/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*
d^7*e^6+27/4*c^2/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*d^7*e^6+63/4*c^2/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan((1/a*c)^(1/
2)*x^2)*e^9*d^4+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x-1)*d^13+3/32*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)
*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c
)^(1/2)))*d^13+3/16*c^4/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)*d^13-3/4*c^4/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)*e*d^12-147/16*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^8-147/32*c^2/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^
(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x
+(a/c)^(1/2)))*d^5*e^8-5/2*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)*d^3*e^10*a*x^3+3/2
*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*d^11*e^2/a*x^3+11/4*c^2/(a*e^4+c*d^4)^4/(c*x
^4+a)*e^9*a*x^2*d^4-3/4*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)*e/a*x^2*d^12-9/4*c^2/
(a*e^4+c*d^4)^4/(c*x^4+a)*d^5*a*x*e^8+3/4*c/(a*e^4+c*d^4)^4/(c*x^4+a)*d*a^2
*x*e^12+3/16*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*
2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^11*e^2+3/
```

```
8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^11*e^2+3/8*c^3/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x+1)*d^11*e^2+27/16*c^3/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^4+27/32*c^3/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*
2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)
*x+(a/c)^(1/2)))*d^9*e^4+21/16*c/(a*e^4+c*d^4)^4*a*(a/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/c)^(1/4)*x-1)*d*e^12
```

maxima [A] time = 2.60, size = 1394, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -3/32*c*(sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqrt(2)*a^(11/4)*
c^(5/4)*d^2*e^11 - c^4*d^13 + 2*sqrt(a)*c^(7/2)*d^11*e^2 - 9*a*c^3*d^9*e^4
+ 36*a^(3/2)*c^(5/2)*d^7*e^6 + 49*a^2*c^2*d^5*e^8 - 30*a^(5/2)*c^(3/2)*d^3*
e^10 - 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(5/4)) + sqrt(2)*(48*sqrt(2)*a^(7/4)*c^(9/4)*d^6*e^7 - 16*sqr
t(2)*a^(11/4)*c^(5/4)*d^2*e^11 + c^4*d^13 - 2*sqrt(a)*c^(7/2)*d^11*e^2 + 9*
a*c^3*d^9*e^4 - 36*a^(3/2)*c^(5/2)*d^7*e^6 - 49*a^2*c^2*d^5*e^8 + 30*a^(5/2
)*c^(3/2)*d^3*e^10 + 7*a^3*c*d*e^12)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1
/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(17/4)*d^13 + 2*s
qrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/4)*c^(13/4)*d^9*e^4 + 36*
sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9/4)*c^(9/4)*d^5*e^8 - 30*
sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(13/4)*c^(5/4)*d*e^12 + 4*s
qrt(a)*c^4*d^12*e + 44*a^(3/2)*c^3*d^8*e^5 - 84*a^(5/2)*c^2*d^4*e^9 + 4*a^(
7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^
(1/4)*c^(17/4)*d^13 + 2*sqrt(2)*a^(3/4)*c^(15/4)*d^11*e^2 + 9*sqrt(2)*a^(5/
4)*c^(13/4)*d^9*e^4 + 36*sqrt(2)*a^(7/4)*c^(11/4)*d^7*e^6 - 49*sqrt(2)*a^(9
/4)*c^(9/4)*d^5*e^8 - 30*sqrt(2)*a^(11/4)*c^(7/4)*d^3*e^10 + 7*sqrt(2)*a^(1
3/4)*c^(5/4)*d*e^12 - 4*sqrt(a)*c^4*d^12*e - 44*a^(3/2)*c^3*d^8*e^5 + 84*a^
(5/2)*c^2*d^4*e^9 - 4*a^(7/2)*c*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))
*c^(5/4)))/(a*c^4*d^16 + 4*a^2*c^3*d^12*e^4 + 6*a^3*c^2*d^8*e^8 + 4*a^4*c*d
^4*e^12 + a^5*e^16) + 12*(3*c^2*d^6*e^7 - a*c*d^2*e^11)*log(e*x + d)/(c^4*d
^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4*e^16) +
 1/4*(10*a*c^2*d^8*e^3 - 40*a^2*c*d^4*e^7 - 2*a^3*e^11 + 6*(c^3*d^7*e^4 - 7
*a*c^2*d^3*e^8)*x^5 + 3*(3*c^3*d^8*e^3 - 14*a*c^2*d^4*e^7 - a^2*c*e^11)*x^4
 + (c^3*d^9*e^2 + 2*a*c^2*d^5*e^6 + a^2*c*d*e^10)*x^3 - (c^3*d^10*e + 2*a*c
^2*d^6*e^5 + a^2*c*d^2*e^9)*x^2 + (c^3*d^11 + 8*a*c^2*d^7*e^4 - 41*a^2*c*d^
3*e^8)*x)/(a^2*c^3*d^14 + 3*a^3*c^2*d^10*e^4 + 3*a^4*c*d^6*e^8 + a^5*d^2*e^
12 + (a*c^4*d^12*e^2 + 3*a^2*c^3*d^8*e^6 + 3*a^3*c^2*d^4*e^10 + a^4*c*e^14)
*x^6 + 2*(a*c^4*d^13*e + 3*a^2*c^3*d^9*e^5 + 3*a^3*c^2*d^5*e^9 + a^4*c*d*e^
13)*x^5 + (a*c^4*d^14 + 3*a^2*c^3*d^10*e^4 + 3*a^3*c^2*d^6*e^8 + a^4*c*d^2*
e^12)*x^4 + (a^2*c^3*d^12*e^2 + 3*a^3*c^2*d^8*e^6 + 3*a^4*c*d^4*e^10 + a^5*
e^14)*x^2 + 2*(a^2*c^3*d^13*e + 3*a^3*c^2*d^9*e^5 + 3*a^4*c*d^5*e^9 + a^5*d
*e^13)*x)
```

mupad [B] time = 5.04, size = 3256, normalized size = 2.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e^8*z^4 + 2
62144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*e^16*z^4 -
 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712*a^5*c^2*d
```

```
^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 58752*a^3*c
^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4, z, k)*((
108*a*c^10*d^19*e^3 + 3888*a^2*c^9*d^15*e^7 - 99576*a^3*c^8*d^11*e^11 + 591
408*a^4*c^7*d^7*e^15 - 79380*a^5*c^6*d^3*e^19)/(256*(a^10*e^24 + a^4*c^6*d^
24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c
^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 39
3216*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16
*z^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^
6*e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*
a^6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^
2*e^4 + 81*c^3*d^4, z, k)*((6912*a^8*c^5*d*e^24 + 4608*a^3*c^10*d^21*e^4 +
154368*a^4*c^9*d^17*e^8 - 331776*a^5*c^8*d^13*e^12 + 5976576*a^6*c^7*d^9*e^
16 - 612864*a^7*c^6*d^5*e^20)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*
e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11
*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 75571
2*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 +
58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^
4, z, k)*((18432*a^5*c^10*d^23*e^5 - 3072*a^4*c^11*d^27*e + 1170432*a^6*c^9
*d^19*e^9 + 2863104*a^7*c^8*d^15*e^13 + 1797120*a^8*c^7*d^11*e^17 - 423936*
a^9*c^6*d^7*e^21 - 506880*a^10*c^5*d^3*e^25)/(256*(a^10*e^24 + a^4*c^6*d^24
 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3
*d^12*e^12 + 15*a^8*c^2*d^8*e^16)) + root(262144*a^10*c*d^4*e^12*z^4 + 3932
16*a^9*c^2*d^8*e^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z
^4 + 65536*a^11*e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*
e^7*z^3 + 755712*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^
6*c*e^10*z^2 + 58752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*
e^4 + 81*c^3*d^4, z, k)*((98304*a^13*c^4*d*e^30 - 32768*a^6*c^11*d^29*e^2 -
 98304*a^7*c^10*d^25*e^6 + 98304*a^8*c^9*d^21*e^10 + 819200*a^9*c^8*d^17*e^
14 + 1474560*a^10*c^7*d^13*e^18 + 1277952*a^11*c^6*d^9*e^22 + 557056*a^12*c
^5*d^5*e^26)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*
d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16
)) + (x*(81920*a^13*c^4*e^31 - 49152*a^6*c^11*d^28*e^3 - 212992*a^7*c^10*d^
24*e^7 - 245760*a^8*c^9*d^20*e^11 + 245760*a^9*c^8*d^16*e^15 + 901120*a^10*
c^7*d^12*e^19 + 933888*a^11*c^6*d^8*e^23 + 442368*a^12*c^5*d^4*e^27))/(256*
(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*
c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) - (x*(12288*a^
4*c^11*d^26*e^2 + 98304*a^5*c^10*d^22*e^6 - 1413120*a^6*c^9*d^18*e^10 - 403
0464*a^7*c^8*d^14*e^14 - 2813952*a^8*c^7*d^10*e^18 + 393216*a^9*c^6*d^6*e^2
2 + 675840*a^10*c^5*d^2*e^26))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4
*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 1
5*a^8*c^2*d^8*e^16))) + (x*(20736*a^8*c^5*e^25 - 576*a^2*c^11*d^24*e - 576*
a^3*c^10*d^20*e^5 + 484992*a^4*c^9*d^16*e^9 + 2468736*a^5*c^8*d^12*e^13 + 4
093632*a^6*c^7*d^8*e^17 - 228672*a^7*c^6*d^4*e^21))/(256*(a^10*e^24 + a^4*c
^6*d^24 + 6*a^9*c*d^4*e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*
a^7*c^3*d^12*e^12 + 15*a^8*c^2*d^8*e^16))) + (x*(216*a*c^10*d^18*e^4 + 2505
6*a^2*c^9*d^14*e^8 - 2160*a^3*c^8*d^10*e^12 + 59616*a^4*c^7*d^6*e^16 + 8661
6*a^5*c^6*d^2*e^20))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20 + 6*
a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*c^2*
d^8*e^16))) + (81*c^9*d^13*e^6 - 2430*a*c^8*d^9*e^10 + 1296*a^3*c^6*d*e^18
+ 3969*a^2*c^7*d^5*e^14)/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*e^20
+ 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15*a^8*
c^2*d^8*e^16)) + (x*(1296*a^3*c^6*e^19 + 81*c^9*d^12*e^7 - 6318*a*c^8*d^8*e
^11 + 5265*a^2*c^7*d^4*e^15))/(256*(a^10*e^24 + a^4*c^6*d^24 + 6*a^9*c*d^4*
e^20 + 6*a^5*c^5*d^20*e^4 + 15*a^6*c^4*d^16*e^8 + 20*a^7*c^3*d^12*e^12 + 15
*a^8*c^2*d^8*e^16)))*root(262144*a^10*c*d^4*e^12*z^4 + 393216*a^9*c^2*d^8*e
^8*z^4 + 262144*a^8*c^3*d^12*e^4*z^4 + 65536*a^7*c^4*d^16*z^4 + 65536*a^11*
e^16*z^4 - 786432*a^8*c*d^2*e^11*z^3 + 2359296*a^7*c^2*d^6*e^7*z^3 + 755712
```

```
*a^5*c^2*d^4*e^6*z^2 + 36864*a^4*c^3*d^8*e^2*z^2 + 18432*a^6*c*e^10*z^2 + 5
8752*a^3*c^2*d^2*e^5*z + 3456*a^2*c^3*d^6*e*z + 1296*a*c^2*e^4 + 81*c^3*d^4
, z, k), k, 1, 4) - ((a^2*e^11 - 5*c^2*d^8*e^3 + 20*a*c*d^4*e^7)/(2*(a*e^4
+ c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^5*(c^3*d^7*e^4 - 7*a*c
^2*d^3*e^8))/(2*a*(a^3*e^12 + c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
) + (3*x^4*(a^2*c*e^11 - 3*c^3*d^8*e^3 + 14*a*c^2*d^4*e^7))/(4*a*(a^3*e^12
+ c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (c*d^2*e*x^2)/(4*a*(a*e^
4 + c*d^4)) - (c*d*e^2*x^3)/(4*a*(a*e^4 + c*d^4)) - (d*x*(c^3*d^10 + 8*a*c^
2*d^6*e^4 - 41*a^2*c*d^2*e^8))/(4*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*
a*c*d^4*e^4)))/(a*d^2 + a*e^2*x^2 + c*d^2*x^4 + c*e^2*x^6 + 2*a*d*e*x + 2*c
*d*e*x^5) + (log(d + e*x)*(36*c^2*d^6*e^7 - 12*a*c*d^2*e^11))/(a^4*e^16 + c
^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.404 $\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\right.}{128\sqrt{2}\,a^{11/4}c^{3/4}}$$

Rubi [A] time = 0.35, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(7\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}-\frac{3d\left(5\sqrt{a}\,e^2+7\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{3d\left(5\sqrt{a}\,e^2+7\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{x\left(18d^2ex+7d^3+15de^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}-\frac{ae^3-cx\left(3d^2ex+d^3+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] (x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(32*a^2*(a + c*x^4)) - (a*e^3 - c*x
*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(8*a*c*(a + c*x^4)^2) + (9*d^2*e*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/
4)) + (3*d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(
11/4)*c^(3/4)) + (3*d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx = -\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} - \frac{\int \frac{-7d^3-18d^2ex-15de^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \frac{21d^3+36d^2ex+15de^2x^2}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \left(\frac{36d^2ex}{a+cx^4}+\frac{21d^3+15de^2x^2}{a+cx^4}\right) \ldots}{32a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\int \frac{21d^3+15de^2x^2}{a+cx^4}\,dx}{32a^2} + \frac{\left(9d^2e\right. \ldots}{\ldots}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{\left(9d^2e\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, \ldots\right.}{16a^2}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{\left(3d\right. \ldots}{\ldots}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(\ldots\right.}{\ldots}$$

$$= \frac{x\left(7d^3+18d^2ex+15de^2x^2\right)}{32a^2\left(a+cx^4\right)} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2} + \frac{9d^2e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{3d\left(7\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.35, size = 388, normalized size = 0.98

$$\frac{\frac{3\sqrt{2}\left(5a^{3/4}de^2-7\sqrt[4]{a}\sqrt{c}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}+\frac{3\sqrt{2}\left(7\sqrt[4]{a}\sqrt{c}d^3-5a^{3/4}de^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{32a^2\left(ae^3-cdx\left(d^2+3dex+3e^2x^2\right)\right)}{c\left(a+cx^4\right)^2}-\frac{6\sqrt[4]{a}d\left(24\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2+7\sqrt{2}\sqrt{c}d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{6\sqrt[4]{a}d\left(-24\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2+7\sqrt{2}\sqrt{c}d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{c^{3/4}}+\frac{8adx\left(7d^2+18dex+15e^2x^2\right)}{a+cx^4}}{256a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] ((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 - c
*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sqrt
[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[c]*d
^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a^(3/4)
*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (
3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^3/(a + c*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 389, normalized size = 0.99

$$\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3}+\frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3}+\frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3}-\frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3}+\frac{15c^2dx^7e^2+18c^2d^2x^6e+7c^2d^3x^5+27acdx^3e^2+30acd^2x^2e+11acd^3x-4a^2e^3}{32\left(cx^4+a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 + 5
*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3
)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^
3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*lo
g(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2*d*x^7*e
^2 + 18*c^2*d^2*x^6*e + 7*c^2*d^3*x^5 + 27*a*c*d*x^3*e^2 + 30*a*c*d^2*x^2*e
 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

maple [A] time = 0.01, size = 470, normalized size = 1.19

$$\frac{e^3x^4}{8\left(cx^4+a\right)^2a}+\frac{3de^2x^3}{8\left(cx^4+a\right)^2a}+\frac{e^3x^4}{8\left(cx^4+a\right)a^2}+\frac{3d^2ex^2}{8\left(cx^4+a\right)^2a}+\frac{15de^2x^3}{32\left(cx^4+a\right)a^2}+\frac{d^3x}{8\left(cx^4+a\right)^2a}+\frac{9d^2ex^2}{16\left(cx^4+a\right)a^2}+\frac{7d^3x}{32\left(cx^4+a\right)a^2}+\frac{9d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{16\sqrt{ac}\,a^2}+\frac{15\sqrt{2}\,de^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{15\sqrt{2}\,de^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{15\sqrt{2}\,de^2\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^3\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^3*x/a/(c*x^4+a)^2+7/32*d^3/a^2*x/(c*x^4+a)+21/256*d^3/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+3/8*e*d^2*x^2/a/(c*x^4+a)^2+9/16*e*d^2/a^2*x^2/(c*x^4+a)+9/16*e*d^2/a^2/(
a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)+3/8*d*e^2*x^3/a/(c*x^4+a)^2+15/32*d*e^
2/a^2*x^3/(c*x^4+a)+15/256*d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1
/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+15/128*
d*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+15/128*d*e^
2/a^2/c/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e^3*x^4/a/(
c*x^4+a)^2+1/8*e^3/a^2*x^4/(c*x^4+a)
```

maxima [A] time = 2.37, size = 392, normalized size = 0.99

$$\frac{15c^2de^2x^7+18c^2d^2ex^6+7c^2d^3x^5+27acde^2x^3+30acd^2ex^2+11acd^3x-4a^2e^3}{32\left(a^2c^3x^8+2a^3c^2x^4+a^4c\right)}+\frac{3d\left(\frac{\sqrt{2}\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-24\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+24\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}\right)}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(15*c^2*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^
3 + 30*a*c*d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2*x
^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*
x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*s
qrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^
(3/4)*c^(1/4)*e^2 - 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1
/4)*e^2 + 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
3/4)))/a^2
```

mupad [B] time = 0.48, size = 721, normalized size = 1.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(1
6*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7)/
(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*e^2
 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4 + 6
3111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e
^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^
5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 -
 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 5
0625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root(26843545
6*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4
147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^
2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^4 + 63111168
*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z +
245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)^2*a^5*c^2*e
*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812
8512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625
*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768*a^6))*root(26843
5456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z
+ 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481
*c^2*d^12, z, k), k, 1, 4)
```

sympy [A] time = 8.01, size = 413, normalized size = 1.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 + _
t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4*e
**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2621
44000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 539688
960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 77328000
*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t*a**3*
c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 60566940
*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*e**8 - 5
8012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15)))) + (-4*a**2*e**3 + 11*a*c
*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x**5 + 18*c
**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2*x**4 + 32*
a**2*c**3*x**8)
```

3.405 $\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}-\frac{(5\ldots}{}$$

Rubi [A] time = 0.33, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\left(21\sqrt{c}\,d^2-5\sqrt{a}\,e^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}c^{3/4}}-\frac{\left(5\sqrt{a}\,e^2+21\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{\left(5\sqrt{a}\,e^2+21\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}c^{3/4}}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + (x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/(3
2*a^2*(a + c*x^4)) + (3*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(3/4)) - ((21*
Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(1
1/4)*c^(3/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3}\,dx &= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}-\frac{\int \frac{-7d^2-12dex-5e^2x^2}{\left(a+cx^4\right)^2}\,dx}{8a} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{\int \frac{21d^2+24dex+5e^2x^2}{a+cx^4}\,dx}{32a^2} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{\int \left(\frac{24dex}{a+cx^4}+\frac{21d^2+5e^2x^2}{a+cx^4}\right)dx}{32a^2} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{\int \frac{21d^2+5e^2x^2}{a+cx^4}\,dx}{32a^2}+\frac{(3de)\int \frac{x}{a+cx^4}\,dx}{4a^2} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{(3de)\,\mathrm{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{8a^2}+\frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)\int \cdots}{64a^2 \cdots} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-\cdots}\cdots}{128\sqrt{2}\,a^{9/4}c^{3/4}} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{\left(\frac{21\sqrt{c}\,d^2}{\sqrt{a}}-5e^2\right)\log\left(\sqrt{a}-\sqrt{2}\cdots\right.}{128\sqrt{2}\,a^{9/4}c^{3/4}\cdots} \\
&= \frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}+\frac{x\left(7d^2+12dex+5e^2x^2\right)}{32a^2\left(a+cx^4\right)}+\frac{3de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}-\frac{\left(21\sqrt{c}\,d^2+5\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\cdots\right.}{64\sqrt{2}\,a^{11/4}c^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.29, size = 358, normalized size = 0.99

$$
\frac{\frac{\sqrt{2}\left(5a^{3/4}e^2-21\sqrt[4]{a}\sqrt{c}\,d^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}}+\frac{\sqrt{2}\left(21\sqrt[4]{a}\sqrt{c}\,d^2-5a^{3/4}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{c^{3/4}}+\frac{32a^2x(d+ex)^2}{\left(a+cx^4\right)^2}-\frac{2\sqrt[4]{a}\left(48\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2+21\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt[4]{a}\left(-48\sqrt[4]{a}\sqrt[4]{c}\,de+5\sqrt{2}\sqrt{a}\,e^2+21\sqrt{2}\sqrt{c}\,d^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{c^{3/4}}+\frac{8ax\left(7d^2+12dex+5e^2x^2\right)}{a+cx^4}}{256a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x^
2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)*d
*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4
) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]
*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt[2]*(
-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5*a^(3/4
)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(25
6*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(d+ex)^2}{\left(a+cx^4\right)^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)^2/(a + c*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.43, size = 356, normalized size = 0.99

$$\frac{5cx^7e^2+12cdx^6e+7cd^2x^5+9ax^3e^2+20adx^2e+11ad^2x}{32\left(cx^4+a\right)^2a^2}+\frac{\sqrt{2}\left(24\sqrt{2}\sqrt{ac}c^2de+21\left(ac^3\right)^{\frac{1}{4}}c^2d^2+5\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3}+\frac{\sqrt{2}\left(24\sqrt{2}\sqrt{ac}c^2de+21\left(ac^3\right)^{\frac{1}{4}}c^2d^2+5\left(ac^3\right)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3}+\frac{\sqrt{2}\left(21\left(ac^3\right)^{\frac{1}{4}}c^2d^2-5\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3}-\frac{\sqrt{2}\left(21\left(ac^3\right)^{\frac{1}{4}}c^2d^2-5\left(ac^3\right)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*c*x^7*e^2 + 12*c*d*x^6*e + 7*c*d^2*x^5 + 9*a*x^3*e^2 + 20*a*d*x^2*e
 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*c^
2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*sqrt
(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/256*
sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*
d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
3*c^3)
```

maple [A] time = 0.01, size = 419, normalized size = 1.16

$$\frac{e^2x^3}{8\left(cx^4+a\right)^2a}+\frac{dex^2}{4\left(cx^4+a\right)^2a}+\frac{5e^2x^3}{32\left(cx^4+a\right)a^2}+\frac{d^2x}{8\left(cx^4+a\right)^2a}+\frac{3dex^2}{8\left(cx^4+a\right)a^2}+\frac{7d^2x}{32\left(cx^4+a\right)a^2}+\frac{3de\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{8\sqrt{ac}\,a^2}+\frac{5\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{5\sqrt{2}\,e^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{5\sqrt{2}\,e^2\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{256\left(\frac{a}{c}\right)^{\frac{1}{4}}a^2c}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^3}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{a}{c}}}\right)}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d^2*x/a/(c*x^4+a)^2+7/32*d^2/a^2*x/(c*x^4+a)+21/256*d^2/a^3*(a/c)^(1/4)
*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+21/128*d^2/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)+1/4*d*e*x^2/a/(c*x^4+a)^2+3/8*d*e/a^2*x^2/(c*x^4+a)+3/8*d*e/a^2/(a*c)^(1/
2)*arctan((1/a*c)^(1/2)*x^2)+1/8*e^2*x^3/a/(c*x^4+a)^2+5/32*e^2/a^2*x^3/(c*
x^4+a)+5/256*e^2/a^2/c/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))+5/128*e^2/a^2/c/(a/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+5/128*e^2/a^2/c/(a/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.18, size = 364, normalized size = 1.01

$$\frac{5ce^2x^7+12cdex^6+7cd^2x^5+9ae^2x^3+20adex^2+11ad^2x}{32\left(a^2c^2x^8+2a^3cx^4+a^4\right)}+\frac{\frac{\sqrt{2}\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-48\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+48\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{3}{4}}}}{256a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^2
 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(
```

```
c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(
a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c
)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21*sqrt
(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*sqrt(c
)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt
(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(2)*a^(1/
4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c)*d*e)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

mupad [B] time = 0.47, size = 676, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^3,x)
```

```
[Out] ((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^2
*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^8
+ 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(268
435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*
z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^
8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*c^3*z^
4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d
*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*e^4
*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 541
9008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c
^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456*a^11*c^3*z
^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*
d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^3*c*d*
e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5
419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481
*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6))*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 30
7200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z,
k), k, 1, 4)
```

sympy [A] time = 4.88, size = 374, normalized size = 1.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 + _
t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 11190
6*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t**3*
a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t**2*
a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*a**5*
c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c**3*d*
*10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*c**2*d*
*9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*a*c**2*d
**8*e**4 + 85766121*c**3*d**12)))) + (11*a*d**2*x + 20*a*d*e*x**2 + 9*a*e**
2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4 + 64*a**3*
c*x**4 + 32*a**2*c**2*x**8)
```

3.406 $\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx$

Optimal. Leaf size=266

$$-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\,t\ldots}{6\ldots}$$

Rubi [A] time = 0.25, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {1855, 1876, 211, 1165, 628, 1162, 617, 204, 275, 205}

$$\frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)}-\frac{21d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{128\sqrt{2}\,a^{11/4}\sqrt[4]{c}}-\frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{21d\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}\,a^{11/4}\sqrt[4]{c}}+\frac{3e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] (x*(d + e*x))/(8*a*(a + c*x^4)^2) + (x*(7*d + 6*e*x))/(32*a^2*(a + c*x^4))
+ (3*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*Sqrt[c]) - (21*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*d*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*d*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/
4)*c^(1/4)) + (21*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(128*Sqrt[2]*a^(11/4)*c^(1/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx &= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} - \frac{\int \frac{-7d-6ex}{\left(a+cx^4\right)^2}\,dx}{8a} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \frac{21d+12ex}{a+cx^4}\,dx}{32a^2} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{\int \left(\frac{21d}{a+cx^4} + \frac{12ex}{a+cx^4}\right)dx}{32a^2} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{1}{a+cx^4}\,dx}{32a^2} + \frac{(3e)\int \frac{x}{a+cx^4}\,dx}{8a^2} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{(21d)\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(21d)\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}} + \frac{(3e)\,\text{Subst}}{} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} + \frac{(21d)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}} + \frac{(21d)\int}{128} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}} \\
&= \frac{x(d+ex)}{8a\left(a+cx^4\right)^2} + \frac{x(7d+6ex)}{32a^2\left(a+cx^4\right)} + \frac{3e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\tan^{-1}(}{64\sqrt{2}a}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 249, normalized size = 0.94

$$\frac{\frac{32a^{7/4}x(d+ex)}{\left(a+cx^4\right)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(8\sqrt[4]{a}e+7\sqrt{2}\sqrt[4]{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{c}} - \frac{21\sqrt{2}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{21\sqrt{2}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] ((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex}{\left(a+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(d + e*x)/(a + c*x^4)^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.32, size = 260, normalized size = 0.98

$$\frac{21\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}d\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^{3}c}-\frac{21\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}d\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^{3}c}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\left(ac^{3}\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^{3}c^{2}}+\frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\left(ac^{3}\right)^{\frac{1}{4}}cd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^{3}c^{2}}+\frac{6cx^{6}e+7cdx^{5}+10ax^{2}e+11adx}{32\left(cx^{4}+a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))
/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1
/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c
^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32*(6*c*
x^6*e + 7*c*d*x^5 + 10*a*x^2*e + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.01, size = 222, normalized size = 0.83

$$\frac{ex^{2}}{8\left(cx^{4}+a\right)^{2}a}+\frac{dx}{8\left(cx^{4}+a\right)^{2}a}+\frac{3ex^{2}}{16\left(cx^{4}+a\right)a^{2}}+\frac{7dx}{32\left(cx^{4}+a\right)a^{2}}+\frac{3e\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{16\sqrt{ac}\,a^{2}}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)}{128a^{3}}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)}{128a^{3}}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{c}}}\right)}{256a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^3,x)
```

```
[Out] 1/8*d*x/a/(c*x^4+a)^2+7/32*d/a^2*x/(c*x^4+a)+21/256*d/a^3*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2)))+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1
)+21/128*d/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)+1/8*e*x^
2/a/(c*x^4+a)^2+3/16*e/a^2*x^2/(c*x^4+a)+3/16*e/a^2/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)
```

maxima [A] time = 2.20, size = 269, normalized size = 1.01

$$\frac{6cex^{6}+7cdx^{5}+10aex^{2}+11adx}{32\left(a^{2}c^{2}x^{8}+2a^{3}cx^{4}+a^{4}\right)}+\frac{3\left(\frac{7\sqrt{2}d\log\left(\sqrt{c}x^{2}+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{7\sqrt{2}d\log\left(\sqrt{c}x^{2}-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-8\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}+\frac{2\left(7\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+8\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}\,c^{\frac{1}{4}}}\right)}{256a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*c
*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/4)*d
 - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7*sqrt(
2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
```

```
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(1/4)))/a^2
```

mupad [B] time = 0.30, size = 315, normalized size = 1.18

$$\frac{\frac{5ex^2}{16a}+\frac{11dx}{32a}+\frac{7cdx^5}{32a^2}+\frac{3cex^6}{16a^2}}{a^2+2acx^4+c^2x^8}+\left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(63de^2+36e^3x-\mathrm{root}\left(268435456a^{11}c^2z^4+4718592a^6ce^2z^2-2709504a^3cd^2ez+194481cd^4+20736ae^4,z,k\right)^2a^5cd\,7168-\mathrm{root}\left(268435456a^{11}c^2z^4+4718592a^6ce^2z^2-2709504a^3cd^2ez+194481cd^4+20736ae^4,z,k\right)a^2cd^2x\,1176+\mathrm{root}\left(268435456a^{11}c^2z^4+4718592a^6ce^2z^2-2709504a^3cd^2ez+194481cd^4+20736ae^4,z,k\right)^2a^5cex\,4096\right)3}{a^6\,2048}\right)\mathrm{root}\left(268435456a^{11}c^2z^4+4718592a^6ce^2z^2-2709504a^3cd^2ez+194481cd^4+20736ae^4,z,k\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^3,x)
```

```
[Out] ((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(1
6*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^3*
x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*
c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(2684354
56*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*
d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z^4 + 4
718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4,
z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c
*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k), k, 1,
 4)
```

sympy [A] time = 1.48, size = 192, normalized size = 0.72

$$\mathrm{RootSum}\left(268435456t^4a^{11}c^2+4718592t^2a^6ce^2-2709504ta^3cd^2e+20736ae^4+194481cd^4,\left(t\mapsto t\log\left(x+\frac{-67108864t^3a^9ce^2-9633792t^2a^6cd^2e-589824ta^4e^4-2765952ta^3cd^4+423360ad^2e^3}{193536ade^4-453789cd^5}\right)\right)\right)+\frac{11adx+10aex^2+7cdx^5+6cex^6}{32a^4+64a^3cx^4+32a^2c^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_t
*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-6710
8864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e**4
- 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 453789*c*
d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4 + 64
*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

3.407 $\int \frac{1}{\left(a+cx^4\right)^3} \, dx$

Optimal. Leaf size=219

$$-\frac{21 \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x+\sqrt{a}+\sqrt{c} x^2\right)}{128 \sqrt{2} a^{11/4} \sqrt[4]{c}}+\frac{21 \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x+\sqrt{a}+\sqrt{c} x^2\right)}{128 \sqrt{2} a^{11/4} \sqrt[4]{c}}-\frac{21 \tan ^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64 \sqrt{2} a^{11/4} \sqrt[4]{c}}+\frac{21 \tan ^{-1}\left(\sqrt{}\right.}{64 \sqrt{2} a^{1}}$$

Rubi [A] time = 0.14, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {199, 211, 1165, 628, 1162, 617, 204}

$$\frac{7 x}{32 a^2 \left(a+c x^4\right)}-\frac{21 \log \left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x+\sqrt{a}+\sqrt{c} x^2\right)}{128 \sqrt{2} a^{11/4} \sqrt[4]{c}}+\frac{21 \log \left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{c} x+\sqrt{a}+\sqrt{c} x^2\right)}{128 \sqrt{2} a^{11/4} \sqrt[4]{c}}-\frac{21 \tan ^{-1}\left(1-\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)}{64 \sqrt{2} a^{11/4} \sqrt[4]{c}}+\frac{21 \tan ^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}+1\right)}{64 \sqrt{2} a^{11/4} \sqrt[4]{c}}+\frac{x}{8 a \left(a+c x^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3),x]
```

```
[Out] x/(8*a*(a + c*x^4)^2) + (7*x)/(32*a^2*(a + c*x^4)) - (21*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*c^(1/4)) - (21*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*c^(1/4)) +
(21*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^
(11/4)*c^(1/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+cx^4\right)^3}\,dx = \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7\int \frac{1}{\left(a+cx^4\right)^2}\,dx}{8a}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{1}{a+cx^4}\,dx}{32a^2}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{\sqrt{a}-\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}}+\frac{21\int \frac{\sqrt{a}+\sqrt{c}x^2}{a+cx^4}\,dx}{64a^{5/2}}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}+\frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}}+\frac{21\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{128a^{5/2}\sqrt{c}}-\frac{21\int \cdots}{128\cdots}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{128\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\log\left(\sqrt{a}+\sqrt{2}\cdots\right)}{128\sqrt{2}\cdots}$$

$$= \frac{x}{8a\left(a+cx^4\right)^2}+\frac{7x}{32a^2\left(a+cx^4\right)}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}+\frac{21\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}-\frac{21\log\cdots}{\cdots}$$

Mathematica [A] time = 0.08, size = 200, normalized size = 0.91

$$\frac{\frac{32a^{7/4}x}{\left(a+cx^4\right)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{c}}}{256a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3),x]
```

```
[Out] ((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + c*x^4)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + c*x^4)^(-3), x]
```

fricas [A] time = 1.46, size = 232, normalized size = 1.06

$$\frac{28\,cx^5 + 84\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\arctan\left(-a^8cx\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}} + \sqrt{a^6\sqrt{-\frac{1}{a^{11}c}} + x^2}\,a^8c\left(-\frac{1}{a^{11}c}\right)^{\frac{3}{4}}\right) + 21\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) - 21\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(-a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) + 44\,ax}{128\left(a^2c^2x^8 + 2\,a^3cx^4 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(28*c*x^5 + 84*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*
arctan(-a^8*c*x*(-1/(a^11*c))^(3/4) + sqrt(a^6*sqrt(-1/(a^11*c)) + x^2)*a^8
*c*(-1/(a^11*c))^(3/4)) + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c)
)^(1/4)*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 +
a^4)*(-1/(a^11*c))^(1/4)*log(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)
```

giac [A] time = 0.36, size = 204, normalized size = 0.93

$$\frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\,\sqrt{2}\left(ac^3\right)^{\frac{1}{4}}\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{7\,cx^5 + 11\,ax}{32\left(cx^4 + a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32*(7*c
*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

maple [A] time = 0.00, size = 158, normalized size = 0.72

$$\frac{x}{8\left(c\,x^4 + a\right)^2a} + \frac{7x}{32\left(c\,x^4 + a\right)a^2} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} - 1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}} + 1\right)}{128a^3} + \frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2 + \left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x + \sqrt{\frac{a}{c}}}{x^2 - \left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,x + \sqrt{\frac{a}{c}}}\right)}{256a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^3,x)
```

```
[Out] 1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/256/a^3*(a/c)^(1/4)*2^(1/2)*ln(
(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1
/2)))+21/128/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+21/128
/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
```

maxima [A] time = 2.70, size = 212, normalized size = 0.97

$$\frac{7\,cx^5+11\,ax}{32\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}+\frac{21\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{256\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

mupad [B] time = 0.10, size = 80, normalized size = 0.37

$$\frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^3,x)
```

```
[Out] ((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atan
((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

sympy [A] time = 0.46, size = 63, normalized size = 0.29

$$\frac{11ax+7cx^5}{32a^4+64a^3cx^4+32a^2c^2x^8}+\mathrm{RootSum}\left(268435456t^4a^{11}c+194481,\left(t\mapsto t\log\left(\frac{128ta^3}{21}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**3,x)
```

```
[Out] (11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootSu
m(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)))
```

3.408 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^3} dx$

Optimal. Leaf size=1352

$$\frac{\log(d+ex)e^{11}}{\left(cd^4+ae^4\right)^3}-\frac{\log\left(cx^4+a\right)e^{11}}{4\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}\,d^2\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}\,\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}\,a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,\right.}{}$$

Rubi [A] time = 1.41, antiderivative size = 1352, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] (c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4)
) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c*x
^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a*e^
4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt
[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4
*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^
4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*S
qrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sq
rt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d^4 + a*e^4)^3 - (
c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*d*e
^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(21*Sqrt[
c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 -
 Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*
e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*d*(21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(
c*d^4 + a*e^4)) - (e^11*Log[a + c*x^4])/(4*(c*d^4 + a*e^4)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(cd^4+ae^4\right)\left(a+cx^4\right)^3} - \frac{ce^4\left(-d^3+d^2ex-de^2x\right.}{\left(cd^4+ae^4\right)^2\left(a+c\right.}\right.$$

$$= \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^8\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{a+cx^4}dx}{\left(cd^4+ae^4\right)^3} - \frac{\left(ce^4\right)\int \frac{-d^3+d^2ex-de^2x^2+e^3x^3}{\left(a+cx^4\right)^2}dx}{\left(cd^4+ae^4\right)^2} + \frac{c}{\ldots}$$

$$= \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2ex+de^2x^2\right)\right)}{4a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)} + \frac{e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^3}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

$$= \frac{cx\left(7d^3-6d^2ex+5de^2x^2\right)}{32a^2\left(cd^4+ae^4\right)\left(a+cx^4\right)} + \frac{ae^3+cx\left(d^3-d^2ex+de^2x^2\right)}{8a\left(cd^4+ae^4\right)\left(a+cx^4\right)^2} + \frac{e^4\left(ae^3+cx\left(d^3-d^2e\right.\right.}{4a\left(cd^4+ae^4\right)^2\left(\right.}$$

Mathematica [A] time = 0.74, size = 835, normalized size = 0.62

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] ((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x
^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e^2
*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4)) -
(2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2
]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5/4)*d
^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 12
```

```
0*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 + 24*a^(1
/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^
6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt
[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^
10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^11*Log[d + e*
x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^2 - 66*a*c^(3
/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 + 45*a^(5/2)*d*
e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) + (S
qrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 66*a*c^(3/2)*d^7*
e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a^(5/2)*d*e^10)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4) - 64*e^11*Lo
g[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)*(a + c*x^4)^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.91, size = 1259, normalized size = 0.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 - 75*sqrt(2
)*a*c^2*d^2*e^3 + 122*(a*c^3)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3
*d^6 - 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2
+ 9*sqrt(2)*a^4*c^2*d^2*e^4 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3 - 6*(a*c^3)^(1/4
)*a^4*c*d*e^5 + sqrt(2)*sqrt(a*c)*a^4*c*e^6) + 1/64*(51*sqrt(2)*sqrt(a*c)*c
^2*d^4*e + 21*(a*c^3)^(1/4)*c^2*d^5 + 75*sqrt(2)*a*c^2*d^2*e^3 + 122*(a*c^3
)^(3/4)*d^3*e^2 + 45*(a*c^3)^(1/4)*a*c*d*e^4)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 6*(a*c^3)^(1/4)*a^3*c
^2*d^5*e + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*e^2 + 9*sqrt(2)*a^4*c^2*d^2*e^4
+ 16*(a*c^3)^(3/4)*a^3*d^3*e^3 + 6*(a*c^3)^(1/4)*a^4*c*d*e^5 + sqrt(2)*sqrt
(a*c)*a^4*c*e^6) + 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*
c^3)^(3/4)*c^2*d^9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2
)*(a*c^3)^(3/4)*a*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45
*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12)
- 1/256*(21*sqrt(2)*(a*c^3)^(1/4)*c^4*d^11 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^
9*e^2 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 - 18*sqrt(2)*(a*c^3)^(3/4)*a
*c*d^5*e^6 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 45*sqrt(2)*(a*c^3)^
(3/4)*a^2*d*e^10)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^1
```

```
2 + 3*a^4*c^4*d^8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12) - 1/4*e^11*log(ab
s(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + e
^12*log(abs(x*e + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a^3
*e^13) + 1/32*(4*a^2*c^2*d^8*e^3 + 16*a^3*c*d^4*e^7 + (5*c^4*d^9*e^2 + 18*a
*c^3*d^5*e^6 + 13*a^2*c^2*d*e^10)*x^7 - 2*(3*c^4*d^10*e + 10*a*c^3*d^6*e^5
+ 7*a^2*c^2*d^2*e^9)*x^6 + (7*c^4*d^11 + 22*a*c^3*d^7*e^4 + 15*a^2*c^2*d^3*
e^8)*x^5 + 8*(a^2*c^2*d^4*e^7 + a^3*c*e^11)*x^4 + 12*a^4*e^11 + (9*a*c^3*d^
9*e^2 + 26*a^2*c^2*d^5*e^6 + 17*a^3*c*d*e^10)*x^3 - 2*(5*a*c^3*d^10*e + 14*
a^2*c^2*d^6*e^5 + 9*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 30*a^2*c^2*d^7*e^
4 + 19*a^3*c*d^3*e^8)*x)/((c*d^4 + a*e^4)^3*(c*x^4 + a)^2*a^2)
```

maple [A] time = 0.02, size = 2098, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^3,x)

[Out] e^11*ln(e*x+d)/(a*e^4+c*d^4)^3-1/4*e^11*ln(c*x^4+a)/(a*e^4+c*d^4)^3+3/8/(a*
e^4+c*d^4)^3/(c*x^4+a)^2*e^11*a^2+33/128*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4
)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/
2)*x+(a/c)^(1/2)))*d^7*e^4+33/64*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^7*e^4+77/128*c/(a*e^4+c*d^4)^3/a*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^3*e^8+5/256*c^2/(a*e^4+c*d^
4)^3/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^9*e^2+5/128*c^2/(a*e^4+c*d^4)^3/a^2
/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^9*e^2+5/128*c^2/(a*e
^4+c*d^4)^3/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^9*e^2
+9/64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-
1)*d^5*e^6+9/64*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x+1)*d^5*e^6+77/128*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e^8+77/256*c/(a*e^4+c*d^4)^3/a*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^3*e^8+9/128*c/(a*e^4+c*d^4)^3/a/(a/c)^(1/4)*2^(1/2)*ln((x^2
-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*d^5*e^6+33/64*c^2/(a*e^4+c*d^4)^3/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/c)^(1/4)*x-1)*d^7*e^4+13/32*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^10*x^7-7
/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^2*e^9*x^6+15/32*c^2/(a*e^4+c*d^4)^3/(
c*x^4+a)^2*d^3*x^5*e^8+7/32*c^4/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^11/a^2*x^5+1/
4*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*x^4*e^7*d^4+13/16*c^2/(a*e^4+c*d^4)^3/(c*
x^4+a)^2*d^5*e^6*x^3+1/4*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*x^4*e^11*a+1/2*c/(a*
e^4+c*d^4)^3/(c*x^4+a)^2*e^7*d^4*a-7/8*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*
e^5*x^2+15/16*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^7*x*e^4+11/32*c^3/(a*e^4+c*
d^4)^3/(c*x^4+a)^2*d^11/a*x-15/16*c/(a*e^4+c*d^4)^3/(a*c)^(1/2)*arctan((1/a
*c)^(1/2)*x^2)*e^9*d^2+45/256/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*ln((x^2-(
a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*
d*e^10+45/128/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)*d*e^10+45/128/(a*e^4+c*d^4)^3/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)*d*e^10+9/32*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^9*e^2/a*x^3-5/1
6*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a*x^2-9/16*c/(a*e^4+c*d^4)^3/(c*x^
4+a)^2*d^2*e^9*a*x^2+19/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^3*a*x*e^8+21/128
*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)*d^11+21/256*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/
4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^11+21/
128*c^3/(a*e^4+c*d^4)^3/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^11-5/8*c^2/(a*e^4+c*d^4)^3/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e
^5*d^6-3/16*c^3/(a*e^4+c*d^4)^3/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e
*d^10-3/16*c^4/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^10*e/a^2*x^6+11/16*c^3/(a*e^4+
c*d^4)^3/(c*x^4+a)^2*d^7/a*x^5*e^4+17/32*c/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d*e^
10*a*x^3+9/16*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^5*e^6/a*x^7+5/32*c^4/(a*e^4
```

```
+c*d^4)^3/(c*x^4+a)^2*d^9*e^2/a^2*x^7+1/8*c^2/(a*e^4+c*d^4)^3/(c*x^4+a)^2*e
^3*d^8-5/8*c^3/(a*e^4+c*d^4)^3/(c*x^4+a)^2*d^6*e^5/a*x^6
```

maxima [A] time = 2.41, size = 1015, normalized size = 0.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] e^11*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12)
 - 1/256*c*(sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1/4)*e^11 - 21*c^3*d^11 + 5*sqr
t(a)*c^(5/2)*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^(3/2)*c^(3/2)*d^5*e^6 - 77*a
^2*c*d^3*e^8 + 45*a^(5/2)*sqrt(c)*d*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1
/4)*e^11 + 21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4 - 18*
a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^10)*log
(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(
21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*
sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*s
qrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 + 24*sq
rt(a)*c^3*d^10*e + 80*a^(3/2)*c^2*d^6*e^5 + 120*a^(5/2)*c*d^2*e^9)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^1
1 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4
 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8
+ 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 - 24*sqrt(a)*c^3*d^10*e - 80*a^(3/2)*c
^2*d^6*e^5 - 120*a^(5/2)*c*d^2*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c
^(5/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c*d^4*e^8 + a^5*e^12) +
1/32*(8*a^2*c*e^7*x^4 + 4*a^2*c*d^4*e^3 + 12*a^3*e^7 + (5*c^3*d^5*e^2 + 13*
a*c^2*d*e^6)*x^7 - 2*(3*c^3*d^6*e + 7*a*c^2*d^2*e^5)*x^6 + (7*c^3*d^7 + 15*
a*c^2*d^3*e^4)*x^5 + (9*a*c^2*d^5*e^2 + 17*a^2*c*d*e^6)*x^3 - 2*(5*a*c^2*d^
6*e + 9*a^2*c*d^2*e^5)*x^2 + (11*a*c^2*d^7 + 19*a^2*c*d^3*e^4)*x)/(a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4 + a^6*e^8 + (a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^
2*e^8)*x^8 + 2*(a^3*c^3*d^8 + 2*a^4*c^2*d^4*e^4 + a^5*c*e^8)*x^4)
```

mupad [B] time = 4.41, size = 2720, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)),x)
```

```
[Out] symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d*
e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^1
1*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(805306368*a
^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^1
2*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*
d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 96522
24*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*
a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*
c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^
4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*
e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a
^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*
d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*
d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 +
268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*
z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c
*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*
```

```
e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306368*a^12*c^2*d^8*
e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3*d^12*z^4 + 2684
35456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a^7*c*d^4*e^6*z^2
+ 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 + 9652224*a^4*c*d^4
*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z + 676881*a*c*d^4*e^4
+ 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*((402653184*a^15*c^4*d*e^22 - 134
217728*a^10*c^9*d^21*e^2 - 134217728*a^11*c^8*d^17*e^6 + 805306368*a^12*c^7
*d^13*e^10 + 1879048192*a^13*c^6*d^9*e^14 + 1476395008*a^14*c^5*d^5*e^18)/(
1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4
+ 6*a^10*c^2*d^8*e^8)) + (x*(335544320*a^15*c^4*e^23 - 201326592*a^10*c^9*d
^20*e^3 - 469762048*a^11*c^8*d^16*e^7 + 134217728*a^12*c^7*d^12*e^11 + 1207
959552*a^13*c^6*d^8*e^15 + 1140850688*a^14*c^5*d^4*e^19))/(1048576*(a^12*e^
16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8
*e^8))) + (211288064*a^12*c^4*d*e^21 - 11010048*a^7*c^9*d^21*e + 20447232*a
^8*c^8*d^17*e^5 + 204472320*a^9*c^7*d^13*e^9 + 514850816*a^10*c^6*d^9*e^13
+ 553123840*a^11*c^5*d^5*e^17)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*
c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(251658240*a^12
*c^4*e^22 - 28311552*a^7*c^9*d^20*e^2 - 67108864*a^8*c^8*d^16*e^6 + 1887436
8*a^9*c^7*d^12*e^10 + 377487360*a^10*c^6*d^8*e^14 + 571473920*a^11*c^5*d^4*
e^18))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d
^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (36962304*a^9*c^4*d*e^20 + 11010048*a^5*c
^8*d^17*e^4 + 57999360*a^6*c^7*d^13*e^8 + 138805248*a^7*c^6*d^9*e^12 + 1413
61152*a^8*c^5*d^5*e^16)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e
^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + (x*(62914560*a^9*c^4*e^21
 - 1806336*a^4*c^9*d^20*e + 2670592*a^5*c^8*d^16*e^5 + 43032576*a^6*c^7*d^1
2*e^9 + 143179776*a^7*c^6*d^8*e^13 + 171732992*a^8*c^5*d^4*e^17))/(1048576*
(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10
*c^2*d^8*e^8))) + (4030464*a^6*c^4*d*e^19 + 576576*a^2*c^8*d^17*e^3 + 50618
24*a^3*c^7*d^13*e^7 + 15959232*a^4*c^6*d^9*e^11 + 17863744*a^5*c^5*d^5*e^15
)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e
^4 + 6*a^10*c^2*d^8*e^8)) + (x*(5242880*a^6*c^4*e^20 + 755136*a^2*c^8*d^16*
e^4 + 6023488*a^3*c^7*d^12*e^8 + 19579200*a^4*c^6*d^8*e^12 + 22240704*a^5*c
^5*d^4*e^16))/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a^11*c*d^4*e^12 + 4*a^
9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8))) + (x*(194481*c^7*d^12*e^7 + 871362*a
*c^6*d^8*e^11 + 970321*a^2*c^5*d^4*e^15))/(1048576*(a^12*e^16 + a^8*c^4*d^1
6 + 4*a^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)))*root(805
306368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11
*c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305715
2*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k), k, 1, 4) + (
(3*a*e^7 + c*d^4*e^3)/(8*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^5*(7*c^3
*d^7 + 15*a*c^2*d^3*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x
^2*(5*c^2*d^6*e + 9*a*c*d^2*e^5))/(16*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (c*e^7*x^4)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2*d^7 +
19*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(3*c^3*d
^6*e + 7*a*c^2*d^2*e^5))/(16*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (e^
2*x^3*(9*c^2*d^5 + 17*a*c*d*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)
) + (e^2*x^7*(5*c^3*d^5 + 13*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a
*c*d^4*e^4)))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (e^11*log(d + e*x))/(a^3*e^12 +
 c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.409 $\int \frac{1}{(d+ex)^2(a+cx^4)^3} dx$

Optimal. Leaf size=1830

result too large to display

Rubi [A] time = 2.78, antiderivative size = 1830, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + c*x^4)^3),x]

[Out] -(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12*
d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*e^
4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(c*d
^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a + c*
x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*c*d^4
 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a + c*x^
4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x
^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^4 - a*e^4
)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) - (c^(1/4)*(
21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a
*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 +
 a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(1
1*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt
[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqr
t[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3
*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(3*Sqrt[c]*d^2*
(3*c*d^4 - a*e^4) + Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (12*c*d^3*e^11*Lo
g[d + e*x])/(c*d^4 + a*e^4)^4 - (c^(1/4)*e^8*(9*c^(3/2)*d^6 - 11*Sqrt[a]*c*
d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) - (c^(1/4)*
(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*
d^4 + a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) - Sqrt[a]
*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*e^8*(9*c^(3/2)*d^6 -
 11*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^
4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - 5*Sqrt[a]*e^2*(3*c*d^4 -
a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2
]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*
e^4) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^3) - (3*c*d^3*e^11*L
og[a + c*x^4])/(c*d^4 + a*e^4)^4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^3}\,dx = \int \left(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^2} + \frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)} + \frac{c\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4\right.\right.}{\left(cd^4\right.} \right.$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{12cd^3e^{11}\log(d+ex)}{\left(cd^4+ae^4\right)^4} + \frac{\left(ce^8\right)\int \frac{3d^2\left(3cd^4-ae^4\right)-2de\left(5cd^4-ae^4\right)x}{a+c}}{\left(cd^4+ae\right.}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{c\left(4ad^3e^3+x\left(d^2\left(cd^4-3ae^4\right)-2de\left(cd^4-ae^4\right)x+e^2\right.\right.}{8a\left(cd^4+ae^4\right)^2\left(a+cx^4\right)^2}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

$$= -\frac{e^{11}}{\left(cd^4+ae^4\right)^3(d+ex)} + \frac{cx\left(7d^2\left(cd^4-3ae^4\right)-12de\left(cd^4-ae^4\right)x+5e^2\left(3cd^4-\right.\right.}{32a^2\left(cd^4+ae^4\right)^2\left(a+cx^4\right)}$$

Mathematica [A] time = 1.55, size = 1115, normalized size = 0.61

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] ((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(7
*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e^2
*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2*(a
 + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^2) +
```

```
 a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2) - (6
*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*S
qrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(9/4)*d^
9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 - 2
40*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^
3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(7*Sqrt[2]*c
^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^3*d^12*e^2 +
 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 + 27*Sqrt[2]*a^
(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9/4)*c^(5/4)*d^
5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt[c]*d^2*e^12 -
80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x] - (3*Sqrt[2]*c^
(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27
*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77
*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/a^(11/4) + (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqr
t[a]*c^3*d^12*e^2 + 33*a*c^(5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2
*c^(3/2)*d^6*e^8 - 135*a^(5/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^
(7/2)*e^14)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(11/4
) - 768*c*d^3*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((d + e*x)^2*(a + c*x^4)^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 2769, normalized size = 1.51

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^3,x)
```

```
[Out] 12*c*d^3*e^11*ln(e*x+d)/(a*e^4+c*d^4)^4-3*c*d^3*e^11*ln(c*x^4+a)/(a*e^4+c*d
^4)^4-e^11/(a*e^4+c*d^4)^3/(e*x+d)+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4
```

```
)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^
4/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^10*e^4+231/256*
c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/
c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^6*e^8+1/2*c^3/(a*e^4+c
*d^4)^4/(c*x^4+a)^2*e^3*d^11+99/256*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^10*e^4+231/128*c^2/(a*e^4+c*d^4)^4/a*(a/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^6*e^8+99/128*c^3/(a*e^4+c*d^4)^4/a^2*(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^10*e^4+81/256*c^2/(a*e^4+c*
d^4)^4/a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^8*e^6+81/128*c^2/(a*e^4+c*d^4)^4/a/
(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^8*e^6+15/128*c^3/(a*e
^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^12*e^
2+15/128*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)*d^12*e^2+81/128*c^2/(a*e^4+c*d^4)^4/a/(a/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)*d^8*e^6+15/256*c^3/(a*e^4+c*d^4)^4/a^2/(a/c)^(1/4)
*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2
)*x+(a/c)^(1/2)))*d^12*e^2+5/2*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^11*d^3*a^2-1
7/32*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^14*a^2*x^3+11/32*c^4/(a*e^4+c*d^4)^4/(
c*x^4+a)^2*d^14/a*x+2*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*x^4*d^7*e^7-13/32*c^2
/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^14*a*x^7+7/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^
2*d^14/a^2*x^5+3*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^7*d^7*a+101/32*c^3/(a*e^
4+c*d^4)^4/(c*x^4+a)^2*e^6*x^3*d^8-17/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^9
*e^5*x^2+29/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^10*x*e^4+53/32*c^3/(a*e^4+
c*d^4)^4/(c*x^4+a)^2*e^10*x^7*d^4-5/8*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^5*e
^9*x^6-19/32*c^3/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^6*x^5*e^8-45/8*c^2/(a*e^4+c*
d^4)^4/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d^5-45/256/(a*e^4+c*d^4)^4
*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*e^14-45/128/(a*e^4+c*d^4)^4*a/(a/c)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*e^14-45/128/(a*e^4+c*d^4)^4*a/(a/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*e^14+9/8*c/(a*e^4+c*d^4)^4/(c
*x^4+a)^2*d*e^13*a^2*x^2-57/32*c/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^2*a^2*x*e^12
-15/8*c^3/(a*e^4+c*d^4)^4/a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^5*d^9-3
/8*c^4/(a*e^4+c*d^4)^4/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e*d^13+405
/256*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^4*e^10+405/128*c/(a*e
^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^4*e^10+40
5/128*c/(a*e^4+c*d^4)^4/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
*d^4*e^10+27/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^2/a*x^3*d^12-3/8*c^2/(a*e
^4+c*d^4)^4/(c*x^4+a)^2*d^5*e^9*a*x^2-5/8*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d
^13*e/a*x^2-39/32*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^6*a*x*e^8+2*c^2/(a*e^4+
c*d^4)^4/(c*x^4+a)^2*x^4*a*d^3*e^11+81/32*c^4/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e
^6/a*x^7*d^8+15/32*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^2/a^2*x^7*d^12+7/8*c^2
/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d*e^13*a*x^6-15/8*c^4/(a*e^4+c*d^4)^4/(c*x^4+a
)^2*d^9*e^5/a*x^6-3/8*c^5/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^13*e/a^2*x^6-45/32*
c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*d^2*a*x^5*e^12+33/32*c^4/(a*e^4+c*d^4)^4/(c
*x^4+a)^2*d^10/a*x^5*e^4+57/32*c^2/(a*e^4+c*d^4)^4/(c*x^4+a)^2*e^10*a*x^3*d
^4+15/8*c/(a*e^4+c*d^4)^4*a/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^13*d-23
1/128*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)
*d^2*e^12+21/128*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a/c)^(1/4)*x-1)*d^14-231/256*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*ln((x^
2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)
))*d^2*e^12+21/256*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c
)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^1
4-231/128*c/(a*e^4+c*d^4)^4*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)*d^2*e^12+21/128*c^4/(a*e^4+c*d^4)^4/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)*d^14
```

maxima [A] time = 3.10, size = 1564, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] 12*c*d^3*e^11*log(e*x + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8
 + 4*a^3*c*d^4*e^12 + a^4*e^16) - 3/256*c*(sqrt(2)*(128*sqrt(2)*a^(11/4)*c^
(5/4)*d^3*e^11 - 7*c^4*d^14 + 5*sqrt(a)*c^(7/2)*d^12*e^2 - 33*a*c^3*d^10*e^
4 + 27*a^(3/2)*c^(5/2)*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^(5/2)*c^(3/2)*d
^4*e^10 + 77*a^3*c*d^2*e^12 - 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(128*sqrt(2)
*a^(11/4)*c^(5/4)*d^3*e^11 + 7*c^4*d^14 - 5*sqrt(a)*c^(7/2)*d^12*e^2 + 33*a
*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 135*a^(5/
2)*c^(3/2)*d^4*e^10 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt
(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(7*sqr
t(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(
2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqr
t(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*s
qrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 + 16*sq
rt(a)*c^4*d^13*e + 80*a^(3/2)*c^3*d^9*e^5 + 240*a^(5/2)*c^2*d^5*e^9 - 80*a^
(7/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(7*sqrt(
2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2 + 33*sqrt(2)
*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*e^6 + 77*sqrt(
2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^4*e^10 - 77*sqr
t(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4)*e^14 - 16*sqrt
(a)*c^4*d^13*e - 80*a^(3/2)*c^3*d^9*e^5 - 240*a^(5/2)*c^2*d^5*e^9 + 80*a^(7
/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^4*d^16
 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6*e^16) +
1/32*(16*a^2*c^2*d^8*e^3 + 80*a^3*c*d^4*e^7 - 32*a^4*e^11 + 3*(5*c^4*d^8*e^
3 + 22*a*c^3*d^4*e^7 - 15*a^2*c^2*e^11)*x^8 + 3*(c^4*d^9*e^2 + 6*a*c^3*d^5*
e^6 + 5*a^2*c^2*d*e^10)*x^7 - (5*c^4*d^10*e + 22*a*c^3*d^6*e^5 + 17*a^2*c^2
*d^2*e^9)*x^6 + (7*c^4*d^11 + 26*a*c^3*d^7*e^4 + 19*a^2*c^2*d^3*e^8)*x^5 +
3*(9*a*c^3*d^8*e^3 + 46*a^2*c^2*d^4*e^7 - 27*a^3*c*e^11)*x^4 + (7*a*c^3*d^9
*e^2 + 26*a^2*c^2*d^5*e^6 + 19*a^3*c*d*e^10)*x^3 - 3*(3*a*c^3*d^10*e + 10*a
^2*c^2*d^6*e^5 + 7*a^3*c*d^2*e^9)*x^2 + (11*a*c^3*d^11 + 34*a^2*c^2*d^7*e^4
 + 23*a^3*c*d^3*e^8)*x)/(a^4*c^3*d^13 + 3*a^5*c^2*d^9*e^4 + 3*a^6*c*d^5*e^8
 + a^7*d*e^12 + (a^2*c^5*d^12*e + 3*a^3*c^4*d^8*e^5 + 3*a^4*c^3*d^4*e^9 + a
^5*c^2*e^13)*x^9 + (a^2*c^5*d^13 + 3*a^3*c^4*d^9*e^4 + 3*a^4*c^3*d^5*e^8 +
a^5*c^2*d*e^12)*x^8 + 2*(a^3*c^4*d^12*e + 3*a^4*c^3*d^8*e^5 + 3*a^5*c^2*d^4
*e^9 + a^6*c*e^13)*x^5 + 2*(a^3*c^4*d^13 + 3*a^4*c^3*d^9*e^4 + 3*a^5*c^2*d^
5*e^8 + a^6*c*d*e^12)*x^4 + (a^4*c^3*d^12*e + 3*a^5*c^2*d^8*e^5 + 3*a^6*c*d
^4*e^9 + a^7*e^13)*x)
```

mupad [B] time = 5.75, size = 3572, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^5
*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(a^
14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c
^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612
736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^
14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 +
 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*
a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5
```

*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*
d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c
^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e
^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 322122547
2*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^
10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5
419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 +
4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*a^13*c^2*d^8*e^8
*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d
^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2
 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3
*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^
2*c*e^8 + 194481*c^3*d^8, z, k)*((23592960*a^14*c^4*e^29 - 11010048*a^7*c^1
1*d^28*e + 33030144*a^8*c^10*d^24*e^5 + 504889344*a^9*c^9*d^20*e^9 + 310326
0672*a^10*c^8*d^16*e^13 + 6799491072*a^11*c^7*d^12*e^17 + 6101139456*a^12*c
^6*d^8*e^21 + 1967652864*a^13*c^5*d^4*e^25)/(1048576*(a^14*e^24 + a^8*c^6*d
^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^
11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(1610612736*a^13*c^2*d^8*e^
8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 +
 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*
d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^
2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^
3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a
^2*c*e^8 + 194481*c^3*d^8, z, k)*((402653184*a^17*c^4*d*e^30 - 134217728*a^
10*c^11*d^29*e^2 - 402653184*a^11*c^10*d^25*e^6 + 402653184*a^12*c^9*d^21*e
^10 + 3355443200*a^13*c^8*d^17*e^14 + 6039797760*a^14*c^7*d^13*e^18 + 52344
91392*a^15*c^6*d^9*e^22 + 2281701376*a^16*c^5*d^5*e^26)/(1048576*(a^14*e^24
 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16
*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(335544320*a^17*
c^4*e^31 - 201326592*a^10*c^11*d^28*e^3 - 872415232*a^11*c^10*d^24*e^7 - 10
06632960*a^12*c^9*d^20*e^11 + 1006632960*a^13*c^8*d^16*e^15 + 3690987520*a^
14*c^7*d^12*e^19 + 3825205248*a^15*c^6*d^8*e^23 + 1811939328*a^16*c^5*d^4*e
^27))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^
20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^1
6))) + (x*(2554331136*a^10*c^8*d^15*e^14 - 144703488*a^8*c^10*d^23*e^6 - 15
4140672*a^9*c^9*d^19*e^10 - 34603008*a^7*c^11*d^27*e^2 + 7659847680*a^11*c^
7*d^11*e^18 + 7556038656*a^12*c^6*d^7*e^22 + 2494562304*a^13*c^5*d^3*e^26))
/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^
4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)))
+ (12681216*a^5*c^10*d^23*e^4 + 127107072*a^6*c^9*d^19*e^8 + 674168832*a^7*
c^8*d^15*e^12 + 1018626048*a^8*c^7*d^11*e^16 - 446201856*a^9*c^6*d^7*e^20 +
 906854400*a^10*c^5*d^3*e^24)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c
*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^
12 + 15*a^12*c^2*d^8*e^16)) + (x*(516096*a^5*c^10*d^22*e^5 - 1806336*a^4*c^
11*d^26*e + 90427392*a^6*c^9*d^18*e^9 + 896090112*a^7*c^8*d^14*e^13 + 19609
06752*a^8*c^7*d^10*e^17 + 1732829184*a^9*c^6*d^6*e^21 + 1183887360*a^10*c^5
*d^2*e^25))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*
c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d
^8*e^16))) + (387072*a^2*c^10*d^22*e^3 + 8004096*a^3*c^9*d^18*e^7 + 4937932
8*a^4*c^8*d^14*e^11 + 49572864*a^5*c^7*d^10*e^15 - 156930048*a^6*c^6*d^6*e^
19 + 125452800*a^7*c^5*d^2*e^23)/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^1
3*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^12
*e^12 + 15*a^12*c^2*d^8*e^16)) + (x*(126360000*a^7*c^5*d*e^24 + 561600*a^2*
c^10*d^21*e^4 + 9609408*a^3*c^9*d^17*e^8 + 75731328*a^4*c^8*d^13*e^12 + 114
991488*a^5*c^7*d^9*e^16 - 80136000*a^6*c^6*d^5*e^20))/(1048576*(a^14*e^24 +
 a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e
^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16))) + (x*(4100625*a^4*c^5*
e^23 + 194481*c^9*d^16*e^7 + 1527012*a*c^8*d^12*e^11 - 167994*a^2*c^7*d^8*e

```
^15 - 13988700*a^3*c^6*d^4*e^19))/(1048576*(a^14*e^24 + a^8*c^6*d^24 + 6*a^
13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^10*c^4*d^16*e^8 + 20*a^11*c^3*d^1
2*e^12 + 15*a^12*c^2*d^8*e^16)))*root(1610612736*a^13*c^2*d^8*e^8*z^4 + 107
3741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*c*d^4*e^12*z^4 + 268435456*
a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3221225472*a^11*c*d^3*e^11*z^
3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a^6*c^3*d^10*e^2*z^2 + 1153105
920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e
*z + 124416000*a^5*c*d*e^9*z + 1138050*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 +
194481*c^3*d^8, z, k), k, 1, 4) + ((c^2*d^8*e^3 - 2*a^2*e^11 + 5*a*c*d^4*e^
7)/(2*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (3*x^8*(5*c^4*
d^8*e^3 - 15*a^2*c^2*e^11 + 22*a*c^3*d^4*e^7))/(32*a^2*(a^3*e^12 + c^3*d^12
 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8)) + (x^5*(7*c^3*d^7 + 19*a*c^2*d^3*e^4
))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (3*x^2*(3*c^2*d^6*e + 7*a
*c*d^2*e^5))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^3*(7*c^2*d^5*e
^2 + 19*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x*(11*c^2
*d^7 + 23*a*c*d^3*e^4))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^6*(
5*c^3*d^6*e + 17*a*c^2*d^2*e^5))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4
)) + (3*e^2*x^7*(c^3*d^5 + 5*a*c^2*d*e^4))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a
*c*d^4*e^4)) + (3*e^2*x^4*(9*c^3*d^8*e - 27*a^2*c*e^9 + 46*a*c^2*d^4*e^5))/
(32*a*(a*e^4 + c*d^4)*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d + c^2*d*
x^8 + c^2*e*x^9 + a^2*e*x + 2*a*c*d*x^4 + 2*a*c*e*x^5) + (12*c*d^3*e^11*log
(d + e*x))/(a^4*e^16 + c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a
^2*c^2*d^8*e^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.410 $\int \frac{1}{(d+ex)^3 \left(a+cx^4\right)^3} \, dx$

Optimal. Leaf size=2204

result too large to display

Rubi [A] time = 3.16, antiderivative size = 2204, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 15, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {6742, 1854, 1855, 1876, 275, 205, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 260}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] -e^11/(2*(c*d^4 + a*e^4)^3*(d + e*x)^2) - (12*c*d^3*e^11)/((c*d^4 + a*e^4)^
4*(d + e*x)) + (c*x*(7*d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - 6*e*(3*c^
2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*x + 10*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)
)/(32*a^2*(c*d^4 + a*e^4)^3*(a + c*x^4)) + (c*(2*a*d^2*e^3*(5*c*d^4 - 3*a*e
^4) + x*(d*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8) - e*(3*c^2*d^8 - 12*a*c*d
^4*e^4 + a^2*e^8)*x + 2*c*d^3*e^2*(3*c*d^4 - 5*a*e^4)*x^2)))/(8*a*(c*d^4 +
a*e^4)^3*(a + c*x^4)^2) + (c*e^4*(12*a*d^2*e^3*(3*c*d^4 - a*e^4) + x*(3*d*(
5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8) - e*(21*c^2*d^8 - 26*a*c*d^4*e^4 + a^
2*e^8)*x + 4*c*d^3*e^2*(7*c*d^4 - 5*a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^4*(a
 + c*x^4)) - (Sqrt[c]*e^9*(55*c^2*d^8 - 40*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*e^4)^5) - (Sqrt[c]*e^5*(21*c^2*
d^8 - 26*a*c*d^4*e^4 + a^2*e^8)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(
c*d^4 + a*e^4)^4) - (3*Sqrt[c]*e*(3*c^2*d^8 - 12*a*c*d^4*e^4 + a^2*e^8)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)^3) - (3*c^(3/4)*d*e
^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c
*d^4 - 5*a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 -
 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*ArcTan[1 - (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^4) - (c^(3/4)*d*(10*
Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4
+ 3*a^2*e^8))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)
*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e
^8 + 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*a*e^4))*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(
4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c*d^4 - 5*a*e^4) + 9*(5*c^2*d^8 - 10*a*c*d^4*e
^4 + a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*
e^4) + 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (6*c*d^2*e^11*(13
*c*d^4 - 3*a*e^4)*Log[d + e*x])/(c*d^4 + a*e^4)^5 - (3*c^(3/4)*d*e^8*(15*c^
2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*d^4 - 5*
a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*(c*d^4 + a*e^4)^5) + (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*e^2*(7*c
*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4
)^4) + (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) - 21*(c^2
*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) + (3*c^(3/4)*d*e^
8*(15*c^2*d^8 - 16*a*c*d^4*e^4 + a^2*e^8 - 2*Sqrt[a]*Sqrt[c]*d^2*e^2*(11*c*
d^4 - 5*a*e^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^5) - (c^(3/4)*d*e^4*(4*Sqrt[a]*Sqrt[c]*d^2*
e^2*(7*c*d^4 - 5*a*e^4) - 9*(5*c^2*d^8 - 10*a*c*d^4*e^4 + a^2*e^8))*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4
 + a*e^4)^4) - (c^(3/4)*d*(10*Sqrt[a]*Sqrt[c]*d^2*e^2*(3*c*d^4 - 5*a*e^4) -
```

```
 21*(c^2*d^8 - 12*a*c*d^4*e^4 + 3*a^2*e^8))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(128*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^3) - (3*c*d^
2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(2*(c*d^4 + a*e^4)^5)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1855

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(x*Pq*(a + b*x
^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*(p
 + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] &
& PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+cx^4\right)^3}\,dx = \int\Bigg(\frac{e^{12}}{\left(cd^4+ae^4\right)^3(d+ex)^3}+\frac{12cd^3e^{12}}{\left(cd^4+ae^4\right)^4(d+ex)^2}+\frac{6cd^2e^{12}\left(13cd^4-3ae^4\right)}{\left(cd^4+ae^4\right)^5(d+ex)}+\frac{c\left(d\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{6cd^2e^{11}\left(13cd^4-3ae^4\right)\log(d+e}{\left(cd^4+ae^4\right)^5}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{c\left(2ad^2e^3\left(5cd^4-3ae^4\right)+x\left(d\left(c\right.\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

$$= -\frac{e^{11}}{2\left(cd^4+ae^4\right)^3(d+ex)^2}-\frac{12cd^3e^{11}}{\left(cd^4+ae^4\right)^4(d+ex)}+\frac{cx\left(7d\left(c^2d^8-12acd^4e^4+3a^2e^8\right.\right.}{}$$

Mathematica [A] time = 2.87, size = 1338, normalized size = 0.61

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + c*x^4)^3),x]
```

```
[Out] ((-128*e^11*(c*d^4 + a*e^4)^2)/(d + e*x)^2 - (3072*c*d^3*e^11*(c*d^4 + a*e^
4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(a^3*e^11*(-96*d^2 + 45*d*e*x - 14*e^2
*x^2) + c^3*d^11*x*(7*d^2 - 18*d*e*x + 30*e^2*x^2) + a*c^2*d^7*e^4*x*(43*d^
2 - 114*d*e*x + 204*e^2*x^2) + a^2*c*d^3*e^7*(288*d^3 - 303*d^2*e*x + 274*d
```

```
*e^2*x^2 - 210*e^3*x^3)))/(a^2*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(-(a^
2*e^7*(6*d^2 - 3*d*e*x + e^2*x^2)) + c^2*d^7*x*(d^2 - 3*d*e*x + 6*e^2*x^2)
+ 2*a*c*d^3*e^3*(5*d^3 - 6*d^2*e*x + 6*d*e^2*x^2 - 5*e^3*x^3)))/(a*(a + c*x
^4)^2) - (6*Sqrt[c]*(7*Sqrt[2]*c^(17/4)*d^17 - 24*a^(1/4)*c^4*d^16*e + 10*S
qrt[2]*Sqrt[a]*c^(15/4)*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 - 176*a^(
5/4)*c^3*d^12*e^5 + 78*Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*
c^(9/4)*d^9*e^8 - 960*a^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7
*e^10 - 770*Sqrt[2]*a^3*c^(5/4)*d^5*e^12 + 1200*a^(13/4)*c*d^4*e^13 - 390*S
qrt[2]*a^(7/2)*c^(3/4)*d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 - 40*a^(17/
4)*e^17)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*Sqrt[c]*(7*
Sqrt[2]*c^(17/4)*d^17 + 24*a^(1/4)*c^4*d^16*e + 10*Sqrt[2]*Sqrt[a]*c^(15/4)
*d^15*e^2 + 50*Sqrt[2]*a*c^(13/4)*d^13*e^4 + 176*a^(5/4)*c^3*d^12*e^5 + 78*
Sqrt[2]*a^(3/2)*c^(11/4)*d^11*e^6 + 220*Sqrt[2]*a^2*c^(9/4)*d^9*e^8 + 960*a
^(9/4)*c^2*d^8*e^9 + 702*Sqrt[2]*a^(5/2)*c^(7/4)*d^7*e^10 - 770*Sqrt[2]*a^3
*c^(5/4)*d^5*e^12 - 1200*a^(13/4)*c*d^4*e^13 - 390*Sqrt[2]*a^(7/2)*c^(3/4)*
d^3*e^14 + 77*Sqrt[2]*a^4*c^(1/4)*d*e^16 + 40*a^(17/4)*e^17)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 1536*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*
Log[d + e*x] - (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) + (3*Sqrt[2]*c^(3/4)*(7*c^4*d^17 - 10*Sqrt[a]*c^(7/2)*d^15*e^2
 + 50*a*c^3*d^13*e^4 - 78*a^(3/2)*c^(5/2)*d^11*e^6 + 220*a^2*c^2*d^9*e^8 -
702*a^(5/2)*c^(3/2)*d^7*e^10 - 770*a^3*c*d^5*e^12 + 390*a^(7/2)*Sqrt[c]*d^3
*e^14 + 77*a^4*d*e^16)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(11/4) - 384*c*d^2*e^11*(13*c*d^4 - 3*a*e^4)*Log[a + c*x^4])/(256*(c*
d^4 + a*e^4)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex)^3 \left(a+cx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^3),x]
[Out] IntegrateAlgebraic[1/((d + e*x)^3*(a + c*x^4)^3), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 1.53, size = 2119, normalized size = 0.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
[Out] 3/64*(23*sqrt(2)*a*c^4*d^6*e - 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 + 30*sqrt(2)*
sqrt(a*c)*a*c^3*d^4*e^3 - 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^2
*e^5 + 65*(a*c^3)^(3/4)*a*c*d^3*e^4 + 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 + 20*
sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e^
2 + 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e + 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90*
```

```
sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 + 148*(a*c^3
)^(3/4)*a^4*c*d^5*e^5 + 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt(
a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 + 10*(a*c^3)^(3/4)*a^5*d*e^9) -
 3/64*(23*sqrt(2)*a*c^4*d^6*e + 65*(a*c^3)^(1/4)*a*c^3*d^5*e^2 - 30*sqrt(2)
*sqrt(a*c)*a*c^3*d^4*e^3 + 7*(a*c^3)^(3/4)*c^2*d^7 - 115*sqrt(2)*a^2*c^3*d^
2*e^5 - 65*(a*c^3)^(3/4)*a*c*d^3*e^4 - 123*(a*c^3)^(1/4)*a^2*c^2*d*e^6 - 20
*sqrt(2)*sqrt(a*c)*a^2*c^2*e^7)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/
4))/(a/c)^(1/4))/(sqrt(2)*sqrt(a*c)*a^3*c^4*d^10 - 25*sqrt(2)*a^4*c^4*d^8*e
^2 - 10*(a*c^3)^(3/4)*a^3*c^2*d^9*e - 80*(a*c^3)^(1/4)*a^4*c^3*d^7*e^3 + 90
*sqrt(2)*sqrt(a*c)*a^4*c^3*d^6*e^4 - 90*sqrt(2)*a^5*c^3*d^4*e^6 - 148*(a*c^
3)^(3/4)*a^4*c*d^5*e^5 - 80*(a*c^3)^(1/4)*a^5*c^2*d^3*e^7 + 25*sqrt(2)*sqrt
(a*c)*a^5*c^2*d^2*e^8 - sqrt(2)*a^6*c^2*e^10 - 10*(a*c^3)^(3/4)*a^5*d*e^9)
+ 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3)^(3/4)*c^3*d^
15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)*(a*c^3)^(3/4)
*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 - 702*sqrt(2)*(
a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c^2*d^5*e^12 +
390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(1/4)*a^4*c*d*e
^16)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^20 + 5*a^4*c^5
*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7*c^2*d^4*e^16
+ a^8*c*e^20) - 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^17 - 10*sqrt(2)*(a*c^3
)^(3/4)*c^3*d^15*e^2 + 50*sqrt(2)*(a*c^3)^(1/4)*a*c^4*d^13*e^4 - 78*sqrt(2)
*(a*c^3)^(3/4)*a*c^2*d^11*e^6 + 220*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^9*e^8 -
 702*sqrt(2)*(a*c^3)^(3/4)*a^2*c*d^7*e^10 - 770*sqrt(2)*(a*c^3)^(1/4)*a^3*c
^2*d^5*e^12 + 390*sqrt(2)*(a*c^3)^(3/4)*a^3*d^3*e^14 + 77*sqrt(2)*(a*c^3)^(
1/4)*a^4*c*d*e^16)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^6*d^
20 + 5*a^4*c^5*d^16*e^4 + 10*a^5*c^4*d^12*e^8 + 10*a^6*c^3*d^8*e^12 + 5*a^7
*c^2*d^4*e^16 + a^8*c*e^20) - 3/2*(13*c^2*d^6*e^11 - 3*a*c*d^2*e^15)*log(ab
s(c*x^4 + a))/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^12*e^8 + 10*a^3*c
^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 6*(13*c^2*d^6*e^12 - 3*a*c*d^2
*e^16)*log(abs(x*e + d))/(c^5*d^20*e + 5*a*c^4*d^16*e^5 + 10*a^2*c^3*d^12*e
^9 + 10*a^3*c^2*d^8*e^13 + 5*a^4*c*d^4*e^17 + a^5*e^21) + 1/32*(30*c^5*d^11
*x^9*e^4 + 42*c^5*d^12*x^8*e^3 + c^5*d^13*x^7*e^2 - 4*c^5*d^14*x^6*e + 7*c^
5*d^15*x^5 + 204*a*c^4*d^7*x^9*e^8 + 294*a*c^4*d^8*x^8*e^7 + 19*a*c^4*d^9*x
^7*e^6 - 28*a*c^4*d^10*x^6*e^5 + 97*a*c^4*d^11*x^5*e^4 + 78*a*c^4*d^12*x^4*
e^3 + 5*a*c^4*d^13*x^3*e^2 - 8*a*c^4*d^14*x^2*e + 11*a*c^4*d^15*x - 594*a^2
*c^3*d^3*x^9*e^12 - 546*a^2*c^3*d^4*x^8*e^11 + 35*a^2*c^3*d^5*x^7*e^10 - 44
*a^2*c^3*d^6*x^6*e^9 + 461*a^2*c^3*d^7*x^5*e^8 + 586*a^2*c^3*d^8*x^4*e^7 +
31*a^2*c^3*d^9*x^3*e^6 - 40*a^2*c^3*d^10*x^2*e^5 + 79*a^2*c^3*d^11*x*e^4 +
40*a^2*c^3*d^12*e^3 - 30*a^3*c^2*x^8*e^15 + 17*a^3*c^2*d*x^7*e^14 - 20*a^3*
c^2*d^2*x^6*e^13 - 1165*a^3*c^2*d^3*x^5*e^12 - 1078*a^3*c^2*d^4*x^4*e^11 +
47*a^3*c^2*d^5*x^3*e^10 - 56*a^3*c^2*d^6*x^2*e^9 + 269*a^3*c^2*d^7*x*e^8 +
304*a^3*c^2*d^8*e^7 - 50*a^4*c*x^4*e^15 + 21*a^4*c*d*x^3*e^14 - 24*a^4*c*d^
2*x^2*e^13 - 567*a^4*c*d^3*x*e^12 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15)/((a^2
*c^4*d^16 + 4*a^3*c^3*d^12*e^4 + 6*a^4*c^2*d^8*e^8 + 4*a^5*c*d^4*e^12 + a^6
*e^16)*(c*x^5*e + c*d*x^4 + a*x*e + a*d)^2)
```

maple [A] time = 0.04, size = 3334, normalized size = 1.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(c*x^4+a)^3,x)
```

```
[Out] -1/2*e^11/(a*e^4+c*d^4)^3/(e*x+d)^2-12*c*d^3*e^11/(a*e^4+c*d^4)^4/(e*x+d)+1
65/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*
x-1)*d^9*e^8+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)*d^13*e^4+165/64*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/
2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a
/c)^(1/2)))*d^9*e^8+75/128*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*ln((
x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/
```

2)))*d^13*e^4+165/32*c^3/(a*e^4+c*d^4)^5/a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/c)^(1/4)*x+1)*d^9*e^8+78*e^11*d^6*c^2/(a*e^4+c*d^4)^5*ln(e*x+d)+5/4*c
^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^3*d^14-39/2*c^2/(a*e^4+c*d^4)^5*ln(c*x^4+a
)*e^11*d^6-129/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a*x^5*e^12+25/16*c^5/
(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13/a*x^5*e^4-125/16*c^2/(a*e^4+c*d^4)^5/(c*x^
4+a)^2*d^3*e^14*a^2*x^3-31/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^7*e^10*a*x^
3+27/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a*x^3+75/8*c^2/(a*e^4+c*d^
4)^5/(c*x^4+a)^2*e^13*a^2*x^2*d^4+15/2*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*
a*x^2*d^8-15/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a*x^2*d^16-141/16*c^2/(a*
e^4+c*d^4)^5/(c*x^4+a)^2*d^5*a^2*x*e^12-85/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^
2*d^9*a*x*e^8-3*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*x^4*a^2*d^2*e^15+6*c^3/(a*e
^4+c*d^4)^5/(c*x^4+a)^2*x^4*a*d^6*e^11-105/16*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)
^2*d^3*e^14*a*x^7+117/16*c^5/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^11*e^6/a*x^7+15/
16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^15*e^2/a^2*x^7-1155/64*c^2/(a*e^4+c*d^
4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^5*e^12-1155/128*
c^2/(a*e^4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)
^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^5*e^12-1155/64*c^2/(a*e^
4+c*d^4)^5*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^5*e^12+105
3/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1
)*d^7*e^10+1053/128*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(
1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^7*e^
10+1053/64*c^2/(a*e^4+c*d^4)^5/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^7*e^10-9/16*c^5/(a*e^4+c*d^4)^5/a^2/(a*c)^(1/2)*arctan((1/a*c)^(1
/2)*x^2)*e*d^16+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/c)^(1/4)*x-1)*d^17+21/256*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(
1/2)*ln((x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+
(a/c)^(1/2)))*d^17+21/128*c^5/(a*e^4+c*d^4)^5/a^3*(a/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)*d^17+225/8*c^2/(a*e^4+c*d^4)^5*a/(a*c)^(1/2)*arc
tan((1/a*c)^(1/2)*x^2)*e^13*d^4-33/8*c^4/(a*e^4+c*d^4)^5/a/(a*c)^(1/2)*arct
an((1/a*c)^(1/2)*x^2)*e^5*d^12+57/32*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^3*x*
e^16+65/8*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^13*a*x^6*d^4-33/8*c^5/(a*e^4+c*
d^4)^5/(c*x^4+a)^2*e^5/a*x^6*d^12-9/16*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e/a^
2*x^6*d^16+45/32*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d*a^2*x^5*e^16-18*e^15*d^2
*c/(a*e^4+c*d^4)^5*ln(e*x+d)*a+5*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^9*x^6*d^
8-65/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^9*x^5*e^8+121/16*c^4/(a*e^4+c*d^4)
^5/(c*x^4+a)^2*d^11*e^6*x^3-27/8*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^5*x^2*d^
12+5/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^13*x*e^4+9*c^4/(a*e^4+c*d^4)^5/(c
*x^4+a)^2*x^4*d^10*e^7-7/16*c^2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^2*x^6+7/
32*c^6/(a*e^4+c*d^4)^5/(c*x^4+a)^2*d^17/a^2*x^5+11/32*c^5/(a*e^4+c*d^4)^5/(
c*x^4+a)^2*d^17/a*x+43/4*c^3/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^7*d^10*a+23/4*c^
2/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^11*d^6*a^2-3/16*c^4/(a*e^4+c*d^4)^5/(c*x^4+
a)^2*d^7*e^10*x^7-9/16*c/(a*e^4+c*d^4)^5/(c*x^4+a)^2*e^17*a^3*x^2-15/4*c/(a
*e^4+c*d^4)^5/(c*x^4+a)^2*e^15*d^2*a^3-15/16*c/(a*e^4+c*d^4)^5*a^2/(a*c)^(1
/2)*arctan((1/a*c)^(1/2)*x^2)*e^17+9/2*c/(a*e^4+c*d^4)^5*a*ln(c*x^4+a)*e^15
*d^2-45/2*c^3/(a*e^4+c*d^4)^5/(a*c)^(1/2)*arctan((1/a*c)^(1/2)*x^2)*e^9*d^8
+75/64*c^4/(a*e^4+c*d^4)^5/a^2*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)*d^13*e^4+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/c)^(1/4)*x-1)*d^11*e^6+117/128*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2
^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*
x+(a/c)^(1/2)))*d^11*e^6+117/64*c^3/(a*e^4+c*d^4)^5/a/(a/c)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^11*e^6+15/64*c^4/(a*e^4+c*d^4)^5/a^2/(a/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^15*e^2+15/128*c^4/(a*e^4+c
*d^4)^5/a^2/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/
(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^15*e^2+15/64*c^4/(a*e^4+c*d^4)^5
/a^2/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d^15*e^2-585/64*c/
(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x-1)*d^3*e
^14-585/128*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*ln((x^2-(a/c)^(1/4)*2^(
1/2)*x+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2)))*d^3*e^14-585/6
4*c/(a*e^4+c*d^4)^5*a/(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c)^(1/4)*x+1)*d

```
^3*e^14+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/c
)^(1/4)*x-1)*d*e^16+231/256*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*ln((x^2
+(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*2^(1/2)*x+(a/c)^(1/2))
)*d*e^16+231/128*c/(a*e^4+c*d^4)^5*a*(a/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)*d*e^16
```

maxima [A] time = 3.99, size = 2198, normalized size = 1.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

```
[Out] -3/256*c*(sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2)*a^(1
5/4)*c^(5/4)*d^2*e^15 - 7*c^5*d^17 + 10*sqrt(a)*c^(9/2)*d^15*e^2 - 50*a*c^4
*d^13*e^4 + 78*a^(3/2)*c^(7/2)*d^11*e^6 - 220*a^2*c^3*d^9*e^8 + 702*a^(5/2)
*c^(5/2)*d^7*e^10 + 770*a^3*c^2*d^5*e^12 - 390*a^(7/2)*c^(3/2)*d^3*e^14 - 7
7*a^4*c*d*e^16)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
3/4)*c^(5/4)) + sqrt(2)*(832*sqrt(2)*a^(11/4)*c^(9/4)*d^6*e^11 - 192*sqrt(2
)*a^(15/4)*c^(5/4)*d^2*e^15 + 7*c^5*d^17 - 10*sqrt(a)*c^(9/2)*d^15*e^2 + 50
*a*c^4*d^13*e^4 - 78*a^(3/2)*c^(7/2)*d^11*e^6 + 220*a^2*c^3*d^9*e^8 - 702*a
^(5/2)*c^(5/2)*d^7*e^10 - 770*a^3*c^2*d^5*e^12 + 390*a^(7/2)*c^(3/2)*d^3*e^
14 + 77*a^4*c*d*e^16)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a)
)/(a^(3/4)*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/
4)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^
(7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2
)*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*
sqrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 + 24
*sqrt(a)*c^5*d^16*e + 176*a^(3/2)*c^4*d^12*e^5 + 960*a^(5/2)*c^3*d^8*e^9 -
1200*a^(7/2)*c^2*d^4*e^13 + 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c
)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)
*sqrt(c))*c^(5/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(21/4)*d^17 + 10*sqrt(2)*a^(3/4
)*c^(19/4)*d^15*e^2 + 50*sqrt(2)*a^(5/4)*c^(17/4)*d^13*e^4 + 78*sqrt(2)*a^(
7/4)*c^(15/4)*d^11*e^6 + 220*sqrt(2)*a^(9/4)*c^(13/4)*d^9*e^8 + 702*sqrt(2)
*a^(11/4)*c^(11/4)*d^7*e^10 - 770*sqrt(2)*a^(13/4)*c^(9/4)*d^5*e^12 - 390*s
qrt(2)*a^(15/4)*c^(7/4)*d^3*e^14 + 77*sqrt(2)*a^(17/4)*c^(5/4)*d*e^16 - 24*
sqrt(a)*c^5*d^16*e - 176*a^(3/2)*c^4*d^12*e^5 - 960*a^(5/2)*c^3*d^8*e^9 + 1
200*a^(7/2)*c^2*d^4*e^13 - 40*a^(9/2)*c*e^17)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(c))*c^(5/4)))/(a^2*c^5*d^20 + 5*a^3*c^4*d^16*e^4 + 10*a^4*c^3*d^12*e^8
 + 10*a^5*c^2*d^8*e^12 + 5*a^6*c*d^4*e^16 + a^7*e^20) + 6*(13*c^2*d^6*e^11
- 3*a*c*d^2*e^15)*log(e*x + d)/(c^5*d^20 + 5*a*c^4*d^16*e^4 + 10*a^2*c^3*d^
12*e^8 + 10*a^3*c^2*d^8*e^12 + 5*a^4*c*d^4*e^16 + a^5*e^20) + 1/32*(40*a^2*
c^3*d^12*e^3 + 304*a^3*c^2*d^8*e^7 - 520*a^4*c*d^4*e^11 - 16*a^5*e^15 + 6*(
5*c^5*d^11*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12)*x^9 + 6*(7*c^5*d^1
2*e^3 + 49*a*c^4*d^8*e^7 - 91*a^2*c^3*d^4*e^11 - 5*a^3*c^2*e^15)*x^8 + (c^5
*d^13*e^2 + 19*a*c^4*d^9*e^6 + 35*a^2*c^3*d^5*e^10 + 17*a^3*c^2*d*e^14)*x^7
 - 4*(c^5*d^14*e + 7*a*c^4*d^10*e^5 + 11*a^2*c^3*d^6*e^9 + 5*a^3*c^2*d^2*e^
13)*x^6 + (7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*
c^2*d^3*e^12)*x^5 + 2*(39*a*c^4*d^12*e^3 + 293*a^2*c^3*d^8*e^7 - 539*a^3*c^
2*d^4*e^11 - 25*a^4*c*e^15)*x^4 + (5*a*c^4*d^13*e^2 + 31*a^2*c^3*d^9*e^6 +
47*a^3*c^2*d^5*e^10 + 21*a^4*c*d*e^14)*x^3 - 8*(a*c^4*d^14*e + 5*a^2*c^3*d^
10*e^5 + 7*a^3*c^2*d^6*e^9 + 3*a^4*c*d^2*e^13)*x^2 + (11*a*c^4*d^15 + 79*a^
2*c^3*d^11*e^4 + 269*a^3*c^2*d^7*e^8 - 567*a^4*c*d^3*e^12)*x)/(a^4*c^4*d^18
 + 4*a^5*c^3*d^14*e^4 + 6*a^6*c^2*d^10*e^8 + 4*a^7*c*d^6*e^12 + a^8*d^2*e^1
6 + (a^2*c^6*d^16*e^2 + 4*a^3*c^5*d^12*e^6 + 6*a^4*c^4*d^8*e^10 + 4*a^5*c^3
*d^4*e^14 + a^6*c^2*e^18)*x^10 + 2*(a^2*c^6*d^17*e + 4*a^3*c^5*d^13*e^5 + 6
*a^4*c^4*d^9*e^9 + 4*a^5*c^3*d^5*e^13 + a^6*c^2*d*e^17)*x^9 + (a^2*c^6*d^18
 + 4*a^3*c^5*d^14*e^4 + 6*a^4*c^4*d^10*e^8 + 4*a^5*c^3*d^6*e^12 + a^6*c^2*d
^2*e^16)*x^8 + 2*(a^3*c^5*d^16*e^2 + 4*a^4*c^4*d^12*e^6 + 6*a^5*c^3*d^8*e^1
```

```
0 + 4*a^6*c^2*d^4*e^14 + a^7*c*e^18)*x^6 + 4*(a^3*c^5*d^17*e + 4*a^4*c^4*d^
13*e^5 + 6*a^5*c^3*d^9*e^9 + 4*a^6*c^2*d^5*e^13 + a^7*c*d*e^17)*x^5 + 2*(a^
3*c^5*d^18 + 4*a^4*c^4*d^14*e^4 + 6*a^5*c^3*d^10*e^8 + 4*a^6*c^2*d^6*e^12 +
 a^7*c*d^2*e^16)*x^4 + (a^4*c^4*d^16*e^2 + 4*a^5*c^3*d^12*e^6 + 6*a^6*c^2*d
^8*e^10 + 4*a^7*c*d^4*e^14 + a^8*e^18)*x^2 + 2*(a^4*c^4*d^17*e + 4*a^5*c^3*
d^13*e^5 + 6*a^6*c^2*d^9*e^9 + 4*a^7*c*d^5*e^13 + a^8*d*e^17)*x)
```

mupad [B] time = 7.78, size = 6280, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^3*(d + e*x)^3),x)
```

```
[Out] symsum(log(root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^1
2*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^3
6*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16
*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^
5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268
435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13
*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8
*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^3
2*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e
^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 81
28512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*
a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8
*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^
6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462
847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^
5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z
+ 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^
23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(268435
4560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 322122547
20*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^
17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^1
9*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c
^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 +
 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 79
1941349376*a^12*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 155544
4924416*a^11*c^4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 8156676096
0*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^
14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z
^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2
+ 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 559382
63040*a^7*c^5*d^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*
a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^2
4*z - 5777473473*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548
810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z -
977636142*a^2*c^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d
^10*e^15 - 15169518*c^6*d^14*e^11, z, k)*(root(2684354560*a^12*c^9*d^36*e^4
*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*
z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 +
 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 1
2079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268
435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d
^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^
8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*
e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z
^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 278520
4224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c
```

^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d
^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^1
7*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2
819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c
^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^
12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*
e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^
6*d^14*e^11, z, k)*((44040192*a^8*c^12*d^31*e^5 - 11010048*a^7*c^13*d^35*e
+ 994050048*a^9*c^11*d^27*e^9 + 13683916800*a^10*c^10*d^23*e^13 + 429360414
72*a^11*c^9*d^19*e^17 + 52628029440*a^12*c^8*d^15*e^21 + 23429382144*a^13*c
^7*d^11*e^25 - 2132803584*a^14*c^6*d^7*e^29 - 3125280768*a^15*c^5*d^3*e^33)
/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^
4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 +
56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + root(2684354560*a^12*c^9*d
^36*e^4*z^5 + 32212254720*a^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^2
8*e^12*z^5 + 2684354560*a^20*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^2
4*z^5 + 56371445760*a^15*c^6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*
z^5 + 12079595520*a^13*c^8*d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^
5 + 268435456*a^11*c^10*d^40*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^
9*c^6*d^20*e^14*z^3 - 79148482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12
*c^3*d^8*e^26*z^3 + 1239810048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^
4*d^12*e^22*z^3 + 83755008*a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16
*e^18*z^3 + 12177506304*a^8*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 -
 2785204224*a^6*c^6*d^18*e^13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120
*a^10*c^2*d^2*e^29*z^2 - 543361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^
5*c^7*d^22*e^9*z^2 + 118499328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d
^14*e^17*z^2 + 123990497280*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^
8*z + 2819286*a*c^8*d^24*e^4*z + 10462847841*a^6*c^3*d^4*e^24*z - 577747347
3*a^4*c^5*d^12*e^16*z - 43509753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*
d^16*e^12*z + 12960000*a^7*c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c
^4*d^6*e^19 + 233280000*a^3*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 1516
9518*c^6*d^14*e^11, z, k)*((402653184*a^19*c^4*d*e^38 - 134217728*a^10*c^13
*d^37*e^2 - 671088640*a^11*c^12*d^33*e^6 - 536870912*a^12*c^11*d^29*e^10 +
3758096384*a^13*c^10*d^25*e^14 + 13153337344*a^14*c^9*d^21*e^18 + 206695301
12*a^15*c^8*d^17*e^22 + 18790481920*a^16*c^7*d^13*e^26 + 10200547328*a^17*c
^6*d^9*e^30 + 3087007744*a^18*c^5*d^5*e^34)/(1048576*(a^16*e^32 + a^8*c^8*d
^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^
11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*
c^2*d^8*e^24)) + (x*(335544320*a^19*c^4*e^39 - 201326592*a^10*c^13*d^36*e^3
 - 1275068416*a^11*c^12*d^32*e^7 - 2952790016*a^12*c^11*d^28*e^11 - 1879048
192*a^13*c^10*d^24*e^15 + 4697620480*a^14*c^9*d^20*e^19 + 12213813248*a^15*
c^8*d^16*e^23 + 13153337344*a^16*c^7*d^12*e^27 + 7784628224*a^17*c^6*d^8*e^
31 + 2483027968*a^18*c^5*d^4*e^35))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*
a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d
^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*
e^24))) - (x*(40894464*a^7*c^13*d^34*e^2 + 276824064*a^8*c^12*d^30*e^6 + 96
8884224*a^9*c^11*d^26*e^10 - 13010731008*a^10*c^10*d^22*e^14 - 53433335808*
a^11*c^9*d^18*e^18 - 71647100928*a^12*c^8*d^14*e^22 - 34313601024*a^13*c^7*
d^10*e^26 + 1837105152*a^14*c^6*d^6*e^30 + 4193255424*a^15*c^5*d^2*e^34))/(
1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4
+ 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56
*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (33914880*a^12*c^5*d*e^32 +
 13713408*a^5*c^12*d^29*e^4 + 225902592*a^6*c^11*d^25*e^8 + 2352070656*a^7*
c^10*d^21*e^12 + 2474606592*a^8*c^9*d^17*e^16 - 21361803264*a^9*c^8*d^13*e^
20 + 88707170304*a^10*c^7*d^9*e^24 - 5526503424*a^11*c^6*d^5*e^28)/(1048576
*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^
10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c
^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(132710400*a^12*c^5*e^33 - 18063
36*a^4*c^13*d^32*e - 2027520*a^5*c^12*d^28*e^5 + 162017280*a^6*c^11*d^24*e^

9 + 4635316224*a^7*c^10*d^20*e^13 + 15604273152*a^8*c^9*d^16*e^17 + 3931866
3168*a^9*c^8*d^12*e^21 + 64184389632*a^10*c^7*d^8*e^25 - 2525073408*a^11*c^
6*d^4*e^29))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9
*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*
d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (320544*a^2*c
^12*d^27*e^3 + 11448000*a^3*c^11*d^23*e^7 + 114031584*a^4*c^10*d^19*e^11 -
213750144*a^5*c^9*d^15*e^15 - 3499271712*a^6*c^8*d^11*e^19 + 9699804864*a^7
*c^7*d^7*e^23 - 933615072*a^8*c^6*d^3*e^27)/(1048576*(a^16*e^32 + a^8*c^8*d
^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^
11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28*a^14*
c^2*d^8*e^24)) + (x*(514944*a^2*c^12*d^26*e^4 + 14314752*a^3*c^11*d^22*e^8
+ 266343552*a^4*c^10*d^18*e^12 + 297948672*a^5*c^9*d^14*e^16 - 2642613120*a
^6*c^8*d^10*e^20 + 1782459648*a^7*c^7*d^6*e^24 + 846599040*a^8*c^6*d^2*e^28
))/(1048576*(a^16*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*
e^4 + 28*a^10*c^6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16
+ 56*a^13*c^3*d^12*e^20 + 28*a^14*c^2*d^8*e^24))) + (194481*c^11*d^21*e^6 +
2430324*a*c^10*d^17*e^10 + 12960000*a^5*c^6*d*e^26 - 5918346*a^2*c^9*d^13*
e^14 - 83522988*a^3*c^8*d^9*e^18 + 71628705*a^4*c^7*d^5*e^22)/(1048576*(a^1
6*e^32 + a^8*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^
6*d^24*e^8 + 56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^
12*e^20 + 28*a^14*c^2*d^8*e^24)) + (x*(12960000*a^5*c^6*e^27 + 194481*c^11*
d^20*e^7 + 2430324*a*c^10*d^16*e^11 - 21081546*a^2*c^9*d^12*e^15 - 22781444
4*a^3*c^8*d^8*e^19 + 105734241*a^4*c^7*d^4*e^23))/(1048576*(a^16*e^32 + a^8
*c^8*d^32 + 8*a^15*c*d^4*e^28 + 8*a^9*c^7*d^28*e^4 + 28*a^10*c^6*d^24*e^8 +
56*a^11*c^5*d^20*e^12 + 70*a^12*c^4*d^16*e^16 + 56*a^13*c^3*d^12*e^20 + 28
*a^14*c^2*d^8*e^24)))*root(2684354560*a^12*c^9*d^36*e^4*z^5 + 32212254720*a
^18*c^3*d^12*e^28*z^5 + 32212254720*a^14*c^7*d^28*e^12*z^5 + 2684354560*a^2
0*c*d^4*e^36*z^5 + 56371445760*a^17*c^4*d^16*e^24*z^5 + 56371445760*a^15*c^
6*d^24*e^16*z^5 + 12079595520*a^19*c^2*d^8*e^32*z^5 + 12079595520*a^13*c^8*
d^32*e^8*z^5 + 67645734912*a^16*c^5*d^20*e^20*z^5 + 268435456*a^11*c^10*d^4
0*z^5 + 268435456*a^21*e^40*z^5 + 45339770880*a^9*c^6*d^20*e^14*z^3 - 79148
482560*a^13*c^2*d^4*e^30*z^3 + 791941349376*a^12*c^3*d^8*e^26*z^3 + 1239810
048*a^7*c^8*d^28*e^6*z^3 - 1555444924416*a^11*c^4*d^12*e^22*z^3 + 83755008*
a^6*c^9*d^32*e^2*z^3 + 81566760960*a^10*c^5*d^16*e^18*z^3 + 12177506304*a^8
*c^7*d^24*e^10*z^3 + 117964800*a^14*c*e^34*z^3 - 2785204224*a^6*c^6*d^18*e^
13*z^2 + 8128512*a^3*c^9*d^30*e*z^2 + 2700933120*a^10*c^2*d^2*e^29*z^2 - 54
3361222656*a^8*c^4*d^10*e^21*z^2 + 1048135680*a^5*c^7*d^22*e^9*z^2 + 118499
328*a^4*c^8*d^26*e^5*z^2 - 55938263040*a^7*c^5*d^14*e^17*z^2 + 123990497280
*a^9*c^3*d^6*e^25*z^2 + 24139215*a^2*c^7*d^20*e^8*z + 2819286*a*c^8*d^24*e^
4*z + 10462847841*a^6*c^3*d^4*e^24*z - 5777473473*a^4*c^5*d^12*e^16*z - 435
09753450*a^5*c^4*d^8*e^20*z - 548810316*a^3*c^6*d^16*e^12*z + 12960000*a^7*
c^2*e^28*z + 194481*c^9*d^28*z - 977636142*a^2*c^4*d^6*e^19 + 233280000*a^3
*c^3*d^2*e^23 - 140556060*a*c^5*d^10*e^15 - 15169518*c^6*d^14*e^11, z, k),
k, 1, 5) - ((2*a^3*e^15 - 5*c^3*d^12*e^3 - 38*a*c^2*d^8*e^7 + 65*a^2*c*d^4*
e^11)/(4*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (3*x^8*(5*a^3*c^2*e^15 -
7*c^5*d^12*e^3 - 49*a*c^4*d^8*e^7 + 91*a^2*c^3*d^4*e^11))/(16*a^2*(a^4*e^16
+ c^4*d^16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) - (
x^5*(7*c^5*d^15 + 97*a*c^4*d^11*e^4 + 461*a^2*c^3*d^7*e^8 - 1165*a^3*c^2*d^
3*e^12))/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (3*x^9*(5*c^5*d^1
1*e^4 + 34*a*c^4*d^7*e^8 - 99*a^2*c^3*d^3*e^12))/(16*a^2*(a^4*e^16 + c^4*d^
16 + 4*a*c^3*d^12*e^4 + 4*a^3*c*d^4*e^12 + 6*a^2*c^2*d^8*e^8)) + (x^2*(c^2*
d^6*e + 3*a*c*d^2*e^5))/(4*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) - (x^3*(5
*c^2*d^5*e^2 + 21*a*c*d*e^6))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) -
(x*(11*c^4*d^15 + 79*a*c^3*d^11*e^4 - 567*a^3*c*d^3*e^12 + 269*a^2*c^2*d^7*
e^8))/(32*a*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) + (x^6*(c^3*d^6*e + 5*a*
c^2*d^2*e^5))/(8*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)) + (x^4*(25*a^3*c*
e^15 - 39*c^4*d^12*e^3 - 293*a*c^3*d^8*e^7 + 539*a^2*c^2*d^4*e^11))/(16*a*(
a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)^2) - (e^2*x^7*(c^3*d^5 + 17*a*c^2*d*e^4)
)/(32*a^2*(a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)))/(a^2*d^2 + a^2*e^2*x^2 + c^

```
2*d^2*x^8 + c^2*e^2*x^10 + 2*a^2*d*e*x + 2*a*c*d^2*x^4 + 2*a*c*e^2*x^6 + 2*
c^2*d*e*x^9 + 4*a*c*d*e*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**3,x)
```

```
[Out] Timed out
```

3.411 $\int \frac{-1+x}{1-x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{1-x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 - x + x^2),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{1-x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x)/(1 - x + x^2),x]
[Out] IntegrateAlgebraic[(-1 + x)/(1 - x + x^2), x]
```

fricas [A] time = 1.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

giac [A] time = 0.29, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(x^2-x+1),x)
[Out] 1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(x^2 - x + 1),x)
```

```
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.11, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2-x+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.412 $\int \frac{-1+x^2}{1+x^3} dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1872, 634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^3),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{1+x^3}\,dx &= \int \frac{-1+x}{1-x+x^2}\,dx\\ &= -\left(\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.03

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^3),x]
[Out] -(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{1+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(1 + x^3),x]
[Out] IntegrateAlgebraic[(-1 + x^2)/(1 + x^3), x]
```

fricas [A] time = 1.39, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

giac [A] time = 0.38, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(x^2-x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3+1),x)
```

```
[Out] -1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/2*ln(x^2-x+1)
```

maxima [A] time = 2.53, size = 28, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3 + 1),x)
```

```
[Out] log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 34, normalized size = 1.06

$$\frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3+1),x)
```

```
[Out] log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.413 $\int \frac{-4+3x}{4-2x+x^2}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x}{4-2x+x^2}\,dx &= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx-\int \frac{1}{4-2x+x^2}\,dx\\ &= \frac{3}{2}\log\left(4-2x+x^2\right)+2\,\mathrm{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,-2+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(4-2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+3x}{4-2x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 3*x)/(4 - 2*x + x^2),x]
[Out] IntegrateAlgebraic[(-4 + 3*x)/(4 - 2*x + x^2), x]
```

fricas [A] time = 1.65, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

giac [A] time = 0.39, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}+\frac{3\ln\left(x^2-2x+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+3*x)/(x^2-2*x+4),x)
[Out] 3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))
```

maxima [A] time = 2.16, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.94

$$\frac{3\,\ln\left(x^2-2\,x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 4)/(x^2 - 2*x + 4),x)
```

```
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+3*x)/(x**2-2*x+4),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.414 $\int \frac{-8+2x+3x^2}{8+x^3}\,dx$

Optimal. Leaf size=32

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1872, 634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-2x+4\right)+\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

```
[Out] ArcTan[(1 - x)/Sqrt[3]]/Sqrt[3] + (3*Log[4 - 2*x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1872

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Dist[q^2/a
, Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*(
a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-8+2x+3x^2}{8+x^3}\,dx &= \frac{1}{2}\int \frac{-8+6x}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\int \frac{-2+2x}{4-2x+x^2}\,dx - \int \frac{1}{4-2x+x^2}\,dx \\
&= \frac{3}{2}\log\left(4-2x+x^2\right) + 2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, -2+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$
\frac{3}{2}\log\left(x^2-2x+4\right) - \frac{\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
[Out] -(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{-8+2x+3x^2}{8+x^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
[Out] IntegrateAlgebraic[(-8 + 2*x + 3*x^2)/(8 + x^3), x]
```

fricas [A] time = 1.66, size = 26, normalized size = 0.81

$$
-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

giac [A] time = 0.33, size = 26, normalized size = 0.81

$$
-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

maple [A] time = 0.00, size = 29, normalized size = 0.91

$$
-\frac{\sqrt{3}\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3} + \frac{3\ln\left(x^2-2x+4\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-8)/(x^3+8),x)
```

```
[Out] -1/3*3^(1/2)*arctan(1/6*(2*x-2)*3^(1/2))+3/2*ln(x^2-2*x+4)
```

maxima [A] time = 2.33, size = 26, normalized size = 0.81

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

mupad [B] time = 0.03, size = 30, normalized size = 0.94

$$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
```

```
[Out] (3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

sympy [A] time = 0.12, size = 36, normalized size = 1.12

$$\frac{3\log\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-8)/(x**3+8),x)
```

```
[Out] 3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

3.415 $\int \frac{2+x}{-1+2x+x^2} dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{-1+2x+x^2}\,dx &= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\ &= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x}{-1+2x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x)/(-1 + 2*x + x^2),x]
```

```
[Out] IntegrateAlgebraic[(2 + x)/(-1 + 2*x + x^2), x]
```

fricas [A] time = 1.61, size = 45, normalized size = 1.00

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

giac [A] time = 0.42, size = 44, normalized size = 0.98

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^2+2*x-1),x)
```

```
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

maxima [A] time = 2.62, size = 35, normalized size = 0.78

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

mupad [B] time = 2.32, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(2*x + x^2 - 1),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

sympy [A] time = 0.11, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+2*x-1),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

3.416 $\int \frac{-4+x^2}{2-5x+x^3}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2055, 632, 31}

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + x
])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2055

```
Int[(u_.)*(P_)*(Q_)^(q_), x_Symbol] :> Module[{gcd = PolyGCD[P, Q, x]}, Int
[u*gcd^(q + 1)*PolynomialQuotient[P, gcd, x]*PolynomialQuotient[Q, gcd, x]^
q, x] /; NeQ[gcd, 1]] /; ILtQ[q, 0] && PolyQ[P, x] && PolyQ[Q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-4+x^2}{2-5x+x^3}\,dx &= \int \frac{2+x}{-1+2x+x^2}\,dx\\
&= -\left(\frac{1}{4}\left(-2+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2}+x}\,dx\right)+\frac{1}{4}\left(2+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2}+x}\,dx\\
&= \frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.93

$$\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-x+\sqrt{2}-1\right)-\left(\sqrt{2}-2\right)\log\left(x+\sqrt{2}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

```
[Out] ((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x])
/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + x^2)/(2 - 5*x + x^3),x]
[Out] IntegrateAlgebraic[(-4 + x^2)/(2 - 5*x + x^3), x]
```

fricas [A] time = 1.41, size = 45, normalized size = 1.00

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{x^2-2\,\sqrt{2}\,(x+1)+2\,x+3}{x^2+2\,x-1}\right)+\frac{1}{2}\,\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2*
log(x^2 + 2*x - 1)
```

giac [A] time = 0.31, size = 44, normalized size = 0.98

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{\left|2\,x-2\,\sqrt{2}+2\right|}{\left|2\,x+2\,\sqrt{2}+2\right|}\right)+\frac{1}{2}\,\log\left(\left|x^2+2\,x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*lo
g(abs(x^2 + 2*x - 1))
```

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}+\frac{\ln\left(x^2+2x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x^3-5*x+2),x)
[Out] 1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

maxima [A] time = 2.33, size = 35, normalized size = 0.78

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\,\log\left(x^2+2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
1)
```

mupad [**B**] time = 0.05, size = 34, normalized size = 0.76

$$\ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

```
[Out] log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
1/2)
```

sympy [**A**] time = 0.12, size = 39, normalized size = 0.87

$$\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(x**3-5*x+2),x)
```

```
[Out] (1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
+ 1)
```

3.417 $\int \frac{2}{-1+4x^2} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(2x)$$

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 207}

$$-\tanh^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[2/(-1 + 4*x^2),x]
```

```
[Out] -ArcTanh[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{2}{-1+4x^2} \, dx &= 2 \int \frac{1}{-1+4x^2} \, dx \\ &= -\tanh^{-1}(2x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 23, normalized size = 3.83

$$2 \left(\frac{1}{4} \log(1-2x) - \frac{1}{4} \log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[2/(-1 + 4*x^2),x]
```

```
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2}{-1+4x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[2/(-1 + 4*x^2),x]
```

```
[Out] IntegrateAlgebraic[2/(-1 + 4*x^2), x]
```

fricas [B] time = 1.39, size = 17, normalized size = 2.83

$$-\frac{1}{2} \log(2x+1) + \frac{1}{2} \log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

giac [B] time = 0.31, size = 15, normalized size = 2.50

$$-\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right)+\frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

maple [B] time = 0.00, size = 18, normalized size = 3.00

$$\frac{\ln(2x-1)}{2}-\frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2-1),x)
```

```
[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)
```

maxima [B] time = 1.06, size = 17, normalized size = 2.83

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

mupad [B] time = 2.27, size = 6, normalized size = 1.00

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/(4*x^2 - 1),x)
```

```
[Out] -atanh(2*x)
```

sympy [B] time = 0.09, size = 15, normalized size = 2.50

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/(4*x**2-1),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

3.418 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
[Out] Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Rubi steps

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.10

$$2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
[Out] 2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
[Out] IntegrateAlgebraic[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1), x]
```

fricas [A] time = 1.16, size = 17, normalized size = 0.81

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

giac [A] time = 0.42, size = 19, normalized size = 0.90

$$-\frac{1}{2}\log(|2x+1|)+\frac{1}{2}\log(|2x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{\ln(2x-1)}{2}-\frac{\ln(2x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-1)-1/(2*x+1),x)
```

```
[Out] 1/2*ln(2*x-1)-1/2*ln(2*x+1)
```

maxima [A] time = 0.94, size = 17, normalized size = 0.81

$$-\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

mupad [B] time = 0.15, size = 6, normalized size = 0.29

$$-\operatorname{atanh}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x - 1) - 1/(2*x + 1),x)
```

```
[Out] -atanh(2*x)
```

sympy [A] time = 0.10, size = 15, normalized size = 0.71

$$\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+2*x)-1/(1+2*x),x)
```

```
[Out] log(x - 1/2)/2 - log(x + 1/2)/2
```

3.419 $\int \frac{x}{(1-x^2)^5}\,dx$

Optimal. Leaf size=13

$$\frac{1}{8(1-x^2)^4}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{8(1-x^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8(1-x^2)^4}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8(x^2-1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^5,x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-x^2)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(1 - x^2)^5,x]
```

```
[Out] IntegrateAlgebraic[x/(1 - x^2)^5, x]
```

fricas [B] time = 1.35, size = 24, normalized size = 1.85

$$\frac{1}{8\left(x^8-4x^6+6x^4-4x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

giac [A] time = 0.33, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/8/(x^2 - 1)^4
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^5,x)
```

```
[Out] 1/8/(x^2-1)^4
```

maxima [A] time = 0.88, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/8/(x^2 - 1)^4
```

mupad [B] time = 2.32, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x^2 - 1)^5,x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

sympy [B] time = 0.12, size = 22, normalized size = 1.69

$$\frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**5,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.420

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\right.$$

Optimal. Leaf size=13

$$\frac{1}{8\left(1-x^2\right)^4}$$

Rubi [B] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 0, integrand size = 73, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(32*(-1 + x)^5) + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(-
1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(25
6*(1 + x)^2),x]
```

```
[Out] 1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
+ 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 + x)
)
```

Rubi steps

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\right.$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{1}{8\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(
256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) -
 5/(256*(1 + x)^2),x]
```

```
[Out] 1/(8*(-1 + x^2)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)
^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1
+ x)^3) - 5/(256*(1 + x)^2),x]
```

```
[Out] IntegrateAlgebraic[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)
^3) + 5/(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1
+ x)^3) - 5/(256*(1 + x)^2), x]
```

fricas [B] time = 1.10, size = 24, normalized size = 1.85

$$\frac{1}{8\left(x^8-4x^6+6x^4-4x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

giac [B] time = 0.31, size = 57, normalized size = 4.38

$$\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

maple [B] time = 0.00, size = 58, normalized size = 4.46

$$\frac{1}{128\,(x-1)^4}-\frac{1}{64\,(x-1)^3}+\frac{5}{256\,(x-1)^2}-\frac{5}{256\,(x-1)}+\frac{1}{128\,(x+1)^4}+\frac{1}{64\,(x+1)^3}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/32/(x-1)^5+3/64/(x-1)^4-5/128/(x-1)^3+5/256/(x-1)^2-1/32/(x+1)^5-3/6
4/(x+1)^4-5/128/(x+1)^3-5/256/(x+1)^2,x)
```

```
[Out] 1/128/(x-1)^4-1/64/(x-1)^3+5/256/(x-1)^2-5/256/(x-1)+1/128/(x+1)^4+1/64/(x+
1)^3+5/256/(x+1)^2+5/256/(x+1)
```

maxima [B] time = 0.94, size = 57, normalized size = 4.38

$$\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(
1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(x
 + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\frac{1}{8\left(x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x +
 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(x
+ 1)^5),x)
```

```
[Out] 1/(8*(x^2 - 1)^4)
```

sympy [B] time = 0.31, size = 22, normalized size = 1.69

$$\frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1/
32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

```
[Out] 1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

3.421 $\int \frac{1+x^6}{-1+x^6} dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Rubi [A] time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{-1+x^6}\,dx &= x+2\int \frac{1}{-1+x^6}\,dx\\ &= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\ &= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x\ldots}\\ &= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\ldots\right.\\ &= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^6}{-1+x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^6)/(-1 + x^6),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^6)/(-1 + x^6), x]
```

fricas [A] time = 1.26, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

giac [A] time = 0.29, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.01, size = 67, normalized size = 0.97

$$x-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(x-1)}{3}-\frac{\ln(x+1)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/(x^6-1),x)
```

```
[Out] x+1/3*ln(x-1)-1/6*ln(x^2+x+1)-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^(1/2))+1/6*l
n(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*ln(x+1)
```

maxima [A] time = 2.01, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

mupad [B] time = 0.10, size = 94, normalized size = 1.36

$$x+\frac{\mathrm{atan}(x\,1\mathrm{i})\,2\mathrm{i}}{3}-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}}-\frac{32\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}-\frac{1}{3}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}}+\frac{32\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6 + 1)/(x^6 - 1),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x+\frac{\log(x-1)}{3}-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/(x**6-1),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

3.422 $\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1593, 1584, 388, 210, 634, 618, 204, 628, 206}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] -
 (2*ArcTanh[x])/3 + Log[1 - x + x^2]/6 - Log[1 + x + x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{1}{x^3}+x^3}{-\frac{1}{x^3}+x^3}\,dx &= \int \frac{x^3\left(\frac{1}{x^3}+x^3\right)}{-1+x^6}\,dx\\
&= \int \frac{1+x^6}{-1+x^6}\,dx\\
&= x+2\int \frac{1}{-1+x^6}\,dx\\
&= x-\frac{2}{3}\int \frac{1}{1-x^2}\,dx-\frac{2}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx-\frac{2}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+
\\
&= x-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\tanh^{-1}(x)+\frac{1}{6}\log\left(1-x+x^2\right)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 78, normalized size = 1.13

$$\frac{1}{6}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+6x+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
```

```
[Out] (6*x - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sq
rt[3]] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])
/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{x^3} + x^3}{-\frac{1}{x^3} + x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-3) + x^3)/(-x^(-3) + x^3),x]
[Out] IntegrateAlgebraic[(x^(-3) + x^3)/(-x^(-3) + x^3), x]
```

fricas [A] time = 1.36, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

giac [A] time = 0.31, size = 68, normalized size = 0.99

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(|x+1|) + \frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="giac")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(abs
(x + 1)) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 67, normalized size = 0.97

$$x - \frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} + \frac{\ln\left(x^2-x+1\right)}{6} - \frac{\ln\left(x^2+x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^3+x^3)/(-1/x^3+x^3),x)
[Out] x-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-1/3*3^(1/2)*arctan(1/3*(2*x+1)*3^
(1/2))+1/3*ln(x-1)-1/3*ln(x+1)+1/6*ln(x^2-x+1)-1/6*ln(x^2+x+1)
```

maxima [A] time = 1.99, size = 66, normalized size = 0.96

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + x - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{6}\log\left(x^2-x+1\right) - \frac{1}{3}\log(x+1) + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x^3+x^3)/(-1/x^3+x^3),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + x - 1/6*log(x^2 + x + 1) + 1/6*log(x^2 - x + 1) - 1/3*log(x +
 1) + 1/3*log(x - 1)
```

mupad [B] time = 0.04, size = 94, normalized size = 1.36

$$x + \frac{\mathrm{atan}(x\,1\mathrm{i})\,2\mathrm{i}}{3} - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{-32+\sqrt{3}\,32\mathrm{i}} - \frac{32\sqrt{3}\,x}{-32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} - \frac{1}{3}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,32\mathrm{i}}{32+\sqrt{3}\,32\mathrm{i}} + \frac{32\sqrt{3}\,x}{32+\sqrt{3}\,32\mathrm{i}}\right)\left(\frac{\sqrt{3}}{3} + \frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x^3 + x^3)/(1/x^3 - x^3),x)
```

```
[Out] x + (atan(x*1i)*2i)/3 - atan((x*32i)/(3^(1/2)*32i - 32) - (32*3^(1/2)*x)/(3
^(1/2)*32i - 32))*(3^(1/2)/3 - 1i/3) - atan((x*32i)/(3^(1/2)*32i + 32) + (3
2*3^(1/2)*x)/(3^(1/2)*32i + 32))*(3^(1/2)/3 + 1i/3)
```

sympy [A] time = 0.25, size = 85, normalized size = 1.23

$$x + \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} - \frac{\log(x^2+x+1)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x**3+x**3)/(-1/x**3+x**3),x)
```

```
[Out] x + log(x - 1)/3 - log(x + 1)/3 + log(x**2 - x + 1)/6 - log(x**2 + x + 1)/6
 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3 - sqrt(3)*atan(2*sqrt(3)*x/3 +
 sqrt(3)/3)/3
```

3.423 $\int \frac{-x+x^3}{6+2x} \, dx$

Optimal. Leaf size=24

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 772}

$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] 4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{6+2x}\,dx &= \int \frac{x\left(-1+x^2\right)}{6+2x}\,dx\\ &= \int \left(4-\frac{3x}{2}+\frac{x^2}{2}-\frac{12}{3+x}\right)dx\\ &= 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.29

$$\frac{1}{2}\left(\frac{x^3}{3}-\frac{3x^2}{2}+8x-24\log(x+3)+\frac{93}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/(6 + 2*x),x]
```

```
[Out] (93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x+x^3}{6+2x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + x^3)/(6 + 2*x),x]
[Out] IntegrateAlgebraic[(-x + x^3)/(6 + 2*x), x]
```

fricas [A] time = 1.48, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

giac [A] time = 0.31, size = 21, normalized size = 0.88

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="giac")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

maple [A] time = 0.00, size = 21, normalized size = 0.88

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\ln(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(6+2*x),x)
[Out] 4*x-3/4*x^2+1/6*x^3-12*ln(x+3)
```

maxima [A] time = 0.63, size = 20, normalized size = 0.83

$$\frac{1}{6}x^3 - \frac{3}{4}x^2 + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
[Out] 1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$4x - 12\ln(x+3) - \frac{3x^2}{4} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(2*x + 6),x)
[Out] 4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

sympy [A] time = 0.08, size = 20, normalized size = 0.83

$$\frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(6+2*x),x)
[Out] x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

3.424 $\int \frac{x+x^3}{-1+x} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 772}

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+x^3}{-1+x} \, dx &= \int \frac{x\left(1+x^2\right)}{-1+x} \, dx \\
&= \int \left(2+\frac{2}{-1+x}+x+x^2\right) dx \\
&= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+x^3}{-1+x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^3)/(-1 + x),x]
```

```
[Out] IntegrateAlgebraic[(x + x^3)/(-1 + x), x]
```

fricas [A] time = 1.35, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

giac [A] time = 0.30, size = 21, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

maple [A] time = 0.00, size = 21, normalized size = 0.81

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(x-1),x)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

maxima [A] time = 0.69, size = 20, normalized size = 0.77

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$2x + 2\ln(x-1) + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3)/(x - 1),x)
```

```
[Out] 2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

sympy [A] time = 0.07, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

3.425 $\int (ac + (bc + d)x)\, dx$

Optimal. Leaf size=17

$$acx + \frac{1}{2}x^2(bc + d)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$acx + \frac{1}{2}x^2(bc + d)$$

Antiderivative was successfully verified.

```
[In] Int[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + ((b*c + d)*x^2)/2
```

Rubi steps

$$\int (ac + (bc + d)x)\, dx = acx + \frac{1}{2}(bc + d)x^2$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.29

$$acx + \frac{1}{2}bcx^2 + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*c + (b*c + d)*x,x]
```

```
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ac + (bc + d)x)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a*c + (b*c + d)*x,x]
```

```
[Out] IntegrateAlgebraic[a*c + (b*c + d)*x, x]
```

fricas [A] time = 0.99, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2cb + \frac{1}{2}x^2d + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

giac [A] time = 0.39, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="giac")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$acx + \frac{(bc + d)\, x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x+1/2*(b*c+d)*x^2
```

maxima [A] time = 0.66, size = 15, normalized size = 0.88

$$acx + \frac{1}{2}\,(bc + d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x, algorithm="maxima")
```

```
[Out] a*c*x + 1/2*(b*c + d)*x^2
```

mupad [B] time = 0.02, size = 17, normalized size = 1.00

$$\left(\frac{d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*c + x*(d + b*c),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

sympy [A] time = 0.06, size = 15, normalized size = 0.88

$$acx + x^2\left(\frac{bc}{2} + \frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*c+(b*c+d)*x,x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

3.426 $\int (dx + c(a + bx))\,dx$

Optimal. Leaf size=24

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{c(a+bx)^2}{2b}+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d*x + c*(a + b*x),x]
[Out] (d*x^2)/2 + (c*(a + b*x)^2)/(2*b)
```

Rubi steps

$$\int (dx+c(a+bx))\,dx=\frac{dx^2}{2}+\frac{c(a+bx)^2}{2b}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.92

$$acx+\frac{1}{2}bcx^2+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d*x + c*(a + b*x),x]
[Out] a*c*x + (b*c*x^2)/2 + (d*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c(a+bx))\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[d*x + c*(a + b*x),x]
[Out] IntegrateAlgebraic[d*x + c*(a + b*x), x]
```

fricas [A] time = 1.00, size = 18, normalized size = 0.75

$$\frac{1}{2}x^2cb+\frac{1}{2}x^2d+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="fricas")
[Out] 1/2*x^2*c*b + 1/2*x^2*d + x*c*a
```

giac [A] time = 0.37, size = 20, normalized size = 0.83

$$\frac{1}{2}dx^2+\frac{1}{2}\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

maple [A] time = 0.00, size = 20, normalized size = 0.83

$$\frac{d\,x^2}{2}+\left(\frac{1}{2}b\,x^2+ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c*(b*x+a),x)
```

```
[Out] 1/2*d*x^2+c*(1/2*b*x^2+a*x)
```

maxima [A] time = 0.57, size = 20, normalized size = 0.83

$$\frac{1}{2}\,dx^2+\frac{1}{2}\,\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + 1/2*(b*x^2 + 2*a*x)*c
```

mupad [B] time = 0.02, size = 17, normalized size = 0.71

$$\left(\frac{d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x + c*(a + b*x),x)
```

```
[Out] x^2*(d/2 + (b*c)/2) + a*c*x
```

sympy [A] time = 0.06, size = 15, normalized size = 0.62

$$acx+x^2\left(\frac{bc}{2}+\frac{d}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c*(b*x+a),x)
```

```
[Out] a*c*x + x**2*(b*c/2 + d/2)
```

3.427 $\int \frac{4+4x}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=22

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {801, 635, 203, 260}

$$-2\log\left(x^2+1\right)-\frac{4}{x}+4\log(x)-4\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] -4/x - 4*ArcTan[x] + 4*Log[x] - 2*Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx &= \int \left(\frac{4}{x^2}+\frac{4}{x}-\frac{4(1+x)}{1+x^2}\right)dx\\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1+x}{1+x^2}\,dx\\
&= -\frac{4}{x}+4\log(x)-4\int \frac{1}{1+x^2}\,dx-4\int \frac{x}{1+x^2}\,dx\\
&= -\frac{4}{x}-4\tan^{-1}(x)+4\log(x)-2\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.09

$$4\left(-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] 4*(-x^(-1) - ArcTan[x] + Log[x] - Log[1 + x^2]/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+4x}{x^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 4*x)/(x^2*(1 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(4 + 4*x)/(x^2*(1 + x^2)), x]
```

fricas [A] time = 0.96, size = 25, normalized size = 1.14

$$-\frac{2\left(2\,x\arctan(x)+x\log\left(x^2+1\right)-2\,x\log(x)+2\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*(2*x*arctan(x) + x*log(x^2 + 1) - 2*x*log(x) + 2)/x
```

giac [A] time = 0.39, size = 23, normalized size = 1.05

$$-\frac{4}{x}-4\,\arctan(x)-2\,\log\left(x^2+1\right)+4\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(abs(x))
```

maple [A] time = 0.01, size = 23, normalized size = 1.05

$$-4\arctan(x)+4\ln(x)-2\ln\left(x^2+1\right)-\frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+4*x)/x^2/(x^2+1),x)
```

```
[Out] -4/x-4*arctan(x)+4*ln(x)-2*ln(x^2+1)
```

maxima [A] time = 1.38, size = 22, normalized size = 1.00

$$-\frac{4}{x}-4\,\arctan(x)-2\,\log\left(x^2+1\right)+4\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -4/x - 4*arctan(x) - 2*log(x^2 + 1) + 4*log(x)
```

mupad [B] time = 0.04, size = 28, normalized size = 1.27

$$4\,\ln(x)-\frac{4}{x}+\ln\left(x-\mathrm{i}\right)\,\left(-2+2\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\,\left(-2-2\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 4)/(x^2*(x^2 + 1)),x)
```

```
[Out] 4*log(x) - log(x + 1i)*(2 + 2i) - log(x - 1i)*(2 - 2i) - 4/x
```

sympy [A] time = 0.13, size = 20, normalized size = 0.91

$$4\log(x) - 2\log\left(x^2 + 1\right) - 4\operatorname{atan}(x) - \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+4*x)/x**2/(x**2+1),x)
```

```
[Out] 4*log(x) - 2*log(x**2 + 1) - 4*atan(x) - 4/x
```

3.428 $\int \frac{24+8x}{x(-4+x^2)}\,dx$

Optimal. Leaf size=17

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {801}

$$5\log(2-x)-6\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 5*Log[2 - x] - 6*Log[x] + Log[2 + x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{24+8x}{x\left(-4+x^2\right)}\,dx &= \int \left(\frac{5}{-2+x}-\frac{6}{x}+\frac{1}{2+x}\right)dx\\ &= 5\log(2-x)-6\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.59

$$8\left(\frac{5}{8}\log(2-x)-\frac{3\log(x)}{4}+\frac{1}{8}\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] 8*((5*Log[2 - x])/8 - (3*Log[x])/4 + Log[2 + x]/8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{24+8x}{x\left(-4+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(24 + 8*x)/(x*(-4 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[(24 + 8*x)/(x*(-4 + x^2)), x]
```

fricas [A] time = 1.41, size = 15, normalized size = 0.88

$$\log(x+2)+5\log(x-2)-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="fricas")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

giac [A] time = 0.33, size = 18, normalized size = 1.06

$$\log(|x+2|) + 5\log(|x-2|) - 6\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="giac")
[Out] log(abs(x + 2)) + 5*log(abs(x - 2)) - 6*log(abs(x))
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$-6\ln(x) + 5\ln(x-2) + \ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((24+8*x)/x/(x^2-4),x)
[Out] 5*ln(x-2)+ln(x+2)-6*ln(x)
```

maxima [A] time = 0.75, size = 15, normalized size = 0.88

$$\log(x+2) + 5\log(x-2) - 6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x^2-4),x, algorithm="maxima")
[Out] log(x + 2) + 5*log(x - 2) - 6*log(x)
```

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$5\ln(x-2) + \ln(x+2) - 6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x + 24)/(x*(x^2 - 4)),x)
[Out] 5*log(x - 2) + log(x + 2) - 6*log(x)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$-6\log(x) + 5\log(x-2) + \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((24+8*x)/x/(x**2-4),x)
[Out] -6*log(x) + 5*log(x - 2) + log(x + 2)
```

3.429 $\int \frac{-1+x^2}{-2x+x^3} dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1593, 446, 72}

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{-2x+x^3} dx &= \int \frac{-1+x^2}{x\left(-2+x^2\right)} dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(-2+x)x} dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{2(-2+x)}+\frac{1}{2x}\right) dx, x, x^2\right) \\
&= \frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{1}{4}\log\left(2-x^2\right)+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

```
[Out] Log[x]/2 + Log[2 - x^2]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x^2}{-2x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)/(-2*x + x^3),x]
[Out] IntegrateAlgebraic[(-1 + x^2)/(-2*x + x^3), x]
```

fricas [A] time = 1.28, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{1}{4}\log\left(x^2\right)+\frac{1}{4}\log\left(\left|x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
[Out] 1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

maple [A] time = 0.01, size = 14, normalized size = 0.74

$$\frac{\ln(x)}{2}+\frac{\ln\left(x^2-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^3-2*x),x)
[Out] 1/4*ln(x^2-2)+1/2*ln(x)
```

maxima [A] time = 0.66, size = 13, normalized size = 0.68

$$\frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
[Out] 1/4*log(x^2 - 2) + 1/2*log(x)
```

mupad [B] time = 2.34, size = 13, normalized size = 0.68

$$\frac{\ln\left(x^2-2\right)}{4}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(2*x - x^3),x)
```

```
[Out] log(x^2 - 2)/4 + log(x)/2
```

sympy [A] time = 0.10, size = 12, normalized size = 0.63

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x^{2}-2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**3-2*x),x)
[Out] log(x)/2 + log(x**2 - 2)/4
```

3.430 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[3*x + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3}\log\left(3x+x^3\right)$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.42

$$\frac{1}{3}\log\left(x^2+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] Log[x]/3 + Log[3 + x^2]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{3x+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(3*x + x^3),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(3*x + x^3), x]
```

fricas [A] time = 1.40, size = 10, normalized size = 0.83

$$\frac{1}{3}\log\left(x^3+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x)
```

giac [A] time = 0.40, size = 13, normalized size = 1.08

$$\frac{1}{3}\,\log\left(3\left|\frac{1}{3}\,x^{3}+x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(3*abs(1/3*x^3 + x))
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{\ln\left(\left(x^{2}+3\right)x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^3+3*x),x)
```

```
[Out] 1/3*ln(x*(x^2+3))
```

maxima [A] time = 0.62, size = 10, normalized size = 0.83

$$\frac{1}{3}\,\log\left(x^{3}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x)
```

mupad [B] time = 2.27, size = 10, normalized size = 0.83

$$\frac{\ln\left(x^{3}+3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(3*x + x^3),x)
```

```
[Out] log(3*x + x^3)/3
```

sympy [A] time = 0.09, size = 8, normalized size = 0.67

$$\frac{\log\left(x^{3}+3x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**3+3*x),x)
```

```
[Out] log(x**3 + 3*x)/3
```

3.431 $\int \frac{a+3bx^2}{ax+bx^3} \, dx$

Optimal. Leaf size=10

$$\log \left(ax + bx^3\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1587}

$$\log \left(ax + bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[a*x + b*x^3]
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{a + 3bx^2}{ax + bx^3} \, dx = \log \left(ax + bx^3\right)$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.10

$$\log \left(a + bx^2\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] Log[x] + Log[a + b*x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + 3bx^2}{ax + bx^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + 3*b*x^2)/(a*x + b*x^3), x]
```

fricas [A] time = 1.16, size = 10, normalized size = 1.00

$$\log \left(bx^3 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] log(b*x^3 + a*x)
```

giac [**A**] time = 0.33, size = 11, normalized size = 1.10

$$\log\left(\left|bx^3 + ax\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
[Out] log(abs(b*x^3 + a*x))
```

maple [**A**] time = 0.00, size = 11, normalized size = 1.10

$$\ln\left(\left(b\,x^2 + a\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*b*x^2+a)/(b*x^3+a*x),x)
[Out] ln(x*(b*x^2+a))
```

maxima [**A**] time = 0.61, size = 10, normalized size = 1.00

$$\log\left(bx^3 + ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
[Out] log(b*x^3 + a*x)
```

mupad [**B**] time = 0.06, size = 10, normalized size = 1.00

$$\ln\left(b\,x^3 + a\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 3*b*x^2)/(a*x + b*x^3),x)
[Out] log(a*x + b*x^3)
```

sympy [**A**] time = 0.13, size = 8, normalized size = 0.80

$$\log\left(ax + bx^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*b*x**2+a)/(b*x**3+a*x),x)
[Out] log(a*x + b*x**3)
```

3.432 $\int \frac{-2+4x}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 801}

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+4x}{-x+x^3}\,dx &= \int \frac{-2+4x}{x\left(-1+x^2\right)}\,dx\\ &= \int \left(\frac{1}{-1+x}+\frac{2}{x}-\frac{3}{1+x}\right)dx\\ &= \log(1-x)+2\log(x)-3\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\log(1-x)+2\log(x)-3\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+4x}{-x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + 4*x)/(-x + x^3),x]
```

```
[Out] IntegrateAlgebraic[(-2 + 4*x)/(-x + x^3), x]
```

fricas [A] time = 0.99, size = 15, normalized size = 0.88

$$-3\,\log\left(x+1\right)+\log\left(x-1\right)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

giac [A] time = 0.36, size = 18, normalized size = 1.06

$$-3\,\log\left(|x+1|\right)+\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
[Out] -3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$2\ln(x)+\ln\left(x-1\right)-3\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+4*x)/(x^3-x),x)
[Out] ln(x-1)-3*ln(x+1)+2*ln(x)
```

maxima [A] time = 0.61, size = 15, normalized size = 0.88

$$-3\,\log\left(x+1\right)+\log\left(x-1\right)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
[Out] -3*log(x + 1) + log(x - 1) + 2*log(x)
```

mupad [B] time = 0.06, size = 15, normalized size = 0.88

$$\ln\left(x-1\right)-3\,\ln\left(x+1\right)+2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x - 2)/(x - x^3),x)
[Out] log(x - 1) - 3*log(x + 1) + 2*log(x)
```

sympy [A] time = 0.13, size = 15, normalized size = 0.88

$$2\log\left(x\right)+\log\left(x-1\right)-3\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+4*x)/(x**3-x),x)
[Out] 2*log(x) + log(x - 1) - 3*log(x + 1)
```

3.433 $\int \frac{4+x}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(4*x + x^3),x]
```

```
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x}{4x+x^3}\,dx &= \int \frac{4+x}{x\left(4+x^2\right)}\,dx \\
&= \int \left(\frac{1}{x}+\frac{1-x}{4+x^2}\right) dx \\
&= \log(x) + \int \frac{1-x}{4+x^2}\,dx \\
&= \log(x) + \int \frac{1}{4+x^2}\,dx - \int \frac{x}{4+x^2}\,dx \\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \log(x) - \frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$
-\frac{1}{2}\log\left(x^2+4\right)+\log(x)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(4*x + x^3),x]
[Out] ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{4+x}{4x+x^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + x)/(4*x + x^3),x]
[Out] IntegrateAlgebraic[(4 + x)/(4*x + x^3), x]
```

fricas [A] time = 1.39, size = 17, normalized size = 0.74

$$
\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

giac [A] time = 0.38, size = 18, normalized size = 0.78

$$
\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="giac")
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$
\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln(x)-\frac{\ln\left(x^2+4\right)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(x^3+4*x),x)
```

```
[Out] 1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

maxima [A] time = 1.64, size = 17, normalized size = 0.74

$$\frac{1}{2}\arctan\left(\frac{1}{2}x\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.91

$$\ln(x)+\ln(x-2i)\left(-\frac{1}{2}-\frac{1}{4}i\right)+\ln(x+2i)\left(-\frac{1}{2}+\frac{1}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(4*x + x^3),x)
```

```
[Out] log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.74

$$\log(x)-\frac{\log\left(x^2+4\right)}{2}+\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**3+4*x),x)
```

```
[Out] log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

3.434 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1587}

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] Log[1 - x^2 + x^4]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x + 2*x^3)/(1 - x^2 + x^4),x]
[Out] IntegrateAlgebraic[(-x + 2*x^3)/(1 - x^2 + x^4), x]
```

fricas [A] time = 1.27, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x^4 - x^2 + 1)
```

giac [A] time = 0.28, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
[Out] 1/2*log(x^4 - x^2 + 1)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
[Out] 1/2*ln(x^4-x^2+1)
```

maxima [A] time = 0.65, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x^4 - x^2 + 1)
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
[Out] log(x^4 - x^2 + 1)/2
```

sympy [A] time = 0.09, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
[Out] log(x**4 - x**2 + 1)/2
```

3.435 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\
&= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx\\
&= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-3+x}{2x+3x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x)/(2*x + 3*x^2 + x^3),x]
[Out] IntegrateAlgebraic[(-3 + x)/(2*x + 3*x^2 + x^3), x]
```

fricas [A] time = 1.53, size = 17, normalized size = 0.81

$$-\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

giac [A] time = 0.37, size = 20, normalized size = 0.95

$$-\frac{5}{2}\,\log\left(|x+2|\right)+4\,\log\left(|x+1|\right)-\frac{3}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.86

$$-\frac{3\ln(x)}{2}+4\ln\left(x+1\right)-\frac{5\ln\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)/(x^3+3*x^2+2*x),x)
[Out] -3/2*ln(x)+4*ln(x+1)-5/2*ln(x+2)
```

maxima [A] time = 0.59, size = 17, normalized size = 0.81

$$-\frac{5}{2}\,\log\left(x+2\right)+4\,\log\left(x+1\right)-\frac{3}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

mupad [B] time = 0.07, size = 17, normalized size = 0.81

$$4\,\ln\left(x+1\right)-\frac{5\,\ln\left(x+2\right)}{2}-\frac{3\,\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3)/(2*x + 3*x^2 + x^3),x)
[Out] 4*log(x + 1) - (5*log(x + 2))/2 - (3*log(x))/2
```

sympy [A] time = 0.13, size = 20, normalized size = 0.95

$$-\frac{3\log\left(x\right)}{2}+4\log\left(x+1\right)-\frac{5\log\left(x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

3.436 $\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$

Optimal. Leaf size=10

$$-\frac{2}{x(x+1)}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1594, 27, 74}

$$-\frac{2}{x(x+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x*(1 + x))
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+4x}{x^2+2x^3+x^4}\,dx &= \int \frac{2+4x}{x^2\left(1+2x+x^2\right)}\,dx\\
&= \int \frac{2+4x}{x^2(1+x)^2}\,dx\\
&= -\frac{2}{x(1+x)}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
```

```
[Out] -2/(x + x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+4x}{x^2+2x^3+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 4*x)/(x^2 + 2*x^3 + x^4),x]
[Out] IntegrateAlgebraic[(2 + 4*x)/(x^2 + 2*x^3 + x^4), x]
```

fricas [A] time = 1.19, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="fricas")
[Out] -2/(x^2 + x)
```

giac [A] time = 0.26, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="giac")
[Out] -2/(x^2 + x)
```

maple [A] time = 0.00, size = 14, normalized size = 1.40

$$-\frac{2}{x}+\frac{2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+4*x)/(x^4+2*x^3+x^2),x)
[Out] -2/x+2/(x+1)
```

maxima [A] time = 0.78, size = 9, normalized size = 0.90

$$-\frac{2}{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x^4+2*x^3+x^2),x, algorithm="maxima")
[Out] -2/(x^2 + x)
```

mupad [B] time = 2.20, size = 10, normalized size = 1.00

$$-\frac{2}{x\,\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)/(x^2 + 2*x^3 + x^4),x)
[Out] -2/(x*(x + 1))
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{2}{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+4*x)/(x**4+2*x**3+x**2),x)
```

```
[Out] -2/(x**2 + x)
```

3.437 $\int \frac{1+x}{-6x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1594, 800}

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{-6x+x^2+x^3}\,dx &= \int \frac{1+x}{x\left(-6+x+x^2\right)}\,dx\\ &= \int \left(\frac{3}{10(-2+x)}-\frac{1}{6x}-\frac{2}{15(3+x)}\right)dx\\ &= \frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{3}{10}\log(2-x)-\frac{\log(x)}{6}-\frac{2}{15}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-6*x + x^2 + x^3),x]
```

```
[Out] (3*Log[2 - x])/10 - Log[x]/6 - (2*Log[3 + x])/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{-6x+x^2+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x)/(-6*x + x^2 + x^3),x]
[Out] IntegrateAlgebraic[(1 + x)/(-6*x + x^2 + x^3), x]
```

fricas [A] time = 1.42, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="fricas")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

giac [A] time = 0.36, size = 20, normalized size = 0.80

$$-\frac{2}{15}\log(|x+3|)+\frac{3}{10}\log(|x-2|)-\frac{1}{6}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="giac")
[Out] -2/15*log(abs(x + 3)) + 3/10*log(abs(x - 2)) - 1/6*log(abs(x))
```

maple [A] time = 0.01, size = 18, normalized size = 0.72

$$-\frac{\ln(x)}{6}+\frac{3\ln(x-2)}{10}-\frac{2\ln(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x^3+x^2-6*x),x)
[Out] 3/10*ln(x-2)-2/15*ln(x+3)-1/6*ln(x)
```

maxima [A] time = 0.65, size = 17, normalized size = 0.68

$$-\frac{2}{15}\log(x+3)+\frac{3}{10}\log(x-2)-\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3+x^2-6*x),x, algorithm="maxima")
[Out] -2/15*log(x + 3) + 3/10*log(x - 2) - 1/6*log(x)
```

mupad [B] time = 0.11, size = 17, normalized size = 0.68

$$\frac{3\ln(x-2)}{10}-\frac{2\ln(x+3)}{15}-\frac{\ln(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2 - 6*x + x^3),x)
[Out] (3*log(x - 2))/10 - (2*log(x + 3))/15 - log(x)/6
```

sympy [A] time = 0.13, size = 20, normalized size = 0.80

$$-\frac{\log(x)}{6}+\frac{3\log(x-2)}{10}-\frac{2\log(x+3)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3+x**2-6*x),x)
[Out] -log(x)/6 + 3*log(x - 2)/10 - 2*log(x + 3)/15
```

3.438 $\int \frac{4x^2+x^3}{x+x^3}\,dx$

Optimal. Leaf size=14

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1593, 1584, 774, 635, 203, 260}

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + x^3)/(x + x^3),x]
```

```
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4x^2+x^3}{x+x^3}\,dx &= \int \frac{4x^2+x^3}{x\left(1+x^2\right)}\,dx \\
&= \int \frac{x(4+x)}{1+x^2}\,dx \\
&= x+\int \frac{-1+4x}{1+x^2}\,dx \\
&= x+4\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx \\
&= x-\tan^{-1}(x)+2\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$2\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + x^3)/(x + x^3),x]
[Out] x - ArcTan[x] + 2*Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*x^2 + x^3)/(x + x^3),x]
[Out] IntegrateAlgebraic[(4*x^2 + x^3)/(x + x^3), x]
```

fricas [A] time = 1.73, size = 14, normalized size = 1.00

$$x-\arctan(x)+2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

giac [A] time = 0.25, size = 14, normalized size = 1.00

$$x-\arctan(x)+2\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

maple [A] time = 0.00, size = 15, normalized size = 1.07

$$x-\arctan(x)+2\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2)/(x^3+x),x)
```

```
[Out] x-arctan(x)+2*ln(x^2+1)
```

maxima [A]   time = 1.56, size = 14, normalized size = 1.00

$$x - \arctan(x) + 2 \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
[Out] x - arctan(x) + 2*log(x^2 + 1)
```

mupad [B]   time = 2.22, size = 14, normalized size = 1.00

$$x + 2 \ln\left(x^2 + 1\right) - \mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 + x^3)/(x + x^3),x)
[Out] x + 2*log(x^2 + 1) - atan(x)
```

sympy [A]   time = 0.10, size = 12, normalized size = 0.86

$$x + 2\log\left(x^2 + 1\right) - \mathrm{atan}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2)/(x**3+x),x)
[Out] x + 2*log(x**2 + 1) - atan(x)
```

3.439 $\int \frac{x+2x^3}{(x^2+x^4)^3} dx$

Optimal. Leaf size=13

$$-\frac{1}{4(x^4+x^2)^2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$-\frac{1}{4(x^4+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/(4*(x^2 + x^4)^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+2x^3}{(x^2+x^4)^3} dx = -\frac{1}{4(x^2+x^4)^2}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.08

$$-\frac{1}{4x^4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] -1/4*1/(x^4*(1 + x^2)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+2x^3}{(x^2+x^4)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(x + 2*x^3)/(x^2 + x^4)^3, x]
```

fricas [A] time = 1.45, size = 16, normalized size = 1.23

$$-\frac{1}{4\left(x^8+2\,x^6+x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
[Out] -1/4/(x^8 + 2*x^6 + x^4)
```

giac [A] time = 0.37, size = 11, normalized size = 0.85

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
[Out] -1/4/(x^4 + x^2)^2
```

maple [B] time = 0.01, size = 30, normalized size = 2.31

$$\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{4\left(x^2+1\right)^2}-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+x)/(x^4+x^2)^3,x)
[Out] -1/4/(x^2+1)^2-1/2/(x^2+1)-1/4/x^4+1/2/x^2
```

maxima [A] time = 0.58, size = 11, normalized size = 0.85

$$-\frac{1}{4\left(x^4+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
[Out] -1/4/(x^4 + x^2)^2
```

mupad [B] time = 2.24, size = 20, normalized size = 1.54

$$-\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*x^3)/(x^2 + x^4)^3,x)
[Out] -1/(4*x^4 + 8*x^6 + 4*x^8)
```

sympy [A] time = 0.13, size = 17, normalized size = 1.31

$$-\frac{1}{4x^8+8x^6+4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+x)/(x**4+x**2)**3,x)
[Out] -1/(4*x**8 + 8*x**6 + 4*x**4)
```

3.440 $\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx$

Optimal. Leaf size=26

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Rubi [A] time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1593, 1584, 43}

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx &= \int \frac{x^2(a+bx)}{cx^2+dx^3}\,dx\\ &= \int \frac{a+bx}{c+dx}\,dx\\ &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx\\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
[Out] IntegrateAlgebraic[(a*x^2 + b*x^3)/(c*x^2 + d*x^3), x]
```

fricas [A] time = 1.51, size = 25, normalized size = 0.96

$$\frac{bdx - (bc - ad)\log\left(dx + c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

giac [A] time = 0.28, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad)\log\left(|dx + c|\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

maple [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a\ln\left(dx + c\right)}{d} - \frac{bc\ln\left(dx + c\right)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

maxima [A] time = 0.72, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad)\log\left(dx + c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

mupad [B] time = 2.23, size = 25, normalized size = 0.96

$$\frac{\ln\left(c + d\,x\right)\,\left(a\,d - b\,c\right)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

sympy [**A**] time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

3.441 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal. Leaf size=6

$$\log(2-x)$$

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1586, 31}

$$\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/(-2*x - x^2 + x^3),x]
[Out] Log[2 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{-2x-x^2+x^3} \, dx &= \int \frac{1}{-2+x} \, dx \\ &= \log(2-x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 4, normalized size = 0.67

$$\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
[Out] Log[-2 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x + x^2)/(-2*x - x^2 + x^3),x]
[Out] IntegrateAlgebraic[(x + x^2)/(-2*x - x^2 + x^3), x]
```

fricas [A] time = 1.49, size = 4, normalized size = 0.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
```

```
[Out] log(x - 2)
```

giac [A] time = 0.28, size = 5, normalized size = 0.83

$$\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
```

```
[Out] log(abs(x - 2))
```

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/(x^3-x^2-2*x),x)
```

```
[Out] ln(x-2)
```

maxima [A] time = 0.57, size = 4, normalized size = 0.67

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
```

```
[Out] log(x - 2)
```

mupad [B] time = 0.02, size = 4, normalized size = 0.67

$$\ln(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^2)/(2*x + x^2 - x^3),x)
```

```
[Out] log(x - 2)
```

sympy [A] time = 0.06, size = 3, normalized size = 0.50

$$\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/(x**3-x**2-2*x),x)
```

```
[Out] log(x - 2)
```

3.442 $\int \frac{1-5x^2}{x^3(1+x^2)}\,dx$

Optimal. Leaf size=20

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 77}

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

```
[Out] -1/(2*x^2) - 6*Log[x] + 3*Log[1 + x^2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-5x}{x^2(1+x)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{6}{x}+\frac{6}{1+x}\right)dx,x,x^2\right)\\ &= -\frac{1}{2x^2}-6\log(x)+3\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{1}{2x^2}+3\log\left(x^2+1\right)-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
```

```
[Out] -1/2*1/x^2 - 6*Log[x] + 3*Log[1 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1-5x^2}{x^3\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 - 5*x^2)/(x^3*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(1 - 5*x^2)/(x^3*(1 + x^2)), x]
```

fricas [A] time = 1.65, size = 25, normalized size = 1.25

$$\frac{6\,x^2 \log\left(x^2+1\right) - 12\,x^2 \log(x) - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="fricas")
[Out] 1/2*(6*x^2*log(x^2 + 1) - 12*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.38, size = 27, normalized size = 1.35

$$\frac{6\,x^2-1}{2\,x^2} + 3\,\log\left(x^2+1\right) - 3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="giac")
[Out] 1/2*(6*x^2 - 1)/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$-6\ln(x) + 3\ln\left(x^2+1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5*x^2+1)/x^3/(x^2+1),x)
[Out] -1/2/x^2-6*ln(x)+3*ln(x^2+1)
```

maxima [A] time = 0.72, size = 20, normalized size = 1.00

$$-\frac{1}{2\,x^2} + 3\,\log\left(x^2+1\right) - 3\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x^2+1)/x^3/(x^2+1),x, algorithm="maxima")
[Out] -1/2/x^2 + 3*log(x^2 + 1) - 3*log(x^2)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.90

$$3\,\ln\left(x^2+1\right) - 6\,\ln(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 1)/(x^3*(x^2 + 1)),x)
[Out] 3*log(x^2 + 1) - 6*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.11, size = 19, normalized size = 0.95

$$-6\log\left(x\right) + 3\log\left(x^2+1\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5*x**2+1)/x**3/(x**2+1),x)
[Out] -6*log(x) + 3*log(x**2 + 1) - 1/(2*x**2)
```

3.443 $\int \frac{2x}{(-1+x)(5+x^2)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 801, 635, 203, 260}

$$-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log(1-x)+\frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x)/((-1 + x)*(5 + x^2)),x]
```

```
[Out] (Sqrt[5]*ArcTan[x/Sqrt[5]])/3 + Log[1 - x]/3 - Log[5 + x^2]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx &= 2\int \frac{x}{(-1+x)\left(5+x^2\right)}\,dx\\
&= 2\int \left(\frac{1}{6(-1+x)}+\frac{5-x}{6\left(5+x^2\right)}\right)dx\\
&= \frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{5-x}{5+x^2}\,dx\\
&= \frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{x}{5+x^2}\,dx+\frac{5}{3}\int \frac{1}{5+x^2}\,dx\\
&= \frac{1}{3}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(5+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.05

$$
2\left(-\frac{1}{12}\log\left(x^2+5\right)+\frac{1}{6}\log(1-x)+\frac{1}{6}\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x)/((-1 + x)*(5 + x^2)),x]
[Out] 2*((Sqrt[5]*ArcTan[x/Sqrt[5]])/6 + Log[1 - x]/6 - Log[5 + x^2]/12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{2x}{(-1+x)\left(5+x^2\right)}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x)/((-1 + x)*(5 + x^2)),x]
[Out] IntegrateAlgebraic[(2*x)/((-1 + x)*(5 + x^2)), x]
```

fricas [A] time = 1.27, size = 27, normalized size = 0.71

$$
\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="fricas")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

giac [A] time = 0.39, size = 28, normalized size = 0.74

$$
\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{1}{6}\log\left(x^2+5\right)+\frac{1}{3}\log\left(|x-1|\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="giac")
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(abs(x - 1))
```

maple [A] time = 0.00, size = 28, normalized size = 0.74

$$
\frac{\sqrt{5}\arctan\left(\frac{\sqrt{5}\,x}{5}\right)}{3}+\frac{\ln\left(x-1\right)}{3}-\frac{\ln\left(x^2+5\right)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x/(x-1)/(x^2+5),x)
```

```
[Out] 1/3*ln(x-1)-1/6*ln(x^2+5)+1/3*5^(1/2)*arctan(1/5*5^(1/2)*x)
```

maxima [A] time = 1.97, size = 27, normalized size = 0.71

$$\frac{1}{3}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - \frac{1}{6}\log\left(x^2+5\right) + \frac{1}{3}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x^2+5),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/6*log(x^2 + 5) + 1/3*log(x - 1)
```

mupad [B] time = 0.16, size = 44, normalized size = 1.16

$$\frac{\ln\left(x-1\right)}{3} - \ln\left(x-\sqrt{5}\,1\mathrm{i}\right)\left(\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right) + \ln\left(x+\sqrt{5}\,1\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{5}\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x)/((x^2 + 5)*(x - 1)),x)
```

```
[Out] log(x - 1)/3 - log(x - 5^(1/2)*1i)*((5^(1/2)*1i)/6 + 1/6) + log(x + 5^(1/2)
*1i)*((5^(1/2)*1i)/6 - 1/6)
```

sympy [A] time = 0.14, size = 31, normalized size = 0.82

$$\frac{\log\left(x-1\right)}{3} - \frac{\log\left(x^2+5\right)}{6} + \frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x/(-1+x)/(x**2+5),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + 5)/6 + sqrt(5)*atan(sqrt(5)*x/5)/3
```

3.444 $\int \frac{2+x^2}{2+x} dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - 2x + 6\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2)/(2 + x),x]
[Out] -2*x + x^2/2 + 6*Log[2 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2}{2+x}\,dx &= \int \left(-2+x+\frac{6}{2+x}\right)dx\\ &= -2x+\frac{x^2}{2}+6\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{x^2}{2} - 2x + 6\log(x+2) - 6$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2)/(2 + x),x]
[Out] -6 - 2*x + x^2/2 + 6*Log[2 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+x^2}{2+x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x^2)/(2 + x),x]
[Out] IntegrateAlgebraic[(2 + x^2)/(2 + x), x]
```

fricas [A] time = 1.16, size = 15, normalized size = 0.88

$$\frac{1}{2}x^2 - 2x + 6\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

giac [A] time = 0.37, size = 16, normalized size = 0.94

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(|x + 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(abs(x + 2))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - 2x + 6\ln\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2)/(x+2),x)
```

```
[Out] -2*x+1/2*x^2+6*ln(x+2)
```

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 - 2\,x + 6\,\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2)/(2+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 2*x + 6*log(x + 2)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$6\,\ln\left(x+2\right) - 2\,x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 2)/(x + 2),x)
```

```
[Out] 6*log(x + 2) - 2*x + x^2/2
```

sympy [A] time = 0.07, size = 14, normalized size = 0.82

$$\frac{x^2}{2} - 2x + 6\log\left(x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)/(2+x),x)
```

```
[Out] x**2/2 - 2*x + 6*log(x + 2)
```

3.445 $\int \frac{1}{(-3+x)(4+x^2)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log(3-x)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-3 + x)*(4 + x^2)),x]
```

```
[Out] (-3*ArcTan[x/2])/26 + Log[3 - x]/13 - Log[4 + x^2]/26
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx &= \frac{1}{13}\int \frac{1}{-3+x}\,dx+\frac{1}{13}\int \frac{-3-x}{4+x^2}\,dx\\ &= \frac{1}{13}\log(3-x)-\frac{1}{13}\int \frac{x}{4+x^2}\,dx-\frac{3}{13}\int \frac{1}{4+x^2}\,dx\\ &= -\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{13}\log(3-x)-\frac{1}{26}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.16

$$-\frac{1}{26}\log\left((x-3)^2+6(x-3)+13\right)+\frac{1}{13}\log(x-3)-\frac{3}{26}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-3 + x)*(4 + x^2)),x]
[Out] (-3*ArcTan[x/2])/26 - Log[13 + 6*(-3 + x) + (-3 + x)^2]/26 + Log[-3 + x]/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3+x)\left(4+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((-3 + x)*(4 + x^2)),x]
[Out] IntegrateAlgebraic[1/((-3 + x)*(4 + x^2)), x]
```

fricas [A] time = 1.46, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="fricas")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

giac [A] time = 0.27, size = 22, normalized size = 0.71

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="giac")
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(abs(x - 3))
```

maple [A] time = 0.00, size = 22, normalized size = 0.71

$$-\frac{3\arctan\left(\frac{x}{2}\right)}{26}+\frac{\ln\left(x-3\right)}{13}-\frac{\ln\left(x^2+4\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-3)/(x^2+4),x)
[Out] -1/26*ln(x^2+4)-3/26*arctan(1/2*x)+1/13*ln(x-3)
```

maxima [A] time = 0.99, size = 21, normalized size = 0.68

$$-\frac{3}{26}\arctan\left(\frac{1}{2}x\right)-\frac{1}{26}\log\left(x^2+4\right)+\frac{1}{13}\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -3/26*arctan(1/2*x) - 1/26*log(x^2 + 4) + 1/13*log(x - 3)
```

mupad [**B**] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln(x-3)}{13}+\ln(x-2\mathrm{i})\left(-\frac{1}{26}+\frac{3}{52}\mathrm{i}\right)+\ln(x+2\mathrm{i})\left(-\frac{1}{26}-\frac{3}{52}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 4)*(x - 3)),x)
[Out] log(x - 3)/13 - log(x - 2i)*(1/26 - 3i/52) - log(x + 2i)*(1/26 + 3i/52)
```

sympy [**A**] time = 0.14, size = 22, normalized size = 0.71

$$\frac{\log(x-3)}{13}-\frac{\log\left(x^2+4\right)}{26}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)/(x**2+4),x)
[Out] log(x - 3)/13 - log(x**2 + 4)/26 - 3*atan(x/2)/26
```

3.446 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal. Leaf size=19

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 72}

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{-2+3x}{x(5+2x)}\,dx,x,x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{5x}+\frac{19}{5(5+2x)}\right)dx,x,x^6\right)\\ &= -\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

```
[Out] (-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+3x^6}{x\left(5+2x^6\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
[Out] IntegrateAlgebraic[(-2 + 3*x^6)/(x*(5 + 2*x^6)), x]
```

fricas [A] time = 1.30, size = 15, normalized size = 0.79

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{2}{5}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
[Out] 19/60*log(2*x^6 + 5) - 2/5*log(x)
```

giac [A] time = 0.41, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6-2)/x/(2*x^6+5),x)
[Out] -2/5*ln(x)+19/60*ln(2*x^6+5)
```

maxima [A] time = 0.47, size = 17, normalized size = 0.89

$$\frac{19}{60}\log\left(2x^6+5\right)-\frac{1}{15}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
[Out] 19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

mupad [B] time = 0.09, size = 13, normalized size = 0.68

$$\frac{19\ln\left(x^6+\frac{5}{2}\right)}{60}-\frac{2\ln(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
[Out] (19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

sympy [A] time = 0.11, size = 17, normalized size = 0.89

$$-\frac{2\log(x)}{5}+\frac{19\log\left(2x^6+5\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**6-2)/x/(2*x**6+5),x)
```

```
[Out] -2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

3.447 $\int \frac{3+2x}{(-2+x)(5+x)}\,dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[2 - x] + Log[5 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)}\,dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right)dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] Log[-2 + x] + Log[5 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3+2x}{(-2+x)(5+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3 + 2*x)/((-2 + x)*(5 + x)),x]
[Out] IntegrateAlgebraic[(3 + 2*x)/((-2 + x)*(5 + x)), x]
```

fricas [A] time = 1.33, size = 9, normalized size = 0.82

$$\log\left(x^2+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
[Out] log(x^2 + 3*x - 10)
```

giac [A] time = 0.29, size = 11, normalized size = 1.00

$$\log(|x+5|) + \log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.82

$$\ln((x-2)(x+5))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+3)/(x-2)/(x+5),x)
[Out] ln((x-2)*(x+5))
```

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\log(x+5) + \log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
[Out] log(x + 5) + log(x - 2)
```

mupad [B] time = 2.22, size = 9, normalized size = 0.82

$$\ln\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3)/((x - 2)*(x + 5)),x)
[Out] log(3*x + x^2 - 10)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$\log\left(x^2 + 3x - 10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
[Out] log(x**2 + 3*x - 10)
```

3.448 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1122, 1166, 203}

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{4+5x^2+x^4}\,dx&=x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\&=x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx\\&=x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{4+5x^2+x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(4 + 5*x^2 + x^4),x]
[Out] IntegrateAlgebraic[x^4/(4 + 5*x^2 + x^4), x]
```

fricas [A] time = 1.48, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

giac [A] time = 0.37, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

maple [A] time = 0.01, size = 13, normalized size = 0.72

$$x + \frac{\arctan(x)}{3} - \frac{8\arctan\left(\frac{x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x)
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

maxima [A] time = 1.01, size = 12, normalized size = 0.67

$$x - \frac{8}{3}\arctan\left(\frac{1}{2}x\right) + \frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

mupad [B] time = 2.22, size = 12, normalized size = 0.67

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(5*x^2 + x^4 + 4),x)
[Out] x - (8*atan(x/2))/3 + atan(x)/3
```

sympy [A] time = 0.14, size = 14, normalized size = 0.78

$$x - \frac{8\,\mathrm{atan}\left(\frac{x}{2}\right)}{3} + \frac{\mathrm{atan}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

3.449 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3} dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {88}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right)dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((1 + x)*(2 + x)^2*(3 + x)^3), x]
```

fricas [B] time = 1.32, size = 83, normalized size = 1.80

$$\frac{18\,x^2-17\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+3\right)+16\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+2\right)+\left(x^3+8\,x^2+21\,x+18\right)\log\left(x+1\right)+100\,x+136}{8\left(x^3+8\,x^2+21\,x+18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

giac [A] time = 0.36, size = 52, normalized size = 1.13

$$\frac{1}{x+2}-\frac{\frac{7}{x+2}+6}{4\left(\frac{1}{x+2}+1\right)^2}+\frac{1}{8}\log\left(\left|-\frac{1}{x+2}+1\right|\right)-\frac{17}{8}\log\left(\left|-\frac{1}{x+2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\ln\left(x+1\right)}{8}+2\ln\left(x+2\right)-\frac{17\ln\left(x+3\right)}{8}+\frac{1}{x+2}+\frac{1}{4\left(x+3\right)^2}+\frac{5}{4\left(x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x+2)^2/(x+3)^3,x)
```

```
[Out] 1/(x+2)+1/4/(x+3)^2+5/4/(x+3)+1/8*ln(x+1)+2*ln(x+2)-17/8*ln(x+3)
```

maxima [A] time = 0.45, size = 46, normalized size = 1.00

$$\frac{9\,x^2+50\,x+68}{4\left(x^3+8\,x^2+21\,x+18\right)}-\frac{17}{8}\log\left(x+3\right)+2\log\left(x+2\right)+\frac{1}{8}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

mupad [B] time = 0.04, size = 45, normalized size = 0.98

$$\frac{\ln\left(x+1\right)}{8}+2\ln\left(x+2\right)-\frac{17\ln\left(x+3\right)}{8}+\frac{\frac{9\,x^2}{4}+\frac{25\,x}{2}+17}{x^3+8\,x^2+21\,x+18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)*(x + 2)^2*(x + 3)^3),x)
```

```
[Out] log(x + 1)/8 + 2*log(x + 2) - (17*log(x + 3))/8 + ((25*x)/2 + (9*x^2)/4 + 1
7)/(21*x + 8*x^2 + x^3 + 18)
```

sympy [A] time = 0.19, size = 46, normalized size = 1.00

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

3.450 $\int \frac{x}{-1+x^2} dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {260}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
[Out] Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
[Out] Log[-1 + x^2]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{-1+x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(-1 + x^2),x]
[Out] IntegrateAlgebraic[x/(-1 + x^2), x]
```

fricas [A] time = 1.39, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1)
```

giac [A] time = 0.27, size = 9, normalized size = 0.75

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.00, size = 14, normalized size = 1.17

$$\frac{\ln(x-1)}{2}+\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x)
[Out] 1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 0.44, size = 8, normalized size = 0.67

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1)
```

mupad [B] time = 0.04, size = 8, normalized size = 0.67

$$\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2 - 1),x)
[Out] log(x^2 - 1)/2
```

sympy [A] time = 0.08, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
[Out] log(x**2 - 1)/2
```

3.451 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 207}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)^2} \, dx &= \frac{x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx\\ &= \frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]
```

```
[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-1+x^2\right)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x^2)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(-1 + x^2)^(-2), x]
```

fricas [B] time = 1.29, size = 34, normalized size = 1.62

$$\frac{\left(x^{2}-1\right)\log\left(x+1\right)-\left(x^{2}-1\right)\log\left(x-1\right)-2\,x}{4\left(x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

giac [A] time = 0.27, size = 25, normalized size = 1.19

$$-\frac{x}{2\left(x^{2}-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

maple [A] time = 0.01, size = 28, normalized size = 1.33

$$-\frac{\ln\left(x-1\right)}{4}+\frac{\ln\left(x+1\right)}{4}-\frac{1}{4\left(x-1\right)}-\frac{1}{4\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x)
```

```
[Out] -1/4/(x-1)-1/4*ln(x-1)-1/4/(x+1)+1/4*ln(x+1)
```

maxima [A] time = 0.45, size = 23, normalized size = 1.10

$$-\frac{x}{2\left(x^{2}-1\right)}+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

mupad [B] time = 2.22, size = 17, normalized size = 0.81

$$\frac{\mathrm{atanh}(x)}{2}-\frac{x}{2\left(x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - 1)^2,x)
```

```
[Out] atanh(x)/2 - x/(2*(x^2 - 1))
```

sympy [A] time = 0.11, size = 20, normalized size = 0.95

$$-\frac{x}{2x^{2}-2}-\frac{\log\left(x-1\right)}{4}+\frac{\log\left(x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)
```

```
[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

3.452 $\int \frac{x^2}{(1+x^2)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x^2)^2} \, dx &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= -\frac{x}{2(1+x^2)} + \frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
```

```
[Out] -1/2*x/(1 + x^2) + ArcTan[x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(1+x^2)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + x^2)^2,x]
[Out] IntegrateAlgebraic[x^2/(1 + x^2)^2, x]
```

fricas [A] time = 1.05, size = 21, normalized size = 1.11

$$\frac{(x^2+1)\arctan(x) - x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

giac [A] time = 0.29, size = 15, normalized size = 0.79

$$-\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.84

$$-\frac{x}{2\,(x^2+1)} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x)
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)
```

maxima [A] time = 1.00, size = 15, normalized size = 0.79

$$-\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.89

$$\frac{\operatorname{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + 1)^2,x)
[Out] atan(x)/2 - x/(2*(x^2 + 1))
```

sympy [**A**] time = 0.10, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

3.453 $\int \frac{1}{2+3x} \, dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+3x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 3*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 + 3*x)^(-1), x]
```

fricas [A] time = 1.39, size = 8, normalized size = 0.80

$$\frac{1}{3}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

giac [A] time = 0.38, size = 9, normalized size = 0.90

$$\frac{1}{3}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
[Out] 1/3*log(abs(3*x + 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\frac{\ln\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x+2),x)
[Out] 1/3*ln(3*x+2)
```

maxima [A] time = 0.45, size = 8, normalized size = 0.80

$$\frac{1}{3}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
[Out] 1/3*log(3*x + 2)
```

mupad [B] time = 0.07, size = 6, normalized size = 0.60

$$\frac{\ln\left(x+\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x + 2),x)
[Out] log(x + 2/3)/3
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log\left(3x+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
[Out] log(3*x + 2)/3
```

3.454 $\int \frac{1}{a^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2+x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + x^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^2 + x^2)^(-1), x]
```

fricas [A] time = 1.35, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="fricas")

[Out] arctan(x/a)/a

giac [A] time = 0.26, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="giac")

[Out] arctan(x/a)/a

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2+x^2),x)

[Out] arctan(x/a)/a

maxima [A] time = 0.99, size = 10, normalized size = 1.00

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a^2+x^2),x, algorithm="maxima")

[Out] arctan(x/a)/a

mupad [B] time = 2.25, size = 10, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a^2 + x^2),x)

[Out] atan(x/a)/a

sympy [C] time = 0.11, size = 20, normalized size = 2.00

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a**2+x**2),x)

[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a

3.455 $\int \frac{1}{a+bx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2}\,dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^2)^(-1),x]
[Out] IntegrateAlgebraic[(a + b*x^2)^(-1), x]
```

fricas [A] time = 1.20, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.38, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.00, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 0.98, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 2.24, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.13, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.456 $\int \frac{1}{2-x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-x+x^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2-x+x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - x + x^2)^(-1),x]
[Out] IntegrateAlgebraic[(2 - x + x^2)^(-1), x]
```

fricas [A] time = 1.44, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

giac [A] time = 0.38, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

maple [A] time = 0.00, size = 17, normalized size = 0.89

$$\frac{2\sqrt{7}\,\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x)
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

maxima [A] time = 0.95, size = 16, normalized size = 0.84

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

mupad [B] time = 0.03, size = 16, normalized size = 0.84

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{\sqrt{7}\,(2x-1)}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x + 2),x)
[Out] (2*7^(1/2)*atan((7^(1/2)*(2*x - 1))/7))/7
```

sympy [A] time = 0.11, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}\,x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

3.457 $\int x^2 (4-x^2)^2 \, dx$

Optimal. Leaf size=22

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 (4-x^2)^2 \, dx &= \int (16x^2-8x^4+x^6) \, dx \\ &= \frac{16x^3}{3}-\frac{8x^5}{5}+\frac{x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^7}{7}-\frac{8x^5}{5}+\frac{16x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(4 - x^2)^2,x]
```

```
[Out] (16*x^3)/3 - (8*x^5)/5 + x^7/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (4-x^2)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(4 - x^2)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(4 - x^2)^2, x]
```

fricas [A] time = 1.13, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7-\frac{8}{5}x^5+\frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

giac [A] time = 0.31, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+4)^2,x)
[Out] 16/3*x^3-8/5*x^5+1/7*x^7
```

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
[Out] 1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

mupad [B] time = 0.02, size = 17, normalized size = 0.77

$$\frac{x^3\,\left(15\,x^4-168\,x^2+560\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 - 4)^2,x)
[Out] (x^3*(15*x^4 - 168*x^2 + 560))/105
```

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$\frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+4)**2,x)
[Out] x**7/7 - 8*x**5/5 + 16*x**3/3
```

3.458 $\int x \left(1-x^3\right)^2 \, dx$

Optimal. Leaf size=22

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {270}

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x \left(1-x^3\right)^2 \, dx &= \int \left(x-2x^4+x^7\right) \, dx\\ &= \frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^8}{8}-\frac{2x^5}{5}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^3)^2,x]
```

```
[Out] x^2/2 - (2*x^5)/5 + x^8/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(1-x^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(1 - x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(1 - x^3)^2, x]
```

fricas [A] time = 1.12, size = 16, normalized size = 0.73

$$\frac{1}{8}x^8-\frac{2}{5}x^5+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$\frac{1}{8}\,x^{8} - \frac{2}{5}\,x^{5} + \frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{1}{8}x^{8} - \frac{2}{5}x^{5} + \frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)^2,x)
```

```
[Out] 1/2*x^2-2/5*x^5+1/8*x^8
```

maxima [A] time = 0.46, size = 16, normalized size = 0.73

$$\frac{1}{8}\,x^{8} - \frac{2}{5}\,x^{5} + \frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{x^{2}\,\left(5\,x^{6}-16\,x^{3}+20\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^3 - 1)^2,x)
```

```
[Out] (x^2*(5*x^6 - 16*x^3 + 20))/40
```

sympy [A] time = 0.06, size = 15, normalized size = 0.68

$$\frac{x^{8}}{8} - \frac{2x^{5}}{5} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)**2,x)
```

```
[Out] x**8/8 - 2*x**5/5 + x**2/2
```

3.459 $\int \frac{-4+5x^2+x^3}{x^2} dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {14}

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+5x^2+x^3}{x^2} dx &= \int \left(5 - \frac{4}{x^2} + x\right) dx \\ &= \frac{4}{x} + 5x + \frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] 4/x + 5*x + x^2/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-4+5x^2+x^3}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-4 + 5*x^2 + x^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[(-4 + 5*x^2 + x^3)/x^2, x]
```

fricas [A] time = 1.30, size = 15, normalized size = 0.94

$$\frac{x^3 + 10\,x^2 + 8}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 10*x^2 + 8)/x
```

giac [A] time = 0.31, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+5*x^2-4)/x^2,x)
```

```
[Out] 4/x+5*x+1/2*x^2
```

maxima [A] time = 0.46, size = 14, normalized size = 0.88

$$\frac{1}{2}x^2 + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 5*x + 4/x
```

mupad [B] time = 0.03, size = 15, normalized size = 0.94

$$\frac{x^3 + 10x^2 + 8}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + x^3 - 4)/x^2,x)
```

```
[Out] (10*x^2 + x^3 + 8)/(2*x)
```

sympy [A] time = 0.07, size = 10, normalized size = 0.62

$$\frac{x^2}{2} + 5x + \frac{4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+5*x**2-4)/x**2,x)
```

```
[Out] x**2/2 + 5*x + 4/x
```

3.460 $\int \frac{-1+x}{3-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {634, 618, 204, 628}

$$\frac{1}{6}\log\left(3x^2-4x+3\right)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(3 - 4*x + 3*x^2),x]
```

```
[Out] ArcTan[(2 - 3*x)/Sqrt[5]]/(3*Sqrt[5]) + Log[3 - 4*x + 3*x^2]/6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{3-4x+3x^2} \, dx &= \frac{1}{6}\int \frac{-4+6x}{3-4x+3x^2} \, dx-\frac{1}{3}\int \frac{1}{3-4x+3x^2} \, dx\\ &= \frac{1}{6}\log\left(3-4x+3x^2\right)+\frac{2}{3}\text{Subst}\left(\int \frac{1}{-20-x^2} \, dx,x,-4+6x\right)\\ &= \frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{5}}\right)}{3\sqrt{5}}+\frac{1}{6}\log\left(3-4x+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{6}\log\left(3x^2-4x+3\right)-\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(3 - 4*x + 3*x^2),x]
[Out] -1/3*ArcTan[(-2 + 3*x)/Sqrt[5]]/Sqrt[5] + Log[3 - 4*x + 3*x^2]/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+x}{3-4x+3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + x)/(3 - 4*x + 3*x^2),x]
[Out] IntegrateAlgebraic[(-1 + x)/(3 - 4*x + 3*x^2), x]
```

fricas [A] time = 1.26, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="fricas")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

giac [A] time = 0.27, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="giac")
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$-\frac{\sqrt{5}\,\arctan\left(\frac{(6x-4)\sqrt{5}}{10}\right)}{15}+\frac{\ln\left(3x^2-4x+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)/(3*x^2-4*x+3),x)
[Out] 1/6*ln(3*x^2-4*x+3)-1/15*5^(1/2)*arctan(1/10*(6*x-4)*5^(1/2))
```

maxima [A] time = 0.98, size = 30, normalized size = 0.81

$$-\frac{1}{15}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\,(3\,x-2)\right)+\frac{1}{6}\log\left(3\,x^2-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x^2-4*x+3),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(5)*arctan(1/5*sqrt(5)*(3*x - 2)) + 1/6*log(3*x^2 - 4*x + 3)
```

mupad [B] time = 2.22, size = 30, normalized size = 0.81

$$\frac{\ln\left(x^2-\frac{4\,x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\,\sqrt{5}\,x}{5}-\frac{2\,\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(3*x^2 - 4*x + 3),x)
```

```
[Out] log(x^2 - (4*x)/3 + 1)/6 - (5^(1/2)*atan((3*5^(1/2)*x)/5 - (2*5^(1/2))/5))/
15
```

sympy [A] time = 0.12, size = 39, normalized size = 1.05

$$\frac{\log\left(x^2-\frac{4x}{3}+1\right)}{6}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{3\sqrt{5}x}{5}-\frac{2\sqrt{5}}{5}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(3*x**2-4*x+3),x)
```

```
[Out] log(x**2 - 4*x/3 + 1)/6 - sqrt(5)*atan(3*sqrt(5)*x/5 - 2*sqrt(5)/5)/15
```

3.461 $\int \left(2+x^3\right)^2 \, dx$

Optimal. Leaf size=14

$$\frac{x^7}{7}+x^4+4x$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {194}

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3)^2,x]
```

```
[Out] 4*x + x^4 + x^7/7
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(2+x^3\right)^2 \, dx &= \int \left(4+4x^3+x^6\right) dx \\ &= 4x+x^4+\frac{x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{x^7}{7}+x^4+4x$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3)^2,x]
```

```
[Out] 4*x + x^4 + x^7/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2+x^3\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(2 + x^3)^2, x]
```

fricas [A] time = 0.97, size = 12, normalized size = 0.86

$$\frac{1}{7}x^7+x^4+4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="fricas")
[Out] 1/7*x^7 + x^4 + 4*x
```

giac [A] time = 0.36, size = 12, normalized size = 0.86

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="giac")
[Out] 1/7*x^7 + x^4 + 4*x
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{7}x^7 + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2)^2,x)
[Out] 4*x+x^4+1/7*x^7
```

maxima [A] time = 0.47, size = 12, normalized size = 0.86

$$\frac{1}{7}\,x^7 + x^4 + 4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2)^2,x, algorithm="maxima")
[Out] 1/7*x^7 + x^4 + 4*x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.93

$$\frac{x\,\left(x^6 + 7\,x^3 + 28\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 2)^2,x)
[Out] (x*(7*x^3 + x^6 + 28))/7
```

sympy [A] time = 0.05, size = 10, normalized size = 0.71

$$\frac{x^7}{7} + x^4 + 4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2)**2,x)
[Out] x**7/7 + x**4 + 4*x
```

3.462 $\int \frac{-4+x^2}{2+x} \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}-2x$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {627}

$$\frac{x^2}{2}-2x$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\begin{aligned} \int \frac{-4+x^2}{2+x} \, dx &= \int (-2+x) \, dx \\ &= -2x+\frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2}-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x^2)/(2 + x),x]
```

```
[Out] -2*x + x^2/2
```

IntegrateAlgebraic [A] time = 0.01, size = 9, normalized size = 0.82

$$\frac{1}{2}(x-2)^2$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-4 + x^2)/(2 + x),x]
```

```
[Out] (-2 + x)^2/2
```

fricas [A] time = 1.28, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="fricas")
[Out] 1/2*x^2 - 2*x
```

giac [A] time = 0.29, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="giac")
[Out] 1/2*x^2 - 2*x
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2 - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-4)/(x+2),x)
[Out] -2*x+1/2*x^2
```

maxima [A] time = 0.45, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2 - 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-4)/(2+x),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*x
```

mupad [B] time = 0.02, size = 6, normalized size = 0.55

$$\frac{x\,\left(x-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 4)/(x + 2),x)
[Out] (x*(x - 4))/2
```

sympy [A] time = 0.06, size = 7, normalized size = 0.64

$$\frac{x^2}{2} - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-4)/(2+x),x)
[Out] x**2/2 - 2*x
```

3.463 $\int \frac{1}{(2+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)*(1 + x^2)),x]
```

```
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)\left(1+x^2\right)}\,dx &= \frac{1}{5}\int \frac{1}{2+x}\,dx+\frac{1}{5}\int \frac{2-x}{1+x^2}\,dx\\ &= \frac{1}{5}\log(2+x)-\frac{1}{5}\int \frac{x}{1+x^2}\,dx+\frac{2}{5}\int \frac{1}{1+x^2}\,dx\\ &= \frac{2}{5}\tan^{-1}(x)+\frac{1}{5}\log(2+x)-\frac{1}{10}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log(x+2)+\frac{2}{5}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)*(1 + x^2)),x]
[Out] (2*ArcTan[x])/5 + Log[2 + x]/5 - Log[1 + x^2]/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2+x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((2 + x)*(1 + x^2)),x]
[Out] IntegrateAlgebraic[1/((2 + x)*(1 + x^2)), x]
```

fricas [A] time = 1.44, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="fricas")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

giac [A] time = 0.37, size = 20, normalized size = 0.80

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="giac")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(abs(x + 2))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{2\arctan(x)}{5}+\frac{\ln\left(x+2\right)}{5}-\frac{\ln\left(x^2+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+2)/(x^2+1),x)
[Out] 2/5*arctan(x)+1/5*ln(x+2)-1/10*ln(x^2+1)
```

maxima [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2}{5}\arctan(x)-\frac{1}{10}\log\left(x^2+1\right)+\frac{1}{5}\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x^2+1),x, algorithm="maxima")
[Out] 2/5*arctan(x) - 1/10*log(x^2 + 1) + 1/5*log(x + 2)
```

mupad [**B**] time = 0.05, size = 25, normalized size = 1.00

$$\frac{\ln(x+2)}{5} + \ln(x-\mathrm{i})\left(-\frac{1}{10} - \frac{1}{5}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{10} + \frac{1}{5}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 2)),x)
```

```
[Out] log(x + 2)/5 - log(x - 1i)*(1/10 + 1i/5) - log(x + 1i)*(1/10 - 1i/5)
```

sympy [**A**] time = 0.14, size = 20, normalized size = 0.80

$$\frac{\log(x+2)}{5} - \frac{\log(x^2+1)}{10} + \frac{2\,\mathrm{atan}(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)/(x**2+1),x)
```

```
[Out] log(x + 2)/5 - log(x**2 + 1)/10 + 2*atan(x)/5
```

3.464 $\int \frac{1}{(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {706, 31, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{1+x}\,dx+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(1+x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((1 + x)*(1 + x^2)),x]
[Out] IntegrateAlgebraic[1/((1 + x)*(1 + x^2)), x]
```

fricas [A] time = 2.12, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

giac [A] time = 0.29, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2}+\frac{\ln\left(x+1\right)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)/(x^2+1),x)
[Out] 1/2*arctan(x)+1/2*ln(x+1)-1/4*ln(x^2+1)
```

maxima [A] time = 1.36, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

mupad [B] time = 2.23, size = 25, normalized size = 1.00

$$\frac{\ln(x+1)}{2} + \ln(x-\mathrm{i})\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \ln(x+1\mathrm{i})\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x + 1)),x)
[Out] log(x + 1)/2 - log(x - 1i)*(1/4 + 1i/4) - log(x + 1i)*(1/4 - 1i/4)
```

sympy [A] time = 0.13, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+1),x)
[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

3.465 $\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx$

Optimal. Leaf size=25

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {801, 635, 203, 260}

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{1+x}{2\left(1+x^2\right)}\right)dx\\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1+x}{1+x^2}\,dx\\
&= -\frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\log(1+x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + x^2)),x]
[Out] ArcTan[x]/2 - Log[1 + x]/2 + Log[1 + x^2]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((1 + x)*(1 + x^2)),x]
[Out] IntegrateAlgebraic[x/((1 + x)*(1 + x^2)), x]
```

fricas [A] time = 1.27, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="fricas")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

giac [A] time = 0.25, size = 20, normalized size = 0.80

$$\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="giac")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{\arctan(x)}{2}-\frac{\ln\left(x+1\right)}{2}+\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(x^2+1),x)
[Out] 1/2*arctan(x)-1/2*ln(x+1)+1/4*ln(x^2+1)
```

maxima [A] time = 1.43, size = 19, normalized size = 0.76

$$\frac{1}{2}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x^2+1),x, algorithm="maxima")
[Out] 1/2*arctan(x) + 1/4*log(x^2 + 1) - 1/2*log(x + 1)
```

mupad [B] time = 2.21, size = 25, normalized size = 1.00

$$-\frac{\ln\left(x+1\right)}{2}+\ln\left(x-\mathrm{i}\right)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)),x)
```

```
[Out] log(x - 1i)*(1/4 - 1i/4) - log(x + 1)/2 + log(x + 1i)*(1/4 + 1i/4)
```

sympy [**A**] time = 0.12, size = 19, normalized size = 0.76

$$-\frac{\log(x+1)}{2}+\frac{\log(x^2+1)}{4}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(x**2+1),x)
```

```
[Out] -log(x + 1)/2 + log(x**2 + 1)/4 + atan(x)/2
```

3.466 $\int \frac{2x+x^2}{(1+x)^2} dx$

Optimal. Leaf size=9

$$\frac{x^2}{x+1}$$

Rubi [A] time = 0.01, antiderivative size = 7, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {683}

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{2x+x^2}{(1+x)^2}\,dx &= \int \left(1-\frac{1}{(1+x)^2}\right)dx\\ &= x+\frac{1}{1+x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 0.78

$$x+\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] x + (1 + x)^(-1)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+x^2}{(1+x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2*x + x^2)/(1 + x)^2,x]
```

```
[Out] IntegrateAlgebraic[(2*x + x^2)/(1 + x)^2, x]
```

fricas [A] time = 1.18, size = 12, normalized size = 1.33

$$\frac{x^2+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (x^2 + x + 1)/(x + 1)
```

giac [A] time = 0.30, size = 8, normalized size = 0.89

$$x + \frac{1}{x+1} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] x + 1/(x + 1) + 1
```

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x+1)^2,x)
```

```
[Out] x+1/(x+1)
```

maxima [A] time = 0.95, size = 7, normalized size = 0.78

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] x + 1/(x + 1)
```

mupad [B] time = 0.02, size = 7, normalized size = 0.78

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2)/(x + 1)^2,x)
```

```
[Out] x + 1/(x + 1)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.56

$$x + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(1+x)**2,x)
```

```
[Out] x + 1/(x + 1)
```

3.467 $\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1166, 203}

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx &= -\left(3\int \frac{1}{1+2x^2}\,dx\right)+4\int \frac{1}{8+2x^2}\,dx\\ &= \tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\tan^{-1}\left(\frac{x}{2}\right)-\frac{3\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] ArcTan[x/2] - (3*ArcTan[Sqrt[2]*x])/Sqrt[2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-10+x^2}{4+9x^2+2x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-10 + x^2)/(4 + 9*x^2 + 2*x^4),x]
```

```
[Out] IntegrateAlgebraic[(-10 + x^2)/(4 + 9*x^2 + 2*x^4), x]
```

fricas [A] time = 1.19, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="fricas")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

giac [A] time = 0.29, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

maple [A] time = 0.01, size = 17, normalized size = 0.77

$$-\frac{3\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)}{2}+\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)/(2*x^4+9*x^2+4),x)
```

```
[Out] arctan(1/2*x)-3/2*2^(1/2)*arctan(2^(1/2)*x)
```

maxima [A] time = 1.58, size = 16, normalized size = 0.73

$$-\frac{3}{2}\sqrt{2}\arctan\left(\sqrt{2}x\right)+\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)/(2*x^4+9*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(sqrt(2)*x) + arctan(1/2*x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.73

$$\mathrm{atan}\left(\frac{x}{2}\right)-\frac{3\,\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 10)/(9*x^2 + 2*x^4 + 4),x)
```

```
[Out] atan(x/2) - (3*2^(1/2)*atan(2^(1/2)*x))/2
```

sympy [A] time = 0.15, size = 20, normalized size = 0.91

$$\mathrm{atan}\left(\frac{x}{2}\right)-\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)/(2*x**4+9*x**2+4),x)
```

```
[Out] atan(x/2) - 3*sqrt(2)*atan(sqrt(2)*x)/2
```

3.468 $\int \frac{31+5x}{11-4x+3x^2} \, dx$

Optimal. Leaf size=37

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {634, 618, 204, 628}

$$\frac{5}{6}\log\left(3x^2-4x+11\right)-\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Int[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
```

```
[Out] (-103*ArcTan[(2 - 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{31+5x}{11-4x+3x^2} \, dx &= \frac{5}{6}\int \frac{-4+6x}{11-4x+3x^2} \, dx+\frac{103}{3}\int \frac{1}{11-4x+3x^2} \, dx \\
&= \frac{5}{6}\log\left(11-4x+3x^2\right)-\frac{206}{3}\text{Subst}\left(\int \frac{1}{-116-x^2} \, dx,x,-4+6x\right) \\
&= -\frac{103\tan^{-1}\left(\frac{2-3x}{\sqrt{29}}\right)}{3\sqrt{29}}+\frac{5}{6}\log\left(11-4x+3x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{5}{6}\log\left(3x^2-4x+11\right)+\frac{103\tan^{-1}\left(\frac{3x-2}{\sqrt{29}}\right)}{3\sqrt{29}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
[Out] (103*ArcTan[(-2 + 3*x)/Sqrt[29]])/(3*Sqrt[29]) + (5*Log[11 - 4*x + 3*x^2])/
6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{31+5x}{11-4x+3x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(31 + 5*x)/(11 - 4*x + 3*x^2),x]
[Out] IntegrateAlgebraic[(31 + 5*x)/(11 - 4*x + 3*x^2), x]
```

fricas [A] time = 1.17, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="fricas")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

giac [A] time = 0.38, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="giac")
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

maple [A] time = 0.00, size = 31, normalized size = 0.84

$$\frac{103\sqrt{29}\,\arctan\left(\frac{(6x-4)\sqrt{29}}{58}\right)}{87}+\frac{5\ln\left(3x^2-4x+11\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((31+5*x)/(3*x^2-4*x+11),x)
[Out] 5/6*ln(3*x^2-4*x+11)+103/87*29^(1/2)*arctan(1/58*(6*x-4)*29^(1/2))
```

maxima [A] time = 1.39, size = 30, normalized size = 0.81

$$\frac{103}{87}\sqrt{29}\arctan\left(\frac{1}{29}\sqrt{29}\,(3x-2)\right)+\frac{5}{6}\log\left(3x^2-4x+11\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x^2-4*x+11),x, algorithm="maxima")
```

```
[Out] 103/87*sqrt(29)*arctan(1/29*sqrt(29)*(3*x - 2)) + 5/6*log(3*x^2 - 4*x + 11)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{5\,\ln\left(x^2-\frac{4\,x}{3}+\frac{11}{3}\right)}{6}+\frac{103\,\sqrt{29}\,\mathrm{atan}\left(\frac{3\,\sqrt{29}\,x}{29}-\frac{2\,\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 31)/(3*x^2 - 4*x + 11),x)
```

```
[Out] (5*log(x^2 - (4*x)/3 + 11/3))/6 + (103*29^(1/2)*atan((3*29^(1/2)*x)/29 - (2
*29^(1/2))/29))/87
```

sympy [A] time = 0.12, size = 44, normalized size = 1.19

$$\frac{5\log\left(x^2-\frac{4x}{3}+\frac{11}{3}\right)}{6}+\frac{103\sqrt{29}\,\operatorname{atan}\left(\frac{3\sqrt{29}\,x}{29}-\frac{2\sqrt{29}}{29}\right)}{87}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((31+5*x)/(3*x**2-4*x+11),x)
```

```
[Out] 5*log(x**2 - 4*x/3 + 11/3)/6 + 103*sqrt(29)*atan(3*sqrt(29)*x/29 - 2*sqrt(2
9)/29)/87
```

3.469 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal. Leaf size=15

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {14}

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2 + x^3)/x^4,x]
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^2+x^3}{x^4} \, dx &= \int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right) \, dx \\
&= \frac{2}{3x^3} - \frac{1}{x} + \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2 + x^3)/x^4,x]
[Out] 2/(3*x^3) - x^(-1) + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+x^2+x^3}{x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + x^2 + x^3)/x^4,x]
[Out] IntegrateAlgebraic[(-2 + x^2 + x^3)/x^4, x]
```

fricas [A] time = 1.34, size = 19, normalized size = 1.27

$$\frac{3 \, x^3 \log(x) - 3 \, x^2 + 2}{3 \, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
[Out] 1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

giac [A] time = 0.27, size = 16, normalized size = 1.07

$$-\frac{3\,x^2-2}{3\,x^3}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x)-\frac{1}{x}+\frac{2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-2)/x^4,x)
[Out] 2/3/x^3-1/x+ln(x)
```

maxima [A] time = 0.97, size = 15, normalized size = 1.00

$$-\frac{3\,x^2-2}{3\,x^3}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
[Out] -1/3*(3*x^2 - 2)/x^3 + log(x)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.87

$$\ln(x)-\frac{x^2-\frac{2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - 2)/x^4,x)
[Out] log(x) - (x^2 - 2/3)/x^3
```

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\log\left(x\right)+\frac{2-3x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-2)/x**4,x)
[Out] log(x) + (2 - 3*x**2)/(3*x**3)
```

3.470 $\int \frac{1+x+x^3}{x^2} \, dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {14}

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/x^2,x]
```

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+x^3}{x^2} \, dx &= \int \left(\frac{1}{x^2} + \frac{1}{x} + x\right) dx \\ &= -\frac{1}{x} + \frac{x^2}{2} + \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{x} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/x^2,x]
```

```
[Out] -x^(-1) + x^2/2 + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x+x^3}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x + x^3)/x^2,x]
```

```
[Out] IntegrateAlgebraic[(1 + x + x^3)/x^2, x]
```

fricas [A] time = 0.90, size = 15, normalized size = 1.00

$$\frac{x^3 + 2\,x\log(x) - 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + 2*x*log(x) - 2)/x
```

giac [A] time = 0.28, size = 14, normalized size = 0.93

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/x + log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{x^2}{2} + \ln(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/x^2,x)
```

```
[Out] -1/x+1/2*x^2+ln(x)
```

maxima [A] time = 0.95, size = 13, normalized size = 0.87

$$\frac{1}{2}x^2 - \frac{1}{x} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/x + log(x)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\ln(x) - \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^3 + 1)/x^2,x)
```

```
[Out] log(x) - 1/x + x^2/2
```

sympy [A] time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^2}{2} + \log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/x**2,x)
```

```
[Out] x**2/2 + log(x) - 1/x
```

3.471 $\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx$

Optimal. Leaf size=11

$$\log\left(x^2+2\right)-\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {446, 72}

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)dx,x,x^2\right)\\
&= -\log(x)+\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + x^2)/(x*(2 + x^2)),x]
[Out] IntegrateAlgebraic[(-2 + x^2)/(x*(2 + x^2)), x]
```

fricas [A] time = 1.30, size = 11, normalized size = 1.00

$$\log\left(x^2+2\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
[Out] log(x^2 + 2) - log(x)
```

giac [A] time = 0.29, size = 13, normalized size = 1.18

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)
[Out] -ln(x)+ln(x^2+2)
```

maxima [A] time = 0.69, size = 13, normalized size = 1.18

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 11, normalized size = 1.00

$$\ln\left(x^2+2\right)-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 2)/(x*(x^2 + 2)),x)
[Out] log(x^2 + 2) - log(x)
```

sympy [A] time = 0.09, size = 8, normalized size = 0.73

$$-\log(x)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
[Out] -log(x) + log(x**2 + 2)
```

3.472 $\int (-3+x)\left(-7+4x^2\right)\,dx$

Optimal. Leaf size=22

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {641}

$$-4x^3+\frac{1}{16}\left(7-4x^2\right)^2+21x$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - 4*x^3 + (7 - 4*x^2)^2/16
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (-3+x)\left(-7+4x^2\right)\,dx &= \frac{1}{16}\left(7-4x^2\right)^2-3\int \left(-7+4x^2\right)\,dx\\ &= 21x-4x^3+\frac{1}{16}\left(7-4x^2\right)^2\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.86

$$x^4-4x^3-\frac{7x^2}{2}+21x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-3+x)\left(-7+4x^2\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + x)*(-7 + 4*x^2),x]
```

```
[Out] IntegrateAlgebraic[(-3 + x)*(-7 + 4*x^2), x]
```

fricas [A] time = 0.52, size = 17, normalized size = 0.77

$$x^4-4x^3-\frac{7}{2}x^2+21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="fricas")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

giac [A] time = 0.36, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="giac")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

maple [A] time = 0.00, size = 18, normalized size = 0.82

$$x^4 - 4x^3 - \frac{7}{2}x^2 + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3)*(4*x^2-7),x)
[Out] x^4-4*x^3-7/2*x^2+21*x
```

maxima [A] time = 0.88, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7}{2}\,x^2 + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x^2-7),x, algorithm="maxima")
[Out] x^4 - 4*x^3 - 7/2*x^2 + 21*x
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$x^4 - 4\,x^3 - \frac{7\,x^2}{2} + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)*(x - 3),x)
[Out] 21*x - (7*x^2)/2 - 4*x^3 + x^4
```

sympy [A] time = 0.06, size = 17, normalized size = 0.77

$$x^4 - 4x^3 - \frac{7x^2}{2} + 21x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)*(4*x**2-7),x)
[Out] x**4 - 4*x**3 - 7*x**2/2 + 21*x
```

3.473 $\int(-2+7x)^3\,dx$

Optimal. Leaf size=11

$$\frac{1}{28}(2-7x)^4$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{1}{28}(2-7x)^4$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 7*x)^3,x]
```

```
[Out] (2 - 7*x)^4/28
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int(-2+7x)^3\,dx=\frac{1}{28}(2-7x)^4$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{28}(7x-2)^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 7*x)^3,x]
```

```
[Out] (-2 + 7*x)^4/28
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(-2+7x)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-2 + 7*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(-2 + 7*x)^3, x]
```

fricas [B] time = 1.46, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4-98x^3+42x^2-8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="fricas")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

giac [**A**] time = 0.22, size = 9, normalized size = 0.82

$$\frac{1}{28}(7x-2)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="giac")
```

```
[Out] 1/28*(7*x - 2)^4
```

maple [**A**] time = 0.00, size = 10, normalized size = 0.91

$$\frac{(7x-2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+7*x)^3,x)
```

```
[Out] 1/28*(-2+7*x)^4
```

maxima [**B**] time = 0.86, size = 19, normalized size = 1.73

$$\frac{343}{4}x^4-98x^3+42x^2-8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)^3,x, algorithm="maxima")
```

```
[Out] 343/4*x^4 - 98*x^3 + 42*x^2 - 8*x
```

mupad [**B**] time = 0.14, size = 9, normalized size = 0.82

$$\frac{(7x-2)^4}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x - 2)^3,x)
```

```
[Out] (7*x - 2)^4/28
```

sympy [**B**] time = 0.06, size = 19, normalized size = 1.73

$$\frac{343x^4}{4}-98x^3+42x^2-8x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+7*x)**3,x)
```

```
[Out] 343*x**4/4 - 98*x**3 + 42*x**2 - 8*x
```

3.474 $\int \frac{-7+4x^2}{3+2x}\,dx$

Optimal. Leaf size=13

$$x^2 - 3x + \log(2x+3)$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {697}

$$x^2 - 3x + \log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] -3*x + x^2 + Log[3 + 2*x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-7+4x^2}{3+2x}\,dx &= \int \left(-3+2x+\frac{2}{3+2x}\right)dx \\ &= -3x+x^2+\log(3+2x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.23

$$x^2 - 3x + \log(2x+3) - \frac{27}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] -27/4 - 3*x + x^2 + Log[3 + 2*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-7+4x^2}{3+2x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-7 + 4*x^2)/(3 + 2*x),x]
```

```
[Out] IntegrateAlgebraic[(-7 + 4*x^2)/(3 + 2*x), x]
```

fricas [A] time = 1.00, size = 13, normalized size = 1.00

$$x^2 - 3\,x + \log\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="fricas")
```

```
[Out] x^2 - 3*x + log(2*x + 3)
```

giac [A] time = 0.36, size = 14, normalized size = 1.08

$$x^2 - 3\,x + \log\left(|2\,x + 3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="giac")
[Out] x^2 - 3*x + log(abs(2*x + 3))
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$x^2 - 3x + \ln\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-7)/(2*x+3),x)
[Out] -3*x+x^2+ln(2*x+3)
```

maxima [A] time = 0.64, size = 13, normalized size = 1.00

$$x^2 - 3\,x + \log\left(2\,x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-7)/(3+2*x),x, algorithm="maxima")
[Out] x^2 - 3*x + log(2*x + 3)
```

mupad [B] time = 2.21, size = 11, normalized size = 0.85

$$\ln\left(x + \frac{3}{2}\right) - 3\,x + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 7)/(2*x + 3),x)
[Out] log(x + 3/2) - 3*x + x^2
```

sympy [A] time = 0.08, size = 12, normalized size = 0.92

$$x^2 - 3x + \log\left(2x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-7)/(3+2*x),x)
[Out] x**2 - 3*x + log(2*x + 3)
```

3.475 $\int \frac{1+x}{(-1+x)x^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {77}

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(-1+x)x^2} \, dx &= \int \left(\frac{2}{-1+x}-\frac{1}{x^2}-\frac{2}{x}\right) dx \\ &= \frac{1}{x}+2\log(1-x)-2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{1}{x}+2\log(1-x)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] x^(-1) + 2*Log[1 - x] - 2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x}{(-1+x)x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x)/((-1 + x)*x^2),x]
```

```
[Out] IntegrateAlgebraic[(1 + x)/((-1 + x)*x^2), x]
```

fricas [A] time = 1.32, size = 18, normalized size = 1.12

$$\frac{2\,x\log\left(x-1\right)-2\,x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*log(x - 1) - 2*x*log(x) + 1)/x
```

giac [A] time = 0.36, size = 16, normalized size = 1.00

$$\frac{1}{x} + 2\log\left(|x-1|\right) - 2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="giac")
```

```
[Out] 1/x + 2*log(abs(x - 1)) - 2*log(abs(x))
```

maple [A] time = 0.01, size = 15, normalized size = 0.94

$$-2\ln(x) + 2\ln\left(x-1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)/(x-1)/x^2,x)
```

```
[Out] 2*ln(x-1)+1/x-2*ln(x)
```

maxima [A] time = 0.59, size = 14, normalized size = 0.88

$$\frac{1}{x} + 2\log\left(x-1\right) - 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/x + 2*log(x - 1) - 2*log(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$\frac{1}{x} - 4\,\mathrm{atanh}\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x^2*(x - 1)),x)
```

```
[Out] 1/x - 4*atanh(2*x - 1)
```

sympy [A] time = 0.10, size = 14, normalized size = 0.88

$$-2\log\left(x\right) + 2\log\left(x-1\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-1+x)/x**2,x)
```

```
[Out] -2*log(x) + 2*log(x - 1) + 1/x
```

3.476 $\int \frac{1}{4x^2+4x^3+x^4} \, dx$

Optimal. Leaf size=27

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1106, 199, 206}

$$\frac{x+1}{2\left(1-(x+1)^2\right)}+\frac{1}{2}\tanh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

```
[Out] (1 + x)/(2*(1 - (1 + x)^2)) + ArcTanh[1 + x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1106

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4x^2+4x^3+x^4} \, dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2} \, dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2} \, dx,x,1+x\right)\\ &= \frac{1+x}{2\left(1-(1+x)^2\right)}+\frac{1}{2}\tanh^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2(x+1)}{x(x+2)}-\log(x)+\log(x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
[Out] ((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4x^2 + 4x^3 + x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4*x^2 + 4*x^3 + x^4)^(-1),x]
[Out] IntegrateAlgebraic[(4*x^2 + 4*x^3 + x^4)^(-1), x]
```

fricas [A] time = 1.21, size = 39, normalized size = 1.44

$$\frac{\left(x^2 + 2\,x\right)\log\left(x + 2\right) - \left(x^2 + 2\,x\right)\log(x) - 2\,x - 2}{4\left(x^2 + 2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
[Out] 1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

giac [A] time = 0.38, size = 27, normalized size = 1.00

$$-\frac{x + 1}{2\left(x^2 + 2\,x\right)} + \frac{1}{4}\log\left(|x + 2|\right) - \frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.89

$$-\frac{\ln(x)}{4} + \frac{\ln\left(x + 2\right)}{4} - \frac{1}{4x} - \frac{1}{4\left(x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^3+4*x^2),x)
[Out] -1/4*ln(x)+1/4*ln(x+2)-1/4/x-1/4/(x+2)
```

maxima [A] time = 0.81, size = 25, normalized size = 0.93

$$-\frac{x + 1}{2\left(x^2 + 2\,x\right)} + \frac{1}{4}\log\left(x + 2\right) - \frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
[Out] -1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

mupad [B] time = 2.23, size = 23, normalized size = 0.85

$$\frac{\mathrm{atanh}\left(x + 1\right)}{2} - \frac{\frac{x}{2} + \frac{1}{2}}{x^2 + 2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2 + 4*x^3 + x^4),x)
```

```
[Out] atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

sympy [A] time = 0.11, size = 24, normalized size = 0.89

$$\frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**3+4*x**2),x)
```

```
[Out] (-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

3.477 $\int \frac{1+x^2}{1+x} \, dx$

Optimal. Leaf size=17

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{x^2}{2} - x + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x),x]
```

```
[Out] -x + x^2/2 + 2*Log[1 + x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+x} \, dx &= \int \left(-1+x+\frac{2}{1+x}\right) dx \\ &= -x+\frac{x^2}{2}+2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.06

$$\frac{1}{2}\left(x^2 - 2x + 4\log(x+1) - 3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x),x]
```

```
[Out] (-3 - 2*x + x^2 + 4*Log[1 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1+x^2}{1+x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(1 + x^2)/(1 + x),x]
```

```
[Out] IntegrateAlgebraic[(1 + x^2)/(1 + x), x]
```

fricas [A] time = 1.58, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 - x + 2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="fricas")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

giac [A] time = 0.36, size = 16, normalized size = 0.94

$$\frac{1}{2}\,x^2 - x + 2\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="giac")
[Out] 1/2*x^2 - x + 2*log(abs(x + 1))
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{x^2}{2} - x + 2\ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x+1),x)
[Out] -x+1/2*x^2+2*ln(x+1)
```

maxima [A] time = 0.74, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 - x + 2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(1+x),x, algorithm="maxima")
[Out] 1/2*x^2 - x + 2*log(x + 1)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.88

$$2\,\ln\left(x+1\right) - x + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x + 1),x)
[Out] 2*log(x + 1) - x + x^2/2
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{x^2}{2} - x + 2\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(1+x),x)
[Out] x**2/2 - x + 2*log(x + 1)
```

3.478 $\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx$

Optimal. Leaf size=18

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx &= \int \left(-3 - \frac{1}{x^2} + \frac{3}{x} + x\right) dx\\ &= \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^2}{2} - 3x + \frac{1}{x} + 3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
[Out] IntegrateAlgebraic[(-1 + 3*x - 3*x^2 + x^3)/x^2, x]
```

fricas [A] time = 1.03, size = 20, normalized size = 1.11

$$\frac{x^3 - 6\,x^2 + 6\,x\log(x) + 2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

giac [A] time = 0.37, size = 17, normalized size = 0.94

$$\frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x^2}{2} - 3x + 3\ln(x) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-3*x^2+3*x-1)/x^2,x)
```

```
[Out] 1/x-3*x+1/2*x^2+3*ln(x)
```

maxima [A] time = 0.73, size = 16, normalized size = 0.89

$$\frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 3*x + 1/x + 3*log(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.89

$$3\,\ln(x) - 3\,x + \frac{1}{x} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
```

```
[Out] 3*log(x) - 3*x + 1/x + x^2/2
```

sympy [A] time = 0.08, size = 15, normalized size = 0.83

$$\frac{x^2}{2} - 3x + 3\log\left(x\right) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

```
[Out] x**2/2 - 3*x + 3*log(x) + 1/x
```

3.479 $\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right) dx$

Optimal. Leaf size=18

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {43}

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In] Int[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right) dx &= \int \left(-7+3x+x^2\right) dx\\
&= -7x+\frac{3x^2}{2}+\frac{x^3}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^3}{3}+\frac{3x^2}{2}-7x$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] -7*x + (3*x^2)/2 + x^3/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{2}\left(3-\sqrt{37}\right)+x\right)\left(\frac{1}{2}\left(3+\sqrt{37}\right)+x\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x),x]
```

```
[Out] IntegrateAlgebraic[((3 - Sqrt[37])/2 + x)*((3 + Sqrt[37])/2 + x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: SyntaxError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="fricas")
[Out] Exception raised: SyntaxError >> Malformed expression
```

giac [A] time = 0.35, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

maple [A] time = 0.00, size = 28, normalized size = 1.56

$$\frac{x^3}{3} + \frac{3x^2}{2} + \left(\frac{3}{2} - \frac{\sqrt{37}}{2}\right)\left(\frac{3}{2} + \frac{\sqrt{37}}{2}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x)
[Out] 1/3*x^3+3/2*x^2+(3/2-1/2*37^(1/2))*(3/2+1/2*37^(1/2))*x
```

maxima [A] time = 1.71, size = 14, normalized size = 0.78

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37^(1/2))*(x+3/2+1/2*37^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3 + 3/2*x^2 - 7*x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\left(2x^2 + 9x - 42\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 37^(1/2)/2 + 3/2)*(x + 37^(1/2)/2 + 3/2),x)
[Out] (x*(9*x + 2*x^2 - 42))/6
```

sympy [A] time = 0.06, size = 14, normalized size = 0.78

$$\frac{x^3}{3} + \frac{3x^2}{2} - 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+3/2-1/2*37**(1/2))*(x+3/2+1/2*37**(1/2)),x)
[Out] x**3/3 + 3*x**2/2 - 7*x
```

3.480 $\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx$

Optimal. Leaf size=23

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1850}

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx &= \int \left(\frac{5}{(1+x)^4} - \frac{3}{(1+x)^2} + \frac{2}{1+x}\right) dx \\ &= -\frac{5}{3(1+x)^3} + \frac{3}{1+x} + 2\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{3}{x+1} - \frac{5}{3(x+1)^3} + 2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] -5/(3*(1 + x)^3) + 3/(1 + x) + 2*Log[1 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4+3x^2+2x^3}{(1+x)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 + 3*x^2 + 2*x^3)/(1 + x)^4,x]

[Out] IntegrateAlgebraic[(4 + 3*x^2 + 2*x^3)/(1 + x)^4, x]
```

fricas [B] time = 1.14, size = 46, normalized size = 2.00

$$\frac{9\,x^2 + 6\left(x^3 + 3\,x^2 + 3\,x + 1\right)\log\left(x+1\right) + 18\,x + 4}{3\left(x^3 + 3\,x^2 + 3\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(9*x^2 + 6*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 4)/(x^3 + 3*x^2
+ 3*x + 1)
```

giac [A] time = 0.37, size = 25, normalized size = 1.09

$$\frac{9\,x^2+18\,x+4}{3\,(x+1)^3}+2\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x + 1)^3 + 2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 22, normalized size = 0.96

$$2\ln\left(x+1\right)-\frac{5}{3\,(x+1)^3}+\frac{3}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2+4)/(x+1)^4,x)
```

```
[Out] -5/3/(x+1)^3+3/(x+1)+2*ln(x+1)
```

maxima [A] time = 0.74, size = 34, normalized size = 1.48

$$\frac{9\,x^2+18\,x+4}{3\left(x^3+3\,x^2+3\,x+1\right)}+2\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2+4)/(1+x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(9*x^2 + 18*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + 2*log(x + 1)
```

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$2\,\ln\left(x+1\right)+\frac{3\,x^2+6\,x+\frac{4}{3}}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2*x^3 + 4)/(x + 1)^4,x)
```

```
[Out] 2*log(x + 1) + (6*x + 3*x^2 + 4/3)/(x + 1)^3
```

sympy [A] time = 0.11, size = 31, normalized size = 1.35

$$\frac{9x^2+18x+4}{3x^3+9x^2+9x+3}+2\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2+4)/(1+x)**4,x)
```

```
[Out] (9*x**2 + 18*x + 4)/(3*x**3 + 9*x**2 + 9*x + 3) + 2*log(x + 1)
```

3.481 $\int \frac{x}{(1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {801, 203}

$$\frac{1}{2(x+1)}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] 1/(2*(1 + x)) + ArcTan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{2(1+x)^2}+\frac{1}{2\left(1+x^2\right)}\right)dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= \frac{1}{2(1+x)}+\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.75

$$\frac{1}{2}\left(\frac{1}{x+1}+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] ((1 + x)^(-1) + ArcTan[x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)^2\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((1 + x)^2*(1 + x^2)),x]
```

```
[Out] IntegrateAlgebraic[x/((1 + x)^2*(1 + x^2)), x]
```

fricas [A] time = 0.87, size = 15, normalized size = 0.94

$$\frac{(x+1)\arctan(x)+1}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*((x + 1)*arctan(x) + 1)/(x + 1)
```

giac [B] time = 0.26, size = 32, normalized size = 2.00

$$-\frac{1}{8}\pi-\frac{1}{2}\pi\left\lfloor-\frac{\pi-4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/8*pi - 1/2*pi*floor(-1/4*(pi - 4*arctan(x))/pi + 1/2) + 1/2/(x + 1) + 1/
2*arctan(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\arctan(x)}{2}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)^2/(x^2+1),x)
```

```
[Out] 1/2/(x+1)+1/2*arctan(x)
```

maxima [A] time = 1.79, size = 12, normalized size = 0.75

$$\frac{1}{2(x+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2/(x + 1) + 1/2*arctan(x)
```

mupad [B] time = 2.22, size = 12, normalized size = 0.75

$$\frac{\mathrm{atan}(x)}{2}+\frac{1}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^2 + 1)*(x + 1)^2),x)
```

```
[Out] atan(x)/2 + 1/(2*(x + 1))
```

sympy [A] time = 0.11, size = 10, normalized size = 0.62

$$\frac{\operatorname{atan}{(x)}}{2}+\frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2/(x**2+1),x)
```

```
[Out] atan(x)/2 + 1/(2*x + 2)
```

3.482 $\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx$

Optimal. Leaf size=29

$$\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log(x+2)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1850}

$$\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx &= \int \left(-20+9x-3x^2+x^3+\frac{47}{2+x}\right)dx\\ &= -20x+\frac{9x^2}{2}-x^3+\frac{x^4}{4}+47\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{x^4}{4}-x^3+\frac{9x^2}{2}-20x+47\log(x+2)-70$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] -70 - 20*x + (9*x^2)/2 - x^3 + x^4/4 + 47*Log[2 + x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{7-2x+3x^2-x^3+x^4}{2+x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x),x]
```

```
[Out] IntegrateAlgebraic[(7 - 2*x + 3*x^2 - x^3 + x^4)/(2 + x), x]
```

fricas [A] time = 1.41, size = 25, normalized size = 0.86

$$\frac{1}{4}x^4-x^3+\frac{9}{2}x^2-20\,x+47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="fricas")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

giac [A] time = 0.28, size = 26, normalized size = 0.90

$$\frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="giac")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(abs(x + 2))
```

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\ln\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+3*x^2-2*x+7)/(x+2),x)
[Out] -20*x+9/2*x^2-x^3+1/4*x^4+47*ln(x+2)
```

maxima [A] time = 0.76, size = 25, normalized size = 0.86

$$\frac{1}{4}\,x^4 - x^3 + \frac{9}{2}\,x^2 - 20\,x + 47\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+3*x^2-2*x+7)/(2+x),x, algorithm="maxima")
[Out] 1/4*x^4 - x^3 + 9/2*x^2 - 20*x + 47*log(x + 2)
```

mupad [B] time = 0.03, size = 25, normalized size = 0.86

$$47\,\ln\left(x+2\right) - 20\,x + \frac{9\,x^2}{2} - x^3 + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 2*x - x^3 + x^4 + 7)/(x + 2),x)
[Out] 47*log(x + 2) - 20*x + (9*x^2)/2 - x^3 + x^4/4
```

sympy [A] time = 0.08, size = 24, normalized size = 0.83

$$\frac{x^4}{4} - x^3 + \frac{9x^2}{2} - 20x + 47\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+3*x**2-2*x+7)/(2+x),x)
[Out] x**4/4 - x**3 + 9*x**2/2 - 20*x + 47*log(x + 2)
```

3.483 $\int \frac{-1+x^3}{-1+x} \, dx$

Optimal. Leaf size=16

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1586}

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{-1+x^3}{-1+x} \, dx &= \int \left(1+x+x^2\right) \, dx \\ &= x+\frac{x^2}{2}+\frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-1 + x),x]
```

```
[Out] x + x^2/2 + x^3/3
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.31

$$\frac{1}{6}\left(2(x-1)^2+9(x-1)+18\right)(x-1)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^3)/(-1 + x),x]
```

```
[Out] ((18 + 9*(-1 + x) + 2*(-1 + x)^2)*(-1 + x))/6
```

fricas [A] time = 1.24, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

giac [A] time = 0.35, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x-1),x)
[Out] x+1/2*x^2+1/3*x^3
```

maxima [A] time = 0.89, size = 12, normalized size = 0.75

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(-1+x),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 + x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.81

$$\frac{x\left(2x^2 + 3x + 6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x - 1),x)
[Out] (x*(3*x + 2*x^2 + 6))/6
```

sympy [A] time = 0.06, size = 10, normalized size = 0.62

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(-1+x),x)
[Out] x**3/3 + x**2/2 + x
```

3.484 $\int \frac{2+2x}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=17

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {801, 203}

$$\frac{1}{x-1}-\frac{1}{(1-x)^2}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
[Out] -(1 - x)^(-2) + (-1 + x)^(-1) + ArcTan[x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x}{(-1+x)^3\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{(-1+x)^3}-\frac{1}{(-1+x)^2}+\frac{1}{1+x^2}\right)dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\int \frac{1}{1+x^2}\,dx\\ &= -\frac{1}{(1-x)^2}+\frac{1}{-1+x}+\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{x+(x-1)^2\tan^{-1}(x)-2}{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
[Out] (-2 + x + (-1 + x)^2*ArcTan[x])/(-1 + x)^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2+2x}{(-1+x)^3\left(1+x^2\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 2*x)/((-1 + x)^3*(1 + x^2)),x]
[Out] IntegrateAlgebraic[(2 + 2*x)/((-1 + x)^3*(1 + x^2)), x]
```

fricas [A] time = 1.24, size = 25, normalized size = 1.47

$$\frac{(x^2 - 2x + 1)\arctan(x) + x - 2}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
[Out] ((x^2 - 2*x + 1)*arctan(x) + x - 2)/(x^2 - 2*x + 1)
```

giac [A] time = 0.24, size = 12, normalized size = 0.71

$$\frac{x-2}{(x-1)^2} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="giac")
[Out] (x - 2)/(x - 1)^2 + arctan(x)
```

maple [A] time = 0.01, size = 16, normalized size = 0.94

$$\arctan(x) - \frac{1}{(x-1)^2} + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+2)/(x-1)^3/(x^2+1),x)
[Out] -1/(x-1)^2+1/(x-1)+arctan(x)
```

maxima [A] time = 1.74, size = 17, normalized size = 1.00

$$\frac{x-2}{x^2 - 2x + 1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
[Out] (x - 2)/(x^2 - 2*x + 1) + arctan(x)
```

mupad [B] time = 0.03, size = 17, normalized size = 1.00

$$\operatorname{atan}(x) + \frac{x-2}{x^2 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 2)/((x^2 + 1)*(x - 1)^3),x)
[Out] atan(x) + (x - 2)/(x^2 - 2*x + 1)
```

sympy [A] time = 0.12, size = 14, normalized size = 0.82

$$\frac{x-2}{x^2 - 2x + 1} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+2*x)/(-1+x)**3/(x**2+1),x)
[Out] (x - 2)/(x**2 - 2*x + 1) + atan(x)
```

3.485 $\int \frac{1}{bx+c(d+ex)^2} dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{bx+c(d+ex)^2} dx &= \int \frac{1}{cd^2+(b+2cde)x+ce^2x^2} dx\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{b(b+4cde)-x^2} dx, x, b+2cde+2ce^2x\right)\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b}\sqrt{b+4cde}}\right)}{\sqrt{b}\sqrt{b+4cde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/(Sqrt[b]*Sqrt[b + 4*c*d*e])])/(Sqrt[b]*Sq
rt[b + 4*c*d*e])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bx + c(d + ex)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(b*x + c*(d + e*x)^2)^(-1), x]
```

fricas [A] time = 1.33, size = 190, normalized size = 4.04

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+2\,c^2d^2e^2+4\,bcde+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x}\right)}{\sqrt{4\,bcde+b^2}}, \frac{2\,\sqrt{-4\,bcde-b^2}\,\arctan\left(\frac{\sqrt{-4\,bcde-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde+b^2}\right)}{4\,bcde+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 2*c^2*d^2*e^2 + 4*b*c*d*e + b^2 + 2*(2*c^2*d*e^3 + b*
c*e^2)*x - sqrt(4*b*c*d*e + b^2)*(2*c*e^2*x + 2*c*d*e + b))/(c*e^2*x^2 + c*
d^2 + (2*c*d*e + b)*x))/sqrt(4*b*c*d*e + b^2), 2*sqrt(-4*b*c*d*e - b^2)*arc
tan(sqrt(-4*b*c*d*e - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e + b^2))/(4*
b*c*d*e + b^2)]
```

giac [A] time = 0.28, size = 48, normalized size = 1.02

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde-b^2}}\right)}{\sqrt{-4\,bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e - b^2))/sqrt(-4*b*c*d*e
- b^2)
```

maple [A] time = 0.01, size = 43, normalized size = 0.91

$$-\frac{2\,\mathrm{arctanh}\left(\frac{2c\,e^2x+2cde+b}{\sqrt{4bcde+b^2}}\right)}{\sqrt{4bcde+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+c*(e*x+d)^2),x)
```

```
[Out] -2/(4*b*c*d*e+b^2)^(1/2)*arctanh((2*c*e^2*x+2*c*d*e+b)/(4*b*c*d*e+b^2)^(1/2
))
```

maxima [A] time = 1.02, size = 68, normalized size = 1.45

$$\frac{\log\left(\frac{2\,ce^2x+2\,cde+b-\sqrt{(4\,cde+b)b}}{2\,ce^2x+2\,cde+b+\sqrt{(4\,cde+b)b}}\right)}{\sqrt{(4\,cde+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] log((2*c*e^2*x + 2*c*d*e + b - sqrt((4*c*d*e + b)*b))/(2*c*e^2*x + 2*c*d*e
+ b + sqrt((4*c*d*e + b)*b)))/sqrt((4*c*d*e + b)*b)
```

mupad [B] time = 0.10, size = 42, normalized size = 0.89

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x\,e^2+2\,c\,d\,e+b}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}\right)}{\sqrt{b}\,\sqrt{b+4\,c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(d + e*x)^2 + b*x),x)
```

```
[Out] -(2*atanh((b + 2*c*d*e + 2*c*e^2*x)/(b^(1/2)*(b + 4*c*d*e)^(1/2))))/(b^(1/2
)*(b + 4*c*d*e)^(1/2))
```

sympy [B] time = 0.30, size = 151, normalized size = 3.21

$$\sqrt{\frac{1}{b\left(b+4cde\right)}}\log\left(x+\frac{-b^2\sqrt{\frac{1}{b(b+4cde)}}-4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^2}\right)-\sqrt{\frac{1}{b\left(b+4cde\right)}}\log\left(x+\frac{b^2\sqrt{\frac{1}{b(b+4cde)}}+4bcde\sqrt{\frac{1}{b(b+4cde)}}+b+2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+c*(e*x+d)**2),x)
```

```
[Out] sqrt(1/(b*(b + 4*c*d*e)))*log(x + (-b**2*sqrt(1/(b*(b + 4*c*d*e))) - 4*b*c*
d*e*sqrt(1/(b*(b + 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2)) - sqrt(1/(b*(b + 4
*c*d*e)))*log(x + (b**2*sqrt(1/(b*(b + 4*c*d*e))) + 4*b*c*d*e*sqrt(1/(b*(b
+ 4*c*d*e))) + b + 2*c*d*e)/(2*c*e**2))
```

3.486 $\int \frac{1}{a+bx+c(d+ex)^2} dx$

Optimal. Leaf size=57

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1981, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{-4ace^2+b^2+4bcde}}\right)}{\sqrt{-4ace^2+b^2+4bcde}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*e*(d + e*x))/Sqrt[b^2 + 4*b*c*d*e - 4*a*c*e^2]])/Sqrt[
b^2 + 4*b*c*d*e - 4*a*c*e^2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+bx+c(d+ex)^2} dx &= \int \frac{1}{a+cd^2+(b+2cde)x+ce^2x^2} dx\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{b^2+4bcde-4ace^2-x^2} dx, x, b+2cde+2ce^2x\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{b^2+4bcde-4ace^2}}\right)}{\sqrt{b^2+4bcde-4ace^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.07

$$\frac{2\tan^{-1}\left(\frac{b+2ce(d+ex)}{\sqrt{4ace^2-b^2-4bcde}}\right)}{\sqrt{4ace^2-b^2-4bcde}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*c*e*(d + e*x))/Sqrt[-b^2 - 4*b*c*d*e + 4*a*c*e^2]])/Sqrt[-
b^2 - 4*b*c*d*e + 4*a*c*e^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx+c(d+ex)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*(d + e*x)^2)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x + c*(d + e*x)^2)^(-1), x]
```

fricas [A] time = 0.77, size = 240, normalized size = 4.21

$$\left[\frac{\log\left(\frac{2\,c^2e^4x^2+4\,bcde+2\,(c^2d^2-ac)e^2+b^2+2\,(2\,c^2de^3+bce^2)x-\sqrt{4\,bcde-4\,ace^2+b^2}\,(2\,ce^2x+2\,cde+b)}{ce^2x^2+cd^2+(2\,cde+b)x+a}\right)}{\sqrt{4\,bcde-4\,ace^2+b^2}}, -\frac{2\,\sqrt{-4\,bcde+4\,ace^2-b^2}\,\arctan\left(-\frac{\sqrt{-4\,bcde+4\,ace^2-b^2}\,(2\,ce^2x+2\,cde+b)}{4\,bcde-4\,ace^2+b^2}\right)}{4\,bcde-4\,ace^2+b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*e^4*x^2 + 4*b*c*d*e + 2*(c^2*d^2 - a*c)*e^2 + b^2 + 2*(2*c^2*d*
e^3 + b*c*e^2)*x - sqrt(4*b*c*d*e - 4*a*c*e^2 + b^2)*(2*c*e^2*x + 2*c*d*e +
 b))/(c*e^2*x^2 + c*d^2 + (2*c*d*e + b)*x + a))/sqrt(4*b*c*d*e - 4*a*c*e^2
+ b^2), -2*sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2)*arctan(-sqrt(-4*b*c*d*e + 4*a
*c*e^2 - b^2)*(2*c*e^2*x + 2*c*d*e + b)/(4*b*c*d*e - 4*a*c*e^2 + b^2))/(4*b
*c*d*e - 4*a*c*e^2 + b^2)]
```

giac [A] time = 0.36, size = 60, normalized size = 1.05

$$\frac{2\,\arctan\left(\frac{2\,cxe^2+2\,cde+b}{\sqrt{-4\,bcde+4\,ace^2-b^2}}\right)}{\sqrt{-4\,bcde+4\,ace^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x*e^2 + 2*c*d*e + b)/sqrt(-4*b*c*d*e + 4*a*c*e^2 - b^2))/sqrt
(-4*b*c*d*e + 4*a*c*e^2 - b^2)
```

maple [A] time = 0.00, size = 61, normalized size = 1.07

$$\frac{2\arctan\left(\frac{2c\,e^2x+2cde+b}{\sqrt{4ac\,e^2-4bcde-b^2}}\right)}{\sqrt{4ac\,e^2-4bcde-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+c*(e*x+d)^2),x)
```

```
[Out] 2/(4*a*c*e^2-4*b*c*d*e-b^2)^(1/2)*arctan((2*c*e^2*x+2*c*d*e+b)/(4*a*c*e^2-4
*b*c*d*e-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c*e^2>0)', see `assume?` for mo
re details)Is 4*a*c*e^2    -4*b*c*d*e-b^2    positive or negative?
```

mupad [B] time = 2.23, size = 82, normalized size = 1.44

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,d\,e}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}+\frac{2\,c\,e^2\,x}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}\right)}{\sqrt{-b^2-4\,c\,d\,b\,e+4\,a\,c\,e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*(d + e*x)^2 + b*x),x)
```

```
[Out] (2*atan((b + 2*c*d*e)/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2) + (2*c*e^2*x)/(4*
a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)))/(4*a*c*e^2 - b^2 - 4*b*c*d*e)^(1/2)
```

sympy [B] time = 0.35, size = 294, normalized size = 5.16

$$-\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{-4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)+\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}\log\left(x+\frac{4ace^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-b^2\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}-4bcde\sqrt{-\frac{1}{4ace^2-b^2-4bcde}}+b+2cde}{2ce^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+c*(e*x+d)**2),x)
```

```
[Out] -sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (-4*a*c*e**2*sqrt(-1/(4*a
*c*e**2 - b**2 - 4*b*c*d*e)) + b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)
) + 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e)/(2*c*
e**2)) + sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e))*log(x + (4*a*c*e**2*sqrt(
-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) - b**2*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b
*c*d*e)) - 4*b*c*d*e*sqrt(-1/(4*a*c*e**2 - b**2 - 4*b*c*d*e)) + b + 2*c*d*e
)/(2*c*e**2))
```

3.487 $\int \frac{x^2}{1+\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Rubi [A] time = 0.19, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {1989, 1127, 1161, 618, 204, 1164, 628}

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}\,x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt[2*(-1 + S
qrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*x)/
Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(
4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*
Sqrt[2*(1 + Sqrt[2])])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1989

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u,
 x]
```

Rubi steps

$$\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx = \int \frac{x^2}{2-2x^2+x^4}\,dx$$

$$= -\left(\frac{1}{2}\int \frac{\sqrt{2}-x^2}{2-2x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2}+x^2}{2-2x^2+x^4}\,dx$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2}\,dx+\frac{\int \frac{\sqrt{2(1+}}{-\sqrt{2}-\sqrt{2(}}}{4\sqrt{2(}}$$

$$= \frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}+\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\text{Subst}\left(\int \frac{}{2(}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}+2x}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}x+x^2\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\ldots$$

Mathematica [C] time = 0.03, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]/(-1 + I)^
(3/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{1+\left(-1+x^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(1 + (-1 + x^2)^2),x]
```

```
[Out] IntegrateAlgebraic[x^2/(1 + (-1 + x^2)^2), x]
```

fricas [A] time = 1.51, size = 247, normalized size = 1.31

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{4}\cdot 2^{\frac{3}{4}}\sqrt{2\sqrt{2}+4}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+\frac{1}{2}\cdot 2^{\frac{1}{4}}\sqrt{-2^{\frac{3}{4}}x\sqrt{2\sqrt{2}+4}+2x^2+2\sqrt{2}}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2^(3/4)*x*sqrt(2*sqrt(2)
 + 4) + 2*x^2 + 2*sqrt(2)) - 1/16*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)
*log(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2)) - 1/4*2^(3/4)*sqrt
(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 1/2*2^(1/4)*sqr
t(2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2*sqrt(2))*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/4*2^(3/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/2*2^(3/4)*x*sqrt(2
*sqrt(2) + 4) + 1/2*2^(1/4)*sqrt(-2^(3/4)*x*sqrt(2*sqrt(2) + 4) + 2*x^2 + 2
*sqrt(2))*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1)
```

giac [A] time = 1.49, size = 147, normalized size = 0.78

$$\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x+2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+2}\arctan\left(\frac{2^{\frac{3}{4}}\left(2x-2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}\right)}{2\sqrt{-\sqrt{2}+2}}\right)-\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2+2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)+\frac{1}{8}\sqrt{2\sqrt{2}-2}\log\left(x^2-2^{\frac{1}{4}}x\sqrt{\sqrt{2}+2}+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x + 2^(1/4)*sqrt(sqrt(2) + 2)
)/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(2*sqrt(2) + 2)*arctan(1/2*2^(3/4)*(2*x - 2
^(1/4)*sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 + 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2)) + 1/8*sqrt(2*sqrt(2) - 2)*log
(x^2 - 2^(1/4)*x*sqrt(sqrt(2) + 2) + sqrt(2))
```

maple [B] time = 0.10, size = 308, normalized size = 1.64

$$\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2}\,\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}-\frac{\left(2+2\sqrt{2}\right)\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)}{4\sqrt{-2+2\sqrt{2}}}+\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^2-\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}-\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^2-\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\,\ln\left(x^2+\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}+\frac{\sqrt{2+2\sqrt{2}}\,\ln\left(x^2+\sqrt{2+2\sqrt{2}}\,x+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+(x^2-1)^2),x)
```

```
[Out] -1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))+1/4*
2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))
/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2
))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^
(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)
-x*(2+2*2^(1/2))^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arct
an((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)*
ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2
)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2-1\right)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+(x^2-1)^2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^2 - 1)^2 + 1), x)
```

mupad [B] time = 2.30, size = 101, normalized size = 0.54

$$\operatorname{atanh}\left(32\,x\,\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\,\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}+2\,\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)+\operatorname{atanh}\left(32\,x\,\left(\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)^3\right)\left(2\,\sqrt{-\frac{\sqrt{2}}{32}-\frac{1}{32}}-2\,\sqrt{\frac{\sqrt{2}}{32}-\frac{1}{32}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 - 1)^2 + 1),x)
```

```
[Out] atanh(32*x*((- 2^(1/2)/32 - 1/32)^(1/2) + (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*
(- 2^(1/2)/32 - 1/32)^(1/2) + 2*(2^(1/2)/32 - 1/32)^(1/2)) + atanh(32*x*((-
 2^(1/2)/32 - 1/32)^(1/2) - (2^(1/2)/32 - 1/32)^(1/2))^3)*(2*(- 2^(1/2)/32
- 1/32)^(1/2) - 2*(2^(1/2)/32 - 1/32)^(1/2))
```

sympy [A] time = 0.52, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+(x**2-1)**2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*_t + x)))
```

3.488 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal. Leaf size=60

$$\frac{x^4}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

Rubi [A] time = 0.14, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2102, 1588}

$$-\frac{5x^6}{(x^4+x+3)^3}+\frac{x^4}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}-\frac{3x}{(x^4+x+3)^3}+\frac{2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*
x^9)/(3 + x + x^4)^4),x]
```

```
[Out] 2/(3 + x + x^4)^3 - (3*x)/(3 + x + x^4)^3 + (5*x^2)/(3 + x + x^4)^3 + x^4/(
3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2102

```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[(Coeff[Pm, x, m]*x^(m - n + 1)*Qn^(p + 1))/((m + n*p + 1)*Coeff[Qn
, x, n]), x] + Dist[1/((m + n*p + 1)*Coeff[Qn, x, n]), Int[ExpandToSum[(m +
 n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
(p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 < 0
] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Rubi steps

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5x^6}{(3+x+x^4)^3}+\frac{1}{6}\int \frac{-90+216x-30x^2}{}$$

$$=\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}-\frac{1}{48}\int \frac{72}{}$$

$$=\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+}$$

$$=-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x}$$

$$=\frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.45

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7
 - 30*x^9)/(3 + x + x^4)^4),x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{\left(3+x+x^4\right)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^
6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4),x]
```

```
[Out] IntegrateAlgebraic[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^
6 + 8*x^7 - 30*x^9)/(3 + x + x^4)^4), x]
```

fricas [A] time = 1.29, size = 65, normalized size = 1.08

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [A] time = 0.40, size = 30, normalized size = 0.50

$$-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{\left(x^4+x+3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="giac")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

maple [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+3)^4,x
)
```

```
[Out] (-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

maxima [A] time = 1.25, size = 65, normalized size = 1.08

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x^9-8*x^7-15*x^6-140*x^5+34*x^4-12*x^3-5*x^2+36*x-15)/(x^4+x+
3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 2.34, size = 27, normalized size = 0.45

$$\frac{-5\,x^{6}+x^{4}+5\,x^{2}-3\,x+2}{\left(x^{4}+x+3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
15)/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [A] time = 0.23, size = 60, normalized size = 1.00

$$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{x^{12}+3x^{9}+9x^{8}+3x^{6}+18x^{5}+27x^{4}+x^{3}+9x^{2}+27x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((30*x**9-8*x**7-15*x**6-140*x**5+34*x**4-12*x**3-5*x**2+36*x-15)/
(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.489

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Rubi [F] time = 0.31, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$

Verification is not applicable to the result.

```
[In] Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 75
*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] -19/(4*(3 + x + x^4)^3) + (3 + x + x^4)^(-2) - (621*Defer[Int][(3 + x + x^4
)^(-4), x])/4 + 684*Defer[Int][x/(3 + x + x^4)^4, x] + 360*Defer[Int][x^2/(
3 + x + x^4)^4, x] + 44*Defer[Int][(3 + x + x^4)^(-3), x] - 320*Defer[Int][
x/(3 + x + x^4)^3, x] - 75*Defer[Int][x^2/(3 + x + x^4)^3, x] + 30*Defer[In
t][x/(3 + x + x^4)^2, x]
```

Rubi steps

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx = 3\int \frac{-47+228x+120}{(3+x+x^4}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3}$$

$$= -\frac{19}{4(3+x+x^4)^3}+\frac{}{(3}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*
x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4} + \frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3} + \frac{30x}{(3+x+x^4)^2} \right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (
42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (
42 - 320*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2, x]
```

fricas [B] time = 1.29, size = 65, normalized size = 2.41

$$-\frac{5x^6 - x^4 - 5x^2 + 3x - 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [B] time = 0.40, size = 197, normalized size = 7.30

$$\frac{1}{195075}x\left(\frac{377432x^2 - 2808656x + 703551}{x^4+x+3} - \frac{255032x^2 - 1829456x + 680601}{x^4+x+3} - \frac{7650(16x^2 - 128x + 3)}{x^4+x+3}\right) - \frac{2(16x^3 - 64x^2 + x + 12)}{51(x^4+x+3)} + \frac{754864x^7 - 2808656x^6 + 469034x^5 + 1321012x^4 - 417584x^3 - 13339729x^2 + 2696430x + 2183454}{390150(x^4+x+3)^2} - \frac{510064x^{11} - 1829456x^{10} + 453734x^9 + 1402676x^8 - 472048x^7 - 13501313x^6 + 4720744x^5 + 3747556x^4 - 10935781x^3 - 30736107x^2 + 10203894x + 4117662}{390150(x^4+x+3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

```
[Out] 1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2 -
 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x + 3
)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^7 -
 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 26964
30*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10 + 45
3734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 3747556*
x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x + 3)^3
```

maple [C] time = 0.03, size = 250, normalized size = 9.26

$$\frac{\left(-255032\,\mathrm{RootOf}(\_Z^4+\_Z+3)^2 + 1829456\,\mathrm{RootOf}(\_Z^4+\_Z+3) - 680601\right)\ln\left(-\mathrm{RootOf}(\_Z^4+\_Z+3)+x\right)}{780300\,\mathrm{RootOf}(\_Z^4+\_Z+3)^3 + 195075} + \frac{2\left(-16\,\mathrm{RootOf}(\_Z^4+\_Z+3)^2 + 128\,\mathrm{RootOf}(\_Z^4+\_Z+3) - 3\right)\ln\left(-\mathrm{RootOf}(\_Z^4+\_Z+3)+x\right)}{51\left(4\,\mathrm{RootOf}(\_Z^4+\_Z+3)^3+1\right)} + \frac{\left(377432\,\mathrm{RootOf}(\_Z^4+\_Z+3)^2 - 2808656\,\mathrm{RootOf}(\_Z^4+\_Z+3) + 703551\right)\ln\left(-\mathrm{RootOf}(\_Z^4+\_Z+3)+x\right)}{780300\,\mathrm{RootOf}(\_Z^4+\_Z+3)^3 + 195075} + \frac{\frac{377432}{195075}x^7 - \frac{1404328}{195075}x^6 + \frac{234517}{195075}x^5 + \frac{660506}{195075}x^4 - \frac{208792}{195075}x^3 - \frac{13339729}{390150}x^2 + \frac{89881}{13005}x + \frac{121303}{21675}}{(x^4+x+3)^2} + \frac{-\frac{32}{51}x^3 + \frac{128}{51}x^2 - \frac{2}{51}x - \frac{8}{17}}{x^4+x+3} + \frac{-\frac{255032}{195075}x^{11} + \frac{914728}{195075}x^{10} - \frac{226867}{195075}x^9 - \frac{701338}{195075}x^8 + \frac{236024}{195075}x^7 + \frac{13501313}{390150}x^6 - \frac{2360372}{195075}x^5 - \frac{1873778}{195075}x^4 + \frac{10935781}{390150}x^3 + \frac{3415123}{43350}x^2 - \frac{188961}{7225}x - \frac{76253}{7225}}{(x^4+x+3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+x
+3)^3+30*x/(x^4+x+3)^2,x)
```

```
[Out] (377432/195075*x^7-1404328/195075*x^6+234517/195075*x^5+660506/195075*x^4-2
08792/195075*x^3-13339729/390150*x^2+89881/13005*x+121303/21675)/(x^4+x+3)^
2+1/195075*sum((377432*_R^2-2808656*_R+703551)/(4*_R^3+1)*ln(-_R+x),_R=Root
Of(_Z^4+_Z+3))+30*(-16/765*x^3+64/765*x^2-1/765*x-4/255)/(x^4+x+3)+2/51*sum
((-16*_R^2+128*_R-3)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_Z^4+_Z+3))+3*(-255032/
585225*x^11+914728/585225*x^10-226867/585225*x^9-701338/585225*x^8+236024/5
85225*x^7+13501313/1170450*x^6-2360372/585225*x^5-1873778/585225*x^4+109357
81/1170450*x^3+3415123/130050*x^2-62987/7225*x-76253/21675)/(x^4+x+3)^3+1/1
95075*sum((-255032*_R^2+1829456*_R-680601)/(4*_R^3+1)*ln(-_R+x),_R=RootOf(_
Z^4+_Z+3))
```

maxima [B] time = 1.20, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6} - x^{4} - 5\,x^{2} + 3\,x - 2}{x^{12} + 3\,x^{9} + 9\,x^{8} + 3\,x^{6} + 18\,x^{5} + 27\,x^{4} + x^{3} + 9\,x^{2} + 27\,x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/
(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 0.05, size = 27, normalized size = 1.00

$$\frac{-5\,x^{6} + x^{4} + 5\,x^{2} - 3\,x + 2}{\left(x^{4} + x + 3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8*
x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [B] time = 0.32, size = 60, normalized size = 2.22

$$\frac{-5x^{6} + x^{4} + 5x^{2} - 3x + 2}{x^{12} + 3x^{9} + 9x^{8} + 3x^{6} + 18x^{5} + 27x^{4} + x^{3} + 9x^{2} + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320*
x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.490 $\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$

Optimal. Leaf size=27

$$\frac{-5x^6+x^4+5x^2-3x+2}{\left(x^4+x+3\right)^3}$$

Rubi [F] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(\frac{-3+10x+4x^3-30x^5}{\left(3+x+x^4\right)^3}-\frac{3\left(1+4x^3\right)\left(2-3x+5x^2+x^4-5x^6\right)}{\left(3+x+x^4\right)^4}\right) dx$$

Verification is not applicable to the result.

```
[In] Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
+ 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] 7/(2*(3 + x + x^4)^3) - (63*x)/(22*(3 + x + x^4)^3) - (12*x^2)/(3 + x + x^4
)^3 - (5*x^3)/(3 + x + x^4)^3 + (3*x^4)/(2*(3 + x + x^4)^3) - (10*x^6)/(3 +
 x + x^4)^3 - 1/(2*(3 + x + x^4)^2) + (5*x^2)/(3 + x + x^4)^2 + (144*Defer[
Int][(3 + x + x^4)^(-4), x])/11 + (828*Defer[Int][x/(3 + x + x^4)^4, x])/11
 + 18*Defer[Int][x^2/(3 + x + x^4)^4, x] - 4*Defer[Int][(3 + x + x^4)^(-3),
 x] - 20*Defer[Int][x/(3 + x + x^4)^3, x]
```

Rubi steps

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = -\left(3\int \frac{(1+4x^3)(2-3x+5x^2+x\right.$$

$$= -\frac{10x^6}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^2}-$$

$$= \frac{3x^4}{2(3+x+x^4)^3}-\frac{10x^6}{(3+x+x^4)^3}-$$

$$= -\frac{5x^3}{(3+x+x^4)^3}+\frac{3x^4}{2(3+x+x^4)^3}-$$

$$= -\frac{12x^2}{(3+x+x^4)^3}-\frac{5x^3}{(3+x+x^4)^3}+$$

$$= -\frac{63x}{22(3+x+x^4)^3}-\frac{12x^2}{(3+x+x^4)^3}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+x^4)^3}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+x^4)^3}$$

$$= \frac{7}{2(3+x+x^4)^3}-\frac{63x}{22(3+x+x^4)^3}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
- 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] (2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4
*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4
*x^3)*(2 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4, x]
```

fricas [B] time = 1.36, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

giac [B] time = 0.45, size = 111, normalized size = 4.11

$$\frac{69136\,x^{7}-147344\,x^{6}-30784\,x^{5}+120988\,x^{4}+137584\,x^{3}+1167854\,x^{2}-100680\,x+18621}{390150\,{\left(x^{4}+x+3\right)}^{2}}-\frac{69136\,x^{11}-147344\,x^{10}-30784\,x^{9}+190124\,x^{8}+197648\,x^{7}+2645788\,x^{6}-72044\,x^{5}+129019\,x^{4}+1580606\,x^{3}+1452132\,x^{2}+887031\,x-724437}{390150\,{\left(x^{4}+x+3\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

```
[Out] 1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 11
67854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 1473
44*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5 + 1
29019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3)^3
```

maple [B] time = 0.02, size = 112, normalized size = 4.15

$$-\frac{-\frac{34568}{195075}x^{7}+\frac{73672}{195075}x^{6}+\frac{15392}{195075}x^{5}-\frac{60494}{195075}x^{4}-\frac{68792}{195075}x^{3}-\frac{583927}{195075}x^{2}+\frac{3356}{13005}x-\frac{2069}{43350}}{\left(x^{4}+x+3\right)^{2}}+\frac{-\frac{34568}{195075}x^{11}+\frac{73672}{195075}x^{10}+\frac{15392}{195075}x^{9}-\frac{95062}{195075}x^{8}-\frac{98824}{195075}x^{7}-\frac{1322894}{195075}x^{6}+\frac{36022}{195075}x^{5}-\frac{129019}{390150}x^{4}-\frac{790303}{195075}x^{3}-\frac{80674}{21675}x^{2}-\frac{32853}{14450}x+\frac{26831}{14450}}{\left(x^{4}+x+3\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2)
/(x^4+x+3)^4,x)
```

```
[Out] -(-34568/195075*x^7+73672/195075*x^6+15392/195075*x^5-60494/195075*x^4-6879
2/195075*x^3-583927/195075*x^2+3356/13005*x-2069/43350)/(x^4+x+3)^2+3*(-345
68/585225*x^11+73672/585225*x^10+15392/585225*x^9-95062/585225*x^8-98824/58
5225*x^7-1322894/585225*x^6+36022/585225*x^5-129019/1170450*x^4-790303/5852
25*x^3-80674/65025*x^2-10951/14450*x+26831/43350)/(x^4+x+3)^3
```

maxima [B] time = 0.98, size = 65, normalized size = 2.41

$$-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-
3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

```
[Out] -(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 + 2
7*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

mupad [B] time = 0.04, size = 27, normalized size = 1.00

$$\frac{-5\,x^{6}+x^{4}+5\,x^{2}-3\,x+2}{{\left(x^{4}+x+3\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 - 3
*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

```
[Out] (5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

sympy [B] time = 0.29, size = 60, normalized size = 2.22

$$\frac{-5x^6 + x^4 + 5x^2 - 3x + 2}{x^{12} + 3x^9 + 9x^8 + 3x^6 + 18x^5 + 27x^4 + x^3 + 9x^2 + 27x + 27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**4
+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

```
[Out] (-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18*
x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```